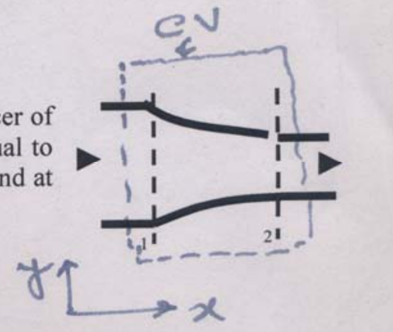
1. The figure represents flow of gasoline (of specific gravity equal to 0.72) through a reducer of weight 25 kg and total volume equal to 0.2 m^3. The velocity at the inlet (diameter equal to 0.4 m) is 3 m/s and at the outlet is 12 m/s. The pressure at the inlet is 58.7 kPa (gage) and at the outlet is 109 kPa (absolute). Calculate the force needed to hold the reducer in place.



$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\,\vec{V}.\vec{dA}$$

Apply x & y components of momentum eq^n

Rx and Ry are the forces needed on the CV to put it in place

$$F_{Sx} + \underbrace{\cancel{F_{Bx}}}_{=0} = \underbrace{\cancel{\frac{\partial}{\partial t}}\int_{CV} u\rho\, d\forall}_{=0\,(SS)} + \int_{CS} u\rho\,\vec{V}.\vec{dA}$$

$$F_{Sy} + F_{By} = \underbrace{\cancel{\frac{\partial}{\partial t}}\int_{CV} v\rho\, d\forall}_{=0\,(SS)} + \int_{CS} v\rho\,\vec{V}.\vec{dA}$$

$$\therefore\ R_x + p_{1g}A_1 - p_{2g}A_2 = u_1\left\{-|\rho V_1 A_1|\right\} + u_2\left\{+|\rho V_2 A_2|\right\}.$$

$$R_x = p_{2g}A_2 - p_{1g}A_1 + (u_2 - u_1)\rho \bar{V}_1 A_1$$

Rx being the force on the reducer as per the cv chosen

$$= (109-101)\times 10^3\,\frac{N}{m^2}\cdot\frac{\pi}{4}(0.2)^2\,m^2 - 58.7\times 10^3\,\frac{N}{m^2}\cdot\frac{\pi}{4}(0.4)^2\,m^2$$

$$+ (12-3)\,\frac{m}{s}\times 0.72\times 10^3\,\frac{kg}{m^3}\cdot \times 3\,\frac{m}{s}\times\frac{\pi}{4}(0.4)^2\,m^2$$

$\underline{R_x = -4.68\text{ kN.}}$ Force to be applied Left.

For the y component

Ry is the force on the reducer

$$R_y - mg - \rho g\forall = \underbrace{\cancel{v_1}}_{=0}\left\{-|\rho V_1 A_1|\right\} + \underbrace{\cancel{v_2}}_{=0}\left\{+|\rho V_2 A_2|\right\}$$

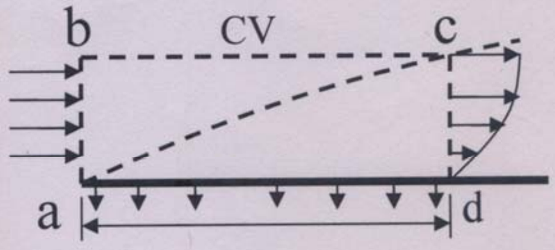
$$R_y = mg + \rho g \forall$$

$$= \left[25\times 9.81 + 0.72\times 1000\times 9.81\times 0.2\right]\text{ N}.$$

$\underline{R_y = 1.66\text{ kN}}$ (force to be applied up ~~right~~).

Please note the CV is defined including the reducer (as in fig.). Therefore Rx and Ry refer to the force on the CV, i.e. the reducer.

1. Consider the steady flow of water past a porous plate with a constant suction velovity of 0.2 mm/s (i.e., V = - 0.2j mm/s). A thin boundary layer grows over the flat plate and the velocity profile at section cd is $\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{1.5}$, where U_∞ is the velocity of approach at section ab and is equal to 3 m/s. Find the mass flow rate across section bc. Given: width of the plate = 1.5m, length = 2m



δ at cd = 1.5 mm

Apply conservation of mass using the cv.

