Heat Transfer (CH21004)

Assignment-1 Solution

Solution 1:

For this case we can safely assume that the hydrodynamic boundary layer is so thin that the entire thermal boundary layer experiences the free
stream velocity.

$$v \approx 0; \quad u = u_\infty$$

For this case, the energy balance simplifies to:

$$u_\infty \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}$$

Using boundary integral:

$$\frac{u_\infty}{\alpha} \int_0^{\delta_t(x)} \frac{\partial T}{\partial x} dy = \int_0^{\delta_t(x)} \frac{\partial^2 T}{\partial y^2} dy$$

$$\frac{u_\infty}{\alpha} \int_0^{\delta_t(x)} \frac{\partial T}{\partial x} dy = -\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

Bringing the differential outside the integral is slightly trickier in this case.

For this, we will first transform the variable T into $\hat{T}$. In terms of $\hat{T}$:

$$\frac{u_\infty}{\alpha} \int_0^{\delta_t(x)} \frac{\partial \hat{T}}{\partial x} dy = -\left.\frac{\partial \hat{T}}{\partial y}\right|_{y=0}$$

Now we can bring the derivative out with the understanding that the lower
limit is constant and the function is zero at the upper limit.

$$\frac{u_\infty}{\alpha}\frac{d}{dx}\int_0^{\delta_t(x)}\hat{T}dy = -\left.\frac{\partial \hat{T}}{\partial y}\right|_{y=0}$$

In terms of scaled $\hat{y}_t$:

$$\frac{u_\infty \delta_t(x)}{\alpha}\frac{d}{dx}\left[\delta_t(x)\int_0^{1}\hat{T}d\hat{y}_t\right] = -\left.\frac{\partial \hat{T}}{\partial \hat{y}_t}\right|_{\hat{y}_t=0}$$

We know :

$$\hat{T} = 1 - \tfrac{3}{2}\hat{y}_t + \tfrac{1}{2}\hat{y}_t^3$$

Therefore,

$$\frac{u_\infty \delta_t(x)}{\alpha}\frac{d}{dx}\left[\delta_t(x)\int_0^{1}(1-\frac{3}{2}\hat{y}_t+\frac{1}{2}\hat{y}_t^3)d\hat{y}_t\right] = \frac{3}{2}$$

Which simplifies to:

$$\frac{d\delta_t(x)^2}{dx} = \frac{8\alpha}{u_\infty}$$

Solving with the boundary condition:

$$\delta_t(x) = 0 \quad \text{at } x = 0$$

Therefore,

$$\delta_t(x) = \sqrt{\frac{8\alpha x}{u_\infty}}$$

Now we can obtain the required Nusselt number correlation:

$$q_w = h.(T_w - T_\infty) = \frac{3}{2}k(T_w - T_\infty)\sqrt{\frac{u_\infty}{8\alpha x}}$$

which gives:

$$\frac{h.x}{k} = \frac{3}{2}\frac{1}{\sqrt{8}}x\sqrt{\frac{u_\infty}{\alpha x}} = 0.53\sqrt{\frac{u_\infty x}{\nu}\frac{\nu}{\alpha}}$$

Hence the required correlation is:

$$\boxed{Nu = 0.53Re^{\frac{1}{2}}Pr^{\frac{1}{2}}}$$

Solution 2:

Dimensional analysis is the mathematical technique of deriving relations between physical quantities by identifying their dimensions. In dimensional analysis, the various physical quantities used in fluid phenomenon can be expressed in terms of fundamental quantities. These fundamental quantities are mass (M), length (L), time (T), and temperature (θ). Forced convection heat transfer coefficient is a function of variables given below in tabular form.

Sl. No.	Variable	Symbol	Dimensions
1	Fluid density	ρ	ML^{-3}
2	Dynamic viscosity of fluid	μ	$ML^{-1}T^{-1}$
3	Fluid velocity	v	LT^{-1}
4	Thermal conductivity of fluid	k	$MLT^{-3}\,\theta^{-1}$
5	Sp. heat of fluid	C_p	$L^2T^{-2}\,\theta^{-1}$
6	Characteristic length of heat transfer area	L	L

Therefore, convective heat transfer coefficient is expressed as

$$h = f(\rho, \mu, V, k, C_p, D) \qquad (1)$$

$$f(h, \rho, \mu, V, k, C_p, D) = 0 \qquad (2)$$

Convective heat transfer coefficient, h is dependent variable and remaining are independent variables.

