Process Dynamics and Control (CH 61016)

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2.

$$A = \begin{bmatrix} a & 0.15 & b & -0.09 \\ -0.01 & c & 0.29 & x \\ d & 3.31 & 0.27 & e \\ 0.22 & f & g & 1.84 \end{bmatrix}$$

RGA matrix (Relative gain array) has a property by which,

sum of each row = 1

& sum of each column = 1.

a) if $x = 1.15$, then

→ for 2nd row

$-0.01 + c + 0.29 + 1.15 = 1$

$\Rightarrow c = -0.43$

→ for 2nd column

$0.15 + c + 3.31 + f = 1$ $(c = -0.43)$

$\Rightarrow f = -2.03$

→ for 4th row

$0.22 + f + g + 1.84 = 1$

$\Rightarrow g = 0.97$

→ for 3^{rd} column

$$b + 0.29 + 0.27 + g = 1 \qquad (g = 0.97)$$

$$\Rightarrow b = -0.53$$

→ for 1^{st} row

$$a + 0.15 + b + (-0.09) = 1$$

$$\Rightarrow a = 1.47$$

→ for 4^{th} column

$$-0.09 + 1.15 + e + 1.84 = 1$$

$$\Rightarrow e = -1.9$$

→ for 3^{rd} row

$$d + 3.31 + 0.27 + e = 1$$

$$\Rightarrow d = -0.68$$

So values are

$a = 1.47$, $b = -0.53$, $c = -0.43$

$d = -0.68$ $e = -1.9$ $f = -2.03$

So, final RGA matrix

$$\begin{bmatrix} 1.47 & 0.15 & -0.53 & -0.09 \\ -0.01 & -0.43 & 0.29 & 1.15 \\ -0.68 & 3.31 & 0.27 & -1.9 \\ 0.22 & -2.03 & 0.97 & 1.84 \end{bmatrix}$$

b.) No, we can not complete the RGA if x is not known.

5). $a \cdot \frac{dy}{dt} + a_0 y = bu(t)$

Taking Laplace transform on both sides, using the initial boundary condition $y(t=0) = 0 = \frac{dy}{dt}(t=0)$,

$$a_1 s \bar{y}(s) + a_0 \bar{y}(s) = b\bar{u}(s)$$

$$\bar{y}(s) = \frac{b\bar{u}(s)}{a_1 s + a_0}$$

For sinusoidal input, $u(t) = A \sin \omega t$

Taking Laplace transform, $\bar{u}(s) = \frac{A\omega}{s^2+\omega^2}$

$$\therefore \bar{y}(s) = \frac{b}{a_1 s + a_0} \frac{A\omega}{s^2+\omega^2} = \frac{A\omega s}{(a_1 s + a_0)(s+i\omega)(s-i\omega)}$$

$$\therefore A_1(s+i\omega)(s-i\omega) + A_2(a_1 s + a_0)(s-i\omega) + A_3(a_1 s + a_0)(s+i\omega) = 1$$

$$\Rightarrow s = i\omega \Rightarrow A_3(i\omega a_1 + a_0)2i\omega = 1$$

$$\Rightarrow A_3 = \frac{1}{2i\omega(i\omega a_1 + a_0)}$$

$$\Rightarrow s = -i\omega \Rightarrow A_2(-i\omega a_1 + a_0)(-2i\omega) = 1$$

$$\Rightarrow A_3 = \frac{1}{2i\omega(i\omega a_1 - a_0)}$$

$$\Rightarrow s = \frac{-a_0}{a_1} \Rightarrow A_1\left(-\frac{a_0}{a_1} + i\omega\right)\left(-\frac{a_0}{a_1} - j\omega\right) = 1$$

$$\Rightarrow A_1 = \frac{1}{\left(\frac{a_0}{a_1}\right)^2 + \omega^2} = \frac{a_1^2}{a_0^2 + a_1^2\omega^2}$$

Taking Laplace Inverse,

$$y(t) = A\omega s\left[\frac{A_1}{a_1}\exp\left(-\frac{a_0}{a_1}t\right) + A_2\exp(-i\omega t) + A_3\exp(i\omega t)\right]$$

$$= A\omega s\left[\frac{A_1}{a_1}\exp\left(-\frac{a_0}{a_1}t\right) + (A_2 + A_3)\cos\omega t + i(A_3 - A_2)\sin\omega t\right]$$

using $a_0 \to 0$

~~$y(t) = \frac{AA_1}{a_1}$~~

$$y(t) = \frac{Ab}{a_1\omega} - \frac{Ab}{a_1\omega}\cos(\omega t)$$

$$\boxed{y(t) = \frac{Ab}{a_1\omega}(1 - \cos(\omega t))}$$

$\therefore$ Difference b/w time period of oscillation.

