The pressure drop per hairpin will be $3.2^2/(2 \times 32.2) = 0.159$ ft of water or 0.07 psi. Unless the velocity is well above 3 fps, the entrance and exit losses may be neglected. Values of $V^2/2g'$ are plotted directly against the mass velocity for a fluid with a specific gravity of 1.0 in Fig. 27 in the Appendix.

The Calculation of a Double Pipe Exchanger. All the equations developed previously will be combined to outline the solution of a double pipe exchanger. The calculation consists simply of computing h_o and h_{io} to obtain U_C. Allowing a reasonable fouling resistance, a value of U_D is calculated from which the surface can be found with the use of the Fourier equation $Q = U_D A \, \Delta t$.

Usually the first problem is to determine which fluid should be placed in the annulus and which in the inner pipe. This is expedited by establishing the relative sizes of the flow areas for both streams. For equal allowable pressure drops on both the hot and cold streams, the decision rests in the arrangement producing the most nearly equal mass velocities and pressure drops. For the standard arrangements of double pipes the flow areas are given in Table 6.2.

TABLE 6.2. FLOW AREAS AND EQUIVALENT DIAMETERS IN DOUBLE PIPE EXCHANGERS

Exchanger, IPS	Flow area, in.^{2}		Annulus, in.	
	Annulus	Pipe	d_e	d'_e
2 × 1¼	1.19	1.50	0.915	0.40
2½ × 1¼	2.63	1.50	2.02	0.81
3 × 2	2.93	3.35	1.57	0.69
4 × 3	3.14	7.38	1.14	0.53

In the outline below, hot- and cold-fluid temperatures are represented by upper and lower case letters, respectively. All fluid properties are indicated by lower case letters to eliminate the requirement for new nomenclature.

Process conditions required:

Hot fluid: T_1, T_2, W, c, s or ρ, μ, k, ΔP, R_{do} or R_{di}
Cold fluid: t_1, t_2, w, c, s or ρ, μ, k, ΔP, R_{di} or R_{do}

The diameter of the pipes must be given or assumed.

A convenient order of calculation follows:

(1) From T_1, T_2, t_1, t_2 check the heat balance, Q, using c at T_{mean} and t_{mean}.

$$Q = WC(T_1 - T_2) = wc(t_2 - t_1)$$

Radiation losses from the exchanger are usually insignificant compared with the heat load transferred in the exchanger.

(2) LMTD, assuming counterflow. (5.14)

(3) T_c and t_c: If the liquid is neither a petroleum fraction nor a hydrocarbon the caloric temperatures cannot be determined through the use of Fig. 17 and Eqs. (5.28) and (5.29). Instead, the calculation of U_C must be performed for the hot and cold terminals giving U_h and U_c from which one may obtain K_c. F_c is then gotten from Fig. 17 or Eq. (5.27). If neither of the liquids is very viscous at the cold terminal, say not more than 1.0 centipoise, if the temperature ranges do not exceed 50 to 100°F, and if the temperature difference is less than 50°F, the arithmetic means of T_1 and T_2 and t_1 and t_2 may be used in place of T_c and t_c for evaluating the physical properties. For nonviscous fluids $\phi = (\mu/\mu_w)^{0.14}$ may be taken as 1.0 as assumed below.

Inner pipe:

(4) Flow area, $a_p = \pi D^2/4$, ft².

(5) Mass velocity, $G_p = w/a_p$, lb/(hr)(ft²).

(6) Obtain μ at T_c or t_c depending upon which flows through the inner pipe. μ, lb/(ft)(hr) = centipoise × 2.42.
From D ft, G_p lb/(hr)(ft²), μ lb/(ft)(hr) obtain the Reynolds number, $Re_p = DG_p/\mu$.

(7) From Fig. 24 in which $j_H = (h_iD/k)(c\mu/k)^{-1/3}(\mu/\mu_w)^{-0.14}$ vs. DG_p/μ obtain j_H.

(8) From c Btu/(lb)(°F), μ lb/(ft)(hr), k Btu/(hr)(ft²)(°F/ft), all obtained at T_c or t_c compute $(c\mu/k)^{1/3}$.

(9) To obtain h_i multiply j_H by $(k/D)(c\mu/k)^{1/3}$ $(\phi = 1.0)$ or

$$\frac{h_iD}{k}\left(\frac{c\mu}{k}\right)^{-1/3}\left(\frac{\mu}{\mu_w}\right)^{-0.14}\frac{k}{D}\left(\frac{c\mu}{k}\right)^{1/3} \times 1.0 = h_i \text{ Btu/(hr)(ft}^2\text{)(°F)} \quad (6.15a)$$

(10) Convert h_i to h_{io}; $h_{io} = h_i(A_i/A) = h_i \times \text{ID/OD}$. (6.5)

Annulus:

(4′) Flow area, $a_a = \pi(D_2^2 - D_1^2)/4$, ft²

$$\text{Equivalent diameter } D_e = \frac{4 \times \text{flow area}}{\text{wetted perimeter}} = \frac{D_2^2 - D_1^2}{D_1} \text{ ft} \quad (6.3)$$

(5′) Mass velocity, $G_a = w/a_a$, lb/(hr)(ft²)

(6′) Obtain μ at T_c or t_c, lb/(ft)(hr) = centipoise × 2.42. From D_e ft, G_a lb/(hr)(ft²), μ lb/(ft)(hr) obtain the Reynolds number,

$$Re_a = \frac{D_e G_a}{\mu}$$

(7′) From Fig. 24 in which $j_H = (h_o D_e/k)(c\mu/k)^{-1/3}(\mu/\mu_w)^{-0.14}$ vs. $D_e G_a/\mu$ obtain j_H.

(8′) From c, μ, and k, all obtained at T_c or t_c compute $(c\mu/k)^{1/3}$.

