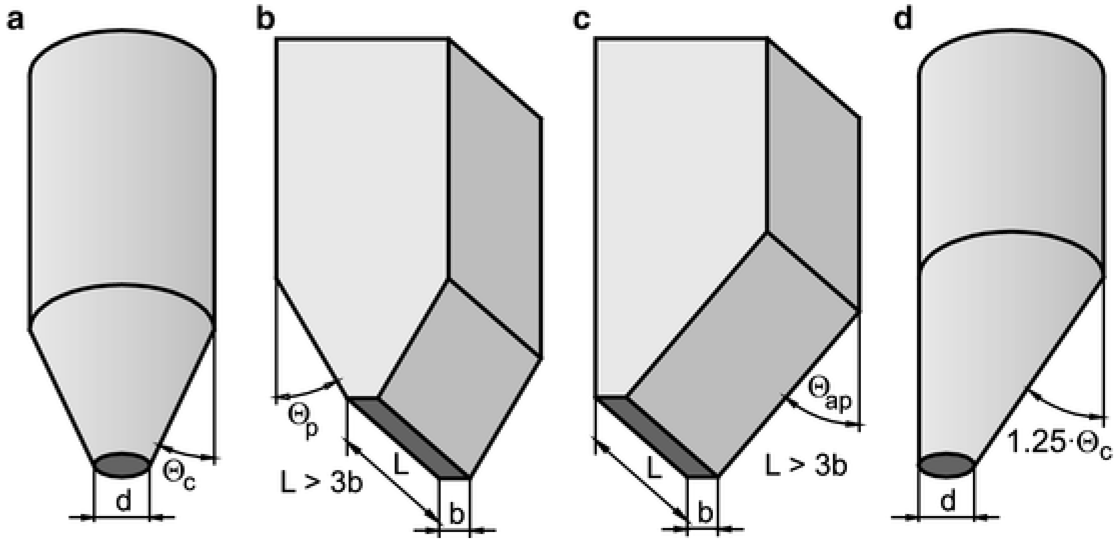
Hopper: Stress distribution
&
Designing



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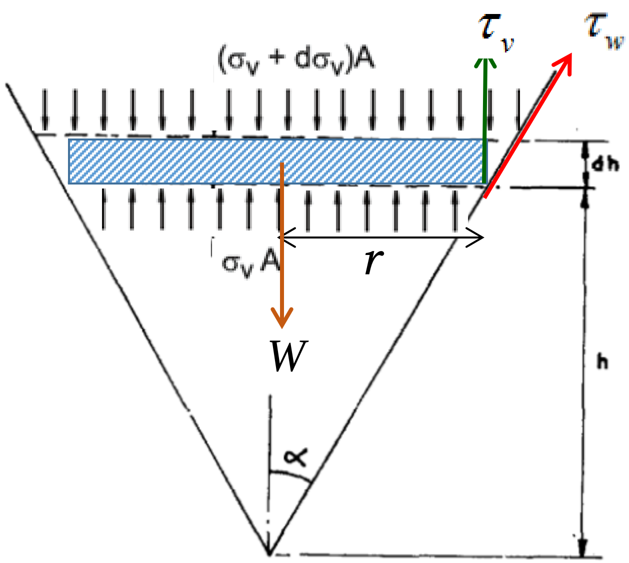


$$A\left(\sigma_v + d\sigma_v\right) + \overset{W}{\overbrace{g\rho_b A dh}} = A\sigma_v + \tau_v U dh$$

$$\frac{d\sigma_v}{dh} = \frac{\tau_v U}{A} - g\rho_b \Rightarrow \qquad \frac{d\sigma_v}{dh} = \frac{2B\sigma_v}{r} - g\rho_b$$

$$\boxed{r = h\tan\alpha}$$

$$\tau_v = B\sigma_v$$

$$B\left(\alpha, \varphi_e, \varphi_x\right)$$

$$\boxed{\frac{d\sigma_v}{dh} - \frac{C\sigma_v}{h} = -g\rho_b, \qquad C = \frac{2B}{\tan\alpha}}$$

