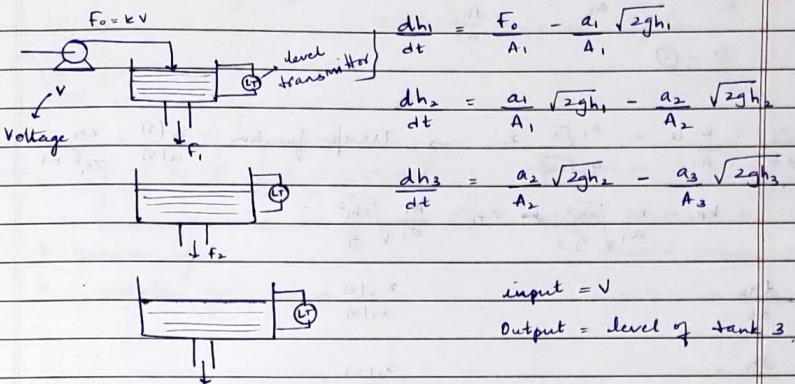


Malluri

 $a_1, a_2, a_3 \rightarrow$ area of outflow pipe $A_1, A_2, A_3 \rightarrow$ area of tank

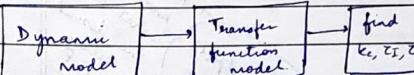
$$\rightarrow \text{PID controller} \quad C = C_b + k_c e + \frac{k_I}{\tau_I} \int e dt + \frac{k_D}{\tau_D} \frac{de}{dt}$$

$$e = 0 \text{ at } ss$$

can be used

to obtain C_b (C = C_b) controller bias

\rightarrow Tuning of PID controller \rightarrow obtain $k_c, \tau_I, \tau_D \rightarrow$ using transfer function model find k_c, τ_I, τ_D

 \rightarrow current to voltage - use resistor \rightarrow replace transform of non-linear equation is not possible

Transfer function model

Define: $x_1 = h_1 - h_1^s$; $x_2 = h_2 - h_2^s$; $x_3 = h_3 - h_3^s$, $u = V - V^s$

(deviation variable) \rightarrow ss values are desired operating conditions

 \rightarrow linearize using Taylor series approx about ss value:-

$$\frac{dh_1}{dt} = \frac{k}{A_1} V - \frac{a_1 \sqrt{2g}}{A_1} \left[\sqrt{h_1^s} + \frac{1}{2\sqrt{h_1^s}} (h_1 - h_1^s) \right] \quad \text{--- (1)}$$

$$\text{ss: } -\frac{k}{A_1} v^s - \frac{a_1 \sqrt{2h_1 s}}{A_1} = 0 \rightarrow \textcircled{2}$$

$\textcircled{1} - \textcircled{2}$

$$\frac{dx_1}{dt} = k u - \frac{a_1 \sqrt{2h_1 s}}{A_1 \sqrt{2h_1 s}} x_1 \rightarrow \text{Transfer function: } \frac{x_1(s)}{u(s)} = \frac{k_{p1}}{z_{p1}s + 1}$$

$$k_{p1} = \frac{k}{a_1 \sqrt{g}} \quad z_{p1} = \frac{A_1}{a_1} \sqrt{\frac{2h_1 s}{g}}$$

$$\frac{dx_2}{dt} = \dots \quad \frac{x_2(s)}{x_1(s)} = \dots$$

$$\frac{dx_3}{dt} = \dots \quad \frac{x_3(s)}{x_2(s)} = \dots$$

$$\text{Process transfer function: } G(s) = \frac{x_3(s)}{u(s)} = \frac{k_{p1} k_{p2} k_{p3}}{(z_{p1}s+1)(z_{p2}s+1)(z_{p3}s+1)}$$

Servo, regulatory $\rightarrow \exists$

\rightarrow Quadruple tank system

$$\frac{dh_1}{dt}$$

$$\frac{dh_2}{dt}$$

$$\frac{dh_3}{dt}$$

$$\frac{dh_4}{dt}$$

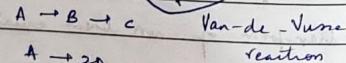
Jacketed CSTR

output = ?

