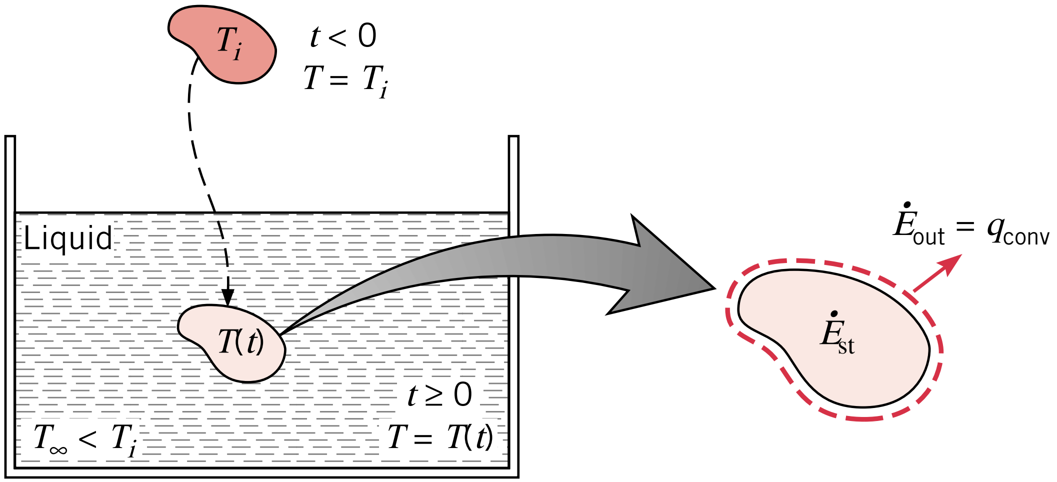
Heat Transfer

CH21004

Transient Heat Conduction



$$-\dot{E}_{out} = \dot{E}_{st}$$

$$-hA_s(T - T_\infty) = \rho V c \frac{dT}{dt}$$

$$\theta \equiv T - T_\infty$$

$$\frac{\rho V c}{hA_s} \frac{d\theta}{dt} = -\theta$$

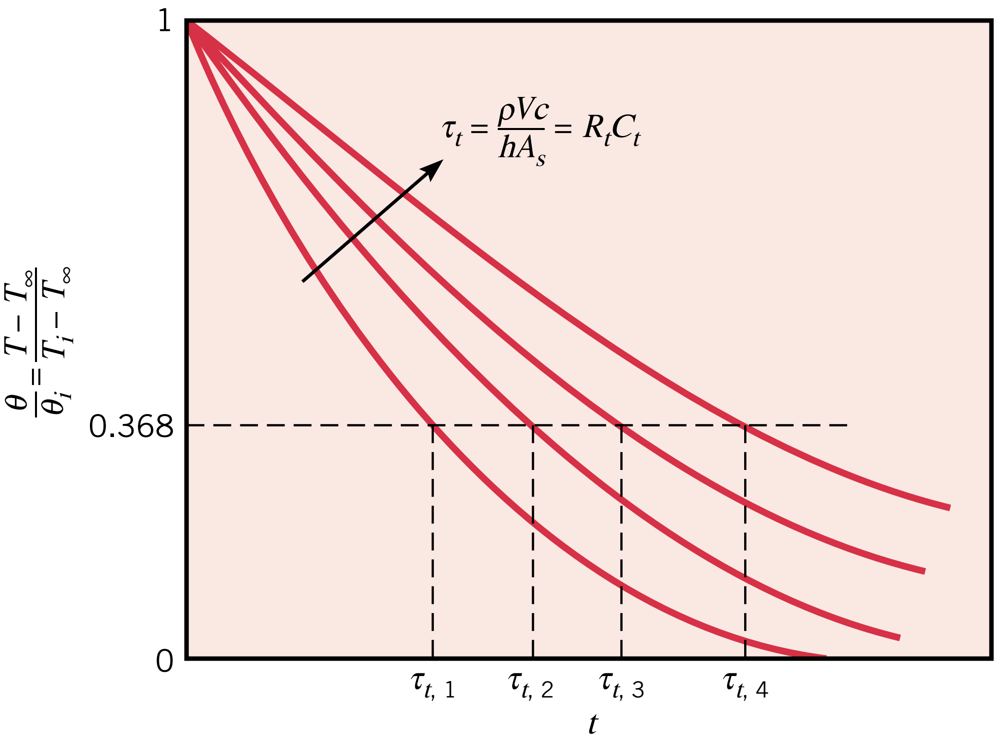
$$\frac{\rho V c}{hA_s} \int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = -\int_0^t dt$$

