# INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

## Department of Chemical Engineering

## Mid-semester (Autumn) Examination 2016-2017

**Subject: Advanced Mathematical Techniques in Chemical Engineering (CH61015)**

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**Remarks:**

1. Attempt ***ALL*** questions. All questions compulsory.
2. Time = 2 h; maximum marks = 80; total number of printed pages = 2.

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**1.** Determine the dimension of the equilibrium solution space of the following set of ordinary differential equations:

$$\frac{dx_1}{dt} = (cos\theta)x_1 - (sin\theta)x_2$$

$$\frac{dx_2}{dt} = (sin\theta)x_1 + (cos\theta)x_2$$

$$\frac{dx_3}{dt} = x_3$$

**...10 marks**

**2.** Determine the bifurcation and sketch the phase portrait for the following family of ordinary differential equations:

$$\frac{dx}{d\theta} = asin\theta$$

**...10 marks**

**3.** Sketch the straight line solutions along the eigenvectors and indicate the direction of time for the following set of ordinary differential equations:

$$\frac{dx_1}{dt} = x_1 + x_2$$

$$\frac{dx_2}{dt} = 2x_2$$

$$\frac{dx_3}{dt} = -x_2 + 4x_3$$

**...10 marks**

**4.** Determine the solution vector using a suitable $LU$-decomposition of the following set of linear equations:

$$2x_1 - x_2 = 0$$

$$-x_1 + 2x_2 - x_3 = 0$$

$$-x_2 + 2x_3 - x_4 = 0$$

$$-x_3 + 2x_4 = 0$$

**...15 marks**

5. A space of polynomial functions $x^n \forall n \in \mathbb{I}^+ \cup 0$ of dimension 3 needs to be expanded using a suitable orthonormal basis. If the inner product in this space is defined as $< p, q >= \int_{-1}^{1} p(x)q(x)dx$ then determine the orthonormal basis functions.

**...15 marks**

6. Prove that the Strum-Liouville operator is a self-adjoint operator.

**...20 marks**

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