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Sturm - Liouville Problem

$$\frac{d}{dx}\left[P(x)\frac{dy}{dx}\right] + \left[q(x) + \lambda\, w(x)\right] y = 0$$

Homogeneous 2nd order ODE with linear BC

$x = a, \quad \alpha_1 y(a) + \beta_1 \frac{dy(a)}{dx} = 0, \quad \alpha_1^2 + \beta_1^2 > 0$

$x = b, \quad \alpha_2 y(b) + \beta_2 \frac{dy(b)}{dx} = 0, \quad \alpha_2^2 + \beta_2^2 > 0$

Constraint :- $P(x), q(x), w(x)$ are real valued continuous funct. including $\frac{dP}{dx}$

$P(x), w(x)$ are +ve over $[a, b]$, $q(x) \leq 0$

$\lambda \rightarrow$ Eigen values (characteristic values)

Corresponding soln's called eigen functions (ϕ)

Non-trivial soln :- $\lambda_1 \rightarrow \phi_1, \quad \lambda_2 \rightarrow \phi_2 \quad \cdots \quad \lambda_n \rightarrow \phi_n$

let λ_n, λ_m be two distinct eigen values $\rightarrow$ $\lambda_n \rightarrow \phi_n$
$\lambda_m \rightarrow \phi_m$

$$(\lambda_m - \lambda_n)\int_a^b \phi_n(x)\,\phi_m(x)\, w(x)\, dx = 0 \rightarrow \text{derive}$$

$$\Rightarrow \int_a^b \phi_n(x)\,\phi_m(x)\, \underbrace{w(x)}_{\text{weight factor}}\, dx = 0 \rightarrow \text{orthogonal.}$$

$\rightarrow$ Consider a set of functions $\{\phi_n(x), n = 0, 1, 2 \cdots\}$ orthogonal wrt weight function $w(x)$

$$f(x) = A_0\phi_0(x) + A_1\phi_1(x) + \cdots + A_n\phi_n(x) + \cdots$$

$$f(x) = \sum_{n=0}^{\infty} A_n \phi_n(x)$$

(Arbitrary function)

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$$\int_a^b f(x)\,\phi_n(x)\,w(x)\,dx = \sum_{n=0}^{\infty} \int_a^b A_n\,\phi_n(x)\,\phi_m(x)\,w(x)\,dx.$$

$$A_n = \frac{\int_a^b \phi_n(x)\,f(x)\,w(x)\,dx}{\int_a^b \phi_n^2(x)\,w(x)\,dx}$$

$$\int_a^b \phi_n^2(x)\,w(x)\,dx = \text{norm of eigen function } \phi_n(x)$$

⇒ <u>Ordinary fourier series</u> :- $\frac{d^2y}{dx^2} + \lambda^2 y = 0$ —— ①

→ eigen value problem.

$$\alpha_1\, y(0) + \beta_1 \frac{dy(0)}{dx} = 0, \quad \alpha_1^2 + \beta_1^2 > 0$$

$$\alpha_2\, y(L) + \beta_2 \frac{dy(L)}{dx} = 0, \quad \alpha_2^2 + \beta_2^2 > 0.$$

for $p(x) = 1$, $q(x) = 0$, $w(x) = 1$

→ eqn ① is S-L problem.

complete orthogonal set in $[0, L]$

→ <u>fourier sine series</u> :- $\frac{d^2y}{dx^2} + \lambda^2 y = 0$

<u>BC</u> :- $\alpha_1\, y(0) = 0$

$\alpha_2\, y(L) = 0.$

$\phi_n(x) = \sin \lambda_n x \Rightarrow \lambda_n = \frac{n\pi}{L}, \quad n = 1, 2, \ldots$

Thus, $f(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L} x\right)$

$$A_n = \frac{\int_0^L f(x) \sin\left(\frac{n\pi}{L} x\right) dx}{\underbrace{\int_0^L \sin^2\left(\frac{n\pi}{L} x\right) dx}_{L/2}}$$

$$A_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi}{L} x\, dx.$$

$f(x)$ is any function

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$\Rightarrow$ If $f(x)$ is odd $\Rightarrow$ $[-L, 0]$ $\xrightarrow{\text{converge to}}$ $-f(-x)$

