Heat Engine Analysis

Wing-Ho Ko

wko1@swarthmore.edu

Outline

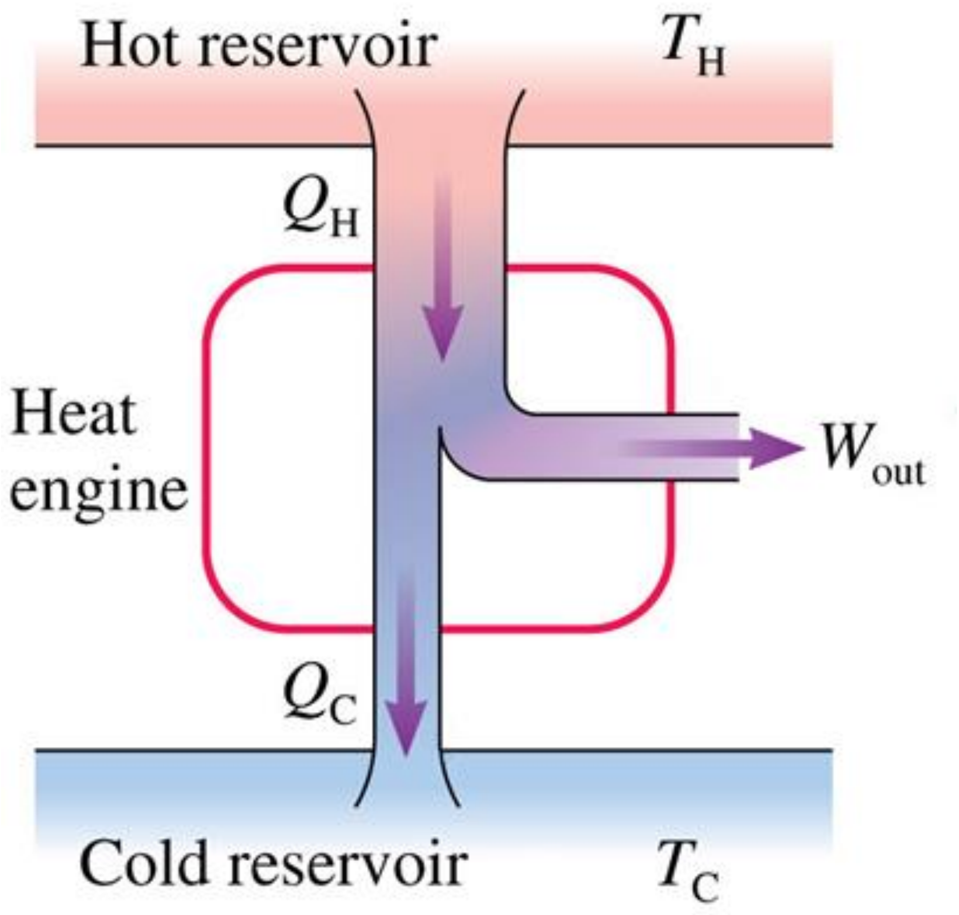
1. Heat engine analysis
2. ~~(?) Refrigerators and heat pump~~

1. Heat engine analysis

Energy-transfer diagram and relevant quantities

- T_H = temperature of hot reservoir
- T_C = temperature of cold reservoir
- Q_H = heat absorbed from hot reservoir
- Q_C = heat released to cold reservoir
- W_{out} = useful work output

*Q_H, Q_C, and W_{out} are values **per cycle**, and are all taken to be **positive**



