**Due Date/Time** October 23 (Wed) 10:30 AM.

**Introduction** This homework contains 5 short questions and 3 long questions. Each short question is worth 2 points while each long question is worth 5 points. Please note that the question order within each section is unrelated to the order in which the relevant concepts were introduced in lectures.

Hints on specific long questions may be included at the end of this document. For details on grading, please refer to the homework guidelines posted separately.

Please **write your name legibly** on your submitted work. This will help us identify your work with you easier.

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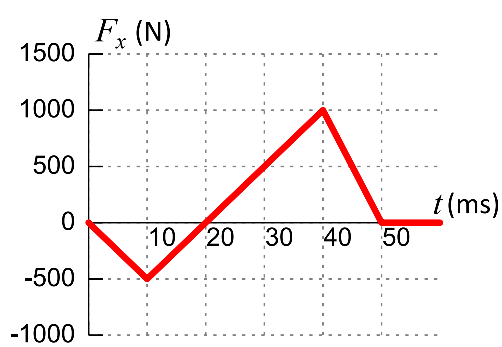
## Short Questions

**Question 1** A 3.0-kg block is initially at rest on a flat, frictionless table but is then subjected to a short burst of force in the horizontal ($x$-) direction as shown in the figure below. Find the velocity of the block after the force has subsumed.




**Solution** Reading from the graph, the impulse imparted on the block over the duration of the burst is:

$$J_x = \frac{1}{2}(-500\ \text{N})(0.020\ \text{s}) + \frac{1}{2}(1000\ \text{N})(0.030\ \text{s}) = 10\ \text{N·s}$$

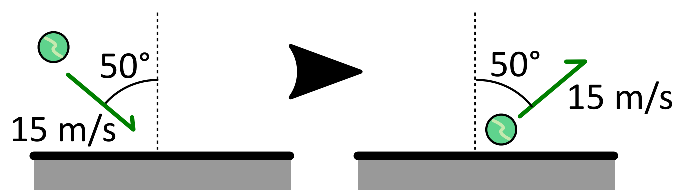
From the situation, the initial momentum of the block is $p_{ix} = 0$ kg·m/s. Thus, by the impulse-momentum theorem,

$$p_{fx} = p_{ix} + J_x = 10\ \text{kg·m/s}$$

Using $p_{fx} = mv_{fx}$ and plugging in $m = 3.0$ kg, we find $v_{fx} = +3.33$ m/s, with the plus sign indicating that the motion is aligned with the $+x$ direction.

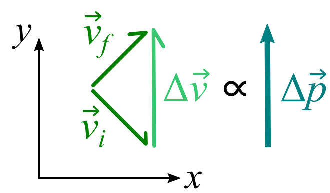
**Question 2** A 60.0 g tennis ball hits the ground at an angle and bounces back. The incoming and outgoing speed of the ball, as well as the angles it makes with the perpendicular, are depicted in the figure below. From the given information, find the impulse that the ground exert on the ball.

**Note:** recall that the impulse is a vector.




**Solution** Align our coordinate system so that the $+y$ direction is up and that the $x$-component of the velocity of the tennis ball is positive. By the impulse-momentum theorem, the impulse on the tennis ball is given by (see also figure for illustration):

$$\vec{J} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i$$




In the present case, the initial and final velocity of the tennis ball are:

$$\vec{v}_i = 15\sin(50°)\hat{\imath} - 15\cos(50°)\hat{\jmath} = (11.5\ \hat{\imath} - 9.64\ \hat{\jmath})\ \text{m/s}$$
$$\vec{v}_f = 15\sin(50°)\hat{\imath} + 15\cos(50°)\hat{\jmath} = (11.5\ \hat{\imath} + 9.64\ \hat{\jmath})\ \text{m/s}$$