Total number of variables, n = 7

Number of fundamental units, m = 4

According to Buckingham's π-theorem, number of π-terms is given by the difference of total number of variables and number of fundamental units.

Number of π-terms = (n-m) = 7-4 = 3

These non-dimensional π-terms control the forced convection phenomenon and are expressed as

$$\mathbf{f(\pi_1, \pi_2, \pi_3) = 0} \qquad (3)$$

Each π-term is written in terms of repeating variables and one other variable. In order to select repeating variables following method should be followed.

- Number of repeating variables should be equal to number of fundamental units involved in the physical phenomenon.
- Dependent variable should not be selected as repeating variable.
- The repeating variables should be selected in such a way that one of the variables should contain a geometric property such as length, diameter or height. Other repeating variable should contain a flow property such as velocity or acceleration and the third one should contain a fluid property such as viscosity, density, specific heat or specific weight.
- The selected repeating variables should not form a dimensionless group.
- The selected repeating variables together must have same number of fundamental dimensions.
- No two selected repeating variables should have same dimensions.

The following repeating variables are selected

1. Dynamic viscosity, μ having fundamental dimensions $[ML^{-1}T^{-1}]$
2. Thermal conductivity, k having fundamental dimensions $[MLT^{-3}\theta^{-1}]$
3. Fluid velocity, V having fundamental dimensions $[LT^{-1}]$
4. Characteristic length, D having fundamental dimensions [L]

Each π-term is expressed as:

$$\boldsymbol{\pi_1 = \mu^a\, k^b, V^c, D^d, h} \tag{4}$$

Writing down each term in above equation in terms of fundamental dimensions

$$M^0L^0T^0\,\theta^0 = (ML^{-1}T^{-1})^a\,(MLT^{-3}\,\theta^{-1})^b\,(LT^{-1})^c\,(L)^d\,MT^{-3}\,\theta^{-1}$$

Comparing the powers of M, we get

$$0 = a+b+1, \quad \text{or,} \quad a+b= -1 \tag{5}$$

Comparing powers of L, we get

$$0 = -a+b+c+d \tag{6}$$

Comparing powers of T, we get

$$0 = -a-3b-c-3 \tag{7}$$

Comparing powers of θ, we get

$$0 = -b-1,$$

$$b=-1 \tag{8}$$

Substituting value of ‘b’ from equation (8) in equation (5), we get

$$a = 0 \tag{9}$$

Substituting values of ‘a’ and ‘b’ in equation (7), we get

$$c = 0 \tag{10}$$

Substituting the values of ‘a’, ‘b’ and ‘c’ in equation (6), we get

$d = 1$

Substituting the values of ‘a’, ‘b’, ‘c’ and ‘d’ in equation (4), we get

$\pi_1 = \mu^{-0}\ k^{-1}, V^0, D^1, h$

$\pi_1 = h\ D / k$ (11)

The second π –term is expressed as

$\boldsymbol{\pi_2 = \mu^a\ k^b, V^c, D^d, \rho}$ (12)

$M^0L^0T^0\ \theta^0 = (ML^{-1}T^{-1})^a\ (MLT^{-3}\ \theta^{-1})^b\ (LT^{-1})^c\ (L)^d\ ML^{-3}$

Comparing the powers of M, we get

$0 = a+b+1,\ a+b= -1$ (13)

Comparing powers of L, we get

$0 = -a+ b+ c +d -3$ (14)

Comparing powers of T, we get

$0 = -a- 3b-c$ (15)

Comparing powers of θ, we get

$0 = -b$,

$b=0$ (16)

Substituting value of ‘b’ from equation (16) in equation (13), we get

$a = -1$ (17)

Substituting values of ‘a’ and ‘b’ in equation (15), we get

$c = 1$ (18)

Substituting the values of ‘a’, ‘b’ and ‘c’ in equation (14), we get

$d = 1$

Substituting the values of ‘a’, ‘b’, ‘c’ and ‘d’ in equation (12), we get

$\pi_2 = \mu^{-1}\, k^0, V^1, D^1, \rho$

$$\pi_2 = \rho\, VD / \mu \qquad (19)$$

The third π –term is expressed as

$$\boldsymbol{\pi_3 = \mu^a\, k^b, V^c, D^d, C_p} \qquad (20)$$

$M^0L^0T^0\, \theta^0 = (ML^{-1}T^{-1})^a\, (MLT^{-3}\, \theta^{-1})^b\, (LT^{-1})^c\, (L)^d\, L^2T^{-2}\, \theta^{-1}$

