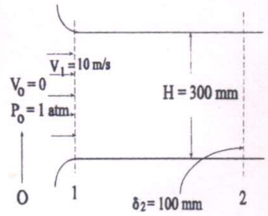
Flow of air (ρ = 1.23kg/m^3) develops in a flat horizontal duct following a well rounded entrance section. The duct height is H = 300 mm. Turbulent boundary layer grows on the duct walls, but the flow is not yet fully developed. Assume that the velocity profile in each boundary layer is given by $u / U = (y / \delta)^{1/7}$. The inlet flow is uniform with V = 10 m/s at section 1. At section 2, the boundary layer thickness on each wall of the channel is δ_2 = 100 mm. Show that for this flow, $\delta^* = \delta / 8$. Evaluate the static **gauge** pressure at section 2. Find the average wall shear stress between the entrance and section 2 at L = 5 m. The region outside the entrance (location O in figure) can be taken to be still air with pressure equal to atmospheric pressure. ~~2+2+2~~



(3)

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \delta\int_0^1 \left(1-\frac{u}{U}\right)d\eta = \delta\int_0^1 \left(1-\eta^{1/7}\right)d\eta$$

$$= \delta\left(\eta - \frac{7}{8}\eta^{8/7}\right)\Big|_0^1$$

$$\boxed{\delta^* = \delta/8}$$

From continuity $V_1 A_1 = V_1 w H = V_2 A_2 = V_2 w (H - 2\delta_2^*)$

$$V_2 = V_1 \frac{H}{H-2\delta_2^*} = 10\frac{m}{s}\,\frac{300\,mm}{(300-25)\,mm} = 10.9\ m/s.$$

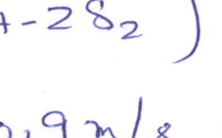
From Bernoulli, since z = const.

$$\frac{p_0}{\rho} + \underbrace{\cancel{\frac{V_0^2}{2}}}_{=0} = \frac{p}{\rho} + \frac{V^2}{2}$$

$$p_{1g} = p_1 - p_0 = -\frac{1}{2}\rho V_1^2 = -\frac{1}{2}\times 1.23\frac{kg}{m^3}\times 10^2\frac{m^2}{s^2}\times\frac{N\,sec^2}{kg\,m}.$$

$$p_{1g} = -61.5\ Pa.$$

$$p_{2g} = p_2 - p_0 = -\frac{1}{2}\rho V_2^2 = -\frac{1}{2}\times 1.23\times(10.9)^2 = -73.1\ Pa.$$



Apply momentum using cv. (half the duct as the cv)

$$F_{Sx} + \underbrace{\cancel{F_{Bx}}}_{=0} = \underbrace{\cancel{\frac{\partial}{\partial t}}}_{=0}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\,\vec{V}\cdot d\vec{A}$$

$$(p_1 - p_2)w\frac{H}{2} - \bar{\tau}_w L = \bar{V}_1\left\{-\rho\bar{V}_1\cdot\frac{H}{2}w\right\} + \underbrace{\int_0^{\delta_2} u\rho u\, w\,dy}_{A}$$

$$+ V_2\left\{+\rho V_2\left(\frac{H}{2} - \delta_2\right)w\right\}.$$

$$A = \rho V_2^2\,\delta_2 w\int_0^1 \eta^{2/7}d\eta = \rho V_2^2\,\frac{7}{9}\delta_2 w$$

$$\therefore \bar{\tau} w L = (p_1 - p_2) w \frac{H}{2} + \rho V_1^2 \frac{H}{2} w - \rho V_2^2 \left(\frac{H}{2} - \frac{2}{9}\delta_2\right) w.$$

$$\bar{\tau} = \frac{1}{L}\left((p_1 - p_2)\frac{H}{2} + \rho V_1^2 \frac{H}{2} - \rho V_2^2 \left(\frac{H}{2} - \frac{2}{9}\delta_2\right)\right)$$

$$\bar{\tau} = \frac{1}{5}\left[11.6 \times 0.15 + 1.23 \times 10^2 \times 0.15 - 1.23 \times (10.9)^2 (0.15 - 0.022)\right]$$

$$\bar{\tau} = 0.3 \ N/m^2$$