ASSIGNMENT-1 (F.M. 50 Marks)

① Overall Mass Balance: 15 Marks

Tank 1: $A_1 \dfrac{dh_1}{dt} = F_1 - F_2.$

Tank 2: $A_2 \dfrac{dh_2}{dt} = F_2 - F_3$

Using deviational variables and in laplace domain.

Tank 1: $A_1 s \bar{h}_1(s) = \bar{F}_1(s) - \bar{F}_2(s)$ ——— ①

Tank 2: $A_2 s \bar{h}_2(s) = \bar{F}_2(s) - \bar{F}_3(s)$ ——— ②

Measured output / Controlled variable : $\bar{h}_2(s)$

Manipulated input / Control variable : $\bar{F}_1(s), \bar{F}_2(s), \bar{F}_3(s)$

(a) Controller (PI) : $G_c = K_c\left(1 + \dfrac{1}{\tau_I s}\right)$

Case 1: $\bar{F}_1(s)$ is the manipulated variable

$\bar{F}_2(s) = \alpha_1 \bar{h}_1(s)$; $\bar{F}_3(s) = \alpha_2 \bar{h}_2(s)$

From ① and ② ; substituting the values of $\bar{F}_2(s)$ and $\bar{F}_3(s)$

$A_1 s \bar{h}_1(s) = \bar{F}_1(s) - \alpha_1 \bar{h}_1(s)$

$$\bar{h}_1(s) = \frac{(1/\alpha_1)}{\left(\frac{A_1}{\alpha_1}\right)s + 1}\bar{F}_1(s) \Rightarrow \bar{h}_1(s) = \frac{(1/\alpha_1)}{(\tau_1 s + 1)}\bar{F}_1(s).$$

where, $\tau_1 = A_1/\alpha_1$

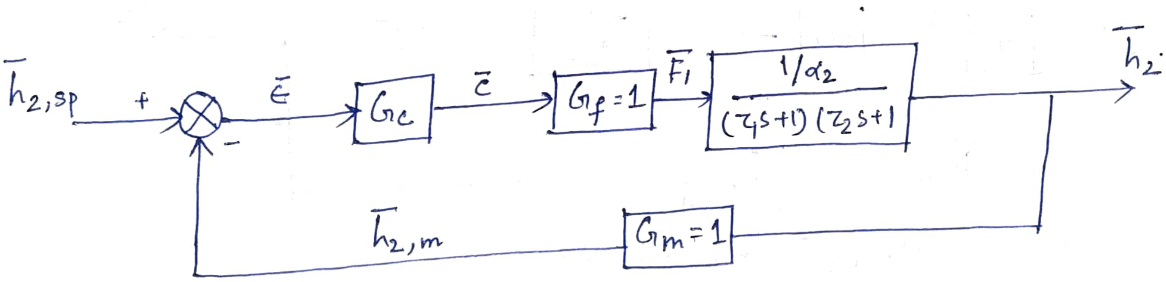
Also, $A_2 s \bar{h}_2(s) = \alpha_1 \bar{h}_1(s) - \alpha_2 \bar{h}_2(s)$

$$\Rightarrow A_2 s \bar{h}_2(s) = \frac{\alpha_1 (1/\alpha_1)}{(\tau_1 s + 1)}\bar{F}_1(s) - \alpha_2 \bar{h}_2(s)$$ 1 Mark

$$\Rightarrow \bar{h}_2(s) = \frac{1/\alpha_2}{(\tau_1 s + 1)\left(\frac{A_2}{\alpha_2}s + 1\right)}\bar{F}_1(s) \Rightarrow \boxed{\bar{h}_2(s) = \frac{(1/\alpha_2)}{(\tau_1 s + 1)(\tau_2 s + 1)}\bar{F}_1(s)}$$ — ③

where, $\tau_2 = \dfrac{A_2}{\alpha_2}$.

Let $\dfrac{(1/\alpha_2)}{(\tau_1 s + 1)(\tau_2 s + 1)} = G_p(s)$ ∴ $\bar{h}_2(s) = G_p(s)\bar{F}_1(s)$



closed-loop block diagram. 1 Mark

Case 2 : $\bar{F}_2(s)$ is the manipulated variable.

