ASSIGNMENT-2

F.M. 50

15 Marks

1. (a) $G(s) = \frac{10}{s(s+5)} \Rightarrow G(s) = 2 \cdot \frac{1}{s} \cdot \frac{1}{0.2s+1}$

Amplitude ratio, $AR = AR_1 \cdot AR_2 \cdot AR_3$

$AR = 2 \cdot \frac{1}{\omega} \cdot \frac{1}{\sqrt{(0.2\omega)^2+1}}$

$\log AR = \log 2 + \log\left(\frac{1}{\omega}\right) + \log\left(\frac{1}{\sqrt{(0.2\omega)^2+1}}\right)$

$\log AR = \log 2 - \log \omega - \frac{1}{2}\log\left[(0.2\omega)^2+1\right]$ — (1)

Low frequency asymptote:

$\omega \to 0 \Rightarrow 0.2\omega \to 0$

from (1), $\boxed{\log AR = \log 2 - \log \omega}$ — (2)

Slope $= -1$.

High frequency asymptote:

$\omega \to \infty \Rightarrow 0.2\omega \to \infty$.

from (1), $\log AR = \log 2 - \log \omega - \log[0.2\omega]$

$\boxed{\log AR = 1 - 2\log \omega}$ — (3)

Slope $= -2$.

Corner frequency:

Equating (2) with (3)

$\log 2 - \log \omega_n = 1 - 2\log \omega_n$

$\Rightarrow \omega_n = 5$

At ω_n, $AR = 0.4$.

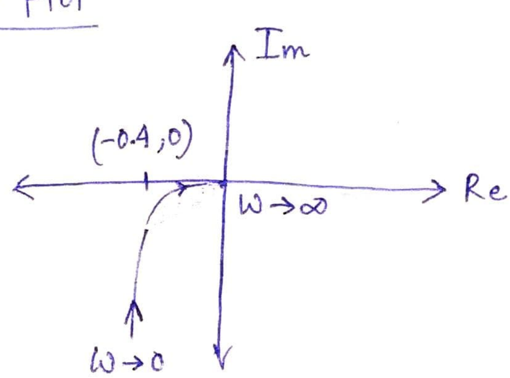
Phase shift, $\phi = \phi_1 + \phi_2 + \phi_3$

$\phi = 0 + \tan^{-1}(-\infty) + \tan^{-1}(-0.2\omega)$

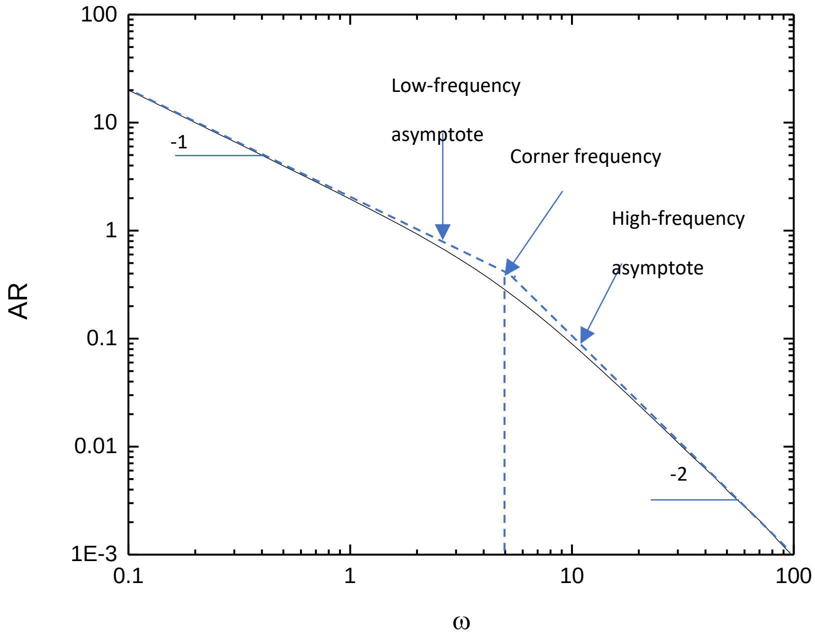
$\Rightarrow \phi = -90^\circ + \tan^{-1}(-0.2\omega)$

$\omega \to 0$	$\Rightarrow 0.2\omega \to 0$	$\Rightarrow \phi \to -90^\circ$	
$\omega \to \infty$	$\Rightarrow 0.2\omega \to \infty$	$\Rightarrow \phi \to -180^\circ$	1 Mark