for all odd functions, expansion of a will work for $[-L, L]$

<u>Fourier cosine series</u>

$$\frac{d^2y}{dx^2} + \lambda^2 y = 0 \qquad \phi_n(x) = \cos \lambda_n x$$

$$\lambda_n = \frac{n\pi}{L}, \quad n = 0, 1, 2 \ldots$$

$$\frac{dy(0)}{dx} = 0, \quad \frac{dy(L)}{dy} = 0.$$

$$f(x) = \sum_{n=0}^{\infty} A_n \cos \frac{n\pi}{L} x$$

Period $= 2L$

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{n\pi x}{L}$$

even function

Norm for $n = 0$ is L

for $n = 1$ to ∞ $\Rightarrow L/2$

$$N_m = \int_0^L \cos^2 \frac{n\pi}{L} x \, dx$$

$$A_0 = \frac{1}{L} \int_0^L f(x) \, dx$$

$$A_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi}{L} x \, dx$$

$n = 1, 2, \ldots$

for even function $\Rightarrow$ expansion works for $[-L, L]$

<u>Fourier Bessel series</u> $\rightarrow$ Heat transfer in cylindrical coordinates

$$r^2 \frac{d^2R}{dr^2} + r\frac{dR}{dr} + (\lambda^2 r^2 - \nu^2) R = 0 \longrightarrow \text{SL problem (eigen value problem)}$$

$$\alpha_1 R(0) + \beta_1 \frac{dR(0)}{dr} = 0, \quad \alpha_1^2 + \beta_1^2 > 0$$

$$\alpha_2 (R(b)) + \beta_2 \frac{dR(b)}{dr} = 0, \quad \alpha_2^2 + \beta_2^2 > 0$$

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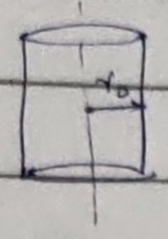
→ $P(r) = r$, $W(r) = r$, $q(r) = -\frac{\nu^2}{r}$ → special case of SL

solid cylinder 

domain :- $[0, r_0]$, $R(0)$ = finite

BC :- (2) $\alpha R(r_0) + \beta \frac{dR(r_0)}{dr} = 0$

Soln :- $R(r) = C_1 J_\nu(\lambda r) + C_2 Y_\nu(\lambda r)$

$r=0$, $R(0)$ = finite → $\boxed{R(r) = C_1 J_\nu(\lambda r)}$ $C_2 = 0$.

BC :- $\alpha C_1 J_\nu(\lambda r_0) + \beta C_1 \frac{d}{dr} J_\nu(\lambda r_0) = 0$

$$\alpha J_\nu(\lambda r) + \frac{d}{dr} J_\nu(\lambda r_0) = 0$$

eigen functions :- $J_\nu(\lambda r)$

eigen value :- λ_n $(n = 1, 2, 3 \ldots)$

⇒ Any arbitrary function $f(r)$ in $[0, r_0]$

$$f(r) = \sum_{n=1}^{\infty} A_n J_\nu(\lambda r)$$

$$A_n = \frac{\int_0^{r_0} f(r) J_\nu(\lambda_n r) r\,dr}{N_n} \qquad N_n = \int_0^{r_0} J_\nu^2(\lambda_n r) r\,dr$$

Separation of variables :- $\frac{1}{\alpha} \frac{\partial T(r,t)}{\partial t} = \nabla^2 T(r,t)$

Homogeneous, istropic, solid, stationary, opaque body, without heat generation, (const ρ, c_p k)

$T(r, t=0) = f(r)$ in R

$k \frac{\partial T}{\partial n}(r,t) + hT(r,t) = 0$ on boundary

~~$T(r,t) = \psi(r)\Gamma(t)$~~ $T(r,t) = \psi(r)\Gamma(t)$

$$\frac{1}{\alpha} \psi(r) \frac{\partial \Gamma(t)}{\partial t} = \Gamma(t) \nabla^2 \psi(r)$$

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$$\frac{1}{\alpha}\frac{1}{T(t)}\frac{dT(t)}{dt} = \frac{1}{\psi(r)}\nabla^2\psi(r) = \text{const } (-\lambda^2)$$

$$T(t) = c_1 e^{-\alpha\lambda^2 t}$$

helmholtz equation $\begin{cases} \nabla^2\psi(r) + \lambda^2\psi(r) = 0 \\ k\frac{\partial}{\partial n}\psi(\underline{r}) + h\psi(\underline{r}) = 0 \end{cases}$ (boundary condition)

