

Experiment-4

Residence Time Distribution in a Tubular Vessel

18CH10071, Anshuman Agrawal

Objectives

1. To construct C curve for pulse input
2. To plot age-distribution curve (E vs t)
3. To calculate average residence time
4. To calculate vessel dispersion number (D/uL) for 3 different flow rates

Theory

The Residence Time Distribution (RTD) is used to determine the deviation from ideal flow patterns in a real reactor. It is also used as an effective diagnostic tool to inspect any possible malfunction in the reactor. The dimensionless form of Fick's second law with $z = x/L$ and $\Theta = t/\tau = t\tau/L$ is as follows:

$$\frac{\partial C}{\partial \Theta} = \left(\frac{D}{uL} \right) \frac{\partial^2 C}{\partial z^2} - \frac{\partial C}{\partial z}$$

Here, D/uL is the vessel dispersion number and is the parameter which measures the extent of axial dispersion.

When $D/uL \rightarrow 0$, negligible dispersion occurring and plug flow occurs.

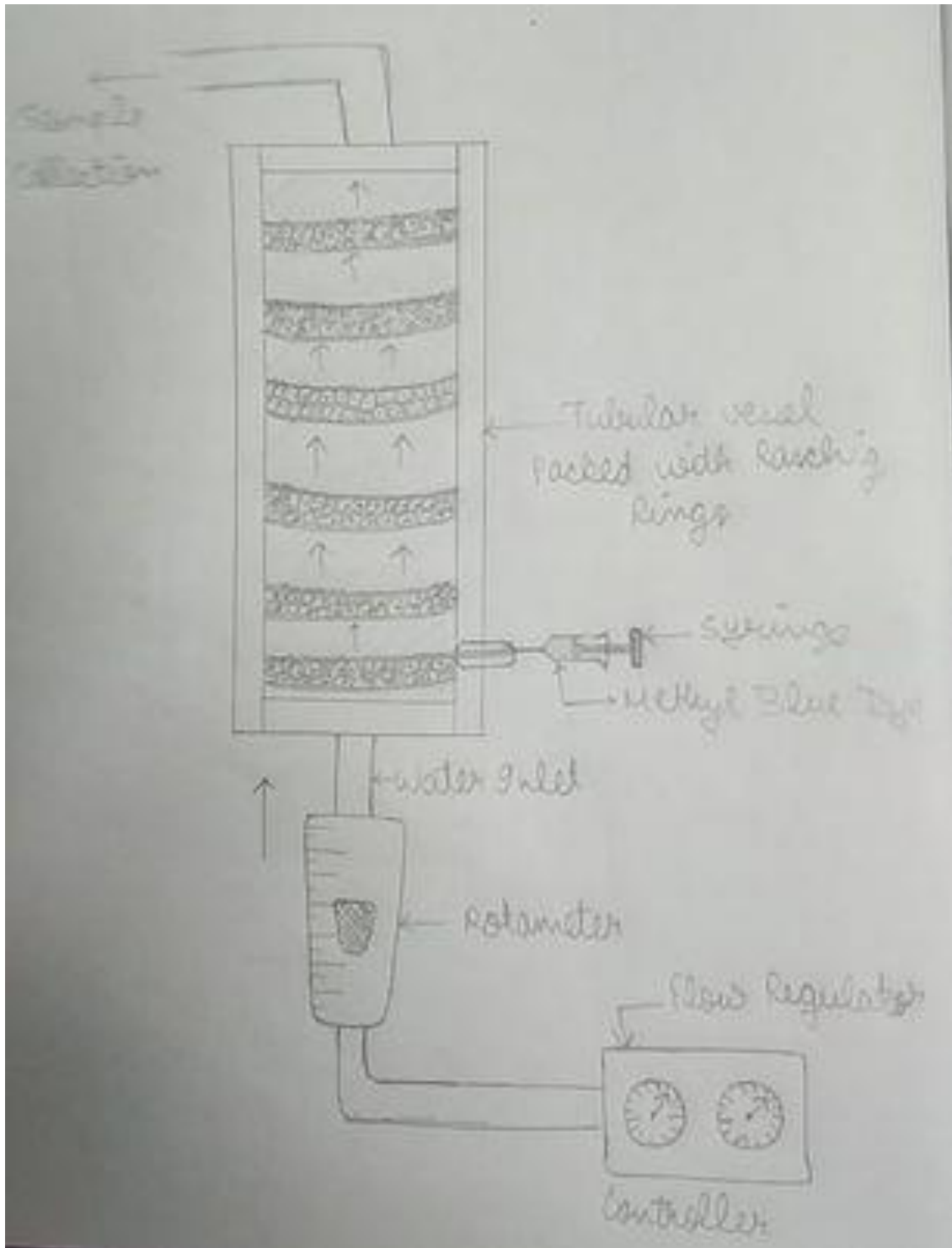
When $D/uL \rightarrow \infty$, large amount of dispersion takes place and mixed flow occurs.

