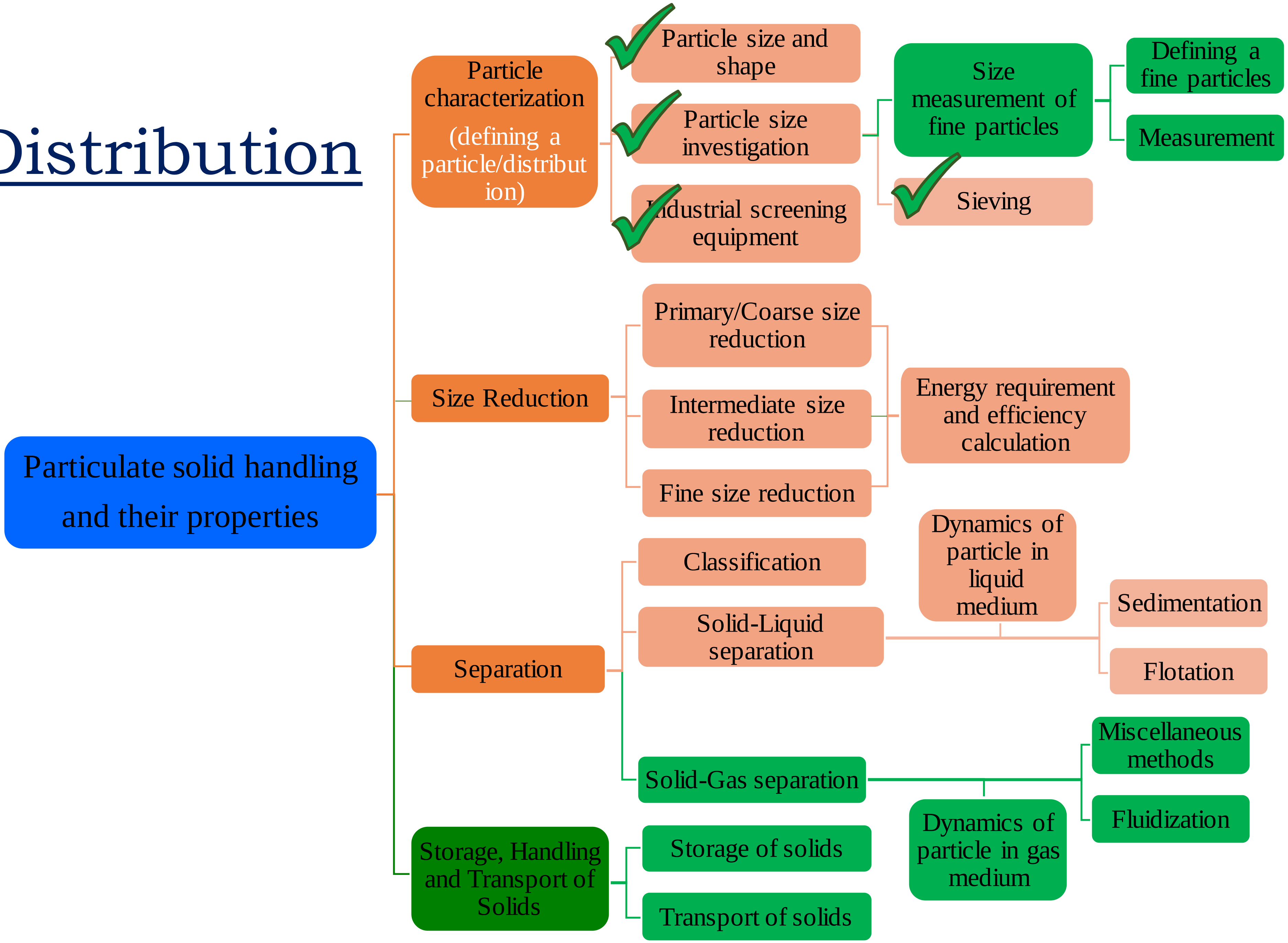
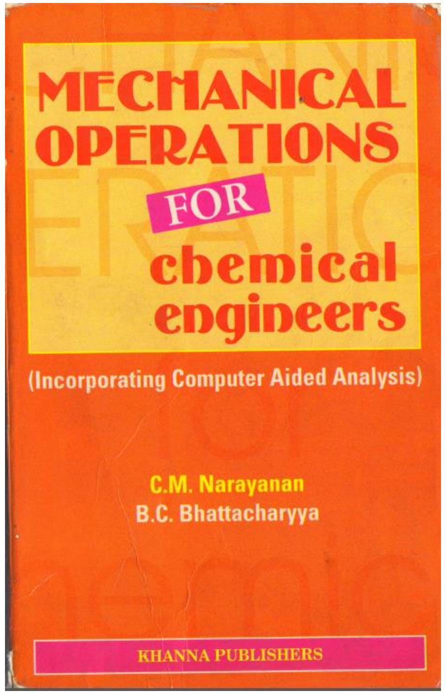
Course Distribution