(9′) To obtain h_o multiply j_H by $(k/D_e)(c\mu/k)^{1/3}$ ($\phi = 1.0$) or

$$\frac{h_o D_e}{k}\left(\frac{c\mu}{k}\right)^{-1/3}\left(\frac{\mu}{\mu_w}\right)^{-0.14}\frac{k}{D_e}\left(\frac{c\mu}{k}\right)^{1/3} \times 1.0 = h_o \text{ Btu/(hr)(ft}^2\text{)(°F)} \qquad (6.15b)$$

Overall coefficients:

(11) Compute $U_C = h_{io}h_o/(h_{io} + h_o)$, Btu/(hr)(ft²)(°F). (6.7)

(12) Compute U_D from $1/U_D = 1/U_C + R_d$. (6.10)

(13) Compute A from $Q = U_D A\, \Delta t$ which may be translated into length. If the length should not correspond to an integral number of hairpins, a change in the dirt factor will result. The recalculated dirt factor should equal or exceed the required dirt factor by using the next larger integral number of hairpins.

Calculation of ΔP. This requires a knowledge of the total length of path satisfying the heat-transfer requirements.

Inner pipe:

(1) For Re_p in (6) above obtain f from Eq. (3.46) or (3.47b).

(2) $\Delta F_p = 4fG^2L/2g\rho^2D$, ft. (3.45)

$\Delta F_p \rho/144 = \Delta P_p$, psi.

Annulus:

(1′) Obtain $D'_e = \dfrac{4\pi(D_2^2 - D_1^2)}{4\pi(D_2 + D_1)} = (D_2 - D_1)$. (6.4)

Compute the frictional Reynolds number, $Re'_a = D'_e G_a/\mu$. For Re'_a obtain f from Eq. (3.46) or (3.47b).

(2′) $\Delta F_a = 4fG^2L/2g\rho^2D'_e$, ft. (6.14)

(3′) Entrance and exit losses, one velocity head per hairpin:

$$\Delta F_l = \frac{V^2}{2g'} \text{ ft/hairpin}$$

$$(\Delta F_a + \Delta F_l)\rho/144 = \Delta P_a, \text{ psi.}$$

There is an advantage if both fluids are computed side by side, and the use of the outline in this manner will be demonstrated in Example 6.1.

Example 6.1. Double Pipe Benzene-Toluene Exchanger. It is desired to heat 9820 lb/hr of cold benzene from 80 to 120°F using hot toluene which is cooled from 160 to 100°F. The specific gravities at 68°F are 0.88 and 0.87, respectively. The other fluid properties will be found in the Appendix. A fouling factor of 0.001 should be provided for each stream, and the allowable pressure drop on each stream is 10.0 psi.

A number of 20-ft hairpins of 2- by $1\frac{1}{4}$-in. IPS pipe are available. How many hairpins are required?

Solution:

(1) Heat balance:

$$\text{Benzene, } t_{av} = \frac{80 + 120}{2} = 100°\text{F} \qquad c = 0.425 \text{ Btu/(lb)(°F)} \qquad \text{(Fig. 2)}$$

$$Q = 9820 \times 0.425(120 - 80) = 167{,}000 \text{ Btu/hr}$$

$$\text{Toluene, } T_{av} = \frac{160 + 100}{2} = 130°\text{F} \qquad c = 0.44 \text{ Btu/(lb)(°F)} \qquad \text{(Fig. 2)}$$

$$W = \frac{167{,}000}{0.44(160 - 100)} = 6330 \text{ lb/hr}$$

(2) LMTD, (see the method of Chap. 3):

Hot fluid		Cold fluid	Diff.	
160	Higher temp	120	40	Δt_2
100	Lower temp	80	20	Δt_1
			20	$\Delta t_2 - \Delta t_1$

$$\text{LMTD} = \frac{\Delta t_2 - \Delta t_1}{2.3 \log \Delta t_2/\Delta t_1} = \frac{20}{2.3 \log 40/20} = 28.8°\text{F} \qquad (5.14)$$

(3) Caloric temperatures: A check of both streams will show that neither is viscous at the cold terminal (the viscosities less than 1 centipoise) and the temperature ranges and temperature difference are moderate. The coefficients may accordingly be evaluated from properties at the arithmetic mean, and the value of $(\mu/\mu_w)^{0.14}$ may be assumed equal to 1.0.

$$T_{av} = \tfrac{1}{2}(160 + 100) = 130°\text{F} \qquad t_{av} = \tfrac{1}{2}(120 + 80) = 100°\text{F}$$

Proceed now to the inner pipe. A check of Table 6.2 indicates that the flow area of the inner pipe is greater than that of the annulus. Place the larger stream, benzene in the inner pipe.

Hot fluid: annulus, toluene

(4′) Flow area,

$D_2 = 2.067/12 = 0.1725$ ft

$D_1 = 1.66/12 = 0.138$ ft

$a_a = \pi(D_2^2 - D_1^2)/4$

$= \pi(0.1725^2 - 0.138^2)/4 = 0.00826 \text{ ft}^2$

Equiv diam, $D_e = (D_2^2 - D_1^2)/D_1$ ft [Eq (6.3)]

$D_e = (0.1725^2 - 0.138^2)/0.138$

$= 0.0762$ ft

Cold fluid: inner pipe, benzene

(4) $D = 1.38/12 = 0.115$ ft

Flow area, $a_p = \pi D^2/4$

$= \pi \times 0.115^2/4 = 0.0104 \text{ ft}^2$

Hot fluid: annulus, toluene

(5′) Mass vel, $G_a = W/a_a$
$= 6330/0.00826 = 767{,}000$ lb/(hr)(ft²)

(6′) At 130°F, $\mu = 0.41$ cp [Fig. 14]
$= 0.41 \times 2.42 = 0.99$ lb/(ft)(hr)

Reynolds no., $Re_a = \dfrac{D_e G_a}{\mu}$
$= 0.0762 \times 767{,}000/0.99 = 59{,}000$

(7′) $j_H = 167$ [Fig. 24]

(8′) At 130°F, $c = 0.44$ Btu/(lb)(°F) [Fig. 2]
$k = 0.085$ Btu/(hr)(ft²)(°F/ft) [Table 4]

$$\left(\frac{c\mu}{k}\right)^{1/3} = \left(\frac{0.44 \times 0.99}{0.085}\right)^{1/3} = 1.725$$

