COMMINUTION LAWS

Several Probabilities

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This article deals with the relationship of both material and crushing mechanism to the exponent of particle size in the conventional comminution law, from the viewpoint of several probabilities. With the aid of this analysis, an empirical equation has been obtained, from which Rittinger's law, Kick's law, and Bond's third theory of comminution can be proved to be one particular case of the general crushing phenomenon.

Comminution has been studied ever since Rittinger's and Kick's laws were proposed in the nineteenth century. These "laws" have continued to be the subject of controversy. Bond's formula is today regarded as the most reliable and useful relation for expressing the interrelationship between particle size and the crushing energy.

As quoted in many textbooks (*7*), the original two laws can be derived from the following general equation:

$$dE = -C\frac{dx}{x^n} \tag{1}$$

where E is the crushing energy, x the particle size, and C a constant. When $n = 1$, integration of this basic equation gives Kick's law:

$$E = C \log x_1/x_2 \tag{2}$$

When $n = 2$, integration gives Rittinger's law:

$$E = C\left(\frac{1}{x_2} - \frac{1}{x_1}\right) \tag{3}$$

Bond's third theory of comminution (*1*) is obtained by substituting $n = 1.5$ in Equation 1, integrating, and rearranging:

$$E = W_i \frac{R^{1/2} - 1}{R^{1/2}} (100/x_2)^{1/2} \tag{4}$$

where W_i is a work index and R is the size reduction ratio, x_1/x_2. Obviously, energy requirements for Bond's law lie between those predicted by the earlier two laws.

Before Bond's law appeared, it was known that Rittinger's law applies primarily in crushing relatively small particles, while Kick's law generally holds for large particles. The evidence indicated that the value of n depends not only on the material crushed but also on the kind of crusher used. Bond's equation made allowance for this variation in the work index, W_i, whose evaluation for any given material involves using an arithmetic mean value based on data accumulated from many kinds of crushers and grinding equipment of different efficiencies. As Bond has indicated, these data are sometimes so widely scattered as to seem unsuitable for taking the arithmetic mean. Nevertheless, taking n as 1.5 proves more workable than either Kick's or Rittinger's laws in correlating many data from various types of crushing devices.

Bond developed his equation using the "crack theory," demonstrated the physical meaning of the n value, and offered an explanation as to why this n value might consistently equal 1.5 for all kinds of material. He showed that crushing energy is absorbed initially by a particle in proportion to its volume—that is, in proportion to x^3. Then, as crack tips form, this energy effectively flows into the new surface, its expansion being proportional to the square of the particle size, x^2. As a result, the mean crushing energy needed to form a new surface is proportional to x raised to an exponent lying between 2 and 3, with 5/2 often proving to be the most satisfactory value. Depending on the nature of crack formation, however, some materials show exponents close to 2; others show n values close to 3. This power must therefore be viewed not as a universal constant, but as a function of the material considered. In fact, in 1957, Holmes (*2*) pointed out that, starting from Kick's law, Bond's third theory should be revised as follows:

$$E = W_i\left(\frac{R^r - 1}{R^r}\right)\left(\frac{100}{x_2}\right)^r \tag{5}$$

where R is the reduction ratio, x_2 is the product size, and r is called a deviation from Kick's law. He demonstrated that r varies from 0.25 to 0.73 for several materials, and that such a concept leads to a more consistent work index, W_i, than that calculated by Bond. Obviously, Bond's equation corresponds to the case where $r = 1/2$ in Equation 5.

There has remained doubt as to whether, for a constant n value, variations in performance among all kinds of crushing and grinding equipment could be adequately reflected by use of different values of W_i. If n is not constant, but varies with the crushing mechanism as well as with the physical characteristics of the material, the usefulness of Equation 1 for design purposes is reduced. An important question, unanswered in past studies, is the meaning of variations of the n value in Equation 1.

If Equation 1 is to be retained, and if n is to be kept constant for a specified material, the dependence of C on particle size and crushing mechanism must be further clarified. This point is discussed here in detail, and important probability functions bearing on the crushing mechanism are introduced.

Internal Mechanics within a Crusher

For simplicity, suppose that a particle in an impact mill is thrown out at a given initial velocity to hit the mill liner or collide with another particle. To analyze the mechanism of crushing, all probabilities involved must be considered: First, whether or not the particle hits an objective. Obviously, if no collision occurs, crushing cannot take place. Second, even if there is a collision, crushing may not occur, since, if the energy of collision fails to produce a stress much larger than the breaking stress of the material, the particle remains intact. The probability of developing collisions of this minimum violence is clearly important. For crushing to take place, therefore, a particle must hit an object (or be hit) hard enough to break. Crushing occurs only when the product of the two probabilities is greater than zero, since they are mutually independent. The nearer the product of the probabilities is to unity, the better the performance of the machine.

