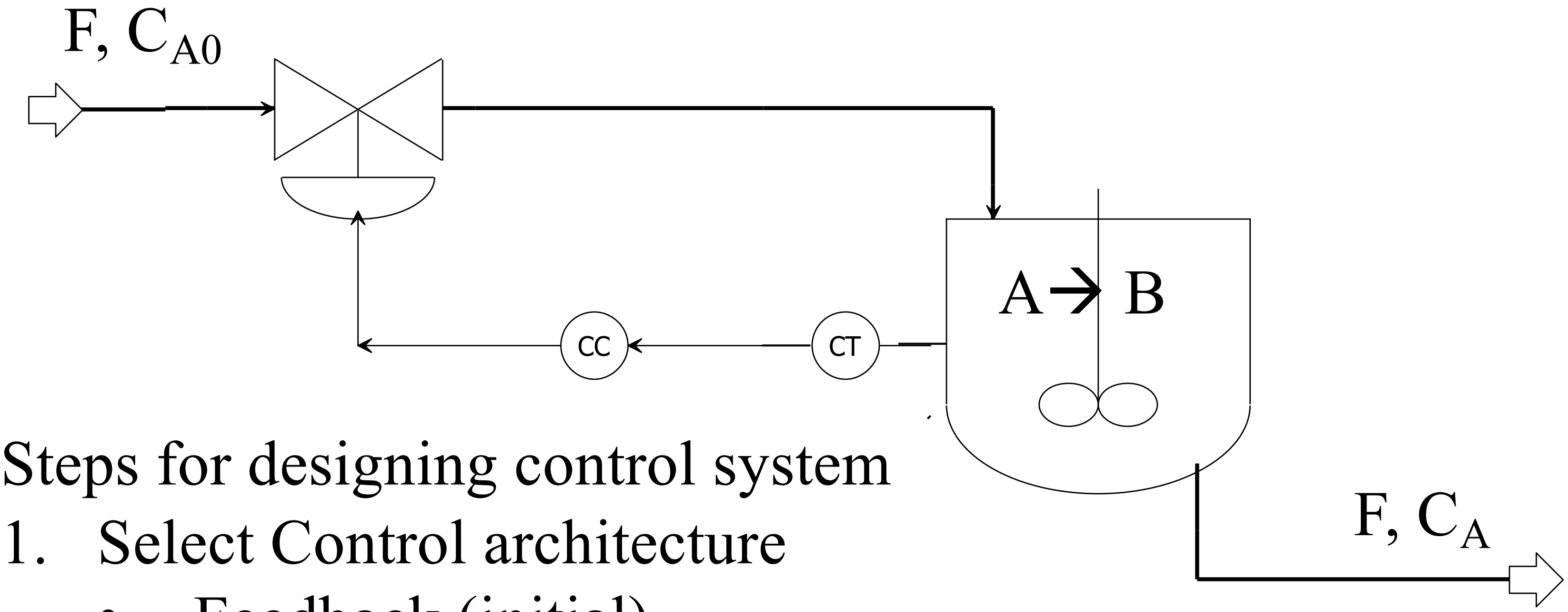
Review
of
Feedback Control System

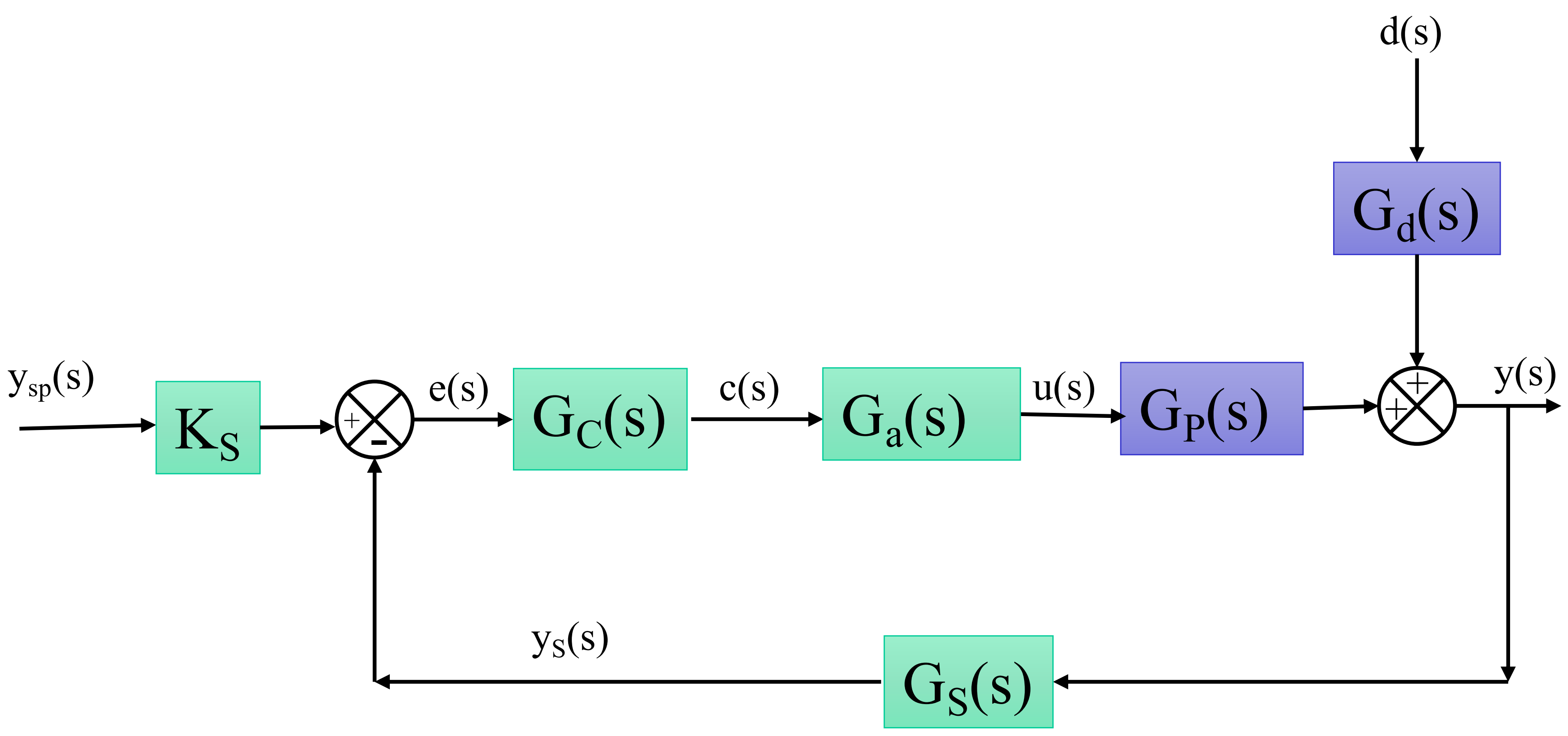
Reactor Control



Steps for designing control system

1. Select Control architecture
   - Feedback (initial)
2. Find Transfer function model for the process
   1. Using model equation
   2. Using Process Data
3. Selecting Controller algorithm
   1. PID controller
   2. Other types

General Feedback Control Loop



Transfer Function Model (Theory)

Reactor:

- $\frac{dC_A}{dt} = \frac{F}{V}(C_{A0} - C_A) - kC_A^2$

The volume of the reactor assumed constant

Linearizing the model and taking Laplace Transform:

$$y(s) = \frac{(C_{A0}^s - C_A^s)\frac{1}{V}}{s + \frac{F^s}{V} + 2kC_A^s} u(s) + \frac{\frac{F^s}{V}}{s + \frac{F^s}{V} + 2kC_A^s} \mathrm{d(s)}$$

