Assignment 3

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Q1.1) a) spherical water droplet

at t, $r = r_S$

C_{As}

@ $r = r_S$

as $r \to \infty$, $C_A = C_{A\infty}$

$W = N_A \cdot 4\pi r^2 = \text{const}$

$J = -D_{AB} \cdot \dfrac{dC_A}{dr}$

Since only the droplet is diffusing

$N_B = 0$

$\therefore N_A = N_A \cdot \dfrac{C_A}{C} + J_A$

$N_A = \dfrac{C}{1 - C_A} \cdot J_A$

$\therefore N_A = -\dfrac{C}{1 - C_A} \cdot D_{AB} \dfrac{dC_A}{dr}$

$$\int_{r_S}^{\infty} \frac{W\, dr}{4\pi r^2} = -\int_{C_{AS}}^{C_{A\infty}} \frac{C \cdot D_{AB}}{1 - C_A}\, dC_A$$

$\therefore \dfrac{W}{4\pi} \dfrac{1}{(+1)} \left(+\dfrac{1}{r_S}\right) = \dfrac{+C \cdot D_{AB}}{1} \ln\left(\dfrac{1-C_{A\infty}}{1-C_{AS}}\right)$

$\boxed{\dfrac{W}{4\pi r_S} = D_{AB} \cdot C \cdot \ln\left(\dfrac{1 - C_{A\infty}}{1 - C_{AS}}\right)}$

②

$$\therefore \boxed{\omega = 4\pi r_s \cdot D_{AB} \cdot c \ln\left(\frac{c - c_{A\infty}}{c - c_{AS}}\right)}$$

Now,

$$\omega = -\frac{d}{dt}\left(\frac{4}{3}\pi r_s^3 \cdot \frac{\rho_{av}}{M_{av}}\right)$$

$$\therefore \cancel{4\pi} D_{AB} \cdot c \cdot \ln\left(\frac{c - c_{A\infty}}{c - c_{AS}}\right) \cdot r_s$$

$$= -\frac{\cancel{4}}{3}\cancel{\pi}\left(\frac{\rho}{M}\right)_{av} \cdot \frac{d(r_s^3)}{dt}$$

$$\therefore \int_0^t \cancel{3} \cdot D_{AB} \cdot c \ln\left(\frac{c - c_{A\infty}}{c - c_{AS}}\right) \cdot \left(\frac{\rho}{M}\right)_{av} dt$$

$$= -\int_{r_{so}}^{r_s} \frac{\cancel{3} r_s^{\cancel{2}}}{\cancel{r_s}} dr_s$$

$$\therefore \boxed{2 D_{AB} \cdot c \cdot \left(\frac{\rho}{M}\right)_{av} \ln\left(\frac{c - c_{A\infty}}{c - c_{AS}}\right) t = r_{so}^2 - r_s^2}$$

(b) Sublimation of cylinderical naphthalen rod.

$$\omega = N_A \cdot 2\pi r \cdot L = \text{const.}$$

$$N_B = 0 \Rightarrow N_A = N_A \cdot \frac{p_A}{P} + J_A$$

$$\therefore N_A = \frac{P}{P - p_A} \cdot J_A$$

$$= -\frac{P \cdot D_{AB}}{P - p_A} \cdot \frac{dc_A}{dr}$$

③

$$\therefore N_A = \frac{-P}{P-P_A} \cdot \frac{D_{AB}}{RT} \cdot \frac{dp_A}{dr}$$

$$\cancel{N_A \frac{RT}{D_{AB} \cdot P}}$$

$$\frac{\omega}{2\pi r L} = \frac{-P \cdot D_{AB}}{RT(P-P_A)} \cdot \frac{dP_A}{dr}$$

$$\therefore \int_{r_c}^{r_c+\delta} \frac{\omega\, dr}{2\pi L r} = -D_{AB} \cdot \frac{P}{RT} \int_{P_{AS}}^{0} \frac{dP_A}{P-P_A}$$