A third element in crushing relates to the propagation of crack tips within a solid, with an assignable probability of its occurrence in any given collision. As is well known, grinding aids (such as aryl alkyl sulfonic acid) can improve the rate of crushing, even without other changes in the internal mechanics of a specified crusher. This improvement is presumably due to acceleration of the rate of crack growths.

The basic Equation 1 should include these three different probabilities, along with the $1/x^n$ term. Since specific surface is inversely proportional to particle size, Equation 1 can be rewritten as follows:

$$\frac{dS}{dE} = C_1 x^N \qquad (1')$$

where $N = n - 2$, a constant, while C_1 is also a constant. Therefore, as discussed above, if Rittinger's law is valid, $n = 2$, and thus $N = 0$. On the other hand, Kick's law, where $n = 1$, gives $N = -1$. Bond's law, where $n = 1.5$, gives $N = -0.5$. E is the energy required to crush a unit weight of particles of the size x.

The equation proposed here is:

$$\frac{dS}{dE} = K(P_c)(P_\sigma)(P_a)x^N \qquad (6)$$

where P_c is the probability of collision, P_σ the probability that the material's breaking stress will be exceeded, P_a the probability of crack tip propagation, and K a constant.

It would be difficult and perhaps impossible to make a rigorous analysis of these probabilities for every kind of crushing device. The purpose of this article, however, is to see whether the simplest crushing mechanics support the basic concept proposed. Given confirmation, then, this approach can be extended to other more complex crushing devices. Eventually, it may be possible to develop a comminution law that can be reliably applied to all types of material and crushers.

Apparatus and Procedure

Figure 1 shows a schematic diagram of the experimental apparatus.

Five ranges of closely classified calcite particles were tested: 10/28, 28/32, 42/48, 80/100, and 100/150 Tyler mesh. The material under study was stored in a bunker, 1, which feeds continuously and at a controlled rate through the vibrating feeder, 2, and to the rotating disk, 5, which is 300 mm. in diameter.

The disk, 5 (see detailed drawing in Figure 2), rotates at 6000 r.p.m., and at this speed accelerates the particles to an initial velocity of 94 meters per second, throwing them out uniformly and tangentially. At 84 mm. from its perimeter, the disk is enclosed by a zigzag steel belt, 20 mm. wide and so designed that the particles always hit it perpendicularly. Larger particles crushed as a result of a single collision with the belt drop by gravity. Smaller ones are swept from the crushing zone by air immediately after collision, so that no further crushing can be expected.

A vacuum pump, 7, is capable of evacuating the entire system, from crusher to bunker, down to 20 mm. of Hg. This makes possible runs at various pressures and thus a study of the effect of adsorption of gas molecules on crack propagation as well as the effect of particle velocity on collision with the crushing belt. Constant pressure is automatically maintained during a given run, and in this work was varied in four convenient stages from 760 to 20 mm. of Hg.

The specific surface of crushed product is determined by an air permeability apparatus, specially designed to handle the coarser than usual product turned out by the experimental crusher. It can measure the specific surface of particles as coarse as 100 microns, or perhaps a little larger. (Usually this type of surface-measuring apparatus is used to measure particles in the 30-micron range.) For comparatively coarse

