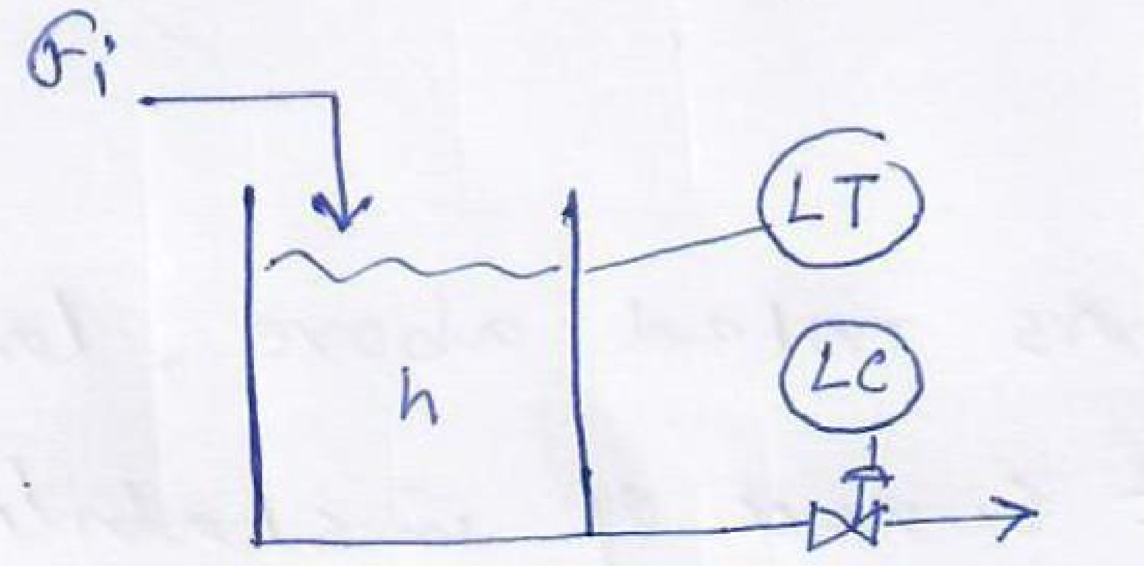
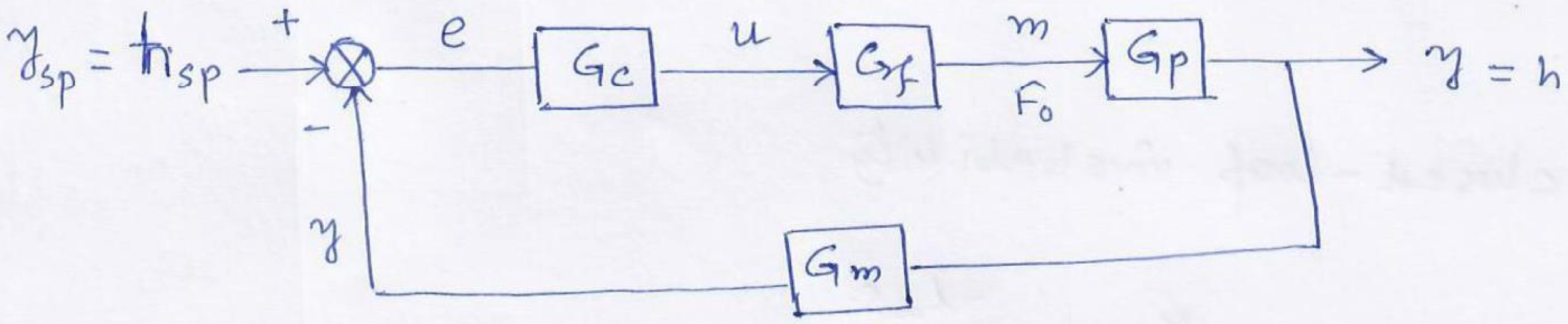
Control of systems with dead time.





