MACROSCOPIC BALANCE

C CET

Fox & McDONALD

FOR A CV

$$\left.\frac{dN}{dt}\right|_{SYST} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall + \int_{CS} \eta \rho \, \vec{V}.\vec{dA} \quad \text{--- (1)}$$

N = EXTENSIVE PROPERTY

$\eta = \frac{N}{M}$, INTENSIVE PROPERTY

$\frac{dN}{dt}$ = TOTAL RATE OF CHANGE OF AN ARBITRARY EXTENSIVE PROP. OF THE SYSTEM

$\frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall$ = TIME RATE OF CHANGE OF THE EXTENSIVE PROP. 'N' WITHIN CV

$\int_{C.S} \eta \rho \, \vec{V}.\vec{dA}$ = NET RATE OF EFFLUX OF THE EXTENSIVE PROPERTY 'N' THROUGH THE CONTROL SURFACES

$$\left.\frac{dN}{dt}\right|_{SYST} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall + \int_{CS} \eta \rho \, \vec{V}\cdot\vec{dA}$$

MASS $\quad N = M, \quad \eta = \frac{N}{M} = 1$

$$\boxed{0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \, \vec{V}\cdot\vec{dA}}$$

CONSERVATION OF MASS

CONTINUITY EQ^N.

STEADY STATE INCOMP. $\quad \rho \int_{CS} \vec{V}\cdot\vec{dA} = 0 \;\Rightarrow\; V_1 A_1 + V_2 A_2 + \dots = 0$

$$\sum V_i A_i = 0$$

MOMENTUM $\quad \eta = \frac{N}{M} = \vec{v} \qquad N = \vec{P} \qquad \frac{d\vec{P}}{dt}$

$$\cancel{\vec{F} = \frac{\partial}{\partial t}\int_{CV} \vec{V} \rho \, d\forall + \int_{CS} \vec{V} \rho \, dA}$$

MOMENTUM

$N = \vec{P}$ $\quad P \equiv$ MOMENTUM

$$\boxed{\vec{F} = \frac{\partial}{\partial t}\int_{CV} \vec{V}\,\rho\, d\forall + \int_{CS} \vec{V}\,\rho\, \vec{V}\cdot\vec{dA}}$$

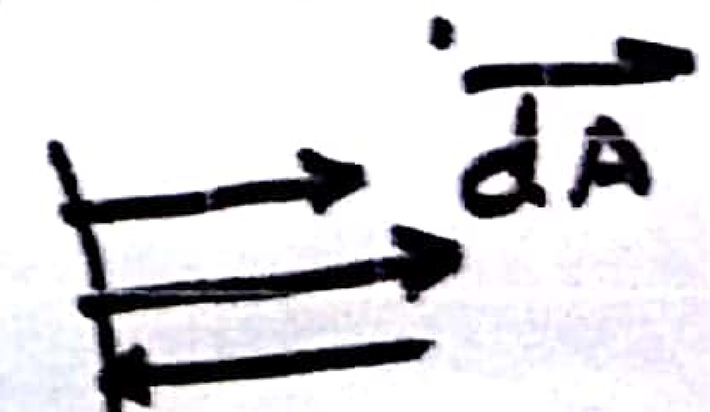
$$\boxed{\vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\,\rho\, d\forall + \int_{CS} \vec{V}\,\rho\, \vec{V}\cdot\vec{dA}}$$

MOMENTUM EQ^N. ✓

→ i) ALL VELOCITIES ARE MEASURED RELATIVE TO THE CV

→ ii) MASS IN ⟶ −ve
MASS OUT ⟶ +ve.

