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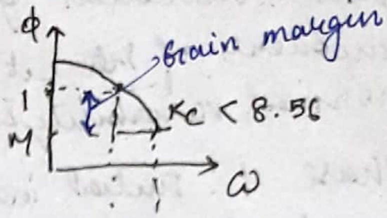
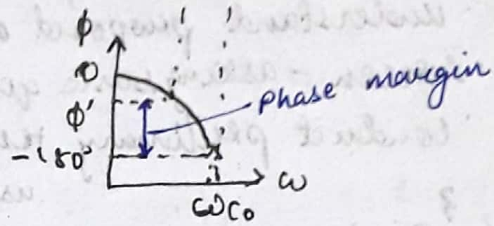
Gain & Phase margin

# Bode stability

$$\phi = 180^\circ \begin{cases} AR > 1 & \text{unstable} \\ = 1 & M.R. \\ < 1 & \text{Stable} \end{cases}$$




$$AR = 1 \begin{cases} \phi < -180^\circ & \text{unstable.} \\ \phi = -180^\circ & M.R \\ > -180^\circ & \text{Stable} \end{cases}$$




$GM = \frac{1}{M}$ [$\log GM = \log 1 - \log M$].

Stability: $M < 1 \Rightarrow GM > 1$

- When process parameters are precisely known:

$$G_{OL} = \frac{K_c e^{-0.1s}}{0.5s+1} \quad ; \quad G_p = \frac{K e^{-t_d s}}{\tau s + 1}$$

GM : 1.4 – 1.7

- " " " " Imprecisely known : GM = 1.7–3.0

Phase margin : 30°–45° – imprecisely known.

Precisely known

Ex : Controller tuning

$$G_p = \frac{e^{-0.15s}}{0.5s+1} \; ; \; G_c = K_c \; ; \; G_f = G_m = 1 \; ; \; GM = 1.7$$

$K_c = ?$ ; $G_{OL} = \frac{e^{-0.15s}}{0.5s+1} \cdot K_c$

$$GM = \frac{1}{M} = \frac{1}{AR} = 1.7.$$

$$G_{OL} = \frac{K_c e^{-0.15j\omega}}{0.5j\omega+1} = \frac{K_c(1-0.5j\omega)}{1+0.25\omega^2} e^{-0.15j\omega}.$$

$\therefore$, $\omega_{co} = 17$ rad.

$$A.R. = \frac{K_c\sqrt{1+0.25\omega^2}}{1+0.25\omega^2} = \frac{K_c}{\sqrt{1+0.25\omega^2}} = \frac{1}{1.7}$$

$$\Rightarrow K_c = \frac{\sqrt{1+0.25\omega^2}}{1.7} = 5.034$$

$$\tan^{-1}(-0.5\omega) + (-0.15\omega) \cdot \frac{180^\circ}{\pi} = -180^\circ.$$

$\omega = 11.6$

$$GM = \frac{\sqrt{1+0.5\omega^2}}{1 K_c} = 1.17 > 1$$

$\Rightarrow AR = 1$ ; $PM = 180° - |\phi'| = 30$

$$\Rightarrow 180 - \left| \tan^{-1}(-1.5\omega) + (-0.1\omega)\frac{180°}{\pi} \right| = 30$$

$\Rightarrow \omega = ?$

**Nyquist stability :**

$$G_{OL} = \frac{0.8K_c}{(5s+1)(10s+1)(15s+1)} e^{-5s}$$

$$AR = \frac{0.8K_c}{\sqrt{(5\omega)^2+1}\left(\sqrt{(10\omega)^2+1}\right)\left(\sqrt{(15\omega)^2+1}\right)}$$

$$\phi = \tan^{-1}(-5\omega) + \tan^{-1}(-10\omega) + \tan^{-1}(-15\omega)$$

$\omega : -\infty$ to $\infty$

OTA

① $\omega : 0 \to \infty$

Starts : $\omega \to 0$ Terminates : $\omega \to \infty$

A

$AR = 0.8K_c$ ; $\phi = 0°$ ; $AR \Rightarrow 0$ , & $\phi = \frac{-3\pi}{2} = -270°$

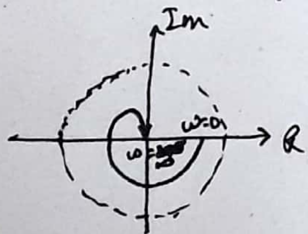



Fig 1.

② $\omega : -\infty$ to $0$.

Starts : $\omega \to -\infty$ : $AR = 0$ ; $\phi = 270°$.

$OA = 0.8K_c$

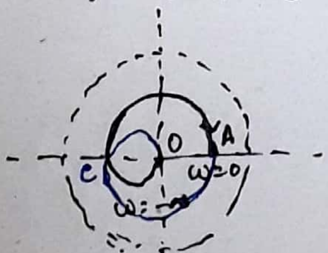



Fig 2.