Time delay is involved in

- transportation of fluid over long distance — Process
- sampling and measurement — Sensor
- developing the actuating signal — FCE
- computing the control action — Controller

Result in unsatisfactory closed-loop performance

Because:

- Affect of disturbance is detected after a significant period of time
- control action calculated at present time step based on old measurement is inadequate.

$$u(t) = u_s + K_c\left[y_{sp} - y(t - t_d)\right]$$

↑ present time (t) — ↑ old ($y(t-t_d)$)

- control action will also take sometime to make its effect felt by the process

- As a result of all these factors noted above, large dead time is a significant source of instability for closed loop response.

Ex. closed-loop instability

$$G_{OL} = \frac{K_c}{0.5s+1} e^{-t_d s}.$$

$AR = AR_1 \cdot AR_2.$

$$1 = \underbrace{\frac{K_c}{\sqrt{(\tau_p \omega_{co})^2+1}}}_{\text{dead-time free part}} \cdot \underbrace{1}_{\text{dead-time part}}$$

$\varphi = \varphi_1 + \varphi_2$

$$-180^\circ = \underbrace{\tan^{-1}(-0.5\,\omega_{co})}_{\text{dead-time free part}} + \underbrace{(-t_d\,\omega_{co}) \times \frac{180}{\pi}}_{\text{dead-time part}}$$

t_d min	ω_{co} rad/min	K_u
0.01	160	80
0.1	17	8.56
1.0	2.3	1.52

Observation

As $t_d \uparrow$

- $K_u \downarrow$. To avoid instability problem K_c must be reduced, which leads to sluggish closed-loop response.

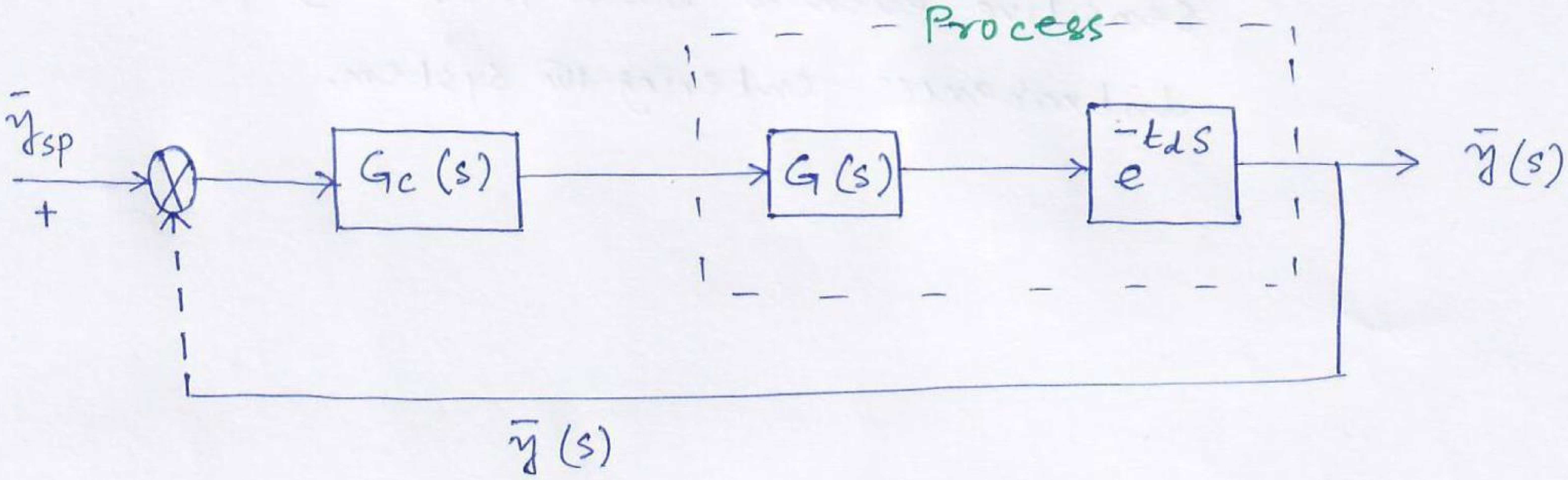
- $\omega_{co} \downarrow$. It implies that closed-loop response will be sensitive even to lower frequency periodic disturbances entering the system.

Dead-time compensator

To reduce the effect of dead-time, dead-time compensator is developed by OJM Smith (also called as Smith predictor).