(9′) $h_o = j_H \dfrac{k}{D_e}\left(\dfrac{c\mu}{k}\right)^{1/3}\left(\dfrac{\mu}{\mu_w}\right)^{0.14}$ [Eq. (6.15b)]

$$= 167 \times \frac{0.085}{0.0762} \times 1.725 \times 1.0$$
$$= 323 \text{ Btu/(hr)(ft}^2\text{)(°F)}$$

Cold fluid: inner pipe, benzene

(5) Mass vel, $G_p = w/a_p$
$= 9820/0.0104 = 943{,}000$ lb/(hr)(ft²)

(6) At 100°F, $\mu = 0.50$ cp [Fig. 14]
$= 0.50 \times 2.42 = 1.21$ lb/(ft)(hr)

Reynolds no., $Re_p = \dfrac{DG_p}{\mu}$
$= 0.115 \times 943{,}000/1.21 = 89{,}500$

(7) $j_H = 236$ [Fig. 24]

(8) At 100°F, $c = 0.425$ Btu/(lb)(°F) [Fig. 2]
$k = 0.091$ Btu/(hr)(ft²)(°F/ft) [Table 4]

$$\left(\frac{c\mu}{k}\right)^{1/3} = \left(\frac{0.425 \times 1.21}{0.091}\right)^{1/3} = 1.78$$

(9) $h_i = j_H \dfrac{k}{D}\left(\dfrac{c\mu}{k}\right)^{1/3}\left(\dfrac{\mu}{\mu_w}\right)^{0.14}$ [Eq. (6.15a)]

$$= 236 \times \frac{0.091}{0.115} \times 1.78 \times 1.0$$
$$= 333 \text{ Btu/(hr)(ft}^2\text{)(°F)}$$

(10) Correct h_i to the surface at the *OD*

$h_{io} = h_i \times \dfrac{\text{ID}}{\text{OD}}$ [Eq. (6.5)]

$$= 333 \times \frac{1.38}{1.66} = 276$$

Now proceed to the annulus.

(11) Clean overall coefficient, U_C:

$$U_C = \frac{h_{io}h_o}{h_{io} + h_o} = \frac{276 \times 323}{276 + 323} = 149 \text{ Btu/(hr)(ft}^2\text{)(°F)} \qquad (6.7)$$

(12) Design overall coefficient, U_D:

$$\frac{1}{U_D} = \frac{1}{U_C} + R_d \qquad (6.10)$$

$R_d = 0.002$ (required by problem)

$$\frac{1}{U_D} = \frac{1}{149} + 0.002$$

$U_D = 115$ Btu/(hr)(ft²)(°F)

Summary

323	h outside	276
U_C	149	
U_D	115	

(13) Required surface:

$$Q = U_D A\,\Delta t \qquad A = \frac{Q}{U_D\,\Delta t}$$

$$\text{Surface} = \frac{167{,}000}{115 \times 28.8} = 50.5 \text{ ft}^2$$

From Table 11 for $1\frac{1}{4}$-in. IPS standard pipe there are 0.435 ft² of external surface per foot length.

$$\text{Required length} = \frac{50.5}{0.435} = 116 \text{ lin ft}$$

This may be fulfilled by connecting three 20-ft hairpins in series.

(14) The surface supplied will actually be $120 \times 0.435 = 52.2$ ft². The dirt factor will accordingly be greater than required. The actual design coefficient is

$$U_D = \frac{167{,}000}{52.2 \times 28.8} = 111 \text{ Btu/(hr)(ft}^2\text{)(°F)}$$

$$R_d = \frac{U_C - U_D}{U_C U_D} = \frac{149 - 111}{149 \times 111} = 0.0023 \text{ (hr)(ft}^2\text{)(°F)/Btu} \qquad (6.13)$$

Pressure Drop

(1′) D'_e for pressure drop differs from D_e for heat transfer.

$$D'_e = (D_2 - D_1) \qquad \text{[Eq. (6.4)]}$$
$$= (0.1725 - 0.138) = 0.0345 \text{ ft}$$
$$Re'_a = \frac{D'_e G_a}{\mu}$$
$$= 0.0345 \times 767{,}000/0.99 = 26{,}800$$
$$f = 0.0035 + \frac{0.264}{26{,}800^{0.42}} = 0.0071 \qquad \text{[Eq. (3.47}b\text{)]}$$

$s = 0.87$, $\rho = 62.5 \times 0.87 = 54.3$ [Table 6]

(2′) $$\Delta F_a = \frac{4fG_a^2L}{2g\rho^2 D'_e}$$
$$= \frac{4 \times 0.0071 \times 767{,}000^2 \times 120}{2 \times 4.18 \times 10^8 \times 54.3^2 \times 0.0345}$$
$$= 23.5 \text{ ft}$$

(3′) $$V = \frac{G}{3600\rho} = \frac{767{,}000}{3600 \times 54.3} = 3.92 \text{ fps}$$
$$F_l = 3\left(\frac{V^2}{2g'}\right) = 3 \times \frac{3.92^2}{2 \times 32.2} = 0.7 \text{ ft}$$
$$\Delta P_a = \frac{(23.5 + 0.7)54.3}{144} = 9.2 \text{ psi}$$

Allowable $\Delta P_a = 10.0$ psi

(1) For $Re_p = 89{,}500$ in (6) above

$$f = 0.0035 + \frac{0.264}{(DG/\mu)^{0.42}} \qquad \text{[Eq. (3.47}b\text{)]}$$
$$= 0.0035 + \frac{0.264}{89{,}500^{0.42}} = 0.0057$$

$s = 0.88$, $\rho = 62.5 \times 0.88 = 55.0$ [Table 6]

(2) $$\Delta F_p = \frac{4fG_p^2L}{2g\rho^2 D}$$
$$= \frac{4 \times 0.0057 \times 943{,}000^2 \times 120}{2 \times 4.18 \times 10^8 \times 55.0^2 \times 0.115}$$
$$= 8.3 \text{ ft}$$
$$\Delta P_p = \frac{8.3 \times 55.0}{144} = 3.2 \text{ psi}$$

Allowable $\Delta P_p = 10.0$ psi

A check of U_h and U_c gives 161 and 138, respectively, and $K_c = 0.17$. From Fig. 17 for $\Delta t_c/\Delta t_h = {}^{20}\!/_{40} = 0.5$, $F_c = 0.43$, whereas in the solution above the arithmetic mean temperatures were used. The arithmetic mean assumes $F_c = 0.50$. However, since the ranges are small for both fluids, the error is too small to be significant. If the ranges of the fluids or their viscosities were large, the error might be considerable for $F_c = 0.43$.

