Ideal Gas Specific Heat; Heat Engine Overview

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Outline

1. Loose ends
2. Specific heat of ideal gas
3. Heat engine overview
4. Ideal-gas heat engine

1. Loose ends

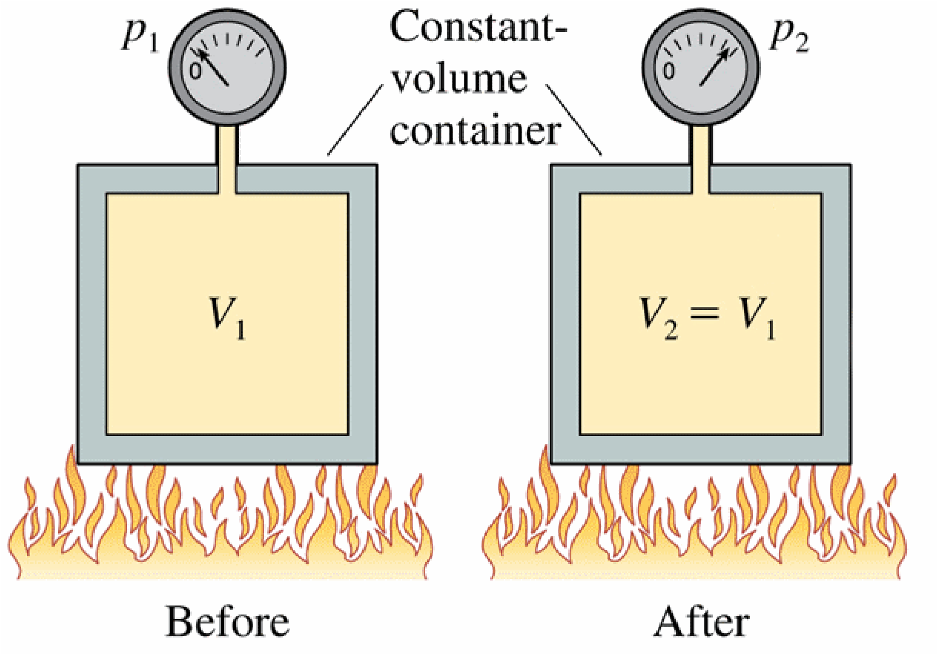
Numerical practice: calorimetry

A 750 g aluminum pan ($c = 900$ J/kg·K) are removed from the stove and plunged into a sink filled with 10.0 L of water ($c = 4190$ J/kg·K) at 20°C. The water temperature quickly rises before it stabilizes at 24.0°C. What was the initial temperature of the pan in °C?

2. Specific heat of ideal gas

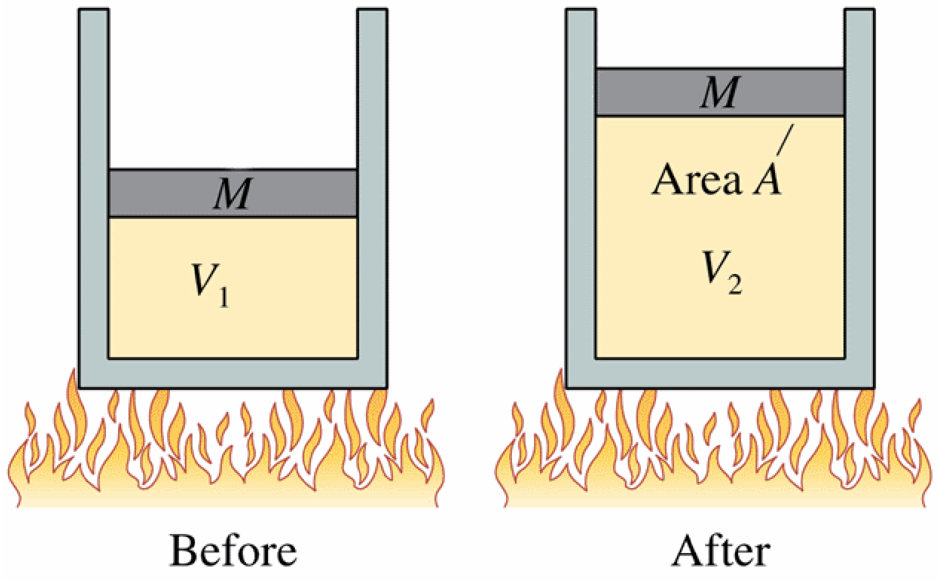
Reminder: isochoric vs isobaric processes