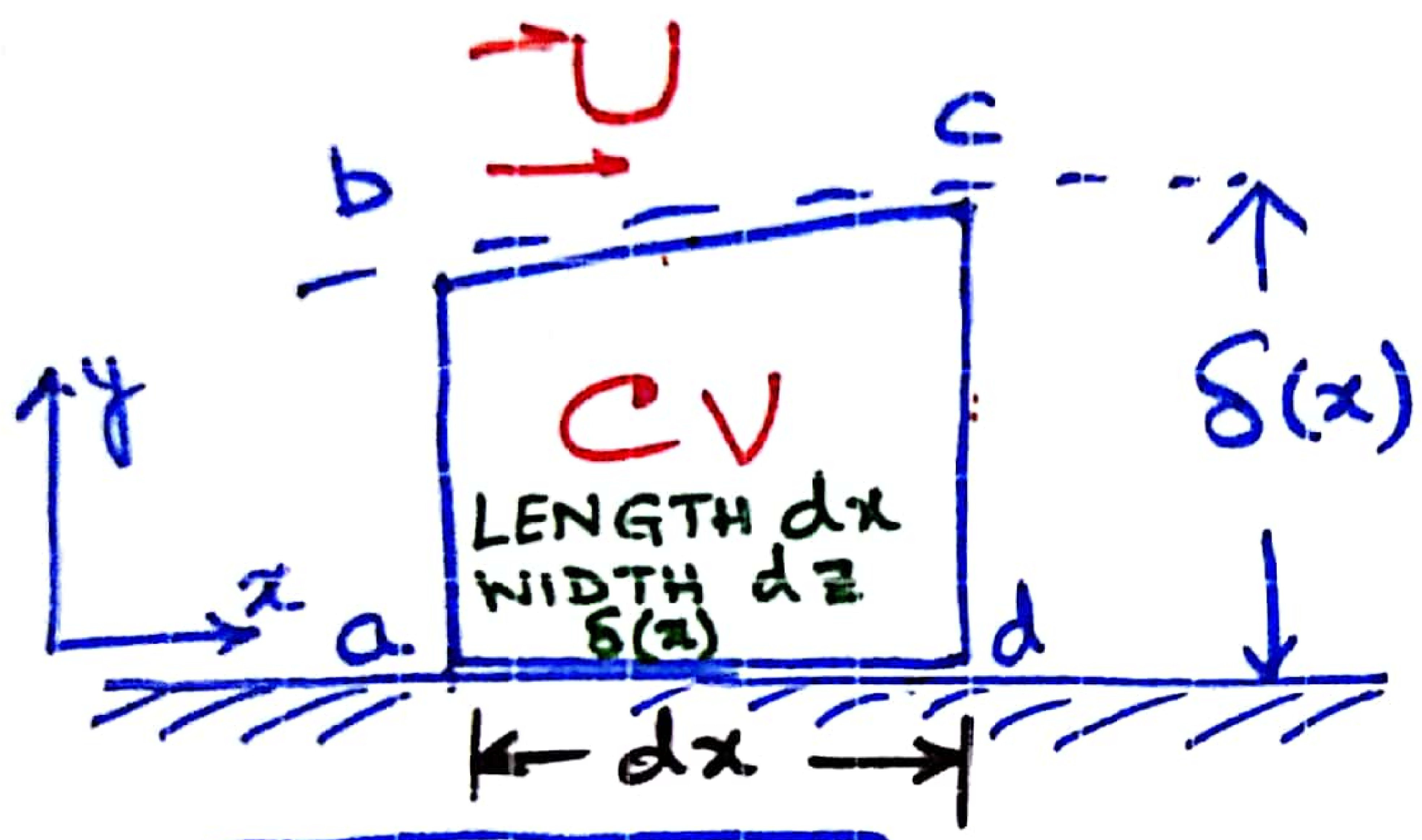
$\vec{V}\cdot\vec{dA}$



MOMENTUM INTEGRAL EQUATION

CONT. EQ^N $\quad 0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot \vec{dA}$

STEADY, 2D FLOW.



