Ideal Gas Specific Heat; Heat Engine Overview

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Info on final exam (1/3)

- **Date, time and venue:** Dec 16, 2 PM – 5 PM in SCI Cunniff
- The final exam will cover the entirety of lectures, homework, and preflights.
  - This corresponds roughly to Knight § 1 – 12 + § 15 – 19
  - I will update the "textbook alignment" document on Moodle for more precise mapping

Info on final exam (2/3)

- There will be 7 short questions (4 points each) and 3 long questions (8 points each) on the final exam
- Other than being 3-hour long, the final exam regulations will be the same as the midterms (one letter-size cheat sheet, etc.)
- Roughly half of the exam will focus on materials covered since midterm 2. The remaining half of the exam will explicitly revisit topics already covered in midterms

Info on final exam (3/3)

- The final exam formula sheets and a mock final exam will be posted over the weekend
- I will leave time on the final day of lecture (Monday Dec 9) to answer your questions
- Extra office hours:
  - Next Thursday (Dec 12), 10:00 AM – 12:00 noon
  - Next Friday (Dec 13), 5:00 PM – 7:00 PM

Outline

1. Loose ends
2. Specific heat of ideal gas
3. Heat engine overview
4. Ideal-gas heat engine

1. Loose ends

Numerical practice: calorimetry

A 750 g aluminum pan ($c = 900$ J/kg·K) are removed from the stove and plunged into a sink filled with 10.0 L of water ($c = 4190$ J/kg·K) at 20°C. The water temperature quickly rises before it stabilizes at 24.0°C. What was the initial temperature of the pan in °C?

$$\rho_\mathrm{w} = 998\ \mathrm{kg/m^3} \quad \Rightarrow \quad \text{10.0 L water weighs 9.98 kg}$$

$$M_\mathrm{Al}\, c_\mathrm{Al}\, \Delta T_\mathrm{Al} + M_\mathrm{w}\, c_\mathrm{w}\, \Delta T_\mathrm{w} = 0$$

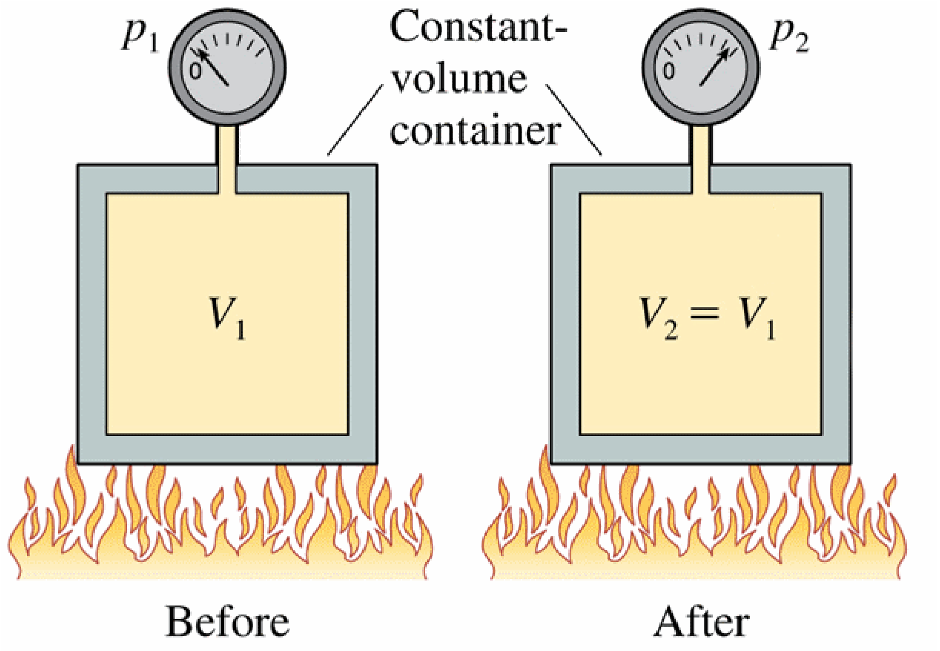
$$\Rightarrow \Delta T_\mathrm{Al} = -\frac{M_\mathrm{w}\, c_\mathrm{w}\, \Delta T_\mathrm{w}}{M_\mathrm{Al}\, c_\mathrm{Al}} = \frac{(9.98)(4190)(4)}{(0.75)(900)} = 248\ \mathrm{K}$$