- Consider the following closed-loop system with:

$$G_f = G_m = 1 \quad \text{.... for simplicity}$$



- Assumption: all the dead-time is caused by the process

$$G_p(s) = G(s) \cdot e^{-t_d s} \equiv \begin{pmatrix}\text{dead-time}\\ \text{free part}\end{pmatrix} \cdot \begin{pmatrix}\text{dead-time}\\ \text{part}\end{pmatrix}$$

- Finding "actual" process output y

$$G_{OL} = \frac{\bar{y}(s)}{\bar{y}_{sp}(s)} = G_p G_f G_m G_c \quad \text{--- general form}$$

$$\bar{y}(s) = G_c(s)\, G(s)\, e^{-t_d s}\, y_{sp}(s) \quad \text{--- Since } G_f = G_m = 1$$

Obviously, it is delayed by t_d time (min say).

Aim: Get output without $e^{-t_d s}$ term.

- Compensator should "ideally" yield:

$$\bar{y}^*(s) = G_c(s)\, G(s)\, y_{sp}(s).$$

To have this, let us consider:

$$\underset{\text{Ideal}}{\bar{y}^*(s)} = \underset{\text{actual}}{\bar{y}(s)} + \underset{\text{added}}{\bar{y}'(s)}.$$

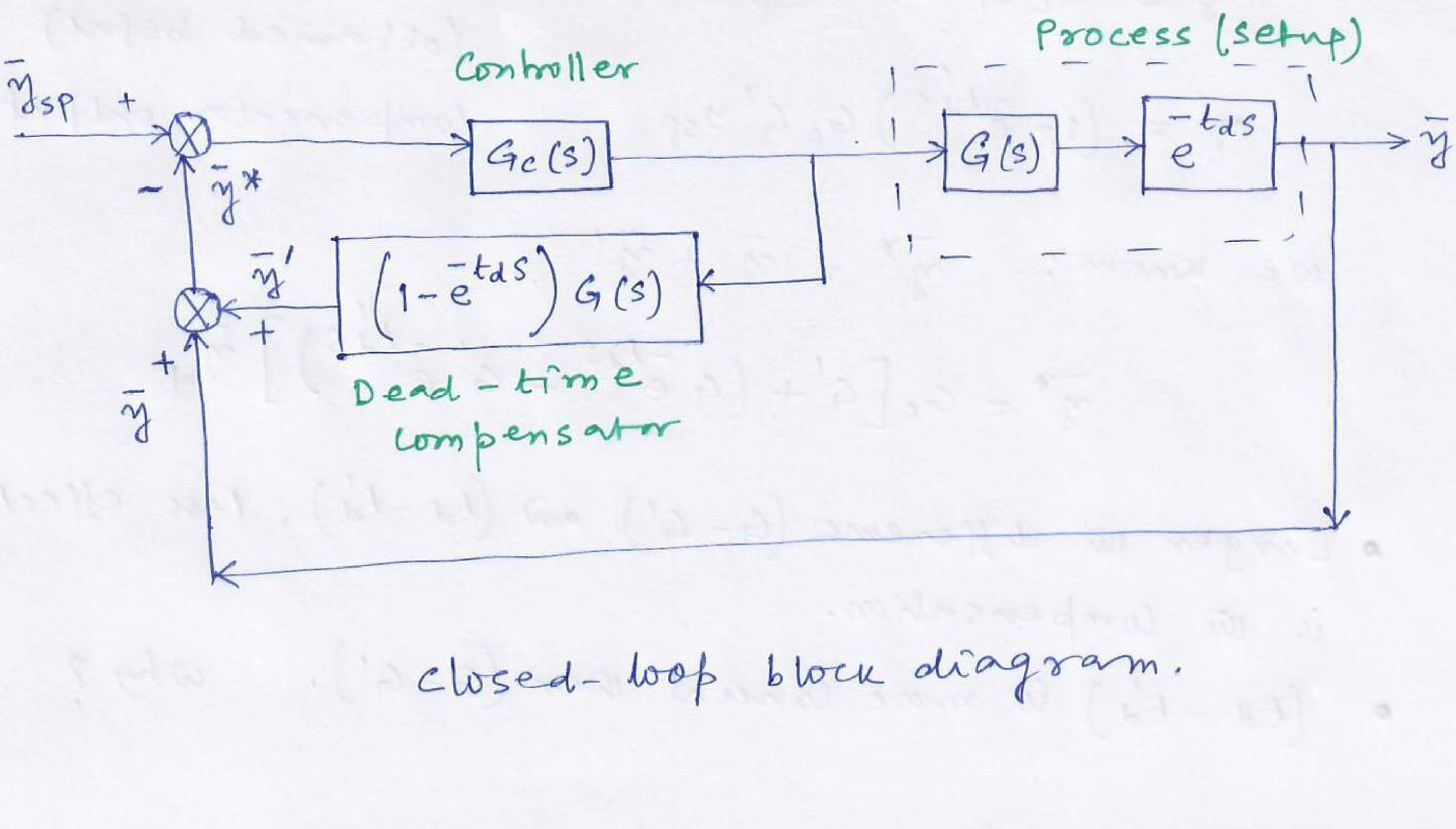
$$\bar{y}'(s) = \bar{y}^*(s) - \bar{y}(s)$$