- Isochoric (V = const.) process:



$$Q = \Delta E_{\text{th}} = \frac{3}{2} nR\Delta T$$

- Isobaric (p = const.) process:



$$Q = \Delta E_{\text{th}} - W = \frac{5}{2} nR\Delta T$$

Molar specific heat for monoatomic ideal gas

- Recall the definition of molar specific heat C: $Q = nC\Delta T$

- For constant volume process,

$$Q = \Delta E_{\mathrm{th}} = \frac{3}{2} nR\Delta T$$

- Thus (for monoatomic gas)…

$$C_V = \frac{3}{2} R$$

- For constant pressure process,

$$Q = \Delta E_{\mathrm{th}} = \frac{5}{2} nR\Delta T$$

- Thus (for monoatomic gas)…

$$C_P = \frac{5}{2} R$$

Relationship between C_P and C_V

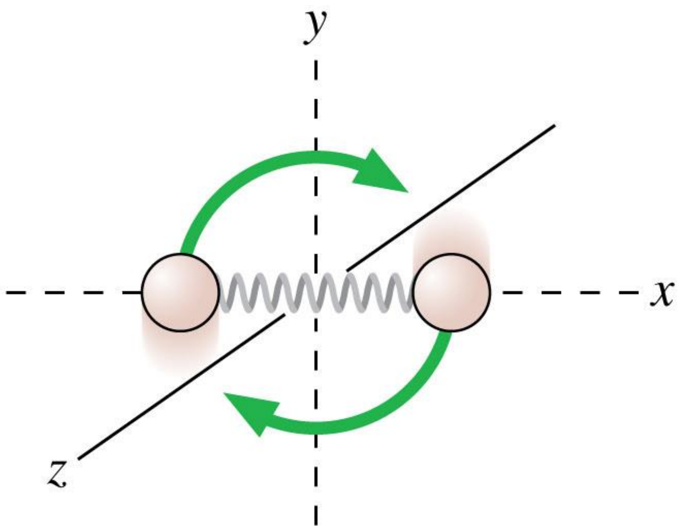
- From $C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R$, we see that $C_P = C_V + R$
- This difference stems from the work done in the constant pressure case, where in $W = -p\Delta V = -nR\Delta T$
- Note that no monoatomic assumption is made. So in general:

$$\boxed{C_P = C_V + R}$$

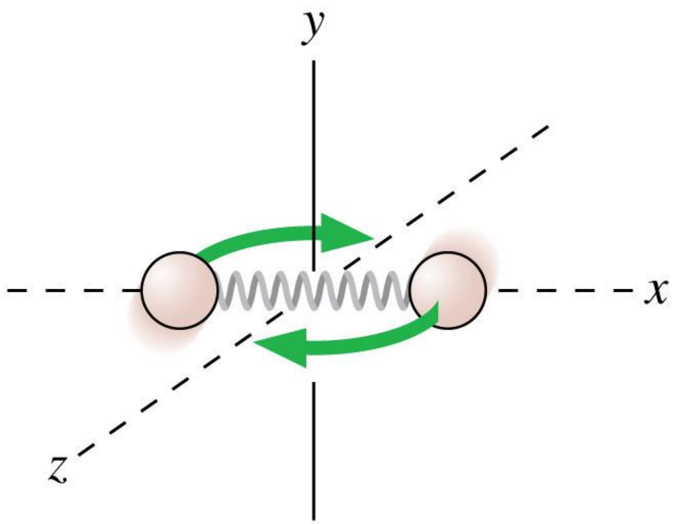
(**all** ideal gas)

Beyond monoatomic gas:* new degrees of freedom

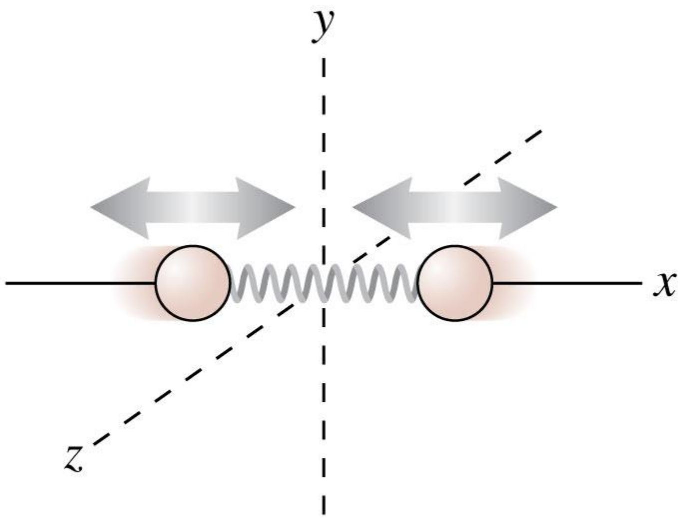
- In addition to an overall translational motion, diatomic molecule can rotate about 2 of its axes and can vibrate about its bond axis