$$\Rightarrow T_{\mathrm{Al},i} = 248 + 24 = 272°\mathrm{C}$$

2. Specific heat of ideal gas

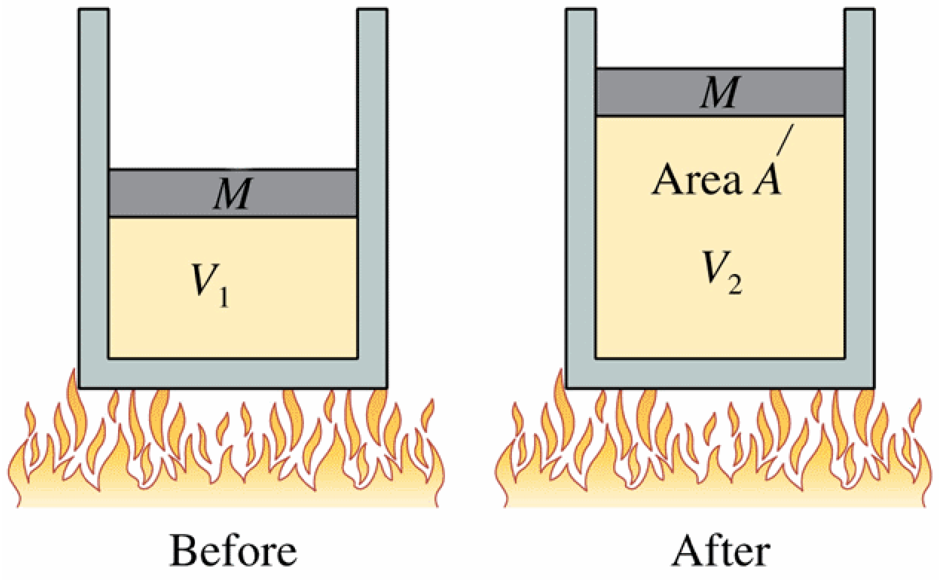
Reminder: isochoric vs isobaric processes

- Isochoric (V = const.) process:



$$Q = \Delta E_{\text{th}} = \frac{3}{2} nR\Delta T$$

- Isobaric (p = const.) process:



$$Q = \Delta E_{\text{th}} - W = \frac{5}{2} nR\Delta T$$

Molar specific heat for monoatomic ideal gas

- Recall the definition of molar specific heat C: $Q = nC\Delta T$

- For constant volume process,

$$Q = \Delta E_{\mathrm{th}} = \frac{3}{2}nR\Delta T$$

- Thus (for monoatomic gas)…

$$C_V = \frac{3}{2}R$$

- For constant pressure process,

$$Q = \Delta E_{\mathrm{th}} = \frac{5}{2}nR\Delta T$$

- Thus (for monoatomic gas)…

$$C_P = \frac{5}{2}R$$

Relationship between C_P and C_V

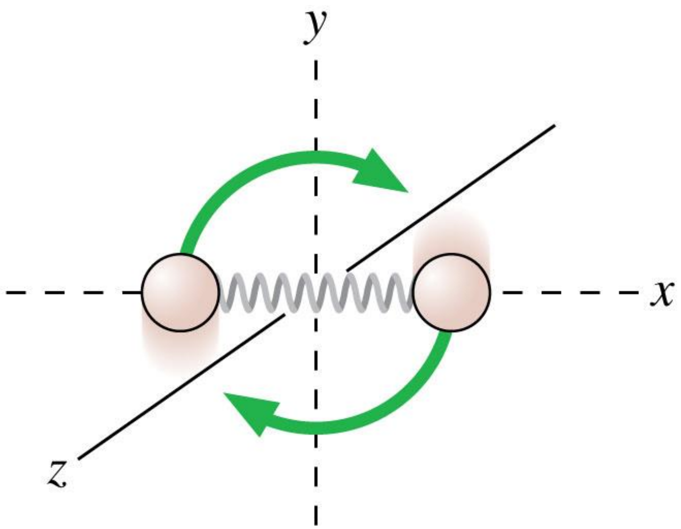
- From $C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R$, we see that $C_P = C_V + R$
- This difference stems from the work done in the constant pressure case, where in $W = -p\Delta V = -nR\Delta T$
- Note that no monoatomic assumption is made. So in general:

$$\boxed{C_P = C_V + R}$$

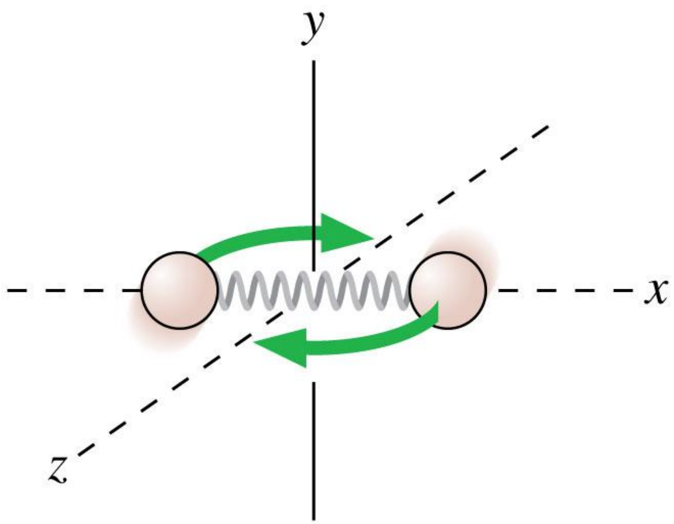
(**all** ideal gas)

Beyond monoatomic gas:* new degrees of freedom

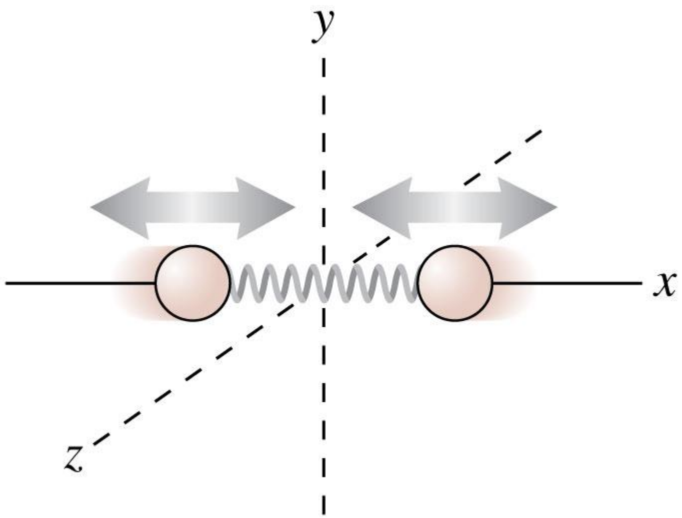
- In addition to an overall translational motion, diatomic molecule can rotate about 2 of its axes and can vibrate about its bond axis