$$= G_c(s)\, G(s)\, (1 - e^{-t_d s})\, \bar{y}_{sp}(s).$$

- Compensator

$$\frac{\bar{y}'(s)}{\bar{y}_{sp}(s)} = G_c\, G\, (1 - e^{-t_d s}) = G_{compensator}$$

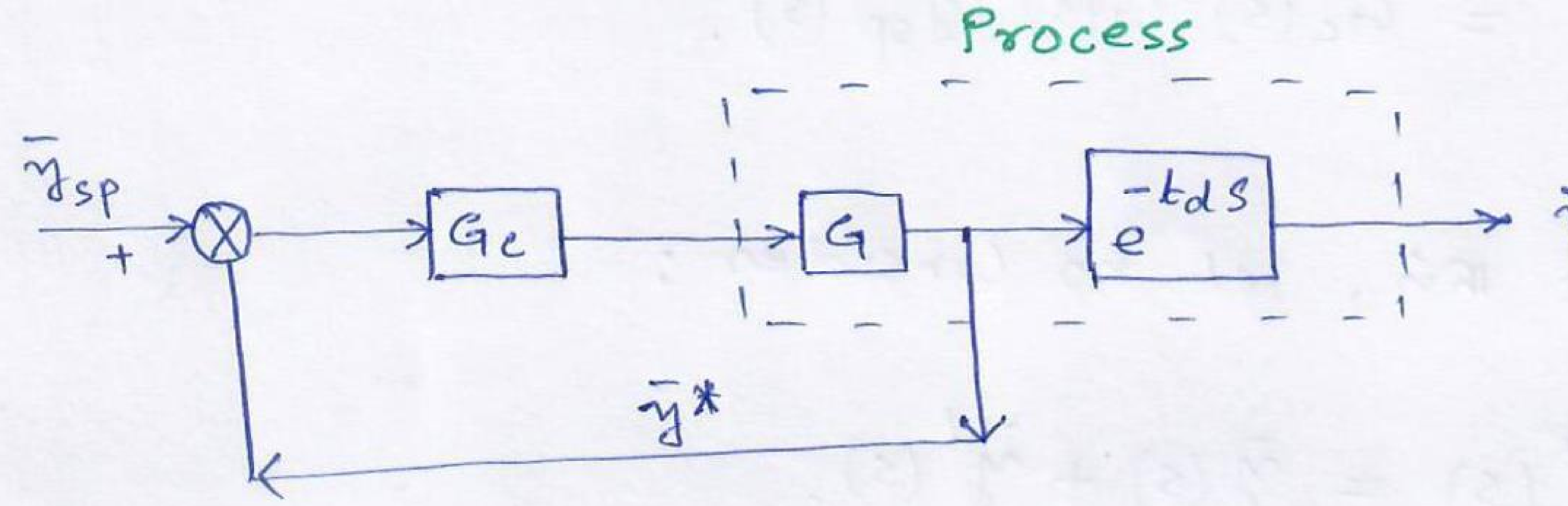
- Block diagram (closed-loop process with compensator).



closed-loop block diagram.

