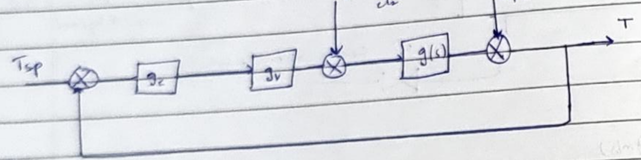
Traditional advanced control strategies

① High level application of PID
- Time delay compensator
- Cascade Control
- split range control → manipulate multiple variables using 1 controller

② Feedforward and ratio control

CASCADE CONTROL

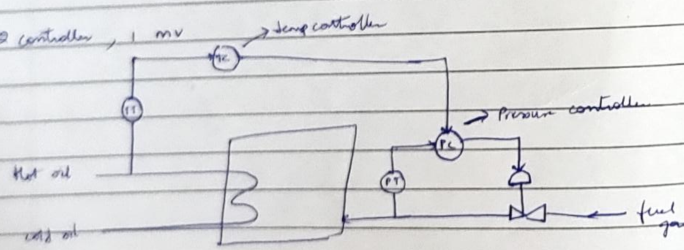
Q



→ regulatory control strategy cannot take care of input disturbances. (fuel gas pressure)

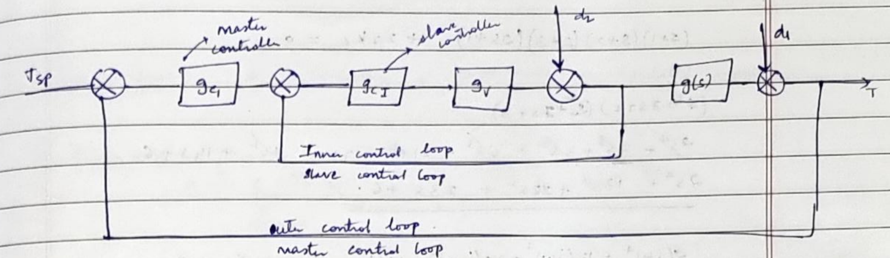
This can be controlled using another controller at the input

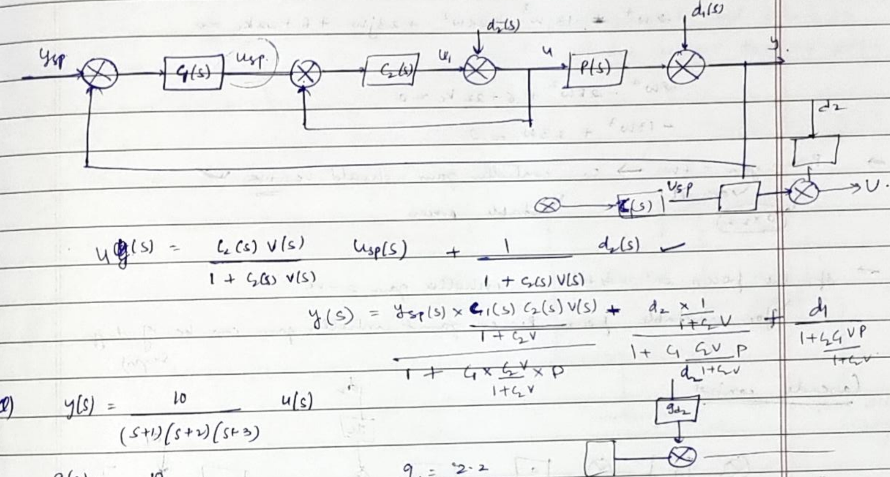
→ 2 controller, 1 mv



cascading temp controller using pressure controller.

(T & PC cascade controller)





$$u(s) = \frac{G_2(s)\,V(s)}{1 + G_2(s)\,V(s)}\,u_{sp}(s) + \frac{1}{1 + G_2(s)\,V(s)}\,d_2(s)$$