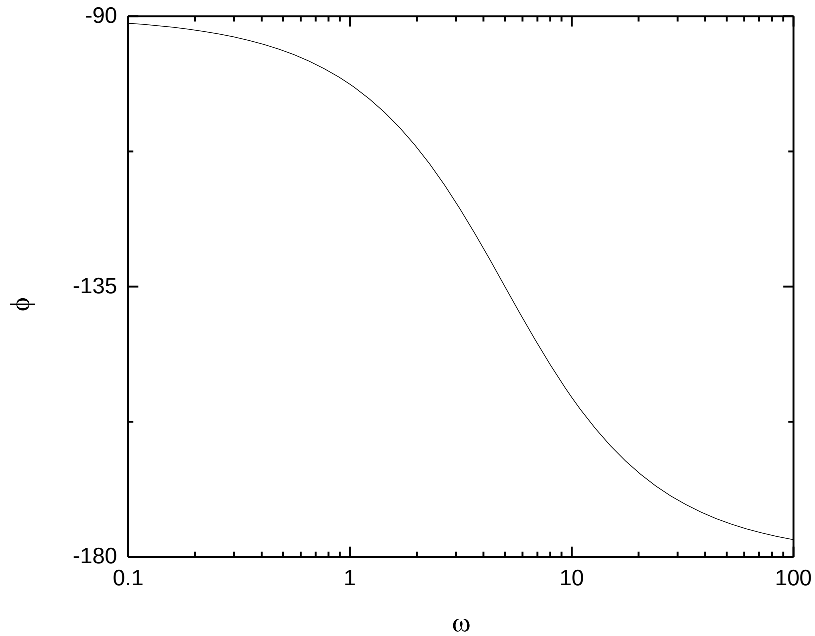
Nyquist Plot



<u>Bode Plot</u>



100
10
1
0.1
0.01
1E-3
AR
Low-frequency
asymptote
Corner frequency
High-frequency
asymptote
-1
-2
0.1
1
10
100
ω



-90
-135
-180
φ
0.1
1
10
100
ω

(b) $G(s) = \dfrac{s+1}{s(s+5)} = 0.2\,(s+1)\cdot\dfrac{1}{s}\cdot\dfrac{1}{0.2s+1}$

Amplitude ratio, $AR = AR_1 \cdot AR_2 \cdot AR_3 \cdot AR_4$

$$AR = 0.2 \cdot \sqrt{\omega^2+1} \cdot \frac{1}{\omega} \cdot \frac{1}{\sqrt{(0.2\omega)^2+1}}$$

$$\log AR = \log 0.2 + \frac{1}{2}\log(\omega^2+1) - \log\omega - \frac{1}{2}\log[(0.2\omega)^2+1]$$

We identify three regions on the frequency scale based on the corner frequency of the constituent transfer functions.

Slope of the overall asymptote for the AR vs ω diagram

Frequency region	Slope of the asymptotes of the individual transfer fn.				Slope of the overall asymptote
	0.2	$s+1$	$\frac{1}{s}$	$\frac{1}{0.2s+1}$	
$0 \le \omega < 1$	0	0	−1	0	−1
$1 \le \omega < 5$	0	1	−1	0	0
$5 \le \omega < \infty$	0	1	−1	−1	−1

Phase shift, $\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4$.

$$\phi = 0 + \tan^{-1}(\omega) + \tan^{-1}(-\infty) + \tan^{-1}(-0.2\omega)$$

$$\phi = -90^\circ + \tan^{-1}(\omega) + \tan^{-1}(-0.2\omega)$$

$\omega \to 0 \Rightarrow \phi \to -90^\circ$ 1

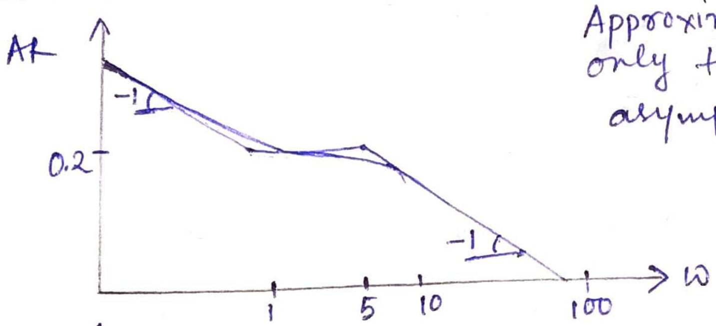
$\omega \to \infty \Rightarrow \phi \to -90^\circ$

For finding ϕ_{max}:

$$\frac{d\phi}{d\omega} = 0 \Rightarrow \frac{1}{1+\omega^2} - \frac{0.2}{1+0.04\omega^2} = 0 \Rightarrow \omega = \sqrt{5}$$

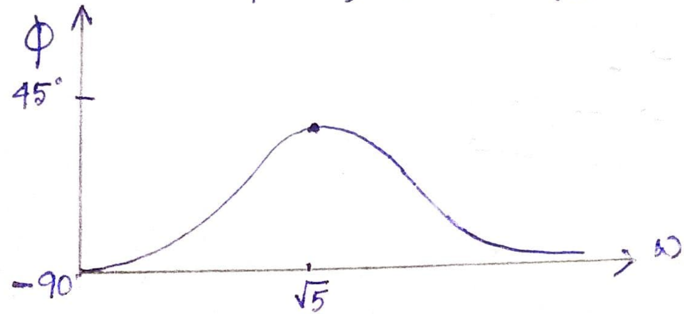
$$\phi_{max} = \phi\big|_{\omega=\sqrt{5}} = -48.2^\circ$$

Bode Plot

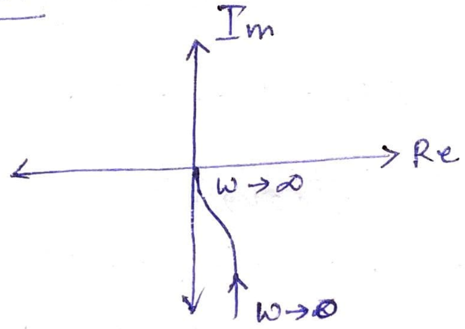


Approximate plot using only the slopes of the asymptotes.

