Date ______ Page ______

# Assignment 4

Varun Madhavan
18CH30031

## Section 1

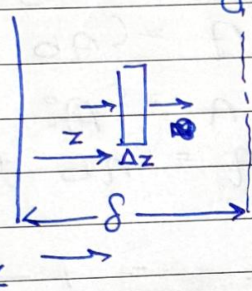
1.1 **Mass transfer in gas phase**

$$N_A = k_G (P_{A1} - P_{A2}) = k_y (y_{A1} - y_{A2})$$
$$= k_c (C_{A1} - C_{A2})$$

**Mass transfer in the liquid phase**

$$N_A = k_x (x_{A1} - x_{A2}) = k_L (C_{A1} - C_{A2})$$

1.2 (a) Film theory




mass conservation : $N_A|_z - N_A|_{z+\Delta z} = 0$

$$\therefore -\frac{dN_A}{dz} = 0 \quad \text{as } \Delta z \to 0$$

$$N_A = -D_{AB} \cdot \frac{dC_A}{dz} \Rightarrow D_{AB} \cdot \frac{d^2C_A}{dz^2} = 0$$

$$\to \frac{d^2 C_A}{dz^2} = 0$$

$$\therefore C_A = k_1 + k_2 \cdot z$$

@ $z = 0$, $C_A = C_{Ai}$,

@ $z = \delta$, $C_A = C_{Ab}$

$$\therefore C_A = C_{Ai} - (C_{Ai} - C_{Ab})\frac{z}{\delta}$$

$$N_A = \frac{D_{AB}}{\delta}(C_{Ai} - C_{Ab})$$

$$\Rightarrow \boxed{K_L = \frac{D_{AB}}{\delta}}$$

(b) Penetration theory

$$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial z^2}$$

Boundary conditions →

at $t = 0$, $C_A = C_{Ab}$ for $z \geq 0$

at $t > 0$, $C_A = C_{Ai}$ for $z = 0$

$C_A = C_{Ab}$ for $z = \infty$

$$\therefore \frac{C_A - C_{Ab}}{C_{Ai} - C_{Ab}} = 1 - \mathrm{erf}(x)$$

$$N_A(t) = -D_{AB}\left(\frac{\partial C_A}{\partial z}\right)_{z=0}$$

$$\therefore N_A(t) = \sqrt{\frac{D_{AB}}{\pi t}}\,(C_{Ai} - C_{Ab})$$

$$\Rightarrow \boxed{K_L = \sqrt{\frac{D_{AB}}{\pi t}}}$$

$$\therefore N_{A\,avg} = 2\sqrt{\frac{D_{AB}}{\pi t_c}}\,.(C_{Ai} - C_{Ab})$$

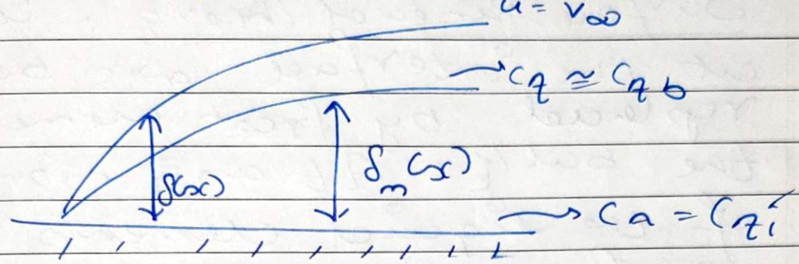
$$\therefore \boxed{K_{L\,avg} = 2\sqrt{\frac{D_{AB}}{\pi t_c}}}$$

(c) Surface Renewal theory:

$$k_L = \sqrt{D_{AB} \cdot S}$$

$S \rightarrow$ surface renewal rate

(d) Boundary layer theory.




③ $Sh_x = \dfrac{K_{L_x} \cdot x}{D_{AB}} = 0.332 (Re_x)^{1/2} (Sc)^{1/3}$

$Sh_{av} = \dfrac{K_{L_{av}} \cdot L}{D_{AB}} = 0.664 (Re_L)^{1/2} (Sc)^{1/3}$

$Re_x = \dfrac{\rho v_\infty \cdot x}{\mu}$

$Re_L = \dfrac{\rho v_\infty \cdot L}{\mu}$

$Sc^{1/3} = \dfrac{\delta}{\delta_m}$

(1.3) <u>Film Theory</u>: Mass transfer from solid surface to flowing liq. Even though the bulk liq. is in turbulent motion, the flow near the wall may be considered to be laminar. Mass transfer occurs at steady state

2

**Penetration Theory**: Unsteady state mass transfer occurs to a liq. element so long as it is in contact with the bubble. Equilibrium exists at the gas-liquid interface.

