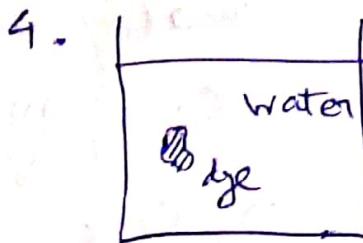
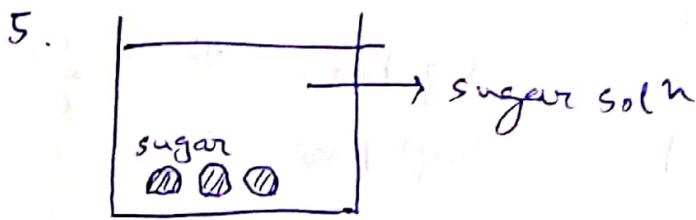


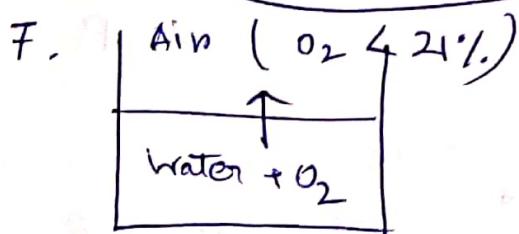
1. Tea leaves  $\rightarrow$  Tea
  2. Distillation Column
  3. Centrifugation
- } Mechanical / Physical



Conc. diff.



Driving force;  
Chemical Potential Difference



$$\mu_i = \left( \frac{\partial G}{\partial n_i} \right)_{T, P}$$

n, T, P

### 8. Osmosis

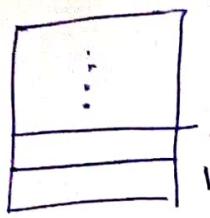
Mass Transfer: Movement of material due to difference in Chemical Potential

Molecular Diffusion: Governed by the kinetic energy of the molecule.

Eddy Diffusion: Enhanced by external force (Creation of packets of fluid)

Mass Transfer  $\rightarrow$  Diffusion

$\rightarrow$  Convective MT



*n components*

Concentration

mass / vol

mole / vol.

$$\rho_i = \frac{m_i}{V}$$

$$c_i = \frac{n_i}{V}$$

$$\sum \rho_i = \rho$$

$$\sum c_i = C$$

$$w_i = \frac{\rho_i}{\rho}$$

$$\text{Liq Phase: } n_i = \frac{c_i}{C}$$

$$\sum w_i = 1$$

$$\text{Vap phase: } J_i = \frac{c_i}{C}$$

$$\rho_i = c_i RT$$

$$\therefore J_i = \frac{\rho_i}{P}$$

$u_i$  > rel. of comp.  $(i)$

Average velocity

Mass

$$u = \frac{\sum \rho_i u_i}{\sum \rho_i}$$

Mole

$$U = \frac{\sum c_i u_i}{\sum c_i}$$

$$\begin{array}{c|cc} & 1 & \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}$$

Reference frame

$$\begin{aligned} \text{Ref velocity} &= 0 \\ &= U \\ &= U \end{aligned}$$

Flux of component  $(i)$

Mass / Area Time of  $(i)$

Moles / Area Time of  $(i)$

$$\text{Ref } v = 0$$

$$\text{Ref } v = U$$

$$\text{Ref } v = U$$

$$N_i = c_i u_i$$

$$J_i = c_i(u_i - U)$$

$$J_i = c_i(u_i - U)$$

$$\text{Ref } v = 0$$

$$n_i = \rho_i u_i$$

$$\text{Ref } v = U$$

$$J_i = \rho_i(u_i - U)$$

$$\text{Ref } v = U$$

$$J_i = \rho_i(u_i - U)$$

Diffusive Flux  $\bar{J}_i = c_i(u_i - u) = c_i u_i - c_i u$

$\therefore N_i$  - convective Flux.

1. Def of MT:

Separation of material / molecules based on chemical potential

$$N_i = \bar{J}_i + c_i u$$

Total flux      Diffusive Flux      Convective Flux

$$N_i = \bar{J}_i + \frac{c_i}{C} (\sum N_i)$$

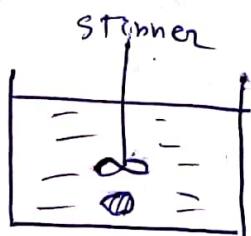
Binary System:

$$N_A = \bar{J}_A + \frac{c_A}{C} (N_A + N_B)$$

Diffusion



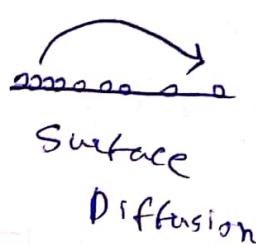
Molecular Diffusion



Eddy Diff  
↓  
Convective MT



Knudsen Diff  
(Pore Diff)



Surface Diffusion

Flux  $\propto$  Conc. gradient

$$\bar{J}_A \propto -\frac{dc_A}{dx}$$

$$\bar{J}_A = -D_{AB} \frac{dc_A}{dx}$$

Ficks Law

Diffusing / Diffusion Co-eff

$D_{AB}$  depends on.



What are the factors

Unit of  $D_{AB}$  : m/s

$$\begin{aligned} & \downarrow \\ & \text{Le / Vol.} \\ & = \frac{n_i}{V} \\ & = C \\ & \frac{c_i}{C} \\ & = \frac{c_i}{C} \end{aligned}$$

$$N_A = -\frac{c_A}{C} (N_A + N_B) = -D_{AB} \frac{dc_A}{dx}$$

$$\mu_i = \mu_i^0 + RT \ln(a_i)$$

$$= \mu_i^0 + RT \ln(\gamma_i c_i)$$

$$\frac{d\mu_i}{dx} = \frac{RT}{\gamma_i c_i} \cancel{\frac{dc_i}{dx}} \therefore \frac{dc_i}{dx} = \frac{c_i}{RT} \frac{d\mu_i}{dx}$$

Proof:  $D_{AB} > D_{BA}$

$$N_A = \frac{c_A}{C} (N_A + N_B) = -D_{AB} \frac{dc_A}{dx}$$

$$N_B = \frac{c_B}{C} (N_A + N_B) = -D_{BA} \frac{dc_B}{dx}$$

$$(N_A + N_B) - \frac{c_A + c_B}{C} (N_A + N_B) = -D_{AB} \frac{dc_A}{dx} - D_{BA} \frac{dc_B}{dx}$$

$$\text{Now, } c_A + c_B = C = \text{const}$$

$$\therefore \frac{dc_A}{dx} + \frac{dc_B}{dx} = 0 \therefore \frac{dc_B}{dx} = -\frac{dc_A}{dx}$$

$$\therefore -D_{AB} \frac{dc_A}{dx} + D_{BA} \frac{dc_A}{dx} = 0$$

$$\boxed{D_{AB} = D_{BA}}$$

$$1) \quad q = -k \frac{dT}{dx} = -\frac{k}{\rho C_p} \frac{d(PC_p T)}{dx}$$

$$2) \quad \tau = -\mu \frac{dv}{dx} = -\mu \frac{d(\rho v)}{dx}$$

$$= -\frac{\mu}{\rho} \frac{d(PV)}{dx} = -\frac{\mu}{\rho} d\left(\frac{mv}{V}\right)$$

$$3) \quad J_A = -D_{AB} \frac{dc_A}{dx} = -D_{AB} \frac{d(n_A/V)}{dx}$$

$$\alpha = \frac{k}{\rho C_p} = \text{Thermal diffusivity}$$

$$\frac{\mu}{\rho} = \nu = \text{momentum diffusivity}$$

$$D_{AB}$$

} Analogous  
(m<sup>2</sup>/s)

## Molecular Diffusivity

### Correlation

### Measurement

Gas Phase (non polar)      liq Phase

→ Based on Kinetic Theory

→ Hershfelder (or Chapman - Enskog)

Feller

### Kinetic Theory

$$D_{AA+} = \frac{1}{3} \lambda u$$

$$\lambda = \text{mean free path} = \frac{KT}{\sqrt{2\pi}\sigma_A^2 P}$$

$$u = \sqrt{\frac{SKNT}{\pi M_A}}$$

$$D_{AA+} = \frac{1}{3} \frac{KT}{\sqrt{2\pi}\sigma_A^2 P} \sqrt{\frac{SKNT}{\pi M_A}} = \frac{2}{3} \frac{T^{3/2}}{\pi^{3/2} \sigma_A^2 P} \left(\frac{k^3 N}{M_A}\right)^{1/2}$$

$$= \frac{2}{3} \left(\frac{k}{\pi}\right)^{3/2} \frac{N^{1/2} T^{3/2}}{P \sigma_A^2} \left(\frac{1}{M_A}\right)^{1/2}$$

$$D_{AB} = \frac{2}{3} \left(\frac{k}{\pi}\right)^{3/2} \frac{N^{1/2} T^{3/2}}{P (\sigma_{AB})^2} \left(\frac{1}{2M_A} + \frac{1}{2M_B}\right)^{1/2}$$

$$\sigma_{AB} > \frac{\sigma_A + \sigma_B}{2}$$

$$\text{Value of } D_{AB} \left( \frac{\text{cm}^2}{\text{s}} \right) = \frac{0.001858 T^{3/2} \left( \frac{1}{M_A} + \frac{1}{M_B} \right)^{1/2}}{P \sigma_{AB}^2 \rho_D}$$

Hershfelder

24. 07. 2019

## Transport Properties:

Thermal Diffusivity	$(\frac{k}{\rho c_p})$
Mass	" $(D_{AB})$
Momentum	" $(\mu/\rho)$

### 1. Analogy

$$2. P_{AAT} =$$

↓

$$D_{AB} = \frac{2}{3} \left( \frac{k}{\pi} \right)^{3/2} N^{1/2} T^{3/2} \left[ \frac{1}{2M_A} + \frac{1}{2M_B} \right]^{1/2}$$

$\underbrace{P \left( \frac{\sigma_A + \sigma_B}{2} \right)^2}_{\text{Hartfelder}} \underbrace{D}_{\text{Lennard Zones potential}}$

Hartfelder

$$\sigma_D = f / \left( \frac{RT}{\epsilon_{AB}} \right)$$

$\epsilon$  = Lennard Zones parameter

$$\epsilon_{AB} = (\epsilon_A \epsilon_B)^{1/2}$$

$$\sigma = 1.18 v_b^{1/3}$$

$$= 0.341 v_c^{1/3}$$

$$= 2.44 (T_c/p_c)^{1/3}$$

$v_b$  = molecular volume at boiling pt  
(usual)

$$\frac{\epsilon}{k} = 0.77 T_c$$

$$= 1.15 T_b$$

Fuller:

$$10^{-3} T^{1.75} \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2}$$

$$D_{AB} = \frac{P}{\left[ (\sum v)_A^{1/3} + (\sum v)_B^{1/3} \right]^2}$$

$v$  = ~~Atomic~~ volume

$\sum v$  = Molecular Vol.