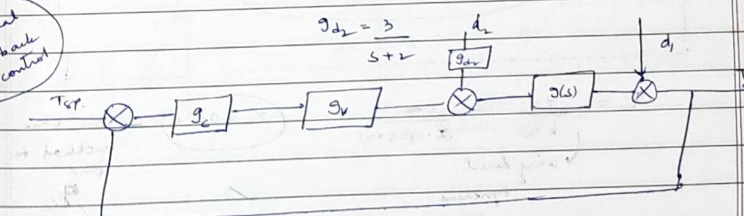
$$y(s) = \frac{y_{sp}(s) \times \dfrac{G_1(s)\,G_2(s)\,V(s)}{1+G_2V}}{1 + G_1 \times \dfrac{G_2V}{1+G_2V} \times P} + \frac{d_2 \times \dfrac{1}{1+G_2V}}{1 + G_1 \dfrac{G_2VP}{1+G_2V}} + \frac{d_1}{1+\dfrac{G_1G_2VP}{1+G_2V}}$$

Q) $y(s) = \dfrac{10}{(s+1)(s+2)(s+3)}\,u(s)$

$g(s) = \dfrac{10}{(s+1)(s+2)(s+3)}$ $\quad g_v = \dfrac{2.2}{2s+1}$

$g_{d_2} = \dfrac{3}{s+2}$

Normal feedback control



$1 + K_c g_v g = 0$

$$1 + K_c\left(\frac{2.2}{2s+1}\right)\left(\frac{10}{(s+1)(s+2)(s+3)}\right) = 0.$$

$K_{cu} = ?$

$s = j\omega$.

$$(2s+1)(s+1)(s+2)(s+3) + K_c(22) = 0.$$

$$(s+1)(s+2)(s+3)(2s+1) + 22k_c = 0$$

$$(s^2+3s+2)(2s^2+7s+3)$$

$$2s^4 + 7s^3 + 3s^2 + 6s^3 + 21s^2 + 9s + 4s^2 + 14s + 6$$

$$2s^4 + 13s^3 + 28s^2 + 23s + 6$$

$$2(j\omega)^4 + 13(j\omega)^3 + 28(j\omega)^2 + 23(j\omega) + 6 + 22k_c = 0$$

$$2\omega^4 - 13j\omega^3 - 28\omega^2 + 23j\omega + 6 + 22k_c = 0$$

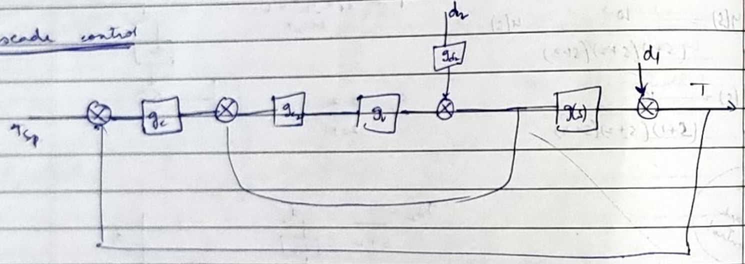
$$2\omega^4 - 28\omega^2 + 6 + 22k_c = 0$$

Q1 $$-13\omega^3 + 23\omega = 0$$

→ Process gain = +ve → so controller gain should be +ve
↳ overall (10×2.2) ↳ stable process

→ If the process is unstable, controller gain = -ve.
↳ for unstable process Process gain & controller gain can be of diff signs)

Cascade control



$g_c = \frac{\tau}{k\tau_c} = \frac{2}{(2.2)(0.5\times2)}$ $\tau_c = 0.5\tau$ → use this method to find g_c → proportional controller

↳ using direct synthesis approach

Cascade controller

Controller: 2 or more

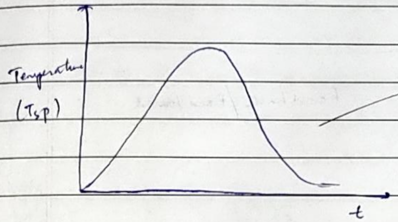
Manipulative input: 1

split range :- 1 controller, 2 or more manipulated input

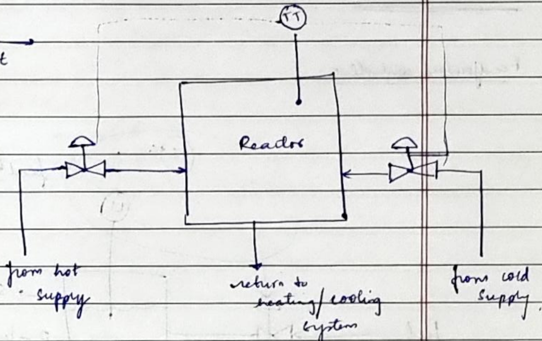
→ Batch reactor

$A \xrightarrow{k_1} B \xrightarrow{k_2} C$ (B desired, C undesired)

