Falkner Skan Transformation

$$\psi = \sqrt{\nu U_\infty x}\; f(\eta)$$

$$\eta = \sqrt{\frac{U_\infty}{\nu x}}\; y.$$

$$\Rightarrow \frac{\partial \eta}{\partial x} = \left(-\frac{1}{2}\sqrt{\frac{U_\infty}{\nu}}\, x^{-3/2}\right) y.$$

$$= \left(-\frac{1}{2}\sqrt{\frac{U_\infty}{\nu x}} \cdot \frac{y}{x}\right)$$

$$\boxed{\frac{\partial \eta}{\partial x} = \left(-\frac{1}{2}\frac{\eta}{x}\right)}$$

$$\frac{\partial \psi}{\partial \eta} = \sqrt{\nu U_\infty x} \cdot \frac{df}{d\eta}.$$

$$\frac{\partial \eta}{\partial y} = \sqrt{\frac{U_\infty}{\nu x}}.$$

$u, \frac{\partial u}{\partial x}, v, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial y^2}$ needs to be expressed in terms of f and η

Now, $u = \frac{\partial \psi}{\partial y}$

$$= \frac{\partial \psi}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}.$$

$$= \sqrt{\nu U_\infty x} \cdot \frac{df}{d\eta} \cdot \sqrt{\frac{U_\infty}{\nu x}}.$$

$$= U_\infty \frac{df}{d\eta}.$$

$$\boxed{u = U_\infty \frac{df}{d\eta}}$$

$$\boxed{v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left[\eta \frac{\partial f}{\partial \eta} - f\right]}$$

$$v = -\frac{\partial \psi}{\partial x}$$

$$= -\frac{\partial}{\partial x}\left[\sqrt{\nu U_\infty x}\; f(\eta)\right]$$

$$= -\left[\sqrt{\nu U_\infty x}\, \frac{\partial f}{\partial x} + \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}} \cdot f\right]$$

$$= -\left[\sqrt{\nu U_\infty x}\, \frac{\partial f}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} + \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\, f\right].$$

$$= -\left[\sqrt{\nu U_\infty x}\, \frac{\partial f}{\partial \eta}\left(-\frac{1}{2}\frac{\eta}{x}\right) + \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\, f\right].$$

$$= \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left[\eta \frac{df}{d\eta} - f\right]$$

$$u = u_\infty \frac{\partial f}{\partial \eta}.$$

$$\frac{\partial u}{\partial x} = u_\infty \frac{\partial}{\partial x}\left(\frac{df}{d\eta}\right)$$

$$= u_\infty \frac{d}{d\eta}\left(\frac{df}{d\eta}\right) \cdot \frac{d\eta}{dx}.$$

$$= U_\infty \frac{d^2 f}{d\eta^2} \cdot \left(-\frac{1}{2}\frac{\eta}{x}\right).$$

$$\boxed{\frac{\partial u}{\partial x} = -\frac{U_\infty}{2x}\eta \cdot \frac{d^2 f}{d\eta^2}}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(u_\infty \frac{df}{d\eta}\right)$$

$$= u_\infty \frac{d}{d\eta}\left(\frac{df}{d\eta}\right) \cdot \frac{d\eta}{dy}.$$

$$\boxed{\frac{\partial u}{\partial y} = U_\infty \sqrt{\frac{U_\infty}{\nu x}} \cdot \frac{d^2 f}{d\eta^2}.}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)$$

$$= \frac{\partial}{\partial \eta}\left(\frac{\partial u}{\partial y}\right) \cdot \frac{\partial \eta}{\partial y}.$$

$$= \frac{d}{d\eta}\left[u_\infty \sqrt{\frac{U_\infty}{\nu x}} \frac{d^2 f}{d\eta^2}\right] \sqrt{\frac{U_\infty}{\nu x}}.$$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial y^2} = \frac{U_\infty^2}{\nu x} \frac{d^3 f}{d\eta^3}}.$$