Q. Consider a process with the following transfer functions:

$$y(s) = \frac{2}{(3s+1)(5s+1)}F(s) + \frac{1}{4s+1}T(s)$$

$$F(s) = \frac{3}{2s+1}u(s) + \frac{1}{s+1}P(s)$$

a) Draw a feedback control block diagram for controlling y by manipulating u and tune a PI controller using Ziegler-Nichols (Z-N) tuning correlations ($K_c = \frac{K_u}{2.2}; \tau_I = \frac{P_u}{1.2}$).

b) Since P changes frequently, the above feedback control strategy is not efficient and a cascade control strategy has to be tried.
   a. Draw a cascade control block diagram
   b. Design the slave controller using direct synthesis approach (reference/desired time constant is 20% of the process time constant)
   c. Design the master controller using Z-N tuning technique

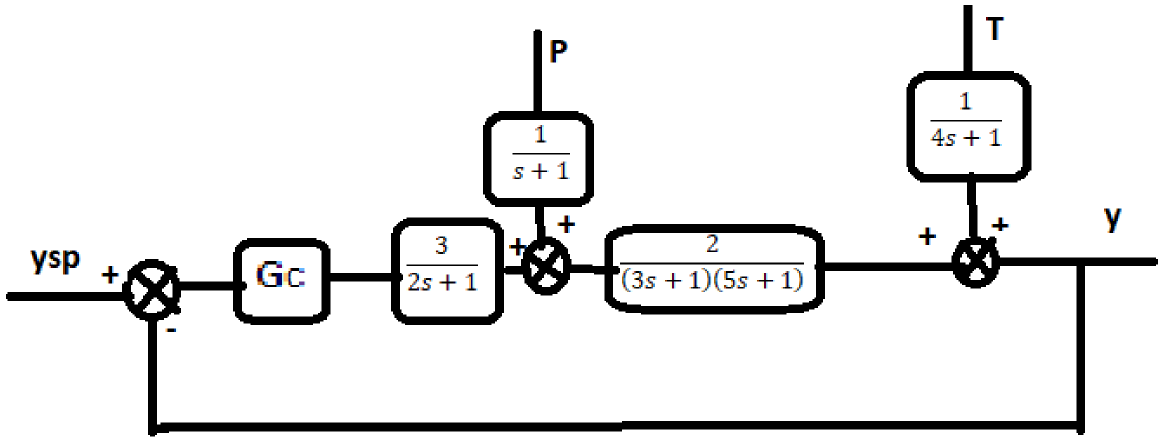
c) The cascade control strategy can not handle the large and frequent changes in T and therefore to be augmented with a feedforward control strategy. T variable is measured and the sensor transfer function is unity. Draw the augmented control block diagram which should include cascade and feedforward control strategy.
   Find the feedforward controller equation (FFC).

d) Is FFC designed above physically realizable? If no, make it physically realizable by converting First order with dead time (FOTD) model.

[0.5+2+0.5+1+2+1+1+2]

Solution

Feedback Control Block diagram



To find the PI controller parameter using Z-N method, we take $G_c = K$ and write the characteristic equation as below.

