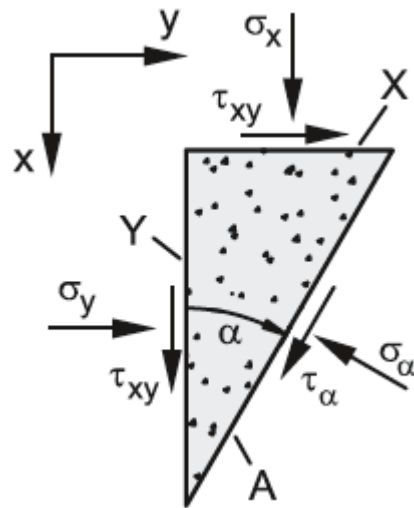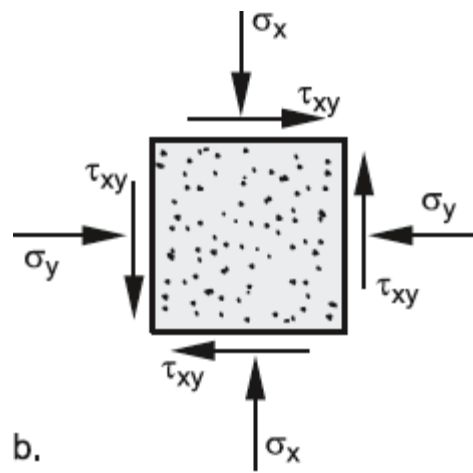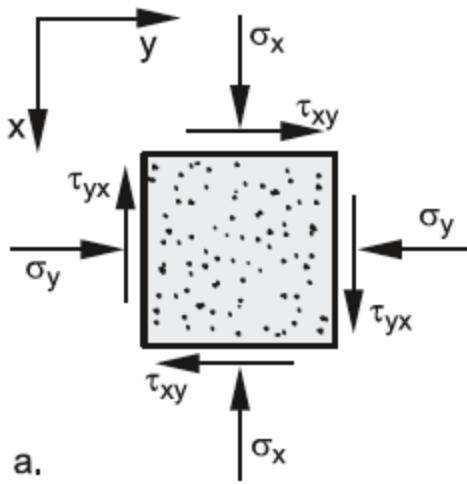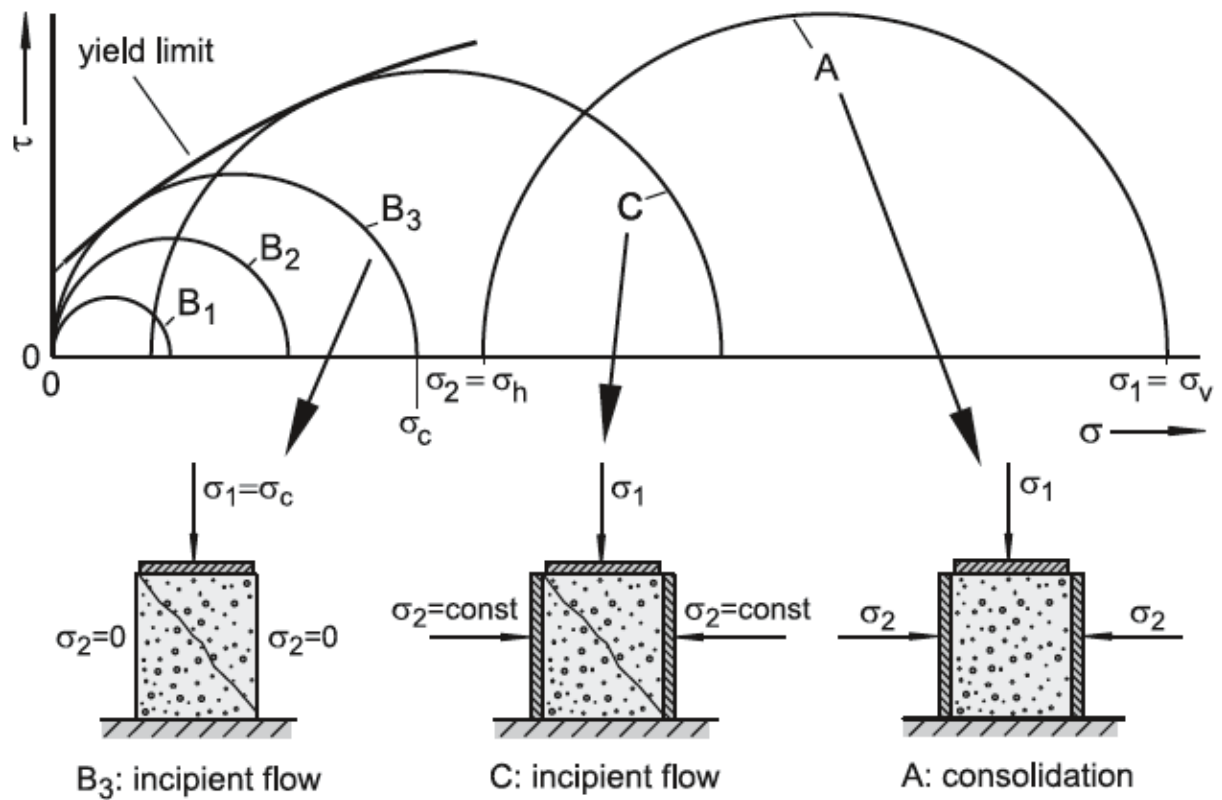$$\sigma_\alpha = \frac{\sigma_v + \sigma_h}{2} + \frac{\sigma_v - \sigma_h}{2}\cos(2\alpha)$$