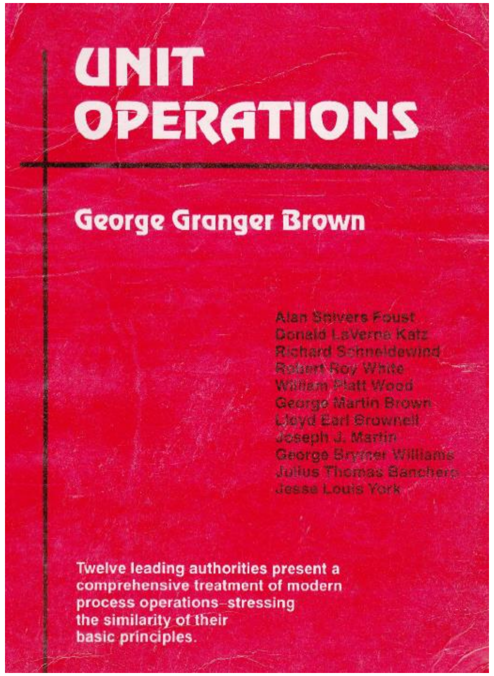


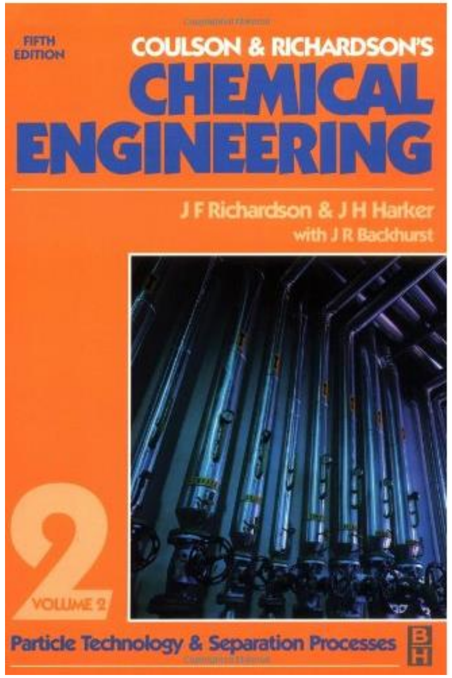
Comminution/ Size Reduction

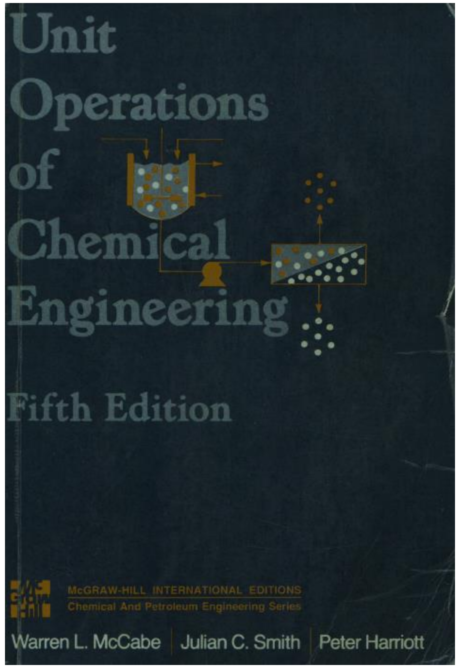
Resource

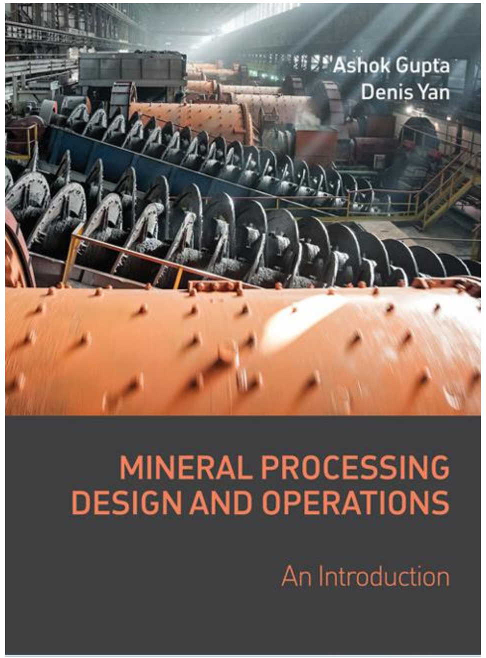
- **Books**







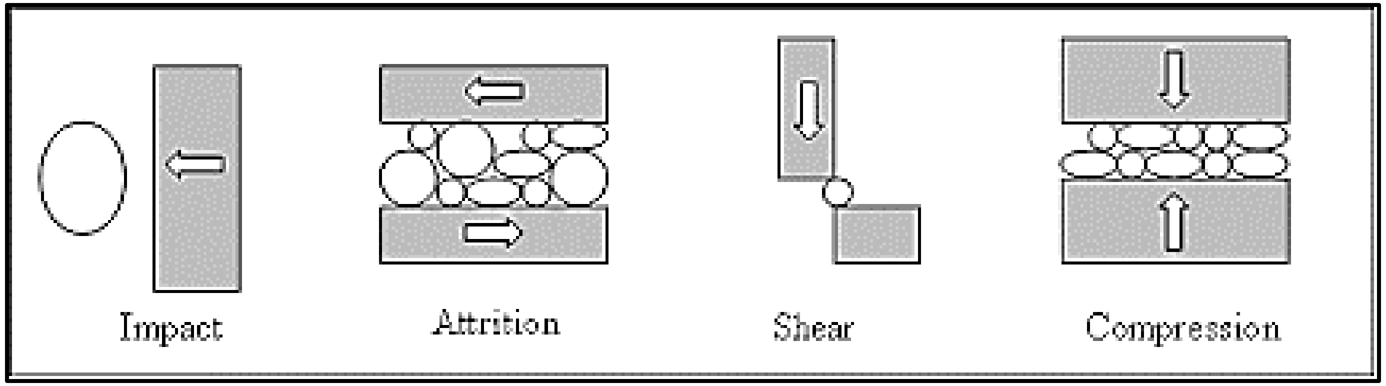




- **Relevant journal papers mentioned in the individual topics**

Introduction

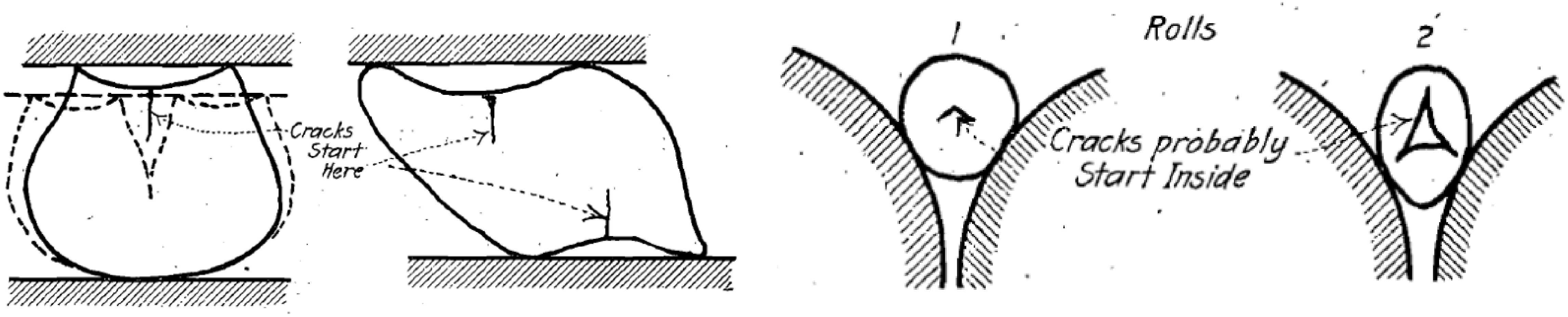
- Comminution is the generic term for size reduction.
- It is an operation that is omnipresent in process industries.
- Solid particles can be reduced in size by compression, shear, impact, attrition(rubbing) or by cutting or tearing.



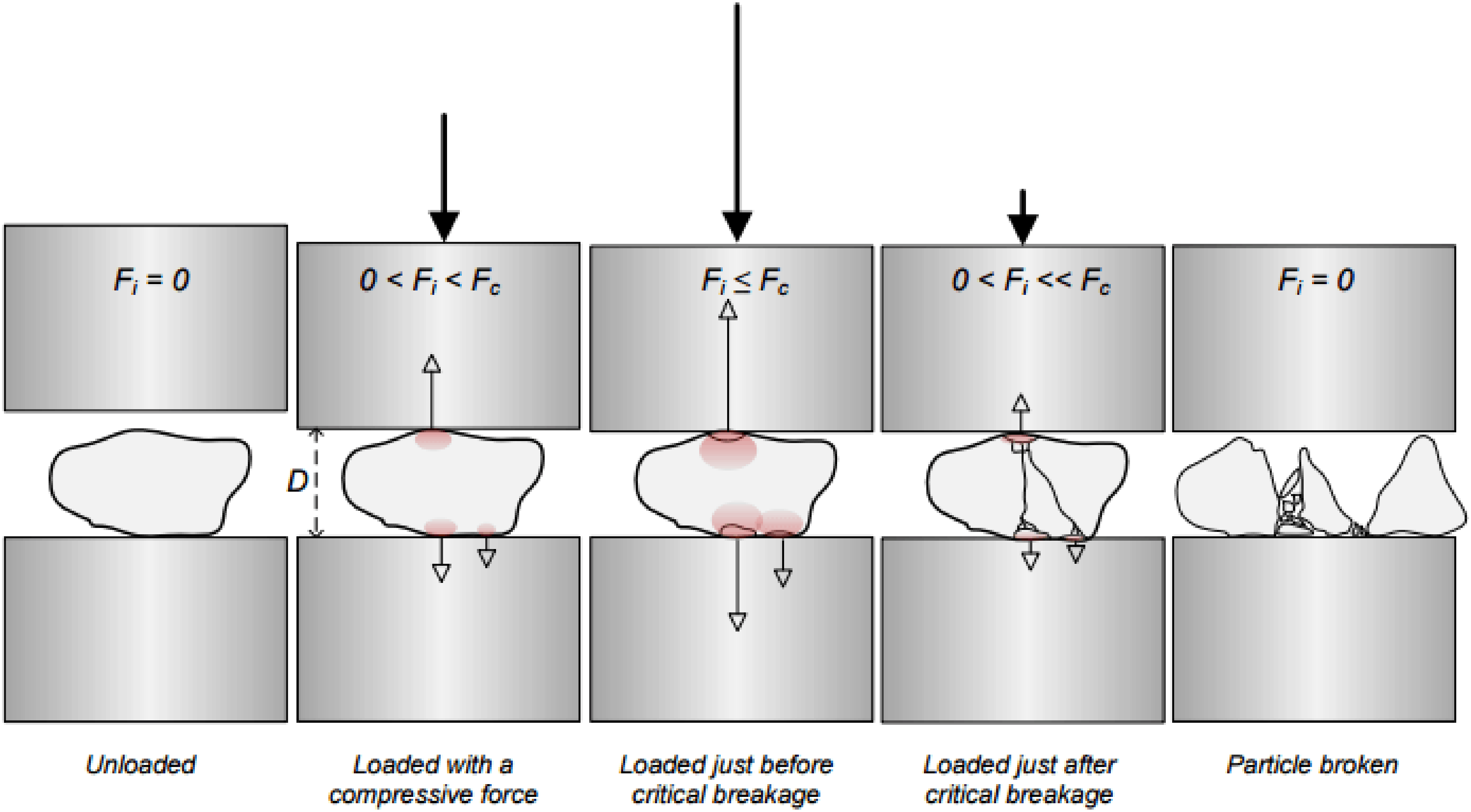
- Compression, impact or shearing loads are used for breaking brittle materials like coal and minerals whereas fibrous materials like wood and asbestos are disintegrated by exerting tearing loads.

The Mechanism of Size Reduction

Stress → Strain → Fracture in lines of Weakness → Released Heat



The force applied may be compression, impact, or shear, and both the magnitude of the force and the time of application affect the extent of grinding achieved.


$F_i = 0$
$0 < F_i < F_c$
$F_i \leq F_c$
$0 < F_i << F_c$
$F_i = 0$
D
Unloaded
Loaded with a compressive force
Loaded just before critical breakage
Loaded just after critical breakage
Particle broken

Energy for size reduction

- For efficient grinding, the energy applied to the material should exceed, by as small a margin as possible, to the minimum energy needed to rupture the material.
- Excess energy is lost as heat and this loss should be kept as low as practicable.
- Grinding is a very inefficient process and it is important to use energy as efficiently as possible.

- Three theories/laws depend upon the basic assumption that the work done/Energy required to produce a change *dL* in a particle of a typical size dimension *L* is a simple power function of *L*

$$dW = -C\frac{dL}{L^n} \quad ......(1)$$

Where, *dW* is the differential work done, *dL* is the change in a typical dimension, L is the characteristic length

Comminution Laws

- The laws on comminution proposed by different authors help us determine the energy consumed in comminution.

Rittinger's Law (1867):

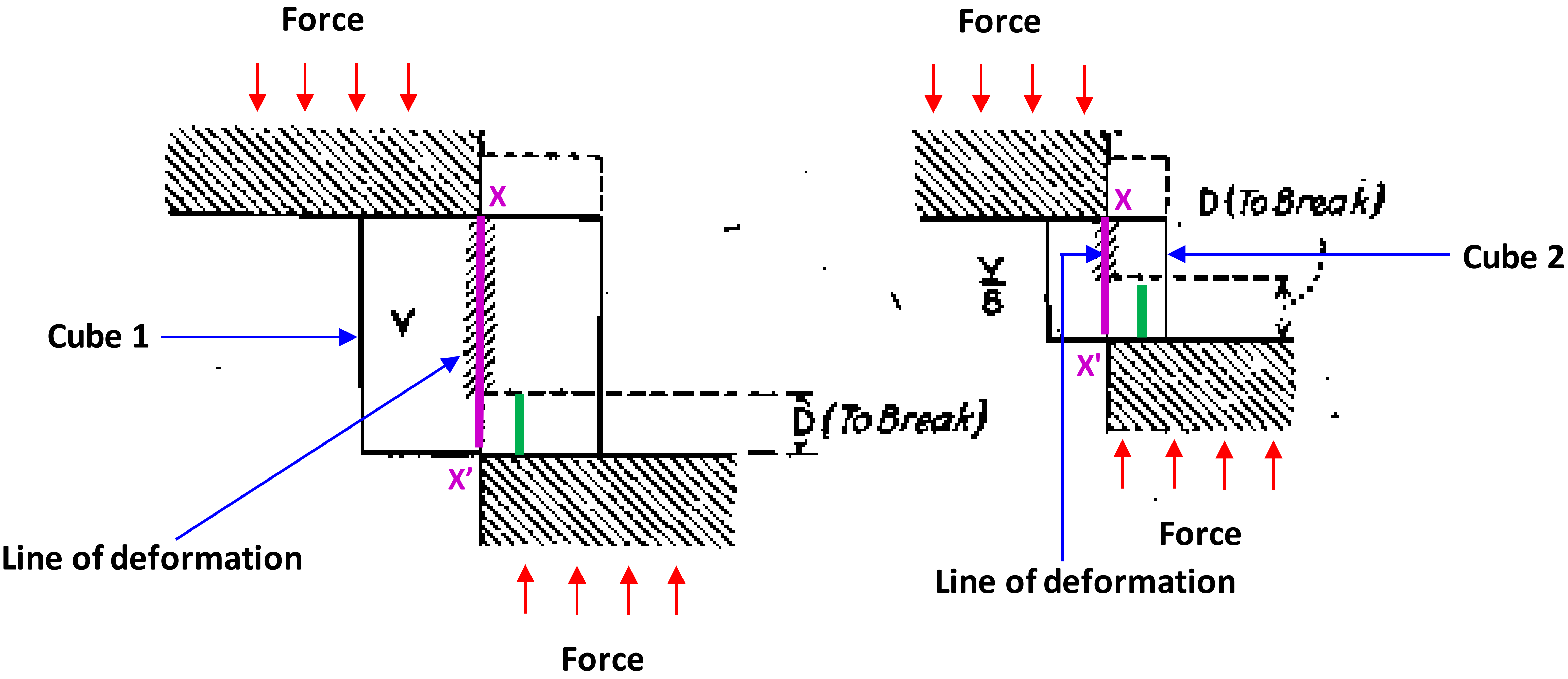
- The law says that "*The work done in crushing is proportional to the surface exposed by the operation*"

$$W = \frac{P}{\dot{m}} = K_R(s_p - s_F) = \frac{6K_R}{\rho_s}\left[\frac{1}{D_{VS,P}} - \frac{1}{D_{VS,F}}\right] = K_R^{'}\left[\frac{1}{D_{VS,P}} - \frac{1}{D_{VS,F}}\right] \quad \text{......(2)}$$

K_R is the Rittinger's constant and $\dfrac{1}{K_R}$ is the Rittinger's number.