$\bar{F}_3(s) = \alpha_2 \bar{h}_2(s)$

From ① and ②

$$A_1 s \bar{h}_1(s) = \bar{F}_1(s) - \bar{F}_2(s)$$

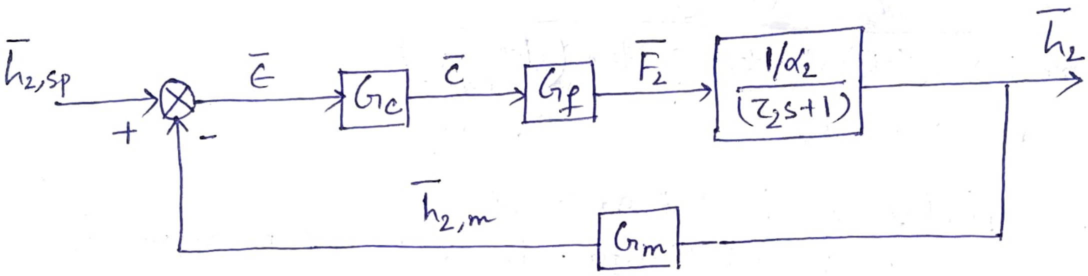
$$\Rightarrow \quad \bar{h}_1(s) = \frac{1}{A_1 s}\bar{F}_1(s) - \frac{1}{A_1 s}\bar{F}_2(s)$$

$$A_2 s \bar{h}_2(s) = \bar{F}_2(s) - \bar{F}_3(s)$$

$$A_2 s \bar{h}_2(s) = \bar{F}_2(s) - \alpha_2 \bar{h}_2(s)$$

1 Mark

$$\bar{h}_2(s) = \frac{(1/\alpha_2)}{\left(\frac{A_2}{\alpha_2}s+1\right)}\bar{F}_2(s) \quad \Rightarrow \quad \boxed{\bar{h}_2(s) = \frac{(1/\alpha_2)}{(\tau_2 s+1)}\bar{F}_2(s)} \quad \text{—④}$$



Closed loop block diagram.

1 Mark

Case 3 : $\bar{F}_3(s)$ is the manipulated variable.

$\bar{F}_2(s) = \alpha_1 \bar{h}_1(s)$

From ① and ②

$$A_1 s \bar{h}_1(s) = \bar{F}_1(s) - \bar{F}_2(s)$$

$$\Rightarrow A_1 s \bar{h}_1(s) = \bar{F}_1(s) - \alpha_1 \bar{h}_1(s) \quad \Rightarrow \quad \bar{h}_1(s) = \frac{(1/\alpha_1)}{(\tau_1 s+1)}\bar{F}_1(s)$$

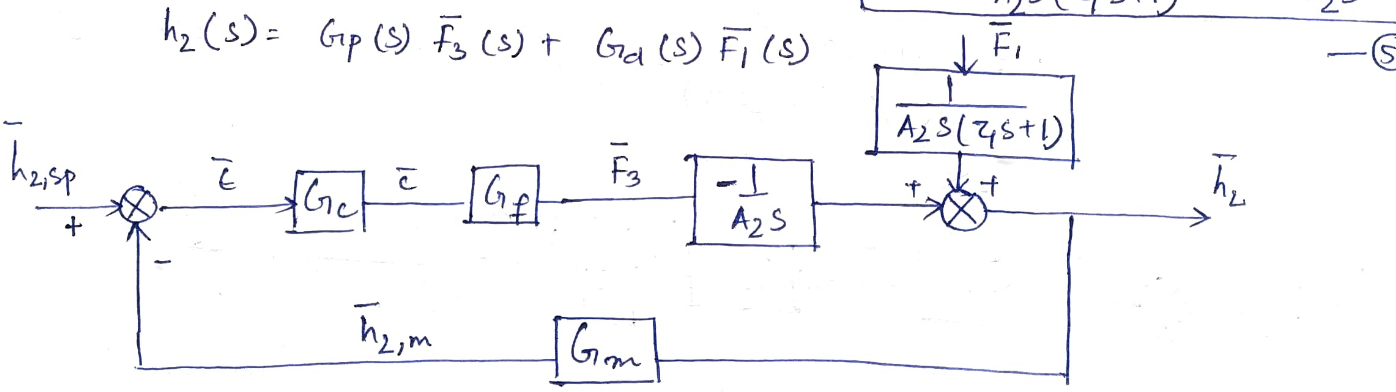
$$A_2 s \bar{h}_2(s) = \bar{F}_2(s) - \bar{F}_3(s)$$