Fig 2 is mirror image of figure 1 w.r.t. real axis.

Stability criteria - A closed loop control system is unstable if the open loop Nyquist plot of the feedback system encircles the point $(-1, 0)$ as the frequency $\omega$ takes any value in between $-\infty$ and $\infty$.

If $K_c = 1 \Rightarrow OA = 0.8 K_c = 0.8 \Rightarrow$ stable.

$= 2 \Rightarrow OA = 1.6 \Rightarrow$ unstable.

Ziegler Nichols Tuning :
(Z - N)

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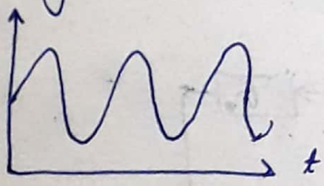
- 1942 - Closed loop or online method.
- Frequency Response Analysis

① Bring the process to the SS.
② P-only ~~[Do not use PI]~~ [Only P controller to be taken]
③ Close the loop.
④ Set point change
⑤ Change $K_c$ until sustained oscillation is achieved



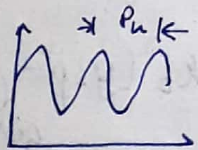
$AR = 1$ ; $\phi = -180°$.

ultimate gain : $K_u$ [value of $K_c$ at which we get sustained oscillation]

⑥ Ultimate period : $P_u = \frac{2\pi}{\omega_{co}}$

| | $K_c$ | $\tau_i$ | $\tau_D$ |
|---|---|---|---|
| P : | $\frac{K_u}{2}$ | – | – |
| PI : | $\frac{K_u}{2.2}$ | $\frac{P_u}{1.2}$ | – |
| PID : | $\frac{K_u}{1.7}$ | $\frac{P_u}{2}$ | $\frac{P_u}{8}$ |

Z-N → Expt. $K_u$



Z-N → Model : $\phi = -180°$ $AR = 1$

$\omega_{co} = ?$ $K_c = K_u = ?$

$P_u = \frac{2\pi}{\omega_{co}}$

$$\begin{array}{r} 11.6 \\ \times .5 \\ \hline 5.80 \end{array}$$

Ex : $G_{OL} = \frac{K_c e^{-0.15s}}{0.5s + 1}$

$G(j\omega) = \frac{K_c\, e^{-0.15j\omega}}{0.5j\omega + 1}$ $\Rightarrow \omega_{co} = \cancel{17}\ 11.6$ rad/min.

$A.R. = \frac{K_c}{\sqrt{1 + (0.5\omega)^2}} = 1$ $\Rightarrow K_u = \sqrt{1 + (\cancel{8.5})^2}$ $(5.8)^2$.

$\phi = \tan^{-1}(-0.5\omega) - 0.15\omega \times \frac{180°}{\pi}$

$P_u = 0.542$ ; $K_u = 5.89$

Remarks:

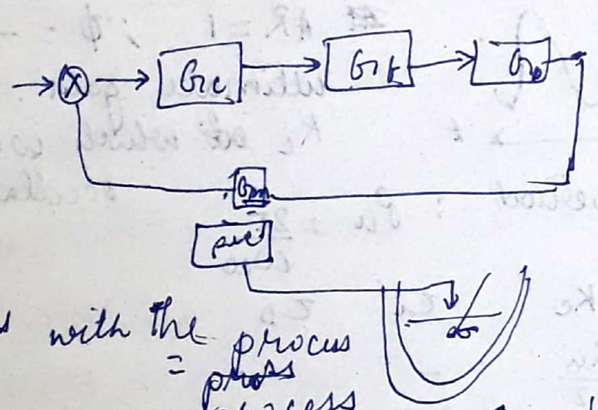
① $K_c$ can't be greater than $K_{cu}$.

② open loop process is unstable.

③ 1st & 2nd order process without $e^{-t_d s}$ deadtime can't be used in this method

④ Z-N method is not used in industrial practice.

Advanced control schemes:

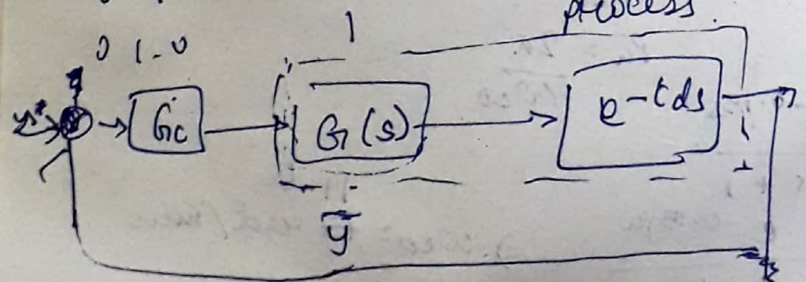


- transported with the process = process
- sampling of meas : sensor
- sampling & measurement.
- developing ~~acting~~ actuating signal DC.

control action.

