Summary: Fourier Transform

Fourier cosine and inverse Fourier cosine transform

$$\boxed{F_c(f) = \hat{f}_c(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(u) \cos \alpha u \, \mathrm{d}u} \quad \boxed{F_c^{-1}(\hat{f}) = f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \hat{f}_c(\alpha) \cos \alpha x \, \mathrm{d}\alpha}$$

Fourier sine and inverse Fourier sine transform

$$\boxed{F_s(f) = \hat{f}_s(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(u) \sin \alpha u \, \mathrm{d}u} \quad \boxed{F_s^{-1}(\hat{f}) = f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \hat{f}_s(\alpha) \sin \alpha x \, \mathrm{d}\alpha}$$

Derivative formula: Assuming that f and f' both goes to 0 as x approaches to ∞

$$\boxed{F_c\{f''(x)\} = -\alpha^2 F_c\{f(x)\} - \sqrt{\frac{2}{\pi}} f'(0)} \quad \boxed{F_s\{f''(x)\} = \sqrt{\frac{2}{\pi}} \alpha f(0) - \alpha^2 F_s\{f(x)\}}$$

Fourier transform

$$\boxed{F(f) = \hat{f}(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty f(u) e^{i\alpha u} \, \mathrm{d}u} \quad \boxed{F^{-1}(\hat{f}) = f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty \hat{f}(\alpha) e^{-i\alpha x} \, \mathrm{d}\alpha}$$

Derivative formula-1: Assuming that f goes to 0 as $|x|$ approaches to ∞

$$\boxed{F\{f'(x)\} = -i\alpha F\{f(x)\}}$$

Derivative formula-2: Assuming that f and f' both go to 0 as $|x|$ approaches to ∞

$$\boxed{F\{f''(x)\} = -\alpha^2 F\{f(x)\}}$$

Convolution property

$$\boxed{(f * g)(x) = \int_{-\infty}^\infty f(y) g(x - y) \mathrm{d}y} \quad \boxed{F\{(f * g)\} = \sqrt{2\pi} F\{f\} F\{g\}}$$