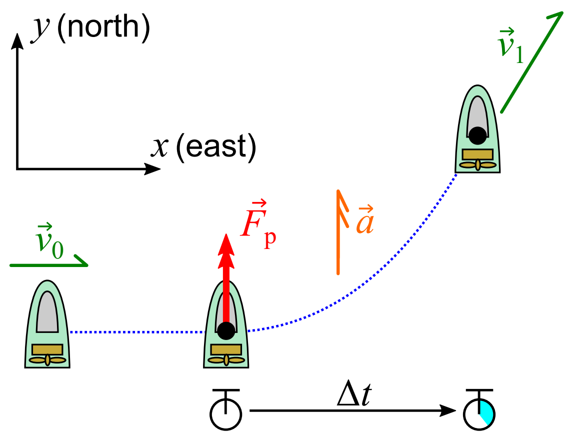
Hence, we see that $\vec{v}_f - \vec{v}_i = (19.3\ \text{m/s})\ \hat{\jmath}$. Together with $m = 0.060$ kg, the impulse on the ball is:

$$\vec{J} = (0.060\ \text{kg})(19.3\ \text{m/s}\ \hat{\jmath}) = (1.16\ \text{N·s})\ \hat{\jmath}$$

Notice that $\vec{J}$ is purely upward, in agreement with the qualitative conclusion from the illustration.

**Question 3** A 500-kg hovercraft (negligible friction/air resistance) is originally facing north but drifting east at 4.0 m/s with its engine shut. At $t = 0$ s the engine starts, which produces a constant force of 800 N pointing north on the hovercraft. Find the velocity of the hovercraft 3.0 s after the engine is turned on. Express your answer in the *cartographic convention*.

**Solution** The situation can be visualized as below, wherein $F_p$ = 800 N, $v_0$ = 5.0 m/s and $\Delta t$ = 3.0 s. The figure also serves to define our coordinate convention.




From Newton's 2nd law, $\vec{a} = \vec{F}_\mathrm{net}/m = \vec{F}_\mathrm{p}/m$. Plugging in $m$ = 500 kg and $F_\mathrm{p}$ = 800 N, we get $\vec{a} = (1.6 \text{ m/s}^2)\hat{\jmath}$.

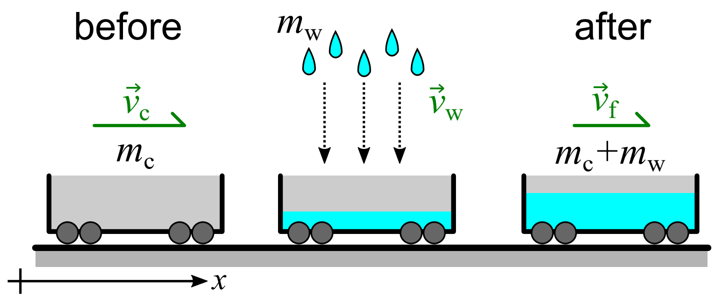
Noting that $\vec{a}$ is a constant, from kinematics we have:

$$\vec{v}_1 = \vec{v}_0 + \vec{a}\Delta t$$

Plugging in $\vec{v}_0 = (4.0 \text{ m/s})\hat{\imath}$, $\vec{a} = (1.6 \text{ m/s}^2)\hat{\jmath}$ and $\Delta t$ = 3.0 s, we find $\vec{v}_1 = (4.0 \text{ m/s})\hat{\imath} + (4.8 \text{ m/s})\hat{\jmath}$. Converting back to cartographic convention, the velocity of the hovercraft is 4.0 m/s east and 4.8 m/s north, or equivalently (via trigonometry) 6.25 m/s at 39.8° east of north.

**Question 4** A 6000 kg open train car is rolling on frictionless rails at 23 m/s when rain starts pouring straight down. A few minutes later, the car's speed is 20 m/s. How much (mass) water has collected in the car?

**Solution** Use the subscript $c$ for car and $w$ for water, and use the subscript $f$ for the final velocity. The situation can be visualized below, where we also define our coordinate convention. In our case, we have $m_c$ = 6000 kg, $v_{cx}$ = 23 m/s, $v_{wx}$ = 0 m/s, and $v_{fx}$ = 20 m/s.