Rotation end-over-end about the z-axis



Rotation end-over-end about the y-axis



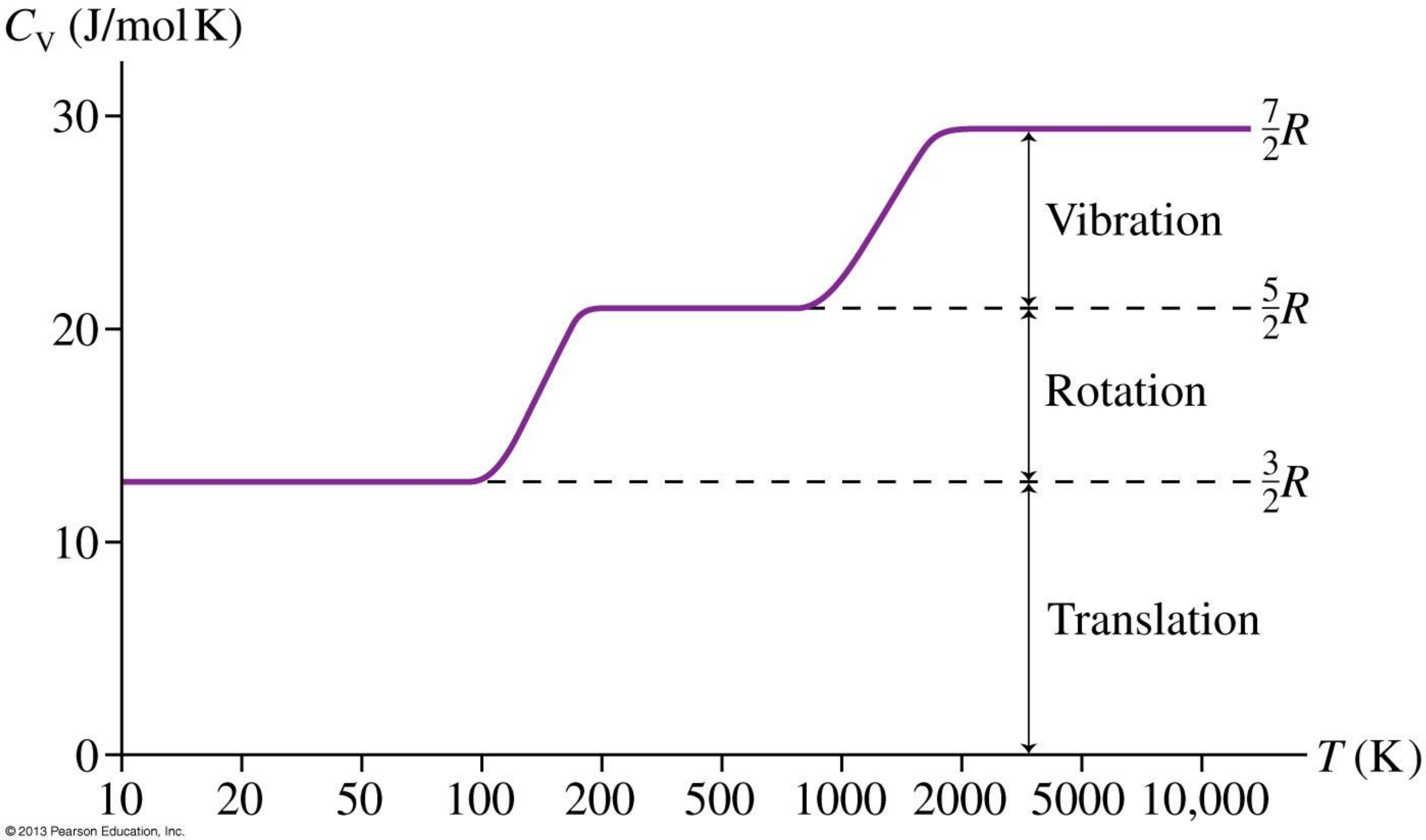
Vibration back and forth along the x-axis

Beyond monoatomic gas: increase in C_V

- Each additional "degree of freedom" can store energy and thus contributes additionally to E_th and hence C_V
- However, these additional degrees of freedom are often "activated" only at higher temperatures
- Similar ideas apply also to more complicated molecules*

* See Knight § 18.4 for more information

Beyond the monoatomic gas: putting all together



Upshot: use $C_V = \frac{5}{2}R$ for diatomic gas at "typical" temperatures $200\text{ K} \lesssim T \lesssim 800\text{K}$

