CHAPTER 6 (II): Mechanical Properties of Materials

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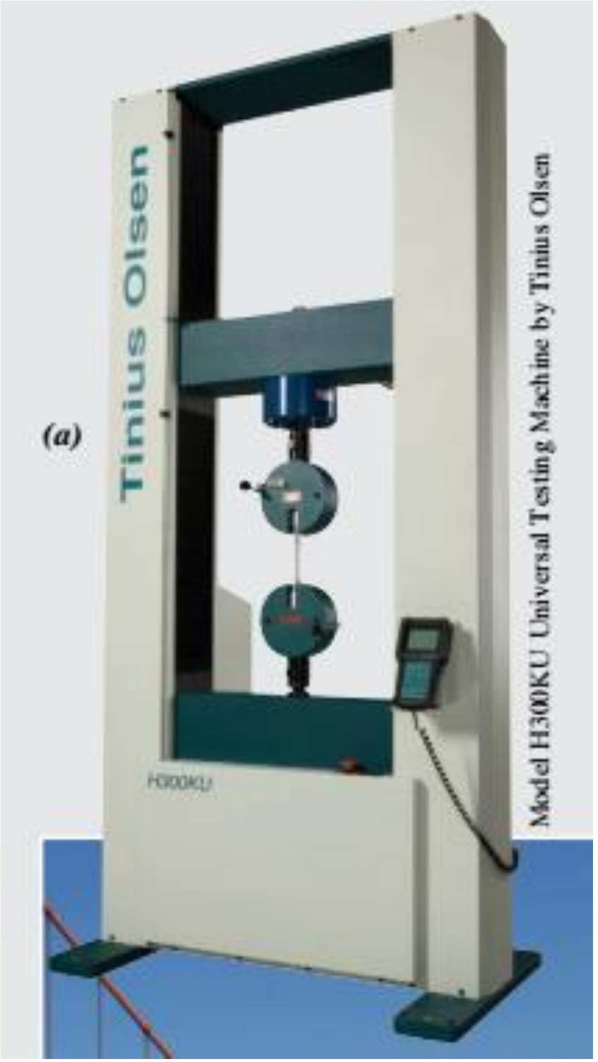
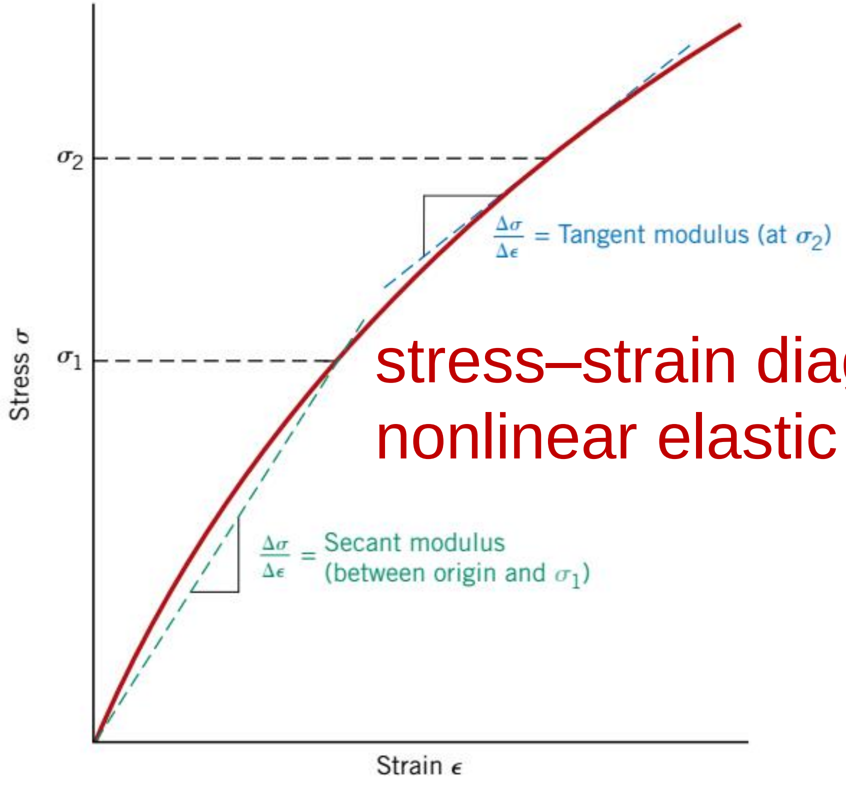


Nonlinear Elastic Behavior

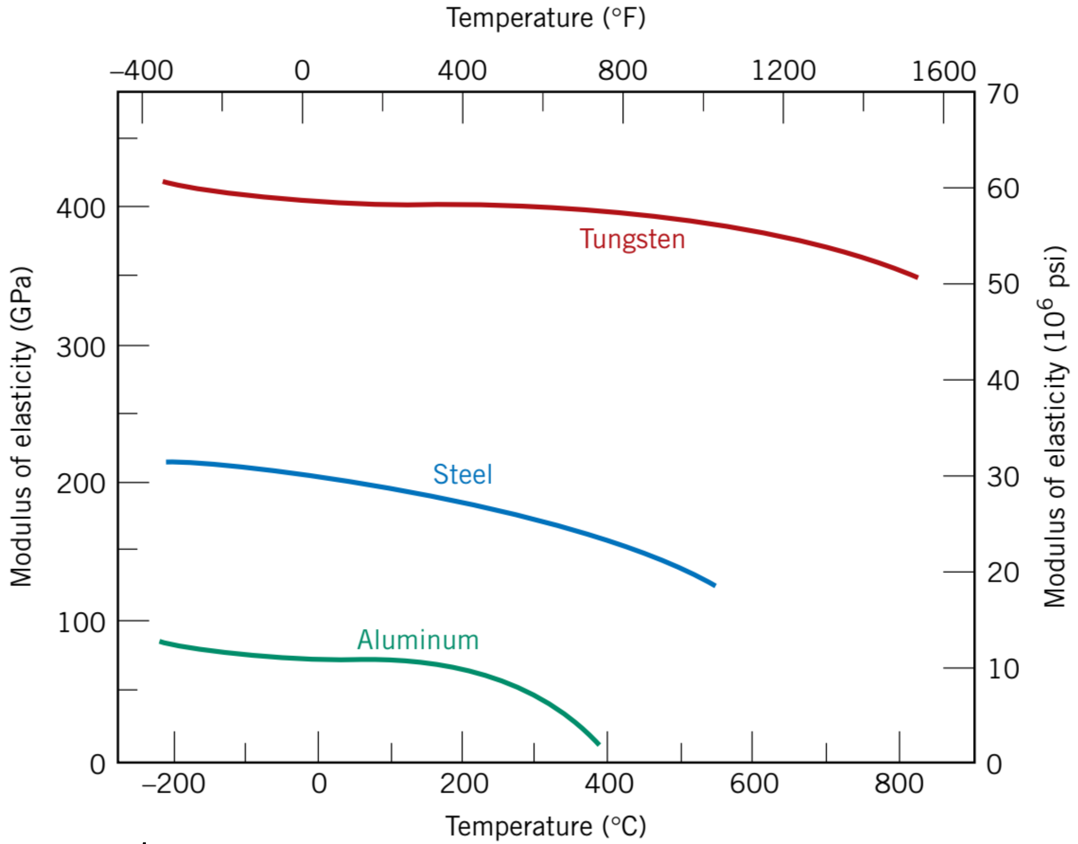
For some materials (i.e., gray cast iron, concrete, and mainly polymers) stress-strain plot is non-linear. What do you do (what is *E*)?

Two approaches

- Tangent modulus – slope of stress-strain curve at a specified stress
- Secant modulus – slope of a line drawn from zero stress to a specified stress



stress–strain diagram showing nonlinear elastic behavior



Modulus of Elasticity ~ Temperature

Anelasticity

We have so far assumed that elastic deformation is **time independent** (i.e. remove load → original shape returns "instantly")

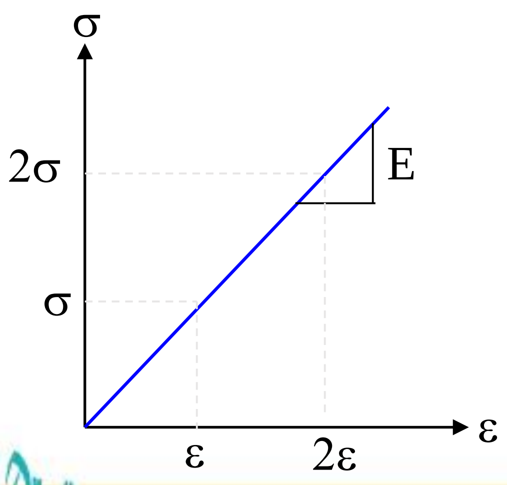
- Most materials also exhibit a time-dependent strain component. In other words deformation continues after removal of applied force. **This is called anelasticity**
- Generally unimportant for metals ; Can be significant for polymers (viscoelastic behavior)