Treat the empty car plus the volume of rain as a system. Observe that there are no external forces with non-zero horizontal component acting on the system. Thus, the horizontal component of the momentum is conserved. Writing down the momentum of the system before and after the rain fall and equating the two, we have:

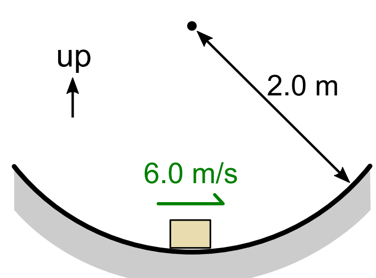
$$m_c v_{cx} + m_w v_{wx} = (m_c + m_w) v_{fx}$$

Since $v_{wx} = 0$ m/s, we get:

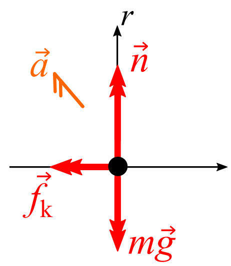
$$m_c + m_w = m_c \left(\frac{v_{cx}}{v_{fx}}\right) = 6000 \text{ kg} \left(\frac{23 \text{ m/s}}{20 \text{ m/s}}\right) = 6900 \text{ kg}$$

Subtracting $m_c$ from $m_c + m_w$, we get $m_w = 900$ kg.

**Question 5** A 2.0 kg block is sliding down a vertical circular track of radius 2.0 m (see figure). It's speed at the bottom of the track is 6.0 m/s. If the coefficient of kinetic friction between the block and the track is 0.40, what is the magnitude of angular acceleration of the block at the bottom of the track?




**Solution** The forces acting on the block can be visualized using the free-body diagram below. We note that the acceleration of the block has both a tangential ($s$) and radial ($r$) component since the block is both slowing down and making turn.




Writing down the Newton's 2nd law and make use of circular kinematics, we have:

$$r\text{-component:} \quad n - mg = ma_r = mv^2/r$$
$$s\text{-component:} \quad -f_k = ma_s = -mr|\alpha|$$

Plugging in $m = 2.0$ kg, $v = 6.0$ m/s, and $r = 2.0$ m to the $r$-component equation, we find $n = 55.6$ N. Using $f_k = \mu_k n$, we get $f_k = 22.2$ N. Finally, plugging the value of $f_k$ into the $s$-component equation, we arrive at $|\alpha| = 5.56$ rad/s$^2$.

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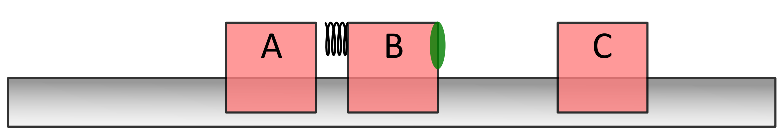
## Long Questions

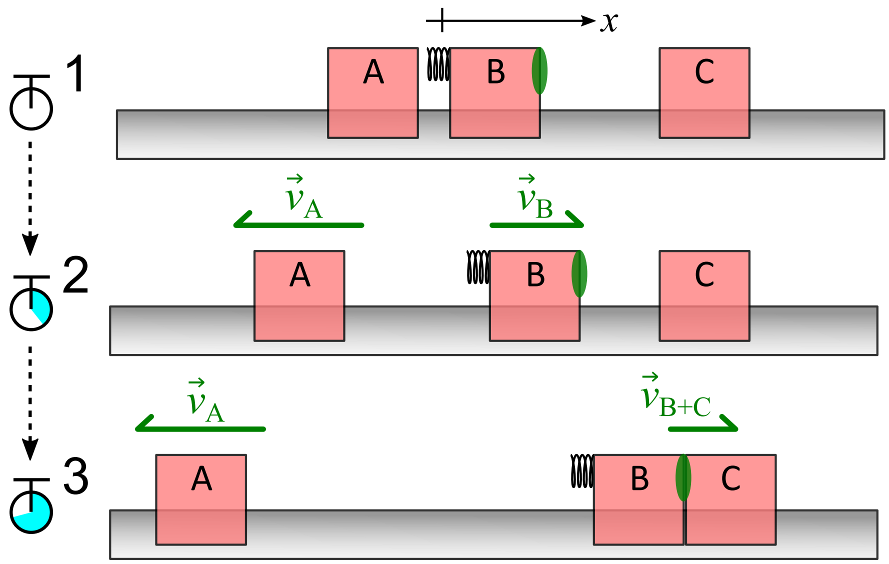
**Question 6 [three gliders]** Three gliders (negligible friction and air resistance) A, B, and C are placed on the same horizontal air-track running left to right. The mass of the gliders A is 200 g. Glider B has a massless spring attached to its left and a putty attached to its right (see illustration below). At an initial time gliders A and B are pressed against each other while both are maintained at rest (meanwhile glider C is also at rest). The two gliders are then released, after which glider B collides with glider C and the two got stuck together.

