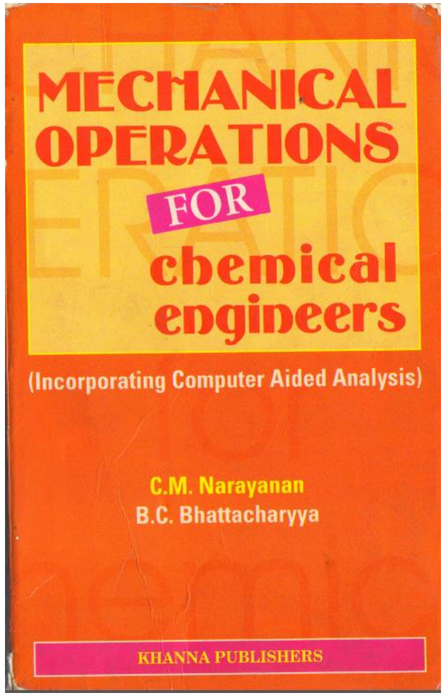
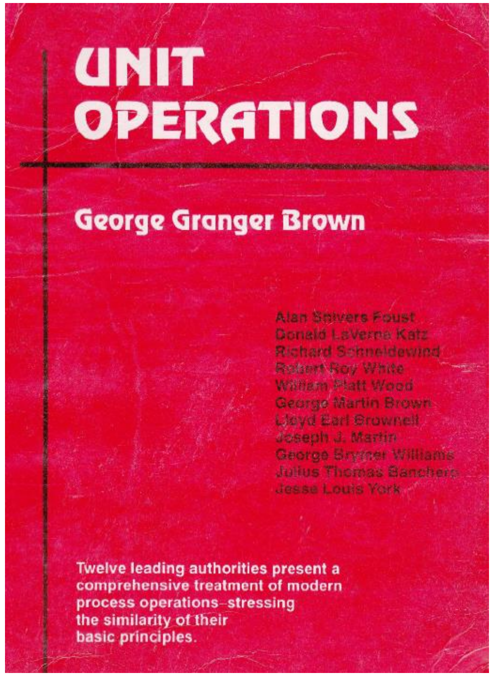
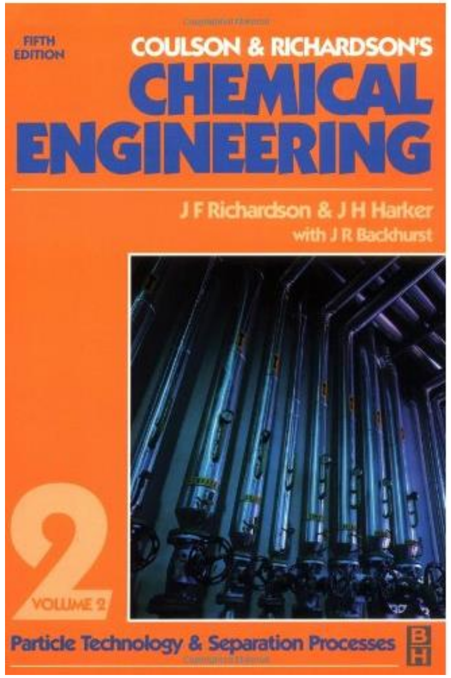
Comminution/ Size Reduction

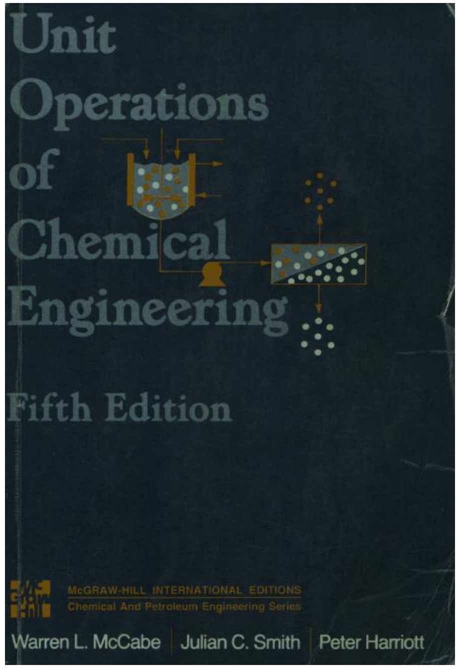
Resource

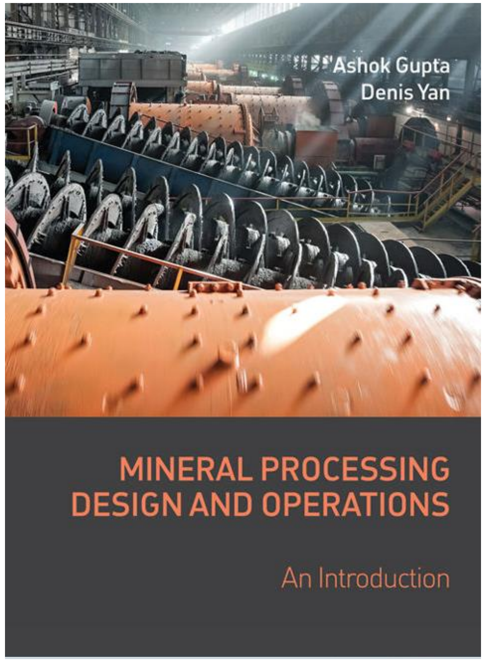
- **Books**







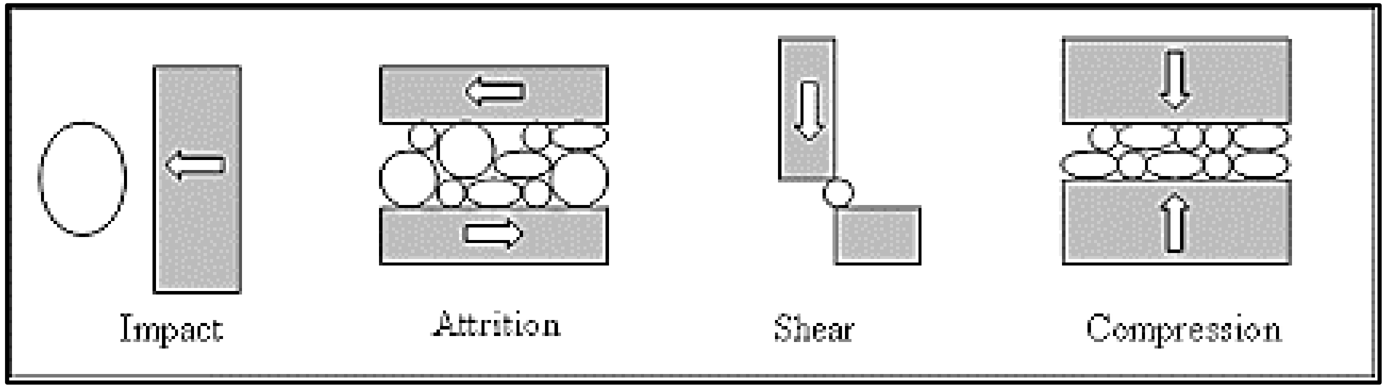




- **Relevant journal papers mentioned in the individual topics**

Introduction

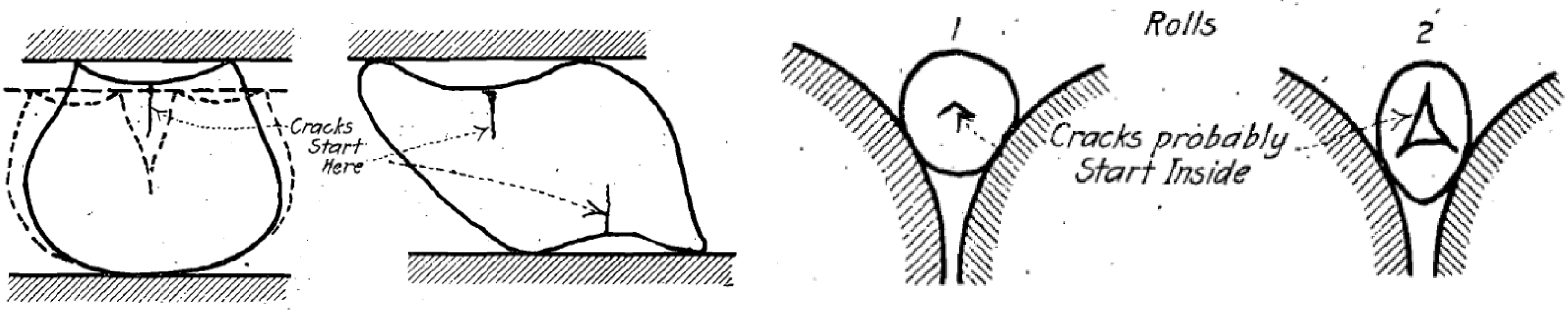
- Comminution is the generic term for size reduction.
- It is an operation that is omnipresent in process industries.
- Solid particles can be reduced in size by compression, shear, impact, attrition(rubbing) or by cutting or tearing.



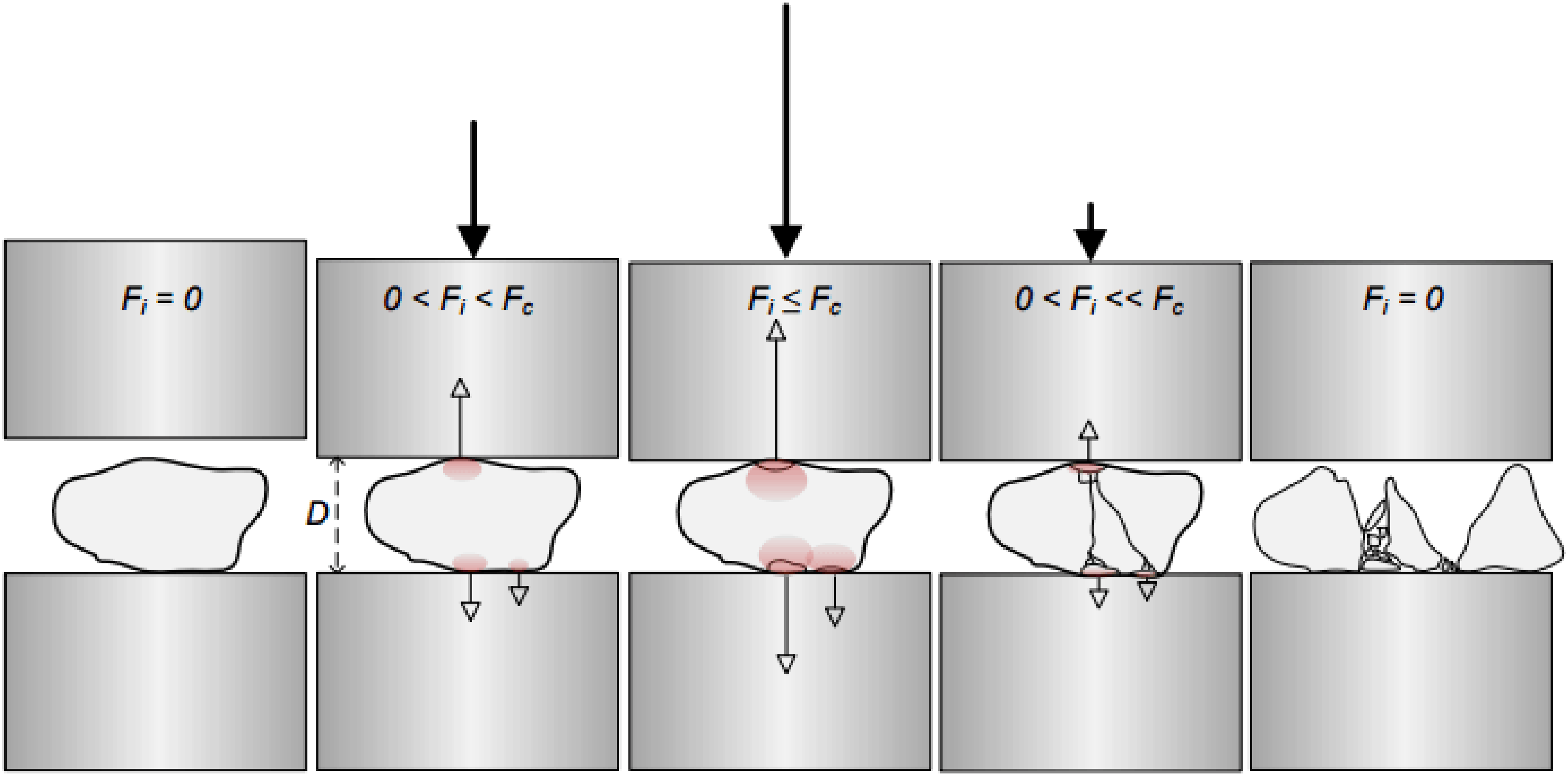
- Compression, impact or shearing loads are used for breaking brittle materials like coal and minerals whereas fibrous materials like wood and asbestos are disintegrated by exerting tearing loads.

The Mechanism of Size Reduction

Stress → Strain → Fracture in lines of Weakness → Released Heat



The force applied may be compression, impact, or shear, and both the magnitude of the force and the time of application affect the extent of grinding achieved.



Unloaded | *Loaded with a compressive force* | *Loaded just before critical breakage* | *Loaded just after critical breakage* | *Particle broken*

Energy for size reduction

- For efficient grinding, the energy applied to the material should exceed, by as small a margin as possible, to the minimum energy needed to rupture the material.
- Excess energy is lost as heat and this loss should be kept as low as practicable.
- Grinding is a very inefficient process and it is important to use energy as efficiently as possible.

- Three theories/laws depend upon the basic assumption that the work done/Energy required to produce a change *dL* in a particle of a typical size dimension *L* is a simple power function of *L*

$$dW = -C\frac{dL}{L^n} \quad \text{......(1)}$$

Where, *dW* is the differential work done, *dL* is the change in a typical dimension, L is the characteristic length

Comminution Laws

- The laws on comminution proposed by different authors help us determine the energy consumed in comminution.

Rittinger's Law (1867):

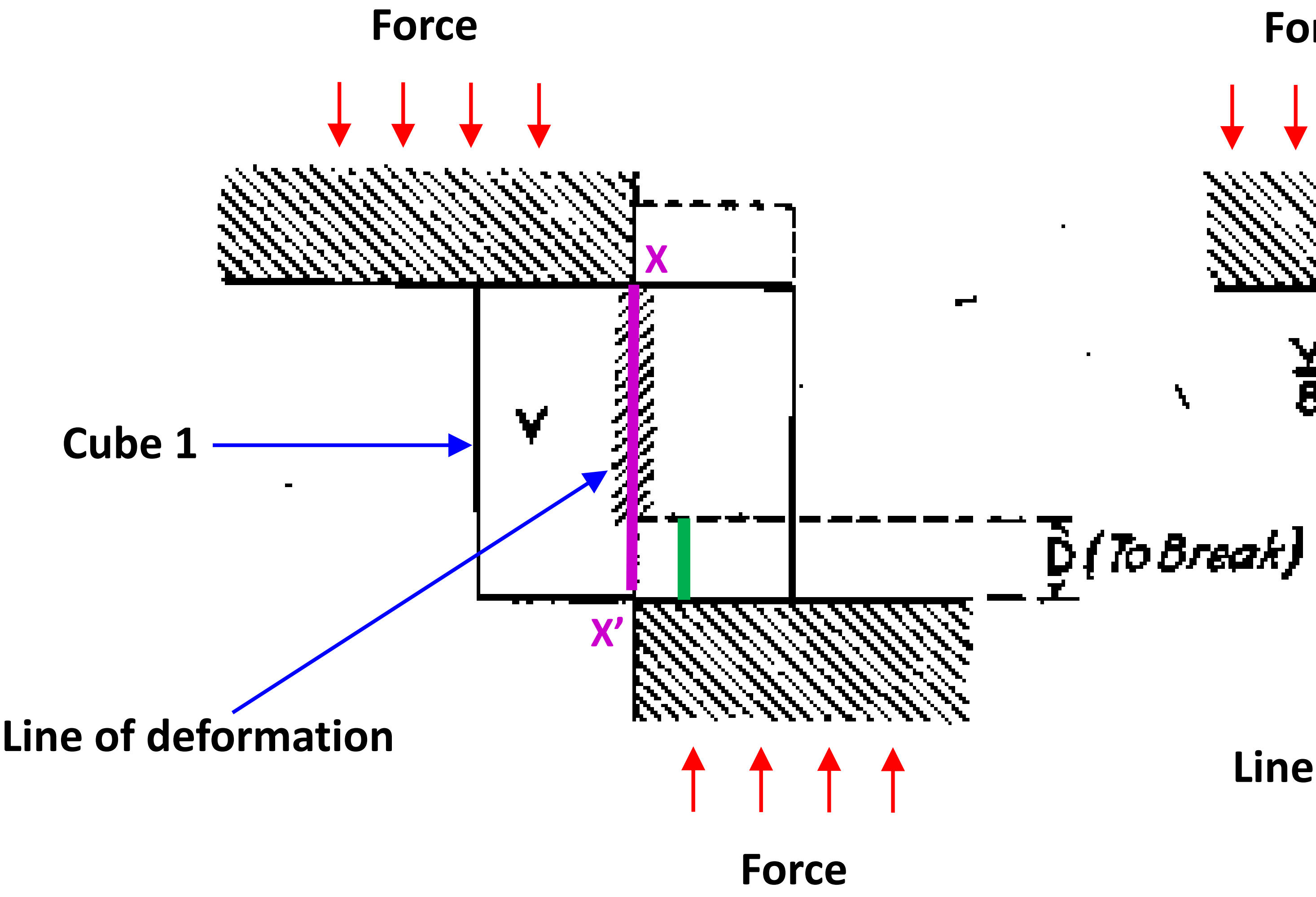
- The law says that "*The work done in crushing is proportional to the surface exposed by the operation*"

$$W = \frac{P}{\dot{m}} = K_R(s_p - s_F) = \frac{6K_R}{\rho_s}\left[\frac{1}{D_{VS,P}} - \frac{1}{D_{VS,F}}\right] = K_R^{'}\left[\frac{1}{D_{VS,P}} - \frac{1}{D_{VS,F}}\right] \quad ......(2)$$

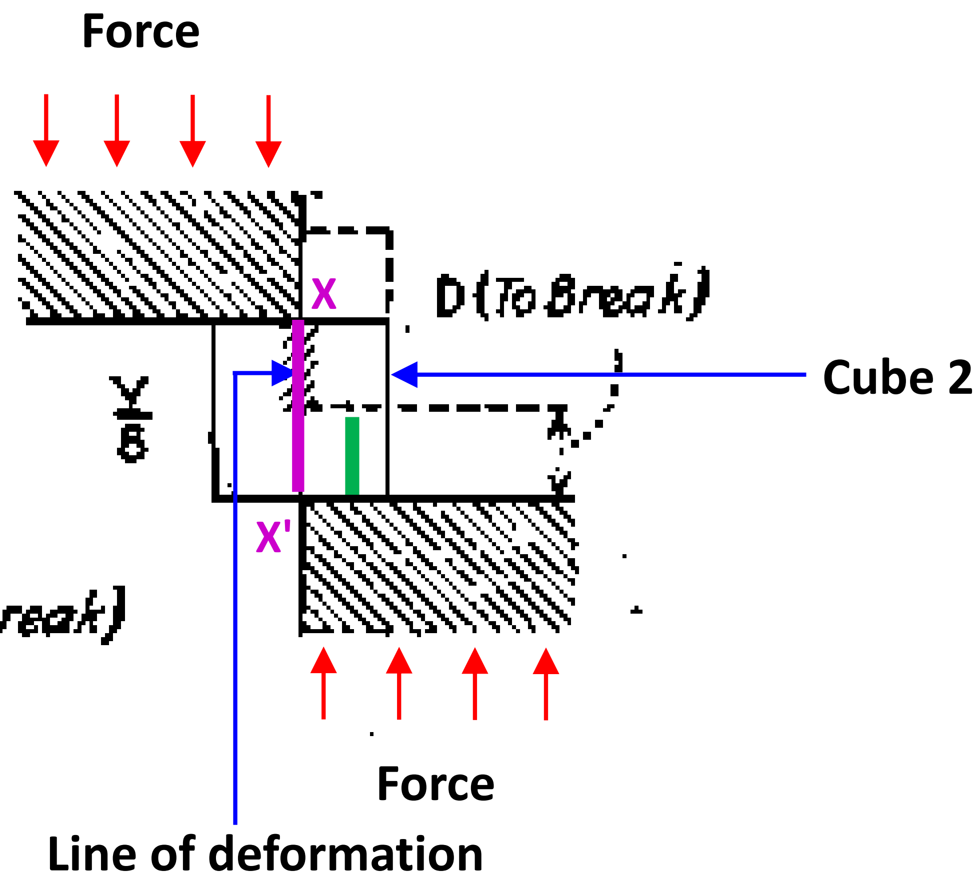
K_R is the Rittinger's constant and $\frac{1}{K_R}$ is the Rittinger's number.

- Rittinger's number is defined as the new surface created per unit mechanical energy absorbed by the metrical being crushed.

- Determined by *drop test* method



Force
X
V
Cube 1
X'
D (To Break)
Line of deformation
Force



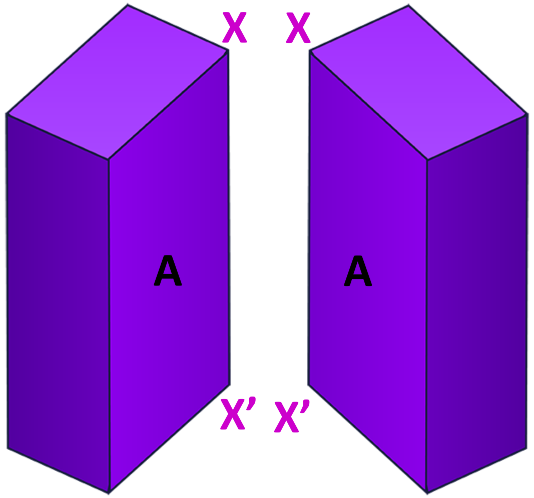
Force
X
D (To Break)
Cube 2
V
8
X'
Force
Line of deformation

After Breakage

Cube 1

Plane of breakage X-X’

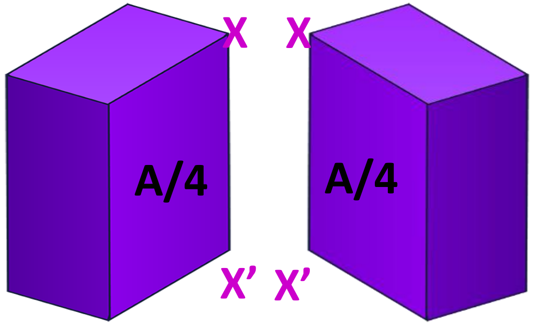
New area created **2A**



Cube 2

Plane of breakage X-X’

New area created 2× A/4



Let area of one side of the cube 1 is A

Cube 1	Cube 2
Area of one section = A	Area of one section = A/4

Let average resistance to shear per sq. in. = F

Cube 1	Cube 2
Distance traveled by the to rapture the cube = D	Distance traveled by the to rapture the cube = D
Energy = F×A×D	Energy = F×A/4×D
Surface Produced = 2 A	Surface Produced = 2 A/4

$$Energy \propto new\ surface\ created$$

Mineral	Rittinger's Number		
	sq in./ft-lb	sq cm/ft-lb	sq cm/kg-cm
Quartz (SiO_2)	37.7	243	17.56
Pyrite (FeS_2)	48.7	314	22.57
Sphalerite (ZnS)	121.0	780	56.2
Calcite ($CaCO_3$)	163.3	1053	75.9
Galena (PbS)	201.5	1300	93.8

Dependency on physical structure

- Marble is a metamorphic rock composed of recrystallized carbonate minerals most commonly ***calcite or dolomite***.
- Chalk is a soft, white, porous, sedimentary carbonate rock, a form of limestone composed of the mineral ***calcite***.
- Chemically there is no difference in them but they differ in their physical structure.