$1 + K.\frac{6}{(2s+1)(3s+1)(5s+1)} = 0$ Or, $30s^3 + 31s^2 + 10s + 1 + 6K = 0$

Put $s = j\omega$, $-30j\omega^3 - 31\omega^2 + 10j\omega + 1 + 6K = 0$

$(-31\omega^2 + 1 + 6K) + j(-30\omega^3 + 10\omega) = 0 + j.0$

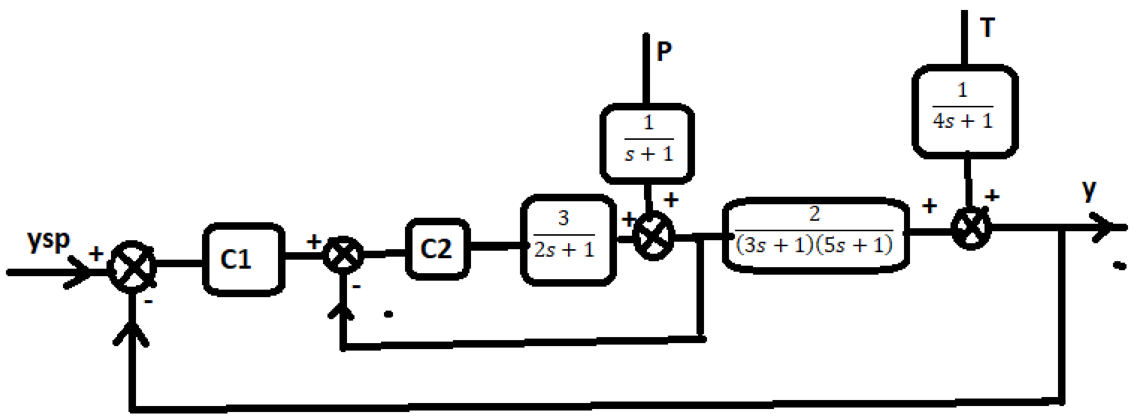
Or, $-30\omega^3 + 10\omega = 0 \;\; or, \;\; \omega = 0, \;\; or\; \omega = \pm\frac{1}{\sqrt{3}}$

$for\; \omega = 0, K = -\frac{1}{6}\;\; which\; is\; not\; feasible\; because\; process\; gain\; is\; positive$

$for\; \omega = \frac{1}{\sqrt{3}}\,, \;\; K = K_u = \frac{14}{9} = 1.55 \;\; and\; P_u = 2\,\pi\,\sqrt{3} = 10.88$

So PI controller parameters are: $K_c = \frac{1.55}{2.2} = 0.704 \;\;\; and\; \tau_I = \frac{10.88}{1.2} = 9.06$

(b) Cascade control block diagram Master controller is C1 and slave controller is C2



Slave controller C2 using DSA: $\tau_r = 0.2 * 2 = 0.4$ and $C2(s) = \frac{1}{\tau_r s}\frac{1}{g_p} = \frac{1}{0.4\,s}.\frac{2s+1}{3} = \frac{2}{1.2}\left(1 + \frac{1}{2s}\right)$

Master Controller C1 using Z-N :

Characteristic equation : $1 + K.\frac{1}{0.4\,s+1}.\frac{2}{(3s+1)(5s+1)} = 0$ or, $6s^3 + 18.2s^2 + 8.4s + 1 + 2K = 0$

Put $s = j\omega$, $-6j\omega^3 - 18.2\omega^2 + 8.4j\omega + 1 + 2K = 0$

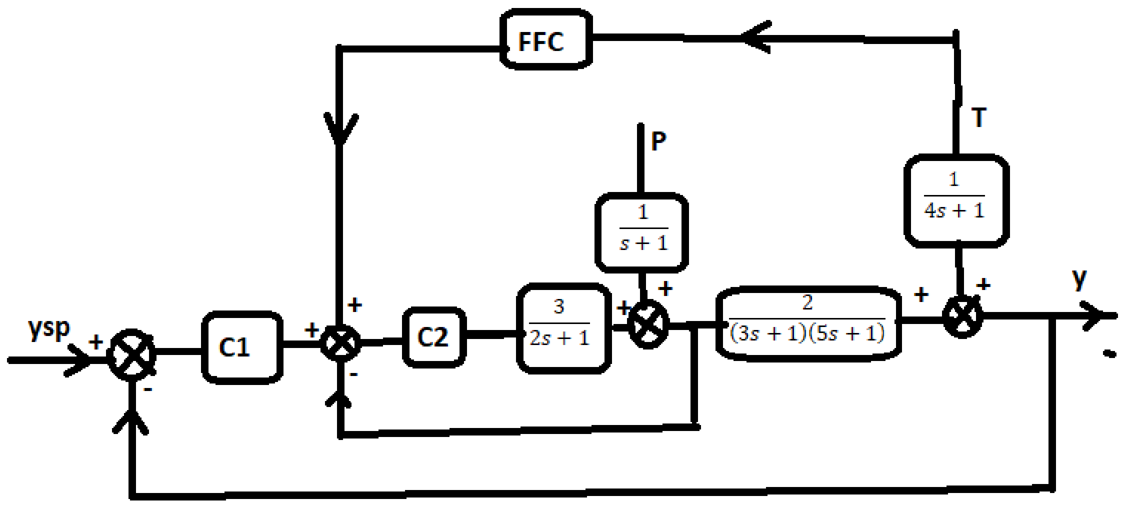
Or, $(-18.2\omega^2 + 1 + 2K) + j(-6\omega^3 + 8.4\omega) = 0 + j.0$