Remarks.

1. The block dig shown below is incorrect. Why?



2. We can have "perfect" compensation if

Process model (G) and dead-time (t_d) are "perfectly known"

3. In practice, G and t_d are approximately known.

Actual	Real
G	G'
t_d	t_d'

$$\bar{y} = G_c G e^{-t_d s} y_{sp} \quad \text{--- actual output (obtained before)}$$

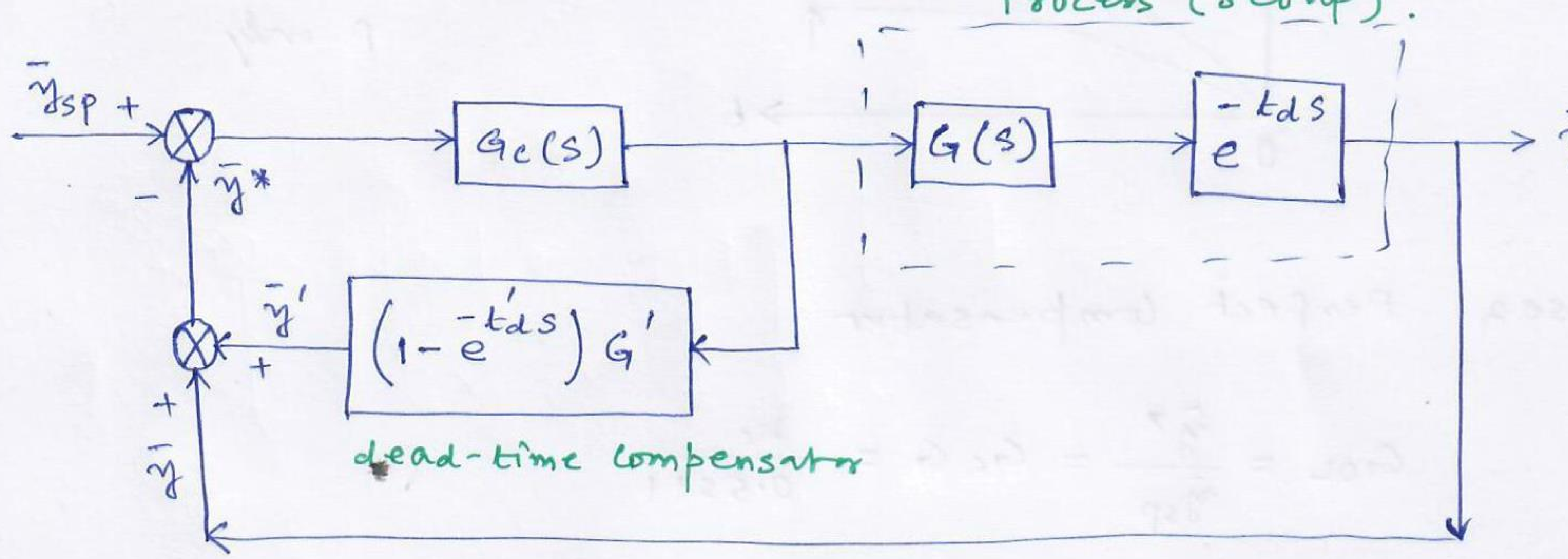
$$\bar{y}' = \left(1 - e^{-t_d' s}\right) G_c G' y_{sp} \quad \text{--- compensator output}$$

we know: $\bar{y}^* = \bar{y} + \bar{y}'$

$$\bar{y}^* = G_c\left[G' + \left(G e^{-t_d s} - G' e^{-t_d' s}\right)\right] \bar{y}_{sp}$$

- Larger the difference $(G - G')$ and $(t_d - t_d')$, less effective is the compensation.
- $(t_d - t_d')$ is more crucial than $(G - G')$. Why?