$$MV = F_j$$

No. of inputs = No. of outputs
 \hookrightarrow DOF = 0

CSTR



desired product = B

Input = $F \cdot \tau \rightarrow$ residence time

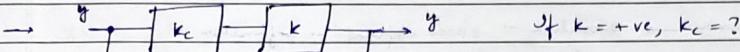
(this will change)

{ Don't require c_C, c_D } the cone

Depending on the control objective, we can reduce the no. of controls are not required (can be neglected)

Input multiplicity system \rightarrow same output can be achieved for multiple different inputs

\Rightarrow gain



K = process gain
 K_c = controller gain

if K changes from +ve to -ve,
 K_c should change accordingly

\rightarrow MATLAB

(Sfunction)

[S-function takes the differential equation in the form of state space model]

\rightarrow [edit spfun.m]

\rightarrow [state space]

$\dot{x} = f(x, u) \rightarrow$ state equation

$y = h(x, u) \rightarrow$ output map

$$\frac{d}{dt} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} u = f_0 \\ x = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \end{bmatrix}$$

\rightarrow Control relevant state-space model

$$\dot{x} = f(x) + g(x) \cdot u \rightarrow \text{non-linear S-S model}$$

$$\dot{x} = Ax + Bu \rightarrow \text{linear S-S model}$$

$$y = CX + DU$$

\rightarrow No. of continuous states = 3 \rightarrow no. of continuous equations

$$h_1, h_2, h_3$$

No. of discrete states = 0

No. of outputs = 1 (h_3)

\hookrightarrow we want to measure the level of 3rd tank only.
 $\text{or } 3(h_1, h_2, h_3) \rightarrow$ to get the info about all 3.

Dir feedthrough \rightarrow while calculating the output whether input is involved or not
 $\text{if input is involved} \rightarrow$ then enter 1 else 0

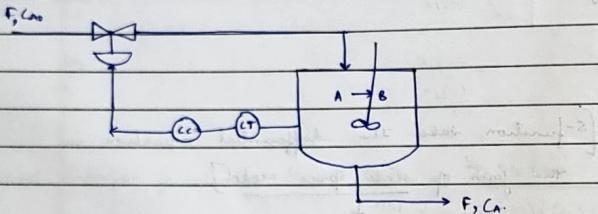
$\Rightarrow x_0 = [] \rightarrow$ initial conditions for the states

→ Code
 $sys = [der1, der2]; \rightarrow \otimes$

→ Simulink

library browser → simulink → user defined → function

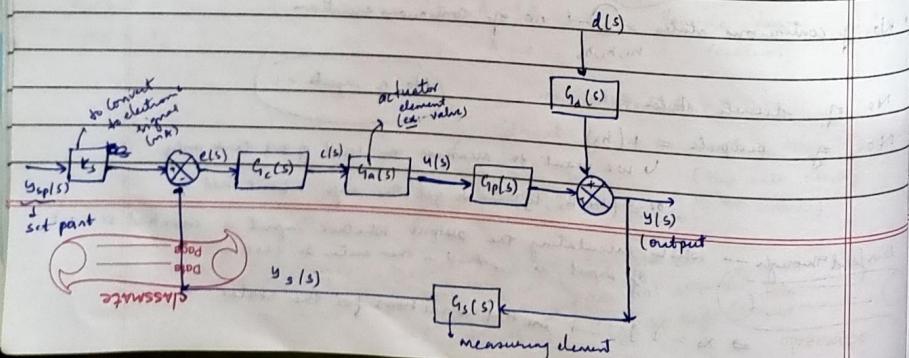
Reactor control



Designing control system

- ① Select control architecture
 - feedback (initial)
- ② find transfer function model of process
 - ① using model equation
 - ② Using process data
- ③ select controller algorithm
 - (a) PID controller
 - (b) Other types.