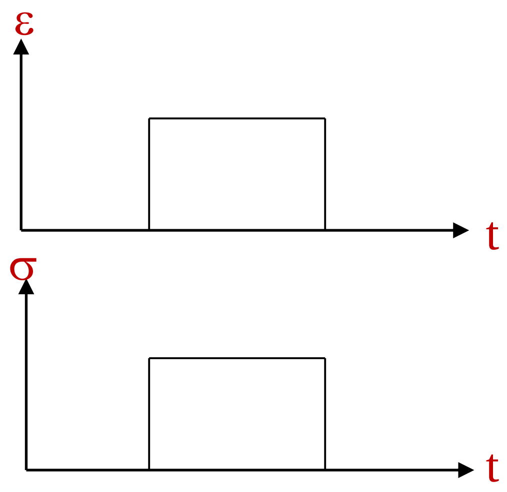
Conditions for Ideal Elasticity

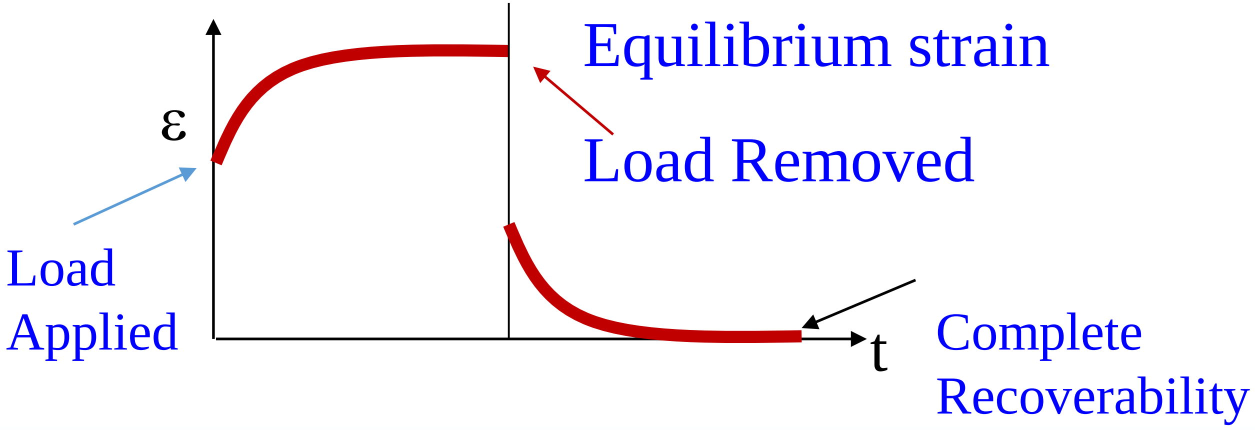
1) Each level of applied stress has a unique equilibrium value of strain
2) ~~The equilibrium response is achieved instantaneously~~
3) The response is linear (doubling the stress doubles the strain)

Conditions for Anelasticity

1) Each level of applied stress has a unique equilibrium value of strain
2) The equilibrium response is achieved only after the passage of sufficient time
3) The response is linear (doubling the stress doubles the strain)







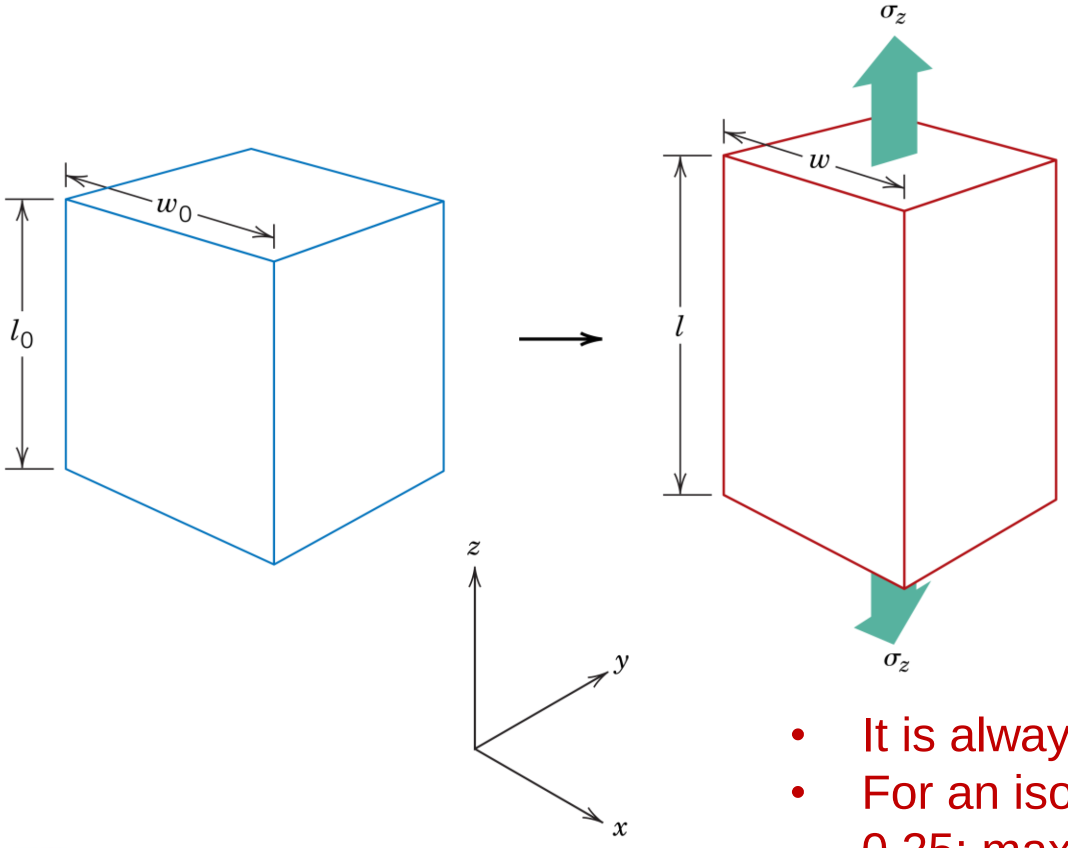
Elastic Properties Of Materials : Poisson's ratio

Poisson's ratio v is defined as the ratio of the lateral and axial strains

Consider the following →

- Elastic elongation in z
- Lateral contraction in x, y

- Can determine the compressive strains ε_x, ε_y from constriction
- If the applied stress is purely along z (uniaxial)
- If the material is isotropic then $\varepsilon_x = \varepsilon_y$



$$\varepsilon_z = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0} > 0$$

$$\varepsilon_x = \frac{w - w_0}{w_0} = \frac{\Delta w}{w_0} < 0$$

Poisson's ratio:

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z}$$

- It is always positive
- For an isotropic material ν should be 0.25; maximum value is 0.5

For isotropic materials the shear (G) and elastic moduli (E) are related through Poisson's ratio:

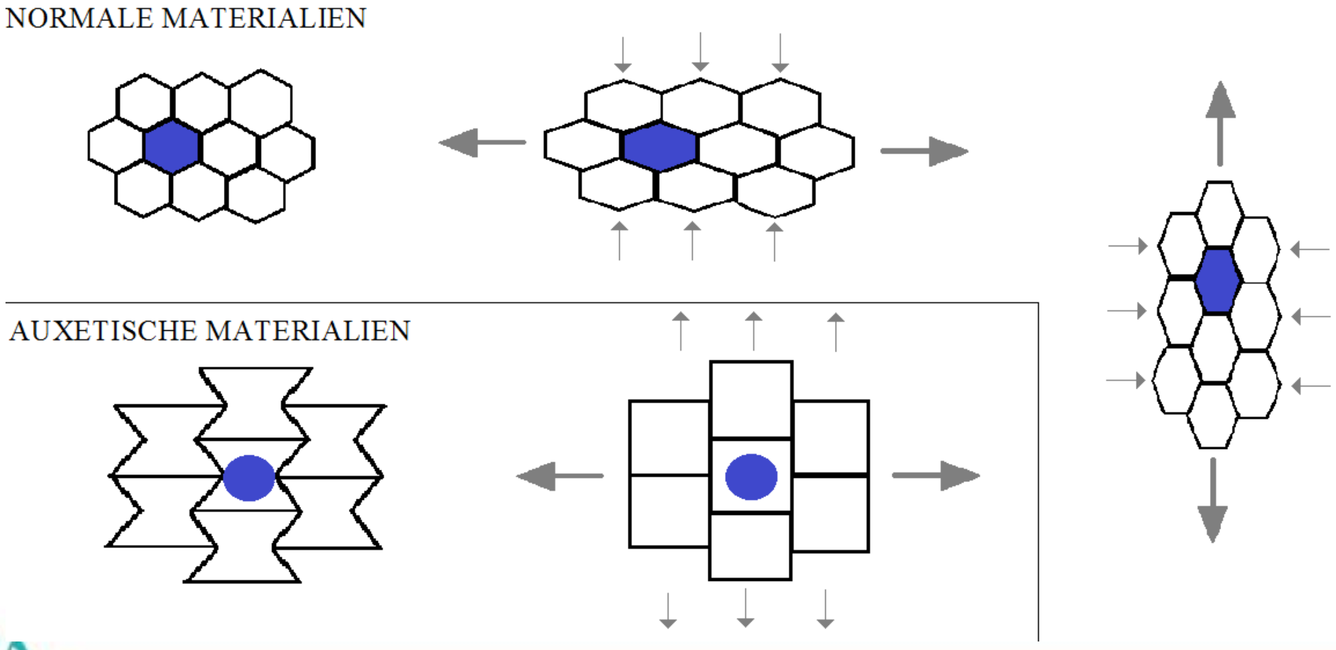
$$\boxed{E = 2G(1+\nu)}$$

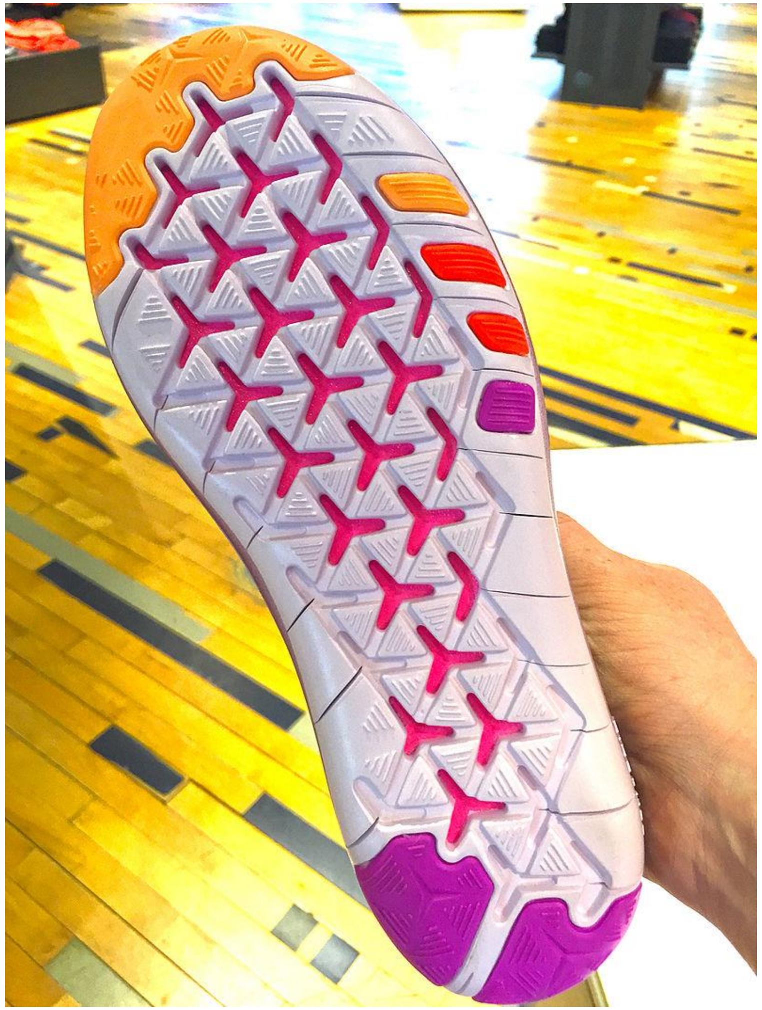
- Isotropic material is that for which measured properties are independent on the direction of measurement
- In most metals G is about $0.4E$

Negative Poisson's ratio : Auxetics material

Some materials (*e.g.*, specially prepared polymer foams), when pulled in tension, actually expand in the transverse direction. In these materials, both ε_x and ε_z are positive, and thus Poisson's ratio is negative. Materials that exhibit this effect are termed *auxetics.*

Auxetics are structures or materials that have a negative Poisson's ratio. When stretched, they become thicker perpendicular to the applied force.





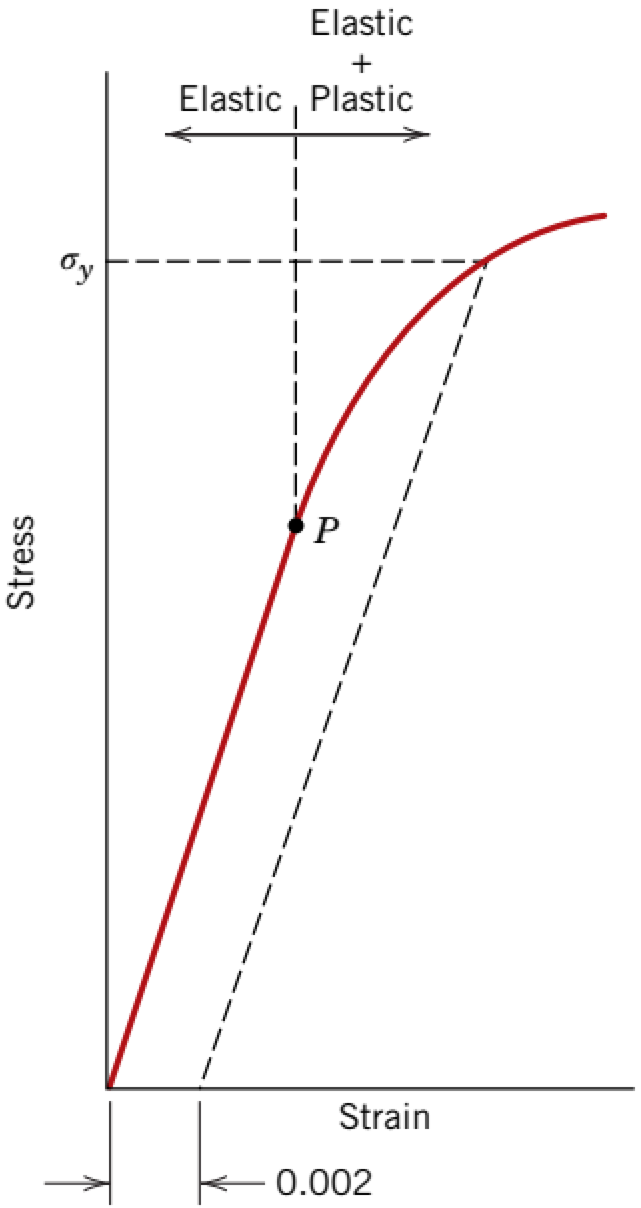
In footwear, auxetic design allows the sole to expand in size while walking or running, thereby increasing flexibility.

Elastic deformation is only part of the story …

- For most metals elastic deformation persists only up to strains of about 0.005. Beyond this *stress is not proportional to strain and permanent non-recoverable **plastic** deformation occurs*
- This transition is generally gradual

- Microscopic picture
  - Breaking bonds with neighboring atoms and forming new bonds with new neighboring atoms after motion
  - This is why the initial state is not recovered … this deformation is accomplished by a process called slip or the motion of the edge dislocations

Mechanical Behavior—Metals

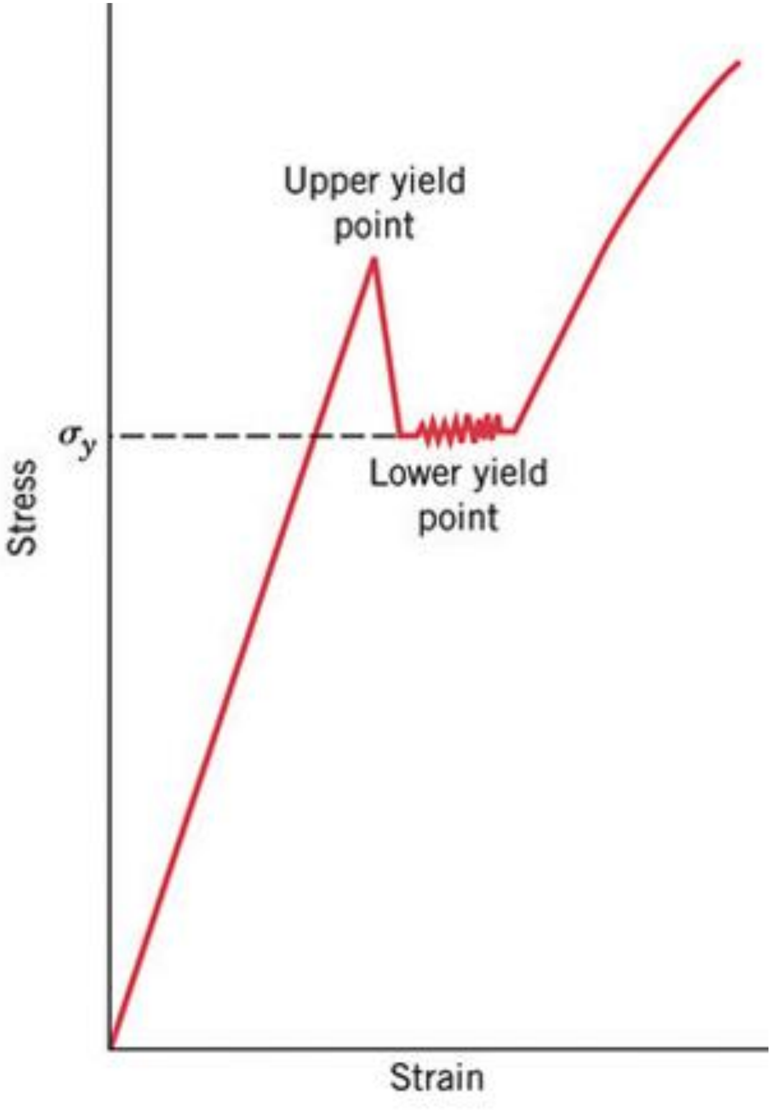


- **Tensile properties**
- When designing structures you **do not** want plastic deformation, so you need to know where the elastic → plastic transition occurs. This point is referred to as the **Yield point** (or where plastic deformation begins)
  - The yield point can be taken as where deviation from linearity is first observed (so called **proportional limit**)
  - But this is not precise … another approach:
    - Draw a straight line parallel to the elastic regime but offset by some value (**typically a strain of 0.002**). Where this intersects the σ–ε **curve** is called the

      yield strength (σ_y)
  - Finally, if the material exhibits nonlinear stress/strain behavior in the elastic regime this is not useful. In that case the yield strength is defined as the stress needed to produce a defined strain (e.g. typically $\varepsilon = 0.005$)

Mechanical Behavior in Metals : **Yield point phenomenon**

Some steels exhibit a very well-defined plastic-elastic transition



At the upper yield point, plastic deformation is initiated with an apparent decrease in engineering stress. Continued deformation fluctuates slightly about some constant stress value, termed the *lower yield point*; stress subsequently rises with increasing strain.

