Mass Transfer
Distillation

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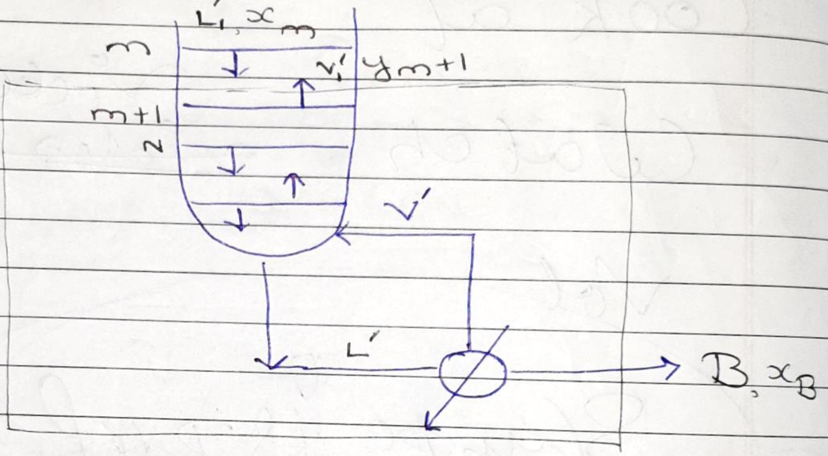
Q.1 a) Stripping section →



$$L'_m = L'_{m+1} = \ldots\ldots = L' \quad (1)$$
$$V'_m = V'_{m+1} = \ldots\ldots = V' \quad (2)$$

Overall material balance
$$L' = V' + B$$

Volatile component balance
$$L' \cdot x_m = V' \cdot y_{m+1} + B \cdot x_B$$

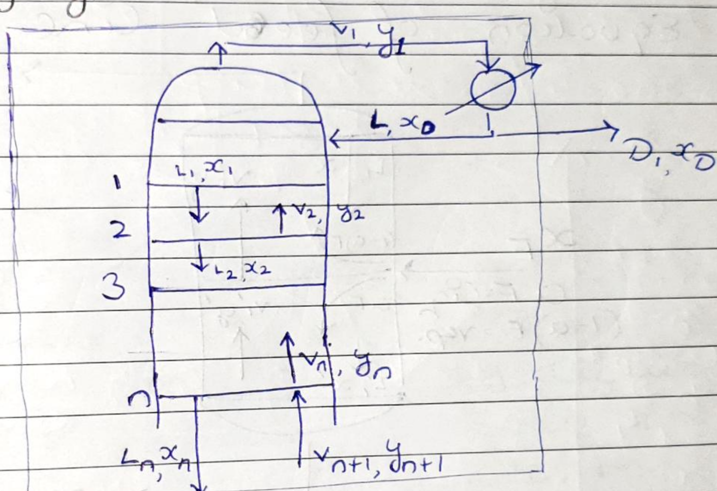
$$\Rightarrow y_{m+1} = \frac{L'}{V'} \cdot x_m - \frac{B}{V'} \cdot x_B$$

$$\therefore y = \frac{L'}{V'} \cdot x - \frac{B}{V'} \cdot x_B$$

$$\Rightarrow \boxed{y = \frac{L'}{L'-B}\cdot x - \frac{B}{L'-B}\cdot x_B}$$

equation of operating line of stripping section

(b) Rectifying section



$V_{n+1} = L_n + D$ (overall mole balance)

$V_{n+1} \cdot y_{n+1} = L_n \cdot x_n + D \cdot x_D$ (mole balance for more volatile component)