Feedback control loop



$$\text{Reactor: } \frac{dc_a}{dt} = \frac{F}{V} (C_{ao} - C_a) - k C_a^2$$

$V \rightarrow \text{const}$

$F \rightarrow \text{manipulative variable } (u(s))$
 linearize and take laplace. left over input \rightarrow disturbance $d(s)$

$$y(s) = \frac{(C_{ao} - C_a^2)/V}{s + \frac{F^2}{V} + 2kC_a^2} u(s) + \frac{F^2/V}{s + F^2/V + 2kC_a^2} d(s)$$

$$y = C_a - C_a^2 \quad u = f - F^2 \quad d = C_{ao} - C_a^2$$

$$\text{Control valve: } u(s) = \frac{K_a}{T_a s + 1} c(s)$$

- Normally first order (specified by vendor)
- Time const specified by vendor
- Gain (calculated from installed characteristics) \rightarrow air to open

Generally
 pneumatic
 control valve
 \rightarrow air to close
 control valve
 \rightarrow air to open

Types of globe valves

Quick opening
 Equal percentage
 Linear

$$\Rightarrow \text{Valve eqn: } Q = C_v f(x) \sqrt{\frac{\Delta P}{S_g}}$$

This will decide which type of valve it is

$$f(x) = x \rightarrow \text{linear}$$

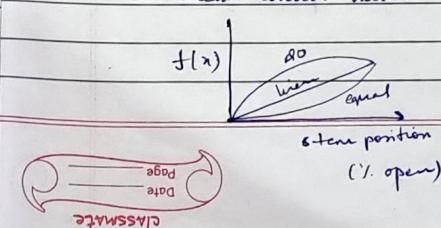
$$f(x) = \sqrt{x} \rightarrow \text{Quick opening}$$

$$f(x) = x^{2-1} \rightarrow \text{equal } \%.$$

Rangability of valve

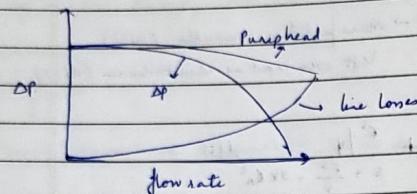
\Rightarrow Inherent control valve.

(pressure drop across the valve is const)



But if flow changes ΔP changes

$\Delta P \propto C^2$

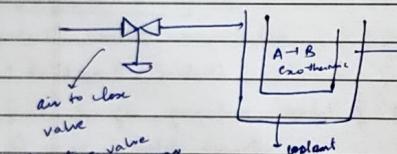


if op changes, valve characteristics change.

→ Installed flow characteristics

→ When to use air to open valve or air to close valve?

- depends on the safety of the process



If the valve fails, the flow will still continue.
But if we use air to open valve, no gas will escape & it will blow off the plane (explosion point)

$$\text{Sensor: } Y_S(s) = \frac{k_s e^{-\tau s}}{z_s s + 1}$$

→ Gain is calculated from range setting

→ Time const & measurement delay is specified by vendor

→ Obtain process data using Specified signal

- step function
- Pulse function
- Sinusoidal function (rarely used)
- Pseudo random Binary sequence (PRBS)

Doublet

(2 pulses opp)

→ Process data includes the actuators and sensor response

→ Evaluate transfer function model from process.

$$Y_p(s) = g_p(s) C(s) + g_i(s) d(s)$$

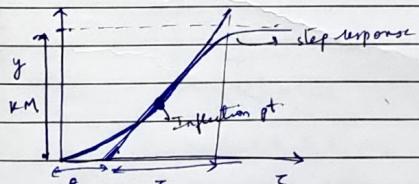
→ 1st order process with dead time (FOTD)

$$g(s) = K \frac{e^{-\theta s}}{zs + 1}$$

→ 4 classical methods (using step response data)

- ① Zeigler Nichols method
- ② Smith's
- ③ Sundaresan & Krishnaswamy
- ④ Nishikawada

→ Z-N method



Draw a tangent at the inflection pt

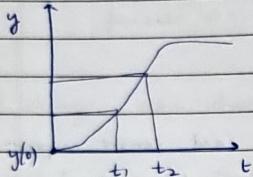
→ Difficult to find inflection point & to draw tangent

Smith's method

$$K = \frac{\Delta \text{output}}{\Delta \text{input}}$$

$$\tau = \frac{3}{2} (t_2 - t_1)$$

$$\theta = t_2 - \tau$$



Sundaresan & Krishnaswamy

$$K = \frac{\Delta \text{output}}{\Delta \text{input}}$$

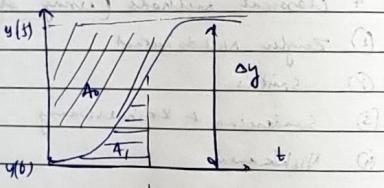
$$\tau = \frac{2}{3} (t_2 - t_1) \quad \theta = 1.3t_1 - 0.29t_2$$