Double Pipe Exchangers in Series-parallel Arrangements. Referring to Example 6.1, it is seen that a calculated pressure drop of 9.2 psi is obtained against an allowable pressure drop of 10.0 psi. Suppose, however, that the calculated pressure drop were 15 or 20 psi and exceeded the available head. How then might the heat load be transferred with the available pressure head? One possibility is the use of a by-pass so that only three-quarters or two-thirds of the fluid flows through the exchanger and the remainder through the by-pass. This does not provide an ideal solution, since the reduced flow causes several unfavorable changes in the design. (1) The reduced flow through the exchanger reduces the mass velocity G_a and the film coefficient h_o. Since both of the coefficients are nearly alike, 323 vs. 276, any sizable reduction in G_a alone decreases U_C by nearly $G_a^{0.8}$. (2) If less liquid circulates through the annulus, it has to be cooled over a longer range than from 160 to 100°F so that, upon

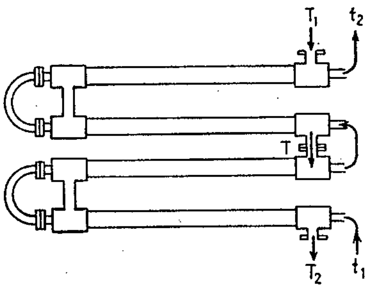

FIG. 6.5. Double pipe exchangers in series.

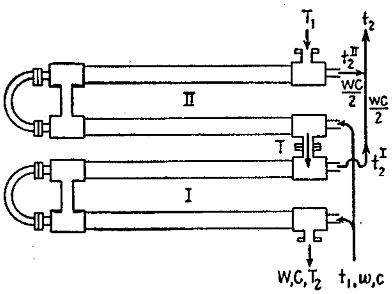

FIG. 6.6. Series—parallel arrangement.

mixing with the by-pass fluid, the process outlet temperature of 100°F results. As an example, the portion circulating through the annulus might have to be cooled over the range from 160 to 85°F depending upon the percentage by-passed. The outlet temperature of 85°F is closer to the inner pipe inlet of 80°F than originally, and the new cold-terminal difference Δt_1 of only 5°F greatly decreases the LMTD. The two effects, decreased U_C and LMTD, increase the required number of hairpins greatly even though the heat load is constant. Reversing the location of the streams by placing the benzene in the annulus does not provide a solution in this case, since the benzene stream is larger than the toluene stream. The possibility of reversing the location of the streams should always be examined first whenever the allowable pressure drop cannot be met.

A solution is still possible, however, even when all the above have failed. When two double pipe exchangers are connected in series, the arrangement is shown in Fig. 6.5. Suppose that the stream which is too large to be accommodated in several exchangers in series is divided in half and each half traverses but one exchanger through the inner pipes in

Fig. 6.6. Dividing a stream in half while keeping the flow area constant produces about *one-eighth* of the series pressure drop, since G and L will be half and the product of G^2L in Eq. (6.14) will be one-eighth. While the film coefficient will also be reduced, the unfavorable temperature difference of by-passing can be circumvented. Where there is a substantial unbalance between the weight flow of the two streams because one operates over a long range and the other over a very short range, the large stream may be divided in three, four, or more parallel streams. In larger services each parallel stream may also flow through several exchangers in series in each parallel bank. The term "parallel streams" should not be confused with "parallel flow." The former refers to the division of the flow of one fluid, while the latter refers to the direction of flow between two fluids.

The True Temperature Difference for Series-parallel Arrangements. The LMTD calculated from T_1, T_2, t_1, and t_2 for the series arrangement will not be the same for a series-parallel arrangement. Half of the pipe fluid enters the upper exchanger II in Fig. 6.6 where the annulus fluid is hot, and half enters the lower exchanger I in which the annulus fluid has already been partially cooled. While exchangers in series do not transfer equal quantities of heat, the series-parallel relationship is even more adverse, the lower exchanger accounting for relatively less of the total heat transfer. If the true temperature difference is called Δt, it will not be identical with the LMTD for the process conditions although both of the exchangers operate in counterflow.

Consider the two exchangers in Fig. 6.6 designated by I and II. The intermediate temperature is T, and the outlets of the parallel streams are designated by t_2^{II} and t_2^{I}. Their mixed temperature is t_2.

For exchanger I, containing half the surface,

$$Q_{\mathrm{I}} = WC(T - T_2) = \frac{UA}{2} \times \mathrm{LMTD_I} \tag{6.16}$$

and

$$\mathrm{LMTD_I} = \frac{(T - t_2^{\mathrm{I}}) - (T_2 - t_1)}{\ln (T - t_2^{\mathrm{I}})/(T_2 - t_1)} \tag{6.17}$$

Substituting in Eq. (6.16),

$$\frac{UA}{2WC} = \frac{(T - T_2)}{(T - t_2^{\mathrm{I}}) - (T_2 - t_1)} \ln \frac{T - t_2^{\mathrm{I}}}{T_2 - t_1}$$

Rearranging,

$$\begin{aligned}\frac{UA}{2WC} &= \frac{(T - T_2)}{(T - T_2) - (t_2^{\mathrm{I}} - t_1)} \ln \frac{(T - t_2^{\mathrm{I}})}{(T_2 - t_1)} \\ &= \frac{1}{1 - (t_2^{\mathrm{I}} - t_1)/(T - T_2)} \ln \frac{T - t_2^{\mathrm{I}}}{T_2 - t_1}\end{aligned} \tag{6.18}$$

Let

$$R^{\mathrm{I}} = \frac{(T - T_2)}{(t_2^{\mathrm{I}} - t_1)} = \frac{wc}{2WC}$$

$$\frac{UA}{2WC} = \frac{R^{\mathrm{I}}}{R^{\mathrm{I}} - 1} \ln \frac{T - t_2^{\mathrm{I}}}{T_2 - t_1} \tag{6.19}$$

