Heat Engine Analysis

Wing-Ho Ko
wko1@swarthmore.edu

Outline

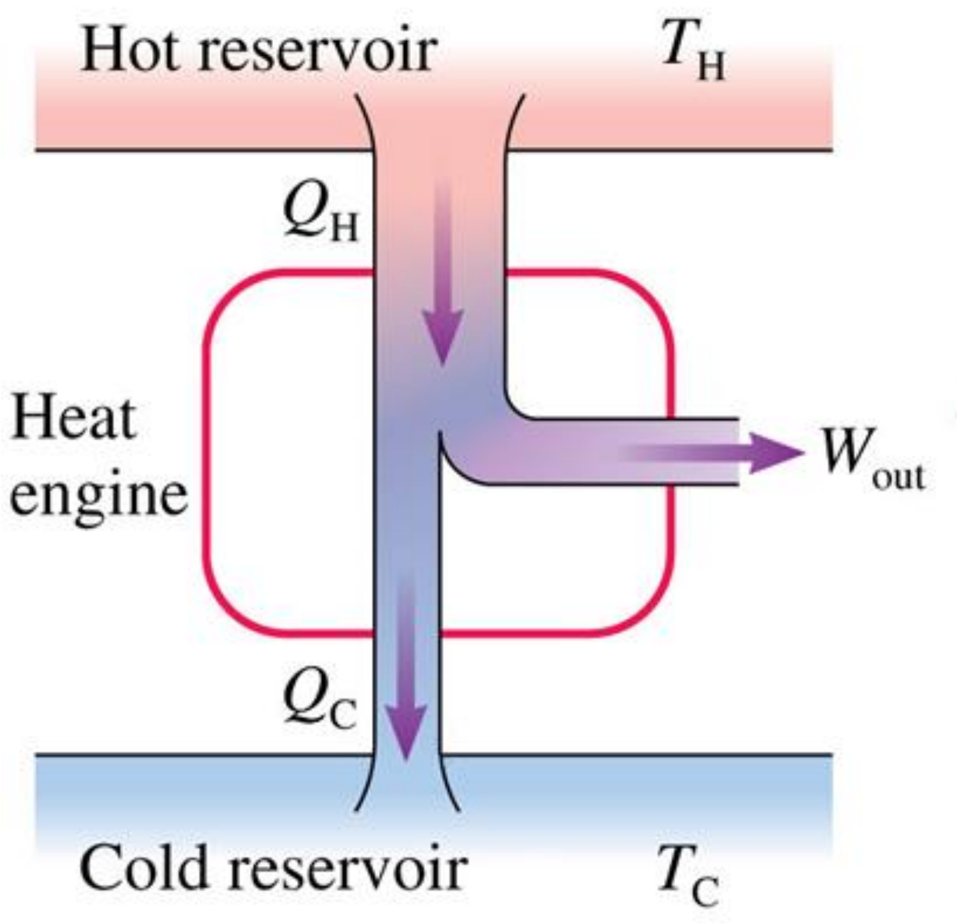
1. Heat engine analysis
2. (?) Refrigerators and heat pump

1. Heat engine analysis

Energy-transfer diagram and relevant quantities

- T_H = temperature of hot reservoir
- T_C = temperature of cold reservoir
- Q_H = heat absorbed from hot reservoir
- Q_C = heat released to cold reservoir
- W_{out} = useful work output

*Q_H, Q_C, and W_{out} are values **per cycle**, and are all taken to be **positive**



