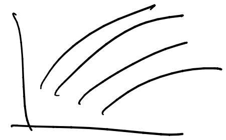
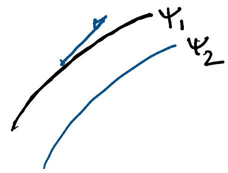
Visualizing Convection

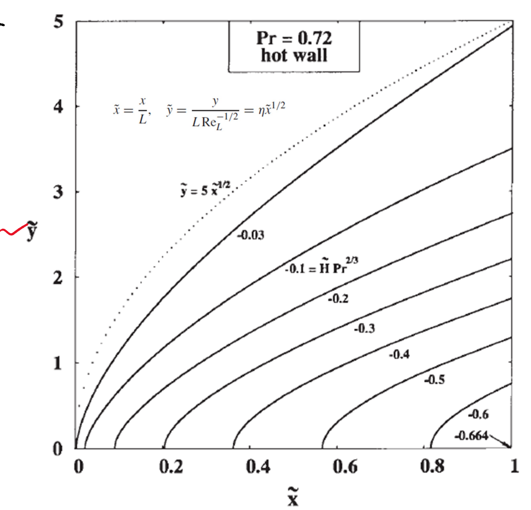
Monday, January 18, 2021 10:26 PM

Stream function

$$u = \frac{\partial \psi}{\partial y} \quad ; \quad v = -\frac{\partial \psi}{\partial x}.$$







Kimura and Bejan → 1984, ✓

$H(x,y)$

Example.

Steady State, 2D, constant property, homogeneous fluid. ✓

$$\rho c \frac{DT}{Dt} = k \nabla^2 T$$

$$\downarrow$$

$$\rho c_p \underbrace{\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right]}_{\text{bulk}} = k \underbrace{\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right]}_{\text{Mol. dif.}} \quad \cdots \quad (1)$$

$$\left[\rho c_p u \frac{\partial T}{\partial x} - k\frac{\partial^2 T}{\partial x^2}\right] + \left[\rho c_p v \frac{\partial T}{\partial y} - k \frac{\partial^2 T}{\partial y^2}\right] = 0$$

$$\frac{\partial}{\partial x}\left[\rho c_p u T - k\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[\rho c_p v T - k\frac{\partial T}{\partial y}\right] = 0. \quad \cdots \quad (2)$$

The heatfunction $H(x,y)$ defines as

$$\frac{\partial H}{\partial y} = \rho c_p u (T - T_{ref}) - k\frac{\partial T}{\partial x}.$$

$$-\frac{\partial H}{\partial x} = \rho c_p v (T - T_{ref}) - k\frac{\partial T}{\partial y}.$$