$\Rightarrow (L_n + D)\cdot y_{n+1} = L_n \cdot x_n + D \cdot x_D$

for const. molal flow $L_1 = L_2 = \dots L_n = L$

and $V_1 = V_2 = \dots V_{n+1} = V$

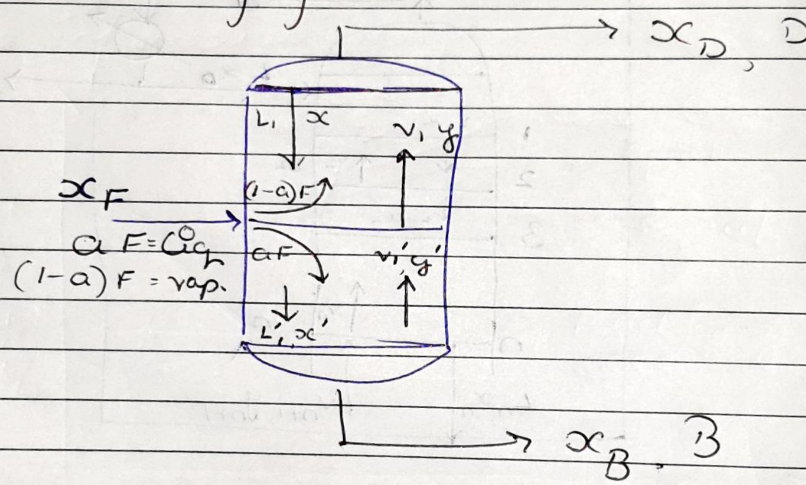
$\rightarrow (L+D)\, y_{n+1} = L x_n + D x_D$

$\Rightarrow y_{n+1} = \frac{L}{L+D} \cdot x_n + \frac{D}{L+D} \cdot x_D$

$\Rightarrow \boxed{y_{n+1} = \left(\frac{R}{R+1}\right) \cdot x_n + \left(\frac{1}{R+1}\right) \cdot x_D}$

equation of operating line

(C) Equation of feed line



Material balance around feed tray →

$L' = L + aF$

$V = V' + (1-a)\, F$

for rectifying section →

$V \cdot y = L \cdot x + D \cdot x_D$ (1)

for stripping section →

$V' \cdot y = L' \cdot x - B x_B$ (2)

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① - ② →

$$(V-V')y = (L-L')x + Dx_D + Bx_B$$

from overall mass balance →

$$F\cdot x_F = D\cdot x_D + B\cdot x_B$$

$$\Rightarrow (1-a)F\,y = -aF\cdot x + F\cdot x_F$$

$$\Rightarrow \boxed{y = -\frac{a}{1-a}\cdot x + \frac{1}{1-a}\,x_F}$$

↳ eqn of feed line

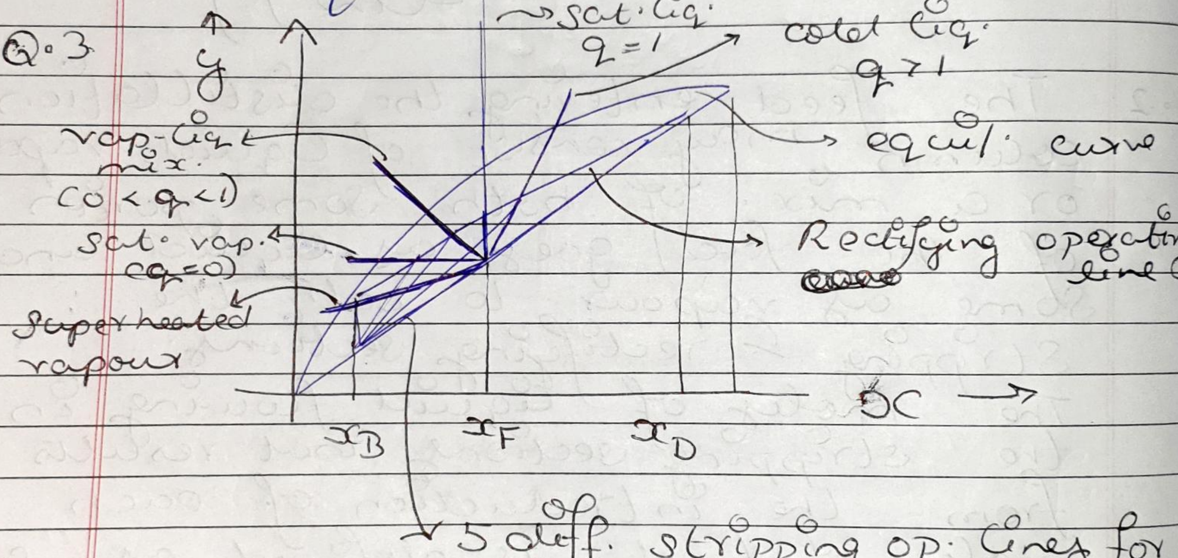
Q.2 The feed entering the distillation column may consist of liquid, vapour or a mix of both. Some portion of the feed goes as liquid and some as vapour to both the stripping & rectifying sections.