**Surface Renewal Theory**: Liq. elements at the interface are being random replaced by fresh elements from the bulk. At any moment each of the liquid elements at the surface has the same probability of being replaced by fresh element. Unsteady state mass transfer occurs.

q (1.4)

| | Mass Transfer | Heat Transfer |
|---|---|---|
| (i) | $Re = \frac{\rho v l}{\mu}$ $= \frac{\text{inertial force}}{\text{viscous force}}$ | Same |
| (ii) | $Sc = \frac{\nu}{D} = \frac{\text{momentum diffusivity}}{\text{molecular diffusivity}}$ | $Pr = \frac{\nu}{\alpha} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$ |
| (iii) | $Sh = \frac{K_L . L}{D}$ $= \frac{\text{convective mass flux}}{\text{diffusive flux across a layer of thickness } l}$ | $Nu = \frac{hL}{K} = \frac{\text{convective flux}}{\text{conduction heat flux}}$ |

(iv) $St_M = \frac{Sh}{Re \cdot Sc} = \frac{\text{convective mass flux}}{\text{flux due to bulk flow of medium}}$ $St_H = \frac{Nu}{Re \cdot Pr} = \frac{\text{convective heat flux}}{\text{heat flux due to bulk flow}}$

(v) $Pe_M = Re \cdot Sc = \frac{\text{flux due to bulk flow}}{\text{diffusive flux}}$ $Pe_H = \frac{\text{heat flux due to bulk flow}}{\text{Conduction flux}} = Re \cdot Pr$

(vi) $J_D = \frac{Sh}{Re (Sc)^{1/3}}$ $J_H = \frac{Nu}{Re \cdot (Pr)^{1/3}}$

Q. 1.5 Reynold's analogy

$$St_M \cdot Sc = 0.023 \, Re^{-0.2}$$

Coulbourn analogy

$$J_D = St_M \cdot Sc^{2/3} = 0.023 \, Re^{-0.2}$$

Q.1.6

| Gas | Driving force | liquid |
| --- | --- | --- |
| $K_C$ | $\Delta C$ | $K_L$ |
| $K_G$ | $\Delta P$ | - |
| - | $\Delta x$ | $K_x$ |
| $K_y$ | $\Delta y$ | - |

$$K_G \cdot RT = \frac{RT}{P} \cdot K_y = K_C$$

$$K_L = \frac{K_x}{C_{av}}$$

Q.1.7 (a) A diffusing through non diffusing B.

<u>Gas phase</u> : $k_G = \dfrac{D_{AB}\, P}{RT\cdot\delta\cdot P_{BM}}$

<u>liq</u> : $k_y = \dfrac{D_{AB}\cdot P^2}{RT\,\delta\cdot P_{BM}}$ . $k_L = \dfrac{D_{AB}}{\delta\cdot x_{BM}}$

(b) Equimolar counter diff

<u>Gas phase</u> : $k_G' = \dfrac{D_{AB}}{RT\cdot\delta}$

$k_y' = \dfrac{D_{AB}\cdot P}{RT\cdot\delta}$ $\quad k_c' = \dfrac{D_{AB}}{\delta}$

<u>liquid phase</u> : $k_x' = \dfrac{D_{AB}}{\delta}\left(\dfrac{\rho}{M}\right)_{avg}$

$k_L' = \dfrac{D_{AB}}{\delta}$

## Section 2

Q.3.1. $Y_{A_1} - Y_{A_2} = \dfrac{P_{A_1}}{P-P_{A_1}} - \dfrac{P_{A_2}}{P-P_{A_2}} = \dfrac{P(P_{A_1}-P_{A_2})}{P_{B_1}\cdot P_{B_2}}$

$\therefore P_{A_1} - P_{A_2} = \dfrac{(Y_{A_1}-Y_{A_2})(P_{B_1}\cdot P_{B_2})}{P}$