Rotation end-over-end about the *z*-axis



Rotation end-over-end about the *y*-axis



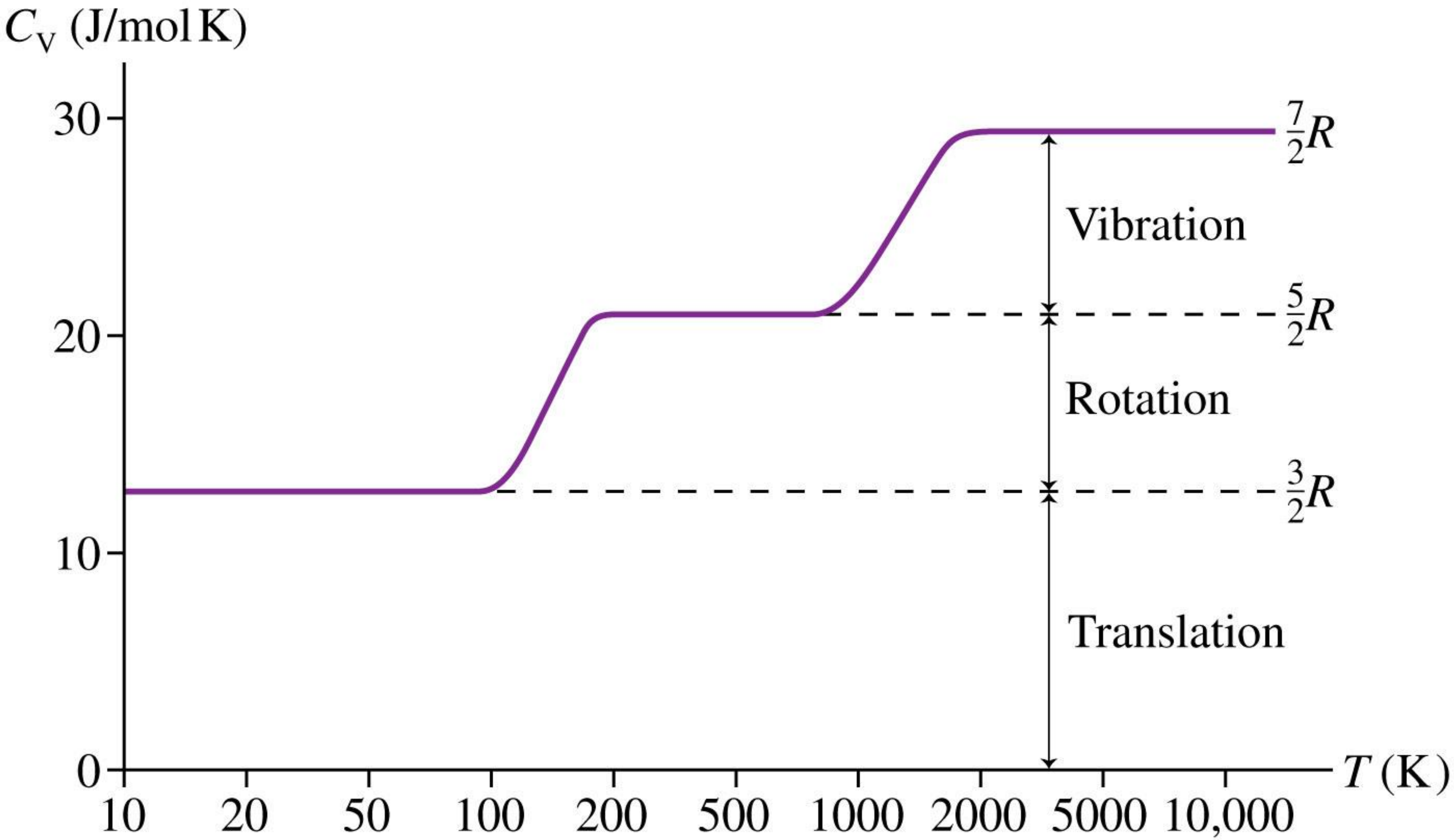
Vibration back and forth along the *x*-axis

Beyond monoatomic gas: increase in C_V

- Each additional "degree of freedom" can store energy and thus contributes additionally to E_th and hence C_V
- However, these additional degrees of freedom are often "activated" only at higher temperatures
- Similar ideas apply also to more complicated molecules*

* See Knight § 18.4 for more information

Beyond the monoatomic gas: putting all together



Upshot: use $C_V = \frac{5}{2} R$ for diatomic gas at "typical" temperatures $200\text{ K} \lesssim T \lesssim 800\text{K}$

