SCIENTIFIC MEASUREMENT AND UNCERTAINTY

PART 1. MAKING A "GOOD" MEASUREMENT AND QUANTIFYING OUR CONFIDENCE IN IT

When we collect data (make measurements), there are usually multiple factors to consider when deciding how to get the most reliable values possible. As a group, we will perform a simple experiment as an example. At the front of the room, your instructor has a pendulum. We want to find the oscillation period of the pendulum. One obvious factor to think about is where in the pendulum cycle is best to measure the period. Let us test if it is better to start and stop the stopwatch at the *top* of the swing or at the *bottom* of the swing.

Your instructor will get the pendulum swinging. With a stopwatch (either on your phone or with the one provided), measure the time for one period starting from the highest position. Repeat the measurement starting from the lowest position.

Record both of these values in your notebook. Also, report your results on the board.

Your instructor will enter the data into a spreadsheet and calculate a class average for both timing methods.

In your notebook, record the class averages. Also note which of the four values you think is the most reliable.

Your instructor will lead a discussion to reflect on these measurements.

READING A DIGITAL SCALE

At your tables, discuss the following and answer the questions.

Suppose you have a metal block and a digital scale that can display mass to one digit after the decimal point (in grams). You put the block on the balance and the display shows 57.4 g.

Clearly, it is reasonable to record the mass of the block as 57.4 grams.

It is possible to change the sensitivity of the digital scale to display mass to two digits past the decimal point (so to 0.01 gram). The second digit after the decimal point can be a 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. Obviously, you cannot predict for sure which digit the display will show at the increased sensitivity.

> Can you say what the *probability* is of the last digit being, say, 6?

At the higher sensitivity, the scale now reads 57.36 g. In principle, increasing the sensitivity (displaying more decimal places) of our scale allows us to know the mass of the block more reliably. This number should be closer to the "real" value of the block's mass.

> Is it possible to design and build an electronic scale that can display an infinite number of decimal places? Explain your answer.

Let us think about what we know about the block's mass in each case based only on the reading of the digital scale.

The display shows:	Inference about the mass of the block:
57.4 g	The mass lies between 57.35 g and 57.45 g.
57.36 g	The mass lies between _____ and _____.
57.362 g	The mass lies between _____ and _____.

In each case, we can say that the mass of the block lies somewhere on an *interval*. The width of the interval decreases as the sensitivity of the scale increases.

READING AN ANALOG SCALE
We could also use an analog scale to measure the mass of our block. There is a needle that is displaced an amount proportional to the mass of the object. At first, you have a scale that has markings every 10 grams.
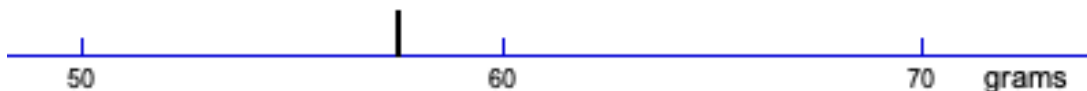
> What do you think is the reading on the scale? Does your value match with the values of others at your table?

> In what *interval* do you think the actual mass of the block lies? Explain how you made this decision.

Next, we replace the scale with one marked every 1 gram.

> What is the reading on the scale now?

> In what *interval* do you think the actual mass of the block lies? Is this interval the same as the previous one or different? Explain why.

We could divide the scale further, but it quickly becomes difficult to read! You will always need to make a judgement about the last digit in any reading.

At this point, your instructor will lead a discussion and introduce some ways of describing the reliability of a measurement.

SUMMARY OF PART 1.

We have talked about three main ways to find the best value for a measurement: a single digital reading, a single analog reading, and the average of multiple readings. The table below summarizes these three methods and how to find the best value and the uncertainty, u, in the value.

| Type of measurement	Set of N readings	Single digital reading	Single analog reading
How best value is found	Average of N trials	Digital reading	Best estimate of value
How uncertainty is found	$u(\bar{x}) = s(\bar{x}) = s(x)/\sqrt{N}$ (standard error or std. deviation of the mean)	$u(x)$ = half of least count	$u(x)$ = judgment

PART 2. COMPARING MEASUREMENTS: HOW DO WE KNOW RESULTS "AGREE"?

In the first part of the lab, we developed ways to quantify the reliability of our measurements. We now need to consider how to tell if two measurements are consistent with one another, or if they are systematically different.

