MS31007: Chapter 8

Electrical Properties

Part II

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Ref. Book Understanding Solids by Richard J. D. Tilley

Content

ISSUES TO ADDRESS...

- How are electrical conductance and resistance characterized?

- What are the physical phenomena that distinguish conductors, semiconductors, and insulators?
- For metals, how is conductivity affected by imperfections, *T*, and deformation?
- For semiconductors, how is conductivity affected by impurities (doping) and *T*?

- Define dielectric constant in terms of permittivity.
- Name and describe the three types of polarization.
- Briefly describe the phenomena of *ferroelectricity* and *piezoelectricity*.
- **What are the foundation of electronic devices**

For a p–n junction, explain the rectification process in terms of electron and hole motions

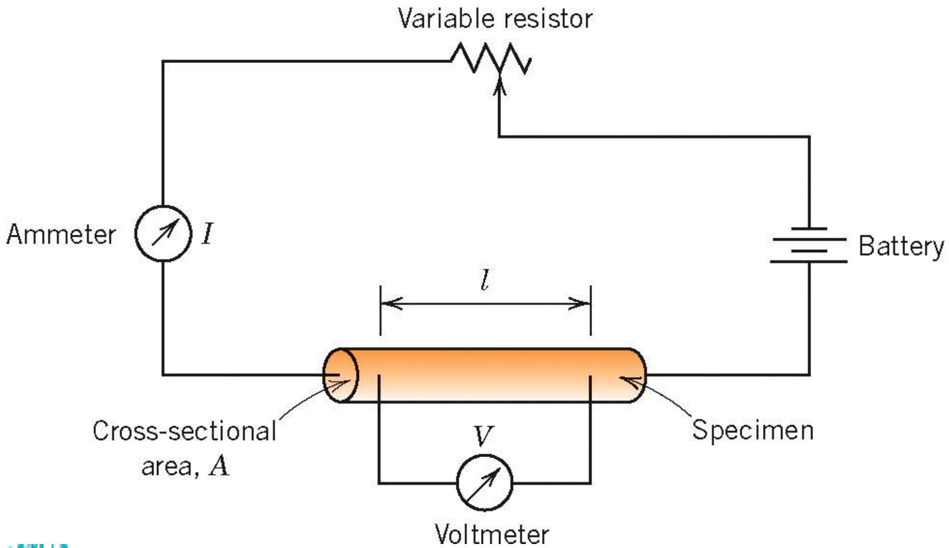
Electrical Conduction

When an electrical potential **V** [volts, J/C] is applied across a piece of material, a current of magnitude ***I*** [amperes, C/s] flows. In most metals, at low values of **V**,

Ohm's law can be described by : **V = I** x **R**

where R is the electrical resistance [ohms, Ω]. R depends on the **intrinsic resistivity ρ** of the material [Ω-m] and on the geometry (length l and area A through which the current passes)

- *R* – resistance, influenced by specimen configuration, often independent of current



Resistivity ρ

$$\rho = \frac{RA}{l} \Longrightarrow \rho = \frac{VA}{lI}, \quad \rho [=] \Omega - m$$

In most materials (e.g. metals), the current is carried by electrons (**electronic conduction**). In ionic crystals, the charge carriers are ions (**ionic conduction**).

Electrical conductivity

Typically the electrical conductivity is used to specify the electrical properties of materials

$$\sigma = \frac{1}{\rho}, \quad \sigma[=](\Omega - m)^{-1}$$

- Conductivity describes the ease with which a material is capable of conducting an electrical current
- Can also express Ohm's law as

$\boxed{J = \sigma E}$ <= another way to state Ohm's law

$J \equiv$ current density $= \dfrac{\text{current}}{\text{surface area}} = \dfrac{I}{A}$ like a flux

$E \equiv$ electric field intensity-potential $= V/\ell$ or $(\Delta V/\Delta \ell)$

$$J = \sigma\,(\Delta V/\Delta \ell)$$

Electron flux conductivity voltage gradient

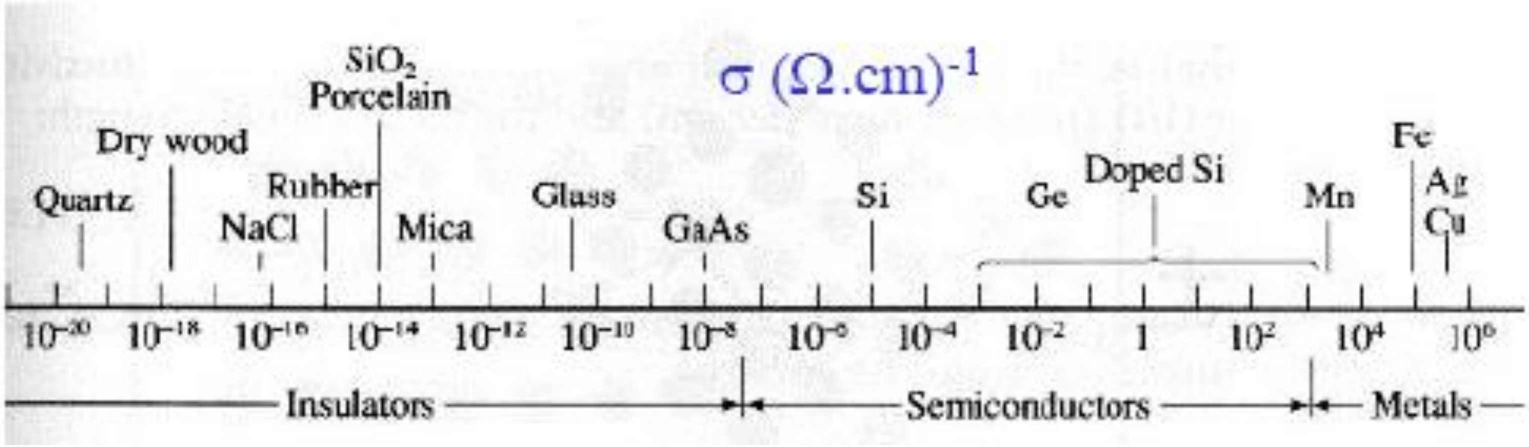
Electrical Properties of Solids

Electronic/ionic conduction: *Electric current results from the motion of electrically charged particles due to forces acting on them from an externally applied E*

- Positive charges – accelerated in field direction,
- Negatively charges – accelerated in opposite direction
- In most solids a current arises from the flow of electrons *(electronic conduction)*
- Ionic solids – a net motion of ions is possible that can produce a current *(ionic conduction)*

Typically materials fall into three categories

– **Conductors** – metals are good conductors ($\sigma \sim 10^{7}$ $(\Omega\text{-m})^{-1}$)
– **Insulators** – poor conductors ($\sigma \sim 10^{-10} – 10^{-20}$ $(\Omega\text{-m})^{-1}$)
– **Semiconductors** – in the middle ($\sigma \sim 10^{-6} – 10^{4}$ $(\Omega\text{-m})^{-1}$)



Conductivity: Comparison

Room T values $(\text{Ohm-m})^{-1} = (\Omega\text{ - m})^{-1}$

METALS	conductors
Silver	6.8×10^{7}
Copper	6.0×10^{7}
Iron	1.0×10^{7}

CERAMICS	
Soda-lime glass	10^{-10}-10^{-11}
Concrete	10^{-9}
Aluminum oxide	$<10^{-13}$

Up to 27 orders of magnitude, possibly widest variation in materials properties

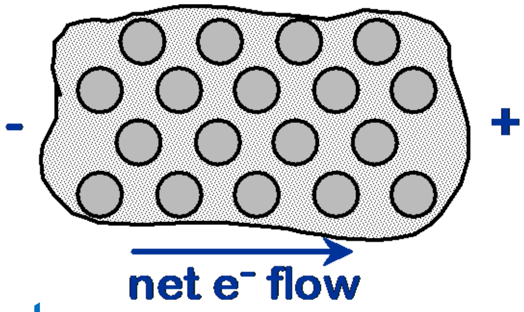
SEMICONDUCTORS	
Silicon	4×10^{-4}
Germanium	2×10^{0}
GaAs	10^{-6}
	semiconductors

POLYMERS	
Polystyrene	$<10^{-14}$
Polyethylene	10^{-15}-10^{-17}
	insulators

Energy Band Structures in Solids

Electrical conductivity in materials are strongly related to number of electrons available for conduction

- Not all electrons will move in a material under an applied potential difference !



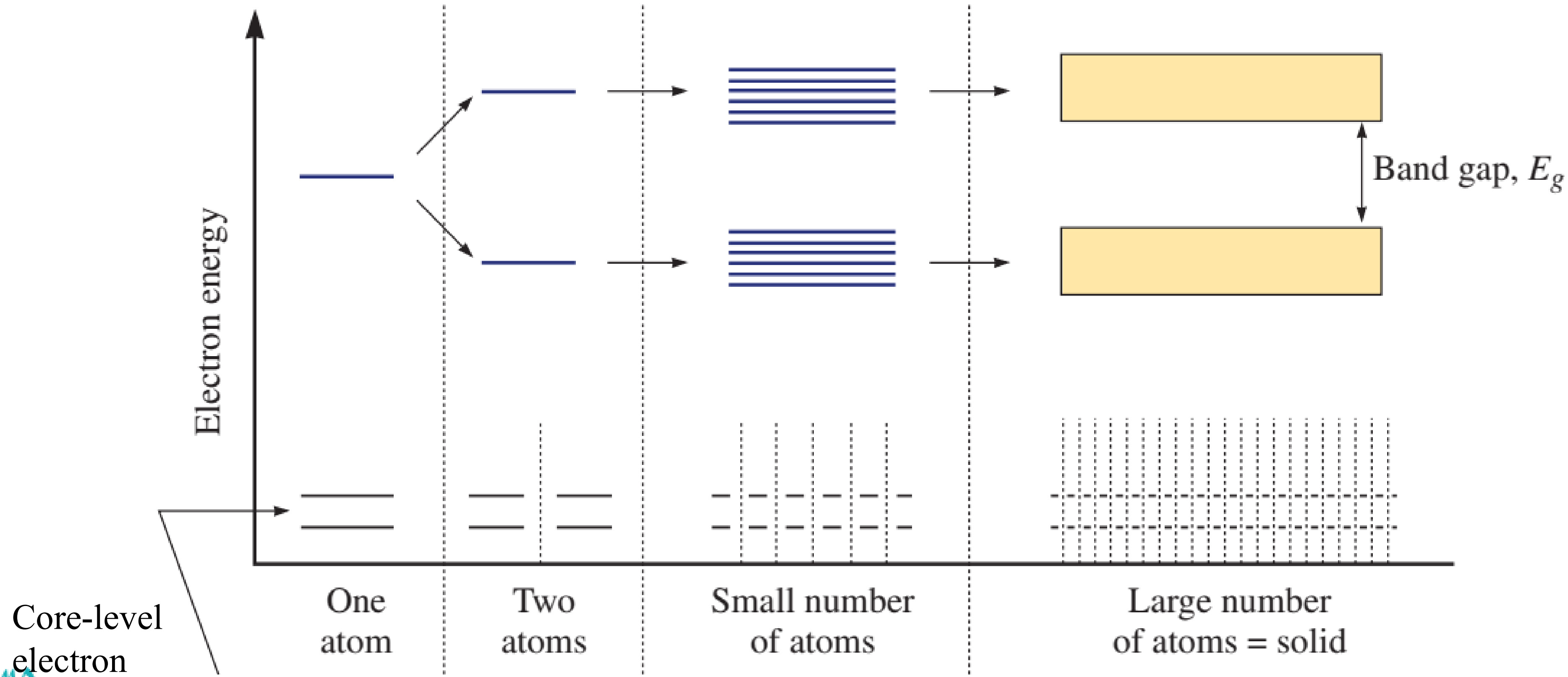
- In an isolated atom electrons occupy well defined energy states, Shells (1,2,3…), Subshells (s,p,d,f…), states in subshells, spin states

- **When atoms come together** to form a solid, a bonded regular atomic arrangement, their valence electrons interact with each other and with nuclei due to Coulombic forces.

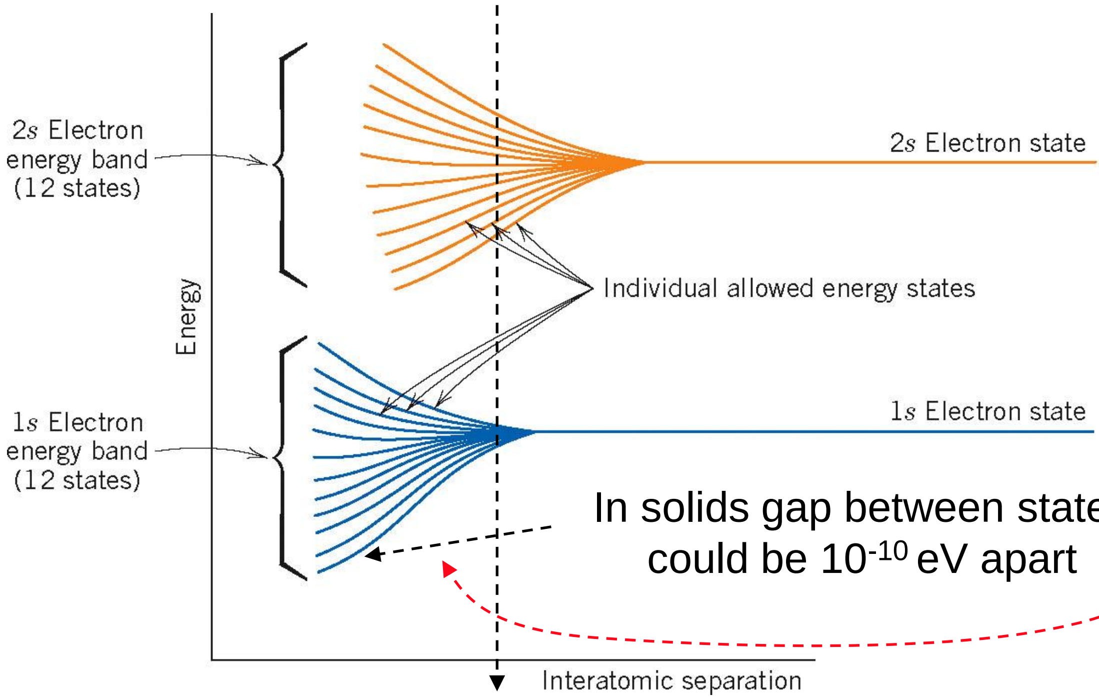
- In addition, two specific quantum mechanical effects happen.
  - First, Heisenberg's uncertainty principle, constraining the electrons to a small volume raises their energy
  - The second effect, due to the Pauli exclusion principle, limits the number of electrons that can have the same energy.