$$I.F. = e^{-\int \frac{Cdh}{h}} = e^{-C\ln h} = e^{\ln h^{-C}} = h^{-C}$$

φ_e : Angle of Internal Friction

φ_x : Angle of Wall Friction

$$\boxed{\sigma_v = \sigma_{v0}\left(\cancel{\frac{h}{h_0}}\right)^{C} + \frac{g\rho_b h}{C-1}\left(1 - \left(\cancel{\frac{h}{h_0}}\right)^{C-1}\right)}$$

$$C > 1$$

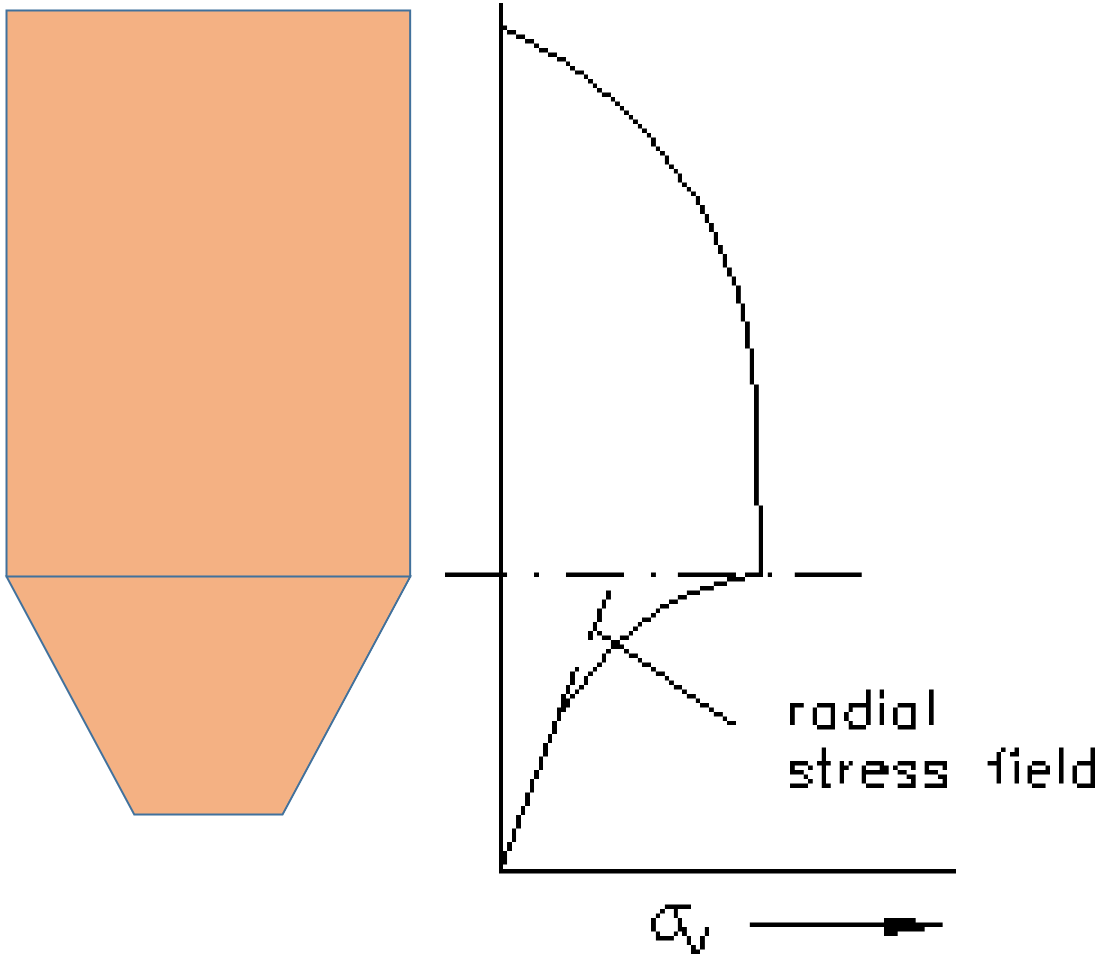
In a hopper, stress decreases as we towards mouth or exit.

