7.1 a) Use a MFR with some particular concentration of A

b) Use a PFR, with low X_A

c) Use a MFR, with high X_A

7.3 For

$A + B \rightarrow R$

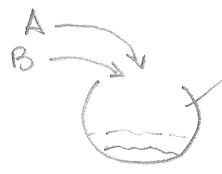
$2A \rightarrow S$

$2B \rightarrow T$

for A use low C_A

for B use low C_B

Thus



drip A and B in slowly

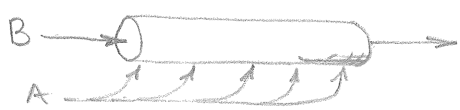
keep X_A high

keep X_B high

Don't have an excess of either A or B

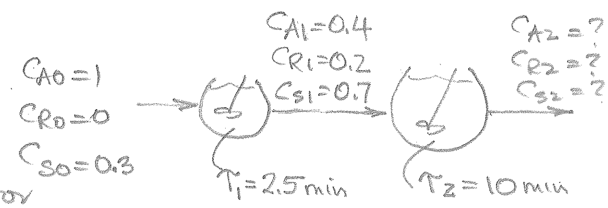
7.5 For

$\left.\begin{array}{l} A + B \rightarrow R \\ 2A \rightarrow S \end{array}\right\}$ keep C_A low, but C_B high



7.7 Given

$A \begin{array}{l} \xrightarrow{1} R, \quad r_R = k_1 C_A^2 \\ \xrightarrow{2} S, \quad r_S = k_2 C_A \end{array}$ and



For mixed flow in the first reactor

$$\tau_1 = \frac{C_{R1} - C_{R0}}{k_1 C_A^2} \quad \therefore \; k_1 = \frac{0.2 - 0}{2.5(0.4)^2} = 0.5$$

Also

$$\tau_1 = \frac{C_{S1} - C_{S0}}{k_2 C_A} \quad \therefore \; k_2 = \frac{0.7 - 0.3}{2.5\,(0.4)} = 0.4$$

For the second reactor

$$\tau_2 = \frac{C_{A1} - C_{A2}}{k_1 C_A^2 + k_2 C_A} = \frac{0.4 - C_{A2}}{0.5\, C_{A2}^2 + 0.4\, C_{A2}} = 10 \quad \therefore \; C_{A2} = 0.0745 \longleftarrow$$

Also

$$\tau_2 = \frac{C_{R2} - C_{R1}}{k_1 C_{A2}^2} \quad \therefore \; C_{R2} = 0.2 + 10(0.5)(0.0745)^2 = 0.2278 \longleftarrow$$

$$\text{and } C_{S2} = 0.7 + 10(0.4)(0.0745) = 0.998 \longleftarrow$$

7.9

$C_{A0}=40 \longrightarrow$ [PFR] $\longrightarrow X_A = 0.9$

$C_R, C_S, \tau = ?$

$A \xrightarrow{1} R \quad r_R = k_1 C_A^2 \quad k_1 = 0.4\ m^3/mol\cdot min$

$A \xrightarrow{2} S \quad r_S = k_2 C_A \quad k_2 = 2\ min^{-1}$

First determine $\varphi(S/A)$ ..

$$\varphi(S/A) = \frac{r_S}{r_R + r_S} = \frac{2C_A}{0.4C_A^2 + 2C_A} = \frac{1}{1+0.2C_A}$$

$$C_S = \int_4^{40} \varphi(S/A)\,dC_A = \int_4^{40} \frac{dC_A}{1+0.2C_A} = \frac{1}{0.2}\ln(1+0.2C_A)\Big|_4^{40} = 5\ln\frac{9}{1.8} = 8$$

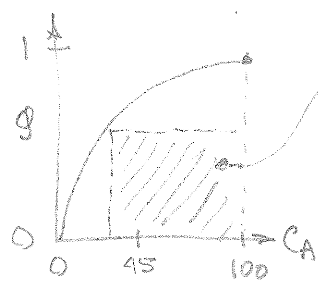
$\therefore$ $C_A = 4$
$C_S = 8$
$C_R = 28$

$$\tau = \int \frac{dC_A}{-r_A} = \int \frac{dC_A}{2C_A + 0.4C_A^2} = \frac{1}{0.4}\int\frac{dC_A}{5C_A + C_A^2} = 2.5\left[\frac{1}{5}\ln\frac{C_A}{C_A+5}\right]_4^{40} = \frac{1}{2}\ln 2 = 0.347$$

7.11 To maximize C_R in a MFR first look at the $\varphi(R/A)$ vs C_A curve

C_A	$\varphi(R/A)$
100	100/105
45	45/50
20	20/25
5	5/10
0	0

... draw the curve ....