$u = u_s + K_c (y_{sp} - y)$ (SO, getting degraded)

| $t_d$ | $K_u$ |
|---|---|
| | 80 |
| 0.01 | 8.56 |
| 0.1 | |
| 1.0 | |



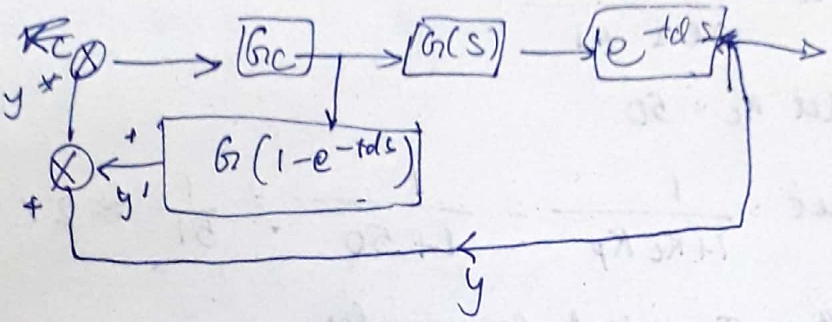
$\bar{y} = G_c G e^{-t_d s} \bar{y}_{sp}$

$\bar{y}^* = G_c G \bar{y}_{sp}$

to get dead time free response.

Adding $y + \bar{y}$ : $y'(s) = \bar{y}^* - \bar{y} = G_c G (1 - e^{-t_d s}) y_{sp}$

Dead time compensator / Smith ~~predictive~~ prediction

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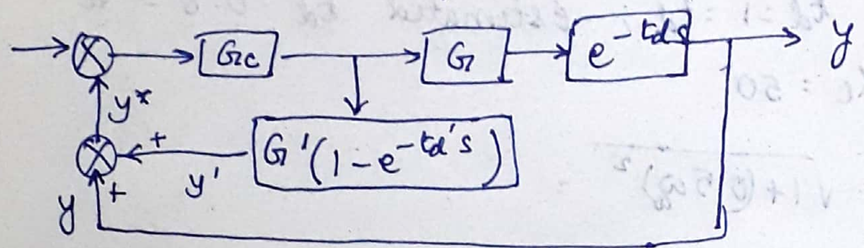
Dead time compensator.




$y^* = y + y' = \left[G_c G e^{-t_d s} + G_c G'(1-e^{-t_d' s})\right] y_{sp}$

$= G_c \left[G' + (G e^{-t_d s} - G' e^{-t_d s})\right] y_{sp}$

① $G = G'$, $t_d = t_d'$

② $G - G'$ $(t_d - t_d')$ → this has detremental effect.

③ If any parameter gets updated with time, then this compensator is ~~called~~ said to be adaptive.

Ex: $G_{OL} = \dfrac{K_c e^{-s}}{0.5s + 1}$

Case 1: No compensator.

$\omega_{co} = ?$ $\quad K_u = ?$

$G(j\omega) = \dfrac{K_c e^{-j\omega}}{0.5j\omega + 1}$

$AR = \dfrac{K_c}{\sqrt{1+(0.5\omega)^2}}$ ; $\phi = -\tan^{-1}(0.5\omega) + (-\omega)\times\dfrac{180}{\pi} = -180$

$\Rightarrow -0.5\omega = \tan\left(\dfrac{180}{\pi}\omega - 180\right)$ & $\omega_{co} = 2.3$

$K_u = 1.52$.

$A = \lim_{s\to 0} s \dfrac{K_c e^{-s}}{0.5s+1}$

offset $= \dfrac{1}{1+K_c K_p} \approx 40\%$ [for $K_c = K_u$]

case 2: Perfect compensation.

$$G_{OL} = \frac{K_c}{0.5s + 1}$$

Let $K_c = 50$

$$\text{offset} = \frac{1}{1+K_c K_p} = \frac{1}{1+50} = \frac{1}{51} \approx 2\%$$

Case 3: Imperfect Compensation.