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

steady flow, incompressible flow, $\vec{V} = -v_0 \hat{j}$ along ad

$$0 = \int_{CS} \rho \vec{V}\cdot d\vec{A} = \int_{ab} \rho \vec{V}\cdot d\vec{A} + \dot{m}_{bc} + \int_{cd} \rho \vec{V}\cdot d\vec{A} + \int_{da} \rho \vec{V}\cdot d\vec{A}$$

$$0 = -\rho U_\alpha w\delta + \dot{m}_{bc} + \int_0^\delta \rho U_\alpha \left[3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{1.5}\right] w\, dy + \rho v_0 w L$$

$$\therefore \dot{m}_{bc} = \rho U_\alpha w\delta - \rho U_\alpha w\delta \int_0^1 \left[3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{1.5}\right] d\left(\frac{y}{\delta}\right) - \rho v_0 w L$$

$$= \rho w\left[U_\alpha\delta - U_\alpha\delta\left\{\frac{3}{2}\left(\frac{y}{\delta}\right)^2 - \frac{2}{2.5}\left(\frac{y}{\delta}\right)^{2.5}\right\}_0^1 - v_0 L\right]$$

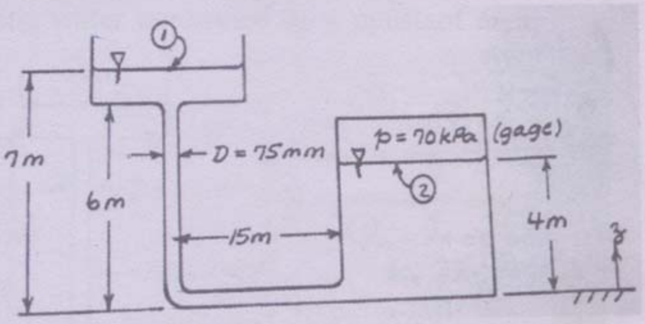
$$= \rho w\left[U_\alpha\delta - U_\alpha\delta\left(\frac{3}{2} - \frac{2}{2.5}\right) - v_0 L\right]$$

$$= \rho w\left[0.3\, U_\alpha \delta - v_0 L\right]$$

$$= 999\,\frac{kg}{m^3} \times 1.5\,m\left(0.3 \times 3\,\frac{m}{s} \times 0.0015\,m - 0.0002\,\frac{m}{s} \times 2\,m\right)$$

$$\dot{m}_{bc} = 1.42\,\frac{kg}{s}$$ (since $\dot{m} > 0$, so flow is out of CV.)

The adjoining figure shows two large reservoirs containing water connected by a constant area, galvanized iron pipe ($\varepsilon/D = 0.002$) that has one right angle bend. The flow can be assumed to be in the fully rough region of the Moody diagram. The surface pressure at the upper reservoir (1 in figure) is atmospheric whereas the pressure (absolute) at the lower reservoir (2 in the figure) surface is 171.3 KPa. The pipe diameter is 75 mm. Assume that the only significant losses occur in the pipe and the bend (L_e/D for the bend is equal to 12). Determine the <u>direction</u> and <u>magnitude</u> of the volume flow rate of water ($\rho = 999$ kg/m^3, kinematic viscosity = 1.1×10^{-6} m^2/s)



Applying energy eq^n ($\alpha = 1$, SS, incompr. flow)

$$\frac{p_1}{\rho} + \underbrace{\cancel{\frac{V_1^2}{2}}}_{=0} + gz_1 = \frac{p_2}{\rho} + \underbrace{\cancel{\frac{V_2^2}{2}}}_{=0} + gz_2 + h_{LT}$$

$$h_{LT} = f\,\frac{L}{D}\,\frac{V^2}{2} + h_{lm}.$$

Assume — i) flow from 1 to 2

ii) Large reservoir $\Rightarrow V_1 = V_2 \simeq 0$.

$$\therefore\ h_{LT_{12}} = -70\times10^{3}\ \frac{N}{m^2} \times \frac{1}{10^3\,kg}\,m^3 + 9.81\ \frac{m}{s^2}\,(7-4)\,m = -40.6\ \frac{m^2}{s^2}$$

h_{LT} cannot be negative. $\therefore$ <u>Flow is from 2 to 1</u>.

$$h_{LT_{21}} = f\,\frac{L}{D}\,\frac{V^2}{2} + f\left(\frac{L_e}{D}\right)_{elbow}\frac{V^2}{2} = f\left(\frac{L}{D} + \frac{L_e}{D}\right)\frac{V^2}{2} = 40.6\ \frac{m^2}{s^2}$$

$\bar{V}$ not known, iteration needed.