The moles of liquid flowing in the stripping sections that results from the introduction of each mole of feed is defined as "q" aka the "q"-value.

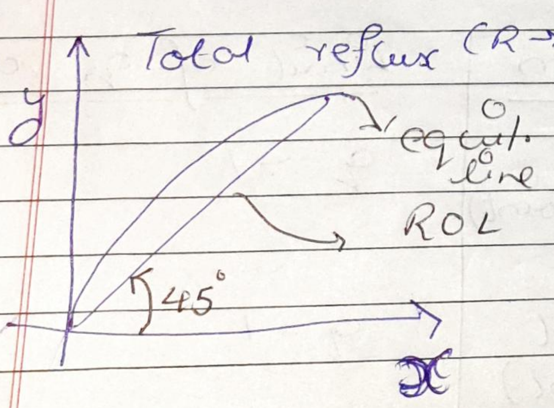
Feed condition	Limit of q-value
① Cold feed (below bubble point)	$q > 1$
② Saturated feed (at bubble point)	$q = 1$

Feed condition	Limit of q-value
(3) feed is partially vaporized	$0 < q < 1$
(4) feed at dew point (sat. vapour)	$q = 0$
(5) Superheated vapour	$q < 0$
(6) feed is a mix. of liq. and vapour at some intermediate temp.	q = fraction of feed that is a liquid.

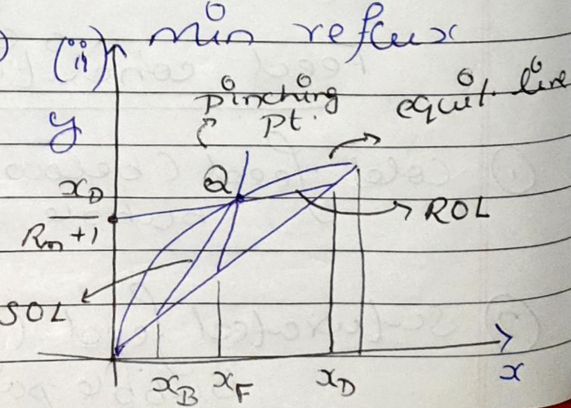
Q.3



(i) Total reflux ($R \to \infty$)



(ii) min reflux



Q4. In Rayleigh distillation, we know that →

$$\int_{W}^{F} \frac{dL}{L} = \int_{x_W}^{x_F} \frac{dx}{y^* - x}, \quad y^* = \alpha x$$

$$\therefore \frac{y^*}{1-y^*} = \alpha \cdot \frac{x}{1-x}$$

$$\Rightarrow \boxed{y^* = \frac{\alpha x}{(1-x) + \alpha x}}$$

$$\ln \frac{F}{W} = \int_{x_W}^{x_F} \frac{dx}{y^* - x}$$

$$= \int_{x_W}^{x_F} \frac{dx}{\frac{\alpha x}{(1-x)+\alpha x} - x}$$

$$= \int_{x_W}^{x_F} \frac{((1-x) + \alpha x)\, dx}{\alpha x - x(1-x) - \alpha x^2}$$

$$= \int_{x_W}^{x_F} \frac{1 - (1-\alpha)}{(1-\alpha)x^2 - (1-\alpha)x} \cdot dx$$

