B Tech Economics
(HS2001)

4

Consumption and Investment Spending

OVERVIEW

This is the first of a sequence of four chapters in Part 2 concerned with how the equilibrium level of income is determined in the simple Keynesian model. This and Chapter 5 are limited to a two-sector model, that is, one that includes only households and businesses. Chapter 6 adds government spending and taxing to produce a three-sector model, and Chapter 7 adds foreign spending to produce a four-sector model.

The simple Keynesian model assumes that the aggregate supply curve is perfectly elastic up to the full employment level of output. This means that the price level in an economy operating below full employment is determined entirely by the height of the aggregate supply curve. The explanation of what determines this position is deferred until Chapter 8; for present purposes, the level of the perfectly elastic portion of this curve is simply taken as given.

With an aggregate supply curve of this kind, the equilibrium level of income is determined solely by aggregate spending. Therefore, the basic question in this simple model is what determines aggregate spending. For the two-sector economy, this is the question of what determines the total of consumption and investment spending. Consumption spending and saving are explained by the level of income; the relationships between spending–saving and the level of income are shown by the consumption function and the saving function. This chapter develops the essentials of these concepts and works through the mechanics of the average and marginal propensities to consume and to save.

Once the apparatus of the consumption and saving functions are understood, it is possible to determine what the equilibrium level of income will be. This chapter presents only the simplest possible theory of income determination—a theory that assumes some given amount of investment spending. In other words, no theory of what determines investment spending is provided here. An aggregate spending function or curve is derived by adding this given amount of investment to the consumption function. Given the resultant aggregate spending curve, the equilibrium level of income is readily identified graphically. Alternatively, the equilibrium level of income is readily identified on another graph which plots the saving function and the given amount of investment spending. The analysis shows why any income level other than that so identified is necessarily a disequilibrium level.

The last part of the chapter examines the crucial distinction between planned and realized quantities in income theory. For the work that follows, it is essential to understand why realized saving and investment are always equal, whatever the level of income, and why planned saving and investment are equal only at the equilibrium level of income.

A basic proposition of Keynesian theory is that the equilibrium level of income and output depends on the economy's aggregate spending for output. If aggregate spending is not sufficient to call forth the level of output that requires the employment of all available workers for its production, unemployment results, and production of goods and services falls below its potential. If aggregate spending is just sufficient, full employment results, and production reaches its potential. If aggregate spending is excessive, inflation also results. However, any level of output—from that which calls for full employment of the labor force to that which imposes idleness on a large part of the labor force—is a possible equilibrium level. Given this wide range of possible equilibrium levels, the actual equilibrium level in any time period is determined by aggregate spending for that period.

In the simplest version of Keynesian theory, the level of output not only depends on aggregate spending, but varies proportionally with it. The output obtained by buyers for spending of, say, $500 billion would increase by 10 percent if total spending increased by 10 percent from $500 billion to $550 billion. This says that the increased spending would not cause any rise in the price level, because a 10 percent increase in spending must result in less than a 10 percent increase in output to the degree that the increased spending is absorbed in paying higher prices for the total of goods and services purchased. This proportional relationship between aggregate spending and output follows from the special kind of aggregate supply curve which Keynes suggested. In contrast to the one in Figure 3-2 (p. 57), this aggregate supply curve resembles the one in Figure 4-1. It has the special characteristic of being perfectly elastic up to the full employment level of output (Y_f). Once at the level of output that can be produced with full employment, the economy runs into a wall and the *AS* curve becomes vertical.

Such a special kind of *AS* curve seems reasonable enough when one recognizes the conditions under which economies were operating at the time that Keynes wrote the *General Theory*. Unemployment was at record high rates in the early thirties. In terms of Figure 4-1, economies were operating far below the output level at which the *AS* curve becomes vertical. Suppliers were

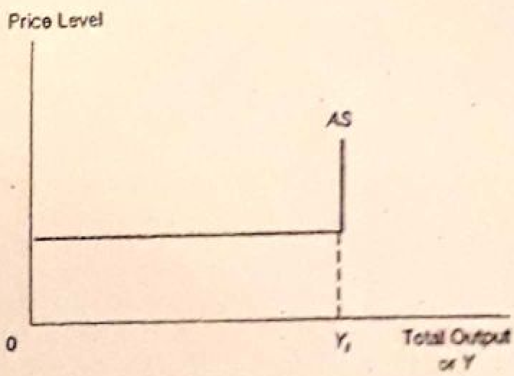

FIGURE 4-1
Aggregate Supply

more than willing to provide huge amounts of labor and other resources to buyers at existing prices. Under such conditions, it is not implausible to expect that increases in spending will exert no upward pressure on the prices of all kinds of goods whose output can be greatly expanded without running into higher per unit costs of production. With prices of goods remaining unchanged, increases in spending are matched by proportional increases in the amount of goods that purchasers get for their increase in spending.

In Chapters 4–7, we develop the simple Keynesian model which assumes that the aggregate supply curve resembles that in Figure 4-1 and further assumes that any change in the level at which the economy operates always leaves the actual level of output below the full employment level. As we have seen, this removes in one step the price level as a variable from the model. In effect, it also eliminates the aggregate supply curve from any active role in the analysis. For this reason, it is possible and customary to develop the simple Keynesian model with no explicit attention given to the supply side. Ordinarily, the supply curve of Figure 4-1 does not appear in the analysis. To answer the basic question of what determines the equilibrium level of output, all one needs is aggregate spending on output. With the price level known from the position of the aggregate supply curve and with that price level unchanging, the level of aggregate spending can be directly converted to a level of output; and any change in the level of aggregate spending can be directly converted to a proportional change in the level of output. Therefore, in this simple model, the problem of identifying the equilibrium level of aggregate output that will be produced is simply one of explaining aggregate spending.

In developing the simple Keynesian model of income and output determination to which this and the following three chapters are devoted, we proceed in the following steps. Chapters 4 and 5 will be limited to an economy in which there are only households and businesses. Government and the rest of the world are assumed nor... for the time being. Aggregate spending ... fore determined by consumption spending ... private domestic investment spending. To exp... what determines aggregate spending in any ... period, we must first develop the essentials of ... theory of consumption spending. This is done ... the section that follows. Then, by at first sim... assuming that investment spending is const... at some fixed dollar amount, we can proce... without further delay to find the level of aggreg... spending. In Chapter 5 we trace the process ... which shifts in the aggregate spending cu... occur in the two-sector economy and thereb... how changes in aggregate output occur in ... simple model of such an economy. Chapte... expands this model to three sectors by ...di... government, and Chapter 7 brings in the rest ... the world to provide the four-sector sim... model.

Because the areas of consumption a... investment spending receive minimum covera... in Part 2, but are nonetheless the areas that pr... ent the major theoretical questions in the fiel... aggregate spending, Part 4 is devoted to ... detailed examination of some of the theoreti... questions in the areas of consumption a... investment spending.

Consumption Spending and the Consumption Function

What determines the aggregate amount of go... purchased by consumers in any time period... the elementary Keynesian model, the real inc... of households basically provides the answe... rise in real income will lead households ... increase the amount of goods purchased ... vice versa. This does not deny that there ... many other less important determinants of ... consumer spending, and some of these will ... considered in Chapter 16. Here the an... is that the aggregate amount of consum...

purchased or the aggregate amount of real consumer spending is determined exclusively by the real income of consumers, that is, by real disposable personal income.

The Consumption Function

To consider how consumption expenditures are related to disposable income, we may begin by positing that consumption expenditures vary directly with disposable income. Second, we can be more specific and say something about how much such expenditures will vary as disposable income varies. Keynes did this in his "fundamental psychological law" which states that "men are disposed, as a rule and on the average, to increase their consumption as their income increases, but not by as much as the increase in their income."[1] In other words, as income increases, consumers will spend part but not all of the increase, choosing instead to save some part of it. Therefore, the total increase in income will be accounted for by the sum of the increase in consumption expenditures and the increase in personal saving, if we simplify by assuming that all consumer income goes into these two uses. Finally, can we be still more specific on the nature of this relationship? Although Keynes placed great confidence in the correctness of his "fundamental psychological law," he advanced with less confidence the argument that a smaller *proportion* of income will be consumed (or a larger *proportion* of income will be saved) as income increases. If this is true, not only will the absolute amount of saving increase with increases in income as indicated by the "fundamental psychological law," but the ratio of saving to income will become greater with increases in income. Keynes felt that this was to be expected, *as a rule*, because, despite the fact that "the satisfaction of the immediate primary needs of a man and his family is usually a stronger motive than the motives toward accumulation," the latter "acquire effective sway when a margin of comfort has been established."[2]

This relationship between consumption and income advanced by Keynes is employed in the simple theory of income determination to be developed here.[3] We will, in other words, proceed on the assumption that the absolute level of consumption varies directly with the level of income and that the fraction of income consumed varies inversely with the level of income.

Theoretical and Empirical Consumption Functions The relationship between consumotion and income that emerges from these particular assumptions is referred to as a theoretical consumption function. As a tool of theory, a consumption function is somewhat analogous to an ordinary market–demand function for a single commodity, such as that shown in Figure 3-1. Just as a theoretical demand curve usually implies that the quantity of a commodity that will be purchased varies inversely with its price, all other things (including income) being unchanged, so the theoretical consumption function here employed holds that aggregate consumption varies directly but not proportionally with consumer *income*, all other things (including prices) being unchanged. The theoretical consumption function that we shall draw, like the theoretical demand function of Figure 3-1, is not derived from actual statistical data. It is nothing more than an attempt to describe in general terms, on the basis of the previously stated assumptions, a typical functional relationship between two variables, all other things being unchanged. Actually, for this purpose, no explicit dollar amounts need be indicated; for example, they are not used in Figure 3-1.

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[1] John Maynard Keynes, *The General Theory of Employment, Interest, and Money*, Harcourt Brace Jovanovich, 1936, p. 96.

[2] *Ibid.*, p. 97.

[3] It may be noted here that the particular relationship between consumption and income advanced by Keynes is perhaps the first statement of what later came to be known as the absolute income hypothesis. This will be examined in Chapter 15 along with the relative income, permanent income, and life-cycle hypotheses.

A distinctly different type of consumption function, the historical or empirical consumption function, will be examined in Chapter 15. Here we may note that the simplest form of an empirical consumption function describes the statistics of income and consumption for each year over a period of years. Because these are recorded quantities, nothing can be assumed to have remained unchanged over the period of years involved. As a result, the actual level of consumption that accompanies the actual level of income for any year reflects every factor that influenced consumption expenditures during that year, not just the year's disposable income. In fact, the nonincome factors may be such as to cause the actual level of consumption in any one year to rise above that of the preceding year, despite a fall in the actual level of disposable income. On the basis of income alone, a decrease in consumption would be expected between these two years, but what otherwise would have been a decrease was more than offset by these other factors that made for a net increase. We will note actual cases like this in Chapter 15 when we look into empirical consumption functions. However, for our present purpose—namely to develop the simple theory of income determination—we need only the theoretical consumption function, which abstracts from all nonincome influences on consumption and posits a relationship between consumption and disposable income that satisfies certain assumptions.