Kick’s Law (1885):

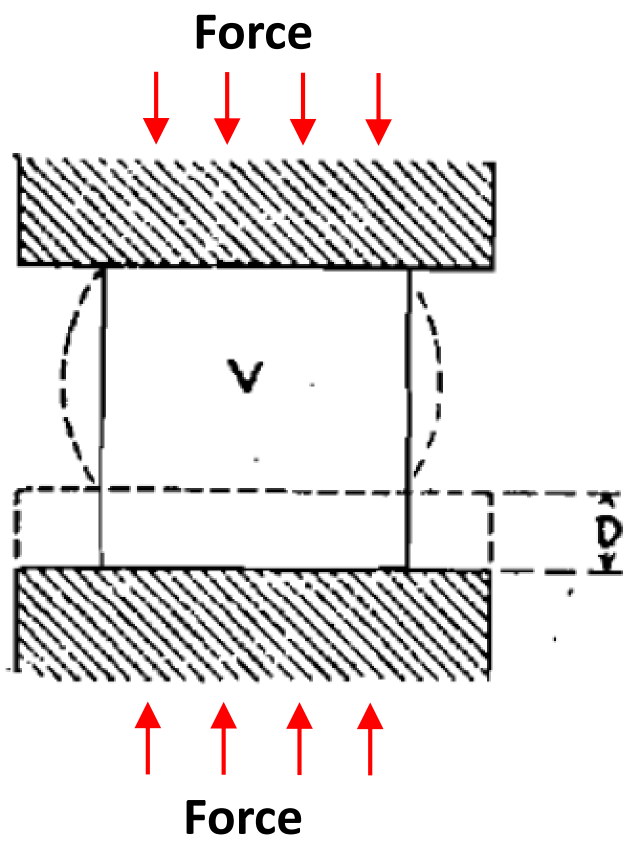
The original form of the law says that “*energy required to produce analogues changes of configuration of geometrically similar bodies varies as the volumes or masses of these bodies*”

- The above statement is not valid when new surfaces are created

The modified Kick’s law that can be applied to crushing states that “*the work required for crushing a given mass of material is constant for the same reduction ratio irrespective of original size*”

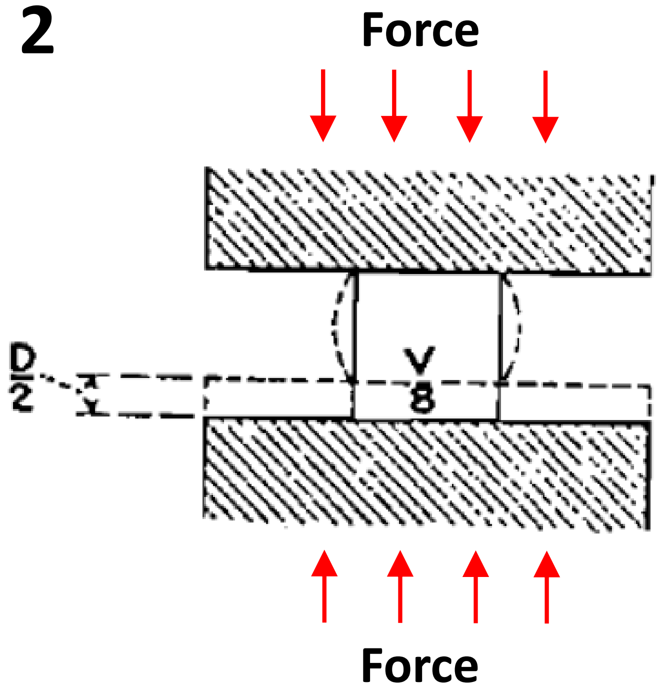
$$W = \frac{P}{\dot{m}} = K_k \ln \frac{D_{VS,\mathrm{F}}}{D_{VS,P}} \qquad \text{……(3)}$$

Cube 1



Area one section = A

Cube 2



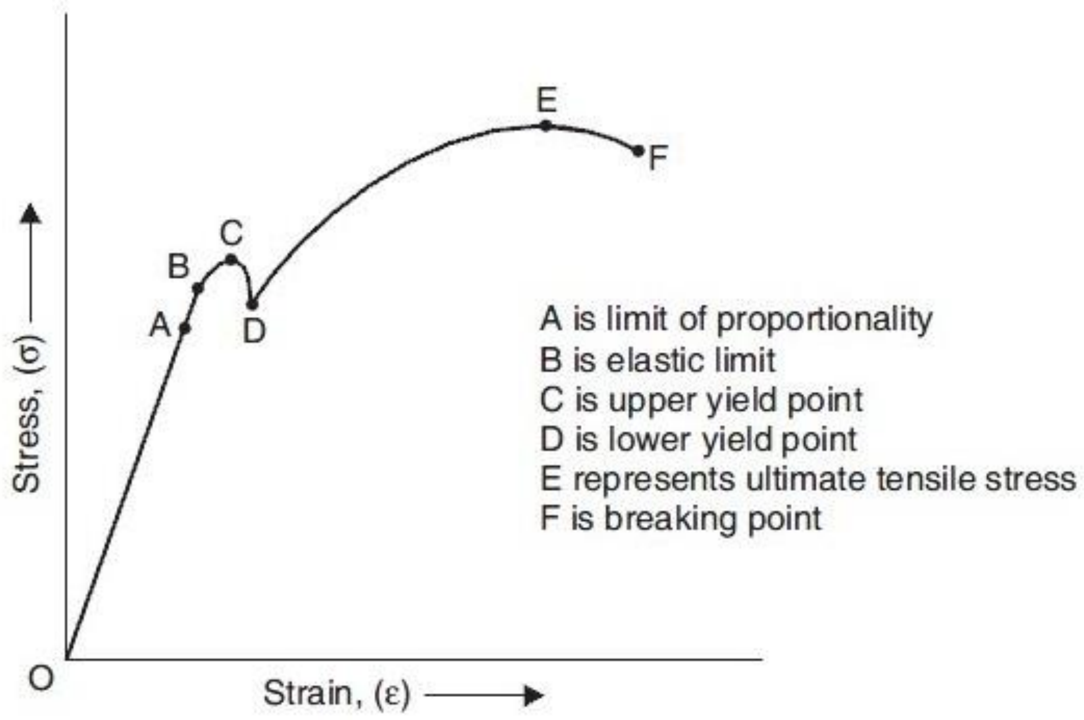
Area one section = A/4

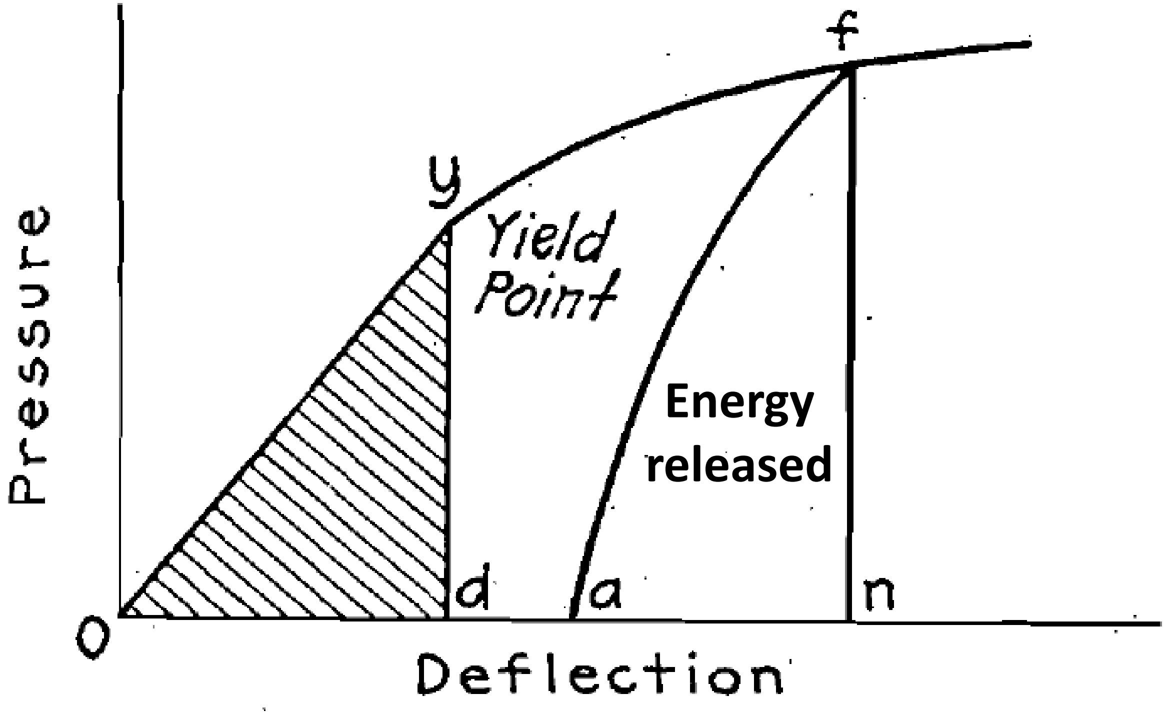
Let the average resistance to shear per sq. in. = F

Energy = FAD

$$Energy \propto Volume$$

Energy = F A/4 D/2 = FAD/8


E
F
C
B
A
D
A is limit of proportionality
B is elastic limit
C is upper yield point
D is lower yield point
E represents ultimate tensile stress
F is breaking point
Stress, (σ)
O
Strain, (ε)


f
y
Yield Point
Energy released
Pressure
O
d
a
n
Deflection

Bond's Law (1952)

- It stats that "*total work useful in breakage that has been applied to a given weight of homogeneous broken material is inversely proportional to the square root of the average size of product particles, directly proportional to the length of crack tips formed and directly proportional to the square root of new surface created*"

Or

- It stats that "*work required to form particle of size **D_p** from very large feed is proportional to the square root of the surface-to-volume ratio of the product*"

$$W = \frac{P}{\dot{m}} = K_b \left[\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right] \qquad \text{......(4)}$$

- Bond constant (K_b) is evaluated by defining **Work index**

- **Work index (Wi)** is defined as the gross energy requirement in kilowatt hours per ton (2000 lb) of feed needed to reduce a very large feed to such a size that 80 percent of the product passes a 100 μm screen.

- If $\boldsymbol{D_p}$ is in millimeters, $\boldsymbol{P}$ in kilowatts, and $\boldsymbol{\dot{m}}$ in tons per hour,

$$K_b = \sqrt{100 \times 10^{-3}} W_i = 0.3162 W_i$$

- If 80 percent of the feed passes a mesh size of $\boldsymbol{D_F}$ millimeters and 80 percent of the product a mesh of $\boldsymbol{D_p}$ millimeters, then

$$W = \frac{P}{\dot{m}} = 0.3162 W_i \left(\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right) \qquad \text{......(5)}$$

- Alternatively we can also write

$$W = \frac{P}{\dot{m}} = W_i \sqrt{\frac{100}{D_P}} \left(1 - \frac{1}{\sqrt{RR}}\right) \qquad \text{......(6)}$$

Where, D_p is in microns and $RR = \frac{D_F}{D_P}$ (Reduction ratio)

Revised Bond's theory

- In 1957, Holmes (2) pointed out that, starting from Kick's law, Bond's third theory should be revised as follows:

$$W = \frac{P}{\dot{m}} = W_i \left(\frac{100}{D_P}\right)^r \left(1 - \frac{1}{(RR)^r}\right) \qquad \text{......(7)}$$

r is called a deviation from Kick's law

He demonstrated that r varies from 0.25 to 0.73 for several materials, and that such a concept leads to a more consistent work index, W_i, than that calculated by Bond.

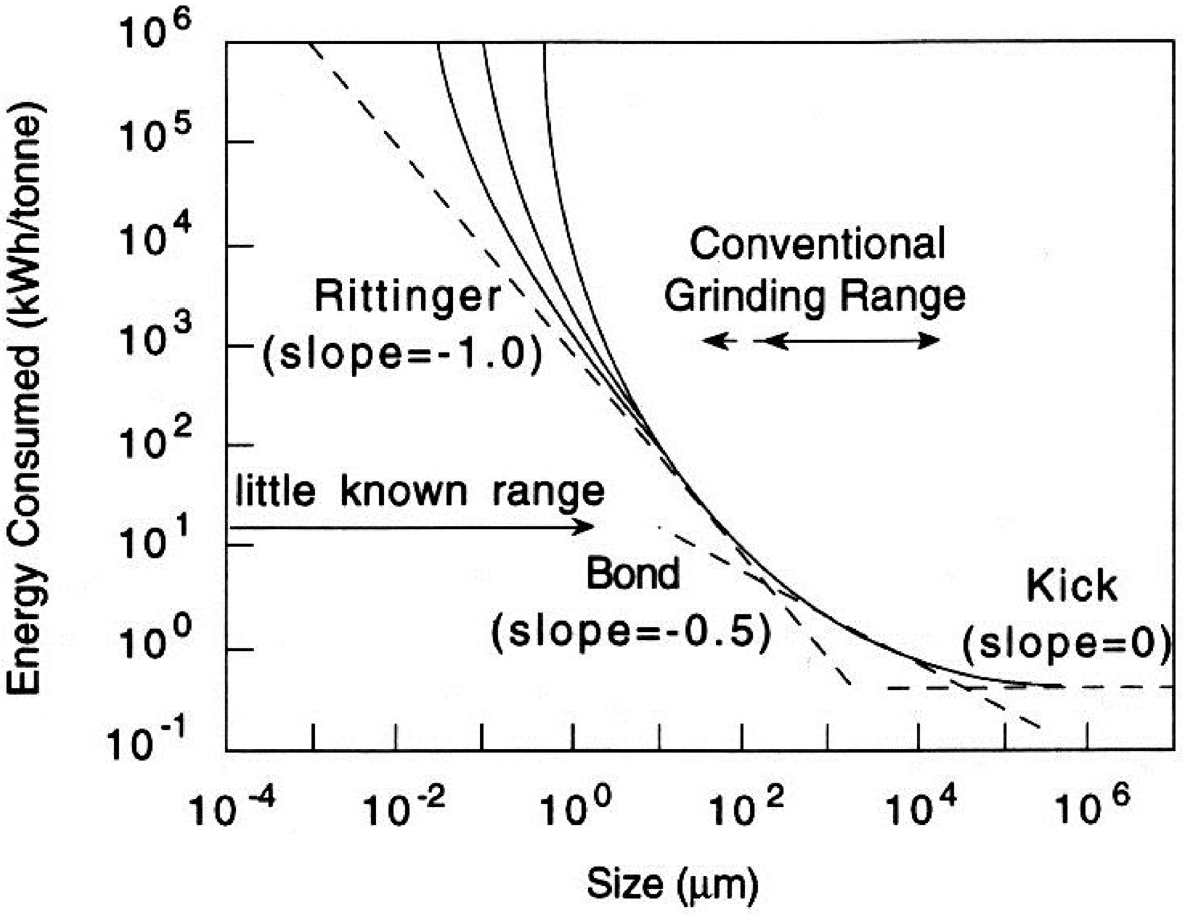
General Form

$$dW = -C\frac{dL}{L^n}$$

n=1 Kick’s Law

n=2 Rittinger’s Law

n=1.5 Bond’s Law



A graph comparing the energy requirement for crushing the particles v/s the particle diameter for the three laws, i.e, Rittinger's law, Kick's law and Bond's law.

Problem 1

Sugar is ground from crystals of which it is acceptable that 80% pass a 500 μm sieve (Standard Sieve No.35), down to a size in which it is acceptable that 80% passes a 88 μm (Standard Sieve No. 170) sieve, and a motor working at 5 HP (working at 90% of its full power) is used for the required throughput. If the requirements are changed such that the grinding is only down to 80% through a 125 μm (No.120) sieve but the throughput is to be increased by 50% would the existing motor have sufficient power to operate the grinder? Assume Bond's law to be valid.

Ans: Yes the existing motor have sufficient power to operate the grinder (5.43 HP would be the energy requirement)

Problem 2

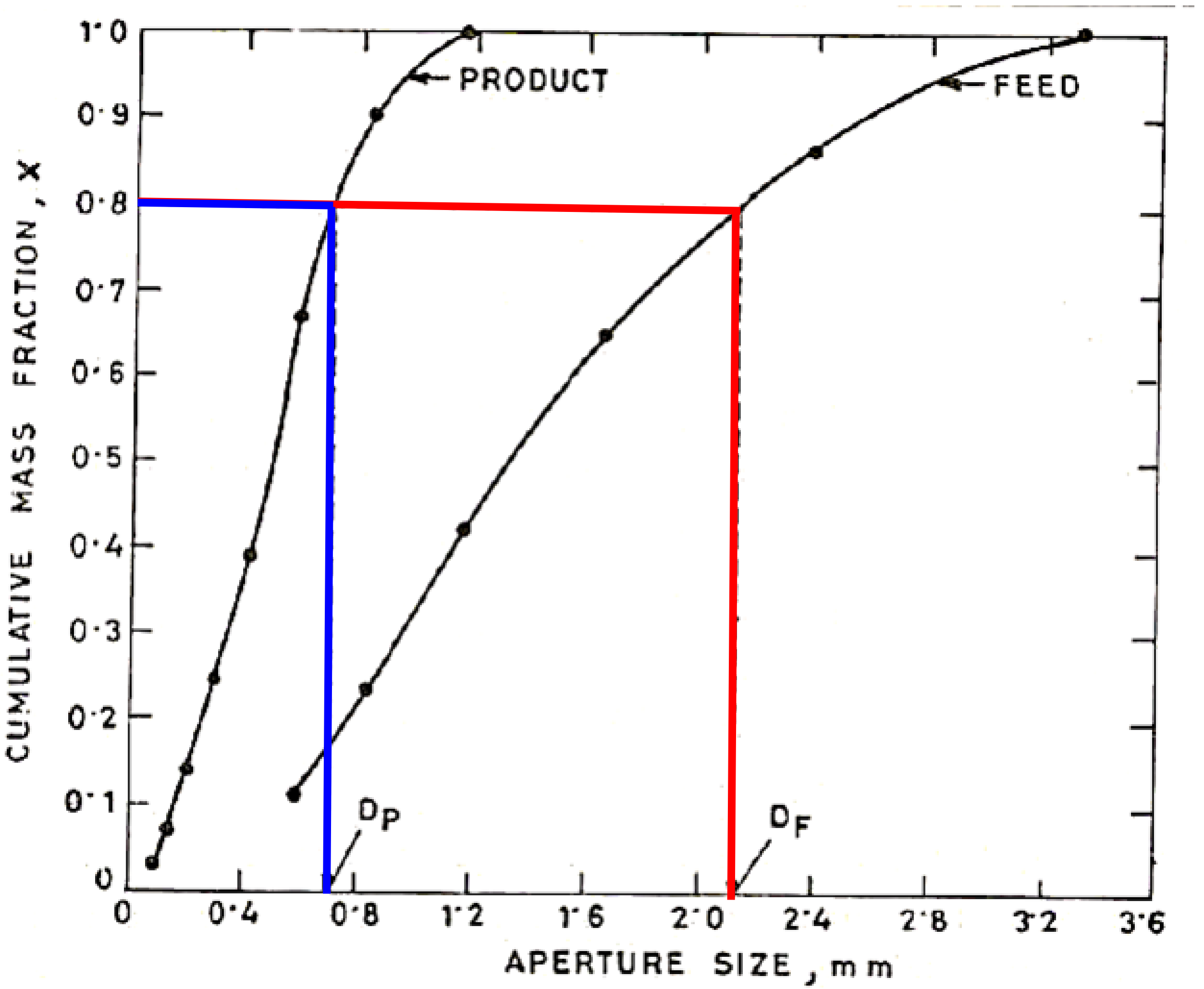
. *A grinder is to be used (which is 8% efficient) to handle 10 tonnes per hour of a siliceous ore (specific gravity = 2.65). The feed and product analysis are given below :*

The grinder costs Rs. 40,000. It operates on a 24 hour basis for 300 days per year and the maintenance, overhead and replacement costs amount to 50% of the power cost. Electricity costs 70p. per kWh. If the machine depreciates on a straight line basis for 10 years, estimate the annual processing cost of the ore if the work index of the ore is 13.57 kWh/tonne.

Screen Size, mm	Feed Mass Fraction	Product Mass Fraction
– 3.327 + 2.362	0.143	0.0
– 2.362 + 1.651	0.211	0.0
– 1.651 + 1.168	0.230	0.0
– 1.168 + 0.833	0.186	0.098
– 0.833 + 0.589	0.120	0.234
– 0.589 + 0.417	0.076	0.277
– 0.417 + 0.295	0.03·	0.149
– 0.295 + 0.208	0.0	0.101
– 0.208 + 0.147	0.0	0.068
– 0.147 + 0.104	0.0	0.044
– 0.104	0.0	0.029

Ans: Annual processing cost 20.656 lakh.

Aperture size, mm	Cumulative Mass Fraction	
	Feed (X_F)	Product (X_P)
3.327	1.0	1.0
2.362	0.857	1.0
1.651	0.646	1.0
1.168	0.416	1.0
0.833	0.230	0.902
0.589	0.110	0.668
0.417	0.034	0.391
0.295	0.0	0.242
0.208	0.0	0.141
0.147	0.0	0.073
0.104	0.0	0.029



Several probabilities involved

1. whether or not the particle hits an objective.

2. whether collision is strong enough to produce a stress larger than the breaking stress of the material.

3. whether crack tips within a solid will propagate and eventually break the solid.

The general equation should include these three different probabilities, along

with $\dfrac{1}{L^n}$

Since specific surface is inversely proportional to particle size, equation 1 can be rewritten as follows:

$$\frac{dS}{dE} = C_1 L^N \qquad \text{......(8)}$$

Where, $N = n\text{-}2$

E is the energy required to crush a unit weight of particles of the size L.

The dependence of C on particle size and crushing mechanism must be further clarified to include the uncertainties.

The equation proposed

$$\frac{dS}{dE} = K(P_c)(P_\sigma)(P_a)L^N \qquad ......(9)$$

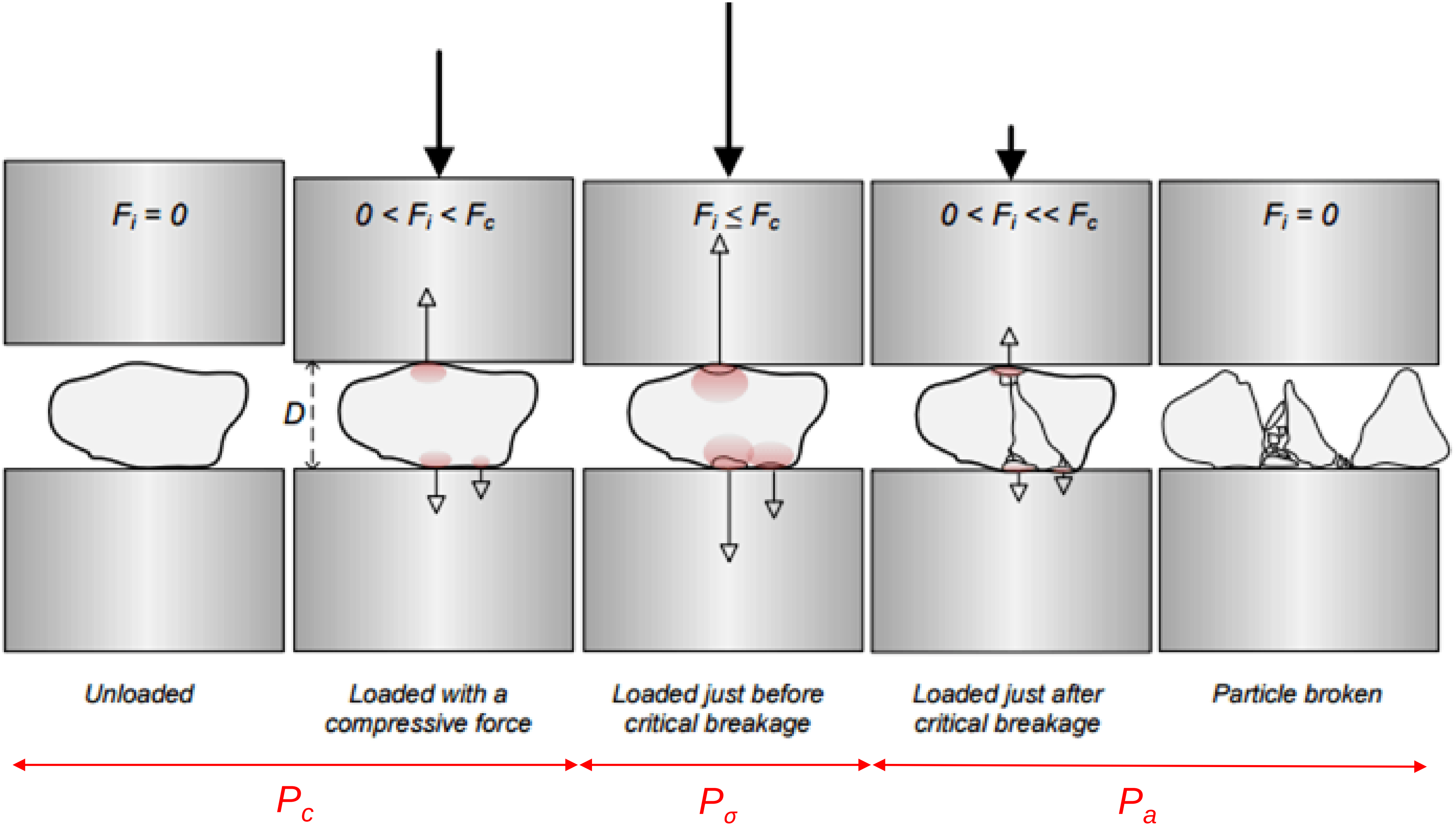
Where,

P_c is the probability of collision,

P_σ the probability that the material’s breaking stress will be exceeded,

P_a the probability of crack tip propagation,

and K is a constant.

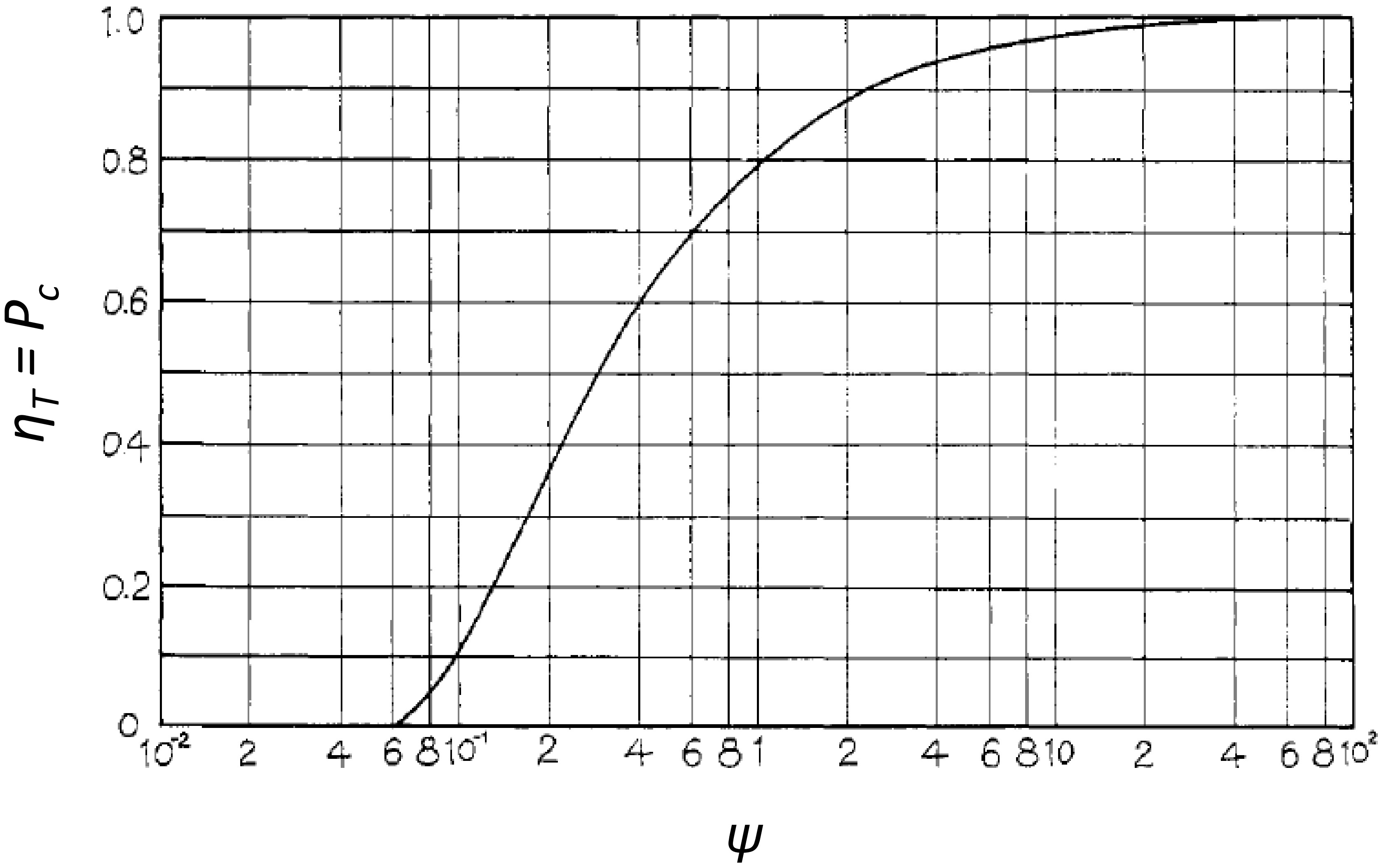

$F_i = 0$
$0 < F_i < F_c$
$F_i \leq F_c$
$0 < F_i << F_c$
$F_i = 0$
D
Unloaded
Loaded with a compressive force
Loaded just before critical breakage
Loaded just after critical breakage
Particle broken
P_c
P_σ
P_a

Probability of collision

The theory governing the striking of an objective by a particle is described by the interrelationship between target efficiency, η_T , and separation number, ψ .

$$\psi = \frac{L^2 v \rho_p}{18 \mu D}$$

Where, *D* is the diameter of the objective and *v* is the particle impact velocity and L is the characteristic length of the particle and μ is the viscosity of the surrounding fluid (mostly air)


$\eta_T = P_c$
1.0
0.8
0.6
0.4
0.2
0
10^{-2} 2 4 6 8 10^{-1} 2 4 6 8 1 2 4 6 8 10 2 4 6 8 10^{2}
ψ

Probability of Exceeding Breaking Stress

According to Hertz , the maximum stress produced in a material at impact is expressed by:

$$\sigma_{\max} = a\left(\frac{m_1 m_2}{m_1 + m_2}\right)^{\frac{1}{5}} \times v^{\frac{2}{5}}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{\frac{3}{5}} \times \left(\frac{1-\beta_1}{\varepsilon_1} + \frac{1-\beta_2}{\varepsilon_2}\right)^{-\frac{4}{5}}$$

where m_1 and m_2 are the masses of the two bodies

r_1 and r_2 the radii of curvature of the bodies at an impact point

ε_1 and ε_2 are the moduli of elasticity; β_1 and β_2 are Poisson's ratio of the bodies.

v is the relative velocity of the particle

Rumpf, H., Chem. Ing. Tech. 31, 323 (1959).