For a closed vessel, the parameter D/uL is calculated from the following:

$$\sigma_{\theta}^2 = 2 \frac{D}{uL} - 2 \left(\frac{D}{uL} \right)^2 (1 - e^{-\frac{uL}{D}})$$

$$\text{Where } \sigma_{\theta}^2 = \frac{\sigma^2}{t^2}, \sigma^2 = \frac{\sum t_i^2 C_i}{\sum C_i} - t^2 \text{ and } t = \frac{\sum t_i C_i}{\sum C_i}$$

Schematic



Observations and Calculations

Length of tubular reactor (L) = 81.5 cm

ID of the tube (d) = 5 cm

*Since concentration is directly proportional to absorbance, they can be used interchangeably.

Flow rate: 10 LPH						
Time t_i (min)	Absorbance (Concentration C_i)	$E_i = C_i/\text{Area}$ under C- curve	$t_i C_i$	t_i^2	$(t_i^2)C_i$	Area
1	0.006	0.00476002	0.006	1	0.006	-
2	0.014	0.0111067	0.028	4	0.056	0.01
3	0.016	0.01269338	0.048	9	0.144	0.015
4	0.018	0.01428005	0.072	16	0.288	0.017
5	0.026	0.02062674	0.13	25	0.65	0.022
6	0.115	0.09123364	0.69	36	4.14	0.0705
7	0.203	0.1610472	1.421	49	9.947	0.159
8	0.225	0.1785006	1.8	64	14.4	0.214
9	0.195	0.15470052	1.755	81	15.795	0.21
10	0.159	0.12614042	1.59	100	15.9	0.177
11	0.132	0.10472035	1.452	121	15.972	0.1455
12	0.113	0.08964697	1.356	144	16.272	0.1225
13	0.083	0.06584689	1.079	169	14.027	0.098
sum (C_i)	1.305	sum ($t_i C_i$)	11.427	sum ($(t_i^2)C_i$)	107.597	
Area under C-curve						1.2605

Flow rate: 20 LPH						
Time t_i (min)	Absorbance (Concentration C_i)	$E_i = C_i/\text{Area}$ under C- curve	$t_i C_i$	t_i^2	$(t_i^2)C_i$	Area
1	0.003	0.003674219	0.003	1	0.003	-
2	0.004	0.004898959	0.008	4	0.016	0.0035
3	0.006	0.007348438	0.018	9	0.054	0.005
4	0.028	0.034292713	0.112	16	0.448	0.017
5	0.175	0.214329455	0.875	25	4.375	0.1015
6	0.158	0.193508879	0.948	36	5.688	0.1665
7	0.113	0.138395591	0.791	49	5.537	0.1355

8	0.107	0.131047152	0.856	64	6.848	0.11
9	0.085	0.104102878	0.765	81	6.885	0.096
10	0.07	0.085731782	0.7	100	7	0.0775
11	0.058	0.071034905	0.638	121	7.018	0.064
12	0.008	0.009797918	0.096	144	1.152	0.033
13	0.006	0.007348438	0.078	169	1.014	0.007
sum (Ci)	0.821	sum (tiCi)	5.888	sum ((ti^2)Ci)	46.038	
Area under C-curve						0.8165

Flow rate: 30 LPH						
Time ti (min)	Absorbance (Concentration Ci)	Ei = Ci/Area under C-curve	tiCi	ti^2	(ti^2)Ci	Area
1	0.003	0.003674219	0.003	1	0.003	-
2	0.004	0.004898959	0.008	4	0.016	0.0035
3	0.006	0.007348438	0.018	9	0.054	0.005
4	0.028	0.034292713	0.112	16	0.448	0.017
5	0.175	0.214329455	0.875	25	4.375	0.1015
6	0.158	0.193508879	0.948	36	5.688	0.1665
7	0.113	0.138395591	0.791	49	5.537	0.1355
8	0.107	0.131047152	0.856	64	6.848	0.11
9	0.085	0.104102878	0.765	81	6.885	0.096
10	0.07	0.085731782	0.7	100	7	0.0775
11	0.058	0.071034905	0.638	121	7.018	0.064
12	0.008	0.009797918	0.096	144	1.152	0.033
13	0.006	0.007348438	0.078	169	1.014	0.007
sum (Ci)	0.821	sum (tiCi)	5.888	sum ((ti^2)Ci)	46.038	
Area under C-curve						0.8165

