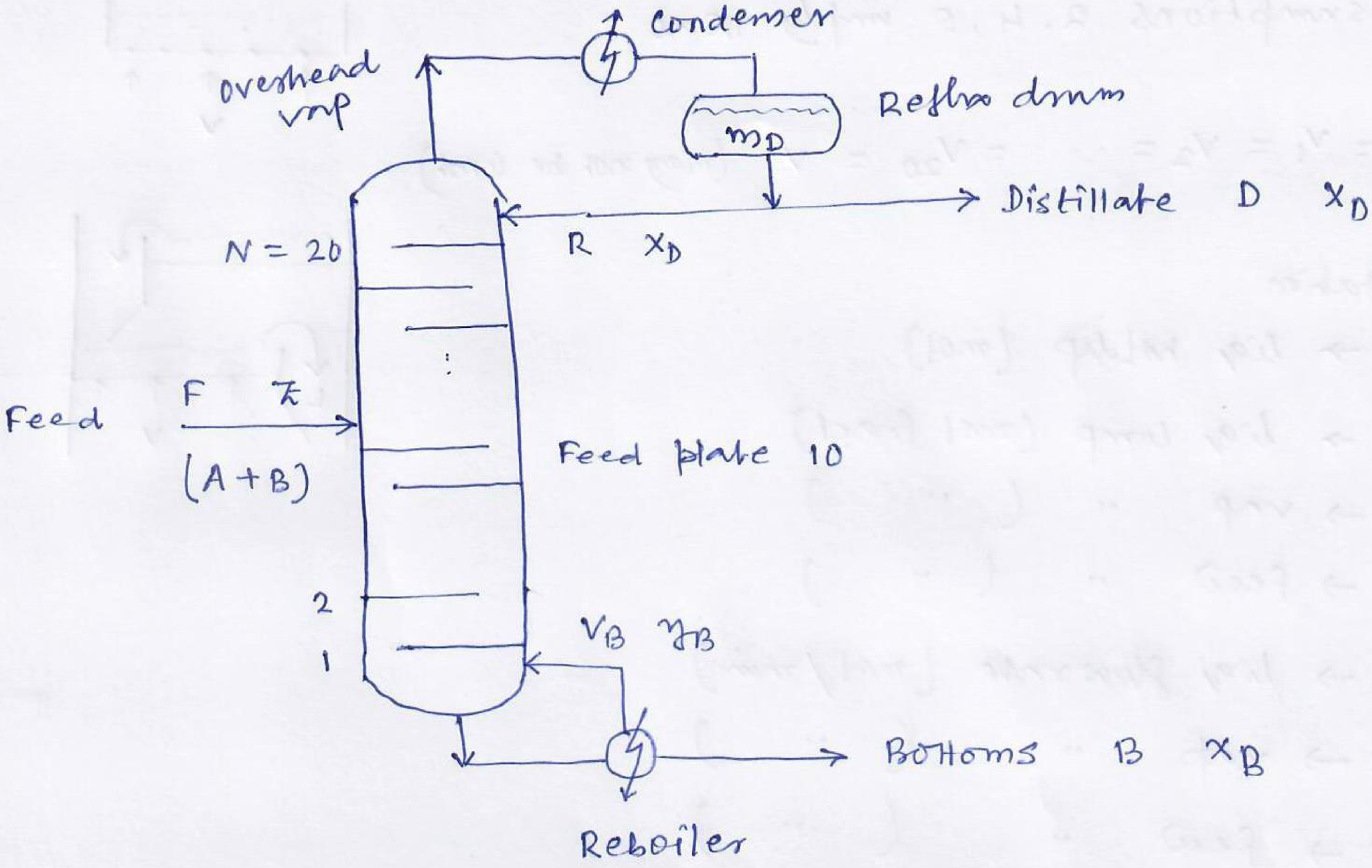
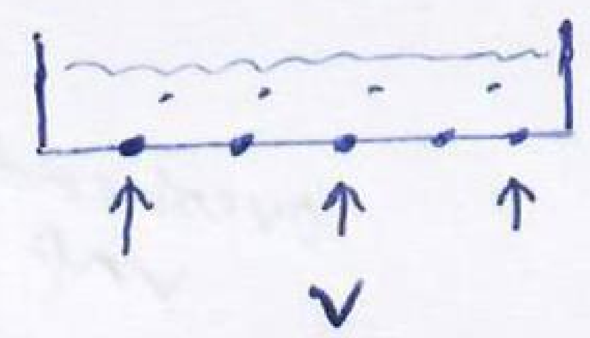
Distillation Column (Binary System)



Assumptions

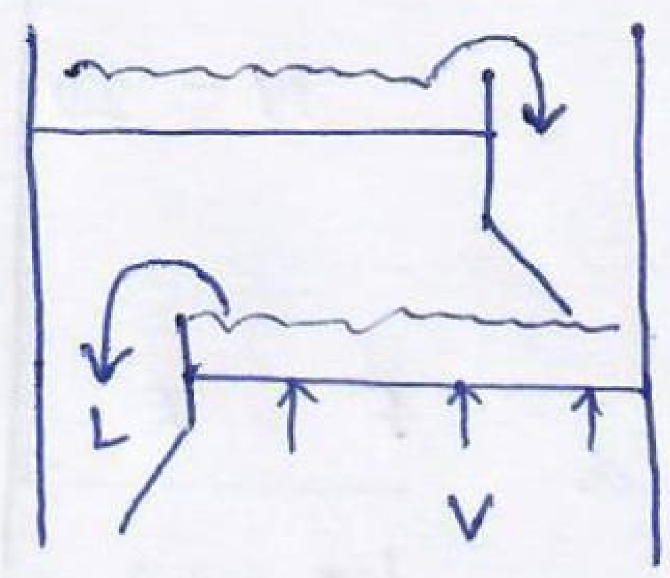
1. Feed is saturated liquid
2. No heat loss (well/perfectly insulated)
3. Each tray is 100% efficient (ideal trays)
4. Negligible vap holdup on each tray ($\rho_v << \rho_L$)
5. Molar heats of vaporization of both components (A and B) are approximately equal. This means that 1 mol of condensing vap releases enough heat to vaporize 1 mol of liquid
6. Perfect mixing on each stage
7. Relative volatility (α) of two components remains const throughout the column.
8. Liquid holdup varies from tray to tray
9. condenser and reboiler dynamics (thermal) is neglected

Tray



Assumptions 2, 4, 5 imply that

$V_B = V_1 = V_2 = \cdots = V_{20} = V$ (may not be const)



Notation

m → liq holdup (mol)

x → liq comp (mol fract)

y → vap " (")

z → feed " (")

L → liq flow rate (mol/min)

V → vap " (")

F → feed " (")

R → reflux " (")