Here are five values for the measured period of a pendulum along with their uncertainties. The data are presented both in a table and as data points with error bars. These error bars graphically represent the size of the uncertainty.

Measurement	Period (s)	Uncertainty (s)
A	1.21	0.02
B	1.403	0.014
C	1.41	0.02
D	1.38	0.10
E	1.52	0.15

Five measurements of pendulum period with error bars representing the measurement uncertainty.
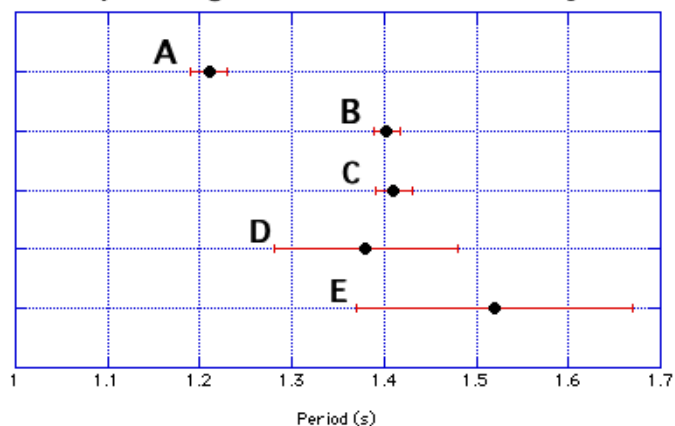
Discuss the following questions at your tables:

On a scale of 1 to 5, how confident are you that measurement A is distinguishable from measurement B? (1 is not at all confident and 5 is very confident.)

On a scale of 1 to 5, how confident are you that measurement A is distinguishable from measurement D? (1 is not at all confident and 5 is very confident.)

On a scale of 1 to 5, how confident are you that measurement C is distinguishable from measurement E? (1 is not at all confident and 5 is very confident.)

With your group, invent at least three different ways to quantify your confidence in the distinguishability of any two measurements.

Your instructor will lead the class in a discussion of your ideas.

Summarize the class discussion in your notebook.

SUMMARY OF PART 2.

The t' statistic is one of several simple ways to quantify the agreement between two measurements.

$$t' = \frac{A - B}{\sqrt{\left(u(A)\right)^2 + \left(u(B)\right)^2}}$$

Generally, the smaller t' is, the more confident you can be that two measurements are consistent. However, it is possible to get a very small t' if one or more of the uncertainties is large. If that is the case, you will want to try reducing the size of the uncertainty.

t' score	Interpretation of measurements
$0<|t'|<1$	Unlikely to be different, but uncertainty may be overestimated
$1<|t'|<3$	Unclear whether they are different.
$3<|t'|$	Likely to be different

PART 3. DESIGNING MEASUREMENTS: PENDULUM PERIOD AND AMPLITUDE

The goal of this final part of the lab is to determine whether the period of a pendulum depends on the maximum amplitude angle of the swing. We will be testing this model for the period, T:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

where L is the length of the string and g is the acceleration due to gravity. (This g a constant, 9.81 m/s². We'll learn what it actually means soon!)

This model assumes that

- The only forces on the pendulum are due to gravity and tension in the string
- The string is massless
- The pendulum bob behaves like a point mass
- The initial angle of amplitude is small

A successful model both explains and predicts a physical behavior, but all have their limitations. One needs to be aware of the assumptions and simplifications made in the model. We will be testing the limitations of assuming that the period does not depend on the initial angle.

Use the materials at your lab station to construct a pendulum. Test out your measurement equipment (stopwatch, protractor, and measuring tape) and think about ways to make the measurements as precisely as you can.

> Discuss with your lab partner(s) the sources of uncertainty in the various measurements and estimate the expected size of those uncertainties. Record both the source and size in your lab notebook.

> In your notebook, write down your group's plan (procedure) to measure periods with the smallest possible uncertainty at amplitudes of 10º and 20º. Describe how you will measure the period and how you will minimize uncertainty using our earlier discussions as a guide.

Carry out your experiment and record the data and results in your notebook.

Write a brief interpretation of your results in your notebook.

Consult with another group. Did their method differ from yours? What results did they obtain?

Based on your experiment, is the given model (that is independent of initial angle) supported?

FINISHING UP

Have your instructor check over your notebook before you leave the lab. You should have responses to all the items in text boxes throughout this document. Look for the "MyLab" post-lab questions on the Moodle page for your lab section and submit your responses by the deadline. If you have kept good notes, this should be pretty easy!