09.2019

$$y \left( \frac{k}{Pc_p} \right) \\ (D_{AB}) \\ (\mu_e)$$

$$KT : D_{AB} \propto T^{3/2}$$

$$\text{Herschfelder} : D_{AB} \propto T^{3/2} \frac{1}{\sqrt{D}}$$

$$\text{Fuller} : D_{AB} \propto T^{1.75}$$

$$D_{AB} \propto \frac{1}{P}$$

$$\text{Phaff} : D_{1m} = \frac{1}{\bar{J}_2^1/D_{12} + \bar{J}_3^1/D_{13}} = \frac{\bar{J}_n^1}{D_{1n}}$$

$$\bar{J}_n^1 = \frac{\bar{J}_n}{1 - \bar{J}_1}$$

$$D_{AB} = \frac{RT}{6\pi n \mu_B}$$

Einstein - Stokes Law  
(based on hydrodynamic theory)

Assumption: Dilute soln

$$\frac{D_{AB} \mu_B}{T} = \text{const.}$$

$\mu_B$  = solvent viscosity

$$\frac{D_{AB} \mu_B}{T} = f(\nu_B) \quad \text{Wilke Chang}$$

$$\frac{D_{AB} \mu_B}{T} = \frac{7.5 \times 10^{-3}}{\nu_A^{0.6}} (\phi_B \mu_B)^{1/2}$$

$\nu$  =  $\text{cm}^3/\text{g mol}$

$\phi_B$  = association factor of the solvent

Haugdalk Caudie

$$D_{AB}^\circ = 13.26 \times 10^{-5} \mu_B^{-1.14} \nu^{-0.589}$$

$\downarrow$  (ininitely dilute soln)  $\downarrow$   $\downarrow$   
 $\downarrow$   $(\text{cm}^2/\text{s})$   $\downarrow$   $\downarrow$   
 $\downarrow$   $c_p$   $\downarrow$   $\downarrow$   
 $\downarrow$   $\text{cm}^3/\text{g mole}$

$$D_{AB} \mu = \left( D_B^{\circ} \mu_A \right)^{\frac{1}{2}} \left( D_A^{\circ} \mu_B \right)^{\frac{1}{2}}$$

$$\left( 1 + \frac{d \ln \frac{D_B^{\circ}}{D_A^{\circ}} \gamma_A}{d \ln \gamma_A} \right)$$

25. 07. 2019

### Estimation of Molecular diffusivity



$$D_{AB} |_{T_2} = D_{AB} |_{T_1} \frac{(2\pi)_{T_1}}{(2\pi)_{T_2}} \left( \frac{T_2}{T_1} \right)^{3/2}$$

$\lambda > d_{\text{pore}}$   $\Rightarrow$  Knudsen Diffusion  
 $\Downarrow$   
 Mean free path      Pore Diffusion

$$Kn = \text{Knudsen No.} = \frac{\lambda}{d_{\text{pore}}}$$

$Kn > 1 \Rightarrow$  Knudsen Diffusion

$Kn < 1 \Rightarrow$  No.



For Molecular Diffusion

$$J_A = -D_{AB} \frac{dC_A}{dn}$$

For Knudsen Diffusion

$$J_A = - \left[ D_{KA} \frac{\epsilon}{\tau} \right] \frac{dC_A}{dn}$$

$\epsilon$  = porosity

$\tau$  = tortuosity

For straight path,  $\tau = 1$

$$D_{KA} \frac{\epsilon}{\tau} = D_{AK}^{\text{eff}}$$

$$D_{KA} = \frac{1}{3} \frac{dU}{dp_{\text{pore}}} \quad U = \sqrt{\frac{8kNT}{\pi M_A}}$$

$$= 4850 d_{\text{pore}} \sqrt{\frac{1}{M_A}}$$

In Case of Combination of Molecular Diffusion  
+ Knudsen Diffusion  
(In case of high conc.)

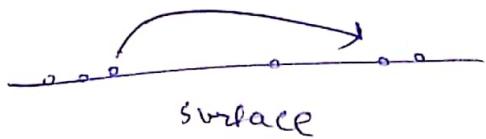
$$J_A = - D_{Ae} \frac{\epsilon}{\tau} \frac{dc_A}{dx}$$

$$\frac{1}{D_{Ae}} = \frac{1 - \alpha J_A}{D_{AB}} + \frac{1}{D_{KA}} \quad \alpha = 1 + \frac{N_B}{N_A}$$

$$\rightarrow J_A = 0 \quad \text{or when } \alpha > 0$$

$$\frac{1}{D_{Ae}} = \frac{1}{D_{AB}} + \frac{1}{D_{KA}}$$

Surface Diffusion :



$$J_A = - D_{AS} \frac{dc_A}{dx}$$

for ~~big~~ fashionable Product  $\rightarrow$  2-3 months

months

## Binary Molecular Diffusion

31.07.2019

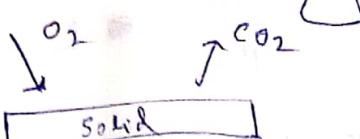
- A + B

- Steady state

- No reaction

- Single Phase Homogeneous

- Constant Geometry

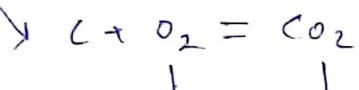


(A ↑ - B ↑) (counter diffusion)

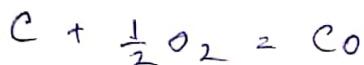
Depletion / Accumulation rate is so slow, it is considered to be S.S.

$$\frac{dN_A}{dn} = \frac{dc_A}{dt} = 0$$

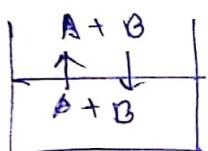
$$\frac{dN_A}{dx} = 0$$



equimolar counter diffusion



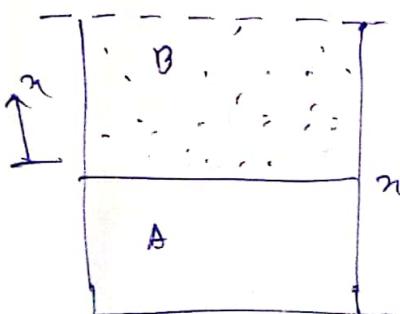
Non-equimolar  $\approx n$



A ↑ - B ↑

$$N_A L_A = N_B L_B$$

Thermally Isolated



$$z=L \quad C_A = C_A2$$

$$z=0 \quad C_A = C_A1$$

CASE - A

A ↑, B ≠

$$\frac{dN_A}{dx} = 0$$

$$J_A = -D_{AB} \frac{dC_A}{dx}$$

$$N_A = \frac{C_A}{c} (N_A + N_B) = -D_{AB} \frac{dC_A}{dx}$$

$\frac{C_A}{c}$

$$\frac{C_B}{c} N_A = - D_{AB} \frac{d C_A}{dx}$$

$$\frac{L - C_A}{c} N_A = - D_{AB} \frac{d C_A}{dx}$$

$$\int_{C_{A1}}^{C_{A2}} \frac{d C_A}{L - C_A} = - D_{AB} \int_0^L \frac{N_A}{D_{AB} c} dx$$

$$- \ln \left( \frac{L - C_{A2}}{L - C_{A1}} \right) = - \frac{N_A}{D_{AB} c} L$$

$$N_A = \frac{D_{AB} c}{L} \ln \left( \frac{L - C_{A2}}{L - C_{A1}} \right)$$

$$= \frac{D_{AB} c}{L} \frac{\frac{C_{A1} - C_{A2}}{C_{A1} - C_{A2}}}{\ln \left( \frac{L - C_{A2}}{L - C_{A1}} \right)}$$

$$= \frac{D_{AB} c}{L} \frac{\frac{C_{A1} - C_{A2}}{(C_{B2} - C_{B1})}}{\ln \left( \frac{C_{B2}}{C_{B1}} \right)}$$

↓

~~C<sub>B1m</sub>~~

$$N_A = \frac{D_{AB} c}{L} \frac{\frac{C_{A1} - C_{A2}}{C_{B1m}}}{C_{B1m}}$$

Gas Phase

$$C = \frac{P}{RT}, \quad C_A = \frac{P_A}{RT}$$

$$\gamma_A = \frac{C_A}{C} = \frac{P_A}{P}$$

Liquid Phase

$$\pi_A = \frac{C_A}{C}$$

$$\frac{dN_A}{dx} = 0 \Rightarrow N_A \neq f(x)$$

$$N_A = - \frac{D_{AB} C}{c - C_A} \frac{dC_A}{dx}$$

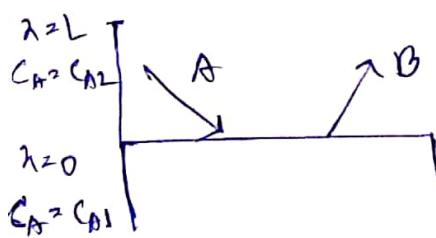
$$N_A \int_b^x dn = - D_{AB} C \int_{C_{A1}}^{C_A} \frac{dC_A}{c - C_A}$$

$$N_A x = - D_{AB} C \ln \left( \frac{c - C_A}{c - C_{A1}} \right)$$

$$\frac{D_{AB} C}{L} \ln \left( \frac{c - C_{A2}}{c - C_{A1}} \right) x = D_{AB} C \ln \left( \frac{c - C_A}{c - C_{A1}} \right)$$

$$\frac{c - C_A}{c - C_{A1}} = \left( \frac{c - C_{A2}}{c - C_{A1}} \right)^{x/L}$$

$$\text{Depletion} = \frac{\text{Mass}}{N_A A}$$



A↑ - B↑

equimolar  $N_A = - N_B$   
Non  $n$   $N_A = - n N_B$

$$N_B = \frac{C_A}{c} (N_A + N_B) \Rightarrow - D_{AB} \frac{dC_A}{dx}$$

equimolar :  $N_A = - N_B \quad N_A + N_B > 0$

$$N_A = - D_{AB} \frac{dC_A}{dx}, \quad (\text{no convective flux})$$

$$N_A = - \frac{D_{AB}}{L} (C_{A2} - C_{A1}) = \frac{D_{AB}}{L} (C_{A1} - C_{A2})$$

