INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Chemical Engineering

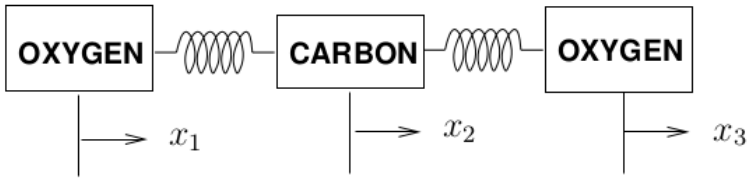
Subject: Process Dynamics and Control (CH61016) - Dynamics

Remarks:

1. Please use a pen and a paper to solve the exam. Show all the steps in detail. Step marking will not be done for the solutions which are not sufficiently detailed.
2. You must scan your solution to a single pdf and attach it in the MS Teams portal.
3. You **must** mention your name and roll number in the answersheet.
4. Time = 1 h 15 min; start time for the exam: 11:15 am; time of closing of the portal: 12:45 pm. Extra 15 minutes have been provided for scanning, pdf conversion, uploading extra. **No submission beyond 12:45 pm shall be evaluated under any circumstances**.

Both questions are compulsory and carry equal marks (15 each).

Question 1: Consider a spring and mass model of a CO_2 molecule as shown in the figure below.



The oxygen molecules have mass m_o and the carbon molecule has mass m_c . The springs have a spring constant k and obey Hooke's law. Using Newton's laws and assuming that the motion is constrained along the x-axis the system of equations describing the displacement of masses is

$$\begin{aligned} \frac{d^2x_1}{dt^2} &= -a(x_1 - x_2) \\ \frac{d^2x_2}{dt^2} &= -b(x_2 - x_1) - b(x_2 - x_3) \\ \frac{d^2x_3}{dt^2} &= -a(x_3 - x_2) \end{aligned}$$

where $a = k/m_o$ and $b = k/m_c$

(a) What is the order of the system? Write the dynamical equation(s) and dynamical variable(s) for the system.

(b) Determine the equilibrium solution(s) for the system. Show all the steps in detail.

(c) Solve the dynamical equation(s) to give the evolution of the system. Show all the steps in

detail. If you use a software/package/online server for the intermediate steps then detail them.
(d) Comment upon the nature of the solution and provide a qualitative sketch of any one of the solution. The graph is not expected to be an exact depiction of the solution but it should capture all the characteristic features of the dynamics.

Question 2: A batch fermentor utilizes a certain yeast for the fermentation process. The total batch operation consists of a production run and a steralization run. During the production run, alcohol is produced. The alcohol and by-products are then drained to empty the vessel. The empty fermentor with the residual yeast is then steralized in UV radiation to make the fermentor yeast-free for the new batch. During the production run, the yeast population follows the continuous logistic growth with the carrying capacity as unity and the growth parameter as α. During the steralization run, the yeast population follows the continuous logistic growth with carrying capacity as unity, the growth parameter as α, and a constant yeast decay rate of β. For this system, sketch the variation of α with β which satisfies the bifurcation condition during the steralization run.

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