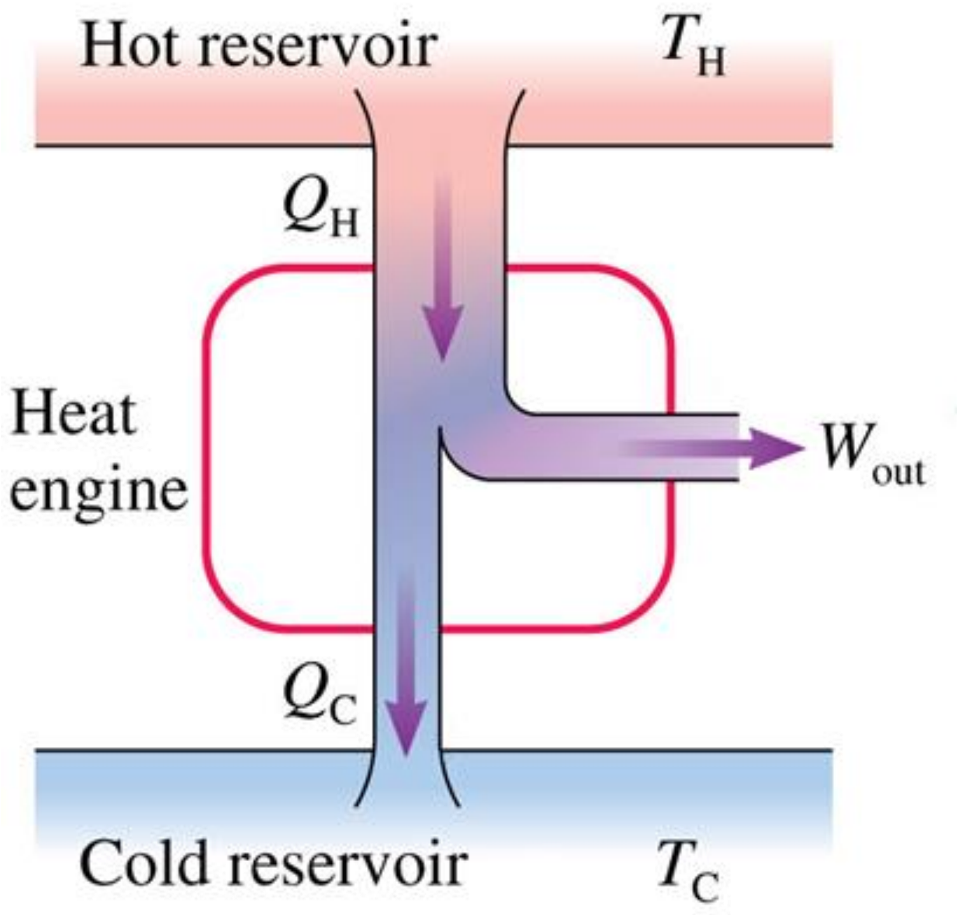
First law of thermodynamics and thermal efficiency

- 1st law of thermodynamics:

$$\boxed{W_{\text{out}} = Q_H - Q_C}$$

- Thermal efficiency:

$$\eta = \frac{W_{\text{out}}}{Q_H}$$



Strategy for solving ideal-gas heat engine problems

1. Visualize the situation using a p-V diagram
2. Obtain p, V, n, T of the ideal gas at one point in the cycle
3. Use the ideal gas law and knowledge of specific processes to obtain p, V, T at the beginning and the end of each process
4. Compute W, Q, and $\Delta \mathrm{E}_{\mathrm{th}}$ for each process. Check for consistency
5. Combine the results to obtain Q_H, Q_C, and W_{out}
6. Compute the thermal efficiency η if asked

Summary: monoatomic ideal gas processes

Process	Definition	W	Q	ΔE_{th}
Isochoric	$V = \text{const.}$	0	$\frac{3}{2}nR\Delta T$	$\frac{3}{2}nR\Delta T$
Isobaric	$p = \text{const.}$	$-p\Delta V \; (= -nR\Delta T)$	$\frac{5}{2}nR\Delta T$	$\frac{3}{2}nR\Delta T$
Isothermal	$T = \text{const.}$	$-nRT\ln(V_f/V_i)$	$nRT\ln(V_f/V_i)$	0
Adiabatic	$Q = 0$ [*]	$\frac{3}{2}(p_f V_f - p_i V_i)$	0	$\frac{3}{2}(p_f V_f - p_i V_i)$

* Consequences: $pV^{\gamma} = \text{const.}$ and $TV^{\gamma-1} = \text{const.}$ $(\gamma = 5/3)$

Setup of our heat engine

1 mole of monoatomic gas is placed in a sealed container. The container is initially at $p_1 = 1$ atm (=101 kPa) and $T_1 = 300$ K. The gas works at a heat engine with the following cycle:

i. The gas undergoes adiabatic compression until it reaches $p_2 = 3$ atm
ii. The gas undergoes isobaric heating until it reaches $T_3 = 600$ K
iii. The gas expands isothermally until its volume returns to the initial
iv. The gas undergoes isochoric cooling until pressure returns to 1 atm

