

Problems on Batch Reactor

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Problem-1

- Liquid A decomposes by first-order kinetics, and in a batch reactor 50% of A is converted in a 5-min run. How much would it take to reach 75% Conversion?

Solution: We have $-\ln(1 - X_A) = kt$

$$-\ln(1 - 0.5) = k \cdot 5, \quad k = 0.1386 \text{ min}^{-1}$$

For 75% conversion, $-\ln(1 - 0.75) = 0.1386 \cdot t$

$t = 10 \text{ mins}$ Ans.

Problem-2

The first-order reversible liquid reaction $A \rightleftharpoons R$, $C_{A0} = 0.5 \text{ mol/L}$, $C_{R0} = 0$, takes place in a batch reactor. After 8 mins, conversion of A is 33.3%, while equilibrium conversion is 66.7%. Find the rate equation.

Solution: $A \rightleftharpoons R$, $C_{A0} = 0.5 \text{ mol/L}$, $C_{R0} = 0$

$$-r_A = k_1 C_A - k_2 C_R = k_1 C_{A0} (1 - X_A) - k_2 C_{R0} (1 + X_A)$$

$$\text{As } C_{A0} X_A = C_{R0} X_R$$

$$-r_R = k_1 C_A - k_2 C_R = k_1 C_{A0} (1 - X_A) - k_2 (C_{R0} + C_{A0} X_A)$$

- $C_{A0} \frac{dX_A}{dt} = k_1 C_{A0} (1-X_A) - k_2 C_{A0} X_A$
- $\int_0^{X_{Af}} \frac{dX_A}{k_1 - X_A(k_1 + k_2)} = \int_0^t dt$
- $t = - \frac{1}{(k_1 + k_2)} \ln \frac{k_1}{k_1 - X_A(k_1 + k_2)}$
- $= - \frac{1}{k_1(1 + k_2/k_1)} \ln \frac{k_1}{1 - X_A(1 + k_2/k_1)}$
- $K_e = \frac{k_1}{k_2} = \frac{C_{Re}}{C_{Ae}} = \frac{C_{R0} + C_{A0}X_{Ae}}{C_{A0}(1 - X_{Ae})} = \frac{X_{Ae}}{(1 - X_{Ae})}$
- $\frac{k_2}{k_1} = \frac{1}{X_{Ae}} - 1 = \frac{1}{0.667} - 1 = 0.4993$

- $t = - \frac{1}{k_1(1+0.4993)} \ln \frac{k_1}{1-X_A(1+0.4993)}$
- Given at $t = 8 \text{ min}$, $X_A = 0.333$
- $k_1 = 0.05767, k_2 = 0.0288$
- $-r_A = 0.05767C_A - 0.0288C_R$ Answer

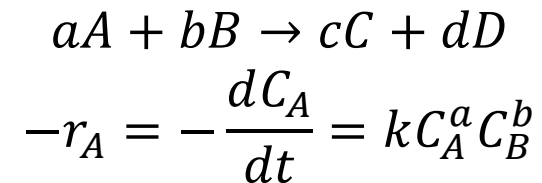
Problem-3

The irreversible reaction $2\text{H}_2 + 2\text{NO} \rightarrow \text{N}_2 + \text{H}_2\text{O}$ is carried out in a constant volume reactor using equimolar amounts of hydrogen and nitric oxide. The reactor data is as follows.

P_t , mm Hg	200	240	280	320	326
$t_{1/2}$, sec	265	186	115	104	67

Find the overall order of the reaction

Solution:



Reaction is performed with a stoichiometric ratios throughout the reaction period.

$$\frac{C_B}{C_A} = \frac{b}{a} \text{ and } -\frac{dC_A}{dt} = kC_A^a \left(\frac{b}{a}C_A\right)^b = k\left(\frac{b}{a}\right)^b C_A^{a+b} = k' C_A^n$$