3. Heat engine overview

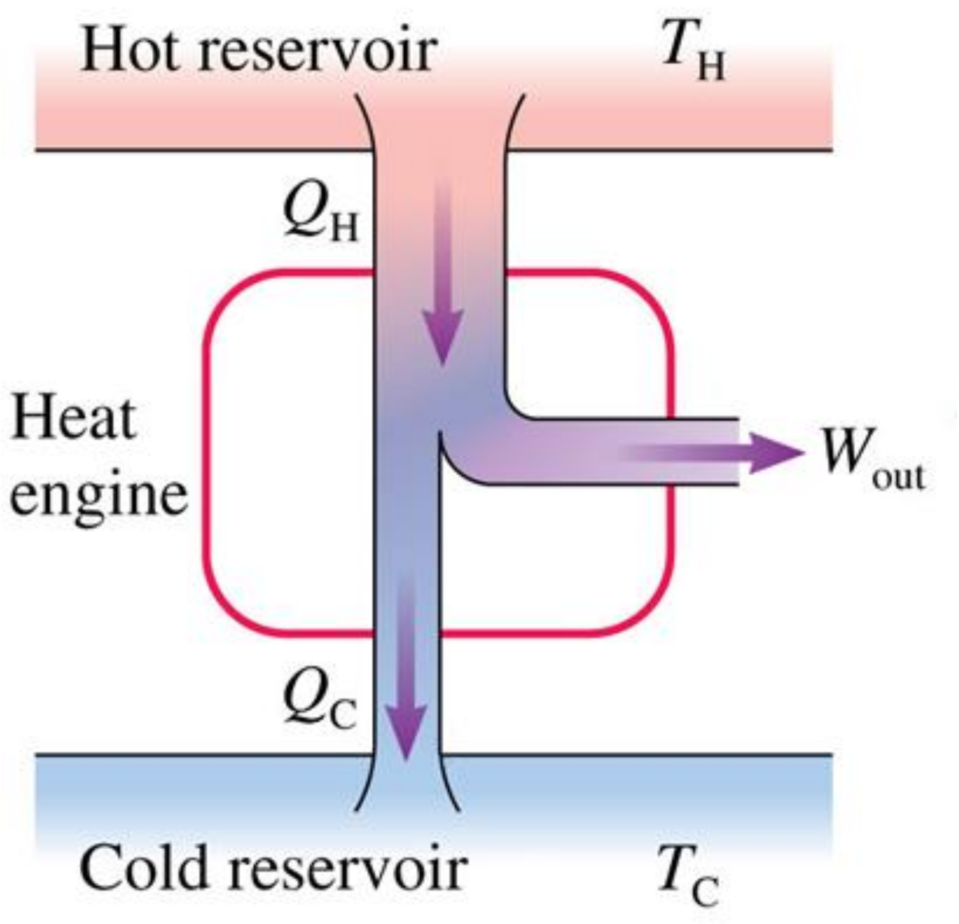
Basic properties of a heat engine

- **Heat engine** converts heat into useful work
- Conceptually, car engines, steam generators, etc. are all heat engine of some kind
- A heat engine operates in **cycles**, i.e., it returns to its initial state periodically
- A heat engine general operates between a **hot reservoir** and a **cold reservoir**

Energy-transfer diagram and relevant quantities

- T_H = temperature of hot reservoir
- T_C = temperature of cold reservoir
- Q_H = heat absorbed from hot reservoir
- Q_C = heat released to cold reservoir
- W_{out} = useful work output

*Q_H, Q_C, and W_{out} are values **per cycle**, and are all taken to be **positive**



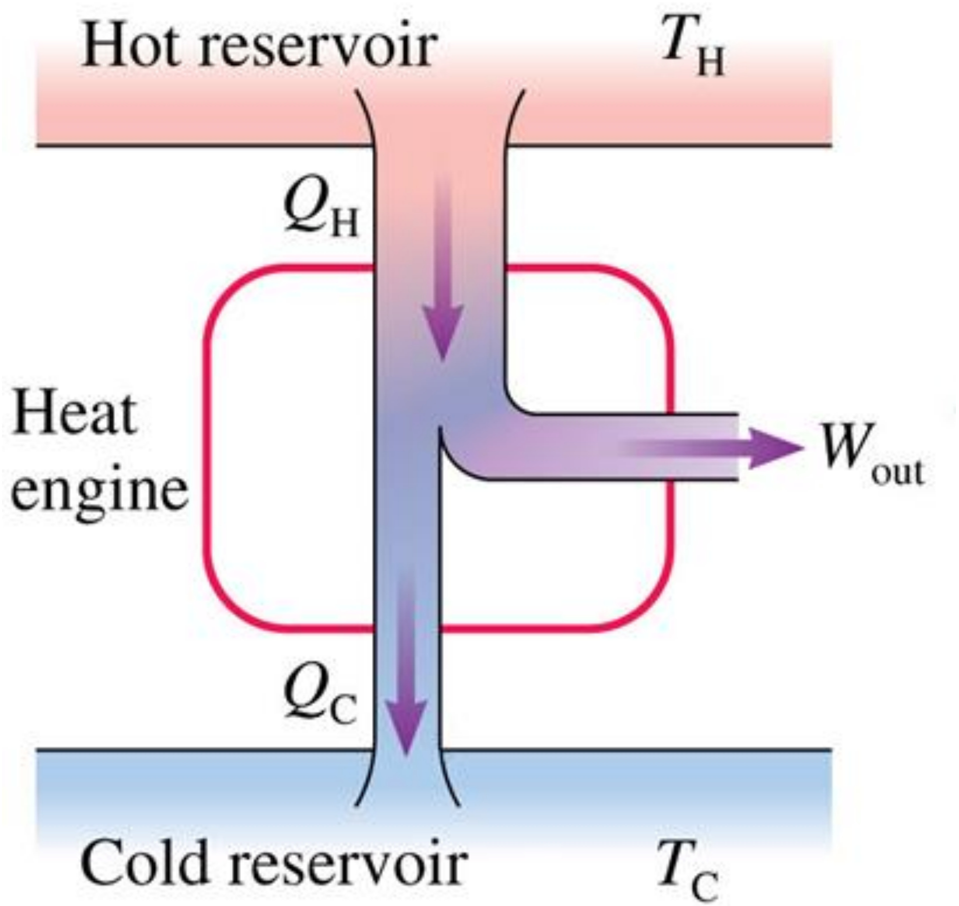
First law of thermodynamics on heat engine