Band Structure of Solids

When atoms come together - the valence electrons of atoms at distinct energy states get split into closely spaced electron states and form wide electron energy bands when they form a solid. These **bands** are separated by **gaps**, where electrons cannot exist.



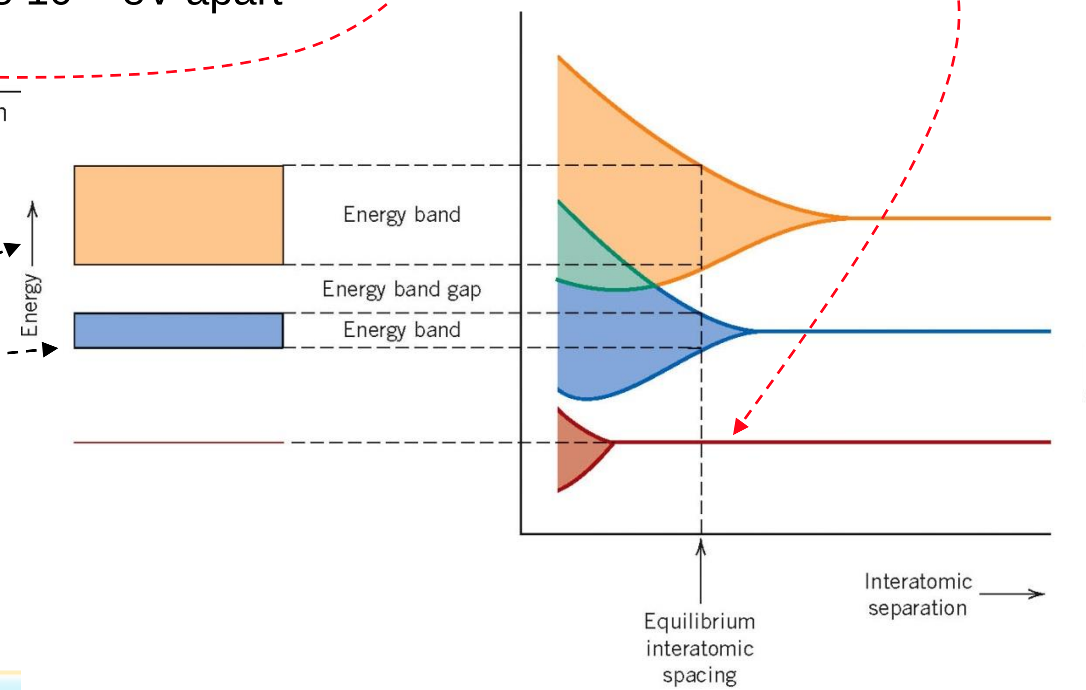
Energy Band Structures in Solids



For N atom solid
s band has N states
p band has $3N$ states

The extend of energy states splitting depends on inter-atomic distance

In solids gap between states could be 10^{-10} eV apart



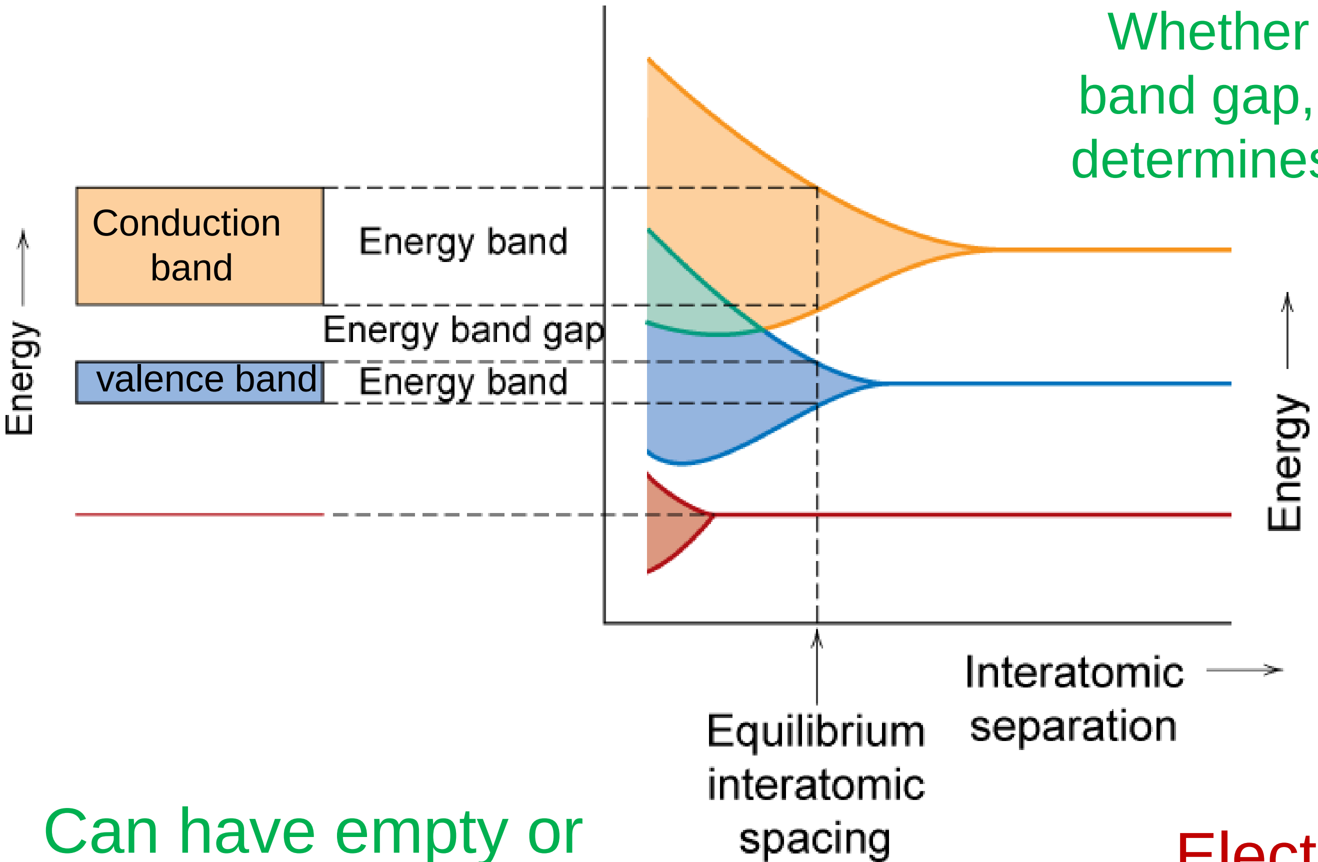
Outermost shells start forming bands as atoms get closer, however core shells may not form bands as still far from each other. Band gaps, depending upon the inter-atomic distances also start forming.

Band Structure

- Valence band – filled – highest partially or completely occupied energy levels
- Conduction band – empty – lowest unoccupied energy levels
- Fermi Energy – highest filled state at 0 K

Why does this matter?

Whether or not there is an energy band gap, and if so, how large it is, determines the electrical properties of materials. WHY?

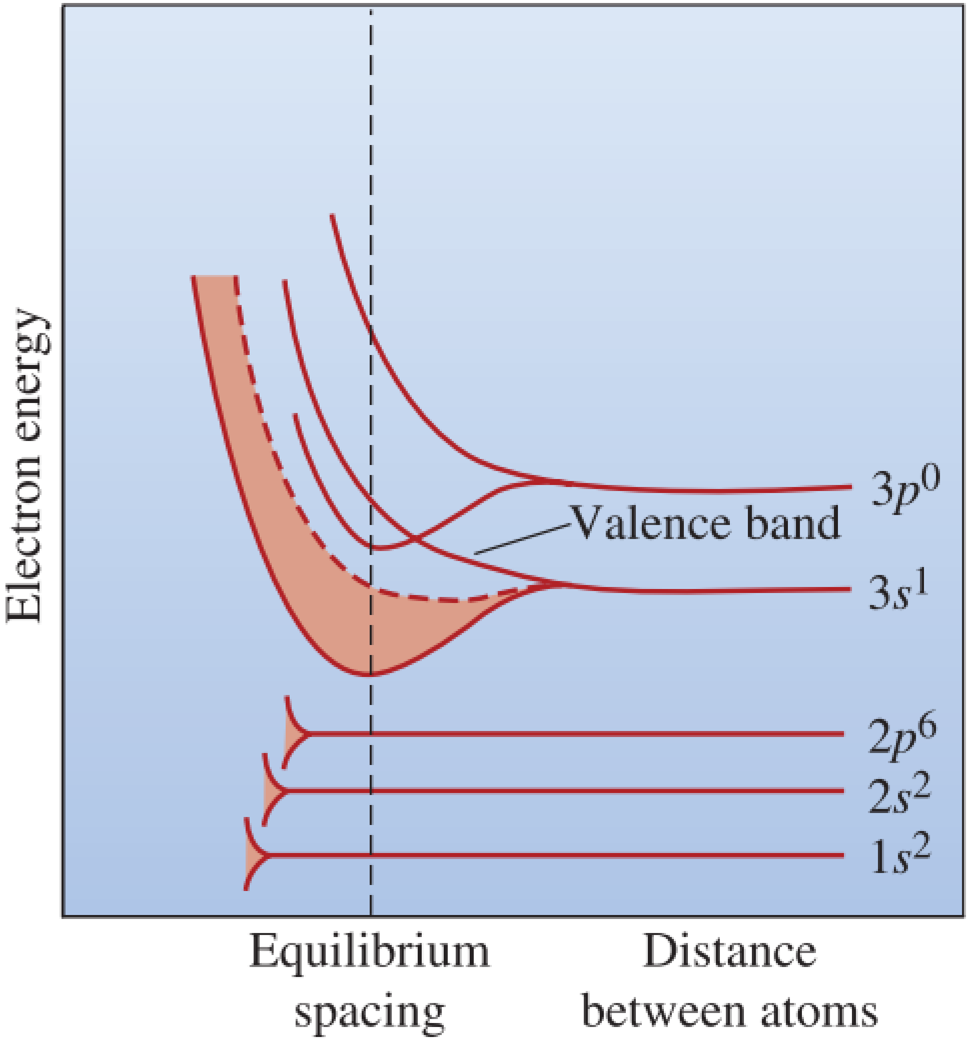


Can have empty or partially filled bands

<u>Electrical properties are a consequence of the band structure!</u>

Band Structure of Sodium

Sodium is a metal and has the electronic structure $1s^2 2s^2 2p^6 3s^1$.



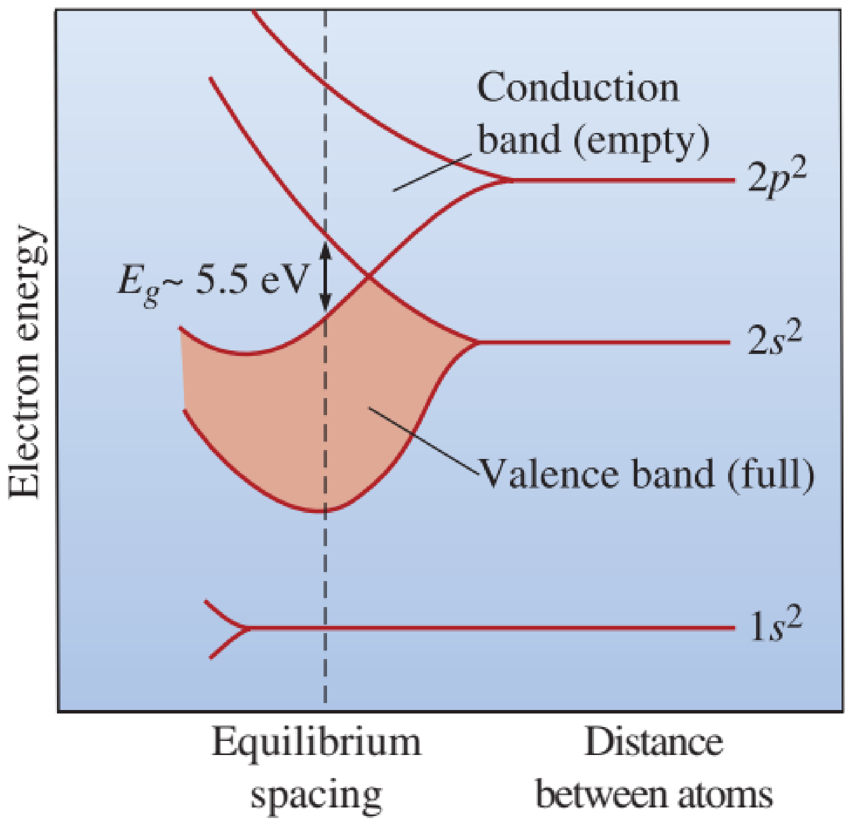
Na and other alkali metals in column ***1A*** of the periodic table have only one electron in the outermost ***s*** level. The **3s** valence band in ***Na*** is half filled and, at absolute zero, only the lowest energy levels are occupied.

The energy levels broaden into bands. The **3s** band, which is only half filled with electrons, is responsible for conduction in Na. Note that the energy levels of the orbitals in the ***1s, 2s***, and ***2p*** levels do not split at the equilibrium spacing for sodium.

Band Structure of Diamond

The elements in Group **4—C (diamond), Si, Ge, and tin**—contain two electrons in their outer *p* shell and have a valence of four. We might expect these elements to have a high conductivity due to the unfilled *p* band, but this behavior is not observed!

These elements are covalently bonded; - the electrons in the outer *s* and *p* bands are rigidly bound to the atoms- which produces a complex change in the band structure.