Accordingly, we develop the following block dig



4. Dead-time may change with time. So, constant t_d may lead to degrade the compensator performance.

Ex. Dead-time Compensator

✓ Process: $G_p(s) = \dfrac{e^{-s}}{0.5s+1} \equiv \dfrac{K_p \cdot e^{-t_d s}}{\tau_p s+1}$ $\quad K_p = 1$, $t_d = 1$, $\tau_p = 0.5$

✓ Controller: $G_c(s) = K_c$ -- P-only

✓ Sensor + FCE: $G_f = G_m = 1$ -- for simplicity

So, $G_{OL} = \dfrac{K_c\, e^{-s}}{0.5s+1}$

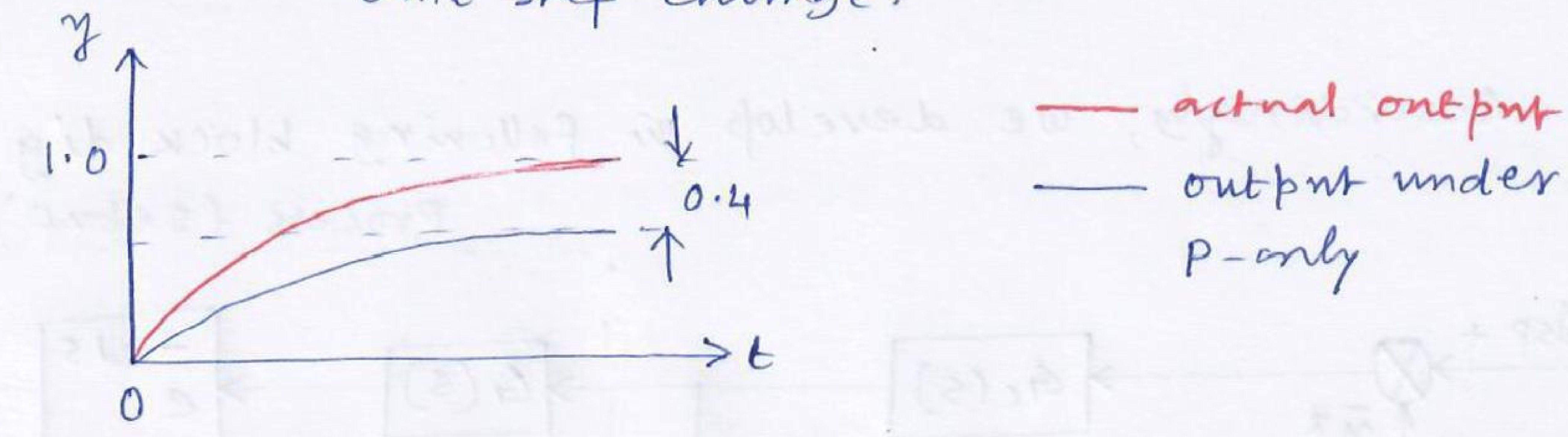
Case 1. No compensator

Determined previously: $K_u = 1.52$ $\quad (K_c < 1.52)$

$\omega_{co} = 2.3$.

$\therefore$ offset $= \dfrac{1}{1+K_p K_c} = 0.4$ $\quad$ (when $K_c = K_u$)

Unit step change.



case 2. Perfect compensator

$$G_{OL} = \frac{\bar{y}^*}{\bar{y}_{sp}} = G_c G = \frac{K_c}{0.5s+1}$$

It has no ω_{co} since for 1st-order system, ϕ varies from 0 to -90^0. Consequently, we can adopt arbitrarily large K_c ($=50$) without any instability problem.

$$\text{offset} = \frac{1}{1+K_pK_c} = \frac{1}{1+50} = 0.0196 \equiv 2\%.$$

case 3. Imperfect compensator

$$G' = G = \frac{1}{0.5s+1}, \quad t_d = 1, \quad t_d' = 0.8.$$

Process with uncompensated dead time $\equiv \dfrac{e^{-0.2s}}{0.5s+1}$

$AR = 1$

$$\frac{K_u}{\sqrt{(0.5\times 9)^2+1}} = 1$$

$\Rightarrow K_u = 4.6$ $(K_c < 4.6)$

$\phi = -180^0$

$$\tan^{-1}(-0.5\,\omega_{co}) - 0.2\,\omega_{co}\,\frac{180}{\pi} = -180^0$$

$\Rightarrow \omega_{co} = 9$

✓ for $K_c = 50$, we will have "unstable" closed-loop response.

✓ we must be conservative in selecting K_c, even if we use dead-time compensator.