$$A_2 s \bar{h}_2(s) = \alpha_1 \bar{h}_1(s) - \bar{F}_3(s)$$

1 Mark

$$A_2 s \bar{h}_2(s) = \frac{1}{(\tau_1 s+1)}\bar{F}_1(s) - \bar{F}_3(s) \quad \Rightarrow \quad \boxed{\bar{h}_2(s) = \frac{1}{A_2 s(\tau_1 s+1)}\bar{F}_1(s) - \frac{1}{A_2 s}\bar{F}_3(s)}$$

$$\bar{h}_2(s) = G_p(s)\bar{F}_3(s) + G_d(s)\bar{F}_1(s) \quad \text{—⑤}$$



Closed loop block diagram. 1 Mark

(b) General form of Closed Loop transfer function.

$$\bar{y} = \frac{G_p G_f G_c}{1 + G_p G_f G_c G_m} \bar{y}_{sp} + \frac{G_d}{1 + G_p G_f G_c G_m} \bar{d} \quad \text{—— (6)}$$

where $\bar{y} \leftarrow$ output variable

$\bar{y}_{sp} \leftarrow$ Set point value of the output variable

$\bar{d} \leftarrow$ disturbance variable / load variable

This can be represented as.

$$\bar{y} = G_{sp} \bar{y}_{sp} + G_{load} \bar{d} \quad \text{—— (7)}$$

G_{sp} and G_{load} are the corresponding closed-loop transfer functions for changes in set point and load variables respectively.

Case 1: $G_p = \dfrac{(1/\alpha_2)}{(\tau_1 s+1)(\tau_2 s+1)}$, $G_f = G_m = 1$, $G_c = k_c\left(1+\dfrac{1}{\tau_I s}\right)$

1 Mark $$\bar{h}_2(s) = \frac{k_c\left(1+\frac{1}{\tau_I s}\right)\frac{(1/\alpha_2)}{(\tau_1 s+1)(\tau_2 s+1)}}{1 + k_c\left(1+\frac{1}{\tau_I s}\right)\cdot\frac{(1/\alpha_2)}{(\tau_1 s+1)(\tau_2 s+1)}} \bar{h}_{2,sp}(s) \quad \text{—— (8)}$$

Case 2: $G_p = \dfrac{(1/\alpha_2)}{(\tau_2 s+1)}$, $G_f = G_m = 1$, $G_c = k_c\left(1+\dfrac{1}{\tau_I s}\right)$

1 Mark $$\bar{h}_2(s) = \frac{k_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{1/\alpha_2}{\tau_2 s+1}\right)}{1 + k_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{1/\alpha_2}{\tau_2 s+1}\right)} \bar{h}_{2,sp}(s) \quad \text{—— (9)}$$

Case 3: $G_p = -\dfrac{1}{A_2 s}$, $G_d = \dfrac{1}{A_2 s(\tau_1 s+1)}$, $G_f = G_m = 1$, $G_c = k_c\left(1+\dfrac{1}{\tau_I s}\right)$

1 Mark $$\bar{h}_2(s) = \frac{k_c\left(1+\frac{1}{\tau_I s}\right)\left(-\frac{1}{A_2 s}\right)}{1 + k_c\left(1+\frac{1}{\tau_I s}\right)\left(-\frac{1}{A_2 s}\right)} \cdot \bar{h}_{2,sp}(s) + \frac{\frac{1}{A_2 s(\tau_1 s+1)}}{1 + k_c\left(1+\frac{1}{\tau_I s}\right)\left(-\frac{1}{A_2 s}\right)} \cdot \bar{F}_i(s) \quad \text{—— (10)}$$