Nishikawa's

$$A_0 = \int [y(\infty) - \Delta y(t)] dt$$

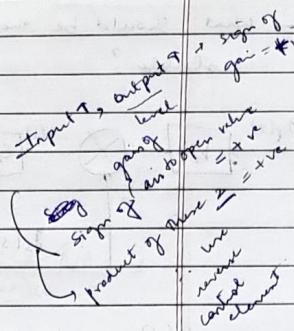
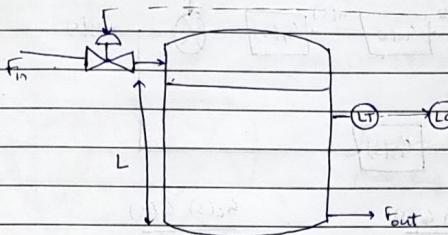
$$A_1 = \int_{t_0}^t \Delta y(t) dt, \quad t_0 = A_0 / \Delta y(\infty)$$

$$\tau = \frac{A_1}{0.368 \Delta y(\infty)} \quad \theta = t_0 - \tau \quad K = \frac{\Delta y(\infty)}{\Delta (\infty)}$$



PID Controller

Direct / Reverse acting controller



Process variable T , controller output T \rightarrow direct acting controller \rightarrow controller
 $PV +$, controller output $+ \rightarrow$ reverse acting \rightarrow sign of controller
 $gain = K_c = +ve$.
 $gain = -ve$.

$$e = \underline{SP} - PV$$

Set point

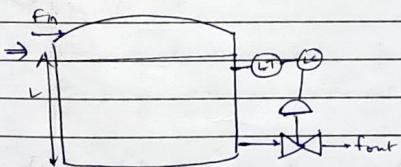
$$\left[\frac{dCout}{dt} = K_c \frac{de}{dt} \right]$$

for direct acting $PV + \equiv e = -ve$.

but $\frac{dCout}{dt} = +ve \therefore K_c = -ve$.

- fail open/fail close
- ↓ valve
- air to open/pneumatically closed

Guidelines of selecting a controller



bigger gain = $-ve \rightarrow$ final $+ve$.

If we use air to open valve \rightarrow gain = $+ve$.

product = $-ve$

\therefore use direct acting controller

action controller

is taking, value

will be in

same direction.

(if the level T

the cont to

(close the valve)

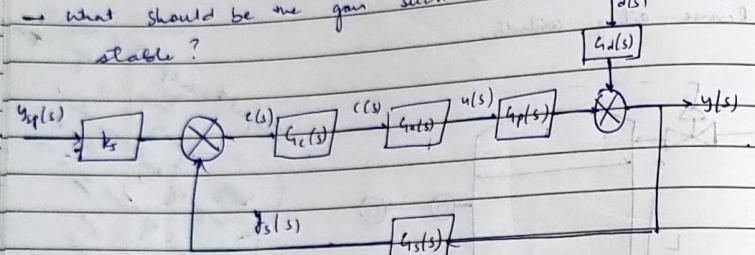
(gain = $+ve$)

(controller

\rightarrow reverse acting

controller

→ What should be the gain such that the closed loop system is stable?



$$\frac{y(s)}{u(s)} = \frac{k_g G_p(s) G_a(s) G_d(s)}{1 + G_p(s) G_a(s) G_d(s) G_c(s)} = \frac{G_c(s) G_d(s)}{1 + G_c(s) G_d(s)}$$

characteristic equation
find the poles.

controller gain = k_c (proportional controller)

Proportional control

Integral control

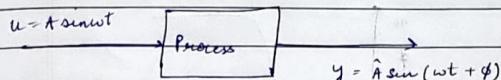
P I control

Closed loop stability

BIBO → bounded input bounded output

- Stability analysis

Frequency Response



$$\text{Amplitude ratio} = \frac{\hat{A}}{A} \cdot e^{j\phi}$$

$$\text{phase lag} = -\phi / \omega \times 360^\circ$$

$$s = j\omega$$

$$G(s) \rightarrow s = j\omega$$

$$G(j\omega) = \text{Re}(s) + j(\text{Im}(s)) = \epsilon(\omega) e^{j\phi(\omega)}$$

$$\epsilon(\omega) = \sqrt{(\text{Re}(\omega))^2 + (\text{Im}(\omega))^2} = |G(\omega)| = AR$$

$$\phi = \text{phase difference} = \arctan\left(\frac{\text{Im}(\omega)}{\text{Re}(\omega)}\right) = \angle G(j\omega)$$

→ Two ways of representing the frequency response.