$\boxed{\delta \quad f^n(x)}$

$$\int_{CS} \rho \vec{V} \cdot \vec{dA} = 0.$$

$$\dot{m}_{ab} + \dot{m}_{bc} + \dot{m}_{cd} + \dot{m}_{ad} = 0.$$

$$\dot{m}_{ad} \approx 0.$$

$$\boxed{\dot{m}_{bc} = -\dot{m}_{ab} - \dot{m}_{cd}.}$$

$$\int_A \rho \vec{V} \cdot d\underline{A}$$

SURF ab LOCATED AT x

$$\dot{m}_{ab} = -\left\{\int_0^{\delta} \rho v_x \, dy\right\} dz. \checkmark$$

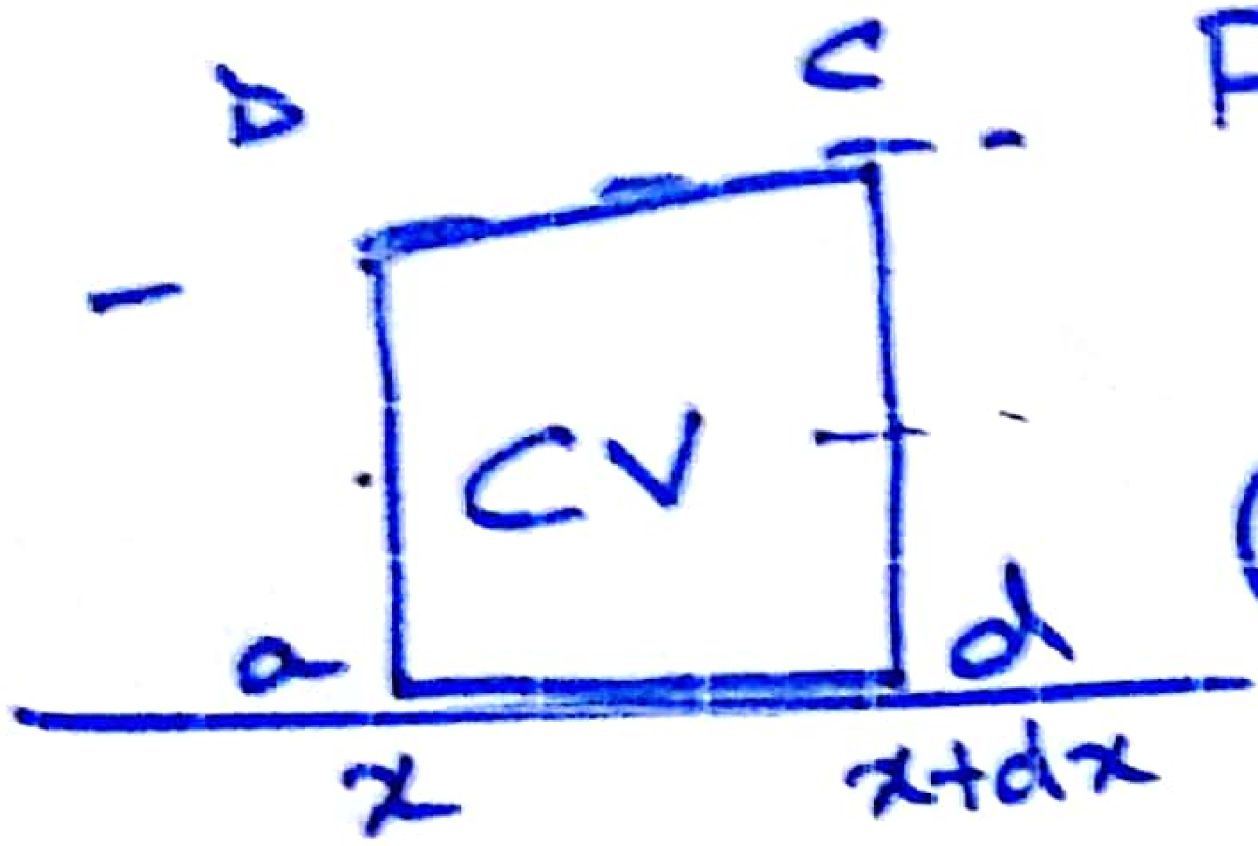
$\frac{cd}{x+dx}$ $\quad \dot{m}_{x+dx} = \dot{m}_x + \left.\frac{\partial \dot{m}}{\partial x}\right|_x dx.$ TAYLOR SER. EXPANSION.

$$\dot{m}_{cd} = \left\{ \int_0^\delta \rho v_x \cancel{dy} + \frac{\partial}{\partial x}\left[\int_0^\delta \rho v_x \, dy\right] dx \right\} dz. \checkmark$$

$$\boxed{\dot{m}_{bc} = -\left\{ \frac{\partial}{\partial x}\left[\int_0^\delta \rho v_x \, dy\right] dx \right\} dz}$$

x COMP OF THE M^2 EQN

$$\cancel{F}_{Sx} + \underbrace{\cancel{F_{Bx}}}_{=0} = \underbrace{\cancel{\frac{\partial}{\partial t}}}_{\substack{=0 \\ SS}} \int_{CV} v_x \rho \, d\forall + \int_{CS} v_x \rho \vec{V} \cdot \vec{dA}$$



$$F_{Sx} = mf_{ab} + mf_{bc} + mf_{cd}.$$

$mf_{ad} = 0$ since. $v_x = 0$ AT 'ad'