$$\Rightarrow \frac{\omega}{2\pi L} \cdot \ln\left(\frac{r_c+\delta}{r_c}\right) = \frac{+D_{AB} \cdot P}{RT} \ln\left(\frac{P}{P_e - P_{AS}}\right)$$

$$\omega = \frac{-d}{dt}\left(4\pi r_s^2 \left(\frac{\rho}{M}\right)_{av}\right)$$

$$\therefore \omega\, dt = -\left(\frac{\rho}{M}\right)_{av} \cdot 4\pi \cdot 2 r_s \cdot dr_s$$

$$\therefore \int_0^t \frac{D_{AB} \cdot P}{RT} \cdot \frac{2\pi L}{\ln\left(\frac{r_c+\delta}{r_c}\right)} \cdot dt = \int_{r_{so}}^{r_s} -\left(\frac{\rho}{M}\right)_{av} 8\pi r_s\, dr_s$$

$$\therefore \frac{D_{AB} \cdot P}{RT} \cdot \frac{\cancel{2\pi} L}{\ln\left(\frac{r_c+\delta}{r_c}\right)} \cdot t = \cancel{8\pi}^{2} \left(\frac{\rho}{M}\right)_{av} \left(r_{so}^2 - r_s^2\right)$$

4

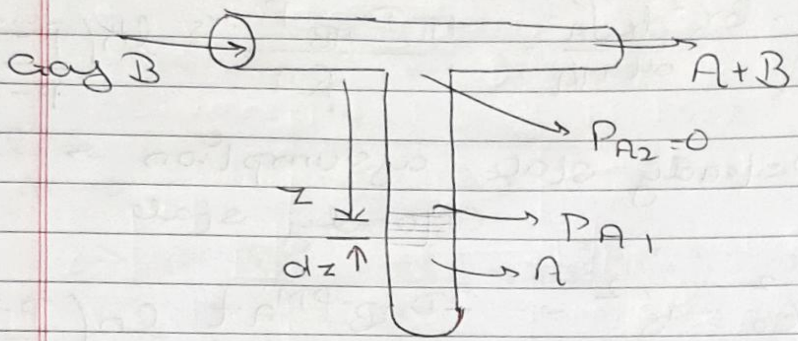
$$\therefore \boxed{r_{c_0}^2 - r_c^0 = \frac{D_{AB} \cdot P \cdot L}{2RT} \cdot \frac{1}{\left(\frac{\rho}{M}\right)_{av} \cdot \ln\left(1+\frac{x}{\delta}\right)}}$$

5

Assignment 3

Q01.2 Stefan's Tube

A → volatile liquid
B → gas which is not soluble in A.



D of A thru non-D B.

~~$N_A = (N_A + N_B)$~~

$$N_A = \frac{D_{AB} \cdot P (P_{A1} - P_{A2})}{RT \cdot z \cdot P_{BM}}$$

$$\underbrace{\frac{a \cdot dz \cdot \rho_A}{M_A}}_{\text{number of moles of A that diffuse out}} = a \cdot N_A \cdot dt = \frac{a \cdot D_{AB} \cdot P(P_{A1} - P_{A2})}{RT \cdot z \cdot P_{BM}} dt$$

limits $z \to z_0 \to z'$
$t \to 0 \to t'$

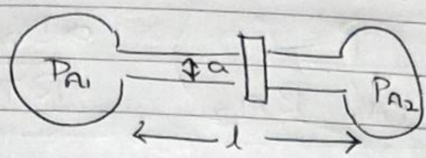
$$\therefore D_{AB} = \frac{RT\, P_{BM}\, \rho_A (z'^2 - z_0^2)}{2P M_A (P_{A1} - P_{A2})\, t'}$$