Similarly for exchanger II

$$Q_{\mathrm{II}} = WC(T_1 - T) = \frac{UA}{2} \times \mathrm{LMTD}_{\mathrm{II}} \tag{6.20}$$

$$\mathrm{LMTD}_{\mathrm{II}} = \frac{(T_1 - t_2^{\mathrm{II}}) - (T - t_1)}{\ln (T_1 - t_2^{\mathrm{II}})/(T - t_1)} \tag{6.21}$$

Let

$$R^{\mathrm{II}} = \frac{T_1 - T}{t_2^{\mathrm{II}} - t_1} = \frac{wc}{2WC}$$

$$\frac{UA}{2WC} = \frac{R^{\mathrm{II}}}{R^{\mathrm{II}} - 1} \ln \frac{T_1 - t_2^{\mathrm{II}}}{T - t_1} \tag{6.22}$$

Since c and C were assumed constant,

$$R^{\mathrm{I}} = R^{\mathrm{II}} = R' = \frac{wc}{2WC} \tag{6.23}$$

Let

$$S^{\mathrm{I}} = \frac{t_2^{\mathrm{I}} - t_1}{T - t_1} \qquad M^{\mathrm{I}} = \frac{T - T_2}{T - t_1}$$

$$M^{\mathrm{I}} = R'S^{\mathrm{I}}$$

Similarly let

$$S^{\mathrm{II}} = \frac{t_2^{\mathrm{II}} - t_1}{T_1 - t_1} \qquad M^{\mathrm{II}} = \frac{T_1 - T}{T_1 - t_1}$$

R' and S are ratios which recur frequently in obtaining the true temperature difference Δt from the LMTD. S is the ratio of the cold fluid range to the maximum temperature span, the latter being the difference between both inlet temperatures, T_1 and t_1.

But

$$M^{\mathrm{II}} = R'S^{\mathrm{II}}$$

$$1 - S^{\mathrm{I}} = \frac{T - t_2^{\mathrm{I}}}{T - t_1} = \frac{T - t_1}{T - t_1} - \frac{t_2^{\mathrm{I}} - t_1}{T - t_1}$$

$$\frac{T - t_2^{\mathrm{I}}}{T_2 - t_1} = \frac{1 - S^{\mathrm{I}}}{1 - R'S^{\mathrm{I}}}$$

and from Eq. (6.19)

$$\frac{UA}{2WC} = \frac{R'}{R' - 1} \ln \frac{1 - S^{\mathrm{I}}}{1 - R'S^{\mathrm{I}}} \tag{6.24}$$

and from Eq. (6.22)

$$\frac{UA}{2WC} = \frac{R'}{R' - 1} \ln \frac{1 - S^{\mathrm{II}}}{1 - R'S^{\mathrm{II}}} \tag{6.25}$$

and equating Eqs. (6.24) and (6.25),

$$\frac{1 - S^{\mathrm{I}}}{1 - R'S^{\mathrm{I}}} = \frac{1 - S^{\mathrm{II}}}{1 - R'S^{\mathrm{II}}}$$

Therefore

$$S^{\mathrm{I}} = S^{\mathrm{II}}$$
$$M^{\mathrm{I}} = M^{\mathrm{II}}$$

Adding Eqs. (6.24) and (6.25),

$$\frac{UA}{WC} = \frac{2R'}{R' - 1} \ln \frac{1 - S^{\mathrm{I}}}{1 - R'S^{\mathrm{I}}} = \frac{2R'}{R' - 1} \ln \frac{T - t_2^{\mathrm{I}}}{T_2 - t_1} \tag{6.26}$$

in which T is the only unknown, and since $M^{\mathrm{I}} = M^{\mathrm{II}}$,

$$\frac{T_1 - T}{T_1 - t_1} = \frac{T - T_2}{T - t_1}$$
$$T^2 - 2t_1T + t_1(T_1 + T_2) - T_1T_2 = 0 \tag{6.27}$$

Equation (6.27) is a quadratic whose solution is

$$T = \frac{2t_1 \pm \sqrt{4t_1^2 - 4t_1[(T_1 + T_2) - 4T_1T_2]}}{2}$$
$$= t_1 \pm \sqrt{(T_1 - t_1)(T_2 - t_1)} \tag{6.28}$$

The minus sign applies when the heating medium is in the pipes. The plus sign applies when the cooling medium is in the pipes.

Substituting for T in Eq. (6.26),

$$\frac{UA}{WC} = \frac{2R'}{R' - 1} \ln \left[\frac{(R' - 1)(T_1 - t_1) + \sqrt{(T_1 - t_1)(T_2 - t_1)}}{R' \sqrt{(T_1 - t_1)(T_2 - t_1)}} \right]$$
$$= \frac{2R'}{R' - 1} \ln \left[\left(\frac{R' - 1}{R'} \right) \left(\frac{T_1 - t_1}{T_2 - t_1} \right)^{1/2} + \frac{1}{R'} \right] \tag{6.29}$$

Δt is the single value for the entire series-parallel arrangement; thus

$$Q = UA\,\Delta t = WC(T_1 - T_2) \tag{6.30}$$
$$\Delta t = \frac{Q}{UA} = \frac{WC}{UA}(T_1 - T_2) \tag{6.31}$$

It is convenient in this derivation to employ a definition for the true temperature difference in terms of the maximum temperature span $T_1 - t_1$:

$$\Delta t = \gamma(T_1 - t_1) \tag{6.32}$$

Equating (6.31) and (6.32),

$$\frac{WC}{UA}(T_1 - T_2) = \gamma(T_1 - t_1)$$
$$\gamma = \frac{WC(T_1 - T_2)}{UA(T_1 - t_1)}$$