$$\tau_\alpha = \frac{\sigma_v - \sigma_h}{2}\sin(2\alpha)$$

a.

b.

$$\sigma_\alpha = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2} \cos 2\alpha + \tau_{xy} \sin 2\alpha$$

$$\tau_\alpha = \frac{\sigma_y - \sigma_x}{2} \sin 2\alpha - \tau_{xy} \cos 2\alpha$$

yield limit

τ

B_3

B_2

B_1

A

C

0

$\sigma_2 = \sigma_h$

σ_c

$\sigma_1 = \sigma_v$

σ

$\sigma_1 = \sigma_c$

$\sigma_2 = 0$ $\sigma_2 = 0$

B_3: incipient flow

σ_1

$\sigma_2 = const$ $\sigma_2 = const$

C: incipient flow

σ_1

σ_2 σ_2

A: consolidation

resultant force

F_R

$\sigma_{pre}{\cdot}A$

area A

$\tau_{pre}{\cdot}A$

v

a. preshear

$\sigma_{sh}{\cdot}A$

$\tau_{sh}{\cdot}A$

v

b. shear to failure

τ, ρ_b

preshear (σ_{pre})

shear to failure ($\sigma_{sh}<\sigma_{pre}$)

τ

steady-state
flow (τ_{pre})

incipient
flow (τ_{sh})

ρ_b

time

c. elastic   plastic

Determination of the yield limit from the measured shear stresses

τ

shear point

yield locus

τ_{sh}

incipient
flow
(failure)

