Consider a fluid reservoir at temperature T_∞ in which an object of temperature $T_\circ$ is immersed at time $t = 0$. The time rate of change of temperature of the object is governed by Newton's law of cooling according to which the rate of change of temperature of a body is proportional to the difference of the body's temperature and the constant surrounding temperature. Hence, the governing equation for the system can be written as follows.

$$\frac{dT}{dt} = -h(T - T_\infty)$$

(a) What is/are the equilibrium temperature(s) of the system.

(b) Solve the above equation analytically to determine the time evolution of the system. Check if you get the same answer for equilibrium state(s) as you got as the answer for the part (a).

(c) Develop the phase portrait for the system using the solutions obtained for part (b).

(d) Without explicitly solving the governing equation, develop the phase portrait.

(e) Analyse the solutions and phase portraits for initial conditions $T_\infty > T_\circ, T_\infty < T_\circ$ and $T_\infty = T_\circ$

(f) Study the effect of the parameter h on the phase portraits.

(g) Comment upon the bifurcation in the system.

(h) Now consider a case when the reservoir temperature is a function of time. Analyze the dynamics of the system for the following time functions of T_∞,

(i) linear decay

(ii) linear rise

(iii) exponential decay

(iv) exponential rise