Comparing the powers of M, we get

$$0 = a+b, \quad \text{or,} \quad a+b= 0 \qquad (21)$$

Comparing powers of L, we get

$$0 = -a+ b+ c +d +2 \qquad (22)$$

Comparing powers of T, we get

$$0 = -a- 3b-c-2 \qquad (23)$$

Comparing powers of θ, we get

$0 = -b -1,$

$$b=-1 \qquad (24)$$

Substituting value of ‘b’ from equation (24) in equation (21), we get

$$a = 1 \qquad (25)$$

Substituting values of ‘a’ and ‘b’ in equation (23), we get

$$c = 0 \qquad (26)$$

Substituting the values of ‘a’, ‘b’ and ‘c’ in equation (22), we get

$d = 0$

Substituting the values of ‘a’, ‘b’, ‘c’ and ‘d’ in equation (12), we get

$\pi_3 = \mu^1\, k^{-1}, V^0, D^0, C_p$

$\pi_3 = \mu\, C_p/k$ (27)

Substituting the values of π_1, π_2, π_3 in equation (3), we get

$f(h\,D / k,\ \rho\, VD / \mu,\ \mu\, C_p/k) = 0$

$h\,D / k = \varphi(\rho\, VD / \mu,\ \mu\, C_p/k)$

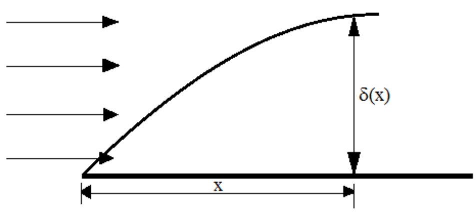
$\boxed{\mathbf{Nu = \varphi(Re, Pr)}}$

The above correlation is generally expressed as

$$\boxed{\mathbf{Nu = C\,(Re)^a\,(Pr)^b}}$$

The constant C and exponents ‘a’ and ‘b’ are determined through experiments.

Solution 3:



$$\underbrace{\int_0^{\delta} u\frac{\partial u}{\partial x}dy}_{\text{Term-1}} + \underbrace{\int_0^{\delta} v\frac{\partial u}{\partial y}}_{\text{Term-2}} = \underbrace{\int_0^{\delta}\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}}_{\text{Term-3}}$$

Term - 2:

$$\int_0^{\delta} v\frac{\partial u}{\partial y} = v\,u\Big|_0^{\delta} - \int_0^{\delta}\frac{\partial v}{\partial y}u\,dy$$

Term -1 + Term-2:

$$\int_0^{\delta} \frac{\partial (u)^2}{\partial x} dy + u_\infty v_\infty$$

Therefore,

$$\int_0^{\delta} \frac{\partial}{\partial x}\left(uu_\infty - u^2\right) dy = \frac{\mu}{\rho} \frac{\partial u}{\partial y}\bigg|_{y=0} \qquad (1)$$

Now,

$$\int_0^{\delta} \frac{\partial}{\partial x}\left(uu_\infty - u^2\right) dy = \frac{\partial}{\partial x}\int_0^{\delta}\left(uu_\infty - u^2\right) dy \text{ (using Libnitz ruel)}$$

From equation (1),

$$\frac{\partial}{\partial x}\int_0^{\delta}\left(uu_\infty - u^2\right) dy = \frac{d}{dx}\int_0^{\delta}\left(uu_\infty - u^2\right) dy = \frac{\mu}{\rho} \frac{\partial u}{\partial y}\bigg|_{y=0}$$

By dividing both sides by ${u_\infty}^2$,

$$\frac{d}{dx}\int_0^{\delta} \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy = \frac{\tau_w}{\rho {u_\infty}^2} \qquad (2)$$

This is known as Von-Karman momentum integral equation.