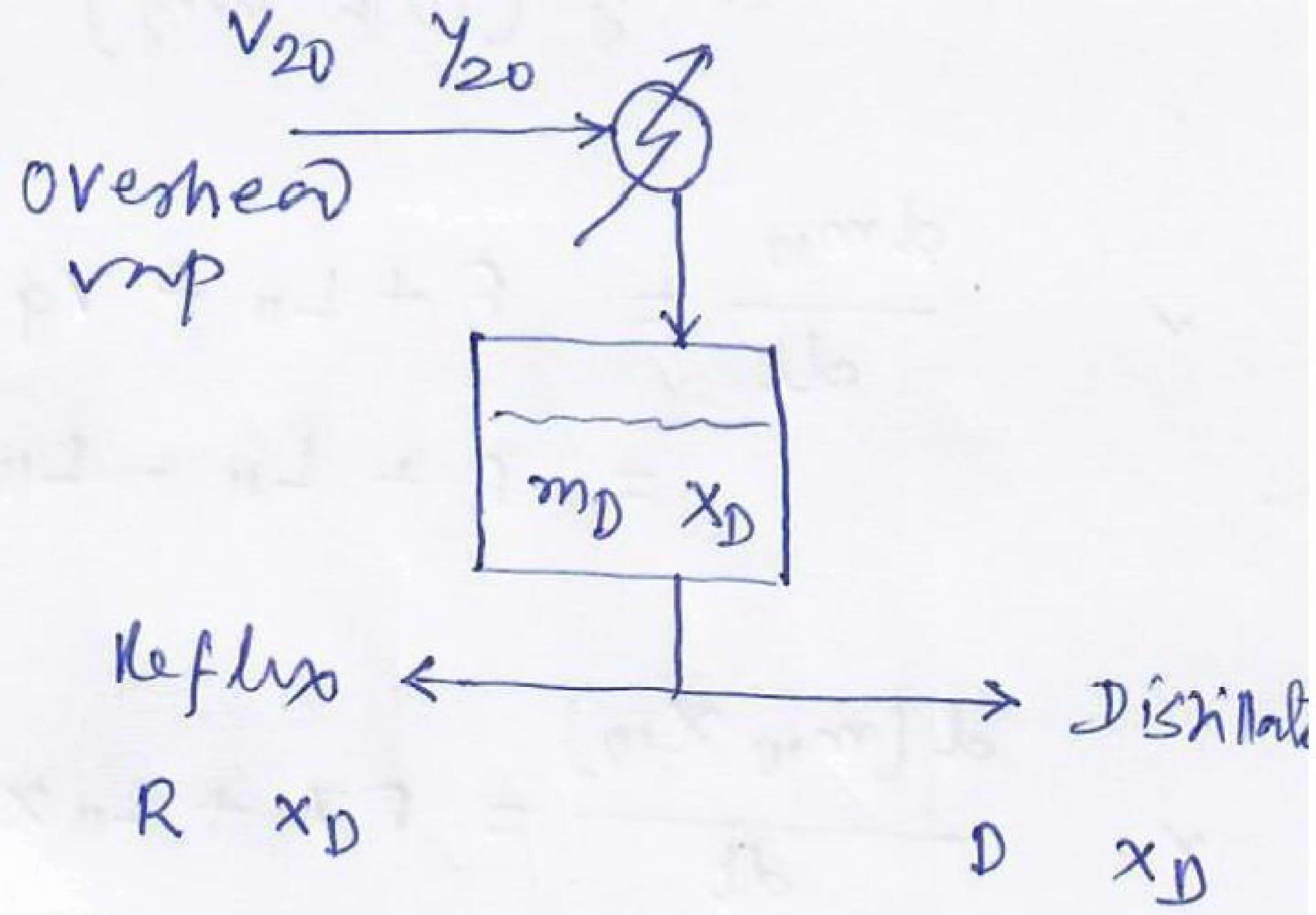
D → Distillate " (")

B → Bottoms " (")

V_B → vap boilup " (")

Model equations

Condenser - reflux drum



✓ Total mass bal.

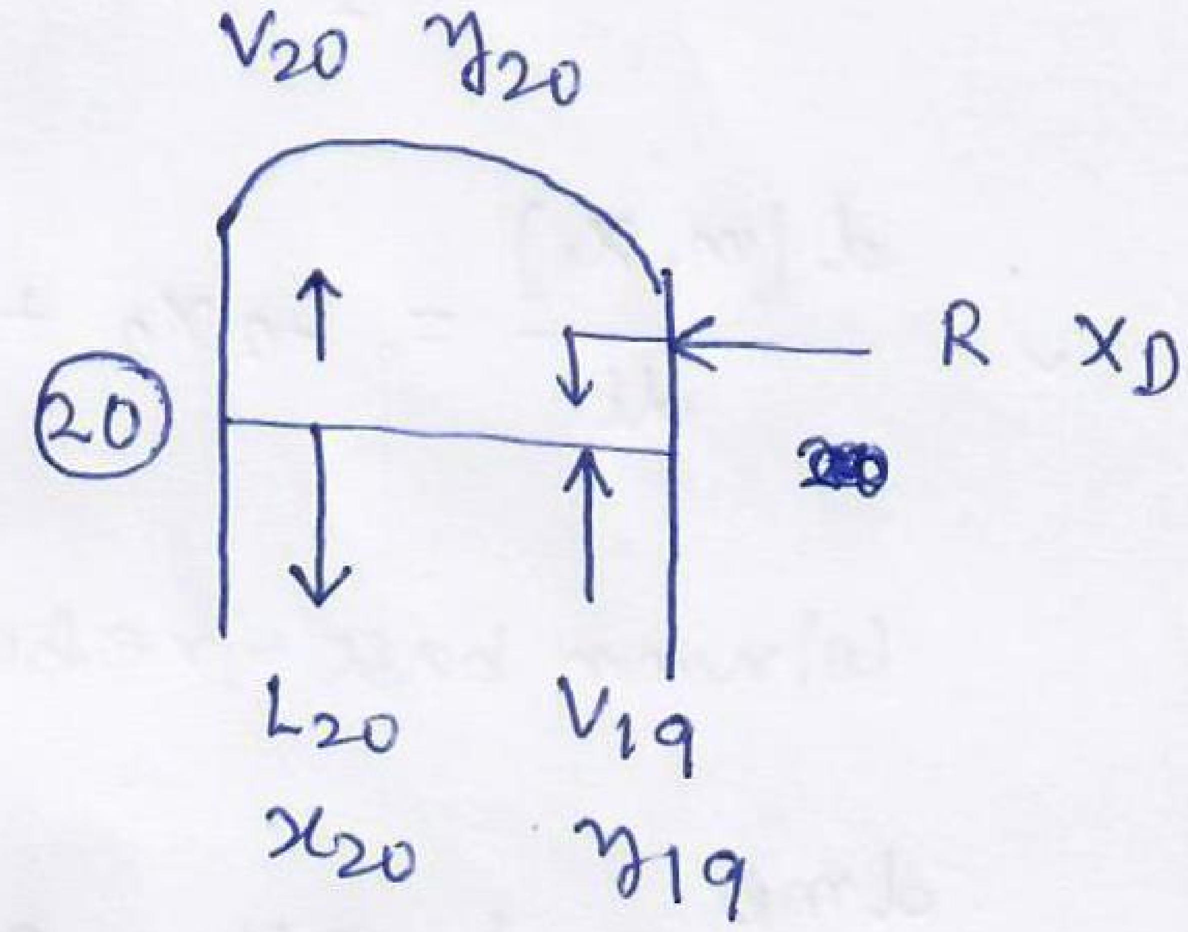
$$\frac{dm_D}{dt} = V_{20} - R - D$$

✓ Comp mass bal.