Input $T = \frac{2\pi}{\omega}$

Response $T = \frac{2\pi}{\omega}$.

$$\therefore \boxed{\text{Input } T - \text{Response } T = \frac{2\pi}{\omega} - \frac{2\pi}{\omega} = 0}$$

3. Transfer function model of a process

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1.318 e^{-3s}}{20s+1} & \dfrac{-e^{-s}}{3s} \\ \dfrac{0.38(18s+1)e^{-2s}}{(27s+1)(10s+1)} & \dfrac{0.36}{s} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

a) Steady state gain matrix.

$$\begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix} = \begin{bmatrix} 1.318 & -1/3 \\ 0.38 & 0.36 \end{bmatrix} \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix}$$

$$\lambda_{11} = \frac{1}{1 - \dfrac{K_{12} K_{21}}{K_{11} K_{22}}} = \frac{1}{1 - \dfrac{(-1/3)(0.36)}{1.318 \times 0.36}}$$

$$= 0.7893$$

$\therefore$ RGA matrix for the system :

$$\Lambda = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix} = \begin{bmatrix} 0.7893 & 0.2107 \\ 0.2107 & 0.7893 \end{bmatrix}$$

b) $G_R = \text{diag}\,(G(s))$

$$= \begin{bmatrix} \dfrac{1.318 e^{-3s}}{20s+1} & 0 \\ 0 & \dfrac{0.36}{s} \end{bmatrix}$$

$$G^{-1} = \frac{1}{g_{11}g_{22} - g_{12}g_{21}} \begin{bmatrix} \frac{0.36}{s} & \frac{e^{-s}}{3s} \\ -\frac{0.38(18s+1)e^{-2s}}{(27s+1)(10s+1)} & \frac{1.318e^{-3s}}{20s+1} \end{bmatrix}$$

$$G_c = G^{-1} G_R$$

$$= \frac{1}{g_{11}g_{22} - g_{12}g_{21}} \begin{bmatrix} \frac{1.318 \times 0.36e^{-3s}}{s(20s+1)} & \frac{0.36e^{-s}}{3s^2} \\ -\frac{0.38 \times 1.318(18s+1)e^{-5s}}{(20s+1)(27s+1)(10s+1)} & \frac{1.318 \times 0.36e^{-3s}}{s(20s+1)} \end{bmatrix}$$

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = G_2 \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

4). State space Model of drug concentration in blood and tissue as

$$\frac{dx_1}{dt} = 0.4x_1 + 0.09x_2 + 0.005x_3 + 0.1u$$

$$\frac{dx_2}{dt} = 0.2x_1 - 0.2x_2$$

$$\frac{dx_3}{dt} = 0.4x_1 - 0.005x_3$$

$$y = x_1$$

a) ⇒ State space Model has been determined,

$$\frac{dx_1}{dt} = -0.4x_1 + 0.09x_2 + 0.005x_3 + 0.1u$$

$$\frac{dx_2}{dt} = 0.2x_1 + 0.2x_2$$

$$\frac{dx_3}{dt} = 0.04x_1 - 0.005x_3$$

$$y = x_1$$

$$\dot{x} = Ax + Bu$$

$$\dot{x} = \begin{bmatrix} -0.4 & 0.09 & 0.005 \\ 0.2 & -0.2 & 0 \\ 0.04 & 0 & -0.005 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix} u$$

Since we have been given pole location

[-0.5, -0.15, -0.1]

we have to define a gain Matrix

$$K = [K_1, K_2, K_3]$$

& Now, we have to equate

$$|sI - A + BK| =$$

$$|sI - A| = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -0.4 & 0.09 & 0.005 \\ 0.2 & -0.2 & 0 \\ 0.04 & 0 & -0.005 \end{bmatrix}$$

$$= \begin{bmatrix} s-0.4 & -0.09 & -0.005 \\ -0.2 & s+0.2 & 0 \\ -0.04 & 0 & s+0.05 \end{bmatrix}$$

$$|sI - A + BK|$$

$$= \begin{bmatrix} s-0.4 & -0.09 & -0.009 \\ -0.2 & s+0.2 & 0 \\ -0.04 & 0 & s+0.5 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix} [K_1 \; K_2 \; K_2] \quad \text{— (1)}$$

Equating the constants with the help

$$(s+0.5)(s+0.15)(s+0.1) = 0 \quad \text{—— (2)}$$

Now solving & Equating with (1) & (2)

Now we can get

$$K = [1.45 \quad 0.5154 \quad 8.7918]$$

$$\boxed{K = [1.45 \quad 0.5154 \quad 8.7918]}$$

4. b)

$$\frac{dx_2}{dt} = 0.2x_1 - 0.2x_2$$

$$\frac{dx_3}{dt} = 0.04x_1 - 0.005x_3$$

$AX + BU$

$$= \begin{bmatrix} 0.2 & 0 \\ 0 & -0.005 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

reduced system has been mentioned.