• Bode plot

log-log AR v/s ω , semi-log ϕ v/s ω .

• Nyquist plot

Plot of $\text{Re}(\omega)$ v/s $\text{Im}(\omega)$ for various values of ω .

Frequency response does not appear explicitly in this plot

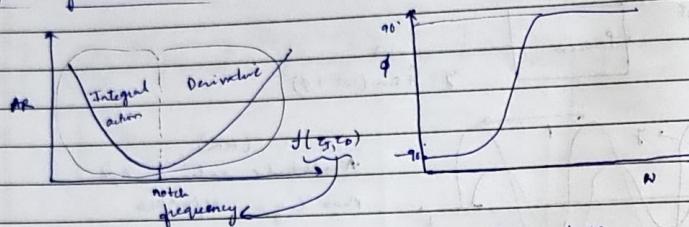
→ steady state gain $\Rightarrow k = \epsilon(0) e^{j\phi(0)}$ → Bode plot

w.r.t. stabilization of process & is faster

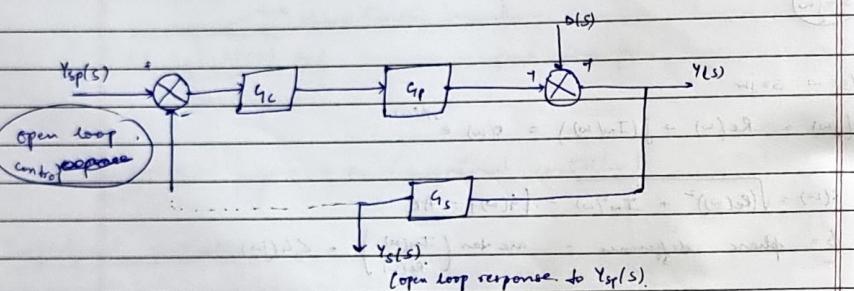
→ Nyquist Diagram

Bode stability

Bode plot of PID controller



By placing the notch frequency we can design PID controller.



→ AR < 1 → at some K_c value

→ if $Y_{sp}(s) = 0$ and complete the loop.

$$\text{Input} = A \sin(\omega t) \quad AR = 1 \leftarrow K_c \uparrow$$

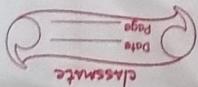
$$\text{Output} = A \sin(\omega t + \theta) \quad \text{Signal will not decay.}$$

(θ = -180° → Signal will remain same in the loop) ✓ Against Design

→ ↑ K_c further, $AR > 1 \rightarrow$ amplitude will go on increasing
↳ the system will become unstable.

→ ↓ $K_c \rightarrow AR < 1 \rightarrow$ amplitude ↓ → oscillations will die. → System is stable

→ ~~unstable~~ System is on the verge of instability when θ = -180°, AR = 1



Bode stability criteria

$K_{cu} =$ ultimate gain → proportional controller

Nyquist stability

PID controller tuning

→ $\tau_I \rightarrow$ high Integral action → means more oscillations

What value of K_c & τ_I should be selected for good performance?

→ Optimization methods use Integral error.

Direct synthesis method

$$G_c = ? \quad \text{unknown}$$

$$Y/Y_{sp} = ?$$

$$\frac{Y}{Y_{sp}} = 1 \rightarrow \text{ideally} \rightarrow G_c \rightarrow \infty$$

$$\Rightarrow G_c = \frac{Y/Y_{sp}}{C(1 - Y/Y_{sp})} \quad (\because \frac{Y}{Y_{sp}} \neq 1 \rightarrow \text{never})$$

① $\Rightarrow \frac{Y}{Y_{sp}}$ can be considered a 1st order process $\rightarrow \left(\frac{Y}{Y_{sp}}\right)_d = \frac{1}{T_c s + 1}$
→ valid for any process without time delay