The line labeled C in Part A of Figure 4-2 is one of many possible theoretical consumption functions that satisfy the specific assumptions stated previously. The other line is a 45° guideline; any point on this line is equidistant from the vertical and horizontal axes. For example, point K on the 45° line represents a value of income, or $Y = 160$, on the horizontal axis and an equal value of consumption plus saving on the vertical axis.

Because by definition any portion of disposable income that is not consumed must be saved, given the consumption function, the guideline, a[illegible]d the hypothetical amounts laid off [illegible] at a glance how [illegible] given level of disp[illegible] income between spending and saving[illegible] example at Y [illegible] 160 (point M), total inc[illegible] given by $MK = 160$. Because consumption [illegible] 140, saving must equal the balance of in[illegible] $MK - ML = LK$ or $160 - 140 = 20$.

The specific consumption function in F[illegible] 4-2 is drawn on the assumption that there is [illegible] level of income at which planned consump[illegible] exactly equal to income. This is referred to [illegible] "break even" level of income; here it occ[illegible] level 80, because the consumption functio[illegible] the guideline at this level. At any higher le[illegible] income, people collectively feel well enou[illegible] to save some part of their aggregate inc[illegible] Above the break-even level, therefore, the [illegible]sumption function lies below the guideline [illegible] the vertical distance between the two [illegible] equals the amount of saving for that lev[illegible] income. At any level of income below 80, pe[illegible] collectively spend more than their aggre[illegible] income. In this situation, the consumption [illegible]tion lies above the guideline and the vertica[illegible] tance between the two lines equals the am[illegible] of dissaving, or the excess of consumption [illegible] income, at that level of income. At the inc[illegible] level of 40, the consumption function is 10 a[illegible] the guideline and the excess of consum[illegible] over income is 10.

Average Propensity to Consume The a[illegible]age consumption-income relationship is def[illegible] by the ratio C/Y for different levels of Y. Fo[illegible] function in Figure 4-2, at Y of 40, we have [illegible] 50, so that $C/Y = 50/40$, or 1.25. At Y of 80 [illegible] have C of 80, so that $C/Y = 80/80$, or 1, the br[illegible] even ratio. At Y of 160, we have C of 140, so $C/Y = 140/160$, or 0.875. The C/Y ratio coul[illegible] computed for any other level of Y in similar f[illegible]ion. However, from what is given, it is appa[illegible] that the ratio of C to Y in this consumption fu[illegible]tion decreases steadily as income increa[illegible] and vice versa. In other words, C increases [illegible] than proportionally with increases in Y, and [illegible] versa. The C/Y ratio is one of two basic ratios

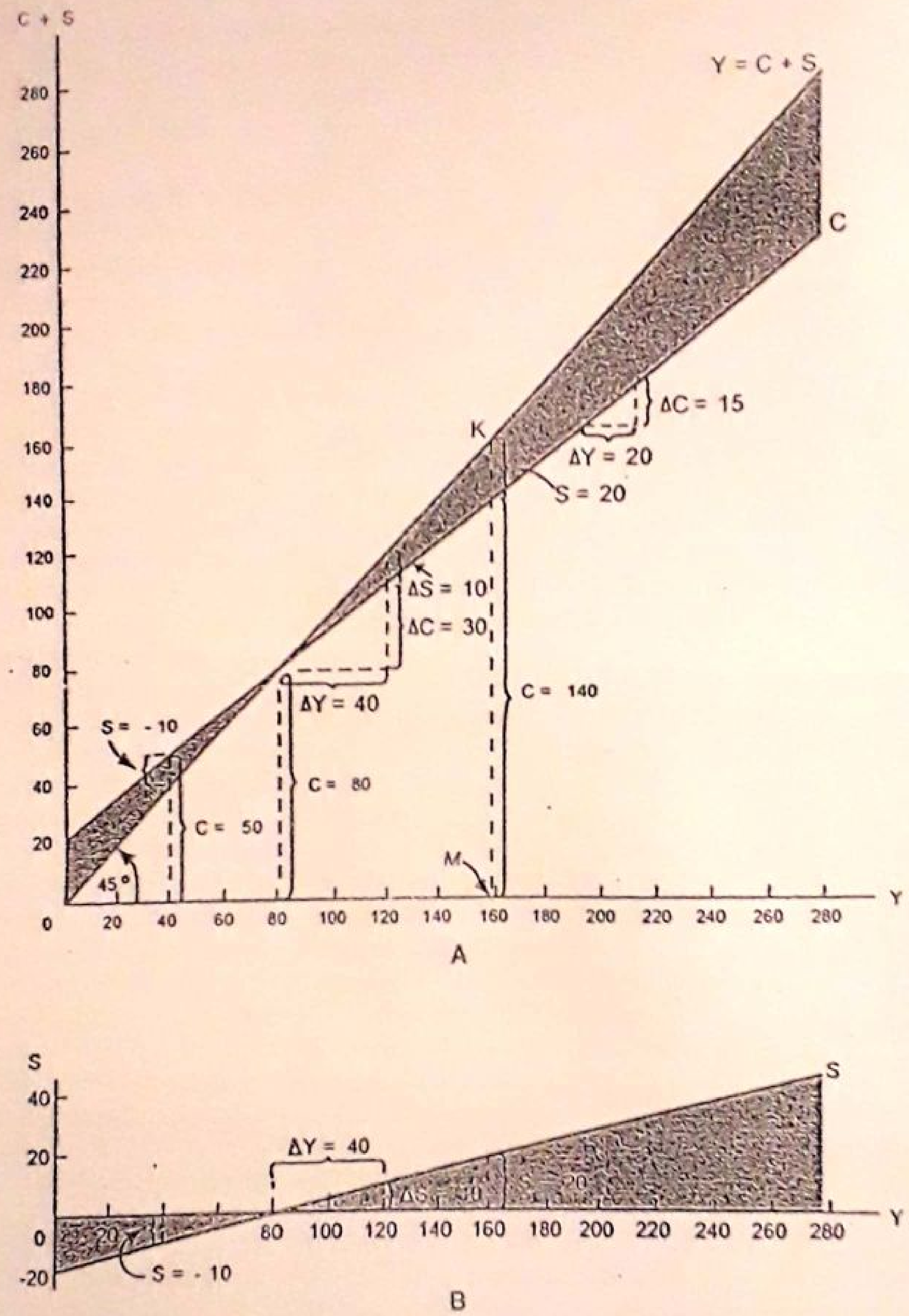

FIGURE 4-2
Consumption and Saving Functions

may be derived from the consumption function and is known as the average propensity to consume, or the APC.

Marginal Propensity to Consume If we know the APC at all levels of disposable income, we know how each level of disposable income will be divided between consumption and saving. Suppose, however, we also want to know how any given *change* in the level of income will be divided between a *change* in consumption and a *change* in saving. The APC will not give us the answer directly, but the slope of the consumption function will. To see this, take any two levels of disposable income and call the difference between them ΔY. Then determine the amount of consumption at each of these two levels and call the difference between these two amounts of consumption ΔC. For example, if we take Y of 200 and Y of 220, ΔY is 20. With Y of 200, C is 170; with Y of 220, C is 185; therefore, ΔC is 15. Expressed as a ratio, $\Delta C/\Delta Y = 15/20 = 3/4$. In Figure 4-2, for any selected change in income taken anywhere along the income axis, the same result will be found; for every change of 4 in Y there will be a change of 3 in C, or a constant ratio of $\Delta C/\Delta Y = 3/4$. Geometrically, it should be clear that this ratio equals the slope of the consumption function. $\Delta C/\Delta Y$ is the second basic ratio derived from the consumption function and is known as the marginal propensity to consume, or the MPC.

Note, however, that only if the consumption function is a straight line like the one in Figure 4-2 will the MPC be the same for any change in income. Any other straight-line consumption function with a slope different from that in Figure 4-2 will indicate an MPC that is larger or smaller than 3/4, but still the same for any change in income. The slope of the consumption function is the geometric representation of the MPC.[4]

Up to this point we have not considered evidence that would indicate whether a consumption function of the type given in Figure 2 is a realistic description of the way that people divide their income between consumption and saving at different levels of disposable income. We will turn to this question in Chapter 15, but for the moment all we have is the hypothesis that the consumption–income relationship exhibits certain properties that may be summarized as follows. The MPC is positive but less than 1, this being Keynes's "fundamental psychological law." The MPC is the same for any change in income, this following from the assumption that the consumption function is a straight line. The APC is infinity at a zero level of income and declines steadily as income rises but is always greater than the MPC, this following from the previous assumptions plus the assumption that consumption remains positive no matter how low the level of income may fall.[5]

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[4]If this is not apparent, keep the present straight-line consumption function anchored in its given position on the vertical axis and visually tilt it upward. The MPC will become greater than 3/4. Tilt it so that it becomes parallel with the guideline (which has a slope of 1) and the MPC becomes 1. Tilt it even more and the MPC exceeds 1. Tilt the consumption function downward from its given position, and the MPC will become less than 3/4. Tilt it so that it becomes parallel with the horizontal axis, and the MPC becomes zero.

[5]Geometrically, at any level of income, the APC is equal to the slope of a line from the origin to the point on the consumption function corresponding to that level of income. If such lines are drawn into Figure 4-2, it will be seen that the slopes of such lines start at infinity at zero income and fall steadily as income increases. Because the MPC equals the slope of the consumption function itself, any straight-line consumption function that cuts the vertical axis above the origin as in Figure 4-2 is a line whose constant slope is at all points less than the slope of a line from the origin to that point. In other words, for such a consumption function we have the property that $C/Y > \Delta C/\Delta Y$ at all levels of income. Note, however, that any straight-line consumption function that intersects the axes at the origin will be one for which $C/Y = \Delta C/\Delta Y$ at all levels of income. Finally, any straight-line consumption function that intersects the vertical axis below the origin will be one in which $C/Y < \Delta C/\Delta Y$ at all levels of income. For nonlinear consumption functions, the relationship between C/Y and $\Delta C/\Delta Y$ are more complex.

The Consumption Function—Equations Mathematically, the straight-line consumption function shown in Figure 4-2 may be described in terms of its intercept with the vertical axis and its slope with the aid of the simple equation of the straight line.[6] In the case of this consumption function, the intercept with the vertical axis indicates that C is 20 when Y is 0. For any level of income above 0, given the slope or the MPC as 3/4, C will be the 20 it would be at zero income *plus* 3/4 of the difference between zero income and any chosen level of income. This may be written in equation form as $C = 20 + \frac{3}{4}Y$. This theoretical consumption function may thus be thought of as the sum of two parts: an amount of consumption that is independent of the level of income, because it is the amount found even at zero income (20), and an amount of consumption that depends on the level of income, because it rises and falls by a constant fraction (3/4) of any rise or fall in income. The first part is commonly described as autonomous consumption, C_a, and the second part as induced consumption. The subscript is used here and elsewhere to designate a variable whose magnitude is autonomous or independent of the level of income.