Fick's 2nd Law  $\frac{d^2 C_A}{dx^2} = 0$

$$C_A = Ax + B$$

$$x=0, C_A = C_{A1} \quad C_{A1} = B$$

$$x=L \quad C_A = C_{A2} \quad C_{A2} = AL + B \quad \therefore A = \frac{C_{A2} - C_{A1}}{L}$$

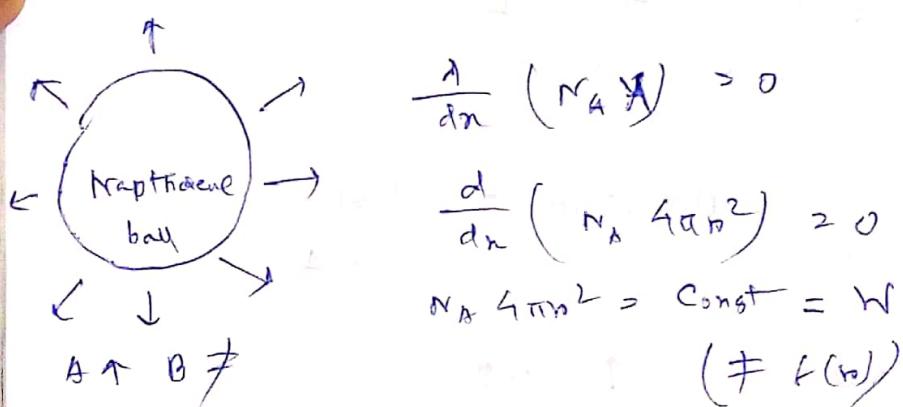
$$C_A = (C_{A2} - C_{A1}) \frac{x}{L} + C_{A1}$$

01.08.2019

$$\frac{C_A - C_{A1}}{C_{A2} - C_{A1}} = \frac{x}{L}$$

### Variable Geometry of System

$$\begin{cases} A \uparrow B \neq \\ A \uparrow B \downarrow \end{cases} \quad N_A = ? \quad C_A = ?$$



$$N_A = \frac{C_A}{C} (N_A + N_B)^0 = - D_{AB} \frac{d C_A}{d x}$$

$$\frac{C - C_A}{C} N_A = - D_{AB} \frac{d C_A}{d x}$$

$$\frac{C - C_A}{C} \cdot \frac{W}{4\pi n^2} = - D_{AB} \frac{d C_A}{d x}$$

$$-\cancel{\frac{C - C_A}{C}} \cancel{\frac{W}{D_{AB}}} - \frac{W}{D_{ABC}} \left( \frac{d n}{4\pi n^2} \right) = \int \frac{d C_A}{C - C_A}$$

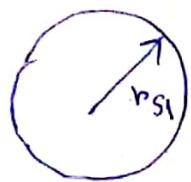
•  $n_S$

$C_{AS}$

$$\Rightarrow \frac{W}{D_{AB} 4\pi} \left( \cancel{\frac{C - C_A}{C}} - \frac{1}{n_S} \right) = - \ln \left( \frac{C - C_{AS}}{C - C_A} \right)$$

$$\frac{C - C_{AS}}{C - C_A} = \exp \left( \frac{W}{D_{AB} 4\pi n_S} \right)$$

$$w = D_{AB} C 4\pi n_s \ln \left( \frac{c - c_{A\infty}}{c - c_{As}} \right)$$



$n_s : n_{S1} \rightarrow n_{S0}$

$$- w_{n_s} = - \frac{d}{dt} \left( \frac{4}{3} \pi r_s^3 \frac{P_A}{M_A} \right)$$

$$= - D_{AB} C 4\pi n_s \ln \left( \frac{c - c_{A\infty}}{c - c_{As}} \right)$$

$$- \cancel{\frac{4}{3} \pi r_s^2} \frac{dn_s}{dt} \frac{r_s}{M_A} = D_{AB} C \ln \left( \frac{c - c_{A\infty}}{c - c_{As}} \right) 4\pi n_s$$

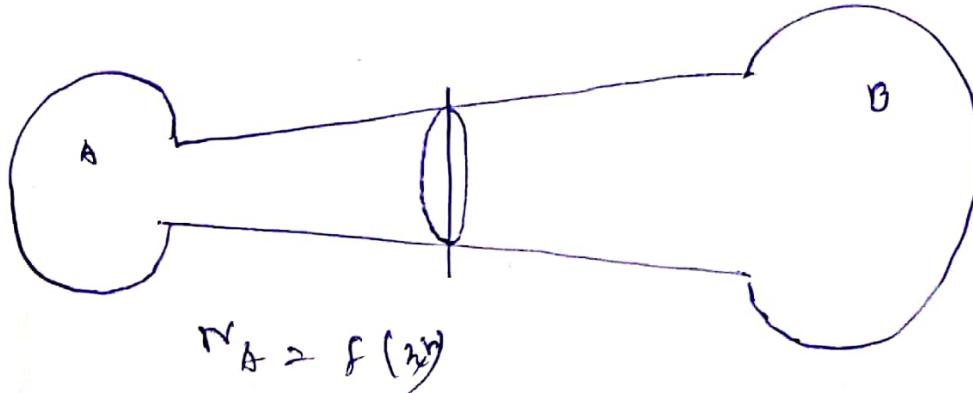
$$- \cancel{\frac{4}{3} \pi r_s^2} dn_s = \frac{D_{AB} C M_A}{P_A} \ln \left( \frac{c - c_{A\infty}}{c - c_{As}} \right) \frac{dt}{r_s}$$

$$n_s = - \frac{D_{AB} C M_A t}{P_A} \ln \left( \frac{c - c_{A\infty}}{c - c_{As}} \right)$$

$$\frac{1}{2} (r_{S0}^2 - r_{S1}^2)$$

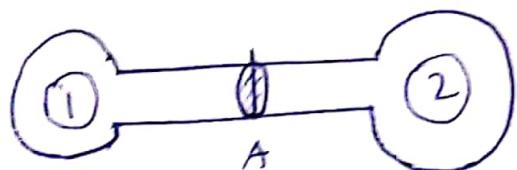
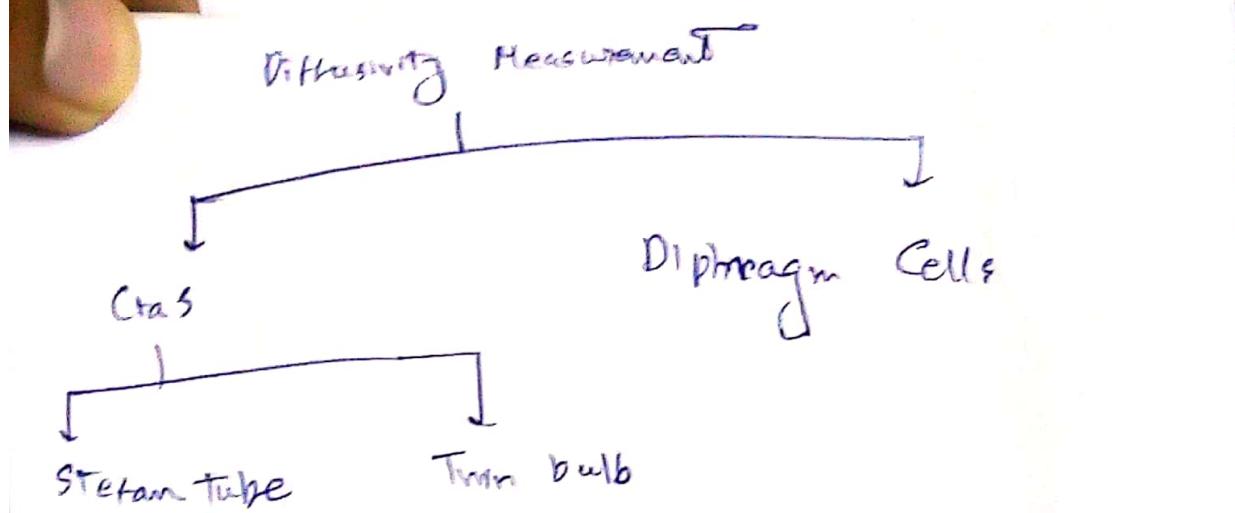
For Complete Sublimation

$$\frac{1}{2} r_{S1}^2 = \frac{D_{AB} C M_A}{P_A} t \ln \left( \frac{c - c_{A\infty}}{c - c_{As}} \right)$$



$$n_A = f(x)$$





$$-\frac{d}{dt} (v_1 C_{A1}) = N_A a = \frac{D_{AB} a}{l} (C_{A1} - C_{A2})$$

$$\frac{d}{dt} (v_2 C_{A2}) = \frac{D_{AB} a}{l} (C_{A1} - C_{A2})$$

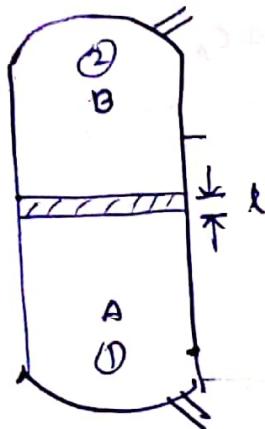
$$-\frac{d}{dt} C_{A1} = \frac{D_{AB} a}{l v_1} (C_{A1} - C_{A2})$$

$$\frac{d}{dt} C_{A2} = \frac{D_{AB} a}{l v_2} (C_{A1} - C_{A2})$$

$$-\frac{d}{dt} (C_{A1} - C_{A2}) = \frac{D_{AB} a}{l} \left( \frac{1}{v_1} - \frac{1}{v_2} \right) (C_{A1} - C_{A2})$$

$$-\ln \left( \frac{C_{A1f} - C_{A2f}}{C_{A10} - C_{A20}} \right) = \frac{D_{AB} a}{l} \left( \frac{1}{v_1} - \frac{1}{v_2} \right) t$$

