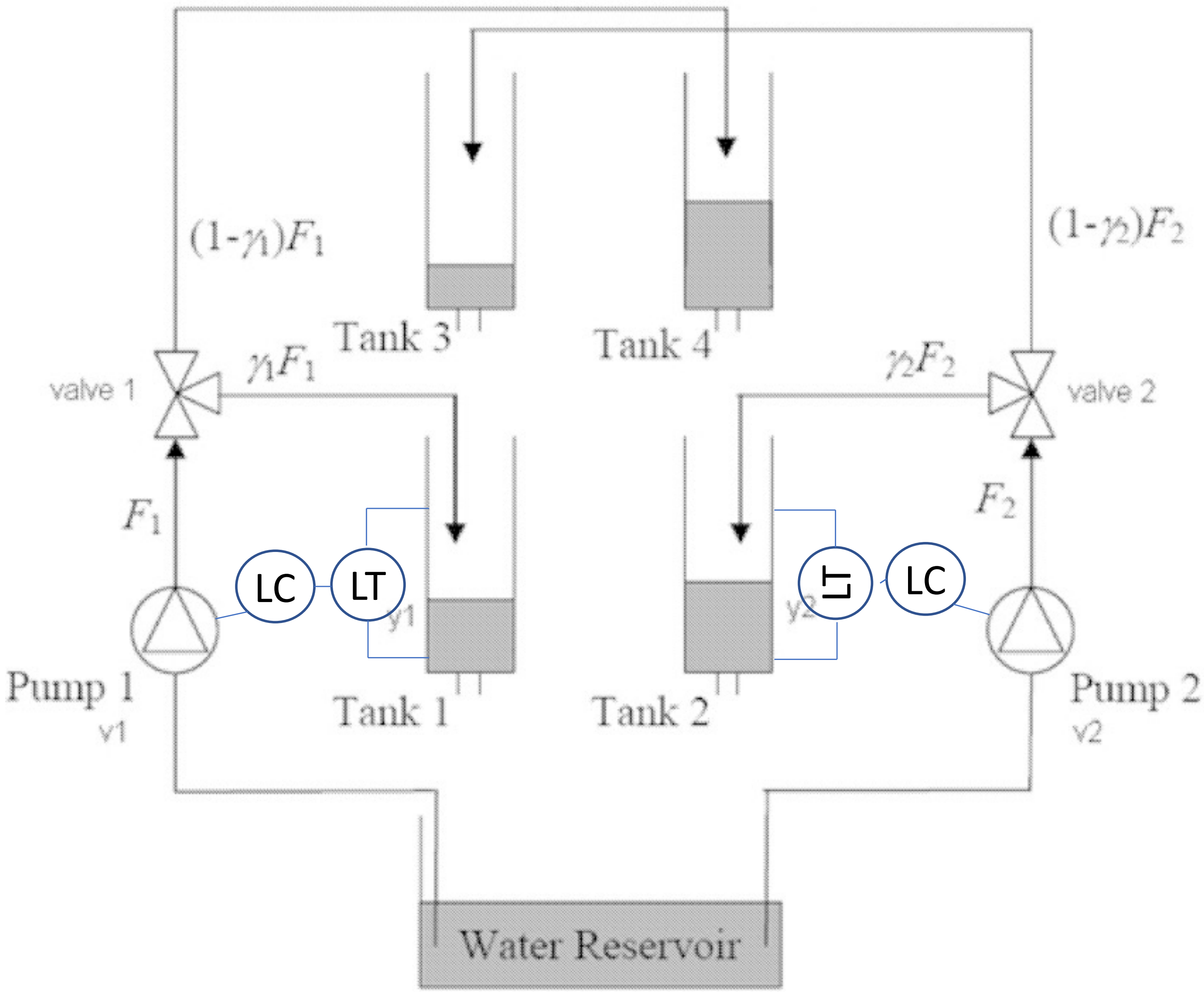
Multivariable Control

Quadruple Tank

MATLAB & SIMULINK

Quadruple Tank problem



Quadruple Tank problem

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3} v_2$$

$$\frac{dh_2}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4} v_1$$

Control Objective

Both the levels of tank1 and tank2 should be controlled by manipulating voltages to the pumps.