In a hopper, near the mouth the vertical stress is proportional to local radius.

$$\boxed{\sigma_v = \frac{g\rho_b h}{C-1} = \frac{g\rho_b r}{(C-1)\tan\alpha}}$$

$$h << h_0,$$

Stress profile



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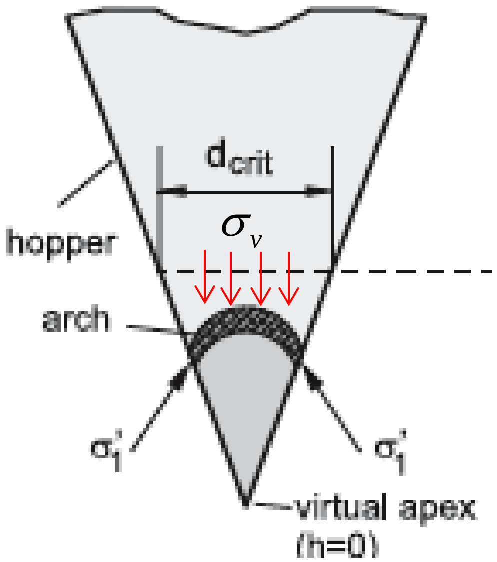
Critical Diameter to avoid Arching



In a hopper, as we go near mouth, the vertical stress decreases, whereas cohesive forces start dominating.

Hence arch formation usually takes place near the mouth of the hopper.

If the **vertical stress** is able to **overcome** the **yield stress of Arch** formed due to cohesive forces, no arch formation will take place.

There exists a critical diameter, above which no arch formation is possible.

Assumptions:

- σ_v plays a role in an indirect way.
- Weight of material above arch is very small as compared the total weight of the material kept in cylindrical section.
- The major normal stress experienced by arch is σ_1'.