- Take lower yield point as the yield strength since it is well defined and insensitive to the testing method

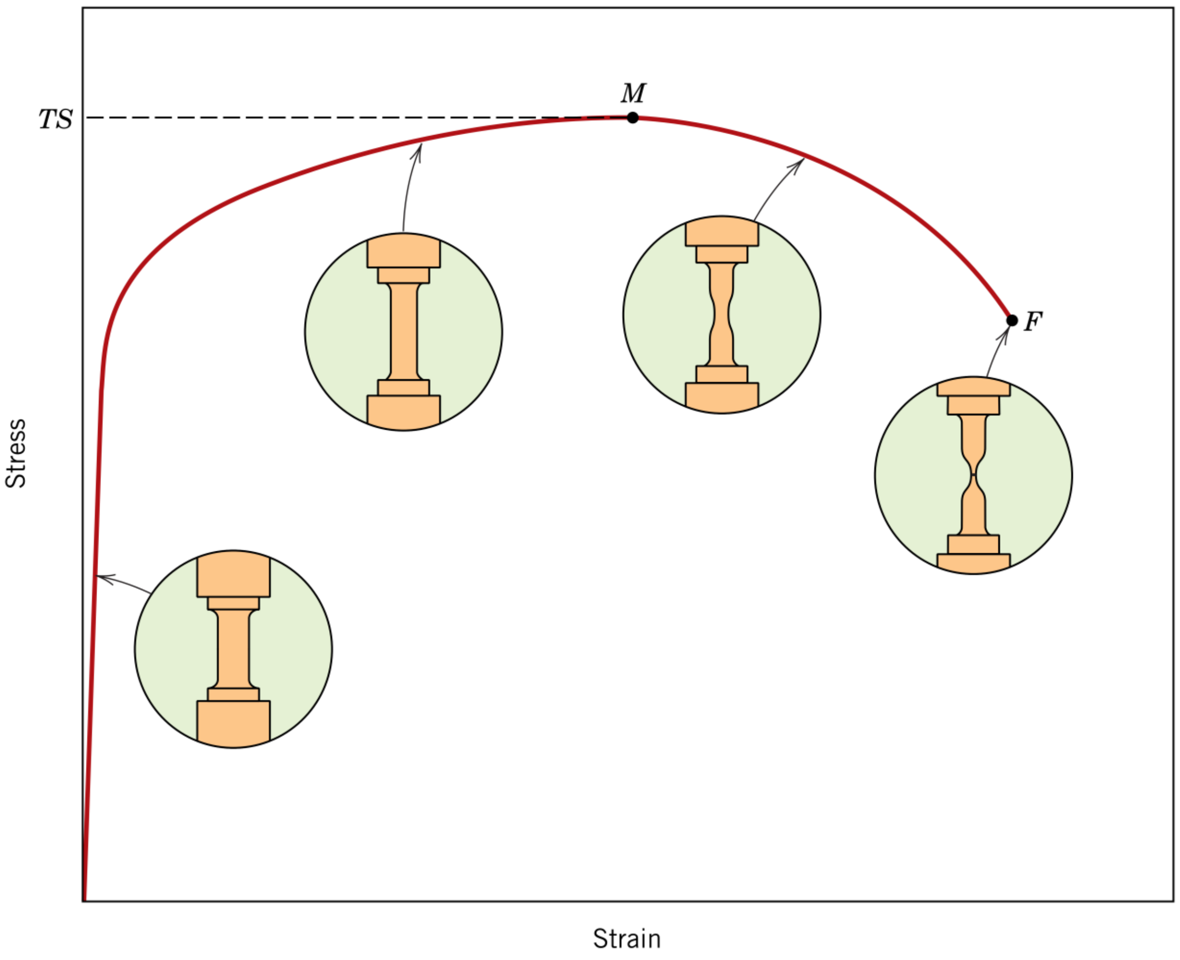
- The magnitude of the yield stress is a measure of the material's resistance to plastic deformation

<u>Range of yield strengths</u>
Low strength aluminum ~ 35 MPa
High-strength steels ~ 1400 MPa

Mechanical Behavior—Metals

What happens when you go into the plastic deformation regime?

- After yielding, the stress to continue plastic deformation increases to a maximum (point M) and then decreases until you reach fracture (point F)



- The **tensile strength** (TS) is the stress at the maximum of the stress/strain curve

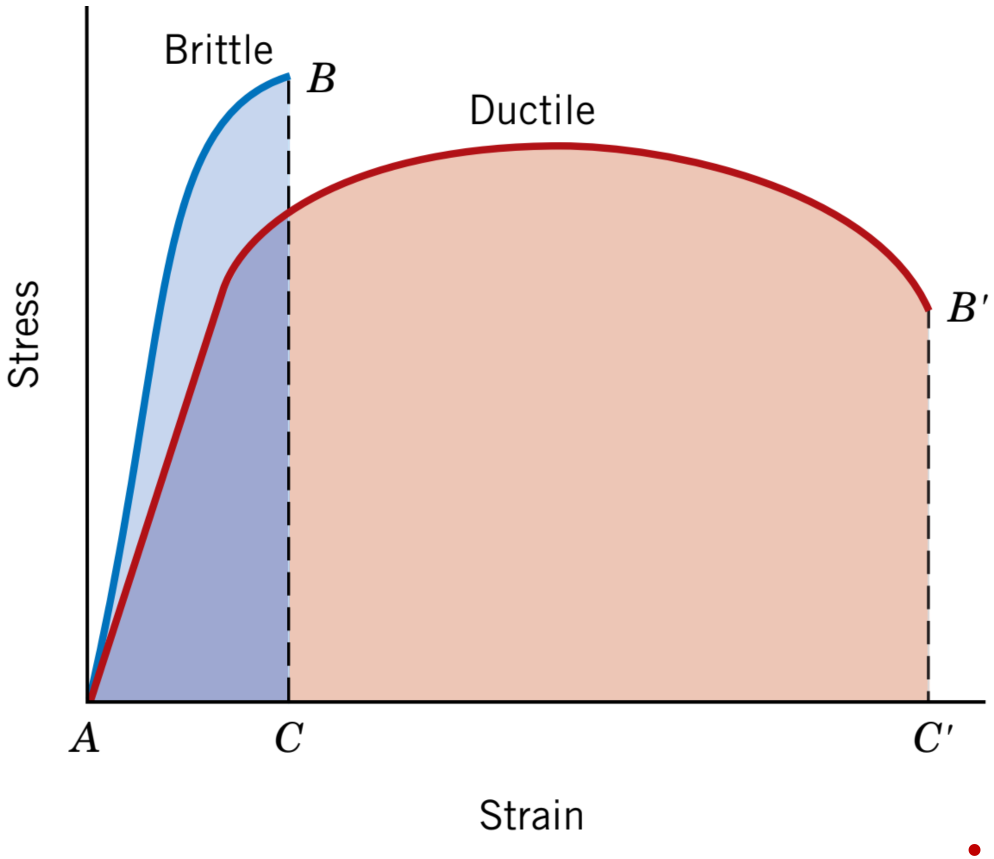
- If the tensile strength at the maximum is maintained, the material will fracture! Why?

- Before the maximum in the stress/strain plot, the stress/deformation is uniform throughout the sample. At point M a constriction (or neck) forms at a spot in the sample. This point acts to concentrate stress, in that subsequent deformation is confined to the neck region, and this is where fracture occurs.

- The **fracture strength** is exactly what it sounds like – the stress at fracture
- Tensile stresses range from 50 – 3000 MPa
- For design purposes, use yield strength and not tensile strength … why?

Ductility

It is a measure of the degree of plastic deformation that has been sustained at fracture. A material that experiences very little or no plastic deformation upon fracture is called brittle



Quantify **ductility** as either percent elongation or percent reduction in area (% plastic strain **at failure**)

$$\%EL = \left(\frac{l_f - l_o}{l_o}\right) \times 100\,\%$$

l_f – length at fracture
l_o – initial length

- Note that percent elongation depends on sample length – "standard" is to use 50 mm long pieces for testing
- %Reduction in area is similar:

$$\%RA = \left(\frac{A_o - A_f}{A_o}\right) \times 100\,\%$$

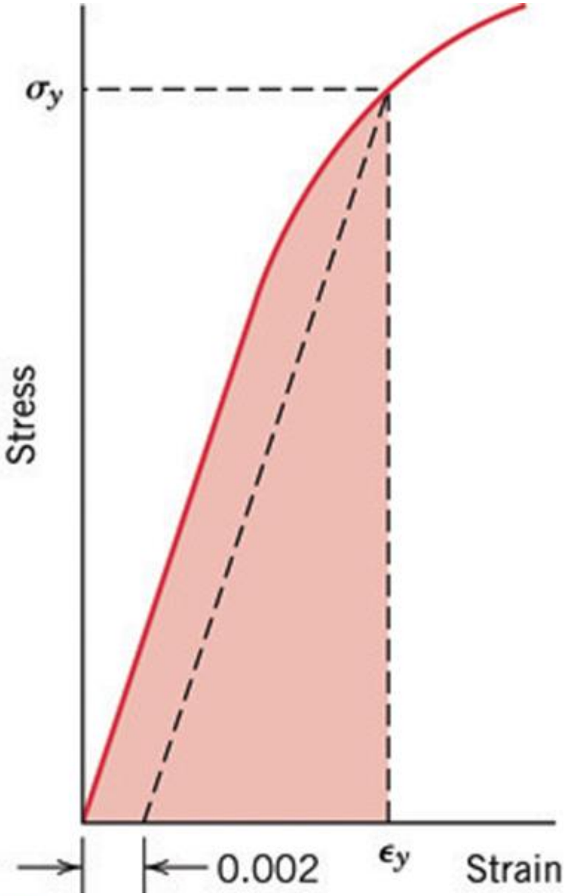
Why do you care about ductility?

- Indicates how much a material will deform plastically before failure
- Specifies degree of allowable deformation during fabrication operation
- One criteria for a brittle material: the fracture strain is less than 5%

Resilience

Capacity of a material to absorb energy when it is deformed elastically. Then upon unloading this energy is recovered

- Define modulus of resilience (U_r)
  - Strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding



- The modulus of resilience is also the area under the stress-strain curve up to the yielding point

How to find U_r ?