Data for simulation:

A_1, A_3 = 28 cm^2

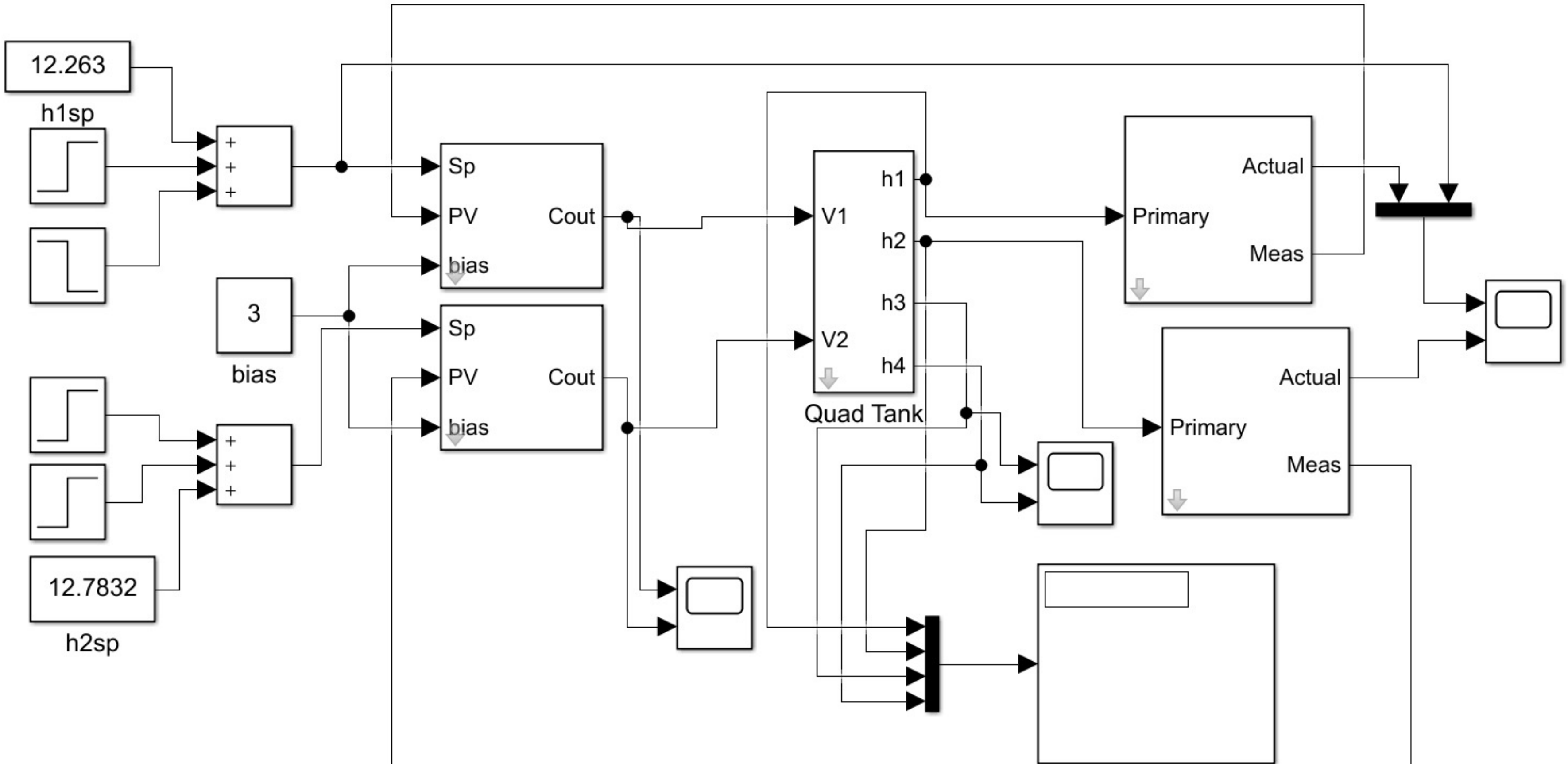
A_2, A_4 = 32 cm^2

a_1, a_3 = 0.071 cm^2

a_2, a_4 = 0.057 cm^2

Case	k1	k2	v1	v2	γ_1	γ_2
1	3.33	3.35	3.0	3.0	0.7	0.6
2	3.14	3.29	3.15	3.15	0.43	0.34
3	3.2	3.32	3.1	3.1	0.5	0.5

Simulink Model of Quad Tank



Let simulate the Model in Simulink

Linearized Equations for Quad Tank

- Introducing the deviation variables $x_i = h_i - h_{is}$ (i=1..4) and $u_j = v_j - v_{js}$ (j=1..2) the linearized equations are
- $\frac{dx_1}{dt} = -\frac{1}{T_1}x_1 + \frac{A_3}{A_1 T_3}x_3 + \frac{\gamma_1 k_1}{A_1}u_1$
- $\frac{dx_2}{dt} = -\frac{1}{T_2}x_2 + \frac{A_4}{A_2 T_4}x_4 + \frac{\gamma_2 k_2}{A_2}u_2$
- $\frac{dx_3}{dt} = -\frac{1}{T_3}x_3 + \frac{(1-\gamma_2)k_2}{A_3}u_2$
- $\frac{dx_4}{dt} = -\frac{1}{T_4}x_4 + \frac{(1-\gamma_1)k_1}{A4}u_1$
- $y_1 = k_m x_1 \qquad y_2 = k_m x_2$
- $T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_{is}}{g}} \qquad i = 1,..,4$