1



Nyquist plot



1

(c) $G(s) = \frac{50}{(s+2)^2} = 50 \cdot \frac{1}{(s+2)} \cdot \frac{1}{(s+2)}$

$= 12.5 \cdot \frac{1}{(0.5s+1)} \cdot \frac{1}{(0.5s+1)}$

Amplitude ratio, $AR = AR_1 \cdot AR_2 \cdot AR_3$

$AR = 12.5 \cdot \frac{1}{\sqrt{(0.5\omega)^2+1}} \cdot \frac{1}{\sqrt{(0.5\omega)^2+1}}$

$\log AR = \log 12.5 + (-)\frac{1}{2}\log[(0.5\omega)^2+1] - \frac{1}{2}\log[(0.5\omega)^2+1]$

$\log AR = \log 12.5 - \log[(0.5\omega)^2+1]$ —— (1)

Low frequency asymptote

$\omega \to 0 \Rightarrow 0.5\omega \to 0$.

(1) $\Rightarrow \boxed{\log AR = \log 12.5}$ —— (2) Slope = 0

High frequency asymptote.

$\omega \to \infty \Rightarrow 0.5\omega \to \infty$

(1) $\Rightarrow \boxed{\log AR = \log 12.5 - 2\log(0.5\omega)}$ —— (3) Slope = -2.

Corner frequency:

from (2) and (3)

$\log 12.5 = \log 12.5 - 2\log(0.5\omega_n)$

$\Rightarrow 0.5\omega_n = 1 \Rightarrow \omega_n = 2$.

1

$AR|_{\omega_n=2} = 12.5$

Phase Shift $\phi = \phi_1 + \phi_2 + \phi_3$

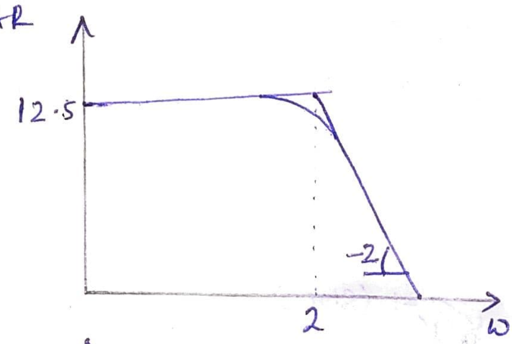
$\phi = 0 + \tan^{-1}(-0.5\omega) + \tan^{-1}(-0.5\omega)$

$\phi = 2\tan^{-1}(-0.5\omega)$

$\omega \to 0 \Rightarrow \phi \to 0$.

$\omega \to \infty \Rightarrow \phi \to -180^\circ$

Bode plot



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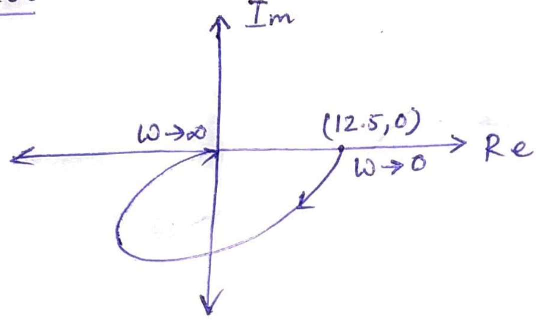
Nyquist plot



1

(d) $G(s) = \dfrac{10e^{-s}}{(s+1)(s+3)} = 3.33 \cdot \dfrac{1}{s+1} \cdot \dfrac{1}{0.33s+1} \cdot e^{-s}$

Amplitude ratio, $AR = AR_1 \cdot AR_2 \cdot AR_3 \cdot AR_4$

$$AR = 3.33 \cdot \frac{1}{\sqrt{\omega^2+1}} \cdot \frac{1}{\sqrt{(0.33\omega)^2+1}} \cdot 1$$

$$\log AR = \log 3.33 - \frac{1}{2}\log[\omega^2+1] - \frac{1}{2}\log[(0.33\omega)^2+1] + 0 \quad \text{--- (1)}$$

We identify three regions on the frequency scale based on corner frequency of the constituent transfer functions.

Slope of the overall asymptote for the AR vs ω diagram

Frequency region	Slope of the asymptote of the individual transfer fn.			Slope of the overall asymptote.
	3.33	$\frac{1}{s+1}$	$\frac{1}{0.33s+1}$	
$0 \le \omega < 1$	0	0	0	0
$1 \le \omega < 3$	0	-1	0	-1
$3 \le \omega < \infty$	0	-1	-1	-2.