This equation for the consumption function tells us everything that Figure 4-2 tells us. Just as we can find C for any level of Y by inspection in Figure 4-2, we can do the same by substituting any level of Y in the equation. Figure 4-2 shows that C is 80 when Y is 80, and the equation shows the same: $C = 20 + \frac{3}{4}(80) = 80$. Similarly, when Y is 160, the figure shows that C equals 140, and the equation shows the same: $C = 20 + \frac{3}{4}(160) = 140$. To find the APC or C/Y at any level of Y, we simply divide the original equation through by Y, or $C/Y = 20/Y + \frac{3}{4}Y/Y$, which is equal to $20/Y + 3/4$. When Y equals 80, $C/Y = 20/80 + 3/4 = 1$. When Y equals 160, $C/Y = 20/160 + 3/4 = 0.875$.

[6] This standard linear equation is $y = a + bx$, where b is the slope of its graph and a is its y intercept (the value of y at the point where $x = 0$; that is, the point at which the graph cuts the y axis).

Because we will not restrict ourselves later to the consumption function with the intercept and slope of the function shown in Figure 4-2, a general equation for the linear consumption function may be given here:

$$C = C_a + cY$$

In this equation C_a is autonomous consumption or the amount of consumption when Y equals 0 and the constant c is the slope of the function or the MPC. If we divide this equation through by Y, we derive the general equation for the APC:

$$APC = \frac{C}{Y} = \frac{C_a}{Y} + c$$

The hypothesis as to the way people divide their income between consumption and saving was summarized above as a set of properties. To satisfy these properties, it is required that C_a be positive and that c be positive but less than 1. This may be verified by noting that if C_a is negative, the proportion of income consumed increases as income increases; if c is greater than 1, the increase in consumption accompanying an increase in income exceeds the increase in income; and if c is negative, there is a decrease in consumption with an increase in income. All these possibilities conflict with the hypothesis.

The Saving Function

Part B of Figure 4-2 shows the saving function, which is the counterpart of the consumption function shown in Part A. In Part A the amount of saving at any level of income is the difference between the consumption function and the guideline (the shaded area). The saving function shown in Part B can therefore be directly derived from Part A.

When income is 80, we see in Part A that consumption is 80 and saving is 0; this is depicted in Part B by the intersection of the saving function with the horizontal axis at income of 80. When income is 40, consumption is 50, and saving is −10; the saving function lies 10 below the horizontal axis at income of 40. When income is 160, consumption is 140, and saving is 20, the saving function lies 20 above the horizontal axis at income of 160.

Average Propensity to Save The saving counterpart to the APC is the average propensity to save, or the APS. Whereas the APC is the ratio of C to Y, the APS is the ratio of S to Y. Because Y itself is devoted to either C or S, it follows that the two ratios, C/Y and S/Y, must add up to 1. Thus when Y is 40, $C/Y = 50/40$, or 1.25, and $S/Y = -10/40$, or -0.25. Similarly, when Y is 160, $C/Y = 140/160$, or 0.875, and $S/Y = 20/160$, or 0.125.

Marginal Propensity to Save There is also a saving counterpart to the MPC. If, instead of looking at the ratio of S to Y at any level of Y, we look at the ratio of the change in S to the change in Y for any change in Y, we have what is termed the **marginal propensity to save, or the MPS.** Given a change in Y, ΔY, then $\Delta S/\Delta Y$ is the ratio of the change in S to the change in Y, just as $\Delta C/\Delta Y$ is the ratio of the change in C to the change in Y. Because ΔY must be devoted to either ΔC or ΔS, the two ratios $\Delta C/\Delta Y$ and $\Delta S/\Delta Y$ must add up to 1.

If the MPC is positive but less than 1 and is the same for any change in income, then it follows by subtraction, because MPS = 1 − MPC, that the MPS must also be positive but less than 1 and that it must also be the same for any change in income. Furthermore if the APC decreases steadily as income rises, then the APS must increase steadily as income rises, because these two ratios also add up to 1 at all levels of income. Finally, if the APC is always greater than the MPC, it follows that the APS is always less than the MPS.[7]

The Saving Function—Equations As is the case for a straight-line consumption function, a straight line saving function can be described in terms of its vertical intercept and its slope with the aid of the equation of the straight line. The derivation of the equation for the saving function is analogous to that for the consumption function. When income is 0, saving is −20. For any level of income above 0, saving is −20 plus 1/4 of the difference between zero income and any chosen level of income. Therefore, $S = -20 + \frac{1}{4}Y$. To derive the equation for the APS, or S/Y, we simply divide through by Y, which gives us $S/Y = -20/Y + \frac{1}{4}Y/Y$, or $S/Y = -20/Y + 1/4$.

The general equation for the linear consumption function was given as $C = C_a + cY$; therefore, the general equation for the linear saving function may be given as

$$S = S_a + sY$$

S_a equals autonomous saving or the amount of saving at the theoretical zero level of income and s equals the marginal propensity to save.[8] If we divide this equation through by Y, we derive the general equation for the APS:

$$\text{APS} = \frac{S}{Y} = \frac{S_a}{Y} + s$$

The hypothesis about how people divide their incomes between consumption and saving was summarized previously with a set of properties. To satisfy these properties, the equation for the saving function, $S = S_a + sY$, requires that the value of S_a be negative and the value of s be positive but less than 1. This may be verified by noting that if S_a is positive, the proportion of income saved decreases as income increases; if s is greater than 1, the increase in saving

[7] Earlier, in describing the relationship between APC and MPC, we saw that for a consumption function of the type given in Figure 4-2, APC > MPC at all levels of income. Since APC + APS = 1 and MPC + MPS = 1, if APC > MPC at all levels of income, it follows that MPS > APS at all levels of income. For example, at $Y = 80$, APC = 1 and MPC = 3/4, whereas APS = 0 and MPS = 1/4. At Y of 160, APC = 7/8 and MPC = 3/4, whereas APS = 1/8 and MPS = 1/4.

[8] S_a equals $-C_a$ and s equals $1 - c$. This can be shown as follows: Because $Y = C + S$, $S = Y - C$. Substituting $C_a + cY$ for C, we have $S = Y - (C_a + cY)$. From this, $S = Y - C_a - cY$, or $S = -C_a + Y - cY$, or $S = -C_a + (1 - c)Y$. Because we have written $S = S_a + sY$, it follows that $S_a = -C_a$ and $s = 1 - c$ as was to be demonstrated.

accompanying any increase in income exceeds the increase in income, and if s is negative, there is a decrease in saving with an increase in income. All these possibilities conflict with the hypothesis.

Determination of the Equilibrium Level of Income and Output

The GNP identity for a two-sector economy was given in Chapter 2 as $C + S = \text{GNP} = C + I$ in which S includes business saving in the form of capital consumption allowances, and I includes business investment spending before deduction of capital consumption allowances. If both of these are measured net of capital consumption allowances, the final product so measured is net national product. Because there is no government in this economy, national income equals net national product.[9] If we further assume that all firms are noncorporate, there are no undistributed profits, and personal income equals national income.[10] Again, because there is no government, there can be no taxes, and all personal income becomes disposable personal income. In this economy, disposable personal income equals net national product; every dollar spent during the time period for either consumption or net investment produces a dollar of disposable personal income. Disposable personal income must be devoted either to personal consumption expenditures or to personal saving. Because disposable personal income equals net national product, personal saving (the amount of unconsumed disposable personal income) must then equal investment (the amount of unconsumed net national product).

If we measure the results for any time period in this economy, we have the following identities:

$$\text{Net National Product} = C + I$$
$$\text{Disposable Personal Income} = C + S$$

Here S and I are understood to be net amounts.

Because net national product and disposable personal income are identical in this economy in any time period, we may refer to them interchangeably, disposable personal income being identical with the value of output and the value of output being identical with disposable personal income. If we designate both by Y, we may write:

$$Y = C + I$$
$$Y = C + S$$

and

$$S = I$$

These are the fundamental accounting identities with which we will work in the two-sector economy. Note that these are the same identities developed in Chapter 2 (except for the substitution of net investment for gross investment and correspondingly net national product for gross national product). As identities, they are composed of the *realized* values for the variables for any time period. Therefore, by our accounting definitions, realized saving is identical with realized investment. However, as we will see, realized investment may be greater or less than the amount of investment *planned* by business people, if the amount of investment planned by business people differs from the amount of saving planned by income recipients. In what follows, we will use the terms *realized* and *planned* to make this distinction. Later in the chapter, we will turn to a more detailed examination of realized and planned investment.

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[9] Strictly, this also requires that business transfer payments be zero, an assumption we make here.

[10] In a two-sector economy in which all firms are noncorporate, personal income would exceed national income by the amount of interest paid by consumers. The easiest way to avoid the complications this factor would otherwise bring into the analysis is to assume that interest paid by consumers is zero.

Equilibrium Income and Output

What determines the economy's consumption and investment expenditures? According to the assumption we have worked with so far, disposable income is the sole determinant of consumption expenditures. What investment expenditures will be depends on factors yet to be considered. However, in order to get started, suppose that these factors are such that business people plan to spend a total of 20 (billion dollars) per time period for additions to plant and equipment and change in inventories. No matter how they may have arrived at these plans, we need only to assume for the time being that the plans are independent of the level of output. In other words, at all levels of output, planned investment expenditures are fixed at 20, or the investment function is simply $I = 20$.[11]

To derive a function or curve which will show aggregate expenditures or aggregate spending at each level of output, we must add together the consumption and investment functions. This is illustrated in Part A of Figure 4-3 by the curve labeled $C + I$. Here the consumption function is the same as that in Figure 4-2 which indicates the amount of planned consumption at each level of output. The investment function, which we have assumed is constant at 20 for all levels of output, is added to the consumption function. The resulting aggregate spending function is read as follows: If aggregate output and therefore aggregate real income were 120, aggregate spending would be 130 (110 + 20), or if aggregate output and therefore aggregate real income were 160, aggregate spending would be 160 (140 + 20).

... amount along the horizontal axis ... different possible level of aggregate out... actual amount of goods and services ... given time period could be any of these am... However, only if the amount of output happ... be 160 will aggregate spending just equal ... gate output.

