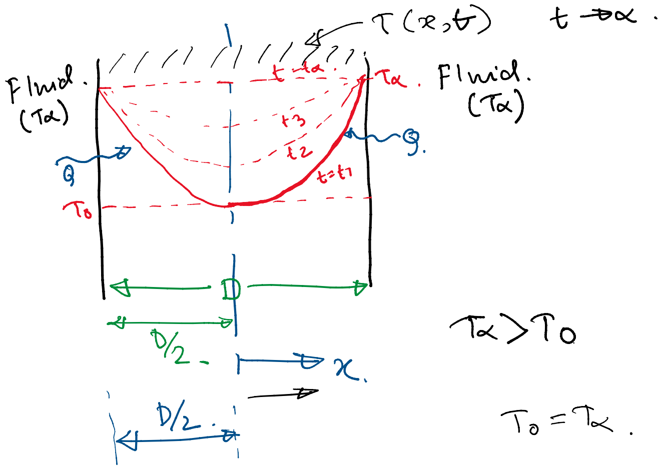
Scaling Analysis

Monday, January 18, 2021 10:25 PM

Problem:
A plate plunged into a highly conducting fluid, such that the surfaces of the plate instantaneously assume the fluid temperature:
Estimating the time needed by the thermal front to penetrate the plate



$T_\infty > T_o$

$T_o = T_\infty$.

Energy equation.

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}.$$

$T(x,t) = ----$

$t \to 5.2$ sec

$t \sim 5$

Order of magnitude

$$\rho c_p \frac{\partial T}{\partial t} \sim \rho c_p \frac{\Delta T}{t} \quad \text{(LHS)}$$

for RHS.

$$k\frac{\partial^2 T}{\partial x^2} \approx k\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) \sim k\,\frac{1}{D/2}\left(\frac{\Delta T}{D/2}\right)$$

$$\sim k\,\frac{\Delta T}{(D/2)^2}.$$

$$\rho c_p \frac{\Delta T}{t} \sim k\,\frac{\Delta T}{(D/2)^2}.$$

$$t \sim \frac{(D/2)^2}{\alpha \cancel{\delta}} \qquad \alpha = k/\rho c_p.$$

Rule. 1. → Define spatial extend.

Rule - 2 → Dominating term.

Rule - 3 → if $c = a + b$. and $o(a) > o(b)$

$$o(c) \sim o(a)$$

Rule - 4 – if $c = a + b$ $\quad o(a) \sim o(b)$.

$$o(c) \sim o(a) \sim o(b)$$

Rule - 5 →

$$p = a \times b$$

then $o(p) \sim o(a) \times o(b)$.

and same for ratio

$r = a/b$ then $o(r) \sim \dfrac{o(a)}{o(b)}$