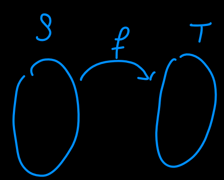
1. Prof. Parag Deshpande.
   - Linear Algebra.
2. This course will be math focused, not ChE focused.
3. Course notes PDF → downloaded. See Good Reads.
4. Review of set theory.

(a) Binary Operator

- if S is a non-empty set & f is a function defined on S, then f is a binary operator on S if



domain co-domain

- range $\subseteq$ co-domain

$$f : S \times S \to \underline{\underline{S}}$$ the result should also lie in S.

"f maps $S \times S$ to S".

f maps every element in $S \times S$ to an element in S

- A binary op. maps every (a, b), where $a, b \in S$, to a unique $c \in S$.

e.g. $S = \{1, 2, 3\}$

1. multiplication by 1 ✓
2. mult. of two elements ✗ (2×3 not in S)

(b) Commutativity

- if $\otimes$ is an operator defined on S, then $\otimes$ is said to be commutative iff -

$$a \otimes b = b \otimes a \quad \forall\, a, b \in S.$$

ⓒ Associativity:

$$a \otimes (b \otimes c) = (a \otimes b) \otimes c.$$

ⓓ Distributivity:

- if $\otimes$ & $\odot$ are defined on S, then $\otimes$ is said to be left distributive over $\odot$ iff

$$a \otimes (b \odot c) = (a \otimes b) \odot (a \otimes c)$$
$$\forall\ a, b, c \in S.$$

- " right distributive

$$(a \odot b) \otimes c = (a \otimes c) \odot (b \otimes c)$$

- if both l & r distr., then we can just say distr.

⑤ Algebraic Structure $(S, \otimes)$

- A set S with an operator (a binary operator). $\otimes$ defined on it.

⑥ Identity element:

- If $\otimes$ is defined on S & $e \in S$ such that

$$e \otimes a = a \quad \forall\ a \in S,$$

then e is the left-identity element

- similarly for right-identity element.

- if both, then just identity element → ★

$$a \otimes e = e \otimes a = a \quad \forall a \in S \;\&\; e \in S.$$

e.g. $S = \mathbb{N}$, $\otimes = +$

note that e must belong to S.

$e \to ?$ → does not exist!

$\because 0 \notin \mathbb{N}$.

⑦ Inverse of an element

- If $\otimes$ is defined on S & $b \in S$, then b is called the left inverse of $a \in S$ iff

$$b \otimes a = e,$$

where $e \in S$ is the identity element.

- $a \otimes b = b \otimes a = e$

⇒ a is the inverse of b

& b is the inverse of a.

8. Group.

- A group is an algebraic structure $(G, \otimes)$ which follows -

associativity ① $a \otimes (b \otimes c) = (a \otimes b) \otimes c \quad \forall a, b, c \in G$

identity element must exist ② $\forall a \in G, \exists b$ s.t. $a \otimes e = e \otimes a = a$

inverse element must exist ③ $\forall a \in G, \exists b$ s.t. $a \otimes b = b \otimes a = e$, $e \in G$.

9. Ring.

- A ring is an algebraic structure $(R, +, *)$, s.t.

associativity ① $a + (b + c) = (a + b) + c \quad \forall a, b, c \in R$

identity element must exist ② $\forall a \in R, \exists b$ s.t. $a + e = e + a = a$

inverse element must exist ③ $\forall a \in R, \exists b$ s.t. $a + b = b + a = e$, $e \in R$.

commutativity of + ④ $a + b = b + a, \forall a, b \in R$.

assoc. of $\times$ ⑤ $(a \times b) \times c = a \times (b \times c) \quad \forall a, b, c \in R$

$\times$ distributes over + ⑥ $\left.\begin{array}{l} a \times (b+c) = (a \times b) + (a \times c) \\ (a+b) \times c = (a \times c) + (b \times c) \end{array}\right\} \forall a, b, c \in R$

identity for $\times$ ⑦ $\forall a \in R \; \exists e$ s.t. $a \times e = e \times a = a$, $e \in R$

comm. of $\times$ ⑧ $\forall a, b \in R$, $a \times b = b \times a$

① → ③ = group
① → ④ = Abellian / comm. group.
① → ⑦ = Ring
① → ⑧ = Field.

1. Associativity of +
2. Identity for +
3. Inverse for +
4. Commutativity of +
5. Associativity of x
6. Identity for x
7. x distributes over +
8. Commutativity of x

1 → 2 => monoid
1 → 3 => group
1 → 4 => abellian group
1 → 7 => rings
1 → 8 => field.

(10) Closure Property

- Given a binary op. ⊗ on S, for $T \subseteq S$, T is said to be closed under ⊗ if $a \otimes b \in T \quad \forall a, b \in T$.

(11) Uniqueness of identity of a group. we say group here, but we really mean algebraic structure.

- Assume that the group has 2 ident. elements e_1 & e_2.

$$\therefore \quad a \otimes e_1 = e_1 \otimes a = a$$
$$\& \quad a \otimes e_2 = e_2 \otimes a = a$$

$\therefore$ if we take $a = e_2$

$$e_2 \otimes e_1 = e_2$$

& if we take $a = e_1$

$$e_2 \otimes e_1 = e_1$$

$\therefore \quad e_2 = e_1.$

(12) Uniqueness of inverse for each element in a group.

- Assume two inverses a^1 & a^2 for a

$$\therefore \quad a^1 \otimes a = a \otimes a^1 = e$$

$$\& \quad a^2 \otimes a = a \otimes a^2 = e$$

$$e \otimes a^2 = a^2$$

$$\Rightarrow (a \otimes a^1) \otimes a^2 = a^2 \qquad (1)$$

$$\& \quad e \otimes a^1 = a^1$$

$$\Rightarrow (a \otimes a^2) \otimes a^1 = a^1 \qquad (2)$$

Since $\otimes$ is associative LHS (1) = LHS (2)

$$\therefore \quad a_1 = a_2.$$

(13) Subgroup.