Transfer function matrix of Quad Tank

- The corresponding transfer function matrix is

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{\gamma_1 C_1}{T_1 s + 1} & \dfrac{(1-\gamma_2) C_1}{(T_1 s + 1)(T_3 s + 1)} \\ \dfrac{(1-\gamma_1) C_2}{(T_2 s + 1)(T_4 s + 1)} & \dfrac{\gamma_2 C_2}{T_2 s + 1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}$$

Where, $C_1 = \frac{T_1 k_1 k_m}{A_1}$ and $C_2 = \frac{T_2 k_2 k_m}{A_2}$

Poles and Zeros of Quad Tank

- **Case 1**

$$G(s) = \begin{bmatrix} \dfrac{4.153}{62.36s+1} & \dfrac{3.036}{(62.36s+1)(22.76s+1)} \\ \dfrac{1.78}{(90.63s+1)(30.09s+1)} & \dfrac{4.554}{90.63s+1} \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 4.153(90.63s+1)(30.09s+1)(22.76s+1) & 3.036(90.63s+1)(30.09s+1) \\ 1.78(62.36s+1)(22.76s+1) & 4.554(30.09s+1)(62.36s+1)(22.76s+1) \end{bmatrix}}{(90.63s+1)(30.09s+1)(62.36s+1)(22.76s+1)}$$

So, poles are

$$p_1 = -\frac{1}{90.63}; \; p_2 = -\frac{1}{30.09}; \; p_3 = -\frac{1}{62.36}; \; p_4 = -\frac{1}{22.76}$$

Zeros of the system

- Since poles of $G^{-1}(s)$ is zeros of $G(s)$,
- $G^{-1}(s) = \frac{Adj\left(G(s)\right)}{Det|G(s)|}$
- So, $Det|G(s)| = 0$ will give poles of $G^{-1}(s)$ and Zeros of G(s).
- For Case 1:
- Zeros are at $\mathrm{z}_1 = -\frac{1}{16.751};\ \ and\ z_2 = -\frac{1}{57.24}$

Poles and Zeros of Quad Tank

	Case 1	Case 2	Case 3
p1	$-\frac{1}{90.63}$	$-\frac{1}{59.82}$	$-\frac{1}{55.37}$
p2	$-\frac{1}{30.09}$	$-\frac{1}{87.6}$	$-\frac{1}{98.19}$
p3	$-\frac{1}{62.36}$	$-\frac{1}{36.88}$	$-\frac{1}{28.2}$
p4	$-\frac{1}{22.76}$	$-\frac{1}{53.91}$	$-\frac{1}{48.19}$
z1	$-\frac{1}{16.751}$	$-\frac{1}{16.93}$	0
z2	$-\frac{1}{57.24}$	$\frac{1}{74.64}$	$-\frac{1}{17.79}$

Multi-Loop vs Multivariable Control

Multi-loop - use of several single-loop controllers (e.g., PID) on pairs of manipulated/controlled variables

Multivariable - make control adjustments decisions *jointly* considering all outputs simultaneously

Multi-loop control configurations are typically used as a base control configuration and reside in the Distributed Control System (DCS).

» e.g., flow control, temperature control, pressure control

Multivariable control configurations typically require additional computational capability, and sit over a base multi-loop control configuration, sending setpoints to the multi-loop controllers.

Multi-Loop vs Multivariable Control

Under the multi-loop control strategy, each controller g_{ci} operates according to:

$$u_i = g_{ci}(y_{di} - y_i) = g_{ci}\varepsilon_i$$

Multivariable controller must decide on u_i, not using only ε_i, but using the entire set, ε_1, ε_2, ..., ε_n;. Thus, the controller actions are obtained from

$$u_1 = f_1(\varepsilon_1, \varepsilon_2, \cdots \varepsilon_n)$$
$$u_2 = f_2(\varepsilon_1, \varepsilon_2, \cdots \varepsilon_n)$$
$$u_3 = f_3(\varepsilon_1, \varepsilon_2, \cdots \varepsilon_n)$$
$$\cdots = \cdots$$
$$u_n = f_n(\varepsilon_1, \varepsilon_2, \cdots \varepsilon_n)$$