To see why this is the case, suppo... business people believe that during a giv... period they can sell 160 in goods and th... produce this amount of goods during the p... With output of 160, disposable income ... 160, and with disposable income of 160, th... sumption function in Figure 4-3 indicate... consumers will spend 140. Adding to this ... that business people will spend, we have ... gate spending of 160 when output is 160 ... *ness people produced aggregate output in the expectation that sales of output wou... 160, and sales turned out to be exactly 1... the plans of both sellers and buyers wer... ized.* Consumers with income of 160 pur... the 140 of consumer goods they plan ... when income is 160, business people pur... the 20 of investment goods they plan to bu... sectors purchase the amounts intende... their purchases match exactly what bu... people intended to sell. In short, the am... goods purchased equals the amount pro... for sale—which is one way of stating the ... tion for equilibrium in the level of output.

In Part A of Figure 4-3, the aggregate s... ing function, $C + I$, intersects the guide... the equilibrium level of output. Aggrega...

---

[11] Although we are assuming for the time being that investment expenditures are not functionally related to any other variable, it is still convenient to use here the term "investment function" to parallel the term "consumption function." In Chapter 18, we will examine several theories of investment spending and will employ "investment functions" that express these specific theories.

[12] Not all goods, of course, are produced for sa... ...ers and to other firms. Parts of some firms' pr... may be intended as additions to their own inv... Such output may be viewed as if it were sales of g... firms to themselves. Thus, some part of estim... of 160 is made up of planned "sales" by firms ...

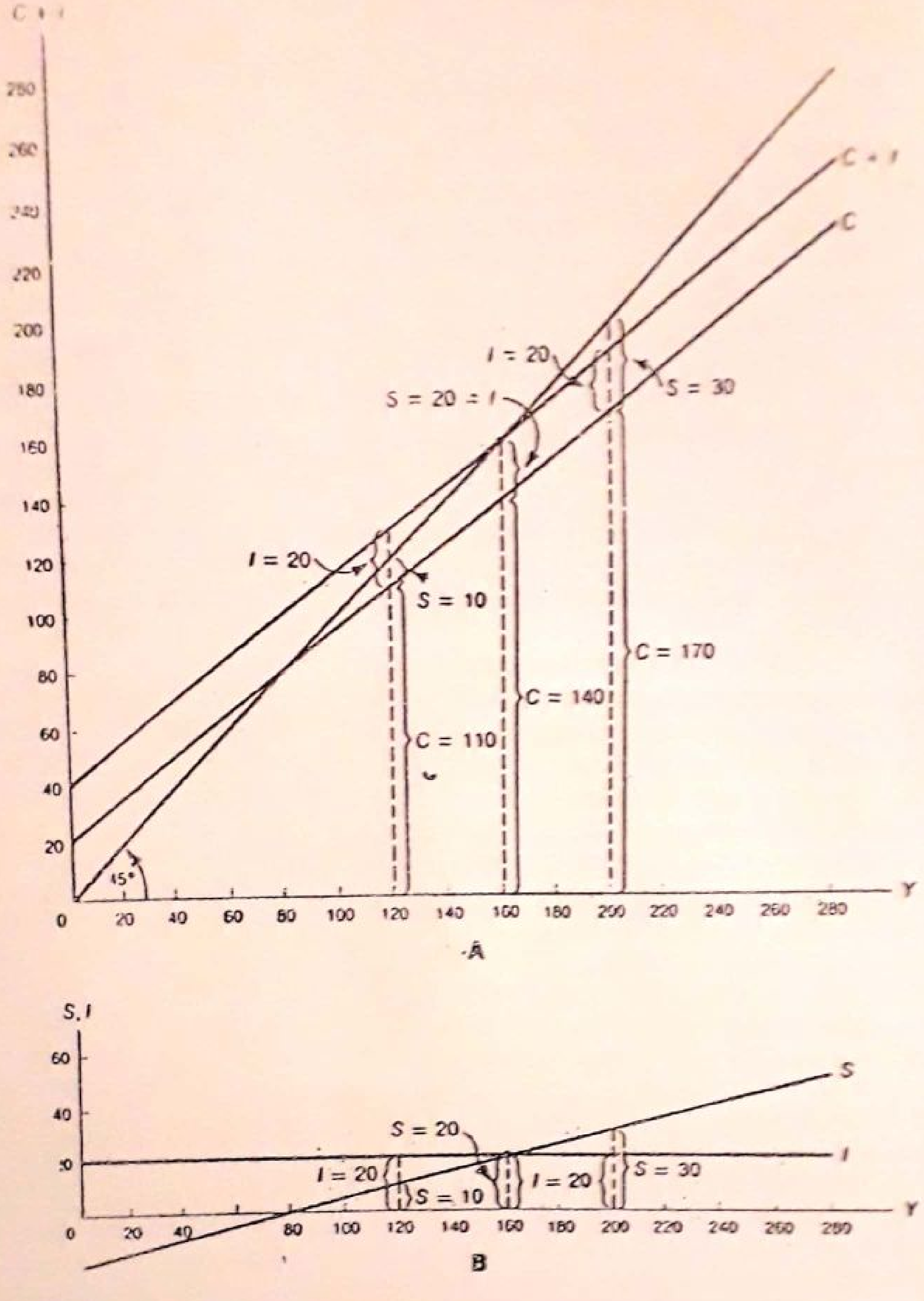

FIGURE 4-3
The Equilibrium Level of Income and Output

put of 160, measured on the horizontal axis is matched by an equal aggregate of spending, 160 (140 + 20), on the vertical axis. Because any point on the guideline is equidistant from both axes, and because the condition for equilibrium in the level of output is that $C + I$ be equal to Y, it follows that the equilibrium level of output must be that level of output at which the aggregate spending function intersects the guideline.

Equilibrium may also be defined as that level of output at which planned saving equals planned investment. In Part B of Figure 4-3, planned investment is shown by an investment curve horizontal to the output axis and 20 above that axis to conform with the assumption that investment is 20 at all levels of output. The construction of the saving curve was explained earlier. It will be seen that planned saving is 20 at the output and income level of 160. Because planned investment is also 20, the amount that business people choose to spend for investment goods exactly matches the amount of income of 160 that income receivers choose *not* to spend for consumer goods. In other words, given that the dollar amount of income generated during a time period is equal to the dollar amount of goods and services produced during that time period, it follows that aggregate spending will have to be equal to that aggregate output of goods and services for the period if each dollar of that period's income that is saved or not spent on consumption is matched by a dollar spent on investment. If planned saving and planned investment are equal, aggregate spending and aggregate output are also equal.

Equilibium Income and Output—Equations

Taking as our condition for equilibrium the equality between $C + I$ and Y, we may also determine this equilibrium income and output by solving the equation $Y = C + I$, in which C and I refer to *planned* consumption and *planned* investment. Solution of this equation identifies that level of output at which planned spending by people on consumption out of the income earned in producing that output will, when added to the planned spending on investment, be just sufficient to purchase the total amount of output actually produced. The amount income receivers plan to spend on consumption at any possible level of income is given for our economy by the equation $C = 20 + \frac{3}{4}Y$. The amount business people plan to spend for investment is assumed to be fixed at 20 and is given by the equation $I = 20$. This gives us the following three equations:

$$Y = C + I \quad [1]$$
$$C = 20 + \tfrac{3}{4}Y \quad [2]$$
$$I = 20 \quad [3]$$

Substituting [2] and [3] into [1] and solving for Y, we have

$$Y = 20 + \tfrac{3}{4}Y + 20$$

or

$$Y = 160$$

Alternatively, we find the equilibrium level of output to be that at which planned saving equals planned investment, or $S = I$. At this particular level of output, the amount of the income earned in producing that output *not* spent on *consumption* will be exactly offset by an amount spent by business people for *investment*. The amount income receivers plan to save at any possible level of income is given by the equation $S = -20 + \frac{1}{4}Y$. The amount business people plan to spend for investment is again assumed to be fixed at 20; that is, $I = 20$. This gives us the following three equations:

$$S = I \quad [1]$$
$$S = -20 + \tfrac{1}{4}Y \quad [2]$$
$$I = 20 \quad [3]$$

Substituting [2] and [3] into [1] and solving for Y, we have

$$-20 + \tfrac{1}{4}Y = 20$$
$$Y = 160$$

This confirms that only when Y equals 160 will aggregate spending equal aggregate output and planned saving equal planned investment. With C, I, and S all referring to planned amounts, we may summarize as follows:

Output:

$$Y = C + I$$
$$160 = 20 + \tfrac{3}{4}(160) + 20$$

Income:

$$Y = C + S;$$
$$160 = 20 + \tfrac{3}{4}(160) + [-20 + \tfrac{1}{4}(160)]$$
$$S = I$$
$$-20 + \tfrac{1}{4}(160) = 20$$

With the consumption function and investment function as given, we may further illustrate why 160 is the only equilibrium level of income and output by taking at random any other level of income and output and showing why it is necessarily a disequilibrium level.

Disequilibrium Income and Output

The actual level of output in any period is the result of the decisions of thousands of business people, and there is no reason to expect their collective decisions to result precisely in the equilibrium output. In our example, suppose these decisions result in output of 200. With an output of 200, disposable income will also be 200. The consumption function indicates that income receivers will now spend 170 on consumption. Adding planned investment of 20 to planned consumption of 170, we have aggregate spending of 190 when output and income are 200. Aggregate spending is clearly insufficient to buy the amount of goods business people expected to sell. What is required for equilibrium with output of 200 is aggregate spending of 200; what is found is aggregate spending of 190, or a deficiency of 10. ...

In Part A of Figure 4-3, this deficiency of aggregate spending is reflected in the difference in height between the aggregate spending function and the guideline. Instead of intersecting the guideline at 200, the aggregate spending function lines below the guideline at 200; the vertical distance between the two lines is the measure of the deficiency of aggregate spending. At this level of output, there is also necessarily disequilibrium between planned saving and planned investment; planned saving is 30 and planned investment is still the unvarying 20. Of the 200 of income earned in the course of producing 200 of output, the 30 that income receivers choose *not* to spend on consumer goods is greater than the 20 that business people choose to spend on investment goods. In Part B of Figure 4-3, the degree to which the saving function is positioned above the investment function at the income level of 200 is the measure of the deficiency of aggregate spending at this level of output.