3. Heat engine overview

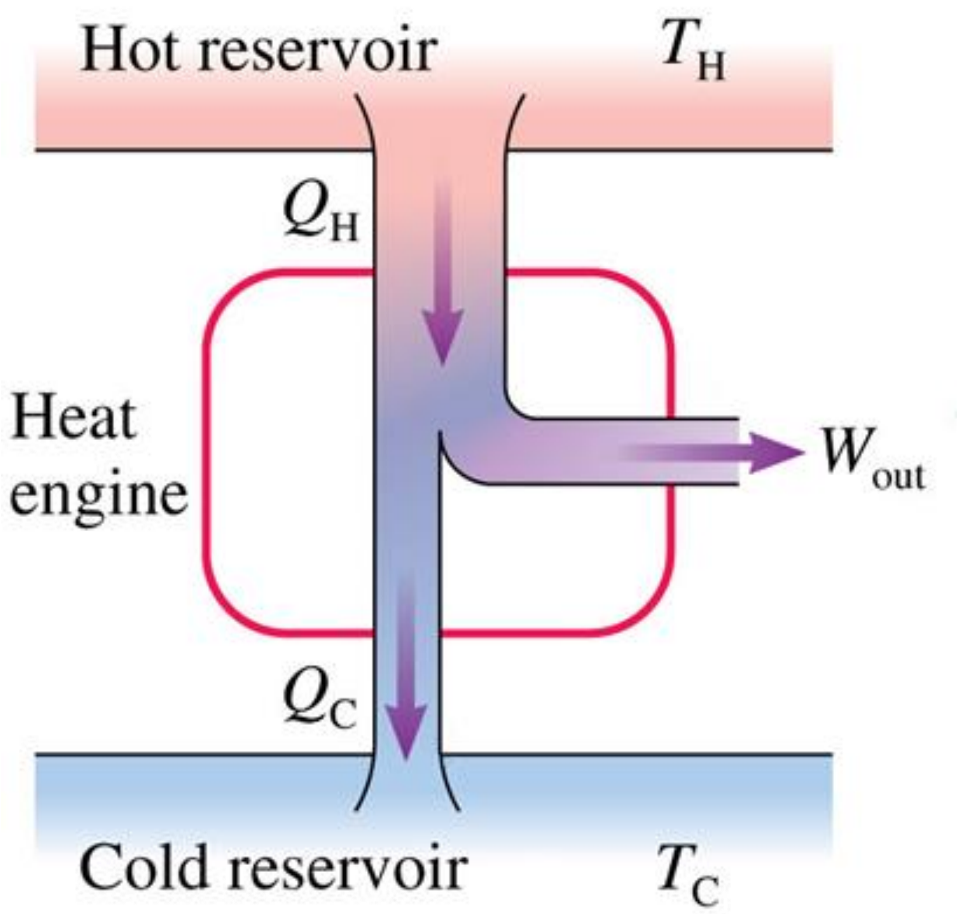
Basic properties of a heat engine

- **Heat engine** converts heat into useful work
- Conceptually, car engines, steam generators, etc. are all heat engine of some kind
- A heat engine operates in **cycles**, i.e., it returns to its initial state periodically
- A heat engine general operates between a **hot reservoir** and a **cold reservoir**

Energy-transfer diagram and relevant quantities

- T_H = temperature of hot reservoir
- T_C = temperature of cold reservoir
- Q_H = heat absorbed from hot reservoir
- Q_C = heat released to cold reservoir
- W_{out} = useful work output

*Q_H, Q_C, and W_{out} are values **per cycle**, and are all taken to be **positive**



First law of thermodynamics on heat engine

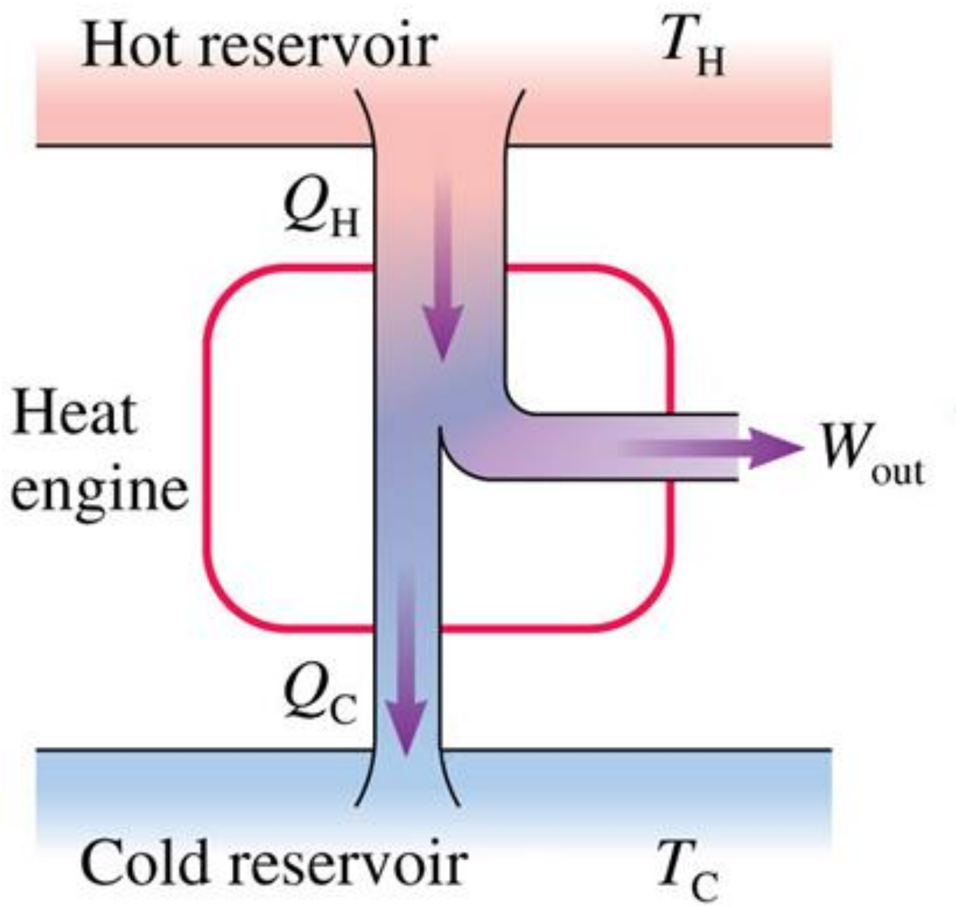
- Relating our definitions,

$$Q = Q_H - Q_C$$
$$W = -W_{\text{out}}$$

(per cycle)