General soln : $T(r,t) = \sum C_m e^{-\alpha\lambda_m^2 t}\psi_m(\underline{r})$

for $C_m \Rightarrow$ IC $= f(\underline{r}) = \sum C_m \cancel{e^{-\alpha\lambda_m^2 t}}^{\,1} \psi_m(r)$ $\quad t=0$

$$C_m = \frac{\int_R f(\underline{r})\,\psi_m(\underline{r})\,dr}{\int_R \psi_m^2(\underline{r})\,dr}$$

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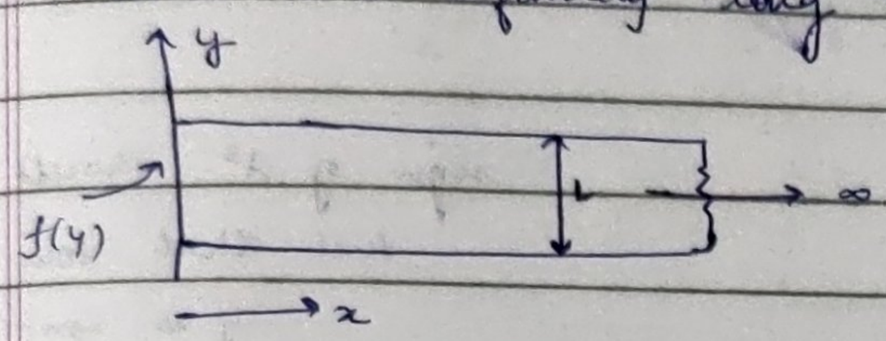
Separation of variables

Examples

① Consider an infinitely long 2D fin of thickness L



Ambient $= T_\infty$

heat transfer coeff $= h =$ very large

Find steady state $T(x,y)$

$k =$ constant $\qquad \dot{q} = 0$.

$$\cancel{-k A_c(x) \frac{dT}{dx}\Big|_x + k A_c(x) \frac{dT}{dx}\Big|_{x+\Delta x} - k A_y \frac{dT}{dy}\Big|_y + k A_y \frac{dT}{dy}\Big|_{y+\Delta y}}$$

$$\cancel{- h A_s (T - T_\infty) = 0}$$

$$\cancel{k A_c \frac{d^2T}{dx^2} + k A_y \frac{d^2T}{dy^2} - hP}$$

$$\cancel{\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0}}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

$\cancel{x=0}$ $\quad T(0,y) = f(y)$

$T(\infty, y) = T_\infty$

$T(x,0) = T_\infty$

$T(x,L) = T_\infty$

because h is very large

$\cancel{x=0 \rightarrow T(0,y) = f(y)}$

$\cancel{x \rightarrow \infty, \quad T = T_\infty \quad \boxed{T(\infty, y) = T_\infty}}$

$\cancel{y=0 \quad -k \frac{\partial T}{\partial y}\Big|_{y=0} = h(T - T_\infty)}$

$\cancel{y = L \quad k \frac{\partial T}{\partial y}\Big|_{y=L} = h(T - T_\infty)}$

$\theta(x,y) = T(x,y) - T_\infty$

$$\boxed{\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0} \longrightarrow \text{homogeneous.}$$

$\theta(0,y) = f(y) - T_\infty = F(y)$

$\theta(\infty, y) = 0$

$\theta(x,0) = 0$

$\theta(x,L) = 0$.

separation of variables can be used if atleast 3 BC are homogeneous & DE is linear and homogeneous

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$$\theta(x,y) = X(x)\,Y(y)$$

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} = 0$$

$$\frac{1}{X}\frac{d^2X}{dx^2} = \underline{\frac{-1}{Y}\frac{d^2Y}{dy^2}} = \text{const} = \pm\lambda^2$$

sign of λ^2 should be such that soln of Y is non-trivial (eigen value problem in homogeneous soln)

$$\frac{d^2Y}{dy^2} + Y\lambda^2 = 0.$$

$$\boxed{Y = c_1 \sin(\lambda y) + c_2 \cos(\lambda y)}$$

$Y \to$ homogeneous soln (S-L problem)

