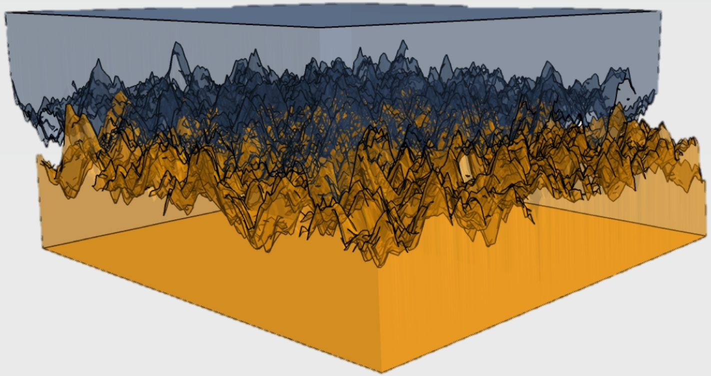
Presentation by Group-3

Mechanical Operations

Topics of Discussion

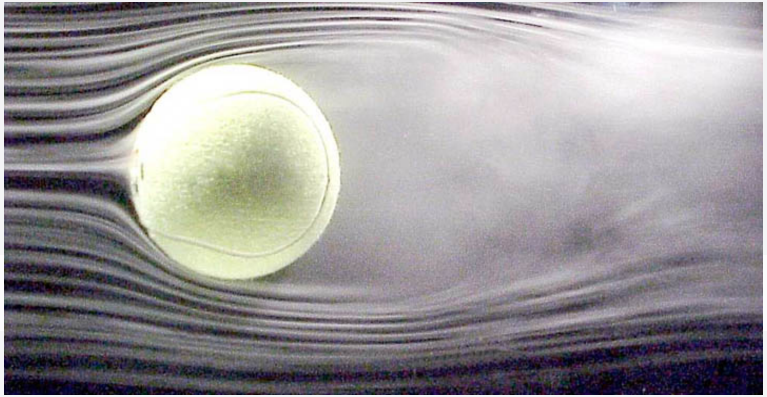
- **Friction in Bulk Solids**
- **Angle of Wall Friction**
  - Principles of Kinematic Wall Friction
  - Measurement of Angle of Wall Friction
  - Static Friction
- **Mohr's Circle**
  - Principal Stresses of a Bulk solid
  - Mohr Circle and its Equation
  - Properties of Mohr Circle
- **Uniaxial Compression Test**
  - Consolidation of Bulk Solids
  - Yield limit curve using Mohr's Circle

Models of Friction



Coulomb Friction Model

$$f_{max} = \mu F_n sign(v)$$



Viscous Friction Model

$$f = -bv$$

Friction in Bulk Solids

Bulk solids obey Coulombic Frictional Behavior against container and process equipment surfaces.

Angle of Wall Friction is the most important parameter that characterises the frictional properties of Bulk solids.

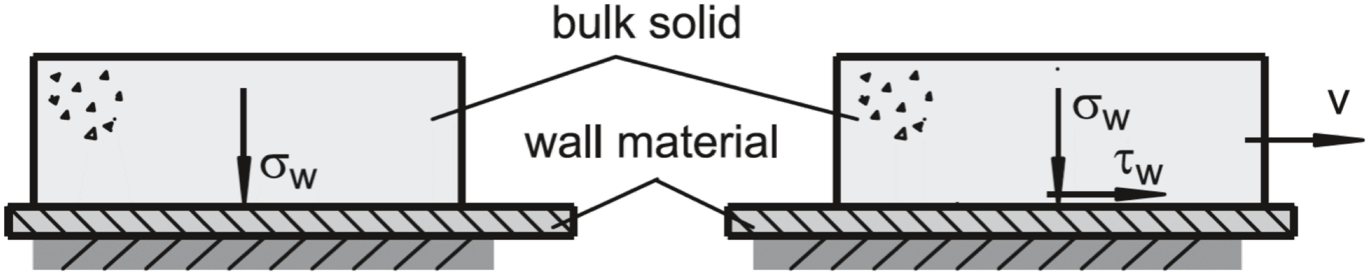
Applications of φ_x

- Silo Design for flow
- Silo Design for Strength
- Design of Chutes and Other Equipment where the bulk solid flows across a solid surface.