- Since the heat engine is cyclic, $\Delta E_{\text{th}} = 0$ after each cycle
- Thus, 1st law implies:

$$\boxed{W_{\text{out}} = Q_H - Q_C}$$

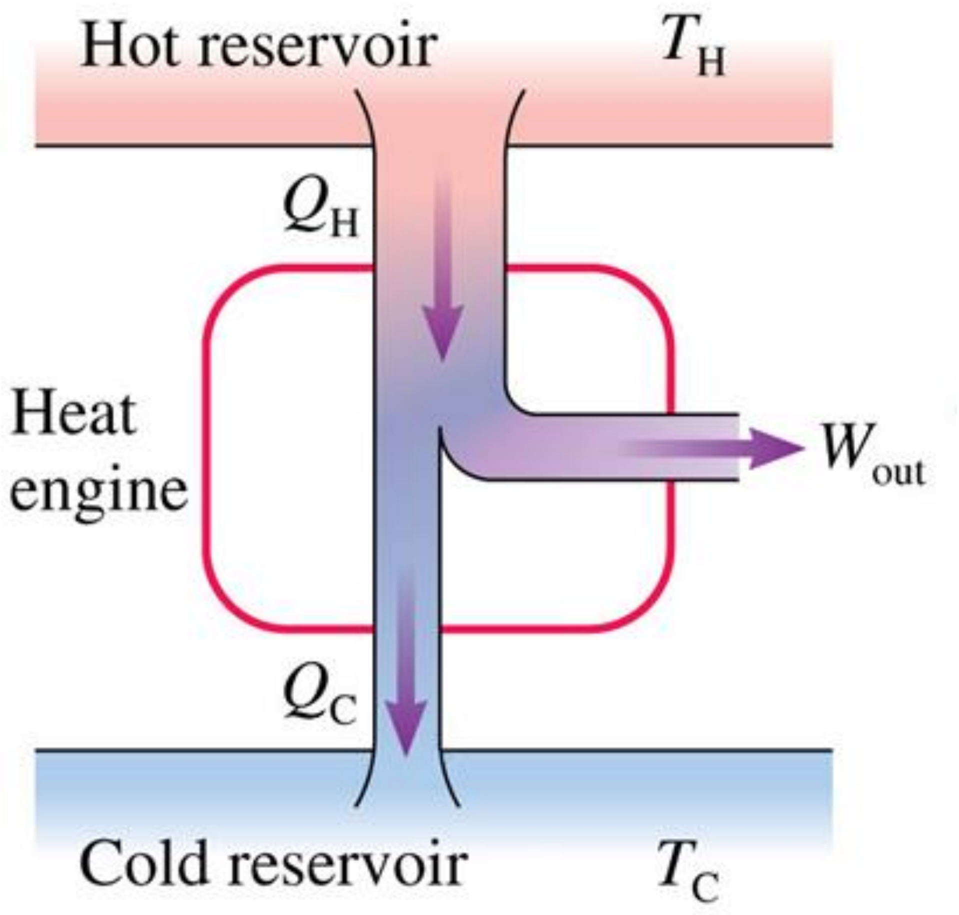


Heat engine efficiency

- The **thermal efficiency** of a heat engine, η, is defined as:

$$\eta = \frac{W_{\text{out}}}{Q_H}$$

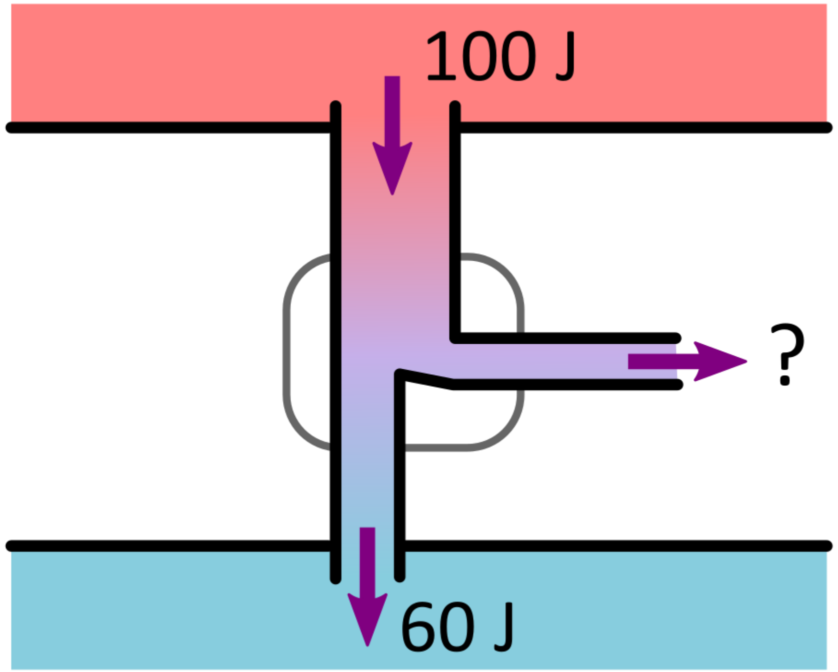
- One important question for heat engine is whether η is limited