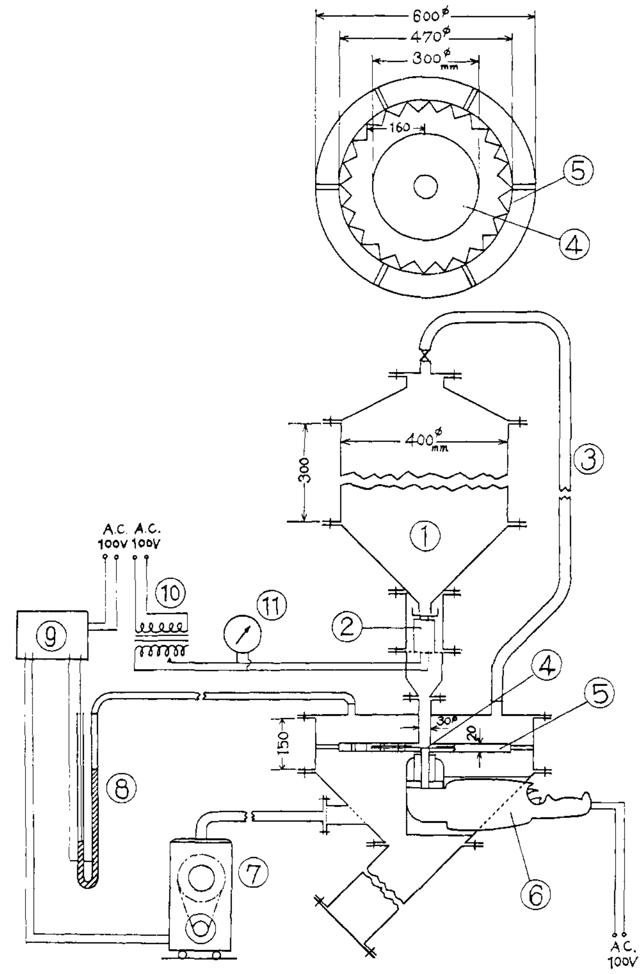

Figure 1. Schematic diagram of experimental apparatus

1. Bunker
2. Vibratory feeder
3. Connecting tube
4. Rotating disk
5. Impact belt
6. Motor
7. Vacuum pump
8. Manometer
9. Relay
10. Voltage regulator
11. Voltmeter

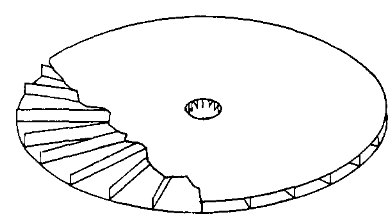

Figure 2. Detail of rotating disk

particles (significantly larger than 100 microns), the initial specific surface cannot be determined by the air permeability method, since pressure drops are not large enough to measure accurately. Instead, in this work, the finished product was carefully screened and the specific surface of the undersized material measured to give the surface increment, ΔS, with reasonable accuracy. Larger particles (greater than 100 microns) were assumed to have passed through the crusher unbroken.

Empirical Results and Correlation

Specific surface increments, ΔS, thus obtained are plotted for each particle-size group against pressure within the crusher (Figure 3). Generally, ΔS increases with decrease in pressure. However, this tendency depends greatly on the initial sizes of the particle. For example, at the lowest pressure (20 mm. of Hg), the smallest particle group can be crushed to the finest range. However, at 1 atm. this same group is crushed to the

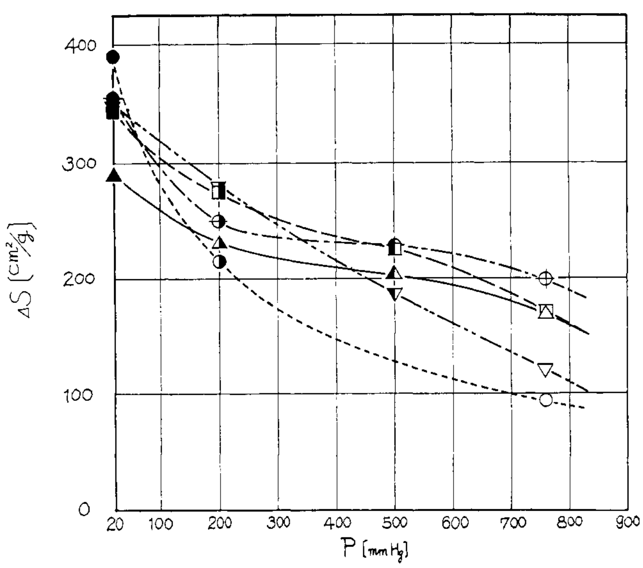

Figure 3. Relation between specific surface increment, ΔS, and pressure within crusher

least degree. The curves intersect in a complicated way, suggesting that several other kinds of variables should be considered in order to correlate the three variables in this figure. (Table I is the key for Figure 1 and other figures as applicable.)

Impact Velocity

A particle in motion with a speed v is resisted by the air according to the second law of motion (neglecting gravity):

$$\frac{dv}{dt} = \frac{dv}{ds}v = -bC_Dv^2 \tag{7}$$

where s is the distance, t the time, and C_D the drag coefficient. b equals $3\ \rho/4\ x\rho_p$, where x is the particle size, and ρ and ρ_p are the density of air and solid, respectively.