Actual $t_d = 1 = t_d$ ; Estimated $t_d = 0.8 = t_d'$

$K_c = 50$

$$K_u = \sqrt{1+(0.5\omega_{co})^2} =$$

$$G_{OL} = \frac{K_c e^{-0.2s}}{0.5s+1}$$ [for uncompensated dead time]

$$y^* = G_c G \left[1 + (1-e^{-t_d' s}) - (1-e^{-t_d s})\right] y_{sp}.$$

$$= G_c G e^{-(t_d - t_d')s} y_{sp}$$

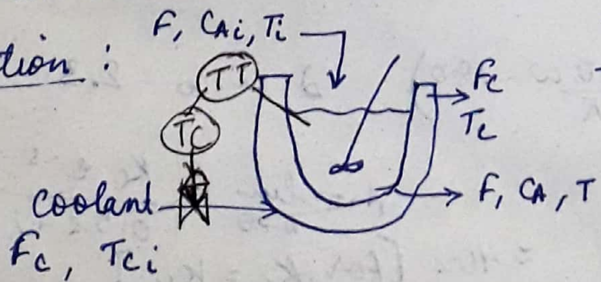
**Multi loop control**

| No. of measurement | No. of MV | Type of control |
|---|---|---|
| 1 | 1 | Feedback control |
| >1 | 1 | Cascade, ratio, override |
| 1 | >1 | split range |

**Cascade control** (Dual loop)

- Primary / master controller
- secondary /slave controller

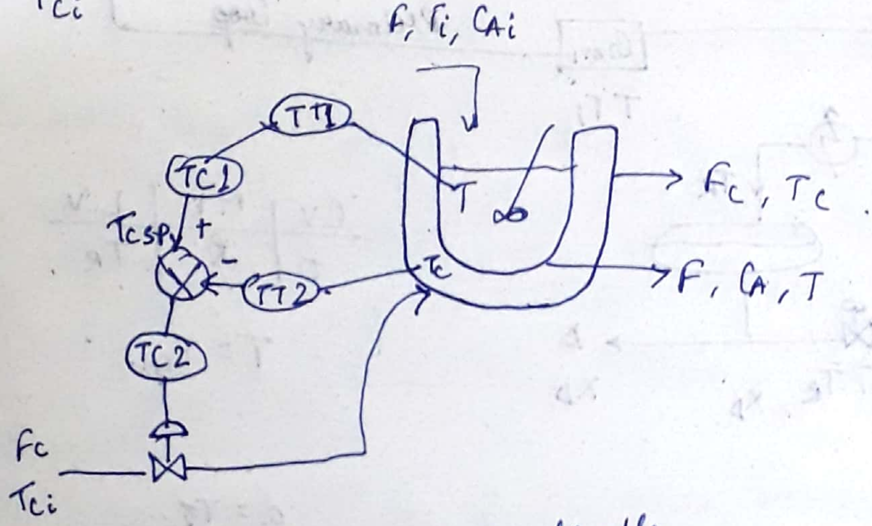
**Motivation** :




- Exothermic
- $T = T_{sp}$

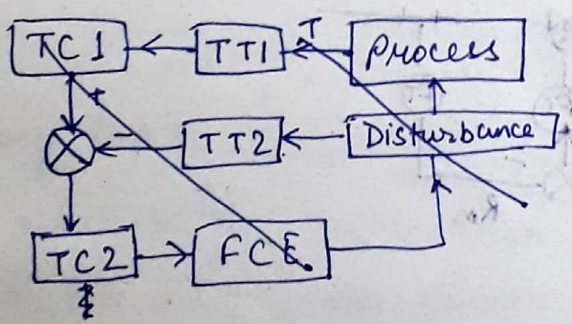
| CV | MV | LV |
|---|---|---|
| T | $F_c$ | $T_i$ <br> $T_{ci}$ |

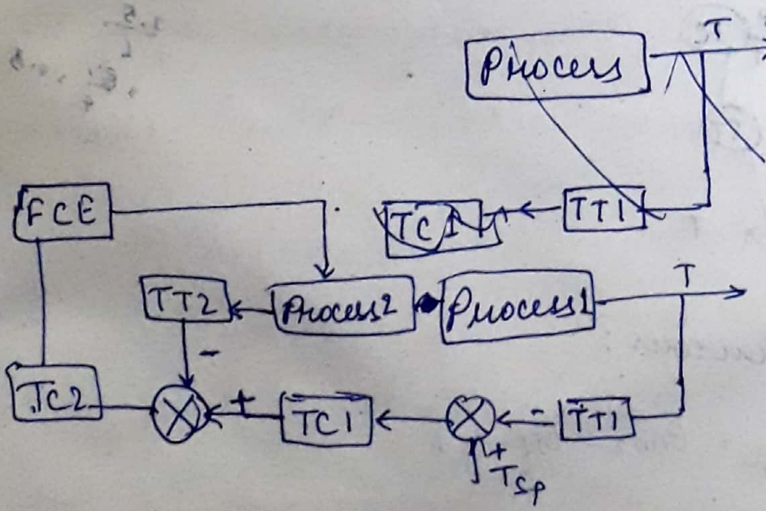
This ~~A~~ feedback control scheme is more effective to reduce the effect of disturbance in $T_i$ & less effective to reduce the effect of disturbance in $T_{ci}$