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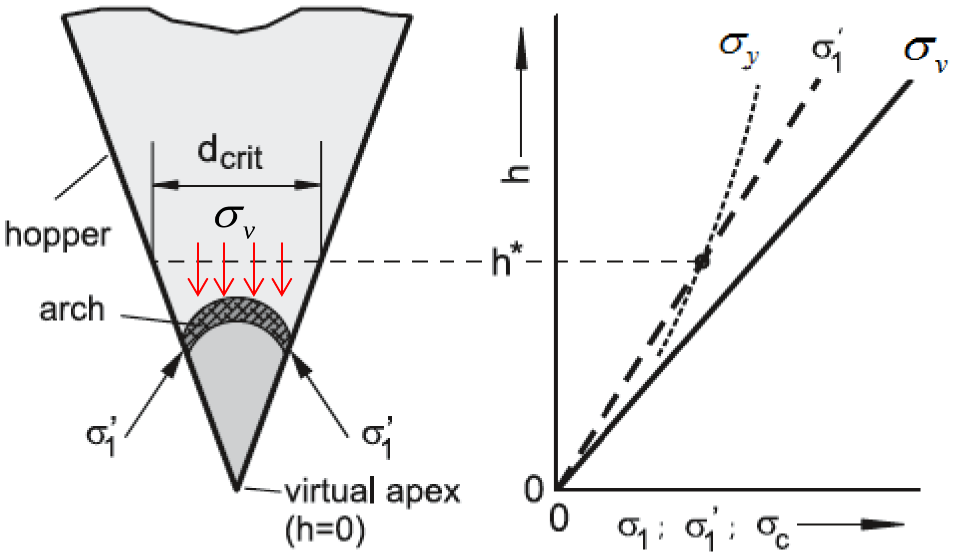
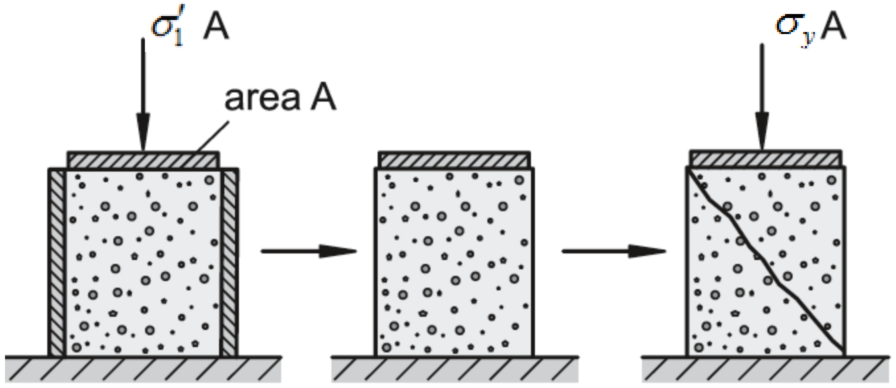
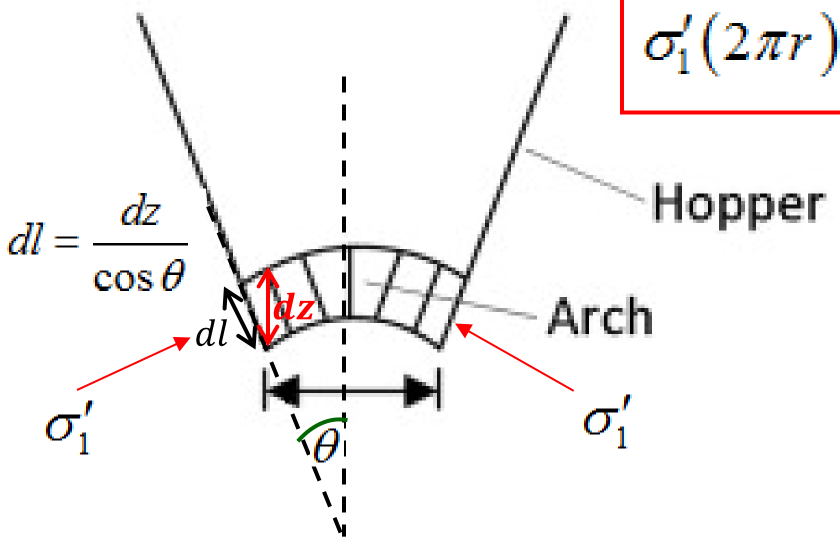
Critical Diameter to avoid Arching



Unconfined Yield Strength





$$\sigma_1'(2\pi r)\,dl\sin\theta = \pi r^2 \rho_b g\,dz \quad \Rightarrow \sigma_1' = \frac{r\rho_b g}{2\tan\theta}$$

$$\sigma_1' > \sigma_y \rightarrow \text{No arch formation}$$
$$\sigma_1' < \sigma_y \rightarrow \text{arch formation}$$

$$\frac{r_{cri}\rho_b g}{2\tan\theta} = \sigma_1' = \sigma_{yc}$$
$$\Rightarrow r_{cri} = \frac{2\sigma_{yc}\tan\theta}{\rho_b g}$$

$$D_{cri} = \frac{4\sigma_{yc}\tan\theta}{\rho_b g}$$

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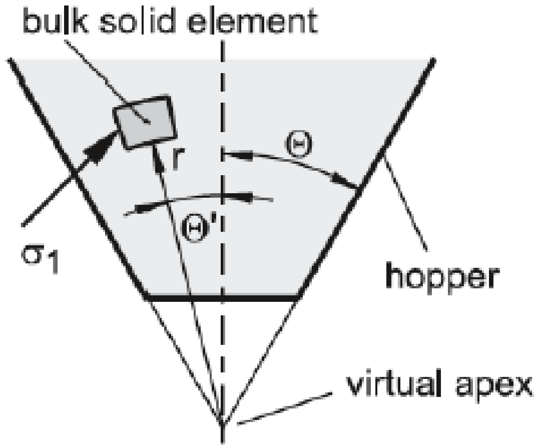
JENIKE's Flow Phase diagram for a hopper