The equations also indicate that income and output of 200 is a disequilibrium level. Here, rather than solving to find what level of Y is the equilibrium level, we assume a given level of Y and find whether it is the equilibrium level. In the following set of equations, we know that the equilibrium level of Y must satisfy Equation [2]. Equations [3] and [4] show that the equilibrium level could not be 200.

$$Y = C + I \qquad [1]$$
$$Y = 20 + \tfrac{3}{4}Y + 20 \qquad [2]$$
$$200 \neq 20 + \tfrac{3}{4}(200) + 20 \qquad [3]$$
$$200 \neq 170 + 20 \qquad [4]$$

In terms of saving and investment, the equilibrium level of Y is that which satisfies Equation [2] in the following set of equations. Could the equilibrium level be 200? Equations [3] and [4] show that it could not.

$$S = I \qquad [1]$$
$$-20 + \tfrac{1}{4}Y = 20 \qquad [2]$$
$$-20 + \tfrac{1}{4}(200) \neq 20 \qquad [3]$$
$$30 \neq 20 \qquad [4]$$

The combination of an aggregate output of 200 and aggregate spending of 190 can mean only one thing. Business as a whole finds its inventories of goods 10 greater than it had planned.[13] If output is maintained at the 200 level period after period and the aggregate spending function remains as given ($20 + \frac{3}{4}Y + 20$), business people will experience an unplanned or involuntary addition of 10 to inventories in each period. Sooner or later, in order to get inventories down to a lower, desired level, business people will lay off workers and cut back output. This in turn will cause income to fall as fast as output. Once output and income are reduced to 160, equilibrium will be restored; aggregate spending will equal aggregate output, and planned saving will equal planned investment.

To consider another disequilibrium situation, suppose that business people err in the opposite direction and estimate that they can sell only 120 in output. If output is 120, income will be 120; if income is 120, planned consumption will be 110. Assuming an unvarying 20 of planned investment, aggregate spending will be 110 + 20, or 130, 10 in excess of aggregate output of 120. In Part A of Figure 4-2, with output at 120, the aggregate spending function at 130 is 10 above the guideline, and its vertical distance above the guideline is a measure of the excess in aggregate spending, just as its vertical distance below the guideline at output of 200 was a measure of the deficiency in aggregate spending. In Part B of Figure 4-3, with output at 200, planned saving is 10, and planned investment is 20. The 10 that income receivers choose *not* to spend for consumer goods is less than the 20 that business people choose to spend for investment goods. The excess of planned investment over planned saving means that aggregate spending must be greater than aggregate output by the amount of this excess. The distance of the investment function above the saving function is the measure of the excess of aggregate spending at this level of output.

The equations also indicate that the income and output level of 120 is a disequilibrium level. The solution to Equation [2] would give us the equilibrium level, and Equations [3] and [4] show that this level could not be 120.

$$
\begin{aligned}
Y &= C + I \\
Y &= 20 + \tfrac{3}{4}Y + 20 \\
120 &\neq 20 + \tfrac{3}{4}(120) + 20 \\
120 &\neq 110 + 20
\end{aligned}
$$

Similarly, in terms of saving and investment, the solution to Equation [2] below would give the equilibrium level. Again Equations [3] and [4] show that this equilibrium level could not be 120.

$$
\begin{aligned}
S &= I \\
-20 + \tfrac{1}{4}Y &= 20 \\
-20 + \tfrac{1}{4}(120) &\neq 20 \\
10 &\neq 20
\end{aligned}
$$

In each period during which output remains at 120 and spending at 130, there must be an unplanned decrease of 10 in inventories held by business people.[14] Sooner or later, in order to stop this unplanned drain of inventories, business people will hire more workers and expand output. If they raise output to the 160 level, equilibrium will be restored.

[13] Planned investment of 20 may include a planned increase in inventories. Perhaps plans call for 15 of net investment in plant and equipment and 5 in additional inventories. The result above would thus become an addition of 15 to plant and equipment and 15 to inventories, the planned addition of 5 plus the unplanned addition of 10.

[14] If plans had called for net investment of 10 in plant and equipment and 10 in inventories per time period, the addition to inventories would be 0. If plans had called for net investment of 20 in plant and equipment and no addition to inventories per time period, the results would be net investment of 20 in plant and equipment and −10 (a decrease) in inventories. In both cases, realized investment, or unconsumed output would be (120 − 110).

Investment—Planned Versus Realized

We have examined three levels of income and output for our simple economy, of which one (160) was the equilibrium level and the other two (120 and 200) were disequilibrium levels. Because income and output are flows, these three levels must all be amounts corresponding to specific time periods. The national income accountant who seeks to measure income and output for these three time periods (the order of which here has no relevance) would summarize the data for the periods as follows:

Period	Realized $C + S = Y$	Realized $= C + I$	Realized $S = I$
A	140 + 20 = 160	= 140 + 20	20 = 20
B	170 + 30 = 200	= 170 + 30	30 = 30
C	110 + 10 = 120	= 110 + 10	10 = 10

Notice that the figures in the accountant's identities reveal nothing about the *planned* investment of business people and nothing about the equilibrium or disequilibrium of the economy at each of these income and output levels. The accountant's identities show only what income and output actually were, how the actual income was divided into realized consumption and realized saving, how the actual output was divided into realized consumption and realized investment, and, from these, the identity between realized saving and realized investment. Whether or not realized investment is equal to, less than, or greater than planned investment cannot be determined from the accountant's identities.

Unlike the accountant, the economist seeks to determine the level of output at which the economy will be in equilibrium. We assume that the economist knows what planned consumption spending and planned investment spending will be at each level of income and output and that he or she therefore knows the aggregate spending function ($C + I$) for the economy. In contrast to the accountant's identities, our economist uses a set of equations that show planned consumption and planned investment for the actual income and output levels in each of these time periods. Realized saving and realized investment figures for each period are repeated in Table 4-1 for easy reference.

Comparing the economist's equations with the accountant's identities, we find, as before, that only when income and output are 160 does planned investment of 20 correspond with realized investment. When income and output are 200, business people discover that, contrary to their plans for investment of 20, realized investment is 30 (consumed output is 170, and the remainder of output or the unconsumed portion of output, 30, equals realized investment). Similarly, when income and output are 120, business people discover, again contrary to their plans for investment of 20, that realized investment is 10 (consumed output is 110, and unconsumed output of 10 equals realized investment). Because realized investment may be described as the sum of planned and unplanned investment, and because planned investment is the constant 20,

TABLE 4-1

Period	Planned $C + S = Y$	Planned $= C + I$	Planned $S = I$	Realized $S = I$
A	140 + 20 = 160	= 140 + 20	20 = 20	20 = 20
B	170 + 30 = 200	> 170 + 20	30 > 20	30 = 30
C	110 + 10 = 120	< 110 + 20	10 < 20	10 = 10

The economist may also express this in equation form as follows

Period	Planned Invest-ment	+	Unplanned Investment	=	Realized Invest-ment	=	Realized Saving
A	20	+	0	=	20	=	20
B	20	+	10	=	30	=	30
C	20	+	− 10	=	10	=	10

In Figure 4-3, the amounts of unplanned investment may be identified as the difference between realized investment and planned investment at each of the three levels of income and output.[15] When unplanned investment is 10, the excess of realized investment over planned investment amounts to an unplanned increase in inventories of 10; some of the goods produced simply are not sold and remain in inventory, even though the producers do not want them to. When unplanned investment is − 10, on the other hand, the excess of planned investment over realized investment amounts to an unplanned decrease in inventories of 10.

In short, if there is any unplanned investment, planned investment will not equal realized investment, and the economy will therefore be at a disequilibrium level of output. However, the identity between realized saving and realized investment is just as consistent with positive or negative [illegible] The [illegible] can tell us nothing [illegible] equilibrium or disequilibrium [illegible] period and therefore nothing [illegible] level of income and output [illegible] ceeding time periods. To determine [illegible] equations that show the [illegible] saving and planned investment.

Therefore, we must work [illegible] saving and investment that at first [illegible] contradictory. In the one definition [illegible] investment (realized) are necessarily [illegible] any time period, in the other [illegible] and investment (planned) [illegible] equal and in fact are typically [illegible] period. In the introduction to national accounting in Chapter 2, we used only [illegible] definition of saving and investment [illegible] have to distinguish between planned [illegible] ized. From now on, however, they [illegible] distinguished. We must avoid saying th[illegible] and investment can be both equal and [illegible] We can, however, flatly and unambiguou[illegible] that realized saving and realized investm[illegible] only be equal, but that planned [illegible] planned investment can be unequal. *Th[illegible] the remainder of this book all reference[illegible] sumption, saving, and investment not sp[illegible] designated as realized quantities [illegible] understood to be planned quantities.*

[15]Realized investment at any level of output is the difference between the *C* curve and the guideline in Part A of Figure 4-3 and the difference between the horizontal axis and the *S* curve in Part B of Figure 4-3. Planned investment is the difference between the *C* curve and the *C* + *I* curve in Part A and the difference between the horizontal axis and the *I* curve in Part B.

[16]A discrepancy can appear only between planned investment and realized investment and not between planned saving and realized saving during a particular time period, because it has been [illegible] income receivers succeed in saving [illegible] they plan to save at each level of [illegible] this assumption and consider [illegible] which business people always [illegible] amount they plan to invest while [illegible] save the amount they plan to [illegible] income. Disequilibrium will then [illegible] planned saving being greater [illegible]

5 Shifts in the Aggregate Spending Function and the Multiplier

OVERVIEW

Chapter 4 provided an explanation of the determination of the equilibrium level of output for a simple two-sector model. This chapter provides an explanation of *changes* in the equilibrium level of output in the same two-sector model.

Any such change is shown to be the result of a shift in the aggregate spending function—that is, an upward or downward shift in the $C + I$ curve—and the dollar amount of the change in the income and output level is ordinarily a multiple of the dollar amount of this shift. The ratio of the change in income and output to the change in the level of the spending function is called the *multiplier*. The primary purpose of this chapter is the development of the basic multiplier mechanism as it is found in the simple two-sector model.

The chapter discusses separately a *temporary* shift and a *permanent* shift in the aggregate spending function. Detailed numerical illustrations are provided to show that although in both cases the change in income and output is a multiple of the amount of the shift in the aggregate spending function, only in the latter case is there a permanent change in the equilibrium level of income and output.

The multiplier is an important concept, and future chapters will expand it beyond the basic form in which it appears in this chapter. However, once the logic of the multiplier is understood in this basic form, students should encounter no great difficulty in following its elaboration in later chapters. To make its logic clear, the latter part of the chapter provides a discussion of the multiplier mechanism, in both verbal and equation form.

has raised the equilibrium level of income and output not merely by 10 but by 40, perhaps surprising at first glance, will be explained in the following pages.

A Temporary Shift in the Aggregate Spending Function

The new equilibrium level in Figure 5-1 was established as the aggregate spending function shifted from $C + I$ to the higher level of $C + I + \Delta I$. We have assumed that this is a permanent shift. However, to understand better the implications of a *permanent* shift, let us first examine the implications of a *temporary* shift in the function. Unlike the results shown in Figure 5-1, if the rise in investment is a temporary one, the rise in income and output will also be temporary. When investment subsequently drops back to its original level, income and output will also eventually drop back to their original level.