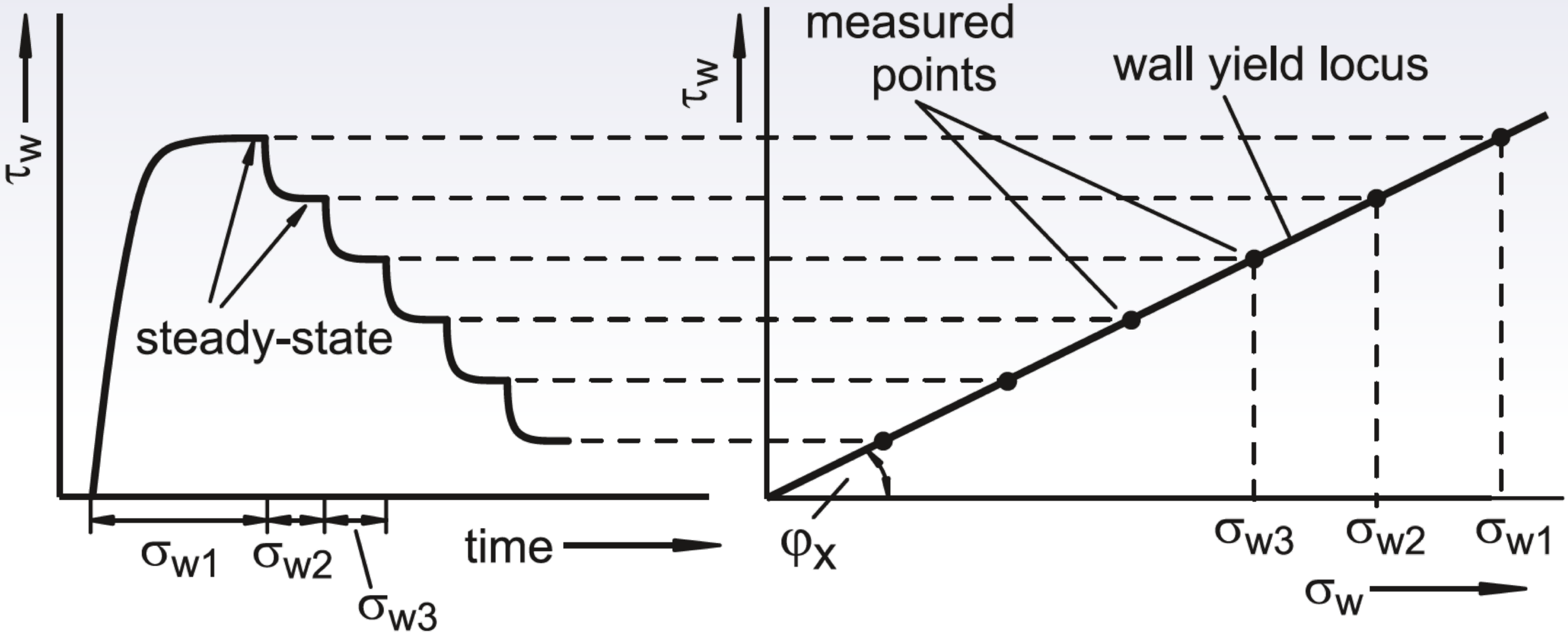
Measurement of Wall Friction

Wall Yield Locus & Kinematic Wall Friction

At the beginning of the shear process the wall shear stress, τ_w, increases with time, the rate of increase of the wall shear stress becomes less until finally a constant wall shear stress is attained (steady-state shear stress).



Principle of Test Setup for a Wall Friction Test



The curve (or line) running through the measured points is called the *wall yield locus*. Since the wall yield locus is based on the shear stresses measured at steady-state conditions, it describes the kinematic friction of the bulk solid.

Angle of Wall Friction

To quantify wall friction, the wall friction angle φ_x, or the coefficient of wall friction μ, are used.

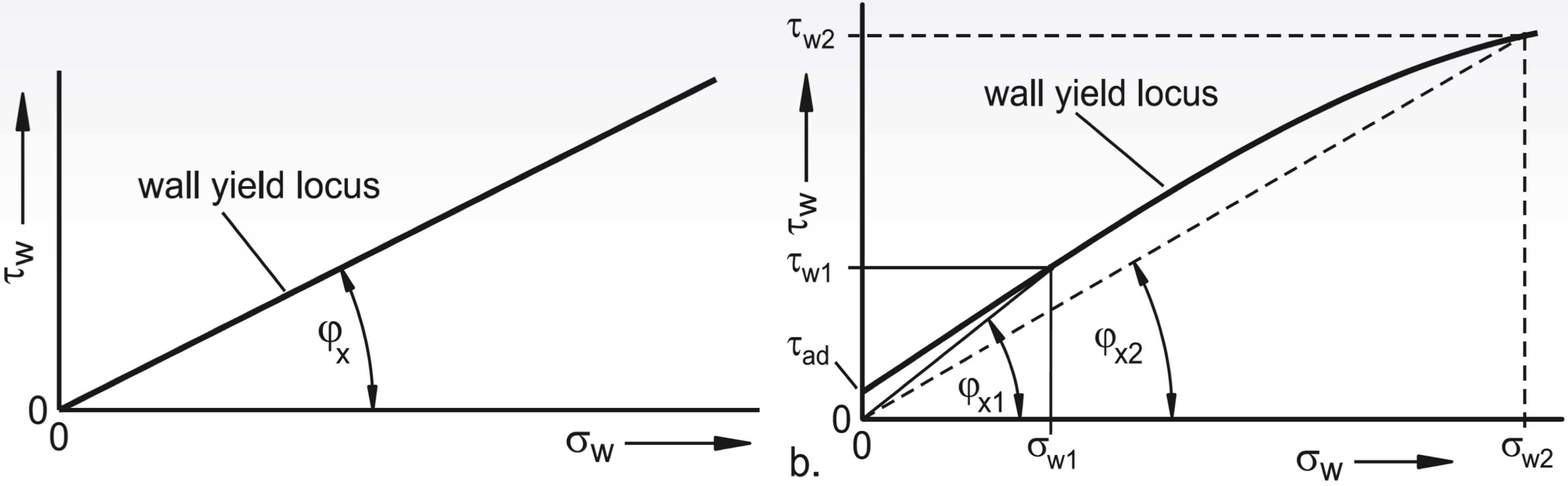
- The coefficient of wall friction μ, is defined as the ratio of wall shear stress τ_w, to wall normal stress, σ_w, for a point on the wall yield locus,

$$\mu = \frac{\tau_w}{\sigma_w}$$

- The wall friction angle, φ_x, is the slope of a line running from the origin of the σ_w, τ_w diagram to a point on the wall yield locus.

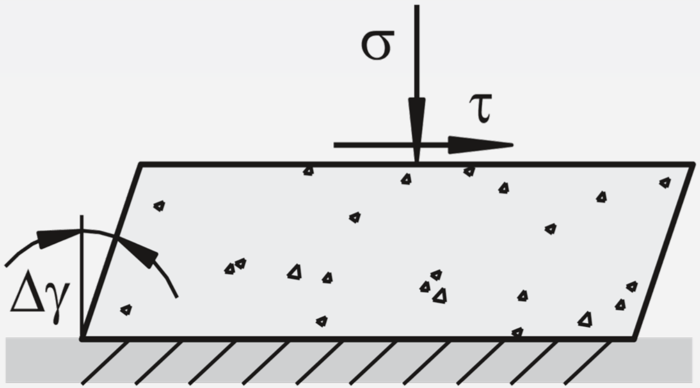
$$\varphi_x = Tan^{-1}\left(\frac{\tau_w}{\sigma_w}\right)$$