(ab) $$mf_{ab} = -\left\{ \int_0^{\delta} v_x \rho v_x \, dy \right\} dz$$

$$mf_{cd} = mf_{ab} + \frac{\partial}{\partial x}(mf_{ab})\,dx$$

$$mf_{cd} = \left\{ \int_0^{\delta} v_x \rho v_x \, dy + \frac{\partial}{\partial x}\left[\int_0^{\delta} v_x \rho v_x \, dy \right] dx \right\} dz.$$

$$mf_{bc} = U\,\dot{m}_{bc}.$$

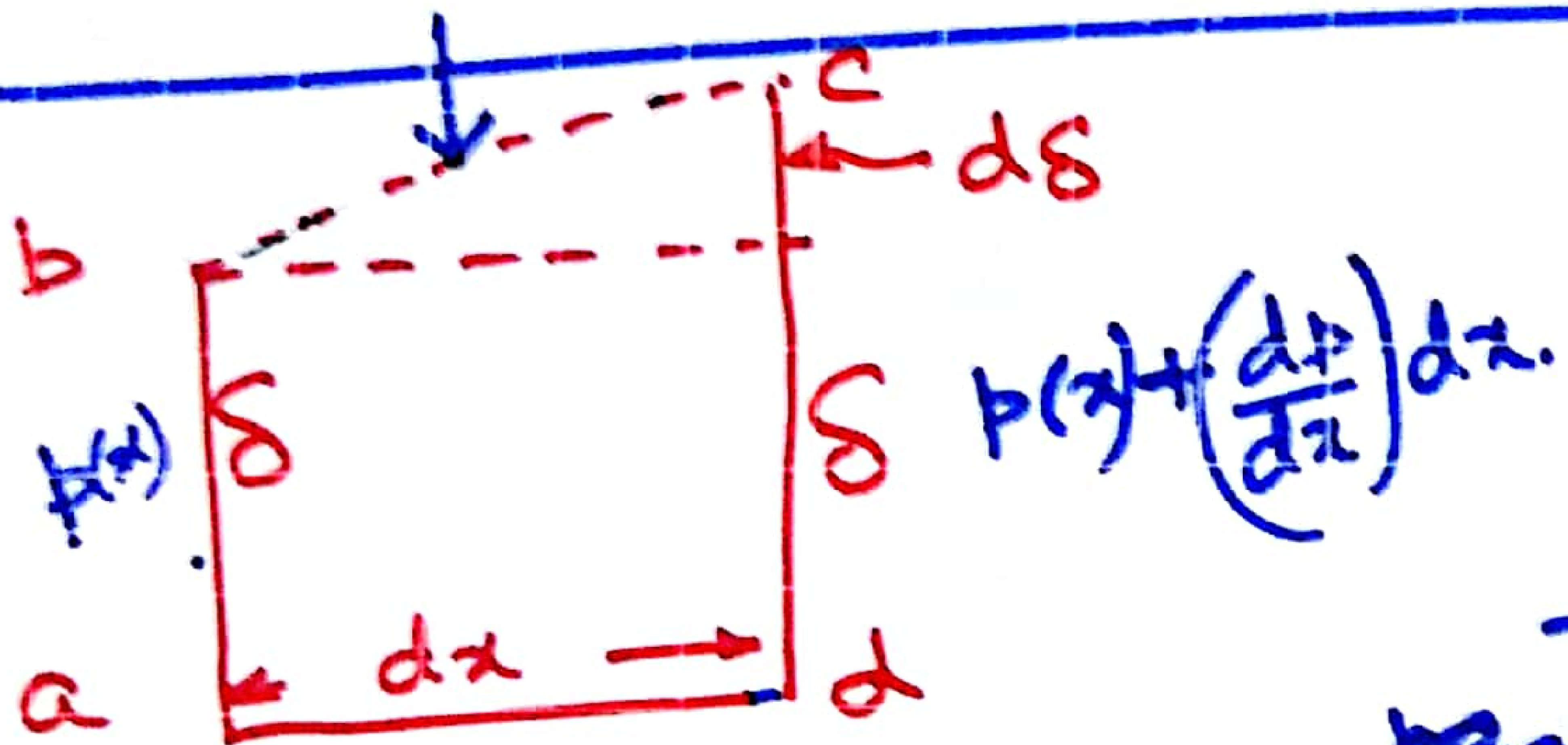
mf_{ab}

mf_{cd}

$$mf_{bc} = -U\left\{\frac{\partial}{\partial x}\left[\int_0^\delta \rho v_x\,dy\right]dx\right\}dz.$$

$$\rightarrow \int_{cs} v_x \rho \vec{V}\cdot d\vec{A} = \left\{\frac{\partial}{\partial x}\left[\int v_x \rho v_x\,dy\right]dx - U\frac{\partial}{\partial x}\left[\int_0^\delta \rho v_x\,dy\right]dx\right\}dz.$$

SURF. FORCES ON CV IN THE x DIRECTN.