$$\frac{d(m_D x_D)}{dt} = V_{20} y_{20} - R x_D - D x_D$$

$$\frac{dx_D}{dt} = \frac{V_{20}}{m_D}(y_{20} - x_D)$$

Top Tray (20)

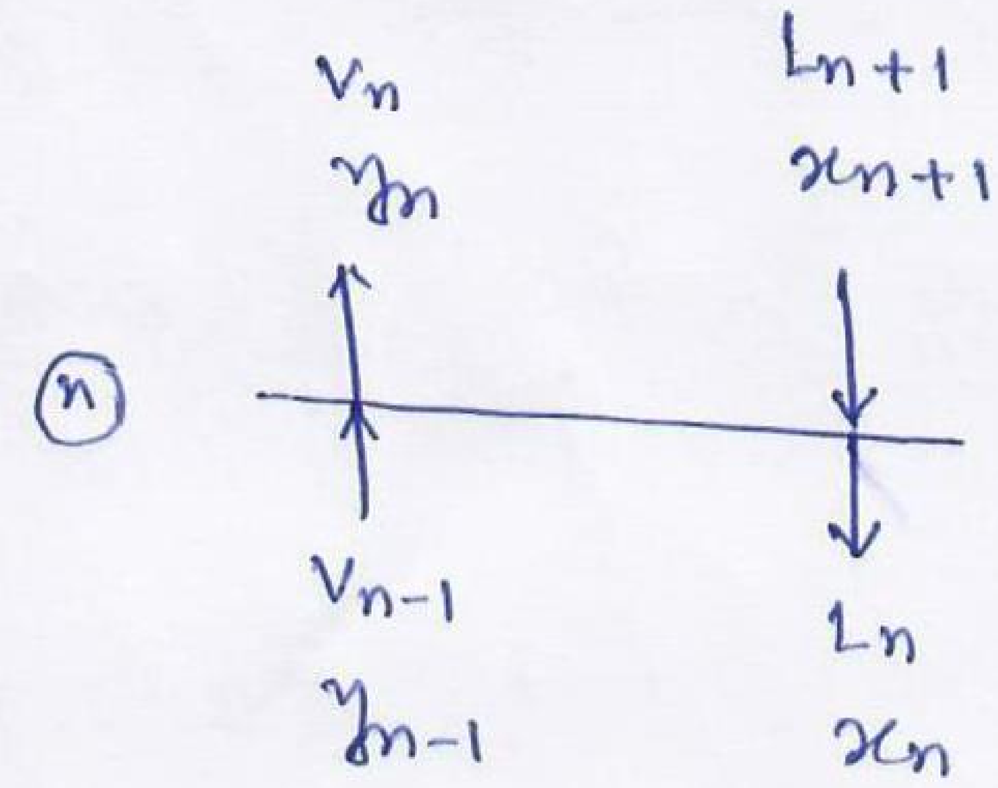


✓ $$\frac{dm_{20}}{dt} = R + V_{19} - L_{20} - V_{20} = R - L_{20}$$

✓ $$\frac{d(m_{20} x_{20})}{dt} = R x_D + V_{19} y_{19} - L_{20} x_{20} - V_{20} y_{20}$$

$$\Rightarrow \quad \frac{dx_{20}}{dt} = \frac{1}{m_{20}}\left[R(x_D - x_{20}) + v_B(y_{19} - y_{20})\right]$$

nth tray (2 to 9 & 11 to 19)

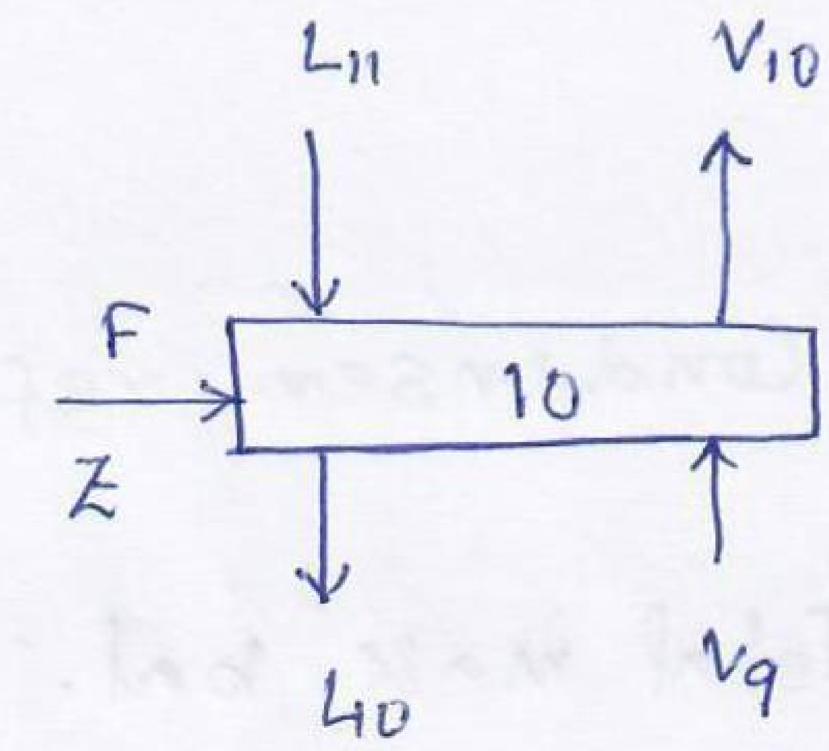


✓ $$\frac{dm_n}{dt} = L_{n+1} + V_{n-1} - L_n - V_n$$

$$= L_{n+1} - L_n$$

✓ $$\frac{d(m_n x_n)}{dt} = L_{n+1} x_{n+1} + V_{n-1} y_{n-1} - L_n x_n - V_n y_n$$

Feed tray (10th tray)

