Calculation of $R_{critical}$ for hopper

In today's class we calculated σ_c (geometrical) $= \sigma_{c,gm} = \dfrac{R\gamma}{\sin(2(\alpha+\phi_w))}$ — (a)

Now Refer fig. 9. of Walker's paper

$$V_w = OC + CT = \frac{\gamma}{\sin\delta} - r\cos(2(\alpha+\beta))$$

$$\sigma_1 = OM = OC + CM = \frac{\gamma}{\sin\delta} + r$$

$$\frac{\sigma_1}{V_w} = \left(\frac{1+\sin\delta}{1-\sin\delta\cos(2(\alpha+\beta))}\right) = X \quad \text{— (1)}$$

(X: a constant)

$V_w = D\bar{V} = \dfrac{D\gamma R}{(\tan\alpha)(C-1)}$ for $C > 1$

closer to mouth $h \ll h_0$

$$\Rightarrow \frac{V_w}{\gamma R} = \left\{\frac{D}{(\tan\alpha)(C-1)}\right\} = Y \quad \text{— (2)}$$

(Y: constant)

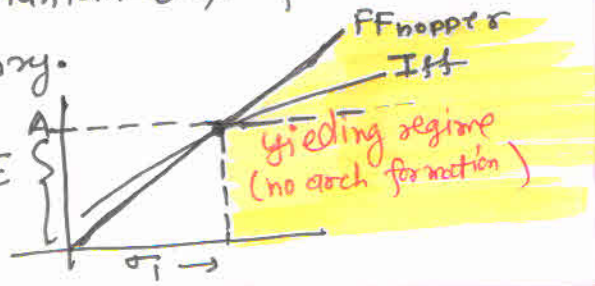
from eqn (1) & (2)

$$\boxed{\frac{\sigma_1}{\gamma R} = XY = \text{constant}} \quad \text{— (b)}$$

from eqn (a) & (b) FF of hopper $= FF_{hopper} = \dfrac{\sigma_1}{\sigma_{c,gm}} = \dfrac{XY\sin(2(\alpha+\phi_w))}{}$

$FF_{hopper} = XY\sin(2(\alpha+\phi_w)) =$ a constant Number $= Z$

Now one should understand that material will flow or yield if the instantaneous flow curve (Iff) is below FF_{hopper} calculated from theory.



A is ordinate (y-value) of intersection point.

*Lower the curve is, less steeper it is, higher the flowability will be $\Rightarrow$

the shaded region in the diagram tells us that in this regime Iff. is below FF_{hopper} curve, which implies material has better flowability than it's theoretical estimate using geometry of the hopper. Hence in the shaded regime, no arch formation will take place. (Remember less steeper the flow curve is, higher the flowability)

Thus no arching → shaded region, wherein.

$\sigma_{c,geometrical} \geq A$

$$\frac{R\gamma}{\sin(2(\alpha+\phi_w))} \geq A$$

$$\Rightarrow \boxed{R_{critical} = \frac{A}{\gamma}\sin(2(\alpha+\phi_w))}$$

So approach for hopper Critical radius determination is as follows:-.

one has to ~~ob~~obtain Iff (instan. flow function) of material from independent experiment.

FF_{hopper} should be calculated using theoretical analysis.

both flow curves (Iff & FF) should be plotted on same plot to get the intersection point-ordinate, which we have said A,

* no arching will occur if,

~~$\sigma_{c,geometrical} > \sigma_{c,material}$~~

☆ Iff flow curve is below FF_{hopper} curve, in other words material has higher flowability as compared to theoretical estimation $\Rightarrow$ $\sigma_{c,geometrical} > A \Rightarrow R \geq \frac{A}{\gamma}\sin(2(\alpha+\phi_w))$

☆☆ more below (or less steeper) your flow curve is, higher will be the flowability