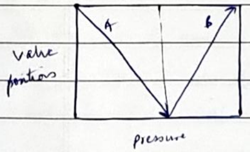
$k_i = A_i e^{-\Delta H_i / RT}$



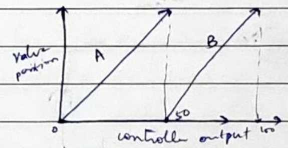
hot stream & cold stream are blended to get this temp profile



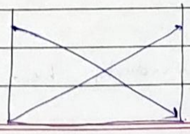
split range



Sequence operation (split range)

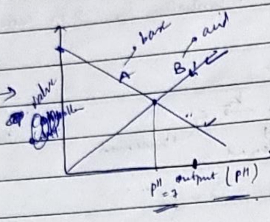


opposition action split range control



Reflux drum

→ pH neutralization reactor
$pH \downarrow$, controller output $\downarrow$

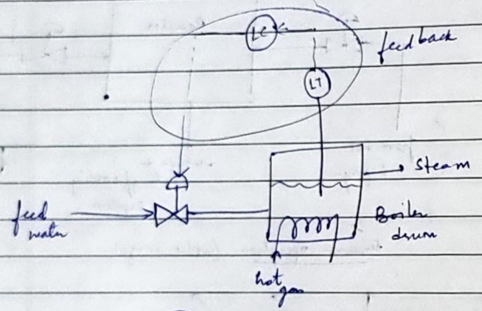


FEEDFORWARD AND RATIO CONTROL

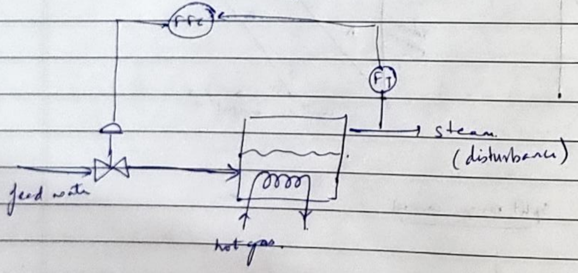
feedback / Feedforward

a. Feed back control

Feedforward controller

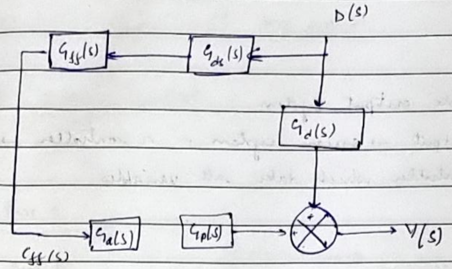


liq level is output



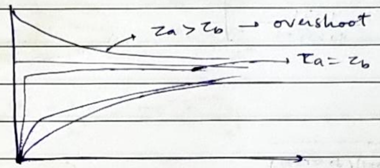
Only FFC strategy is never applied in an industry becoz it cannot solve servo problems.

→



~~$Y(s) = D(s)\, G_{ds}(s)\, G_{ff}(s)\, G_a(s)\, G_p(s) + D(s)\, G_d(s) = 0$~~

→ $\dfrac{Y(s)}{U(s)} = G_2(s) = \dfrac{\tau_a s + 1}{\tau_b s + 1}$



→ Combined FF & FB control

If we use a pure FFC strategy, there is no stability issue
If we have combination of FF & FB → stability will come into picture
Adding FFC will not hamper the stability of the overall control loop.

Ratio control

$R = \dfrac{u}{d}$ — u → mv, d → disturbance

Multivariable control

→ multiple input, multiple output system

- ① n input, n output → square system – n controller → multi loop
- (or) single controller which take all variables

Qudruple tank system

K_m: measurement gain

Q. ⇒ find poles & zeros for a multivariable system

$$G(s) = \begin{bmatrix} \dfrac{4.153}{62.36s+1} & \dfrac{3.036}{(62.36s+1)(22.76s+1)} \\ \dfrac{1.78}{(90.63s+1)(30.07s+1)} & \dfrac{4.534}{90.63s+1} \end{bmatrix}$$

$$G(s) = \frac{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}{\underbrace{(62.36s+1)(22.76s+1)(90.63s+1)(30.07s+1)}_{\text{will give poles}}}$$

→ Poles of $G^{-1}(s)$ are zeros of $G(s)$

$$G^{-1}(s) = \frac{Adj\,|G(s)|}{Det\,|G(s)|}$$

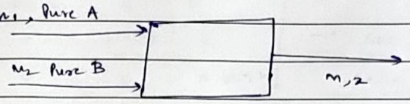
$\boxed{Det\,|G(s)| = 0 \text{ will give poles of } G^{-1}(s) \text{ and zeros of } G(s)}$

→ non-minimum phase system → designing the control strategy is difficult

Right hand pole is zero, time delay.