You record the entire process on a video camera but forgot to include a standard length scale for calibration. When you analyze the motion using a computer, you find that once A and B are released, glider A moves 15 pixels per frame to the left while glider B is moves 10 pixels per frame to the right. You find also that once gliders B and C got stuck together, they move 6 pixels per frame to the right.

Based on what you find from the video analysis, can you determine the mass of gliders B and C?



**Solution** The situation can be visualized as below, where we also define our coordinate convention and identify 3 distinct phases of motion denoted by 1, 2, and 3. Letting $v_0$ to be the speed that corresponds to 1 pixel per frame, we know $v_{Ax} = -15v_0$, $v_{Bx} = 10v_0$, and $v_{B+C,x} = 6v_0$. Moreover, we are given $m_A = 200$ g and we need to find $m_B$ and $m_C$.




There are multiple ways to set up the equations that would solve for $m_B$ and $m_C$. The most efficient approach is perhaps to treat the 3 gliders together as a system. For each phase of the motion, the expression of the momentum (or rather its $x$-component) of the system is:

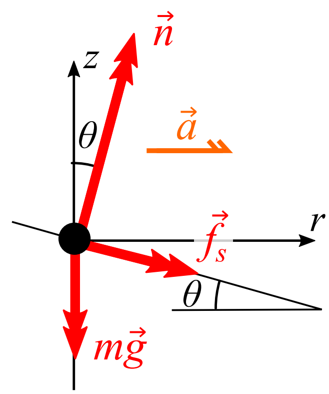
$$P_{1x} = 0$$
$$P_{2x} = m_A v_{Ax} + m_B v_{Bx}$$
$$P_{3x} = m_A v_{Ax} + (m_B + m_C) v_{B+C,x}$$

By momentum conservation $P_{1x} = P_{2x} = P_{3x}$. From $P_{1x} = P_{2x}$ we find $m_B = 300$ g, and from $P_{1x} = P_{3x}$ we find $m_B + m_C = 500$ g, which in turn gives $m_C = 200$ g.

**Question 7 [race track]** A certain bicycle race track is essentially a circular loop of radius 150 m. The track is banked inward (i.e., towards the center of the circle) at a 15° angle with the horizontal. The static coefficient of friction between the bicycle and the track is 0.6. If the biker and bicycle together has a mass of 80 kg, what is the highest speed that the biker may ride before he/she starts inevitably slipping outward?

**Solution** Before we draw the free-body diagram it is worth pondering about the magnitude and direction of friction. In the absence of friction, when the speed of the rider becomes too high, inertia will result in the rider slipping outward. Since friction is there to oppose slipping, we see that friction must point downward along the slope (and hence has an inward component) in this case. Moreover, when the speed of the rider is maximal, the friction between the rider and the track must also be maximal.

With this insight we adopt the $rsz$-coordinate system and draw the free-body diagram for the rider at maximum speed (in the diagram $\theta = 15°$), where we also depict the acceleration $\vec{a}$. Notice that $\vec{a}$ is purely horizontal since the circle that the rider makes is also purely horizontal.




Also from the previous discussion, we may set $f_s = \mu_s n$, where $\mu_s = 0.6$. Then, from Newton's Second Law we get:

$$\begin{aligned} r\text{-component:} \quad & n\sin\theta + \mu_s n\cos\theta = ma \\ z\text{-component:} \quad & n\cos\theta - \mu_s n\sin\theta - mg = 0 \end{aligned}$$

We note that in the second equation the only unknown is $n$. Solving, we found $n = 967$ N. Plugging this into the first equation, we obtain $a = 10.1$ m/s$^2$.