$N_a \times a$

$$P = P_{A_1}' + P_{B_1}' = P_{A_2}' + P_{B_2}'$$

$\Rightarrow P_{A_1}$

Q. 1.3



$$N_A = (N_A + N_B) \cdot \frac{P_A}{P_B} - D_{AB} \cdot \frac{dc_A}{dx}$$

$$N_A = -N_B$$

$$\Rightarrow N_A = -D_{AB} \frac{dc_A}{dx}$$

$$\Rightarrow N_A \cdot \int_0^l dx = -\frac{D_{AB}}{RT} \int_{P_{A_1}}^{P_{A_2}} dP$$

$$\Rightarrow N_A = \frac{-D_{AB}}{RT \cdot l} \cdot (P_{A_2} - P_{A_1})$$

$$\boxed{N_A = \frac{+D_{AB} (P_{A_1} - P_{A_2})}{RTl}}$$

$$-V_1 \frac{dc_{A_1}}{dt} = a \cdot N_A$$

$a \to$ inner cross-sectional area

$$\Rightarrow \frac{-V_1}{RT} \cdot \frac{dP_{A_1}}{dt} = a \cdot N_A$$

$$\text{IIIly} \quad \frac{V_2}{RT} \cdot \frac{dP_{A_2}}{dt} = a \cdot N_A$$

$$\therefore \frac{d}{dt}(P_{A_1} - P_{A_2}) = a N_A \cdot RT \left(\frac{1}{-V_1} - \frac{1}{V_2} \right)$$

$$\boxed{\frac{d}{dt}(P_{A_1} - P_{A_2}) = -a N_A RT \left(\frac{1}{V_1} + \frac{1}{V_2} \right)}$$

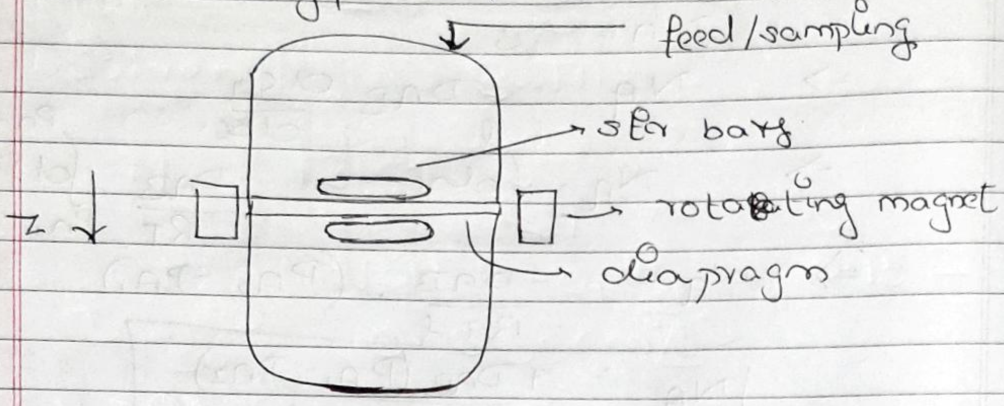
$$= \frac{-a \cdot D_{AB} (P_{A_1} - P_{A_2})}{RTl} \left(\frac{1}{V_1} + \frac{1}{V_2} \right)$$

$$\therefore \int_{P}^{P_{A_1}' - P_{A_2}'} \frac{d(P_{A_1} - P_{A_2})}{(P_{A_1} - P_{A_2})} = -\frac{a \cdot D_{AB}}{RTl} \left(\frac{1}{V_1} + \frac{1}{V_2} \right) \int_0^{t'} dt$$

$$\therefore \boxed{\ln\left(\frac{P}{P'_{A_1}-P'_{A_2}}\right)=\frac{aD_{AB}}{RTl}\cdot\left(\frac{1}{V_1}+\frac{1}{V_2}\right)\cdot t'}$$

↳ solve for D_{AB}

Q. 1.4 Diagphram cell



a = area of cross section

$\varepsilon \rightarrow$ porosity of diapragm

$\tau \rightarrow$ tortousity factor $= \dfrac{\text{length of averge diffusion path}}{\text{thickness of the diapragm}}$

if the solution is dilute, we can neglect the bulk flow term.