Changes in the level of income and output take place over time. To trace the process set into motion by a temporary increase in investment spending, we may split up the time interval required for the system to reestablish equilibrium into a series of shorter, numbered time periods. Let us assume that in Period 1 we have the original equilibrium position described in the previous chapter and shown in Figure 5-1: aggregate output is 160 and aggregate spending, the sum of consumption spending of 140 and investment spending of 20, is also 160. In Period 2, we upset this equilibrium by introducing an increase in investment spending of 10, or a rise in investment spending from 20 to 30. We assume that business people do not attempt to anticipate changes in the demand for their output, but instead follow a simple rule of thumb of producing in each period an output equal to their sales in the preceding period. Given this behavior by business people, output in Period 2 will be equal to sales of Period 1; that is, output will be 160. Aggregate spending in Period 2, however, is found to be 170, because in this period we have

[illegible] investment spending [illegible] the excess of aggr[illegible] spending of [illegible] absorbed in Period 2 [illegible] unplanned [illegible] of inventories.

In Period 3, business people expand ag[illegible] gate output to 170, the figure for total sal[illegible] Period 2. Output of 170 in Period 3 gene[illegible] disposable income of 170 during this pe[illegible] Given the consumption function, $C = 20 +$ [illegible] consumption spending in Period 3 will be 1[illegible] (7.5 greater than in Period 2). If investi[illegible] spending now drops back to its original lev[illegible] 20 following its temporary rise to 30 in Peri[illegible] we find in Period 3 consumption spending of 1[illegible] and investment spending of 20, or aggre[illegible] spending of 167.5. Because output in Peri[illegible] is 170, there is now a *deficiency* of aggre[illegible] spending of 2.5, which is reflected in an unplar[illegible] increase in inventories. In Period 4, busi[illegible] people reduce aggregate output to 167.5, total for sales in Period 3. This means a c[illegible] sponding decline in income to 167.5, and consumption function indicates that consu[illegible] tion in turn will be 145.6 in Period 4. Spendir[illegible] Period 4 is accordingly 145.6 plus 20 for inv[illegible] ment (an aggregate of 165.6). Because outp[illegible] Period 4 is 167.5, there is again a deficienc[illegible] spending—now equal to 1.9—which agai[illegible] reflected in an unplanned increase in invento[illegible] In Period 5, there is a further reduction in ou[illegible] and again a deficiency of spending but smaller than that of Period 4. In this way, the l[illegible] of output declines period by period until Pe[illegible] n, assumed to be the last in what is actually infinite number of periods. In Period n, outp[illegible] 160, income is correspondingly 160, consu[illegible] tion spending is 140, and investment spen[illegible] is 20—or aggregate spending of 160 is equ[illegible] aggregate output of 160. The system has retur[illegible] to the same equilibrium position from which started in Period 1.

Table 5-1 gives the period-by-period deta[illegible] the process just described. In this table, C [illegible] I indicate the values for consumption and inv[illegible] ment spending in the original equilibrium Period 1. The change in consumption spend[illegible]

TABLE 5-1

A Temporary (One-Period) Increase in Investment Spending

Period	C + I + ΔC + ΔI =	Aggregate Spending	> = <	Aggregate Output	= Y + ΔY	Realized Investment	Planned Investment
1	140 + 20 + 0.0 + 0 =	160.0	=	160.0	= 160 + 0.0	20.0	20
2	140 + 20 + 0.0 + 10 =	170.0	>	160.0	= 160 + 0.0	20.0	30
3	140 + 20 + 7.5 + 0 =	167.5	<	170.0	= 160 + 10.0	22.5	20
4	140 + 20 + 5.6 + 0 =	165.6	<	167.5	= 160 + 7.5	21.9	20
5	140 + 20 + 4.2 + 0 =	164.2	<	165.6	= 160 + 5.6	21.4	20
6	140 + 20 + 3.2 + 0 =	163.2	<	164.2	= 160 + 4.2	21.0	20
7	140 + 20 + 2.4 + 0 =	162.4	<	163.2	= 160 + 3.2	20.8	20
8	140 + 20 + 1.8 + 0 =	161.8	<	162.4	= 160 + 2.4	20.6	20
9	140 + 20 + 1.3 + 0 =	161.3	<	161.8	= 160 + 1.8	20.5	20
.	. . . .	.		.	. . .	.	
.	. . . .	.		.	. . .	.	
.	. . . .	.		.	. . .	.	
n	140 + 20 + 0.0 + 0 =	160.0	=	160.0	= 160 + 0.0	20.0	20
$n + 1$	140 + 20 + 0.0 + 0 =	160.0	=	160.0	= 160 + 0.0	20.0	20
	30.0 10				40.0		

between Period 1 and any following period is shown by ΔC, and the change in investment spending between Period 1 and any following period is shown by Δ*I*. Total consumption spending in any period is then given by C + ΔC, total investment spending by *I* + Δ*I*, and aggregate spending by their sum. In the same way, Y indicates aggregate output in Period 1, ΔY the change in output between Period 1 and any other period, and Y + ΔY the aggregate output for any period. The last two columns in the table show *realized* investment, which equals saving, and *planned* investment for each period. Planned investment for each period is the same as *I* + Δ*I*. Realized investment or saving (or unconsumed output) in any period is the difference between that period's output and its consumption, or (Y + ΔY) − (C + ΔC).

In any time period, aggregate spending may be equal to, greater than, or less than aggregate output. Or what is the same thing, in any time period, planned investment may be equal to, greater than, or less than realized investment.

The level of output is an equilibrium level in Period 1, because aggregate spending of 160 is just equal to aggregate output of 160 and planned investment of 20 is just equal to realized investment of 20. The equilibrium of Period 1 is upset in Period 2 by the rise in planned investment, which is a rise in aggregate spending. Equilibrium is not restored until Period *n*, because in all intervening periods aggregate spending is either greater or less than aggregate output or—what is the same thing—planned investment is either greater or less than realized investment.

Disequilibrium occurs through all these periods, despite the fact that the cause of disequilibrium—the rise in investment spending—is limited to Period 2 alone. This one-period rise in investment spending, however, produces the changing level of output in all these later periods by initiating a series of changes in consumption spending starting in Period 3. Thus, ΔC of 7.5 in Period 3 results from ΔY of 10 in Period 3, which results from Δ*I* of 10 in Period 2. Similarly, ΔC of 5.6 in Period 4 results from ΔY of 7.5 in Period 4.

which results from ΔC of 7.5 in Period 3, which results from ΔY of 10 in Period 3, which results from ΔI of 10 in Period 2. In other words, the ΔI of Period 2 initiates a process in which ΔC in each period is 3/4 (the MPC) of ΔY in that period. Furthermore, given that ΔC is equal to 3/4 of ΔY of that period and that ΔY of each period is equal to $\Delta C + \Delta I$ of the preceding period, the fact that the increase in investment (ΔI) is limited to Period 2 alone means that ΔY and ΔC become smaller each period until eventually, in Period n, ΔC becomes zero.[2] Because ΔI is also zero in Period n, ΔY becomes zero, and the level of income and output is back to the equilibrium found in Period 1, or 160.[3]

Although Table 5-1 shows that the equilibrium level of output established in Period n is the same as that of Period 1, it is important to note that the *cumulative* addition to income and output over the time interval in which the process works itself out is, in the present case, four times the size of the initiating increase in investment spending in Period 2. This cumulative addition to income and output is 40, as shown at the bottom of the column headed ΔY in Table 5-1. The 40 is composed of cumulative additions of 30 to consumption and 10 to investment, as shown at the bottom of the columns headed ΔC and ΔI in the table. Therefore, although the one-period injection of investment spending does not lift income and output to a permanently higher level, the cumulative effect in the present case is a flow of income and output four times the amount of the one-period injection of investment spending.

A Permanent Shift in the Aggregate Spending Function

If the aggregate spending function shifts up and remains at the new higher level period after period, the original equilibrium level of income and output will be replaced by a new equilibrium level. This is the result shown in Figure 5-1.

To describe the process by which the system moves to a higher equilibrium level of output as a result of such a sustained increase in aggregate spending requires only that we extend the description of the process for a temporary increase in aggregate spending. In Table 5-1, ΔI of Period 2 called forth ΔY of 10 in Period 3. Because we then assumed that investment spending returned to its original level in Period 3, ΔY of Period 4 (equal only to ΔC of 7.5 of Period 3) dropped below ΔY of Period 3, which was equal to ΔI of 10 of Period 2. In the present case, however, with a permanent rise in investment spending of 10, we have ΔI of 10 as well as ΔC of 7.5 in Period 3, so that ΔY of Period 4 (equal to ΔC of 7.5 plus ΔI of 10) is 17.5, greater than ΔY of 10 in Period 3. From ΔY of 17.5 in Period 4, we get ΔC of 13.1 in Period 4, so ΔY in Period 5 is 23.1 (the sum of ΔC of 13.1 and the constant ΔI of 10). With investment sustained at the new higher level, ΔY in each period continues to rise above ΔY of the preceding period until a new equilibrium is established with ΔY of 40.

Table 5-2 records the period-by-period details of this process. The column headings are the same as those in Table 5-1. ΔI starts at zero in Period 1, becomes 10 in Period 2, and re...

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[2] For a one-period increase in investment spending of \$1, the *differences* between each period's aggregate spending and spending in the original period are given by the series $1, c, c^2, c^3, c^4, \ldots, c_n$. Because c is less than 1, the differences become smaller and smaller. After the passage of an infinite number of time periods represented by n, c_n becomes infinitely small, so that income returns to its original equilibrium level.

[3] Actually, the cumulative addition to investment spending does not occur in Period 2 as is suggested by the column headed ΔI. An increase in investment of 10 was planned for Period 2, but the realized increase in investment was 0. As shown by the next to last column, realized investment in Period 2 is 20, the same as in Period 1. The *realized* increases in investment may be identified in this column as the differences between the indicated values and 20—that is, 2.5 in Period 3, 1.9 in Period 4, and so forth. The sum of these changes will total 10, equal to the increase in planned investment of 10 shown in Period 2.