Wall yield locus

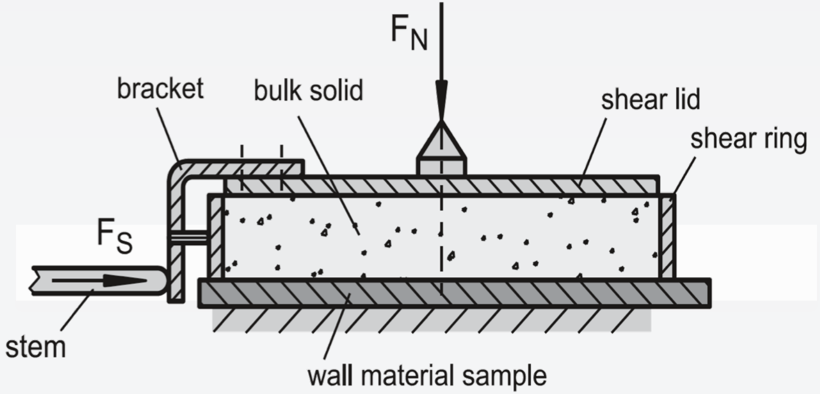


Values $\tau_{ad} > 0$ are found for bulk solid which can adhere at vertical walls due to large adhesive forces (e.g. moist clay, where the adhesive forces due to liquid bridges are dominant).

Measurement of Kinematic Friction



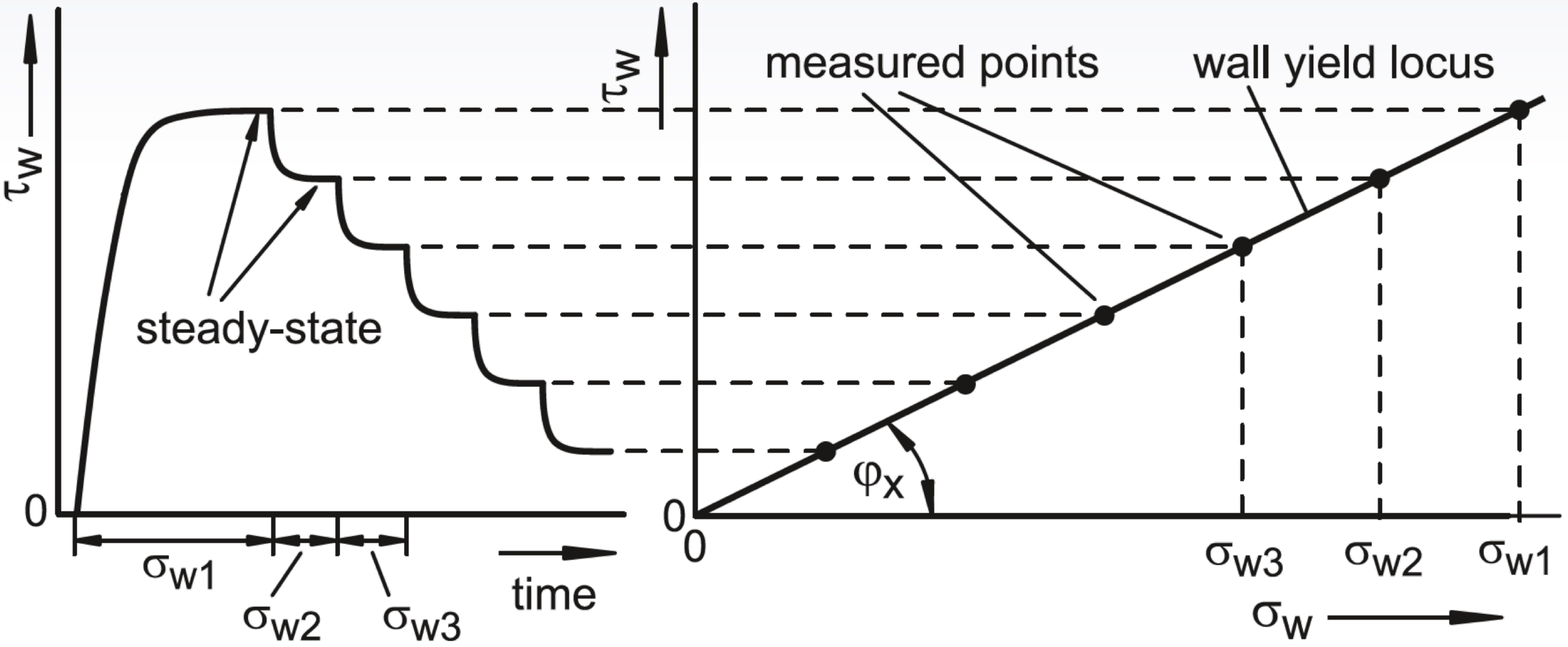
Principle of the shear deformation in a translational shear tester



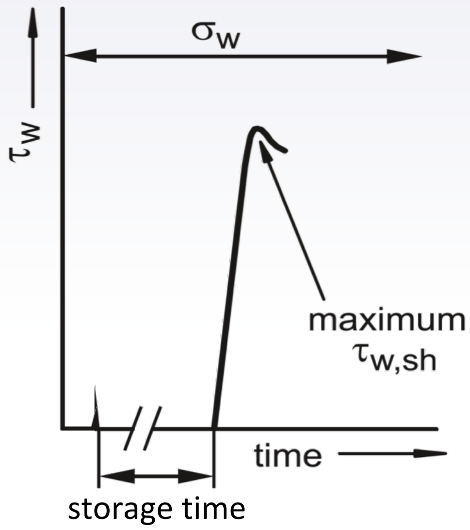
Wall Friction Measurement with the Jenike shear tester