Finally, we relate $a$ to the speed of the biker $v$ using the uniform circular motion formula:

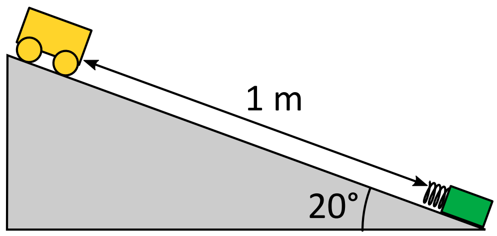
$$a = \frac{v^2}{r}$$

here $r = 150$ m is the radius of the track. Plugging in, we get $v = 39.0$ m/s.

## Question 8 [cart and spring]

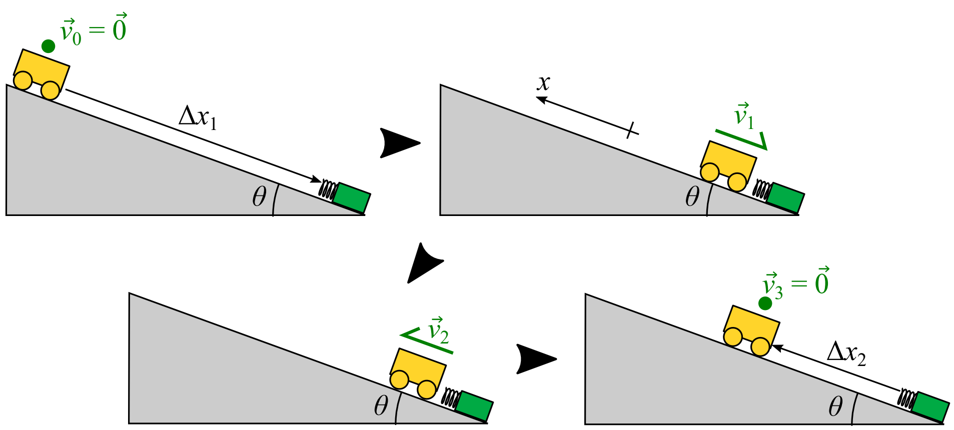
A 200 g toy cart is placed initially at rest on a 20° incline of negligible friction (also neglect air resistance). At the bottom of the incline is a spring anchored firmly to the ground. The distance between the cart and the spring is initially 1.0 m (see figure below).

As the toy cart slides down the incline it eventually hits the spring, which propels it back to 60 cm up along the incline before the cart slides back down again. Determine the impulse that *the cart exert on the spring* over the duration when the cart is in contact with the spring and got bounced back the first time. You may take the impulse approximation in your calculations.




**Note.** There is no "magnitude of" in the question. In other words, you need to specify the impulse as a vector.

**Solution** We visualize the situation in the figure below, where we define various quantities and also establish our coordinate convention. Note that $\vec{v}_1$ denotes the velocity of the cart right before it hits the spring while $\vec{v}_2$ denotes the velocity of the cart right after it leaves the spring. From the given information, we know $\theta = 20°$, $\Delta x_1 = -1.0$ m and $\Delta x_2 = +0.6$ m.




When the cart is moving up and down the slope, it undergoes constant acceleration motion. Moreover, since the slope is frictionless, the acceleration of the cart is simply $a_x = -g \sin\theta = -3.35$ m/s$^2$ (the minus sign comes from our coordinate choice). Next, from the standard formula for constant acceleration motion,

$$v_{1x}^2 - v_{0x}^2 = 2a_x \Delta x_1$$
$$v_{3x}^2 - v_{2x}^2 = 2a_x \Delta x_2$$

(Note that all quantities appearing above are signed). Plugging in the known values and solving, we find $v_{1x} = -2.59$ m/s while $v_{2x} = 2.01$ m/s.

Next, using the impulse-momentum theorem, the impulse $J_{c,x}$ imparted *on the cart by the spring* is, with $m = 0.200$ kg,

$$J_{c,x} = \Delta p_{c,x} = m(v_{2x} - v_{1x}) = +0.919 \text{ N·s}$$

By Newton's Third Law, the impulse $J_{sp,x}$ imparted *on the spring by the cart* is related to $J_{c,x}$ via $J_{sp,x} = -J_{c,x}$. Thus, we find $J_{sp,x} = -0.919$ N·s. Put differently, the impulse is 0.919 N·s *down along the incline*.