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Sample calculation for 10 LPH flow rate :-

∴ Absorbance and concentration are related as $C = \frac{A}{E \cdot d}$, where E and d are constants depending on the sample and apparatus.

∴ $C \propto A$, and we are taking ratios, A can be used interchangeably for C.

$\Sigma C_i = 1.305$, $\Sigma t_i C_i = 11.427$, $\Sigma t_i^2 C_i = 107.597$

∴ $E_i = \frac{C_i}{\text{Area under C-curve}}$ Area under C-curve is calculated in an approximate way by taking trapezoids and summing up their areas.

∴ Area under C-curve = 1.2605

∴ $Q = 10 \text{ LPH}$, density = 1000 kg/m^3

∴ cross-section area = $\frac{\pi D^2}{4} = \frac{\pi}{4} \times 0.05^2 = 1.96 \times 10^{-3} \text{ m}^2$

velocity (u) = $\frac{10 \times 10^{-3}}{3600 \times 1.96 \times 10^{-3}} = 1.42 \times 10^{-3} \text{ m/s}$

$\bar{t} = \frac{\Sigma t_i C_i}{\Sigma C_i} = \frac{11.427}{1.305} = 8.756 \text{ min}$

$$\frac{\Sigma t_i^2 C_i}{\Sigma C_i} = \frac{107.597}{1.305} = 82.45 \text{ min}^2$$

$$\sigma^2 = \frac{\Sigma t_i^2 C_i}{\Sigma C_i} - \bar{t}^2 = 82.45 - 8.756^2$$

$$\sigma^2 = 5.7823 \text{ min}^2$$

$$\sigma_\theta^2 = \frac{0.07547}{5.7823} = 2 \frac{D}{uL} - 2 \left(\frac{D}{uL} \right)^2 \left(1 - e^{-\frac{uL}{D}} \right)$$

solving this $\frac{D}{uL} = 0.286306$ 0.039291559

$$Re = \frac{\rho u d}{\mu} = \frac{1000 \times 1.42 \times 10^{-3} \times 0.05}{10^{-3}} = 71$$

Sample calculation for flow rate = 20 LPH :-

$$\sum C_i = 0.821, \sum t_i C_i = 5.888, \sum t_i^2 C_i = 46.038$$

Area under C-curve : 0.8165.

$$Q = 20 \text{ LPH, density} = 1000 \text{ kg/m}^3$$

$$\text{cross-sectional area} = 1.96 \times 10^{-3} \text{ m}^2$$

$$\text{velocity} = \frac{20 \times 10^{-3}}{3600 \times 1.96 \times 10^{-3}} = 2.84 \times 10^{-3} \text{ m/s}$$

$$\bar{t} = \frac{\sum t_i C_i}{\sum C_i} = \frac{5.888}{0.821} = 7.172 \text{ min}$$

$$\frac{\sum t_i^2 C_i}{\sum C_i} = \frac{46.038}{0.821} = 56.08 \text{ min}^2$$

$$\sigma^2 = \frac{\sum t_i^2 C_i}{\sum C_i} - \bar{t}^2 = 56.08 - 7.172^2 = 5.3624 \text{ min}^2$$

$$\sigma_\theta^2 = \frac{0.10425}{5.3624} = \frac{2D}{uL} - 2\left(\frac{D}{uL}\right)^2 (1 - e^{-uL/D})$$

$$\text{solving this } \frac{D}{uL} = 0.047397024$$

$$Re = \frac{\rho u d}{\mu} = \frac{1000 \times 2.84 \times 10^{-3} \times 0.05}{10^{-3}} = 142$$