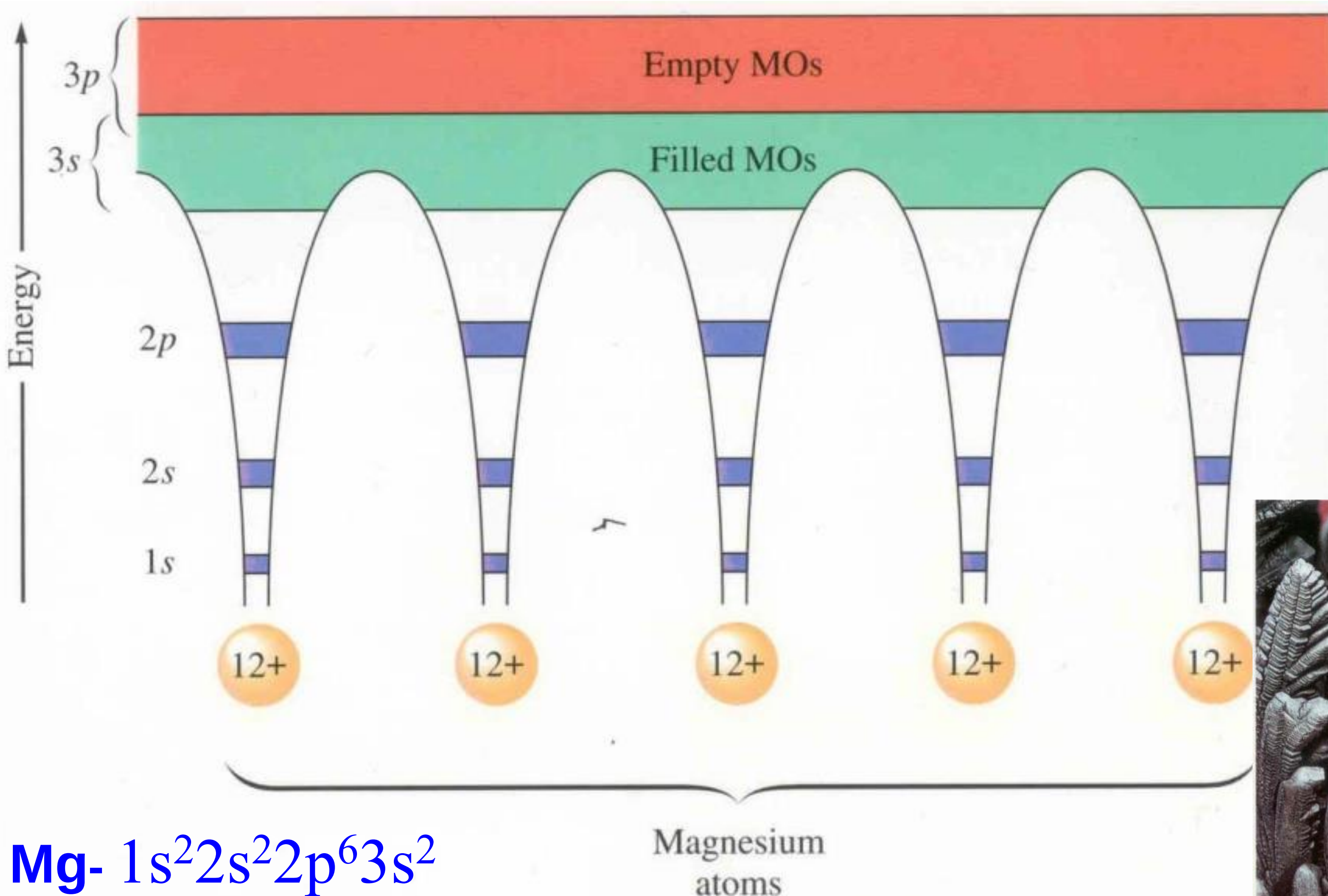


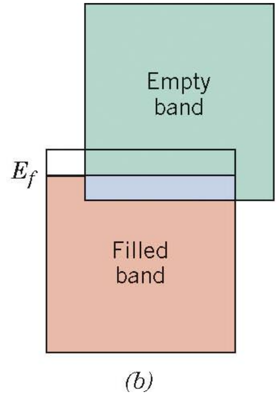
When C atoms are brought together to form solid **diamond**, the **2*s* and 2*p*** levels interact and produce two bands. Each hybrid band can contain 4*N* electrons, Since there are only 4*N* electrons available,

- The **valence** band is completely full
- **Conduction** band is empty at 0 K
- A large **energy gap** (E_g, 5.5 eV)

Electrical conductivity of Diamond is less than 10^{-18} ohm^{-1} cm^{-1}

Magnesium metal





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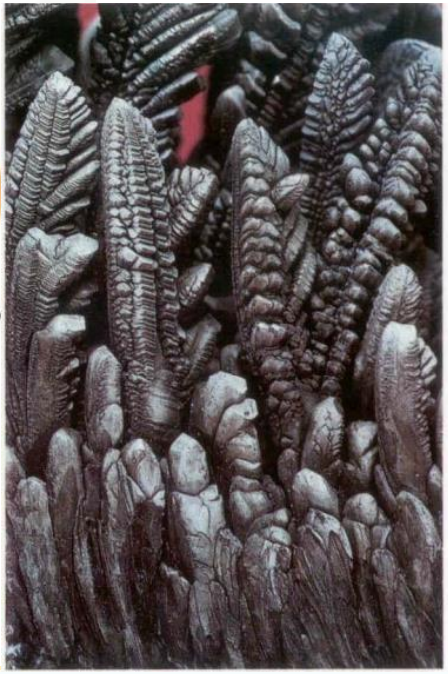
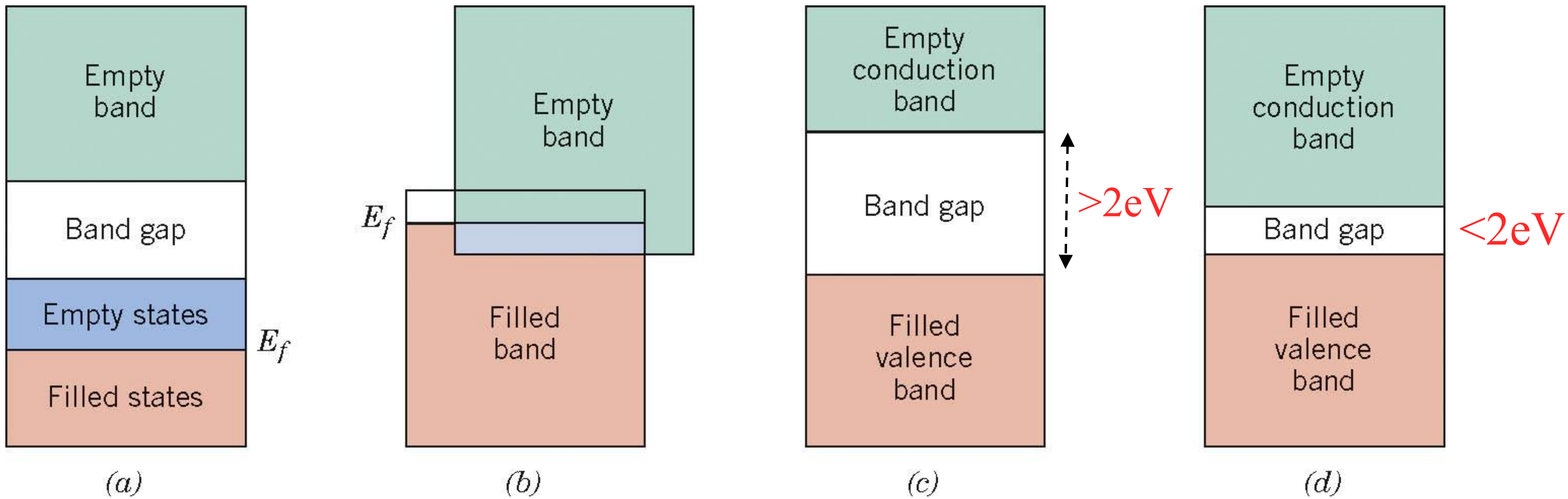


Mg- $1s^2 2s^2 2p^6 3s^2$

Mg has this band structure. Each isolated Mg atom has two 3*s* electrons. However, when a solid is formed, the 3*s* and 3*p* bands overlap.

Band Structure defines Electrical Properties

Metal/Semi-metal **Insulator** **Conductor**



Metals,

Single **s** valence electron
N atoms => 4s band capable of 2N electrons
=> but only one **s** electron means half filled band structure.

EX: Cu

Metals/semimetals

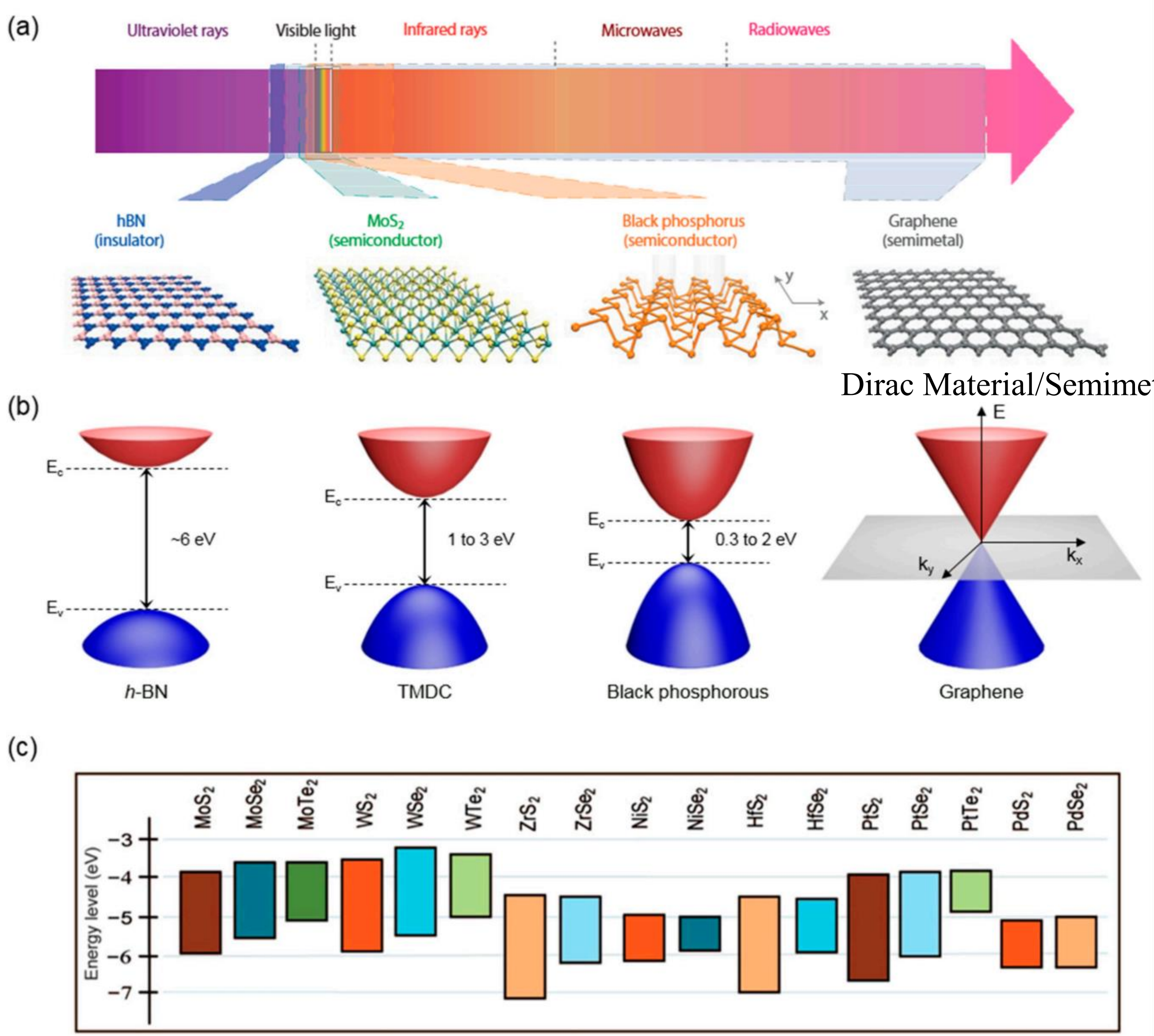
Overlap between an empty band and a filled band (e.g. **Mg-** $1s^2 2s^2 2p^6 3s^2$

Mg has this band structure. Each isolated Mg atom has two 3*s* electrons. However, when a solid is formed, the 3*s* and 3*p* bands overlap.

The valence band is filled, and no more electrons can be added (Pauli's principle).

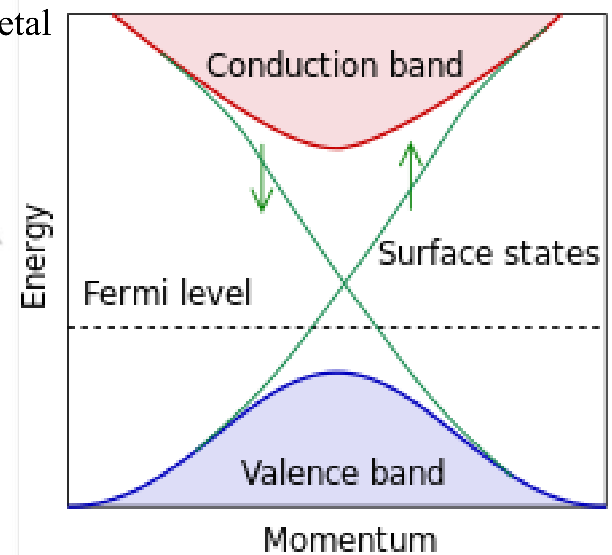
- The difference between the two band structures lies in the magnitude of the energy gap
- Large energy difference (band gap) – the material is an insulator (Large? > 2.0 eV (3.2×10^{-19} J))
- Small (< 2.0 eV) band gap – semiconductor
- Wide-band Gap Semiconductor (~ 2- 4 eV)

Band Structure of Two-Dimensional materials



Topological Insulator

is a material that behaves as an insulator in its interior but whose surface contains conducting states, meaning that electrons can only move along the surface of the material.



An idealized band structure for a topological insulator. The Fermi level falls within the bulk band gap which is traversed by topologically-protected spin-textured Dirac surface states.