- Rittinger's number is defined as the new surface created per unit mechanical energy absorbed by the metrical being crushed.

- Determined by *drop test* method

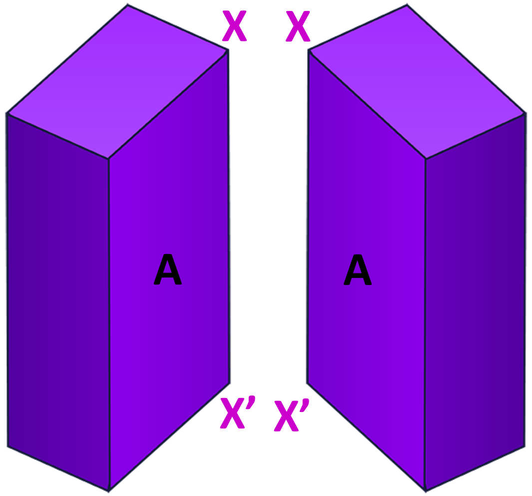

Force
Force
Cube 1
V
X
X'
D (To Break)
Line of deformation
Force
Force
X
D (To Break)
Cube 2
V
8
X'
Line of deformation
Force

After Breakage

Cube 1

Plane of breakage X-X'

New area created **2A**

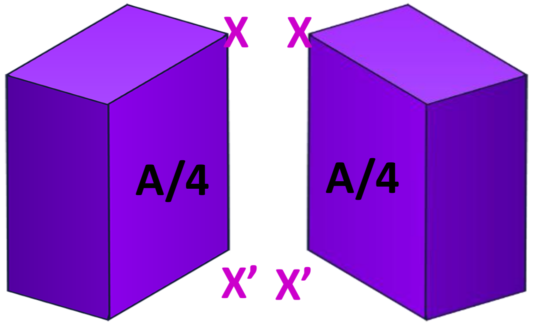


Let area of one side of the cube 1 is A

Cube 2

Plane of breakage X-X'

New area created 2× A/4



Cube 1

Cube 2

Area of one section = A

Area of one section = A/4

Let average resistance to shear per sq. in. = F

Distance traveled by the to rapture the cube = D

Distance traveled by the to rapture the cube = D

Energy = F×A×D

Energy = F×A/4×D

Surface Produced = 2 A

Surface Produced = 2 A/4

$$Energy \propto new\ surface\ created$$

Mineral	Rittinger's Number		
	sq in./ft-lb	sq cm/ft-lb	sq cm/kg-cm
Quartz (SiO_2)	37.7	243	17.56
Pyrite (FeS_2)	48.7	314	22.57
Sphalerite (ZnS)	121.0	780	56.2
Calcite ($CaCO_3$)	163.3	1053	75.9
Galena (PbS)	201.5	1300	93.8

Dependency on physical structure

- Marble is a metamorphic rock composed of recrystallized carbonate minerals most commonly ***calcite or dolomite***.

- Chalk is a soft, white, porous, sedimentary carbonate rock, a form of limestone composed of the mineral ***calcite***.

- Chemically there is no difference in them but they differ in their physical structure.

Kick's Law (1885):

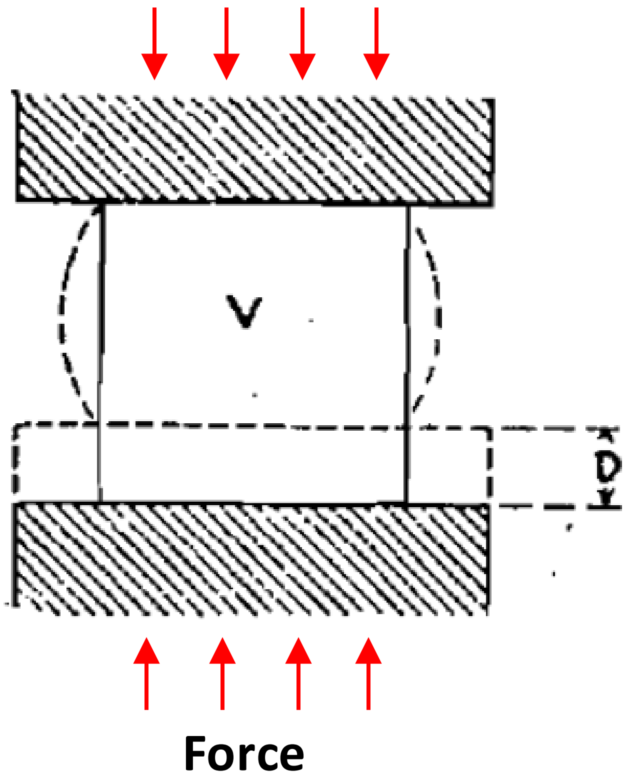
The original form of the law says that "*energy required to produce analogues changes of configuration of geometrically similar bodies varies as the volumes or masses of these bodies*"

- The above statement is not valid when new surfaces are created

The modified Kick's law that can be applied to crushing states that "*the work required for crushing a given mass of material is constant for the same reduction ratio irrespective of original size*"

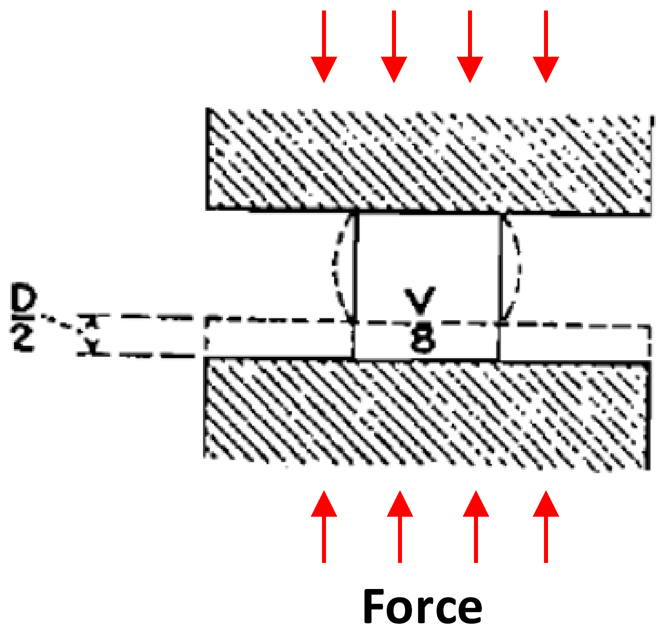
$$W = \frac{P}{\dot{m}} = K_k \ln \frac{D_{VS,F}}{D_{VS,P}} \qquad \text{......(3)}$$

Cube 1



Area one section = A

Cube 2



Area one section = A/4

Let the average resistance to shear per sq. in. = F

Energy = FAD

$$Energy \propto Volume$$

Energy = F A/4 D/2 = FAD/8

Bond’s Law (1952)

- It stats that “*total work useful in breakage that has been applied to a given weight of homogeneous broken material is inversely proportional to the square root of the average size of product particles, directly proportional to the length of crack tips formed and directly proportional to the square root of new surface created*”

Or

- It stats that “*work required to form particle of size* $\boldsymbol{D_p}$ *from very large feed is proportional to the square root of the surface-to-volume ratio of the product*”

$$W = \frac{P}{\dot{m}} = K_b \left[\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right] \qquad \text{......(4)}$$

- Bond constant (K_b) is evaluated by defining **Work index**
- **Work index (*Wi*)** is defined as the gross energy requirement in kilowatt hours per ton of feed needed to reduce a very large feed to such a size that 80 percent of the product passes a 100 μm screen.

$K_b = f(W_i, \lambda)$

- If ***E*** in kilowatts, and ***ṁ*** in tons per hour,

a) $K_b = \sqrt{100 W_i} = 10 W_i$ When ***D_p*** is in microns

b) $K_b = \sqrt{100 \times 10^{-3} W_i} = 0.3162 W_i$ When ***D_p*** is in millimeters

- If 80 percent of the feed passes a mesh size of ***D_F*** millimeters and 80 percent of the product passes a mesh size of ***D_p*** millimeters, then

$$W = \frac{E}{\dot{m}} = 0.3162 W_i \left(\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right) \qquad ......(5)$$

$$W = K_b \left[\frac{1}{\sqrt{D_p}} - \cancel{\frac{1}{\sqrt{D_F}}} \right] \to 0$$

$W = W_i$ when $D_F \to \infty$ and $D_p = 100\ \mu m \to$ 80% of the product

$$W_i = K_b \times \left[\frac{1}{\sqrt{100}} \right]$$

D_p is in μm.

$D_p =$

$K_b = 10\, W_i$ when D_p is in μm

$K_b = 0.316\, W_i$ when D_p is in mm

- Alternatively we can also write

$$W = \frac{P}{\dot{m}} = W_i \sqrt{\frac{100}{D_P}} \left(1 - \frac{1}{\sqrt{RR}}\right) \qquad \text{......(6)}$$

$$W = K_b \left[\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right]$$

$$= \frac{K_b}{\sqrt{D_P}} \left[1 - \frac{1}{\sqrt{\frac{D_F}{D_P}}} \right]$$

Where, D_p is in microns and $\underline{RR = \frac{D_F}{D_P}}$ (Reduction ratio)

Revised Bond's theory

- In 1957, Holmes (2) pointed out that, starting from Kick's law, Bond's third theory should be revised as follows:

$$W = \frac{P}{\dot{m}} = W_i \left(\frac{100}{D_P}\right)^r \left(1 - \frac{1}{(RR)^r}\right) \qquad \ldots\ldots(7)$$