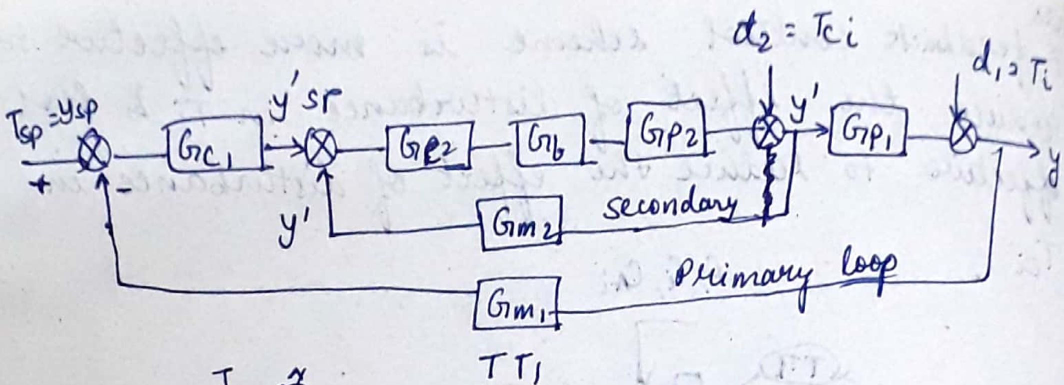


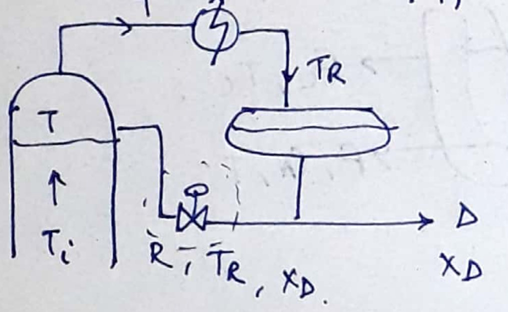

| Process | Controller |
|---|---|
| Process I reactor | TC1 (master) |
| Process II jacket | TC2 (secondary) |

Cascade control is ~~used~~ preferred over simple feedback control when there is disturbance in the manipulated stream.



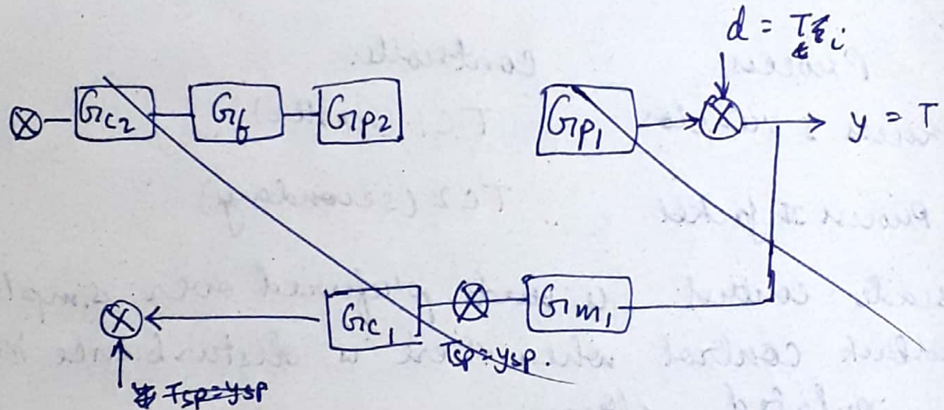



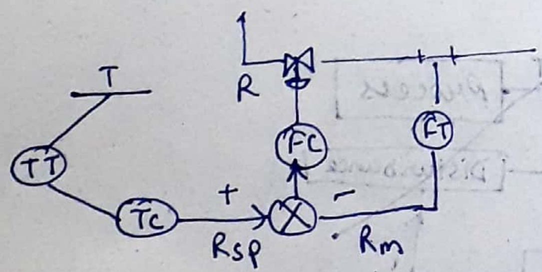


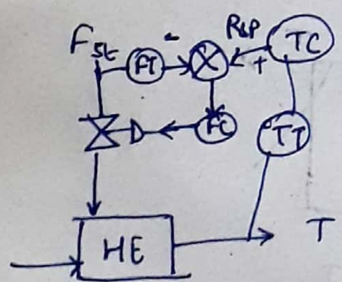

| CV | MV | LV |
|---|---|---|
| T | R | $T_R$ . |

$T = T_{sp}$.




