Flow of gas through microchannel

Knudsen No. $K_n = \dfrac{\lambda}{L}$

λ_{N_2} at 1 bar = 70 nm $\Rightarrow$ $K_n > 10^{-1}$ can be rarely obtained in a channel of μm characteristic dimension.

When

$K_n \lesssim 10^{-2}$ Continuum flow with no slip boundary condition

$10^{-2} < K_n \lesssim 10^{-1}$,, ,, ,, SLIP ,, ,,

$10^{-1} < K_n \lesssim 10$ Transition flow (Burnett equation)

$K_n > 10$ Free molecular flow

Slip boundary condition

(*) The sublayer at the interface, few mean free paths thick is referred as Knudsen layer.

(*) When K_n is small (< 0.1), the ~~continuum~~ average velocity and the wall is continuous and equal to temperature of gas molecules near wall. The knudsen layer is insignificant. The velocity and temperature of wall, bulk flow can be extrapolated to the Surface and the continuum solution for bulk flow is low. equilibrium between is low. equilibrium is low established. In that case,

(*) When K_n is large, collision frequency be established. In that wall and near-wall cannot be the wall and near-wall velocity, one mean free path away from the wall the tangential velocity, one mean free path away from the wall

$= u_{wall} + \lambda \left. \dfrac{\partial u}{\partial y} \right|_{wall}$ = Slip velocity under 1st order slip

$= u_{wall} + \left. \dfrac{\partial u}{\partial y} \right|_{wall}$ = Slip velocity under 1st order slip flow condition (Taylor series expand higher order terms and ignoring higher order terms

Additional considerations in slip flow

Interactions between gas and solid molecules.

Tangential momentum accommodation coefficient

$$\sigma_v = \frac{\tau_i - \tau_r}{\tau_i - \tau_w}$$

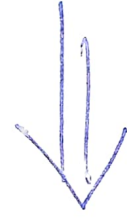τ_i = tangential momentum of incoming molecules

τ_r = " " " " reflected "

τ_w = " " " " the wall $= 0$

$$\Rightarrow \text{A reduction in velocity after collision} \Downarrow$$

Expression by Maxwell

For monatomic gases

$$u_{gas} - u_{wall} = \frac{2 - \sigma_v}{\sigma_v} \frac{\mu \sqrt{\pi}}{\rho \sqrt{2 R T_w}} \frac{\partial u}{\partial y}$$

for an isothermal wall

$$\boxed{u_{gas} - u_{wall} = \frac{2 - \sigma_v}{\sigma_v} \lambda \frac{\partial u}{\partial y}}$$

T_w is the wall temperature

A slip length L_s can be defined such that the slip velocity

$$u_s = u_w + L_s \frac{\partial u}{\partial y}$$

First order slip for flow between two infinite parallel plates

$\longrightarrow U$

Held Stationary

(y, x axes)

- Steady, fully developed flow
- (✱) translation invariance of the set-up along x and z direction.
  $\Rightarrow$ velocity field can only depend on y
- (✱) Driving force points to x-direction.
  $\Rightarrow$ only x-component of velocity is non-zero.

For ~~slip~~ boundary condition

$$\Rightarrow \mu \frac{\partial^2 u}{\partial y^2} = 0$$

$$\Rightarrow u = c_1 y + c_2$$

$$\frac{u}{U} = \frac{y}{h} \qquad \text{or} \qquad \bar{u} = \tilde{y}$$

For SLIP Boundary conditions
the governing equation remains same
B.C.s On the lower wall $(\tilde{y}=0, y=0)$, $\bar{u}_w = 0$, and $\frac{\partial \bar{u}_s}{\partial n} = \frac{\partial \bar{u}_s}{\partial y}$

On the upper wall, $\bar{u}_w = 1$

n is the unit normal vector
$\equiv \frac{\tilde{y}}{h}$ for lower wall

and $= -\frac{\tilde{y}}{h}$ for upper wall

$$\frac{\partial \bar{u}_s}{\partial n} = -\frac{\partial \bar{u}_s}{\partial \tilde{y}}$$

$$\alpha = \frac{2-\sigma_v}{\sigma_v} K_n \qquad \text{where } K_n = \frac{\lambda}{h}$$
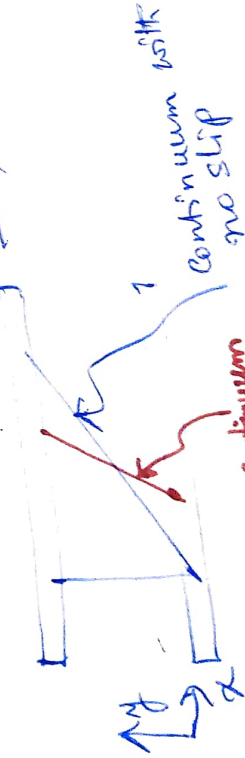
Application of boundary condition will lead to

$$\bar{u} = \frac{1}{1+2\alpha}\tilde{y} + \frac{\alpha}{1+2\alpha}$$

$$C_1 = \frac{1}{1+2\alpha}$$

$$C_2 = \frac{\alpha}{1+2\alpha}$$

(diagram: continuum no slip / continuum with slip flow)

the non-dimensional volume flow rate per unit depth in the z-direction $\perp$ to paper

$$Q = \int_0^1 \bar{u}\, d\tilde{y} = 0.5$$

RHS value is the same as the non-dimensional volume flow solution with no-slip boundary condition.