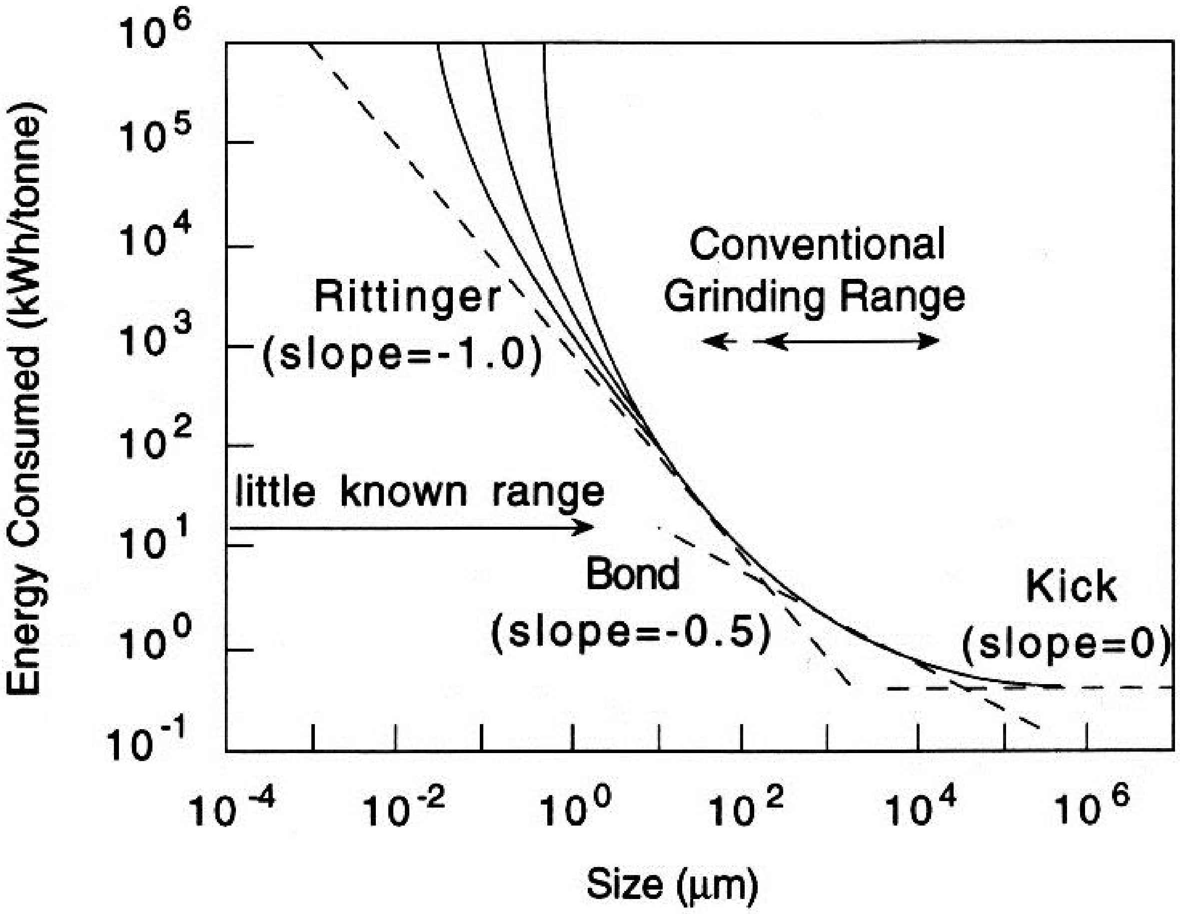
r is called a deviation from Kick's law

He demonstrated that r varies from 0.25 to 0.73 for several materials, and that such a concept leads to a more consistent work index, W_i, than that calculated by Bond.

General Form

$$dW = -C\frac{dL}{L^n}$$

n=1	Kick's Law
n=2	Rittinger's Law
n=1.5	Bond's Law



A graph comparing the energy requirement for crushing the particles v/s the particle diameter for the three laws, i.e, Rittinger's law, Kick's law and Bond's law.

Problem 1

$P_{max} = 5/0.9 = 5.55$

$P_2 = 5.242$

Sugar is ground from crystals of which it is acceptable that 80% pass a 500 μm sieve (D_{F_1}) (Standard Sieve No.35), down to a size in which it is acceptable that 80% passes a 88 (D_{P_1}) μm (Standard Sieve No. 170) sieve, and a motor working at 5 HP (working at 90% of its full power) is used for the required throughput. If the requirements are changed such that the grinding is only down to 80% through a 125 μm (D_{P_2}) (No.120) sieve but the throughput is to be increased by 50% would the existing motor have sufficient power to operate the grinder? Assume Bond's law to be valid.

$P_2 < P_{max}$

Ans = Yes

Solution

- First case (before the requirement change)

Let m_1 (ton/hr) be the initial throughput

D_p = 88 µm (88 × 10^{-6} m) (Average diameter of product from the grinder)

D_F = 500 µm (500 × 10^{-6} m) (Average diameter of feed from the grinder)

Energy supplied E_1 = 5 HP

Reduction ratio (RR)= $\dfrac{D_p}{D_F} = \dfrac{500}{88}$

Using equation 3 we can find the work required for breaking the particles

$$W = \frac{E_1}{m_1} = W_i \sqrt{\frac{100}{D_P}} \left(1 - \frac{1}{\sqrt{RR}}\right)$$

$$\frac{E_1}{m_1} = \frac{5}{m_1} = W_i \sqrt{\frac{100}{88}} \left(1 - \sqrt{\frac{88}{500}}\right) \qquad \text{......(4)}$$

- Second case (after the requirement change)

  Throughput is increased by 50%

$$m_2 = 1.5\, m_1$$

D_p = 125 μm (125 × 10^{-6} m) (Average diameter of product from the grinder)

So,

$$\frac{E_2}{m_2} = \frac{E_2}{1.5\, m_1} = W_i\sqrt{\frac{100}{125}}\left(1-\sqrt{\frac{125}{500}}\right) \quad ......(5)$$

Note: Work index (W_i) will remain unaltered as the system remains same

Dividing equation 5 by equation 4 we get

$$\frac{E_2}{1.5\, m_1}\times\frac{m_1}{5} = \frac{W_i\sqrt{\frac{100}{125}}\left(1-\sqrt{\frac{125}{500}}\right)}{W_i\sqrt{\frac{100}{88}}\left(1-\sqrt{\frac{125}{500}}\right)} \Rightarrow \quad E_2 = 5.43\, HP$$

Full power of the motor = $\frac{5}{0.9} = 5.55\ HP > 5.43\ HP$

So the motor would be expected to have sufficient power to pass the 50% increased throughput.

The motor has to run $\frac{5.43}{5.55} \times 100 \approx 98\%$ of its full power to meet the requirement (Ans)

Problem 2

A grinder is to be used (which is 8% efficient) to handle 10 tonnes (1 tonne= 1000 kg) per hour of an ore (specific gravity = 2.65). The feed and product analysis are given below

Screen Size, mm		Mass Fraction	
		Feed	Product
-3.327	+2.362	0.143	0.0
-2.362	+1.651	0.211	0.0
-1.651	+1.168	0.230	0.0
-1.168	+0.833	0.186	0.098
-0.833	+0.589	0.120	0.234
-0.589	+0.417	0.076	0.277
-0.417	+0.295	0.034	0.149
-0.295	+0.208	0.0	0.101
-0.208	+0.147	0.0	0.068
-0.147	+0.104	0.0	0.044
-0.104		0.0	0.029

The grinder costs Rs. 40,000. It operates on a 24 hour basis for 300 days per year and the miscellaneous cost amount to 50% of the power cost. Electricity costs 70 p. per kWh. If the machine depreciates on a straight line basis for 10 years, estimate the annual processing cost of the ore if the work index of the ore is 13.57 kWh/ tonne.

Solution

As the work index of the ore is given we can use the Bond's law to estimate the power consumption in grinding the ore

Given, W_i = 13.57 kWh/ tonne

$$\dot{m} = 10\ tonnes / hr$$

Using equation 2

$$\frac{E}{\dot{m}} = 0.3162 \times W_i \left(\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right)$$

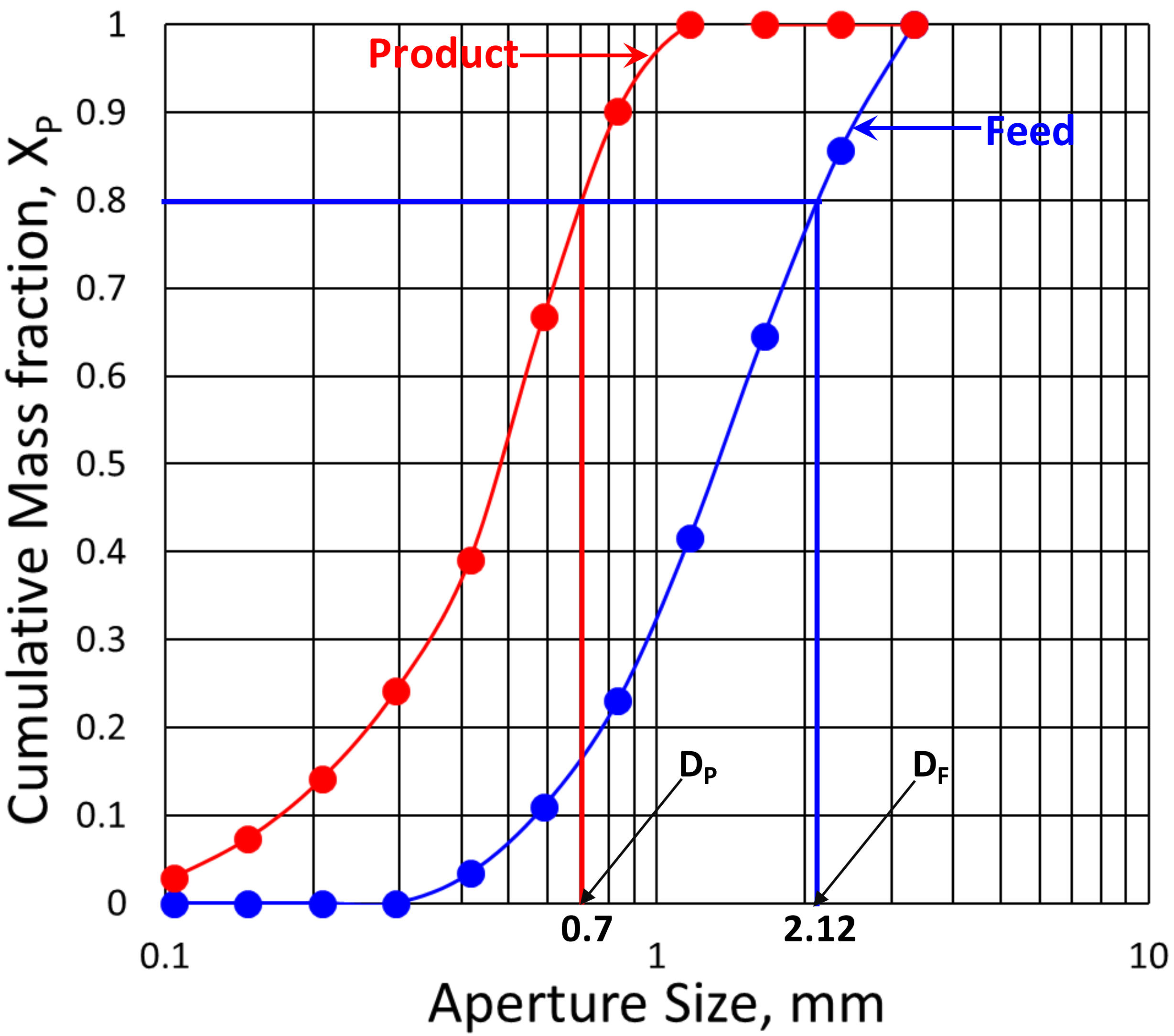
- D_p and D_F (in mm) are the aperture size through with 80% of the material (both product and feed) pass through (as per the definition of work index)

- So, 80% of the feed particle are of size less then D_F and 80% of the product particle are of size less then D_P

- Cumulative analysis of the particle size distribution will give us relevant information

Cumulative analysis

Undersize aperture, mm	Cumulative mass fraction	
	Feed (X_F)	Product (X_p)
3.327	1.0	1.0
2.362	0.857	1.0
1.651	0.646	1.0
1.168	0.416	1.0
0.833	0.230	0.902
0.589	0.110	0.668
0.417	0.034	0.391
0.295	0.0	0.242
0.208	0.0	0.141
0.147	0.0	0.073
0.104	0.0	0.029



From graph we have D_P = 0.7 mm and D_F = 2.12 mm

Substituting in equation 2 we get

$$\frac{E}{\dot{m}} = 0.3162 \times W_i \left(\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right)$$

$$\frac{E}{10} = 0.3162 \times 13.57 \left(\frac{1}{\sqrt{0.7}} - \frac{1}{\sqrt{2.12}} \right) \quad \Rightarrow \quad E = 21.816\, kW / hr$$

Total power consumption = $\dfrac{E}{Efficiency} = \dfrac{21.816}{0.08} = 272.696\, kW / hr$

Power consumption in a year = 272.696 × 24 × 300 =19,63,413.6 kwh

Annual power cost = 19,63,413.6 × 0.70 ≈ Rs. 13.744 lakh

Miscellaneous cost ≈ 13.744 × 0.50 ≈ Rs. 6.872 lakh

Annual depreciation cost = 40,000/10 = Rs. 4000

So, the total annual processing cost of the ore is ≈ (13.744 + 6.872 + 0.04) lakh
≈ **Rs. 20.656 lakh** (Ans)

Several probabilities involved

1. whether or not the particle hits an objective.