Now, we have been given the pole placement as $[-0.05, -0.5]$

using ackermann's approach.

$$K = [0\ 0]\ [B', AB]\ \phi(A) \quad \text{---} \quad (1)$$

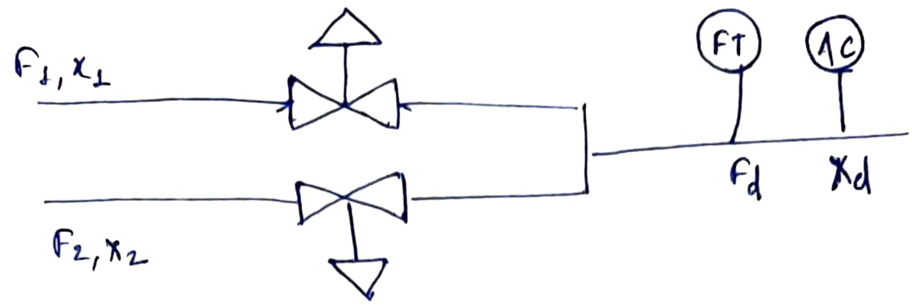
$\therefore$ characteristic equation is given by.

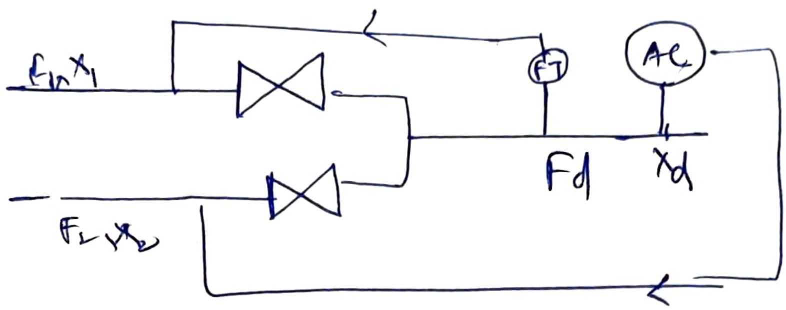
$$(s+0.05)(s+0.5) = 0$$

Now, $\phi(A) = (A+0.5)(A+0.05)$

Now eq^n two eq^n with the help of eq^n (3) & doing the Matrix OP in MATLAB, we get the values of K.

1.





consider $F_1, F_2 \longrightarrow$ input

$F_d, x_d \longrightarrow$ output

let F_1 is ~~constr~~ control to F_d and

F_2 is control to x_d

equation are as follow

$$F_d = F_1 + F_2 \quad \text{---} \quad (1)$$

$$x_d = \frac{x_1 f_1 + x_2 f_2}{f_d} \quad \text{—} \quad (2)$$

for open loop

$$\frac{df_d}{df_1} = 1 \quad \Big| \text{open loop}$$

now closed $F_2 - x_d$ loop and open $F_d - F_1$ loop eq ① become

$$0.5F_d = 0.8F_1 + 0.1F_2$$

$$\therefore F_2 = 5F_d - 8F_1 \quad \text{——— ③}$$

put ③ in eq ①

$$F_1 + (5F_d - 8F_1) = F_d$$

$$4F_d = 7f_1$$

$$\therefore \left.\frac{dF_d}{dF_1} = \frac{7}{4}\right|_{\text{Closed loop}}$$

Relative gain is given by

$$\lambda = \frac{1}{7/4} = \frac{4}{7}$$

Relative gain array is given by

$$RGA = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 4/7 & 3/7 \\ 3/7 & 4/7 \end{bmatrix} \begin{bmatrix} F_d \\ X_d \end{bmatrix}$$

$$RGA = \begin{bmatrix} \boxed{0.571} & 0.42857 \\ 0.42857 & \boxed{0.571} \end{bmatrix}$$

From above RGA array it can be seen that best pair is $F_1 - F_d$ and $F_2 - X_d$.

6. Let the transfer function be :

$$\frac{\bar{y}(s)}{\bar{u}(s)} = \frac{K.(x_1 s+1)(x_2 s+1)\ldots(x_q +1)}{(Z_{p_1} s+1)(Z_{p_2} s+1)\ldots(Z_p s+1)}$$

This is for order (p,q)

$$\frac{\bar{y}(s)}{\bar{u}(s)} = \frac{k(x_1 s+1)\ldots(x_q s+1)}{(Z_{p_1} s+1)\ldots(Z_p s+1)}$$

Now, $\bar{u}(s) = \frac{A}{s}$ for step input

$$\therefore \bar{y}(s) = \frac{k(x_1 s+1)\ldots(x_q s+1)}{(Z_{p_1} s+1)\ldots(Z_{p_p} s+1)} \cdot \frac{A}{s}$$

Here, poles are roots of denominator, & zeros are roots of numerator,

At poles, $\bar{y}(s)$ goes to infinity. whereas at zeroes, $\bar{y}(s)$ goes to 0.

Therefore stability depends on $\bar{y}(s)$.