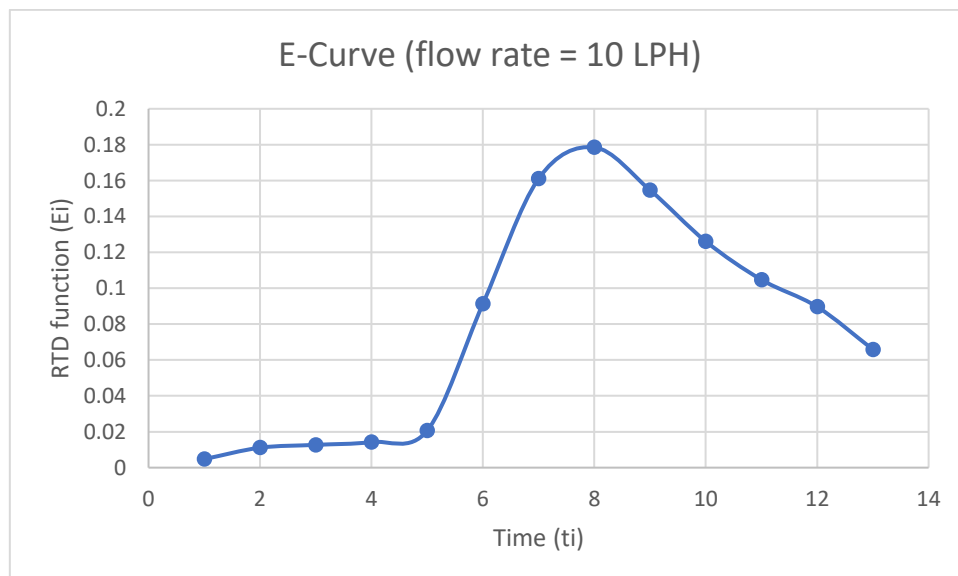
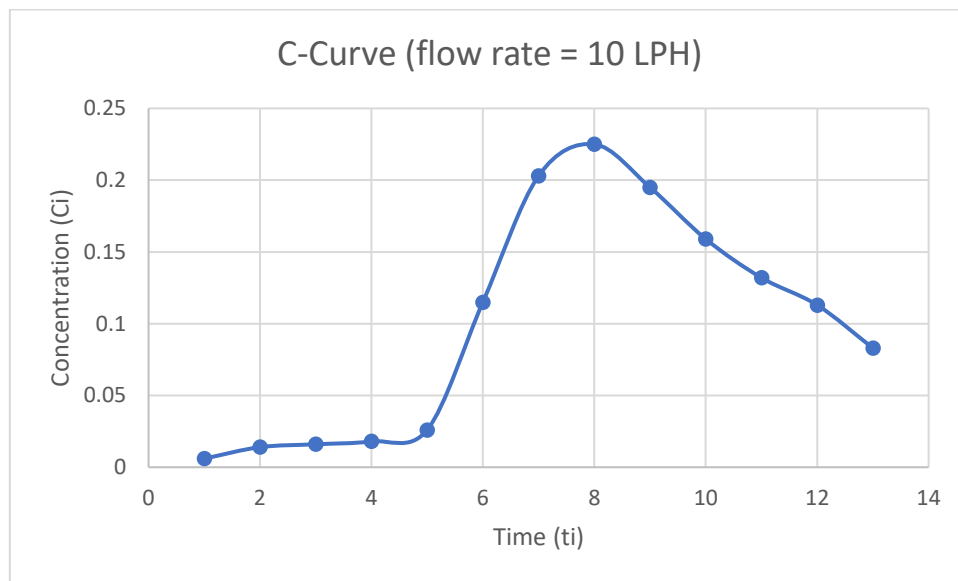
\therefore the concentration values for flow rate = 30 LPH DAY 27 are same as flow rate = 20 LPH,

$\sum C_i, \sum t_i C_i, \sum t_i^2 C_i, \sigma_\theta^2$, area under C-curve, \bar{t} and $\frac{D}{uL}$ are same values. $Re = 3 \times 71 = 213$.