(c) Static gain of the system refers to the ratio of change in output ~~data~~ and change in input at steady state condition. It can be evaluated at $\boxed{G(s\to 0)}$

Case 1 $$G(s) = \frac{k_c\left(1+\frac{1}{\tau_I s}\right)\frac{(1/\alpha_2)}{(\tau_1 s+1)(\tau_2 s+1)}}{1 + k_c\left(1+\frac{1}{\tau_I s}\right)\cdot\frac{(1/\alpha_2)}{(\tau_1 s+1)(\tau_2 s+1)}} \quad [\because \text{From (8)}]$$

$$= \frac{k_c(\tau_I s+1)(1/\alpha_2)}{(\tau_1 s+1)(\tau_2 s+1)\tau_I s + k_c(\tau_I s+1)(1/\alpha_2)}$$

1 Mark

$G(s\to 0) = 1$ $\therefore$ Closed loop static gain $= 1$. (between h_2, $\bar{h}_{2,sp}$)

Case 2:

$$G(s) = \frac{k_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{1/\alpha_2}{\tau_2 s+1}\right)}{1+k_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{1/\alpha_2}{\tau_2 s+1}\right)}$$ [From (9)]

1 Mark

$$= \frac{k_c(\tau_I s+1)(1/\alpha_2)}{(\tau_2 s+1)\tau_I s + k_c(\tau_I s+1)(1/\alpha_2)}$$

$G(s \to 0) = 1.$ $\therefore$ Closed loop static gain = 1 (between $\bar{h}_2, \bar{h}_{2,sp}$)

Case 3:

between $\bar{h}_2$ and $\bar{h}_{2,sp}$ [From (10)]

$$G(s) = \frac{k_c\left(1+\frac{1}{\tau_I s}\right)\left(-\frac{1}{A_2 s}\right)}{1+k_c\left(1+\frac{1}{\tau_I s}\right)\left(-\frac{1}{A_2 s}\right)}$$

1 Mark

$$= \frac{k_c(\tau_I s+1)(-1)}{(A_2 s)(\tau_I s) + k_c(\tau_I s+1)(-1)}$$

$G(s \to 0) = 1$ $\therefore$ Closed loop static gain = 1

between $\bar{h}_2$ and $\bar{F}_1$

$$G(s) = \frac{\frac{1}{A_2 s(\tau_1 s+1)}}{1+k_c\left(1+\frac{1}{\tau_I s}\right)\left(-\frac{1}{A_2 s}\right)}$$

$$= \frac{1}{A_2 s(\tau_1 s+1)} \cdot \frac{(\tau_I s)(A_2 s)}{[(\tau_I s)(A_2 s) + k_c(\tau_I s+1)(-1)]}$$

$G(s \to 0) = 0$ $\therefore$ Closed loop static gain = 0

(d) Case 1:

$$G_{sp} = \frac{k_c\left(1+\frac{1}{\tau_I s}\right)\frac{(1/\alpha_2)}{(\tau_1 s+1)(\tau_2 s+1)}}{1+k_c\left(1+\frac{1}{\tau_I s}\right)\cdot\frac{(1/\alpha_2)}{(\tau_1 s+1)(\tau_2 s+1)}}$$ [comparing (7) and (8)]

1 Mark

Case 2:

$$G_{sp} = \frac{k_c\left(1+\frac{1}{\tau_I s}\right)\frac{(1/\alpha_2)}{(\tau_2 s+1)}}{1+k_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{1/\alpha_2}{\tau_2 s+1}\right)}$$ [comparing (7) and (9)]

1 Mark

Case 3:

$$G_{sp} = \frac{k_c\left(1+\frac{1}{\tau_I s}\right)(-1/A_2 s)}{1+k_c\left(1+\frac{1}{\tau_I s}\right)(-1/A_2 s)}$$ [comparing (7) and (10)]

1 Mark

$$G_{load} = \frac{\frac{1}{A_2 s(\tau_1 s+1)}}{1+k_c\left(1+\frac{1}{\tau_I s}\right)\left(-\frac{1}{A_2 s}\right)}$$

(2) For a feedback control system the characteristic equation is give by. 10 Marks

$1 + G_p G_f G_c G_m$.