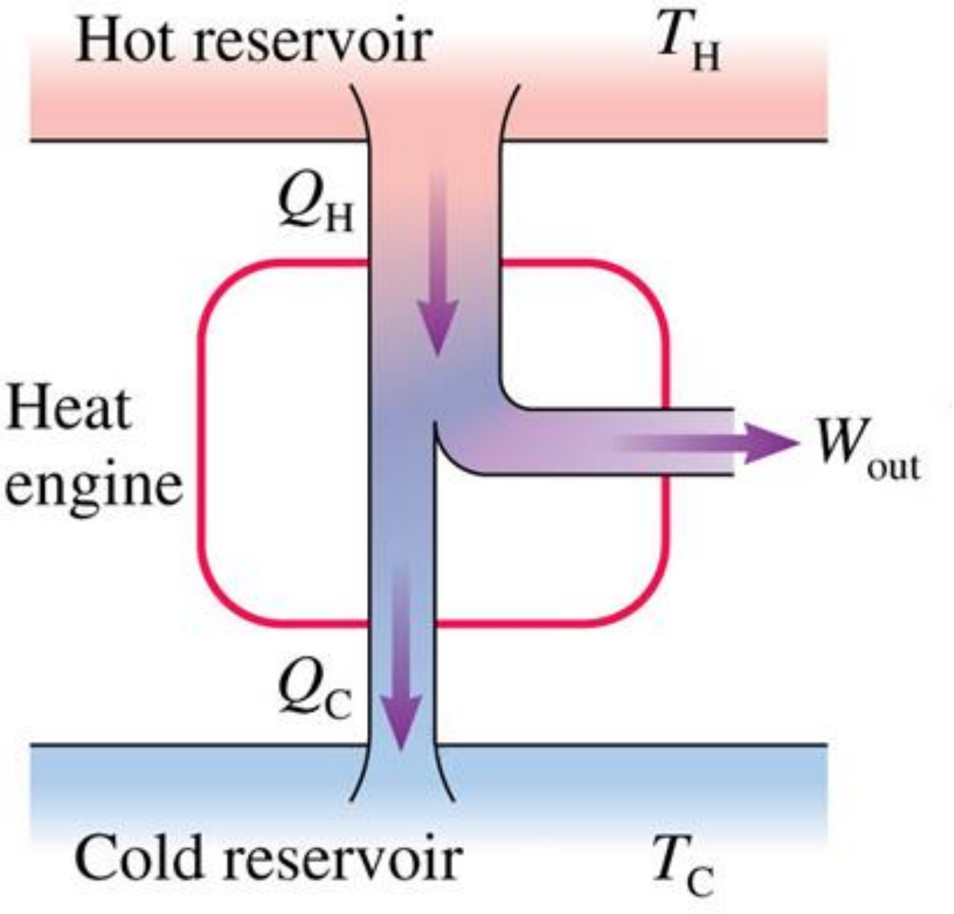
First law of thermodynamics and thermal efficiency

- 1st law of thermodynamics:

$$\boxed{W_{\text{out}} = Q_H - Q_C}$$

- Thermal efficiency:

$$\eta = \frac{W_{\text{out}}}{Q_H}$$



Strategy for solving ideal-gas heat engine problems

1. Visualize the situation using a p-V diagram
2. Obtain p, V, n, T of the ideal gas at one point in the cycle
3. Use the ideal gas law and knowledge of specific processes to obtain p, V, T at the beginning and the end of each process
4. Compute W, Q, and $\Delta\mathrm{E}_{\mathrm{th}}$ for each process. Check for consistency
5. Combine the results to obtain Q_H, Q_C, and W_{out}
6. Compute the thermal efficiency η if asked

Reminder: monoatomic ideal gas processes

Process	Definition	W	Q	ΔE_{th}
Isochoric	V = const.	0	$\frac{3}{2}nR\Delta T$	$\frac{3}{2}nR\Delta T$
Isobaric	p = const.	$-p\Delta V$ $(= -nR\Delta T)$	$\frac{5}{2}nR\Delta T$	$\frac{3}{2}nR\Delta T$
Isothermal	T = const.	$-nRT\ln\left(V_f/V_i\right)$	$nRT\ln\left(V_f/V_i\right)$	0
Adiabatic	$Q = 0$ [*]	$\frac{3}{2}\left(p_fV_f - p_iV_i\right)$	0	$\frac{3}{2}\left(p_fV_f - p_iV_i\right)$