- Relating our definitions,

$$Q = Q_H - Q_C$$
$$W = -W_{\text{out}}$$
(per cycle)

- Since the heat engine is cyclic, $\Delta E_{\text{th}} = 0$ after each cycle
- Thus, 1st law implies:

$$\boxed{W_{\text{out}} = Q_H - Q_C}$$

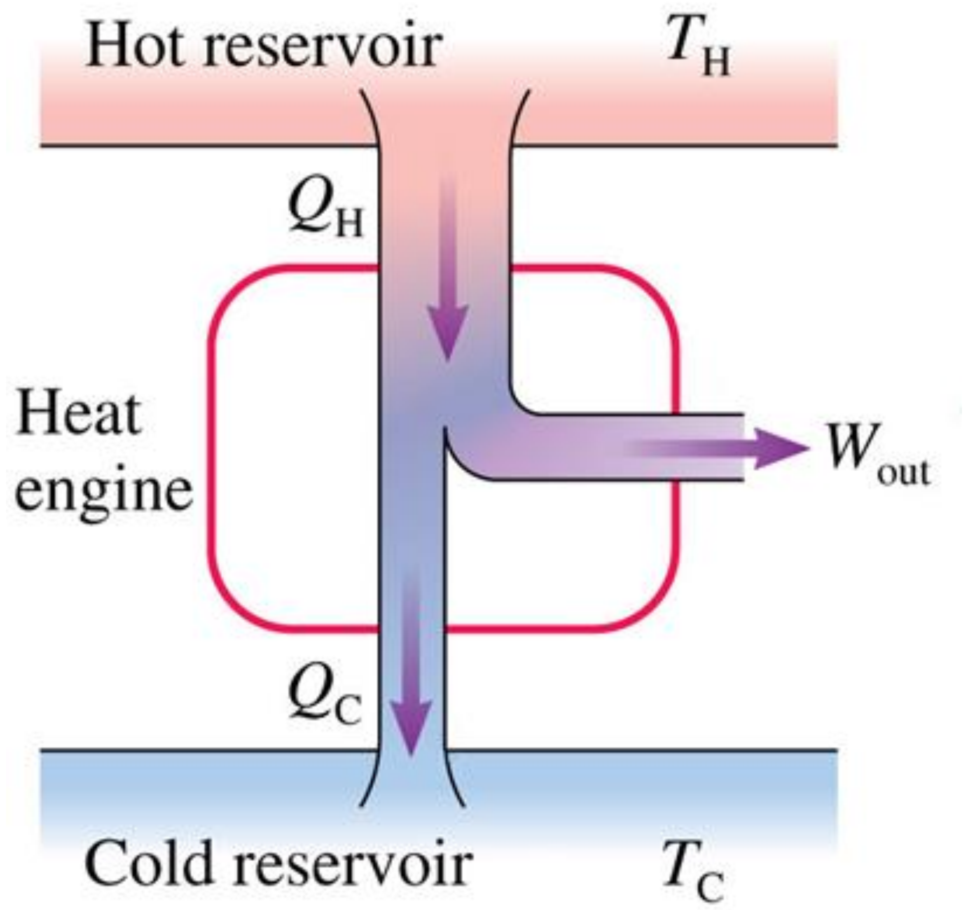


Heat engine efficiency

- The **thermal efficiency** of a heat engine, η, is defined as:

$$\eta = \frac{W_{\text{out}}}{Q_H}$$

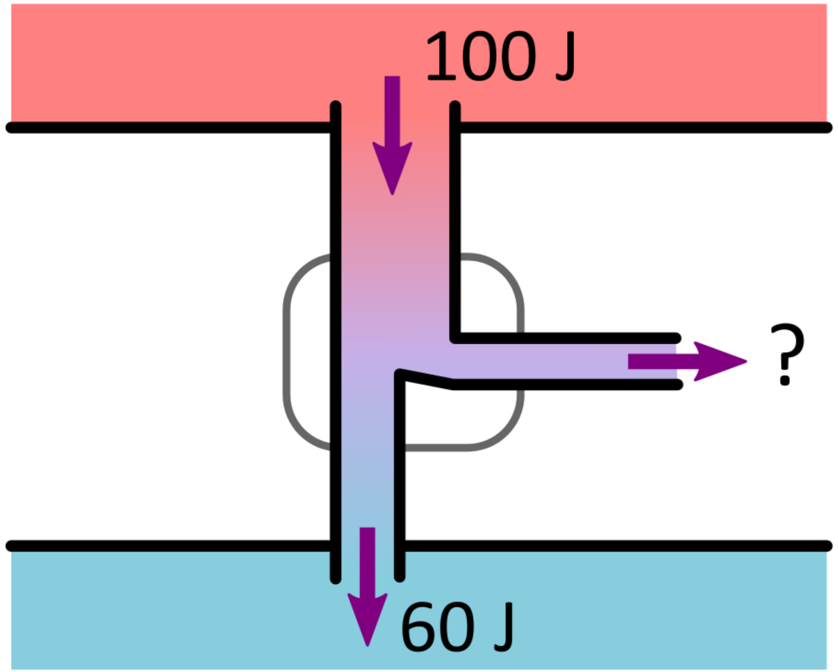
- One important question for heat engine is whether η is limited