1

Phase shift, $\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4$

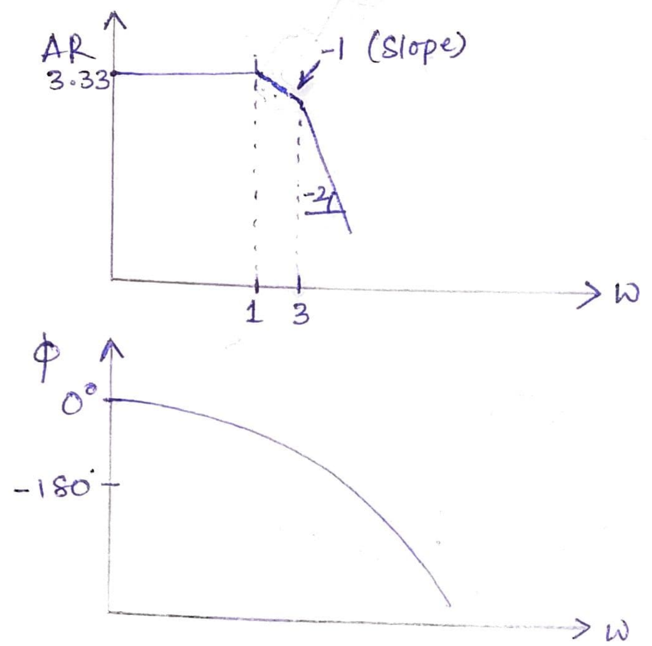
$$\phi = 0 + \tan^{-1}(-\omega) + \tan^{-1}(-0.33\omega) - 1 \times \frac{180°}{\pi} \times \omega$$

$$\phi = \tan^{-1}(-\omega) + \tan^{-1}(-0.33\omega) - 57.32° \times \omega$$

$\omega \to 0 \quad \Rightarrow \quad \phi = 0^\circ$

$\omega \to \infty \quad \Rightarrow \quad \phi \to -\infty$

Bode Plot



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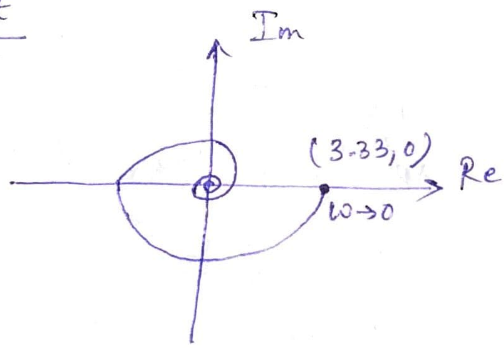
Nyquist plot



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(e) $G(s) = \dfrac{1}{s^2+3s+1} = \dfrac{1}{(2.62s+1)} \cdot \dfrac{1}{(0.38s+1)}$

Amplitude ratio, $AR = AR_1 \cdot AR_2$.

$$AR = \frac{1}{\sqrt{(2.62\omega)^2+1}} \cdot \frac{1}{\sqrt{(0.38\omega)^2+1}}$$

$$\log AR = -\frac{1}{2}\log\left[(2.62\omega)^2+1\right] - \frac{1}{2}\log\left[(0.38\omega)^2+1\right]$$

We identify three regions on the frequency scale based on their corner frequencies.

Slope of the overall asymptote for the AR vs ω diagram.

Frequency region	Slope of asymptote of individual transfer function		Slope of the overall asymptote
	$\frac{1}{2.62s+1}$	$\frac{1}{0.38s+1}$	
$0 \le \omega < 0.38$	0	0	0
$0.38 \le \omega < 2.62$	-1	0	-1
$2.62 \le \omega < \infty$	-1	-1	-2.

1

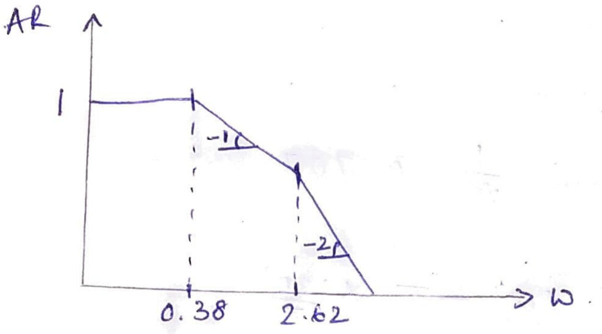
Phase Shift, $\phi = \phi_1 + \phi_2$

$$\phi = \tan^{-1}(-0.38\omega) + \tan^{-1}(-2.62\omega)$$

$\omega \to 0 \Rightarrow \phi = 0°$

$\omega \to \infty \Rightarrow \phi = -180°$

Bode Plot



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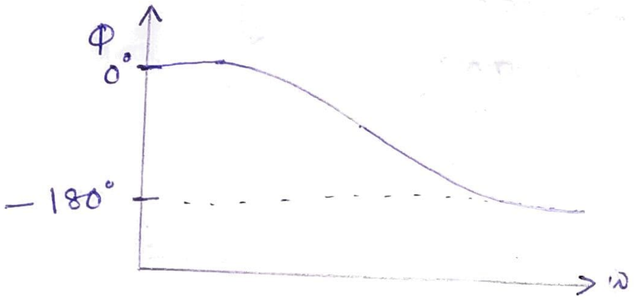
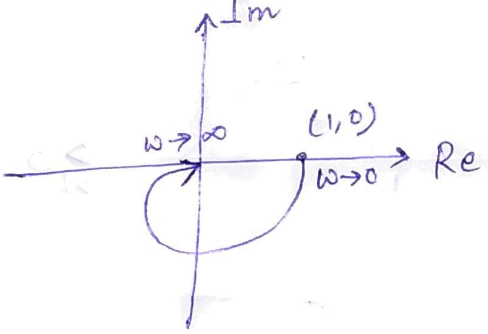


Nyquist plot



1

10 Marks

2. Bode Stability criterion: A feedback control system is unstable if the amplitude ratio (AR) of the corresponding open-loop transfer function is larger than 1 at the cross over frequency.