$$\sigma_v = \frac{g\rho_b h}{C-1} = \frac{g\rho_b r}{(C-1)\tan\alpha}$$

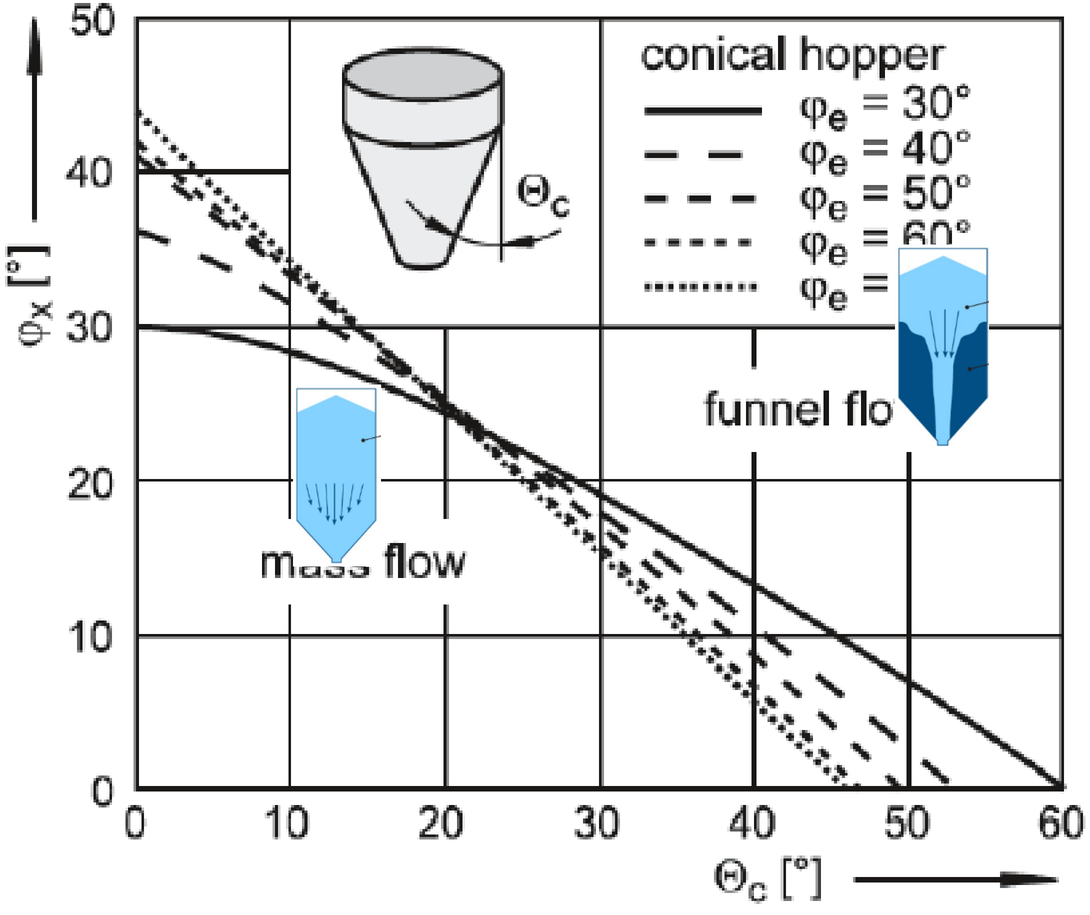


$$\sigma_1 > \sigma_{y,local} \Rightarrow Mass\ flow$$
$$\sigma_1 < \sigma_{y,local} \Rightarrow No\ flow$$

$$\sigma_1 = rg\rho_b s(\Theta', \Theta, \varphi_x, \varphi_e).(1+\sin\varphi_e)$$

φ_e : Angle of Internal Friction

φ_x : Angle of Wall Friction



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