Since $M = (T_1 - T_2)/(T_1 - t_1)$, define $P' = (T_2 - t_1)/(T_1 - t_1)$ and $UA/WC = M/\gamma$; then

$$P' + M = 1 \qquad \text{or} \qquad M = 1 - P'$$

Substituting in Eq. (6.29),

$$\frac{UA}{WC} = \frac{2R'}{R' - 1} \ln\left[\left(\frac{R' - 1}{R'}\right)\left(\frac{1}{P'}\right)^{1/2} + \frac{1}{R'}\right] \tag{6.33}$$

or

$$\frac{1 - P'}{\gamma} = 2\left(\frac{R'}{R' - 1}\right) \ln\left[\left(\frac{R' - 1}{R'}\right)\left(\frac{1}{P'}\right)^{1/2} + \frac{1}{R'}\right] \tag{6.34}$$

If developed in a generalized manner it can be shown that, for *one series hot stream* and *n parallel cold streams*, Eq. (6.34) becomes

$$\frac{1 - P'}{\gamma} = 2.3\frac{nR'}{R' - 1} \log\left[\left(\frac{R' - 1}{R'}\right)\left(\frac{1}{P'}\right)^{1/n} + \frac{1}{R'}\right] \tag{6.35a}$$

where

$$R' = \frac{T_1 - T_2}{n(t_2 - t_1)}$$

For *one series cold stream* and *n parallel hot streams*,

$$\frac{1 - P''}{\gamma} = 2.3\frac{n}{1 - R''} \log\left[(1 - R'')\left(\frac{1}{P''}\right)^{1/n} + R''\right] \tag{6.35b}$$

where

$$P'' = \frac{T_1 - t_2}{T_1 - t_1} \qquad \text{and} \qquad R'' = \frac{n(T_1 - T_2)}{t_2 - t_1}$$

Example 6.2. Calculation of the True Temperature Difference. A bank of double pipe exchangers operates with the hot fluid in series from 300 to 200°F and the cold fluid in six parallel streams from 190 to 220°F. What is the true temperature difference Δt?

$$P' = \frac{T_2 - t_1}{T_1 - t_1} = \frac{200 - 190}{300 - 190} = 0.091 \qquad R' = \frac{T_1 - T_2}{n(t_2 - t_1)} = \frac{300 - 200}{6(220 - 190)} = 0.558$$

Substituting in Eq. (6.35*a*) and solving, $\gamma = 0.242$.

$$\Delta t = 0.242(300 - 190) = 26.6°\text{F} \tag{6.32}$$

The LMTD would be 33.7°F, and an error of 27 per cent would be introduced by its use.

Exchangers with a Viscosity Correction, ϕ. For heating or cooling fluids, the use of Fig. 24 with an assumed value of $(\mu/\mu_w)^{0.14} = 1.0$ also assumes a negligible deviation of fluid properties from isothermal flow. For nonviscous fluids the deviation from isothermal flow during heating or cooling does not introduce an appreciable error in the calculation of the heat-transfer coefficient. When the pipe-wall temperature differs appreciably from the caloric temperature of the controlling fluid and the controlling fluid is viscous, the actual value of $\phi = (\mu/\mu_w)^{0.14}$ must be taken into account. To include the correction, t_w may be determined by Eq. (5.31) or by (5.32) from uncorrected values of h_o/ϕ_a and h_{io}/ϕ_p, which are then corrected accordingly by multiplication by ϕ_a and ϕ_p respectively. The corrected coefficients where $\phi \neq 1.0$ are

$$h_o = \left(\frac{h_o}{\phi_a}\right)\phi_a \tag{6.36}$$

$$h_{io} = \left(\frac{h_{io}}{\phi_p}\right)\phi_p \tag{6.37}$$

Similarly for two resistances in series employing the viscosity corrections for deviation from the isothermal the clean overall coefficient is again

$$U_C = \frac{h_{io}h_o}{h_{io} + h_o} \tag{6.38}$$

Example 6.3. Double Pipe Lube Oil–Crude Oil Exchanger. 6,900 lb/hr of a 26°API lube oil must be cooled from 450 to 350°F by 72,500 lb/hr of 34°API mid-continent crude oil. The crude oil will be heated from 300 to 310°F.

A fouling factor of 0.003 should be provided for each stream, and the allowable pressure drop on each stream will be 10 psi.

A number of 20-ft hairpins of 3- by 2-in. IPS pipe are available. How many must be used, and how shall they be arranged? The viscosity of the crude oil may be obtained from Fig. 14. For the lube oil, viscosities are 1.4 centipoises at 500°F, 3.0 at 400°F, and 7.7 at 300°F. These viscosities are great enough to introduce an error if $(\mu/\mu_w)^{0.14} = 1$ is assumed.

Solution:

(1) Heat Balance:
Lube oil, $Q = 6900 \times 0.62(450 - 350) = 427{,}000$ Btu/hr
Crude oil, $Q = 72{,}500 \times 0.585(310 - 300) = 427{,}000$ Btu/hr

(2) Δt:

Hot Fluid		Cold Fluid	Diff.	
450	Higher temp	310	140	Δt_2
350	Lower temp	300	50	Δt_1
			90	$\Delta t_2 - \Delta t_1$

It will be impossible to put the 72,500 lb/hr of crude into a single pipe or annulus, since the flow area of each is too small. Assume, as a trial, that it will be employed in two parallel streams.

$$\Delta t = 87.5°\text{F} \tag{6.35a}$$

(3) Caloric temperatures:

$$\frac{\Delta t_c}{\Delta t_h} = \frac{50}{140} = 0.357 \qquad K_c = 0.43 \qquad F_c = 0.395 \qquad \text{(Fig. 17)}$$

$$T_c = 350 \times 0.395(450 - 350) = 389.5°\text{F} \qquad (5.28)$$

$$t_c = 300 \times 0.395(310 - 300) = 304°\text{F} \qquad (5.29)$$

Proceed now to the inner pipe.