Conduction in Metal

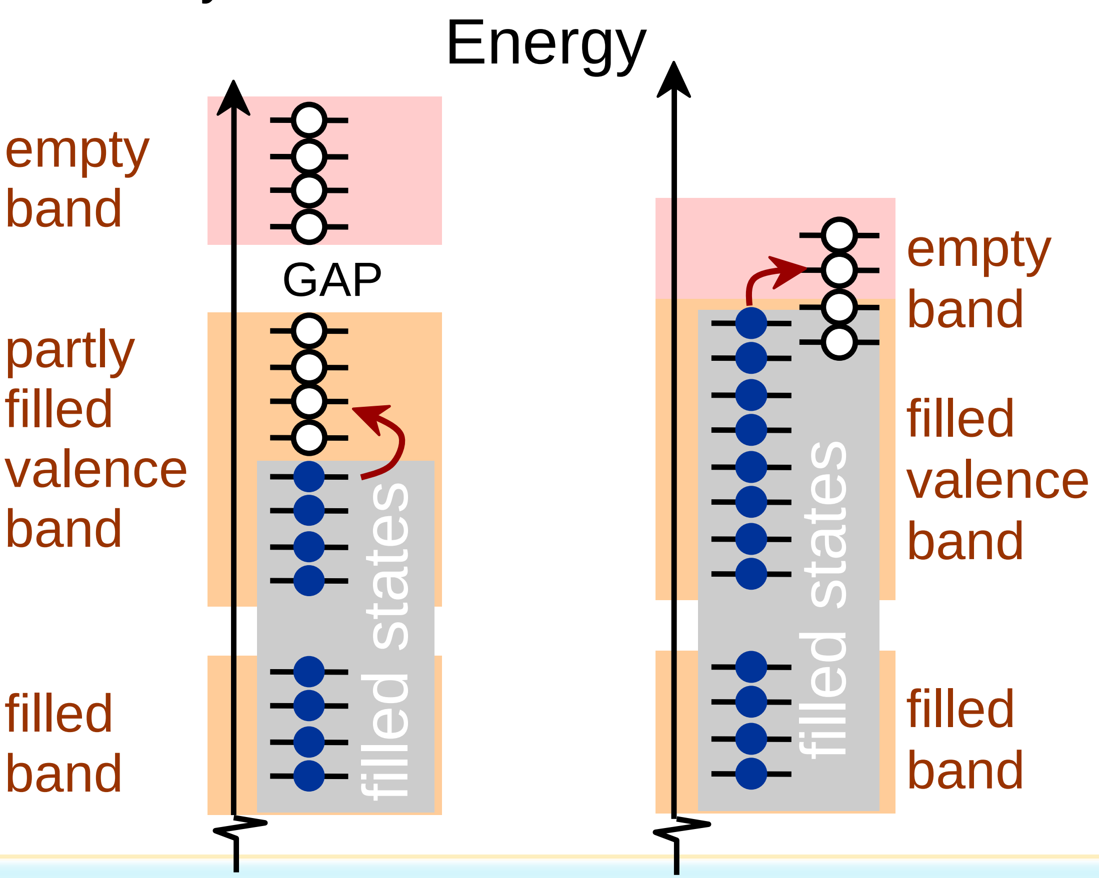
- In **metals**, highest occupied band is partially filled or bands overlap.
- Conduction occurs by promoting electrons into conducting states, that starts right above the Fermi level. The conducting states are separated from the valence band by an infinitesimal amount (~10^{-10} eV).
- Energy provided by an electric field is sufficient to excite many electrons into conducting states. => High conductivity.

- Metals (Conductors):

-- Thermal energy puts many electrons into a higher energy state.

- Energy States:

-- for metals nearby energy states are accessible by thermal fluctuations.



Semiconductors ~ Insulators

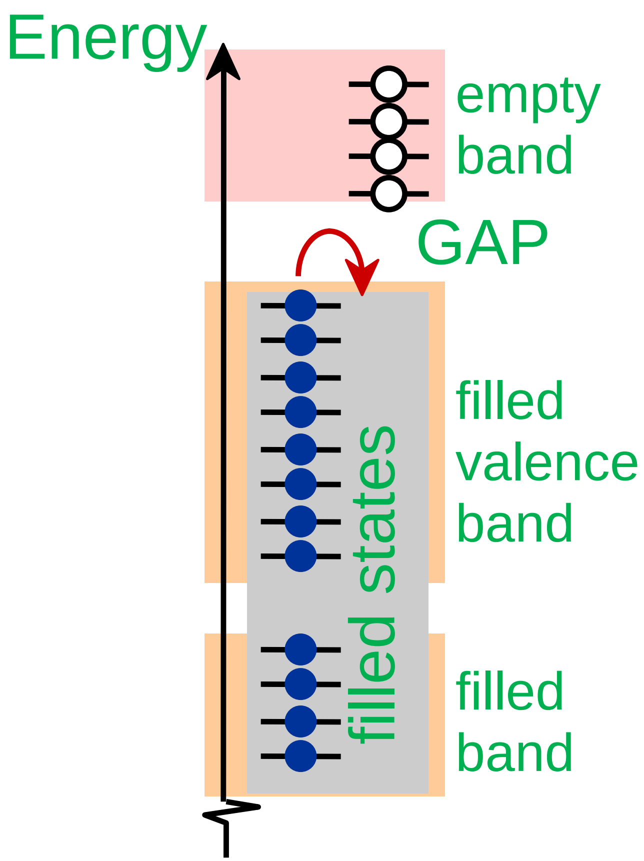
In semiconductors and insulators, electrons have to jump/move across the band gap into conduction band to find conducting states above E_f

- In Semiconductors electrons can reach the conduction band at ordinary temperatures, where in insulators they cannot.
- E_g is too large for insulators to have thermally or optically exited electrons promote to the conduction band.
- The probability that an electron reaches the conduction band : $\mathrm{Exp}(-E_g/2kT)$ where E_g is the band gap.
- If this probability is $< 10^{-24}$ one would not find a single electron in the conduction band in a solid of 1 cm^3. Remember $N_{Av} \sim 10^{24}$
- This requires $E_g/2kT > 55$. At room temperature, $2kT = 0.05$ eV, **E_g > 2.8 eV corresponds to an insulator.**
- An electron promoted into the conduction band leaves a **hole** (+ charge) in the valence band, that can also participate in conduction. Holes exist in metals as well, but are more important in semiconductors and insulators.

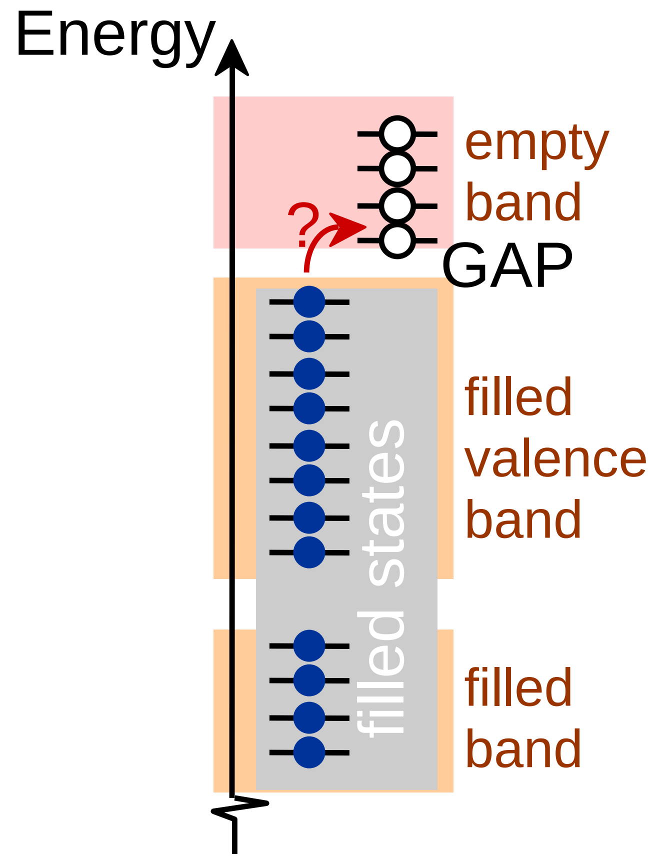
Energy States: Insulators & Semiconductors

- Insulators:
  -- Higher energy states not accessible due to gap (> 2.0 eV, ideally > 4 eV).

- Semiconductors:
  -- Higher energy states separated by smaller gap (< 2 eV).

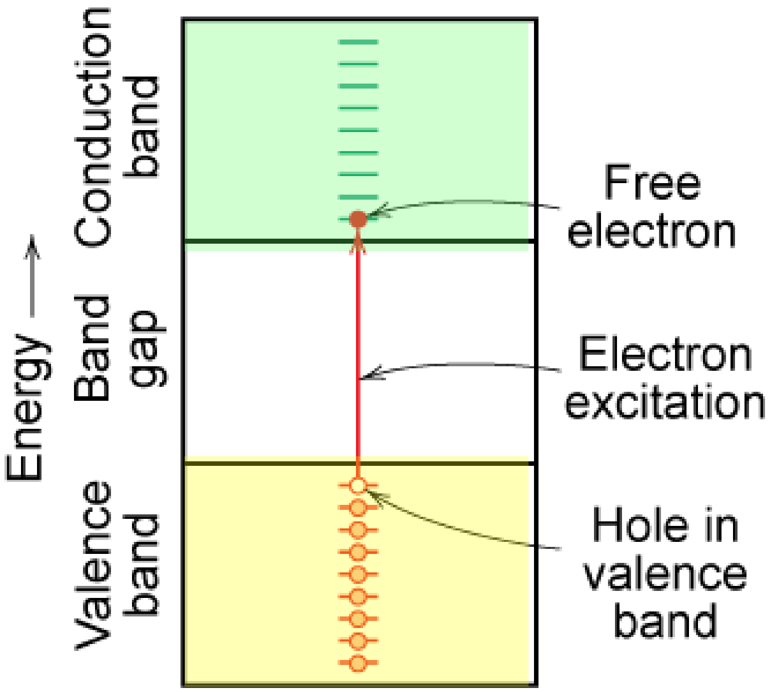


dielectrics, or nonconductors



Wide bandgap semiconductor (2-4 eV)

Charge Carriers



- Two charge carrying mechanisms

Electron – negative charge

Hole – equal & opposite positive charge

- Move at different speeds - drift velocity

Higher temp. promotes more electrons into the conduction band

$\therefore \quad \sigma\uparrow$ as $T\uparrow$

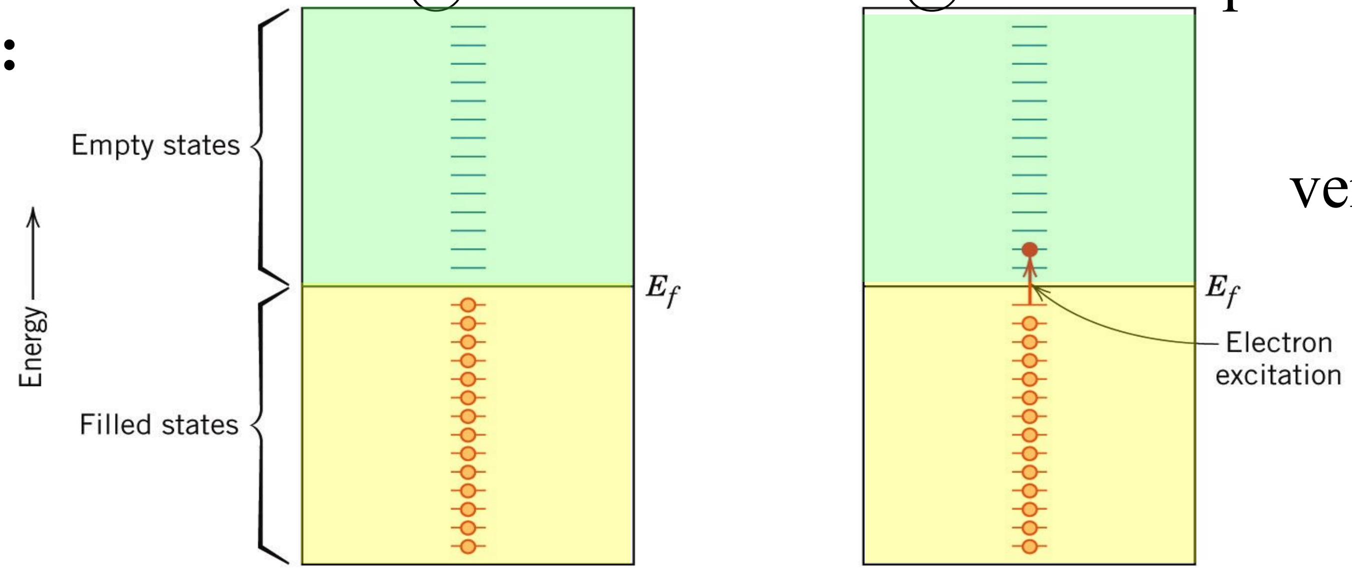
Electrons scattered by impurities, grain boundaries, etc.