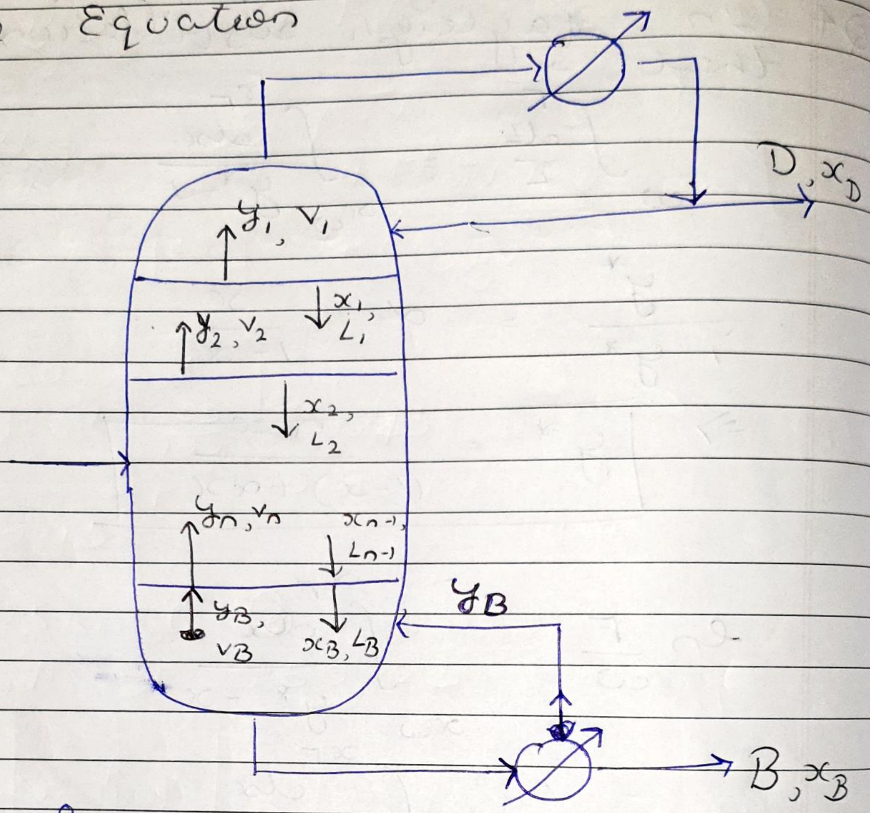
$$= \frac{1}{1-\alpha} \left[\ln\left(\frac{1-x_F}{1-x_W}\right) - \ln\left(\frac{x_F}{x_W}\right)\right] - \ln\left(\frac{1-x_F}{1-x_W}\right)$$

$$\therefore (1-\alpha) \cdot \ln\left(\frac{F}{W}\right) = \alpha \ln\left(\frac{1-x_F}{1-x_W}\right) - \ln\left(\frac{x_F}{x_W}\right)$$

$$\therefore \boxed{\ln\left(\frac{F \cdot x_F}{W \cdot x_W}\right) = \alpha \ln\left(\frac{F(1-x_F)}{W(1-x_W)}\right)}$$

Q.E.D.

Q.5 Fenske Equation



At equil → $y_1 = K_1 x_1$

for total condenser $y_1 = x_D$

$$\Rightarrow \boxed{x_D = K_1 \cdot x_1}$$

material balance of top tray + reflux system

$$V_2 = L_1 + D$$

for total reflux $D = 0$

$$\therefore \boxed{V_2 = L_1}$$

Component material balance →

$$V_2 \cdot y_2 = L_1 \cdot x_1 + D \cdot x_D$$

For min-trays condition →

$$y_2 = x_1$$

$y_2 = K_2 \cdot x_2$ at equil.

$\Rightarrow x_1 = K_2 \cdot x_2$

$\therefore x_D = K_1 \cdot K_2 \cdot x_2$

continuing similarly for entire column →

$$x_D = K_1 \cdot K_2 \cdots K_n \cdot K_B \cdot x_B$$

Similarly $x_D' = K_1' \cdot K_2' \cdots K_n' \cdot K_B' \cdot x_B'$

$$\Rightarrow \frac{x_D}{x_D'} = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdots \alpha_n \cdot \alpha_B \cdot \frac{x_B}{x_B'},$$

where α_i = relative volatility

$$\Rightarrow \frac{x_D}{x_D'} = \alpha_{avg}^{N_{min}} \cdot \frac{x_B}{x_B'},$$

assuming $\alpha_i = \alpha_{avg}$ $\forall$ i stages

$$\Rightarrow N_{min} = \frac{\ln\left(\frac{x_D \cdot x_B'}{x_D' \cdot x_B}\right)}{\ln(\alpha_{avg})}$$

$$= \frac{\ln\left(\frac{(D x_D)(B x_B')}{(D x_D')(B x_B)}\right)}{\ln(\alpha_{avg})}$$

$D \cdot x_D = (FR_D) \cdot F x_{LK}$

↳ amount of light key in feed

↓ fractional recovery of light key component

$\therefore B \cdot x_B = (1 - FR_D) \cdot F x_{LK}$

$$\Rightarrow \boxed{N^{o}_{min} = \frac{\ln\left(\frac{FR_D \cdot FR'_B}{(1-FR_D)(1-FR'_B)}\right)}{\ln(\alpha_{avg})}}$$