$G_m = G_f = 1$, $G_c = K_c$ ($\because$ proportional controller)

(a) $G_p(s) = \frac{10}{2s-1}$

Characteristic equation : $1 + \frac{10K_c}{2s-1}$.

To find the roots of the characteristic equation.

$$1 + \frac{10K_c}{2s-1} = 0 \Rightarrow 2s - 1 + 10K_c = 0$$

$$\Rightarrow s = -\frac{10K_c + 1}{2}$$

A feedback control system is said to be stable if all the roots of its characteristic equation have negetive real parts.

To produce stable closed loop response $-\frac{10K_c+1}{2} < 0$

$\Rightarrow \boxed{K_c > \frac{1}{10}}$ 2

(b) $G_p(s) = \frac{2}{0.1s+1}$

Characteristic equation : $1 + \frac{2K_c}{0.1s+1}$

$$1 + \frac{2K_c}{0.1s+1} = 0 \Rightarrow 0.1s + 1 + 2K_c = 0 \Rightarrow s = -\frac{(2K_c+1)}{0.1}$$

$\therefore$ $\boxed{\text{Any value of } K_c}$ will result in stable closed loop response. 2

(c) $G_p(s) = \frac{10}{2s^2+3s-4}$

Characteristic equation: $1 + \frac{10K_c}{2s^2+3s-4}$

$$1 + \frac{10K_c}{2s^2+3s-4} = 0 \Rightarrow 2s^2 + 3s + (10K_c - 4) = 0$$

$$s = \frac{-3 \pm \sqrt{9 - 8(10K_c - 4)}}{4}$$

To produce stable closed loop response $-8(10K_c - 4) < 0$

$\Rightarrow \boxed{K_c > 2/5}$ 2

(d) $G_p(s) = \frac{1}{3s^3+2s^2+s-5}$

Characteristic equation: $1 + \frac{K_c}{3s^3+2s^2+s-5}$

$$1 + \frac{K_c}{3s^3+2s^2+s-5} = 0 \Rightarrow 3s^3 + 2s^2 + s + (K_c - 5) = 0.$$

From Routh - Hurwitz criterion.

3	1
2	$(K_c - 5)$
$\frac{2-3(K_c-5)}{2}$	

$$\frac{2-3(K_c-5)}{2} > 0 \Rightarrow K_c < 5 + \frac{2}{3} \Rightarrow K_c < \frac{17}{3}$$

Also, $K_c - 5 > 0 \Rightarrow K_c > 5$

$\therefore \boxed{5 < K_c < \frac{17}{3}}$ 2

(e) $G_p(s) = \dfrac{1}{10s^3+2s^2+s-5}$

Characteristic equation : $1 + \dfrac{k_c}{10s^3+2s^2+s-5}$

$1 + \dfrac{k_c}{10s^3+2s^2+s-5} = 0 \Rightarrow 10s^3+2s^2+s+(k_c-5)=0$.

From Routh-Hurwitz stability criterion.

$k_c - 5 > 0 \Rightarrow k_c > 5$

10	1
2	(k_c-5)
$\dfrac{2-10(k_c-5)}{2}$	
k_c-5	

$\dfrac{2-10(k_c-5)}{2} > 0$

$\Rightarrow k_c < 5 + \dfrac{2}{10} \Rightarrow k_c < \dfrac{52}{10}$

$\therefore \boxed{5 < k_c < \dfrac{52}{10}}$ 2

3 $G_p = \dfrac{1}{(s+1)(2s+1)}$, $G_c = k_c$, $G_f = G_m = 1$. 10 Marks

Characteristic equation : $1 + G_p G_c G_f G_m$

$1 + \dfrac{k_c}{(s+1)(2s+1)} = 0$ to find the roots of the characteristic equation