(a) $G_{OL} = \dfrac{1}{s-1}$

Let $s = j\omega$.

$$G_{OL} = \frac{1}{j\omega - 1} = \frac{1}{-1+j\omega} \cdot \frac{-1-j\omega}{-1-j\omega} = \frac{-1-j\omega}{1+\omega^2}$$

$$G_{OL} = \frac{-1}{1+\omega^2} - j\frac{\omega}{1+\omega^2}$$

$$AR = \sqrt{\frac{1}{(1+\omega^2)^2} + \frac{\omega^2}{(1+\omega^2)^2}} = \frac{\sqrt{1+\omega^2}}{(1+\omega^2)} = \frac{1}{\sqrt{1+\omega^2}}$$

$$\log AR = -\frac{1}{2}\log\left[1+\omega^2\right] \quad \text{——} \ (1)$$

Low frequency asymptote:

$\omega \to 0 \quad \Rightarrow \quad \log AR = 0 \quad \text{—(2)} \quad \Rightarrow AR = 1 \quad ; \text{ slope} = 0.$

High frequency asymptote.

$\omega \to \infty \quad \Rightarrow \quad \log AR = -\log \omega \quad \text{—(3)} \quad ; \text{ slope} = -1.$

Corner frequency:

from (2) and (3)

$-\log \omega_n = 0 \quad \Rightarrow \quad \omega_n = 1.$

$AR\big|_{\omega_n = 1} = \frac{1}{\sqrt{2}} = 0.707.$

Phase shift

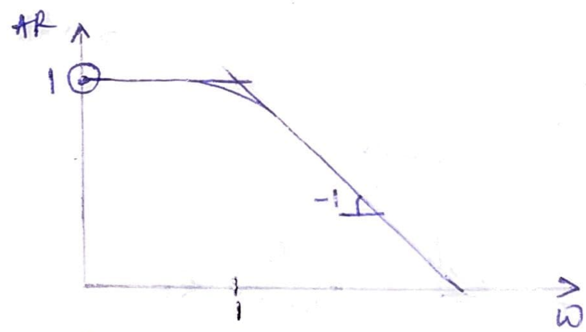
$\phi = \tan^{-1}(\omega) - \pi \times \frac{180^\circ}{\pi}$

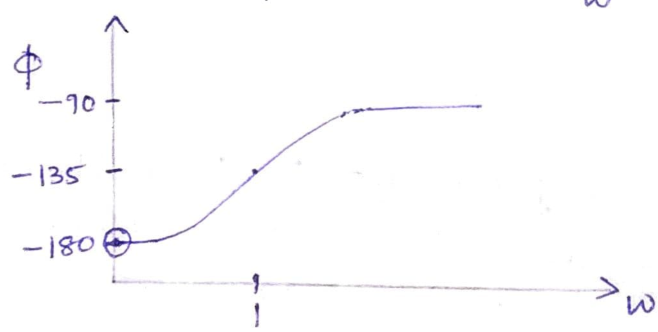
$\omega \to 0 \quad \Rightarrow \quad \phi \to -180^\circ$

$\omega \to \infty \quad \Rightarrow \quad \phi \to -90^\circ$

$\omega \to \omega_n = 1 \quad \phi \to -135^\circ$

Bode Plots.





$\omega_{co} \to$ Cross over frequency : Frequency when the phase lag is 180°

1

$AR\big|_{\omega_{co}} = 1 \quad \leftarrow$ The system lies on the stability limit

1 0.5

Bode criterion leads to rigorous conclusions only when AR and ϕ of the corresponding open loop transfer functions decrease continuously as ω increases.

As phase shift (ϕ) is not a monotonically decreasing function of ω, Bode stability criterion may lead to erroneous conclusions.

(b) $G_{OL} = \frac{10 e^{-3s}}{4s+1} = 10 \cdot \frac{1}{4s+1} \cdot e^{-3s}$.

$AR = AR_1 \cdot AR_2 \cdot AR_3$

$AR = 10 \cdot \frac{1}{\sqrt{(4w)^2+1}} \cdot 1 \Rightarrow \log AR = \log 10 + (-1)\frac{1}{2}\log [(4w)^2+1]$

$\Rightarrow \log AR = 1 - \frac{1}{2} \log [(4w)^2+1]$ — ①

Low frequency asymptote.