Where, $y = C_A - C_A^s$; $u = F - F^s$; $d = C_{A0} - C_{A0}^s$

Transfer Function Model (Theory)

Control Valve : $u(s) = \frac{K_a}{\tau_a s+1} c(s)$

- Normally first order (specified by vendor)
- Time Constant specified by vendor
- Gain (calculated from installed characteristics)

Sensor : $y_s(s) = \frac{K_s e^{-\alpha s}}{\tau_s s+1}$

- Gain calculated from range setting
- Time constant & measurement delay specified by vendor

Transfer Function Model (Empirical)

- Obtain Process data using specified signal
  - Step function
  - Pulse function
  - Sinusoidal function (rarely used)
  - Pseudo Random Binary Sequence (PRBS)
- Process data includes the actuator and sensor response
- Evaluate Transfer function model from Process data: $y_s(s) = g_p(s)c(s) + g_d(s)d(s)$

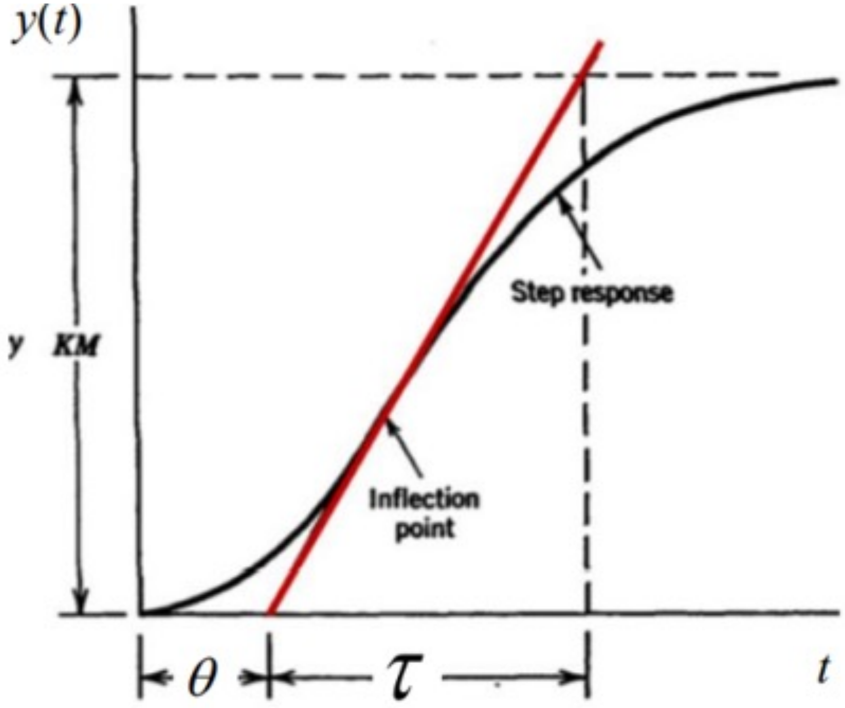
Transfer Function from Process Data

- Normally, chemical process transfer functions are approximated by First Order with Dead Time (FODT) model, $g(s) = \frac{Ke^{-\theta s}}{\tau s+1}$

- Four classical methods (using step response data)
  - Ziegler-Nichols method
  - Smith's method
  - Sundaresan and Krishnaswamy method
  - Nishikawa's method

Transfer function from Process Data

Ziegler-Nichols method (1942) consists of applying a tangent line to the curve at an inflection point, to determine the system's gain, time, and delay constants, as shown in Figure.



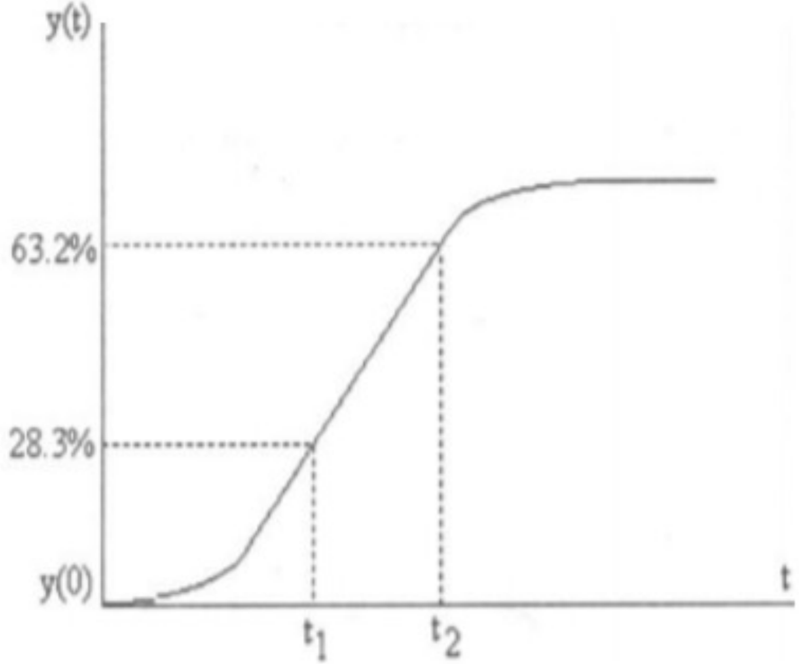
The inflection point is defined where the curve changes direction and the derivative is equal to zero.

Drawback:

Difficult to find inflection Point and to draw tangent.

Transfer function from Process Data

Smith's Method (1972)proposed that the values of θ and τ be selected in such a way that the model and the real responses coincide in two points that present a high rate of variation.



The two time points are at 28.3% 63.2 % of change in output respectively.

So from figure, after finding t_1 and t_2, we can calculate

$$K = \frac{\Delta output}{\Delta input} \quad \tau = \frac{3}{2}(t_2 - t_1) \quad \text{and} \quad \theta = t_2 - \tau$$

Transfer function from Process Data

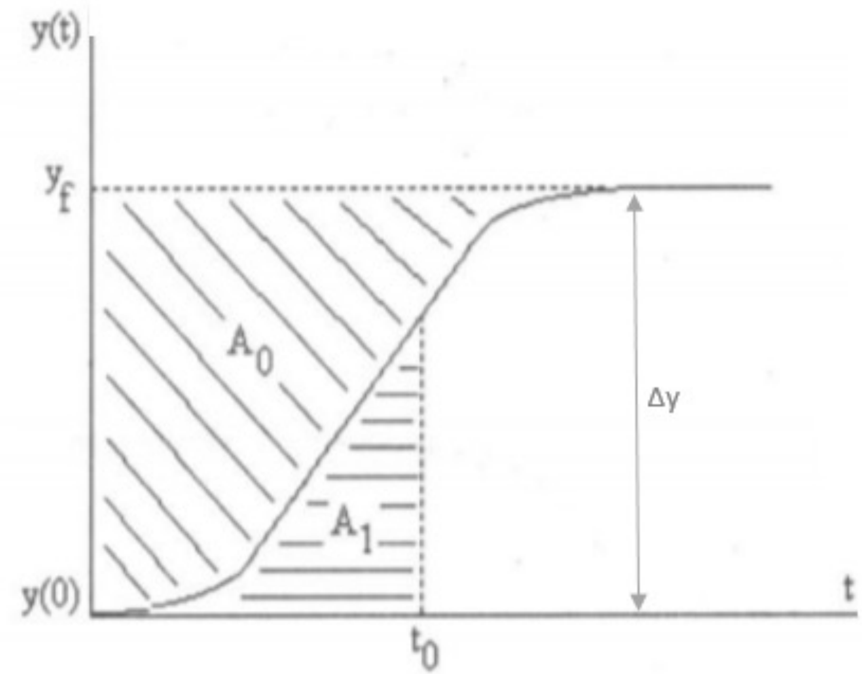
Sundaresan and Krishnaswamy Method:

This is similar to smith's method but the two time points t_1 and t_2 are based on 35.3% and 85.3% of change in output response respectively. So, model parameters are calculated as,

$$K = \frac{\Delta output}{\Delta input}\ ;\ \tau = \frac{2}{3}(t_2 - t_1);\ \theta = 1.3t_1 - 0.29t_2$$

Transfer function from Process Data

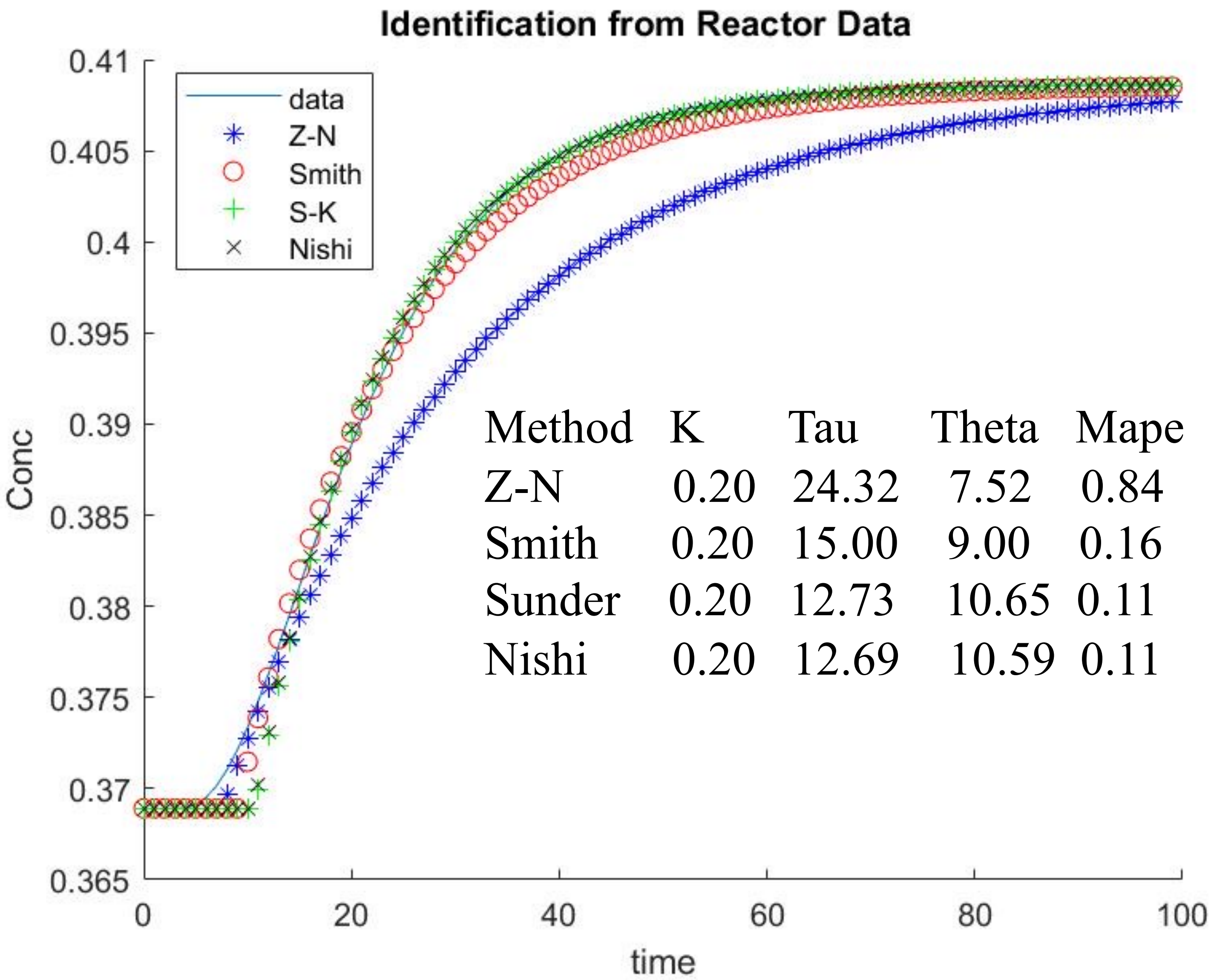
Nishikawa (2007) method determine the values of the constants using the calculation of the curve areas as shown in Figure.



$$A_0 = \int_0^{\infty} \big(\Delta y(\infty) - \Delta y(t)\big)dt$$

$A_1 = \int_0^{t_0} \Delta y(t)dt$ where, $t_0 = \frac{A_0}{\Delta y(\infty)}$

$\tau = \frac{A_1}{0.368\,\Delta y(\infty)}$; $\theta = t_0 - \tau$; $K = \frac{\Delta y(\infty)}{\Delta c(\infty)}$


Identification from Reactor Data
data
Z-N
Smith
S-K
Nishi
Conc
time
0.41
0.405
0.4
0.395
0.39
0.385
0.38
0.375
0.37
0.365
0
20
40
60
80
100
Method K Tau Theta Mape
Z-N 0.20 24.32 7.52 0.84
Smith 0.20 15.00 9.00 0.16
Sunder 0.20 12.73 10.65 0.11
Nishi 0.20 12.69 10.59 0.11

Transfer function from Process Data

Method of Moments method

- can be used to determine model parameters from the output response generated using arbitrary input function.

- is based on the definition of the Laplace transform of the impulse response *g(t)* of a system, which is its transfer function $G(s) = \int_0^\infty e^{-st} g(t)\,dt$

As the n-th-order moment of a function f(x) is defined by

$$M_n(f) = \int_0^\infty x^n f(x)\,dx$$

Transfer function from Process Data

It can be noticed that the first two derivatives of G(s) w.r.t s

$G'(s) = -\int_0^\infty te^{-st} g(t)\, dt$ & $G''(s) = \int_0^\infty t^2 e^{-st} g(t)\, dt$

are related to the moments of the impulse response function by $G(0) = \int_0^\infty g(t)\, dt$; $G'(0) = -\int_0^\infty t\, g(t)\, dt$; and

$G''(0) = \int_0^\infty t^2 g(t)\, dt$;

- Thus G(O), -G'(O), G"(O) are respectively the zero-, first- and second-order moments of the impulse response *g(t)*.
- Note that the above three integrals can be calculated by using the measured output response.

Transfer function from Process Data

- Note that in this method, it is possible to use any type of input. We have, in general

$$Y(s) = G(s)\ U(s)\ ;\ Y'(s) = G'(s)\ U(s) + G(s)\ U'(s)$$

$$Y''(s) = G''(s)\ U(s) + G(s)\ U''(s) + 2\ G'(s)\ U'(s)$$

- from which the following equations are deduced

$$Y(0) = G(0)\ U(0)\ ;\ Y'(0) = G'(0)\ U(0) + G(0)\ U'(0)$$

$$Y''(0) = G''(0)\ U(0) + G(0)\ U''(0) + 2\ G'(0)\ U'(0)$$

These quantities can be calculated by the following equations:

$$U(0) = \int_0^\infty u(t)dt\ ;\ \ U'(0) = -\int_0^\infty t\,u(t)dt\ ;$$

$$U''(0) = \int_0^\infty t^2 u(t)\,dt\ ;\ \ Y(0) = \int_0^\infty y(t)\,dt;$$

$Y'(0) = -\int_0^\infty t\,y(t)\,dt;$ and $Y''(0) = \int_0^\infty t^2 y(t)\,dt;$

Transfer function from Process Data

FODT model: $\mathrm{G}(s) = \frac{Ke^{-\theta s}}{\tau s+1}$

So,

$G(0) = K = Y(0)/U(0);$

$G'(0) = -(\tau + \theta) = \{Y'(0) - G(0)U'(0)\}/U(0)$

$G''(0) = K(2\tau^2 + 2\tau\theta + \theta^2)$

$$= \{\mathrm{Y}''(0) - \mathrm{G}(0)\mathrm{U}''(0) - 2\mathrm{G}'(0)\mathrm{U}'(0)\}/\mathrm{U}(0);$$

Solving the above equations, K, θ and τ may be obtained.