2. whether collision is strong enough to produce a stress larger than the breaking stress of the material.

3. whether crack tips within a solid will propagate and eventually break the solid.

The general equation should include these three different probabilities, along with $\dfrac{1}{L^n}$

Since specific surface is inversely proportional to particle size, equation 1 can be rewritten as follows:

$$\frac{ds}{dE} = C_1 L^N \qquad \text{......(8)}$$

Where, $N = n-2$

E is the energy required to crush a unit weight of particles of the size L.

The dependence of C on particle size and crushing mechanism must be further clarified to include the uncertainties.

The equation proposed

$$\frac{ds}{dE} = K(P_c)(P_\sigma)(P_a)L^N \qquad \text{......(9)}$$

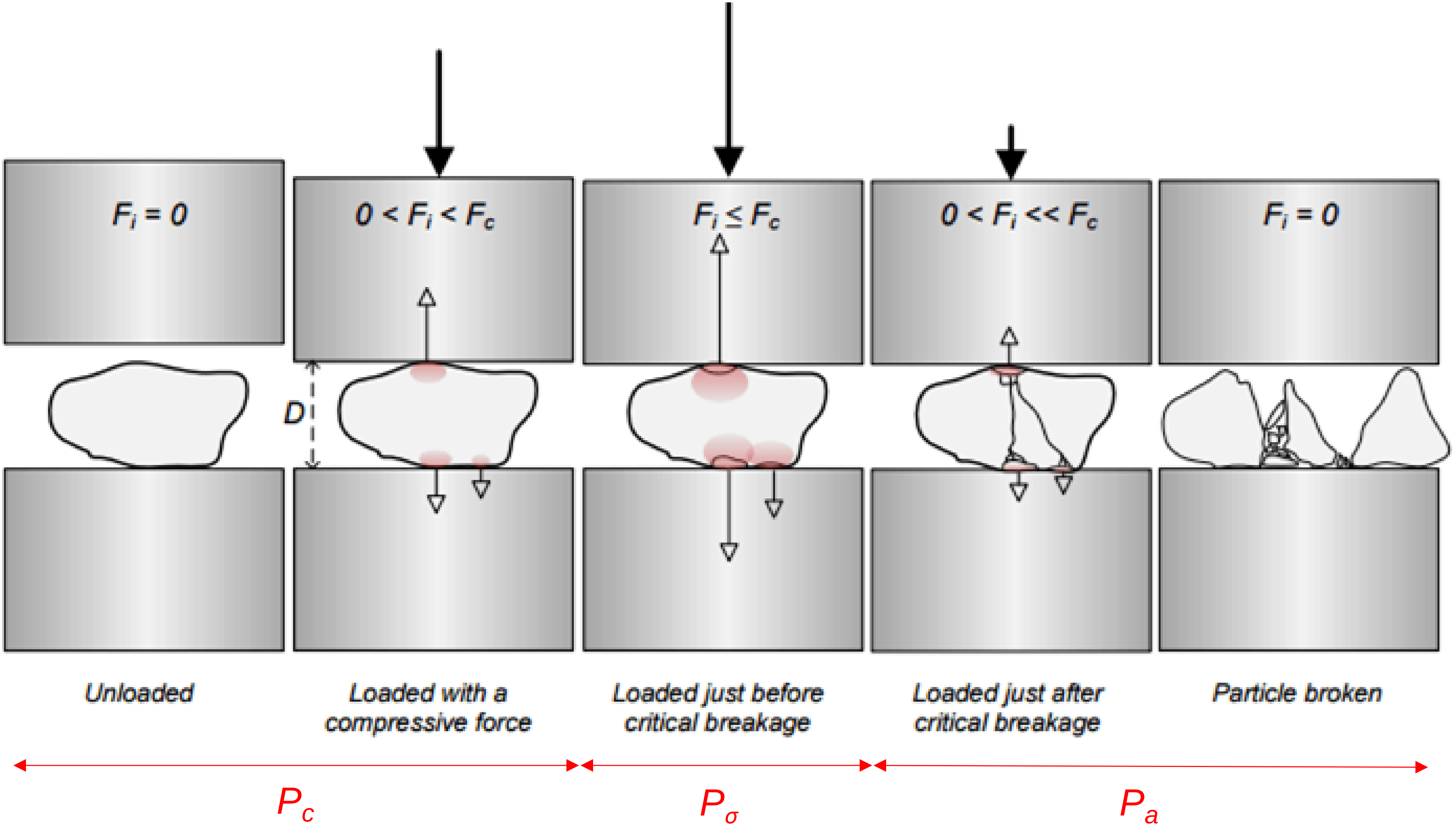
Where,

P_c is the probability of collision,

P_σ the probability that the material's breaking stress will be exceeded,

P_a the probability of crack tip propagation,

and K is a constant.



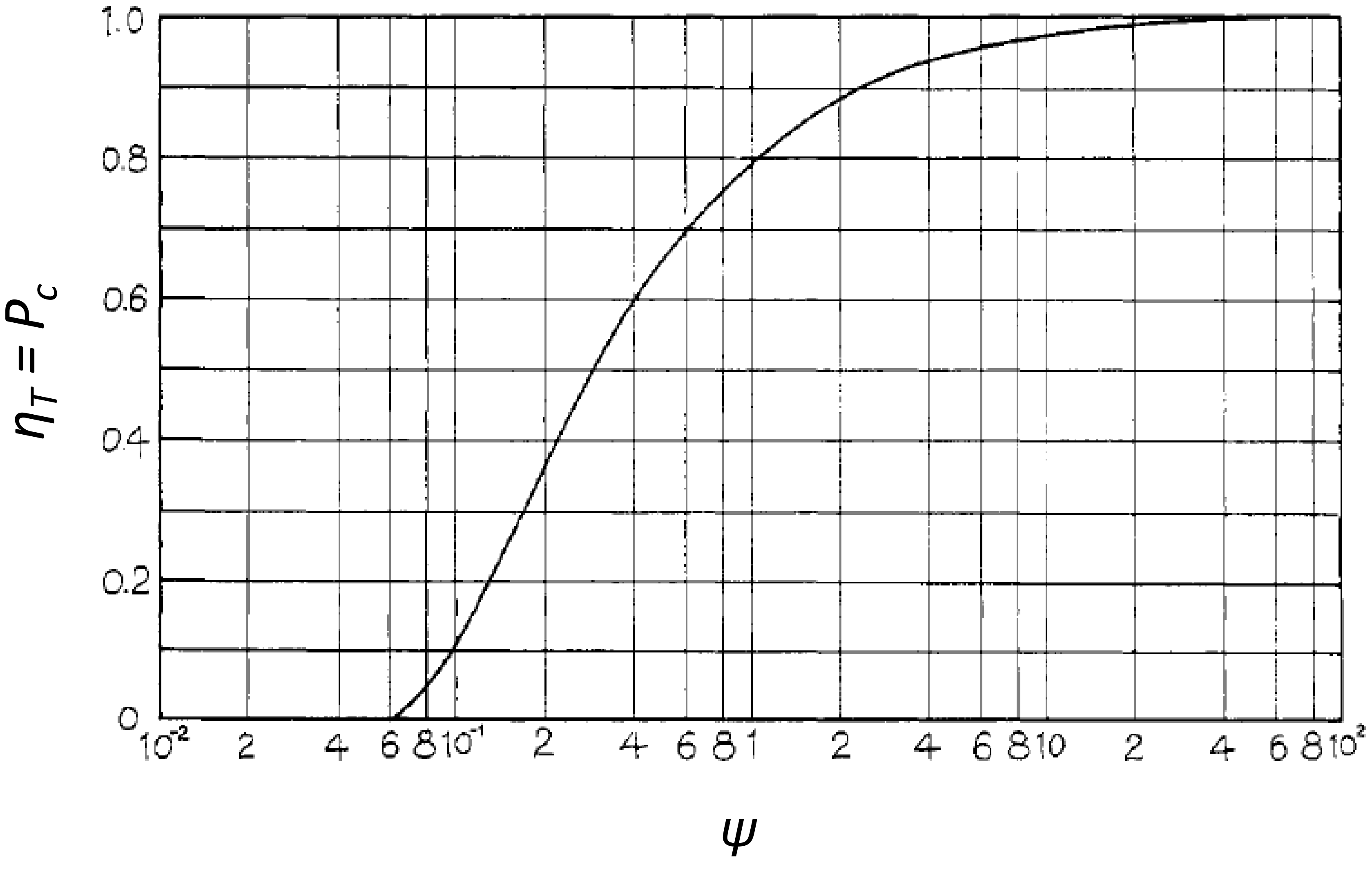
$F_i = 0$
$0 < F_i < F_c$
$F_i \leq F_c$
$0 < F_i << F_c$
$F_i = 0$
D
Unloaded
Loaded with a compressive force
Loaded just before critical breakage
Loaded just after critical breakage
Particle broken
P_c
P_σ
P_a

Probability of collision

The theory governing the striking of an objective by a particle is described by the interrelationship between target efficiency, η_T , and separation number, ψ .

$$\psi = \frac{L^2 v \rho_p}{18 \mu D}$$

Where, D is the diameter of the objective and v is the particle impact velocity and L is the characteristic length of the particle and μ is the viscosity of the surrounding fluid (mostly air)



$\eta_T = P_c$
1.0
0.8
0.6
0.4
0.2
0
10^{-2} 2 4 6 8 10^{-1} 2 4 6 8 1 2 4 6 8 10 2 4 6 8 10^{2}
ψ

Probability of Exceeding Breaking Stress

According to Hertz , the maximum stress produced in a material at impact is expressed by:

$$\sigma_{\max} = a\left(\frac{m_1 m_2}{m_1 + m_2}\right)^{\frac{1}{5}} \times v^{\frac{2}{5}}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{\frac{3}{5}} \times \left(\frac{1-\beta_1}{\varepsilon_1} + \frac{1-\beta_2}{\varepsilon_2}\right)^{-\frac{4}{5}}$$

where m_1 and m_2 are the masses of the two bodies

r_1 and r_2 the radii of curvature of the bodies at an impact point

ε_1 and ε_2 are the moduli of elasticity; β_1 and β_2 are Poisson's ratio of the bodies.

v is the relative velocity of the particle

Rumpf, H., Chem. Ing. Tech. 31, 323 (1959).