$w \to 0 \Rightarrow 4w \to 0 \Rightarrow \log AR = 1 \Rightarrow AR = 10$

Slope = 0

High frequency asymptote:

$w \to \infty \Rightarrow \log AR = 1 - \log [4w]$

Slope = -1

Corner frequency:

$1 - \log(4w_n) = \log 10 = 1 \Rightarrow 4w_n = 1 \Rightarrow w_n = \frac{1}{4}$

$AR\Big|_{w_n = \frac{1}{4}} = \frac{10}{\sqrt{2}} = 7.07$

$\phi = \tan^{-1}(-4w) - 3w \cdot \frac{180}{\pi}$ — ②

$w \to 0 \Rightarrow \phi = 0°$

$w \to \infty \Rightarrow \phi = -\infty$

Crossover frequency:

From ②, $\tan^{-1}(-4w_{co}) - 3w_{co} \cdot \frac{180°}{\pi} = -180°$

$\tan^{-1}(-4w_{co}) - 171.97 w_{co} = -180°$

$\Rightarrow w_{co} = 0.65$ rad/min (approx) 1

AR at crossover frequency:

$AR\Big|_{w_{co}} = \frac{10}{\sqrt{(4w_{co})^2+1}} = \frac{10}{\sqrt{(4 \times 0.65)^2+1}} = 3.59$ 1

$\because AR\Big|_{w_{co}} = 3.59 > 1$

Hence, the system is unstable. 0.5

(c) $G_{OL} = \frac{5e^{-5s}}{(2s+1)(s+1)} = 5 \cdot \frac{1}{2s+1} \cdot \frac{1}{s+1} \cdot e^{-5s}$.

$AR = AR_1 \cdot AR_2 \cdot AR_3 \cdot AR_4$

$AR = 5 \cdot \frac{1}{\sqrt{(2w)^2+1}} \cdot \frac{1}{\sqrt{w^2+1}} \cdot 1$

$\log AR = \log 5 - \frac{1}{2} \log [(2w)^2+1] - \frac{1}{2} \log [w^2+1]$

$\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4$

$\phi = 0 + \tan^{-1}(-2w) + \tan^{-1}(-w) - 5w \cdot \frac{180°}{\pi}$

Cross over frequency:

$$\tan^{-1}(-2\omega_{co}) + \tan^{-1}(-\omega_{co}) - 5\omega_{co} \cdot \frac{180^\circ}{\pi} = -180^\circ$$

$\omega_{co} = 0.412$ rad/min (approx) 1

Amplitude ratio at ω_{co}

$$AR\big|_{\omega_{co}} = \frac{5}{\sqrt{(2\omega_{co})^2+1}\,\sqrt{(\omega_{co})^2+1}} = 3.57$$ 1

$\therefore \; AR\big|_{\omega_{co}} = 3.57 > 1.$

Hence, the system is unstable 0.5

(d) $$G_{OL} = \frac{1}{0.2s^2+0.8s-1}$$

$s = j\omega.$

$$G_{OL} = \frac{1}{-0.2\omega^2+0.8\omega j-1} = \frac{1}{(-0.2\omega^2-1)+j\,0.8\omega} \times \frac{(-0.2\omega^2-1)-j0.8\omega}{(-0.2\omega^2-1)-j0.8\omega}$$

$$= \frac{-1-0.2\omega^2}{(-1-0.2\omega^2)^2+(0.8\omega)^2} - j\,\frac{0.8\omega}{(-1-0.2\omega^2)^2+(0.8\omega)^2}$$

$$AR = \sqrt{\frac{(-1-0.2\omega^2)^2+(0.8\omega)^2}{[(-1-0.2\omega^2)^2+(0.8\omega)^2]^2}}$$

$$AR = \frac{1}{\sqrt{(-1-0.2\omega^2)^2+(0.8\omega)^2}}$$

$$\phi = -\pi + \tan^{-1}\left(\frac{0.8\omega}{1+0.2\omega^2}\right)$$

Cross over frequency:

$$\left(\frac{180}{\pi}\right)(-\pi) + \tan^{-1}\left(\frac{0.8\omega_{co}}{1+0.2\omega_{co}^2}\right) = -180^\circ$$

$\omega_{co} = 0$ 1

AR at ω_{co}

$AR\big|_{\omega_{co}} = 1$ 1

$\therefore$ The system lies on the stability limit 0.5

3. (a) $G_{OL} = \frac{1}{s-1}$

10 Marks

From problem 2(a)

$AR|_{w_{co}} = 1 = M$

Gain Margin $= \frac{1}{M} = 1$ —① 1

Phase Margin $= 180° - |\phi'|$

$\phi' \rightarrow$ phase lag at the frequency for which $AR = 1$.

$AR = 1$ at $w = w_{co}$

$\phi' = \phi|_{w_{co}} = -180°$

∴ Phase Margin $= 0°$ —② 1

From ① and ②

The system lies on the stability limit. 0.5

(b) $G_{OL} = \frac{10e^{-3s}}{4s+1}$

From problem 2(b)

$AR|_{w_{co}} = 3.59 = M$

Gain Margin $= \frac{1}{M} = \frac{1}{3.59} = 0.279$

Gain Margin < 1. —① 1

$AR = \frac{10}{\sqrt{(4w)^2+1}} = 1 \Rightarrow (4w)^2 + 1 = 100$

$\Rightarrow w = 2.49$ rad/min

$AR = 1$ at $w = 2.49$.