By integrating for $n \neq 1$ gives

$$(C_A^{1-n} - C_{A0}^{1-n}) = k'(n-1)t$$

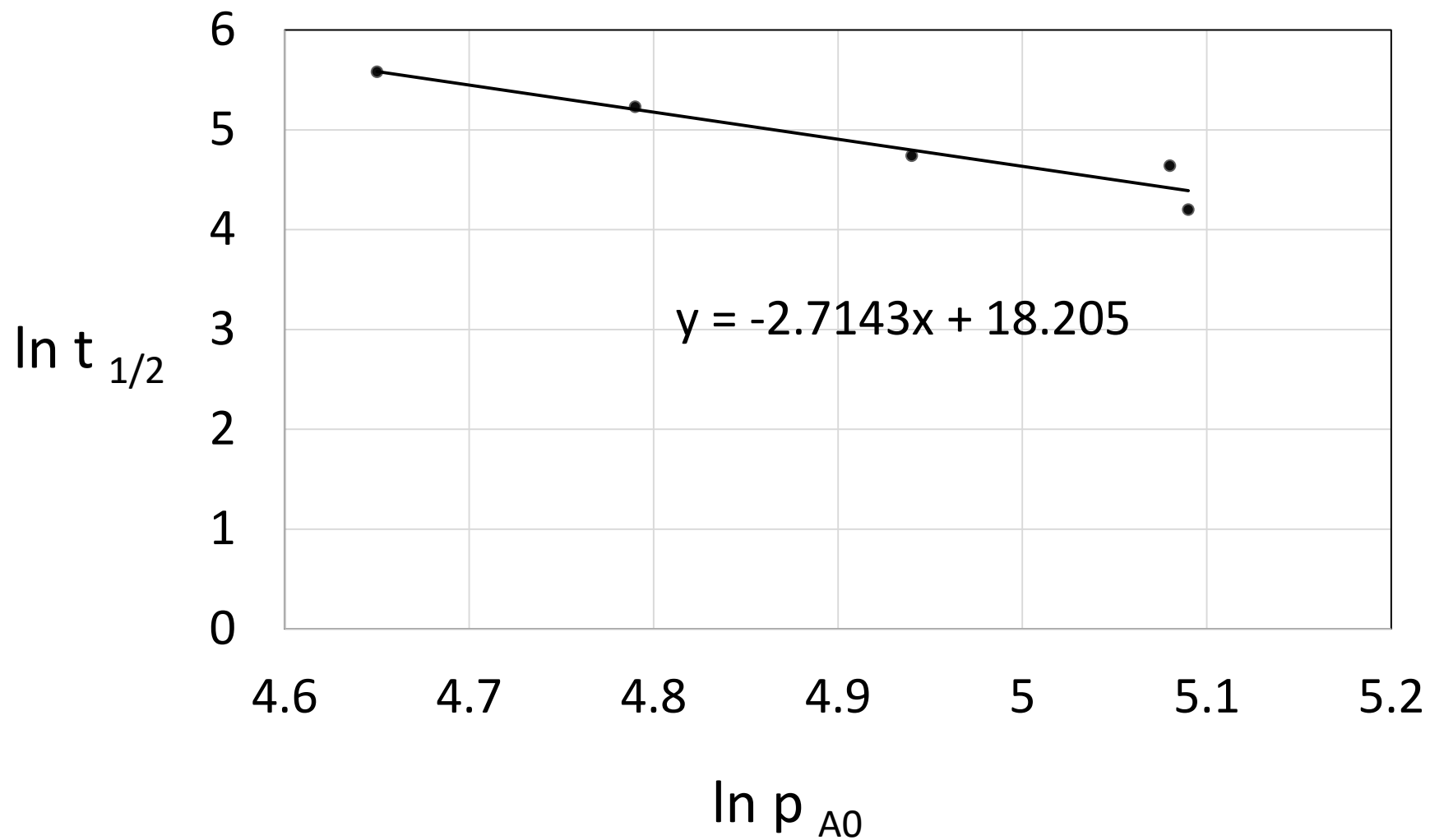
For half-life period: $t = \frac{1}{2}$, $C_A = \frac{C_{A0}}{2}$ and $t_{\frac{1}{2}} = \frac{0.5^{1-n}-1}{k(n-1)} C_{A0}^{1-n}$ (44)

$$t_{\frac{1}{2}} = \frac{2^{n-1}-1}{k'(n-1)} C_{A0}^{1-n}$$

- $\ln(t_{1/2}) = \ln \left[\frac{2^{n-1}-1}{k'(n-1)(RT)^{1-n}} \right] + (1-n) \ln(p_{A0})$

- As H_2 and NO are using in equimolar amounts $p_{A0} = p_T/2$

$\ln(p_{A0})$	4.65	4.79	4.94	5.08	5.09
$\ln(t_{1/2})$	5.58	5.23	4.74	4.64	4.20



$$\text{Slope} = -2.714 = 1 - n$$

$$\text{So, } n = 3.714 \approx 4$$

Problem-4

The aqueous reaction $A \rightarrow R + S$ is carried out in a batch reactor with $C_{A0}=0.183\text{mol/l}$, $C_{R0}=0$ and $C_{S0}=55\text{ mol/l}$. The reactor data is as follows:

t, min	0	36	65	100	160	∞
$C_A, \text{mol/l}$	0.183	0.145	0.122	0.103	0.0795	0.0494

Find a rate equation for this reaction.