$$Y(x,0) = 0$$

$$Y(x,L) = 0$$

$$\boxed{0 = c_2}$$

$$0 = c_1 \sin\lambda L$$

$$\lambda L = n\pi$$

$$\lambda_n = \frac{n\pi}{L} \qquad n = 1, 2, \ldots$$

$$Y = A_n \sin\lambda_n y$$

$$\lambda_n = \frac{n\pi}{L},\ n = 1, 2$$

$$\rightarrow \frac{d^2X}{dx^2} - \lambda^2 X = 0$$

$$X = c_1 e^{\lambda x} + c_2 e^{-\lambda x}$$

$$X(0,y) = F(y)$$

$$X(\infty,y) = 0$$

$$\boxed{c_1 = 0}$$

$$\boxed{F(y) = c_2}$$

$$X = B_n e^{-\lambda_n x}$$

$$\theta(x,y) = B_n e^{-\lambda_n x} \times A_n \sin\lambda_n y$$

$$\theta(x,y) = \sum_{n=1}^{\infty} C_n e^{-\lambda_n x} \sin\lambda_n y$$

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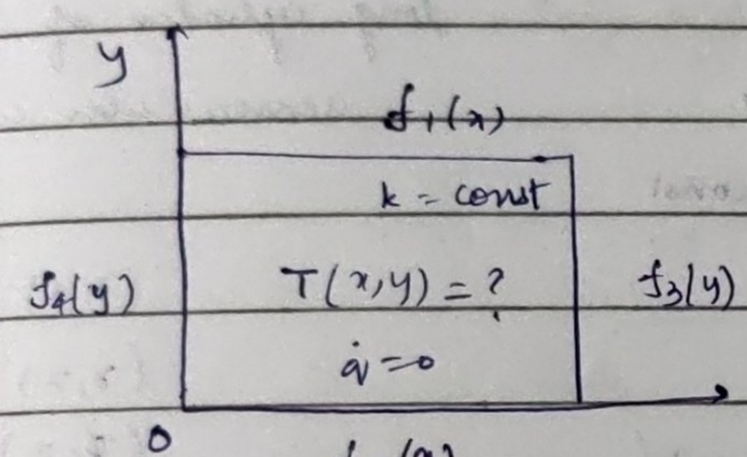
$\theta(0,y) = F(y)$

$$F(y) = \sum_{n=0}^{\infty} c_n \sin \lambda_n y.$$

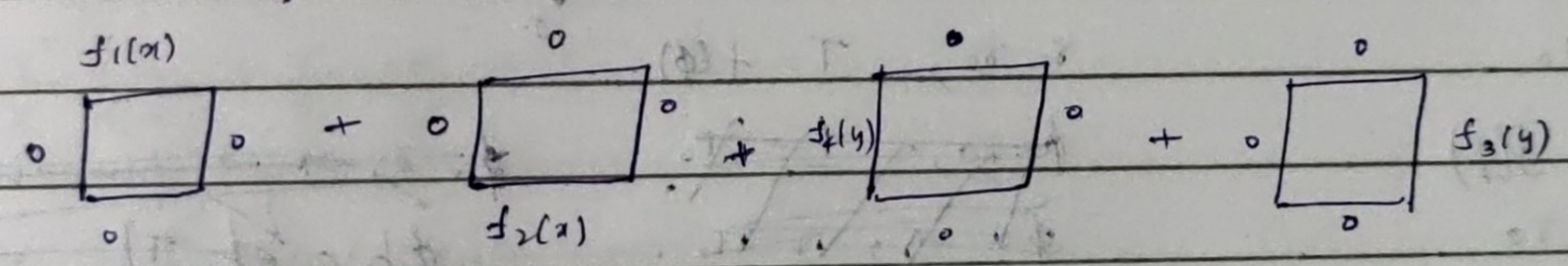
$$\int_0^L F(y) \sin \lambda_n y \, dy = c_n \int_0^L \sin^2 \lambda_n y \, dy.$$

$$\boxed{c_n = \frac{2}{L} \int_0^L F(y) \sin \lambda_n y \, dy}$$

Q2)