## Diaphragm Cell



$$T = 0, \quad C_{A1} = C_{A10}, \quad C_{B1} = C_{B10} = 0$$

$$C_{A2} = C_{A20} = 0$$

$$C_{B2} = C_{B20}$$

$$T = T_f, \quad C_{A1} = C_{A1f}$$

$$C_{B1} = C_{B1f}$$

$$N_{A2} = \frac{1}{V_1} - \frac{D_{AB} a e}{l c} (C_{A1} - C_{A2})$$

$$-\frac{d}{dt} (C_{A1} - C_{A2}) = \frac{D_{AB} a}{l c} e \left( \frac{1}{V_1} - \frac{1}{V_2} \right) (C_{A1} - C_{A2})$$

$$\text{Cell Const} = \frac{a}{l} \frac{e}{c} \left( \frac{1}{V_1} - \frac{1}{V_2} \right) = \frac{D_{AB} K_{\text{cell}}}{(C_{A1} - C_{A2})}$$

w  $\propto$   $\Delta$  Driving force ( $\Delta$  D.F)

w  $\propto$  a

w  $\propto$  a  $\Delta$  DF  $\propto$  N<sub>A</sub>  $\propto$   $\Delta$  DF

$$N_A = (MTC) \Delta DF$$

Mass Transfer Coefficient

$$N_A = K_C (\Delta C_A)$$

$$N_A = K_G (\Delta n_A)$$

$$\begin{aligned} \text{Gas phase} &= K_L (\Delta C_A) \\ &\quad (\text{Liq phase}) \end{aligned}$$

$$\begin{aligned} &= K_x (\Delta n_A) \\ &\quad (\text{Gas phase}) \end{aligned}$$

$$N_A = K_p \Delta P_A$$

$$= K_G \Delta P_A$$

$$\begin{aligned} N_A &= K_C \Delta C_A \\ &= K_J \Delta J_A \\ &= K_G \Delta P_A \\ &\xrightarrow{\text{Gas}} \end{aligned}$$

$$\begin{aligned} N_A &= K_L \Delta C_A \\ &= K_n \Delta C_A \\ &\xrightarrow{\text{Eqv}} \end{aligned}$$

$$cK_C = K_J$$

$$K_G RT = K_C$$

$$\textcircled{1} A \uparrow, B \neq \quad K_C, K_J, K_G, K_L, K_n \neq$$

$$\textcircled{2} A \uparrow, B \downarrow \quad - - - - -$$

$$\textcircled{3} A \uparrow, B \neq$$

$$N_A = \left[ \frac{D_{AB} C}{L C_{BLm}} \right] \left( \frac{C_{A1} - C_{A2}}{\Delta C_A} \right)$$

$$K_C = \frac{D_{AB} C}{L C_{BLm}}$$

$$K_L = \frac{D_{AB} C}{L C_{BLm}}$$

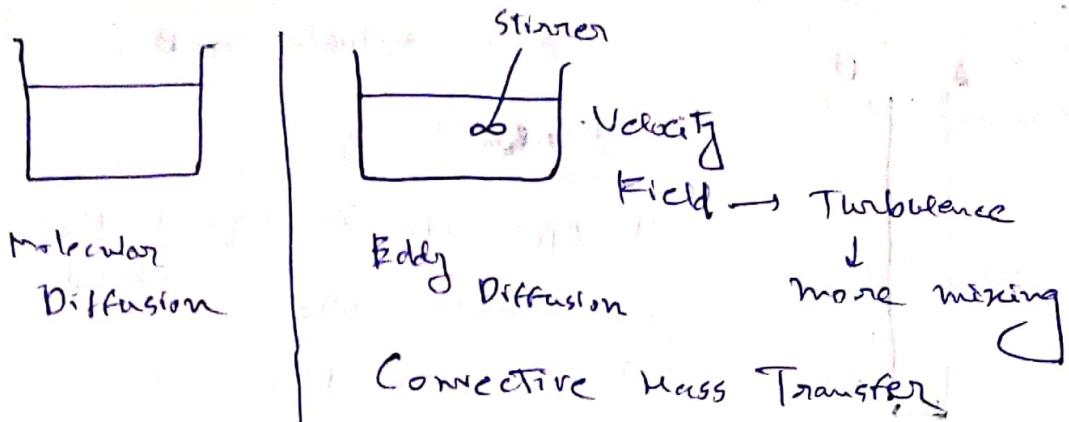
~~$$N_A = \frac{D_{AB} C}{L C_{BLm}} C (J_{A1} - J_{A2})$$~~

$$K_J = \frac{D_{AB} C^2}{L C_{BLm}}$$

$$K_n = \frac{D_{AB} C^2}{L C_{BLm}}$$

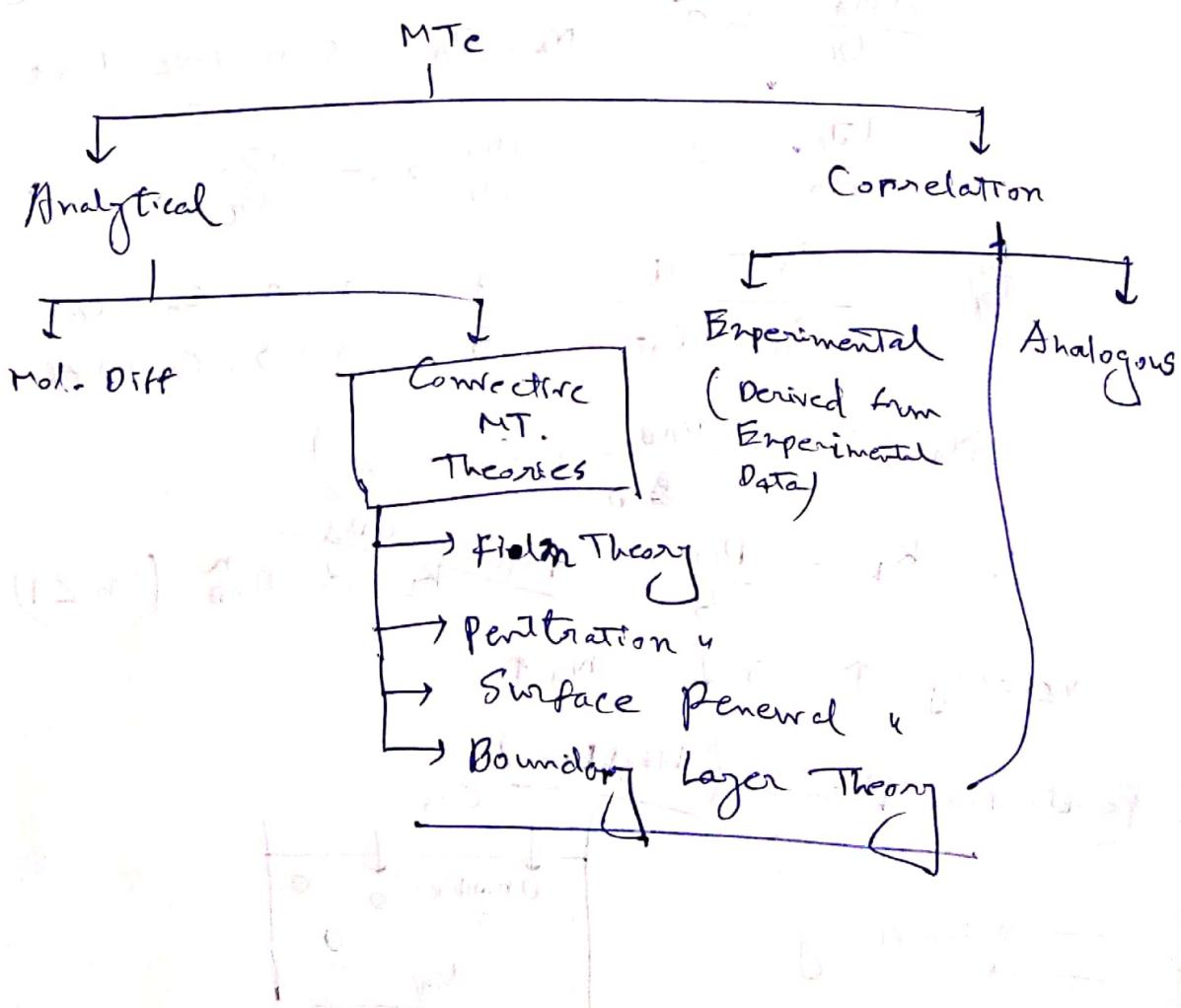
$$N_A = \frac{D_{AB} P}{L P_{BLm}} (P_{A1} - P_{A2}) \frac{1}{RT}$$

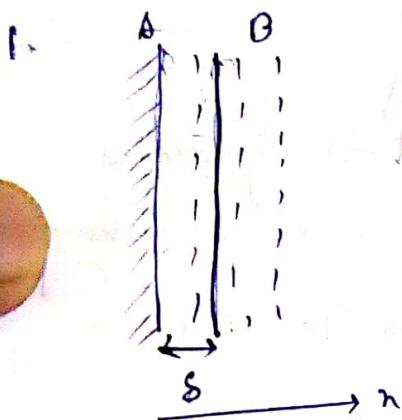
$$K_G = \frac{D_{AB} P}{L P_{BLm} RT}$$



$$N_A = (MTC) (\Delta D.F.)$$

$$\begin{aligned} N_A &= K_C (\Delta C_b) \xrightarrow{\text{approx}} K_L (\Delta C_g) \\ &= K_d (\Delta \gamma_A) = K_n (\Delta \gamma_g) \\ &\Rightarrow K_p (\Delta P_A) \end{aligned}$$





A is soluble in B.