Q) Stream :




Closed loop behaviour :

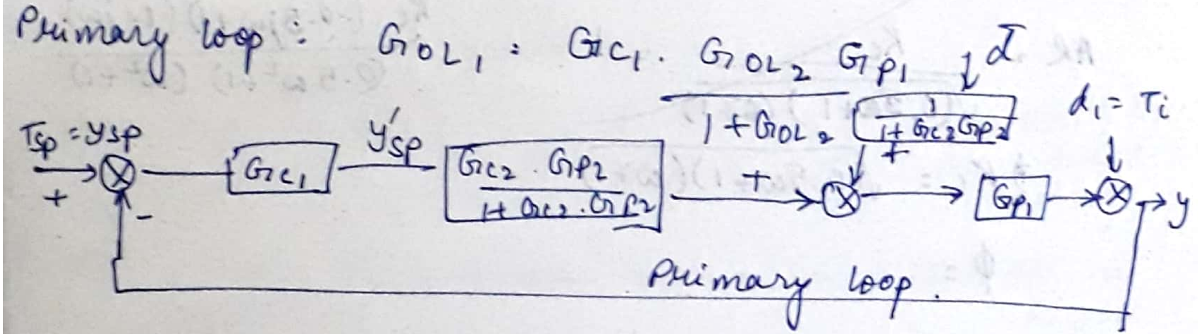
Assumed $G_{m_1} = G_{m_2} = G_f = 1$

Secondary loop : $G_{OL_2} : G_{C_2} \cdot G_{P_2}$

CE : $1 + G_{OL_2} = 0$ .

CLTF : $$y' = \frac{G_{C_2} \cdot G_{P_2}}{1 + G_{C_2} \cdot G_{P_2}} y'_{sp} + \frac{1}{1 + G_{C_2} \cdot G_{P_2}} d_2,$$

Primary loop : $G_{OL_1} : G_{C_1} \cdot \frac{G_{OL_2}}{1+G_{OL_2}} G_{P_1}$




Remarks :

① secondary ~~sf~~ is faster than Primary as there is change in order.

② S : P, PI
P : PI, PID

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pb : cascade control

$$G_{P_1} = \frac{1}{(0.5s+1)(s+1)} \; ; \; G_{m_1} = G_{m_2} = G_f = 1$$

$$G_{P_2} = \frac{1}{1.5s+1} \; ;$$ Primary controller = PID
Secondary " = P

Secondary loop : $G_{OL_2} : G_{C_2} \cdot G_{P_2} : \frac{K_{C_2}}{1.5s+1}$

Any value of $K_c$ can be chosen.

Let $K_{C_2} = 5$ ($K_{C_2}$ is ~~sto~~ chosen such that it has proper coordination with primary controller.

$$G_{OL_1} : G_{C_1} G_{P_1} \cdot \frac{G_{P_2} \cdot G_{C_2}}{1 + G_{C_2} \cdot G_{P_2}} =$$

$$= \frac{K_{C_1}\left(1 + \frac{1}{\tau_i s} + \tau_D s\right)}{(0.5s+1)(s+1)} \cdot \frac{\frac{K_{C_2}}{1.5s+1}}{1 + \frac{K_{C_2}}{1.5s+1}}$$

$$= \frac{K_{C_1}\left(1 + \frac{1}{\tau_i s} + \tau_D s\right)}{(0.5s+1)(s+1)} \cdot \frac{K_{C_2}}{1.5s + K_{C_2} + 1}$$

$$G_{cl} = \cancel{\frac{K_c}{\tau s + 1}}$$

$$G_{OL} = \frac{K_c}{(0.5s+1)(s+1)} ; \frac{K_c}{(0.5j\omega+1)(j\omega+1)}$$

$$\frac{K_c(-0.5j\omega+1)(1-j\omega)}{(0.5\omega^2+1)(\omega^2+1)}$$

$$AR = \frac{K_c}{\sqrt{(0.5\omega+1)(\omega+1)}} = 1$$

$$\Rightarrow K_c = \sqrt{(0.5\omega+1)(\omega+1)}$$

$\phi =$

$$= \frac{\frac{5}{6}K_{c1}}{(0.5s+1)(s+1)(0.5s+1)}$$ [P-only to be taken]

$AR = 1 \; ; \; \phi = -180^\circ$.

$\Rightarrow \omega_{co} = 3.734 \; ; \; K_u = 13.48$

$P_u = \frac{2\pi}{\omega_{co}}$

PID: $K_c = \frac{K_u}{1.7} = ?$

$\tau_i = \frac{P_u}{2} = ?$

$\tau_D = \frac{P_u}{8} = ?$

Min^m ISE

**Override / constraint control**

During the normal operation of the plant or at start up or shut down conditions, there may arise some abnormal situations which may lead to destruction of the equipment and/or operating personal. In such a situation, a special type of switch is used, one of which is HSS (high selector switch). It prevents to exceed the upper limit constraint.