$N_A = k_G(P_{A_1} - P_{A_2}) = \dfrac{D_{AB}\cdot P}{RT\,\delta\cdot P_{BM}}(P_{A_1} - P_{A_2})$

classmate
Date
Page

$$k_y = \frac{D_{AB} \cdot P_{B_1} P_{B_2}}{RT \cdot \delta \cdot P_{BM}} = K_G \cdot \frac{P_{B_1} P_{B_2}}{P}.$$

Q.3.2 $P = 1.2$ bar $= 1.185$ atm $= 900.3$ mm Hg
$T = 300$ K , $R = 0.0831 \ \frac{m^3 \ bar}{Kmol \ K}$

(a) (i) $k_y = P \cdot K_G = 2.4 \times 10^{-6} \times 900.3$
$= 2.161 \times 10^{-3} \ \frac{Kmol}{s \cdot m^2 \cdot \Delta y}$

(ii) $P_{A_1} = 0.0877$ bar
$P_{B_1} = P - P_{A_1} = 1.1123$ bar
$P_{A_2} = 0 \Rightarrow P_{B_2} = 1.2$ bar

$$k_y = \frac{K_G \cdot P_{B_1} P_{B_2}}{P} = \frac{2.4 \times 10^{-6} \times 834.5 \times 900.3}{900.3}$$

$$\boxed{k_y = 2.003 \times 10^{-3} \ \frac{kmol}{s \cdot m^2 \cdot \Delta y}}$$

(iii) $K_C = K_G RT = 0.0449$ m/s

(b) $K_G = 2.4 \times 10^{-6} \ \frac{k \ mol}{m^2 \cdot s \cdot mm \ Hg} = 2.541 \times 10^{-5} \ \frac{lbmol}{s \cdot ft^2 psi}$

(ii) $K_G = 1.345 \ \frac{lb \ mol}{ft^2 \cdot h \cdot atm}$

(iii) $K_G = 4.55 \times 10^{-2} \ \frac{lb \ mol}{ft^2 \cdot h \cdot inch \ Hg}$.

Thickness of stagnant film

$$\delta = \frac{D_{AB} \cdot P}{K_G \cdot RT \ln(P_{BM})}$$

$$D_{AB} = 0.102 \times \left(\frac{1.013}{1.185}\right)\left(\frac{300}{273}\right)^{1.75} = 1.03 \times 10^{-5} \ \frac{m^2}{s}$$

$$P_{BM} = \frac{P_{B2} - P_{B1}}{\ln(P_{B2}/P_{B1})} = 1.1556 \text{ bar}$$

$$K_G = 1.8 \times 10^{-3} \frac{\text{Kmol}}{m^2 \cdot s \cdot \text{bar}}$$

$$\therefore \boxed{\delta = 0.24 \text{mm}}$$

Q.33. $$Re = \frac{2r \cdot \rho \cdot v}{\mu} = r \times \frac{2 \times 1.1 \times 5}{1.92 \times 10^{-5}} = 5.73 \times 10^{5} r$$

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{1.92 \times 10^{-5}}{1.1 \times 6.92 \times 10^{-6}} = 2.522$$

$$Sh = 2 + 0.6 \times Re^{0.5} \cdot Sc^{1/3}$$
$$= 2 + 618 r^{1/2}$$

$\rightarrow$ $P/P_{Bm} \cong 1$

$$Sh = \frac{K_G \cdot P_{BM} \cdot RT \cdot 2r}{P \cdot D_{AB}} = \frac{K_G \times 0.0831 \times 318 \times 2r}{6.92 \times 10^{-6}}$$
$$= 7.644 \times 10^{6} r \, K_G$$

$$K_G = \frac{2.616 \times 10^{-7}}{r} + \frac{8.085 \times 10^{-5}}{\sqrt{r}}$$

$$\rightarrow \frac{-d}{dt}\left(\frac{4}{3}\pi r^3 \cdot \frac{\rho}{M}\right) = 4\pi r^2 \cdot K_G (P_{AV} - P_{A0})$$

$$\Rightarrow -\frac{dr}{dt} = \frac{M}{\rho} \cdot K_G \cdot P_{AV}$$
$$= \frac{3.3867 \times 10^{-11}}{r} + \frac{1.0467 \times 10^{-8}}{\sqrt{r}}$$

$$\therefore \boxed{t = 3.84 h}$$

Q34 $M$ = mass of undissolved solid at time t.
Mass of solid already dissolved = 20 - m
mass of solution = 50 + (20 - m)

$\therefore$ volume = $\frac{520 - m}{1000}$

$\therefore C = \frac{20 - m}{520 - m} \times 1000$

$C_S = 80 \ \frac{kg}{m^3}$.