Step 1: draw the p-V diagram that depict the cycle

Step 1: visualize with p-V diagram

Step 2: find p, V, n, T of initial state

- Given:

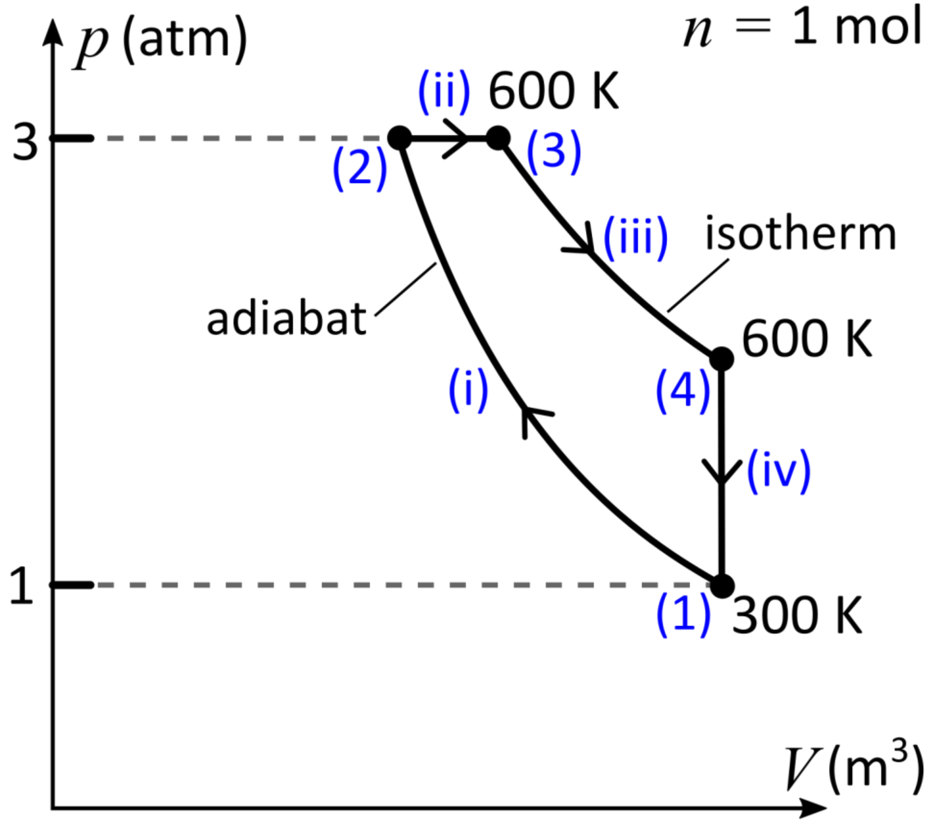
  $n = 1$ mole

  $p_1 = 1$ atm $= 101$ kPa

  $T_1 = 300$ K

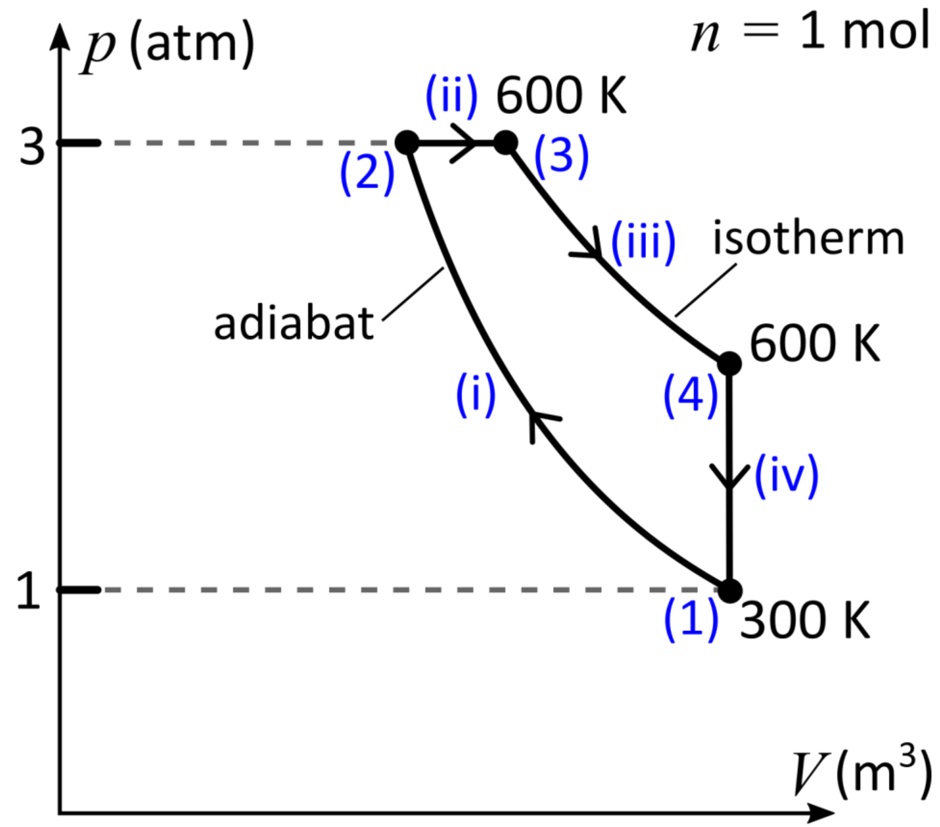
- Want:

  $V_1 = ???$



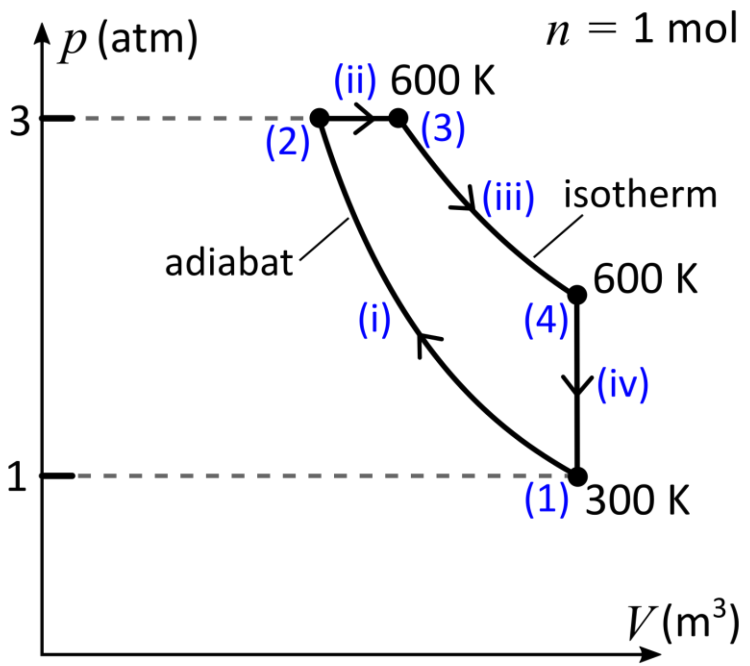
Step 3: find p, V, T at endpoints of processes

- $p_4 = ???$
- $V_3 = ???$
- $V_2 = ???$
- $T_2 = ???$



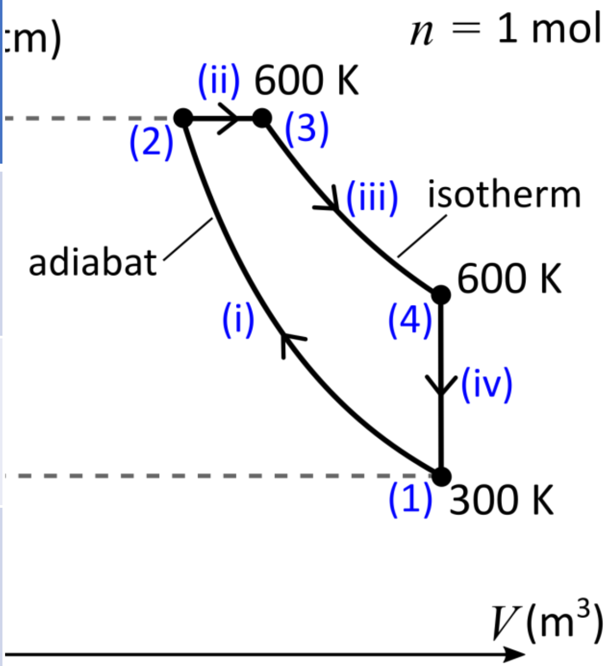
Step 4: compute W, Q, and ΔE_{th}

Stage	W	Q	ΔE_{th}
(i)			
(ii)			
(iii)			
(iv)			