Week 13 preflight Q2

$$W_{\text{out}} = Q_H - Q_C = 40\text{ J}$$

$$\eta = W_{\text{out}}/Q_H = 0.4$$



Your turn: heat engine efficiency

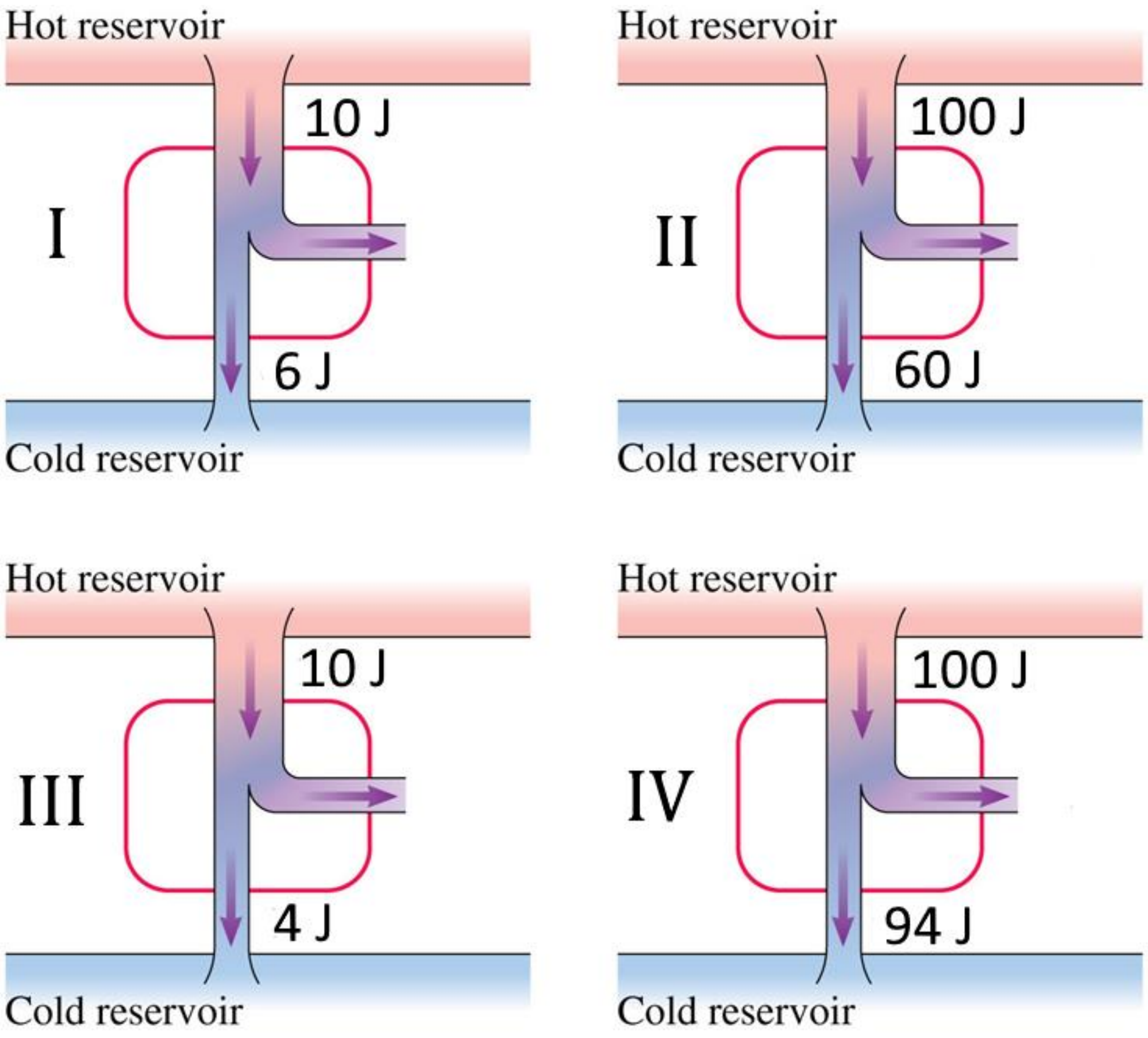
Rank the following heat engines by thermal efficiency, from largest to smallest