$$N_A = (N_A+N_B)x_A - D_{AB}\left(\frac{\rho}{M}\right)_{av}\frac{dx_A}{dz}$$

put $N_B = 0$

$$\rightarrow N_A = -\frac{D_{AB}(\rho/M)_{av}}{1-x_A}\frac{dx_A}{dz}$$

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Date
Page

$$\cancel{N_A = N_A \cdot \frac{C_A}{C} - D_{AB}\frac{dC_A}{dx}}$$

$$\therefore \cancel{N_A = \frac{-D_{AB}}{1-C_A/C}\cdot\frac{dC_A}{dx}}$$

$N_A = (N_A+N_B)\frac{Y_A}{C} - D_{AB}\frac{dC_A}{dx}$ $\quad \frac{C_A}{C} \cong 0$

$$\therefore N_A = -D_{AB}\frac{dC_A}{dx}$$

$$\therefore N_A = \frac{-D_{AB}}{\tau l}\cdot\left(C_{A_1}' - C_{A_2}'\right)$$

conc. at time t

$$-V_1\frac{dC_{A_1}'}{dt} = a\cdot N_A\cdot\varepsilon$$

$$V_2\frac{dC_{A_2}'}{dt} = a\cdot\varepsilon\cdot N_A$$

$$\therefore \frac{d(C_{A_1}' - C_{A_2}')}{dt} = -a\varepsilon N_A\left(\frac{1}{V_1}+\frac{1}{V_2}\right)$$

$$\therefore \int_{C_{A_{10}}-C_{A_{20}}}^{C_{A_{1t}}-C_{A_{2t}}} \frac{d(C_{A_1}' - C_{A_2}')}{C_{A_1}' - C_{A_2}'} = \int_0^t \frac{+a\varepsilon D_{AB}}{\tau l}\left(\frac{1}{V_1}+\frac{1}{V_2}\right)dt$$

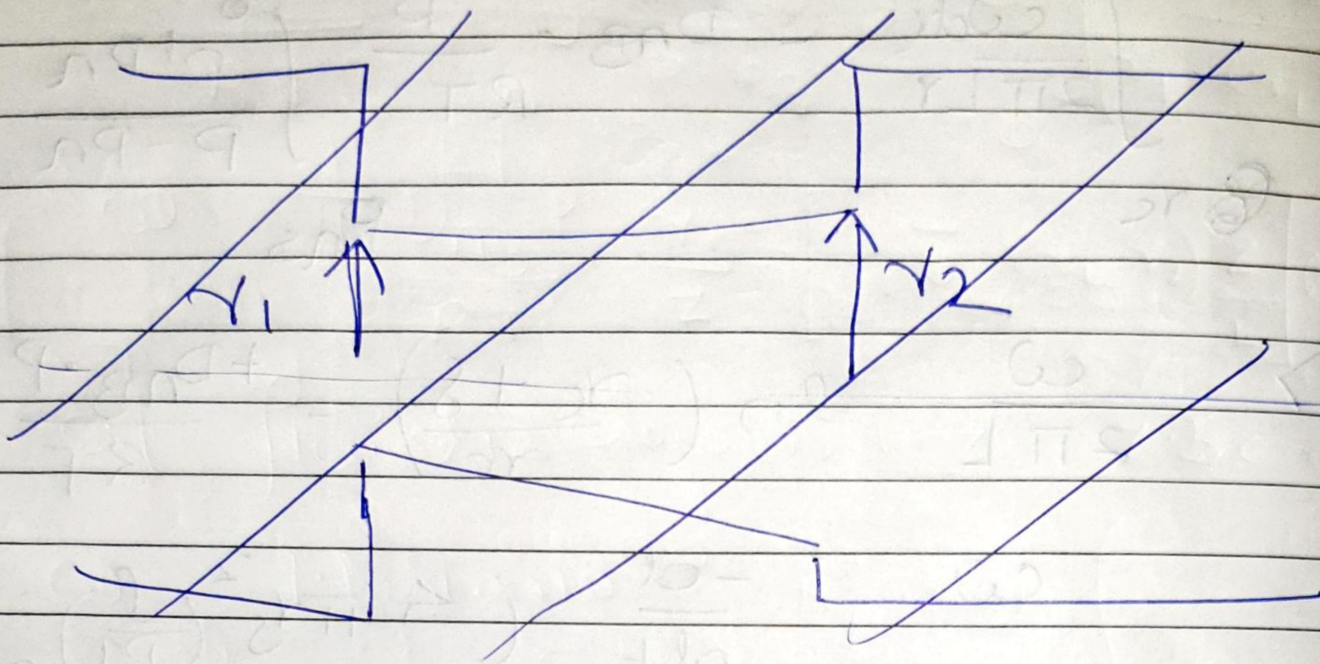
$$\therefore \ln\left(\frac{C_{A_{1t}} - C_{A_{2t}}}{C_{A_{10}} - C_{A_{20}}}\right) = \frac{a\varepsilon D_{AB}}{\tau l}\cdot t\left(\frac{1}{V_1}+\frac{1}{V_2}\right)$$