Integrating Equation 7 gives:

$$s = -\frac{1}{b}\int_{v_i}^{v}\frac{dv}{C_Dv} = -\frac{1}{b}\int_{\mathrm{Re}_i}^{\mathrm{Re}}\frac{d(\mathrm{Re})}{C_D\mathrm{Re}} \tag{8}$$

where Re is the Reynolds number, expressed by $(xv\rho/\mu)$, where μ is the viscosity of air and subscript i refers to the "initial" state. If C_D is assumed constant, then:

$$s = \frac{1}{bC_D}\log\left(\frac{\mathrm{Re}_i}{\mathrm{Re}}\right) \tag{9}$$

Since the initial velocity and Reynolds number for the particle and C_D are known, the velocity with which the particle hits the objective after traversing a distance of 160 mm. can be calculated according to Equation 9.

For example, when $P = 500$ mm. of Hg at room temperature, ρ is found to be 0.776 kg. per cu. meter. For the particle group of $x = 0.032$ cm., b is calculated to be 6.71×10^{-1} according to the definition of Equation 7. The initial velocity, v_i, is 94 meters per second, since the disk rotates at 6000 r.p.m. Therefore, the viscosity of air being independent of pressure,

$$\mathrm{Re}_i = \frac{3.2 \times 10^{-4} \times 94 \times 0.776}{1.83 \times 10^{-5}} = 1275$$

The drag coefficient, C_D, is found to be 0.455, corresponding to this Reynolds number, Re_i. By substituting these values in Equation 9, Re = 1140, from which v is calculated to be 84 meters per second. Similarly, the impact velocity, v, can be calculated for each run.

Probability of Collision

The theory governing the striking of an objective by a particle is described by the interrelationship between target efficiency, η_T, and separation number, ψ. The separation number is the dimensionless group $x^2v\rho_p/18\mu D$, where D is the size of the objective. This interrelationship is discussed in many textbooks—e.g., (*4*)—and is graphically shown in Figure 4. The curve refers to the belt—i.e., the objective or target, whose width is D. Target efficiency relates to the fraction of particles headed toward a target which actually strike that target. The target efficiency is defined as the probability of collision and is designated P_c here.

The present apparatus is designed so that the target efficiency is unity over the ranges of empirical conditions used. In other words, all particles hit the target, which is the belt.

When the belt is removed, the wall becomes the target. In this case, the probability that some particles might be swept from the apparatus by the airstream without hitting the target (wall) increases and η_T, therefore, decreases (since the trajectory of the particles in this case is greater). This can qualitatively check the validity of adopting the target efficiency as the

Table I. Key of Empirical Run

Calcite Particle		x [cm] 0.012	0.016	0.032	0.055	0.114
P [mmHg]	20	●	▼	⨁	■	▲
	200	◑	⧩	⊕	◨	◮
	500	◐	⧨	⊕	◧	◭
	760	○	▽	⊕	□	△

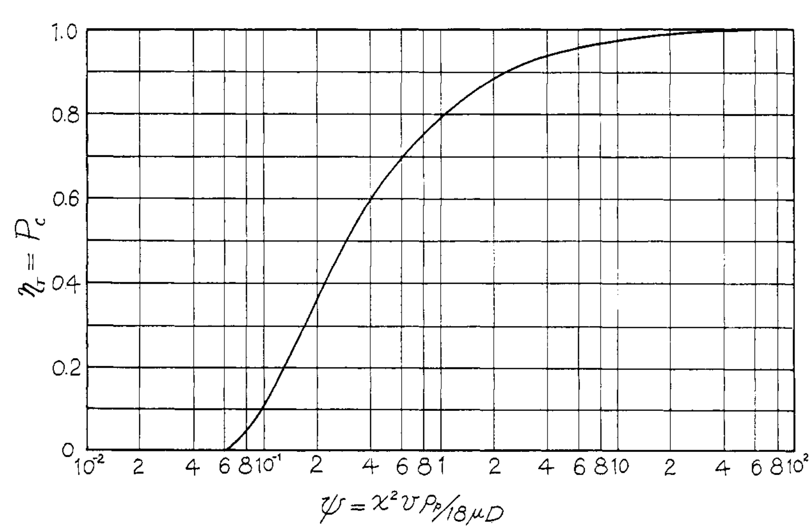

Figure 4. Target efficiency *vs.* separation number in regard to belt

probability, P_c. As a matter of fact, in actual runs, when the belt was removed, ΔS did become smaller. Part of this effect was caused by a lower P_σ, but there still was an effect due to P_c.