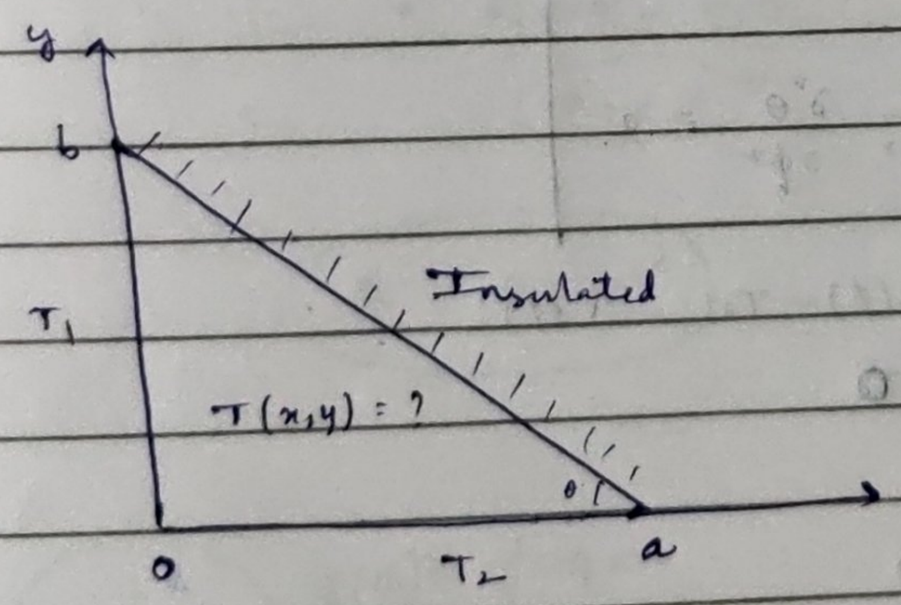


→ split & take one NH at a time



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Q)



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

$T(x,0) = T_2$

$T(0,y) = T_1$

k = const

$\dot{q} = 0.$

$\frac{x}{a} + \frac{y}{b} = 1 \rightarrow \frac{\partial T}{\partial x} = 0, \quad \frac{\partial T}{\partial y} = 0.$

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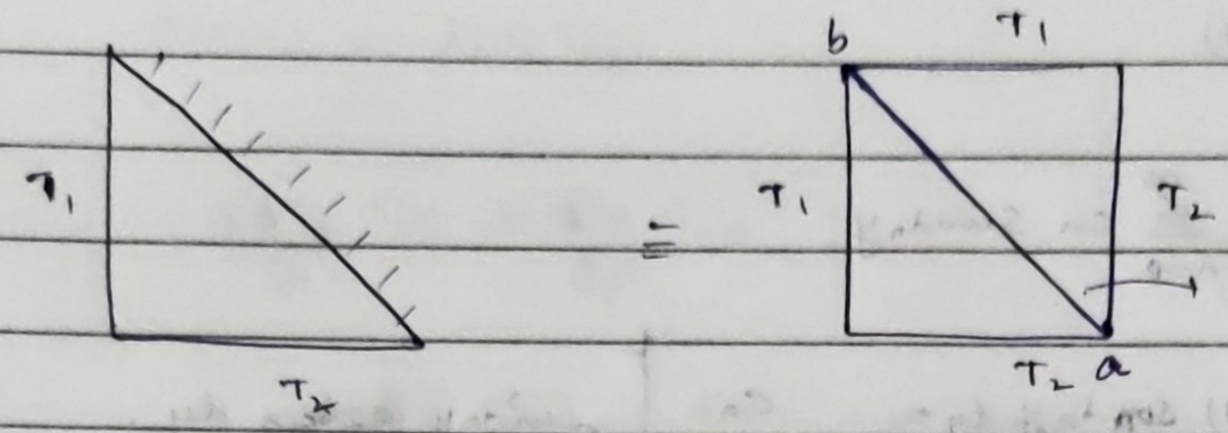
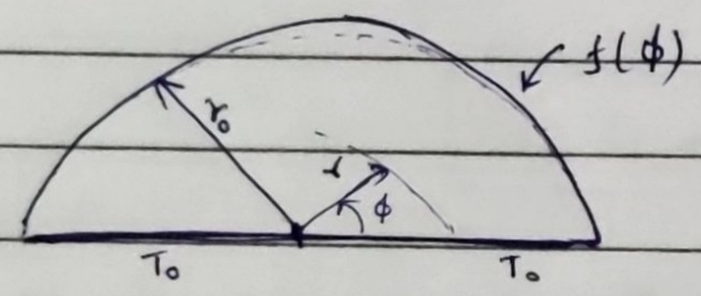


Thermal symmetry acts as insulated boundary.

thermal symmetry (plane of symmetry)

Q)



long cylinder of semicircular cross section

$\dot{q} = 0$

$k = \text{const}$

2D steady state

(r, ϕ)

(r, z)

(ϕ, z) ✗

$T(r, \phi) = ?$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \cancel{\frac{\partial^2 T}{\partial z^2}} = 0$$

$T(0, \phi) = T_0$

$T(r_0, \phi) = f(\phi)$

$T(r, 0) = T_0$

$T(r, \pi) = T_0$

$r = r_0, \quad T = f(\phi)$

$r = 0, \quad T = T_0$

$\theta = T - T_0$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \theta}{\partial \phi^2} = 0$$