$d_o = 0.6 mm$, no. of particles $= 6.574 \times 10^7$

$\therefore m' = $ mass of 1 particle $= \frac{m}{6.574 \times 10^7}$

$d_p = 2.211 \times 10^{-4} \ m^{1/3}$

area of 1 particle $= \pi d_p^2 = \pi \times (2.211 \times 10^{-4} m^{1/3})^2$

total area $= 10.1 \ m^{2/3} = a$

$Sc = \frac{\mu}{D\rho} = 653$

$Re = N \cdot d_p^2 \frac{\rho}{\mu} = 0.09976 \ m^{2/3}$

$\therefore Sh = 2 + 0.44 \times (0.09976 \ m^{2/3})(653)^{0.385}$

$= 2 + 1.67 \ m^{0.336} = \frac{K_L \cdot d_p}{D}$

$\Rightarrow K_L = (6.784 \times 10^{-6})(2 + 1.67 m^{0.336}) \ m^{-1/3}$

$\frac{-dm}{dt} = K_L \cdot a \cdot (C_S - C)$

integrating, $\boxed{t = 15 \text{ min}}$

Q.3.5 $Q = \pi r^2 \cdot v = 13.2\ \frac{lit}{s}$

$$\boxed{Q = 1.032 \times 10^{-5}\ m^3/s}$$

$l$ = length of the jet = 0.05 m

$t_c = l/v$

$$K_{L,av} = 2 \cdot \sqrt{\frac{D_{AB}}{\pi \cdot t_c}} = 2\sqrt{\frac{D_{AB} \cdot v}{\pi l}}$$

area of contact = $2\pi r L$

$C_{Ai}^{\circ} = 1.03 \times 0.1136 = 0.117\ \frac{kmol}{m^3}$

$C_{Ab} = 0$

rate of absorption of $H_2S$ $= 4.42 \times 10^{-4}\ \frac{g}{s}$

$= 1.3 \times 10^{-8}\ \frac{kmol}{s}$

$$1.3 \times 10^{-8} = 2\pi l \cdot \sqrt{\frac{Q}{\pi v}} = \sqrt{\frac{D_{AB} \cdot v}{\pi l}}\ (0.117 - 0)$$

$$\boxed{D_{AB} = 1.17 \times 10^{-7}\ \frac{m^2}{s}}$$

Q.3.6 $t_c = \frac{d_b}{v_b} = \frac{1 cm}{20\ \frac{cm}{s}} = 0.05 s$

$$K_L = 2 \cdot \left(\frac{D_{AB}}{\pi t_c}\right)^{1/2} = 0.0236\ \frac{cm}{s}$$

Residence time $= \frac{0.3}{0.2} = 1.5 s$

Vol. of bubble $= 0.5236\ cm^3$

area of bubble $= 3.1416\ cm^2$

$C_S = 1.493 \times 10^{-5}\ \frac{gmol}{cm^3}$

classmate

Date
Page

$C_b = 0$

Amount of $CO_2$ absorbed from a single bubble during residence time = 1.5 s

$= K_L \times (\text{area}) \times (C_s - C_b) \times 1.5$

$= 1.66 \times 10^{-6}$ gmol

Average number of bubbles formed per minute $= \frac{15}{0.5236} = 28.65$ per min.

Rate of absorption of $CO_2$ from the bubble per min.

$= \frac{1.66 \times 10^{-6} \text{ gmol per bubble}}{28.65 \text{ bubbles per min}}$

$= \boxed{4.757 \times 10^{-5} \frac{\text{g mol}}{\text{min}}}$

Q.3.7 $P = 1040 x^*$

at $P = 2$ atm, $x^* = 0.00122$

Molec. wt of soln $= 4 + 0.00122 + 18(1 - 0.00122)$

$= 18.03 \simeq 18$

Moles of soln/$m^3$ $= \frac{997}{18} = 55.4 \frac{\text{Kmol}}{m^3}$

moles of solute per $m^3$ of soln

$C_s = 55.4 \times 0.00122$

$= 0.0676 \frac{\text{Kmol}}{m^3}$

$C = 2.3\ g\ \frac{CO_2}{lit} = 0.0523\ \frac{Kmol}{m^3}$

volume rate of input of water

= volume rate of output of soln

= 1 lit/min

= $1.667 \times 10^{-5}\ m^3/s$

Absorption of $CO_2$

= $1.667 \times 10^{-5} \times 0.0523$

= $8.718 \times 10^{-7}$ Kmol/s

$V = 8$ lit $= 0.008\ m^3$

$\bar{a} = 80\ m^2/m^3$

rate of absorption of $CO_2$ at steady state

$= V \cdot \bar{a} \cdot K_L \cdot (C_S - C)$ $= 8.718 \times 10^{-7}\ \frac{m}{s}$