Probability of Exceeding Breaking Stress

The probability of exceeding the breaking stress of material is proposed as

$$P_\sigma = \left(1 - \frac{\sigma_a}{\sigma_{\max}}\right)^r$$

where *r* is an empirical constant that can be determined from experimental data.

When $\sigma_a = \sigma_{max}$, $P_\sigma = 0$, and when σ_{max} is much greater than σ_a , then P_σ approaches unity, which corresponds to an actual crushing phenomenon.

Probability of crack tip propagation

$$P_a \propto e^{-kP}$$

P is the pressure within the crusher and k is a constant which depends on the crushing mechanism

Final Correlation

$$\Delta S = \Delta E \times K \times P_c \times \left(1 - \frac{\sigma_a}{\sigma_{\max}}\right)^m \times e^{-kP} \times L^N \qquad \text{......(10)}$$

Crushing Efficiency

- The ratio of surface energy created by crushing to the energy absorbed by the solid is the ***crushing efficiency*** and is denoted by η_c

- If $\boldsymbol{e_s}$ is the surface energy per unit area, s_p and s_F are the specific surface areas of product and feed, respectively, then the the **energy absorbed by a unit mass of the material (W_n)**

$$W_n = \frac{e_s(s_p - s_F)}{\eta_c} \qquad \text{...........(11)}$$

- The surface energy created by the fracture is small in comparison with the total mechanical energy stored in the material at the time of rupture.

Ref. McCabe, Smith

- Rest are converted into heat during the time of rapture.
- Crushing efficiency range between 0.1 to 2 %.
- The energy absorbed by the solid $\boldsymbol{W_n}$ is less than that fed to the machine.
- The ratio of energy absorbed to the energy input is η_m, the mechanical efficiency.
- The energy input to the machine per unit mass (***W)*** is

$$W = \frac{W_n}{\eta_m} = \frac{e_s(s_p - s_F)}{\eta_m \eta_c} \qquad \text{...........(12)}$$

- So the power required by the machine is

$$P = W\dot{m} = \frac{\dot{m}e_s(s_p - s_F)}{\eta_m \eta_c}$$

Where, $\dot{m}$ is the mass flow rate of the feed.

- If the sauter diameter of the feed and product is $D_{VS,F}$ and $D_{VS,P}$ respectively then

$$P = \frac{6\dot{m}e_s}{\eta_m \eta_c \rho_s}\left(\frac{1}{D_{VS,P}} - \frac{1}{D_{VS,F}}\right) \qquad \text{..........(13)}$$

$$D_{VS} = \frac{1}{\sum \frac{n_i x_i}{d_{avgi}}}$$

Problem 3

A material is crushed in a Blake jaw crusher such that the average size of particle is reduced from 50 mm to 10 mm with the consumption of energy of 13.0 kW/(kg/s). What would be the consumption of energy needed to crush the same material of average size 75 mm to an average size of 25 mm :

a) assuming Rittinger's law applies?

b) assuming Kick's law applies?

c) assuming Bonds's law applies?

Which of these results would be regarded as being more reliable and why ?

Ans: a) 4.33 kJ/kg; b) 8.88 kJ/kg; c) 6.28 kJ/kg

Which result is reliable discussed in the class.

Different stages of size reduction

- Coarse size reduction (Feed size 50 to 250mm or more)
  - Equipment used are **Crushers** (Ex: Jaw crusher, Gyraroty crusher, Crushing rolls *etc.*)

- Intermediate size reduction (Feed size in the range of 25 to 75mm)
  - Equipment used are **Grinders** (Ex: Hammer mills, Rolling- compression mills, Attrition mills, Tumbling mills *etc.*)

- Fine size reduction (Feed size in the range of 5 to 15mm)
  - Equipment used are **Ultrafine grinders** (Ex: Fluid energy mills, Agitated mills *etc.*)

Size-Reduction Equipment

Size reduction equipment is divided into crushers, grinders, ultrafine grinders, and cutting machines.

- **Crushers** (coarse size reduction) do the heavy work of breaking large pieces of solid materials into small lumps.
    - **Primary Crusher** breaks into 50-250mm lumps.
    - **Secondary Crusher** breaks into ~ 6mm particles.

- **Grinders** (Intermediate size reduction) reduce crushed feed to powder.

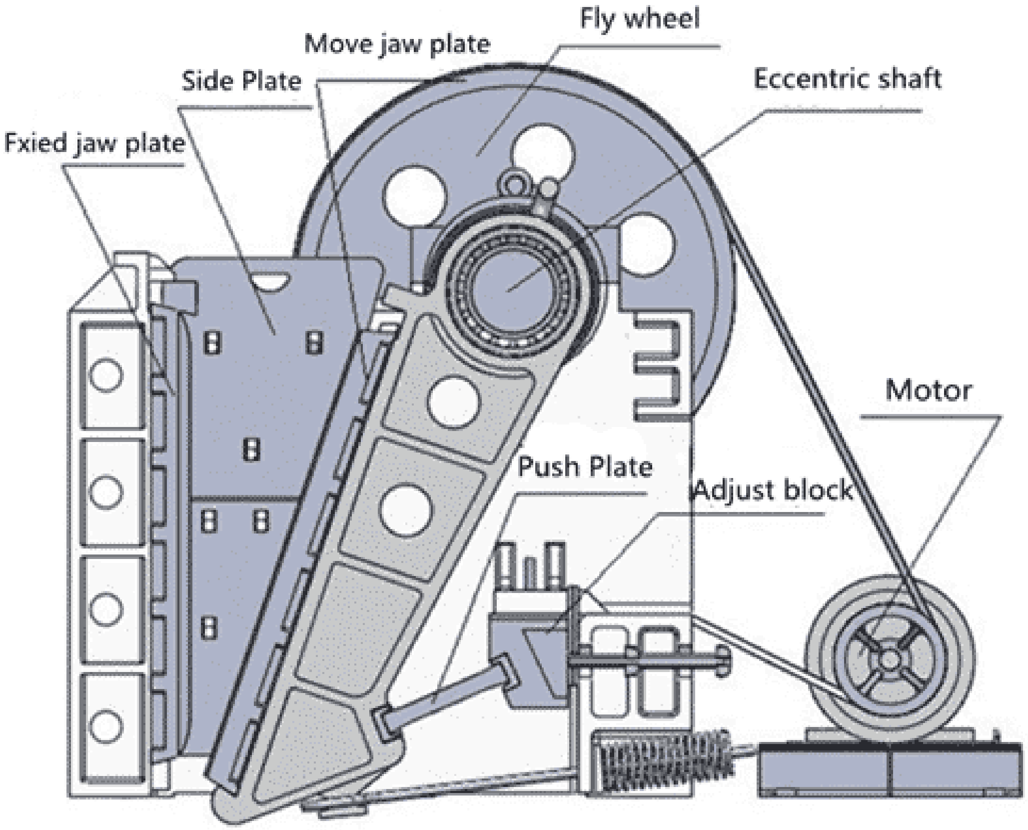
- **Ultrafine grinder** the product size is typically 1-50µm.

- **Cutters** give particles of definite shape and size, 2-10 mm in length.

SIZE REDUCTION EQUIPMENTS

- **Crushers (coarse and fine)**
    - Jaw crushers
    - Gyratory crushers
    - Crushing rolls
- **Grinders (intermediate and fine)**
    - Hammer mills; impactors
    - Rolling-compression mills
    - Attrition mills
    - Tumbling mills
- **Ultrafine grinders**
    - hammer mills with internal classification
    - Fluid-energy mills
    - Agitated mills
- **Cutting machines**
    - Knife cutters; dicers; slitters

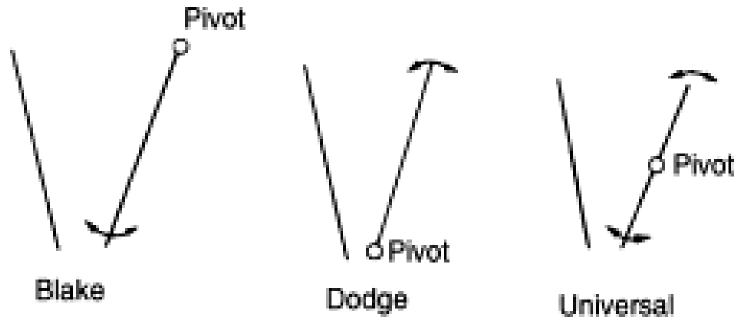
Jaw Crusher


Fly wheel
Move jaw plate
Side Plate
Eccentric shaft
Fxied jaw plate
Motor
Push Plate
Adjust block

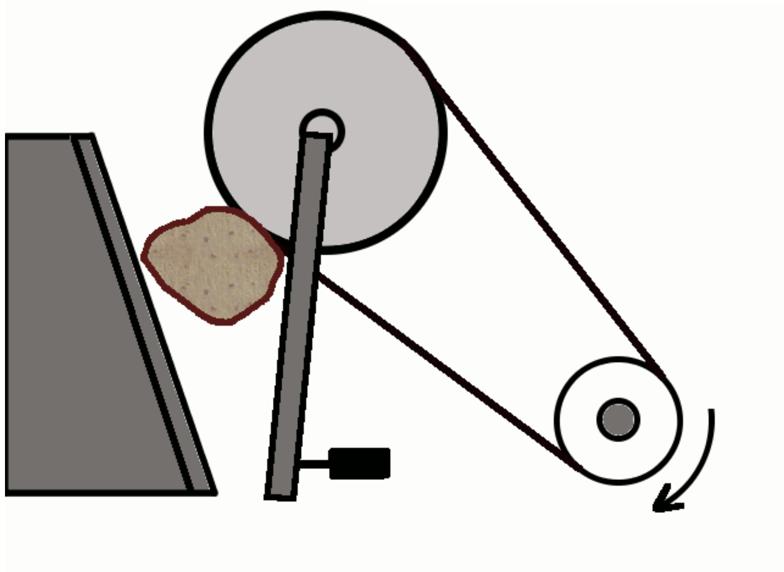
In a **jaw crusher** feed is admitted between two jaws, set to form a V open at the top. One jaw, the fixed, or anvil, jaw, is nearly vertical and does not move; other, the swinging jaw, reciprocates in a horizontal plane. It makes an angle of 20^{o} to 30^{o} with the anvil jaw.It is driven by an eccentric so that it applies great compressive force to lumps caught between the jaws. The jaws open and close 250 to 400 times per minute.

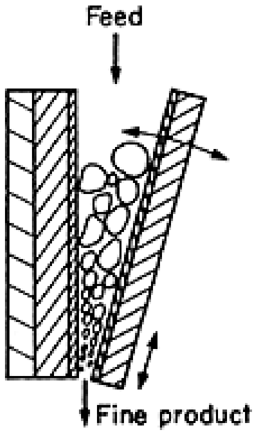
Jaw crushers are classified on the basis of the position of the pivoting of the swing jaw

1. **Blake crusher**-the swing jaw is fixed at the upper position
2. **Dodge crusher**-the swing jaw is fixed at the lower position
3. **Universal crusher**-the swing jaw is fixed at an intermediate position

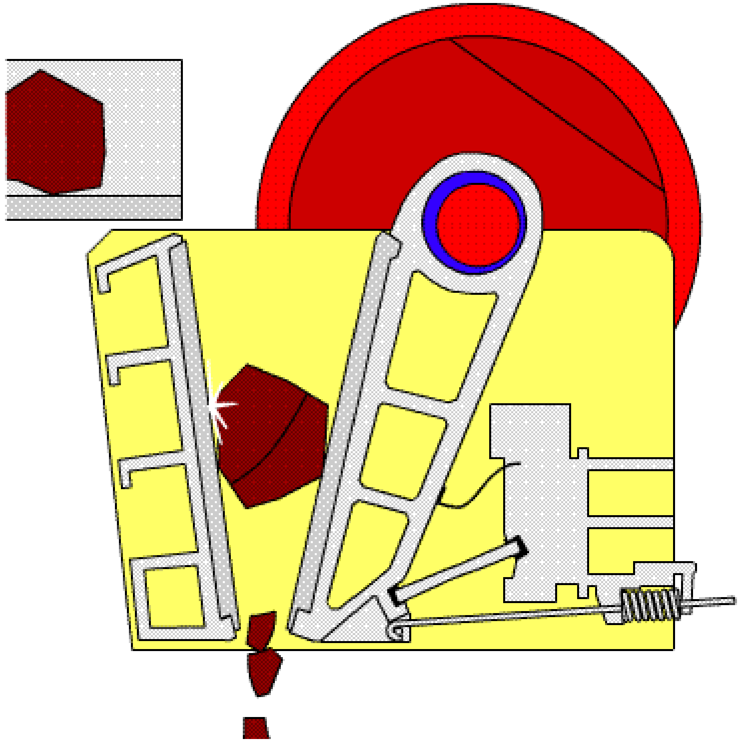


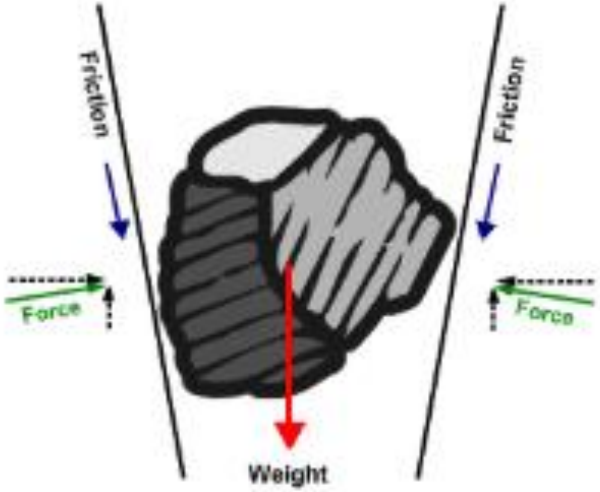
The most common type of jaw crusher is the ***Blake crusher***.
In this machine an eccentric drives a pitman connected to two toggle plates, one of which is pinned to the frame and the other to the swinging jaw.





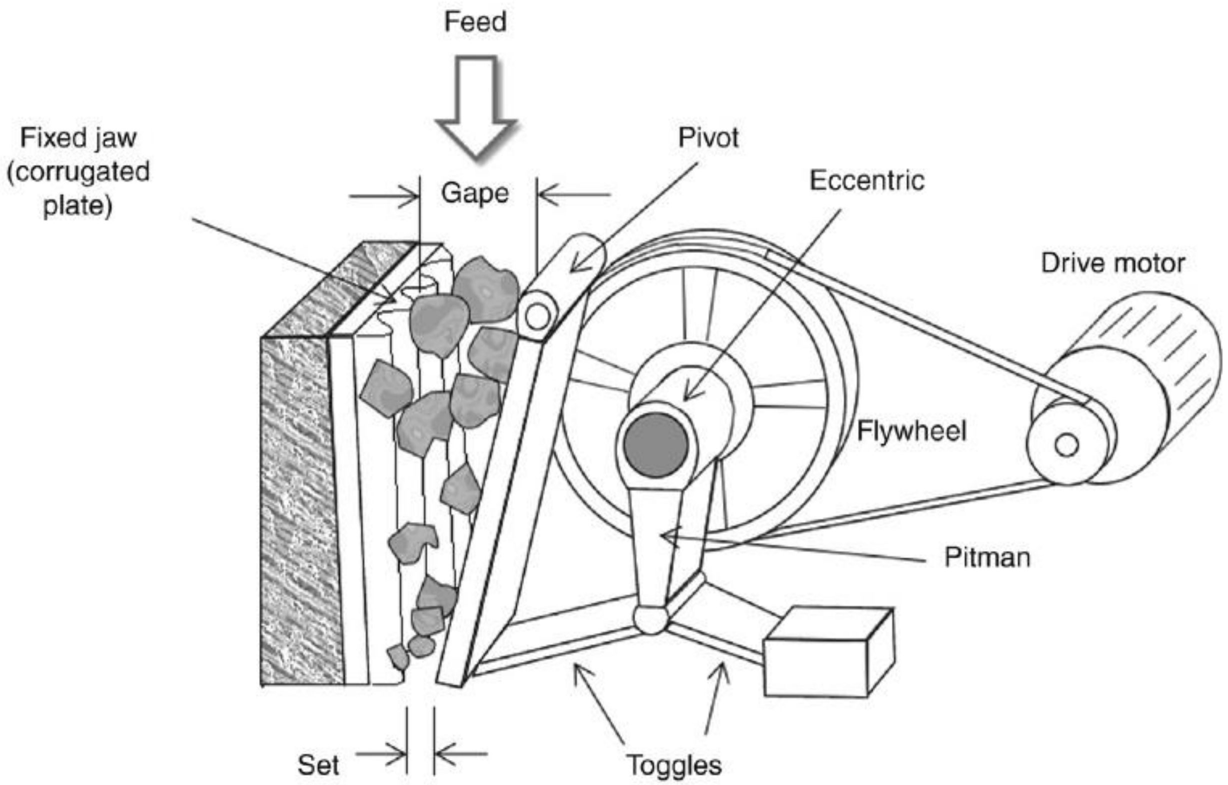
Single-toggle Jaw Crusher



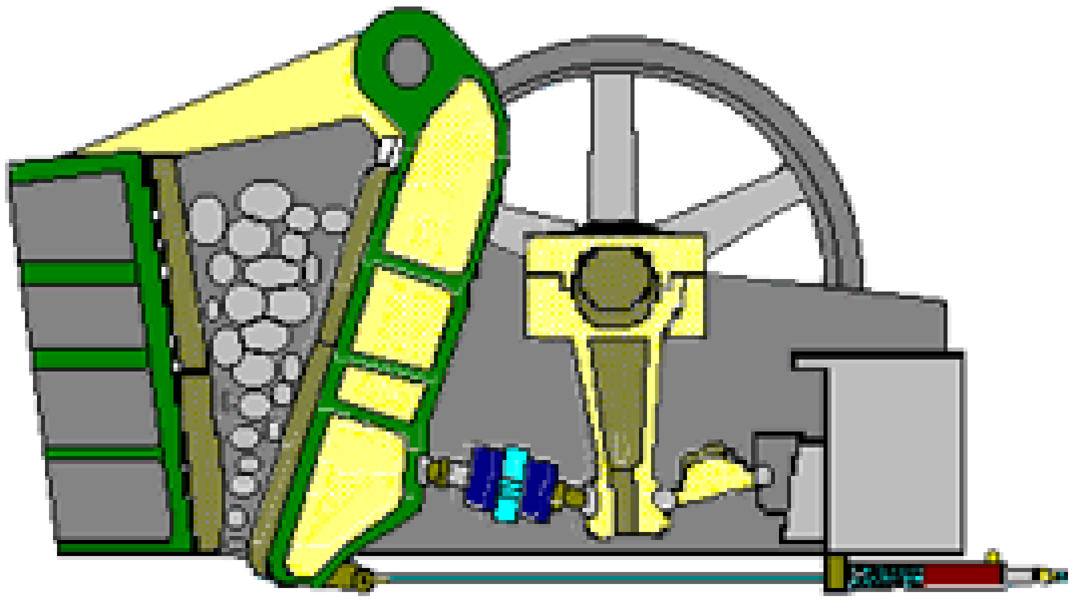


Slippage Principle

Double-toggle Jaw Crusher



Double-Toggle Jaw Crusher.



Largest particle size = $0.9 \times$ Gape

Jaw crusher performance

Crusher Type	Size (mm)				Reduction Ratio		Power (kW)		Toggle Speed (rpm)	
	Gape (mm)		Width (mm)							
	Min	Max	Min	Max	Range	Average	Min	Max	Min	Max
Blake, double toggle	125	1600	150	2100	4:1/9:1	7:1	2.25	225	100	300
Single toggle	125	1600	150	2100	4:1/9:1	7:1	2.25	400	120	300
Dodge	100	280	150	28	4:1/9:1	7:1	2.25	11	250	300

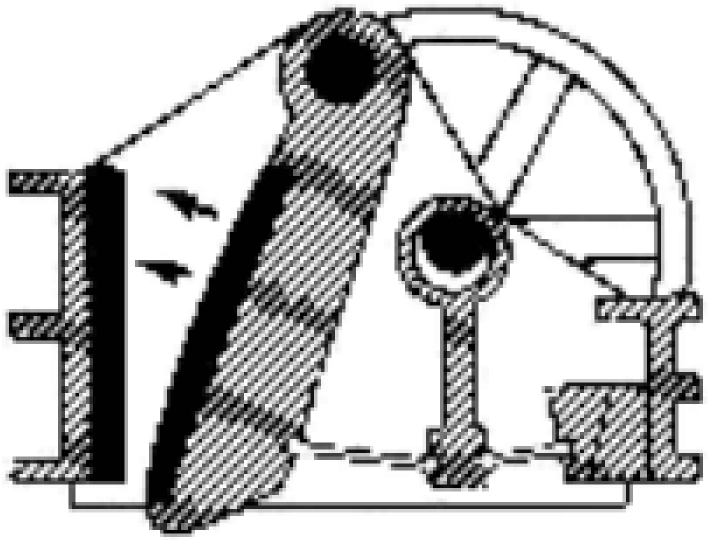
The factors of importance in designing the size of primary crushers, such as a jaw crusher, are:

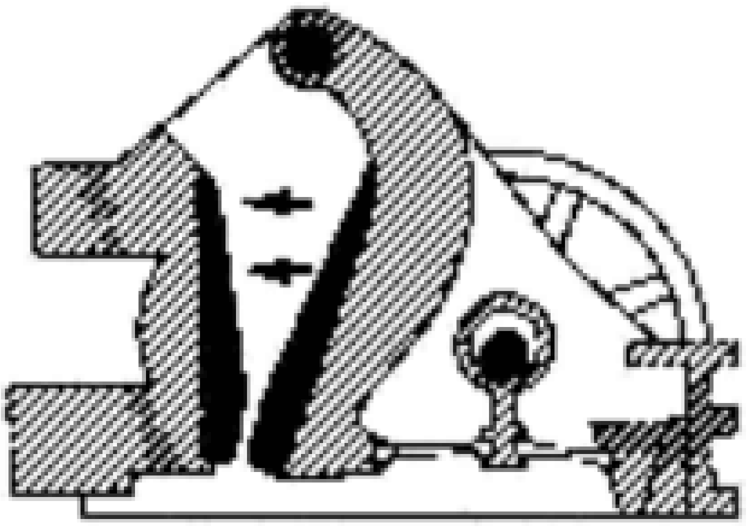
$$\text{Vertical height of crusher} \approx 2 \times \text{Gape}$$

$$\text{Width of jaw} > 1.3 \times \text{Gape}$$

$$< 3.0 \times \text{Gape}$$

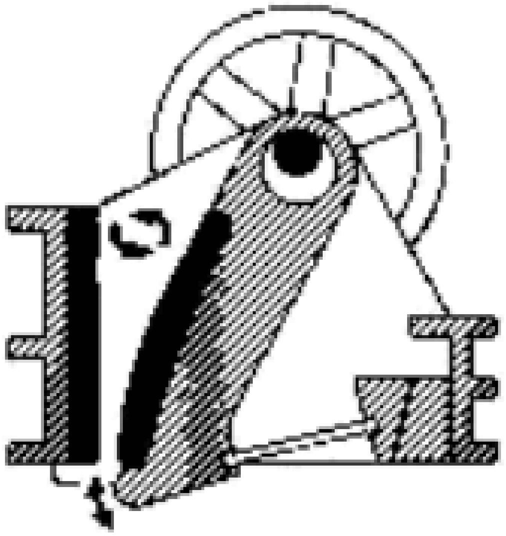
$$\text{Throw} = 0.0502\,(\text{Gape})^{0.85}$$

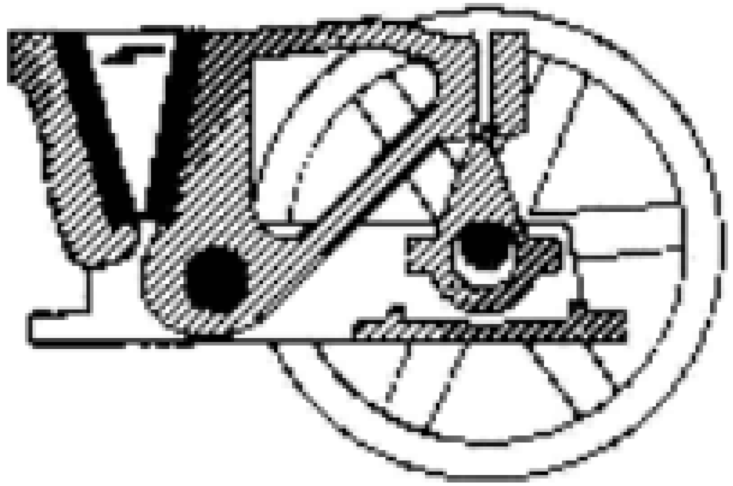




Blake (Double Toggle) Originally the standard jaw crusher used for primary and secondary crushing of hard, tough abrasive rocks. Also for sticky feeds. Relatively coarse slabby product, with minimum fines.

Overhead Pivot (Double Toggle) Similar applications to Blake. Overhead pivot; reduces rubbing on crusher faces, reduces choking, allows higher speeds and therefore higher capacities. Energy efficiency higher because jaw and charge not lifted during cycle.





