Heat Transfer

CH21004

Spatial effect

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$T(x, 0) = T_i$$

and the boundary conditions are

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$

$$T = T(x, t, T_i, T_\infty, L, k, \alpha, h)$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h[T(L, t) - T_\infty]$$

$$\theta^* \equiv \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}$$

$$\frac{\partial^2 \theta^*}{\partial x^{*2}} = \frac{\partial \theta^*}{\partial Fo}$$

$$x^* \equiv \frac{x}{L}$$

$$\theta^*(x^*, 0) = 1$$

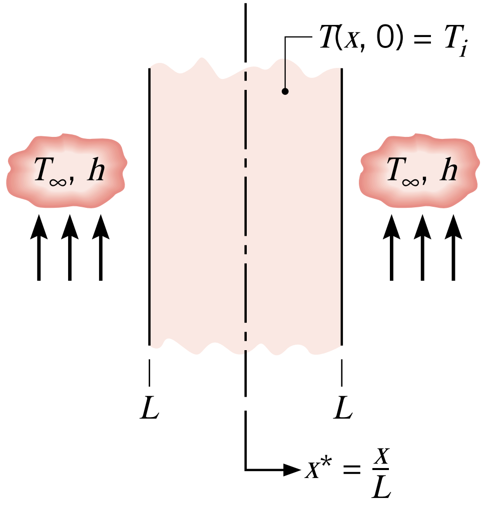
$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=0} = 0$$

$$t^* \equiv \frac{\alpha t}{L^2} \equiv Fo$$

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=1} = -Bi\,\theta^*(1, t^*)$$

Spatial effect

$$\theta^* = f(x^*, Fo, Bi)$$



$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo) \cos(\zeta_n x^*)$$

$$C_n = \frac{4 \sin \zeta_n}{2\zeta_n + \sin(2\zeta_n)}$$

$$\zeta_n \tan \zeta_n = Bi$$

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$$

$$t = p\Delta t$$

$$\left.\frac{\partial T}{\partial t}\right|_{m,\,n} \approx \frac{T_{m,\,n}^{p+1} - T_{m,\,n}^{p}}{\Delta t}$$

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$$

$$\frac{1}{\alpha}\frac{T^{p+1}_{m,n} - T^p_{m,n}}{\Delta t} = \frac{T^p_{m+1,n} + T^p_{m-1,n} - 2T^p_{m,n}}{(\Delta x)^2} + \frac{T^p_{m,n+1} + T^p_{m,n-1} - 2T^p_{m,n}}{(\Delta y)^2}$$

$$T^{p+1}_{m,n} = Fo(T^p_{m+1,n} + T^p_{m-1,n} + T^p_{m,n+1} + T^p_{m,n-1}) + (1 - 4Fo)T^p_{m,n}$$

Explicit form

$$Fo = \frac{\alpha\,\Delta t}{(\Delta x)^2}$$

$$T^{p+1}_m = Fo(T^p_{m+1} + T^p_{m-1}) + (1 - 2Fo)T^p_m$$

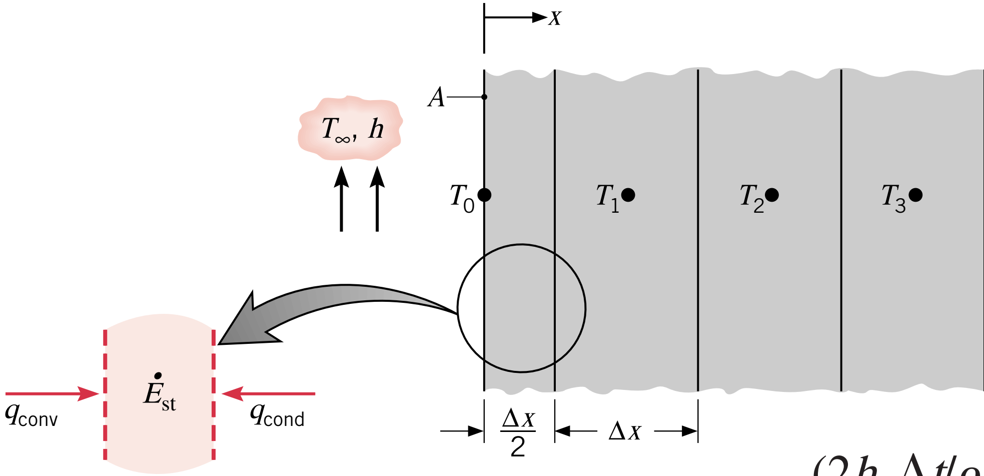
$(1 - 2Fo) \geq 0$, or

$$Fo \leq \frac{1}{2}$$

$(1 - 4Fo) \geq 0$, or

$$Fo \leq \frac{1}{4}$$

$$\dot{E}_{in} + \dot{E}_g = \dot{E}_{st}$$



$$hA(T_\infty - T_0^p) + \frac{kA}{\Delta x}(T_1^p - T_0^p) = \rho c A \frac{\Delta x}{2} \frac{T_0^{p+1} - T_0^p}{\Delta t}$$