Band Theory for Metals and Semiconductors

@ 0 K

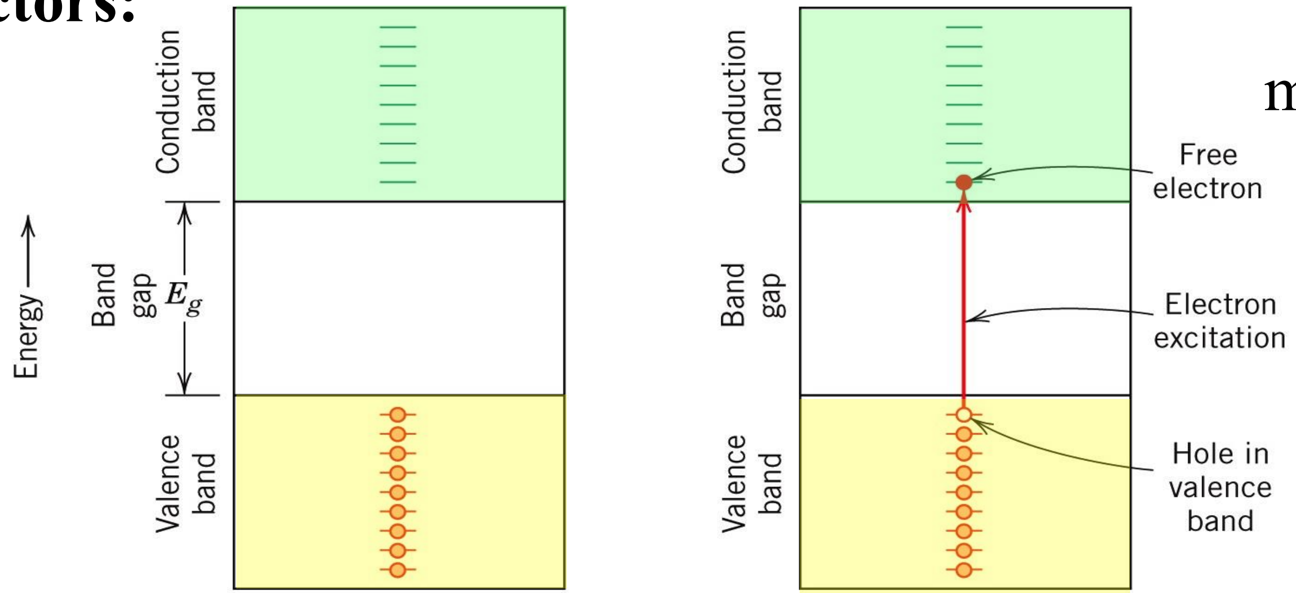
@ room temp

Metals:



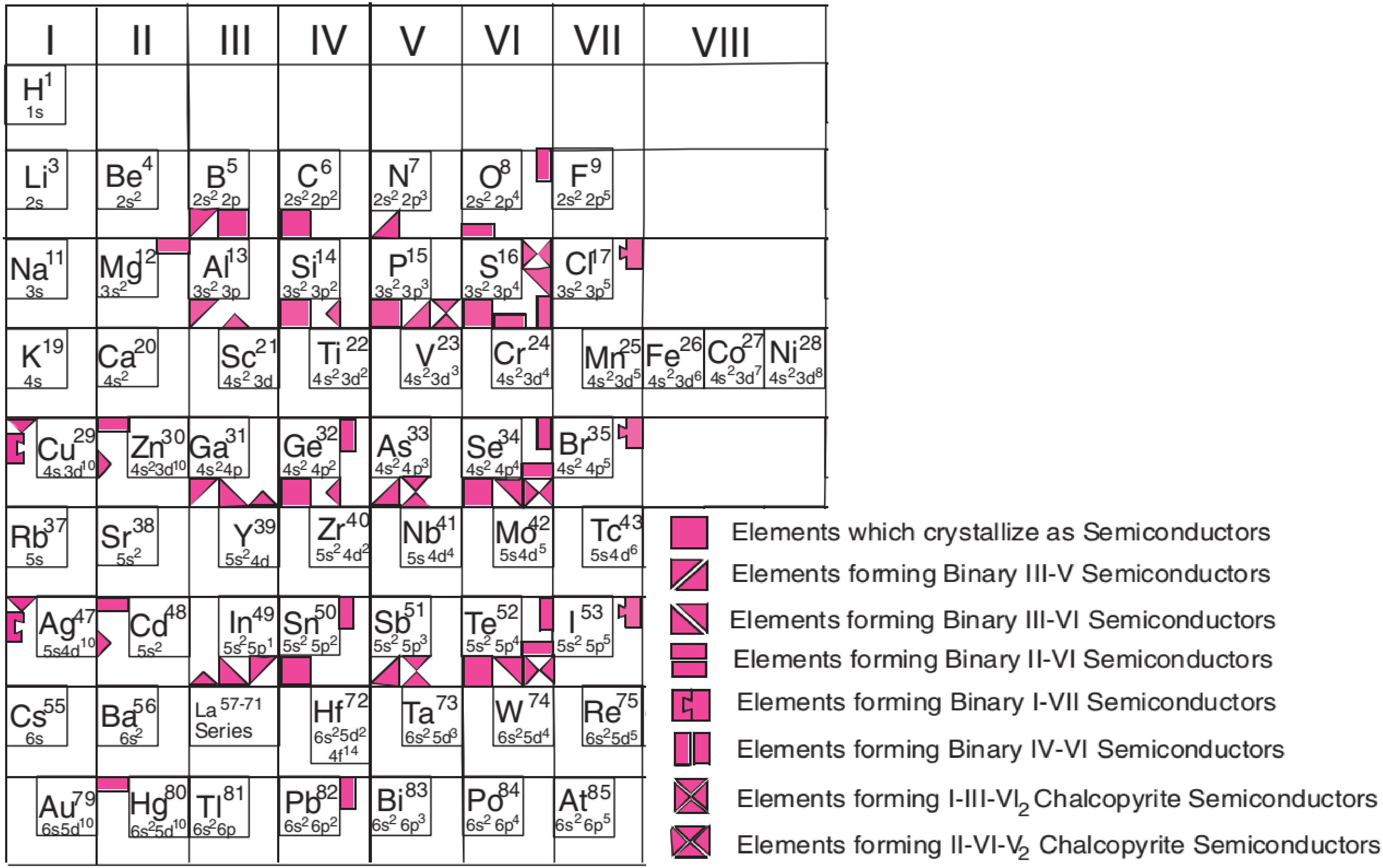
very large **n**

Semiconductors:



modest **n**

Periodic Table of "Semiconductor-Forming" Elements



Electron Mobility

The **force** acting on the electron is ***-eE***, where e is the electric charge by a uniform electric field E. As long as the electric field is present, in the absence of obstacles the electron is expected to speed up continuously in an electric field.

- So, is the case in vacuum (e.g. inside a TV tube) **or in a perfect crystal**
- In a real solid, electrons motion are hindered by defects (dislocations, impurities, vacancies, etc), and even thermal vibrations of atoms. Electrons **scatter** by collisions with imperfections and due to atomic thermal vibrations.
- "frictional forces" => resistance => a net **drift velocity** of electron motion is established:

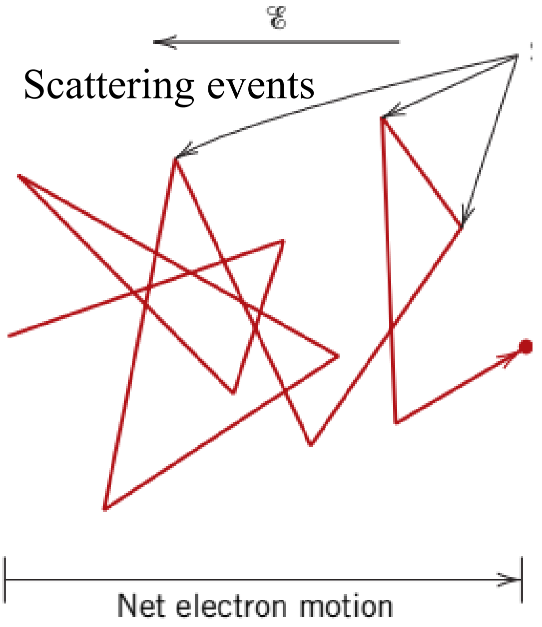
$$\boxed{\begin{aligned} v_d &= \mu_e E \\ \sigma &= n\,|e|\,\mu_e \end{aligned}}$$

v_d drift velocity [m/s]

μ e- mobility [m^2/Vs]

n no of free electrons

$|e|$ charge of an e- [C]



Electrical conductivity is proportional to number of free electrons and electron mobility:

$$\mu_{metal} << \mu_{semi}$$

$$\sigma_{metal} >> \sigma_{semi}$$

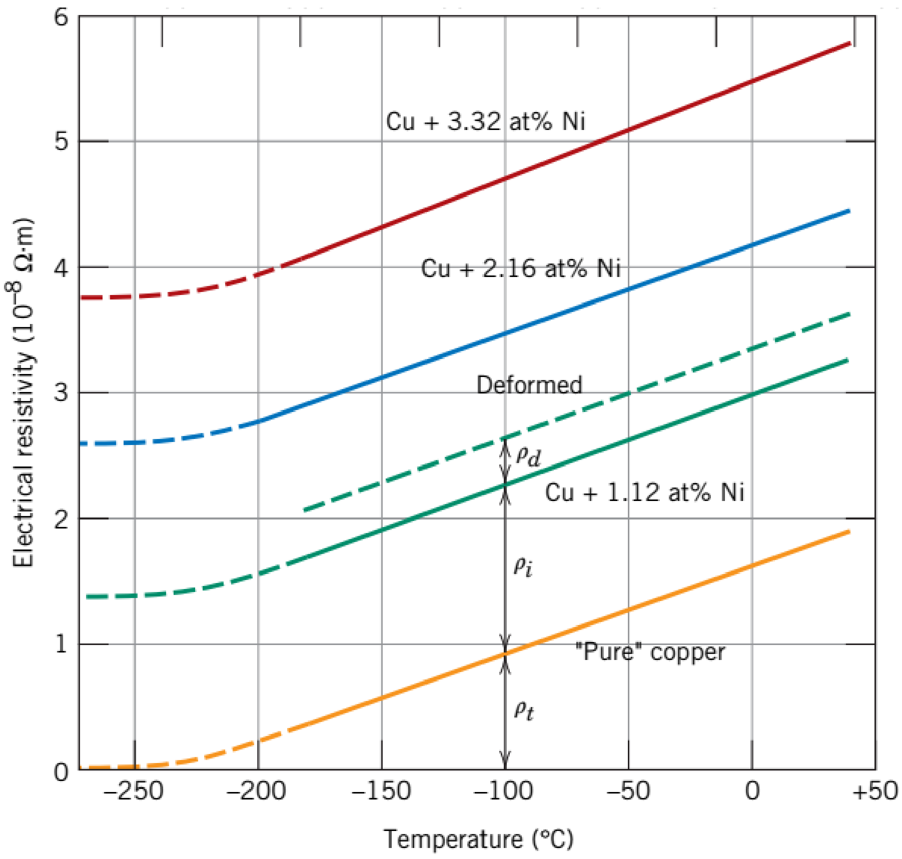
Metals: Resistivity vs T, Impurities

Imperfections increase resistivity

- -- grain boundaries
- -- dislocations
- -- impurity atoms
- -- vacancies

These act to scatter electrons so that they take a less direct path.

- Resistivity increases with:
  - -- temperature
  - -- wt% impurity



Matthiessen's rule—

for a metal, total electrical resistivity equals the sum of thermal, impurity, and deformation contributions

$$\rho = \rho_{thermal} + \rho_{impurity} + \rho_{deformation}$$

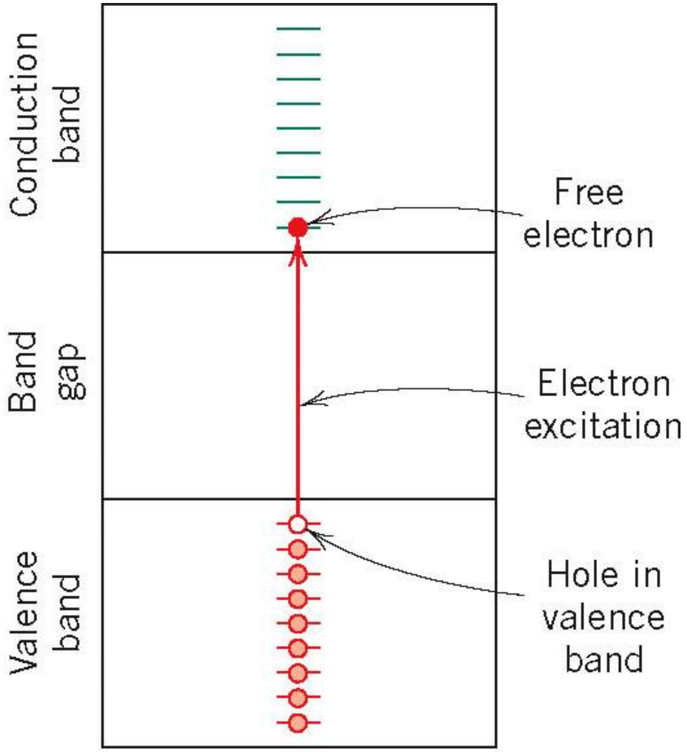
The electrical resistivity versus T for Cu and three Cu–Ni alloys, one of which has been deformed. Thermal, impurity, and deformation contributions to the resistivity are indicated at –100°C

Semi-conductivity

Semiconductor do have a lower conductivity than metals but unique properties of semiconductors make them very useful materials.

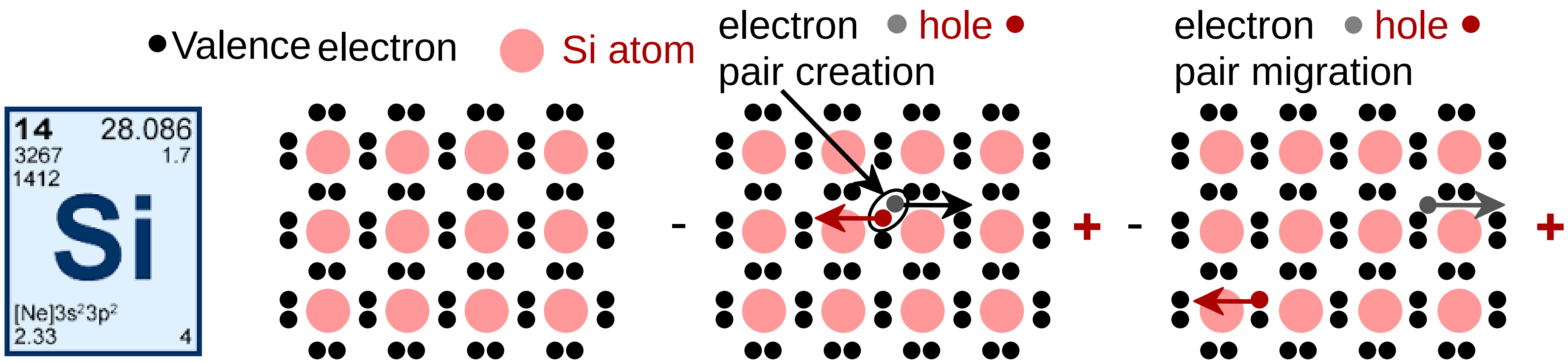
Electrical properties of semiconductors are very sensitive to the presence of impurities:

- **Intrinsic semiconductors** - electrical conductivity is based on the electronic structure of pure material.
- **Extrinsic semiconductors** - electrical conductivity is dictated by impurity atoms.