Area under the stress–strain curve represents energy absorption per unit volume (in cubic meters or cubic inches) of material

$$U_r = \int_0^{\varepsilon_y} \sigma d\varepsilon$$

If the stress-strain behavior is linear

$$U_r = \frac{1}{2}\sigma_y \varepsilon_y$$

or

$$U_r = \frac{1}{2}\sigma_y\left(\frac{\sigma_y}{E}\right) = \frac{\sigma_y^2}{2E}$$

ε_y is the strain at yielding

U_r [=] Pa (which is also J/m^3)

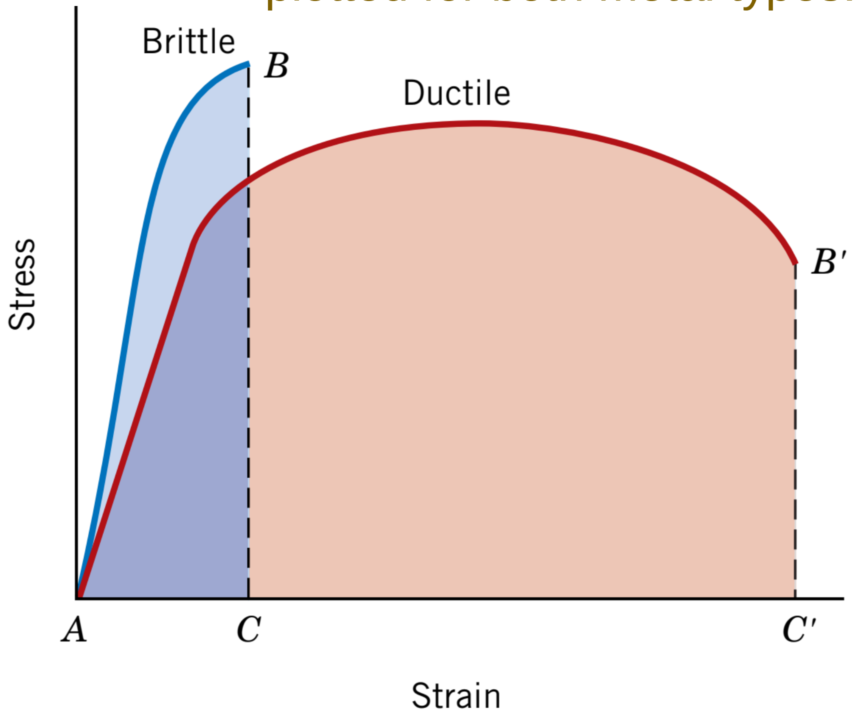
What makes something resilient? Applications?

Toughness

Toughness : (or more specifically, *fracture toughness*) is a property that is indicative of a material's resistance to fracture when a crack (or other stress-concentrating defect) is present

- This word is used in many contexts.
- It is generally a measure of the ability of a material to absorb energy up to the fracture point
  - Compare to ductility (elastic deformation)
  - Toughness is dependent on the sample geometry and load application
  - So toughness is taken to be the area under a stress-strain curve up to fracture
  - The units are the same as for resilience (i.e., energy per unit volume of material)
  - A "Tough" material is strong and ductile (why?)

The stress–strain curves are plotted for both metal types.



Hence, even though the brittle metal has higher yield and tensile strengths, it has a lower toughness than the ductile one, as can be seen by comparing the areas *ABC* and *AB′C′*

- For a metal to be tough, it must display both strength and ductility.

Room-Temperature Mechanical Properties (in Tension) for Various Materials

Material	Yield Strength MPa	Yield Strength ksi	Tensile Strength MPa	Tensile Strength ksi	Ductility, %EL [in 50 mm (2 in.)][a]
			Metal Alloys[b]		
Molybdenum	565	82	655	95	35
Titanium	450	65	520	75	25
Steel (1020)	180	26	380	55	25
Nickel	138	20	480	70	40
Iron	130	19	262	38	45
Brass (70 Cu–30 Zn)	75	11	300	44	68
Copper	69	10	200	29	45
Aluminum	35	5	90	13	40
			Ceramic Materials[c]		
Zirconia (ZrO_2)[d]	—	—	800–1500	115–215	—
Silicon nitride (Si_3N_4)	—	—	250–1000	35–145	—
Aluminum oxide (Al_2O_3)	—	—	275–700	40–100	—
Silicon carbide (SiC)	—	—	100–820	15–120	—
Glass–ceramic (Pyroceram)	—	—	247	36	—
Mullite ($3Al_2O_3$–$2SiO_2$)	—	—	185	27	—
Spinel ($MgAl_2O_4$)	—	—	110–245	16–36	—
Fused silica (SiO_2)	—	—	110	16	—
Magnesium oxide (MgO)[e]	—	—	105	15	—
Soda–lime glass	—	—	69	10	—

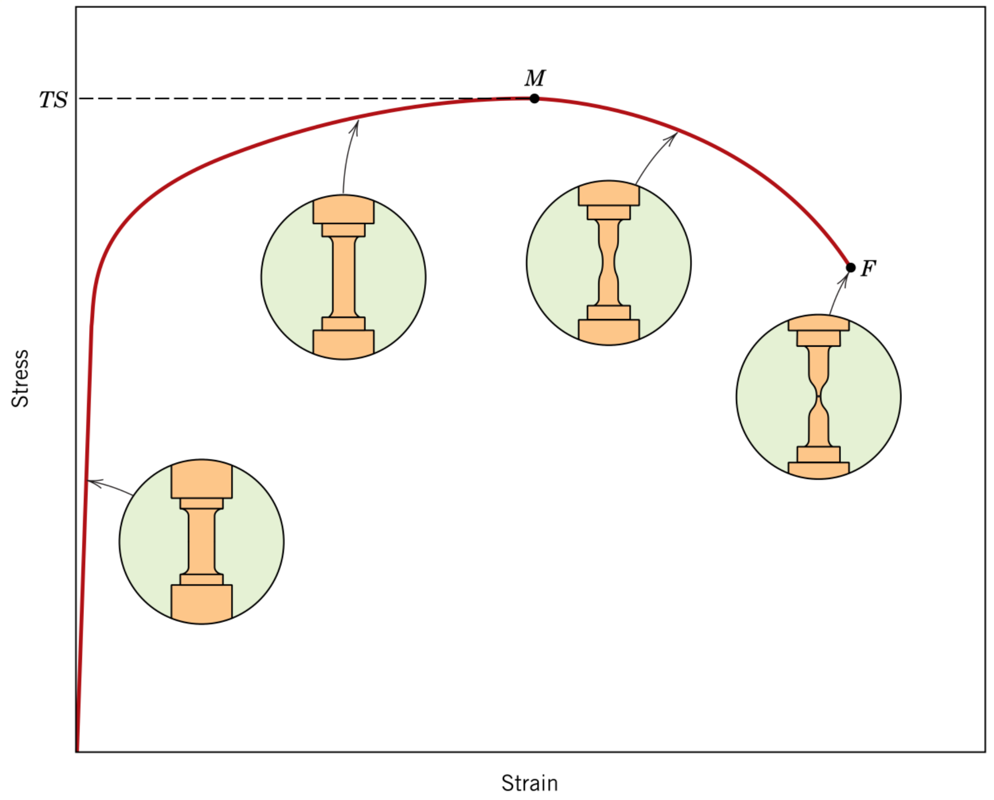
Room-Temperature Mechanical Properties (in Tension) for Various Materials

Material	Yield Strength MPa	Yield Strength ksi	Tensile Strength MPa	Tensile Strength ksi	Ductility, %EL [in 50 mm (2 in.)][a]
Polymers					
Nylon 6,6	44.8–82.8	6.5–12	75.9–94.5	11.0–13.7	15–300
Polycarbonate (PC)	62.1	9.0	62.8–72.4	9.1–10.5	110–150
Poly(ethylene terephthalate) (PET)	59.3	8.6	48.3–72.4	7.0–10.5	30–300
Poly(methyl methacrylate) (PMMA)	53.8–73.1	7.8–10.6	48.3–72.4	7.0–10.5	2.0–5.5
Poly(vinyl chloride) (PVC)	40.7–44.8	5.9–6.5	40.7–51.7	5.9–7.5	40–80
Phenol-formaldehyde	—	—	34.5–62.1	5.0–9.0	1.5–2.0
Polystyrene (PS)	25.0–69.0	3.63–10.0	35.9–51.7	5.2–7.5	1.2–2.5
Polypropylene (PP)	31.0–37.2	4.5–5.4	31.0–41.4	4.5–6.0	100–600
Polyethylene—high density (HDPE)	26.2–33.1	3.8–4.8	22.1–31.0	3.2–4.5	10–1200
Polytetrafluoroethylene (PTFE)	13.8–15.2	2.0–2.2	20.7–34.5	3.0–5.0	200–400
Polyethylene—low density (LDPE)	9.0–14.5	1.3–2.1	8.3–31.4	1.2–4.55	100–650

True Stress and Strain

In stress-strain plots beyond the maximum (M) it appears that the stress is decreasing between M and the fracture point F

- **But it is actually not – any guess as to why?**
  - It is due to necking, which causes the cross-sectional area to decrease with increasing load
  - The plots appear the way they do because we have up to now normalized quantities based on the initial cross-sectional area