### Film Theory

- ① Applicable To liquid Part solid
- ② Steady state
- ③ Convective Flux = 0

$$\Rightarrow (N_A + N_B) \frac{C}{C} = 0$$

•  $n = s$ ,  $C_A = C_{AB}$   
 $\lambda = 0$ ,  $C_A = C_{AS}$

Transfer ~~resist~~ Resistance ( $\frac{1}{K_L}$ ) will lie in the film

(5) without film ( $n = s$ ), mol. Diff. occurs  $\therefore$  Fick's law applicable

$$\frac{dN_A}{dx} = 0$$

$$N_A - J_A = \text{convective flux} = 0$$

$$\therefore \frac{dJ_A}{dx} > 0$$

$$\frac{dN_A}{dx} = - \frac{dJ_A}{dx}$$

$$\frac{C_A - C_{Ai}}{C_{Al} - C_{Ai}} = \frac{\lambda}{s}$$

$$\lambda = 0, C_A = C_{Ai}$$

$$\lambda = s, C_A = C_{Al}$$

$$N_A = \frac{D_{AB}}{s} (C_{Ai} - C_{Al})$$

$$\therefore K_L = \frac{D_{AB}}{s} \cdot \frac{\text{Experimentally}}{K_L \propto D_{AB}^n (nL)}$$

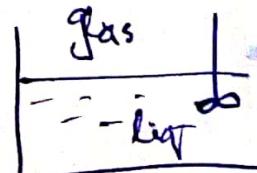
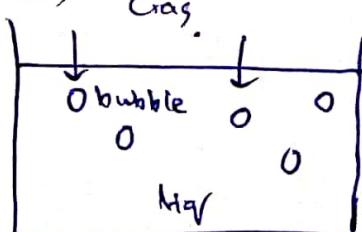
velocity  $\uparrow$   $s \downarrow$   $N_A \uparrow$

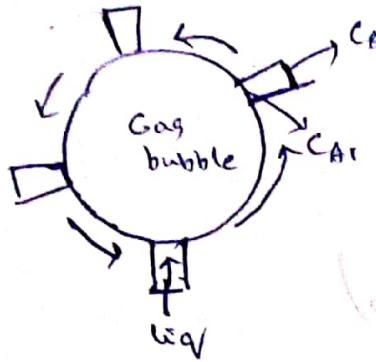
### Penetration Theory

#### (Higbie)

Gas flowing

— liq —

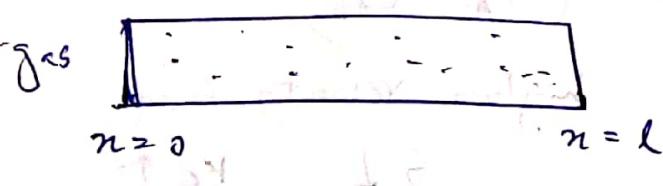




- ① Applicable to Gas-Liq system
- ② UnSteady State
- ③ MT occurs during Contact Time

(4) Contact Time is so low that that liq element behave as infinity slab

$$\begin{aligned}
 N_T &= k_L \Delta C_A \Rightarrow k_L \Delta C_A \\
 &= k_d \Delta d_A \Rightarrow k_d = \Delta n_d \\
 &= k_p \Delta P_A
 \end{aligned}$$



$$\frac{\partial^2 C_A}{\partial x^2} = - \frac{\partial C_A}{\partial t}$$

$$C_A(0, x) = C_{AB}$$

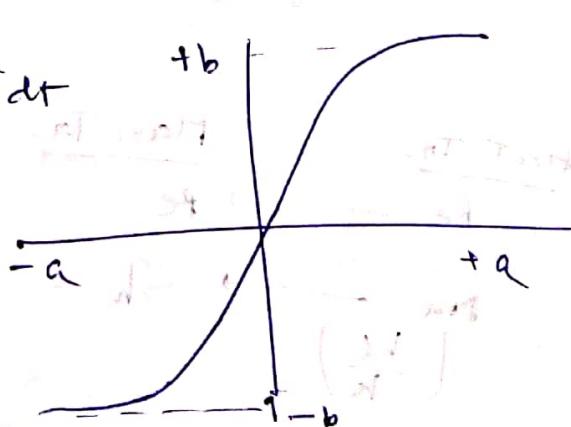
$$C_A(t, 0) = C_{Ai}$$

$$C_A(T, x) = C_{AB}$$

$$\frac{C - C_{AB}}{C_{Ai} - C_{AB}} = 1 - e^{-\eta}$$

$$\eta = \frac{l}{2\sqrt{D_{AB}t}}$$

$$\text{enf} = \frac{1}{\sqrt{\pi}} \int_{+\infty}^{-\infty} e^{-t^2} dt$$

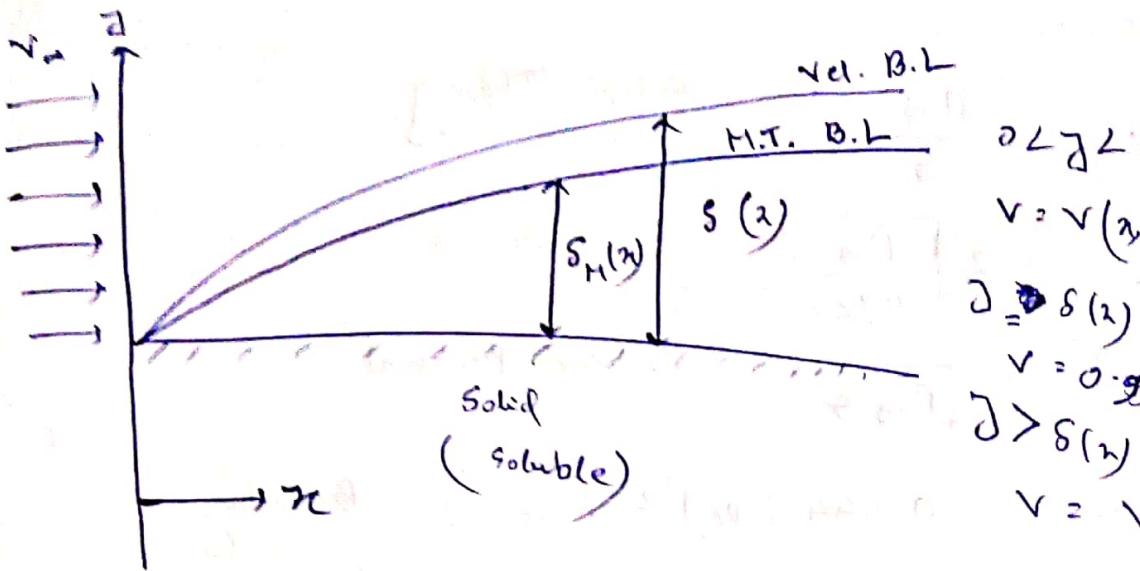


$$N_a(t) = - D_{AB} \frac{dc_A}{dx}$$

$$= \sqrt{\frac{D_{AB}}{\pi t}} (C_{Ai} - C_{AB})$$

কলা কলা। প্রজে  
জাতে করে কাজ  
[পৌত্র করে এসে]

মানা হিজ গন্ম) আব ছি বাঁচে পে? করে দে?  
ধৰণ কৈ, মিজের মেরে দেন না? করে দে?  
"আ" (কাজ দেন) মনা! তেমর ছাঁ দেখ গুন! আ  
শীট" (হিজ দেন) আর মন্ত করে ন কেউ,  
শীট" আচ, আর মন্ত করে ন কেউ? মিজ  
মিজে ধানাদের দেন দেন? মিজ  
শীট" কি করে করে? আর মন্ত করে  
শীট" আচ, আর মন্ত করে? মিজ  
মিজে ধানাদের দেন দেন? মিজ  
শীট" কি করে করে? আর মন্ত করে



$$0 < \delta < S(x)$$

$$V = V(x, z)$$

$$\delta = \delta(x)$$

$$V = 0.99 V_\infty$$

$$\delta > S(x)$$

$$V = V_\infty$$

$$\delta = 0, C_A = C_{Ai}$$

$$\delta = S_M(x) \quad C_A = C_{Ai} - (C_{Ai} - C_{Ab}) 0.99$$

$$\delta > S_M(x) \quad C_A = C_{Ab}$$

$$\frac{K_C x}{D_{AB}} = 0.322 \left( \frac{\pi V_\infty P}{\mu} \right)^{1/2} \left( \frac{\gamma}{D_{AB}} \right)^{1/3}$$

$$Sh|_x = 0.322 (Re_x)^{1/2} Sc^{1/3}$$

$$Sh|_{av} = 0.644 Re^{1/2} Sc^{1/3}$$

$$Sh = f(Re, Sc)$$

Film, Penetration, S.R  $\rightarrow$  Implicit

B.L  $\rightarrow$  explicit

$$\rightarrow Sh = f(Re, Sc)$$

19.08.2019

$$N_A = K_C \Delta C_A = K_L \Delta C_A$$

$$K_D \Delta \bar{A}_A = K_L \Delta \bar{n}_A$$

$$K_P \Delta P_A$$

$$K_c = \frac{D_{AB}}{S} \quad \text{Film Theory}$$

$$\Rightarrow 2 \sqrt{\frac{D_{AB}}{\pi k_c}} \quad \text{Penetration in}$$

$$\Rightarrow \sqrt{D_{AB} S} \quad \text{Surface Renewal}$$

$$Sh = 0.644 (Re)^{1/2} (Sc)^{1/3} \quad \text{Boundary Layer in}$$

$$Sh = f(Re, Sc)$$

$$Sh = \frac{\text{Conv. H.T.}}{\text{mol diff H.T.}} \quad \text{Conv. H.T.} \quad \text{mol diff H.T.}$$

$$\left\{ \frac{K_c l}{D_{AB}} \right\} \approx Nu$$

$$Re = \frac{LV_a P}{\mu}$$

$$Sc = \frac{V}{D_{AB}} \approx Pe$$

$$Pe_{HT} = Re Pe$$

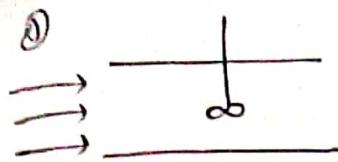
$$Pe_{MT} = Re Sc \quad St_{HT} = \frac{Nu}{Re Pe} \quad St_{MT} = \frac{Nu}{Pe}$$

$$St_{MT} = \frac{Sh}{Re Sc}$$

MTC — Correlations

① Experimental Data &  
Dimensional Analysis

② Analogous H.T.  
Correlation



Nature of A and B  
Velocity  
Material Properties ( $\rho, \mu$ )  
Transport Properties ( $d, D_{AB}, V$ )  
Diameter of pipe

Parameter	$w_A$	$N_A$	$K_C$

$$Sh = f(Re, Sc)$$

Find f (function)

$$(2) Nu = f(Re, Pr)$$

$$Sh = f(Re, Sc)$$

Reynold's Analogy

$$St_H = St_M \Rightarrow \frac{f}{2} \quad f = \text{friction factor}$$

\* valid when  $Sc, Pr \approx 1$

$$St_H Pr^{2/3} \Rightarrow St_M Sc^{2/3} = \frac{f}{2}$$

$$\bar{\rho}_H = \bar{\rho}_M$$

Couhoun factor

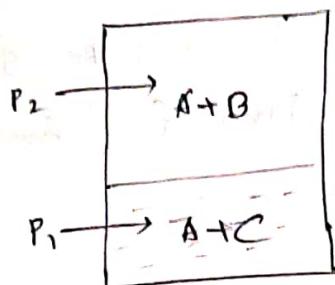
~~Chilton~~ Couhoun-Chilton  
Analogy

$Sh = 2$  when ~~Air~~ is stagnant, naphthalene is sublimed.

