# Indian Institute of Technology Kharagpur

End-Spring Semester 2017 – 18

Date of Examination: 27-04-2018 Session: AN Duration 3 hrs Full Marks 50
Subject Number: CH30012 Subject: Transport Phenomena
Department: Chemical Engineering
Specific charts, graph paper, log book etc., required: No. All questions carry equal marks.
Specific Instructions: Assume and clearly write any assumption and data that you feel are missing.

1. A spherical naphthalene ball of initial size of 0.03 m is hanged in a closet. One has to evaluate the time it takes for the naphthalene ball to sublimate completely. The following information are known. The concentration of naphthalene in the air is very small, and the low mass flux conditions exist so that the Chilton-Coulburn analogy between heat and mass transfer is applicable. Both the air and naphthalene vapor are ideal gases, the temperatures are same everywhere and there are no radiation effects. The molar mass of naphthalene is 128.2 kg/kmol with a density equal to 1100 kg/m$^3$. The dry air properties at 1 atm and 25$^o$ C are $\rho = 1.184$ kg/m$^3$. $c_p$ = 1007 J/Kg.K and $\alpha = 2.141 \times 10^{-5}$ m$^2$/s, the mass fraction of naphthalene on the air side of the surface is $4.8 \times 10^{-4}$. You may use the appropriate value of Nusselt number for this case.

2. The one-dimensional flow of oil between two large parallel plates is to be analyzed at steady state with constant properties and in absence of any significant body forces in the direction of flow. The top plate is moving to the right (in the + x direction), that sustains the flow (pure Couette flow) with the origin of the coordinate system located at the bottom plate. The distance between the two plates is L and there is significant viscous dissipation because of the high viscosity of the oil and the large plate velocity. Note that in this case the dissipation function can simply be expressed as $\mu(dv_x/dy)^2$. The bottom plate is insulated and the top plate is maintained at a constant temperature of $T_o$ with conduction the prevailing mode of heat transfer.

a) Derive an expression for the velocity distribution in the oil.

b) Derive an expression for the temperature distribution in the oil.

c) Locate the maximum temperature in the flow domain.

d) Evaluate the heat that is to be removed from the upper plate to maintain its temperature constant.

3. A steel strip emerges from a steel mill at a speed of 20 m/s and at 1200 K. Its length and thicknesses are L = 100m and t = 0.003m, respectively, and its density and specific heat are 7900 kg/m$^3$ and 640 J/kg.K. Accounting for the heat transfer from the top and bottom surfaces only and neglecting radiation and strip conduction effects, determine the initial time rate of change of the strip temperature at 1 m from the leading edge and at the trailing edge (L = 100m). Determine the distance from the leading edge at which the minimum cooling rate is achieved. Given: for air: v = 76.4x10 - 6 m$^2$ /s, k = 0.0549 W/m.K, Pr = 0.702, $T_\infty$ = 300K.

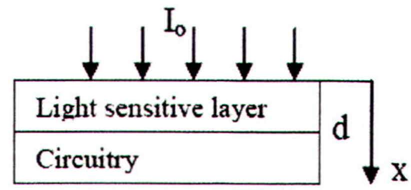
4. Two large flat porous horizontal plates are separated by a relatively small distance L. The upper plate at y = L is at a temperature $T_1$ and the lower one at y = 0 is to be maintained at a lower temperature $T_o$. To reduce the amount of heat that must be removed from the lower plate, an ideal gas at $T_o$ is blown upward through both the plates at a steady rate. Develop an expression for the temperature distribution and the amount of heat $q_o$ that must be removed from the cold plate per unit area as a function of the fluid properties and gas flow rates. Use $\emptyset = {\rho\, C_p\, v_y\, L}/{k}$ where ρ is the density, $c_p$ is the heat capacity, $v_y$ is the upward velocity of air through the

plates and k is the thermal conductivity of air to present your result in a compact form.

5. A semiconductor photodiode consists of a light sensitive layer of thickness, d, coupled to electronic circuitry that measures the current produced in that layer. Electrons are generated at a rate, $\dot{M}$, by light striking the sensitive layer and are directly proportional to the light intensity. The light gets absorbed in the layer so the intensity varies exponentially with depth.

$$I = I_o \exp(-\alpha x) \qquad and \qquad \dot{M}\,(mol/m^3\,s) = m_o I$$

Once generated, the electrons diffuse toward the circuitry (i.e., all generated electrons must flow through the interface at x = d) and a signal is issued proportional to the electron flux at d.




(a) Determine the concentration profile of electrons, $C_e$, in the light sensitive layer. You may assume flow of electrons is governed by a law similar to Fick's law. The diffusivity of electrons in the light sensitive layer is $D_{es}$.

(b) All photodiodes have a dark current, $I_d$, due to electrons formed by random thermal means, that is basically noise. The dark current density is proportional to the volume of light sensitive material, $I_d = I_{no} V$, where V is the volume and $I_{no}$ is a constant. Develop an expression for the signal to noise ratio.