$\Rightarrow 0.008 \times 80 \times K_L \times (0.067 - 0.0523)$

$= 8.718 \times 10^{-7}$ m/s

$\Rightarrow \boxed{K_L = 8.903 \times 10^{-5}\ \frac{m}{s}}$

(a) $\delta = \frac{D_{AB}}{K_L} = 0.0216$ mm

(b) $t_c = \frac{4 D_{AB}}{\pi \cdot K_L^2} = 0.03085$

(c) $S = \frac{K_L^2}{D_{AB}} = 4.13\ s^{-1}$

Q.3.4 (Treybal)

$$\frac{t_i^o - t_L}{t_i^o - t_o} = 0.7857e^{-5.1213n}$$

$$n = \frac{2 \cdot D_{AB} \cdot L}{3 \cdot \delta^2 u_y^2} = \boxed{\frac{2 \cdot L}{3 P_{ep} \cdot \delta}}$$

## Section 3

(3.1) $Sh = 0.664\, Re_L^{1/2} \cdot Sc^{1/3}$

$$Sh = \frac{K_L \cdot L}{D_{AB}}$$

$$Re = \frac{\rho v L}{\mu} = \frac{20 \times 0.2}{1.5 \times 10^{-5}} = 266666$$

$$Sc = \frac{\mu}{\rho \cdot D_{AB}} = \frac{1.5 \times 10^{-5}}{6 \times 10^{-6}} = 2.5$$

$\therefore \boxed{K_L = 0.014 m/s}$

rate of loss of naphthalene $= K(C_{Ai}^o - C_{A\infty})$

$= 0.014(1 \times 10^{-5} - 0)$

$= 1.4024 \times 10^{-7}\ \frac{kmol}{m^2 \cdot s}$

rate of loss per meter width $= (1.4024 \times 10^{-7}) \times 0.2$

$= \boxed{2.8048 \times 10^{-8}\ \frac{kmol}{m \cdot s}}$

Q.3.2. $Re = \frac{\rho v D}{\mu} = \frac{6 \times 10^{-3} \times 3 \times 4.1}{2.7 \times 10^{-5}} = 2733$

$Sc = \frac{\vartheta}{D} = \frac{2.7 \times 10^{-5}}{4.1 \times 9.04 \times 10^{-6}} = 0.728$

classmate
Date
Page

$Sh = 0.43 + (0.532) Re^{0.5} Sc^{0.31}$

$\therefore \frac{K_L \cdot L}{D_{AB}} = 0.43 + (0.532) \times (2733)^{1/2} \times (0.728)^{0.31}$

$\therefore \boxed{\frac{K_L \cdot L}{D_{AB}} = 25.635}$

~~$D_{AB}$~~

$$D_{AB} = \frac{10^{-3} \cdot T^{1.75} \cdot \left(\frac{1}{M_A} + \frac{1}{M_B}\right)^{0.5}}{P\left[(\Sigma v_A)^{1/3} + (\Sigma v_B)^{1/3}\right]^2}$$

$= 9.04 \times 10^{-6} \ \frac{m^2}{s}$

$\therefore \boxed{K_L = 0.0386 \ \frac{m}{s}}$

rate of sublimation per unit area $= N_A = K_L (C_A - C_{A\infty})$

$C_A = \frac{P}{RT} = \frac{53.32 \times 10^{+3}}{8.314 \times 316}$

$\therefore \boxed{N_A = 0.783 \ \frac{mol}{m^2 s}}$

Q.3.3 $N_A = K_c (C_{AS} - C_{A\infty})$

$Sh = 0.62 \times Re^{1/2} \times Sc^{1/3}$

$$\frac{K_c D}{D_{AB}} = 0.62 \times \left(\frac{D^2 \cdot \omega \cdot \rho}{\mu}\right)^{1/2} \times \left(\frac{\mu}{\rho \cdot D_{AB}}\right)^{1/3}$$