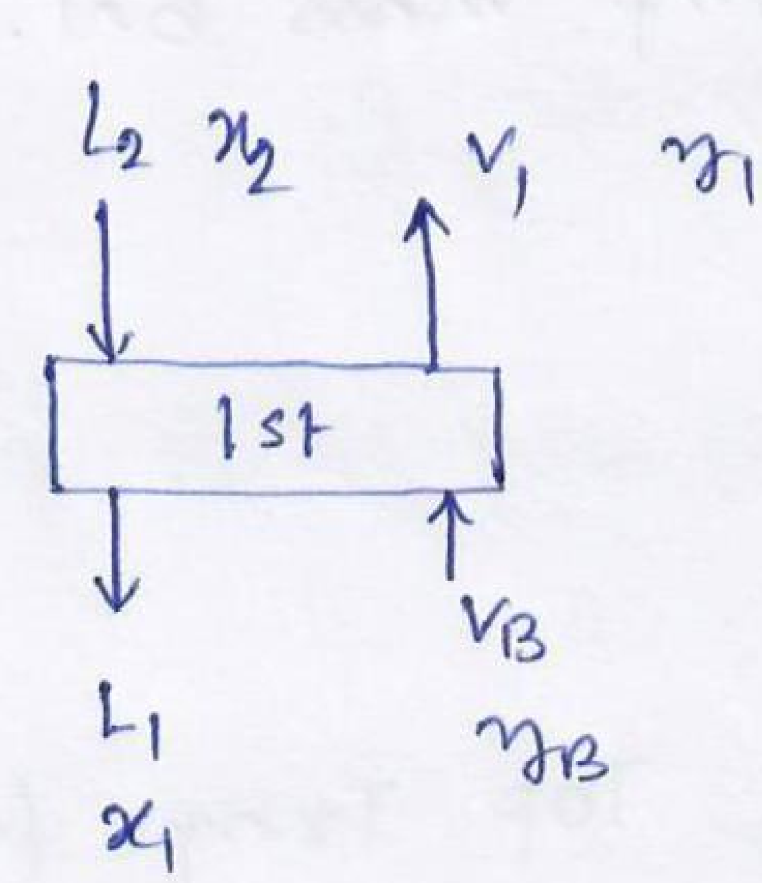


✓ $$\frac{dm_{10}}{dt} = F + L_{11} + V_9 - L_{10} - V_{10}$$

$$= F + L_{11} - L_{10}$$

✓ $$\frac{d(m_{10} x_{10})}{dt} = FZ + L_{11} x_{11} + V_9 y_9 - L_{10} x_{10} - V_{10} y_{10}$$

Bottom tray (1st tray).



✓ $$\frac{dm_1}{dt} = L_2 + V_B - L_1 - V_1$$

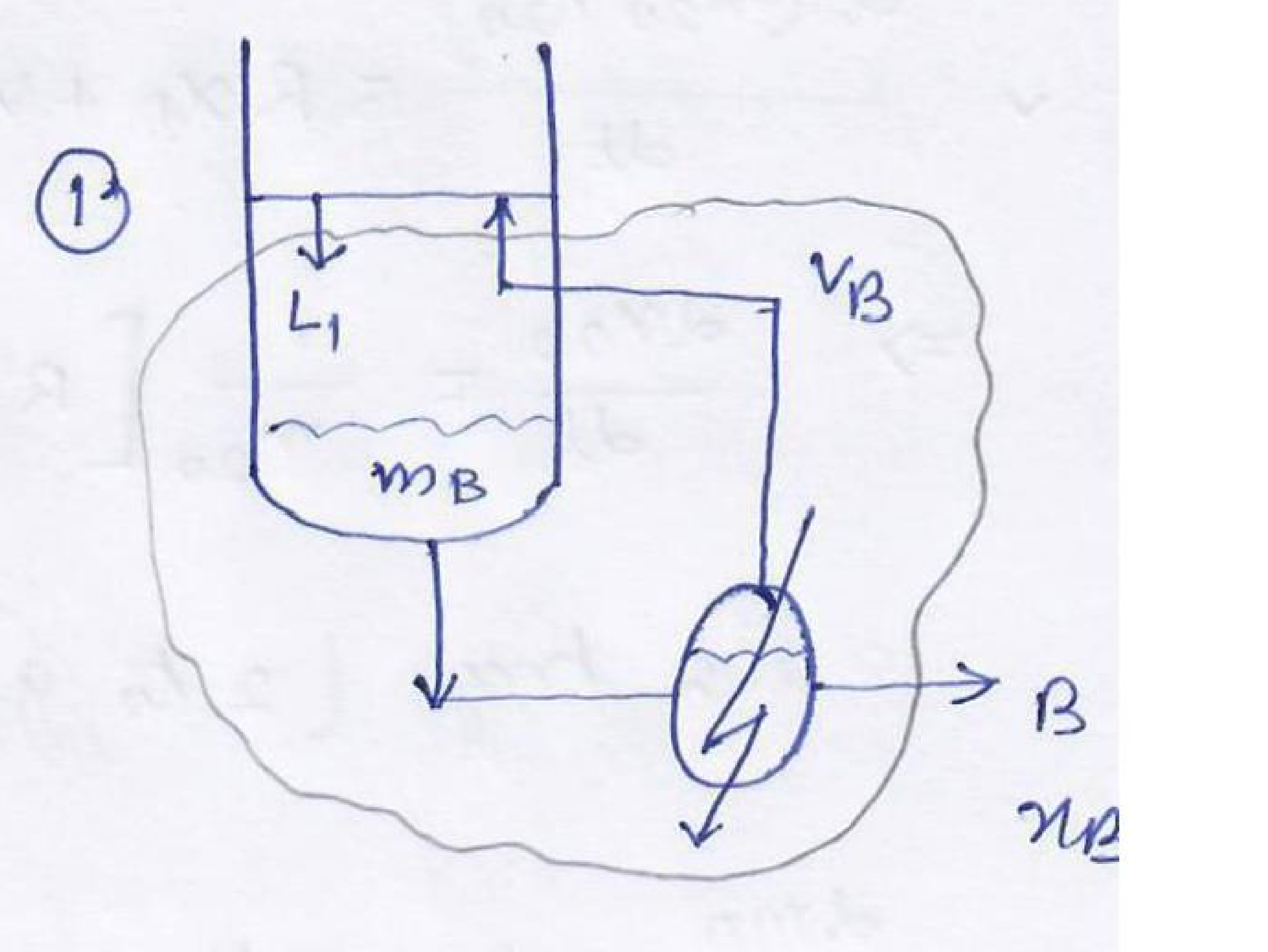
$$= L_2 - L_1$$

✓ $$\frac{d(m_1 x_1)}{dt} = L_2 x_2 + V_B y_B - L_1 x_1 - V_1 y_1$$

Column base-reboiler system

$$\frac{dm_B}{dt} = L_1 - V_B - B$$

$$\frac{d(m_B x_B)}{dt} = L_1 x_1 - V_B y_B - B x_B$$



Relative volatility (α)

$$\alpha_{ij} = \frac{K_i}{K_j}$$

α_{ij} = relative volatility of comp i wrt j

K = vap-liq equilibrium coefficient

$$K_i = \frac{y_i}{x_i} \quad ; \quad K_j = \frac{y_j}{x_j}$$

For a binary system:

$$x_i + x_j = 1 \quad \text{... liq}$$

$$y_i + y_j = 1 \quad \text{... vap.}$$

$$\therefore \alpha_{ij} = \frac{K_i}{K_j} = \frac{y_i / x_i}{y_j / x_j} = \frac{y_i / x_i}{(1-y_i)/(1-x_i)}$$

$$\Rightarrow \boxed{y_i = \frac{\alpha_{ij} x_i}{1 + (\alpha_{ij} - 1) x_i}}$$

✓ Ex: $\alpha_{ij} = 2$ for 1-propanol/ethanol system

Tray hydraulics — Francis-weir formula

$$L_n = L_{n0} + \frac{m_n - m_{n0}}{\beta} \qquad 0 \rightarrow ss$$

β = hydraulic time constant (3-6 sec/tray).

vap boil-up

$$V_B = \frac{Q_R}{\lambda_B}$$

$Q_R \rightarrow$ reboiler duty

$\lambda \rightarrow$ latent heat

Simulation of Distillation Model.

holdup $m \longrightarrow$ total mole bal.

liq comp $x \longrightarrow$ comp mole bal.