$$'' = \beta\cdot D_{AB}\cdot t$$

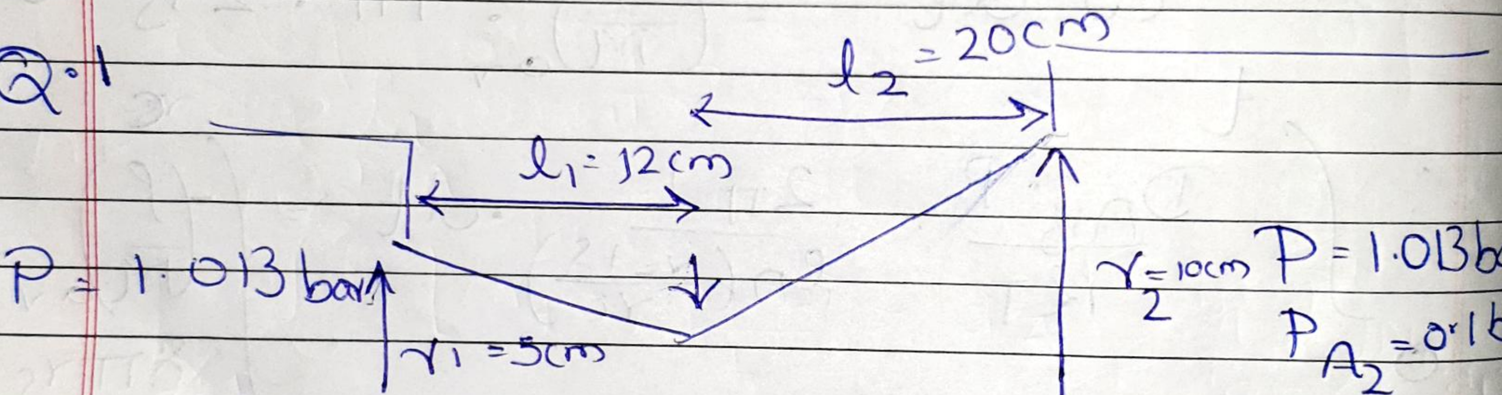
cell constant

solve for D_{AB}

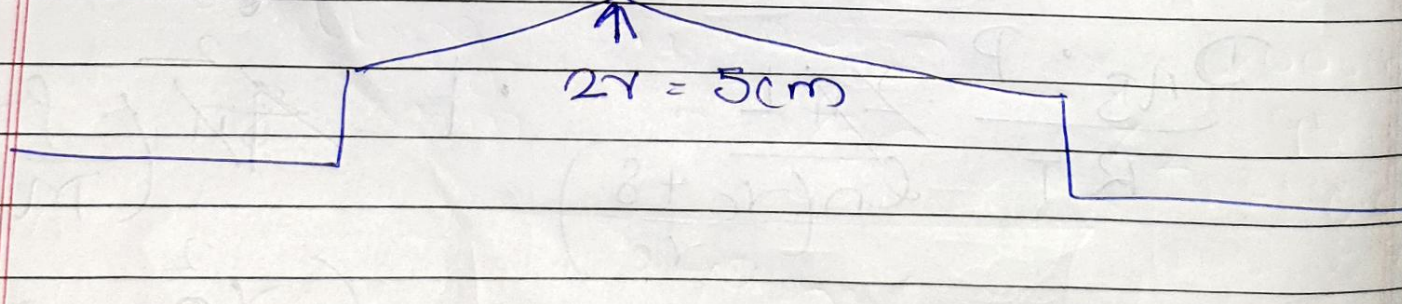
Section 2



Q. 1



$P_{A_1} = 0.3$ bar



(10)

Using the condition of continuity of rate of transport

$$\omega = \frac{\pi D_{AB}}{RT} \cdot \frac{r_1 \cdot r}{l_1} (P_{A_1} - P_{An})$$

$$= \frac{\pi \cdot D_{AB}}{RT} \cdot \frac{r_2 \cdot r}{l_2} (P_{An} - P_{A2})$$

where $P_{An} = \dfrac{r_1 \cdot l_2 \cdot P_{A_1} + r_2 l_1 \cdot P_{A2}}{r_1 l_2 + r_2 l_1}$

$$\therefore \omega = \frac{\pi \cdot D_{AB}}{RT} \cdot \frac{r_1 r_2 \cdot r}{r_1 l_2 + r_2 l_1} (P_{A1} - P_{A2})$$

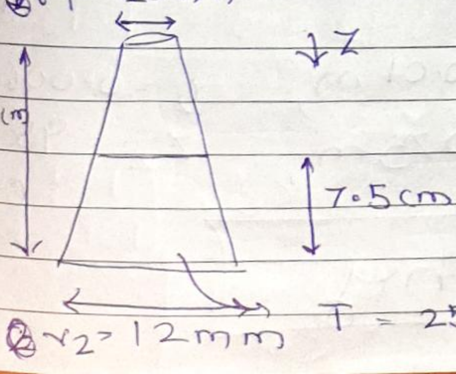
$$\therefore \left[\frac{dP_B}{dz}\right]_n = -\left[\frac{dP_A}{dz}\right]_n = \left[\frac{\omega}{\pi r^2} \cdot \frac{RT}{D_{AB}}\right]_n$$