$G_{OL} = \cancel{\frac{K_c}{\tau s + 1}}$

$G_{OL} = \frac{K_c}{(0.5s+1)(s+1)}$ ; $\frac{K_c}{(0.5j\omega+1)(j\omega+1)}$

$\frac{K_c(-0.5j\omega+1)(1-j\omega)}{(0.5\omega^2+1)(\omega^2+1)}$

$AR = \frac{K_c}{\sqrt{(0.5\omega+1)(\omega+1)}} = 1$

$\Rightarrow K_c = \sqrt{(0.5\omega+1)(\omega+1)}$

$\phi =$

$= \frac{\frac{5}{6}K_{c_1}}{(0.5s+1)(s+1)(0.5s+1)}$ [P-only to be taken]

$AR = 1$ ; $\phi = -180°$.

$\Rightarrow \omega_{co} = 3.784$ ; $K_u = 13.48$

$P_u = \frac{2\pi}{\omega_{co}}$

PID : $K_c = \frac{K_u}{1.7} = ?$

$\tau_i = \frac{P_u}{2} = ?$

$\tau_D = \frac{P_u}{8} = ?$

Min$^m$ ISE

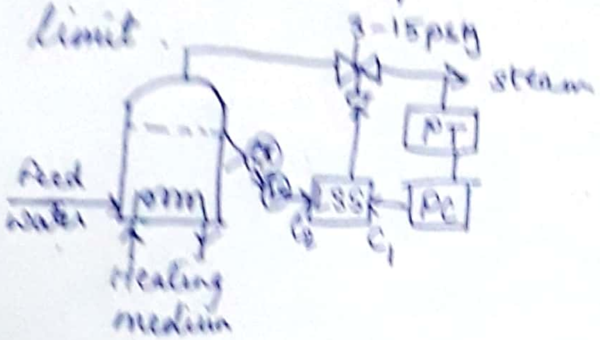
**Override / constraint control**

During the normal operation of the plant or at start up ~~or~~ shut down conditions, there may arise some abnormal situations which may lead to destruction of the equipment and/or operating personal. In such a situation, a special type of switch is used, one of which is HSS (high selector switch). It prevents to exceed the upper limit.
constraint

LSS: Low selector switch: prevents to exceed the lower limit.

Ex: Boiler

$P = P_{sp}$

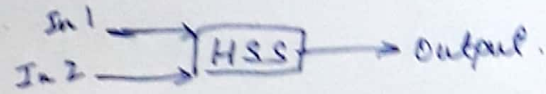



LSS = min$(C_1, C_2)$ ; P ↑ ope ↑ . h ↑ opening ↑ } Both $C_1$ & $C_2$ are direct acting

h ↑ ⇒ P ↓ ⇒ $T_{bp}$ ↓ ⇒ more vaporization ⇒ decrease in height.

LSS output = $C_1$ $(C_1 < C_2)$

- $h < h_L$ = $C_2 = 0 =$ 3psig
- LSS = $C_2$ ⇒ level controller overrides the pressure controller.

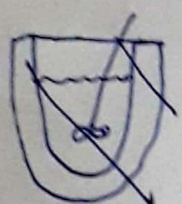



if In 1 > In 2, HSS o/p = In 1.
else if In 1 < In 2, HSS o/p = In 2.
end.

## Split range control

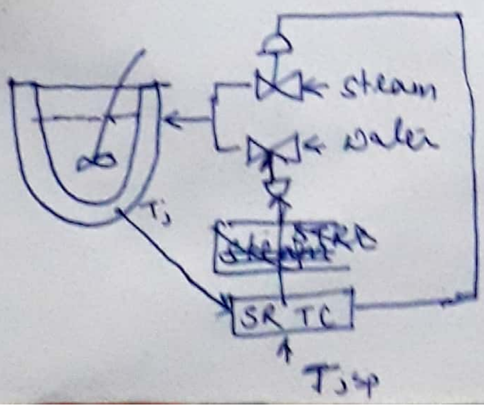
No. of measurements = 1
No. of MV > 1

Ex: Non isothermal batch reactor



Start : 15°C
End : 110°C

Ex 2: pH reactor



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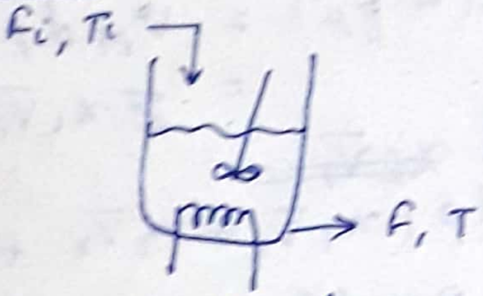
## Feed~~back~~forward Control

Motivation :




FBC :
$T = T_{sp}$

| CV | MV |
|---|---|
| T | Q |

$$PI : Q = Q_s + K_c e + \frac{K_c}{\tau_i}\int e\,dt$$

$Q_s, K_c, \tau_i \Rightarrow$ Fixed

$e = T_{sp} - T$

$T_i \uparrow, T \uparrow, Q \downarrow$

FBC acts after the effect of disturbance has been felt by the process.