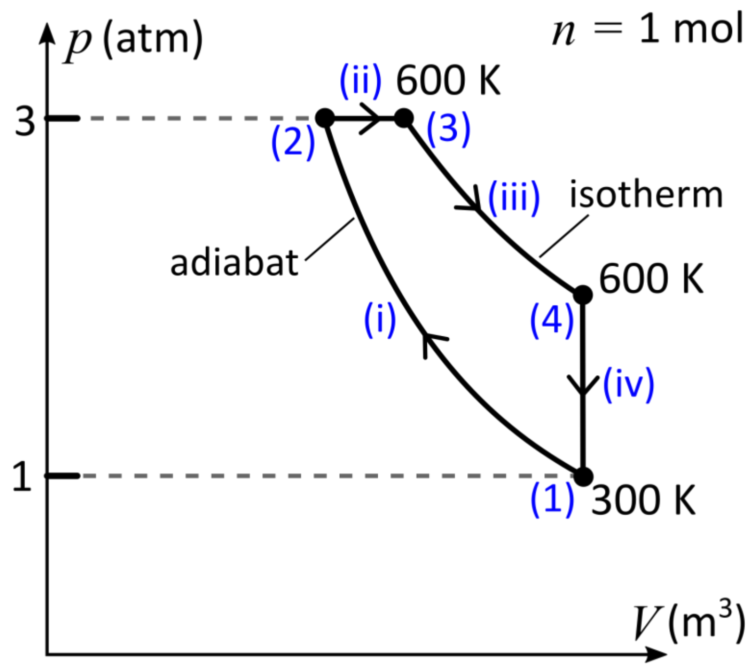
Step 4’: check for consistency

Stage	W	Q	ΔE_{th}	1st law
(i)				
(ii)				
(iii)				
(iv)				
Σ				



Step 5: compute W_{out}, Q_H, and Q_C

Stage	W	Q	ΔE_{th}
(i)			
(ii)			
(iii)			
(iv)			



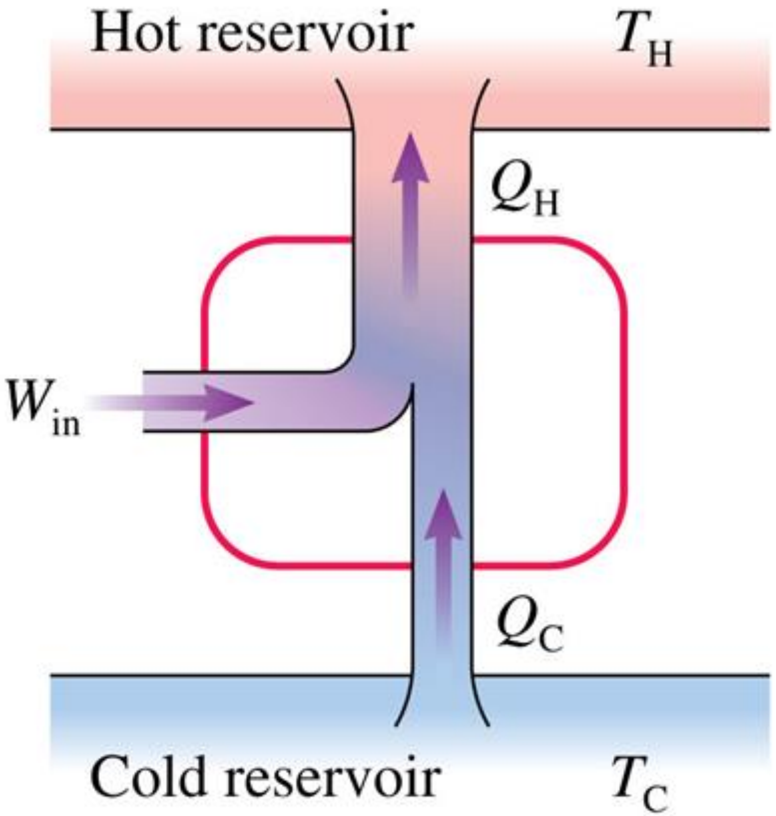
$W_{\text{out}} = ???$, $Q_H = ???$, $Q_C = ???$