TABLE 5-2

A Permanent Increase in Investment Spending

Period	$C + I + \Delta C + \Delta I =$	Aggregate Spending	$\gtreqless$	Aggregate Output	$= Y + \Delta Y$	Realized Investment	Planned Investment
1	140 + 20 + 0.0 + 0 =	160.0	=	160.0	= 160 + 0.0	20.0	20
2	140 + 20 + 0.0 + 10 =	170.0	>	160.0	= 160 + 0.0	20.0	30
3	140 + 20 + 7.5 + 10 =	177.5	>	170.0	= 160 + 10.0	22.5	30
4	140 + 20 + 13.1 + 10 =	183.1	>	177.5	= 160 + 17.5	24.4	30
5	140 + 20 + 17.3 + 10 =	187.3	>	183.1	= 160 + 23.1	25.8	30
6	140 + 20 + 20.5 + 10 =	190.5	>	187.3	= 160 + 27.3	26.8	30
7	140 + 20 + 22.9 + 10 =	192.9	>	190.5	= 160 + 30.5	27.6	30
8	140 + 20 + 24.7 + 10 =	194.7	>	192.9	= 160 + 32.9	28.2	30
9	140 + 20 + 26.0 + 10 =	196.0	>	194.7	= 160 + 34.7	28.7	30
⋮	⋮	⋮			⋮	⋮	⋮
n	140 + 20 + 30.0 + 10 =	200.0	=	200.0	= 160 + 40.0	30.0	30
$n + 1$	140 + 20 + 30.0 + 10 =	200.0	=	200.0	= 160 + 40.0	30.0	30

10 in each succeeding period. As before, ΔY for any period is the sum of ΔI plus ΔC for the preceding period. In comparing the value of ΔY period by period, note that as ΔY becomes larger and larger, the change in ΔY becomes progressively smaller and in Period n becomes zero. In this period, ΔY stabilizes at 40, and the sum of $Y + \Delta Y$ stabilizes at 200. With aggregate output at 200 income is 200, and with income at 200, the consumption function ($C = 20 + \frac{3}{4}Y$) indicates consumption spending of 170. Investment spending is given at 30, so aggregate spending in Period n is 200 and equal to aggregate output. Similarly, as shown in the last two columns of Table 5-2, in Period n realized investment reaches 30 and is equal to planned investment of 30. Viewed in either way, in Period n a new equilibrium is established.

The same period-by-period detail recorded in Table 5-2 may be seen graphically in Figure 5-2. This graph shows only the change in Y and in $C + I$ of 40 between the original equilibrium and the new equilibrium; it does not show the various possible absolute amounts of Y or $C + I$ as is done in Figure 5-1. Specifically, in terms of that earlier figure, Figure 5-2 shows in greatly enlarged form only the small portion of the 45° line and the $C + I + \Delta I$ line running from 160 to 200—from the original equilibrium to the new equilibrium—on both axes.

Paralleling the description of Table 5-2, the initial equilibrium is upset by ΔI of 10 in Period 2 (line segment AB). Because output responds to increased spending with a one-period lag, Y rises by 10 in Period 3 (line segment BD). In Period 3, ΔY of 10 calls forth ΔC of 7.5 (line segment DE). This increase in spending leads to an increase in output of equal amount or ΔY of 7.5 (line segment EF). It is important to recognize here that ΔY is *now* 17.5 above the *original* equilibrium level because it first went up by 10 (B to D) and *now* by another 7.5 (E to F). That ΔY of 7.5 calls forth ΔC of 5.6 (line segment FG). Here ΔC is *now* 13.1 above the original amount because it first went up by 7.5 (D to E) and *now* by another 5.6 (F to G). The process moves ahead in this way period by period until the new equilibrium is established with ΔY of 40, as shown at the top of the diagram and with ΔI of 10 and ΔC of 30 as shown on the left side of the diagram.

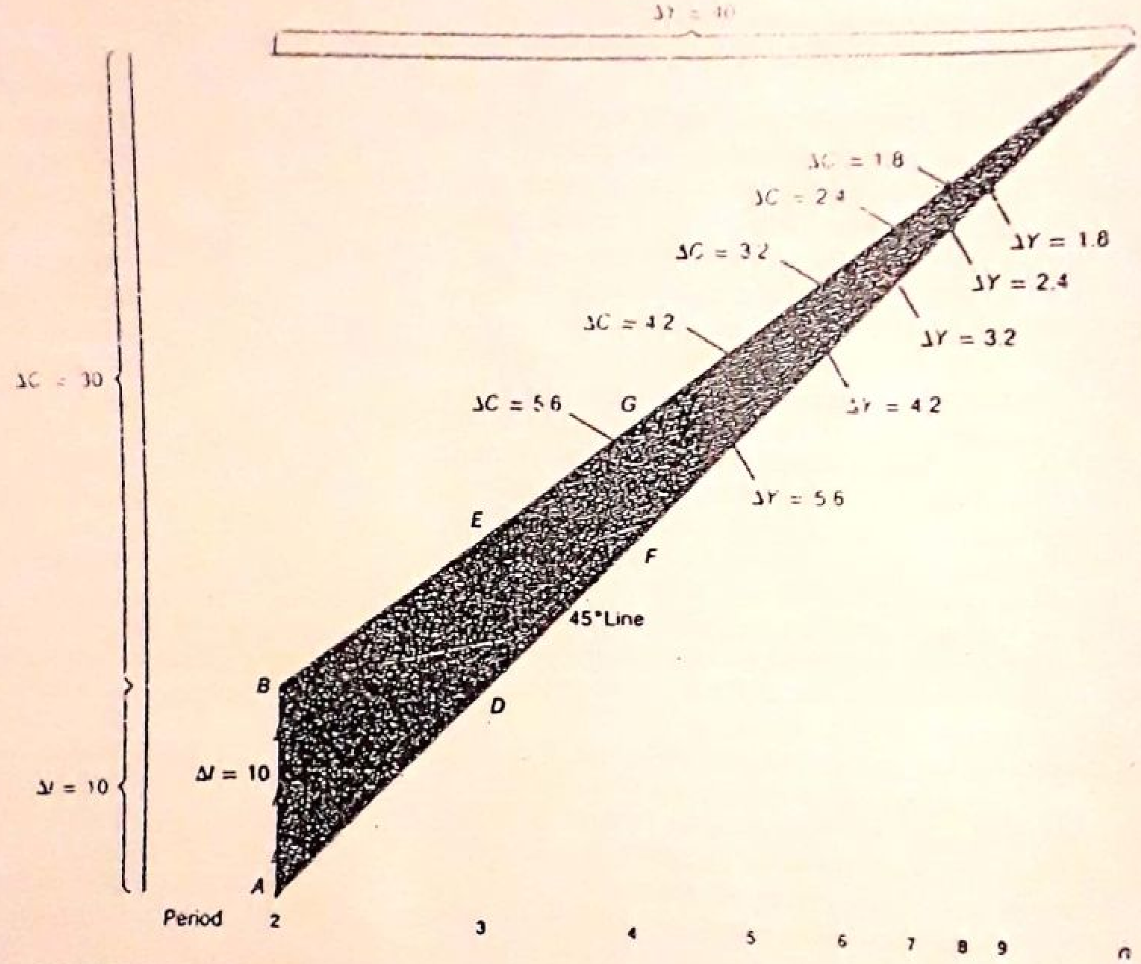

FIGURE 5-2
The Period-by-Period Changes in Income Resulting from a Change in Investment

The Multiplier—A Shift in the Aggregate Spending Function

In terms of Figure 5-1 and Table 5-2, a permanent upward shift in the aggregate spending function results in a movement of income and output to new equilibrium level that is higher than the original equilibrium level by some multiple of the upward shift in the aggregate spending function. The value of this multiple is known as the *multiplier* and represents the number by which the shift in the aggregate spending function must be multiplied to determine the change in the level income and output required to establish a new equilibrium.[4] Under present assumptions, this multiple is 4.

Why do Figure 5-1 and Table 5-2 show a multiplier of 4 and not some other? The reason is that income receivers choose to spend 3/4 of any change in income (MPC = 3/4) on consumption or that they choose to save, or not spend on consumption, 1/4 of any change in income (MPS = 1/4). With MPS = 1/4, only when income and output have risen by 40 will income receivers devote an additional 10 of their higher income to saving. Then $\Delta S = \Delta I$ 10 = 10, and a new equilibrium level of income and output is established.

For any given shift in the aggregate spending function, the change in income required to reestablish equilibrium is entirely dependent on the value of the MPC or the MPS. For example, still assuming that $\Delta I = 10$, we can simply determine the new equilibrium if MPC = 4/5 and MPS = 1/5. Instead of ΔY of 40 as before, we will now have ΔY of 50, because it is only when ΔY is 50 that ΔS will be 10 and therefore equal to ΔI of 10. Accordingly, until Y has risen by 50 (until ΔY equals 50), aggregate spending will exceed aggregate output and investment will exceed saving, forcing a further rise in income and output. By the same reasoning, if the MPC were 2/3 and the MPS 1/3, the rise in income would be 30.[5]

These and various other combinations of MPC and MPS are all possible and plausible. Although many combinations are not plausible, a particular pair of these combinations can help clarify the multiplier mechanism. One of these is MPC = 0 and MPS = 1. The rise in Y necessary to reestablish equilibrium in this case will be exactly equal to the permanent rise in I. If ΔI in Period 2 and in each subsequent period is 10, ΔY in Period 3 and in each subsequent period will be 10 also, because, if MPS = 1, ΔS will be 10 as soon as ΔY is 10. Therefore, $\Delta S = \Delta I$ in Period 3, and the new equilibrium level is immediately established at a level of income and output exactly 10 above the original level. The increase in investment does not lead to an increase in income larger than itself, because with MPS = 1, the rise in income of 10 in Period 3 does not lead to a rise in consumption spending. Instead, the income receivers choose to devote all the increase in income of Period 3 to saving (MPS = 1). There being no induced rise in consumption spending in Period 3, ΔY of Period 4 and each subsequent period is simply equal to ΔI of 10 for each such period. Unlike the situation in Table 5-2, in which only a part of the enlarged income stream of Period 3 was diverted into saving, here all ΔY of Period 3 leaks out of the spending stream in Period 3, and the expansion process ends as quickly as it began. In a formal sense, the value of the multiplier is 1, but this is a far cry from the earlier results in which the multiplier was 4 (MPC = 3/4 and MPS = 1/4).

The other combination is MPC = 1 and MPS = 0. Starting off as before with ΔI of 10 in Period 2 (and in each subsequent period), none of the

[4] Although it was made famous by the role it plays in Keynes' *General Theory*, the term was coined by another British economist, R.F. Kahn. Kahn's multiplier was an *employment* multiplier, measuring the ratio of the increment of total employment associated with a given increment of employment in the capital goods industries. Keynes' multiplier is an *investment* multiplier, the ratio of the increment to total income associated with a given increment in investment. (See John Maynard Keynes, *The General Theory of Employment, Interest, and Money*, Harcourt Brace Jovanovich, 1936, pp. 113–15.)

