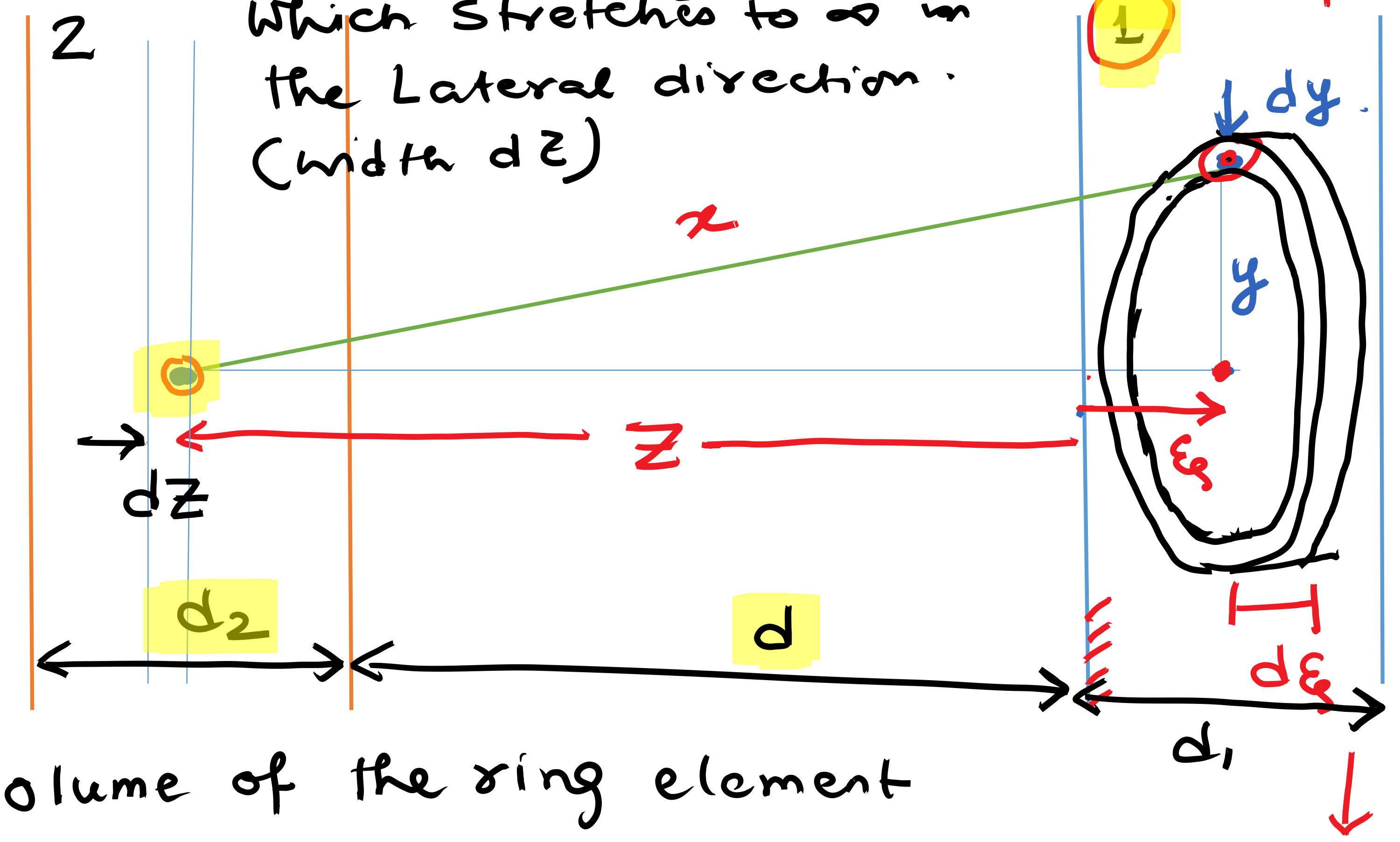
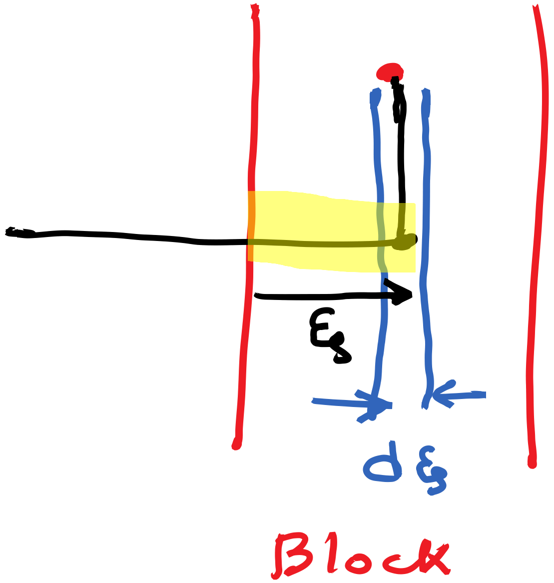
28.02.2022 / Lect 19

