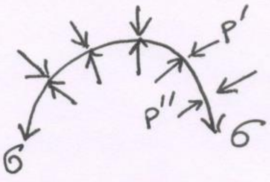
Surface tension in spherical cap



Effect of surface tension is to reduce the size of the sphere, unless it is opposed by a sufficiently great difference between pressures P'' and P'.

Increase in radius by dr changes the interfacial area by dA where $dA = d(4\pi r^2) = 8\pi r\, dr$, and interfacial energy by dU, where $dU = \sigma\, dA$

Force is coefficient of proportionality between increase/decrease of energy (δU) of the system and small displacement. (Force)(displacement) defines the energy.

work = (Force)(displacement) or $F = \pm \frac{dU}{dx}$

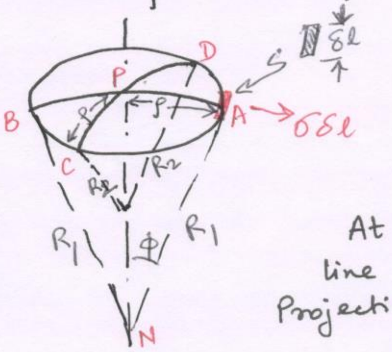
$\Rightarrow U = \pm \int F\, dx$

$\Rightarrow F(r) = \sigma(8\pi r)$. $\quad P = \frac{F}{A} = \frac{2\sigma}{r}$ (Laplace Pressure)

$= P'' - P'$

→ Squeeze of droplet

Non-Spherical cap



P is a point on the surface

AB and CD are arbitrary pair of orthogonal lines, drawn along the surface.

R_1 and R_2 are the radii of curvature at P.

At point 'A', an element δl of the boundary line is subjected to a force $\sigma \delta l$.

Projection of this force along PN $= \sigma \delta l \sin\phi$

$= \sigma \delta l \left(\frac{\rho}{R_1}\right)$ when ϕ is small.

Similar contribution of all points A, B, C, D

$$= \sigma \delta l \left[\frac{2\rho}{R_1} + \frac{2\rho}{R_2}\right] = 2\rho\sigma\delta l\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

where r_1, r_2 are the principal radii of curvature, and follows Euler theorem $\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{R_1} + \frac{1}{R_2}$

The forces due to surface tension to be balanced by the pressure forces for mechanical equilibrium.

$$(P'' - P')\pi\rho^2 = \int_0^{\frac{2\pi\rho}{4}} \left\{2\rho\sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right\}\delta l = \sigma\left[\frac{1}{r_1} + \frac{1}{r_2}\right]\pi\rho^2$$

To avoid repetition of forces, integration was performed over one quarter of a revolution.

$P'' - P' = 2\frac{\sigma}{r_m}$, where r_m is the mean radius of curvature. $\quad \frac{1}{r_m} = \frac{1}{2}\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$

P'' is the pressure on concave side.