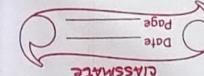
$\tilde{G} \rightarrow$ process transfer function

$$\tilde{G} = k \quad \left(\frac{Y}{Y_{sp}}\right)_d = \frac{1}{T_c s + 1}$$

$$G_c = \frac{\frac{1}{T_c s + 1}}{k(1 - \frac{1}{T_c s + 1})}$$

$$G_c = \frac{1}{k T_c s}$$

$$G_c = \frac{K_I}{s} \quad \text{— Integral controller}$$



$$\tilde{G} = \frac{k}{(z_1 s + 1)(z_2 s + 1)} \rightarrow G_C = \frac{z_1 z_2}{k \tau_c} \left(\frac{z_1 z_2 s^2 + (z_1 + z_2) s + 1}{(z_1 + z_2) s} \right) \rightarrow PID$$

z_i = tuning parameter

$$\Rightarrow \tilde{G}(s) = \frac{k_p e^{-\theta s}}{\tau_p s + 1}$$

$$(1 + \frac{1}{\tau_I s} + z_0 s) \rightarrow PID$$

$$\begin{aligned} Y(s) &= \frac{1}{\tau_c s + 1} \\ Y_{sp}(s) &= \end{aligned}$$

$$G_C = \frac{z_p s + 1}{k_p e^{-\theta p s} \times \tau_c s} = \frac{z_p s + 1}{k_p e^{-\theta p s} \tau_c s}$$

$$\frac{e(s)}{e(s)} = \frac{z_p s + 1}{k_p \tau_c s} e^{\theta p s} \quad (\times) \text{ cannot use this}$$

Not physically realisable

$$e = y_{sp} - y$$

$$\rightarrow \text{use } e^{-\theta s} = 1 - \theta s \quad G_C = k_C \left(1 + \frac{1}{\tau_I s} \right)$$

$$a) \quad g = \frac{2 e^{-s}}{(10s+1)(5s+1)}$$

$$e^{-s} = 1 - s \quad \frac{y}{Y_{sp}} = \frac{e^{-s}}{z_c s + 1}$$

$$g = \frac{2(1-s)}{(10s+1)(5s+1)}$$

$$g = \frac{s}{(10s+1)(5s+1)}$$

$$G_C = \frac{(10s+1)(5s+1)}{s + 5}$$

$$\frac{y}{Y_{sp}} = \frac{e^{-s}}{s + 5}$$

$$G_C(s) = \frac{e^{-s}}{\tilde{G}(z_c + 1)s} = \frac{(10s+1)(5s+1)}{2(z_c + 1)s} \rightarrow PID$$

$$\begin{cases} K_C = ? \\ \tau_I = ? \\ \tau_D = ? \end{cases}$$

loop
Open control

$$y_{sp} \rightarrow [A] \rightarrow [J] \rightarrow y$$

$$\text{for } y = y_{sp} \quad \text{ideally} \quad g(s) = \frac{1}{J(s)}$$

$$y = g(s) g(s) y_{sp}$$

Generally, $y \neq y_{sp}$

$$\therefore \text{we consider } \frac{y(s)}{y_{sp}(s)} = \frac{1}{(s + 1)^n}$$

$$\therefore g(s) = \frac{1}{g(s)} \frac{1}{(s + 1)^n} = \frac{f(s)}{g(s)} \quad \text{actual transfer function of the process}$$

$\rightarrow g(s)$ is estimated as $g_m(s)$

$$g(s) = \frac{f(s)}{g_m(s)}$$

$f(s) \rightarrow$ filter. It should be chosen in such a way that
• $g(s)$ should not become improper
• process dynamic characteristics (\rightarrow no. of zeros $>$ no. of poles)
are not violated

Non-invertible \rightarrow TF containing right hand plane poles \rightarrow dead time

$$g_m(s) = \frac{k e^{-\theta s}}{1 + s^2}$$

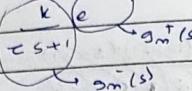
$$\frac{1}{g_m(s)} = \frac{(z_c + 1)e^{-\theta s}}{k} \rightarrow \text{non-invertible}$$

because of dead time.