Material	*Band Gap (eV)*	*Electron Mobility* ($m^2/V\cdot s$)	*Hole Mobility* ($m^2/V\cdot s$)	*Electrical Conductivity (Intrinsic)* $(\Omega\cdot m)^{-1}$
		Elemental		
Ge	0.67	0.39	0.19	2.2
Si	1.11	0.145	0.050	3.4×10^{-4}
		III–V Compounds		
AlP	2.42	0.006	0.045	—
AlSb	1.58	0.02	0.042	—
GaAs	1.42	0.80	0.04	3×10^{-7}
GaP	2.26	0.011	0.0075	—
InP	1.35	0.460	0.015	2.5×10^{-6}
InSb	0.17	8.00	0.125	2×10^{4}
		II–VI Compounds		
CdS	2.40	0.040	0.005	—
CdTe	1.56	0.105	0.010	—
ZnS	3.66	0.060	—	—
ZnTe	2.40	0.053	0.010	—

Conduction in Terms of Electron and Hole Migration



Before excitation   Excitation   Applied electric field

Si (E_g = 1.1 eV) one out of every 10^{13} atoms contributes an electron to the conduction band at RT

Since both electrons and holes conduct, the conductivity of an intrinsic semiconductor is

$$\sigma = n|e|\mu_e + p|e|\mu_h$$

Electrons are more mobile than holes, $\mu_e > \mu_h$

In an intrinsic semiconductor, **n = p=n$_i$**

$$\sigma = n|e|(\mu_e + \mu_h) = p|e|(\mu_e + \mu_h) = n_i|e|(\mu_e + \mu_h)$$

Conductivity of intrinsic semiconductors increase with temperature (different from metals!)

Intrinsic Semiconductors

Number of electrons in conduction band increases exponentially with T

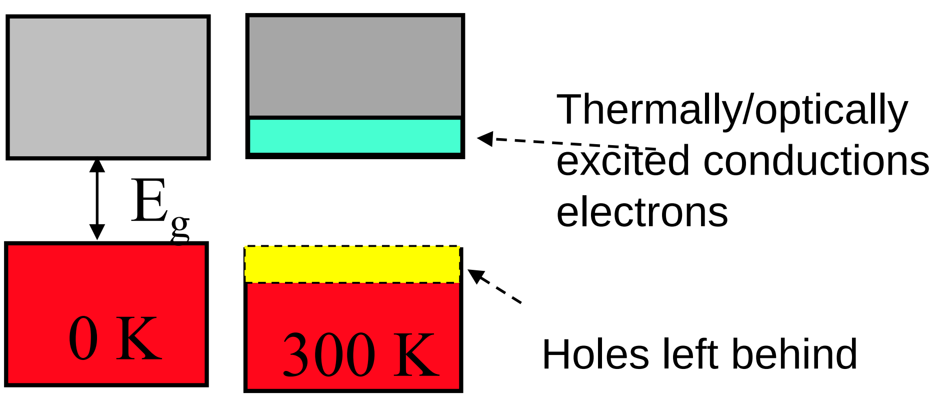
$$n_i = N_c \exp\left(\frac{-E_g}{2k_B T}\right) \qquad p_i = N_v \exp\left(\frac{-E_g}{2k_B T}\right)$$

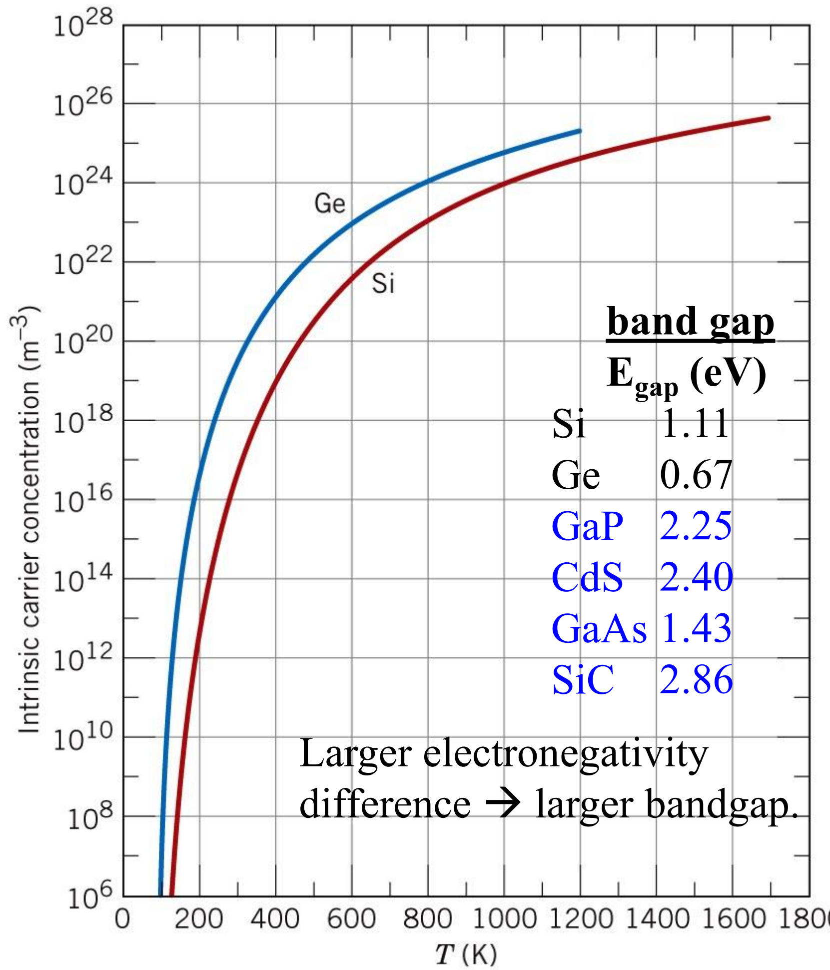
where $N_c = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2}$

$$\boxed{n_i = p_i = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} \exp\left(\frac{-E_g}{2k_B T}\right)}$$

n (and p) increase exponentially with temperature, whereas μ_e and μ_h decrease (about linearly) with temperature.

$$\sigma = n|e|\mu_e + p|e|\mu_h$$

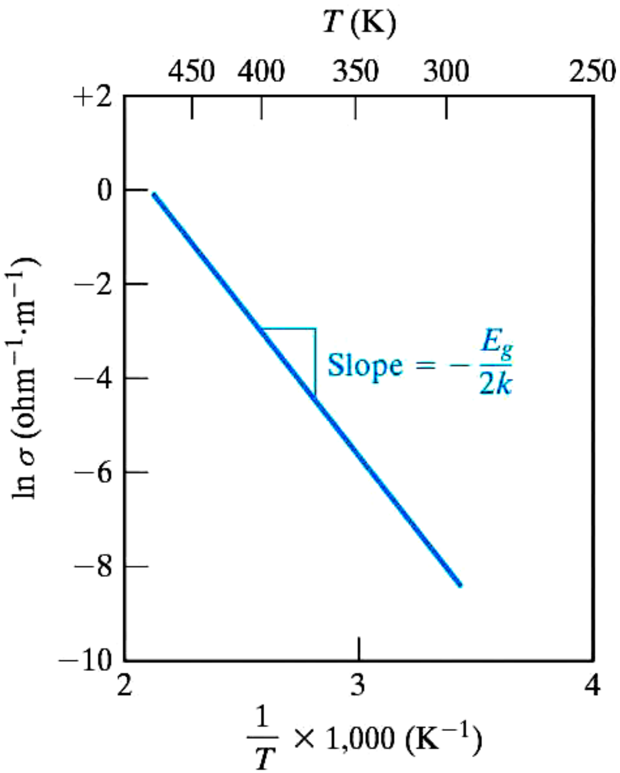




Intrinsic Semiconductors

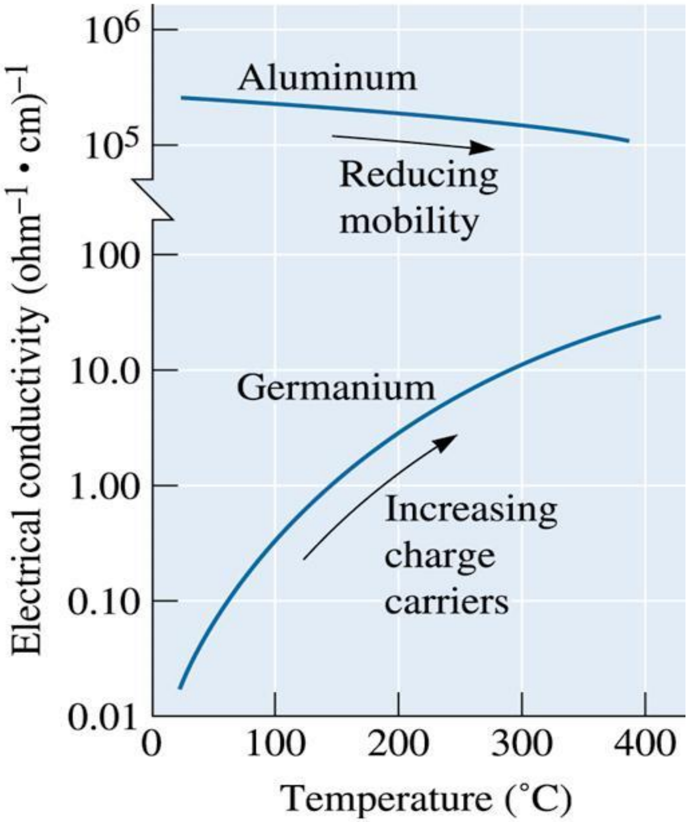
The total conductivity can be expressed as

$$\sigma = \sigma_0 \exp\left(\frac{-E_g}{2k_B T}\right)$$



$$\ln \sigma = \ln \sigma_0 - \frac{E_g}{2k_B T}$$

The gradient of a plot of conductivity versus 1/T will yield a value for the **thermal band gap**



Optical band gap= $E_g = h\nu_g$

where $h\nu_g$ is the energy of the photon required to promote an electron from the valence band and create a hole in its place.

Extrinsic Semiconductors

Extrinsic semiconductors - electrical properties (conductivity) is dictated by impurity atoms. Example: Si is considered to be extrinsic at room T if impurity concentration is one atom per 10^{12}

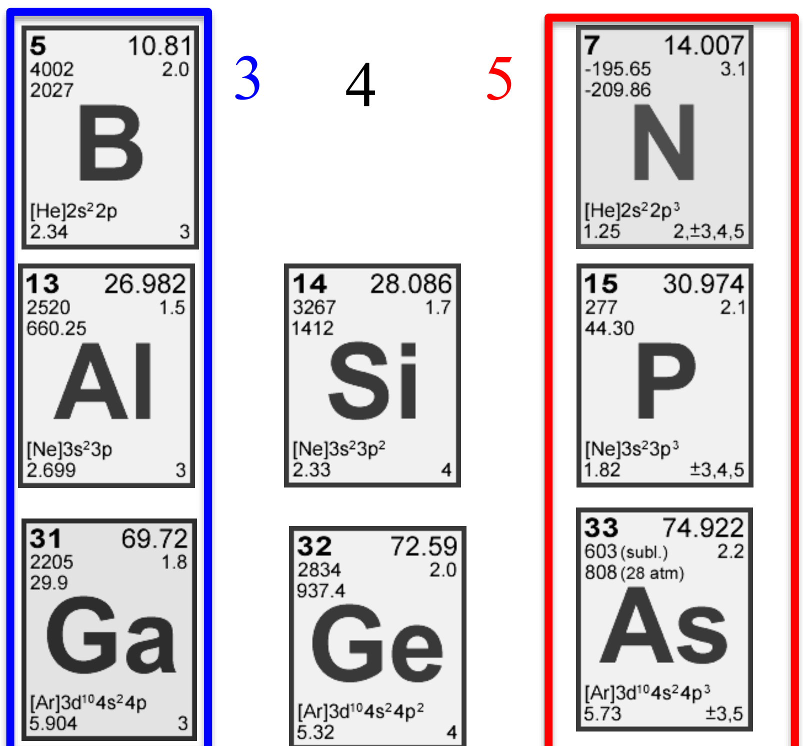
An extrinsic semiconductor may have different concentrations of holes and electrons.

It is called **p-type** if p >> n and **n-type** if n >> p.

– Two common methods of doping are diffusion and ion implantation.