$$x^2 = y^2 + (z+\xi)^2$$

Thin Slice of Block 2 Which Stretches to $\infty$ in the Lateral direction. (width dz)



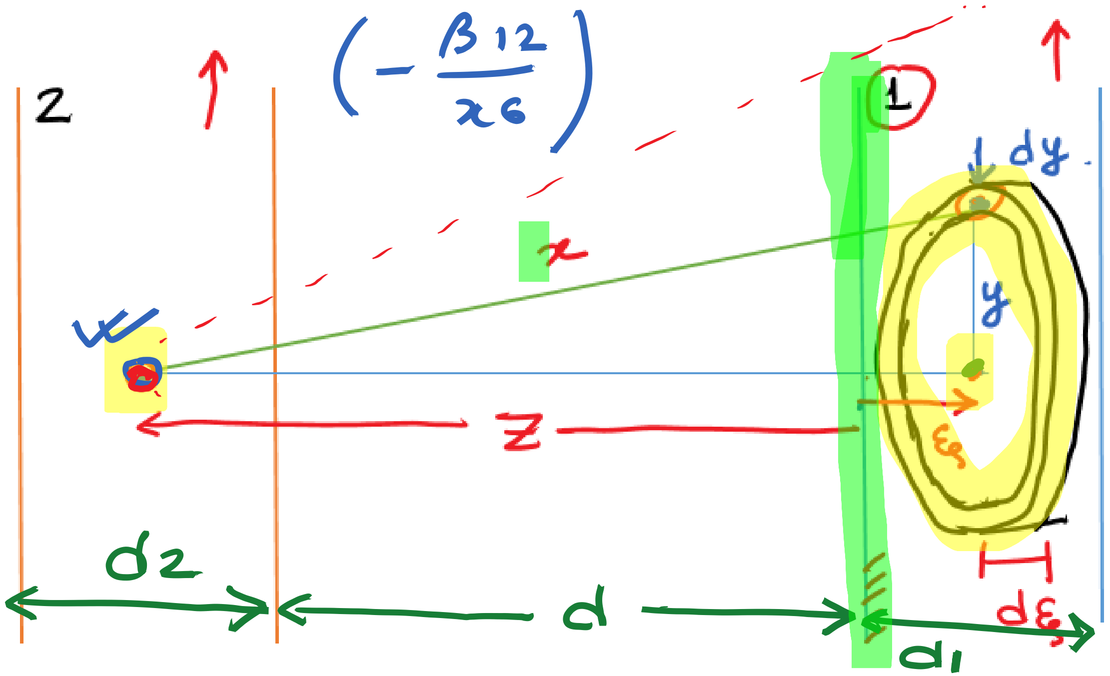




Volume of the ring element

$$dV = 2\pi y \, dy \, d\xi$$

No of molecules present in unit volume.

$$\frac{\text{No.}}{\text{Vol}} = \frac{\text{No.}}{\text{mole}} \times \frac{\text{mole}}{\text{vol.}} = \frac{\text{No.}}{\text{mole}} \times \frac{\text{mole}}{\text{mass}} \times \frac{\text{mass}}{\text{volume}} = \frac{N_A}{M_1} \cdot \rho_1$$




No. of molecules present in the ring $= (2\pi y\, dy\, d\xi)\left(\frac{\rho_1 N_A}{M_1}\right)$

Total Energy of interaction between one molecule of 2 with all molecules of 1 is:

$$\Rightarrow (2\pi y\, dy\, d\xi)\left(\frac{\rho_1 N_A}{M_1}\right)\left(-\frac{B_{12}}{x^6}\right) \leftarrow ??$$

The total energy of Interaction of one molecule of 2 with ALL molecules of 1

$$\phi'' = \int_{\xi=0}^{\xi=d_1}\int_{y=0}^{y=\infty}\left(-\frac{B_{12}}{x^6}\right)\left(\rho_1\frac{N_A}{M_1}\right)(2\pi y\, dy\, d\xi)$$