* Consequences: pV^{γ} = const. and $TV^{\gamma-1}$ = const. $(\gamma = 5/3)$

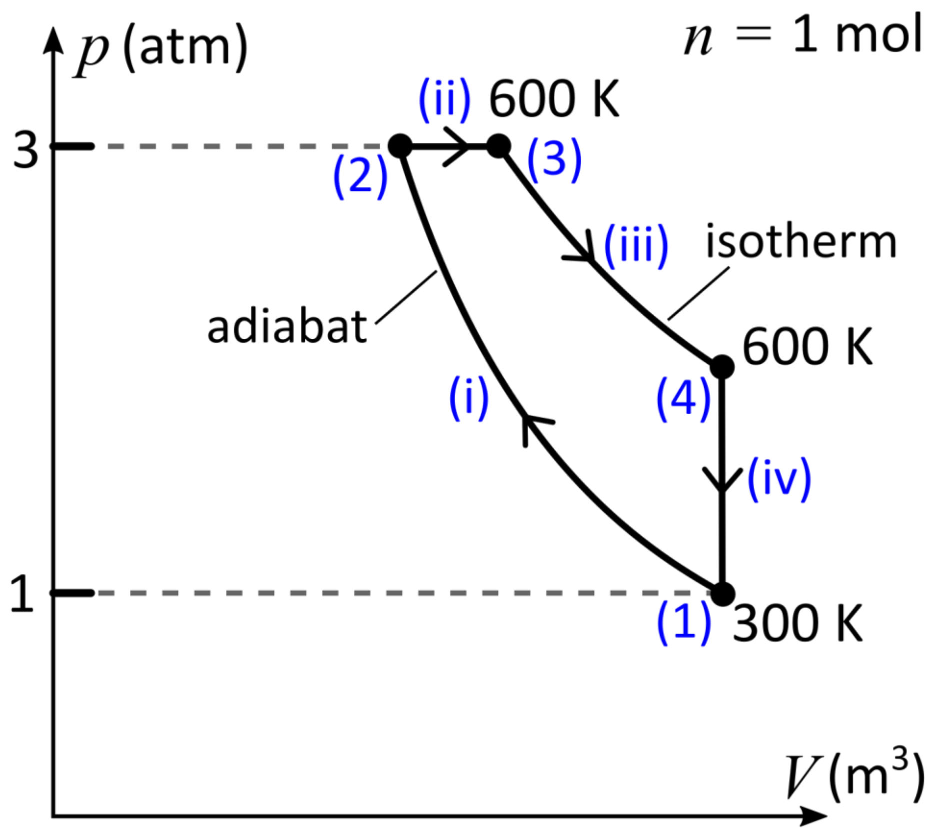
Setup of our heat engine

1 mole of monoatomic gas is placed in a sealed container. The container is initially at $p_1 = 1$ atm (=101 kPa) and $T_1 = 300$ K. The gas works at a heat engine with the following cycle:

i. The gas undergoes adiabatic compression until it reaches $p_2 = 3$ atm
ii. The gas undergoes isobaric heating until it reaches $T_3 = 600$ K
iii. The gas expands isothermally until its volume returns to the initial
iv. The gas undergoes isochoric cooling until pressure returns to 1 atm

Step 1: draw the p-V diagram that depict the cycle

Step 1: visualize with p-V diagram



Step 2: find p, V, n, T of initial state

- Given:

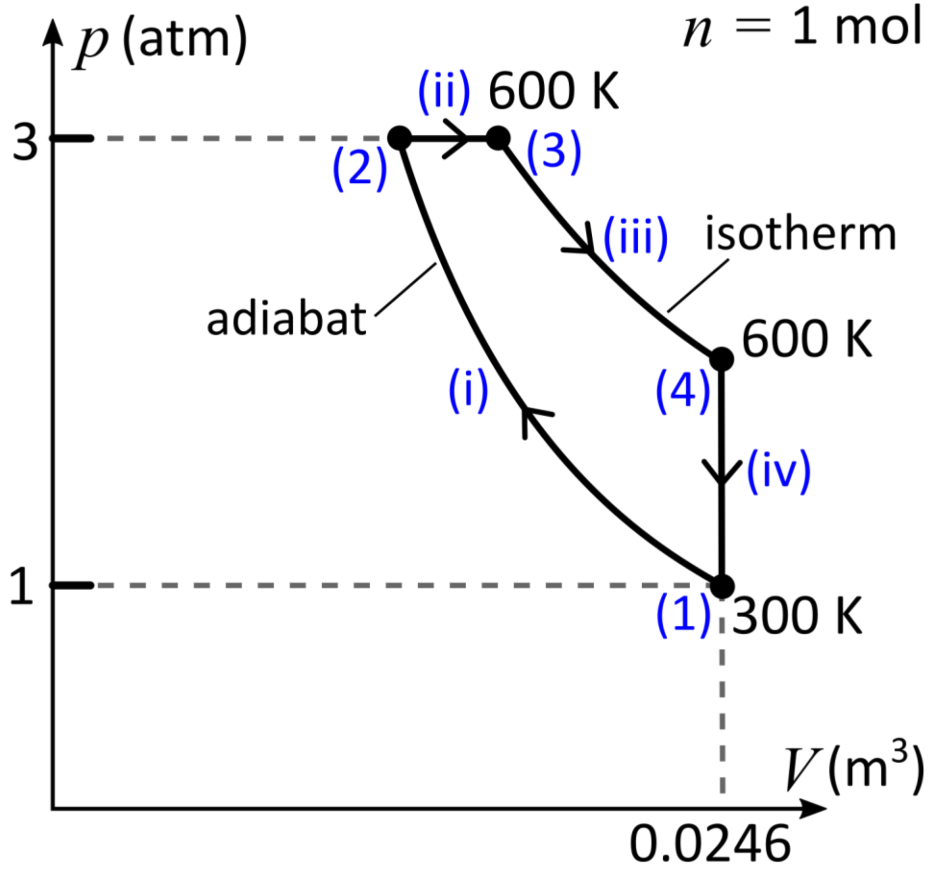
  $n = 1$ mole

  $p_1 = 1$ atm $= 101$ kPa

  $T_1 = 300$ K

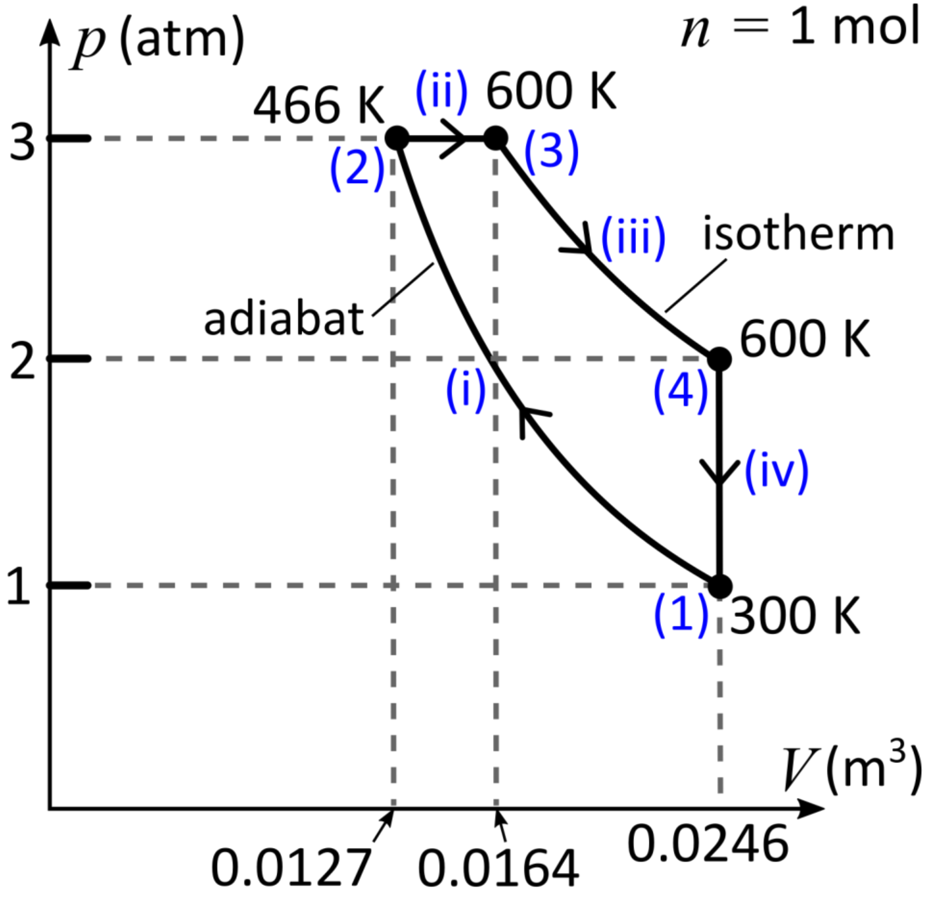
- Ideal gas law:

$$V_1 = \frac{nRT}{p} = 0.0246\ \text{m}^3$$



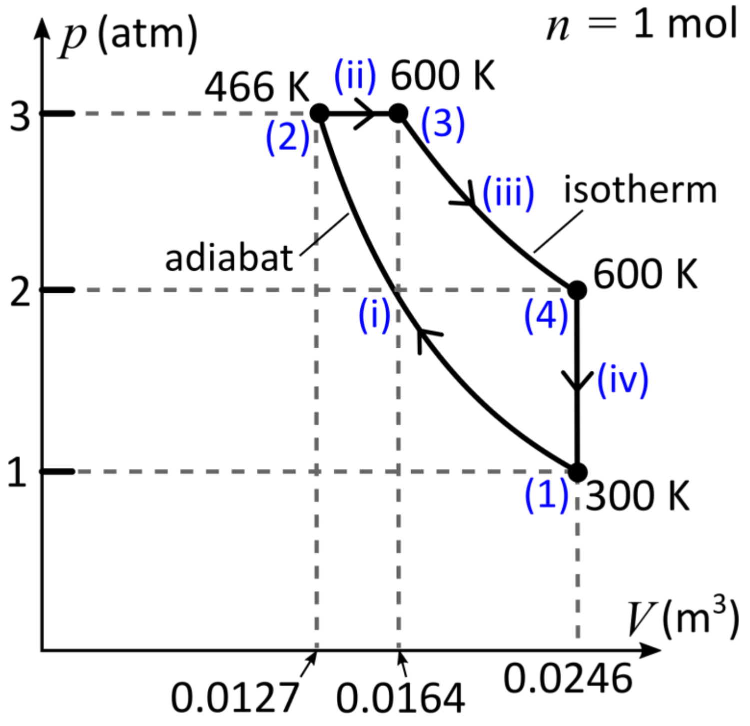
Step 3: find p, V, T at endpoints of processes

- $p_4/T_4 = p_1/T_1 \;\Rightarrow\; p_4 = 2$ atm
- $p_3V_3 = p_4V_4 \;\Rightarrow\; V_3 = 0.0164\ \mathrm{m}^3$
- $p_2{V_2}^\gamma = p_1{V_1}^\gamma \;\Rightarrow\; V_2 = 0.0127\ \mathrm{m}^3$
- $p_2V_2 = nRT_2 \;\Rightarrow\; T_2 = 466$ K