$\theta(r_0, \phi) = f(\phi) - T_0 = F(\phi)$

$\theta(0, \phi) = 0$

$\theta(r, 0) = 0$

$\theta(r, \pi) = 0$

$$\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right)\right) + \frac{1}{r^2}\frac{\partial^2 \theta}{\partial \phi^2} = 0$$

$\theta(r, \phi) = R(r)\,\Psi(\phi)$

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$$\psi \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + \frac{R}{r^2}\frac{\partial^2\psi}{\partial\phi^2} = 0$$

$$\frac{r}{R}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + \frac{1}{\psi}\frac{\partial^2\psi}{\partial\phi^2} = 0$$

$$\frac{r}{R}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) = -\frac{1}{\psi}\frac{\partial^2\psi}{\partial\phi^2} = \pm\lambda^2$$

$+\lambda^2$

$$-\frac{1}{\psi}\frac{\partial^2\psi}{\partial\phi^2} = \lambda^2$$

$$\frac{\partial^2\psi}{\partial\phi^2} + \lambda^2\psi = 0$$

$$\psi_n = A_n \sin\lambda_n\phi$$

$$\lambda_n = \frac{n\pi}{\pi}$$

$\lambda_n = n$

$$\frac{r^2}{R}\frac{d^2R}{dr^2} + \frac{r}{R}\frac{dR}{dr} - \lambda^2 = 0$$

cauchy euler eqn.

$r^{+\lambda}, r^{-\lambda} \rightarrow$ linear combination

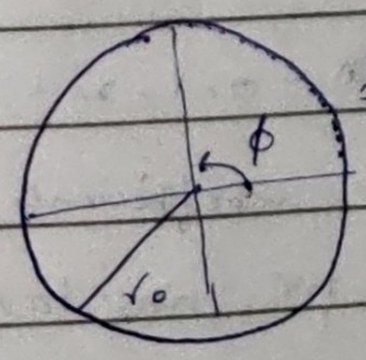
$$R = A_1 r^{\lambda} + A_2 r^{-\lambda}$$

$R = 0,\ r = 0$

$A_2 = 0$

$$R = A_1 r^{\lambda}$$

Q)



$\dot{q} = 0$

$k = \text{const}$

steady state $T(x,y) = ?$

$$\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)\right) + \frac{1}{r^2}\frac{\partial^2\theta}{\partial\phi^2} = 0$$

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$T(r_0, \phi) = f(\phi)$

$T(r, 0) = T(r, 2\pi)$

$\cancel{\left.\frac{\partial T}{\partial \phi}\right|_{\phi=0} = \left.\frac{\partial T}{\partial \phi}\right|_{\phi=2\pi}}$ $\quad \left.\frac{\partial T}{\partial \phi}\right|_{\phi=0} = \left.\frac{\partial T}{\partial \phi}\right|_{\phi=2\pi}$

$T(0, \phi) = \text{finite}$

$\Rightarrow$ $T(r, \phi) = T(r, \phi + 2\pi)$

$\frac{1}{r}\frac{\partial T(r, \phi)}{\partial \phi} = \frac{1}{r}\frac{\partial T(r, \phi + 2\pi)}{\partial \phi}$ } periodic BC

$r^2 \frac{d^2R}{dr^2} + r\frac{dR}{dr} - \lambda^2 R = 0$ | $\frac{d^2\varphi}{d\phi^2} + \lambda^2 \varphi = 0$

$R = B_1 r^{\lambda} + B_2 r^{-\lambda}$ | $\varphi = A_1 \sin\lambda\phi + A_2 \cos\lambda\phi$

$A_1 \sin\lambda\phi + A_2 \cos\lambda\phi = A_1[\sin(\lambda(\phi + 2\pi))] + A_2 \cos(\lambda(\phi + 2\pi))$

$\lambda A_1 \cos\lambda\phi - A_2 \lambda \sin\lambda\phi = \lambda A_1 \cos(\lambda(\phi + 2\pi)) - A_2 \lambda \sin(\lambda(\phi + 2\pi))$

$A_1(\lambda \cos\lambda\phi - \lambda \cos(\lambda(\phi + 2\pi))) + A_2(\lambda \sin\lambda(\phi + 2\pi) - \lambda \sin\lambda\phi) = 0$

$A_1(\sin\lambda\phi - \sin\lambda(\phi + 2\pi)) + A_2(\cos\lambda\phi - \cos(\lambda\phi + 2\pi)) = 0.$

$\lambda = n, \; n = 0, 1, 2 \ldots$

determinants of coefficients should vanish for non-trivial soln.

