Convection

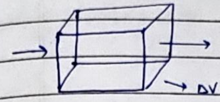
Thermal Boundary layer

Necessity?

→ Control volume is analogous to open system

→ Control mass → closed system

$\frac{\partial(\ )}{\partial t}$, $\frac{d(\ )}{dt}$, $\frac{D(\ )}{Dt}$



$c_i = \lim_{\Delta V \to 0} \frac{m_i}{\Delta V} = \lim_{\Delta V \to 0} \frac{n_i M_i}{\Delta V}$ n_i = no: of particles, M_i = molecular weight

concentration of individual particle

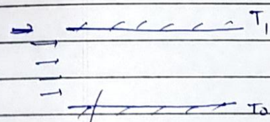
$\rho = \lim_{\Delta V \to 0} \frac{m}{\Delta V} = \lim_{\Delta V \to 0} \frac{\sum n_i M_i}{\Delta V}$

momentum $= \lim_{\Delta V \to 0} \frac{\sum v_{ix} n_i M_i}{\Delta V}$

e = energy density $= \lim_{\Delta V \to 0} \frac{E}{\Delta V} = \lim_{\Delta V \to 0} \frac{\frac{1}{2} m_i v_j^2}{\Delta V}$

v_j = fluctuating component

$v_j = v_i - \bar{v}$ → avg velocity



$T_1 > T_0$

temp inside can be greater than T_1

$$\underbrace{\frac{\partial(\rho v)}{\partial t}}_{\text{acceleration}} = -\underbrace{\left[\frac{\partial}{\partial x}(\rho v u) + \frac{\partial}{\partial y}(\rho u v)\right]}_{\text{Through bulk flow}} - \underbrace{\left[\frac{\partial}{\partial x}(q_x'') + \frac{\partial}{\partial y}(q_y'')\right]}_{\text{conduction}} + \underbrace{q'''}_{\text{heat generation}}$$

$$\underbrace{- \left[u g_x + v g_y\right] - \left[\frac{\partial(pu)}{\partial x} + \frac{\partial(pv)}{\partial y}\right]}_{\text{Net work done.}} + \mu\phi$$

simplification

$$\rho\frac{De}{Dt} = q''' + \rho(\bar{v}\cdot g) - \nabla(p\bar{v}) - \nabla q'' + \mu\phi$$

$e = \hat{u} + \frac{1}{2}v^2 \rightarrow$ KE $\quad g = -\nabla\phi \quad H = U + PV$

internal energy $\quad h = \hat{u} + p\frac{1}{\rho}$

Neglect KE $\rightarrow e = \hat{u}$

$q'' = -k\nabla T$

$\bar{v} = -\nabla\phi$

$$\rho\frac{D(e+\phi)}{Dt} = q''' - \nabla(p\bar{v}) - \nabla q'' + \mu\phi$$

$$\rho\frac{Dh}{Dt} = q''' + \nabla\cdot(k\nabla T) + \frac{DP}{Dt} + \mu\phi$$

$dh = Tds + vdp \rightarrow$ general expression

$dh = c_p dT \rightarrow$ only valid for ideal gases.

$ds = f(T, P)$

$\rightarrow ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$ use Maxwell's theorem.

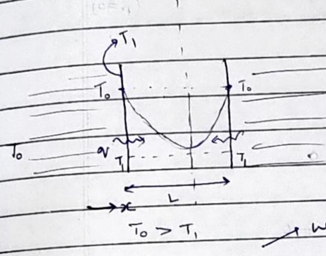
$\hookrightarrow \left(\frac{\partial v}{\partial T}\right)_p dT$.

$$\rho c_p\frac{DT}{Dt} = \nabla\cdot(k\nabla T) + q''' + \beta T\frac{DP}{Dt} + \mu\phi$$

$\mu\phi = -\nabla\cdot(\tau\cdot\vec{v})$

Assumption :- Newtonian fluid.
KE is neglected.
For ideal gas, $\beta = 1/T$
For incompressible liq, $\beta = 0$.

Scaling analysis — Identify the boundary.



solid rod immersed in a liquid
scaling analysis

$T_0 > T_1$

time required for the rod to reach temp T_0 at the center
(There is symmetry)

$\rho c_p\frac{\partial T}{\partial t} = \nabla\cdot(k\nabla T)$ $\quad k =$ const, no velocity, $x = L/2$

$\rho c_p\frac{\partial T}{\partial t} = -k\frac{\partial^2 T}{\partial x^2}$ $\quad \rho c_p\frac{\Delta T}{t} \sim 4k\frac{\Delta T}{L^2}$

$x \sim O(L/2) \rightarrow$ boundary

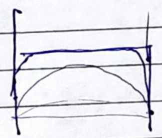
$t \sim \frac{\rho c_p L^2}{4k}$ $\quad T \sim O(\Delta T)$

$t \sim O(t)$

$\frac{\partial T}{\partial x} = 0$, $x = L/2$

$t \sim O\left(\frac{L^2}{4\alpha}\right)$ → time at which it feels the heat at the center.