Solve by maximizing the area of the rectangle

$$\left[\varphi(R/A) = \frac{0.4C_A^2}{0.4C_A^2 + 2C_A} = C_A/(C_A+5)\right]$$

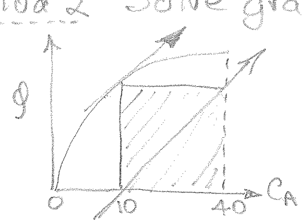
Method 1 Solve analytically

$$C_R = \varphi(-\Delta C_A) = \frac{C_A}{C_A+5}(40 - C_A)$$

$$\frac{dC_R}{dC_A} = \frac{(40-2C_A)(C_A+5) - (40C_A - C_A^2)}{(C_A+5)^2} = 0$$

-- or $C_A = 10$
$C_R = 20$
$C_S = 10$

Method 2 Solve graphically by maximizing the shaded rectangle



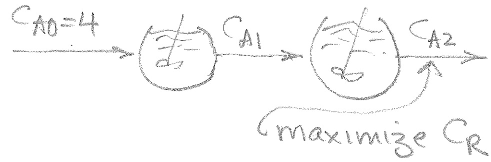
For τ:

$$\tau = \frac{C_{A0} - C_{Af}}{-r_A} = \frac{C_{A0} - C_{Af}}{0.4C_{Af}^2 + 2C_A} = \frac{40-10}{0.4(10)^2 + 2(10)}$$

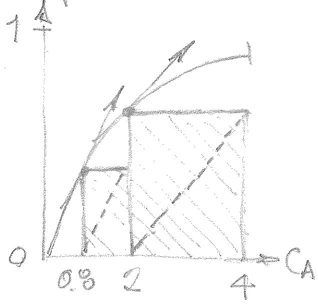
$$= \frac{30}{60} = \frac{1}{2}\ min$$

7.13

$$A \begin{matrix} \nearrow R \\ \searrow S \end{matrix} \quad \left.\begin{matrix} r_R = C_A \\ r_S = 1 \end{matrix}\right\} \quad \varphi\left(\frac{R}{R+S}\right) = \frac{C_A}{C_A+1}$$



First plot φ vs C_A



The shaded areas of the two rectangles gives C_R. We maximize this by trial and error. This gives

$$C_{A1} = 2 \quad \text{and} \quad C_{A2} = 0.8$$

$$\therefore \tau_1 = \frac{V_1}{v} = \frac{C_{A0}-C_{A1}}{-r_{A1}} = \frac{C_{A0}-C_{A1}}{1+C_{A1}} = \frac{4-2}{1+2} = \frac{2}{3}$$

$$\text{and} \quad \tau_2 = \frac{V_2}{v} = \frac{C_{A1}-C_{A2}}{-r_{A2}} = \frac{2-0.8}{1+0.8} = \frac{2}{3}$$

$$\therefore \frac{V_1}{V_2} = 1$$

P.S. I'm not sure that this is the best solution.

and

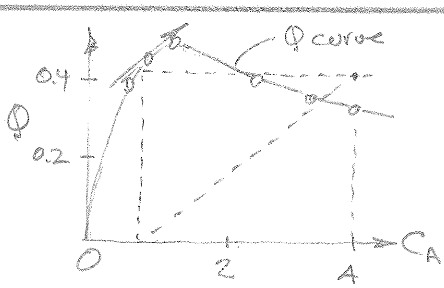
$$C_{R1} = \varphi \cdot \Delta C_A = \frac{C_{A1}}{C_{A1}+1}(C_{A0}-C_{A1}) = \frac{2}{2+1}(4-2) = 1.33$$

$$C_{R2} = \frac{C_{A2}}{C_{A2}+1}(C_{A1}-C_{A2}) = \frac{0.8}{0.8+1}(2-0.8) = 0.533$$