Hot fluid: annulus, lube oil

(4′) Flow area, $D_2 = 3.068/12 = 0.256$ ft

$D_1 = 2.38/12 = 0.199$ ft

$a_a = \pi(D_2^2 - D_1^2)/4$

$= \pi(0.256^2 - 0.199^2)/4 = 0.0206$ ft²

Equiv diam, $D_e = (D_2^2 - D_1^2)/D_1$ [Eq. (6.3)]

$= (0.256^2 - 0.199^2)/0.199 = 0.13$ ft

(5′) Mass vel, $G_a = W/a_a$

$= 6900/0.0206 = 335{,}000$ lb/(hr)(ft²)

(6′) At 389.5°F, $\mu = 3.0$ cp

$= 3.0 \times 2.42 = 7.25$ lb/(ft)(hr) [Fig. 14]

$Re_a = D_e G_a/\mu$ [Eq. (3.6)]

$= 0.13 \times 335{,}000/7.25 = 6{,}000$

If only 2 hairpins in series are required, L/D will be $2 \times 40/0.13 = 614$. Use $L/D = 600$.

(7′) $j_H = 20.5$ [Fig. 24]

(8′) At $T_c = 389.5°$F, $c = 0.615$ Btu/(lb)(°F) [Fig. 4]

$k = 0.067$ Btu/(hr)(ft²)(°F/ft) [Fig. 1]

$$\left(\frac{c\mu}{k}\right)^{1/3} = \left(\frac{0.615 \times 7.25}{0.067}\right)^{1/3} = 4.05$$

(9′) $h_o = j_H \dfrac{k}{D_e}\left(\dfrac{c\mu}{k}\right)^{1/3}\phi_a$ [Eq. (6.15)]

$$\frac{h_o}{\phi_a} = \frac{20.5 \times 0.067 \times 4.05}{0.13} = 42.7 \text{ Btu/(hr)(ft}^2\text{)(°F)}$$

$$t_w = t_c + \frac{h_o/\phi_a}{h_{io}/\phi_p + h_o/\phi_a}(T_c - t_c) \qquad \text{[Eq. (5.31)]}$$

$$= 304 + \frac{42.7}{297 + 42.7}(389.5 - 304) = 314°\text{F}$$

$\mu_w = 6.6 \times 2.42 = 16.0$ lb/(ft)(hr) [Fig. 14]

$\phi_a = (\mu/\mu_w)^{0.14}$

$= (7.25/16.0)^{0.14} = 0.90$ [Fig. 24]

$h_o = \dfrac{h_o}{\phi_a}\phi_a$ [Eq. (6.36)]

$= 42.7 \times 0.90 = 38.4$

Cold fluid: inner pipe, crude oil

(4) Flow area, $D = 2.067/12 = 0.172$ ft

$a_p = \pi D^2/4$

$= \pi \times 0.172^2/4 = 0.0233$ ft²

Since two parallel streams have been assumed, $w/2$ lb/hr will flow in each pipe.

(5) Mass vel, $G_p = w/a_p$

$$= \frac{72{,}500}{2 \times 0.0233}$$

$= 1{,}560{,}000$ lb/(hr)(ft²)

(6) At 304°F, $\mu = 0.83$ cp

$= 0.83 \times 2.42 = 2.01$ lb/(ft)(hr) [Fig. 14]

$Re_p = DG_p/\mu$

$= 0.172 \times 1{,}560{,}000/2.01 = 133{,}500$

(7) $j_H = 320$ [Fig. 24]

(8) At $t_c = 304°$F, $c = 0.585$ Btu/(lb)(°F) [Fig. 4]

$k = 0.073$ Btu/(hr)(ft²)(°F/ft) [Fig. 1]

$$\left(\frac{c\mu}{k}\right)^{1/3} = \left(\frac{0.585 \times 2.01}{0.073}\right)^{1/3} = 2.52$$

(9) $h_i = j_H \dfrac{k}{D}\left(\dfrac{c\mu}{k}\right)^{1/3}\phi_p$ [Eq. (6.15a)]

$$\frac{h_i}{\phi_p} = \frac{320 \times 0.073 \times 2.52}{0.172} = 342 \text{ Btu/(hr)(ft}^2\text{)(°F)}$$

(10) $\dfrac{h_{io}}{\phi_p} = \dfrac{h_i}{\phi_p} \times \dfrac{\text{ID}}{\text{OD}}$

$= 342 \times 2.067/2.38 = 297$

Now proceed from (4′) to (9′) to obtain t_w.

$\mu_w = 0.77 \times 2.42 = 1.86$ [Fig. 14]

$\phi_p = (\mu/\mu_w)^{0.14}$

$= (2.01/1.86)^{0.14} = 1.0$ nearly [Fig. 24]

$h_{io} = \dfrac{h_{io}}{\phi_p}\phi_p$ [Eq. (6.37)]

$= 297 \times 1.0 = 297$

(11) Clean overall coefficient, U_C:

$$U_C = \frac{h_{io}h_o}{h_{io} + h_o} = \frac{297 \times 38.4}{297 + 38.4} = 34.0 \text{ Btu/(hr)(ft}^2\text{)(°F)} \qquad (6.38)$$

(12) Design overall coefficient, U_D:

$$\frac{1}{U_D} = \frac{1}{U_C} + R_d \qquad (6.10)$$

$R_d = 0.003 + 0.003 = 0.006$ (hr)(ft²)(°F)/Btu
$U_D = 28.2$

Summary

38.4	h outside	297
U_C	34.0	
U_D	28.2	

(13) Surface:

$$A = \frac{Q}{U_D\,\Delta t} = \frac{427{,}000}{28.2 \times 87.5} = 173 \text{ ft}^2$$

External surface/lin ft, $a'' = 0.622$ ft (Table 11)

$$\text{Required length} = \frac{173}{0.622} = 278 \text{ lin ft}$$

This is equivalent to more than six 20-ft hairpins or 240 lin feet. Since two parallel streams are employed, use eight hairpins or 320 lin. feet. The hairpins should have the annuli connected in series and the tubes in two parallel banks of four exchangers.

The corrected U_D will be $U_D = Q/A\,\Delta t = 427{,}000/320 \times 0.622 \times 87.5 = 24.5$.

The corrected dirt factor will be $R_d = 1/U_D - 1/U_C = 1/24.5 - 1/34.0 = 0.0114$.