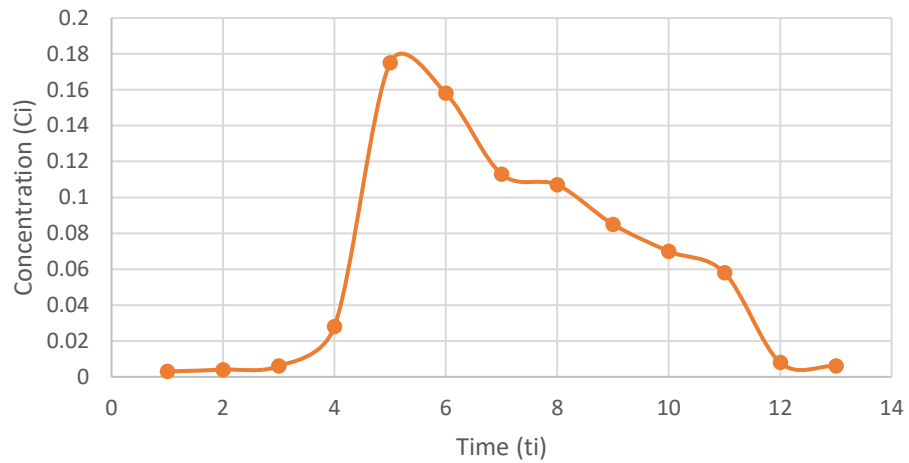
Results

Flow Rate (LPH)	Velocity (m/s)	Reynold's number	Average Residence Time t (min)	D/uL
10	0.00142	71	8.756	0.039291559
20	0.00284	142	7.172	0.047397024
30	0.00426	213	7.172	0.047397024

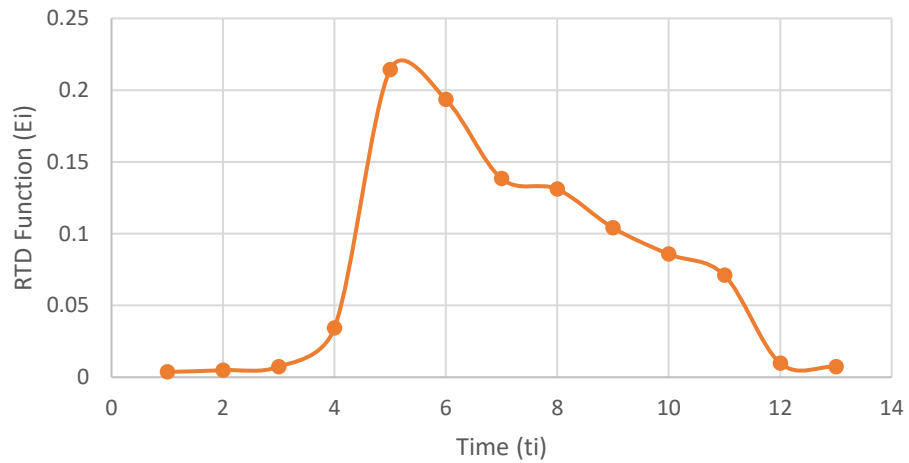
Plots



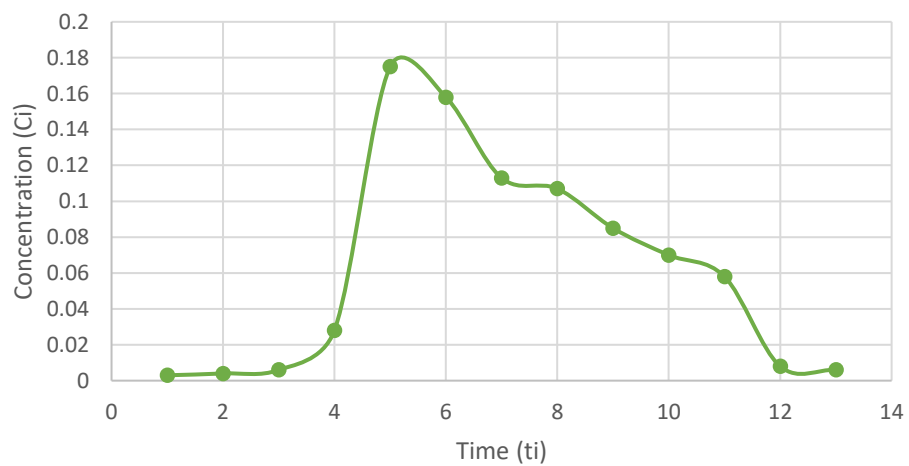
C-Curve (flow rate = 20 LPH)