$$T_0^{p+1} = \frac{2h\,\Delta t}{\rho c\,\Delta x}(T_\infty - T_0^p) + \frac{2\alpha\,\Delta t}{\Delta x^2}(T_1^p - T_0^p) + T_0^p$$

$$(2h\ \Delta t/\rho c\ \Delta x) = 2(h\ \Delta x/k)(\alpha\ \Delta t/\Delta x^2) = 2\ Bi\ Fo$$

$$Bi = \frac{h\,\Delta x}{k}$$

$$T_0^{p+1} = 2Fo(T_1^p + Bi\,T_\infty) + (1 - 2Fo - 2Bi\,Fo)T_0^p$$

$$1 - 2Fo - 2Bi\,Fo \geq 0$$

$$Fo(1 + Bi) \leq \frac{1}{2}$$

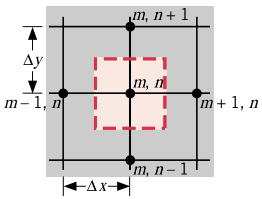
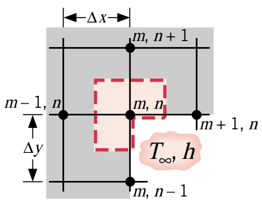
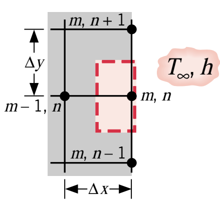
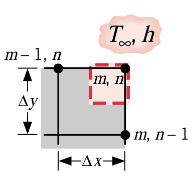
$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$$

$$\frac{1}{\alpha}\frac{T_{m,n}^{p+1} - T_{m,n}^{p}}{\Delta t} = \frac{T_{m+1,n}^{p+1} + T_{m-1,n}^{p+1} - 2T_{m,n}^{p+1}}{(\Delta x)^2} + \frac{T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1} - 2T_{m,n}^{p+1}}{(\Delta y)^2}$$

$$(1 + 4Fo)T_{m,n}^{p+1} - Fo(T_{m+1,n}^{p+1} + T_{m-1,n}^{p+1} + T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1}) = T_{m,n}^{p}$$

Implicit form

Table 5.3 Transient, two-dimensional finite-difference equations ($\Delta x = \Delta y$)

Configuration	(a) Explicit Method: Finite-Difference Equation	(a) Explicit Method: Stability Criterion	(b) Implicit Method
	$T_{m,n}^{p+1} = Fo(T_{m+1,n}^{p} + T_{m-1,n}^{p} + T_{m,n+1}^{p} + T_{m,n-1}^{p}) + (1 - 4Fo)T_{m,n}^{p}$ (5.76) 1. Interior node	$Fo \leq \frac{1}{4}$ (5.80)	$(1 + 4Fo)T_{m,n}^{p+1} - Fo(T_{m+1,n}^{p+1} + T_{m-1,n}^{p+1} + T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1}) = T_{m,n}^{p}$ (5.92)
	$T_{m,n}^{p+1} = \frac{2}{3}Fo(T_{m+1,n}^{p} + 2T_{m-1,n}^{p} + 2T_{m,n+1}^{p} + T_{m,n-1}^{p} + 2Bi\,T_{\infty}) + (1 - 4Fo - \frac{4}{3}Bi\,Fo)T_{m,n}^{p}$ (5.85) 2. Node at interior corner with convection	$Fo(3 + Bi) \leq \frac{3}{4}$ (5.86)	$(1 + 4Fo(1 + \frac{1}{3}Bi))T_{m,n}^{p+1} - \frac{2}{3}Fo \cdot (T_{m+1,n}^{p+1} + 2T_{m-1,n}^{p+1} + 2T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1}) = T_{m,n}^{p} + \frac{4}{3}Bi\,Fo\,T_{\infty}$ (5.95)
	$T_{m,n}^{p+1} = Fo(2T_{m-1,n}^{p} + T_{m,n+1}^{p} + T_{m,n-1}^{p} + 2Bi\,T_{\infty}) + (1 - 4Fo - 2Bi\,Fo)T_{m,n}^{p}$ (5.87) 3. Node at plane surface with convection[a]	$Fo(2 + Bi) \leq \frac{1}{2}$ (5.88)	$(1 + 2Fo(2 + Bi))T_{m,n}^{p+1} - Fo(2T_{m-1,n}^{p+1} + T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1}) = T_{m,n}^{p} + 2Bi\,Fo\,T_{\infty}$ (5.96)
	$T_{m,n}^{p+1} = 2Fo(T_{m-1,n}^{p} + T_{m,n-1}^{p} + 2Bi\,T_{\infty}) + (1 - 4Fo - 4Bi\,Fo)T_{m,n}^{p}$ (5.89) 4. Node at exterior corner with convection	$Fo(1 + Bi) \leq \frac{1}{4}$ (5.90)	$(1 + 4Fo(1 + Bi))T_{m,n}^{p+1} - 2Fo(T_{m-1,n}^{p+1} + T_{m,n-1}^{p+1}) = T_{m,n}^{p} + 4Bi\,Fo\,T_{\infty}$ (5.97)

[a]To obtain the finite-difference equation and/or stability criterion for an adiabatic surface (or surface of symmetry), simply set Bi equal to zero.