Say,

$$\frac{u}{u_\infty} = a_0 + a_1\left(\frac{y}{\delta}\right) + a_2\left(\frac{y}{\delta}\right)^2 + a_3\left(\frac{y}{\delta}\right)^3$$

Boundary Conditions:

$$i)\, at\ y = 0, u = 0$$
$$ii)\, at\ y = \delta, u = u_\infty$$
$$iii)\, at\ y = \delta, \frac{\partial u}{\partial y} = 0$$
$$iv)\, at\ y = 0, \frac{\partial^2 u}{\partial y^2} = 0$$

We get,

$$\frac{u}{u_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

$$\hat{u} = \frac{3}{2}\hat{y}_t - \frac{1}{2}\hat{y}_t^{\,3}$$

$$\text{where, } \hat{u} = \frac{u}{u_\infty} \text{ and, } \hat{y}_t = \frac{y}{\delta}$$

The RHS of equation (2) become,

$$\frac{\tau_w}{\rho u_\infty^{\ 2}} = \frac{\mu}{\rho u_\infty^{\ 2}} \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{3}{2}\frac{\mu \frac{u_\infty}{\delta}}{\rho u_\infty^{\ 2}}$$

Now putting this velocity relation in equation (2) and taking $\frac{y}{\delta} = \eta$, we get,

$$\frac{\delta^2}{2} = \frac{3}{2}\frac{\mu x}{\rho u_\infty (0.1363)} + C$$

Boundary conditions:

$$At, x = 0, \delta = 0, \therefore\ C = 0$$

$$\boxed{\frac{\delta}{x} = 4.64\,\mathrm{Re}_x^{-\frac{1}{2}}}$$

or, $$\boxed{\delta(x) = 4.64x\,\mathrm{Re}_x^{-\frac{1}{2}}}$$

Similar to velocity profile, we may assume temperature profile as below

$$\hat{T} = a + b\hat{y}_t + c\hat{y}_t^2 + d\hat{y}_t^3$$

Boundary conditions:

$$\text{at } \hat{y}_t = 0, \quad \hat{T} = 1$$

$$\text{at } \hat{y}_t = 1, \quad \hat{T} = 0$$

$$\text{at } \hat{y}_t = 0 \quad \frac{\partial^2 \hat{T}}{\partial \hat{y}_t^2} = 0$$

$$\text{at } \hat{y}_t = 1, \quad \frac{\partial \hat{T}}{\partial \hat{y}_t} = 0$$

where,

$$\hat{T} = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\hat{y}_t = \frac{y}{\delta_t(x)}$$

Then, the temperature profile become,

$$\hat{T} = 1 - \frac{3}{2}\hat{y}_t + \frac{1}{2}\hat{y}_t^3$$

Now,

$$\begin{aligned} q_w &= -k\frac{\partial T}{\partial y}\bigg|_{y=0} \\ &= -k\frac{(T_w - T_\infty)}{\delta}\frac{\partial \hat{T}}{\partial \hat{y}_t}\bigg|_{\hat{y}_t=0} \\ &= \frac{3}{2}k\frac{(T_w - T_\infty)}{\delta} \end{aligned}$$

By putting the expression of δ [i.e., δ(x)], we get

$$q_{\mathrm{w}} = \frac{3}{2}k\frac{(T_{\mathrm{w}} - T_\infty)}{4.64x}\mathrm{Re}_x^{\frac{1}{2}}$$

$$= 0.323\frac{k}{x}(T_{\mathrm{w}} - T_\infty)\mathrm{Re}_x^{\frac{1}{2}}$$

Using Newton's law of cooling,

$$q_w = h_x(T_w - T_\infty)$$

Comparing,

$$h_x = 0.323\frac{k}{x}\mathrm{Re}_x^{\frac{1}{2}}$$

$$\boxed{\frac{h_x.x}{k} = 0.323\,\mathrm{Re}_x^{\frac{1}{2}}}$$

Solution 4:

$$\frac{\partial (Tu)}{\partial x} + \frac{\partial (Tv)}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

Integrating this equation over the thermal boundary layer thickness i.e., $\delta_t(x)$,

$$\int_0^{\delta_t(x)} \frac{\partial (Tu)}{\partial x} dy + \int_0^{\delta_t(x)} \frac{\partial (Tv)}{\partial y} dy = \int_0^{\delta_t(x)} \alpha \frac{\partial^2 T}{\partial y^2} dy$$

Simplifying,

$$\int_0^{\delta_t(x)} \frac{\partial (Tu)}{\partial x} dy + (Tv)\Big|_{y=\delta_t(x)} = -\alpha \frac{\partial T}{\partial y}\Big|_{y=0}$$