Correction: $\boldsymbol{G'(0) = -K(\tau + \theta)}$

SODT model from step response

- $G(s) = \frac{Ke^{-\theta s}}{\tau^2 s^2 + 2\xi\tau s + 1}$ with $\xi > 0.707$

Method (Rangaiah & Krishnaswamy)

1. Calculate the process steady-state gain from the magnitude of the step input and that of the corresponding response;
2. Find t_1, t_2 *and* t_3 from the response data corresponding to 14%, 55%, and 91% of the actual response;

SODT model from step response

3. Calculate $\alpha, \beta\ and\ \xi$ using the following equations

$$\alpha = \frac{t_3 - t_2}{t_2 - t_1}$$

$$\beta = \ln\left(\frac{\alpha}{2.485 - \alpha}\right)$$

$$\xi^2 = 0.50906 + 0.51743\beta - 0.076284\beta^2 + 0.041363\beta^3 \\ -0.0049224\beta^4 + 0.00021234\beta^5$$

for $1.2323 < \alpha < 2.485$

SODT model from step response

4. Calculate τ *and* θ using the following equations

$$\frac{t_2 - t_1}{\tau} = 0.85818 - 0.62907\xi + 1.2897\,\xi^2 - 0.36859\,\xi^3 + 0.038891\,\xi^4$$

$$\frac{t_2 - \theta}{\tau} = 1.392 - 0.52536\,\xi + 1.2991\,\xi^2 - 0.36014\,\xi^3 + 0.037605\,\xi^4$$

Non-linear Regression

This method can be used for any form of linear or non-linear models.

For example, process having transfer function of one zero 2 pole system

$$\frac{y(s)}{u(s)} = G(s) = \frac{K(\tau_3 s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

For Step input of magnitude M in u gives,

$$y(t) = KM\left(1 + \frac{\tau_3 - \tau_1}{\tau_1 - \tau_2} e^{-\frac{t}{\tau_1}} + \frac{\tau_3 - \tau_2}{\tau_2 - \tau_1} e^{-\frac{t}{\tau_2}}\right)$$

The above equation can be regressed to get the model parameters $K, \tau_1, \tau_2, \tau_3$

Matlab Implementation

Step response data:

t	0	1	2	3	4	5	6	7	8	9	10
u	1	1	1	1	1	1	1	1	1	1	1
y	0	0.058	0.217	0.36	0.488	0.6	0.692	0.772	0.833	0.888	0.925

Use command

mdl = procest(data, Type)

Where, data should stored in iddata format.

Type =‘P2DUZ’ [2 poles, delay, U underdamped, Z zero]

```
>> u=[0;0;0;u];  y=[0;0;0;y]; % for step changes
>> z = iddata(y, u, 1);  % y and u should be column vectors
>> mdl= procest(z,'P2DUZ');
```

‘procest’ can work for step as well as arbitrary input changes.

PID Controller

The acronym *PID* stands for:

- P - Proportional
- I - Integral
- D - Derivative

PID Controllers:

- greater than 90% of all control implementations
- dates back to the 1930s
- very well studied and understood
- optimal structure for first and second order processes (given some assumptions)
- always first choice when designing a control system

PID Control Algorithm

$$c(t) = c_0 + K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(\zeta) d\zeta + \tau_D \frac{de(t)}{dt} \right]$$

$e(t)$- the error from setpoint [$e(t) = y_{sp} - y_s$].

K_c- the controller gain is a tuning parameter and largely determines the controller aggressiveness.

τ_I- the reset time is a tuning parameter and determines the amount of integral action.

τ_D- the derivative time is a tuning parameter and determines the amount of derivative action.

Transfer Function for a PID Controller

Ideal PID : $G_c(s) = \frac{c(s)}{e(s)} = K_c\left[1 + \frac{1}{\tau_I s} + \tau_D s\right]$

Real PID : $G_c(s) = \frac{c(s)}{e(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right)\left(\frac{\tau_D s}{\beta \tau_D s + 1}\right)$

Derivative Kick:

To avoid sudden jump of output due to setpoint change, sensor output is used in place of error term, i.e,

$$c(t) = c_0 + K_c\left[e(t) + \frac{1}{\tau_I}\int_0^t e(\zeta)d\zeta - \tau_D \frac{dy_s(t)}{dt}\right]$$

Digital Equivalent of PID Controller

$$\int_0^{\infty} e(t)\,dt \approx \sum_{i=1}^{n} e(i\,\Delta t)\,\Delta t$$

- The trapezoidal approximation of the integral.

$$\frac{d\,e(t)}{dt} \approx \frac{e(t) - e(t-\Delta t)}{\Delta t}$$

- Backward difference approximation of the first derivative

Digital Version of PID Control Algorithm

$$c(t) = c_0 + K_c\left[e(t) + \frac{\Delta t}{\tau_I}\sum_{i=1}^{n} e(i\,\Delta t) + \tau_D\,\frac{e(t) - e(t-\Delta t)}{\Delta t}\right]$$

$$n = \frac{t}{\Delta t}$$

Derivation of the Velocity Form of the PID Control Algorithm

$$c(t) = c_0 + K_c\left[e(t) + \frac{\Delta t}{\tau_I}\sum_{i=1}^{n} e(i\,\Delta t) + \tau_D\,\frac{e(t) - e(t-\Delta t)}{\Delta t}\right]$$

$$c(t-\Delta t) = c_0 + K_c\left[e(t-\Delta t) + \frac{\Delta t}{\tau_I}\sum_{i=1}^{n-1} e(i\,\Delta t) + \tau_D\,\frac{e(t-\Delta t) - e(t-2\Delta t)}{\Delta t}\right]$$

$$\Delta c(t) = K_c\left[e(t) - e(t-\Delta t) + \frac{\Delta t\, e(t)}{\tau_I} + \tau_D\left(\frac{e(t) - 2e(t-\Delta t) + e(t-2\Delta t)}{\Delta t}\right)\right]$$

Velocity Form of PID Controller

$$\Delta c(t) = K_c \left[e(t) - e(t-\Delta t) + \frac{\Delta t \;\; e(t)}{\tau_I} + \tau_D \left(\frac{e(t) - 2e(t-\Delta t) + e(t-2\Delta t)}{\Delta t} \right) \right]$$

$$c(t) = c(t-\Delta t) + \Delta c(t)$$

- Note the difference in proportional, integral, and derivative terms from the position form.
- Velocity form is the form implemented on DCSs.