Q. 6 (i) $F = 50$ kmol

Q. 7.10 $x_F = 1 - \frac{55}{100} = 0.45$

$\therefore x_F = 0.45$

$\alpha = 4.15$

(a) $D = 25$ kmol

$\Rightarrow W = F - D = 50 - 25 = 25$ kmol.

$$\ln\left(\frac{x_F \cdot F}{x_W \cdot W}\right) = \alpha \ln\left(\frac{F(1-x_F)}{W(1-x_W)}\right)$$

$$\Rightarrow \ln\left(\frac{0.45 \times 50}{x_W \times 25}\right) = 4.15 \times \ln\left(\frac{50 \times 0.55}{25 \times (1-x_W)}\right)$$

$\Rightarrow \boxed{x_W = 0.218}$

Material balance of volatile component

$25 \times y_D + 25 \times 0.218 = 50 \times 0.45$

$\Rightarrow \boxed{y_D = 0.682}$

= composition of collected distillate

(b) $y_{D,av} = x_D = 0.72$

$50 = D + W$ (1)

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benzene material balance

$50 \times 0.45 = W \cdot x_W + D \times 0.72$

$\Rightarrow 22.5 = W x_W + 0.72(50 - W)$

$\Rightarrow W = \dfrac{13.5}{0.72 - x_W}$

$\ln\left(\dfrac{\frac{50}{13.5}}{0.72 - x_W} \cdot \dfrac{0.45}{x_W}\right) = 4.15 \ln\left[\dfrac{\left(\frac{50}{13.5}\right)}{0.72 - x_W} \cdot \dfrac{0.55}{1 - x_W}\right]$

$\Rightarrow \ln\left(\dfrac{3.6 - 5 \cdot x_W}{3 \cdot x_W}\right) = 4.15 \ln\left[\dfrac{2.04\,(0.72 - x_W)}{1 - x_W}\right]$

$\Rightarrow \boxed{x_W = 0.309}$

$\therefore W = \dfrac{13.5}{0.72 - 0.309} = \boxed{32.85\text{ kmol}}$

$D = F - W = 50 - 32.85 = \boxed{17.15\text{ kmol}}$

(H)
Q 7.15 $x_D = 0.96$, $x_W = 0.04$, $x_F = 0.45$,
$P = 101.3\text{ kPa}$, $R = 1.5$

(a) By plotting the equil. line, ROL & SOL on a graph sheet, we make steps →

$\therefore$ Total ideal trays $= 7.8 \cong 8$

$\therefore$ Actual no. of ideal trays $= 8 - 1 = \boxed{7}$
1 stage
reboiler

(b) No. of real trays = $\dfrac{\text{ideal trays}}{\text{efficiency of tray}}$

$$= \frac{7}{0.4} = 17.5$$

$$\cong \boxed{18}$$

∴ The optimum tray for feed injection = $\boxed{6}$ (∵ the 6th stage line intersects the feed line in the plot)

(c) from the graph for each ideal tray we get →

n	1	2	3	4	5	6	7	8
x_n	0.91	0.84	0.73	0.61	0.47	0.31	0.15	0.04
T_n	65.8	67	68.8	71	73.8	77.7	86	93.5

(d) No. of ideal trays = $22 \times 0.4 = 8.8$
$\cong 9$

∵ ideal trays are different from before ∴ we require more steps between the equil. and operating lines.

To gain more steps, either equil. line can be manipulated by choosing diff. operating pressure or the operating lines can be changed.