$$\theta_i \equiv T_i - T_\infty$$

Lumped Capacitance Method

$$\frac{\rho V c}{h A_s} \ln \frac{\theta_i}{\theta} = t$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{h A_s}{\rho V c}\right) t\right]$$



$$\tau_t = \left(\frac{1}{h A_s}\right)(\rho V c) = R_t C_t$$

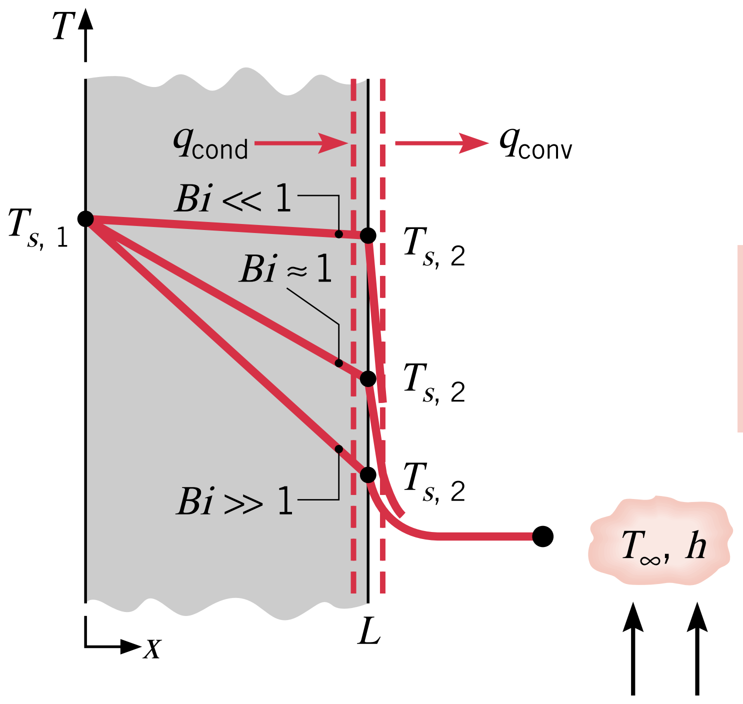
Lumped Capacitance Method

$$Q = \int_0^t q\, dt = hA_s \int_0^t \theta\, dt$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V c} \right) t \right]$$