$\omega = \frac{20}{60} \times 2\pi = \frac{2\pi}{3}$ rad/sec

$$\therefore K_C = 0.62 \times D_{AB} \left(\frac{\omega \times \rho}{\mu}\right)^{1/2} \times \left(\frac{\mu}{\rho \cdot D_{AB}}\right)^{1/3}$$

$$\therefore \boxed{K_C = 8.973 \times 10^{-4} \frac{cm}{s}}$$

$$N_A = 8.973 \times 10^{-4} \times 0.003$$
$$= 2.692 \times 10^{-6} \frac{g}{cm^2 \cdot s}$$

$$\omega_A = N_A \cdot A = 2.692 \times 10^{-6} \times \frac{\pi d^2}{4}$$

$$\therefore \boxed{\omega_A = 1.9 \times 10^{-5} \frac{gm}{sec}}$$

Q.34 $N_A = K_C \cdot (C_{AS} - C_{A\infty}) = \frac{\omega_A}{A}$

$$C_{AS} = \frac{P_{AS}}{RT} = \frac{4.8 \times 10^3}{8.314 \times 293}$$

$$C_{AS} = 1.946 \frac{mol}{m^3}.$$

$$C_{A\infty} \to 0$$

$$\therefore K_C = \frac{\omega_A}{A(C_{AS} - C_{A\infty})} = \frac{2.29 \times 10^{-4}}{10^{-3} \times 1.946}$$

$$\therefore \boxed{K_C = 0.118 \ m/s}$$

classmate
Date
Page

Q.3.5 $Sc = \frac{\nu}{D_{AB}}$ $\nu_{air} = 1.56 \times 10^{-5} \frac{m^2}{s}$

(i) methanol in air

$$D_{AB} = \frac{10^{-3} \cdot T^{1.75} \cdot \left(\frac{1}{M_A} + \frac{1}{M_B}\right)^{1/2}}{P\left((\Sigma \vartheta_A)^{1/3} + (\Sigma \vartheta_B)^{1/3}\right)}$$

$\therefore D_{AB} = 1.59 \times 10^{-5} \frac{m^2}{s}$.

$\vartheta_A = 31.23$, $\vartheta_B = 20.1$

$\therefore Sc = \frac{1.56 \times 10^{-5}}{1.59 \times 10^{-5}} \cong 1$

(ii) methanol in $H_2O$

$D_{AB} = 13.26 \times 10^{-5} \times \mu_B^{-1.14} \times \vartheta_A^{-0.589}$ $\hookrightarrow 31.23$

$D_{AB} = 1.74 \times 10^{-5} \frac{cm^2}{s}$

$\therefore Sc = \frac{0.9 \times 10^{-6}}{1.74 \times 10^{-5} \times 10^{4}} = 517$

$\therefore \boxed{Sc = 517}$

Q.3.6 $Sh = \frac{K_L \cdot L}{D_{AB}} = 0.023 \times Re^{0.8} \times Sc^{1/3}$

$Sc = \frac{\nu}{D_{AB}}$.

Q.3.7 Oxygen in air

$$D_{AB} = \frac{10^{-3} \cdot T^{1.75} \cdot \left(\frac{1}{M_A} + \frac{1}{M_B}\right)^{1/2}}{P\left((\Sigma v_A)^{1/3} + (\Sigma v_B)^{1/3}\right)^2}$$

$$= \frac{10^{-3} \times 300^{1.75} \times \left(\frac{1}{32} + \frac{1}{24}\right)^{1/2}}{1 \times \left((20.1)^{1/3} + (10.9)^{1/3}\right)^2}$$

$$D_{AB} = 0.227 \frac{cm^2}{s}$$

$$\therefore Sc = \frac{\nu}{D_{AB}} = \frac{1.562 \times 10^{-5}}{2.27 \times 10^{-5}}$$

$$\boxed{Sc = 0.688}$$

Oxygen in water

$$D_{AB} = 1.5 \times 10^{-9} \frac{m^2}{s}$$

$$\therefore Sc = \frac{\nu}{D_{AB}} = \frac{0.89 \times 10^{-6}}{1.5 \times 10^{-9}}$$

$$\boxed{Sc = 593.33}$$