Overhead Eccentric (Single Toggle) Originally restricted to sampler sizes by structural limitations. Now in same size of Blake which it has tended to supersede, because overhead eccentric encourages feed and discharge, allowing higher speeds and capacity, but with higher wear and more attrition breakage and slightly lower energy efficiency.

Dodge Bottom pivot gives closer sized product than Blake,but Dodge is difficult to build in large sizes, and is prone to choking. Generally restricted to laboratory used.

Jaw Crusher Capacity Estimation

The capacity of jaw crushers is a measure of the mass or volume of crushed material produced in unit time of operation.

Mathematically, the capacity can be expressed by the general formula

$$Q = (\mathrm{W}, L, L_{\max}, L_{\min}, L_T, \nu, \theta, K)$$

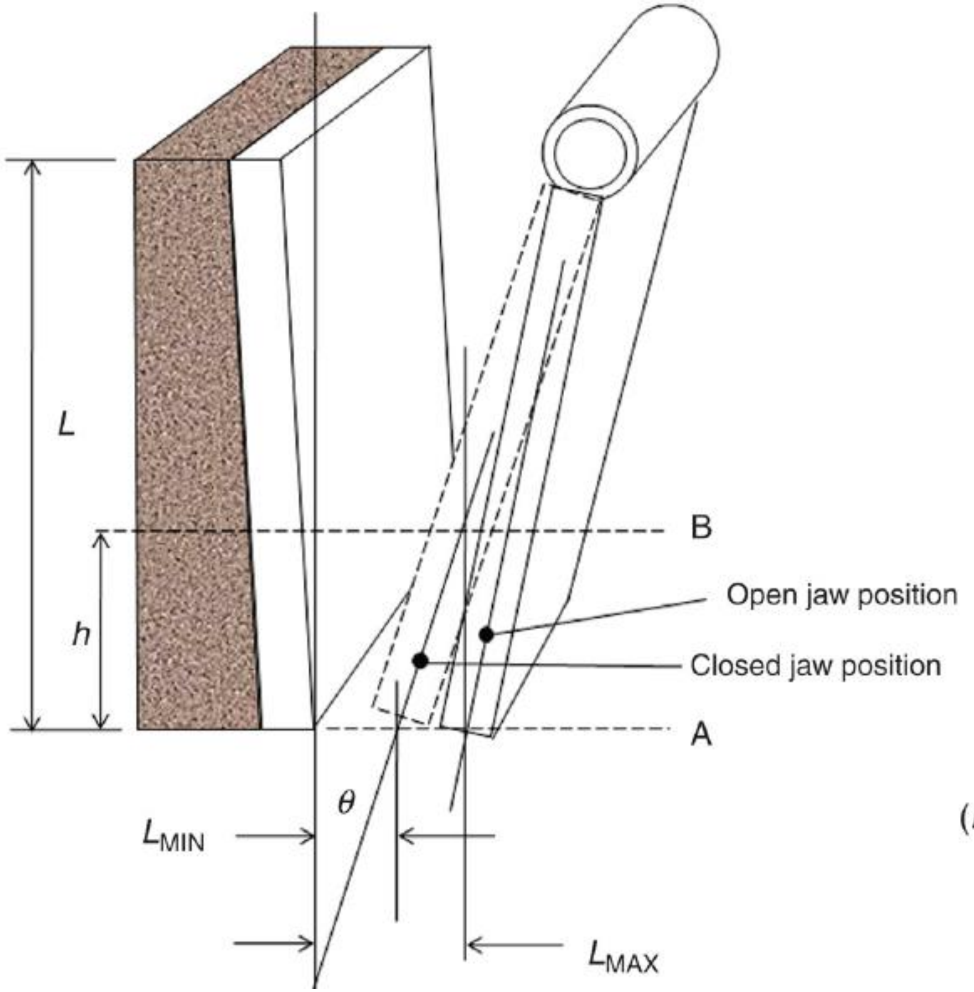
Where, Q = capacity
W = width
L = height (or depth of jaws)
L_{MAX} = set maximum (open set)
L_{MIN} = set minimum (closed set)
L_T = length of stroke or throw
ν = frequency (revolutions rpm = cycles per unit time)
K = constant related to machine characteristics
θ = jaw angle

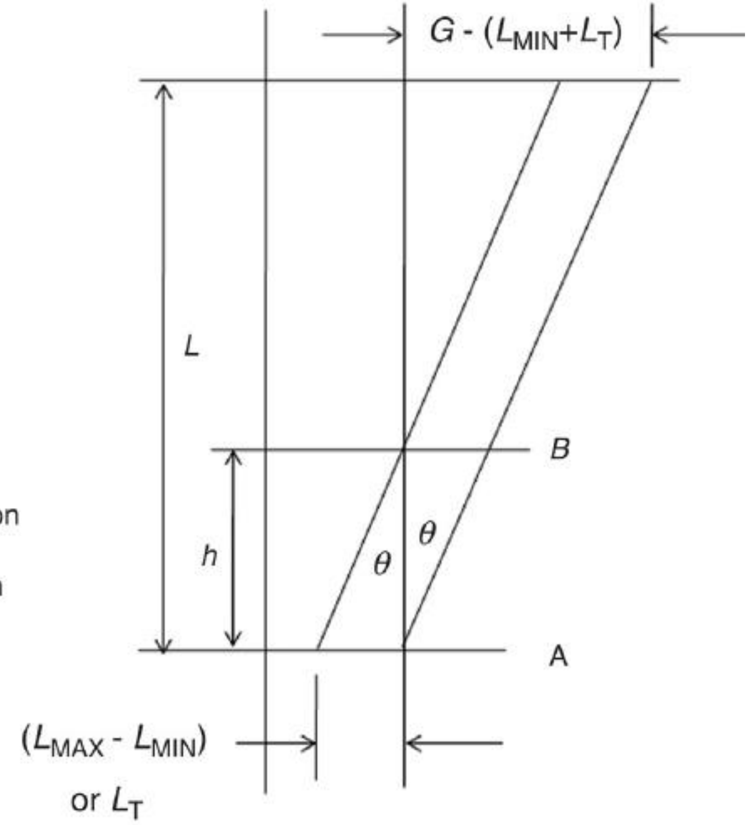
- The mechanism of movement of rocks down the crusher chamber determines the capacity of jaw crushers.
- The movement can be visualised as a succession of wedges (jaw angles) that reduce the size of particles progressively by compression until the smaller particles pass through the crusher in a continuous procession.
- The frequency of opening and closing of the jaws, therefore, exerts a significant action on capacity.

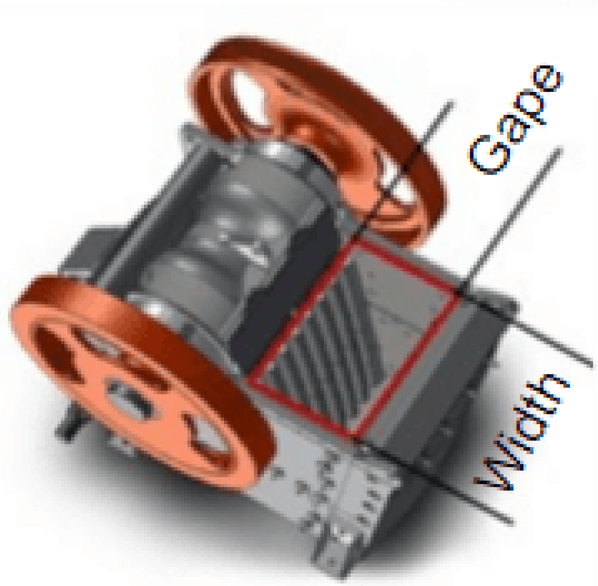
Rose and English Method

Rose and English determined the capacity of a jaw crusher by considering the time taken and the distance travelled by the particles between the two plates after being subjected to repeat crushing forces between the jaws.

Rose HE, English JE. Trans IMM 1967;76:C32







Jaw Crusher Operating Geometry. The Motion of the Jaw is Approximately Parallel

- Let, The distance, *h*, between A and B is equal to the distance the particle would fall during half a cycle of the crusher eccentric.

- Time for half a cycle is (30/ν) (ν is the number of cycles per minute)

$$h = \frac{1}{2} g (30 / \nu)^2$$

$$\nu = 66.4 / h^{0.5}$$

- for a fragmented particle to fall a distance h in the crusher, the frequency must be less than ν

The distance h can be expressed in terms of L_{MIN} and L_{MAX}, provided the angle between the jaws, θ, is known

$$h = \frac{(L_{\max} - L_{\min})}{\tan\theta}$$

with increasing frequency of the toggle movement the production increased up to a certain value but decreased with a further increase in frequency

During comparatively slower jaw movements and frequency, Rose and English derived the capacity, Q_S,

$$Q_s = 60 L_T \nu W (2L_{\min} + L_T)\left(\frac{R}{R-1}\right)$$

OR

$$Q_s \propto \nu$$

If other parameters are constant

Where, L_T = throw

ν = frequency (cycle/min)

W = width of jaw plates (m)

L_{MIN} = closed set

R = machine reduction ratio (gape/set) and

Q_S = capacity (slow frequency) in terms of volume of material product per hour

At faster movement of the jaws where the particle cannot fall the complete distance, h, during the half cycle, Q_F was found to be inversely proportional to frequency

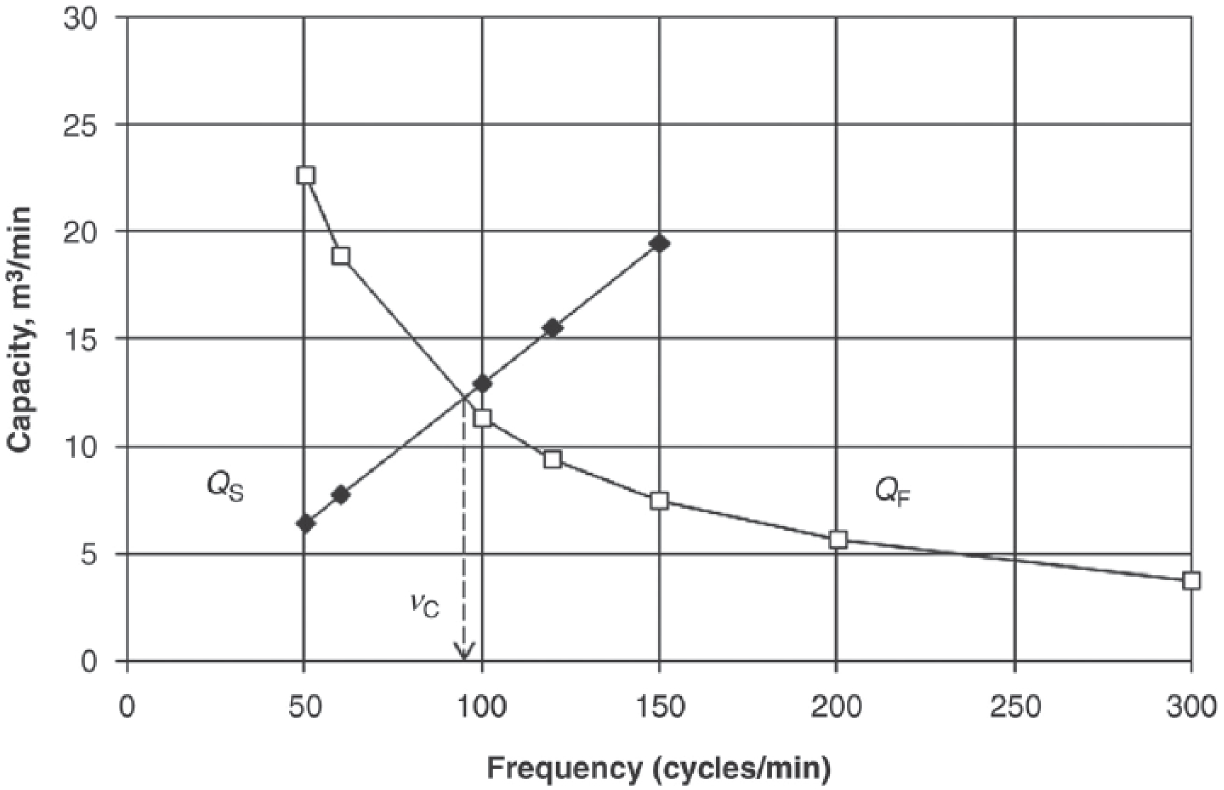
$$Q_F = 132435W(2L_{\min} + L_T)\left(\frac{1}{\nu}\right)$$

OR

$$Q_F \propto \frac{1}{\nu}$$

If other parameters are constant

where Q_F = capacity (fast frequency) in terms of volume of material product per hour.



Change of Capacity with Frequency of the Jaw Plate

L_T = 0.228 m, W = 1.2 m, L_{MIN} = 0.10 m, R = 10, G = 1 m.

Factors not considered

Change of bulk density of the feed as it passes down the crusher

Bulk density depends on

1. the size distribution of the feed,
2. the break-down characteristics which could be different for hard, brittle and friable particle
3. the packing characteristics,
4. the initial density of the ore,
5. surface characteristics of the particles.

Rose and English defined the packing characteristics, P_K

$$P_K = \left(\frac{d_{\max} - d_{\min}}{d_{mean}} \right)$$

P_K is considered a size distribution function and is related to capacity by some function $f(P_K)$.

As the particles decrease in size, while being repeatedly crushed between the jaws, the amount of material discharged for a given set (throw, L_T)increases.

$$\beta = \left(\frac{set\ opening(L_T)}{mean\ \ feed\ size} \right)$$

The capacity is then depend on some function which may be written as ƒ(β)

The maximum capacity, Q_M, of the crusher will be

$$Q_M = 2820 L_T^{0.5} W (2L_{\min} + L_T) \left(\frac{R}{R-1} \right)^{0.5} \rho_s f(\mathrm{P}_K) f(\beta) S_C \qquad (t/\mathrm{h})$$

S_C denotes a parameter related to the surface characteristics and ρ_S is the density of the particle

For actual crusher speeds, the actual crusher capacity is given by

$$Q_A = Q_M \frac{v}{v_c} \qquad for\ v < v_c$$

$$Q_A = Q_M \frac{v_c}{v} \qquad for\ v > v_c$$

Critical Operating Speed

From Fig

$$\tan\theta = \left[\frac{G-(L_{\min}+L_T)}{L}\right]$$

and

$$\tan\theta = \frac{L_T}{h}$$

Hence,

$$\frac{L_T}{h} = \left[\frac{G-(L_{\min}+L_T)}{L}\right]$$

As per the design consideration L ≈ 2G

Substituting *h* from equation 1 we get

$$v^2 = 4414.5\left[\frac{G-(L_{\min}+L_T)}{2G}\right]\frac{1}{L_T}$$

Now if

$$R = \frac{G}{L_{\min}}$$

We get

$$v^2 = 4414.5\left[\frac{RL_{\min}-L_{\min}-L_T}{2RL_{\min}}\right]\frac{1}{L_T}$$

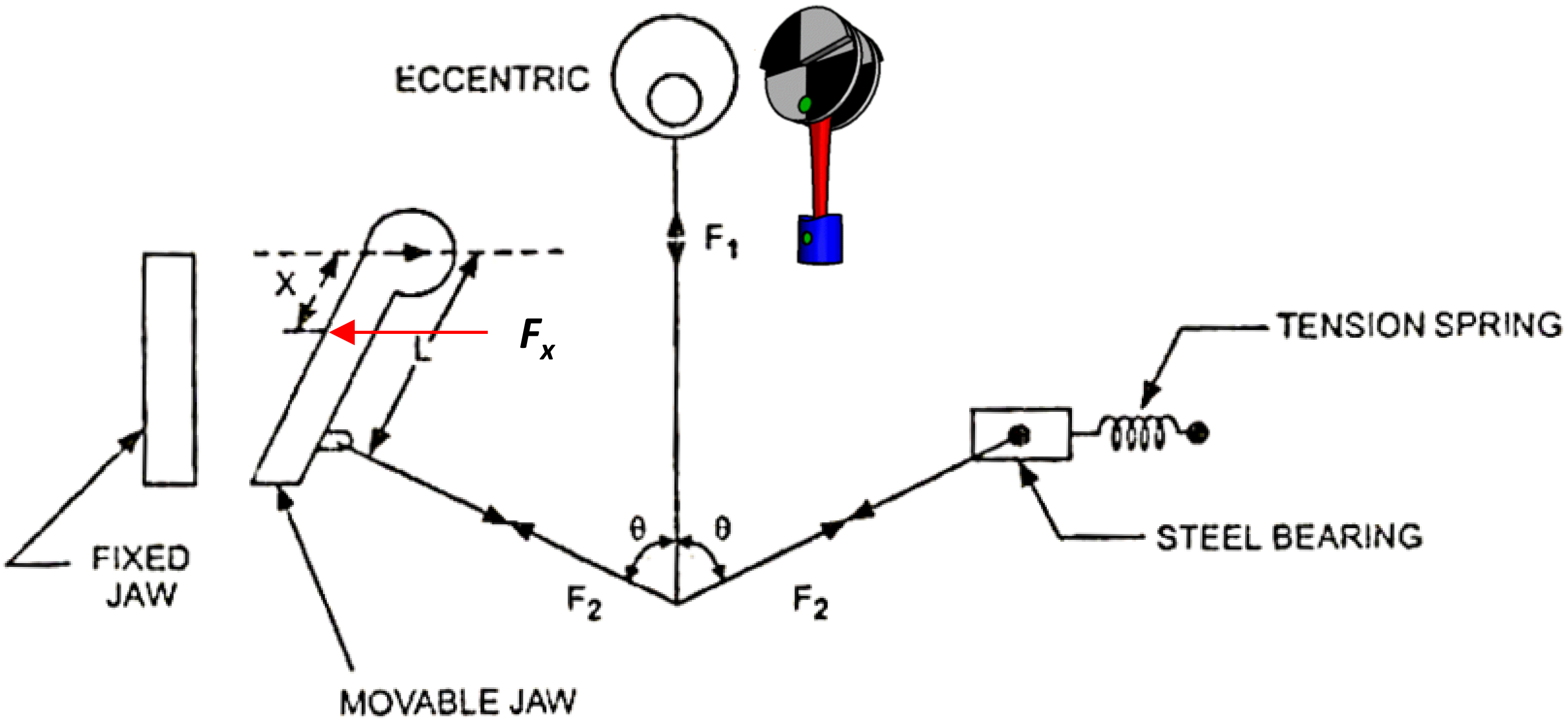
Now for jaw crushers

$$L_T = 0.0502G^{0.85}$$

Substituting and neglecting the square term

$$v_c = 47L_T^{-0.5}\left(\frac{R-1}{R}\right)^{0.5}$$

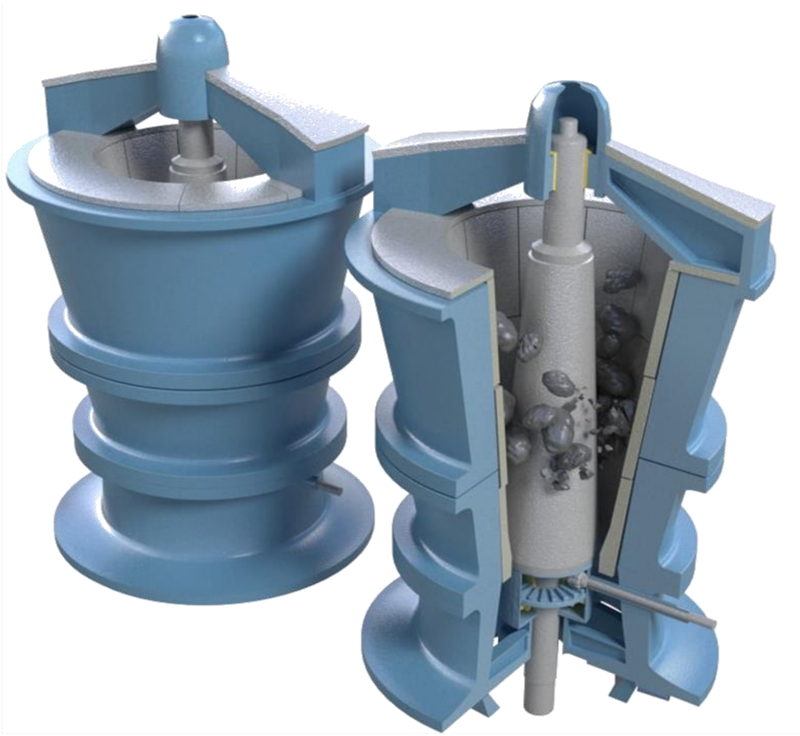
Force on the moving jaw

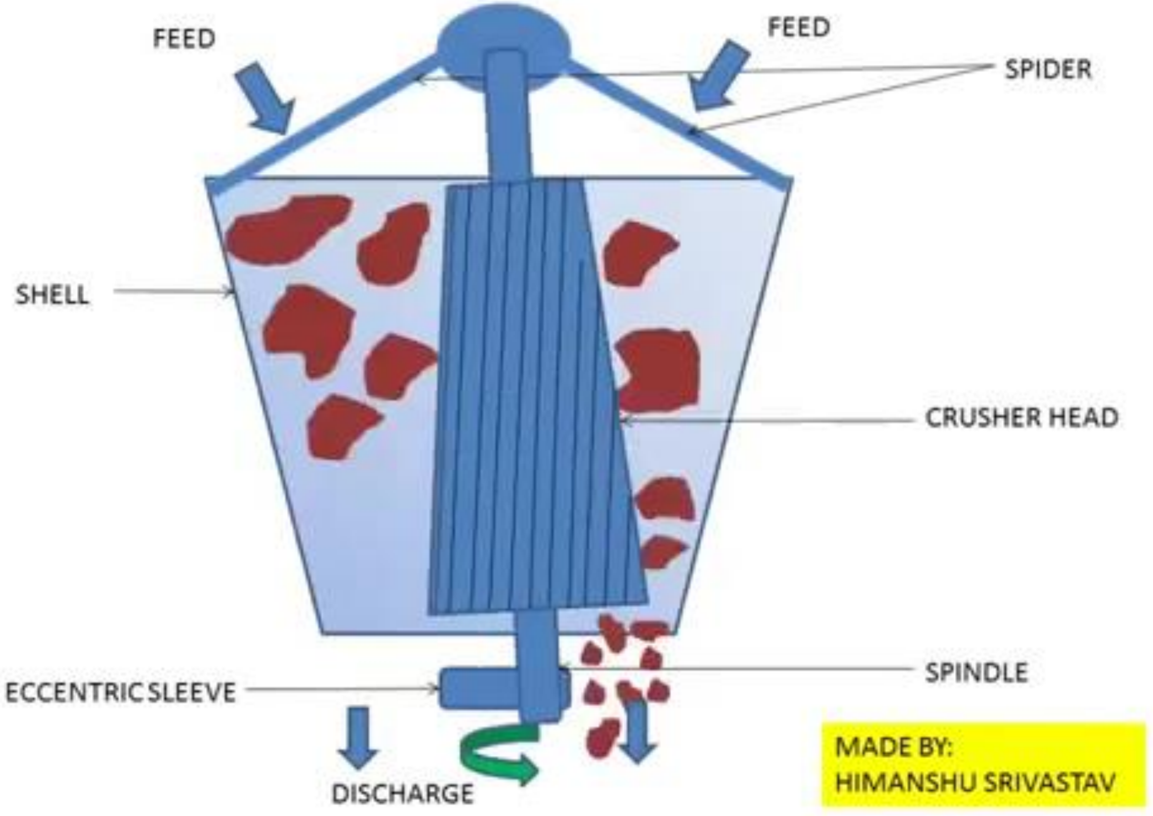


$$F_x = (F_1 L \tan\theta) / 2x$$

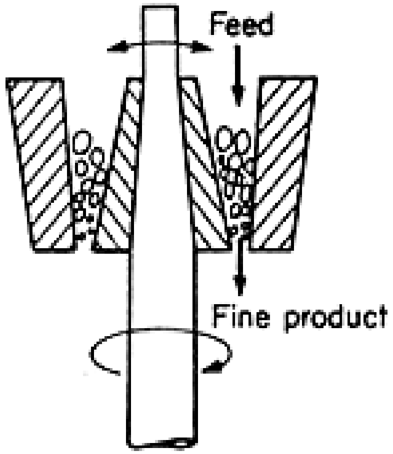
Force Balance on the movable jaw of a Double-toggle (Blake) Jaw Crusher

Gyratory Crusher





It consists essentially of a fixed crushing surface around the axis of which gyrates a movable crushing surface which has the shape of a conical in the erect position. The material to be crushed is fed into the downward converging annular space between these two crushing surfaces. It may be looked upon as a jaw crusher wound around a vertical axis through the mid point of the swing jaw shaft.



Parts of a gyratory crusher -

1. **Plate**: made of cast steel and heavily ribbed as it forms the support of the driving seats.

2. **Spider**: made of cast iron in small machine and of hard steel in large machines, since it is only two armed and is supporting the breaking head, it must be heavy and strongly formed to prevent any wear.

3. **Spindle**: it carries the maximum load and is therefore built of great strength and capable of withstand continuous shock or load.

4. **Suspension bearing**: it is a split steel nut with a downward taper and fits the sleeve of the spindle. It rests on the wearing rings of the spider.

5. **Gear**: it is made mostly of cast steel.