$(s+1)(2s+1) + k_c = 0$

$2s^2 + 3s + (k_c+1) = 0$

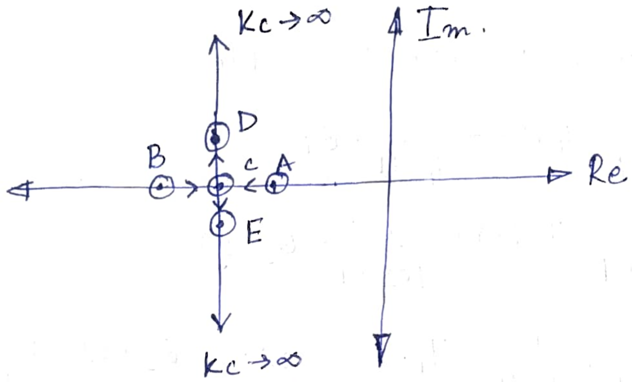
$s = \dfrac{-3 \pm \sqrt{9-8(k_c+1)}}{4}$. $\therefore$ Roots are $\dfrac{-3 \pm \sqrt{9-8(k_c+1)}}{4}$

Let the two roots be represented as r_1 and r_2

We vary the value of k_c in the range $0 \le k_c < \infty$ and evaluate the values of r_1 and r_2.

Then we plot all the roots in the complex plane.

k_c	r_1	r_2
0	$-\frac{1}{2}$ A	-1 B
$\frac{1}{8}$	$-\frac{3}{4}$ C	$-\frac{3}{4}$ C
$\frac{1}{4}$	$-\frac{3}{4} + j\frac{1}{4}$ D	$-\frac{3}{4} - j\frac{1}{4}$ E
1		
⋮		



(a) AC, BC ← overdamped

(b) Point C ← critically damped

(c) CD, CE ← underdamped

5 Marks

4. $G_p = \dfrac{1}{s^2+3s+1}$, $G_c = K_c$, $G_m = G_f = 1$.

Closed loop transfer function for set point change (suppose)

$$G_{sp} = \frac{G_p G_f G_c G_m}{1 + G_p G_f G_c G_m}$$

$$= \frac{K_c/(s^2+3s+1)}{1 + \dfrac{K_c}{(s^2+3s+1)}} = \frac{K_c}{s^2+3s+(K_c+1)}$$

$$= \frac{K_c/(K_c+1)}{\dfrac{1}{K_c+1}s^2 + \dfrac{3}{K_c+1}s + 1}$$

Generalized form of 2nd order transfer function

$$G = \frac{K_p}{\tau^2 s^2 + 2\tau\zeta s + 1}$$

Comparing the above two equations.

$$\tau^2 = \frac{1}{K_c+1} \Rightarrow \tau = \frac{1}{\sqrt{K_c+1}}$$

$$2\tau\zeta = \frac{3}{K_c+1} \Rightarrow \zeta = \frac{3}{2}\frac{1}{\sqrt{K_c+1}} \quad \text{— (1)}$$

Also, it is given that decay ratio $= \frac{1}{4}$.

$$\therefore \exp\left[\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}\right] = \frac{1}{4}$$

$$\Rightarrow \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 1.386 \Rightarrow \zeta = 0.215. \quad \text{— (2)}$$

$\therefore$ From (1) and (2)

$$\frac{3}{2}\frac{1}{\sqrt{K_c+1}} = 0.215$$

$$\Rightarrow \boxed{K_c = 47.67}$$

Case I

(5) Using Q_2 as the manipulated Variable

10 Marks

$$\bar{T}_2(s) = \frac{1}{4s+1}\bar{T}_1(s) + \frac{0.2}{4s+1}\bar{Q}_1(s)$$

$$\bar{T}_3(s) = \frac{1}{10s+1}\bar{T}_2(s) + \frac{0.1}{10s+1}\bar{Q}_2(s)$$

$$= \frac{1}{10s+1}\cdot\frac{1}{4s+1}\bar{T}_1(s) + \frac{0.1}{10s+1}\bar{Q}_2(s) + \frac{1}{10s+1}\cdot\frac{0.2}{4s+1}\bar{Q}_1(s) \quad \text{—(1)}$$

$\bar{T}_1(s)$ is the disturbance Variable.