For instance, the slope of ROL can be reduced by decreasing the reflux ratio

∴ Process can be done by changing some existing param.

iii) $x_D = 0.95$, $x_W = 0.04$, $\alpha_{avg} = 2.5$

from the plot, no. of trays (ideal) = $\boxed{7}$

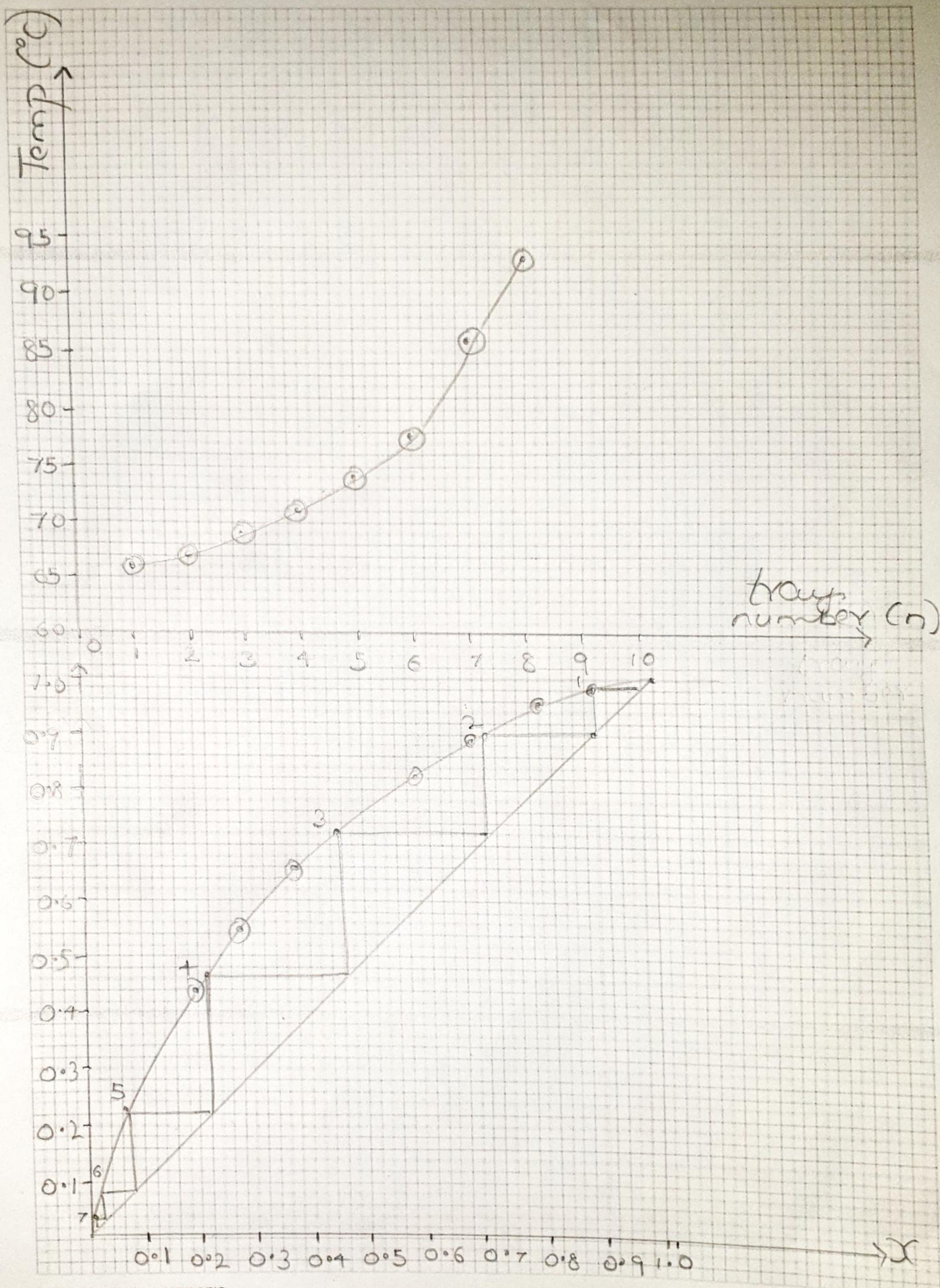
∴ from Fenske eqn

$$N_m + 1 = \text{no. of ideal trays}$$

$$= \frac{\log\left(\frac{x_D(1-x_W)}{x_W(1-x_D)}\right)}{\log(\alpha_{av})}$$

$$= \frac{\log\left(\frac{0.95(1-0.04)}{0.04(1-0.95)}\right)}{\log(2.5)}$$

$$= \boxed{6.7} \cong 7$$ (diff. due to const. α_{avg} assump.).


Temp (°C)
95
90
85
80
75
70
65
60
tray number (n)
0
1
2
3
4
5
6
7
8
9
10
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x