$L \rightarrow$ Francis weir

$V \rightarrow V_1 = V_2 = \cdots = V_{20} = V_B = \dfrac{Q_R}{\lambda_B}$

$y \rightarrow$ vap-liq equilibrium $y = \dfrac{\alpha x}{1+(\alpha-1)x}$.

Degrees of freedom (Distillation).

No. of eqns	Origin
N + 1	VLE $y_i = \dfrac{\alpha_{ij} x_i}{1 + (\alpha_{ij} - 1) x_i}$ for $y_1, y_2, \ldots, y_N, y_B$
N	$L_n = L_{n0} + \dfrac{m_n - m_{n0}}{\beta}$ for $L_1, L_2, \ldots, L_N$
N + 2	Total mole bal. for $m_1, m_2, \ldots, m_N, m_D, m_B$
N + 2	Comp mole bal. for $x_1, x_2, \ldots, x_N, x_D, x_B$

Total $= 4N + 5 = E$

No. of variables	Type
N + 2	$x_1, x_2, \ldots, x_N, x_D, x_B$
N + 1	$y_1, y_2, \ldots, y_N, y_B$
N + 2	$m_1, m_2, \ldots, m_N, m_D, m_B$
N	$L_1, L_2, \ldots, L_N$
6	F, z, D, R, B, V_B

Total $= 4N + 11 = V$ $\qquad \therefore f = V - E = 6$.

How to make $f = 0$?

(i) No. of load variables $= 2$ F, z

(ii) No. of control equations $= 4$

CV	MV
x_D	R
x_B	V_B (or Q_R)
m_D	D
m_B	B

$\therefore \; f = 0.$

Questions.

1. Why there is no energy bal considered ?
2. Why this is called an ideal distillation column ?
3. What is the physical significance of β ?
4. Why there is no y_D calculated ?
5. why liq holdup varies when the tray geometry is fixed ?
6. Why all vap flow rates are same ?

Laplace Transform

$$L[f(t)] = \bar{f}(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

	Time function	Laplace transform
1.	$f(t) = 1$	$L[f(t)] = \frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	t^2	$\frac{2!}{s^3}$
4.	t^n	$\frac{n!}{s^{n+1}}$
5.	e^{-at}	$\frac{1}{s+a}$
6.	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
7.	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
8.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
9.	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
10.	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
11.	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
12.	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
13.	$e^{-at} \sinh \omega t$	$\frac{\omega}{(s+a)^2-\omega^2}$

Laplace transform of derivatives

$$L\left[\frac{df(t)}{dt}\right] = s\bar{f}(s) - f(0)$$

$$L\left[\frac{d^2 f}{dt^2}\right] = s^2\bar{f}(s) - sf(0) - f'(0)$$

$$L\left[\frac{d^n f(t)}{dt^n}\right] = s^n\bar{f}(s) - s^{n-1}f(0) - s^{n-2}f'(0) \cdots - sf^{n-2}(0) - f^{n-1}(0)$$

$$= s^n\bar{f}(s)$$ .. when $f \equiv$ deviation variable

i.e., $f(0) = 0$

(initially the process is at ss).

Laplace transform of integrals

$$L\left[\int_0^t f(t)\,dt\right] = \frac{1}{s}\bar{f}(s).$$

Final value theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0}\left[s\bar{f}(s)\right]$$