The velocity at the outer skin of the thermal boundary layer may be obtained by integrating the continuity equation over the thickness of the thermal boundary layer:

$$\int_0^{\delta_t(x)} \frac{\partial u}{\partial x} dy + \int_0^{\delta_t(x)} \frac{\partial v}{\partial y} dy = 0$$

$$\int_0^{\delta_t(x)} \frac{\partial u}{\partial x} dy + v\Big|_{\delta_t(x)} = 0$$

Substituting:

$$\int_0^{\delta_t(x)} \frac{\partial (Tu)}{\partial x} dy - T_\infty \int_0^{\delta_t(x)} \frac{\partial u}{\partial x} dy = -\alpha \frac{\partial T}{\partial y}\Big|_{y=0}$$

Simplifying:

$$\int_0^{\delta_t(x)} \frac{\partial}{\partial x}[u(T - T_\infty)]dy = -\alpha \frac{\partial T}{\partial y}\bigg|_{y=0}$$

By using Leibnitz rule:

$$\frac{d}{dx}\int_0^{\delta_t(x)} [u(T - T_\infty)]dy = -\alpha \frac{\partial T}{\partial y}\bigg|_{y=0}$$

We will use the following transformations for non-dimensionalization:

$$\begin{aligned}
\hat{T} &= \frac{T - T_\infty}{T_w - T_\infty} \\
\hat{y}_t &= \frac{y}{\delta_t(x)} \\
\hat{u} &= \frac{u}{u_\infty} \\
dy &= \delta_t(x) d\hat{y}_t \\
\frac{\partial}{\partial y} &= \frac{\partial}{\partial \hat{y}_t}\frac{d\hat{y}_t}{dy} = \frac{1}{\delta_t(x)}\frac{\partial}{\partial \hat{y}_t}
\end{aligned}$$

Using the above transformations:

$$\frac{d}{dx}\int_0^1 \left[\hat{u}u_\infty \hat{T}(T_w - T_\infty)\delta_t(x)\right] d\hat{y}_t = -\alpha.\frac{1}{\delta_t(x)}\left(\frac{\partial}{\partial \hat{y}_t}\left[\hat{T}(T_w - T_\infty) + T_\infty\right]\right)_{\hat{y}_t=0}$$

Simplifying:

$$u_{\infty}\frac{d}{dx}\int_0^1[\hat{u}\hat{T}.\delta_t(x)]d\hat{y}_t=-\frac{\alpha}{\delta_t(x)}\frac{\partial\hat{T}}{\partial\hat{y}_t}\bigg|_{\hat{y}_t=0}$$

With additional simplification:

$$\frac{d}{dx}\left[\delta_t(x)\int_0^1\hat{u}\hat{T}d\hat{y}_t\right]=-\frac{\alpha}{u_{\infty}\delta_t(x)}\frac{\partial\hat{T}}{\partial\hat{y}_t}\bigg|_{\hat{y}_t=0}$$

Again, we shall assume scaling behaviour for dimensionless temperature profile similar to previous case and we get:

$$\hat{T}=1-\frac{3}{2}\hat{y}_t+\frac{1}{2}\hat{y}_t^3$$

substituting $\hat{T}$ from equation and $\hat{u}$ from equation into equation, we obtain:

$$\frac{d}{dx}\left[\delta_t(x)\int_0^1\left(\frac{3}{2}\hat{y}-\frac{1}{2}\hat{y}^3\right)\left(1-\frac{3}{2}\hat{y}_t+\frac{1}{2}\hat{y}_t^3\right)d\hat{y}_t\right]=-\frac{\alpha}{u_{\infty}\delta_t(x)}\left(-\frac{3}{2}\right)$$

Let,

$$\frac{\hat{y}}{\hat{y}_t}=\frac{\delta_t(x)}{\delta(x)}$$

$$\frac{\hat{y}}{\hat{y}_t}=\frac{\delta_t(x)}{\delta(x)}\equiv\psi(\nu,\alpha)$$

$$\hat{y}=\psi\hat{y}_t$$

Substituting:

$$\frac{d}{dx}\left[\delta_t(x)\int_0^1\left(\frac{3}{2}\psi\hat{y}_t-\frac{1}{2}\psi^3\hat{y}_t^3\right)\left(1-\frac{3}{2}\hat{y}_t+\frac{1}{2}\hat{y}_t^3\right)d\hat{y}_t\right]=\frac{3\alpha}{2u_{\infty}\delta_t(x)}$$