6. **Counter shaft bearing**: it is made as a integral part of the bottom plate and is made extra long and capable of adjustment to take up wear on the gear and

Operation of Gyratory Crusher

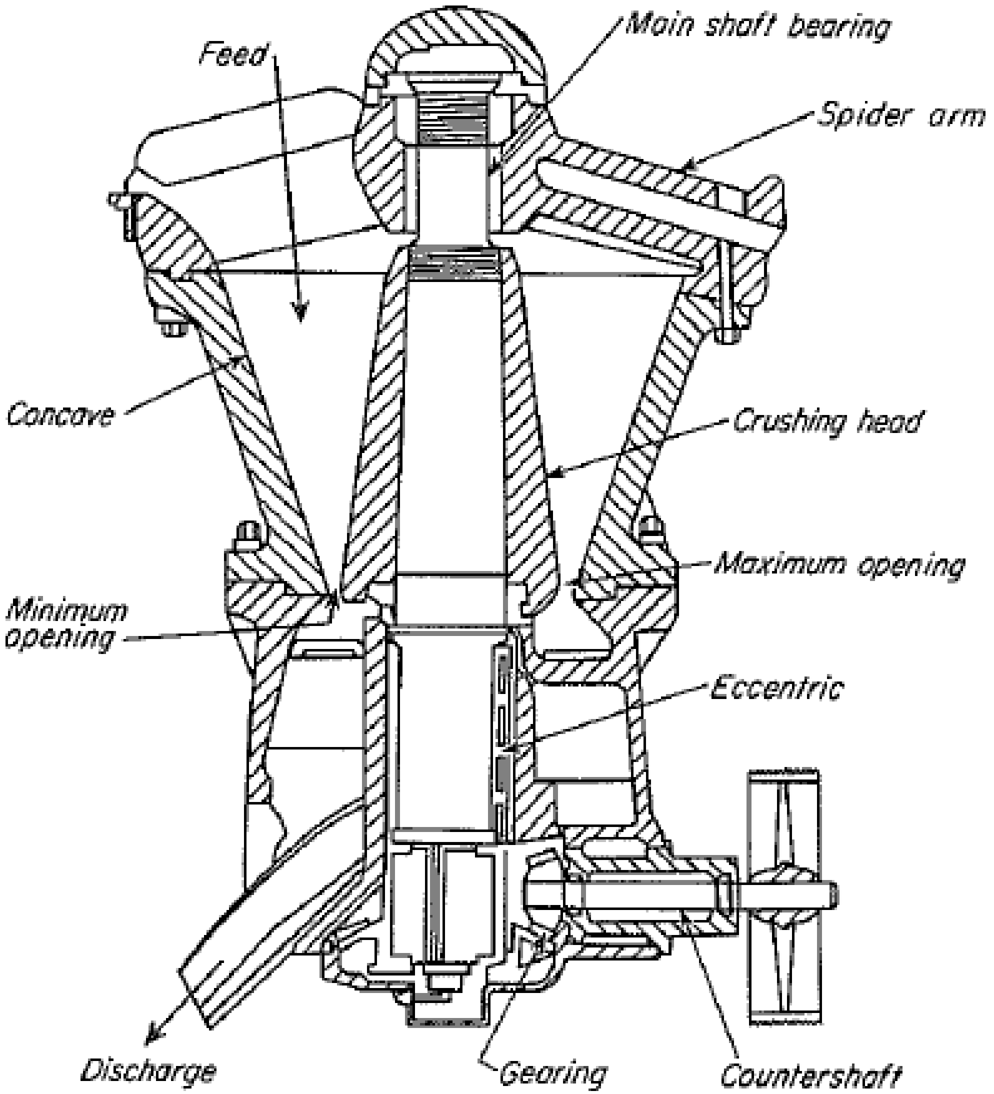
The crusher can be of short spindle, long spindle or fixed spider type. The capacity varies with different types of materials and operations. The capacity is expressed in terms of reduction tons.
Adjustments of gyratory crusher are a) width of discharge opening, b)throw and c) speed.

a) Width of the discharge opening is made by raising or lowering the spindle by means of adjustment nuts. In fixed spindle type the adjustment is made by changing the annular *. Range of adjustment is limited because of change in nip angle.

b) Throw is adjustable by changing the eccentric sleeves.

c) Reduction ratio varies between 3 and with an average of 6.

- ❖ Size of a gyratory crusher = gape x mantle diameter



Size distribution: The crushed product in gyratory crusher is finer. On an average 10% material is larger than open set whereas in jaw crusher its 25%.

Comparison of Jaw and Gyratory Crusher -

1. Cost: gyratory crushers cost 29 to 72% of that of jaw crusher per ton of feed i.e. for same hourly reduction capacity and same reduction ratio, same gape.

2. Installation:
a. cost proportional to weight of the machine
b. foundation – stronger in jaw crusher
c. housing more in gyratory
d. head room more in gyratory
e. vertical height more in gyratory

3. Feeding: easier in gyratory crusher than jaw crusher.

4. Efficiency: reduction per unit energy more in gyratory. Idle HP 40-50% in jaw and 30% in gyratory crusher.

5. Maintenance: higher cost in gyratory, labor cost several times more in gyratory

6. Feed: jaw crusher can take very hard feed, less choking, low angle of nip, so feed not thrown out.

7. Set: easily adjustable in jaw crusher.

8. Lubrication: higher lubricant quantity in gyratory but easily controllable.

Selection: The capacity of gyratory crusher for the same gape is several times more than in jaw crusher, so the selection is on the basis of capacity.

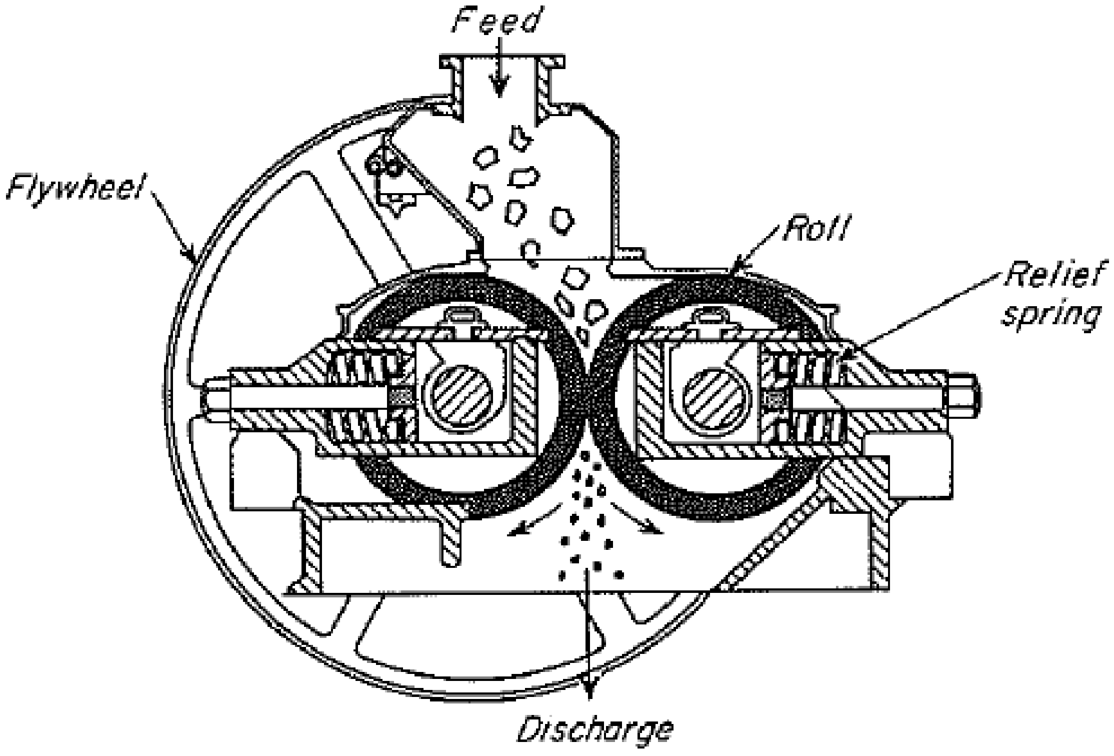
If hourly capacity is tons/gape in sq is
< 0.115 select jaw crusher
> 0.115 select gyratory crusher

Assume gape D=10”
So theoretical length of L= 1.7 D = 17”
L x D = 170 sq. inch
Jaw crusher capacity 10 TPH
So, ratio is 10/170 i.e. < 0.115 hence select jaw crusher.
A gyratory crusher of the same gape size has a capacity of 33 TPH
Ratio= 33/170 > 0.115

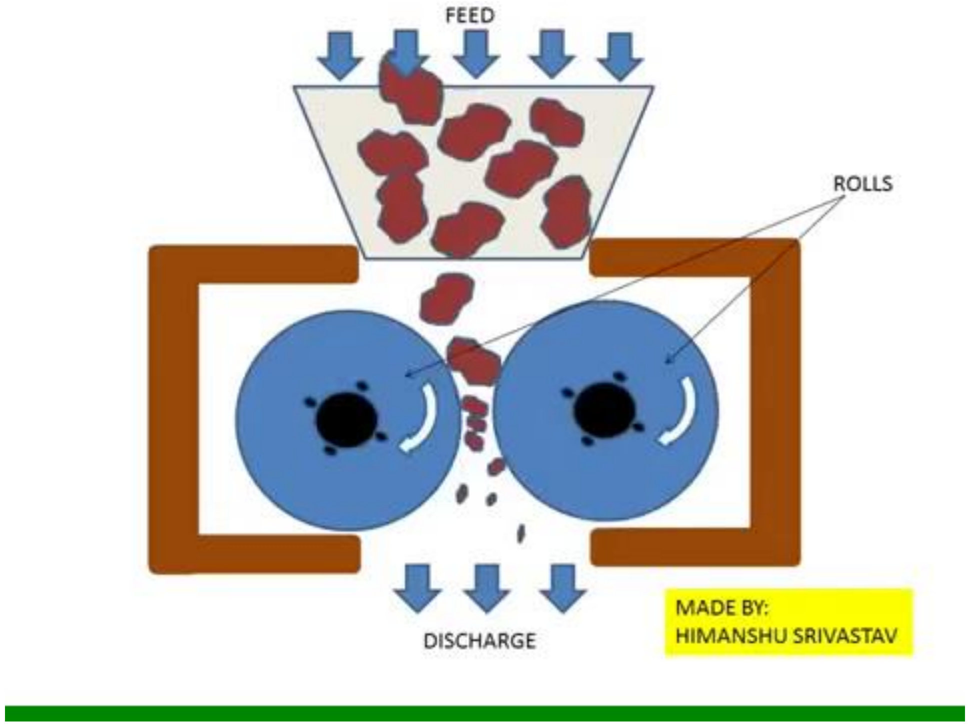
**TPH=Tons Per Hour*

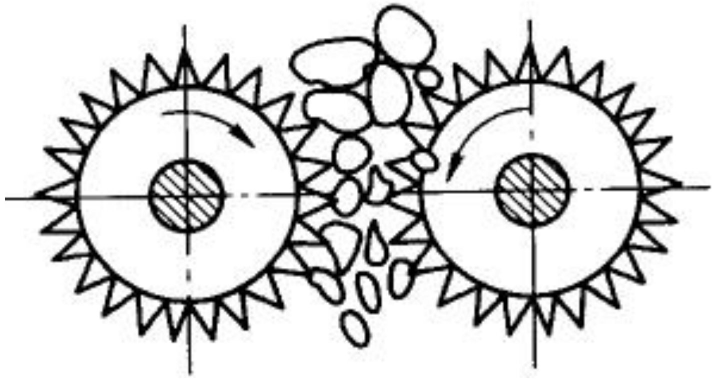
Roll Crusher

Rolls are generally used in the secondary crushing stage or stages. They are adopted to feed 2" or 3" maximum size for crushing to as fine as 10-12 mesh (1.0 to 1.5 mm) though the more popular practice is to feed 1" or less and final product is 5 to 10 mm. Nowadays rolls are being replaced by cone crushers.

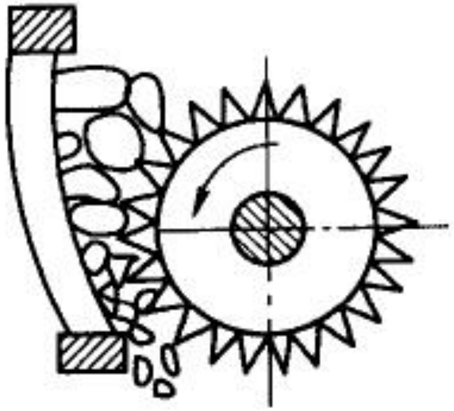


Smooth-roll crusher

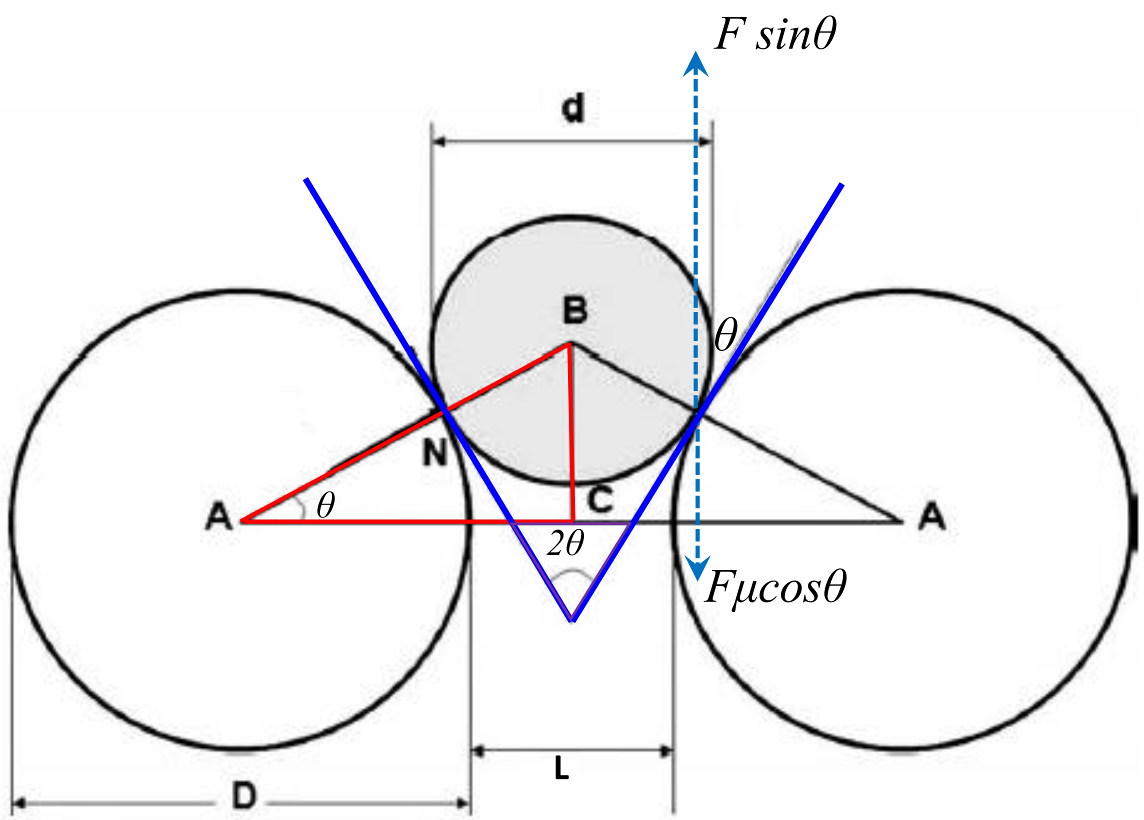




Double-toothed roll crusher



Single-toothed roll crusher



Roll Crusher Geometry

The radius of the roll

$$R = \frac{L - d\cos\theta}{2(\cos\theta - 1)}$$

To estimate the radius *R* of the roll, the nip angle, *2θ* is required

Compressive force, *F*, exerted by the rolls on the particle just prior to crushing, operating normal to the roll surface

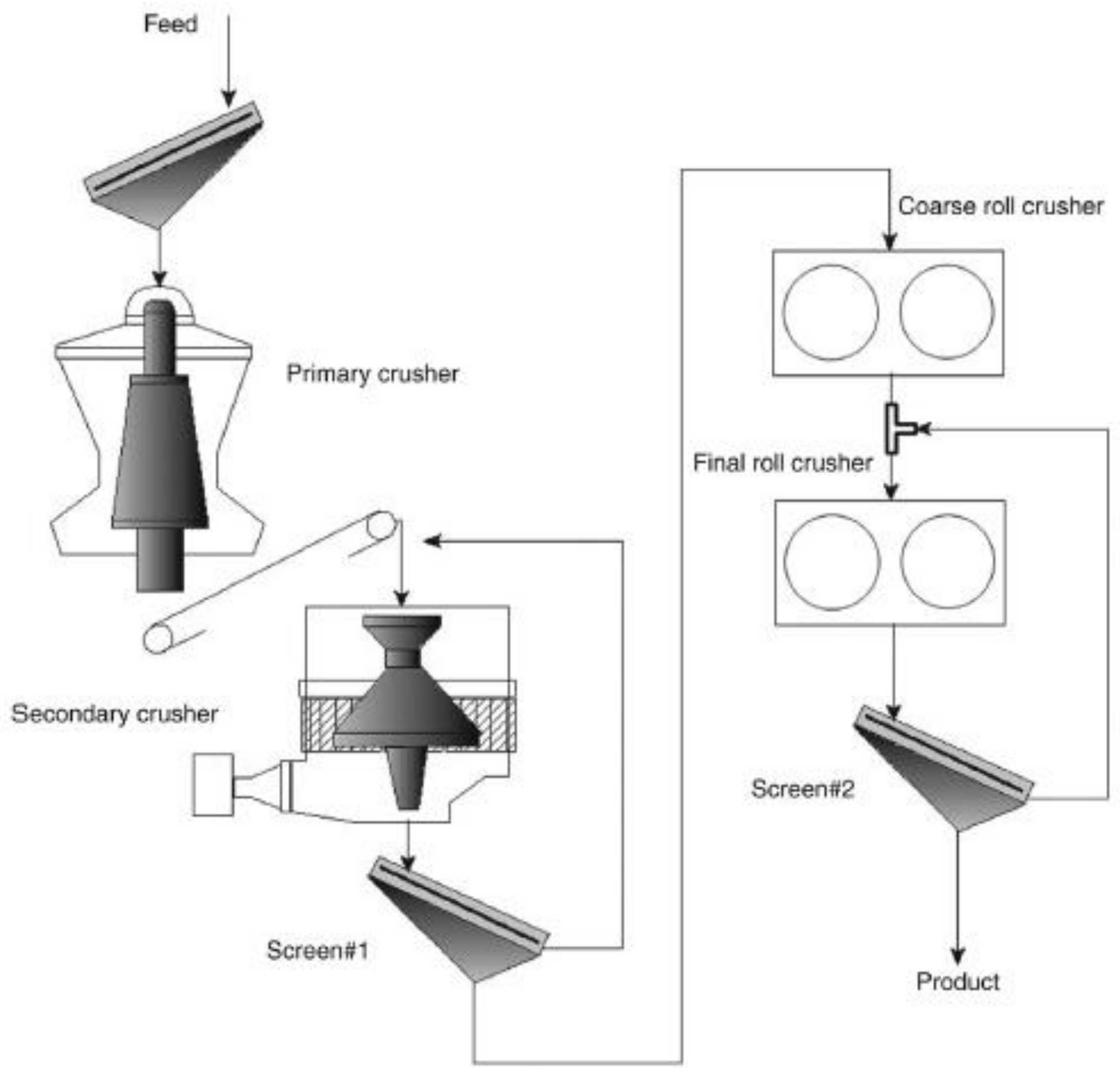
$$F \sin\theta = F\mu\cos\theta \qquad \text{or} \qquad \theta = \tan^{-1}\mu$$

According to Wills , the roll speed *v* (rpm) is related to the kinetic coefficient of friction of the revolving rolls, μ_K, by the relation

$$\mu_K = \frac{(1+1.12v)}{(1+6v)}\mu$$

Wills BA. Mineral processing technology. 2nd ed Oxford, New York Pergamon Press; 1981.

Roll Crusher Circuit



Capacity of Roll Crushers

$$Q = \pi 60 D W \nu L \rho_B \quad t/h$$

Where,
D = diameter of roll (m)
W = width of roll (m)
ν = speed (rpm)
L = set or distance between rolls (m)
ρ_B = bulk SG of the mineral (t/m^3)

Assumption:

- Particles are continuously fed from a height
- Rolls are kept full all the time
- Product is in the form of a continuous ribbon having the width of the roll and thickness equal to the set.

Wills BA. Mineral processing technology. 2nd ed Oxford, New York Pergamon Press; 1981.

The actual production will depend on the 'ribbon factor, R_F' given by the expression

$$R_F = 0.0095 \frac{Q}{v_P L W}$$

where Q = feed rate (t/h)
v_P = peripheral speed of the roll (m/s)

modified equation introducing an efficiency factor, ε and expressing capacity as

$$Q = 3600 \varepsilon W v_P L \rho_B \quad t/h$$

where
ρ_B = bulk density of the feed material (t/m3)
ε= efficiency factor which has a value between 0.15 and 0.30, depending on the roll gap or product size

Problem 4

A smooth surfaced roll crusher had a roll diameter of 910 mm. Its suitability to crush an ore at 10.0 t/h was being examined. Preliminary examination showed that the kinetic friction factor was 0.36 when the speed of revolution was 33 rpm. The average diameter of particles fed to the crusher was 200 mm and the S.G. of the ore was 2.8.

Estimate:

1. the distance between the rolls,
2. the angle of nip,
3. the width of the rolls.

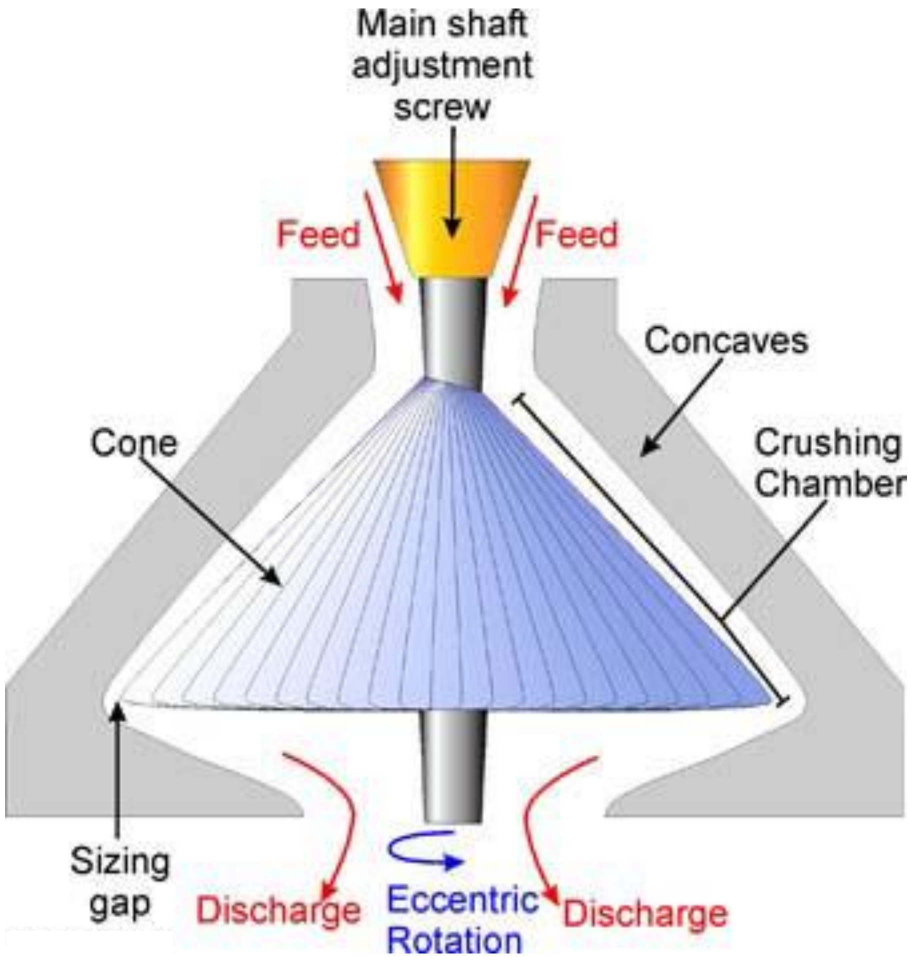
Cone Crusher

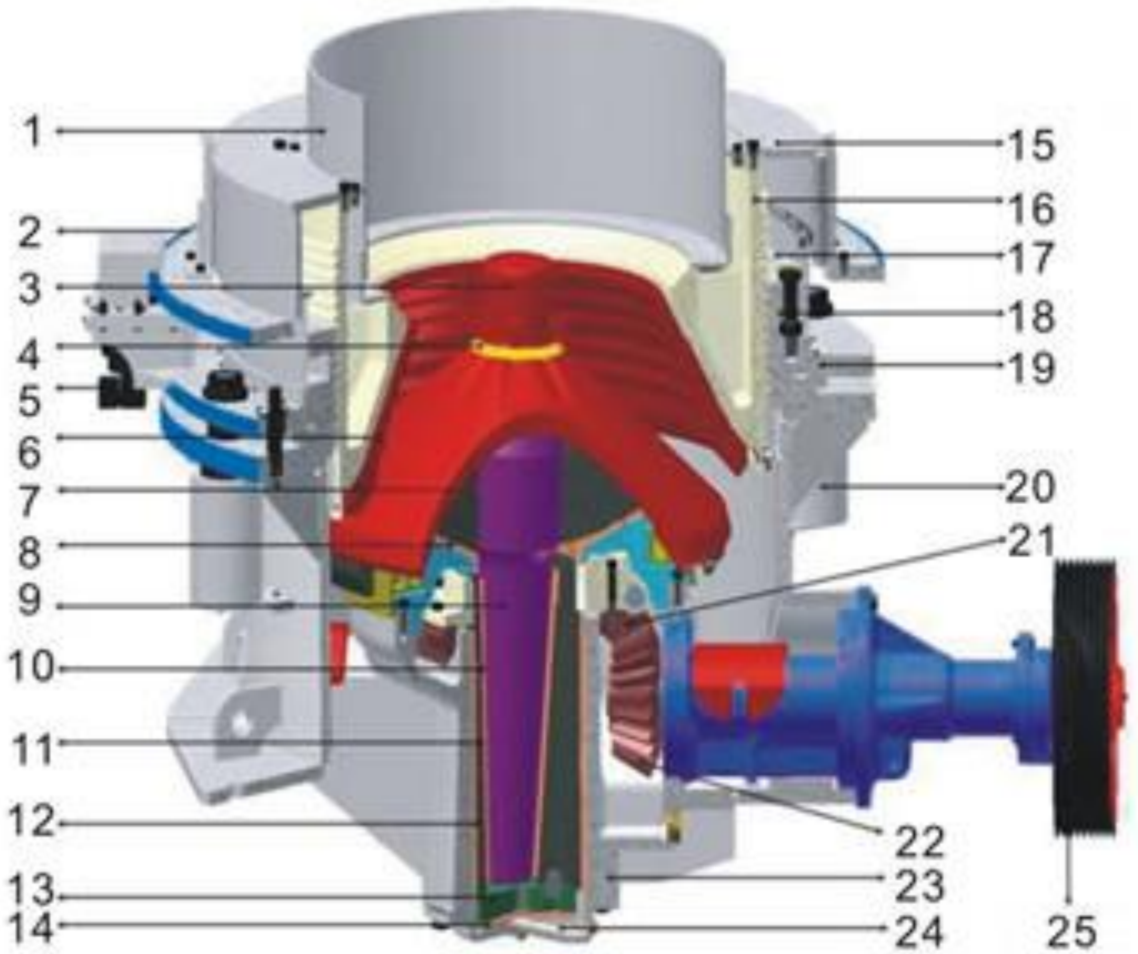
Cone crusher is an intermediate crusher, very similar in construction to the gyratory. The essential difference lies in the shape of crushing head which is very much flared. Another difference is that the spindle is supported on the bottom eccentric bearing. The head gyrates within a truncated cone which is flared outwards in the crushing chamber to allow more volume or area of crushing and more discharge area. The depth of cone is 1/3 rd of the diameter at the base. The throw of the head is 5-9 times the setting as against in gyratory. The large throw allows easy discharge of material. The speed of machine is kept high to crush the material at least twice during its passage. In cone crusher feed opening = 2 x gape.