→ time it takes to travel $L/2$



$a = b + c$ | $a = b \times c$

$o(b) > o(c)$ | $a(a) \sim o(b) \cdot \sim o(c)$

$\sim o(a) \sim o(b)$

→ stream line

→ Temperature map → analogous to pressure in momentum balance.

→ <u>Heat function</u>

2D flow, homogeneous fluid, no viscous heat, steady state / incompressible ($\beta = 0$)

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

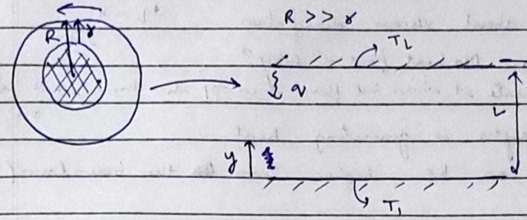
$$+\left(\rho C_p v \frac{\partial T}{\partial y} - k \frac{\partial^2 T}{\partial y^2} \right) + \left(\rho C_p u \frac{\partial T}{\partial x} - k \frac{\partial^2 T}{\partial x^2} \right) = 0$$

$$\frac{\partial}{\partial y}\left(\rho C_p v T - k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial x}\left(\rho C_p u T - k \frac{\partial T}{\partial x} \right) = 0$$

$$\left.\begin{aligned} -\frac{\partial H}{\partial x} &= \rho C_p u (T - T_{ref}) - k \frac{\partial T}{\partial x} \\ \frac{\partial H}{\partial y} &= \rho C_p v (T - T_{ref}) - k \frac{\partial T}{\partial y} \end{aligned}\right\} \text{heat function}$$

analogous to stream function

Q)



bearing

Couette ⊥ no pressure drop.

→ Taylor Couette flow

τ_{xy}

No additional bulk flow.

$$\rho C_p \cancel{\frac{DT}{Dt}}^{\,0} = k \nabla^2 T + \underbrace{\phi \mu}_{\tau(\nabla \cdot (\tau \vec{v}))} + \cancel{q'''}^{\,0} + \beta T \cancel{\frac{Dp}{Dt}}^{\,0}$$

Fully developed thermal profile → no heat flow in x direction

τ_{yx} ~ y direction of force

$$0 = k \frac{d^2 T}{dy^2} + \frac{d}{dy}[\tau_{yx} \times u) \qquad \tau_{yx} = \mu \frac{du}{dy}$$

$$0 = k \frac{d^2 T}{dy^2} + \mu \cancel{\frac{d^2 u}{dy^2}} + \mu \left(\frac{du}{dy} \right)^2$$

0 linear velocity profile.

BC :- $y = 0, \; T = T_1$

$y = L, \; T = T_L$

$$\frac{du}{dy} = \frac{u}{L}$$

$$\theta = \frac{T(y) - T_1}{T_L - T_1} \qquad \eta = \frac{y}{L} \qquad k \frac{d^2 T}{dy^2} + \mu \left(\frac{u}{L} \right)^2$$

$$\frac{d^2\theta}{dy^2} = \frac{1}{T_L - T_1} \frac{d^2 T}{dy^2}$$

$$k (T_L - T_1) \frac{d^2 \theta}{L^2 \, d\eta^2} + \frac{\mu}{L^2} U^2 = 0.$$

$$q''\Big|_{y=L} = -k \frac{dT}{dy}\Big|_{y=L}$$ → heat flux at the moving plate

$$\frac{d^2\theta}{d\eta^2} + \frac{\mu U^2}{k(T_L - T_1)} = 0$$

$$q'' = \frac{-k(T_L - T_1)}{L} \left(-\frac{Br}{2} \right)$$

$$\frac{d^2\theta}{d\eta^2} = -\frac{\mu U^2}{k(T_L - T_1)} = -Br$$

$n = \eta - \eta^2$

$2\eta - \eta^2$

$2-1$

Brinkman

Br → will tell about viscous dissipation

$Br = 2$ → $q'' = 0$ → No heat flux → why?

at $y = 1$, $T = T_2$ → rate at which heat flows is in opp direction → which will cancel out.

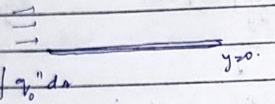
friction between the layers is generating heat

∴ temp inside can be higher than ~~the~~ the boundaries (T_1, T_2).

$$Br = \frac{\mu u^2}{k \Delta T}$$

$$Br = \frac{\mu \left(\frac{u}{L}\right) u}{\frac{k \Delta T}{L}}$$

Boundary layer



$F = \int_A \tau_0 \, dA$ — force exerted by the stream on the plate

$q' = \int_A q_0'' \, dA$ → resistance to the transfer of heat from the plate to the stream

$$\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0} \qquad q_0'' = k \left.\frac{dT}{dy}\right|_{y=0}$$

→ $q = h(T_1 - T_2)$

$$h = \frac{-k\left(\frac{dT}{dy}\right)_{y=0}}{\Delta T}$$