These elements have one less valence e- relative to Si

When present as impurities, they will create lots of extra holes called “p-type”

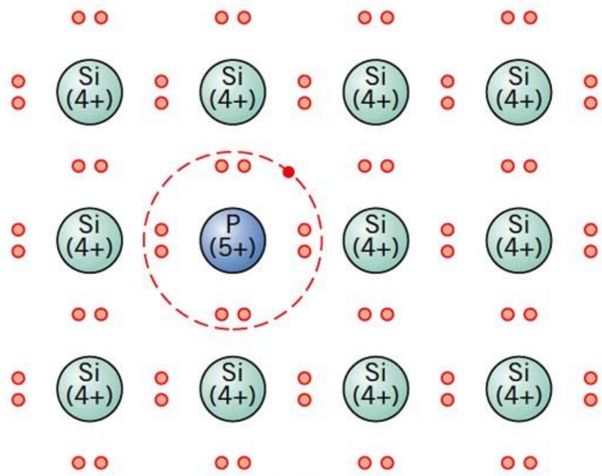


These elements have one more valence e- relative to Si

When present as impurities, they will create lots of extra mobile e- called “n-type”

Extrinsic Semiconductors: n-type

Extrinsic: electrical behavior is determined by impurities that introduce excess e or h



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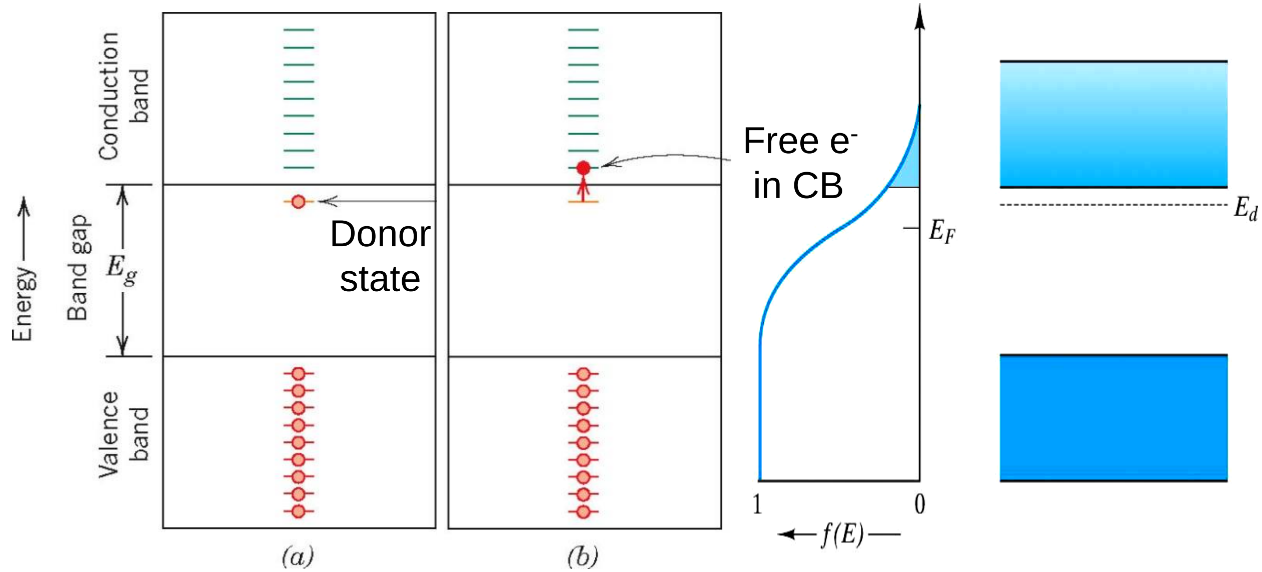
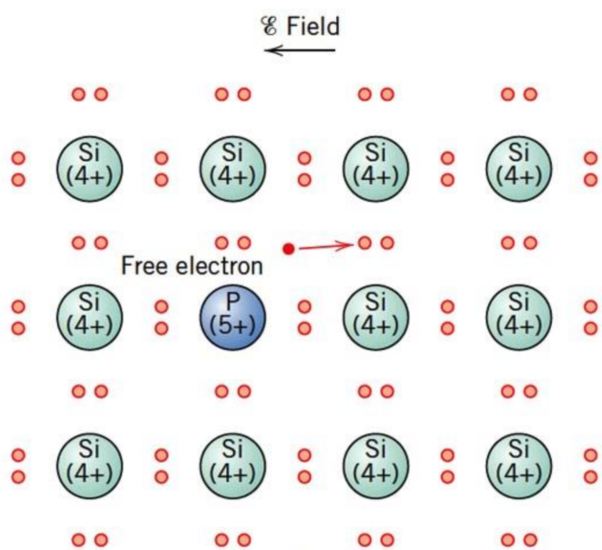




(b)

The hole created in donor state is far from the valence band and is immobile upon the promotion of the donated electron to the conduction band. **n >>p**

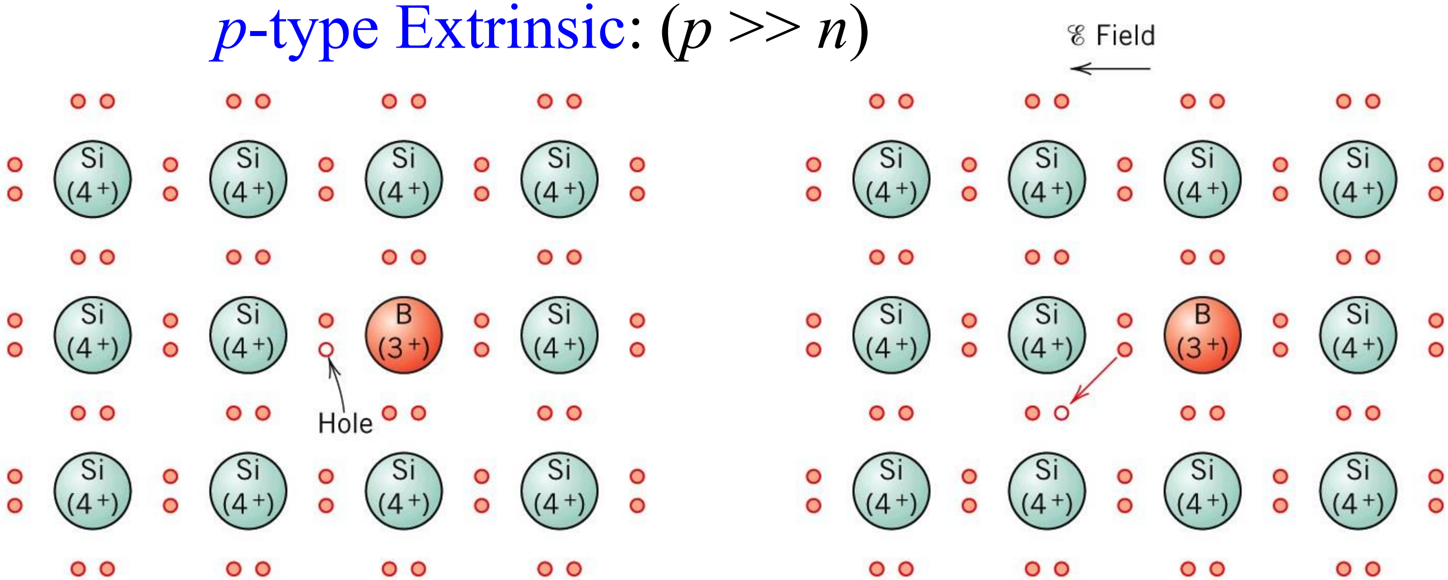
Conduction occurs mainly by the donated electrons (thus n-type)

$$\sigma \sim N_D |e| \mu_e$$

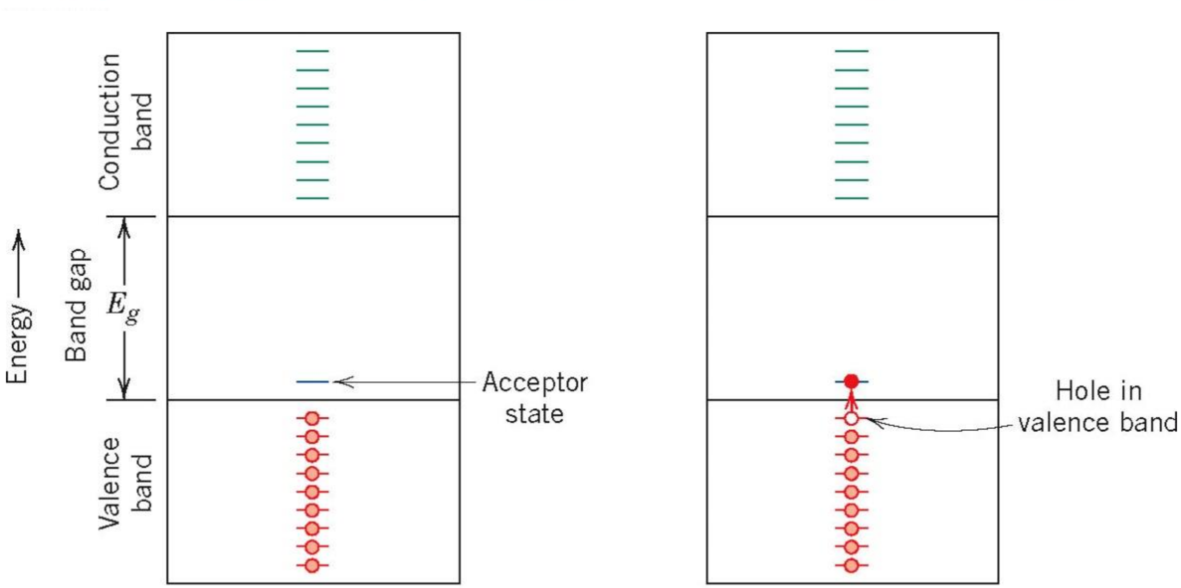
Impurities which produce extra conduction *e* are called **donors**, $N_D = N_{Phosphorus} \sim n$

Extrinsic Semiconductors: p-type

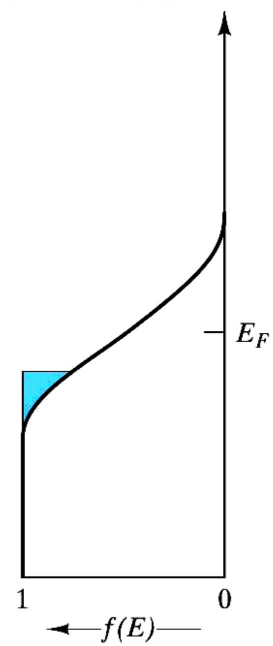
p-type Extrinsic: ($p >> n$)

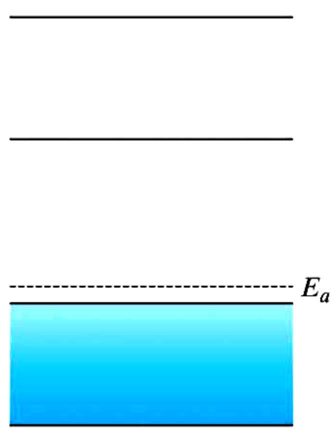


$$\sigma \approx p|e|\mu_h$$



(a) (b)

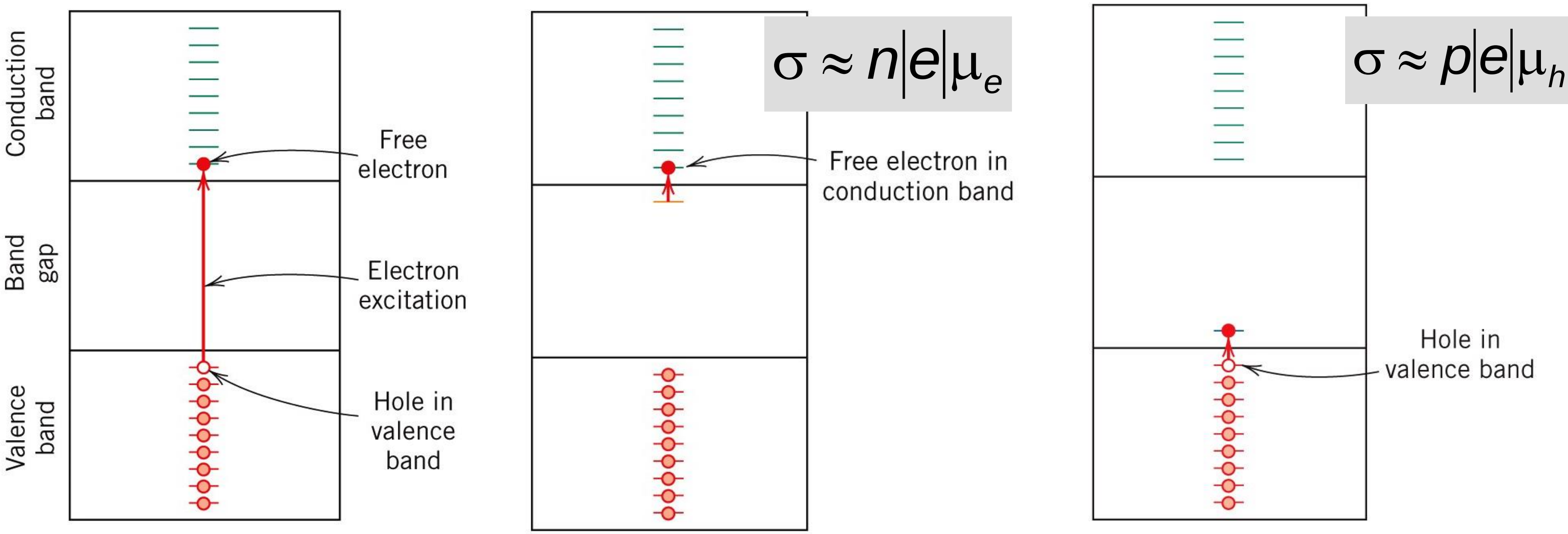




Distribution of carriers

Intrinsic vs. Extrinsic Semiconductors

Intrinsic | Extrinsic n-type | Extrinsic p-type



Law of Mass Action: at constant T and equilibrium condition **the product of –ve free electron conc. and the +ve hole conc. is a constant**

$$\boxed{n\,p = n_i^2}$$

As a doped crystal must remain electrically neutral:

Magnitude of total -ve charge density = +ve charge density

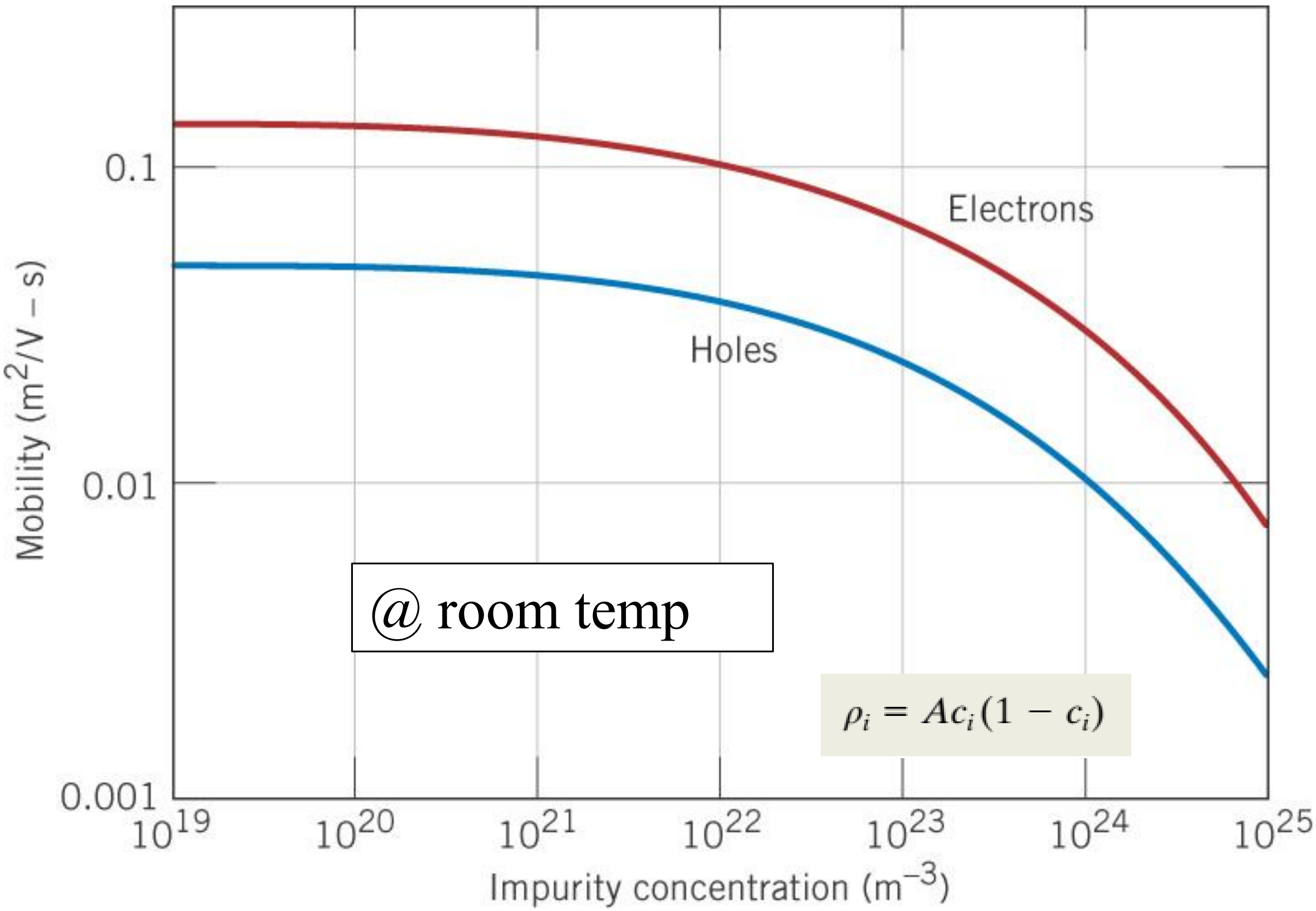
$$\boxed{n + N_a = p + N_d}$$

At high temperatures when an n-type semiconductor contains only completely ionized donors, the number of electrons is equal to the number of donors (same for holes) is given by the equilibrium equation:

$$n\ (\text{n-type}) \sim N_d$$

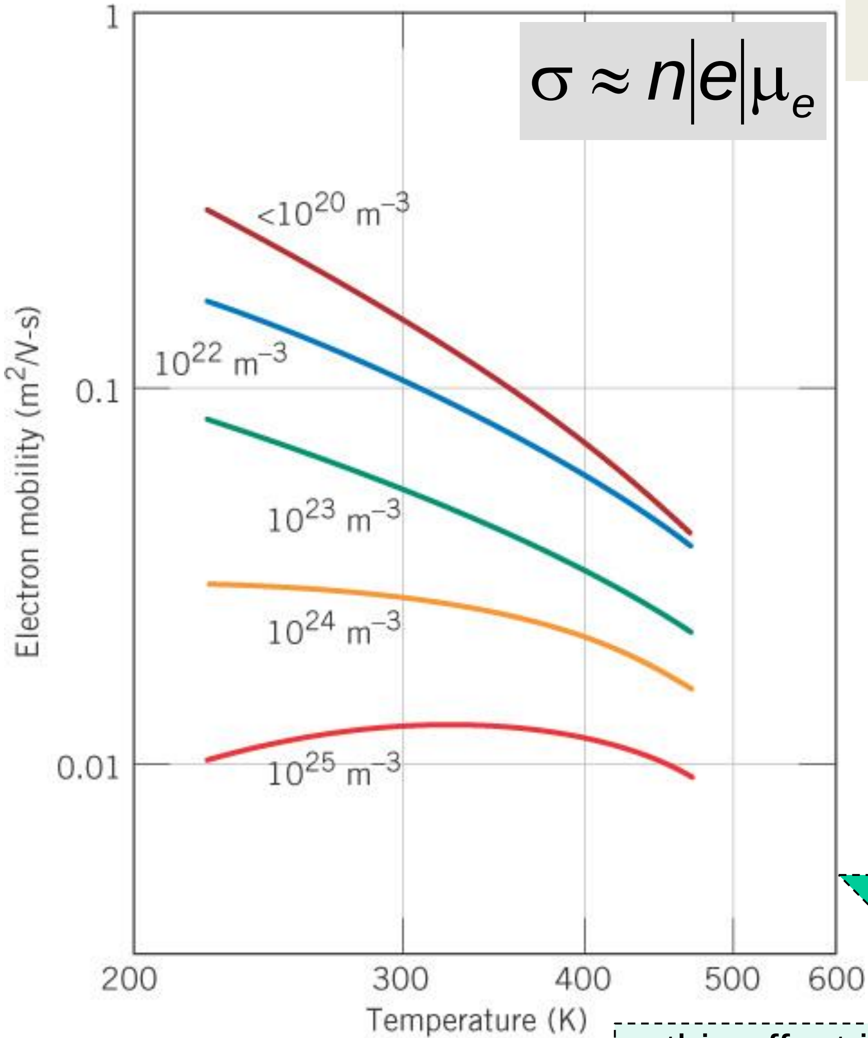
$$p\ (\text{n-type}) \sim \frac{n_i^2}{N_d}$$

Mobility vs. Impurity concentration



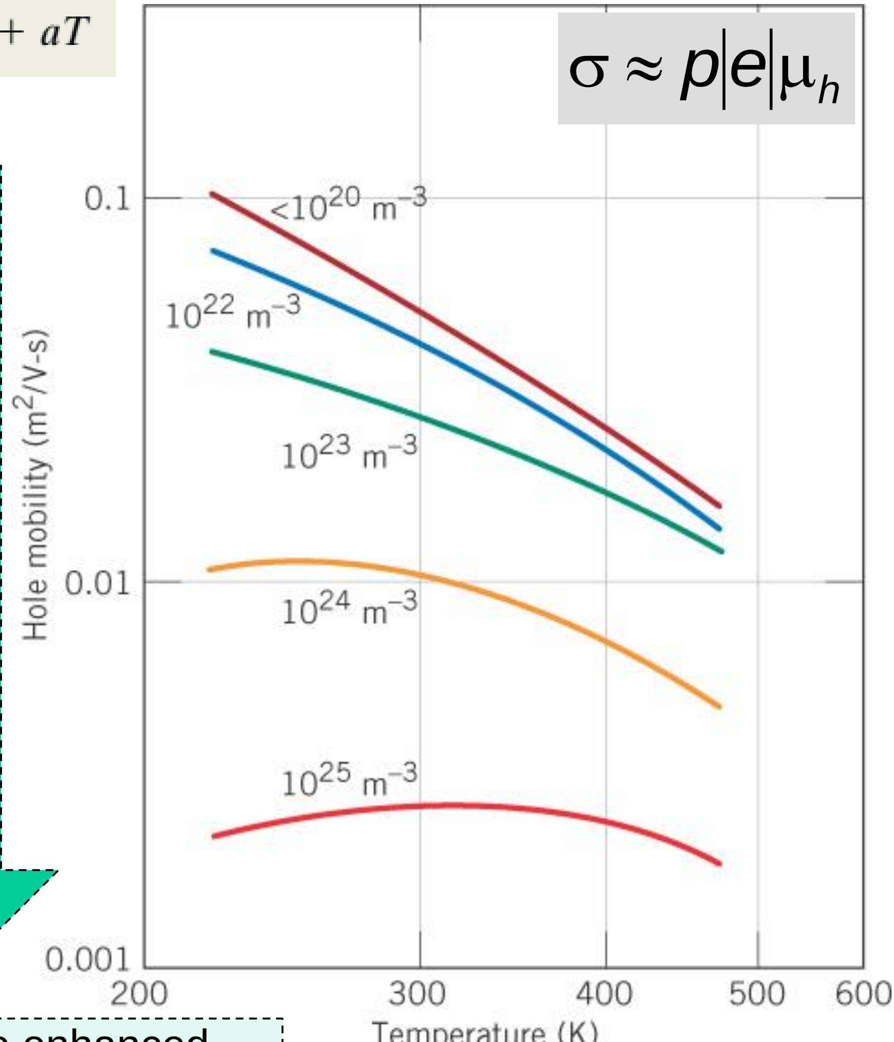
For silicon, dependence of room-temperature electron and hole mobilities (logarithmic scale) on dopant concentration (logarithmic scale).

Mobility vs. Temperature



(a)

$$\rho_t = \rho_0 + aT$$

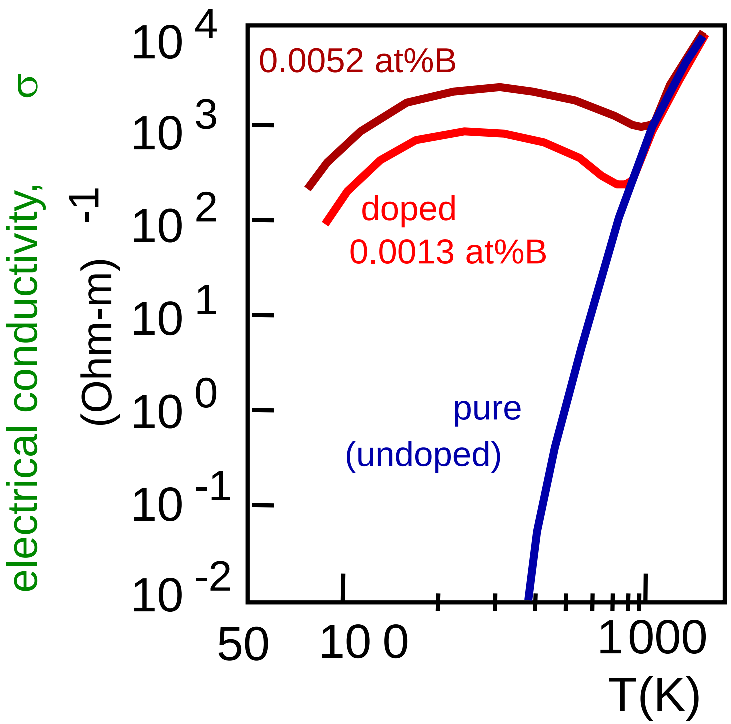


(b)

this effect is due to enhanced thermal scattering of the carriers.

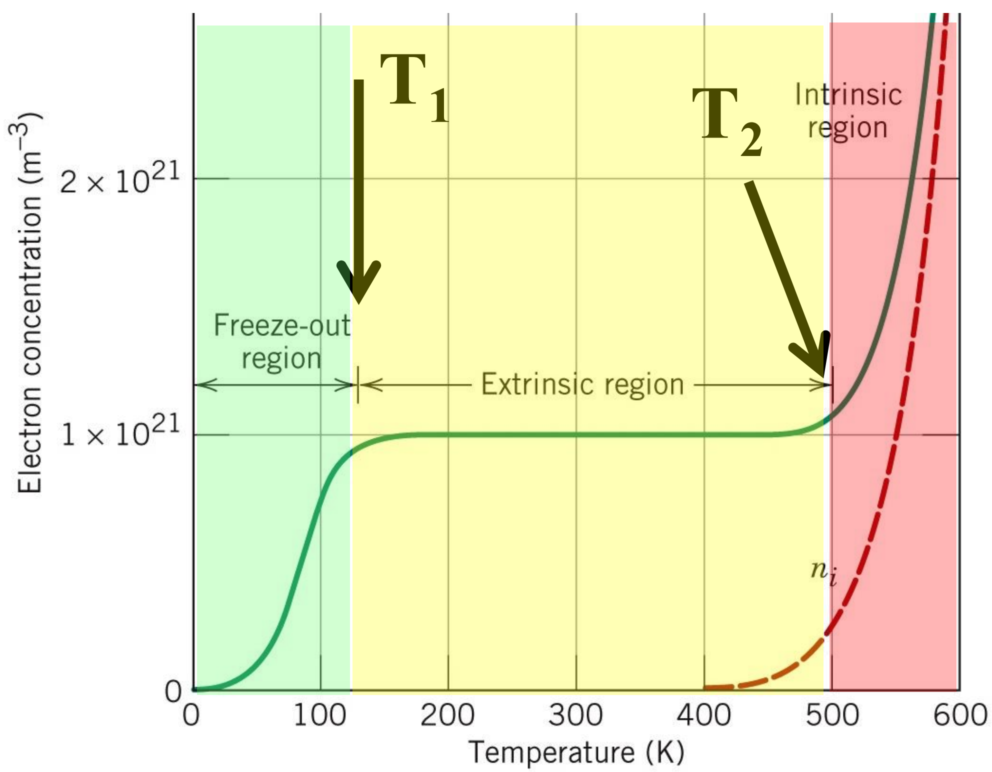
Temperature dependence of (a) electron and (b) hole mobilities for silicon that has been doped with various donor and acceptor concentrations. Both sets of axes are scaled logarithmically.

- Data for Doped Silicon:
  - -- σ increases doping
  - -- reason: imperfection sites lower the activation energy to produce mobile electrons.



Extrinsic doping level: $10^{21}/m^3$ of a *n*-type donor impurity (such as P).

1) $T<T_1$: Freeze-out region, thermal energy is not high enough to excite electron from donor state to CB
2) $T_1<T<T_2$: Extrinsic region, thermal energy is high enough to excite electron from donor state to CB
3) $T>T_2$: Intrinsic region, thermal energy is high enough to excite electron from VB to CB



Summary

- Electrical conductivity and resistivity are:
  -- material parameters.
  -- geometry independent.
- Electrical resistance is:
  -- a geometry and material dependent parameter.
- Conductors, semiconductors, and insulators...
  -- differ in accessibility of energy states for
  conductance electrons.
- For metals, conductivity is increased by
  -- reducing deformation
  -- reducing imperfections
  -- decreasing temperature.
- For pure semiconductors, conductivity is increased by
  -- increasing temperature
  -- doping (e.g., adding B to Si (*p*-type) or P to Si (*n*-type).

Summary

Energy Band Structures and Bonding

(metals, semiconductors, insulators)

- Relation to atomic bonding:
  - **Insulators** – valence electrons are tightly bound to (or shared with) the individual atoms – strongest ionic (partially covalent) bonding. Remember electro-negativity.
  - **Semiconductors** - mostly covalent bonding somewhat weaker bonding. Sharing of electrons.
  - **Metals** – valence electrons form an "electron gas" that are not bound to any particular ion.