Cone crushers are usually denoted by the diameters of the bottom of breaking head in inches or feet. Capacities range up to 150 TPH with ¼" closed set to 900 TPH with 2 ½" closed set. RPM of the mills varies between 435-700 and power up to 300 HP.

To prevent damage to the crushing surface due to tramp iron etc the bowl is usually held by spring which release when hard material falls inside the crusher.

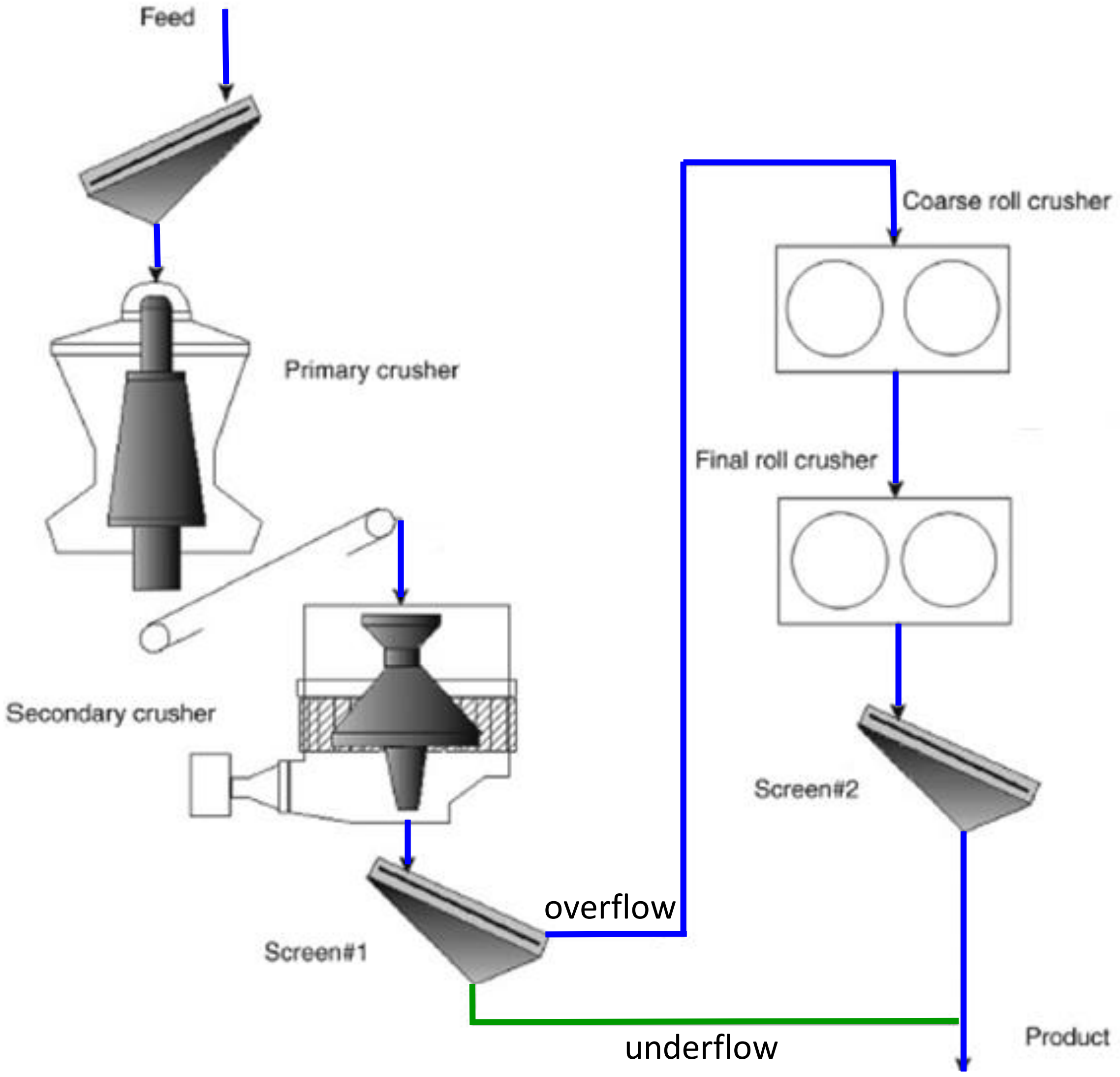
Cone Crusher Video: [*https://www.youtube.com/watch?v=RGbNv88r5Wc*](https://www.youtube.com/watch?v=RGbNv88r5Wc)





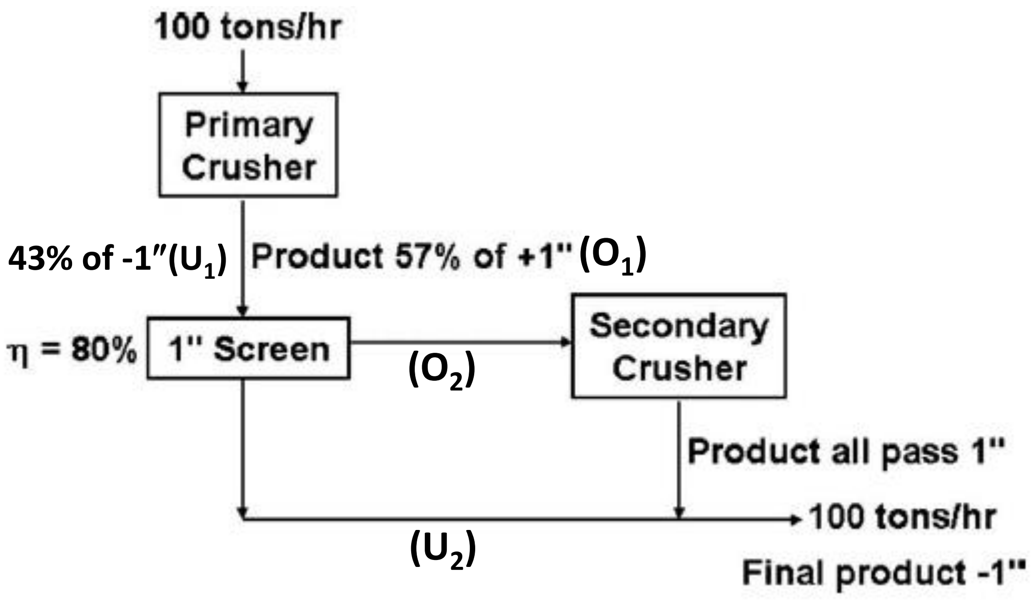
1. Feed Hopper
2. Drive Ring Pinion
3. Distribute Plate
4. Pressure Ring
5. Hydraulic Moto
6. Concave
7. Mantle
8. Socket Liner
9. Main Shaft
10. Eccentric Bushing
11. Eccentric
12. Main Frame Bushing
13. Upper Thrust Plate
14. Bottom Thrust Plate
15. Adjusting Cap
16. Adjusting sets
17. Locked Ring
18. Locked Cylinder
19. Supporting Sets
20. Tramp Release Cylinder
21. Gear
22. Pinion
23. Main Frame
24. Main Frame Bottom Cov
25. Pulley

Open Circuit



Problem 5

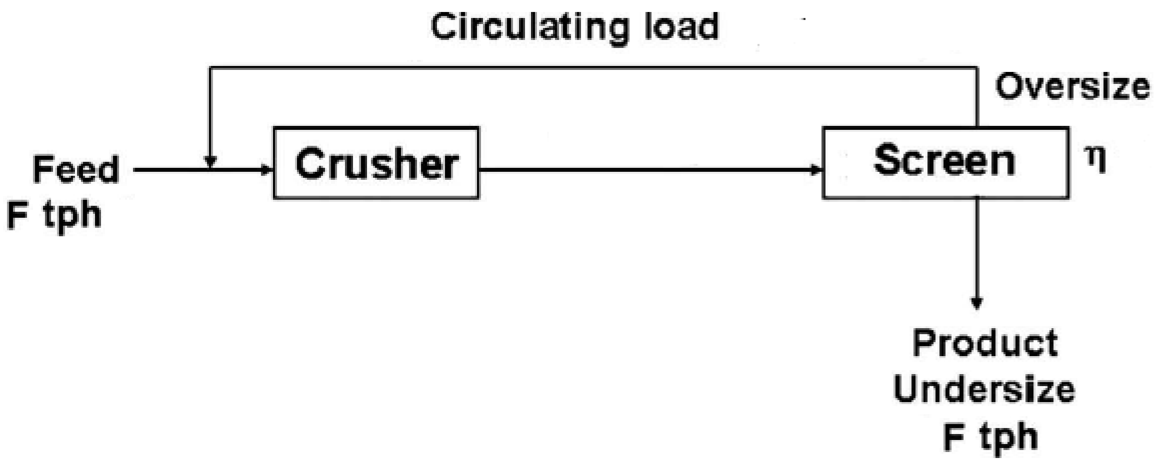
Crushed product of a primary crusher having 57% of +1″ is sent to 1″ screen. The overflow of the screen is again crushed in a secondary crusher to –1″ size and sent along with underflow of the screen as final product. Draw the crushing circuit with all details. Calculate the rate of the material crushed in secondary crusher if the effectiveness of the screen is 80% based on oversize material and the feed rate to a primary crusher is 100 tons/hr.



Ans : 71.25 tons/hr

Close Circuit

- There are two types of closed circuit crushing operations called **regular** and **reverse**.
- In regular type, the new feed is fed to the crusher, crusher product is fed to the screen and the overflow from the screen is fed back to the same crusher. The quantity of the overflow material fed back to the crusher is called **circulating load**.



Closed circuit crushing details at equilibrium when screen efficiency is η

- The underflow from the screen is the required final product and is equal to the new feed.

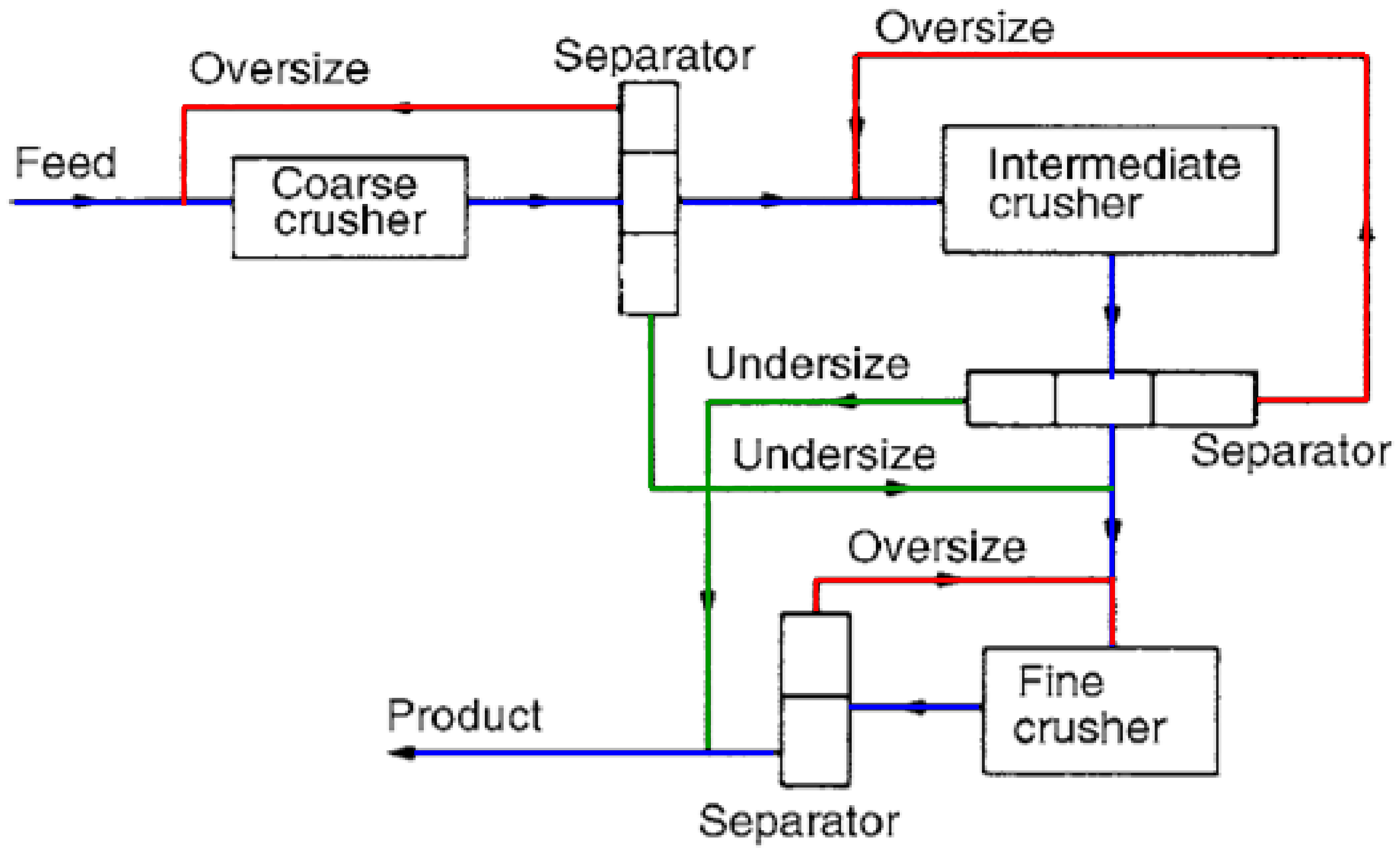
- This regular type is usually employed when the feed contains less percentage of undersize material.

- Initially the quantity of final product produced is less than the quantity of the feed material.

- As the operation proceeds further the quantity of the final product gradually increases and will be equal to the quantity of the feed material after some time. After attaining this equilibrium condition, the quantity of the final product is always equal to the quantity of the feed material and the circulating load is constant

- The circulating load, expressed as a percentage of the quantity of feed material is called **percent circulating load**

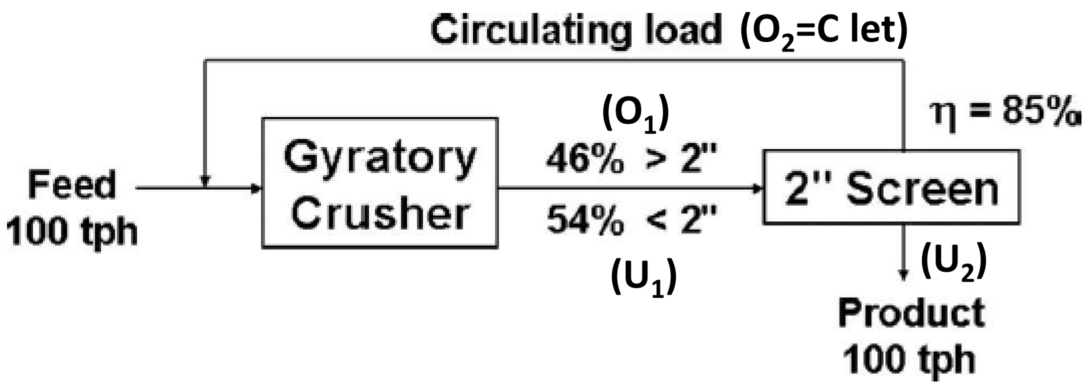
Flow diagram for closed circuit grinding system



Problem 6

The details of crushing plant employing gyratory crusher are as follows:
2″ square screen is in closed circuit with crusher
New feed to crusher=100 T.P.H
Crusher product contains 54% <2 and 46% > 2 fed to screen
Screen efficiency=85% (based on oversize)

Draw the flow diagram and find circulating load.
ii) Also find circulating load if the screen efficiency is based on undersize.



Ans : i) 117.95 tons/hr
ii) 117.86 tons/hr

Advantages of Close Circuit Operation-

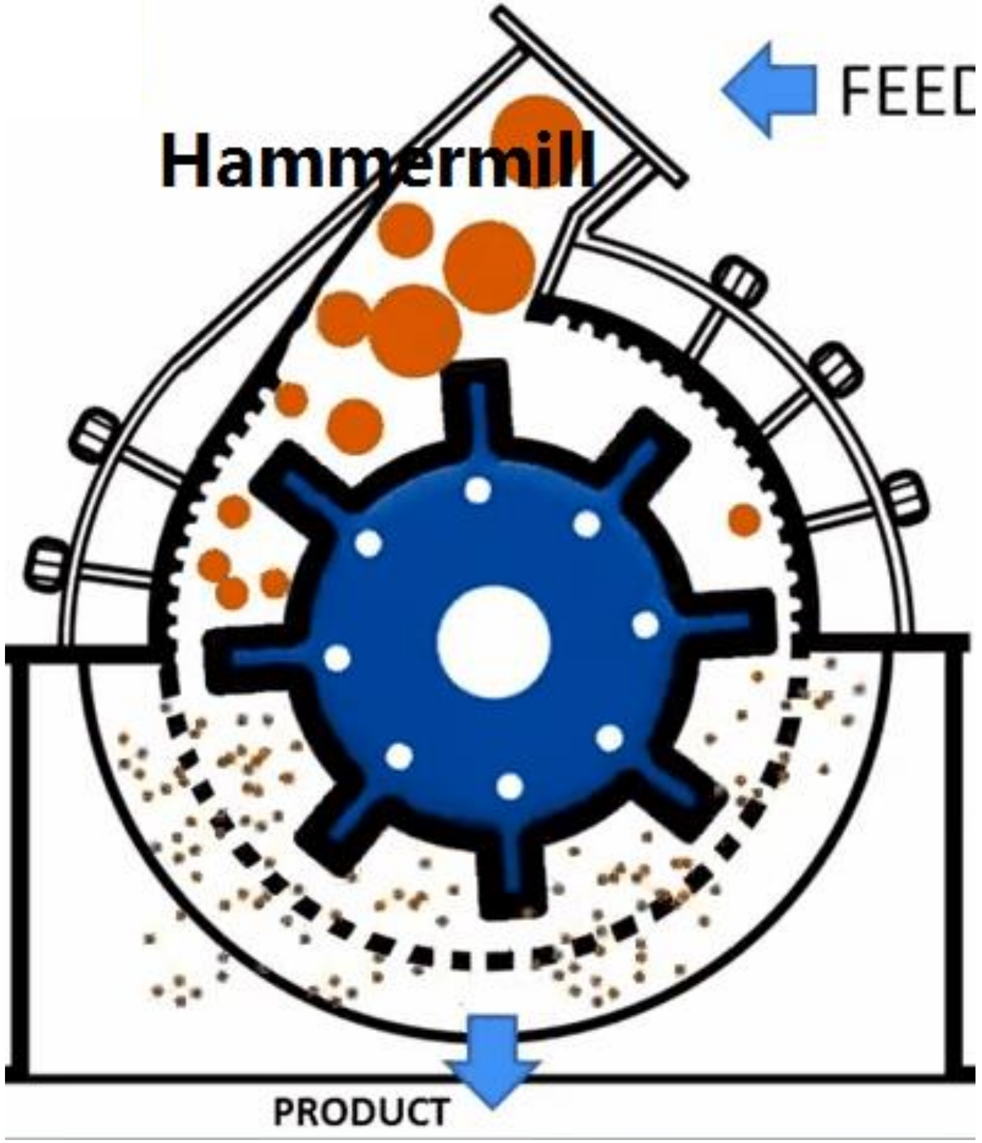
1) increases the quantity of finished material and capacity
2) reduces steel consumption
3) the size distribution is more uniform
4) power consumption decreases

The advantages of close circuit grinding arise from

i) reduction in the mean size of the material,
ii) marked increase in the near finished material in the feed,
iii) marked decrease in proportion of finish size in the feed, iv) more rapid travel (resistance time is low), hence shorter time of pass,
v) uniformity of size and
vi) presence of near finished material in mill.

Hammer Mill

- The hammer mill or swing hammer crusher or impactor is used as a primary crusher for reducing runoff quarry material to a size of less than 1” or as a secondary crusher to crush to ¾” size feed or less.
- It is almost confined to softer, easily crushable such as phosphates, gypsum, barite, asbestos rock, and cement rock and hard limestone (being the hardest rock commonly ground).
- The mill is particularly useful for clayey material that would clog reciprocating type primary crusher.
- It is also used for crushing bituminous coal for coke ovens, power plant.
- The hammers are suspended by pins between heavy steel dishes which are spaced by suitable spacers. The shaft is carried on heavy bearings.
- A heavy flywheel is mounted on one side of the shaft; the other end is fitted to the drive pulley or directly to the motor.



Force in hammer mill breaking

Let **H** be the hammer hitting a falling particle. If **M** is the change in momentum of the particle **b** in the direction of the velocity of hammer, the average force of blow = $\mathbf{F_{av} = M/t}$, where **t** is the time taken by the particle to reach a maximum velocity i.e.

$$F_{max} = 1.5\,W\,\frac{u}{g\,t}$$

Where W is weight of the hammer, u= velocity of the hammer, 1.5 is experimentally determined constant, t for highly elastic substance is 0.001 sec and for inelastic substance 0.002 sec. So,

$$F_{max} = 1.5\,\frac{W\,u}{32.2 * 0.002} = 23\ Wu\ lb$$

The operating variables are,

a) diameter and configuration of the hammer

b) speed

c) number of hammer

d) size of hammer

e) spacing of anvil and break plate

f) height of feed

g) spacing of grate bar

Size of product depends on,

a) characteristics of rock
b) size of feed
c) type of mill
d) grid spacing
e) hammer weight

Capacity of hammer mill is given by

$$Q = \frac{K.D^2 Ln^2}{3600\,(R-1)}\ \text{tons/hr}$$

Where D = rotor diameter, m, L = length of rotor, m
N = rpm, R = reduction ratio
K = constant depending upon the material (ranging from 4 to 6.2)

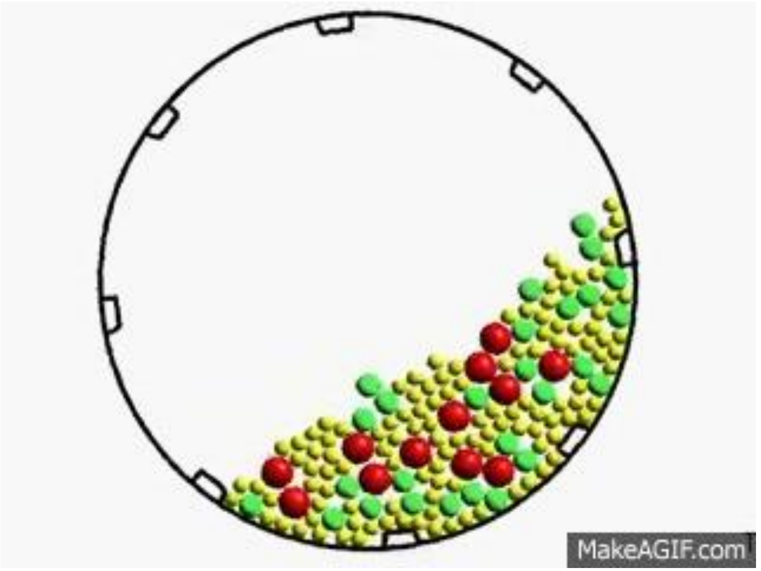
Tumbling Mills

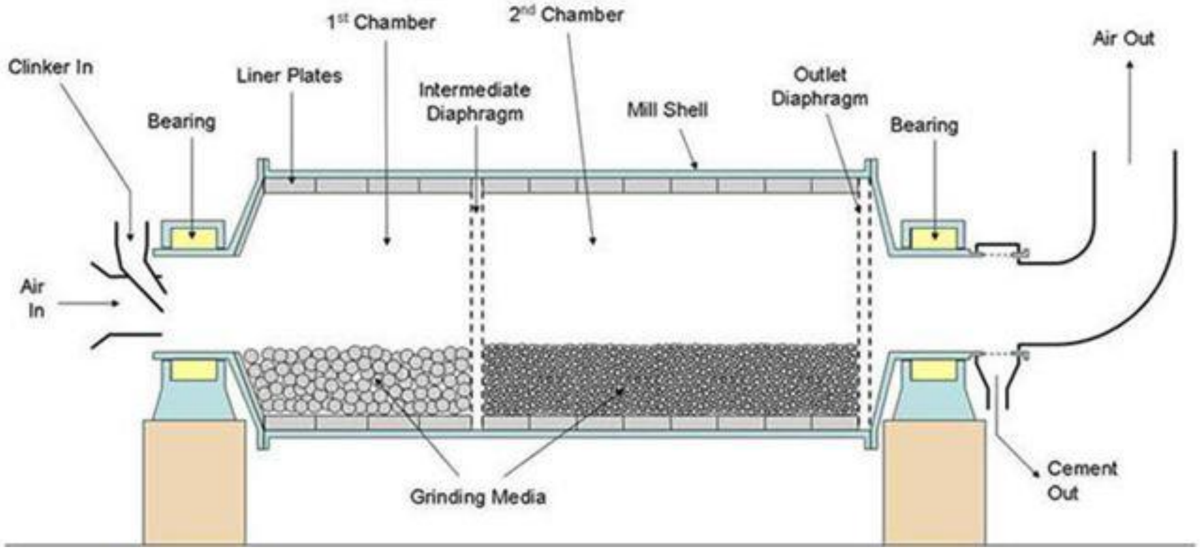
- In a tumbling mill the grinding of the material is affected by tumbling action of media which may be balls, rods or pebbles contained in a steel, rubber or stone lined steel shell.
- The **media used** in the charge generally describes a tubular mill. Thus, the medium could be **steel or cast iron balls** when the mill is designated as a ***ball mill***, or it could be steel rods where the mill is known as a ***rod mill***.
- The ball mill differs from the tube mill by its shortness in length and as a rule doesn't exceed the diameter.
- In conventional **ball mills** large balls are used on a coarse feed to produce comparatively coarse product.
- The **tube mills** are long compared to ball mills, L/D > 2.
- The compartment mill is a combination of the above types, consisting of a cylindrical shell divided into a number of sections by perforated partitions.
- Coarse grinding takes place at one end where coarse media is used and finishing is given at the other end with smaller balls.