$$P_{An} = \frac{5\times10^{-2} \times 20\times10^{-2} \times 0.3 + 10\times10^{-2} \times 12\times10^{-2} \times 0.1}{5\times10^{-2} \times 20\times10^{-2} + 10\times10^{-2} \times 12\times10^{-2}}$$

$$= 0.19 \text{ bar}$$

$$\therefore \boxed{\omega = 2.26 \times 10^{-6} \ \frac{mol}{s}}$$

Q.2 Diffusion through a tapered region



(the question says diameters are 12, 20 mm, but the have actually taken these to be the radii)

T = 25°C = 298K

(11)

$$\omega = \pi r^2 N_A$$

$$N_A = \frac{D_{AB} \cdot P}{RT(P-P_A)} \cdot \frac{dP_A}{dz} \quad (1)$$

$$r = r_1 - \frac{r_1 - r_2}{L} z \quad (2)$$

$$\frac{\pi \cdot D_{AB} \cdot P}{RT} \cdot \int_0^{P_A^v} \frac{dP_A}{P-P_A} = \omega \int_0^z \frac{dz}{\left(r_1 - (r_1-r_2)\frac{z}{L}\right)^2}$$

$$\omega = \frac{\pi \cdot D_{AB} \cdot P}{RT} \cdot \frac{r_1}{z} \cdot \left[r_1 - (r_1-r_2)\frac{z}{L}\right] \ln\left(\frac{P}{P-P_A^v}\right)$$