Correction for Derivative Kick

- Derivative kick occurs when a setpoint change is applied that causes a spike in the derivative of the error from setpoint.
- Derivative kick can be eliminated by replacing the approximation of the derivative based on the error from setpoint with the negative of the approximation of the derivative based on the measured value of the controlled variable, i.e.,

$$-\tau_D \frac{y_s(t) - 2y_s(t-\Delta t) + y_s(t-2\Delta t)}{\Delta t}$$

Correction for Aggressive Setpoint Tracking

- For certain process, tuning the controller for good disturbance rejection performance results in excessively aggressive action for setpoint changes.
- This problem can be corrected by removing the setpoint from the proportional term. Then setpoint tracking is accomplished by integral action only.

$$K_c\left[e(t)-e(t-\Delta t)\right] \text{ substituted for by } K_c\left[y_s(t-\Delta t)-y_s(t)\right]$$

The Three Versions of the PID Algorithm Offered on DCS's

- (1) The original form in which the proportional, integral, and derivative terms are based on the error from setpoint

$$\Delta c(t) = K_c \left[e(t) - e(t-\Delta t) + \frac{\Delta t\, e(t)}{\tau_I} + \tau_D \left(\frac{e(t) - 2e(t-\Delta t) + e(t-2\Delta t)}{\Delta t} \right) \right]$$

The Three Versions of the PID Algorithm Offered on DCSs

- (2) The form in which the proportional and integral terms are based on the error from setpoint while the derivative-on-measurement is used for the derivative term.

$$\Delta c(t) = K_c \left[e(t) - e(t-\Delta t) + \frac{\Delta t \, e(t)}{\tau_I} - \tau_D \left(\frac{y_s(t) - 2y_s(t-\Delta t) + y_s(t-2\Delta t)}{\Delta t} \right) \right]$$

The Three Versions of the PID Algorithm Offered on DCS's

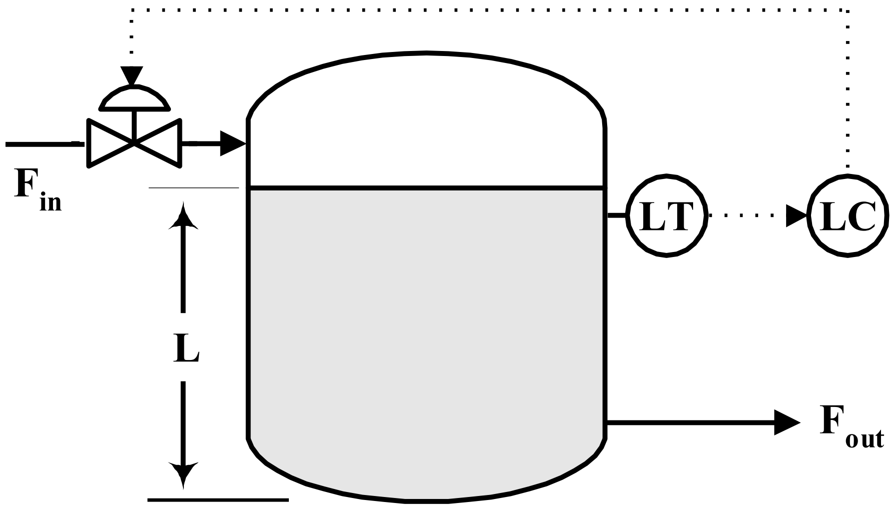
- (3) The form in which the proportional and derivative terms are based on the process measurement and the integral is based on the error from setpoint.

$$\Delta c(t) = K_c \left[y_s(t-\Delta t) - y_s(t) + \frac{\Delta t\, e(t)}{\tau_I} - \tau_D \left(\frac{y_s(t) - 2y_s(t-\Delta t) + y_s(t-2\Delta t)}{\Delta t} \right) \right]$$

Guidelines for Selecting Direct and Reverse Acting PID's

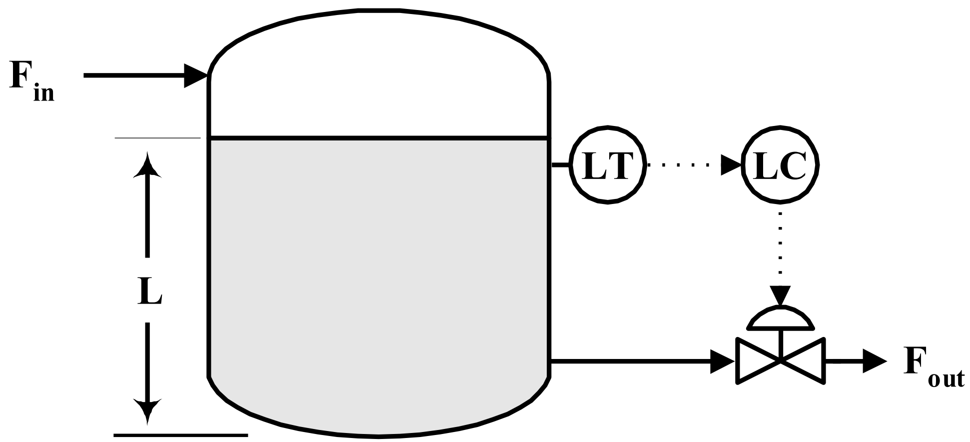
- Consider a direct acting final control element to be positive and reverse to be negative.
- If the sign of the product of the final control element and the process gain is positive, use the reverse acting PID algorithm.
- If the sign of the product is negative, use the direct acting PID algorithm
- If control signal goes to a control valve with a valve positioner, the actuator is considered direct acting.

Level Control Example



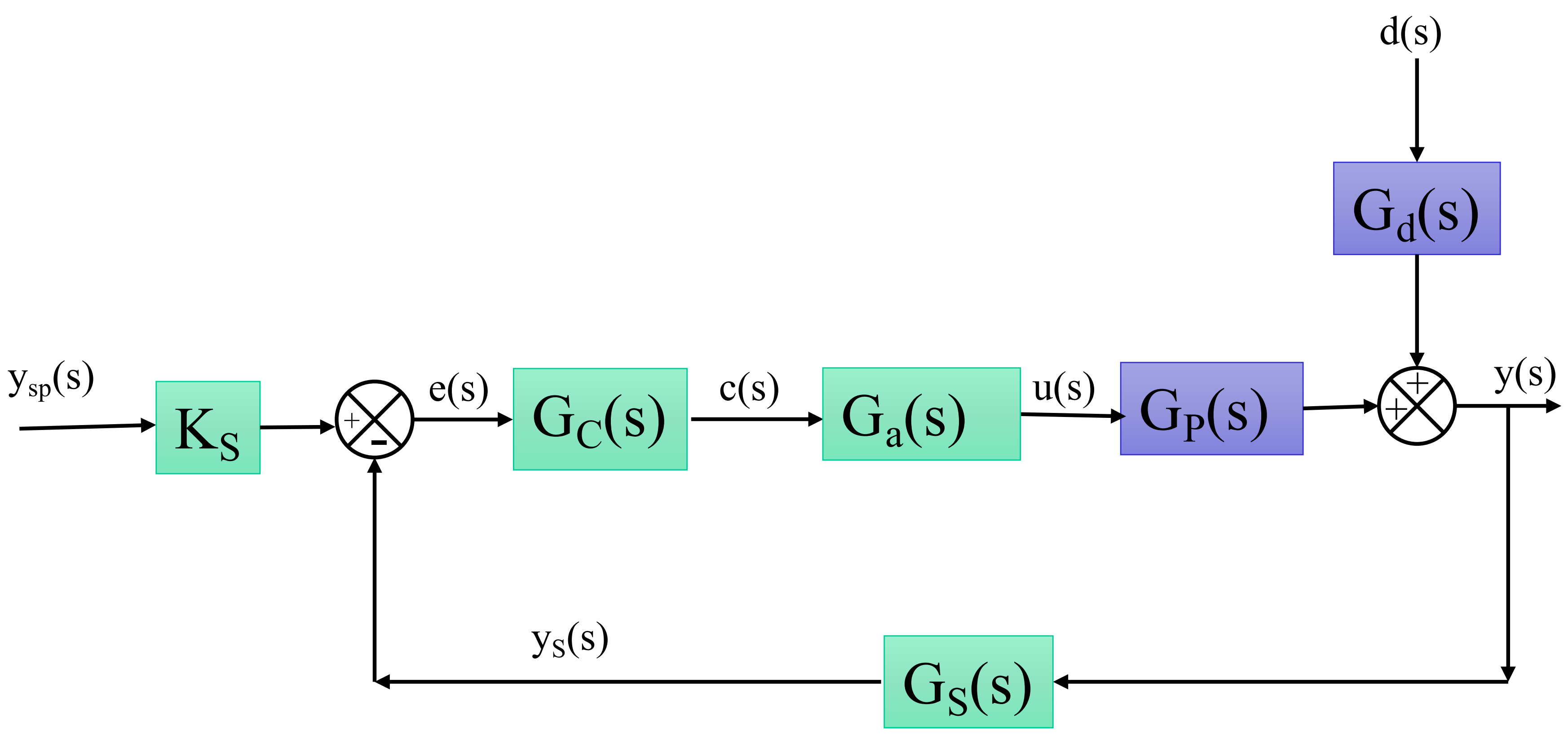
- Process gain is positive because when flow in is increased, the level increases.
- If the final control element is direct acting, use reverse acting PID.
- For reverse acting final control element, use direct acting PID.