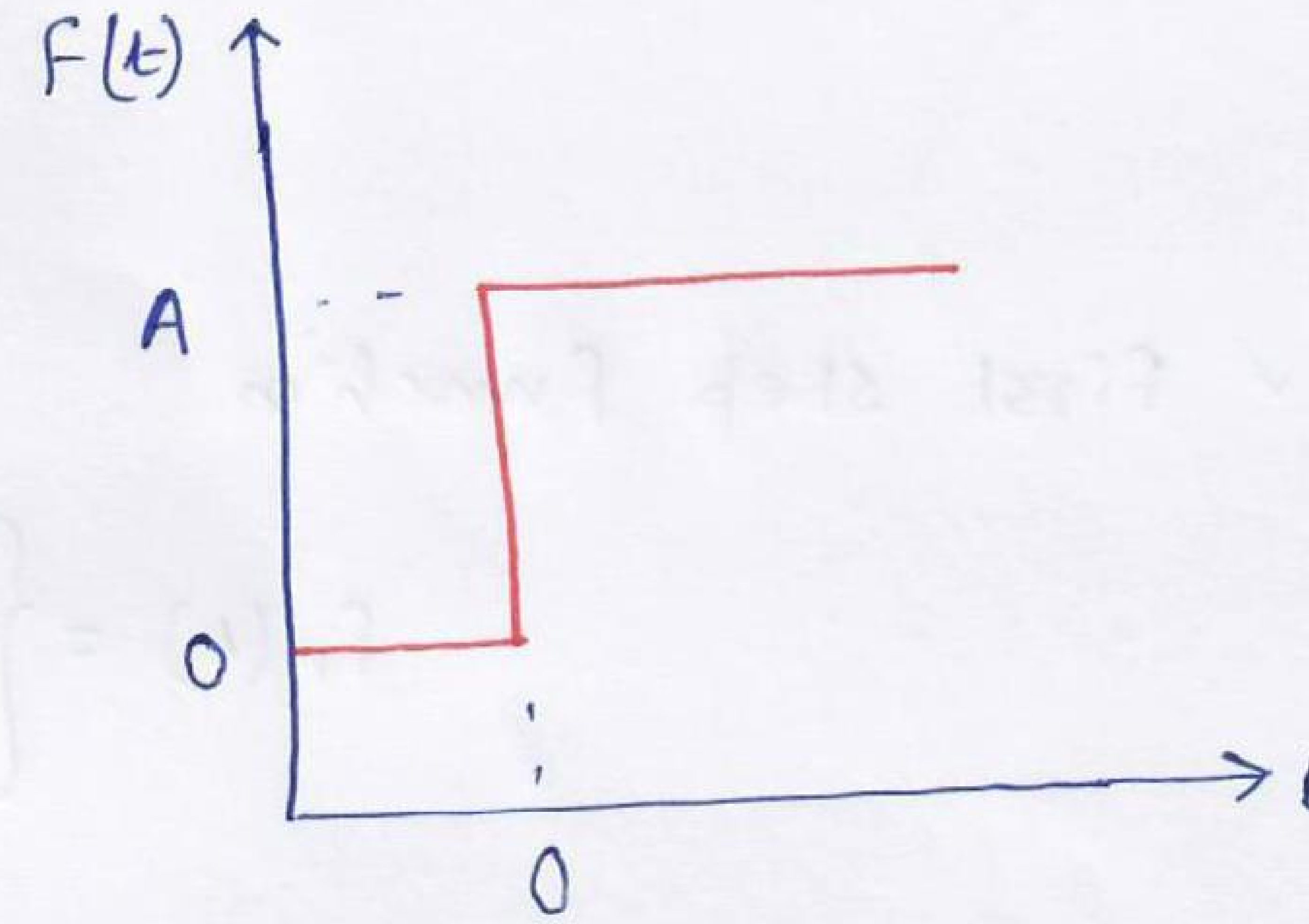
Initial value theorem

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty}\left[s\bar{f}(s)\right]$$

Ideal forcing functions.

1. Ideal step function

$$f(t) = \begin{cases} 0 & \text{when } t<0 \\ A & \text{when } t \geq 0 \end{cases}$$



$A \equiv$ magnitude

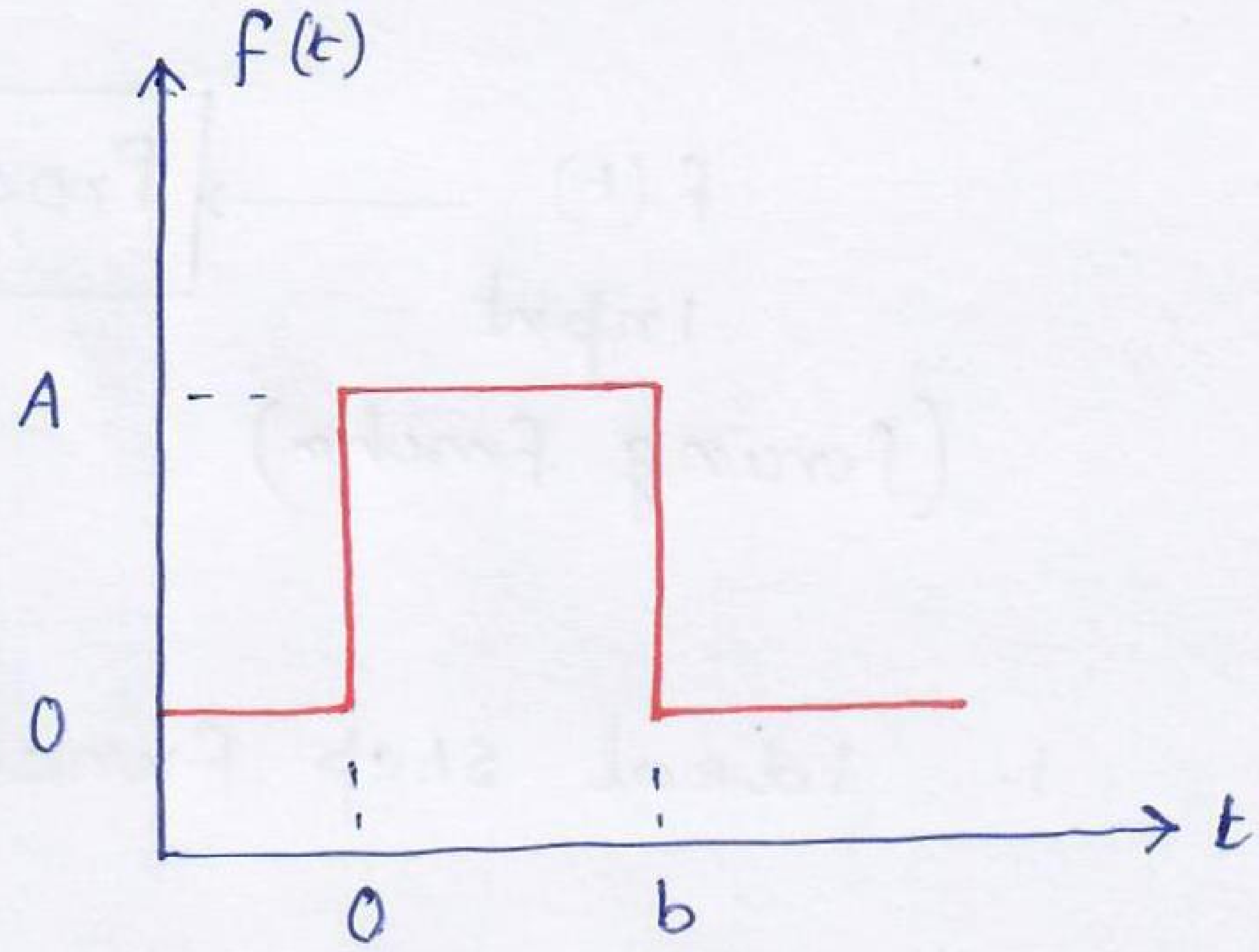
$$\boxed{L[f(t)] = \frac{A}{s}}$$

$$\because \bar{f}(s) = \int_0^\infty A e^{-st}\,dt = -\frac{A}{s}\left[e^{-st}\right]_0^\infty = \frac{A}{s}.$$

- Unit step change when $A = 1$ & $\bar{f}(s) = \frac{1}{s}$.
- Ex: open the steam valve with a certain % such that f changes by A units.

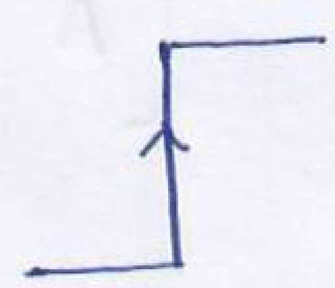
2. Ideal rectangular pulse function

$$f(t) = \begin{cases} 0 & \text{when } t<0 \\ A & \text{when } 0<t<b \\ 0 & \text{when } t>b \end{cases}$$



✓ First step function

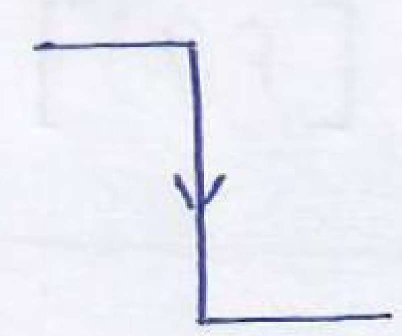
$$f_1(t) = \begin{cases} 0 & \text{when } t<0 \\ A & \text{when } t>0 \end{cases}$$



