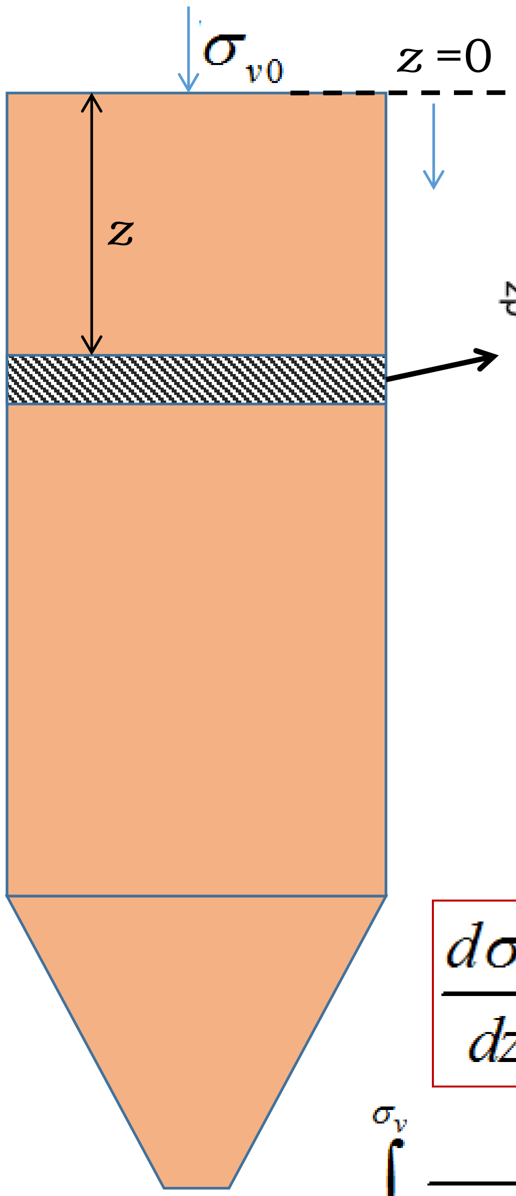
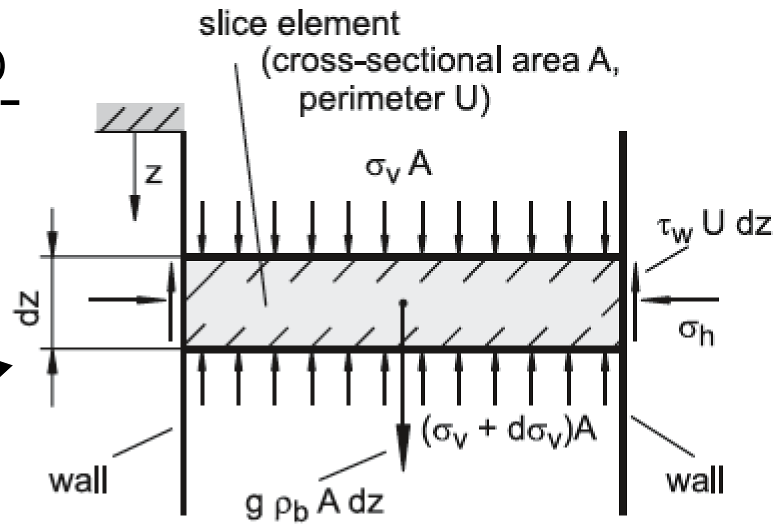




Force Balance for slice element

$$A\cancel{\sigma_v} + g\,\rho_b\,A\,dz = A(\cancel{\sigma_v} + d\sigma_v) + \tau_w U\,dz$$

$$\frac{d\sigma_v}{dz} = g\rho_b - \frac{\tau_w U}{A}$$

$$\boxed{K = \sigma_h / \sigma_v}$$

Angle of Wall Friction

$$\tan\varphi_x = \tau_w / \sigma_h$$

K: Lateral Stress Ratio (Principle stress ratio)

We assume ***K*** is constant throughout the material.

In reality *K* varies through out the material.

Janssen's equation

$$\boxed{\frac{d\sigma_v}{dz} = g\rho_b - \frac{\sigma_v K U \tan\varphi_x}{A}}$$

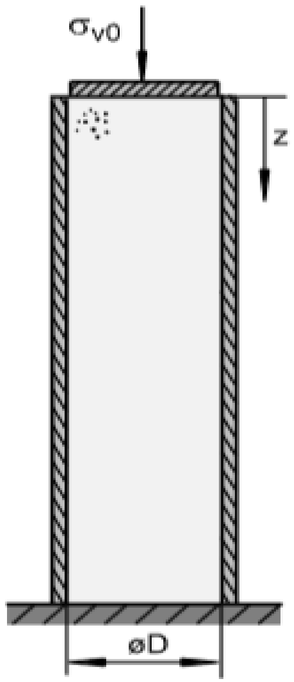
$$\int_{\sigma_{v0}}^{\sigma_v} \frac{d\sigma_v}{g\rho_b - (\sigma_v K U \tan\varphi_x / A)} = \int_0^z dz$$

$$\boxed{\sigma_v = \frac{g\,\rho_b A}{K\,\tan\varphi_x\,U} + \left[\sigma_{v0} - \frac{g\,\rho_b A}{K\,\tan\varphi_x\,U}\right] \cdot e^{\frac{-K\tan\varphi_x U z}{A}}}$$