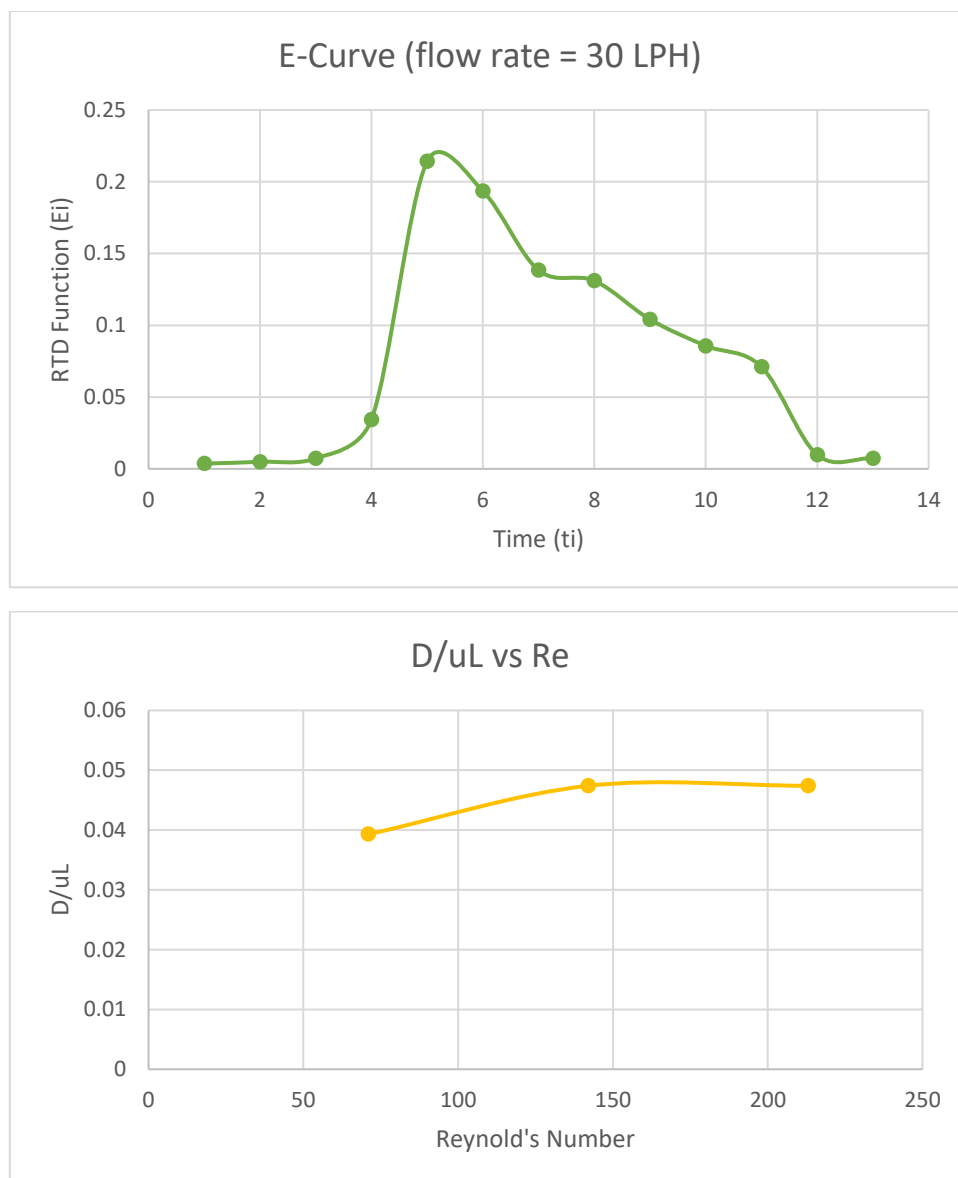


E-Curve (flow rate = 20 LPH)



C-Curve (flow rate = 30 LPH)





Discussion

The probability that reactant molecules/ atoms undergo reaction inside a flow reactor is judged by the residence time distribution. There has been a certain measurement error while measuring absorbance for 20 LPH and 30 LPH flow rates because they are the exact same values. Although there is only slight change in the Reynold's number, exactly same results are not expected to come up for different flow rates. The spectrophotometer was possibly not calibrated properly which might have resulted in such anomalous readings. We need to ensure that the sample is collected as much as possible, in exact 1-minute intervals because a few milliseconds here and there can cause erroneous measurements. The E-curve and C-curve for

different flow rates are seen to have same shape due to them being related by a constant which is the area under the C-curve. The area under the C-curve is calculated by considering trapezoids formed by each segment of the plot. This will give an approximate estimation of the area and is probably the closest one can get to have minimum error. The flow rates for this experiment can be increased a bit to ensure more distinct results. Since we are dealing on the hour scale of flow rates, therefore a small change of 10 litres doesn't create a very big difference as is evident from the D/uL vs Re plot.