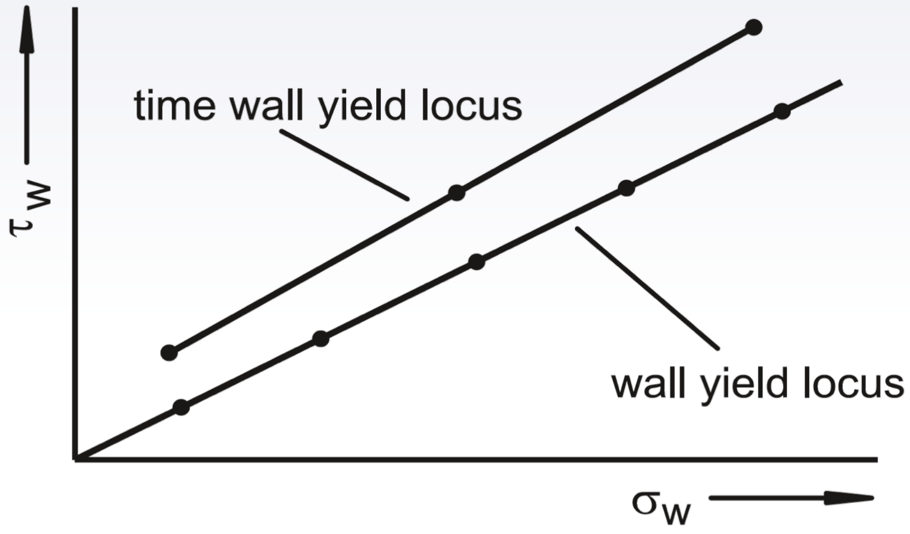
We ensure a thinner specimen of the bulk solid, for two reasons:

- So that internal friction of the bulk solid doesn't dominate.
- The normal force exerted by the wall on the material is $(F_n + \text{mg})$ and the mg gravitational force can be neglected if the thickness is reduced.



Static Friction





The time wall yield locus defines the wall shear stress, τ_w, required to initiate flow of a bulk solid across the surface of a wall after the bulk solid has been stored at rest under the wall normal stress σ_w for a specific period of time on this surface. In analogy to a wall yield locus, wall friction angles can be determined from a time wall yield locus; these wall friction angles are static wall friction angles, $\varphi_{x,st}$.

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Stresses in Bulk Solids: 2-Dimensional

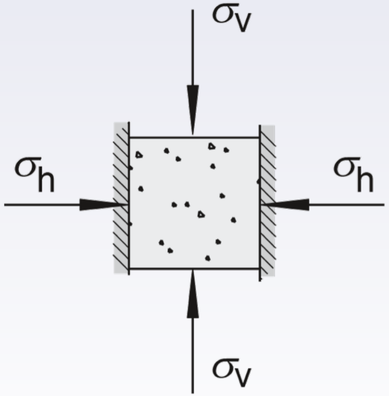
Bulk Solid Element

- Assumptions: Infinite filling height & frictionless internal walls
- Lateral stress ratio, K

$$k = \lambda = \frac{\sigma_h}{\sigma_v}$$

- Typical values of K are ***between 0.3 and 0.6,*** means $\boldsymbol{\sigma_h < \sigma_v}$

- ***No shear stresses*** are exerted on the ***top or bottom surface*** of the bulk solid element i.e., the shear stresses, $\boldsymbol{\tau}$
- ***No shear stresses*** are acting at the ***lateral walls***, since the walls were assumed frictionless.
- Thus ***only the normal stresses shown are acting on the bulk solid*** from outside.

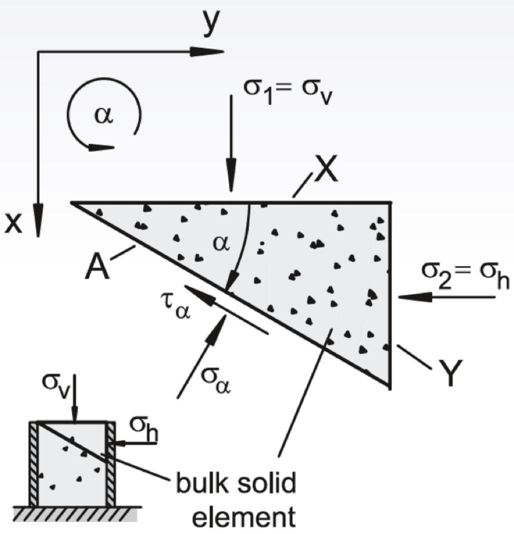


Principal Stress

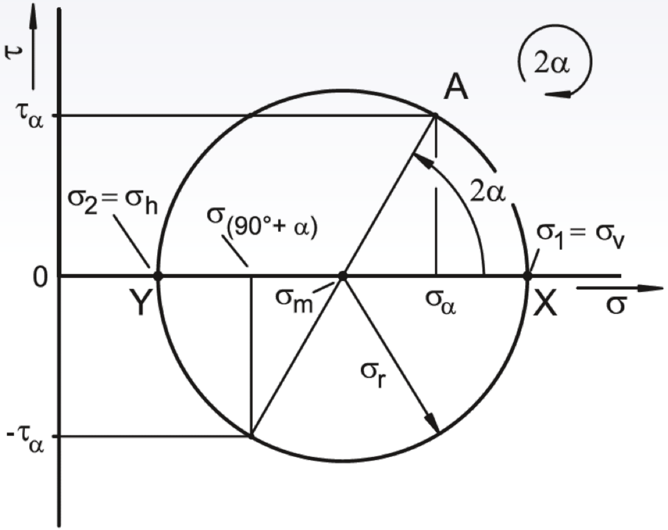
- If on a given plane of the bulk solid, ***only*** either $\boldsymbol{\sigma}$ or $\boldsymbol{\tau}$ will act ***but not both***
- In general, exclusively acting ***extreme normal stresses*** on a plane are called as ***Principal stresses*** and corresponding Planes as ***Principal Planes***

Expressions of σα and τα

- ***Equilibrium of forces*** on a volume element with ***triangular cross-section*** cut from the bulk solid element
- Normal stress, **σα**, and the Shear stress, **τα**, acting on a plane inclined by an arbitrary angle **α**