Probability of Exceeding Breaking Stress

According to Hertz (*5*), the maximum stress produced in a material at impact is expressed by:

$$\sigma_{\max} = a\left(\frac{m_1 m_2}{m_1 + m_2}\right)^{1/5} \times v^{2/5}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{3/5} \times \left(\frac{1-\nu_1}{E_1} + \frac{1-\nu_2}{E_2}\right)^{-4/5} \quad (10)$$

where m_1 and m_2 are the masses of the two bodies, r_1 and r_2 the radii of curvature of the bodies at an impact point, and E_1 and E_2 and ν_1 and ν_2 the moduli of elasticity and Poisson's ratios of the bodies, respectively. The relative velocity is v, but since the target is fixed, v becomes the impact velocity of the particle. a is a constant.

In the apparatus of Figure 1, E and ν are constants. The radius, r, of the target, which has a flat surface, is infinite, and the radius of any given size fraction of irregularly shaped particles is assumed constant. Similarly, the mass, m_1, of the crushing belt is taken as infinite. Since m_2 is proportional to the volume of the particles, or to x^3, Equation 10 becomes:

$$\sigma_{\max} = a' v^{2/5} x^{3/5} \quad (11)$$

where x is the particle size and a' is another constant. If the particle hits the objective at an angle to the normal direction of an objective surface, the velocity component ($v \cos \theta$) must be taken into account instead of v. The apparatus of Figure 1 is designed so that θ is almost 0°. Since v can be calculated as described above, $\sigma_{\max}/a'$ is readily obtained for each run of a definite particle size by using Equation 11.

From this value, σ_a/a' can be estimated, where σ_a is the breaking stress of a material as follows.

For crushing to occur, $\sigma_{\max}$ must be equal to or greater than σ_a and this condition gives a critical value that defines whether crushing is carried out. In terms of ΔS, if $\sigma_{\max} \geq \sigma_a$, $\Delta S \geq 0$. But if $\sigma_{\max} < \sigma_a$, then $\Delta S = 0$. Therefore, extrapolating the curves ΔS *vs.* $\sigma_{\max}/a'$ for each particle size to the point where $\Delta S = 0$ gives the point where $\sigma_{\max}$ just equals σ_a, or $\sigma_{\max}/a = \sigma_a/a$ (Figure 5). In this way, σ_a/a' is found to be 1.8×10^{-2}. The value of σ_a/a' should be independent of particle size for this range of particles and is thus a constant. From this value, $\sigma_a/\sigma_{\max}$ for each experimental condition can be calculated. The probability of exceeding the breaking stress of material proposed by the author is:

$$P_\sigma = \left(1 - \frac{\sigma_a}{\sigma_{\max}}\right)^m \quad (12)$$

where m is an empirical constant that can be determined from experimental data. When $\sigma_a = \sigma_{\max}$, $P_\sigma = 0$, and when $\sigma_{\max}$ is much greater than σ_a, then P_σ approaches unity, which corresponds to an actual crushing phenomenon.

Probability of Crack Propagation

When ΔS is plotted against $(1 - \sigma_a/\sigma_{\max})$, the series of curves shown in Figure 6 results. Inspection shows that each point lies roughly along the parallel curves for constant pressure. If the pressure within the apparatus influences only impact velocity, all the points should lie on a single curve regardless of the pressure. (The effect of air pressure is accounted for in

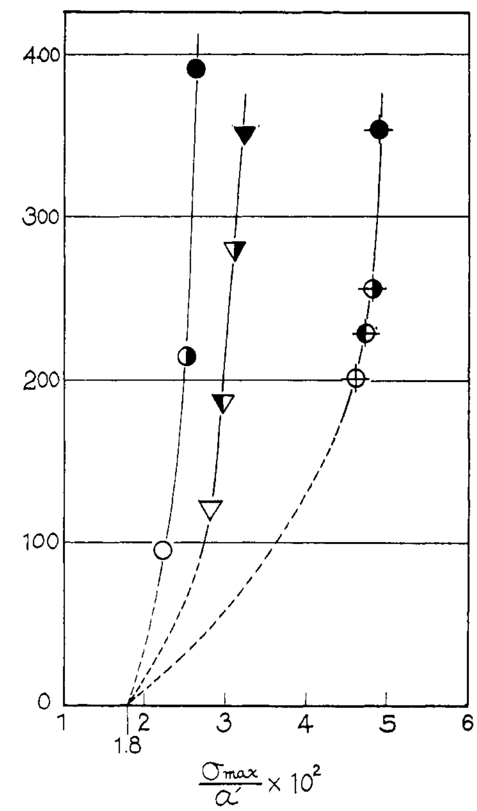

Figure 5. ΔS vs. $\sigma_{\max}/a'$ for estimating σ_a/a'