$\phi' = \phi|_{w=2.49} = \tan^{-1}(-4\times2.49) - 3\times2.49\times\frac{180}{\pi}$

$= -84.266 - 428.21$

$= -512.48°$

∴ phase Margin $= 180° - |-512.48|$

$= -332.5$ 1

phase Margin < 0 —②

From ① and ②

The system is unstable. 0.5

(c) $G_{OL} = \dfrac{5e^{-5s}}{(2s+1)(s+1)}$

From problem 2(c)

$AR|_{\omega_{co}} = 3.57 = M$

Gain Margin $= \dfrac{1}{M} = \dfrac{1}{3.57} = 0.28$ 1

Gain Margin < 1. — ①

$AR = \dfrac{5}{\sqrt{4\omega^2+1}\sqrt{\omega^2+1}} = 1.$

$\Rightarrow \quad 25 = (4\omega^2+1)(\omega^2+1) = 4\omega^4 + 4\omega^2 + \omega^2 + 1$

$\Rightarrow \quad 4\omega^4 + 5\omega^2 - 24 = 0$

$\Rightarrow \quad \omega = 1.38$ rad/min.

$AR = 1$ at $\omega = 1.38$

$\phi' = \phi|_{\omega=1.38} = \tan^{-1}(-2\times1.38) + \tan^{-1}(-1.38) - 5\times1.38\times\dfrac{180^\circ}{\pi}$

$= -519.5^\circ$

$\therefore$ phase margin $= 180^\circ - |\phi'|$

$= -339.5$ 1

phase margin < 0 — ②

From ① and ②

The system is unstable. 0.5

(d) $G_{OL} = \dfrac{1}{0.2s^2 + 0.8s - 1}$

From problem 2(d)

$AR|_{\omega_{co}} = 1 = M$

Gain Margin $= \dfrac{1}{M} = 1$ — ① 1

$AR = 1$ at $\omega = \omega_{co}$

$\phi' = \phi|_{\omega_{co}} = -180^\circ$

Phase margin $= 0$ — ② 1

From ① and ②

The system lies on stability limit 0.5

4. **Nyquist Stability criterion:**

If the open loop Nyquist plot of a feedback
10 Marks system encircles the point $(-1,0)$ as the frequency ω taken any value from $-\infty$ to $+\infty$, the closed loop response is unstable.

(a) $G_{OL} = \dfrac{1}{s-1}$

From problem 2(a)

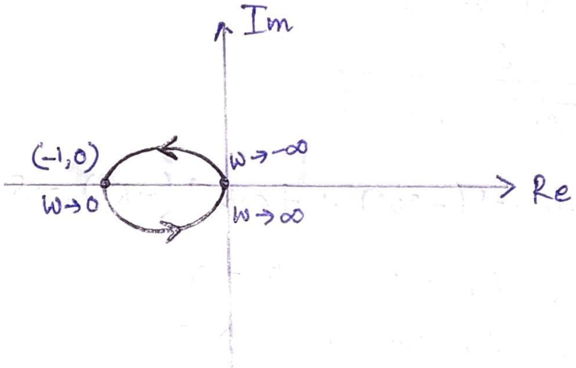
$$AR = \frac{1}{\sqrt{1+\omega^2}}$$

$$\phi = \tan^{-1}(\omega) - \pi \times \frac{180^\circ}{\pi}$$

At $\omega = 0$, $AR = 1$, $\phi = -180^\circ$

At $\omega \to \infty$, $AR \to 0$, $\phi \to -90^\circ$ 1

At $\omega \to -\infty$, $AR \to 0$, $\phi \to -270^\circ$



1

The point $(-1,0)$ lies on the Nyquist plot

$\therefore$ The system is on stability limit 0.5

(b) $G_{OL} = \dfrac{10e^{-s}}{4s+1}$

From problem 2(b)

$$AR = \frac{10}{\sqrt{(4\omega)^2+1}}$$

$$\phi = \tan^{-1}(-4\omega) - 3\omega \cdot \frac{180}{\pi}$$

At $\omega = 0$, $AR = 10$, $\phi = 0$ —— (1)

$\omega \to \infty$ $AR \to 0$, $\phi \to \infty$ —— (2) 1

At $\phi = -180^\circ$, $\omega = ?$ $\Rightarrow \omega = \omega_{co}$

$$\tan^{-1}(-4\omega) - 3\omega \cdot \frac{180}{\pi} = -180$$

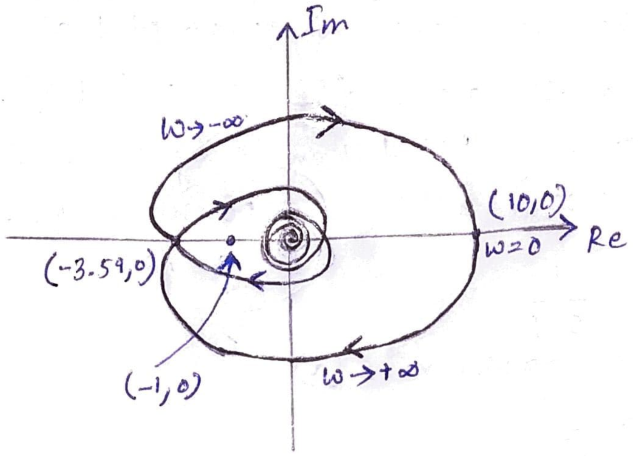
$\omega_{co} = 0.65$

At $\omega = \omega_{co}$, $AR = 3.59$

$\therefore$ $\omega = 0.65$, $AR = 3.59$, $\phi = -180^\circ$ —— (3)

$\omega \to -\infty$ $AR \to 0$, $\phi \to \infty$ —— (4)

From ①, ②, ③ and ④



1

The point (-1,0) is encircled
∴ The system is unstable.