Probability of Exceeding Breaking Stress

The probability of exceeding the breaking stress of material is proposed as

$$P_\sigma = \left(1 - \frac{\sigma_a}{\sigma_{\max}}\right)^r$$

where *r* is an empirical constant that can be determined from experimental data.

When $\sigma_a = \sigma_{max}$, $P_\sigma = 0$, and when σ_{max} is much greater than σ_a , then P_σ approaches unity, which corresponds to an actual crushing phenomenon.

Probability of crack tip propagation

$$P_a \propto e^{-kP}$$

P is the pressure within the crusher and k is a constant which depends on the crushing mechanism

Final Correlation

$$\Delta s = \Delta E \times K \times P_c \times \left(1 - \frac{\sigma_a}{\sigma_{\max}}\right)^m \times e^{-kP} \times L^N \qquad \text{......(10)}$$

Crushing Efficiency

- The ratio of surface energy created by crushing to the energy absorbed by the solid is the ***crushing efficiency*** and is denoted by η_c

- If $\boldsymbol{e_s}$ is the surface energy per unit area, s_p and s_F are the specific surface areas of product and feed, respectively, then the the **energy absorbed by a unit mass of the material** ($\boldsymbol{W_n}$)

$$W_n = \frac{e_s(s_p - s_F)}{\eta_c} \qquad ....(11)$$

- The surface energy created by the fracture is small in comparison with the total mechanical energy stored in the material at the time of rupture.

Ref. McCabe, Smith

- Rest are converted into heat during the time of rapture.
- Crushing efficiency range between 0.1 to 2 %.
- The energy absorbed by the solid $\boldsymbol{W_n}$ is less than that fed to the machine.
- The ratio of energy absorbed to the energy input is η_m, the mechanical efficiency.
- The energy input to the machine per unit mass (***W)*** is

$$W = \frac{W_n}{\eta_m} = \frac{e_s(s_p - s_F)}{\eta_m \eta_c} \quad \text{...........(12)}$$

➢ So the power required by the machine is

$$P = W\dot{m} = \frac{\dot{m}e_s(s_p - s_F)}{\eta_m \eta_c}$$

Where, $\dot{m}$ is the mass flow rate of the feed.

➢ If the sauter diameter of the feed and product is $D_{VS,F}$ and $D_{VS,P}$ respectively then

$$P = \frac{6\dot{m}e_s}{\eta_m \eta_c \rho_s}\left(\frac{1}{D_{VS,P}} - \frac{1}{D_{VS,F}}\right) \qquad \text{...........(13)}$$

$$D_{VS} = \frac{1}{\sum \frac{n_i x_i}{d_{avgi}}}$$

Problem 3

A material is crushed in a Blake jaw crusher such that the average size of particle is reduced from 50 mm to 10 mm with the consumption of energy of 13.0 kW/(kg/s). What would be the consumption of energy needed to crush the same material of average size 75 mm to an average size of 25 mm :

a) Assuming Rittinger's law applies?
b) Assuming Kick's law applies?
c) Assuming Bonds's law applies?

Which of these results would be regarded as being more reliable and why ?

Solution:

Given

When

D_p = 10 mm (Average diameter of product from the crusher)

D_F = 50 mm (Average diameter of feed from the crusher)

W= 13.0 kW/(kg/s)

Find

D_p = 25 mm (Average diameter of product from the crusher)

D_F = 75 mm (Average diameter of feed from the crusher)

W= ??

a) **Rittinger's Law (1867)**

The law says that "*The work done in crushing is proportional to the surface exposed by the operation*"

$$W_R = K_R(s_p - s_F) = \frac{6K_R}{\rho_s}\left[\frac{1}{D_P} - \frac{1}{D_F}\right] = K_R^{'}\left[\frac{1}{D_P} - \frac{1}{D_F}\right] \qquad .....(6)$$

K_R is the Rittinger's constant and $\frac{1}{K_R}$ is the Rittinger's number.

Case 1:

Using equation 6

$$W_R = K_R^{'}\left[\frac{1}{D_P} - \frac{1}{D_F}\right]$$

$$13.0 = K_R^{'}\left[\frac{1}{10} - \frac{1}{50}\right]$$

$$\Rightarrow \quad K_R^{'} = 162.5 \, \frac{kJ}{kg\ mm^{-1}}$$

Case 2:

Using equation 6

$$W_R = 162.5\left[\frac{1}{25} - \frac{1}{75}\right] = 4.33 \text{ kJ/kg}$$

b) **<u>Kick's law Law (1867)</u>**

The modified Kick's law that can be applied to crushing states that "*the work required for crushing a given mass of material is constant for the same reduction ratio irrespective of original size*"

$$W_K = K_k \ln \frac{D_F}{D_P} \qquad .....(7)$$

Case 1:

Using equation 7

$$W_K = K_k \ln \frac{D_F}{D_P}$$

$$13.0 = K_k \ln \frac{50}{10}$$

$$\Rightarrow \quad K_K = 8.08 \frac{kJ}{kg}$$

Case 2:

Using equation 7

$$W_K = 8.08 \ln \frac{75}{25} = 8.87 \frac{kJ}{kg}$$

b) **Bonds's Law (1952)**

Case 1:

Using equation 1

$$W_B = K_b\left[\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}}\right]$$

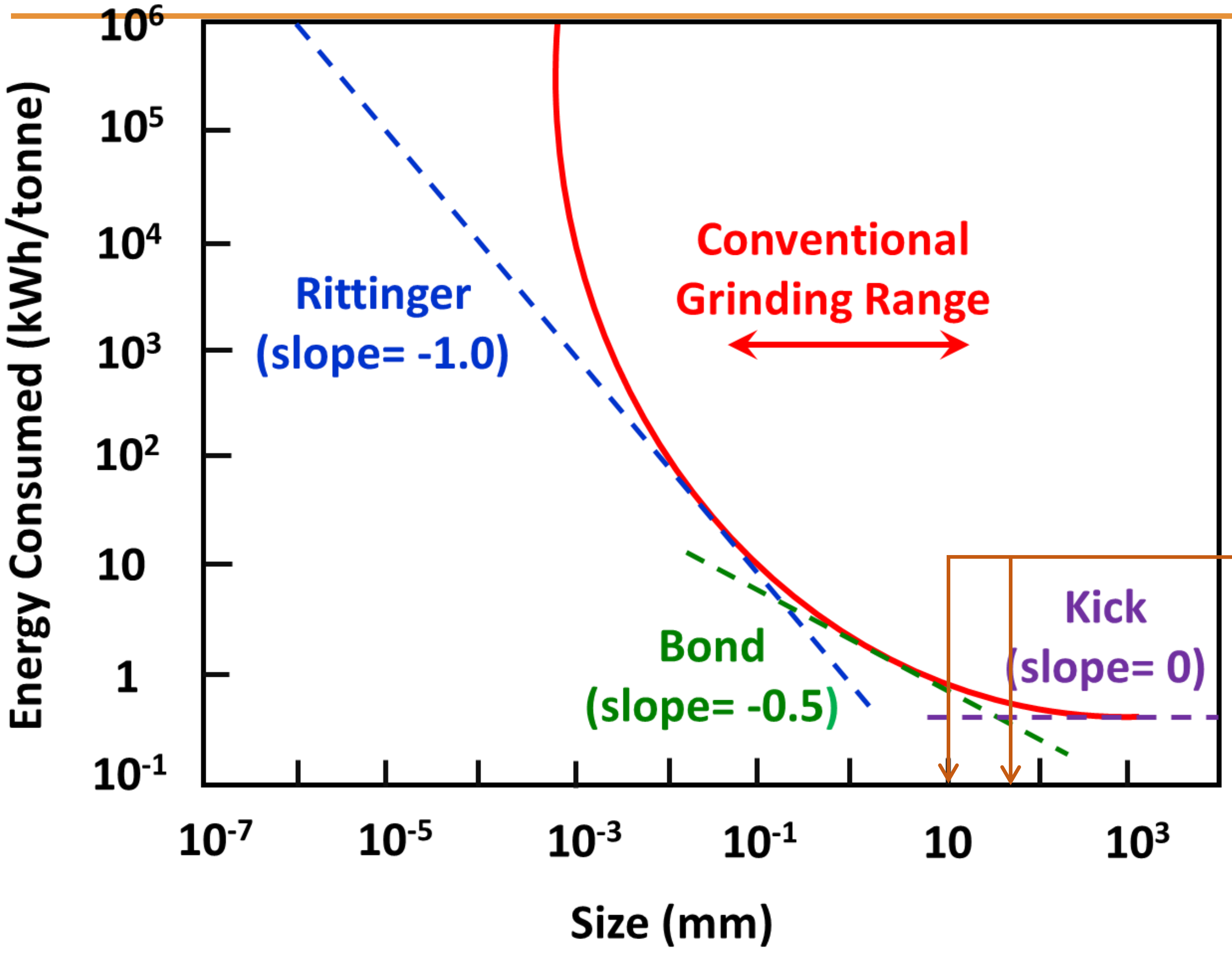
$$13.0 = K_b\left[\frac{1}{\sqrt{10}} - \frac{1}{\sqrt{50}}\right]$$

$$\Rightarrow \quad K_b = 74.36\,\frac{kJ}{kg\ mm^{-1/2}}$$

Case 2:

Using equation 2

$$W_B = K_b\left[\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}}\right] = 74.36\left[\frac{1}{\sqrt{25}} - \frac{1}{\sqrt{75}}\right] = 6.28\ \frac{kJ}{kg}$$



Consumption of energy using Bond's law / Kick's law will give more accurate estimate for the given problem

Given particle range

Different stages of size reduction

- Coarse size reduction (Feed size 50 to 250mm or more)
  - Equipment used are **Crushers** (Ex: Jaw crusher, Gyraroty crusher, Crushing rolls *etc.*)

- Intermediate size reduction (Feed size in the range of 25 to 75mm)
  - Equipment used are **Grinders** (Ex: Hammer mills, Rolling- compression mills, Attrition mills, Tumbling mills *etc.*)

- Fine size reduction (Feed size in the range of 5 to 15mm)
  - Equipment used are **Ultrafine grinders** (Ex: Fluid energy mills, Agitated mills *etc.*)