Step 6: compute η

2. Refrigerators and heat pumps

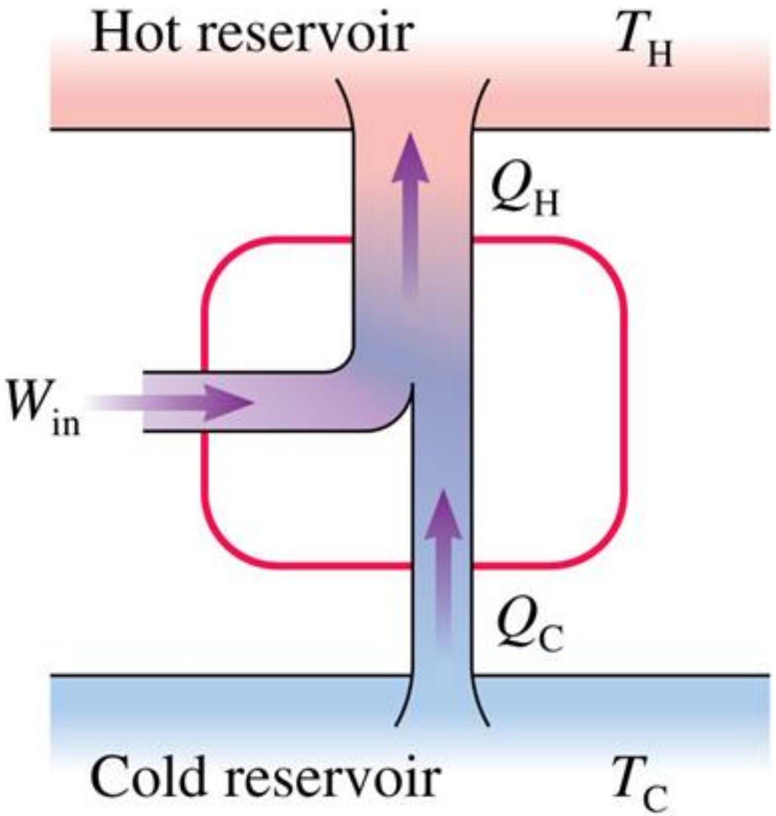
Heat engine in reverse: refrigerators and heat pumps

- **Refrigerators** remove heat from the cold reservoir
- **Heat pumps** deposit heat into the hot reservoir
- Both refrigerators and heat pumps operate in **cycles**, and their energy-transfer diagrams look the same



A first glimpse of the second law of thermodynamics

- One classic formulation of the **second law of thermodynamics** is that heat cannot spontaneously flow from a cold object to a warm object
- Hence, both refrigerators and heat pumps require **external work input** to operate