Pressure Drop

(1′) $D'_e = (D_2 - D_1)$ [Eq. (6.4)]
$= (0.256 - 0.198) = 0.058$ ft

$Re'_a = D'_e G_a/\mu$
$= 0.058 \times 335{,}000/7.25 = 2680$

$$f = 0.0035 + \frac{0.264}{2680^{0.42}} = 0.0132 \qquad \text{[Eq. (3.47b)]}$$

$s = 0.775$, $\rho = 62.5 \times 0.775 = 48.4$ [Fig. 6]

(2′) $$\Delta F_a = \frac{4fG_a^2L_a}{2g\rho^2D'_e}$$

$$= \frac{4 \times 0.0132 \times 335{,}000^2 \times 320}{2 \times 4.18 \times 10^8 \times 48.4^2 \times 0.058} = 16.7 \text{ ft}$$

(3′) $$V = \frac{G_a}{3600\rho} = \frac{335{,}000}{3600 \times 48.4} = 1.9 \text{ fps}$$

$$\Delta F_l = 8\left(\frac{V^2}{2g'}\right) = 8\left(\frac{1.9^2}{2 \times 32.2}\right) = 0.45 \text{ ft}$$

$$\Delta P_a = \frac{(16.7 + 0.45) \times 48.4}{144} = 5.8 \text{ psi}$$

Allowable $\Delta P_a = 10.0$ psi

(1) For $Re_p = 133{,}500$ in (6) above

$$f = 0.0035 + \frac{0.264}{133{,}500^{0.42}} = 0.005375 \qquad \text{[Eq. (3.47b)]}$$

$s = 0.76$, $\rho = 62.5 \times 0.76 = 47.5$ [Fig. 6]

Halves of the tube fluid will flow through only four exchangers.

(2) $$\Delta F_p = \frac{4fG_p^2L_p}{2g\rho^2D}$$

$$= \frac{4 \times 0.005375 \times 1{,}560{,}000^2 \times 160}{2 \times 4.18 \times 10^8 \times 47.5^2 \times 0.172} = 25.7 \text{ ft}$$

$$\Delta P_p = \frac{25.7 \times 47.5}{144} = 8.5 \text{ psi}$$

Allowable $\Delta P_p = 10.0$ psi

If the flow had not been divided, the pressure drop would be nearly eight times as great, or about 60 psi.

PROBLEMS

6.1. What is the fouling factor when (*a*) $U_C = 30$ and $U_D = 20$, (*b*) $U_C = 60$ and $U_D = 50$, and (*c*) $U_C = 110$ and $U_D = 100$? Which do you consider reasonable to specify between two moderately clean streams?

6.2. A double pipe exchanger was oversized because no data were available on the rate at which dirt accumulated. The exchanger was originally designed to cool 13,000 lb/hr of 100 per cent acetic acid from 250 to 150°F by heating 19,000 lb/hr of butyl alcohol from 100 to 157°F. A design coefficient $U_D = 85$ was employed, but during initial operation a hot-liquid outlet temperature of 117°F was obtained. It rose during operation at the average rate of 3°F per month. What dirt factor should have been specified for a 6-month cleaning cycle?

6.3. O-xylene coming from storage at 100°F is to be heated to 150°F by cooling 18,000 lb/hr of butyl alcohol from 170 to 140°F. Available for the purpose are five 20-ft hairpin double pipe exchangers with annuli and pipes each connected in series. The exchangers are 3- by 2-in. IPS. What is (*a*) the dirt factor, (*b*) the pressure drops? (*c*) If the hot and cold streams in (*a*) are reversed with respect to the annulus and inner pipe, how does this justify or refute your initial decision where to place the hot stream?

6.4. 10,000 lb/hr of 57°API gasoline is cooled from 150 to 130°F by heating 42°API kerosene from 70 to 100°F. Pressure drops of 10 psi are allowable with a minimum dirt factor of 0.004. (*a*) How many 2½- by 1¼-in. IPS hairpins 20 ft long are required? (*b*) How shall they be arranged? (*c*) What is the final fouling factor?

6.5. 12,000 lb/hr 26°API lube oil (see Example 6.3 in text for viscosities) is to be cooled from 450 to 350°F by heating 42°API kerosene from 325 to 375°F. A pressure drop of 10 psi is permissible on both streams, and a minimum dirt factor of 0.004 should be provided. (*a*) How many 20-ft hairpins of 2½- by 1¼-in. IPS double pipe are required? (*b*) How shall they be arranged, and (*c*) what is the final dirt factor?

6.6. 7,000 lb/hr of aniline is to be heated from 100 to 150°F by cooling 10,000 lb/hr of toluene with an initial temperature of 185°F in 2- by 1-in. IPS double pipe hairpin exchangers 15 ft long. Pressure drops of 10 psi are allowable, and a dirt factor of 0.005 is required. (*a*) How many hairpin sections are required? (*b*) How shall they be arranged? (*c*) What is the final dirt factor?

6.7. 24,000 lb/hr of 35°API distillate is cooled from 400 to 300°F by 50,000 lb/hr of 34°API crude oil heated from an inlet temperature of 250°F. Pressure drops of 10 psi are allowable, and a dirt factor of 0.006 is required. Using 20-ft hairpins of 4- by 3-in. IPS (*a*) how many are required, (*b*) how shall they be arranged, and (*c*) what is the final fouling factor?

6.8. A liquid is cooled from 350 to 300°F by another which is heated from 290 to 315°F. How does the true temperature difference deviate from the LMTD if (*a*) the hot fluid is in series and the cold fluid flows in two parallel counterflow paths, (*b*) the hot fluid is in series and the cold fluid flows in three parallel-flow–counterflow paths, (*c*) The cold-fluid range in (*a*) and (*b*) is changed to 275 to 300°F.

6.9. A fluid is cooled from 300 to 275°F by heating a cold fluid from 100 to 290°F. If the hot fluid is in series, how is the true temperature difference affected by dividing the hot stream into (*a*) two parallel streams and (*b*) into three parallel streams?

6.10. 6330 lb/hr of toluene is cooled from 160 to 100°F by heating amyl acetate from 90 to 100°F using 15-ft hairpins. The exchangers are 2- by 1¼-in. IPS. Allowing 10 psi pressure drops and providing a minimum dirt factor of 0.004 (*a*) how many hairpins are required, (*b*) how shall they be arranged, and (*c*) what is the final dirt factor?