Week 13 preflight Q2

$$W_{\text{out}} = Q_H - Q_C = 40\text{ J}$$

$$\eta = W_{\text{out}}/Q_H = 0.4$$



Your turn: heat engine efficiency

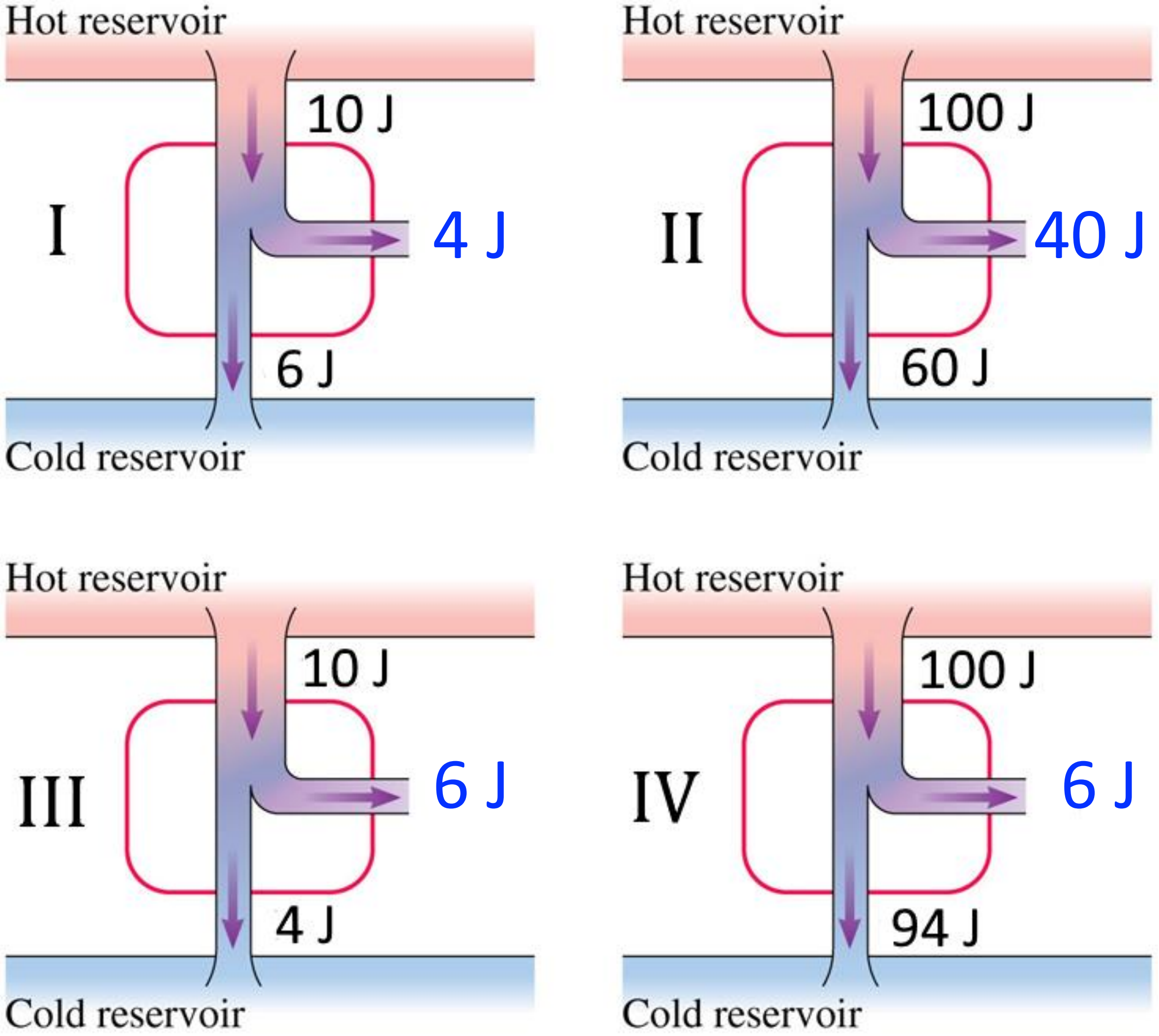
Rank the following heat engines by thermal efficiency, from largest to smallest