Level Control Example



- Process gain is negative because when flow out is increased, the level decreases.
- If the final control element is direct acting, use direct acting PID.
- For reverse acting final control element, use reverse acting PID.

General Feedback Control Loop



Closed Loop Transfer Functions

- From the general feedback control loop and using the properties of transfer functions, the following expressions can be derived:

$$\frac{y(s)}{y_{sp}(s)} = \frac{K_S G_p(s)\ G_a(s)\ G_c(s)}{G_p(s)\ G_a(s)\ G_c(s)\ G_s(s) + 1}$$

$$\frac{y(s)}{d(s)} = \frac{G_d(s)}{G_p(s)\ G_a(s)\ G_c(s)\ G_s(s) + 1}$$

Characteristic Equation

- Since setpoint tracking and disturbance rejection have the same denominator for their closed loop transfer functions, this indicates that both setpoint tracking and disturbance rejection have the same general dynamic behavior.
- The roots of the denominator determine the dynamic characteristics of the closed loop process.
- The **characteristic equation** is given by:

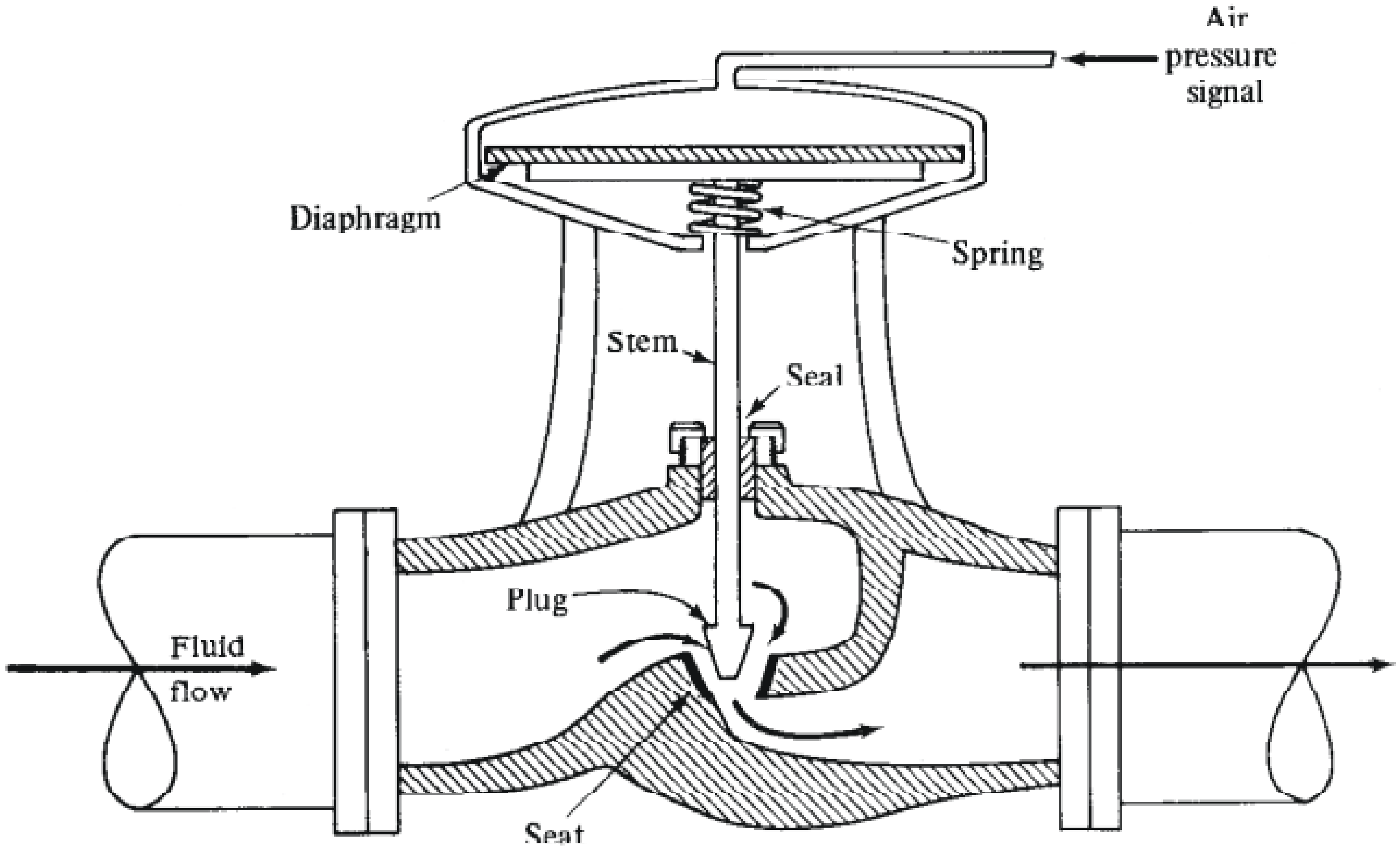
$$G_p(s)\, G_a(s)\, G_c(s)\, G_s(s) + 1 = 0$$