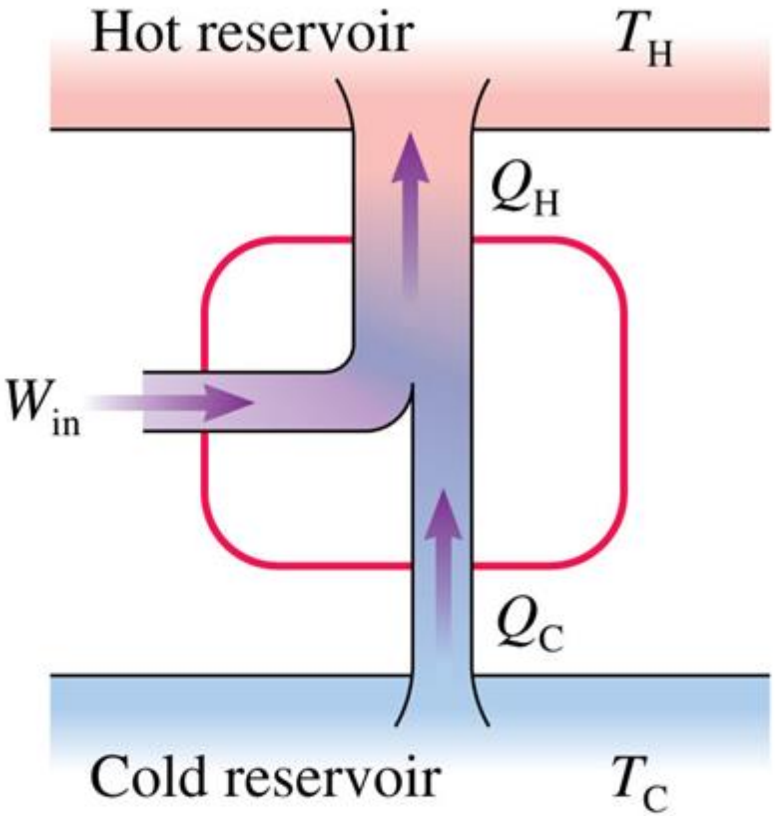


Coefficient of performance K

- The performance of refrigerators can be assessed by the **coefficient of performance** K:

$$K = \frac{Q_C}{W_{\text{in}}}$$

- As with heat engine efficiency η, we want to know if there are limits to K



Your turn: Coefficient of performance K

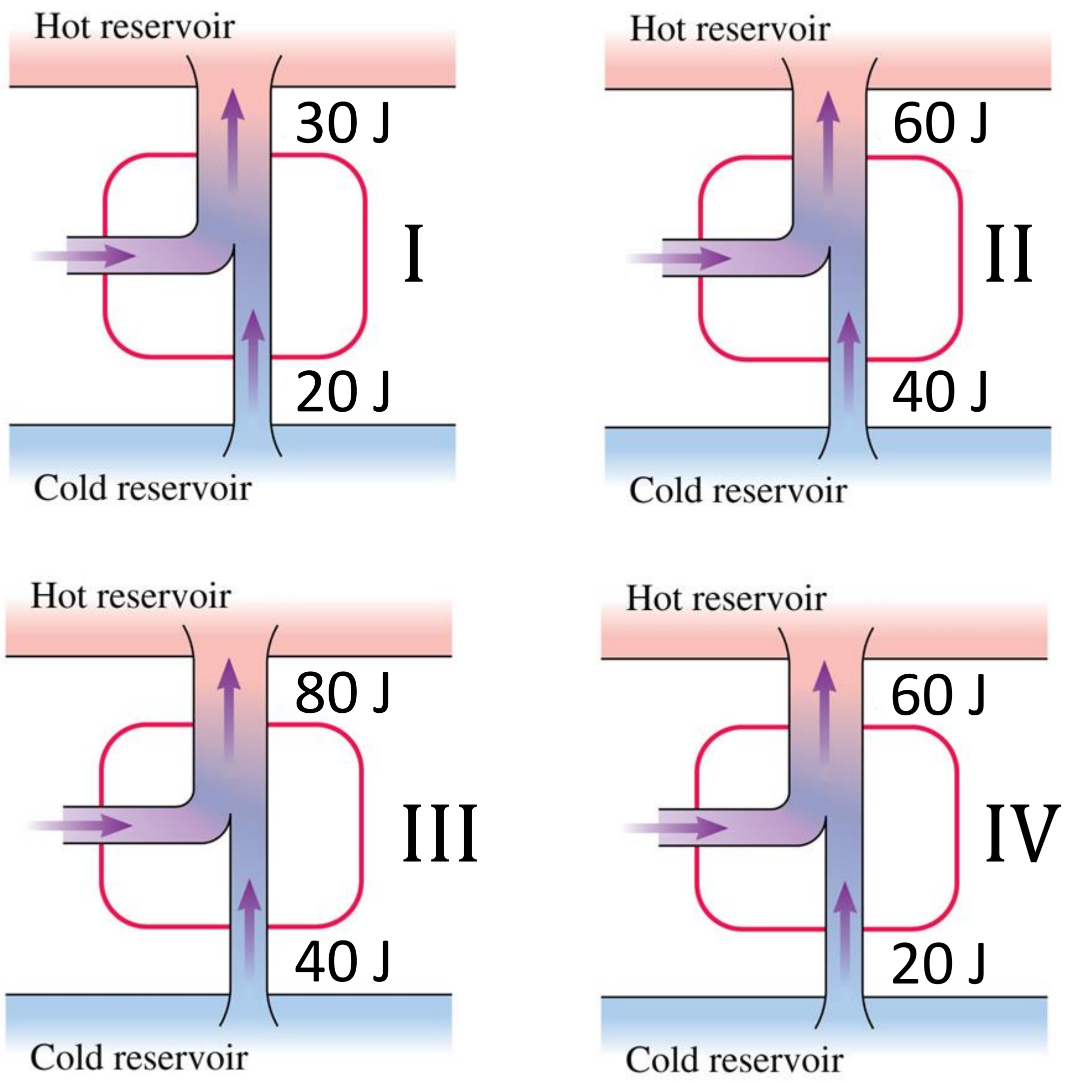
Rank the following heat engines by coefficients of performance, from largest to smallest