$L = 21$ m, $L_e/D = 12$,

Choose f from the fully rough region.

$L_e/D = 0.0002$,

$f \simeq 0.023$ (Moody diagram)

$$\bar{V} = \left[\frac{2h_{LT}}{f\left(\frac{L}{D} + \frac{L_e}{D}\right)}\right]^{1/2} = \left[\frac{2\times40.6}{0.023\left(\frac{21}{0.075} + 12\right)}\right]^{1/2} = 3.48\ m/s.$$

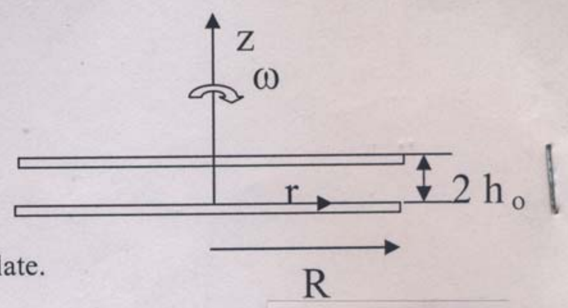
With $\nu = 1.1\times10^{-6}\ m^2/s$ $\quad Re = \frac{VD}{\nu} = \frac{3.48\times75\times10^{-3}}{1.1\times10^{-6}} = 2.37\times10^{5}$

$f = 0.024$ Substitute, recalculate & check......

$$Q = VA = 3.48\ \frac{m}{s} \times \frac{\pi}{4}(0.075)^2\,m^2 = 0.0154\ m^3/s.$$

<u>$Q = 0.0154\ m^3/s.$</u>

A parallel plate viscometer consists of a stationary, circular plate over which another identical plate rotates as shown in the figure. Fluid is placed in the gap and the torque on the lower plate is measured. Simplify Navier Stokes equation, clearly stating all assumptions with proper justification. Show that $V_\theta = \omega rz/(2h_o)$ can be a solution for this situation. Using this expression for V_θ obtain an expression for viscosity of the liquid in terms of the measured torque on the lower plate.



$V_r = V_z = 0$, $\frac{\partial}{\partial t} = 0$ (SS), $\frac{\partial}{\partial \theta} = 0$ (symmetry)

NS eq^n

r comp. $\Rightarrow \frac{\partial p}{\partial r} = \frac{\rho v_\theta^2}{r}$, $\quad$ z: $\Rightarrow \frac{\partial p}{\partial z} = 0$.

θ: $\Rightarrow \frac{\partial}{\partial r}\left[\frac{1}{r}\left\{\frac{\partial}{\partial r}(r v_\theta)\right\}\right] = -\frac{\partial^2 v_\theta}{\partial z^2}$.

$v_\theta = \frac{\omega r z}{2h_0}$

$\therefore \frac{\partial^2 v_\theta}{\partial z^2} = 0 = \frac{\partial}{\partial r}\left[\frac{1}{r}\left\{\frac{\partial}{\partial r}(r v_\theta)\right\}\right]$ or $-\frac{v_\theta}{r^2} + \frac{1}{r}\frac{dv_\theta}{dr} + \frac{d^2 v_\theta}{dr^2}$

$\therefore$ NS eq^n is satisfied, and $v_\theta = \frac{\omega r z}{2h_0}$ is a solution.

$\tau = \mu \frac{\partial v_\theta}{\partial z} = \mu \frac{\omega r}{2h_0}$.

Shear force $= \frac{\mu \omega r}{2h_0} 2\pi r\, dr \quad 2$

Torque $= \int_0^R \frac{\mu \omega r}{2h_0} 2\pi r\, dr \cdot r$

$T = \frac{\mu \omega \pi R^4}{4 h_0}$.

$\Rightarrow \mu = \frac{4 h_0 T}{\omega \pi R^4}$