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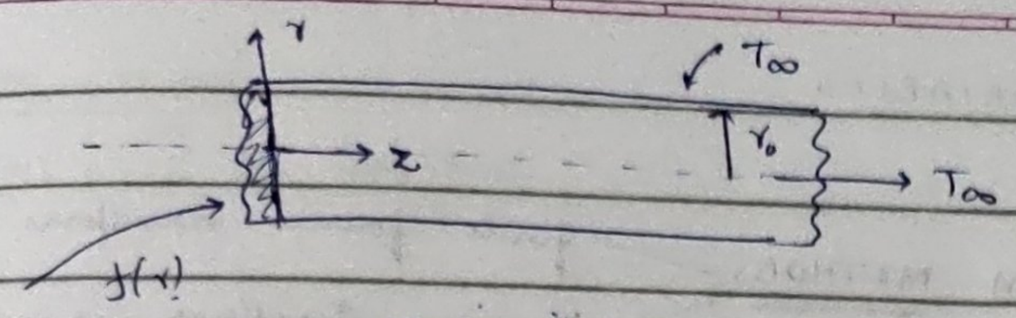
Q)



h = large

semi-infinite cylinder of radius r_0.

$\dot{q} = 0$

k = const

$$\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right) + \frac{\partial^2 T}{\partial z^2} = 0$$

~~$T(0,z) = T_\infty$~~
~~$T(r_0,z) = T_\infty$~~
~~$T(r,0) = f(r)$~~
~~$T(r,\infty) = T_\infty$~~

$$\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right)\right) + \frac{\partial^2 \theta}{\partial z^2} = 0.$$

$\theta(0,z)$ = finite

$\theta(r_0,z) = 0$

$\theta(r,0) = f(r)$

$\theta(r,\infty) = 0$.

$\theta(0,z) + \frac{\partial \theta}{\partial r} = 0$ → can be written

$\theta = R(r)\,Z(z)$

$$\frac{d^2 Z}{dz^2} - \lambda^2 Z = 0.$$

$Z(\infty) = 0$

$Z = B_1 e^{-\lambda z}$.

$\frac{dR(0)}{dr} = 0$ | $R(0)$ = finite

$R(r_0) = 0$

$R = A_1 J_0(\lambda_n r)$

where λ_n is the roots

$J_0(\lambda_n r_0) = 0$.

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→ SEPARATION OF VARIABLES

→ INTEGRAL TRANSFORM METHODS → finite fourier transform → rectangular coordinate
→ Hankel transform → cylindrical coordinate
→ Laplace transform

(a) laplace transform

① 2D PDE $\xrightarrow{\text{FFT}}$ ODE $T(x)$

$T(x,y)$ ← take inverse ← solve

FINITE FOURIER TRANSFORM

Any $f(x)$ which is piecewise differentiable in $[0, L]$ can be expanded in sine series.

$$f(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}\right)x$$

$$\boxed{A_n = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{n\pi}{L}\right)x \, dx}$$

$$\Rightarrow f(x) = \frac{2}{L}\sum_{n=1}^{\infty}\left[\int_0^L f(x') \sin\left(\frac{n\pi}{L}x'\right)dx'\right] \sin\left(\frac{n\pi}{L}\right)x$$

rearrange. let $K_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L}x$. → Kernel of transformation

$\bar{f}_n = \int_0^L f(x) K_n(x) \, dx$ → finite fourier transform of $f(x)$

$\cancel{K_n(x) = \frac{2}{n\pi}}$ $f(x) = \sum_{n=1}^{\infty} K_n(x) \bar{f}_n$ ⟹ Inversion formula

normalized eigen value

$\frac{\text{eigen function}}{\sqrt{N_n}}$

norm of eigen function

for $\sin \frac{n\pi}{L}x$ it is $L/2$ Norm

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Cosine series

$$f(x) = \sum_{n=0}^{\infty} A_n \cos \frac{n\pi}{L} x$$

$$K_n(x) = \frac{1}{\sqrt{N_m}} \cos \frac{n\pi}{L} x$$

$N_m = L \quad n=0$

$N_m = L/2 \quad n=1,2,\ldots$

→ STL

$$\frac{d^2y}{dx^2} + \lambda^2 y = 0$$

$$\alpha_1 y(0) + \beta_1 \frac{dy(0)}{dx} = 0, \quad \alpha_1^2 + \beta_1^2 > 0$$

$$\alpha_2 y(L) + \beta_2 \frac{dy(L)}{dx} = 0, \quad \alpha_2^2 + \beta_2^2 > 0$$