A. (II) > (III) = (IV) > (I)

B. (II) = (IV) > (I) = (III)

C. (III) > (I) = (II) > (IV)

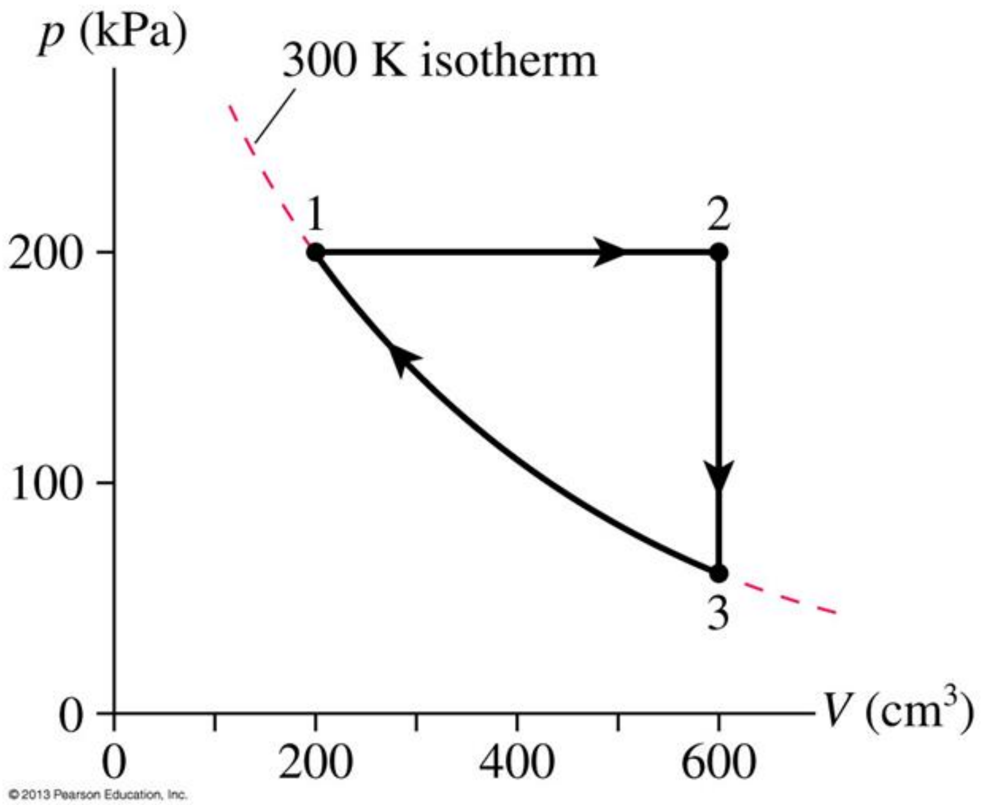
D. (IV) > (II) > (I) > (III)



4. Ideal-gas heat engine

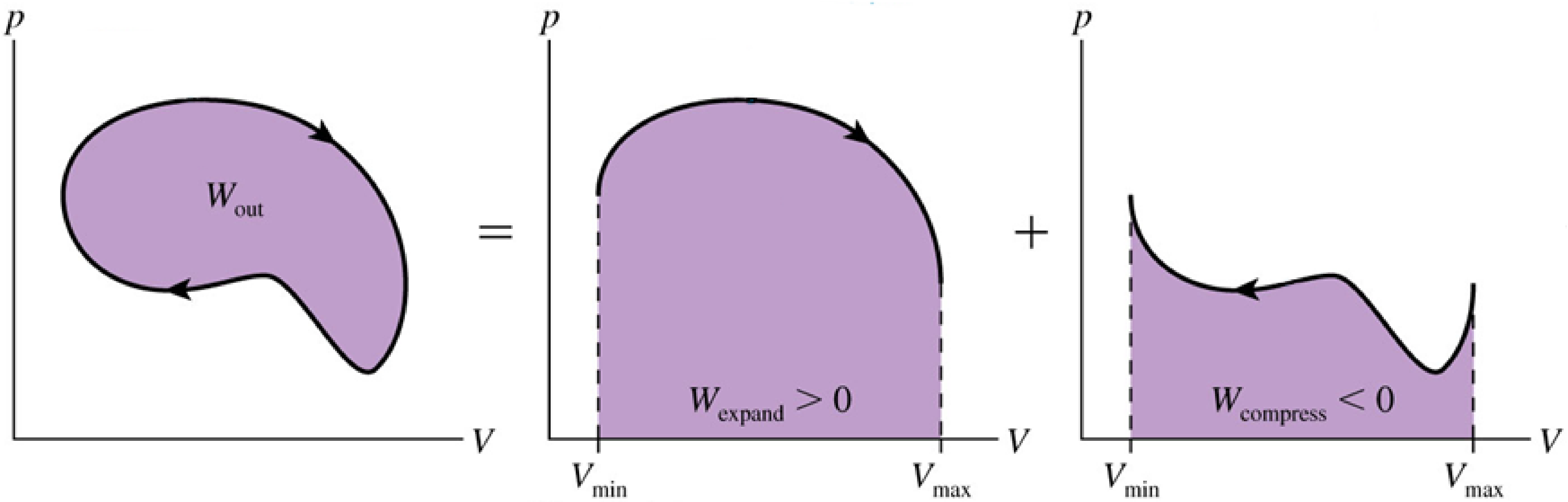
Ideal-gas heat engine

- For simplicity our heat engine will use monoatomic ideal gas as its working substance
- We will put together several ideal gas processes to form a cycle
- As before, p-V diagram will be central to our analysis