Or, $-6\omega^3 + 8.4\omega = 0$ $\quad this\ gives\ \omega = 0\ or\ \omega = \pm\sqrt{1.4}$

Or, $K = K_u = 12.24$ $\quad and\ P_u = \frac{2\pi}{\sqrt{1.4}} = 5.31$

So, PI controller parameter $K_c = 5.56$ $\quad and\ \tau_I = 4.42$

(c) Feedforward-cascade control block diagram



$$y(s) = \frac{C1(s)gs(s)g_1(s)}{1+C1(s)gs(s)g_1(s)}ysp(s) + \frac{FFC(s)g_1(s)gs(s)+g_{d1}(s)}{1+C1(s)gs(s)g_1(s)}T(s) + \frac{g_1(s)g_{d2}(s)}{1+C1(s)gs(s)g_1(s)}P(s)$$

Where, $g_1(s) = \frac{2}{(3s+1)(5s+1)}$ $\quad gs(s) = \frac{C2(s)g_2(s)}{1+C2(s)g_2(s)}$ $\quad g_2(s) = \frac{3}{2s+1}$ $\quad g_{d1}(s) = \frac{1}{4s+1}$ $\quad g_{d2}(s) = \frac{1}{s+1}$

And C1 and C2 is designed in (b).

To design the FFC, y should be zero for any changes in T

i.e. $FFC(s)g_1(s)gs(s) + g_{d1}(s) = 0$

$$FFC(s) = -\frac{g_{d1}(s)}{g_1(s)gs(s)} = -0.5\frac{(3s+1)(5s+1)(0.4s+1)}{4s+1}$$

(d) the FFC designed is not physically realizable.

$$g_1(s)gs(s) = g_p(s) = \frac{2}{(0.4s+1)(3s+1)(5s+1)} = \frac{A}{0.4s+1} + \frac{B}{3s+1} + \frac{C}{5s+1}$$

$A|_{s=-\frac{1}{0.4}} = 0.0268 \quad B|_{s=-\frac{1}{3}} = -3.4615 \quad C|_{s=-\frac{1}{5}} = 5.4348$

$g_p'(s) = -\frac{0.4A}{(0.4s+1)^2} - \frac{3B}{(3s+1)^2} - \frac{5C}{(5s+1)^2}$ or, $g_p'(0) = -0.4A - 3B - 5C = -16.8$

$g_p''(s) = \frac{0.32A}{(0.4s+1)^3} + \frac{18B}{(3s+1)^3} + \frac{50C}{(5s+1)^3}$ or, $g_p''(0) = 0.32A + 18B + 50C = 209.44$

$(\tau + \theta) = -\frac{g_p'(0)}{g_p(0)} = \frac{16.8}{2} = 8.4$ and $\tau^2 = \frac{g_p''(0)}{g_p(0)} - (\tau+\theta)^2 = \frac{209.44}{2} - 8.4^2 = 34.16$

So, $\tau = 5.8447 \quad and\ \theta = 2.5553$ therefore, $g_p(s) \approx \frac{2e^{-2.5553s}}{5.8447s+1}$

And $FFC(s) = -\frac{g_{d1}(s)}{g_p(s)} = -0.5\frac{5.8447s+1}{4s+1}e^{2.5553s} \approx -0.5\frac{8.4s+1}{4s+1}$

Simplest way to make it physically realizable is to neglect 0.4s+1 as 0.4 << 5 and add the two time constants i.e,

$$FFC(s) = -0.5\frac{8s+1}{4s+1}$$