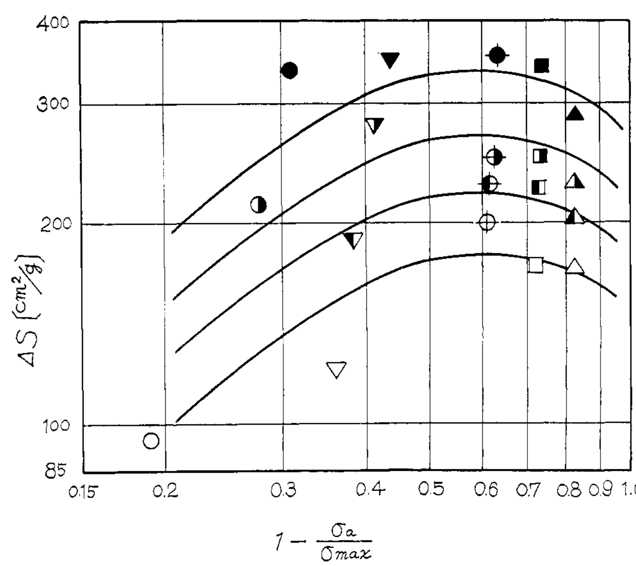

Figure 6. Logarithmic plot of ΔS vs. $[1 - (\sigma_a/\sigma_{\max})]$ as a function of pressure

$\sigma_{\max}$.) However, as discussed above, adsorption of gas molecules affects the propagation of cracks formed within a material. It seems reasonable to assume that ΔS is related to the pressure, P (partial pressure of some specified gas molecules, mm. of Hg), by an exponential function. This is illustrated in Figure 7, which is a cross plot from Figure 6 made for a value of $(1 - \sigma_a/\sigma_{\max})$ of 0.5. There are, of course, an infinite number of such cross plots possible, but they all give the same slope. From the value of the slope of the curve in Figure 7, we can say that

$$\Delta S \text{ is proportional to } e^{-kP} \quad (13)$$

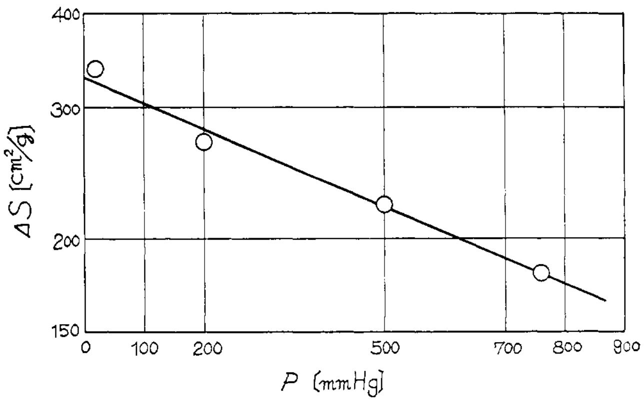

Figure 7. Semilogarithmic plot of ΔS vs. pressure, P, at definite value of $[1 - (\sigma_a/\sigma_{\max})]$ in Figure 6

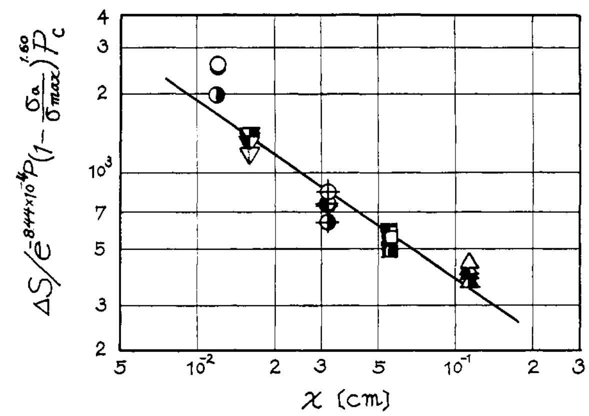

Figure 9. Final plot of $\Delta S/e^{-kP}[1 - (\sigma_a/\sigma_{\max})]^{1.60} \times P_c$ vs. particle size

where k, a constant, is found to be 8.44×10^{-4}. This exponential function ranges from 0 to unity, and can be regarded as the third probability, P_a.

Final Correlation

The three desired probabilities—P_c, P_σ, and P_a—have been determined numerically above. Thus, the probability of collision, P_c, is given by the target efficiency, η_T, which equals unity in the experiment described here. The probability of crack propagation, P_a, is given by $e^{-8.44 \times 10^{-4}P}$, and the probability of exceeding the breaking stress of the structure, P_σ, is the function, $(1 - \sigma_a/\sigma_{\max})^m$. The power m can be calculated by using all these data.

