INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

End Spring Semester Examination, 2015-2016

Subject : Advanced Mathematical Techniques in Chemical Engineering

Subject No.: CH 61015

Date: *Time*: 3 Hrs *Full Marks*: 80

Instructions : Answer **all Questions**. Assume any missing data suitably. Closed notes/books

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1. A water soluble solute is dissolved from the wall (wall concentration c_w) of a rectangular channel of half height h and length L through which water is flowing under laminar flow condition. We want to solve the concentration field within mass transfer boundary layer. The governing equation and relevant boundary conditions are

$$u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial y^2}$$

at x=0, c=0; at y=0, c= c_w ;at y=∞, c=0. Last boundary condition signifies that beyond boundary layer, solute is convected away by flowing water and hence its concentration is nil. The velocity field within the boundary layer is $u = \frac{3u_0}{h}y$. u_0 is cross section average velocity.

(i) Make the above system (equation and boundary conditions) non-dimensional in terms of Reynolds number, Schmidt number, geometries like, equivalent diameter, channel half height and length.

(ii) Find the concentration profile with mass transfer boundary layer using **similarity solution technique.** **(15)**

2. Solve the above problem using **integral method of solution.** Only the boundary condition at y=∞, c=0 should be replaced by y= δ , c=0, where, δ is the thickness of mass transfer boundary layer at any x. The velocity field is turbulent and it can be approximated as $u=u_0$ within mass transfer boundary layer. Obtain an expression of length averaged Sherwood number. **(15)**

3. Solve the following transient heat conduction equation with a source term **completely (using Green's function method)**,

$$\rho c_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + x$$

At t=0, T=T_0; At x=0, $-k\frac{\partial T}{\partial x} = 0$; At x=L, T=$T_\infty$. **(20)**

4. Prove that 0^{th} order Bessel functions are orthogonal to each other. **(10)**

5. Completely solve **using separation of variables**: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$,

at x=0, $\frac{\partial u}{\partial x} = 1$; at x=1, u=0; at y=0, $\frac{\partial u}{\partial x} = 0$; at y=1, u =2. **(20)**

-----------*END & Best Wishes*----------