In terms of ψ,

$$\int_0^1 \left(\frac{3}{2}\psi\hat{y}_t - \frac{1}{2}\psi^3\hat{y}_t^3\right)\left(1 - \frac{3}{2}\hat{y}_t + \frac{1}{2}\hat{y}_t^3\right) d\hat{y}_t = \left[\frac{3}{20}\psi - \frac{3}{280}\psi^3\right] \equiv \zeta(\psi)$$

Substituting from above and with some rearranging:

$$2\delta_t(x)\frac{d\delta_t(x)}{dx} = \frac{3\alpha}{u_\infty \xi(\psi)}$$

Solution of this equation with the boundary condition at x=0, δ_t(x)=0, gives:

$$\delta_t(x) = \sqrt{\frac{3\alpha x}{u_\infty \xi(\psi)}}$$

Now, we need to look back to check whether our solution contradicts our initial assumption that the ratio of the two boundary layer thickness is a function of two diffusities or not. It can be seen from equation and equation, that this is indeed the case. Hence, our solution is consistent with our assumption. Now we need to obtain a closed form expression for ψ.

$$\begin{aligned}
\psi &\equiv \frac{\delta_t(x)}{\delta(x)} \\
&= \sqrt{\frac{3\alpha x}{u_\infty \xi(\psi)}} \Big/ 4.64\sqrt{\frac{\nu x}{u_\infty}} \\
&= \frac{\sqrt{3}}{4.64}\sqrt{\frac{\alpha}{\nu}}\frac{1}{\sqrt{\zeta(\psi)}} \\
&= 0.373\sqrt{\frac{\alpha}{\nu}}\left[\frac{3}{20}\psi - \frac{3}{280}\psi^3\right]^{-0.5}
\end{aligned}$$

Mathematically,

$$0 < \psi \leq 1.0$$

In this range:

$$\frac{3}{20}\psi - \frac{3}{280}\psi^3 \approx \frac{3}{20}\psi$$

Using this simplification,

$$\psi = 0.373\sqrt{\frac{\alpha}{\nu}}\frac{1}{\sqrt{3\psi/20}}$$

Solving for ψ:

$$\psi = 0.975\sqrt[3]{\frac{\alpha}{\nu}} \approx \sqrt[3]{\frac{\alpha}{\nu}}$$

The quantity ν/α is one of the most important dimensionless number used in heat transfer an is known as Prandtl Number (Pr). In terms of Prandtl number, the thermal boundary layer thickness is given by:

$$\psi \equiv \frac{\delta_t(x)}{\delta(x)} = \frac{1}{\sqrt[3]{\mathrm{Pr}}}$$

$$\delta_t(x) = \frac{\delta(x)}{\sqrt[3]{\mathrm{Pr}}} = \frac{4.64x}{\sqrt[3]{\mathrm{Pr}}\sqrt{Re_x}}$$

Now, by heat balance,

$$\begin{aligned} q_w &= -k\frac{\partial T}{\partial y}\bigg|_{y=0} \\ &= -k\frac{(T_w - T_\infty)}{\delta_t}\frac{\partial \hat{T}}{\partial \hat{y}_t}\bigg|_{\hat{y}_t=0} \\ &= \frac{3}{2}k\frac{(T_w - T_\infty)}{\delta_t} \\ &= \frac{3}{2}k\frac{(T_w - T_\infty)\sqrt[3]{\mathrm{Pr}}\sqrt{Re_x}}{4.64x} \\ &= 0.323\frac{k}{x}(T_w - T_\infty)\sqrt[3]{\mathrm{Pr}}\sqrt{Re_x} \end{aligned}$$

Using Newton's law of cooling:

$$q_w = h_x(T_w - T_\infty)$$

Comparing,

$$\begin{aligned} h_x &= 0.323\frac{k}{x}(Pr)^{\frac{1}{3}}.Re_x^{\frac{1}{2}} \\ \frac{h_x.x}{k} &= 0.323(Pr)^{\frac{1}{3}}.Re_x^{\frac{1}{2}} \\ \end{aligned}$$

$$\boxed{Nu_x = 0.323(Pr)^{\frac{1}{3}}Re_x^{\frac{1}{2}}}$$