In Figure 8, each point corresponding to the runs with the smaller particle sizes (the two left-hand curves) gives a reasonable value of $m = 1.60$ from the slope of the straight lines. On the other hand, for the larger particles, most of the data tend to lump because larger particles do not differ much in impact velocity, the effect of air pressure being relatively less. In order to eliminate the effects of all probabilities, the values of ΔS divided by their product are plotted against the particle size, x (see Figure 9). The final correlation is given by the following expression:

$$\Delta S = 74(e^{-8.44 \times 10^{-4}P})P_c(1 - \sigma_a/\sigma_{\max})^{1.60}(1/x^{0.76}) \qquad (14)$$

The power of the particle size is found to be 0.76, being

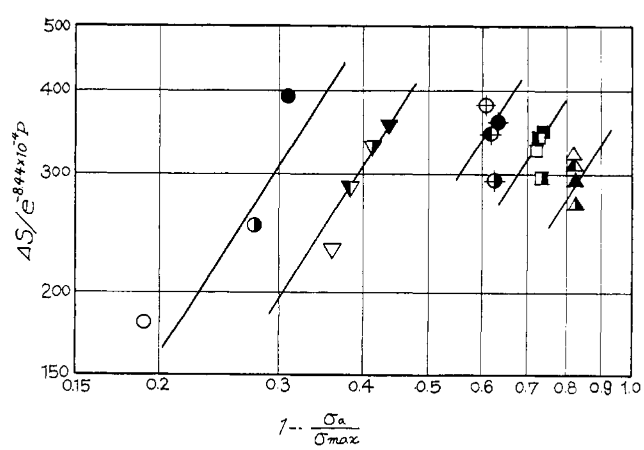

Figure 8. Logarithmic plot of $\Delta S/e^{-kP}$ vs. $[1 - (\sigma_a/\sigma_{\max})]$ as a function of particle size, x

constant for the present material, and P_c and $\sigma_{\max}$ also involve the effect of particle size.

Discussion

Since the crushing described here is carried out in a single stage, over-all crushing energy has been considered to equal the energy required by the rotating disk to accelerate a unit weight of particles to the initial velocity, 94 meters per second. However, the net energy consumed in crushing is a fraction of this, and the difference must be regarded as waste energy. The fraction appears to depend, in this case, on several probabilities, including particle size. Hence, the term dS/dE can be replaced by $\Delta S/\Delta E = \Delta S/\Phi E_A$, where E_A is the energy for acceleration of particles and Φ is the fractional or net energy actually spent on crushing.

If this relation is used, Equation 1′ becomes

$$\Delta S = C_2 \Phi E_A x^N = C_2' \Phi x^N \qquad (15)$$

This equation is obviously comparable with empirical Equation 14, and

$$\Phi = P_c(1 - \sigma_a/\sigma_{\max})^{1.60}(e^{-8.44 \times 10^{-4}P}) \qquad (16)$$

The value of N equals -0.76 in Equation 14; hence n in Equation 1 is found to be 1.24 for the calcite used in the present experiment. Both N and n are constants for any material, being independent of the internal mechanics of the crusher, and constant C in Equation 1 also involves the effect of particle size on the crushing mechanism.

It appears, therefore, that C in Equation 1 is a function not only of particle size but of the material being ground and of predictable mill characteristics. Only when this is recognized does N prove to be a constant, independent of particle size. It is this finding which seems to have been overlooked in past work.

The major points of this discussion are illustrated in Figure 10. Assuming P_c is unity, and $P = 760$ mm. of Hg, ΔS is calculated from the empirical Equation 14 over a wide range of particle sizes. In this case, the particle size, x, influences the terms $x^{-0.76}$ and $(1 - \sigma_a/\sigma_{\max})^{1.60}$ according to Equation 11, and the remaining factors in the equation are constant. Values of ΔS thus calculated and divided by these constants are plotted as the ordinate against the particle size, as the abscissa.