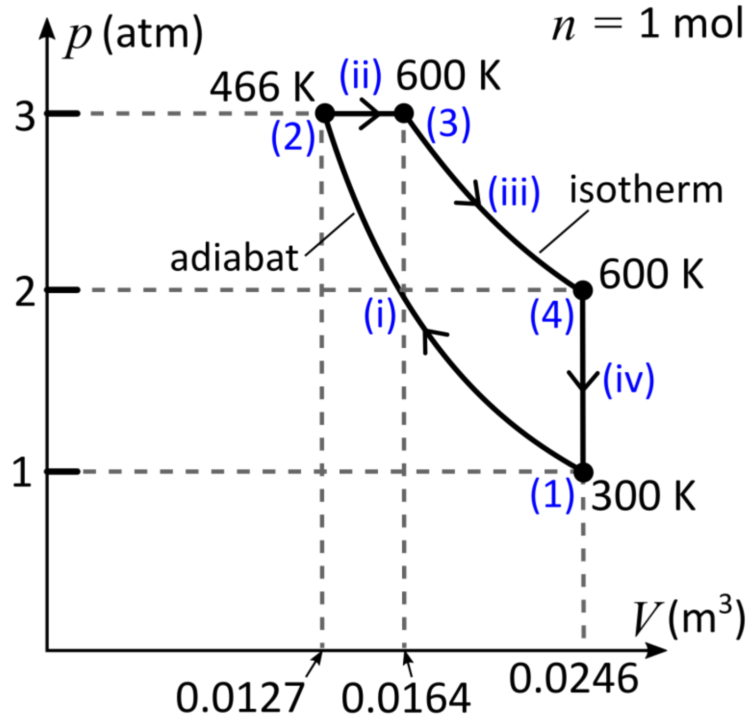
Step 4: compute W, Q, and ΔE_{th}

Stage	W	Q	ΔE_{th}
(i)	$\frac{3}{2}nR(T_2 - T_1)$	0	$\frac{3}{2}nR(T_2 - T_1)$
(ii)	$-nR(T_3 - T_2)$	$\frac{5}{2}nR(T_3 - T_2)$	$\frac{3}{2}nR(T_3 - T_2)$
(iii)	$-nRT_3 \ln(V_4/V_3)$	$nRT_3 \ln(V_4/V_3)$	0
(iv)	0	$\frac{3}{2}nR(T_1 - T_4)$	$\frac{3}{2}nR(T_1 - T_4)$



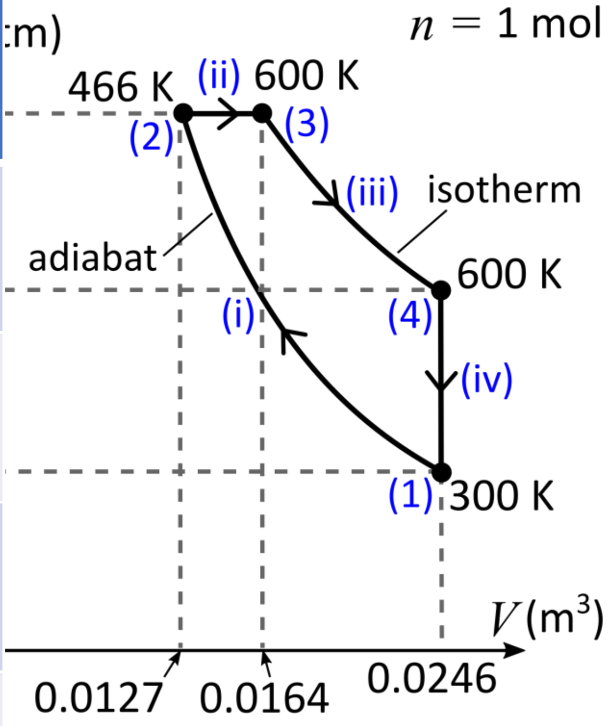
Step 4: compute W, Q, and ΔE_{th}

Stage	W	Q	ΔE_{th}
(i)	2060 J	0	2060 J
(ii)	−1120 J	2790 J	1680 J
(iii)	−2020 J	2020 J	0
(iv)	0	−3740 J	−3740 J