The “true” stress accounts for this by normalizing the instantaneous load with the instantaneous cross-sectional area

$$\sigma_T = \frac{F}{A_i}$$

And analogously

$$\varepsilon_T = \ln\left(\frac{l_i}{l_o}\right)$$

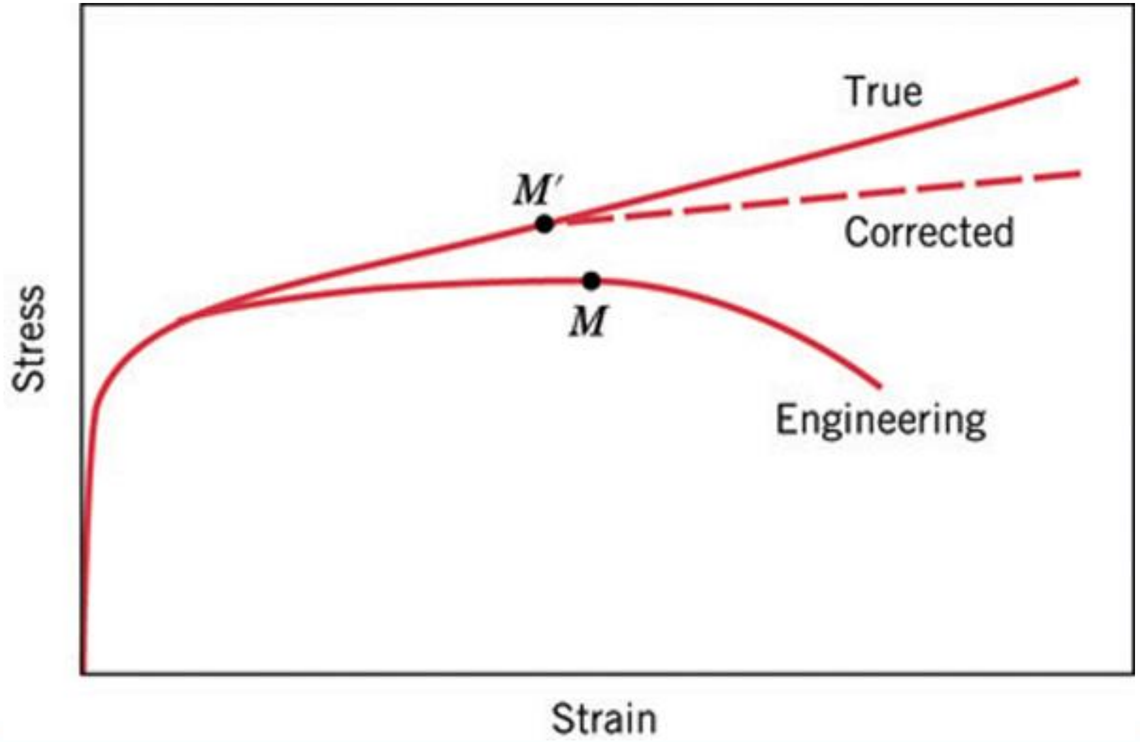
True Stress and Strain

So if there is no volume change during deformation

$$\boxed{A_i l_i = A_o l_o} \Rightarrow \left.\begin{aligned} \sigma_T &= \sigma(1+\varepsilon) \\ \varepsilon_T &= \ln(1+\varepsilon) \end{aligned}\right\}$$

Valid up to the onset of necking, beyond this point true stress and strain should be computed from actual load, cross-sectional area, and gauge length measurements.

Plot comparing the various stresses and strains



What's this! Upon neck formation the stress becomes more complex (not strictly uniaxial). This corrected stress accounts for that.

Does the plot make sense based on this?

True Stress and Strain

For some metals and alloys; We can describe stress-strain properties between onset of plastic deformation and necking via power-law

$$\sigma_T = k\varepsilon_T^n$$

n < 1, see Table

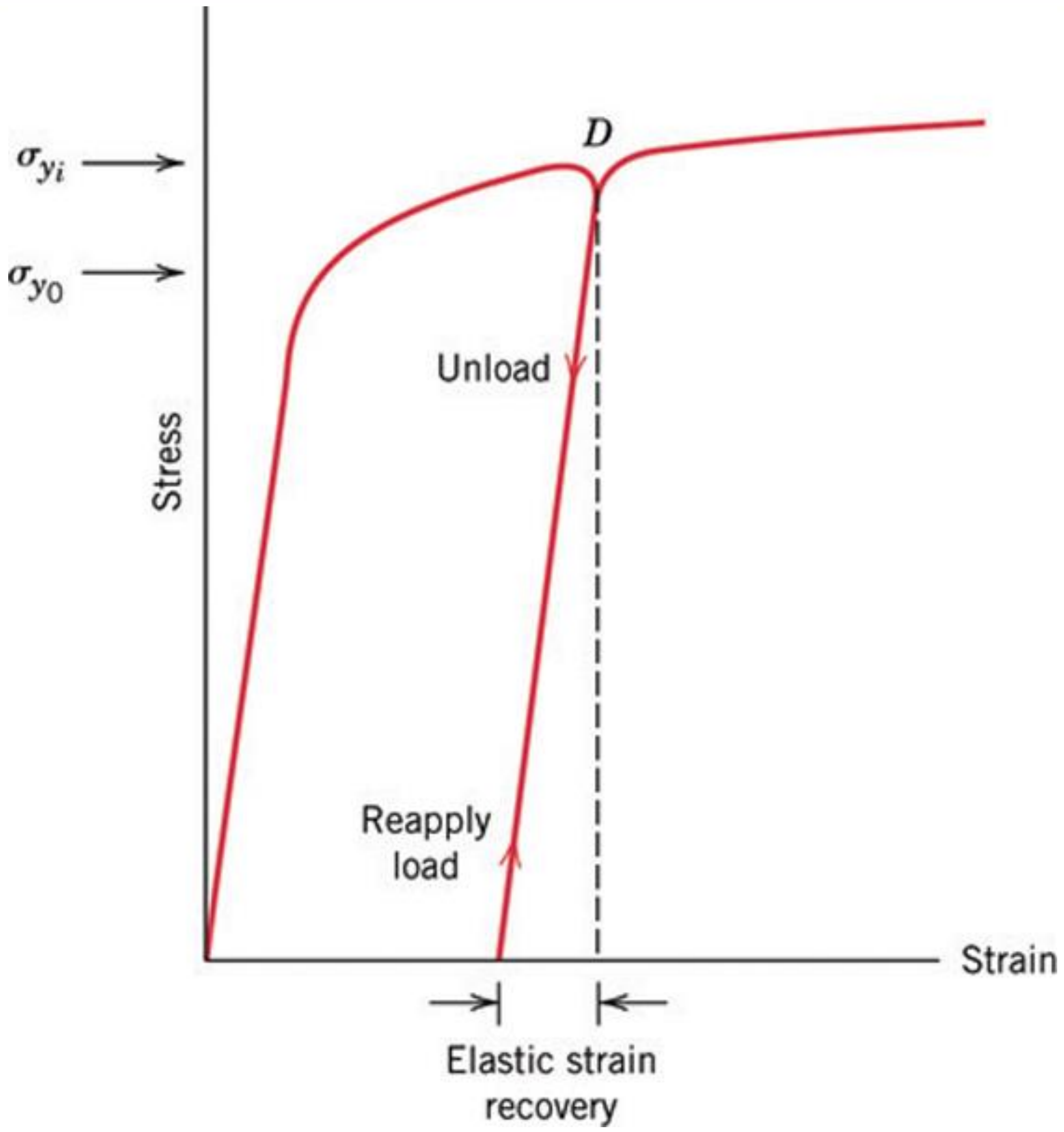
K and *n* are constants; these values vary from alloy to alloy and also depend on the condition of the material; The parameter *n* is often termed the *strain-hardening exponent* and has a value less than unity.

		K	
Material	***n***	***MPa***	***psi***
Low-carbon steel (annealed)	0.21	600	87,000
4340 steel alloy (tempered at 315°C)	0.12	2650	385,000
304 stainless steel (annealed)	0.44	1400	205,000
Copper (annealed)	0.44	530	76,500
Naval brass (annealed)	0.21	585	85,000
2024 aluminum alloy (heat-treated—T3)	0.17	780	113,000
AZ-31B magnesium alloy (annealed)	0.16	450	66,000

Elastic Recovery after Plastic Deformation

Upon release of the load during a stress-strain test some fraction of the total deformation is recovered as elastic strain

- When the load is removed the curve traces a line nearly parallel to the elastic portion of the **σ–ε curve**
- The magnitude of the elastic strain recovered during unloading is the ***Strain recovery***
- If the load is reapplied, follow the same trajectory, and yielding will again occur at the stress level where unloading began
- See a similar phenomenon at fracture



Mechanical Properties of Ceramics!!

The most important difference in ceramics compared to metals

- Ceramics fracture with very little energy absorption and typically do not deform plastically – why?

❖ So do you think tensile testing is useful for ceramics? Why or why not?