**EQUATION OF CONTINUITY (Cartesian, cylindrical and spherical coordinates)**

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

**EQUATION OF MOTION (Cartesian and Cylindrical coordinates)**

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

**INTEGRAL EQUATIONS**

$$\frac{dN}{dt}\Big|_{system} = \frac{\partial}{\partial t}\int_{CV} \eta\, \rho\, d\forall + \int_{CS} \eta\, \rho \vec{V}.\vec{dA} \qquad \delta^* = \int_0^\delta \left(1 - \frac{v_x}{U}\right) dy, \qquad \theta = \int_0^\delta \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right) dy$$

$$\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \frac{dU}{dx} \qquad \delta = \frac{5.0x}{\sqrt{Re_x}} \ (laminar\ flow) \quad \delta = \frac{0.37x}{(Re_x)^{\frac{1}{5}}} \ (turbulent\ flow) \qquad \frac{\overline{v_z}}{U} = \left(\frac{y}{R}\right)^{\frac{1}{7}}$$

Laminar Flow: $C_f = \frac{0.664}{\sqrt{Re_x}}$ $\quad C_D = \frac{1.328}{\sqrt{Re_L}}$ Turbulent Flow: $C_f = \frac{0.0594}{(Re_x)^{\frac{1}{5}}}$ $\quad C_D = \frac{0.0742}{(Re_L)^{\frac{1}{5}}}$

*For Mixed Flow,* $\quad C_{D\,Turb} = \frac{0.074}{(Re_L)^{\frac{1}{5}}} - \frac{1740}{Re_L}, \quad 10^5 < Re < 10^7, \quad C_{D\,Turb} = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1610}{Re_L}, \ Re > 10^7$

STOKES LAW $\qquad F = 3\pi\mu V d$

**ENERGY EQUATION (in all coordinate systems)**

$$\rho \hat{C}_p \left( \frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \mu \phi_v + \dot{Q}$$

$$\rho \hat{C}_p \left( \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] + \mu \phi_v + \dot{Q}$$

$$\rho \hat{C}_p \left( \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right) = k \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right] + \mu \phi_v + \dot{Q}$$

$Sc = \frac{\mu}{\rho D_{AB}}$ $Sh = \frac{h_m l}{D_{AB}}$ $St = \frac{Nu}{Re*Pr}$ $Fo = \frac{\alpha t}{l^2}$ $Bi = hl/k_s$ $Le = \frac{\alpha}{D_{AB}}$ $C_f = \frac{\tau_w}{\frac{1}{2}\rho v^2}$ $C_D = \frac{F_D/A_P}{\frac{1}{2}\rho v^2}$

**Flow over flat plate**

For laminar flow ($0.6 \leq \Pr \leq 60$ & $Re \leq 5\times10^5$) $\delta = \frac{5x}{\sqrt{Rex}}$ $C_{fx} = \frac{0.664}{\sqrt{Rex}}$

$Nu_x = 0.332 Re_x^{1/2} \Pr^{1/3}$ $\overline{Nu} = 0.664 Re_x^{1/2} \Pr^{1/3}$

For turbulent flow ($0.5 \leq \Pr \leq 60$ & $Re \geq 10^7$)

$\delta = \frac{0.37x}{Rex^{1/5}}$ $C_{fx} = \frac{0.0577}{Rex^{1/5}}$ $C_D = \frac{0.455}{\log(Re_l)^{2.88}}$

$Nu_x = 0.0296 Re_x^{4/5} \Pr^{1/3}$ $\overline{Nu_l} = 0.037 Re_l^{4/5} \Pr^{1/3}$

For mixed flow

$C_f = \frac{0.072}{Re_l^{1/5}} - \frac{1740}{Re_l}$ $Re \leq 10^7$ $C_D = \frac{0.455}{\log(Re_l)^{2.88}} - \frac{1610}{Re_l}$ $Re \geq 10^7$

$\overline{Nu_l} = \left(0.037 Re_l^{\frac{4}{5}} - 871\right) \Pr^{\frac{1}{3}}$ $5 \times 10^5 \leq Re \leq 10^7$

Reynold's Analogy ($Pr = 1.0$) $St = \frac{C_{fx}}{2}$, $St = Nu/(Re.\Pr)$

Chilton Coulburn Analogy $St \times Pr^{2/3} = \frac{C_{fx}}{2}$ $For\ 0.5 \leq Pr \leq 50$

**SPECIES BALANCE EQUATIONS**

$$\rho \left( \frac{\partial \omega_A}{\partial t} + v_x \frac{\partial \omega_A}{\partial x} + v_y \frac{\partial \omega_A}{\partial y} + v_z \frac{\partial \omega_A}{\partial z} \right) = \rho D_{AB} \left[ \frac{\partial^2 \omega_A}{\partial x^2} + \frac{\partial^2 \omega_A}{\partial y^2} + \frac{\partial^2 \omega_A}{\partial z^2} \right] + r_A$$

$$\rho \left( \frac{\partial \omega_A}{\partial t} + v_r \frac{\partial \omega_A}{\partial r} + \frac{v_\theta}{r} \frac{\partial \omega_A}{\partial \theta} + v_z \frac{\partial \omega_A}{\partial z} \right) = \rho D_{AB} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \omega_A}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \omega_A}{\partial \theta^2} + \frac{\partial^2 \omega_A}{\partial z^2} \right] + r_A$$

$$\rho \left( \frac{\partial \omega_A}{\partial t} + v_r \frac{\partial \omega_A}{\partial r} + \frac{v_\theta}{r} \frac{\partial \omega_A}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial \omega_A}{\partial \phi} \right) = \rho D_{AB} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \omega_A}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial \omega_A}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 \omega_A}{\partial \phi^2} \right] + r_A$$