A. (II) = (III) > (I) = (IV)

B. (III) > (II) = (IV) > (I)

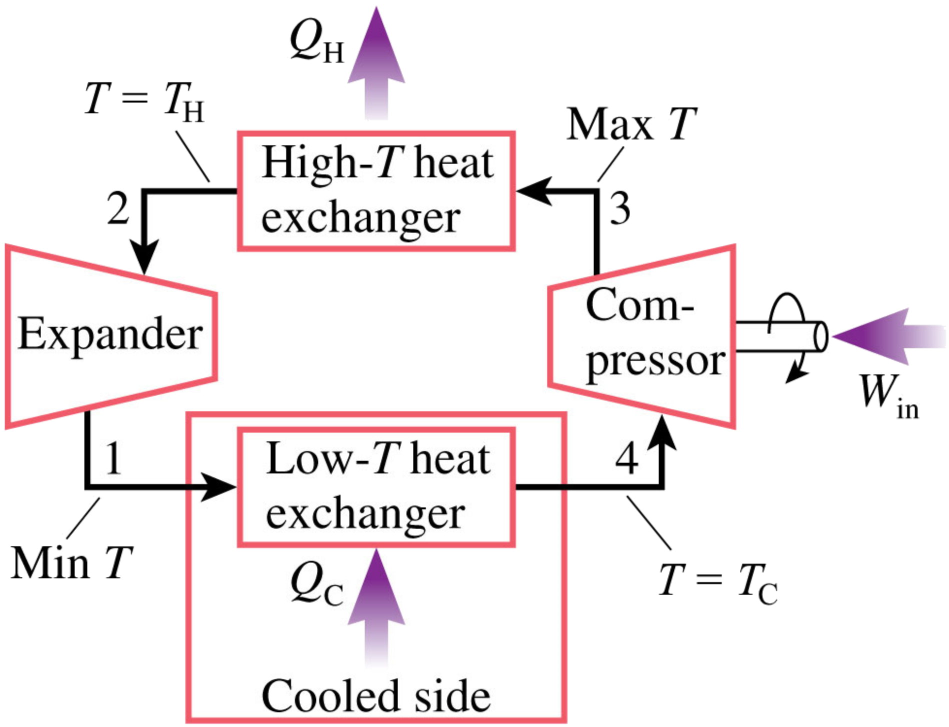
C. (IV) > (III) > (II) = (I)

D. (I) = (II) > (III) > (IV)



Refrigerators from ideal gas processes

- A refrigerator has to **remove** heat from cold reservoir and **deposit** heat into hot reservoir. Thus…
  - The refrigerator may need to reach $T > T_\mathrm{H}$ and $T < T_\mathrm{C}$ for heat transfer to happen
  - To do so the refrigerator has to rely on adiabatic processes



Your turn: refrigerator analysis—reverse Brayton cycle

A refrigerator extracts heat from a cold reservoir at 200 K and exhaust it to a hot reservoir at 300 K. It does so by running the reverse Brayton cycle (adiabatic compression → isobaric cooling → adiabatic expansion → isobaric heating) between 1 atm and 4 atm, using 1 mole of helium as working substance.

(a) Sketch the p-V diagram that depicts the cycle. Indicate on the diagram instants when T_{C}, T_{H}, $T_{\min}$, and $T_{\max}$ are attained.

(b) Determine the highest temperature $T_{\max}$ and the lowest temperature $T_{\min}$ that the substance would reach during a cycle