Size-Reduction Equipment

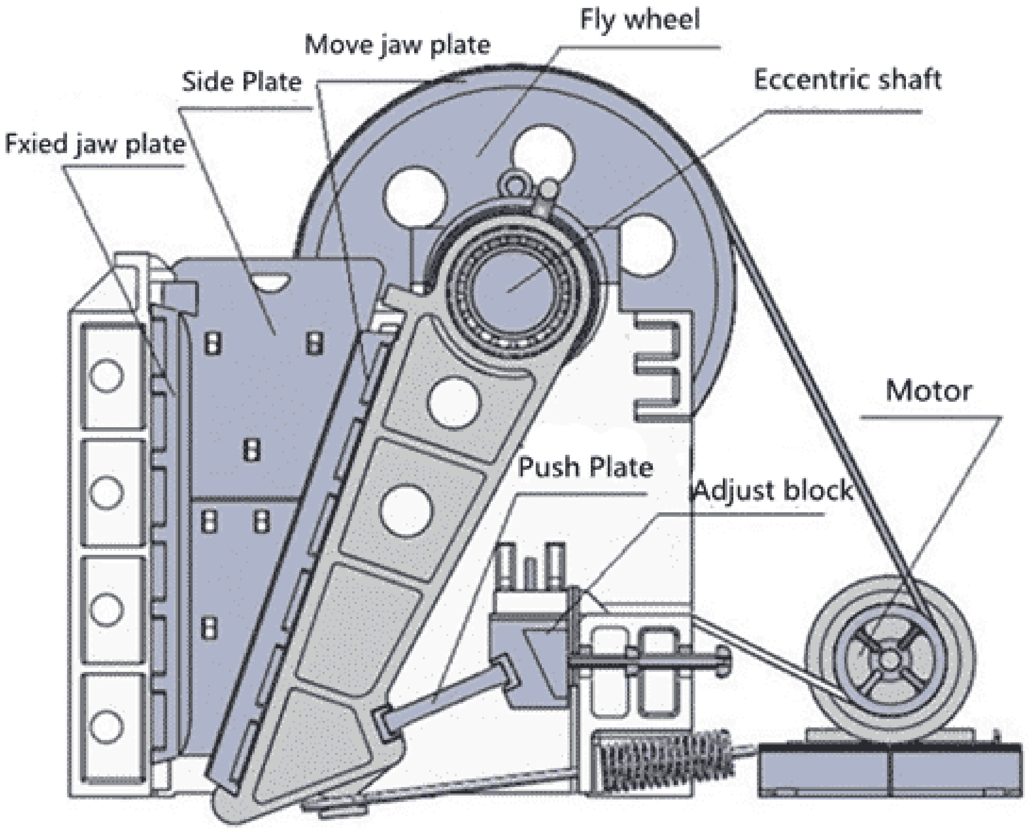
Size reduction equipment is divided into crushers, grinders, ultrafine grinders, and cutting machines.

- **Crushers** (coarse size reduction) do the heavy work of breaking large pieces of solid materials into small lumps.
  - **Primary Crusher** breaks into 50-250mm lumps.
  - **Secondary Crusher** breaks into ~ 6mm particles.
- **Grinders** (Intermediate size reduction) reduce crushed feed to powder.
- **Ultrafine grinder** the product size is typically 1-50µm.
- **Cutters** give particles of definite shape and size, 2-10 mm in length.

SIZE REDUCTION EQUIPMENTS

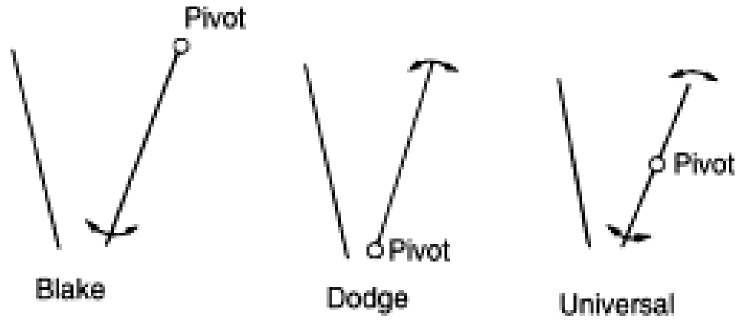
- **Crushers (coarse and fine)**
  - Jaw crushers
  - Gyratory crushers
  - Crushing rolls
- **Grinders (intermediate and fine)**
  - Hammer mills; impactors
  - Rolling-compression mills
  - Attrition mills
  - Tumbling mills
- **Ultrafine grinders**
  - hammer mills with internal classification
  - Fluid-energy mills
  - Agitated mills
- **Cutting machines**
  - Knife cutters; dicers; slitters

Jaw Crusher


Fly wheel
Move jaw plate
Side Plate
Eccentric shaft
Fxied jaw plate
Motor
Push Plate
Adjust block

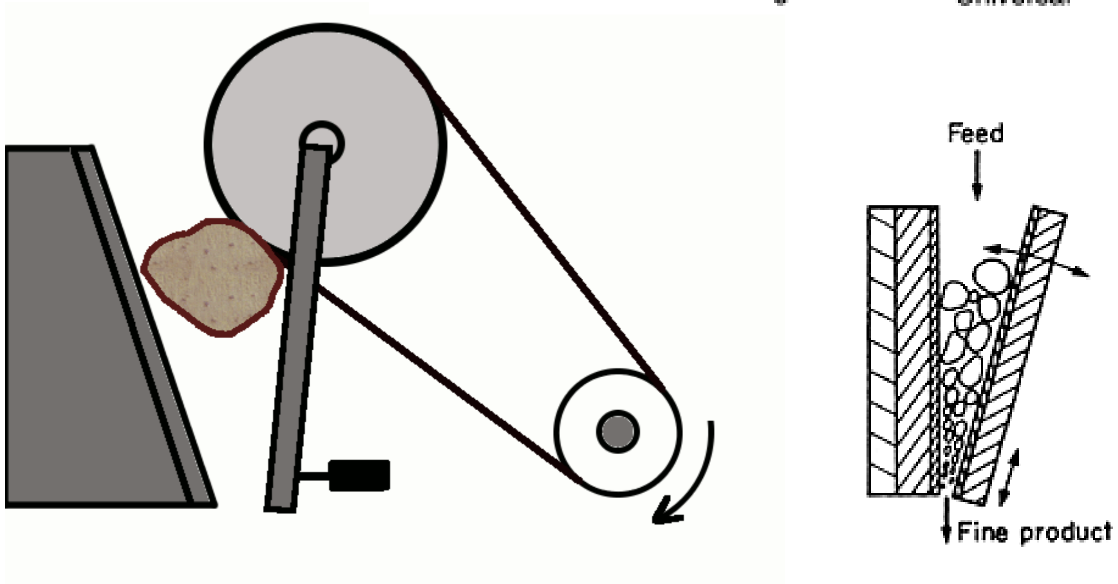
In a **jaw crusher** feed is admitted between two jaws, set to form a V open at the top. One jaw, the fixed, or anvil, jaw, is nearly vertical and does not move; other, the swinging jaw, reciprocates in a horizontal plane. It makes an angle of 20^o to 30^o with the anvil jaw.It is driven by an eccentric so that it applies great compressive force to lumps caught between the jaws. The jaws open and close 250 to 400 times per minute.

Jaw crushers are classified on the basis of the position of the pivoting of the swing jaw

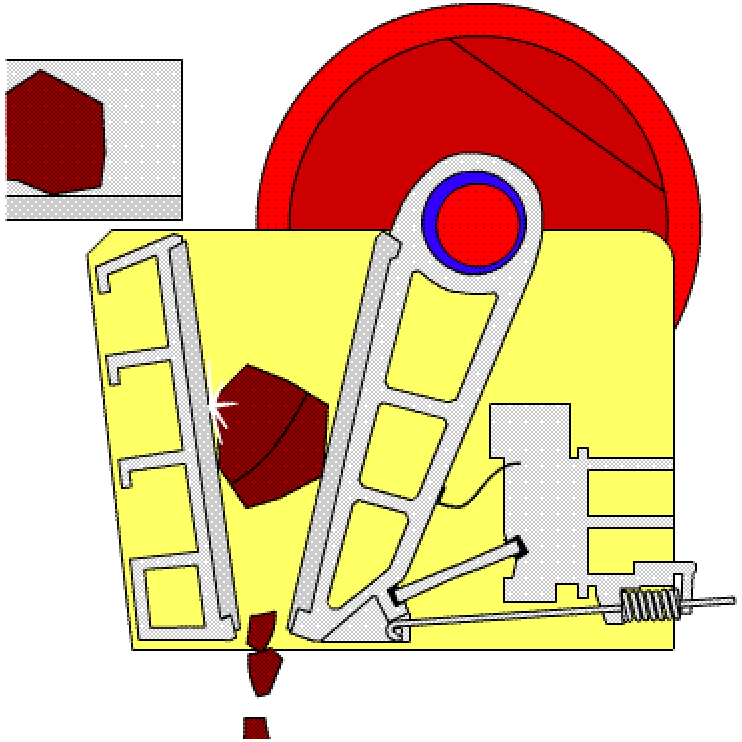


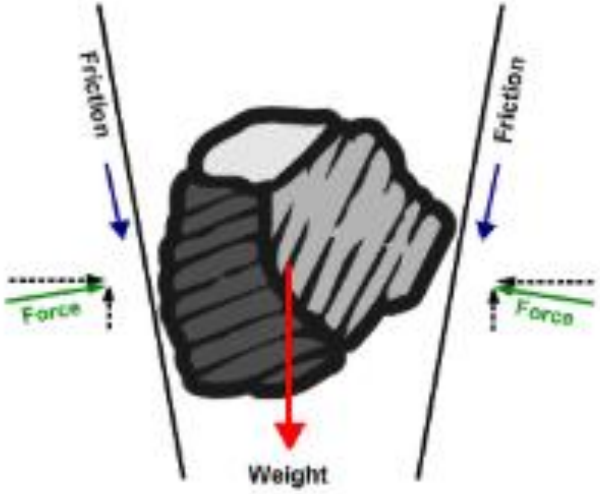
1. **Blake crusher**-the swing jaw is fixed at the upper position
2. **Dodge crusher**-the swing jaw is fixed at the lower position
3. **Universal crusher**-the swing jaw is fixed at an intermediate position

The most common type of jaw crusher is the ***Blake crusher***.
In this machine an eccentric drives a pitman connected to two toggle plates, one of which is pinned to the frame and the other to the swinging jaw.