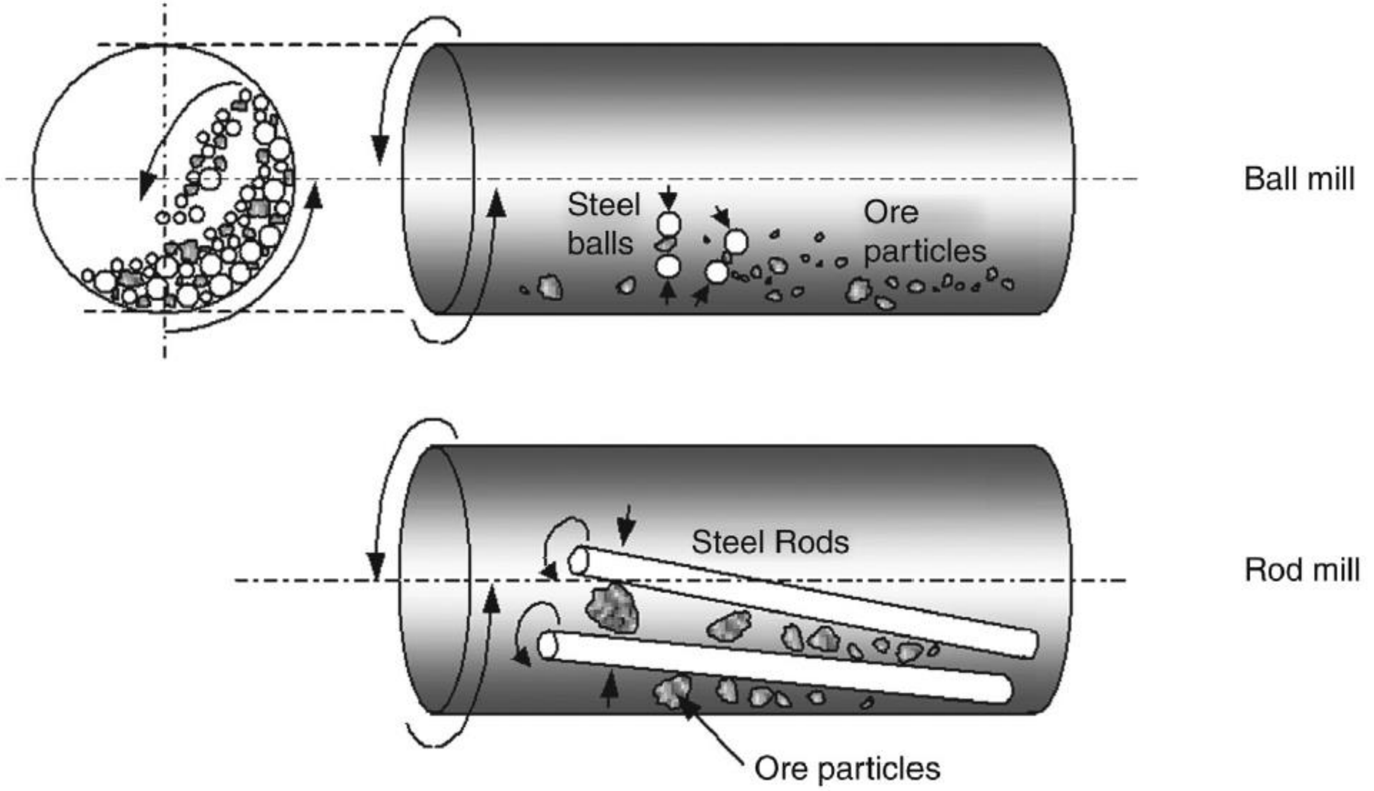
- Due to rotation of the mills the bigger balls tend to collect at the larger diameter portion towards the feed end and the small balls at the discharge end where the diameter is relatively small. It is thus counterpart of compartmental mills.

- Another type is **rod mill** which delivers a more uniform and granular product. The pebble is a fluid with ceramic pebbles as the grinding media and may be lined with ceramic or any other non-metallic liner.

- **Autogenous mills** are one of the latest developments and are gradually coming into use in different industries where the specific gravity of the material is high and contamination with the grinding media is not desired. In principle the balls are replaced by large lumps of the ore itself which act as media.

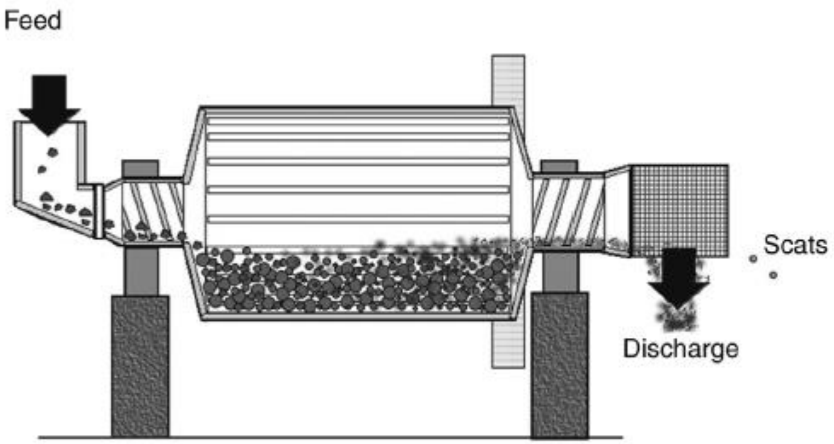




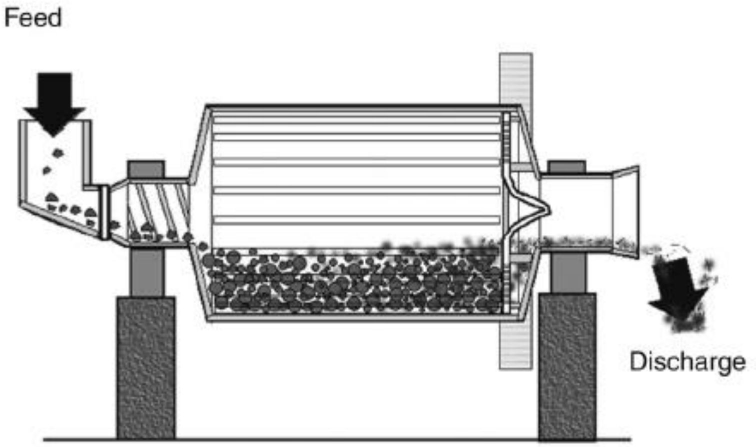


Mechanism of Crushing in Tubular Mills

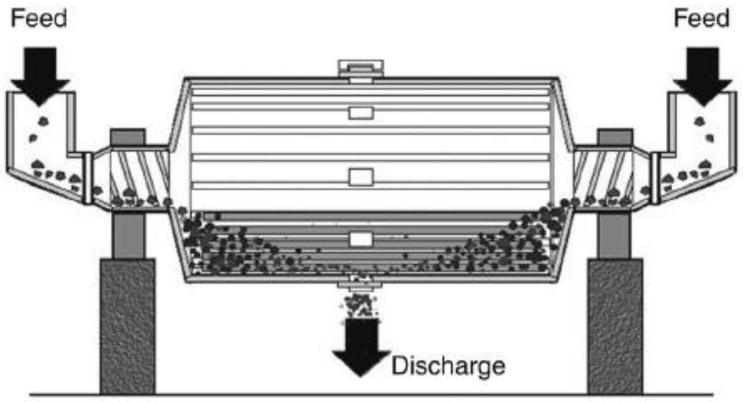
Tubular Mill Types based on discharge



A: Overflow discarge mill



B: Diaphragm or grate discharge mill



C: Centre-periphery discharge mill

Basic design parameters of the tubular mills

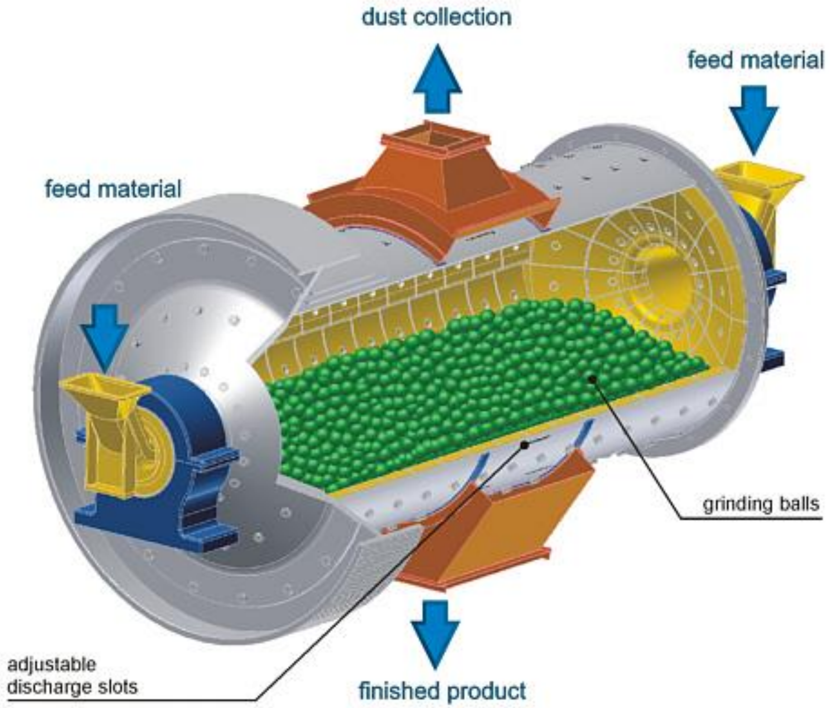
1. **Size:** diameter × length,
2. **Feed system:** one hopper feed, diameter 40–100 cm at 30° to 60° entry angle and top of feed hopper at least 1.5 m above the center line of the mill for ease of entry of feed,
3. **Feeder:** single or double helical scoop feeder or a spout feeder,
4. **Discharge system:** one exit unit, about 5–110 cm lower than the center line for overflow mills.

Tumbling mill characteristics

Parameter	Rod Mill	Ball Mill	Autogenous Mill
Length/diameter ratio*	1.4–1.8	0.5–3.5	0.25 to 0.5:1
Feed size	2.5 cm max	−1.9 cm −1.25 cm to 0.9 cm	Course ore Normal ore
Reduction ratio	15:1 to 20:1	20: 1 to 200:1	

*Multicompartment mills: length/diameter ≈ 5.

Ball Mills


dust collection
feed material
feed material
grinding balls
adjustable
discharge slots
finished product

Operations

According to Morrell, the size reduction is proportional to the ball mass and surface area and that

Impact breakage $\propto M_B$

Attrition breakage $\propto 1/S_B$, where M_B is the ball mass and S_B is the ball surface area.

The size reduction will depend on:

1. the charge characteristics (mass, volume, hardness, density, size distribution),
2. the characteristics of the grinding media (mass, density, number, ball size distribution),
3. speed of rotation of the mill,
4. slurry density when wet grinding is adopted.

Morrell S., in Comminution: theory and practice, In: Kawatra SK, editor, Littleton, Colorado: SME; 1992. pp. 369–380.

Charge characteristics

1. Charge Volume

$$J_R = \frac{M_R/\rho_S}{V_M} \times \frac{1}{1-\varphi}$$

$$J_B = \frac{M_B/\rho_B}{V_M} \times \frac{1}{1-\varphi}$$

J_R = the fraction of the mill volume occupied by the bulk rock charge and

J_B = the fraction of the mill volume occupied by the bulk ball charge.

φ *is the* pore volume to its total volume of feed and balls

Masses of the rocks and balls were M_R and M_B respectively

V_M is the internal volume of the mill

φ *is the* porosity of the bed containing crushed rock and crushing medium (balls)

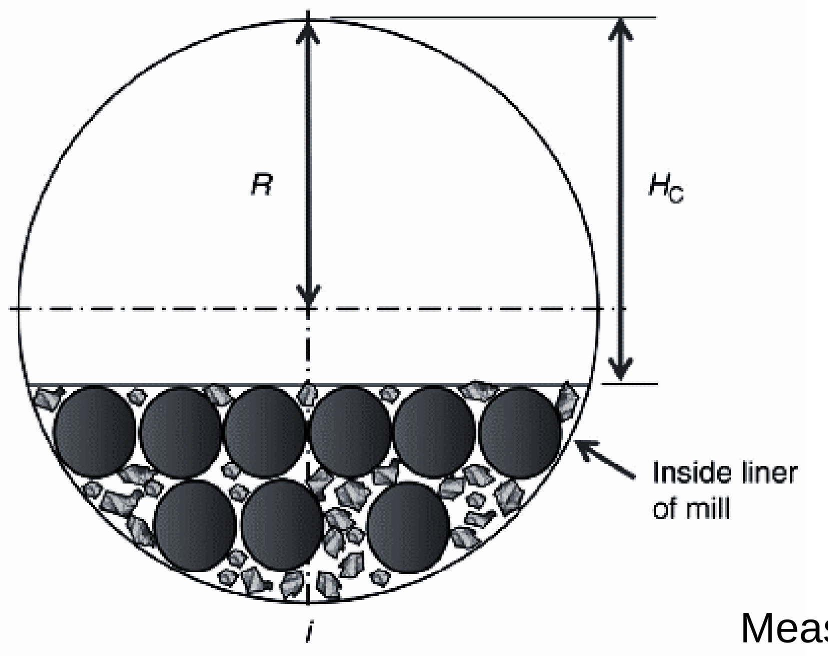
In practice, the preferred ratio of $J_R/J_B \sim 0.4$

2. Charge Height

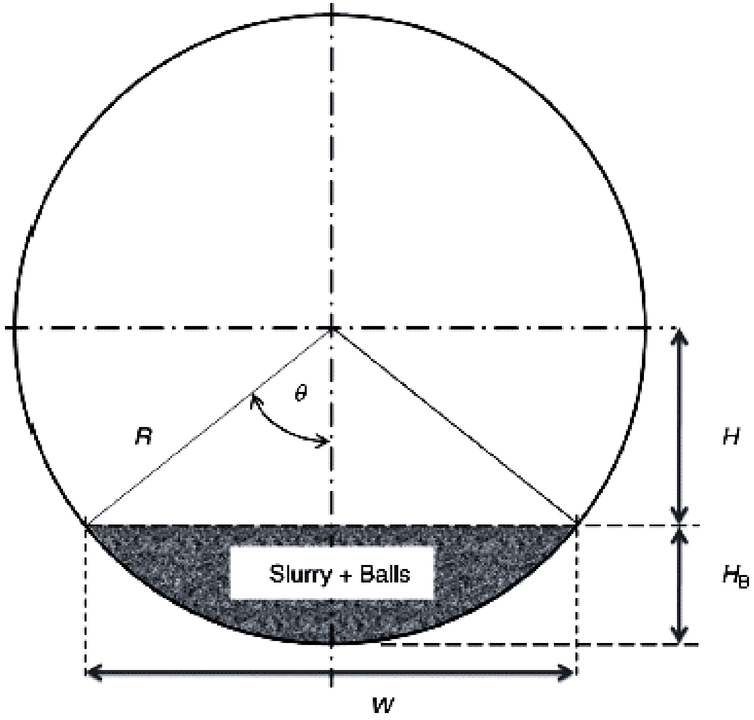
Measurement of the charge height within a mill is a convenient method to estimate the charge volume.

As a general rule:

1. For over-flow ball mills the charge should not exceed 45% of the mill volume,
2. For grate discharge mills the charge should occupy about 50% of the mill volume.



Measurement of Bed Depth



A statistical relationship

$$\text{Charge\%} = 113 - (63H_c/R)$$

The bed height, H_B, is taken from the bottom of the mill. This height will be given by

$$H_B = R(1 - \cos\theta) \quad \text{and} \quad W = 2\sin\theta$$

It is difficult to measure the angle θ in practice

From simple geometry, the segment of a shaded part of the circle is given by the equation

$$A_C = R^2 \cos^{-1}\left(\frac{H}{R}\right) - H\sqrt{R^2 - H^2}$$

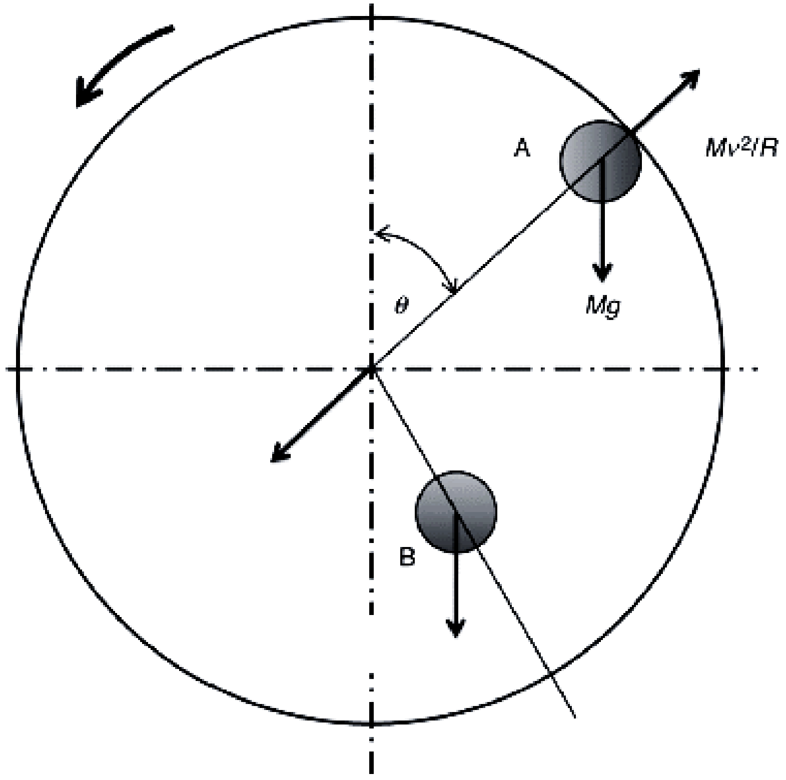
Where $$H = R - H_B$$

and $$W = 2\sqrt{R^2 - H^2}$$

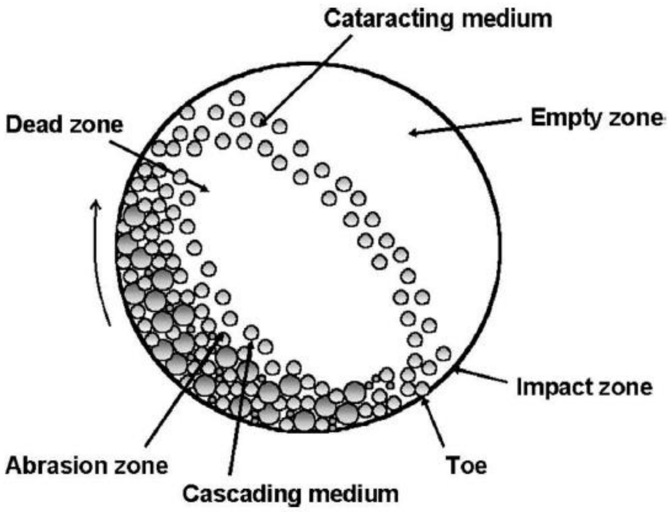
Since the cross sectional area of the mill is πR^2, the volume fraction filled by the charge would be

$$J_B = \frac{A_C}{\pi R^2}$$

Mill Rotation and Critical Speed



Equilibrium Forces on a Ball Held Against the Mill Liner
Due to the Mill Rotation



If we shall only consider position A where at equilibrium

$$mg\cos\theta = \frac{Mv^2}{(R-r)}$$

Where, *M* = mass of the ball
g = acceleration due to gravity (m/s^2)
u = angle that the ball subtends with the vertical
v = linear velocity (m/s) of the ball and
R, *r* = radii of the mill and ball, respectively

At the rotational speed ω, $v = 2\pi(R - r)\omega/60$ rpm

Substituting

$$\cos\theta = \frac{[2\pi(R-r)\omega]^2}{9.81(R-r)60^2}$$

At u = 0, cos θ=1, the force of gravity tending to pull the ball off the wall will be at a maximum and the speed required to overcome this force is known as the critical speed.

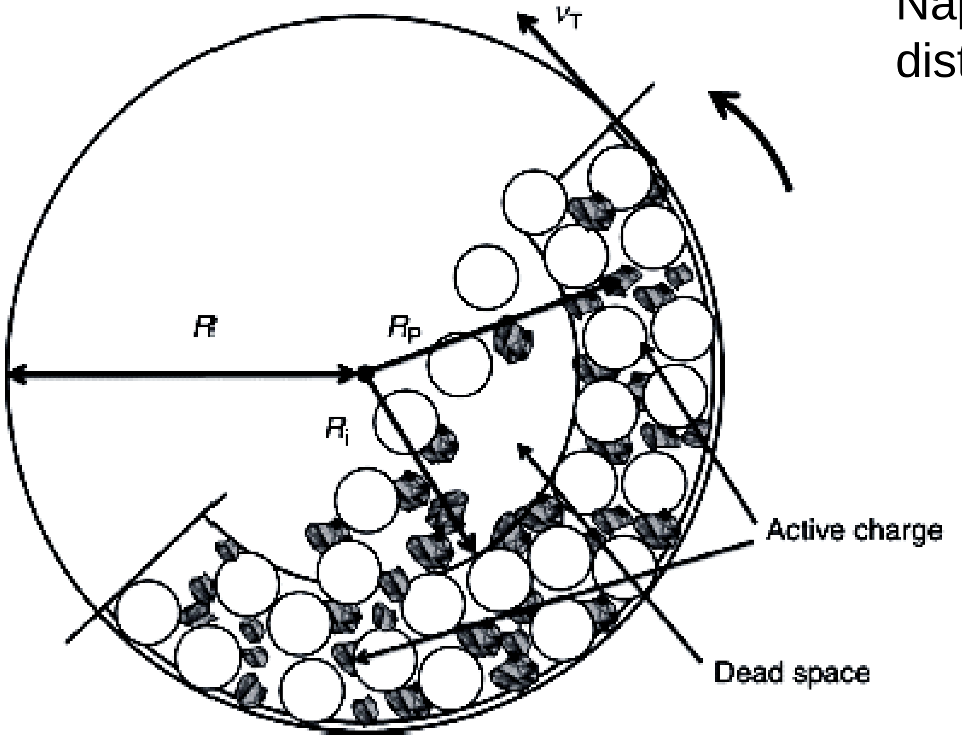
Denoting the critical speed as v_c,

$$v_c = \frac{42.3}{\sqrt{(D-d)}} \quad revs/\min$$

If D and d are taken in feet, the same expression is reduced to

$$v_c = \frac{76.65}{\sqrt{(D-d)}} \quad revs/\min$$

The simplified consideration does not cover the entire charge profile (position B) that is in motion within the tumbling mill particularly as the velocity of particles at the periphery and nearer the shell wall will be different from those nearer the centre.



Napier-Munn et al. derived the rotational speed of a particle at a distance R_P from the centre as

$$\omega_p = \frac{v_N R(R_P - \zeta R_i)}{R_P(R - \zeta R_i)}$$

where v_N = the normalized tangential velocity = v_R/v_T

v_R, v_T = the tangential velocities at position R_P and the inside liner surface

R_P = the radial distance of any particle P located in the active region of the charge

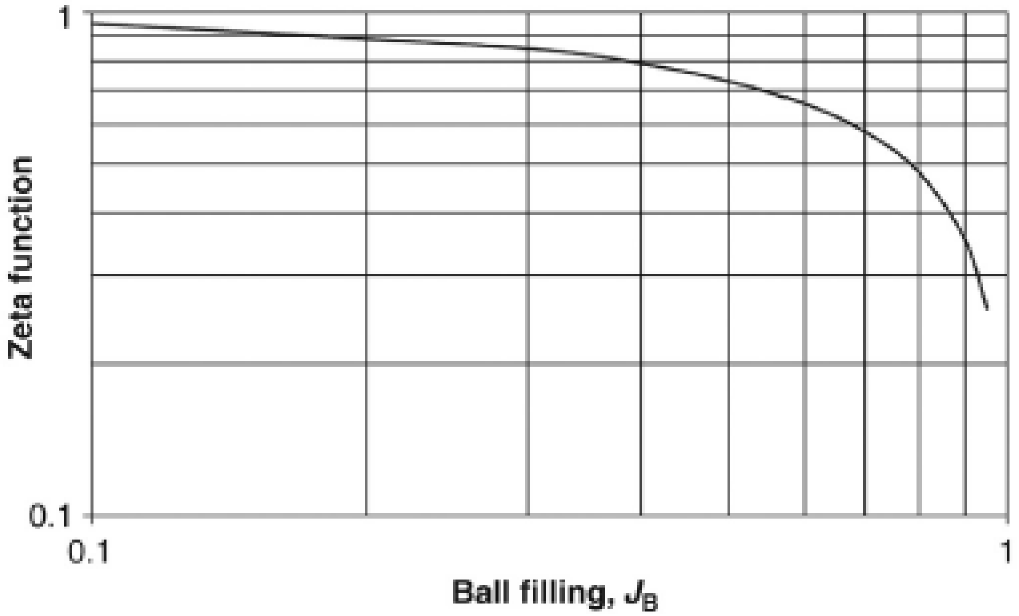
R_i = the radial distance to the inner radius of the active charge in the mill

Napier-Munn TJ, Morrell S, Morrison RD, Kojovic T. JKMRC monograph series in mining and mineral processing. University of Queensland, Brisbane 1999;2.

The term ζ is a function of the volumetric filling of the mill, J_B

$$\log \zeta = 0.4532 \log(1 - J_B)$$

It can be seen that as J_B approaches 1, ω_P approaches v_N.



Relation Between J_B and ζ from equation

Problem 7

In a ball mill of 2000mm diameter, 100mm diameter steel balls are being used for grinding at a speed of 15 rpm. At what speed will the mill have to run if the 100mm balls are replaced with 50mm balls, all the other conditions remaining same?

Ans: Operating speed of the mill = 14.79 rpm

Estimation of Mill Capacity

The capacity of a ball mill depends on its

- dimensions
- the type of mill (overflow or grate discharge)
- the speed at which the mill rotates
- the mill loading
- the product size required from a given feed size
- the work index
- the mill shaft power and specific gravity of the rock

Bond expressed the relation by an empirically derived equation

$$Q = \frac{P_M}{E} \quad t/h$$

Where,
P_M = the mill power (kW) and
E = the energy required fro crushing (kWh/t) (From Bond’s law)

In deriving the value of P_M

1. **Bond Method**

From the results of a large number of observations, Bond established that the power drawn by a mill was

- directly proportional to the length of the mill
- a function of the mill speed
- a function of the total mass of the grinding media plus the rock charged
- a function of the feed characteristics and
- a function of the work index of the material

Bond found that the mill power did not vary linearly with speed but varied linearly with a factor F_C,

$$F_c = 100\phi_c\left[1 - \frac{0.1}{2^{9-10\phi_c}}\right]$$

ϕ_C = fraction of theoretical speed at which the mill is operated

Using this relationship, with a large number of laboratory and industrial mills, Bond proposed the following empirical equation to compute the *shaft power* for mills as

$$P_S = 7.33 J_B \phi_c (1 - 0.937 J_B) \rho_b L D^{2.3} \left(1 - \frac{0.1}{2^{9-10\phi_c}}\right)$$

where P_S = power at the mill shaft in kW for L, D in meters and ρ_b in t/m^3.

Bond empirically determined the power required for wet grinding and expressed it in terms of unit mass of the grinding media (M_B) as

$$\frac{P_M}{M_B} = 15.6 D^3 \phi_c (1 - 0.937 J_B) \left(1 - \frac{0.1}{2^{9-10\phi_c}}\right) \quad kW/t$$

The work of Austin et al. indicates that Bond's Equation does not adequately describe mill shaft power when the mill diameters were less than 2.4 m. They found that

$$\frac{P_M}{M_B} = \frac{13.0D^{0.5}(\phi_c - 0.1)(1 - 0.937J_B)}{(1 + 5.95J_B^5)(1 + \exp[15.7(\phi_c - 0.94)]}\quad kW/t$$

where D is in meters and M_B is in metric tones.