Feedback Control Analysis

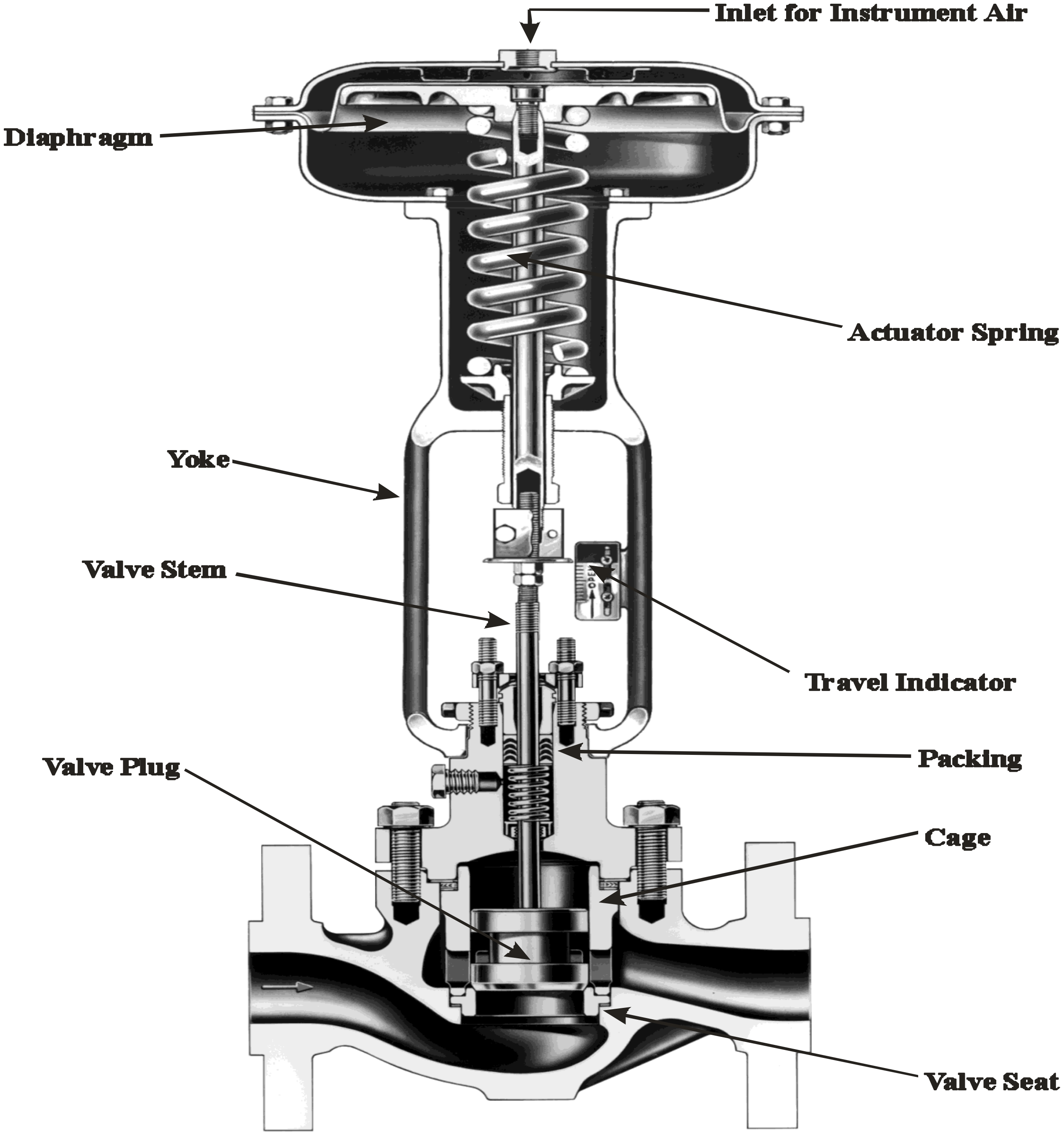
- The loop gain ($K_c K_a K_p K_s$) should be positive for stable feedback control.
- An open-loop unstable process can be made stable by applying the proper level of feedback control.

Thank You

Typical Globe Control Valve



Cross-section of a Globe Valve

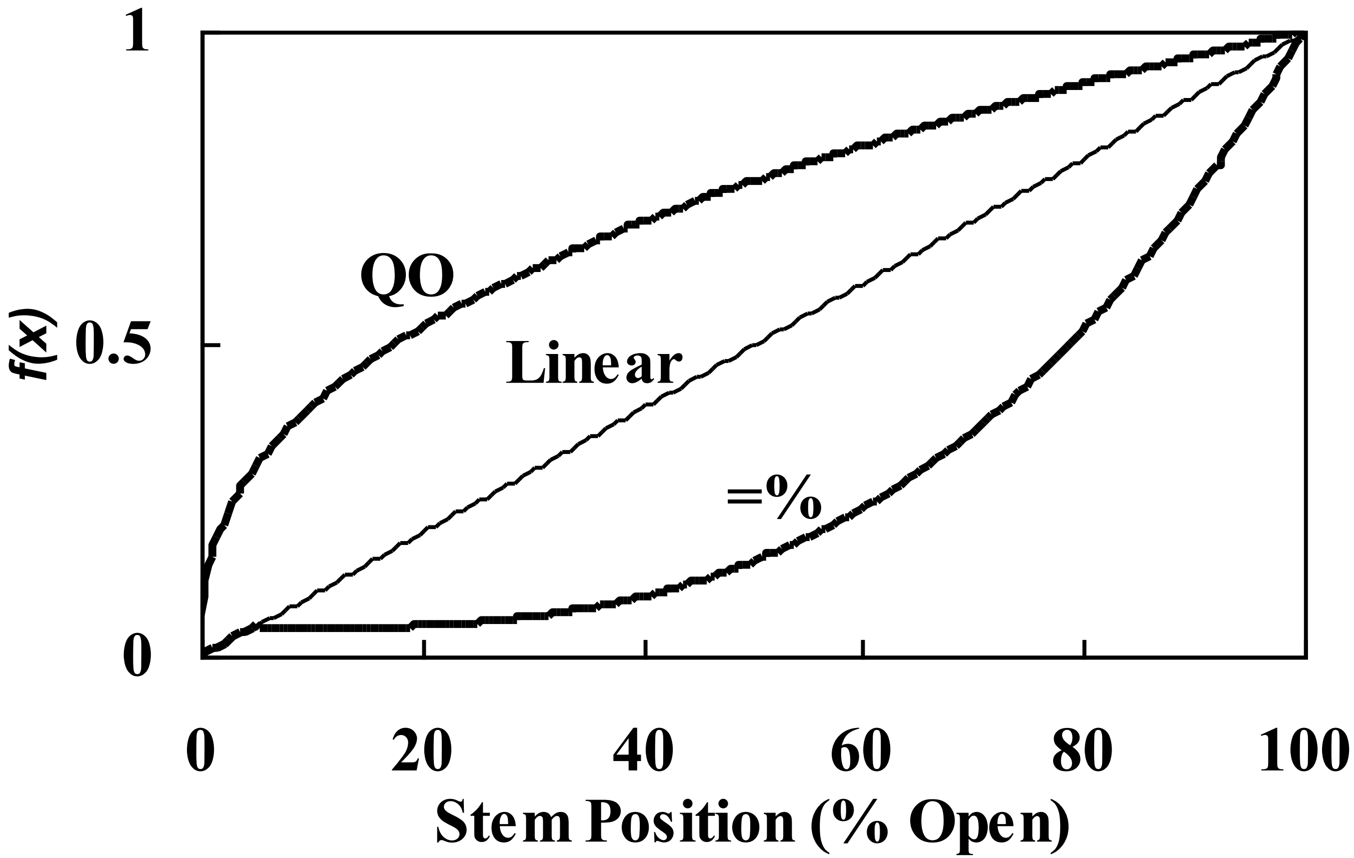


Types of Globe Valves

- Quick Opening- used for safety by-pass applications where quick opening is desired
- Equal Percentage- used for about 90% of control valve applications since it results in the most linear installed characteristics
- Linear- used when a relatively constant pressure drop is maintained across the valve