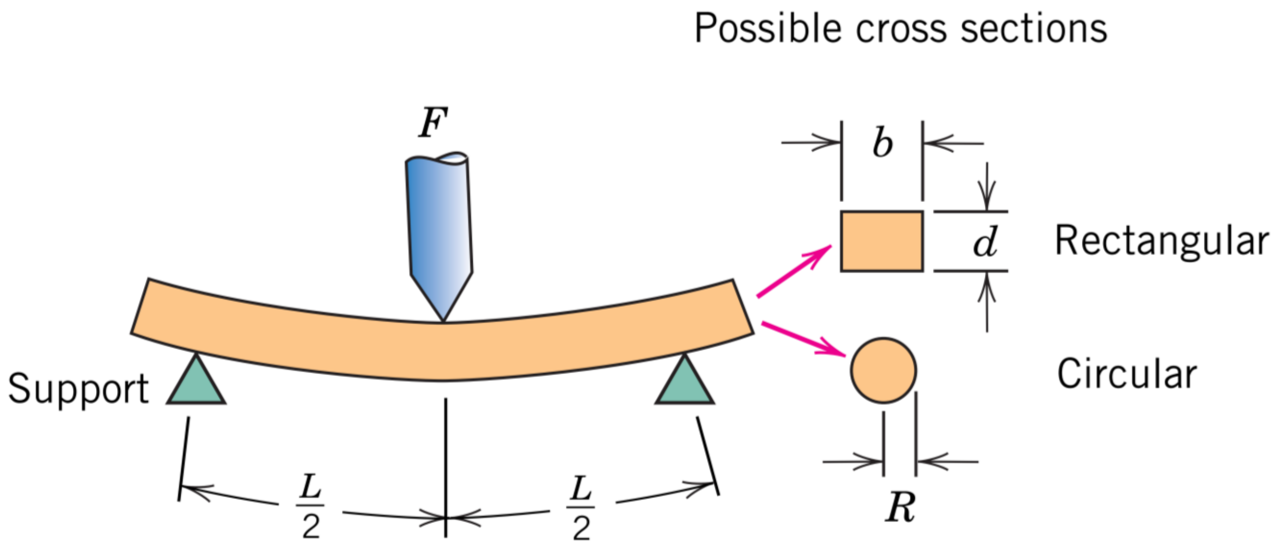
Three reasons not to measure tensile properties of ceramics

1. Hard to prepare ceramics with desired size, shape
2. Brittle! Hard to secure ends without breaking them
3. Typically fail at ~0.1% strain (need to perfectly align sample)

- No – measure what is called the *flexural strength*

Ceramics – Flexural Strength

A three point loading method for measuring the Stress–Strain behavior and **flexural strength** of brittle ceramics



$$\sigma = \text{stress} = \frac{Mc}{I}$$

where M = maximum bending moment
c = distance from center of specimen to outer surface
I = moment of inertia of cross section
F = applied load

L – distance between support points

The stress at fracture using this flexure test is known as the flexural strength, *modulus of rupture, fracture strength,* or *bend strength,* an important mechanical parameter for brittle ceramics

Home Assignment

Flexural strength for

	M	c	I	
Rectangular	$\frac{FL}{4}$	$\frac{d}{2}$	$\frac{bd^3}{12}$	$\boxed{\sigma_{fs} = \frac{3F_f L}{2bd^2}}$
Circular	$\frac{FL}{4}$	R	$\frac{\pi R^4}{4}$	$\boxed{\sigma_{fs} = \frac{F_f L}{\pi R^3}}$

Ceramics – Flexural Strength

Sample is under both tension and compression

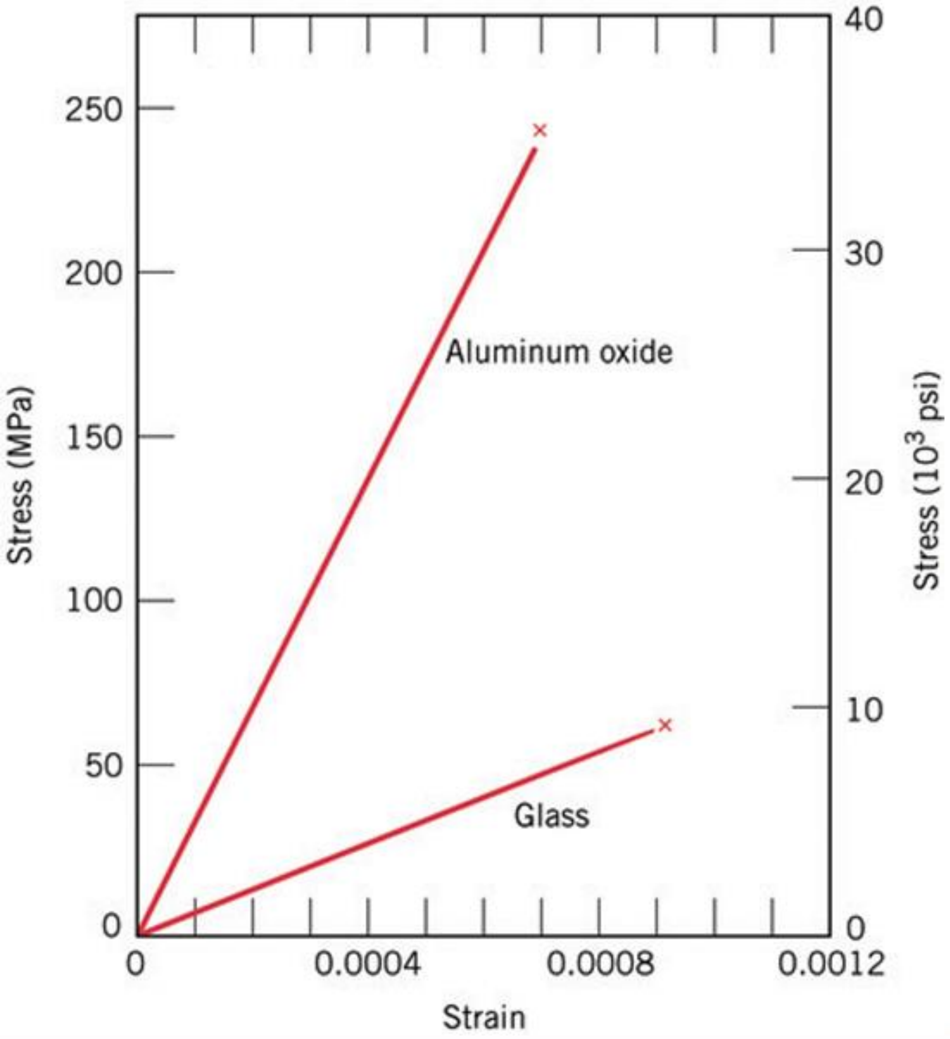
- The value of σ_{fs} depends on specimen size – bigger samples, increase chance of cracks (lower flexural strength)

- the magnitude of flexural strength for a specific ceramic material is greater than its fracture strength measured from a tensile test.

Ceramics – Elastic Behavior

The elastic stress–strain behavior for ceramic materials using these flexure tests is similar to the tensile test results for metals: a linear relationship exists between stress and strain.

- Note values on x-axis



What do the low strain values mean?

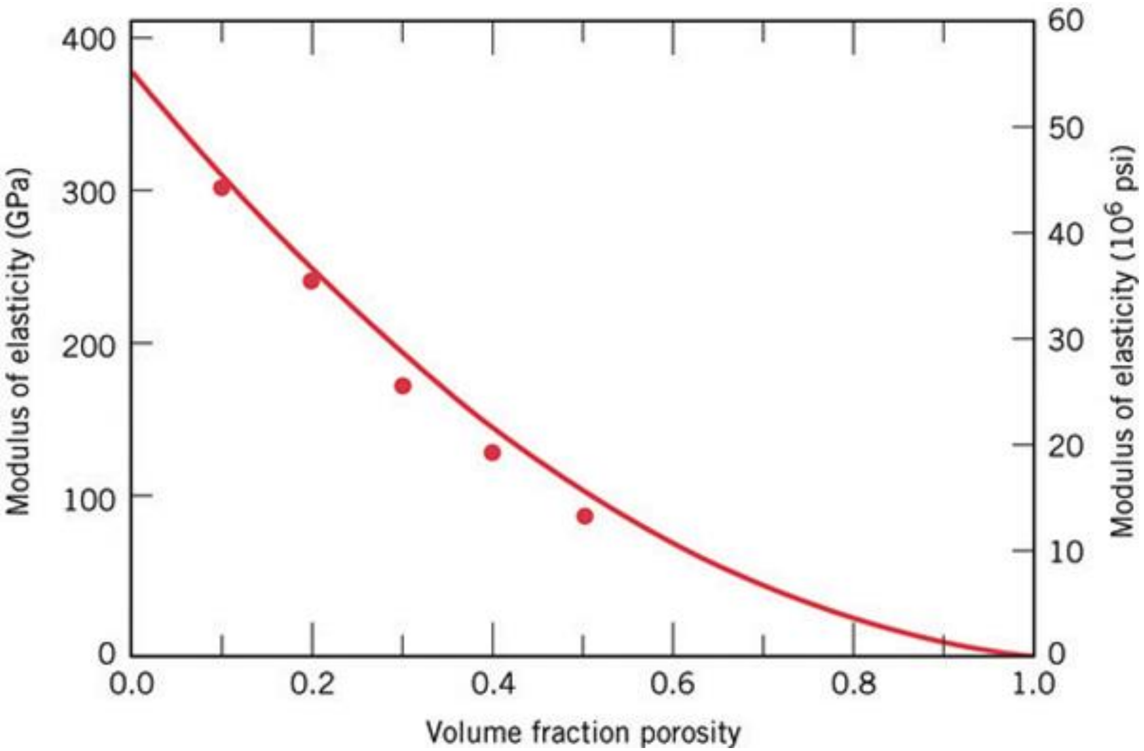
How do they compare to metals?