Solution: $-r_A = k_1 C_A - k_2 C_R C_S = k_1 C_{A0} (1-X_A) - k_2 (C_{A0} X_A)(C_{S0} + C_{A0} X_A)$

As $C_R = C_{A0} X_A$ and $C_S = C_{S0} + C_{A0} X_A$

At equilibrium, $C_{Ae} = C_{A0} (1-X_{Ae})$, $C_{Re} = C_{A0} X_{Ae}$ and $C_{Se} = C_{S0} + C_{A0} X_{Ae}$

- $X_{ae} = (C_{A0} - C_{Ae})/C_{A0} = (0.1823 - 0.0494)/0.1823 = 0.73$

$$C_{Re} = C_{A0} X_{Ae} = 0.1823 \times 0.73 = 0.1331$$

and $C_{Se} = C_{S0} + C_{A0} X_{Ae} = 55 + 0.1823 \times 0.73 = 55.1331$

At equilibrium, $-r_A = 0$

$$\frac{k_1}{k_2} = \frac{C_{Re} C_{Se}}{C_{Ae}} = \frac{0.1331 \times 55.1331}{0.0494} = 149.06$$

$$k_1 = 149.06 k_2$$

$$C_{A0} \frac{dX_A}{dt} = k_1 C_{A0} (1 - X_A) - k_2 (C_{A0} X_A) (C_{S0} + C_{A0} X_A)$$

Neglecting higher power of X_A

$$\frac{dX_A}{dt} = k_1(1 - X_A) - k_2X_A(C_{S0} + C_{A0}X_A)$$

$$\frac{dX_A}{dt} = 149.06k_2 - 204.06 k_2X_A$$

$$\int_0^{X_A} \frac{dX_A}{149.06 - 204.06 X_A} = k_2 \int_0^t dt$$

$$\ln \left[\frac{149 - 204 X_A}{149} \right] = -204k_2t \quad \text{At } t=36 \text{ min, } X_A=0.203$$

$$k_2 = 4.45 \times 10^{-5} \text{ lit/mol.min and } k_1 = 6.6 \times 10^{-3} / \text{min}$$

$$-r_A = 6.6 \times 10^{-3} C_A - 4.45 \times 10^{-5} C_R C_S \quad \text{Ans.}$$

Problem-5

A zero-order homogeneous gas reaction $A \rightarrow rR$ proceeds in a constant volume bomb, with 20% inerts, and the pressure rises from 1 to 1.3 atm in 2 min. If the same reaction takes place in a constant pressure batch reactor, what is the fractional volume change in 4-min if feed is at 3 atm and consists of 40% inerts ?

Solution: $A \rightarrow rR$ is zero order reaction

0.8 moles A + 0.2 moles inerts = 0.8 r moles R and 0.2 moles inerts

For constant volume reactor, pressure rises, $\Delta n = r-1$, $a=1$

$$\bullet -r_A = \frac{dC_A}{dt} = k, -r_A = \frac{dp_A}{RTdt} = k$$

$$-\frac{1}{RT} \int_{p_{A0}}^{p_A} dp_A = kt, -\frac{1}{RT} (p_A - p_{A0}) = kt$$

$$-\frac{1}{RT}(p_A - p_{A0}) = kt$$

We have $p_A - p_{A0} = -\frac{a}{\Delta n}(\pi - \pi_0)$

$$-\frac{a}{\Delta n RT}(\pi - \pi_0) = kt$$

$$-\frac{a}{(r-1)RT}(\pi - \pi_0) = kt \text{ or, } -\frac{1}{(r-1)RT}(1.3 - 1) = k.2$$

$$\frac{0.15}{(r-1)RT} = k$$

Constant pressure batch is variable volume reactor

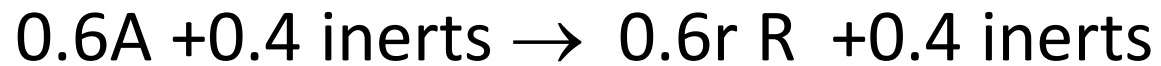
Design equation is $t = C_{A0} \int_0^{X_A} \frac{dX_A}{(1 + \varepsilon_A X_A)(-r_A)}$

$$C_{A0} \int_0^{X_A} \frac{dX_A}{(1+\epsilon_A X_A)} = kt \quad \text{As } (-r_A) = k$$

$$\frac{C_{A0}}{\epsilon_A} \ln(1 + \epsilon_A X_A) = kt$$

$$\frac{C_{A0}}{\epsilon_A} \ln \frac{V}{V_0} = kt$$

Stoichiometry:



$$\epsilon_A = \frac{0.6(r - 1)}{1}$$

$$p_{A0} = 0.6 \times \pi = 0.6 \times 3 = 1.8 \text{ atm}$$

$$C_{A0} = \frac{1.8}{RT}$$

$$\frac{1.8}{RT0.6(r-1)} \ln \frac{V}{V_0} = kt = \frac{0.15}{(r-1)RT} \times 4$$

$$\ln \frac{V}{V_0} = \frac{0.15}{3} \times 4 = 0.2$$

$$\frac{V}{V_0} = 1.221$$

$$\text{Fractional volume change} = \frac{\Delta V}{V_0} = 0.221 \text{ Ans}$$