σ_{sh}

loading with σ_{sh}

σ

σ_1

σ_{sh}

presheared specimen

loading with σ_{sh}

σ_{sh} τ_{sh}

shear (to failure)
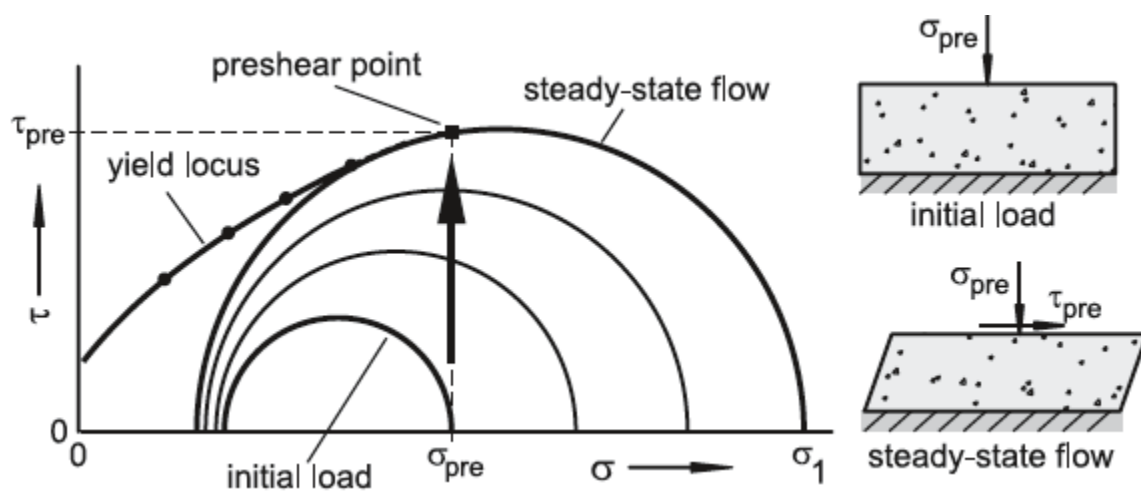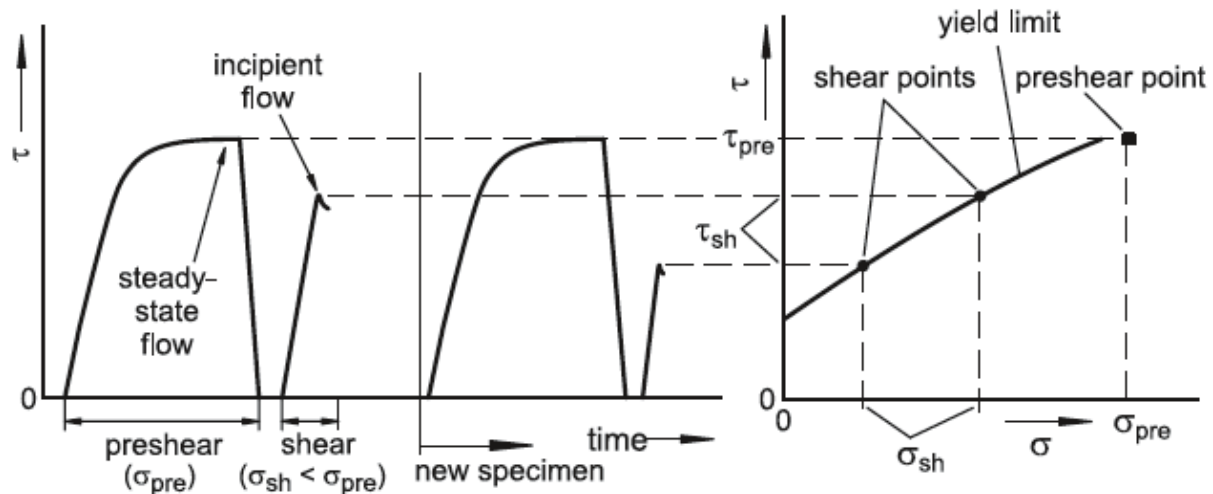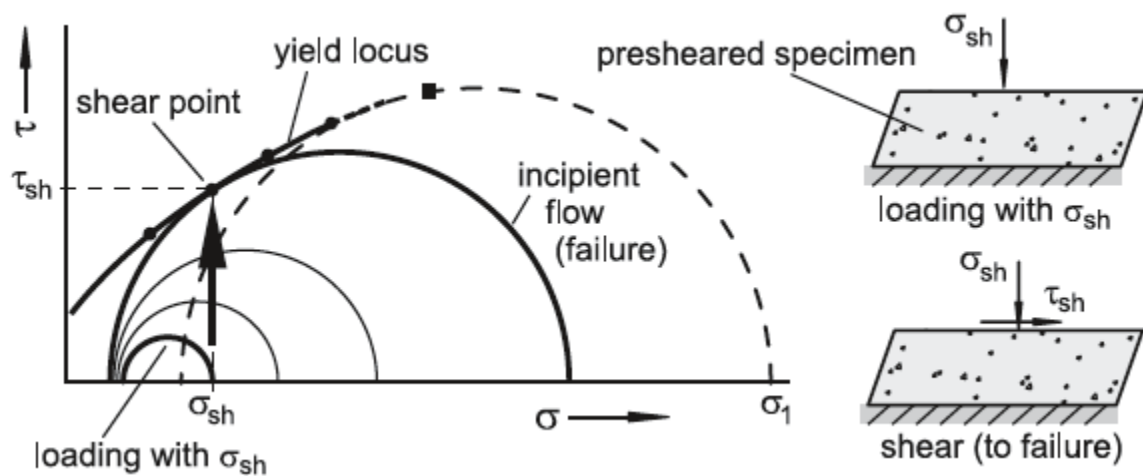
Yield locus and flow properties

- slope angle of the linearized yield locus, φ_{lin}
- effective angle of internal friction, φ_e (slope of the effective yield locus)
- angle of internal friction at steady-state flow, φ_{sf}
- bulk density, ρ_b

bulk solid

wall material

measured points

wall yield locus

τ_w

φ_x

σ_{w3} σ_{w2} σ_{w1}

σ_w

σ_w

σ_w

τ_w

v

shear points
(incipient flow)

preshear point
(steady-state flow)

yield locus

end point of the yield locus

τ

0

0   σ_2 σ_c σ σ_1

$\sigma_c A$

$\sigma_1 A$

preshear point (horizontal plane)

principal stress plane

τ_{pre}

yield locus

horizontal plane

σ_1

σ_{pre}

τ_{pre}

α

τ

2α

0

0

σ_{pre}

σ

σ_1

steady-state flow