$$Sh = \frac{K_C \Delta C_A}{\frac{P_{AB}}{S} \Delta C_A} =$$

MF

- Binary system
- Standy state
- No reaction
- homogeneous / single phase



Cas - liqu

Liq - Cas

Liq - Liqu

$$c_{A2b} = f(c_{A1b})$$

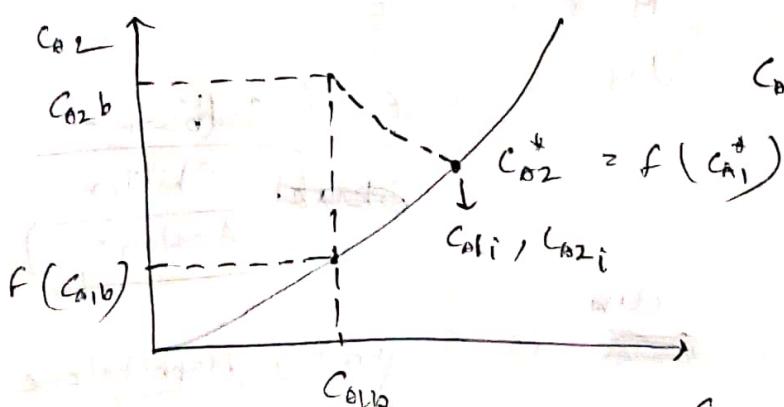
→ Equilibrium  
No MT

$$c_{A2b} \neq f(c_{A1b}) \rightarrow \text{No Equilibrium}$$

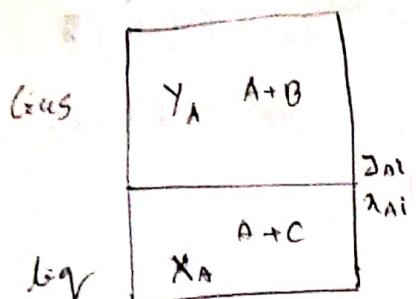
MT

$$c_{A2b} > f(c_{A1b})$$

MT: Phase 2 > Phase 1

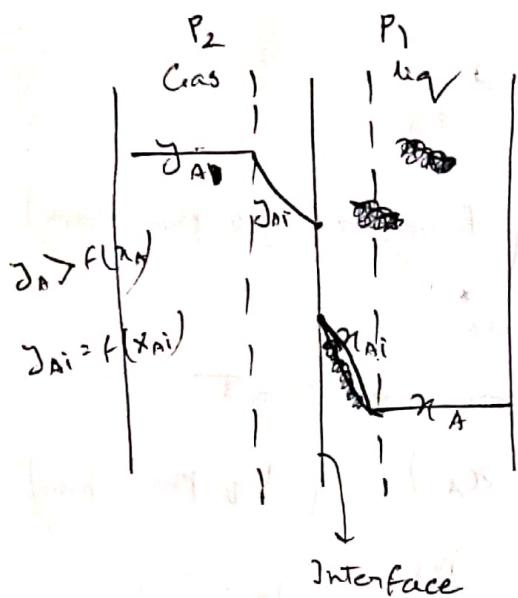


$$c_{A2b} > f(c_{A1b}) : \text{MT: Phase 2} \rightarrow \text{Phase 1}$$



$$J_A = f(x_A) \quad \text{At Equilibrium}$$

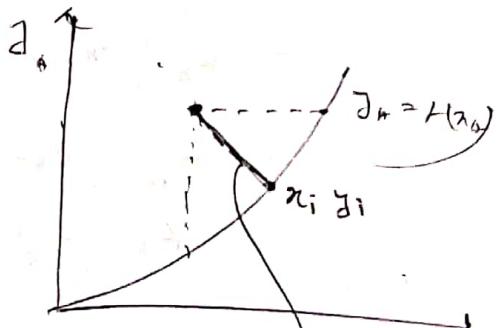
Whitman: (Hypothesis)  
Two Field Theory /  
Two Resistance Theory



① A is moving from bulk phase to interface  
→ film on both side  
Produces all the resistance to the MT.

② Establishing equilibrium at interface  
→ No. Resistance

l<sub>iq</sub> → g<sub>as</sub>: leaching  
gas → l<sub>iq</sub>: Adsorption

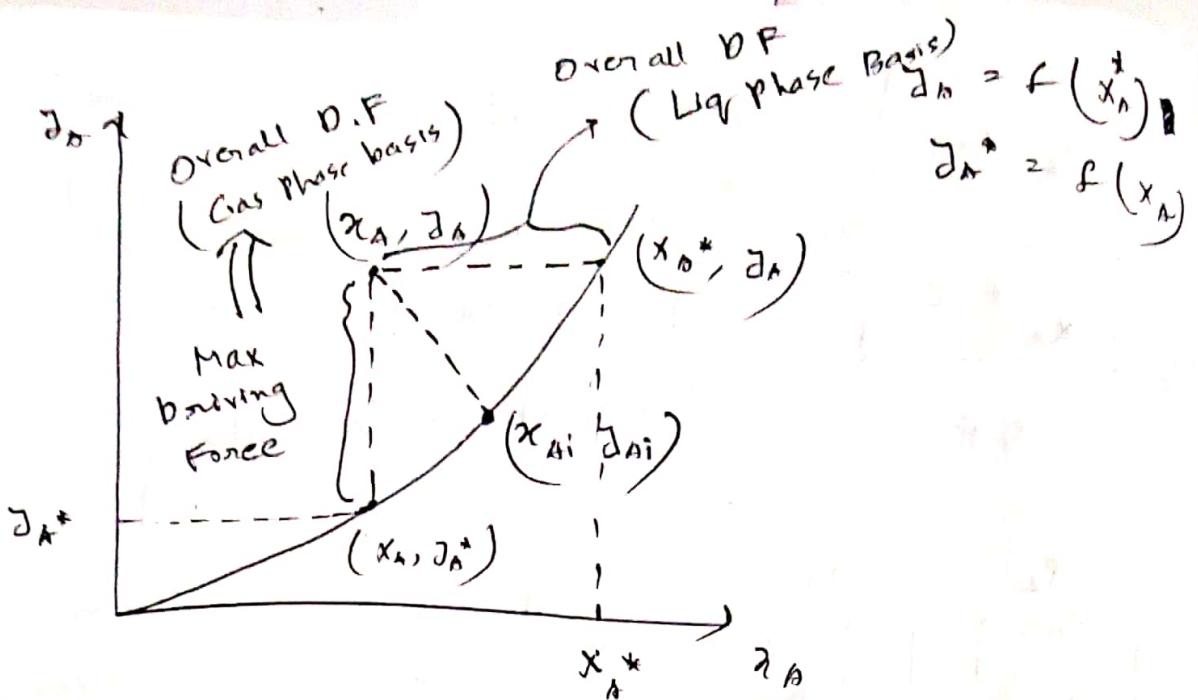


Gas  $N_A = k_d \Delta J_A = k_d (J_A - J_{Ai})$

Liquid  $N_A = k_n \Delta x_A = k_n (x_{Ai} - x_A)$

$$\frac{J_A - J_{Ai}}{x_A - x_{Ai}} = - \frac{k_n}{k_d}$$

Slope of the line



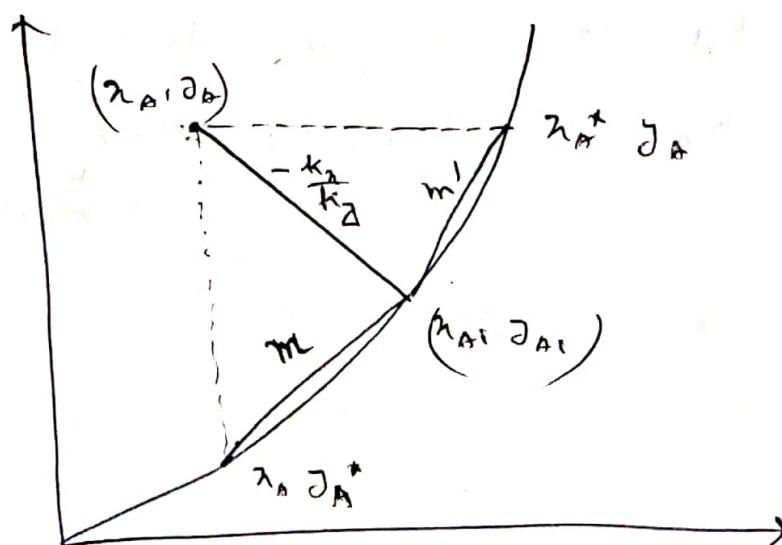
$N_A \propto$  (Overall Driving force)  $\rightarrow$  (gas phase basis)

$$N_A = k_g (D_A - D_A^*)$$

$K_D \equiv$  Overall Mass Transfer Co-efficient

$$N_A = K_D (x_{A,i}^* - x_A) \quad (\text{Liqr phase basis})$$

$$\begin{aligned} N_A &= k_g (D_A - D_{A,i}) & \frac{N_A}{k_g} &= D_A - D_{A,i} \\ &= k_n (x_{A,i} - x_A) & \frac{N_A}{k_n} &= x_{A,i} - x_A \\ &= K_D (D_A - D_A^*) & \frac{N_A}{K_D} &= D_A - D_A^* \\ &= K_n (x_A^* - x_A) & \frac{N_A}{K_n} &= x_A^* - x_A \end{aligned}$$



$$\begin{aligned}
 \frac{N_A}{K_g} &= \frac{\bar{J}_A - \bar{J}_{Ai} + \bar{J}_{Ai} - \bar{J}_A^*}{\bar{J}_A^*} \\
 &= \frac{\frac{N_A}{K_g}}{\bar{J}_A^*} + \frac{\bar{J}_{Ai} - \bar{J}_A^*}{\bar{J}_A^*} \left( \frac{\bar{x}_{Ai} - \bar{x}_A}{\bar{x}_{Ai} - \bar{x}_A^*} \right) \\
 &= \frac{N_A}{K_g} + m \frac{N_A}{K_n} \xrightarrow{\text{M.T resistance by gas phase to overall M.T resistance}}
 \end{aligned}$$

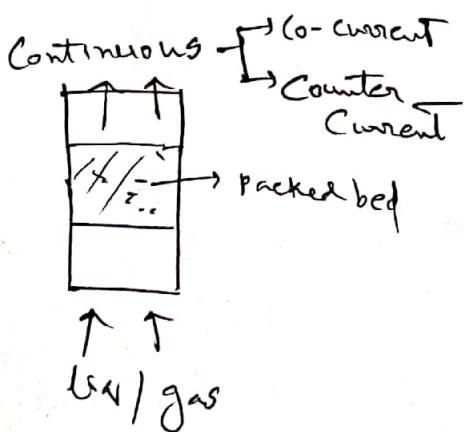
M.T resistance by gas phase to overall M.T resistance  
 M.T resistance by liquid phase to overall M.T resistance

Overall Mass Transfer Resistance (gas phase basis)

$$\begin{aligned}
 \frac{N_A}{K_n} &= \bar{x}_A^* - \bar{x}_{Ai} + \bar{x}_{Ai} - \bar{x}_A \\
 &= \left( \frac{\bar{x}_A^* - \bar{x}_{Ai}}{\bar{x}_A - \bar{x}_{Ai}} \right) (\bar{J}_A - \bar{J}_{Ai}) + \frac{N_A}{K_n} \\
 &= \frac{1}{m} \left( \frac{N_A}{K_g} \right) + \frac{N_A}{K_n}
 \end{aligned}$$

$$\boxed{\frac{1}{K_n} = \frac{1}{m} \frac{1}{K_g} + \frac{1}{K_n}}$$

### M.T. Equipments



Stagewise



$N_A \Rightarrow$  local MTC  $\Rightarrow$  Interfacial Concentration.