$$\therefore C_{R\,max} = 1.86 \ \frac{mol}{lit}$$

7.15

$$A \begin{matrix} \nearrow R \\ \rightarrow S \\ \searrow T \end{matrix} \quad \left.\begin{matrix} r_R = 1 \\ r_S = 2C_A \\ r_T = C_A^2 \end{matrix}\right\} \quad \varphi(S/A) = \frac{r_S}{-r_A} = \frac{2C_A}{(1+C_A)^2}$$



a) For plug flow go to $C_{Af}=0$ for maximum area

graphical integration

$$C_S = \int_0^4 \frac{2C_A}{(1+C_A)^2}\,dC_A \cong (0.3984)4 = 1.59$$

b) For mixed flow let's solve analytically. Note we could also solve graphically.

$$C_S = \varphi(-\Delta C_A) = \frac{2C_A}{(1+C_A)^2}(4-C_A)$$

differentiating & setting to zero gives

$$\frac{dC_S}{dC_A} = \frac{[2(4-C_A)+2C_A(-1)](1+C_A)^2 - 2C_A(4-C_A)2(1+C_A)}{(1+C_A)^4} = 0$$

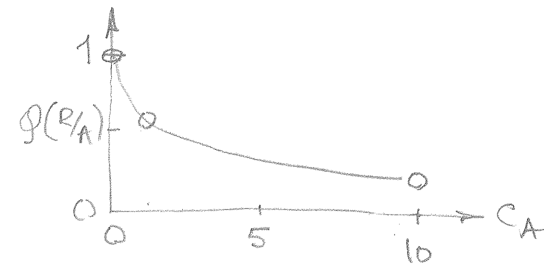
$$\therefore C_A(\text{at } C_{S\,opt}) = \frac{2}{3}$$

$$\varphi\left(\text{at } C_A = \tfrac{2}{3}\right) = 2\left(\tfrac{2}{3}\right) \Big/ \left(1+\tfrac{2}{3}\right)^2 = \frac{12}{25}$$

$$\therefore C_{S\,max} = \frac{12}{25}\left(4-\frac{2}{3}\right) = 1.6$$

7.17 For reactions in parallel first evaluate & draw the φ vs C_A curve

C_A	$\varphi(R/A) = \dfrac{16 C_A^{1/2}}{16 C_A^{1/2} + 12 C_A + C_A^2}$
10	0.185
1	0.55
0	1.0



The highest value for comes where $C_A = 0$ and where $\varphi = 1$

$\therefore$ operate the mixed flow reactor at $X_A \to 1$. Here

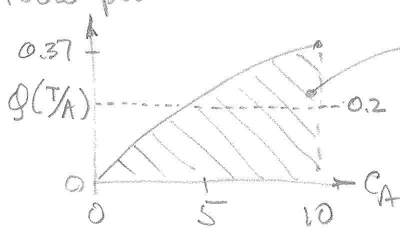
$C_{A0} = 10$, $C_{R0} = 0$, $C_{S0} = 0$ $\longrightarrow$

$\longrightarrow$ $C_{Af} = 0$, $C_{Rf} = 10$, $C_{Sf} = 0$, $V \to \infty$ $\longleftarrow$

7.19 First find the φ vs C_A curve, then decide which reactor type to use

C_A	$\varphi(T/A) = \dfrac{C_A^2}{16 C_A^{1/2} + 12 C_A + C_A^2}$	$\dfrac{1}{-r_A} = \dfrac{1}{16 C_A^{1/2} + 12 C_A + C_A^2}$
10	0.37	0.0037
9	0.34	0.0042
4	0.167	0.0104
1	0.03	0.03
0.1		0.16

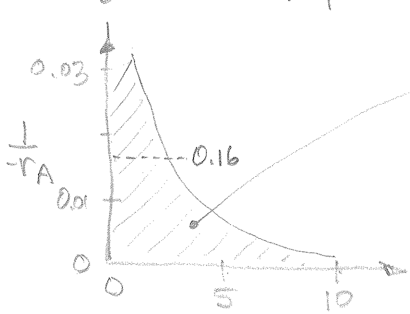
Now plot



$C_R = \begin{pmatrix}\text{shaded}\\ \text{area}\end{pmatrix} = \int \varphi \, dC_A = 0.2(10-0) = 2 \ \dfrac{\text{mol}}{\text{lit}}$ $\longleftarrow$