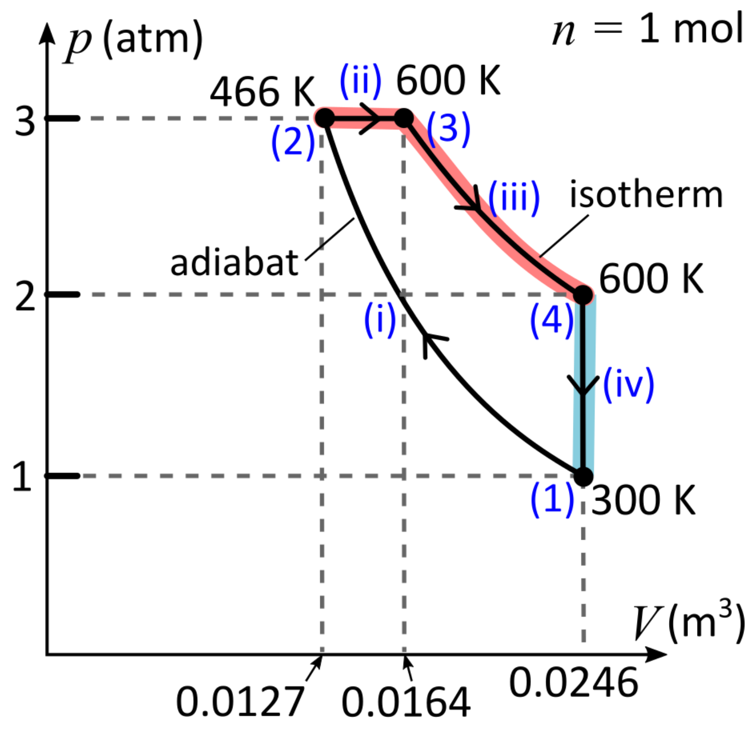
Step 4’: check for consistency

Stage	W	Q	ΔE_{th}	1st law
(i)	+ve	0	+ve	0
(ii)	−ve	+ve	+ve	0
(iii)	−ve	+ve	0	0
(iv)	0	−ve	−ve	0
$\sum$	−1080 J (< 0)	1080 J (> 0)	0	



Step 5: compute W_{out}, Q_H, and Q_C

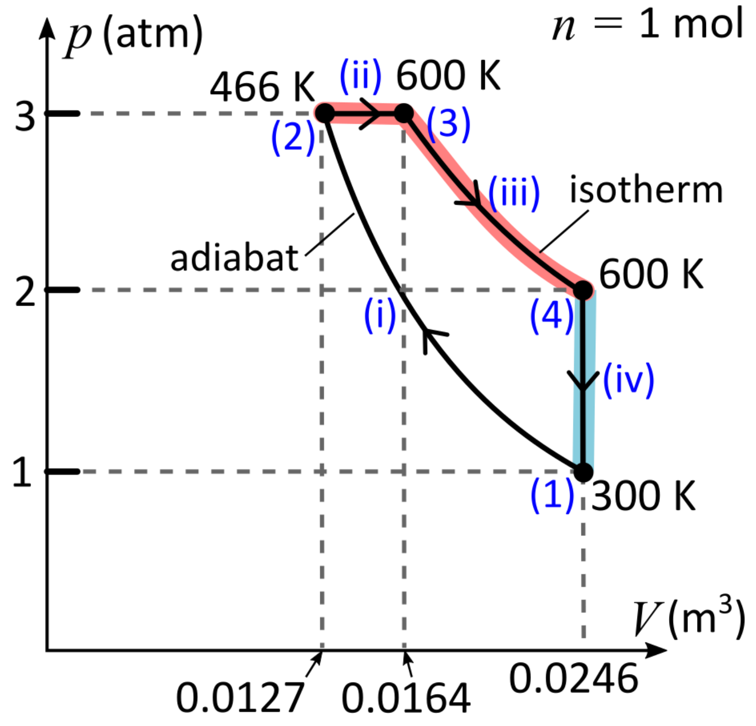
Stage	W	Q	ΔE_{th}
(i)	2060 J	0	2060 J
(ii)	−1120 J	2790 J	1680 J
(iii)	−2020 J	2020 J	0
(iv)	0	−3740 J	−3740 J



$W_{\mathrm{out}} = -\sum_a W_a = 1080$ J, $Q_H = Q_{\mathrm{ii}} + Q_{\mathrm{iii}} = 4810$ J, $Q_C = -Q_{\mathrm{iv}} = 3740$ J

Step 6: compute η

- $W_{\text{out}} = 1080$ J
- $Q_H = 4810$ J
- $Q_C = 3740$ J



$$\Rightarrow \eta = \frac{W_{\text{out}}}{Q_H} = 0.22$$