Overall MTC  $\Rightarrow$  Two basis  $\begin{matrix} \xrightarrow{\text{Gas phase}} \\ \xrightarrow{\text{Liq phase}} \end{matrix}$

Gas phase basis  $K_D, K_A, K_C$

Liq -  $K_x, K_{OL}$

$N_A =$  Overall MTC  $\times$  deviation from Equilibrium

Overall MT R / = Liq phase resistance

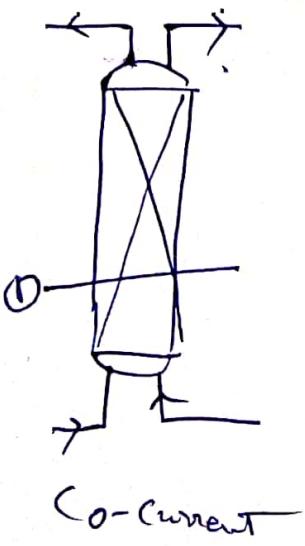
+ Gas phase resistance

$$\frac{1}{K_y} = \frac{1}{k_D} + \frac{m}{k_{n'}}$$

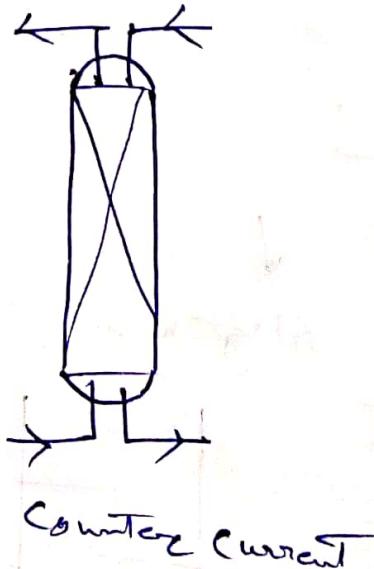
$$\frac{1}{K_A} = \frac{1}{k_x} + \frac{1}{m' k_y}$$

### MT Equipment

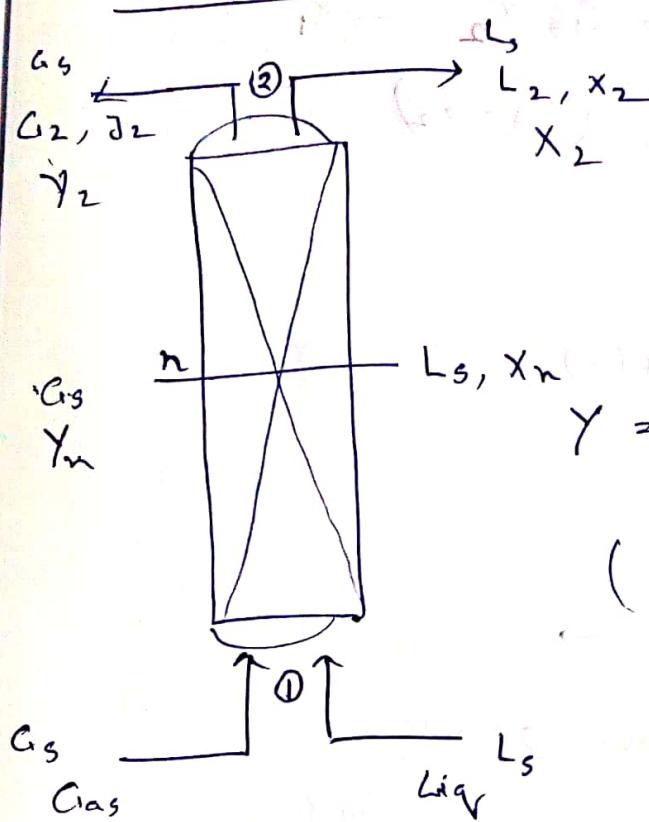
Continuous  
operation



Stage wise  
operation



## Co-current Continuous Operation



$G_s \geq$  All the moles  
Except solute which  
is being transferred

$$G_s = G(1-y)$$

$y = \frac{\text{Mole fraction of A}}{\text{Mole fraction of B}}$

(in a Binary Mixture)

$$y = \frac{J}{1-J}$$

$$G_s \text{ moles/time}$$

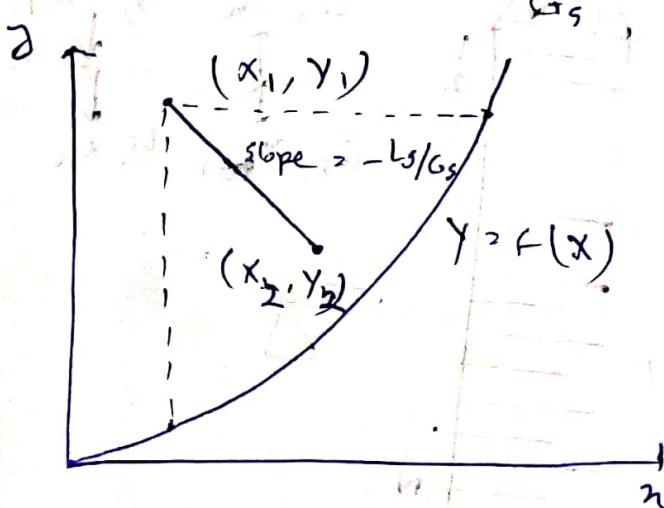
$$L_s \text{ moles/time}$$

$$J_1 \quad Y_1$$

$$x_1 \quad X_1$$

$$G_s y_1 + L_s x_1 - G_s y_n - L_s x_n = 0$$

$$y_1 - y_n = \pm \frac{L_s}{G_s} (x_1 - x_n) \quad \text{operating line equation}$$



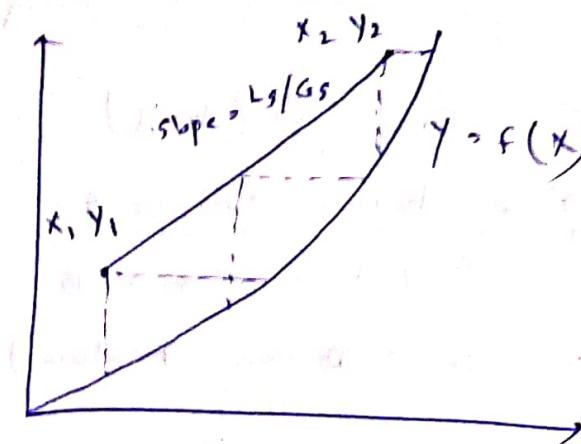
$$\Rightarrow y = f(x)$$

$$y_1 - y_2 = -\frac{L_s}{G_s} (x_1 - x_2)$$

For Counter Current

$$G_s Y_1 + L_s X_2 - G_s Y_2 - L_s X_1 = 0$$

$$Y_1 - Y_2 = \frac{L_s}{G_s} (X_1 - X_2)$$



Co - Current Continuous Operation

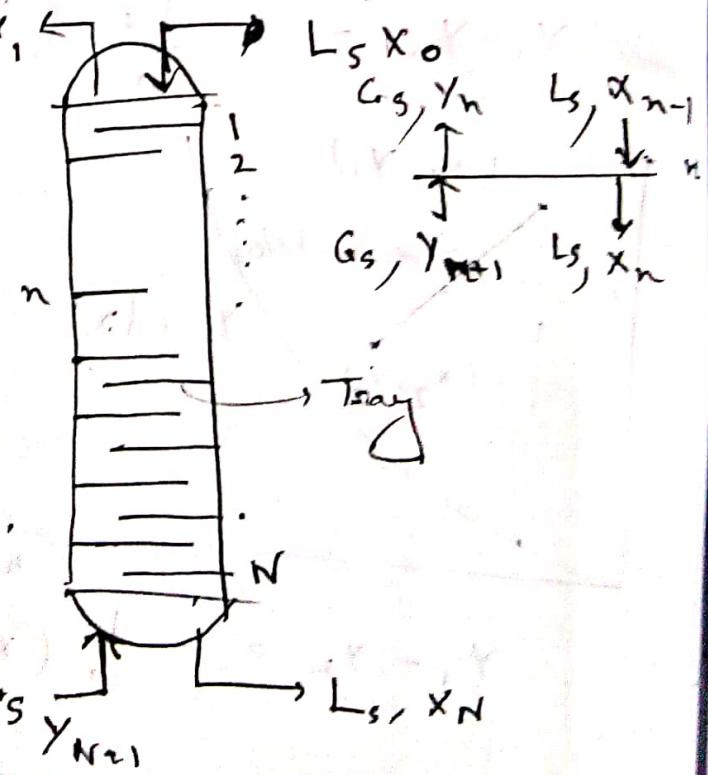
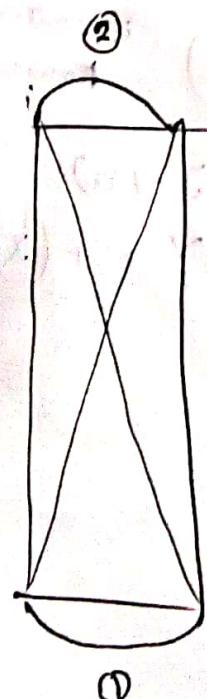
$$Y_n - Y_1 = \frac{L_s}{G_s} (x_n - x_1)$$