Several other methods like ***Nordberg (Metso) Method, Rose and Sullivan Method*** *etc.* are also available for the estimation of $\mathbf{P_M}$

Austin LG, Klimpel RR, Luckie PT. Process engineering of size reduction: ball milling. New York: SME/AIME; 1984.

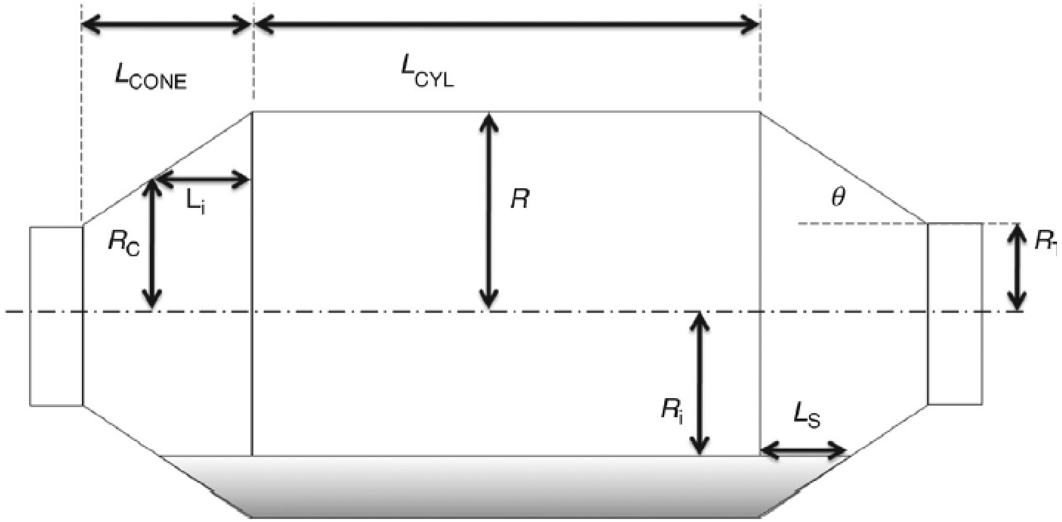
Theoretical Mill Power

Morrell approach

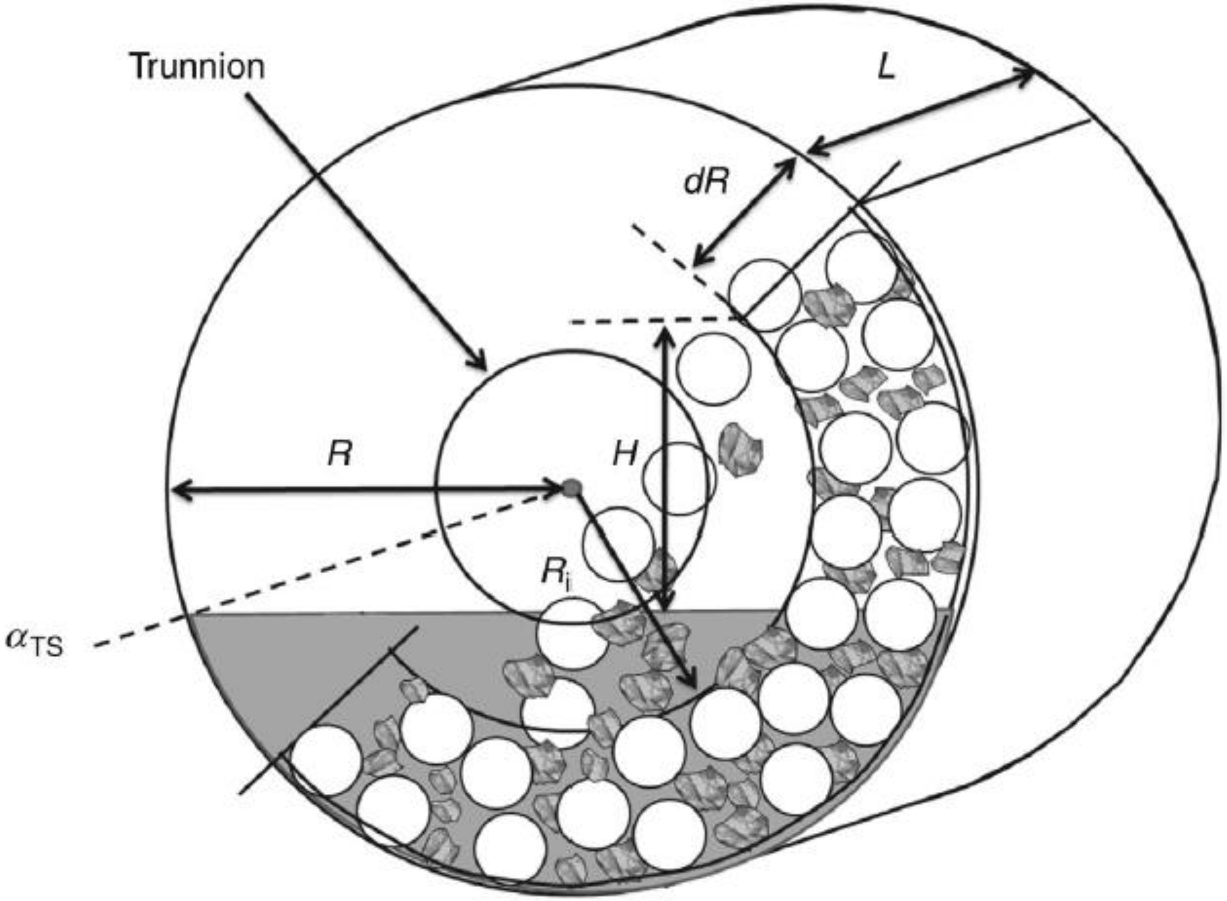
Assumptions

- The power drawn was related to the transference of kinetic and potential energies from the rotating mill to the grinding media and charge which was translated to kinetic and potential energies of the charge as it moved within a rotating mill
- The heat and sound energy produced was neglected.
- In wet grinding the slurry in the center did not affect the torque of the mill shaft as the load was distributed evenly around the center.
- The mass of the slurry influenced the friction between the charge and the mill lining and therefore affected the torque, but its magnitude was small and therefore neglected.
- The total power was considered as the sum of the power required at the cylindrical section plus the power required by the two conical end sections plus the power to rotate an empty mill.

Morrell S. Trans Inst Min Metall 1996;105. C43, C54.



Schematic Diagram of a Ball Mill



Position of Balls, Solids and Slurry in a Wet Overflow Ball Mill.

No load power $= 1.68[D^{2.5}\phi_c(0.667 L_{CONE} + \mathrm{L}_{CYL})]^{0.82} \quad kW$

where D, L_{CONE} and L_{CYL} are in meter.

Power required for cylindrical section of Ball mills: Grate Mill

The mass flow rate through the element would be

$$Q_M = v_T \rho_C L dR$$

where v_T = tangential velocity of a particle located at any distance R from the center (m/s) and

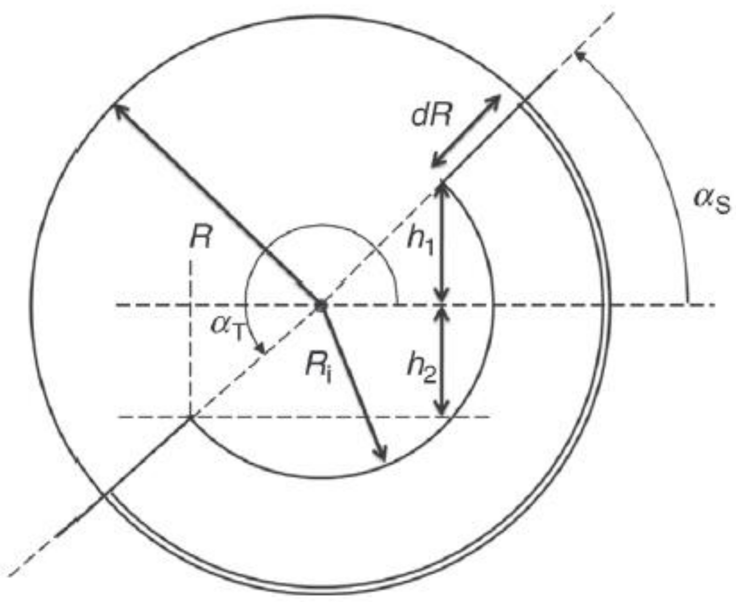
ρ_C = density of the total charge (t/m^3).

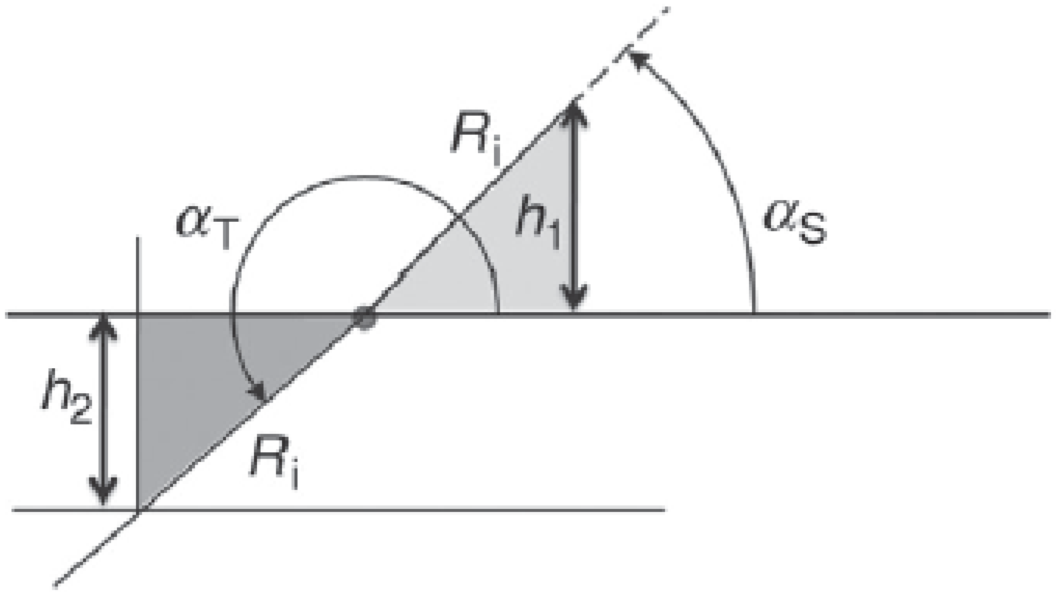
The associated rate of potential energy would be

$$PE = v_T \rho_C L dRgH$$

H is the height to which particles and are raised

From Figures $$H = h_1 + h_2 = R_i(\sin\alpha_s - \sin\alpha_T)$$





The rate of imparting kinetic energy (KE) to the particles would be

$$KE = \frac{1}{2}(v_T \rho_C LdR)v_T^2 = \frac{v_T^3 \rho_C ldR}{2}$$

Hence, the rate at which the total energy, E_T, is generated will be

$$E_T = v_T \rho_C LdRgH + \frac{v_T^3 \rho_C ldR}{2}$$

At any radial distance, R_P, in the charge, the tangential velocity is given by

$$v_T = 2\pi R_P \omega_P$$

Where, ω_P = is the rate of rotation of a particle at the radial distance, R_P, in the charge in revolutions/s.

Napier-Munn et al. derived the rotational speed of a particle at a distance R_P from the centre as

$$\omega_p = \frac{v_N \mathrm{R}(\mathrm{R}_P - \zeta \mathrm{R}_i)}{\mathrm{R}_P (R - \zeta R_i)}$$

where $$\log \zeta = 0.4532 \log(1 - J_B)$$

and J_B = volumetric filling of the mill

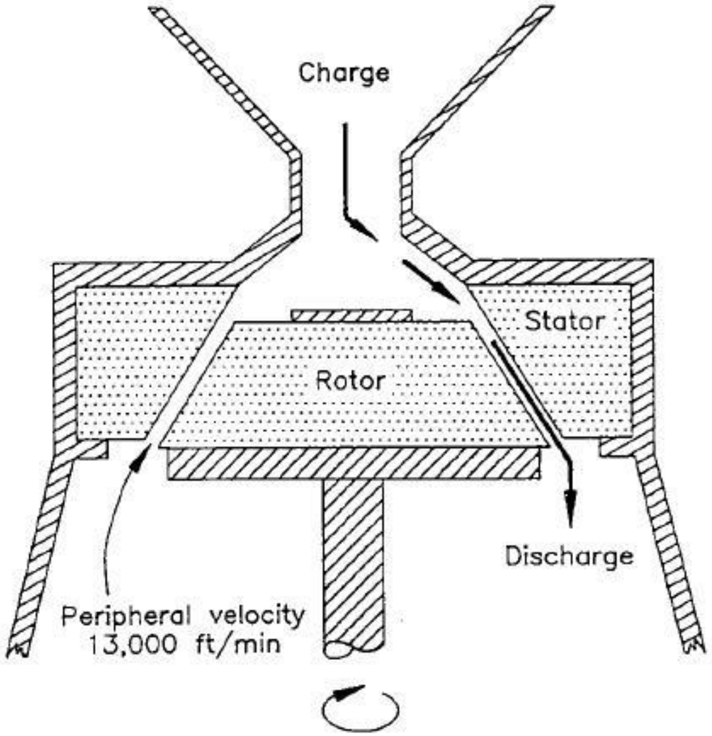
Substituting this value of v_T in terms of ω_P, $R^/$ and H and integrating between the limits R_i and R gives the power, P_{CYL}, for the cylindrical portion of the mill as

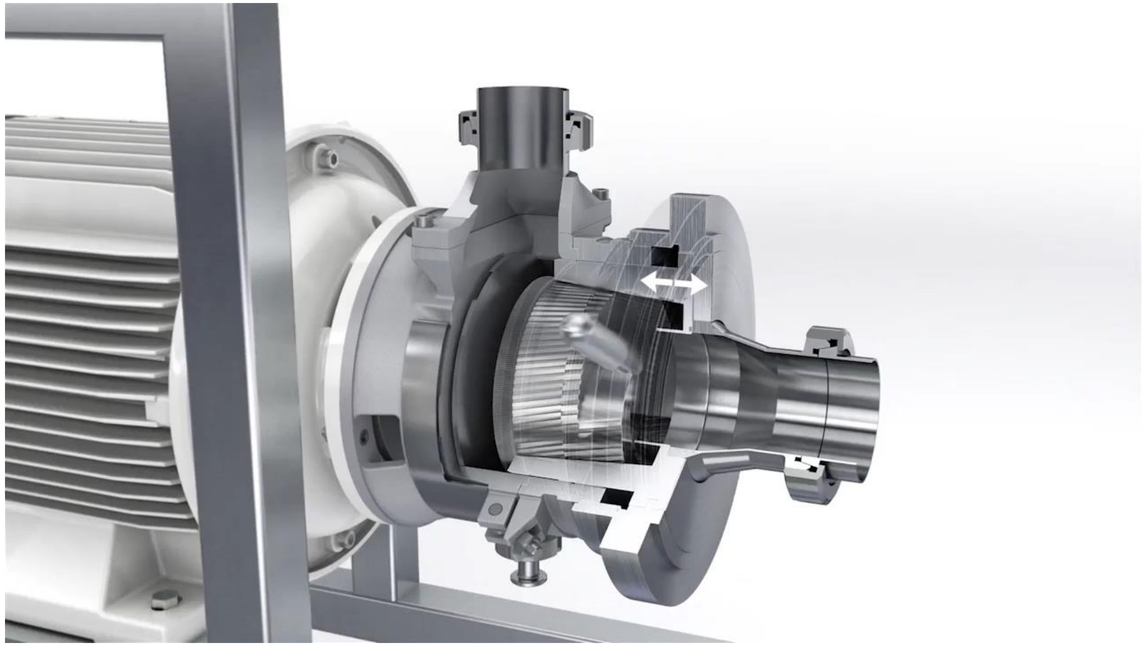
$$P_{CYL} = \frac{\pi g L \rho_C \omega R}{3(R - \zeta R_i)}\left[2R^3 - 3\zeta R^2 R_i + R_i^3(3\zeta - 2)\right](\sin\alpha_s - \sin\alpha_T) + L\rho_C\left[\frac{\omega R \pi}{(R - \zeta R_i)}\right]^3\left[(R - \zeta R_i)^4 - R_i^4(\zeta - 1)^4\right]$$

where P_{CYL} is in kW for L, R and R_i in meters, ρ_C in t/m^3 and the fraction of the mill volume filled, ζ,

Ultrafine Grinder

Colloid mill





Working principle

- Colloid mill works on the principle of sharing

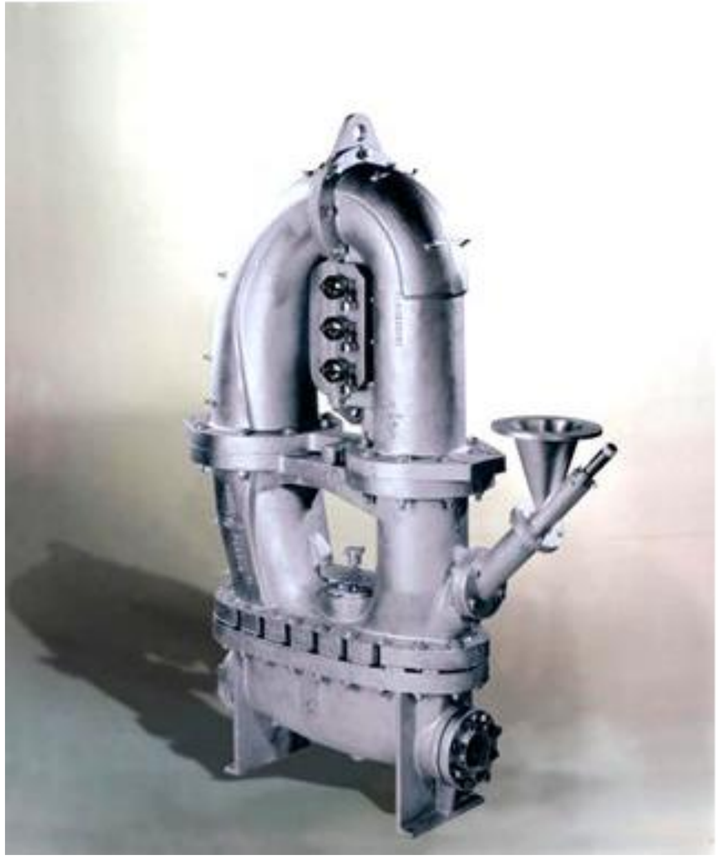
- The main function of the colloid mill is to ensure a breakdown of agglomerate or in the case of emulsions to produce droplets of fine size around 1 micron.

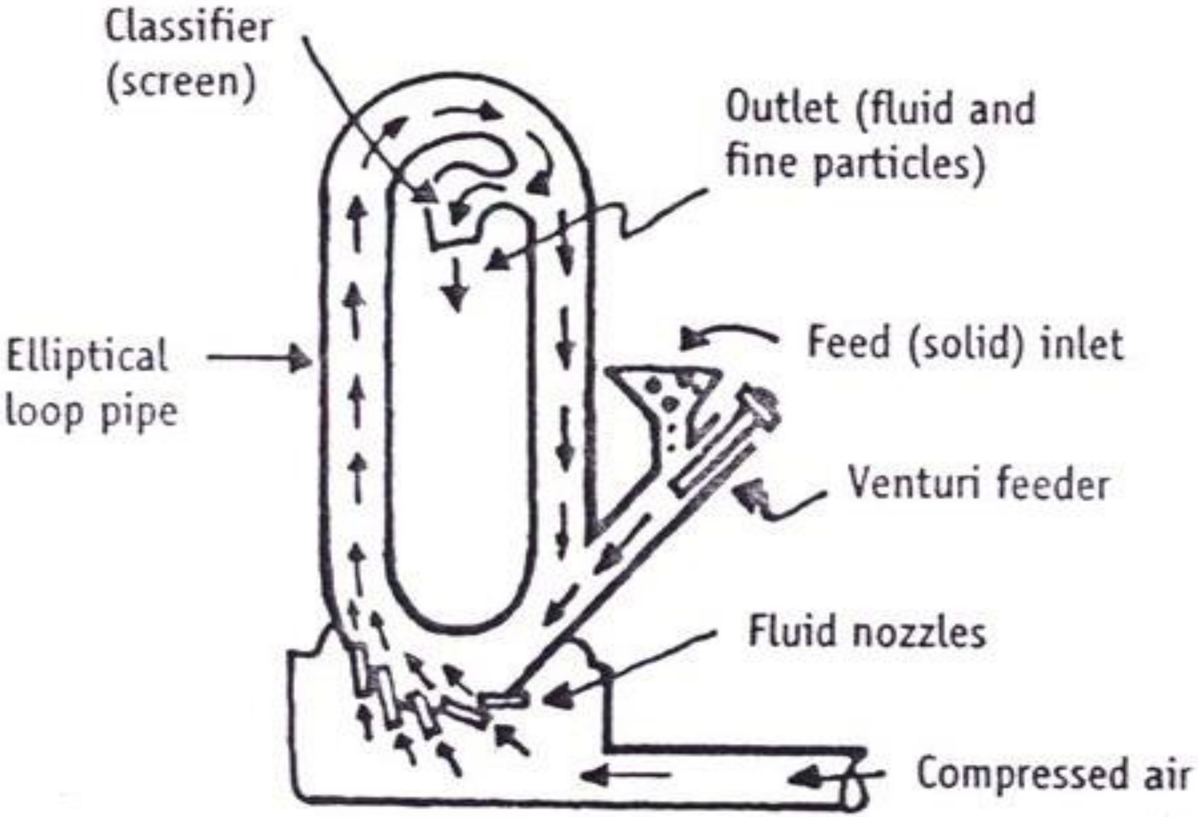
- Rotational speed of the rotor varies from 3000-20,000 rpm with the spacing between the rotor and stator capable of fine adjustment varying from 0.001 inch to 0.005 inch depending on the size of the equipment.

- The feed being forced through the narrow clearance by centrifugal action taking a spiral path.

- Most of the colloid mills are fitted with water jackets and it is also necessary to cool the materials before and after passing through the mill.

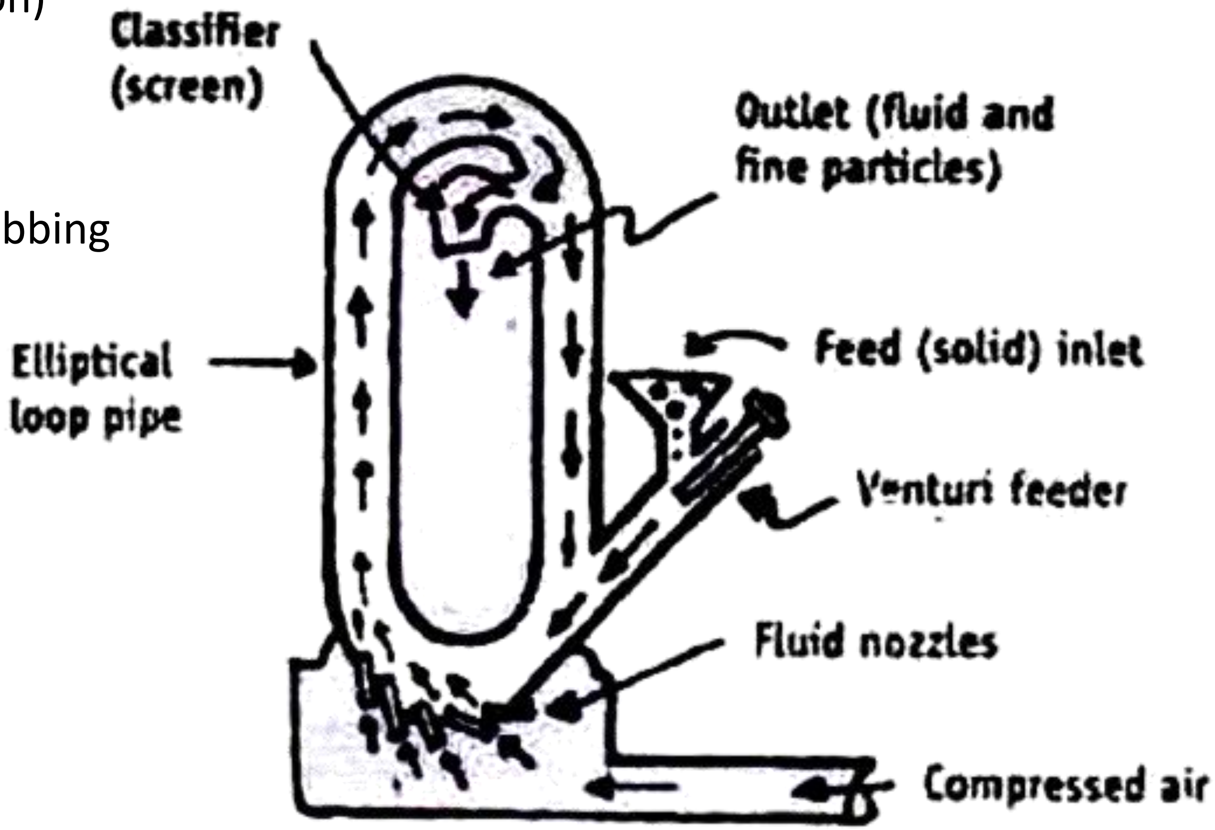
Fluid energy mill





Fluid energy mill

- Particle suspended in high velocity gas stream (fluidization)
- Elliptical or circular path.
- Size reduction $\Rightarrow$ inter particle interaction + striking + rubbing
- Coarse particle remain in circulation
- Feed ~ 1 ton/hr (not sticky)
- Product $\Rightarrow$ 0.5 to 10 μm
- 1 to 4 kg of steam or 6 to 9 kg of air per kg of product



Wet vs. Dry Grinding

There are two types of grinding practices i.e. “wet” and “dry” grinding. The selection of the process is mainly governed by type of material and use of material. If water changes physical or chemical properties of the material, dry grinding is a must. Conversely, if the finished material is to be used as a suspension in water and classified by hydraulic method, then we prefer wet grinding.

Wet grinding has got the following advantages:

1. Lower power consumption per ton of product (15-20%)
2. Higher capacity per cubic foot of mill volume
3. Elimination of dust problem
4. Use of simple handling methods such as pumps, pipes etc
5. Wet screening or hydraulic classification can be used for closer product control.

The following two points are in favor of dry grinding:

1. lower wear rate of mill liners and grinding media compared to wet grinding
2. Higher percentage of fines in mill product. In many cases, this is desirable e.g. cement, talcum powder, pulverized coal *etc.*