✓ Second step function

$$f_2(t) = \begin{cases} 0 & \text{when } t<b \\ A & \text{when } t>b \end{cases}$$

or $(t-b)>0$



$$= -f_1(t-b)$$

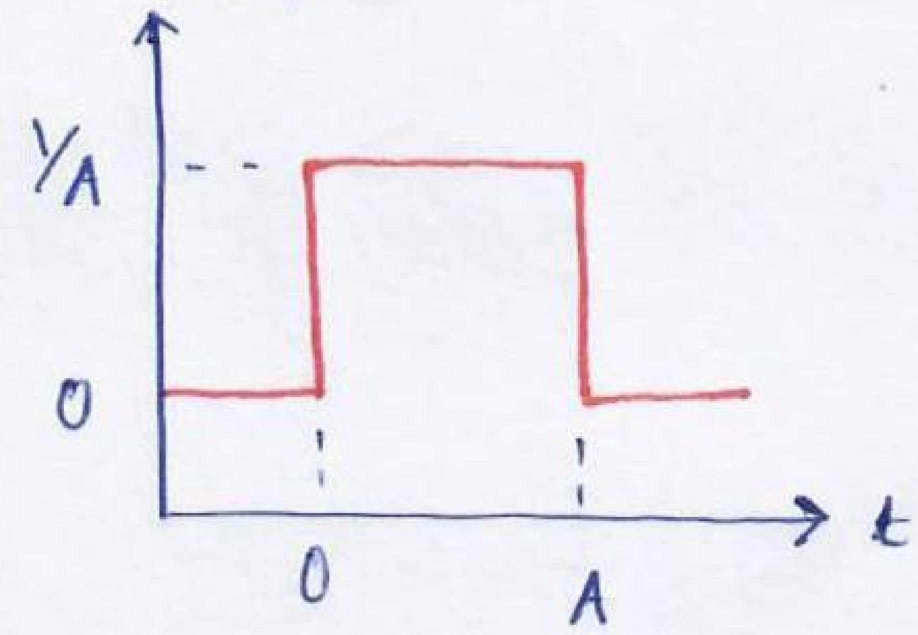
$$\therefore \quad f(t) = f_1(t) + f_2(t) = f_1(t) - f_1(t-b).$$

$$\boxed{\bar{f}(s) = \frac{A}{s} - \frac{A}{s} e^{-bs} = \frac{A}{s}\left(1-e^{-bs}\right)}$$

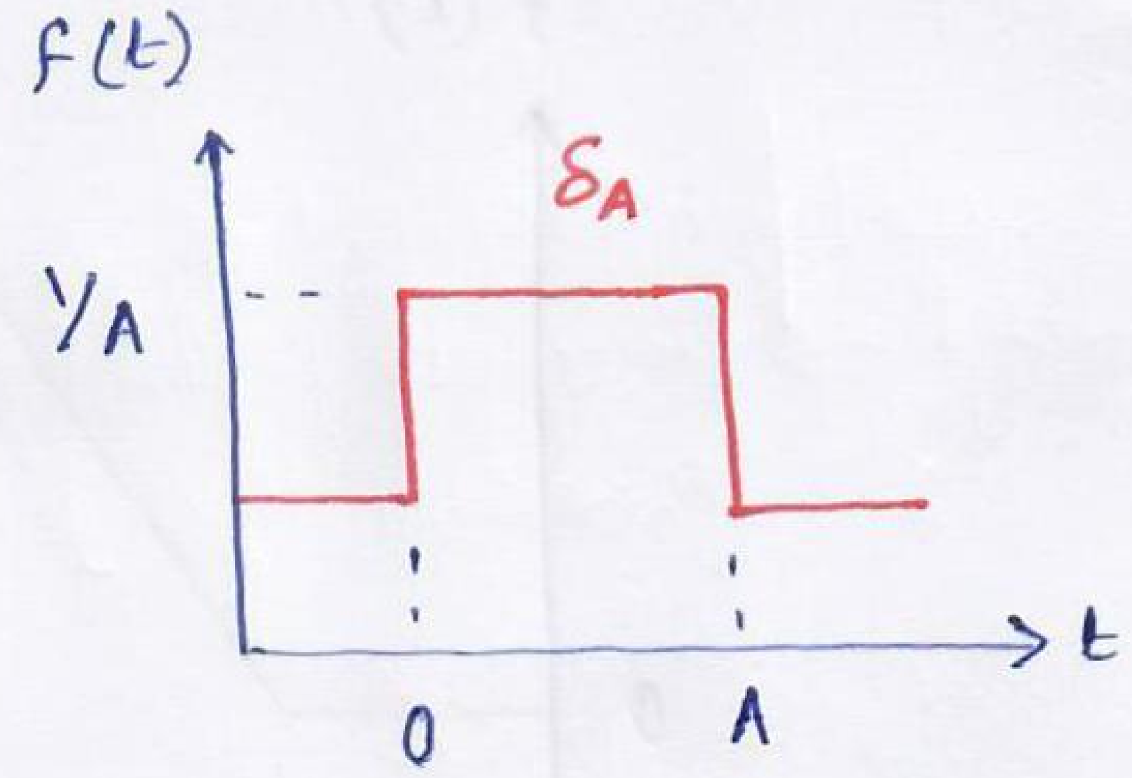
Unit pulse (Area = 1).