Mohr Stress Circle

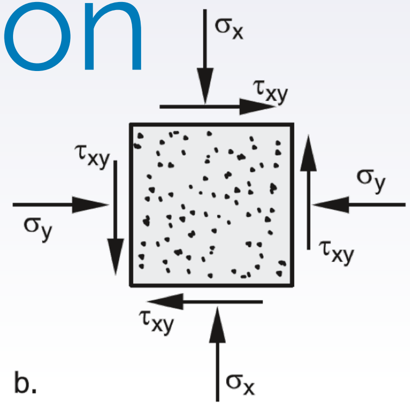


$$\sigma_\alpha = \frac{\sigma_v + \sigma_h}{2} + \frac{\sigma_v - \sigma_h}{2}\cos(2\alpha)$$

$$\tau_\alpha = \frac{\sigma_v - \sigma_h}{2}\sin(2\alpha)$$

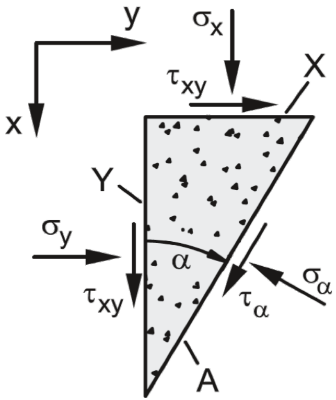
Derivation of the equation

Consider the ***length of the hypotenuse to be l*** and assume thickness to be uniform through out the element to cancel out the areas. So, lengths of the sides are $\boldsymbol{x = lcos(\alpha)}$ and $\boldsymbol{y = lsin(\alpha)}$



Force balancing in x and y – directions:

$$\tau_{xy} l \sin(\alpha) + \sigma_y l \cos(\alpha) = \tau_\alpha \sin(\alpha) l + \sigma_\alpha \cos(\alpha)\, l$$
$$\tau_{xy} l \cos(\alpha) + \sigma_x l \sin(\alpha) = -\tau_\alpha \cos(\alpha) l + \sigma_\alpha \sin(\alpha)\, l$$



Solving,

$$\sigma_\alpha = \sigma_y cos^2\alpha + \sigma_x sin^2\alpha + 2\tau_{xy} cos\alpha\ sin\alpha$$
$$\tau_\alpha = \sigma_y cos\alpha\ sin\,\alpha - \sigma_x cos\alpha\ sin\alpha\ - \tau_{xy} cos^2\alpha + \tau_{xy} sin^2\alpha$$

From the trigonometric relations/ identities:

$$\sin 2\alpha = 2 \sin\alpha \cdot \cos\alpha\,; \quad sin^2\alpha + cos^2\alpha = 1;$$
$$\cos 2\alpha = 2cos^2\alpha - 1 = 1 - 2sin^2\alpha = \ cos^2\alpha - sin^2\alpha$$

Derivation of the equation

$$\sigma_\alpha = \sigma_y\left(\frac{1+\cos 2\alpha}{2}\right) + \sigma_x\left(\frac{1-\cos 2\alpha}{2}\right) + \tau_{xy}\sin 2\alpha$$

$$\tau_\alpha = \sigma_y\left(\frac{sin\, 2\alpha}{2}\right) - \sigma_x\left(\frac{sin\, 2\alpha}{2}\right) - \tau_{xy}\cos 2\alpha$$

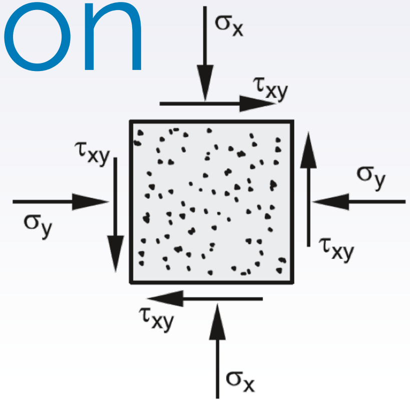
$$\Rightarrow\ \sigma_\alpha = \left(\frac{\sigma_y+\sigma_x}{2}\right) + \cos 2\alpha\left(\frac{\sigma_y-\sigma_x}{2}\right) + \tau_{xy}\sin 2\alpha$$

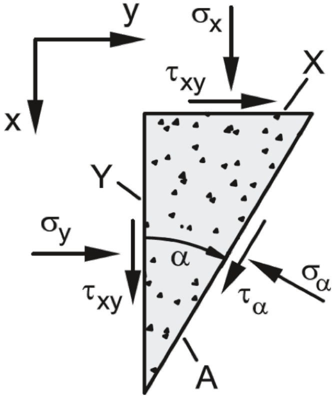
$$\Rightarrow\ \tau_\alpha = sin\, 2\alpha\left(\frac{\sigma_y-\sigma_x}{2}\right) - \tau_{xy}\cos 2\alpha$$

$$\Rightarrow \left(\sigma_\alpha - \frac{\sigma_y+\sigma_x}{2}\right)^2 + {\tau_\alpha}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + {\tau_{xy}}^2$$

This is of the form $(\boldsymbol{x}-\boldsymbol{x_0})^2 + (\boldsymbol{y}-\boldsymbol{0})^2 = \boldsymbol{R}^2$

i.e., ***circle whose centre lies on the x-axis***.