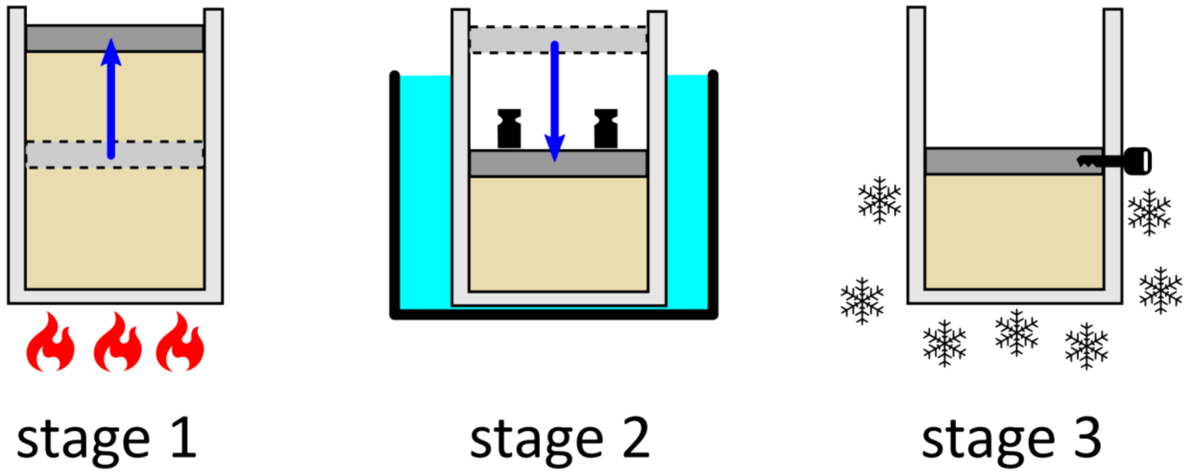


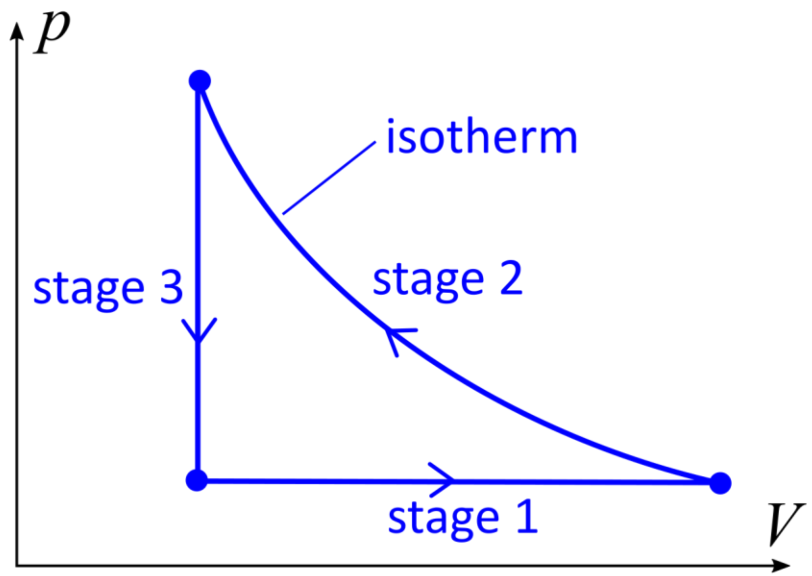
Work output in an ideal-gas heat engine

- For an ideal-gas heat engine, the work output per cycle can be read off from the area enclosed by the cycle:



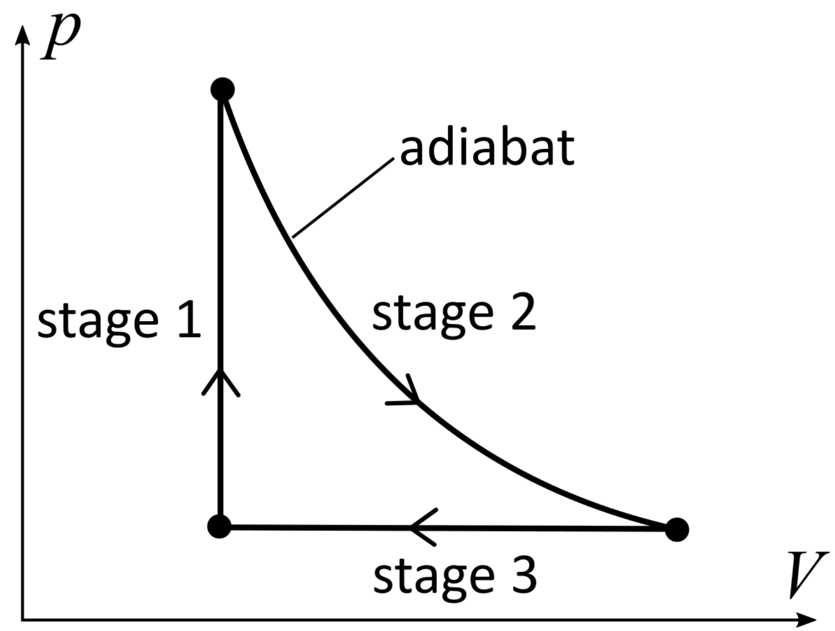
Week 13 preflight Q1





Week 13 preflight Q3

Stage	W	ΔE_{th}	Q
1	0	+ve	+ve
2	−ve	−ve	0
3	+ve	−ve	−ve



Your turn: ideal-gas heat engine

Which of the following p-V diagram represent an ideal-gas heat engine?

A. (I) only

B. $\boxed{\text{(I) and (II)}}$

C. (I), (II), and (III)

D. All of the above

(III) is not outputting work

(IV) is not a cycle