$$Q = (\rho V c)\theta_i \left[1 - \exp\left(-\frac{t}{\tau_t} \right) \right]$$

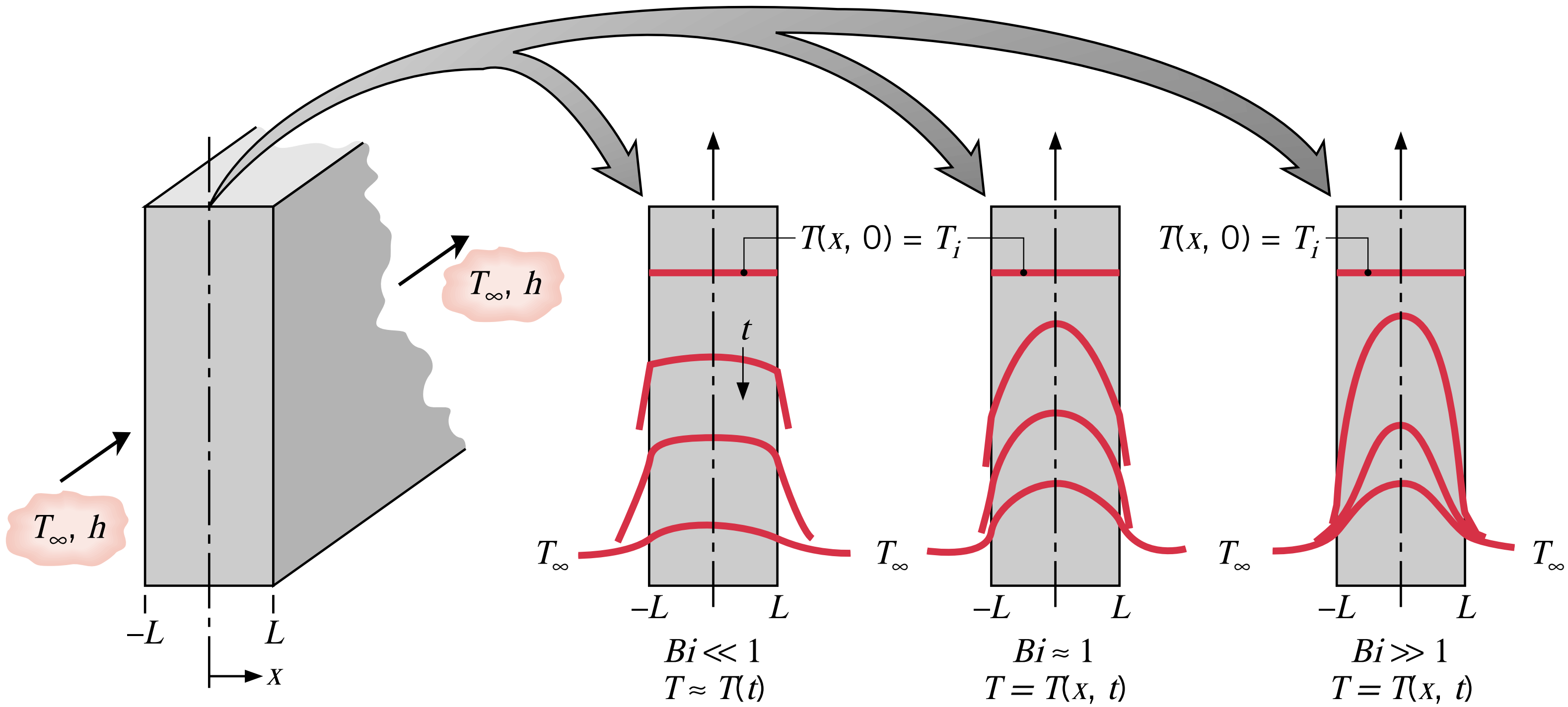
Validity of the Lumped Capacitance Method



$$\frac{kA}{L}(T_{s,1} - T_{s,2}) = hA(T_{s,2} - T_\infty)$$

$$\frac{T_{s,1} - T_{s,2}}{T_{s,2} - T_\infty} = \frac{(L/kA)}{(1/hA)} = \frac{R_{cond}}{R_{conv}} = \frac{hL}{k} \equiv Bi$$

Bi = Biot number



$$\boxed{Bi = \frac{hL_c}{k} < 0.1}$$

$$L_c \equiv V/A_s,$$

$$\boxed{\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right]}$$

$$\frac{hA_s t}{\rho Vc} = \frac{ht}{\rho c L_c} = \frac{hL_c}{k}\frac{k}{\rho c}\frac{t}{L_c^2} = \frac{hL_c}{k}\frac{\alpha t}{L_c^2}$$

$$\frac{hA_s t}{\rho V c} = Bi \cdot Fo$$

$$Fo \equiv \frac{\alpha t}{L_c^2}$$

Fourier number

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V c}\right) t\right]$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-Bi \cdot Fo)$$