Mohr Circle Properties

- Its center is located at σ = σ_m = $\frac{(\sigma_v+\sigma_h)}{2}$ and **τ = 0**.
- The radius of the circle is σ_R = $\sqrt{(\frac{(\sigma_v-\sigma_h)}{2})^2+\tau_{xy}{}^2}$.
- Normal stresses defined through the points of intersection with X-axis are called the principal stresses, whereby ***major principal stress is*** σ_1 and the ***minor principal stress is*** σ_2 and the respective planes where those stresses are acting are called major and minor ***principal planes*** respectively. $\boldsymbol{\sigma_1 = \sigma_m + \sigma_R}$ **;** $\boldsymbol{\sigma_2 = \sigma_m - \sigma_R}$
- The stresses acting on a ***cutting plane that is rotated at an angle α*** to the first cutting plane are found at a point on the ***Mohr circle rotated by an angle 2α*** relative to the point representing the stresses in the first cutting plane, where the angle **2α** on the Mohr stress circle is measured in the ***opposite direction*** from the angle **α** between the cutting planes in the bulk solid

Mohr Circle Properties

- $\boldsymbol{\sigma_y}$ is obtained by substituting **α= α+90°** ***in*** $\boldsymbol{\sigma_x}$, because they're always perpendicular to each other where one is the maximum and other is the minimum stress
- τ_α is maximum when $\sigma = \sigma_m$ i.e., $\boldsymbol{\tau_{max}} = \boldsymbol{\sigma_R} = \frac{\sigma_1 - \sigma_2}{2}$
- α_p = angle where principal stresses are found, α_s = angle where maximum shear stress is found; $\tan(2\alpha_p) = \tan(2\alpha_s) = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \times \left(\frac{\sigma_x - \sigma_y}{-2\tau_{xy}}\right) = -1$ i.e., ***principal and maximum shear stresses occur at 45°to each other***

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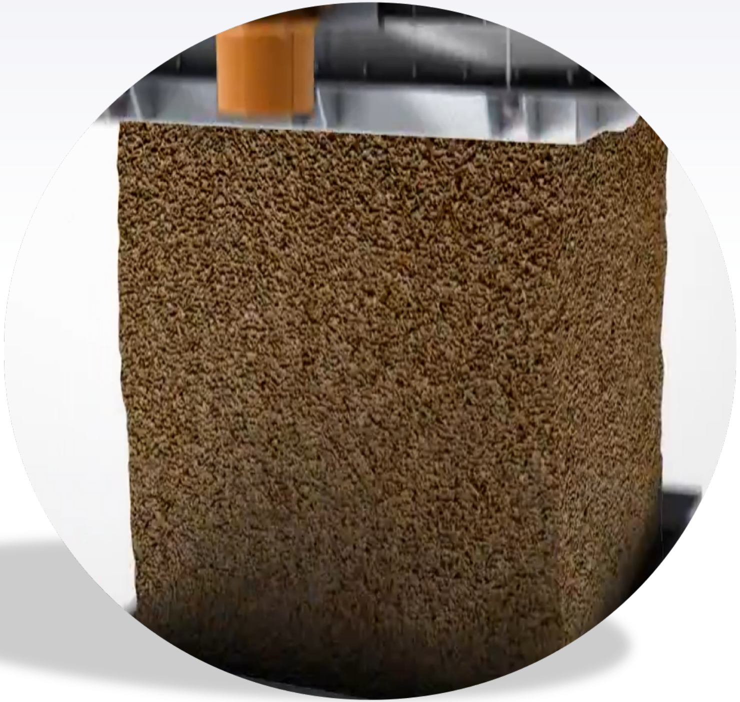
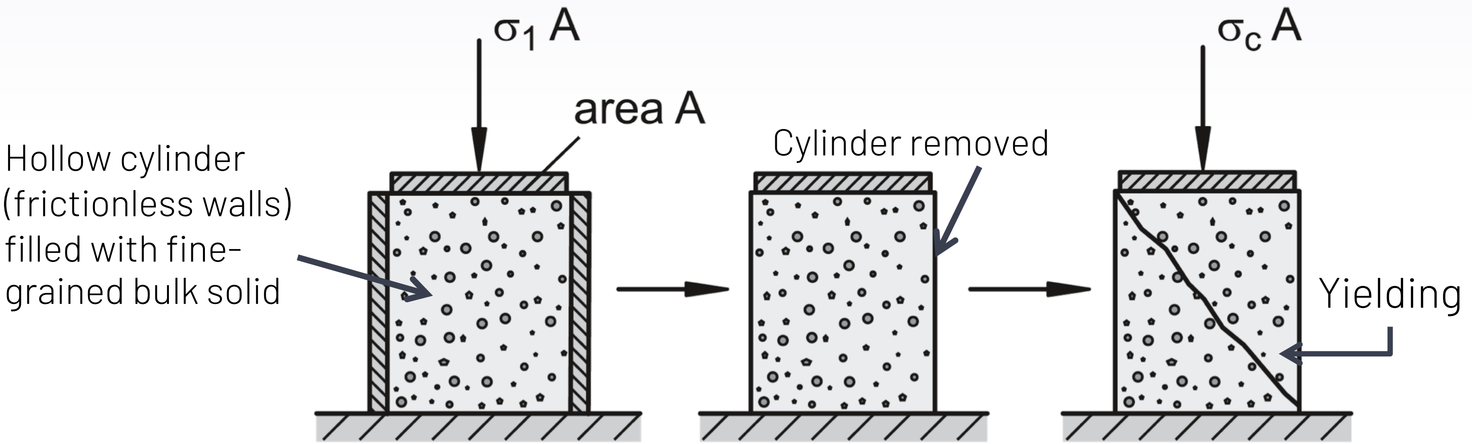
Consolidation of Bulk Solids



Consolidation of Bulk Solids

"Flowing" means that a bulk solid is deformed plastically due to the loads acting on it. The magnitude of the load necessary for flow is a measure of flowability.