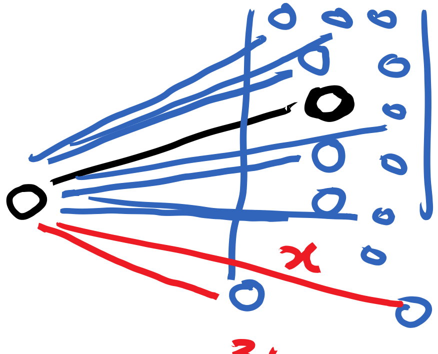
$d\xi$ from 0 to $d_1$

$dy$ from 0 to $\infty$

$$\phi'' = \int_{\xi=0}^{\xi=d_1}\int_{y=0}^{y=\infty}\left(-\frac{\beta_{12}}{x^6}\right)\left(\rho_1\frac{N_A}{M_1}\right)\left(2\pi y\,dy\,d\xi\right).$$

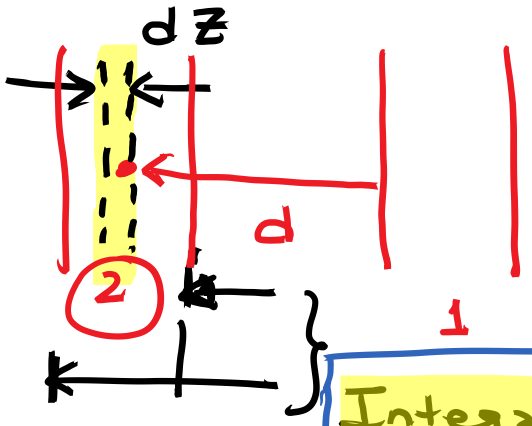
$$= \frac{-2\rho_1 N_A \pi \beta_{12}}{M_1}\int_{\xi=0}^{\xi=d_1}\int_{y=0}^{y=\infty}\frac{y\,dy\,d\xi}{\left[(z+\xi)^2+y^2\right]^3}$$

$$\int_{y=0}^{y=\infty}\frac{y\,dy}{\left[(z+\xi)^2+y^2\right]^3} = \frac{1}{4}\cdot\frac{1}{(z+\xi)^4}$$




$$\frac{1}{4}\int_{\xi=0}^{\xi=d_1}\frac{d\xi}{(z+\xi)^4} = \frac{1}{12}\left[\frac{1}{z^3}-\frac{1}{(z+d_1)^3}\right]$$

$$\phi'' = -\frac{\rho_1 N_A \pi \beta_{12}}{6 M_1}\left[\frac{1}{Z^3} - \frac{1}{(Z+d_1)^3}\right]$$




No. of molecules per unit volume of material 2 is $= \left(\rho_2 \frac{N_A}{M_2}\right)$

volume of this thin Area $A\,dZ$.

$dZ \to$ width

$A \to$ Cross Sectional Area

Integration in Z, then we cover all mol of 2

$\therefore$ Total no. of molecules with in this thin slice

$$\left(\rho_2 \frac{N_A}{M_2}\right) \cdot (A\,dZ)$$

Total No. of molecules of 2 per unit Cross Sectional Area =

$$\left(\rho_2 \frac{N_A}{M_2}\right) \cdot dZ$$

Energy of Interaction of all molecules of 2 present within the thin slice of material with ALL molecules of 1 per unit Cross Sectional Area:

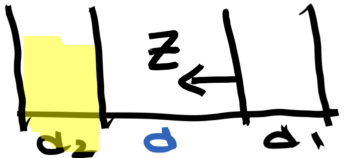
$$\boxed{\phi'' \cdot \frac{\rho_2 N_A}{M_2}\, dZ} \qquad \phi'' \to -\frac{\beta_{12}}{x6}$$

Assumption: Additive *

$$= -\frac{\rho_1 N_A \pi \beta_{12}}{6 M_1}\left[\frac{1}{Z^3} - \frac{1}{(Z+d_1)^3}\right] \cdot \frac{\rho_2 N_A}{M_2}\, dZ$$