Perfect control can't be achieved by the FBC (theoretically)

FFC : → Measured L.V.
→ Take perfect action beforehand

FBC acts after the fact in a compensatory manner
FFC " before " " " an anticipatory "

$F_i = F =$ constant $\Rightarrow V =$ constant

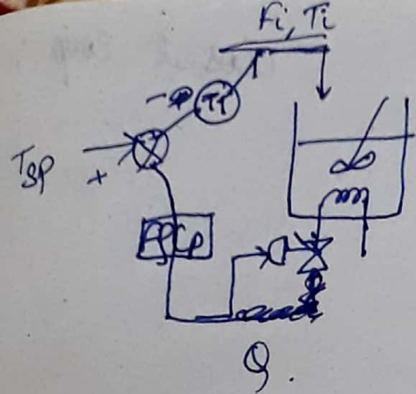
Energy :

$$\frac{d}{dt}\left(\rho C_p (T - T_{ref})V\right) = F_i \rho C_p (T_i - T_{ref}) - F_0 \rho C_p (T - T_{ref}) + Q$$

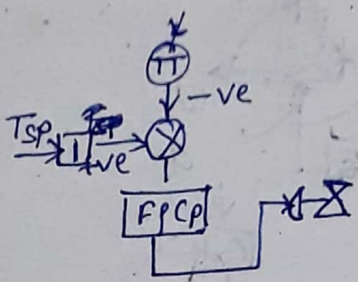
$$\Rightarrow \rho C_p V \frac{dT}{dt} = F_i \rho C_p T_i - F_0 \rho C_p T + Q$$

static : $Q = F\rho C_p (T_{sp} - T_i)$ $\left[\frac{dT}{dt} = 0\right]$

Scanned by CamScanner




$Q$.




Dynamic : ~~$V\rho C_p\, s\, \bar{T}(s) = F_i \rho C_p \bar{T}_i(s) - F\rho C_p \bar{T}(s) + \frac{Q}{s}$~~

$$\tau \frac{dT}{dt} + T = T_i + \frac{Q}{F\rho C_p} \quad ; \quad \tau = \frac{V}{F}$$

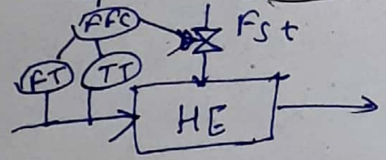
$$\bar{Q} = ((\tau s + 1)\bar{T} - \bar{T}_i)\, F\rho C_p$$

$$\tau s T + T = \bar{T}_i + \frac{\bar{Q}}{F\rho C_p}$$

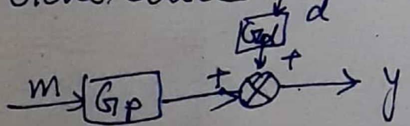
$\leftarrow m$ $\quad \leftarrow G_{sp}$

$$\therefore \bar{Q}_{sp} = ((\tau s + 1)\bar{T}_{sp} - \bar{T}_i)\, F\rho C_p \leftarrow \text{FFC}$$

Ex1 : HE :




Generalized FFC :




$$y_{sp} = G_p m + G_d d$$

$$m = \frac{1}{G_p}[y_{sp} - G_d d]$$

Comparing : $y_{sp} = \bar{Q}_{sp}$ $\quad = \frac{G_d}{G_p}\left[\frac{y_{sp}}{G_d} - d\right]$.

$$F\rho C_p = \frac{G_d}{G_p}$$

$$\therefore G_c = \frac{G_d}{G_p} \quad ; \quad G_{sp} = \frac{1}{G_d}$$

$$\tau s + 1 = \frac{1}{G_d}$$

& $G_p = \frac{K_p}{\tau_p s + 1}$ ; $G_d = \frac{K_d}{\tau_d s + 1}$

$$G_{sp} = \frac{1}{G_d} = \frac{\tau_d s + 1}{K_d} \quad ; \quad G_c = \frac{G_d}{G_p} = \frac{K_d}{K_p}\left(\frac{\tau_p s + 1}{\tau_d s + 1}\right)$$

→ It can't provide perfect control as it depends on Gd & Gp which further depends on process modelling & perfect modelling can't be achieved.

→ FFC: $F\rho C_p$ ; FBC : $Q = Q_s + K_c e$

Closed loop :