[5] While we may ask such questions and give such reasons to illustrate the principle, it should be noted that if we were to assume some value other than 3/4 for the MPC (or 1/4 for the MPS) the original equilibrium would not have been 160. With an MPC of 4/5, the original equilibrium income and output would have been 200, equal to the aggregate spending function, $20 + \frac{4}{5}Y + I$, in which I equals 20. From this original equilibrium, ΔI of 10 would result in ΔY of 50 before equilibrium was restored with ΔS of 10 equal to ΔI of 10; ΔY would be 50 as described above, but the rise in Y would have been from an original equilibrium with Y = 200 to a new equilibrium with $Y + \Delta Y = 250$.

additional income of 10 [illegible] income receivers in Period 3 is diverted from the spending stream. Therefore, the entire 10 of ΔY of Period 3 appears on the market as spending for consumption goods. In Period 3, $\Delta C = 10$, $\Delta I = 10$, and therefore in Period 4 $\Delta Y = 20$. In Period 4, all ΔY of that period appears as spending for consumption goods so that $\Delta C = 20$, $\Delta I = 10$, and therefore in Period 5, $\Delta Y = 30$. No new equilibrium would be possible in this case, and income would rise without limit. Equilibrium requires that $\Delta S = \Delta I$. But because all ΔY of any period is devoted to ΔC and none is diverted to ΔS, ΔS remains zero period after period and can never equal ΔI as required for equilibrium. Period after period, investment exceeds saving and aggregate spending exceeds aggregate output. In this special situation, we quickly have to drop our assumption that output expands proportionally with aggregate spending. This assumption is not too unreasonable for an economy with substantial unemployment of people and machines; but with spending growing without limit, unemployment would be quickly eliminated. As full employment approached, output would cease expanding proportionally with spending; once all available resources were fully utilized, output would not expand at all. Aggregate spending would continue to rise, but the rising expenditures would mean only continuously rising prices paid for an amount of output temporarily at its physical maximum. Such is the consequence of the assumption of an increase in I with MPC = 1 and MPS = 0.

In all the examples above, we have assumed that the aggregate spending function shifts upward. Shifts in the opposite direction are equally possible. In such cases, the multiplier works to produce a multiple contraction of income and output instead of a multiple expansion. With a downward shift in the aggregate spending function, our concern is not how much income will rise before equilibrium is restored but how much it will fall before equilibrium is restored. With MPC = 3/4 and MPS = 1/4, if investment spending falls from 20 to 10 ($\Delta I = -10$), the drop

[illegible]
[illegible] will be 40. The reasoning
before income will drop until
equals investment. Because invest[illegible]
from 20 to 10 (by -10), saving must
from 20 to 10 (by -10), to restore
Given MPS = 1/4, only when income
by 40 will saving be reduced by 10.
receivers increase saving by 1/4 of
to income, they decrease saving by 1/4
reduction in income. From the origi[illegible]
rium of 160, the system reaches a
rium level with income and output
to 120. Observe that the downwa[illegible]
is 4 for the same reason that the up[illegible]
plier is 4: because MPC = 3/4 and M[illegible]
If we assume different values for the M[illegible]
the MPS, the results for a downward sh[illegible]
aggregate spending function parallel th[illegible]
described, differing only in the size of th[illegible]
plier. With MPC = 2/3 and MPS = 1/3,
ward shift in the aggregate spending fun[illegible]
10 ($\Delta I = -10$) will produce a decline in
and output of 30.

Because the MPC or the MPS deter[illegible]
multiplier and because the multiplier d[illegible]
the size of the increase or decrease
and output that will follow any given
downward shift in the aggregate spend[illegible]
tion, the practical importance of the MP[illegible]
MPS is great. Given the variability of that
of the aggregate spending function m[illegible]
investment spending, the degree of insta[illegible]
the entire economic system depends
extent on the values of the MPC and the
business spending for plant and
increases in one period and decreases
there is a direct impact on the level of
and output within each period. This
source of instability. Yet whatever th[illegible]
of investment spending, a relatively
a relatively high MPS will tend to
instability in the economy than will
high MPC and a relatively low MPS.

Of the two extreme cases we disc[illegible]
lier, the one in which MPC = 1 and M[illegible]

produce extreme instability in income and output, because any variability in investment spending from one period to the next will be greatly magnified by continuously rising induced consumption spending. At the other extreme, when MPC = 0 and MPS = 1, the instability in income and output will be far less, because the variability of investment spending from one period to the next will not be magnified at all by induced consumption spending. Any explanation of fluctuations in the level of income and output involves far more than just the variability of investment spending and the values of the MPC and the MPS. Nonetheless, these values play a vital role in explaining the amplitude of the upward and downward movements in income and output during business cycles.

The Multiplier—Equations

To determine the equilibrium level of income and output in the two-sector economy, we used the equation

$$Y = C + I \qquad [1]$$

Once given the consumption function and the investment function, the equation could be readily solved:

$$= 20 + \tfrac{3}{4}Y + 20$$
$$160 = 140 + 20$$

If we retain this same consumption function but assume an upward shift in the investment function, the equilibrium level of income and output will increase. Because any change in Y—that is ΔY—must be equal to $\Delta C + \Delta I$, we have the following equation, the solution to which gives us the new equilibrium level of income and output. (The column headings in Tables 5-1 and 5-2 were derived from this equation.)

$$Y + \Delta Y = C + I + \Delta C + \Delta I \qquad [2]$$
$$160 + \Delta Y = 140 + 20 + \Delta C + \Delta I$$

Subtracting Equation [1] from Equation [2], we have another equation, the solution to which indicates the *change* in the level of income necessary to produce the new equilibrium level of income.

$$\Delta Y = \Delta C + \Delta I \qquad [3]$$

The consumption function ($C = C_a + cY$) indicates that consumption spending (C) rises or falls by an amount equal to the MPC or c (here ¾) times the change in income. That is, it says that $\Delta C = c\Delta Y$ or, in the present example, $\Delta C = \tfrac{3}{4}\Delta Y$. Substituting in Equation [3], we have the following:

$$\Delta Y = c\Delta Y + \Delta I$$
$$\Delta Y - c\Delta Y = \Delta I$$
$$\Delta Y(1 - c) = \Delta I$$
$$\Delta Y = \frac{1}{1 - c}\Delta I$$

and

$$\frac{\Delta Y}{\Delta I} = \frac{1}{1 - c}$$

or

$$\Delta Y = \tfrac{3}{4}\Delta Y + \Delta I$$
$$\Delta Y - \tfrac{3}{4}\Delta Y = \Delta I$$
$$\Delta Y(1 - 3/4) = \Delta I$$
$$\Delta Y = \frac{1}{1 - \tfrac{3}{4}}\Delta I$$

and

$$\frac{\Delta Y}{\Delta I} = \frac{1}{1 - \tfrac{3}{4}} = 4$$

If $\Delta I = 10$, $\Delta Y = 40$; the new equilibrium level of income and output will be 40 above the original level. Because $\Delta C = \tfrac{3}{4}\Delta Y$ and $\Delta Y = 40$, $\Delta C = 30$. The rise in income and output is divided between a rise in consumption of 30 and a rise in investment of 10.

Given any change in investment (ΔI), the change in income and output necessary to re-establish equilibrium is known as soon as the multiplier is known. The multiplier, in turn, is known as soon as the MPC is known. As we have just seen, the general expression for the multiplier is

$$\frac{\Delta Y}{\Delta I} = \frac{1}{1 - MPC}$$

in other words, the multiplier is the reciprocal of 1 minus the MPC; the larger the value of the MPC, the larger will be the value of the multiplier. This clearly agrees with our intuitive notion that the rise in income induced by a given rise in investment will be larger if a larger fraction of additional income is spent on consumption and will be smaller if a smaller fraction of additional income is spent on consumption—that is, the rise in income will be larger or smaller for a larger or smaller MPC.

There is a second approach to the determination of the multiplier. To determine the equilibrium level of income and output, we earlier used $Y = C + I$ and

$$S = I \qquad [1]$$

Whereas $Y = C + I$ focuses on the equality between aggregate output and aggregate spending, $S = I$ focuses on the equality between saving and investment. Once given the saving function and the investment function, the equation could be readily solved for the equilibrium level of income and output as follows:

$$\begin{aligned} S &= I \\ -20 + \tfrac{1}{4}Y &= 20 \\ \tfrac{1}{4}Y &= 40 \\ Y &= 160 \end{aligned}$$

The further development of the approach in terms of saving and investment is exactly parallel with the preceding development in terms of consumption and investment. Assuming an upward

[illegible] in the investment function, the new [illegible] level of income and output is that at [illegible]

$$S + \Delta S = I + \Delta I$$

Because $S = I$, we may subtract $S =$ [illegible] Equation [2] which gives us

$$\Delta S = \Delta I$$

The solution to this equation indicates the c[illegible] in the level of income necessary to produ[illegible] new equilibrium level of income, given a [illegible]fied change in the investment function.

The saving function ($S = S_a + sY$) ind[illegible] that saving rises or falls by an amount eq[illegible] the MPS or s (here $\tfrac{1}{4}$) times the rise or [illegible] income. That is, $\Delta S = s\Delta Y$. Substituting in[illegible]tion [3], we have

$$s\Delta Y = \Delta I$$

$$\Delta Y = \frac{1}{s}\Delta I$$

$$\frac{\Delta Y}{\Delta I} = \frac{1}{s}$$

If, as before, $\Delta I = 10$ then $\Delta Y = 40$; the equilibrium level of income and output will [illegible] above the original level. Given any chan[illegible] investment (ΔI), the change in income and o[illegible] necessary to restore equilibrium is know[illegible] soon as the multiplier is known. The multipl[illegible] turn, is known as soon as the MPS is kn[illegible] Therefore, we have as a second general ex[illegible]sion for the multiplier

$$\frac{\Delta Y}{\Delta I} = \frac{1}{MPS}$$

Because $MPS = 1 - MPC$, $1/MPS$ as derived is exactly equal to our earlier equ[illegible] for the multiplier, $1/(1 - MPC)$. Thus, the [illegible]plier is the reciprocal of 1 minus the MPC [illegible] reciprocal of the MPS.

Simple Income Determination— A Concluding Note

In this chapter we have examined the theory of income determination under some highly simplifying assumptions—hence the use of the adjective "simple." Some of these assumptions will be dropped in later chapters. Naturally, the more that are dropped, the more complicated the theory becomes and the closer it comes to describing the infinitely complex process by which income and output change in the real world.

There is no doubt, however, that the simple theory outlined in this chapter sheds considerable light on the aggregate economic process in the world about us; it explains things that are not immediately apparent. One may properly argue that it takes no high level of economic sophistication to recognize that output will be increased in an economy with idle workers and idle machines if either business people or consumers or both step up their spending for goods and services. However, the completely unsophisticated will probably expect output to increase by the amount of this initial increase in spending. But merely by introducing the concepts of the consumption function, the marginal propensity to consume, and the multiplier, even the simple theory makes very clear the secondary consequences that produce an increase in income and output larger than the initial increase in investment or consumer spending.

Our simple theory not only shows the process by which an increase in autonomous spending will raise income and output by a multiple of that increase, but also indicates what determines the size of this multiple (admittedly under very restrictive assumptions). Although this simple theory cannot begin to explain the actual fluctuations in the level of income and output in the real world, it can explain an important truth, a truth necessary but not sufficient to an understanding of these fluctuations. To understand this is to understand the essence of what the simple theory has to tell us.