Potential Energy of Int. between ALL molecules of 2 with all molecules of 1 per unit Cross Sectional Area:

$$G^{LW}_{12} = \int_{Z=d}^{Z=d+d_2} -\frac{\rho_1 \rho_2 N_A^2 \pi \beta_{12}}{6 M_1 M_2}\left[\frac{1}{Z^3} - \frac{1}{(Z+d_1)^3}\right] dZ$$



$$G_{12}^{LW} = \int_{z=d}^{z=d+d_2} -\left(\frac{\rho_1 \rho_2 \pi^2 N_A^2 \beta_{12}}{M_1 M_2}\right)\left(\frac{1}{6\pi}\right)\left(\frac{1}{z^3} - \frac{1}{(z+d)^3}\right) dz$$

$$A_{12} = \frac{\rho_1 \rho_2 \pi^2 N_A^2 \beta_{12}}{M_1 M_2} = \underline{\text{Hamaker Constant}}$$

(Material Pair dependent Constant)

$$G_{12}^{LW} = \int_{z=d}^{z=d+d_2} \frac{A_{12}}{6\pi}\left(\frac{1}{z^3} - \frac{1}{(z+d)^3}\right) dz$$

$$\Rightarrow G_{12}^{LW} = -\frac{A_{12}}{12\pi}\left[\frac{1}{(z+d_1)^2} - \frac{1}{z^2}\right]_{d}^{d+d_2}$$

$$G_{23}^{LW} = -\frac{A_{23}}{12\pi}\left(\cdots\cdots\right)$$

$$\Rightarrow G_{12}^{LW} = -\frac{A_{12}}{12\pi}\left[\frac{1}{(d_1+d_2+d)^2} + \frac{1}{d^2} - \frac{1}{(d+d_1)^2} - \frac{1}{(d+d_2)^2}\right]$$

If the two blocks are thick $d_1 \to \infty$

$d_2 \to \infty$

$$\boxed{G_{12}^{LW} = -\frac{A_{12}}{12\pi d^2}}$$

$$-\frac{B_{12}}{x^6}$$

Similar

↳ Both Based on Attractive VdW Interaction.

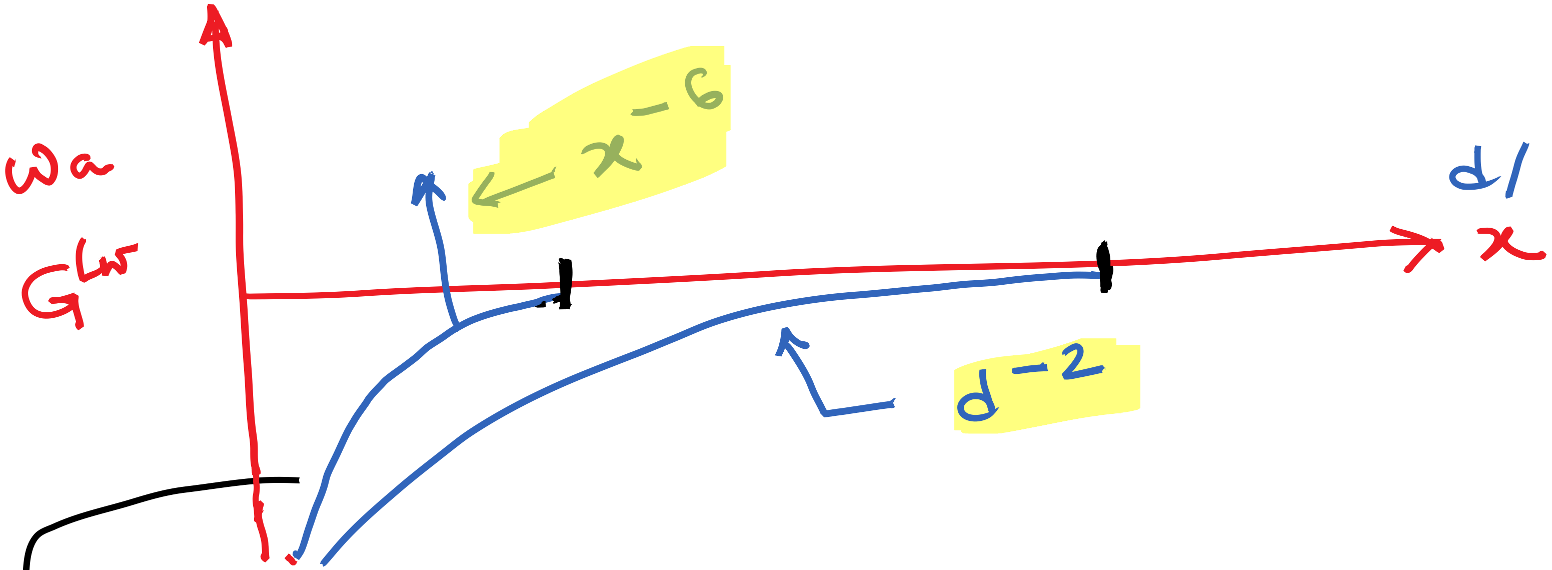
Are they Similar ??

