INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid-Autumn Semester Examination, 2016-2017

Subject: Advanced Fluid Dynamics *Subject No.*: CH 61011

Time: 2Hrs **No. of Students: 92** *Full Marks*: 30

Instructions:

1. All Questions are compulsory.
2. Clearly write your Name, Roll No., Subject Name, and Subject Number on the Answer Book.
3. **Also, all sub parts of each question MUST be together.**
4. Be Precise with your answers. Long, redundant answers can potentially fetch zero!
5. **No doubts will be clarified in the examination hall.**

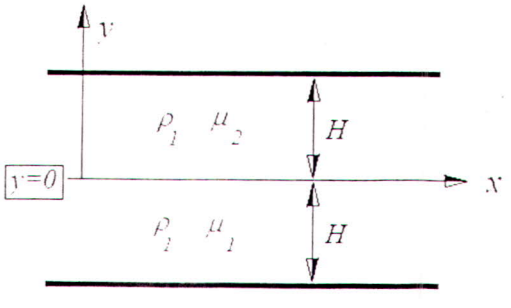
PART A

1. A high pressure drop flow is occurring in a capillary that is held perfectly horizontal. The liquid flow occurs in the direction of its central axis. No helical flow. This system is used to measure the applied stress-deformation profile of the flowing liquid and its viscosity. Other than the dimensions of the tube, the only tangibly measurable quantity of the flow is the volumetric flow rate.
   (a) Identify the stress tensor that will help in measurement of the viscosity of the fluid (1 mark)
   (b) write a deformation tensor with all its components that represents the flow, clearly indicating those that have finite positive or negative values and those that are zero (3 marks)
   (c) derive an expression between applied stress and deformation which includes the volumetric flow rate (7 marks)

2. A system under external forces undergo rotation, translation and self-deformation. In one case, the response of the material is viewed with a coordinate system that is stationary and in the other case the coordinate moves along the translational path of the material. Clearly mention, the type of derivative that one would use to describe the system with its mathematical expression. How is the response of the material differ in each case. **Do not answer in more than one paragraph** (3+1=4 marks)

PART B

1. Two viscous, incompressible, immiscible fluids of same density but different viscosities flow in separate layer between two infinitely wide parallel flat, horizontal plates as shown in the figure. The interface between the fluids is perfectly flat. The flow is driven by a constant favorable pressure gradient $\frac{dp}{dx}$. Derive expressions for the velocity profiles of the fluid layers. Assume both fluids to be Newtonian and the flow to be steady state.



You may want to use the following equations. However the simplifications and all assumptions must be clearly shown.

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}\right)+\rho g_y$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\left(\frac{\partial \tau_{zx}}{\partial x}+\frac{\partial \tau_{zy}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z}\right)+\rho g_z$$

(marks: 9)

2. Derive the Prandtle Boundary Layer Equations, stating all assumptions (marks 4)
3. What are the two constituent components of general τij (with justification)? (marks: 2)