Countercurrent

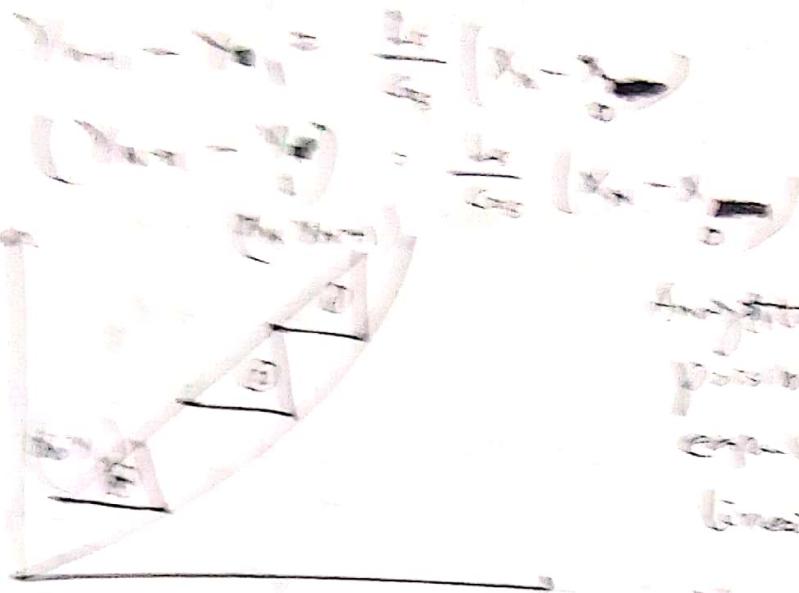
$$G_s Y_{n+1} + L_s X_{n-1} = G_s Y_n + L_s X_n$$

$$Y_{n+1} - Y_n = \frac{G_s}{L_s} (x_n - x_{n-1})$$

$$Y_n - Y_1 = \frac{L_s}{G_s} (x_n - x_1)$$



$$G_1 Y_1 + G_2 Y_2 = G_1 Y_1 + G_2 X_0$$



adjusted set of  
points  $\Rightarrow$  Eqn of  
equilibrium is  
linear.

Method  
of  
least  
squares

$$F \rightarrow Y_1, Y_2$$

$$\begin{aligned} Y_{n+1} - Y_n &= \frac{k}{G_2} \left( \frac{Y_n}{\alpha} - x_0 \right) \\ \text{or } \frac{Y_{n+1} - Y_n}{k} &= \frac{1}{G_2} x_0 \quad \Rightarrow \bar{Y}_{n+1} - \frac{k}{G_2} x_0 \\ &= \bar{Y}_n - \bar{k} x_0 \end{aligned}$$

$$Y_{n+1} - \bar{Y}_n - Y_1 = -\bar{k} x_0 = Y_1 - \bar{k} x_0$$

at  $n=0$

$$\therefore n=1$$

$$Y_2 = Y_{n+1}$$

$$Y_2 = Y_1$$

$$Y_{n+1} = \left( \frac{\alpha x_0 - Y_1}{1 - \bar{k}} \right) \bar{Y}_n + Y_1$$

$$\therefore Y_{n+1} = \frac{Y_1 - \bar{k} x_0}{1 - \bar{k}}$$

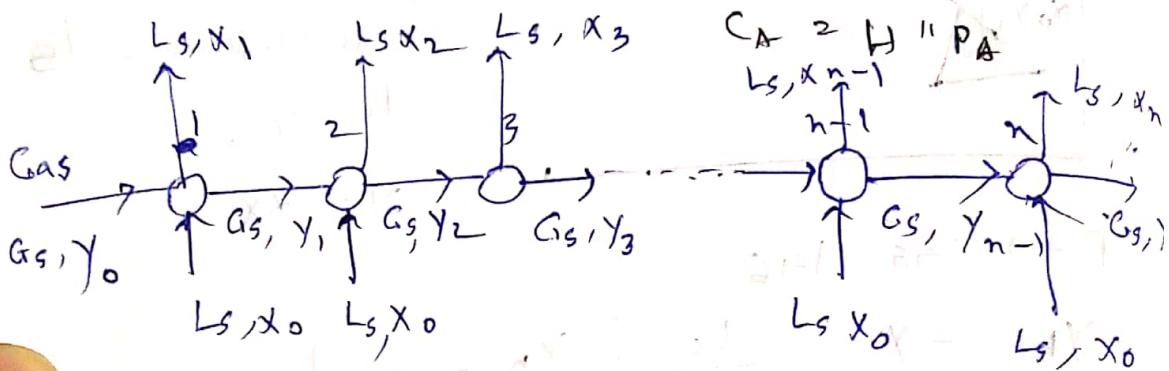
$$N = \frac{Y_0}{\bar{k}} \left( \left( \frac{Y_{n+1} - \bar{k} x_0}{Y_1 - \bar{k} x_0} \right) \left( 1 - \bar{k} \right) + \frac{1}{\bar{k}} \right)$$

number  
of

$$N = \log \left( \frac{x_0 - y_{n+1}/\alpha}{x_0 - y_n/\alpha} \right) \left( 1 - \frac{1}{\alpha} \right)^{-1} + \bar{\delta}$$

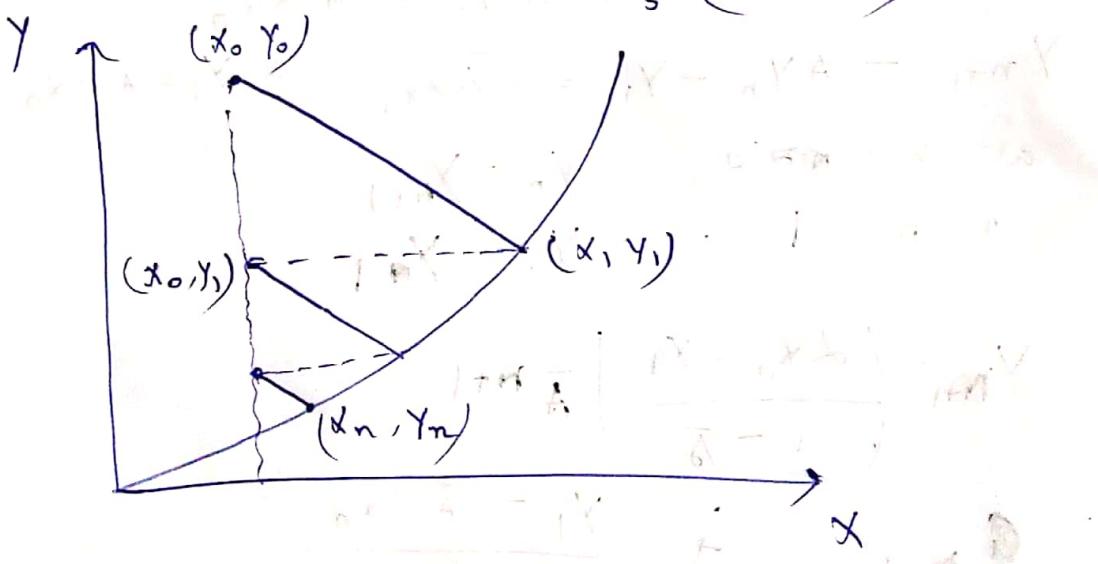
Dalton's Law, Raoult's law  $\bar{J}_A = m \bar{c}_A$

$$\begin{aligned} P_A &= H' X_A \\ &= H' C_A \end{aligned}$$



$$G_s y_{n+1} + L_s x_0 = G_s y_n + L_s x_n$$

$$y_{n+1} = y_n + \frac{L_s}{G_s} (x_n - x_0)$$

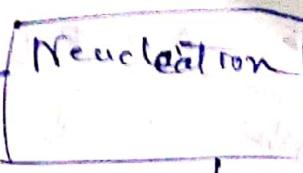


### 1. Crystallization:

Separation of solute from solution to solid phase.

Solution :

$$\boxed{\text{Supersaturated}} \quad \boxed{c > c^*}$$



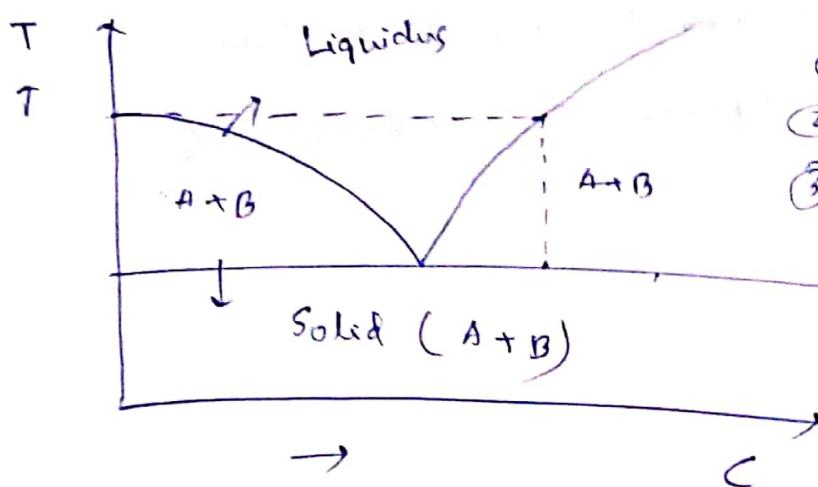
$$\bullet A \rightarrow A \\ \text{Cu} \rightarrow \text{Cs} \\ K = \frac{Cs}{Cs}$$

**Pat**  
CSD  
Crystal size  
Distribution

Growth of  
Crystals

Degree of Supersaturation

$$S = \frac{c - c^*}{c}$$



- ① Cooling
- ② Evaporation
- ③ Altering Equin. Curve

Nucleating Agent

Nucleation

homogeneous

heterogeneous

$$l = \text{size}$$

Nucleation Rate,

$$B_0 = \lim_{L \rightarrow 0} \frac{d}{dt} \left( \frac{N(L)}{\sqrt{V}} \right)$$

$$= \frac{d}{dt} \left( \frac{N(0)}{\sqrt{V}} \right)$$

$$B_0 \propto (K, M_T, S^n)$$

found by Experiments

Order of Crystallisation

$$n = (0, 5)$$

$$n = 2 \text{ (mostly)}$$

$$D \Rightarrow A = \rho_a L^2$$

Characteristic Length

$$V = \rho_a L^3$$

$$\text{Rate of Growth} \cdot \frac{dL}{dt} = G$$

$$G = K_2 S^m$$

$$B_0 = k_1 N_T S^n$$

Competing Process

McCabe  $\alpha L$  Law

$$G \neq f(L)$$

Growth rate doesn't depend on size

Final size depends on Residence Time