Application of Janssen's equation

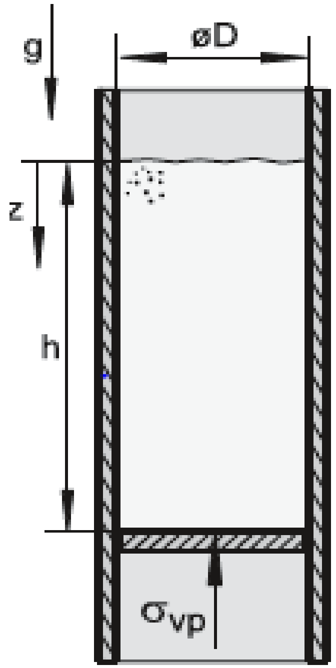
Example 1 (by group 1)

Consider that the wall of the cylindrical silo (shown in figure below) is made of a material, which can bear the maximum normal stress up to 4.4kPa. Assume that the only possible source of damage of wall is the horizontal normal stress exerted by the bulk solid on the wall (and not due to the frictional forces/shear stress present between bulk-solid and wall). There is no surcharge stress, $\sigma_{v0}=0$. Calculate the maximum height allowed for the silo to avoid any damage to silo-wall.



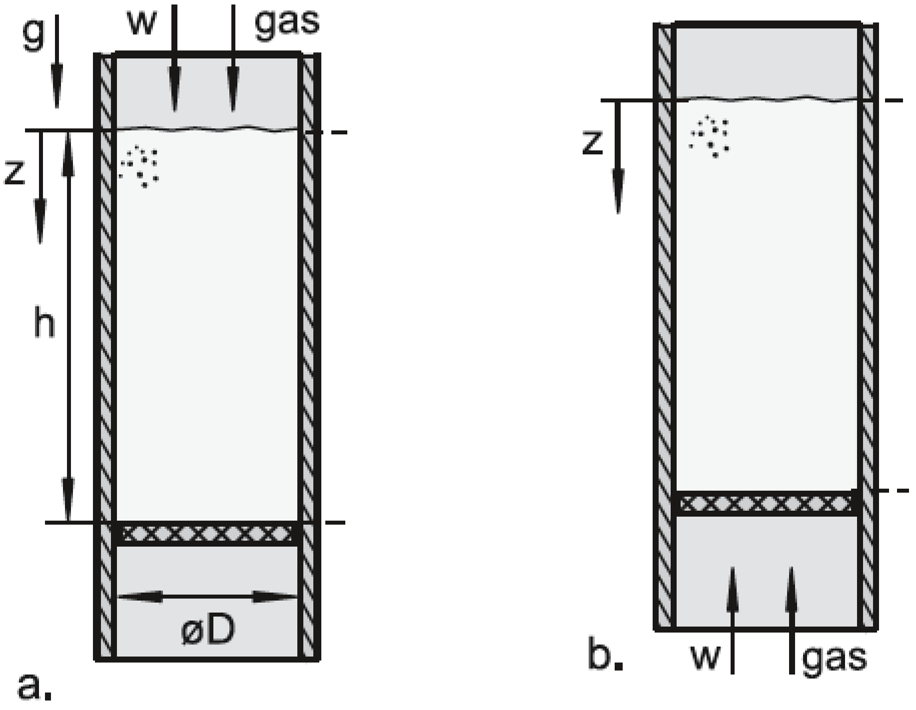
Example 2 (by group 2)

Consider a case wherein a column of bulk solid should be pushed upwards in a vertical channel, the wall shear stress on bulk solids then start acting downwards. Find out the expression for stress as a function of depth.



Example 3 (by group 2)

If a gas flows through a stationary packing of bulk solid, the stresses in the bulk solid are influenced by the pressure gradient. This effect can be taken into account for a setup as shown in figure below by adding pressure gradient, dp/dz.



$$A\sigma_v + g\,\rho_b A\,dz - \left(\frac{dp}{dz}\right)A\,dz = A(\sigma_v + d\sigma_v) + \tau_w U\,dz$$

Summary

- Within a cylindrical section of a silo, both vertical and horizontal normal stress are increasing function of depth, and saturates to a maximum value and very large depth.

$$\boxed{\sigma_v = \frac{g\,\rho_b A}{K\ \tan\varphi_x\, U} + \left[\sigma_{v0} - \frac{g\,\rho_b A}{K\ \tan\varphi_x\, U}\right]\cdot e^{\frac{-K\tan\varphi_x U z}{A}}}$$

- The frictional forces applied by wall on material is very crucial, which makes bulk solids different than liquids.

- The above expression works fine in practical sense, however more rigorous derivations are available with less assumption.

Next Class: Stress distribution in Hopper

Hoppers