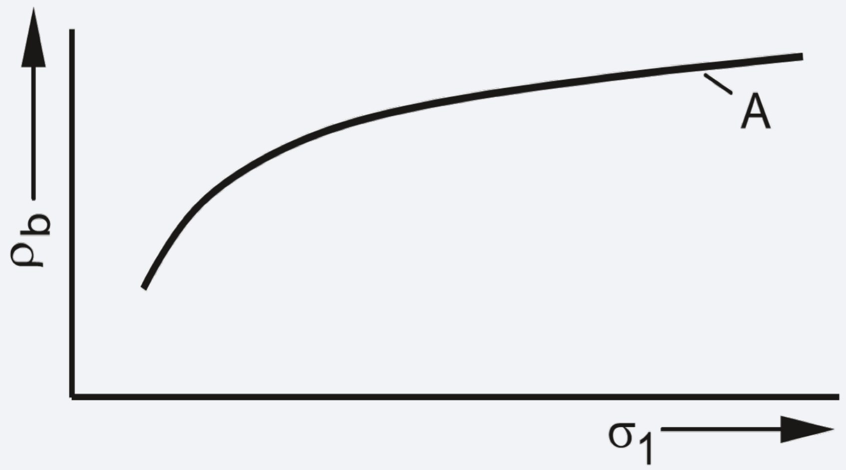


Confined State | **Unconfined State** | **Incipient Flow (plastic deformation with decrease of bulk density)**

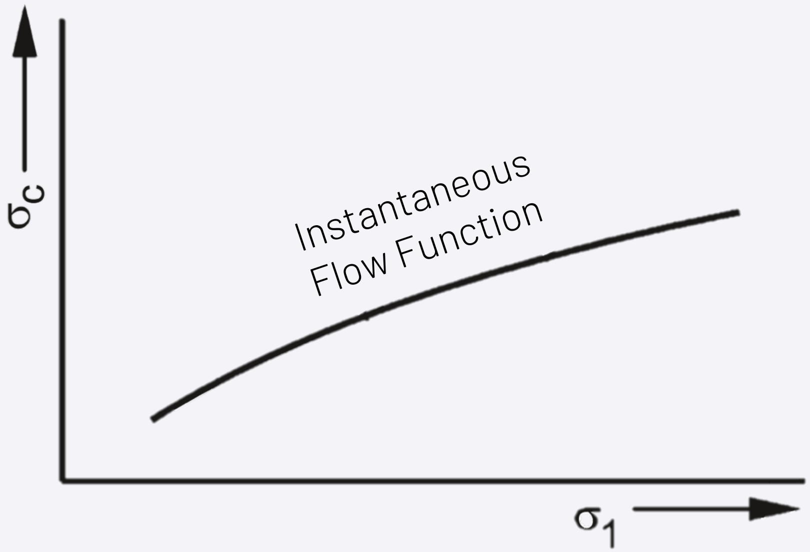
$\sigma_1 \equiv$ Consolidation stress (or) major principal stress

$\sigma_c \equiv$ Compressive Strength $\equiv$ Cohesive Strength $\equiv$ Unconfined Yield Strength

Experiment Analysis



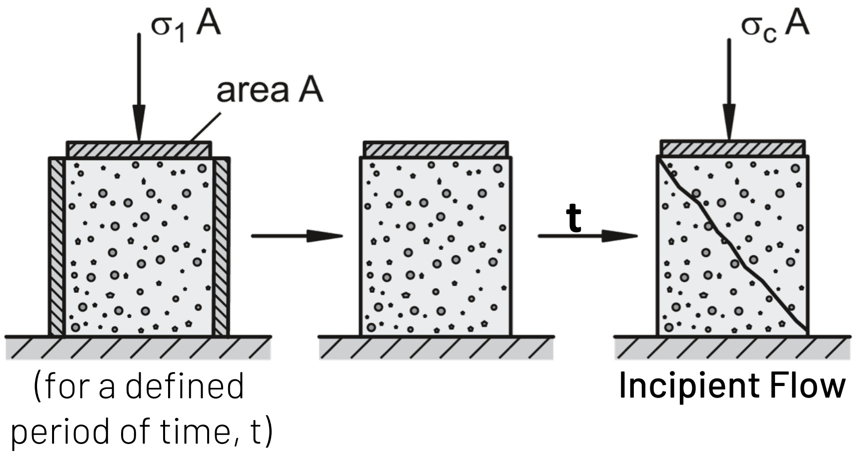
Dependence of Bulk Density ρ_b on Consolidation Stress σ_1