$$\frac{1}{g_m(s)} = \frac{(z_c + 1)(z_c + 2)}{1 + s^2} - 1$$

$$N(s) =$$

$$J_m(s) = g_m^-(s) g_m^+(s)$$



$g_m^-(s)$ is invertible part

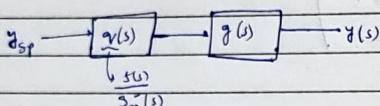
$g_m^+(s)$ is non-invertible

$$g(s) = \frac{f(s)}{g_m^-(s)}$$

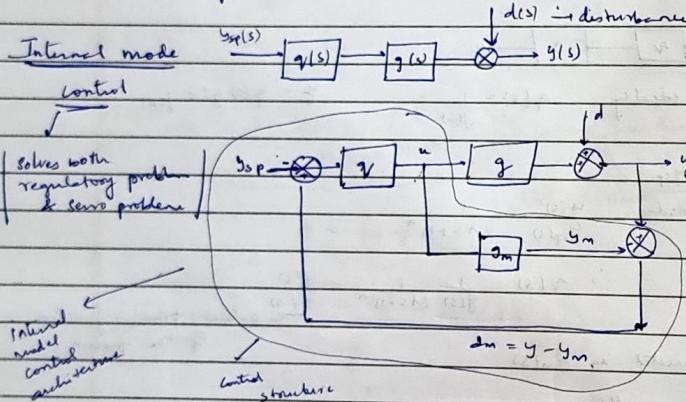
open

loop control law \rightarrow only for servo control (no disturbance)

Open loop control is always stable if the process is stable.



If we have disturbance, the open control law fails.



$$y(s) = \frac{g(s) v(s)}{1 + g(s)(g(s) - g_m(s))} y_{sp}(s) + \frac{1 - g_m(s)v(s)}{1 + g(s)(g(s) - g_m(s))} d(s)$$

Identify: $v(s) = \frac{1}{g_m(s)}$ $\rightarrow y = y_{sp} \rightarrow$ it can take care of servo control
 \rightarrow if $d(s)$ changes \rightarrow takes care of disturbance
 \rightarrow also there is no change in the process variable \rightarrow takes care of regulatory control

$$\rightarrow J_m(s) = g_m^-(s) g_m^+(s)$$

$$v_I(s) = \frac{1}{g_m^-(s)}$$

$$f(s) = \frac{1}{(1s+1)^n}$$

$$v(s) = v_I(s) f(s) = \frac{f(s)}{g_m^-(s)}$$

(λ, n are the parameters)

$$a) g(s) = \frac{1}{10s+1} e^{-2s}$$

No process/model mismatch $\rightarrow J_m(s) = f(s)$

$$g_m^-(s) = \frac{1}{10s+1} \quad g_m^+(s) = e^{-3s}$$

$$v_I(s) = 10s+1$$

$$v(s) = v_I(s) f(s) = \frac{10s+1}{1s+1} \rightarrow \text{lead-lag TF.}$$

lead-lag element is available.

$$y(s) = g_m^+(s) f(s) \frac{v_I(s)}{y_{sp}(s)} = \frac{e^{-3s}}{1s+1} y_{sp}(s)$$

a has advantages

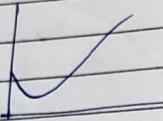
$$b) g(s) = \frac{-9s+1}{(15s+1)(3s+1)}$$

$$g_m^-(s) = \frac{-9s+1}{(15s+1)(3s+1)} \quad g_m^+(s) = \frac{-9s+1}{9s+1}$$

$$v_I(s) = v_I(s) f(s) = \frac{(15s+1)(3s+1)}{1s+1} \frac{1}{1s+1}$$

$$y(s) = g_m^+(s) f(s) y_{sp}(s) = \frac{-9s+1}{(15s+1)(3s+1)} y_{sp}(s)$$

→ If we have right hand plane zero, we will have inverse response



PI D controller

$$J_C(s) = \frac{g(s)}{1 - g_m(s)g(s)}$$

a) FODT system using PI controller

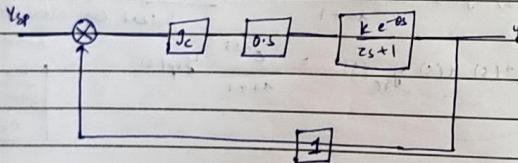
Control valve gain = 0.5

measurement device gain = 1

Method

	K_C	Z_I
Z-N	0.3426	14.2546
IMC-PI	0.2	10.0

Z-N PI relation: $K_C = 0.45 K_u$, $Z_I = P_u / 1.2$

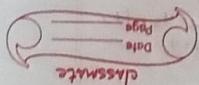


Find K , Z , θ

$g_C(s)$ = known

$g_m(s)$ = known

$$\begin{aligned} y &= g_C(0.5) \frac{k_C e^{-0.5s}}{2s+1} \\ y_{sp} &= \frac{1 + g_m(6s)k_C e^{-0.5s}}{2s+1} \end{aligned}$$