$$\bar{f}(s) = \frac{1}{A}\int_0^A e^{-st}\, dt$$

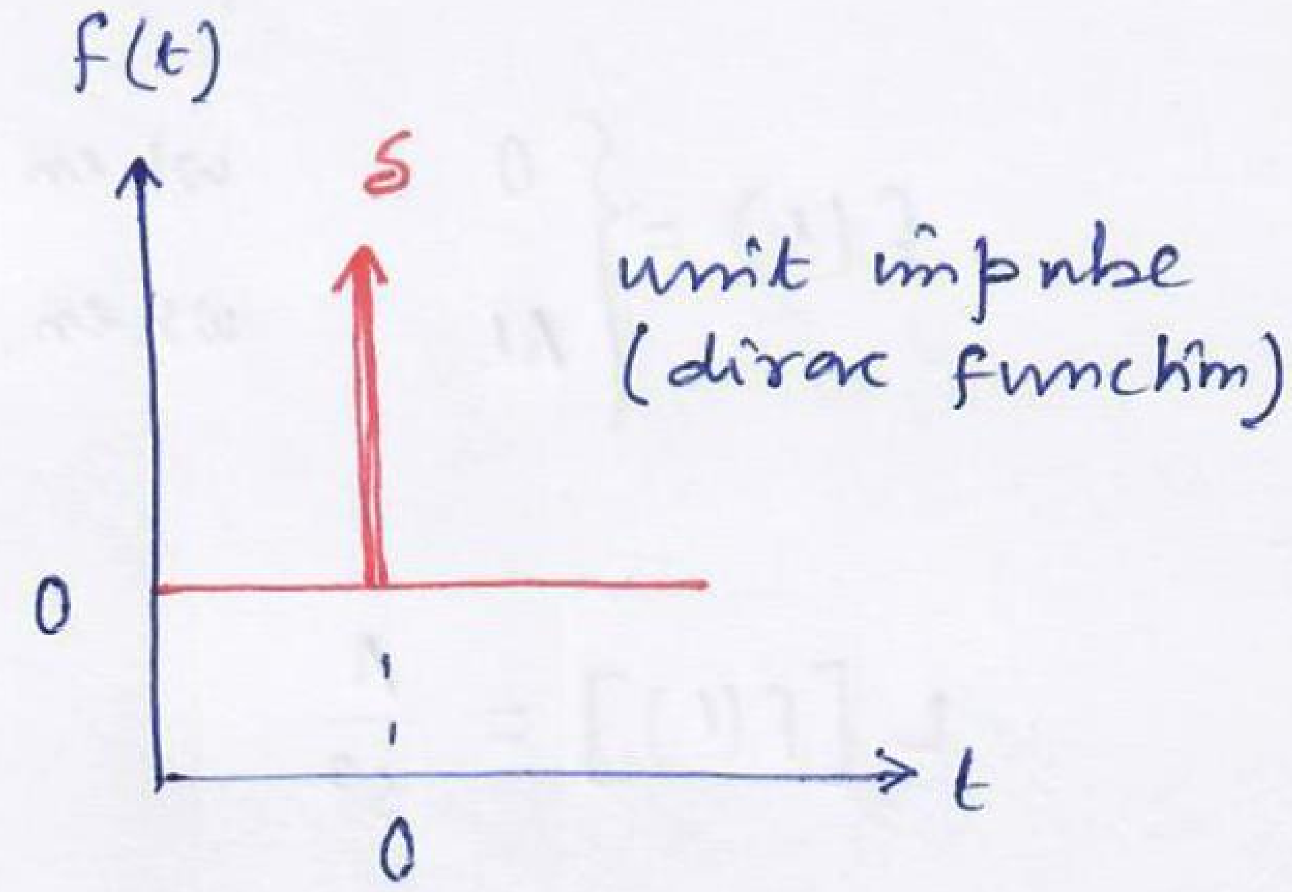
$$= \frac{1-e^{-sA}}{sA} = L\left[\delta_A(t)\right]$$



3. Ideal impulse function



Unit pulse



Unit impulse

Unit impulse (Area = 1)

$$\boxed{\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1}$$

Again, $\delta = \lim_{A \to 0} \delta_A(t)$

$$L[\delta(t)] = L\left[\lim_{A \to 0} \delta_A(t)\right] = \int_0^{\infty} \lim_{A \to 0} \delta_A(t)\, e^{-st}\, dt$$

$$= \lim_{A \to 0} \frac{1 - e^{-sA}}{sA} = 1$$

$$\therefore \quad \boxed{L[\delta(t)] = 1}$$

Impulse function of area A

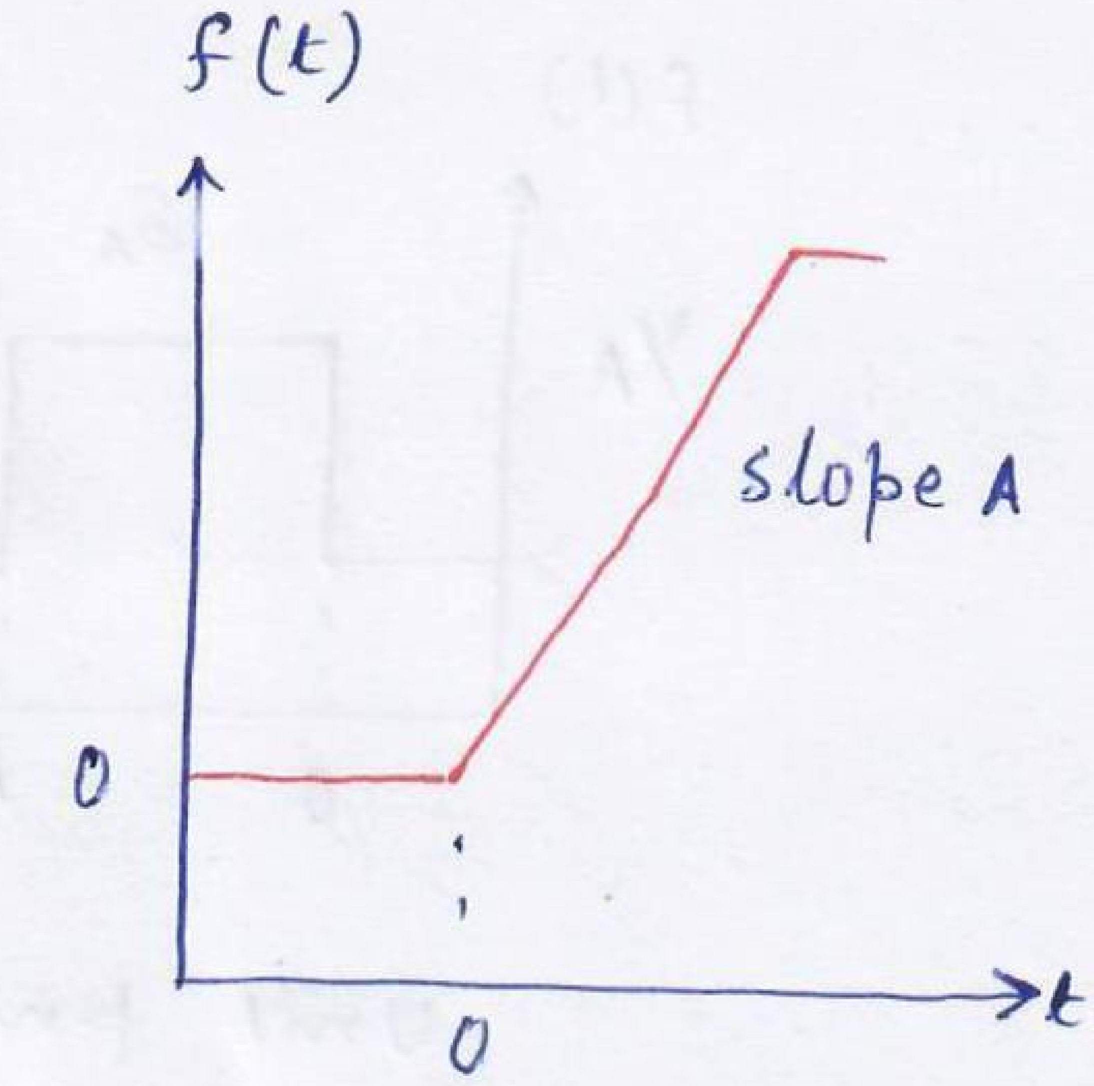
$$F(t) = A\,\delta(t)$$

$$L[f(t)] = A.$$

Ex. Open the steam valve at $t = 0$ and instantly (as soon as physically possible thereafter) return to the initial position.

4. Ideal ramp function

$$f(t) = \begin{cases} 0 & \text{when } t < 0 \\ At & \text{when } t > 0 \end{cases}$$



$$L\left[f(t)\right] = \frac{A}{s^2}$$

Ex. Open the steam valve such that f increases linearly. Ramp ends when the valve is fully open.