Interaction analysis for multi-loop control

→ Multivariable process control

→ 

$M = M_1 + M_2$

$x = \dfrac{M_1}{M_1 + M_2}$

2 loops: control x by manipulating M_1 → $MV = M_2$

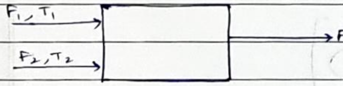
$\left.\dfrac{\partial m}{\partial m_1}\right|_{\text{both loops open}} = 1$ | $\left.\dfrac{\partial m}{\partial m_1}\right|_{\text{other loop is closed}} = 1$

→ $x^* = \dfrac{M_1}{M_1 + M_2}$, $M_2 = \dfrac{M_1}{x^*} - M_1$

$M = M_1 + \dfrac{M_1}{x^*} - M_1 = \dfrac{M_1}{x^*}$

→ $\left.\dfrac{\partial m}{\partial m_1}\right|_{\text{other loop closed}} = \dfrac{1}{x^*}$

→ $\lambda_{11} = x^*$

→ 

F → volumetric flow rate

Analysis of RGA

$$\lambda = \frac{\Delta y}{\Delta y + \Delta y_r} = \frac{K_d}{K_d + K_r}$$

$\lambda = 1$, → $K_r = 0$

→ $\lambda < 1$, means K_r is finite and in the same direction to K_d

→ $\lambda > 1$ → $|K_d| > |K_r|$ K_r is acting in opp direction to K_d.

→ $\lambda_{11} = 1$ → $\frac{k_d}{k_d + k_r}$ ($k_r = 0$)

↳ no interaction from other loops.

→ $0 < \lambda_{11} < 1$ → $k_d \uparrow$, $k_r \uparrow$

→ $\lambda_{11} = 0$ → assumed loop pairing is incorrect.

→ $\lambda_{11} > 1$ → $k_d \uparrow$, $k_r \downarrow$

→ $\lambda_{11} < 0$

⇒ RGA

	u_1	u_2	u_3	u_4
y_1	0.931	0.15	0.08	-0.164
y_2	-0.011	-0.429	0.286	1.154
y_3	-0.135	3.314	-0.27	-1.91
y_4	0.215	-2.03	0.9	1.917

→ avoid -ve values

→ choose values close to 1

loop pairing should ~~...~~ ~~...~~ min interaction

⇒ Singular value analysis

relation between CN & linear independency.

→ CN↑, independency ↓

→ if k is ill-conditioned → Pairing is ill conditioned

→ RGA = $\begin{bmatrix} -2.43 & 3.024 & 0.413 \\ 1.2211 & -0.7617 & 0.5407 \\ 2.2165 & -1.2623 & 0.0458 \end{bmatrix}$

$y_1 \to u_2$
$y_2 \to u_3$
$y_3 \to u_1$

} better pairing

$y_1 \to u_2$
$y_2 \to u_1$ X
$y_3 \to u_3$ ~~0.0458~~ is very low

$y_1 \to u_2$
$y_2 \to u_1$
$y_3 \to u_1$ → not possible (Same variable)

$W = \begin{bmatrix} 0.5714 & 0.3762 & 0.7292 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

↓ find highest Mag of

$\Sigma = \begin{bmatrix} 1.618 & 0 & 0 \\ 0 & 1.143 & 0 \\ 0 & 0 & 0.0097 \end{bmatrix}$ → CN is very high

$CN = \frac{1.618}{0.0097}$

} CN is very high unstable loop

↓

Whatever loop pairing you consider, loop will be unstable.

→ 3 control loops → System is unstable

∴ design 2 control loops.

→ ~~Find the~~ no. of

→ Find the possible combinations ~~a~~ to get the 2 control loops.