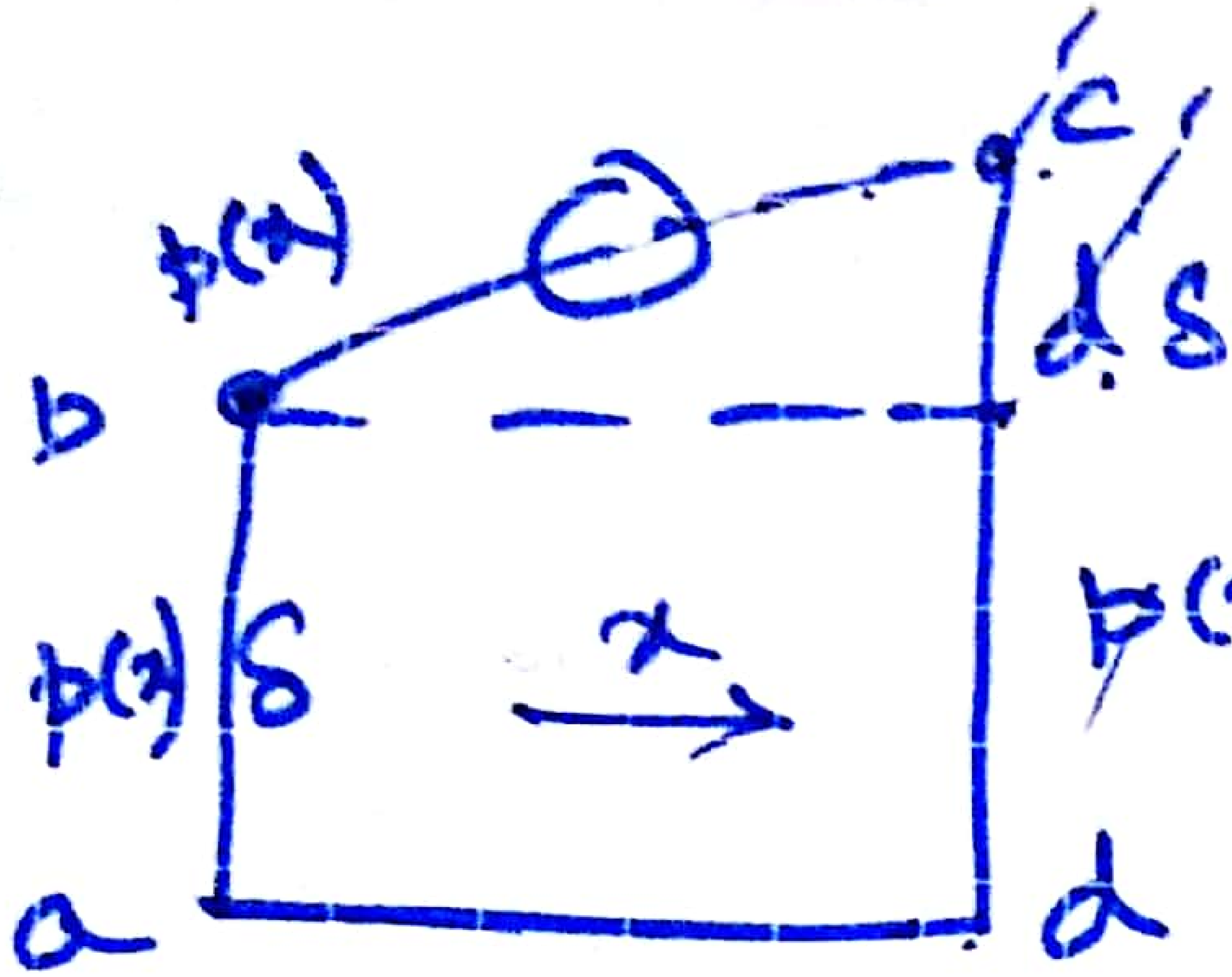


$F_{s_x} \rightarrow$ PRESSURE

$F_{s_x} \rightarrow$ SHEAR. $\rightarrow \propto$ VEL. GRAD

ab: $F_{ab} = p\,\delta\,dz$

cd: $F_{cd} = \left(p + \frac{dp}{dx}\Big|_x dx\right)(\delta + d\delta)\,dz$



$$p(x) + \left(\frac{dp}{dx}\right) dx.$$

Pr on BC

$$\text{Av. Pr} = p(x) + \frac{1}{2} \frac{dp}{dx}\Big| dx$$

<u>AREA</u> $d\delta dz$

$$F_{bc} = \left[p(x) + \frac{1}{2} \frac{dp}{dx}\Big| dx \right] d\delta dz.$$

<u>ad</u>

$$F_{ad} = -\tau_w \, dx \, dz$$

τ_w = WALL SHEAR STRESS

MI EQN

$$\boxed{\frac{\tau_w}{\rho} = \frac{d}{dx}\left(U^2\Theta\right) + \delta^* U \frac{dU}{dx}.}$$