$$G_C = k_C \left(1 + \frac{1}{Z_I s} \right)$$

$$K_C = \frac{1}{K} \left(\frac{T}{\theta + T_C} \right)$$

Z-N method

$$\rightarrow K_C = 0.3426 \quad K_u = \frac{0.3426}{6.45}$$

$$P_u = 1.2 \times 14.2546$$

$P_u = \frac{2\pi}{\omega_{co}}$ → corner frequency. ($\phi = -180^\circ$)
 ω_{co} → find ω_{co} .

$$\omega_{co} = 0.3673$$

$$\phi = -\tan^{-1}(\omega_c) - \omega_{co}\theta.$$

(get θ)

$$K_u = \frac{1}{A_R C}$$

$$A_R = \frac{k_0 \cdot 5}{\sqrt{1 + Z^2 \omega_{co}^2}}$$

for IMC PI :-

$$Z_I = Z$$

$$K_C = \frac{C}{K(A+B)}$$

find Z from here. $Z = 10 = Z_I$

$$Z = 10$$

$$-\pi = \tan^{-1}(3.673) - \omega_{co} \times \theta.$$

$$+\pi = +1.364 + 0.367 \times 0.$$

$$\theta = \frac{5.026 \times 180}{\pi} = 286.83^\circ$$

$$K = 10,$$

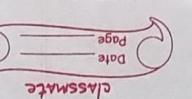
$$\rightarrow \text{PID for } \text{sound} \rightarrow f(s) = \frac{1}{(As+1)^n}$$

$$\Rightarrow f(s) = \frac{Ns+1}{(As+1)^n}$$

$$\text{IMC eqn.} \quad y(s) = \frac{(1 - g(s)g_m(s)) d(s)}{1 + g(s)(g(s) - g_m(s))}$$

$$\approx \frac{1 - g(s)g(s)}{1 + g(s)(g(s) - g_m(s))} g(s) L(s)$$

for disturbance.



$$y(s) = \frac{1 - v(s) g(s)}{1 + v(s) (g(s) - g_m(s))}$$

$$y(s) = (1 - v(s) g(s)) g(s) L(s)$$

$y = f(z)$
should be calculated
using this

$$(Q1) \quad g(s) = \frac{2}{(5s+1)(2s-1)}$$

$$v(s) = \frac{(5s+1)(2s-1) \times ys+1}{2(As+1)^2}$$

$$P_u = y_2$$

$$f(s) = \frac{ys+1}{(As+1)^2}$$

$$y_1 = \frac{ys+1}{(As+1)+1}$$

$$y = \left[\left(A\left(\frac{1}{2}\right) + 1 \right)^2 - 1 \right]_2$$

$$y = \left(\frac{\lambda^2 + 2A}{4} \right)^2$$

$$y = \frac{\lambda^2 + 2A}{2}$$

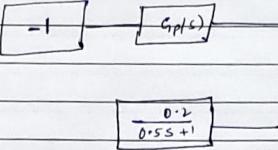
$$g_L(s) = \frac{1}{2} \left[\frac{(5s+1)(2s-1)(ys+1)}{\lambda^2 s^2 + (2A-y)s} \right]$$

$$g_C(s) = \frac{1}{2} \left[\frac{(5s+1)(ys+1)}{\frac{\lambda^2 s}{2}} \right] \quad \text{PID}$$

(~~cancel~~)

$$(Q3) \quad g_p(s) = \frac{-s-7}{(s+1)(1.5s+1)}$$

control valve



gain = $4^\circ C$
 1 kg/m^2

steam control valve.

for temp measurement device
gain = $\frac{100 - 0}{20 - 4}$
output
input