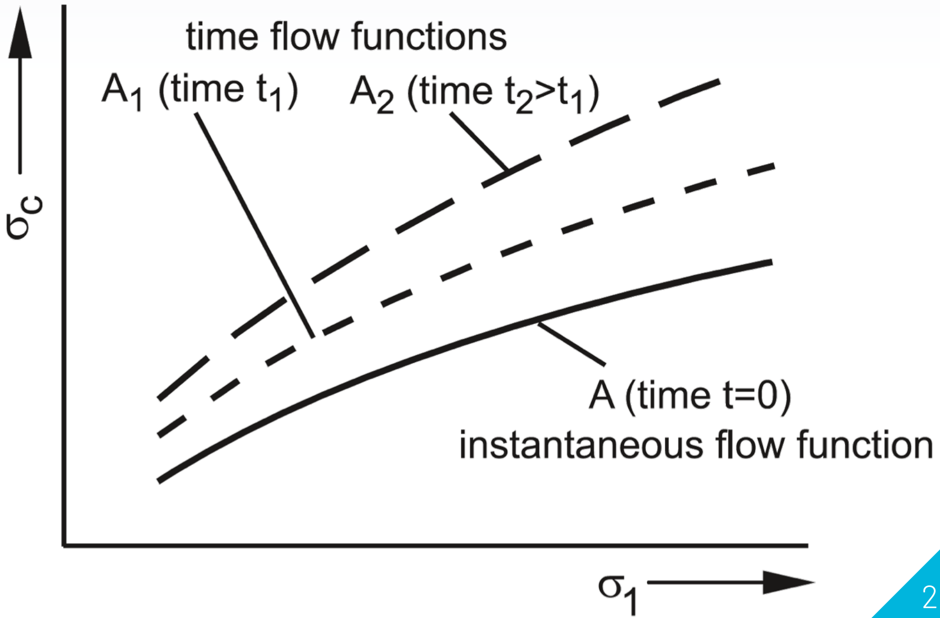


unconfined yield strength, σ_c *vs* consolidation stress, σ_1

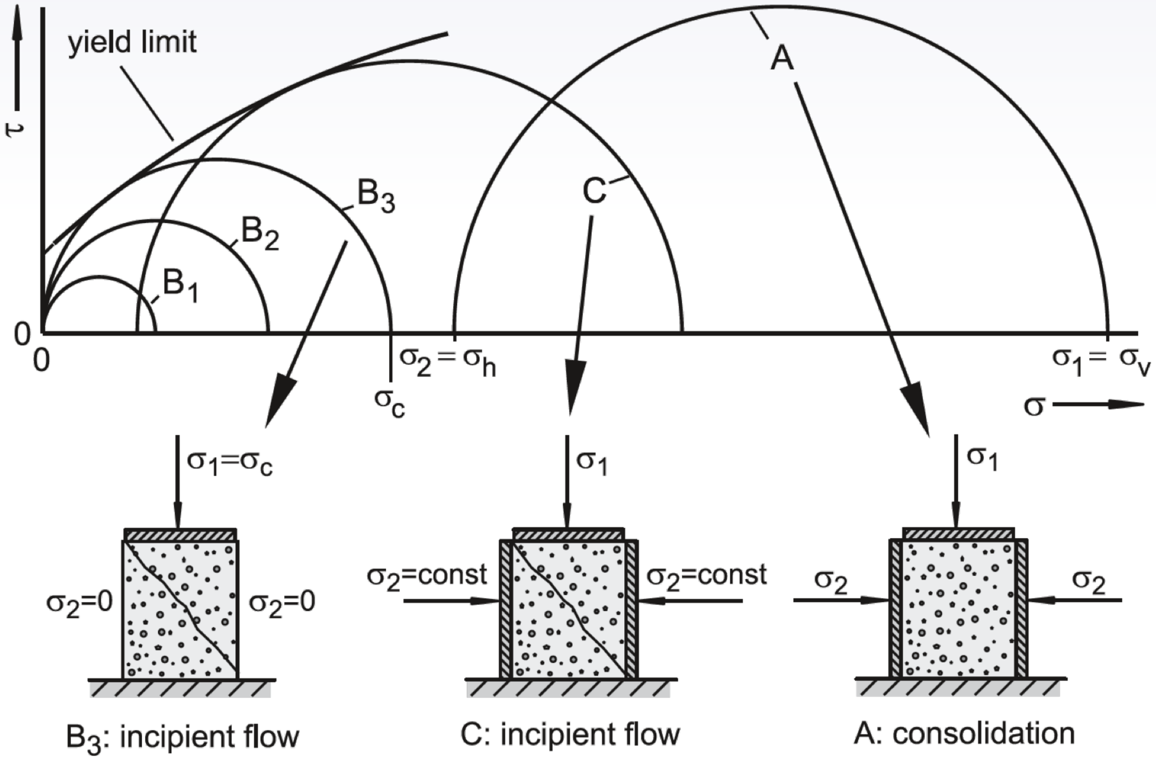
Time Consolidation

Some bulk solids **increase in strength** if they are stored for a period of time at rest under a compressive stress (e.g. in a silo or an IBC). This effect is called ***time consolidation*** or ***caking***.



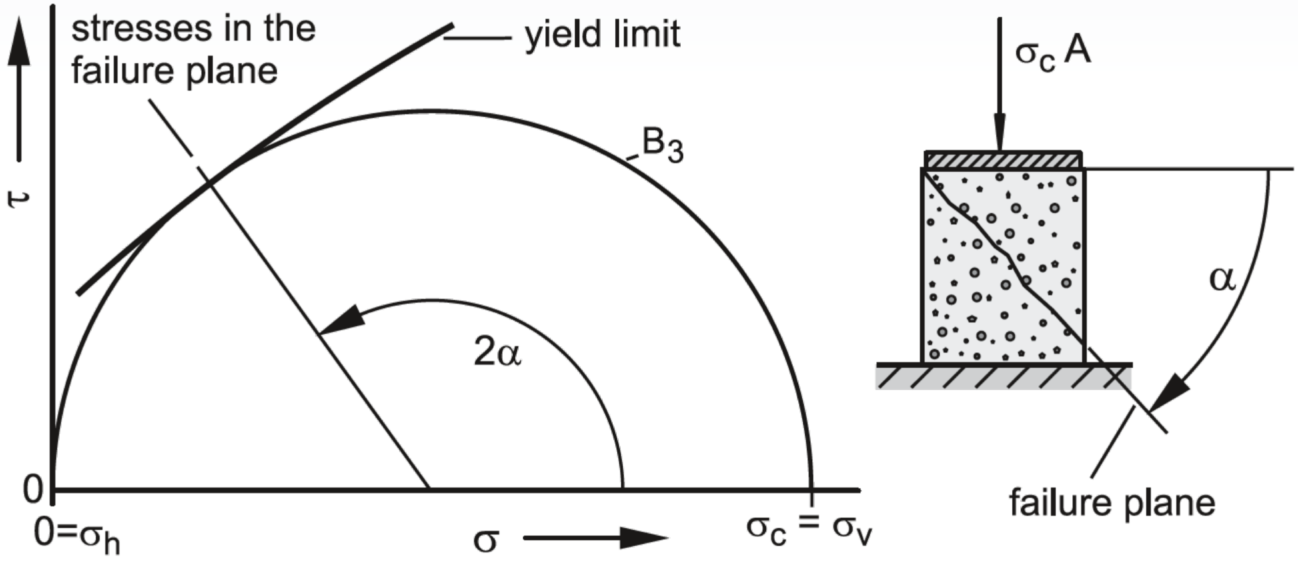


Representation of stresses using Mohr stress circles



Yield Limit and the Failure Plane

- For every normal stress, σ, **Yield Limit** gives a shear stress, τ, which is necessary to initiate flow
- Yield limit is the **envelope** of all stress circles that indicate **failure** of a bulk solid sample



THANK YOU!

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