$$\Theta = \int_0^\delta \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right) dy \qquad \delta^* = \int_0^\delta \left(1 - \frac{v_x}{U}\right) dy$$

i) ODE

ii) WALL SHEAR STRESS < TURB. ✓ / LAM. ✓

USE OF MI EQ^N — ZERO PR. GRAD FLOW.

FLOW OVER A FLAT PLATE

$$\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \cancel{\frac{dU}{dx}}$$

$= 0.$

U IS A CONST.

$$\frac{\tau_w}{\rho} = U^2 \frac{d}{dx}\theta$$

$$\frac{\tau_w}{\rho} = U^2 \frac{d\theta}{dx}$$

$$\tau_w = \rho U^2 \frac{d}{dx}\int_0^\delta \frac{v_x}{U}\left(1-\frac{v_x}{U}\right)dy$$

$$\tau_w = \rho U^2 \frac{d\delta}{dx} \int_0^1 \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right) d\eta \qquad \frac{y}{\delta} \equiv \eta$$

$$\tau_w = \rho U^2 \frac{d\delta}{dx} \left[\int_0^1 \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right) d\eta \right] \qquad dy = \delta \, d\eta$$

$$\tau_w = \rho U^2 \frac{d\delta}{dx} \cdot \beta \qquad \beta = \text{CONST.}$$

$$\boxed{\frac{v_x}{U} = f^n(\eta)}$$

$$\frac{v_x}{U} = a + b\eta + c\eta^2$$

AT $y = 0$ $\quad v_x = 0.$ $\qquad \eta = 0 \quad \frac{v_x}{U} = 0$

AT $y = \delta$ $\quad v_x = U$ $\qquad \eta = 1 \quad \frac{v_x}{U} = 1$

AT $y = \delta$ $\quad \frac{\partial v_x}{\partial y} = 0.$ $\qquad \eta = 1 \quad \frac{d\, v_x/U}{d\eta} = 0.$

$$\frac{v_x}{U} = 2\eta - \eta^2 \qquad \eta = y/\delta.$$

$$\frac{\tau_w}{\rho} = U^2 \frac{d}{dx}\theta =$$

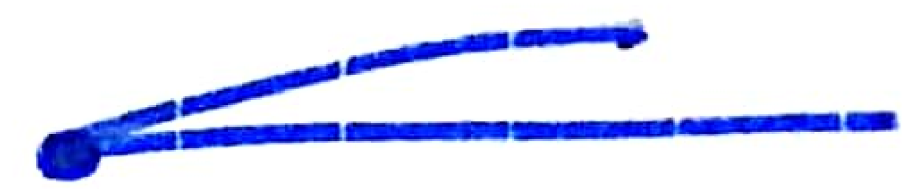
$$\tau_w = \rho U^2 \frac{d\delta}{dx}\left[\int_0^1 \frac{v_x}{U}\left(1-\frac{v_x}{U}\right)d\eta\right]$$

NEWTONIAN FLUID.

$$\tau_w = \mu \left.\frac{\partial v_x}{\partial y}\right|_{y=0} = \frac{2\mu U}{\delta} = \rho U^2 \frac{d\delta}{dx}\left[\int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\,d\eta\right]$$

$$\frac{2\mu}{\delta \rho U} = \frac{2}{15}\frac{d\delta}{dx}$$

$$\frac{\delta^2}{2} = \frac{15\mu}{\rho U} x + C.$$



AT $x = 0$ $\delta = 0 \Rightarrow c = 0.$

$$\boxed{\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}}$$

BLASIUS SOL^N $\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{2\mu(U/\delta)}{\frac{1}{2}\rho U^2} = \frac{4\mu}{\rho U \delta}.$$

$$C_f = \frac{0.73}{\sqrt{Re_x}}$$

$$C_f = \frac{0.664}{\sqrt{Re_x}}$$