Inherent Valve Characteristics

Valve Eqn. $Q = C_V\, f(x) \sqrt{\frac{\Delta p}{s.g}}$



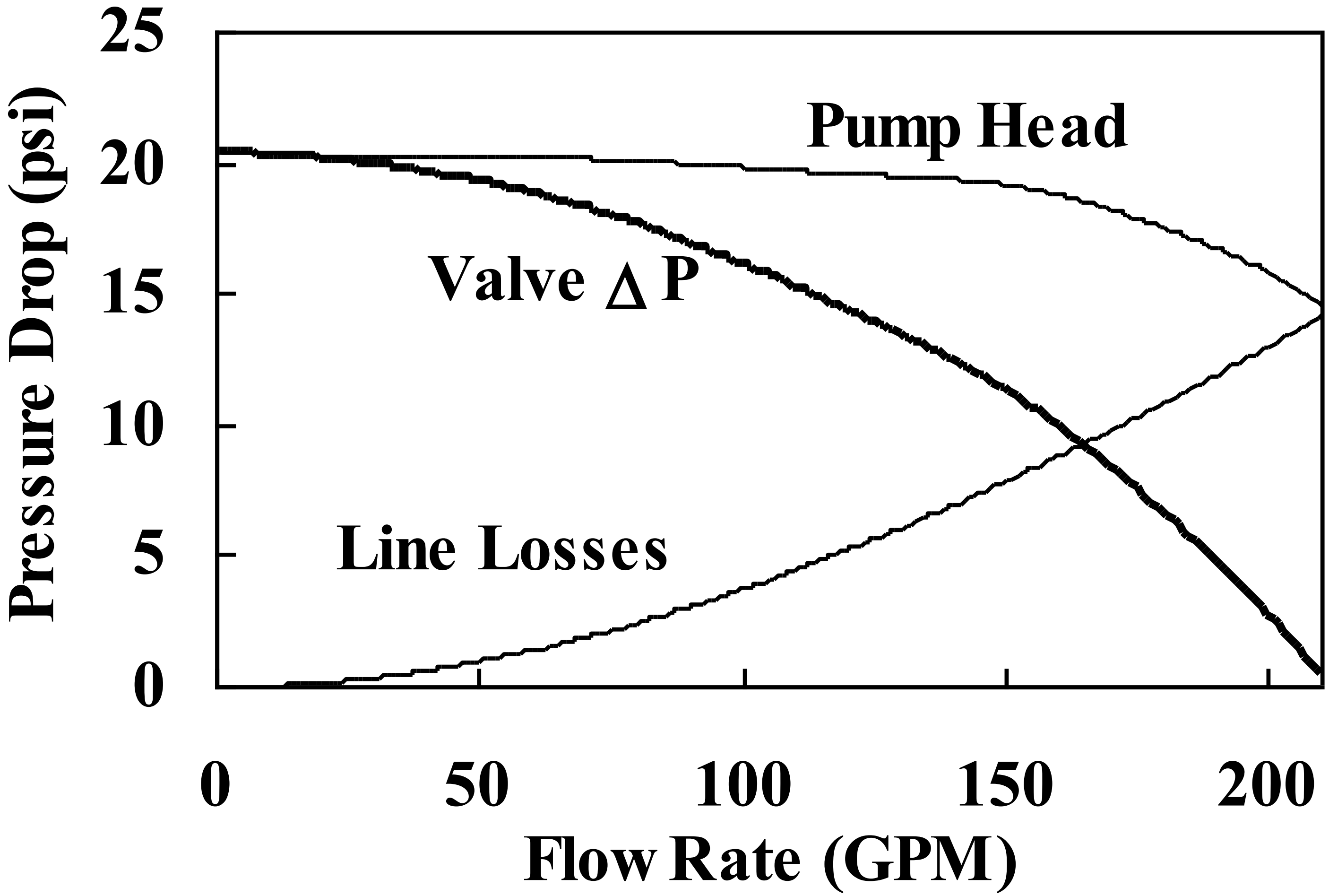
Linear:
$f(x) = x$

Quick opening
$f(x) = \sqrt{x}$

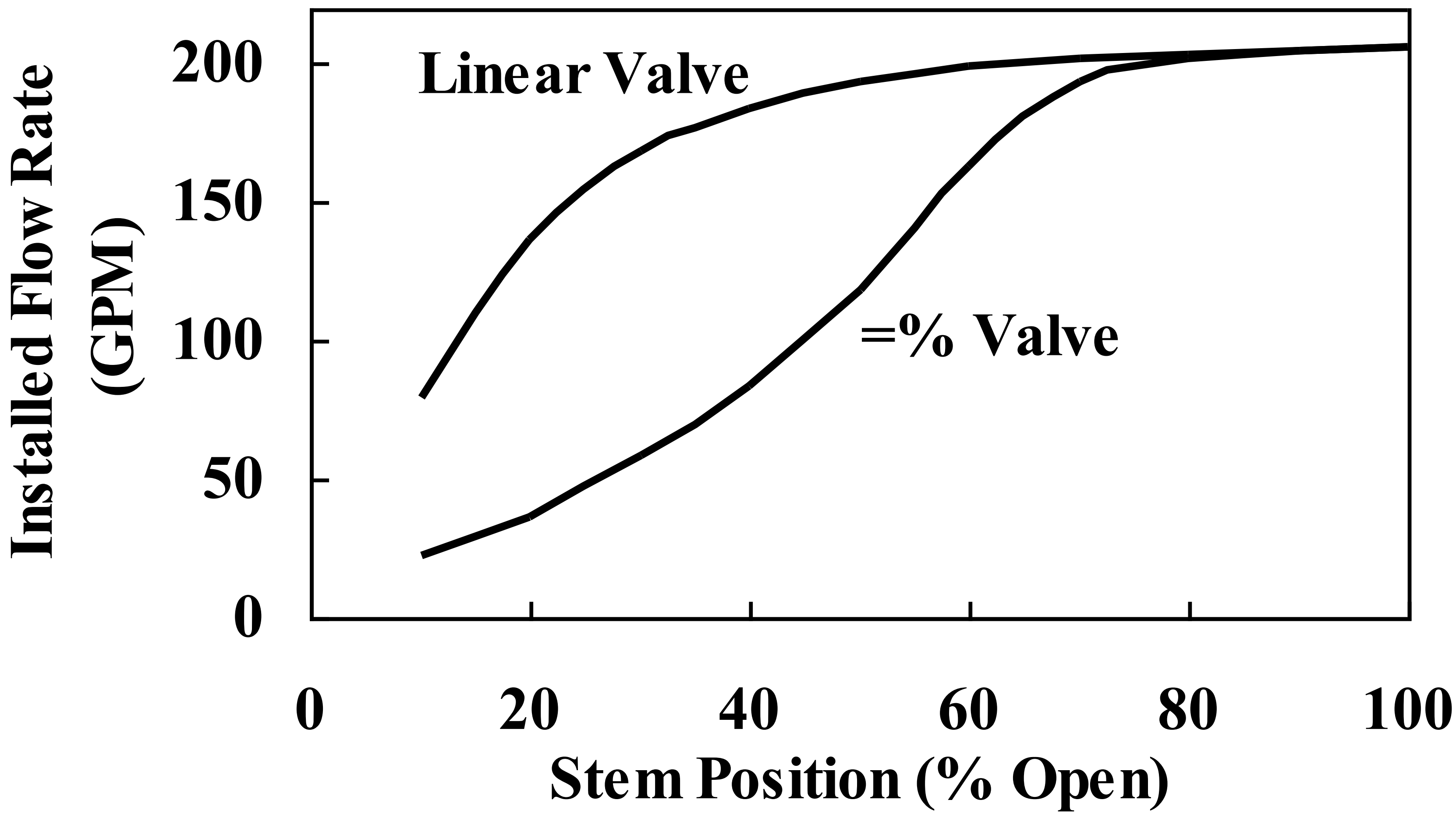
Equal %
$f(x) = R^{x-1}$

R varies 20 to 50

Pressure Drop vs. Flow Rate



Installed Flow Characteristic



Control Valve Design Procedure

- Evaluate C_v at the maximum and minimum flow rate using the flow equation for a valve.
- Determine which valves can effectively provide the max and min flow rate remembering that, in general, the valve position should be greater than about 15% open for the minimum flow rate and less than 85% open for the maximum flow rate.
- Choose the smallest valve that meets the above criterion for the minimum capital investment or choose the largest valve to allow for future throughput expansion.

Additional Information Required to Size a Control Valve

- C_V versus % open for different valve sizes.
- Available pressure drop across the valve versus flow rate for each valve. Note that the effect of flow on the upstream and downstream pressure must be known.

Valve Deadband

- It is the maximum change in instrument air pressure to a valve that does not cause a change in the flow rate through the valve.
- Deadband determines the degree of precision that a control valve or flow controller can provide.
- Deadband is primarily affected by the friction between the valve stem and the packing.

Valve Actuator Selection

- Choose an air-to-open for applications for which it is desired to have the valve fail closed.
- Choose an air-to-close for applications for which it is desired to have the valve fail open.

Back