The logistic growth model for the population of a species included a *carrying capacity* parameter. Now imagine a population which goes to extinction if the initial population is below a certain number *i.e.* there exists a *critical threshold* for the population. The features of such a population dynamics are:

- upper limit on population based on the carrying capacity
- exponential growth during initial condition and saturation later ($i.e$ logistic growth)
- extinction when the initial population is below the *critical threshold*

Population dynamics of such a system is modelled by the equation given below.

$$\frac{dx}{dt} = -rx\left(1 - \frac{x}{\lambda_1}\right)\left(1 - \frac{x}{\lambda_2}\right)$$

where $r$ is a non-zero positive growth parameter, $\lambda_1$ and $\lambda_2$ are carrying capacity and critical threshold, respectively, such that $0 < \lambda_2 < \lambda_1$.

(a) What is/are the equilibrium population(s) of the system.

(b) Solve the above equation analytically to determine the time evolution of the system. Check if you get the same answer for equilibrium population(s) as you got as the answer for the part (a).

(c) Develop the phase portrait for the system using the solutions obtained for part (b).

(d) Without explicitly solving the governing equation, develop the phase portrait.

(e) Analyse the solutions and phase portraits for initial conditions $x_0 > \lambda_1, x_0 < \lambda_1$, $x_0 > \lambda_2, x_0 < \lambda_2$

(f) Comment upon the bifurcation in the system.

(g) Determine the population(s) at which any one solution curve in the phase portrait exhibits inflection.