They are interconnected

Manifestation of the Same Effect.

⇒ Interaction was scaling as $\frac{1}{x^6}$ between two molecules.

⇒ Between two Surfaces it Scales $\frac{1}{d^2}$




$W_a = -\frac{B_{12}}{x^6}$

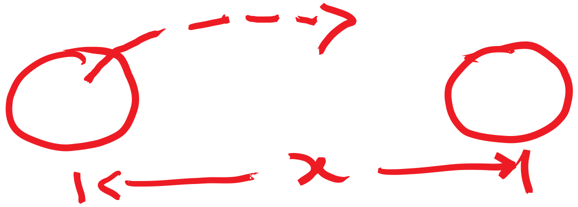
$G^{lw}_{12} = -\frac{A_{12}}{12\pi d^2}$

As the Sepn. distance increases, Wa will → 0, more rapidly, then $G^{lw}_{12}$.

More rapidly → 0 then $\frac{1}{d^2}$ → Longer tail.

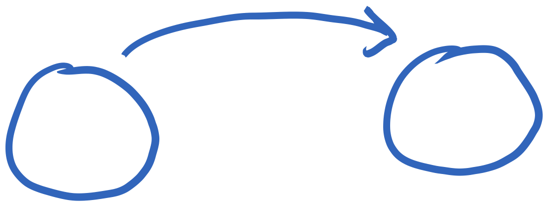
($\frac{1}{x^6}$)

As $x \to \infty$, $d \to \infty$, Both $W_a \to 0$, $G^{lw}_{12} \to 0$



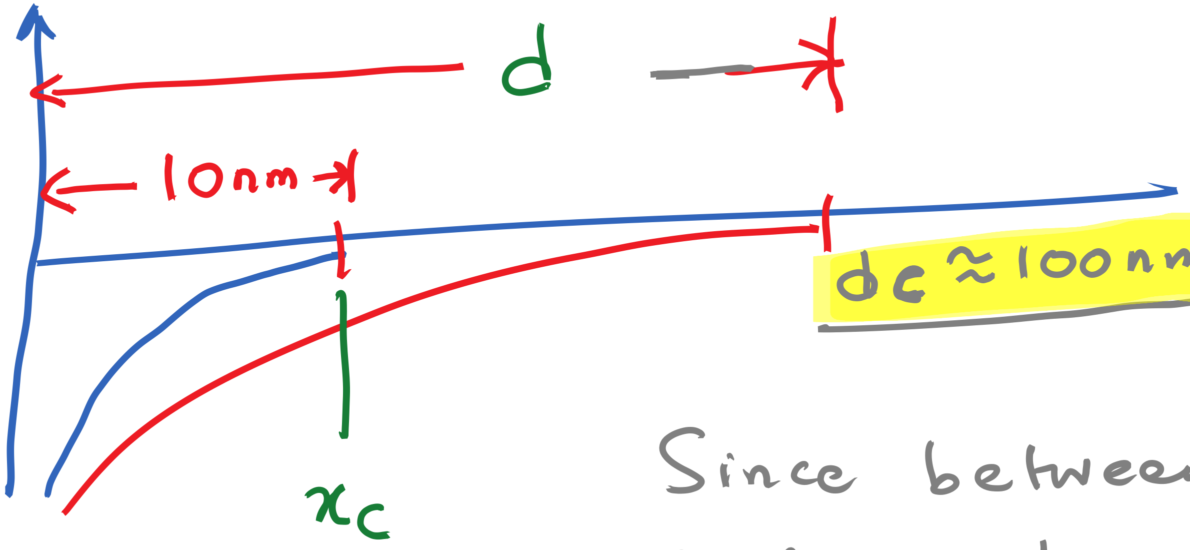

Far away, there is no interaction

If x is large, $W_a \to 0$



For vdw interaction, stretches $\approx$ 10 nm between two particles

$\boxed{x_c \approx 10\,nm}$




$\boxed{d_c \approx 100\,nm}$


Since between two surfaces the scaling changes to $\frac{1}{d^2}$, so the interaction becomes slightly longer range

Manifestation of vdw interaction between two surfaces can be felt over longer seperation distance.