A. (II) > (III) = (IV) > (I)

B. (II) = (IV) > (I) = (III)

C. (III) > (I) = (II) > (IV)

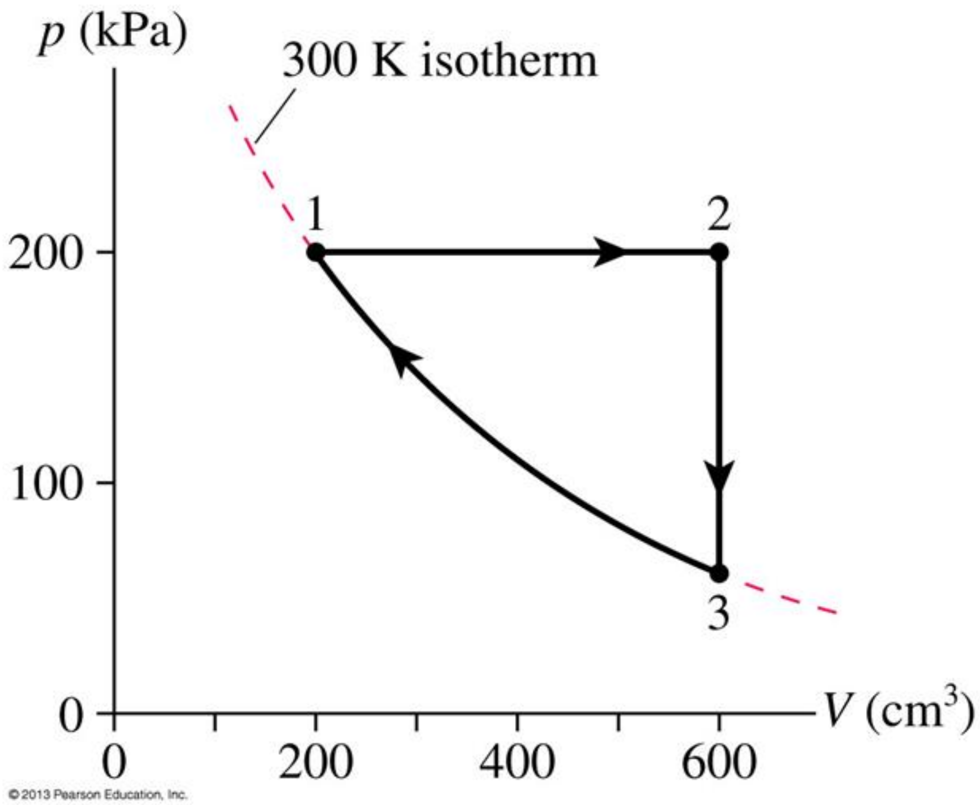
D. (IV) > (II) > (I) > (III)



4. Ideal-gas heat engine

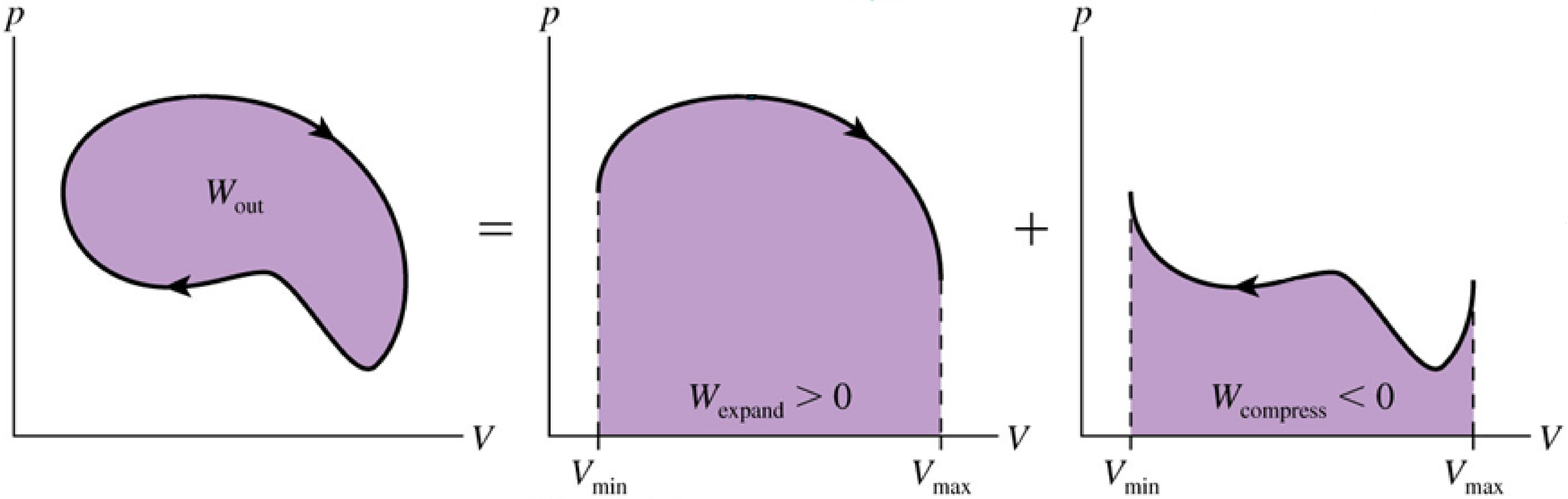
Ideal-gas heat engine

- For simplicity our heat engine will use monoatomic ideal gas as its working substance
- We will put together several ideal gas processes to form a cycle
- As before, p-V diagram will be central to our analysis