[Note that plug flow is best since it gives most T]

Next, to find τ plot $-1/r_A$ vs C_A



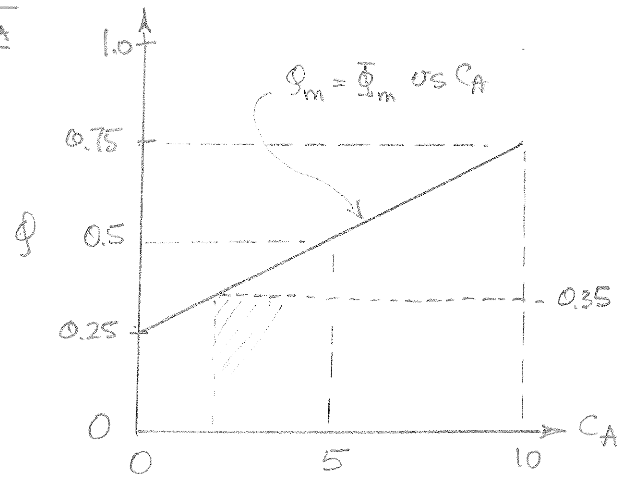
$\tau = \int_{10}^{0} \dfrac{dC_A}{-r_A} = 0.016(10) = 0.16 \text{ min}$

$\therefore V = \tau v = 0.16(1) = 0.16 \, m^3$

$= 160 \text{ lit}$ $\longleftarrow$

7.21 First let us calculate φ, the instantaneous fractional yield. This is obtained directly from the mixed flow run because $\varphi_m = \Phi_m$

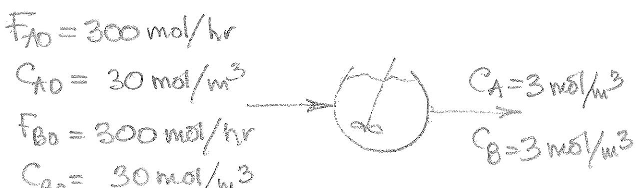
C_{A0}	C_A	C_R	$\varphi = \Phi = \frac{C_R}{C_{A0}-C_A}$
100	90	7	7/10 = 0.7
100	80	13	0.65
100	70	18	0.60
100	60	22	0.55
100	50	25	0.50
100	40	27	0.45
100	30	28	0.40
100	20	28	0.35
100	10	27	0.30
100	0	25	25/100 = 0.25



So for mixed flow with $C_{A0} = 100$

$$C_{Rf} = \Phi(-\Delta C_A) = \varphi(-\Delta C_A) = 0.35(100-20) = 28$$

7.23



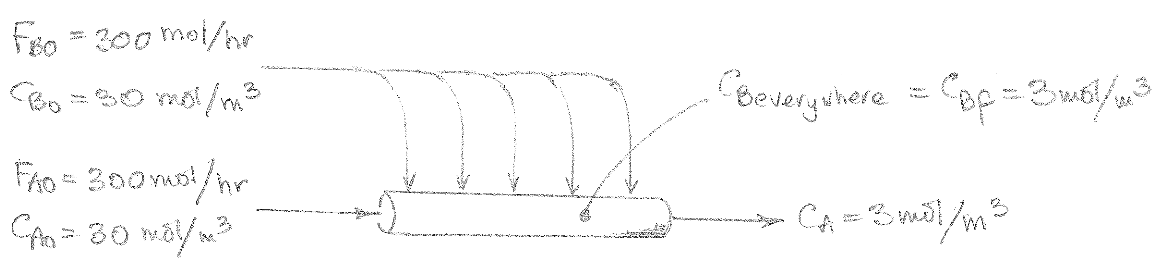
$$\varphi(R/A) = \frac{50C_A}{50C_A + 100C_B} = \frac{1}{3}$$

$$\varphi(S/A) = \frac{100C_B}{50C_A + 100C_B} = \frac{2}{3}$$

$$\therefore C_R = \varphi(R/A)(-\Delta C_A) = \frac{1}{3}(30-3) = 9 \text{ mol/m}^3 \quad \therefore R/S = 1/2$$

$$V = \frac{F_{A0}}{C_{A0}} \cdot \frac{C_{A0} - C_A}{\frac{1}{20}C_A + \frac{1}{10}C_B} = \frac{300}{30}\ \frac{30-3}{3/20 + 3/10} = 600 \text{ lit}$$

