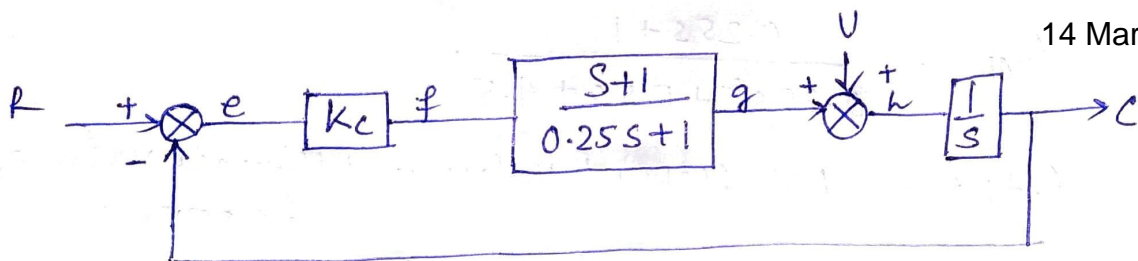1.

$e = R - c$

$f = k_c e = k_c (R-c)$

$g = \dfrac{(s+1) \, k_c \, (R-c)}{0.25s+1}$

$h = U + g$

$\quad = \dfrac{(s+1) \, k_c \, (R-c)}{0.25s+1} + U$

$c = \dfrac{1}{s} \times \dfrac{(s+1) \, k_c \, (R-c)}{(0.25s+1)} + \dfrac{U}{s}$

$c + \dfrac{(s+1) \, k_c \, c}{s(0.25s+1)} = \dfrac{(s+1) \, k_c \, R}{s(0.25s+1)} + \dfrac{U}{s}$

$c = \dfrac{(s+1) k_c}{s(0.25s+1) + (s+1) k_c} R + \dfrac{(0.25s+1)}{s(0.25s+1) + (s+1) k_c} U$

$\therefore \quad \dfrac{c}{U} = \dfrac{0.25s+1}{s(0.25s+1) + (s+1) k_c}$

$$\boxed{\dfrac{c}{U} = \dfrac{0.25s+1}{0.25s^2 + (1+k_c)s + k_c}} = G_{cL} \quad \text{5 Marks}$$

From the closed loop Transfer function.

$\tau^2 = \dfrac{0.25}{k_c}$ 2 Marks

$2\xi\tau = \dfrac{1+k_c}{k_c}$

$\tau = \dfrac{0.5}{\sqrt{k_c}}$

$2\xi\tau = \dfrac{\xi}{\sqrt{k_c}} = \dfrac{1+k_c}{k_c} \Rightarrow \xi = \dfrac{1+k_c}{\sqrt{k_c}}$ 2 Marks

Given : $\xi = 3$

$\therefore \quad \dfrac{1+k_c}{\sqrt{k_c}} = 3$

2 Marks

$\Rightarrow \quad 1 + k_c^2 + 2k_c = 9 k_c$

$\Rightarrow \quad k_c^2 - 7 k_c + 1 = 0 \quad \Rightarrow \boxed{k_c = 6.85, \ 0.146}$

Desirable to minimize offset

Let,

$k_c = 4.5$

$$G_{OL} = \frac{0.25s + 1}{0.25s^2 + 5.5s + 4.5}$$ · 1 Mark

Offset = New setpoint value − Ultimate value of Response.

$$= 0 - \lim_{t \to \infty} \frac{1}{s} \cdot \frac{0.25s + 1}{0.25s^2 + 5.5s + 4.5}$$

$$= 0 - \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{0.25s + 1}{0.25s^2 + 5.5s + 4.5}$$

$$= 0 - \frac{1}{4.5} = \boxed{-0.22}$$ 2 Marks

6 Marks

2. $G_p = \frac{1}{2s - 1}$, $G_m = G_f = 1$, $G_c = k_c$

For a feedback control system, the characteristic equation is given by.

$1 + G_p G_f G_c G_m$

Characteristic equation:

$$1 + \frac{k_c}{2s - 1} = \frac{2s + (k_c - 1)}{2s - 1}$$

Roots of the characteristic equation.

$$\frac{2s + (k_c - 1)}{2s - 1} = 0 \quad \Rightarrow \quad 2s + (k_c - 1) = 0$$

$$s = \frac{1 - k_c}{2}$$ 3 Marks

A feedback control system is said to produce stable response if all the roots of its characteristic equation have negative real parts.

∴ $\frac{1 - k_c}{2} < 0$

$$\Rightarrow \boxed{k_c > 1}$$ 3 Marks