Ceramics – Porosity

Many ceramics are made from powders – form ceramic disks, other objects by high temperature/pressure treatment

- This results in porosity (holes, void spaces → pores)
- Porosity generally has a negative effect on mechanical properties

Example 1 – effect on elastic modulus



$$\boxed{E = E_o\left(1 - 1.9P + 0.9P^2\right)}$$

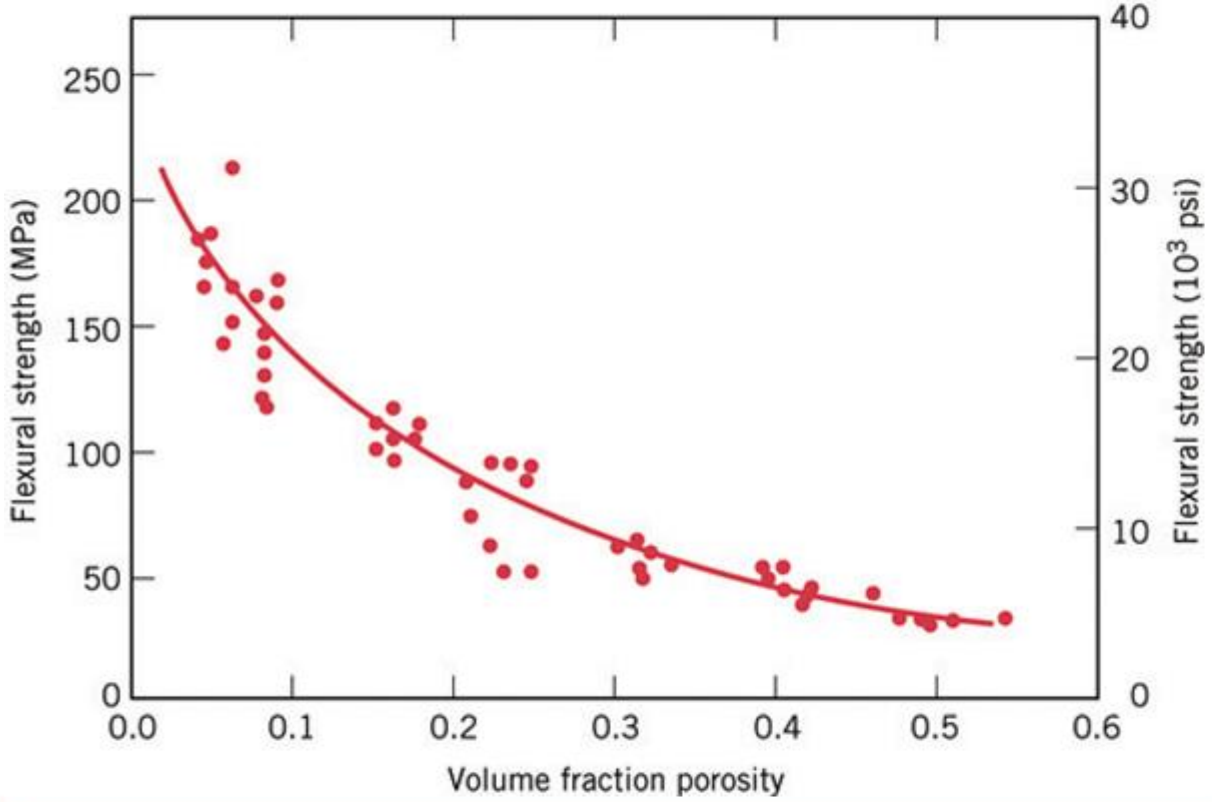
E_o – modulus of non porous material

The influence of porosity on strength is rather dramatic;
10 vol% porosity often decreases the flexural strength by 50% from the measured value for the nonporous material.

Two reasons porosity deteriorates mechanical properties

1. Pores reduce cross-sectional area across which a load is applied
2. Pores act as “stress concentrators”

Example 2 – flexural strength



$$\sigma_{fs} = \sigma_o \exp(-nP)$$

σ_o – strength of non porous material

n is experimental constants

P= volume fraction porosity

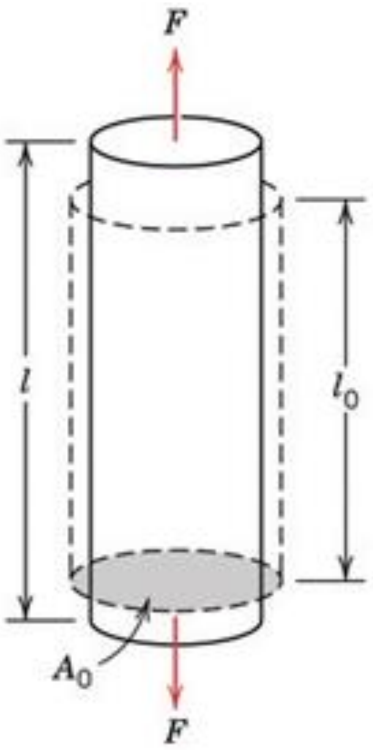
Summary of Mechanical Properties

Property	*Symbol*	*Measure of*
Modulus of elasticity	E	Stiffness—resistance to elastic deformation
Yield strength	σ_y	Resistance to plastic deformation
Tensile strength	TS	Maximum load-bearing capacity
Ductility	%EL, %RA	Degree of plastic deformation at fracture
Modulus of resilience	U_r	Energy absorption—elastic deformation
Toughness (static)	—	Energy absorption—plastic deformation
Hardness	e.g., HB, HRC, HV, HK	Resistance to localized surface deformation
Flexural strength	σ_{fs}	Stress at fracture (ceramics)
Relaxation modulus	$E_r(t)$	Time-dependent elastic modulus (polymers)

Example:

Consider a brass rod under tensile stress:

If d_o = 10 mm what load is needed to produce a 2.5 x 10^{-3} mm change in diameter (you may assume the deformation is elastic)



What is the problem asking for? ***F!***

Have d_o, Δd, and know the material of construction – what can we find first?

$$\varepsilon_x = \frac{\Delta d}{d_o} = \frac{-2.5\times10^{-3}\,mm}{10\,mm} = -2.5\times10^{-4}$$

ε_d

ε_z $$\varepsilon_z = \frac{\Delta l}{l_o}$$ Don't know either of these -- What about Poisson's ratio?

$$\nu = \frac{-\varepsilon_x}{\varepsilon_z} \Rightarrow \varepsilon_z = \frac{-\varepsilon_x}{\nu}$$ Table 3.1 ν_{Brass} = 0.34

$$\varepsilon_z = \frac{-\varepsilon_x}{\nu} = \frac{-2.5\times10^{-4}}{0.34} = 7.35\times10^{-4}$$

- <u>Example:</u>

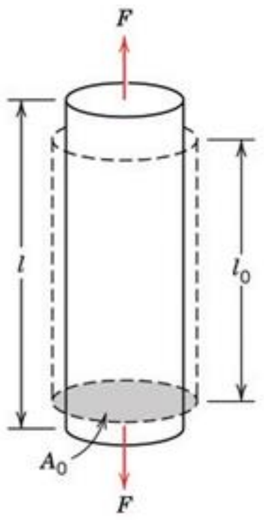
Ok, what next (have ε_z, ε_x)?

$$\sigma = E\varepsilon_z$$

Table 3.1 – E_{brass} = 97 GPa

$$\sigma = 97GPa \times \left(7.35 \times 10^{-4}\right) = 71.3MPa$$

Almost done → what is left to do?



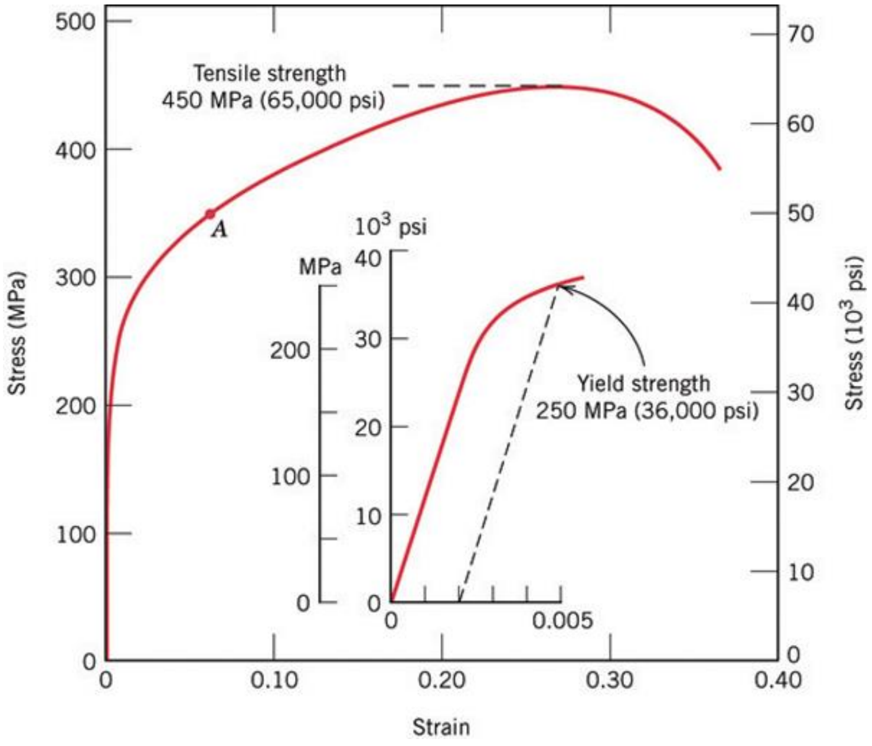
$$\sigma = \frac{F}{A_o} \Rightarrow F = A_o \sigma$$

$$A_o = \pi\left(\frac{10mm}{2}\frac{1m}{1000\,mm}\right)^2 = \pi\left(\frac{10 \times 10^{-3}m}{2}\right)^2$$

$$F = \pi\left(\frac{10 \times 10^{-3}m}{2}\right)^2 \left(71.3MPa\right) = 5600\,N$$

Assignment

- <u>Example</u> Given the plot below for brass, calculate :



a) Modulus of elasticity
b) Yield strength (strain offset of 0.002)
c) Maximum load that can be sustained by a cylindrical specimen with d_o = 12.8 mm
d) The change in length of a specimen originally 250 mm long subjected to a tensile stress of 345 MPa