The curve illustrates the importance of particle size. In the regions on the right, representing coarse particles, the slope is almost -0.8, corresponding very closely with Kick's law, as previously explained. As the particles become smaller, the

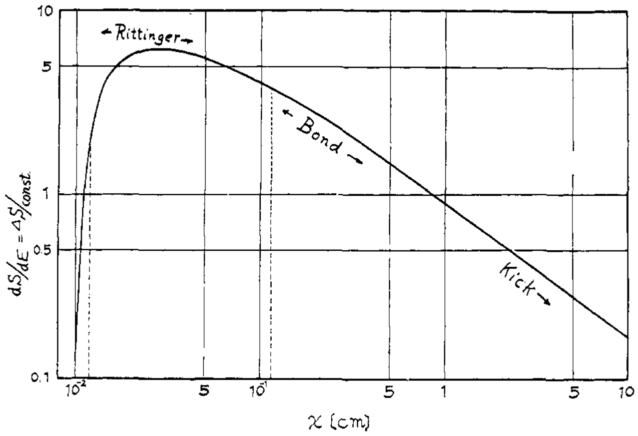

Figure 10. Over-all effect of particle size on dS/dE
ΔS/constant

curve develops a maximum where the slope, of course, is zero. Here it is supposed that Rittinger's law holds. In between, where the slope is -0.5, Bond's third theory of comminution may be applicable. More careful study indicates that the actual behavior results from the interaction of x^N and one of the probabilities—i.e., $(1 - \sigma_a/\sigma_{max})^m$, in this example, which is also a function of particle size. The rapid falling off of the curve to the left of the maximum means that a "limit" is being approached. In other words, for a given mechanism, crushing cannot be expected below a certain particle size. The existence of a crushing limit has been pointed out in previous publications by the author (*6*) and others (*3*, *5*).

Depending on the effect of size on P_c, the target efficiency, the curve should be varied in form. From this consideration, it is suggested that, depending on the material crushed and the crusher used, the range to which a particular law—say, Rittinger's—is applicable can become wider or smaller than the present example shows. This effect is in accord with previous experience.

The empirical equation proposed in this paper, of course, applies only to the experimental apparatus studied. It nevertheless suggests that the crushing and grinding mechanism should be analyzed along the lines discussed here involving various kinds of crushing probabilities.

Conclusions

Several probabilities are introduced and correlated with the conventional comminution law. The empirical equation obtained with a simple impact crusher involves three kinds of probabilities and provides a good interpretation of the dependency of the exponent of particle size and crushing mechanism. Even though the equation applies only to the present case, the basic concept described should be considered in analyzing actual crushing and grinding equipment. If this concept is established for a variety of machines, the conventional comminution law becomes more useful and valuable in designing such machines. Furthermore, such a final correlation could be a good criterion for selecting crushing equipment on the basis of quantitative computations.

Acknowledgment

T. Kohata and A. Suzuki are thanked for their valuable assistance in experiments, calculations, and discussions, and P. C. McCurdy for his great help in writing up this paper.

Nomenclature

a, a'	= constants
b	= $3\rho/4x\rho_P$
C	= constant
C_1	= constant
C_D	= drag coefficient, dimensionless
E	= crushing energy, kg.-m./kg.
E_A	= energy for acceleration of particles, kg.-m./kg.
E_1, E_2	= modulus of elasticity
K, k	= constants
m	= constant
m_1, m_2	= mass of particle and objective
N	= $n - 2$, constant
n	= constant
P	= pressure within crusher, mm. Hg
P_a	= probability of crack propagation, dimensionless
P_c	= probability of collision, dimensionless
P_σ	= probability of stress and structure of material, dimensionless
R	= reduction ratio (dimensionless) = x_1/x_2
Re	= $(xv\rho/\mu)$, Reynolds No., dimensionless
r	= deviation from Kick's law
r_1, r_2	= radius of curvature of the two bodies at impact point, cm.
S	= specific surface of particles, sq. cm./g.
ΔS	= specific surface increment, sq. cm./g.
s	= particle trajectory, meters
t	= time, sec.
v	= velocity of particle at impact, meters/sec.
v_i	= initial velocity of particle, meters/sec.
W_i	= work index proposed by Bond, kw-hr./ton
x	= particle size, cm.

Greek Letters

η_T	= target efficiency, dimensionless
θ	= angle, degrees
μ	= viscosity of air, kg./m. sec.
ν_1, ν_2	= Poisson's ratios of two bodies, dimensionless
ρ'	= density of air, kg./cu. meter
ρ_p	= density of particle, kg./cu. meter
σ_a	= breaking stress of particle, kg./sq. meter
σ_{max}	= maximum stress produced within material, kg./sq. meter
Φ	= fractional energy, dimensionless
Ψ	= separation number, dimensionless

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Received for review May 15, 1964
Resubmitted February 28, 1966
Accepted June 13, 1966