

Indian Institute of Technology, Kharagpur
Date: FN/AN Time: 2 hrs Full Marks : 60 ; No. Of students: 100
Autumn/Spring Semester, 2008-2009; Department of Chemical Engg.
Sub. No.: CH61015; M. Tech & B.Tech;
Sub. Name: Advanced Mathematical Methods in Chemical Engineering

Instructions:

1. Attempt all questions.

2. Closed book and notes

1. For n-dimensional real space and three vectors X, Y, Z, prove **from definitions** (in both cases) that

$$(i) \langle X, Y \rangle < \frac{1}{2} [\|X\|^2 + \|Y\|^2]$$

$$(ii) \|X - Z\|^2 + \|Y - Z\|^2 > 2 \langle X - Z, Y - Z \rangle \quad 8$$

2. Consider two continuous functions $f=(x+y)$ and $g=xy$ such that $0 \leq x \leq 1$ and $1 \leq y \leq 2$. If the following equation is satisfied,

$$d^2(f, g) + \langle f, g \rangle + \|f\|^2 - k \|g\|^2 = 0$$

Find the value of k ? 5

3. For a continuous function, $f(x,y)=ax-y$ where, $0 \leq x, y \leq 1$, evaluate the value of a such that f is orthonormal? 4

4. Consider the vectors $X=[1 \ a \ 2]^T$ and $Y=[-1 \ b \ 3]^T$. What is the relation between a and b such that

- (i) X and Y, each of both orthonormal. 10
- (ii) X-Y is orthonormal
- (iii) X+Y is orthonormal
- (iv) X and Y are orthogonal

5. Consider the function, $y = f(x) = \sin(x + \frac{\pi}{2})$. The domain of x is $(0, \frac{\pi}{2})$.

Check whether f is onto, into and one-to-one if (i) $E=[0,1.57]$ (ii) $E=[0.3,1.4]$. 4

6. Find the third vector such that with the vectors, $u_1 = [1 \ 2 \ -1]^T$ and $u_2 = [2 \ 1 \ -1]^T$, it forms a basis set. Do these vectors form orthogonal set? If no, find an orthogonal - orthonormal set of vectors. 8

7. Prove that if the eigenvalues are simple, the eigenvectors of a square matrix form a basis set in R^n space. 8

8. A chemical engineering system has the following dynamics:

$$\frac{dx}{dt} - x(1 + B^2 x^2) + ABx = 0, \text{ where, } 0 < x < 1; \text{ Parameters } A$$

and B are in the range of $(0, \infty)$. Find the condition on the parameters for which the steady state of the system has the unique solution. 5

9. A system is mathematically represented as follows,

$$\frac{dx}{dt} = 3(x - y) \text{ and } \frac{dy}{dt} = -x^2 y + \mu x, \text{ where the parameter } \mu \text{ is real}$$

positive. Obtain the steady states and check the stability of the steady states and examine the possibility of occurrence any bifurcation 8