↳ choose the loops whose $CN \approx 1$

Controller design based on state space model

→ state space model

(1) state variable

(i) Non linear SSM

control affine form
$$\dot{x} = f(x) + g(x)u \rightarrow \text{state eqn}$$
input is represented explicitly.
$$y = h(x,u) \rightarrow \text{output eqn}$$

$x = [x_1 \; x_2 \; x_3 \ldots x_n]^T$

$u = [u_1 \; u_2 \ldots u_m]^T$ (i/p system)

$y = [y_1 \; y_2 \ldots y_p]^T$ Overall dynamics depends on n.

(2) linear SSM

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

X – state variable vector
U – input vector
Y – output vector

} in deviation form.

A, B – matrices.

→ (i) Develop dynamic model
$$\dot{x} = f(x,u) \rightarrow (1)$$

(ii) define control objective
$$y = h(x,u) \rightarrow (2)$$

(iii) control-affine form
$$\dot{x} = f(x) + g(x)u$$
$$y = h(x,u)$$

(iv) linear SSM → linearize around x_s, u_s

~~$f(x_s, u_s) = 0, h(x_s, u_s) = 0$~~

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

Dynamic model : $\dot{x} = f(x,u) \rightarrow (1)$

$y = h(x,u) \rightarrow (2)$

Use taylor series approx around (x^s, u^s)

$$\dot{x}_1 = f_1(x_1^s, x_2^s, \ldots, x_n^s, u_1^s, u_2^s, \ldots u_m^s) + \frac{\partial f_1}{\partial x_1}(x_1 - x_1^s) + \frac{\partial f_1}{\partial x_2}(x_2 - x_2^s) + \ldots + \frac{\partial f_1}{\partial x_n}(x_n - x_n^s) + \ldots$$

$\dot{x}_n$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \vdots & & \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_m} \end{bmatrix}}_{B} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

→ Transfer function model → is only for linear equation

→ $\dot{X} = AX + BU$ take laplace
$Y = CX + DU$

$$sX(s) = AX(s) + BU(s)$$
$$Y(s) = CX(s) + DU(s)$$

$(sI - A)X(s) = BU(s)$; i.e. $X(s) = (sI - A)^{-1} BU(s)$

$$Y(s) = [C(sI-A)^{-1}B + D]\,U(s)$$

$$U = [\underset{MV}{M} \;\; \underset{LV}{L}]$$

$$\boxed{Y(s) = G_p(s)\,M(s) + G_L(s)\,L(s)}$$

Example :- Van De Vusse reactor

$A \to B \to C$

maximize this

Realization from TF

→ If I have TF model, can I get the SS M

Controllable canonical form

non-rational → time delay associated with TF

$$g(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \dots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}$$

rational TF

Consider rational TF
if time delay represent separate that part

$\cancel{\frac{Y(s)}{Z(s)}\ \frac{Z(s)}{U(s)}}$ $\cancel{b_0 s^n + b_1 s}$

Jordan canonical form

→ Observer canonical form
↳ matrix = transpose of matrix in controllable canonical form

Designing the control system

① Controllability

At time t_0, if it is possible by means of an unconstrained control vector to transfer the system from any initial state x_0 to any other state in a finite interval of time
(whatever input I give, system can be controlled)

$\dot{x} = AX + BU$

state transition ; control influence

system :- $\dot{X} = AX + BU$

↳ linear ODE

$$X(t) = e^{At} X(0) + \int_0^t e^{A(t-\tau)} B\, U(\tau)\, d\tau$$

at SS $X(t_1) = 0$
(X is deviation variable)

$$X(0) = -\int_0^{t_1} e^{A(t-\tau)} B\, u(\tau)\, d\tau$$

⇒ $[B \quad AB \quad \dots \quad A^{n-1}B]$

↳ rank should be n to solve
↳ no. of states of the system

⇒ P = no. of outputs

Observability

whether the system is observable or not

n states

$\dot{X} = AX + BU$

$Y = CX + DU$

$y = f(x_1, x_2)$ → we will measure x_1, x_2 to get the output.
↳ $n-2$ state info is not available.

Principle of duality

S_1: $\dot{X} = AX + BU$
$Y_1 = CX$

S_2: $\dot{z} = A^T z + C^T V$
$Y_2 = B^T Z$

State space controller design

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

check whether the states, outputs are controllable or not

→ control signal : $U = \underbrace{-k}_{-k \,\rightarrow\, \text{feedback}} X$