Let us neglect the disturbance $\bar{Q}_1(s)$

$$\bar{T}_3(s) = \frac{0.1}{10s+1}\bar{Q}_2(s) + \frac{1}{(10s+1)(4s+1)}\bar{T}_1(s)$$

$$= G_p(s)\bar{Q}_2(s) + G_d(s)\bar{T}_1(s)$$

$G_c = K_c$, $G_m = G_f = 1$.

Closed loop response :

$$\bar{T}_3(s) = \frac{G_p G_f G_c G_m}{1+G_p G_f G_c G_m}\bar{T}_{3,sp}(s) + \frac{G_d}{1+G_p G_f G_c G_m}\bar{T}_1(s)$$

For 10% step change in the value of disturbance.

$$\bar{T}_3(s) = \frac{G_d}{1+G_p G_f G_c G_m}\frac{0.1\,T_{1,ss}}{s}$$

where $T_{1,ss}$ is the steady state value of disturbance T_1

$$\bar{T}_3(s) = \frac{\frac{1}{(10s+1)(4s+1)}}{1+\frac{0.1}{10s+1}.K_c}\frac{0.1\,T_{1,ss}}{s}$$

$$= \frac{1}{(\cancel{10s+1})(4s+1)}\frac{(\cancel{10s+1})}{10s+1+0.1K_c}\cdot\frac{0.1\,T_{1,ss}}{s} \qquad 2$$

$$\text{Offset} = 0 - \lim_{s\to 0} s.\bar{T}_3(s)$$

$$= 0 - \lim_{s\to 0}\frac{1}{(4s+1)(10s+1+0.1K_c)}\cdot 0.1\,T_{1,ss}$$

$$= -0.1\,T_{1,ss}\frac{1}{1+0.1K_c} \qquad 2$$

Minimum offset is obtained at maximum K_c value at $K_c = 100$.

$$\text{Minimum offset} = \boxed{-0.0091\,T_{1,ss}} \quad \text{—(2)} \qquad 1$$

Case 2

using Q_1 as the manipulated variable

From ①,

$$\bar{T}_3(s) = \frac{1}{10s+1} \cdot \frac{1}{4s+1}\, \bar{T}_1(s) + \frac{0.1}{10s+1}\, \bar{Q}_2(s) + \frac{1}{10s+1} \cdot \frac{0.2}{4s+1}\, \bar{Q}_1(s)$$

$\bar{T}_1(s)$ is the disturbance variable.

Let us neglect the disturbance $\bar{Q}_2(s)$.

$$\bar{T}_3(s) = \frac{0.2}{(10s+1)(4s+1)}\, \bar{Q}_1(s) + \frac{1}{(10s+1)(4s+1)}\, \bar{T}_1(s)$$

$$= G_p(s)\, \bar{Q}_1(s) + G_d(s)\, \bar{T}_1(s).$$

Closed loop response for 10% step change in the value of disturbance.

$$\bar{T}_3(s) = \frac{\frac{1}{(10s+1)(4s+1)}}{1 + \frac{0.2K_c}{(10s+1)(4s+1)}} \cdot \frac{0.1\, T_{1,ss}}{s}.$$

$$= \frac{1}{(4s+1)(10s+1) + 0.2K_c} \cdot \frac{0.1\, T_{1,ss}}{s}.$$ 2

$$\text{Offset} = 0 - \lim_{s \to 0} s \cdot \bar{T}_3(s)$$

$$= 0 - \frac{1}{1 + 0.2K_c} \cdot 0.1\, T_{1,ss}$$ 2

Minimum offset is obtained at $K_c = 100$.

Minimum offset $= \boxed{-0.0048\, T_{1,ss}}$ —③ 1

From ② and ③ it can be concluded that employing the minimum offset criterion we select Q_1 as the manipulated variable.