at fluid depth ($z = 7.5$ cm),

$$\boxed{\omega = 1.58 \times 10^{-10} \ \frac{\text{kg mol}}{\text{sec}}}$$

$$\frac{\rho_A}{M_A} \pi r^2 \cdot \frac{dz}{dt} = \omega$$

$$\int_{z_1}^{z_2} z\left[r_1 - \frac{(r_1-r_2)}{L} \cdot z\right] dz = \frac{D_{AB} \cdot P}{RT} \cdot \frac{M_A}{\rho_A} \cdot r_1 \ln\left(\frac{P}{P-P_A^v}\right) \cdot t$$

Putting $r_1 = 0.01$ m, $r_2 = 0.006$ m,

$z_1 = 7.5$ cm $= 0.075$ m, $z_2 = 9.5$ cm (+2 cm) $= 0.095$ m

$$\rightarrow \boxed{t = 79 \text{ hrs}}$$

Q.3 Diffusion from a sphere

$4g \rightarrow 1g$

$T = 45^\circ C, \quad P = 1.013$ bar

$D_{AB} = 0.92 \times 10^{-6} \dfrac{m^2}{s} \qquad \rho = 1140 \dfrac{kg}{m^3}$

$P_{AS} = P_{sub} = 0.8654$ mmHg $= 0.00114$ atm

initial mass $= 4g \Rightarrow r_{s0} = 0.943 \times 10^{-2}$

final mass $= 1g \Rightarrow r_s' = 0.594 \times 10^{-2}$ m

$$r_{s0}^2 - r_s'^2 = \frac{2 D_{AB} \cdot P \cdot M_A \cdot t'}{RT \rho_A} \ln\left(\frac{P - \overset{\rightarrow 0}{P_{A\infty}}}{P - P_{AS}}\right)$$

substituting, we get

$$\boxed{t' = 219.5 \text{ hr}}$$

Q.4
$$\frac{r_{c_1}^2}{2} \ln\left(1 + \frac{\delta}{r_{c_1}}\right) - \frac{r_{c_2}^2}{2} \ln\left(1 + \frac{\delta}{r_{c_2}}\right) + \frac{\delta}{2}\left[(r_{c_1} - r_{c_2}) - \delta \ln\left(\frac{r_{c_1} + \delta}{r_{c_2} + \delta}\right)\right]$$
$$= D_{AB} \cdot \frac{P}{RT} \cdot \frac{M_A}{\rho_A} \ln\left(\frac{P}{P - P_{AS}}\right) \cdot t$$

Initial mass $= 10g, \quad \rho = 1.14 \dfrac{gm}{cm^3}$

$\Rightarrow \boxed{r_{c_1} = 0.3736 \text{ cm}}$

$\frac{}{}$lly final volume $= \dfrac{\pi r_{c_1}^2 L}{2} = 4.386 \text{ cm}^3$

(13)

$\therefore \boxed{r_{c_2} = 0.2642 \text{ cm}}$

given that $\delta = 3\text{mm}$, $D_{AB} = 0.0692 \frac{cm^2}{s}$,
$P = 1 \text{ atm}$, $P_{AS} = 0.00114 \text{atm}$

substituting, we get →

$$\boxed{t = 10.35h}$$

Section 3

Q.3.1] $r_S = 2\text{mm}$ $\quad P_{AS} = 0.555 \text{ mm Hg}$
$T = 318 \text{ K}$ $\quad = 7.33 \times 10^{1} \text{ Pa}$
$D_{AB} = 6.92 \times 10^{-6} \frac{m^2}{sec}$
$P = 1 \text{atm}$.