0.5

(c) $G_{OL} = \dfrac{5e^{-5s}}{(2s+1)(s+1)}$

From problem 2(c)

$$AR = \frac{5}{\sqrt{4\omega^2+1}\,\sqrt{\omega^2+1}}$$

$$\phi = \tan^{-1}(-2\omega) + \tan^{-1}(-\omega) - 5\omega \cdot \frac{180}{\pi}$$

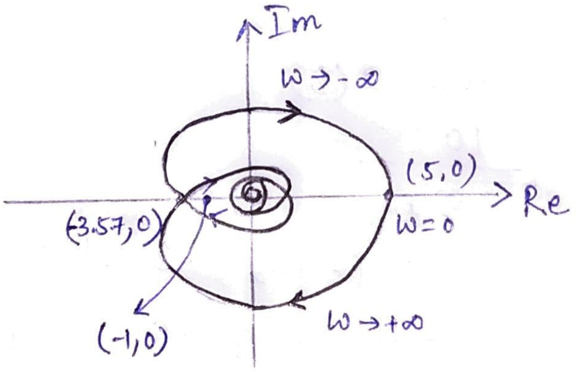
At $\omega = 0$, $AR = 5$, $\phi = 0$

1

$\omega \to \infty$, $AR \to 0$, $\phi \to \infty$

$\omega \to \omega_{co}$, $AR \to 3.57$, $\phi \to -180°$
$= 0.412$

$\omega \to -\infty$, $AR \to 0$, $\phi \to \infty$



1

The point (-1,0) is encircled
∴ The system is unstable.

0.5

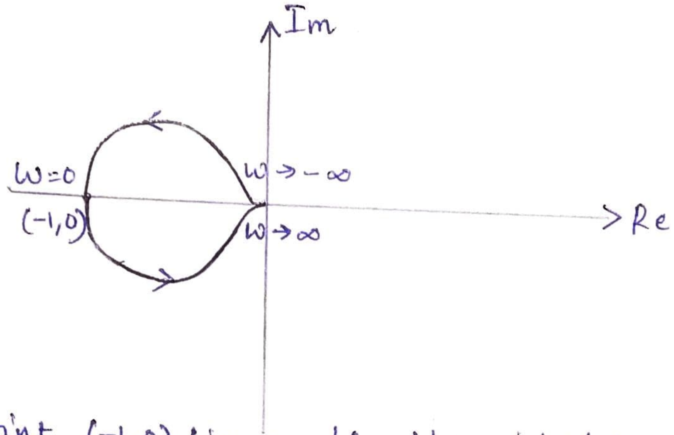
(d) $G_{OL} = \dfrac{1}{0.2s^2 + 0.8s - 1}$

from problem 2(c)

$$AR = \frac{1}{\sqrt{(-1-0.2\omega^2)^2 + (0.8\omega)^2}}$$

$$\phi = -\pi + \tan^{-1}\left(\frac{0.8\omega}{1+0.2\omega^2}\right)$$

At $\omega = 0$, $AR = 1$, $\phi = -180°$
$\omega \to \infty$, $AR \to 0$, $\phi \to -180°$ 1
$\omega \to -\infty$, $AR \to 0$, $\phi \to -180$



1

The point $(-1,0)$ lies on the Nyquist plot
∴ The system lies on the stability limit. 0.5

5 Marks

5.
$$G_p = \frac{10 \cdot e^{-0.1s}}{2s+1}$$
$$= 10 \cdot \frac{1}{2s+1} \cdot e^{-0.1s}$$

$$AR = AR_1 \cdot AR_2 \cdot AR_3$$
$$= 10 \cdot \frac{1}{\sqrt{(2\omega)^2+1}}$$

$$\phi = \phi_1 + \phi_2 + \phi_3$$
$$= 0 + \tan^{-1}(-2\omega) - 0.1\omega \times \frac{180°}{\pi}$$

crossover frequency, ω_{co}
$$\tan^{-1}(-2\omega_{co}) - 0.1\omega_{co} \times \frac{180}{\pi} = -180$$
$$\tan^{-1}(-2\omega_{co}) - 5.73\,\omega_{co} = -180$$
$$\omega_{co} = 16 \text{ rad/min} \quad (\text{approx})$$

$$AR|_{\omega_{co}} = \frac{10}{\sqrt{(2 \times 16)^2 + 1}} = 0.312 = M$$

K_u = Ultimate gain $= \frac{1}{M} = 3.201$ 1

P_u = Ultimate period of sustained oscillation
$$= \frac{2\pi}{\omega_{co}} = \frac{2\pi}{16} = 0.39 \text{ min/cycle.}$$ 1

Ziegler - Nichols settings: (for PID)

$K_c = K_u/1.7 = 1.88$ 1

$\tau_I = P_u/2 = 0.195$ min 1

$\tau_D = P_u/8 = 0.048$ min 1