Question B

2. The velocity profile in a laminar boundary layer flow at zero pressure gradient is to be approximated by the linear expression, $u/U=\eta$; $\eta = y/\delta$. Use the momentum integral equation with this profile to obtain an expression for the ratio, δ/x, and the skin friction coefficient, C_f in terms of Reynold's number.

(B)

The MI eqⁿ for flat plate, incompr. flow, zero pr. grad.

$$\tau_w = \rho U^2 \frac{d\delta}{dx} \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right) d(y/\delta)$$

$$= \rho U^2 \beta \frac{d\delta}{dx}$$

given $\frac{u}{U} = \eta$

Using the given velocity profile

$$\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right) d\eta = \int_0^1 \eta(1-\eta)\, d\eta = \left[\frac{1}{2}\eta^2 - \frac{1}{3}\eta^3\right]_0^1 = \frac{1}{6} = \beta$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\mu U}{\delta} \left.\frac{\partial(u/U)}{\partial(y/\delta)}\right|_{y/\delta=0} = \frac{\mu U}{\delta} \left.\frac{d(u/U)}{d\eta}\right|_{\eta=0} = \frac{\mu U}{\delta}$$

Substituting for β and τ_w

$$\frac{\mu U}{\delta} = \rho U^2 \frac{d\delta}{dx}\left(\frac{1}{6}\right).$$

Integrating $\frac{\delta^2}{2} = \frac{6\mu}{\rho U} x + c.$

B.C. $\delta = 0$ at $x = 0$. $\Rightarrow c = 0$.

$$\therefore \frac{\delta}{x} = \sqrt{\frac{12\mu}{\rho U x}}$$

$$\boxed{\frac{\delta}{x} = \frac{3.46}{\sqrt{Re_x}}}$$

And $$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{\mu U/\delta}{\frac{1}{2}\rho U^2} = \frac{2\mu}{\rho U \delta} = \frac{2\mu}{\rho U x} \cdot \frac{x}{\delta}$$

$$\boxed{C_f = \frac{0.577}{\sqrt{Re_x}}}$$