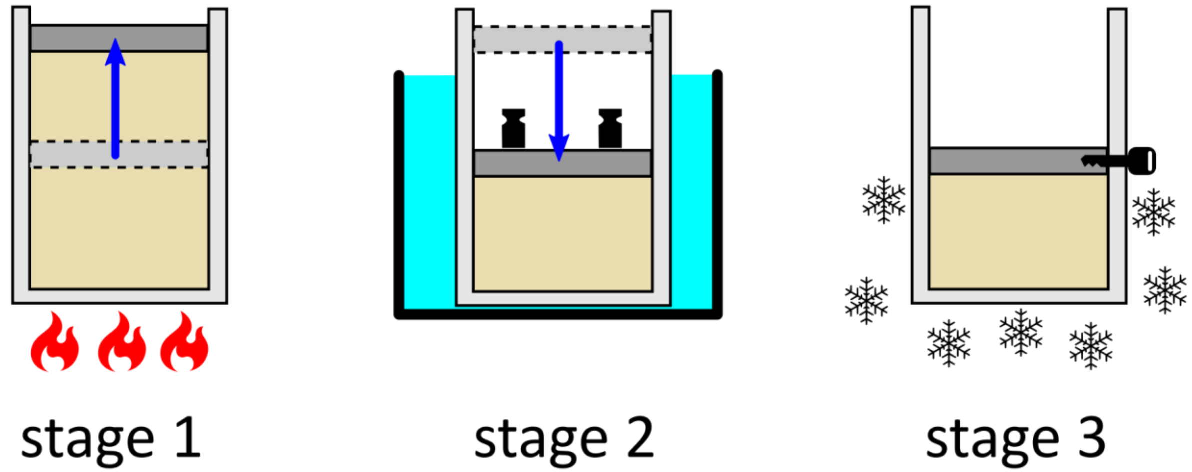


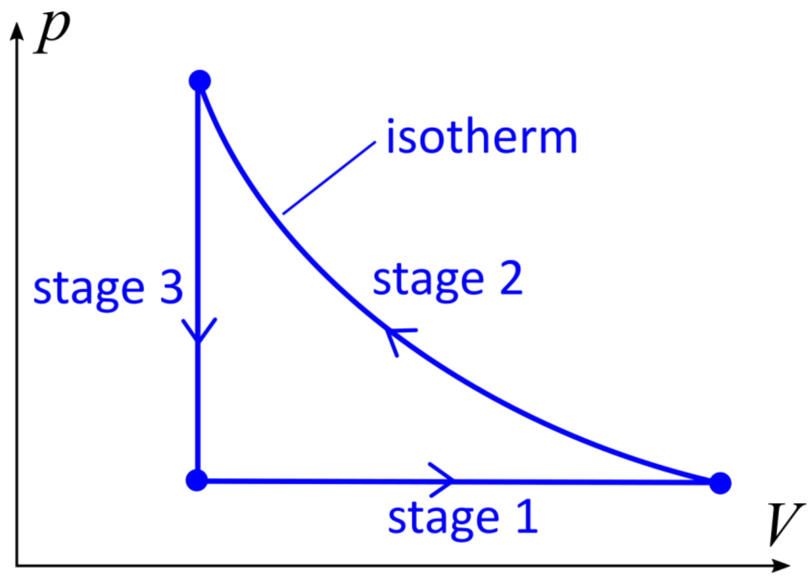
Work output in an ideal-gas heat engine

- For an ideal-gas heat engine, the work output per cycle can be read off from the area enclosed by the cycle:



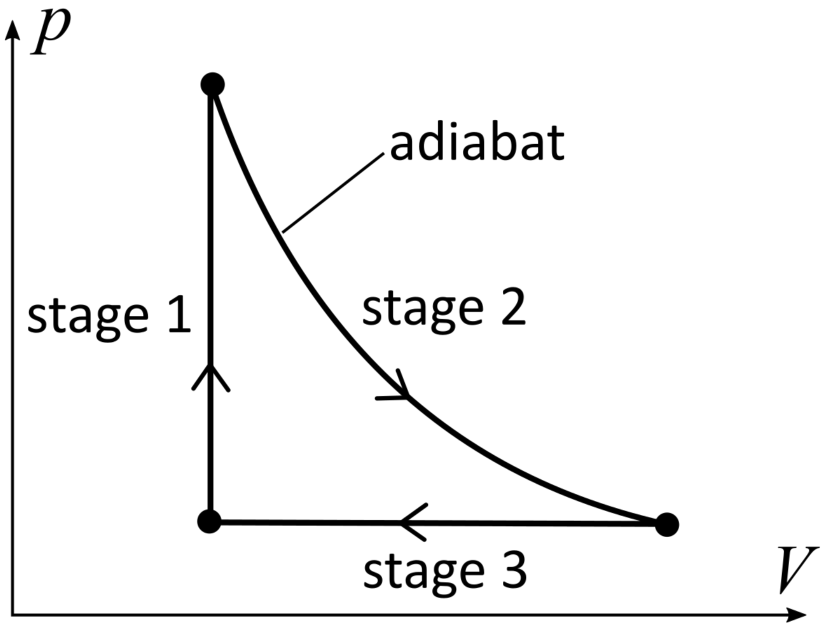
Week 13 preflight Q1





Week 13 preflight Q3

Stage	W	ΔE_{th}	Q
1	0	+ve	+ve
2	−ve	−ve	0
3	+ve	−ve	−ve



Your turn: ideal-gas heat engine

Which of the following p-V diagram represent an ideal-gas heat engine?

A. (I) only

B. (I) and (II)

C. (I), (II), and (III)

D. All of the above