Class Test

QUESTIONS WITH ANSWERS

Indian Institute of Technology, Kharagpur

Time: 40 minutes Full Marks: 10 Autumn Semester 2018, Deptt: CH/MF/PH.
Sub. No. MA 20101 Subject Name: Transform Calculus

Question 1. Evaluate the integral

$$f(t) = \int_0^\infty \frac{\sin(tx)}{x(a^2+x^2)}dx$$

ANS: $\boxed{f(t) = \frac{\pi}{2a^2}(1-e^{-at})}$

Question 2. Solve the initial value problem:

$$y''(t) + 2y'(t) + 5y(t) = 0, \quad t > 0$$
$$y(0) = 1, \quad y'(0) = 0$$

ANS: $\boxed{y(t) = e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t}$

Question 3. Let $f(t) = [t]$, $t > 0$ be the function of the greatest integer value less than or equal to t. That is, if n is a positive integer such that $n \le t < n+1$, then $f(t) = n$. Find the Laplace transform of f.
(Hint: Write f as a series of suitable unit step functions, i.e., $f(t) = H(t-1)+H(t-2)+H(t-3),\ldots$, where $H(t-t_0)$ is 1 for $t \ge t_0$, otherwise 0)

ANS: $\boxed{\frac{e^{-s}}{s(1-e^{-s})}}$

Question 4. Solve the initial value problem:

$$y''(t) + 2y'(t) + 2y(t) = \delta(t-\pi), \quad t > 0$$
$$y(0) = 0, \quad y'(0) = 0$$

ANS: $\boxed{y(t) = H(t-\pi)e^{-(t-\pi)}\sin(t-\pi)}$

Question 5. Solve the integro-differential equation:

$$f(t) = a\sin(t) + 2\int_0^t f'(\tau)\sin(t-\tau)d\tau, \quad f(0) = 0.$$

ANS: $\boxed{f(t) = ate^t}$

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