$\phi_n(x) \rightarrow$ eigen function

$$f(x) = \sum_{n=0}^{\infty} A_n \phi_n(x)$$

$$A_n = \frac{\int_0^L f(x)\phi_n(x)\,dx}{\underbrace{\int_0^L \phi_n^2(x)\,dx}_{N_m}} = \frac{1}{N_m}\int_0^L f(x)\phi_n(x)\,dx$$

$$K_n(x) = \frac{1}{\sqrt{N_m}} \phi_n(x)$$

$$\bar{f}_n = \int_0^L f(x)\, K_n(x)\, dx$$

$$f(x) = \sum_{n=0}^{\infty} \bar{f}_n K_n(x)$$

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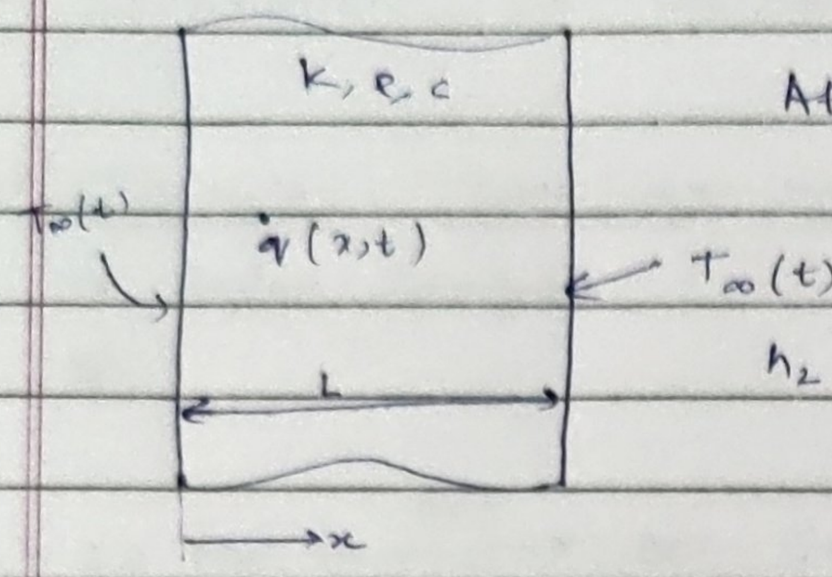
Q)



At time $t=0$, $T(x,t) = T_i(x)$

1D unsteady state heat conduction with space, time dependent $\dot{q}$ and environment temperature

[Consider h_1, h_2 high]

$$\frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} = \frac{\partial^2 T(x,t)}{\partial x^2} + \frac{\dot{q}(x,t)}{k}$$

(Cannot solve using separation of variables)

$T(x,0) = T_i(x)$

$T(0,t) = T_\infty(t)$

$T(L,t) = T_\infty(t)$

define transformation of $T(x,t)$

$$\bar{T}_n(t) = \int_0^L T(x,t)\, K_n(x)\, dx$$

Inversion $T(x,t) = \sum_{n=1}^{\infty} \bar{T}_n(t)\, K_n(x)$

(how to get kernel)?

convert it into STL → linear homogeneous DE.

take homogeneous DE, homogeneous BC

move non-homogeneity from PDE & BC

$$\frac{1}{\alpha}\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial x^2}$$

$\psi(x,0) = T_i(x)$

$\psi(0,t) = 0$

$\psi(L,t) = 0$

$\psi(x,t) = X(x)\,\Gamma(t)$

get eigen function from this & then get kernel from this

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$$\frac{d^2X}{dx^2} + \lambda^2 x = 0$$

$$X(0) = 0$$

$$X(L) = 0.$$

$$X(x) = c_1 \sin \lambda x + c_2 \cos \lambda x.$$

$$c_2 = 0.$$

$$\sin(\lambda L) = 0.$$

$$\lambda L = n\pi$$

$$\lambda = \frac{n\pi}{L}$$

~~let, $k_n(x) = \sqrt{\frac{2}{L}} \int_0^L \sin \frac{n\pi}{L} x$~~

let, $k_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$

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