7.25



Determine $\varphi(R/A) = \frac{50C_A}{50C_A + 100C_B} = \frac{C_A}{C_A + 2C_B}$

7.25 (continued)

$$\therefore\ C_{Rf} = \int_{C_{Af}}^{C_{A0}} \varphi(R/A)\, dC_A = \int_{3}^{30} \frac{C_A}{C_A + 2\underbrace{C_B}_{=3}}\, dC_A = \Big[C_A - 6\ln(C_A+6)\Big]_3^{30} = 18.68$$

$$C_{Sf} = \int_{C_{Af}}^{C_{A0}} \varphi(S/A)\, dC_A = \int_{3}^{30} \frac{2C_B}{C_A + 2\underbrace{C_B}_{=3}}\, dC_A = \Big[6\ln(C_A+6)\Big]_3^{30} = 8.32$$

So $\dfrac{C_{Rf}}{C_{Sf}} = \dfrac{18.68}{8.32} \cong 2.25$

Finally $V = \dfrac{F_{A0}}{C_{A0}} \displaystyle\int_{C_{Af}}^{C_{A0}} \frac{dC_A}{-r_A} = \frac{300}{30} \int_{30}^{300} \frac{dC_A}{50C_A + 100(3)} = \Big[(10)\left(\tfrac{1}{50}\right)\ln(50C_A + 300)\Big]_3^{30}$

$= 0.2773\ m^3 = 277.3\ lit.$

7.27 a) Villeneuve's question – What is the result of a single battle?

Let F = the number of French ships, and let B = the number of British ships. Then according to the problem statement

$$\left.\begin{aligned} -\frac{dF}{dt} &= kB \\ -\frac{dB}{dt} &= kF \end{aligned}\right\}$$ dividing one by the other to eliminate the time variable (as with multiple reactions) we see $\dfrac{dF}{dB} = \dfrac{B}{F}$ ---- (1)

Separating and integrating gives

$$\int_{F_0}^{F} F\,dF = \int_{B_0}^{B} B\,dB \quad \text{---- or} \quad F_0^2 - F^2 = B_0^2 - B^2$$

If we start with $F_0 = 33$ & $B_0 = 27$, then at the end of the battle $B = 0$. Thus replacing in (1) gives

$F^2 = 33^2 - 27^2 = 360$ --- or $F = 19 - \varepsilon$ ships

7.27 (continued) Nelson's question - What is the result of 2 battles in succession?

Let the British fight f ships in the 1st battle, then $F_0 - f$ ships in the 2nd. Applying Eq(i) to the two battles we have

For the 1st battle : $B_0^2 - B_1^2 = f^2 - 0$ this means that the f ships are all destroyed

For the 2nd battle $B_1^2 - B_2^2 = (F_0 - f)^2 - 0$

Combining gives

$$B_0^2 - B_2^2 = (F_0 - f)^2 + f^2 \quad \text{--------} \quad \text{(ii)}$$

To maximize B_2 take $\frac{dB_2}{df} = 0$ from which we find $f = \frac{F_0}{2}$.
Thus the British should fight ½ the French fleet in each battle. So for $f = 16$ or 17 we find that the British are left with

$$B_2 = \sqrt{184}\,, \text{ or 13 to 14 ships}$$

The difference in the two answers shows how math. affected European history.

Comment Equation (i) represents the SQUARE FIREPOWER LAW. This says that the strength of a force is proportional to the square of its firepower, and it applies to battles with cooperative action. Thus 4 toughs working together as a team can take on 16 others -- one at a time.

The British war office had a couple of mathematicians on their staff who in essence did the above calculation. The French military probably were more "practical", and didn't spend money on "useless" activities.

It would be interesting to consider historical battles (Thermopolae, Jutland, Coral sea, guerrila warfare) from the standpoint of this law. For more on this subject see the section on *Mathematics & Warfare* in "World of Mathematics" by James Newman. This problem has many interesting extensions: suppose K values are unequal, suppose a 3rd party fights both (crazy solution), how to minimize the total killing but still achieve a decision, etc.