We have derived the following relation for evaporation of naphthalene ball →

$$\omega = 4\pi r_S \cdot \frac{D_{AB} \cdot P}{RT} \ln\left(\frac{P - P_{A\infty}}{P - P_{AS}}\right)$$

$$= \frac{4\pi \times 2\times10^{-3} \times 6.92 \times 10^{-6} \times 1.013\times10^{5}}{8.314 \times} \times \ln\left(\frac{1.013\times10^{5} - 0}{1.013\times10^{5} - 7.33\times10^{1}}\right)$$

$$\boxed{\omega = 4.86 \times 10^{-12} \frac{\text{kgmol}}{\text{sec}}}$$

Q 3.3]

H_2O

$T = 283K$

$z \uparrow$

$C_{A2} = 0.02 \frac{mol}{m^3}$

4mm

$C_{A1} = 0.1 \frac{mol}{m^3}$

case 1 : water film.

$D_{AB} = 13.26 \times 10^{-5} \times (\mu_B)^{-1.14} \times (V_A)^{-0.589}$

ethanol

H_2O

$\mu_B = 1.4 cP$.

CH_3CH_2OH or C_2H_6O

$V_A = 2V_C + 6V_H + V_O$

$= 2 \times 14.8 + 6 \times 3.7 + 7.4$.

$\therefore \boxed{V_A = 59.2 \frac{cm^3}{(gm\,mol)}}$

$\therefore \boxed{D_{AB} = 8.166 \times 10^{-6} \frac{cm^2}{s}}$

Assume dilute solⁿ $N_A + N_B = 0$.

$\Rightarrow N_A = \frac{D_{AB}}{L} (C_{A1} - C_{A2})$

$= \frac{8.166 \times 10^{-6} \times 10^{-4}}{4 \times 10^{-3}} \times (0.1 - 0.02)$

$\boxed{N_A = 1.63 \times 10^{-8} \frac{mol}{m^2 s}}$

take $C_A = C_1 z + C_2$.

boundary conditions $C_A(z=0) = C_{A1}$

$C_A(z = l) = C_{A2}$

$$\therefore \boxed{C_A = \frac{C_{A2} - C_{A1}}{l} \cdot z + C_{A1}}$$

$$\boxed{C_A = -20z + 0.1}$$

Diffusion in air

$$N_A = -\frac{D_{AB} \cdot C}{l} \ln\left(\frac{C - C_{A2}}{C - C_{A1}}\right)$$

$$D_{AB} = 1.32 \times 10^{-5} \ \frac{m^2}{s}$$

assuming ideal gas $C = \frac{P}{RT} = \frac{10^5}{8.314 \times 283}$

$$\boxed{C = 43.05 \ \frac{mol}{m^3}}$$

$C_{A1} = 0.1 \Rightarrow P_{A1} = 0.1\,RT$

$C_{A2} = 0.02 \Rightarrow P_{A2} = 0.02\,RT$

$$N_A = -\frac{1.32 \times 10^{-5} \times 43.05}{4 \times 10^{-3}} \times \ln\left(\frac{43.05 - 0.1}{43.05 - 0.02}\right)$$

$$\boxed{N_A = 2.64 \times 10^{-4} \ \frac{m^2}{s}}$$

(concentration profile would of course be same)

$\ggg N_{A\,water}$ much faster diffusion.

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Date
Page

Q 3.4] $P = 10^5 \frac{N}{m^2}$

$P_{A0} = 13000 \frac{N}{m^2}$ $P_{Al} = 6500 \frac{N}{m^2}$.

$l = 2mm$

$D_{AB} = 1.87 \times 10^{-5} \frac{m^2}{s}$

$$N_A = \frac{D_{AB} \cdot P}{RTl} \cdot \ln\left(\frac{P - P_{AL}}{P - P_{A0}}\right)$$

$$\boxed{N_A = 5.93 \times 10^{-5} \frac{kg\,mol}{m^2 s}}$$

$$\omega_A = N_A \cdot \underbrace{A}_{=1m^2}$$

$$\boxed{\omega_A = 5.93 \times 10^{-5} \frac{kg \cdot mol}{m^2 \cdot s}}$$