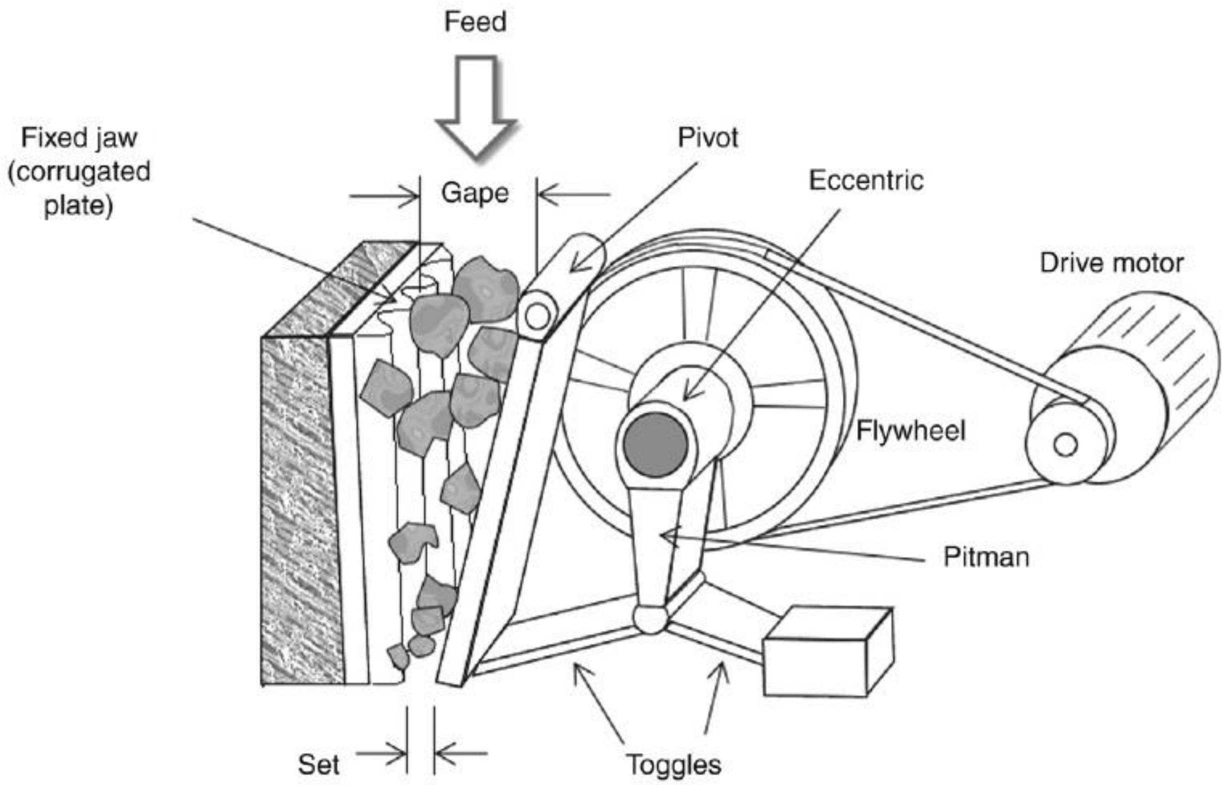


Single-toggle Jaw Crusher



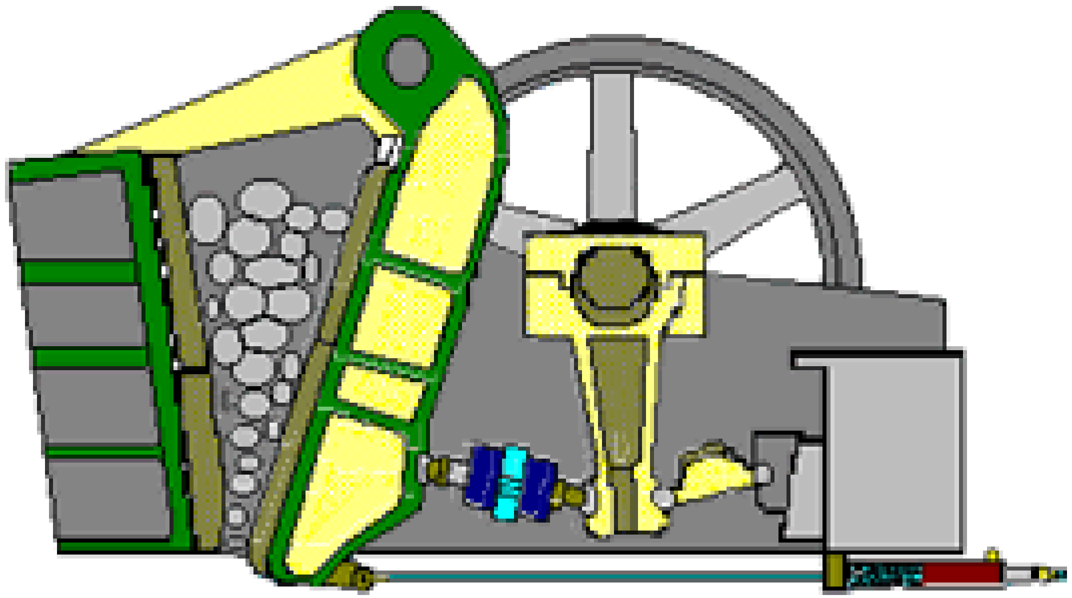


Slippage Principle



Double-Toggle Jaw Crusher.

Double-toggle Jaw Crusher



Largest particle size = 0.9 × Gape

Jaw crusher performance

Crusher Type	Size (mm)				Reduction Ratio		Power (kW)		Toggle Speed (rpm)	
	Gape (mm)		Width (mm)							
	Min	Max	Min	Max	Range	Average	Min	Max	Min	Max
Blake, double toggle	125	1600	150	2100	4:1/9:1	7:1	2.25	225	100	300
Single toggle	125	1600	150	2100	4:1/9:1	7:1	2.25	400	120	300
Dodge	100	280	150	28	4:1/9:1	7:1	2.25	11	250	300

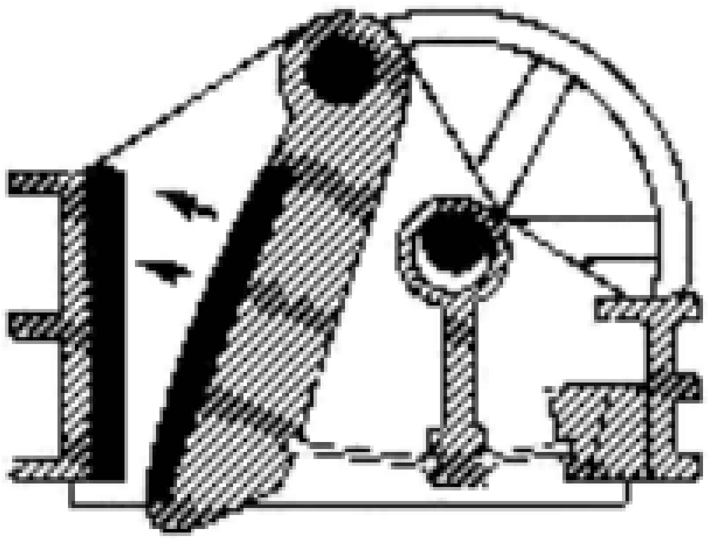
The factors of importance in designing the size of primary crushers, such as a jaw crusher, are:

$$\text{Vertical height of crusher} \approx 2 \times \text{Gape}$$

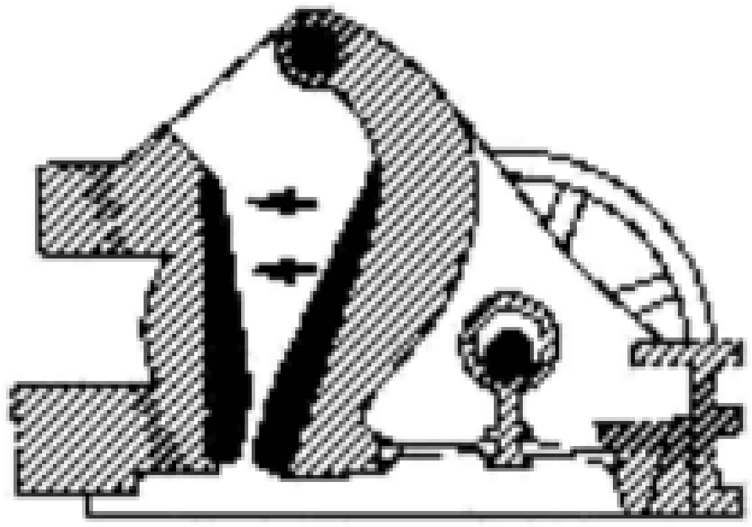
$$\text{Width of jaw} > 1.3 \times \text{Gape}$$

$$< 3.0 \times \text{Gape}$$

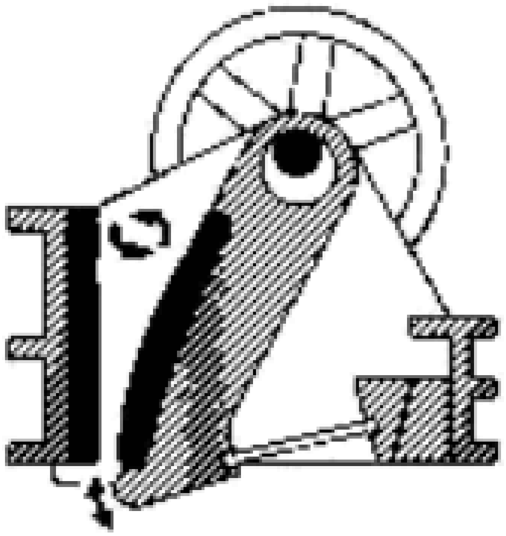
$$\text{Throw} = 0.0502\,(\text{Gape})^{0.85}$$

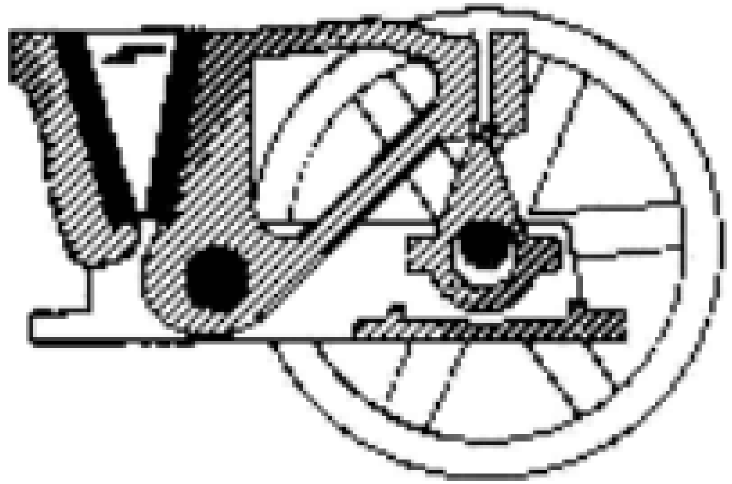


Blake (Double Toggle) Originally the standard jaw crusher used for primary and secondary crushing of hard, tough abrasive rocks. Also for sticky feeds. Relatively coarse slabby product, with minimum fines.



Overhead Pivot (Double Toggle) Similar applications to Blake. Overhead pivot; reduces rubbing on crusher faces, reduces choking, allows higher speeds and therefore higher capacities. Energy efficiency higher because jaw and charge not lifted during cycle.

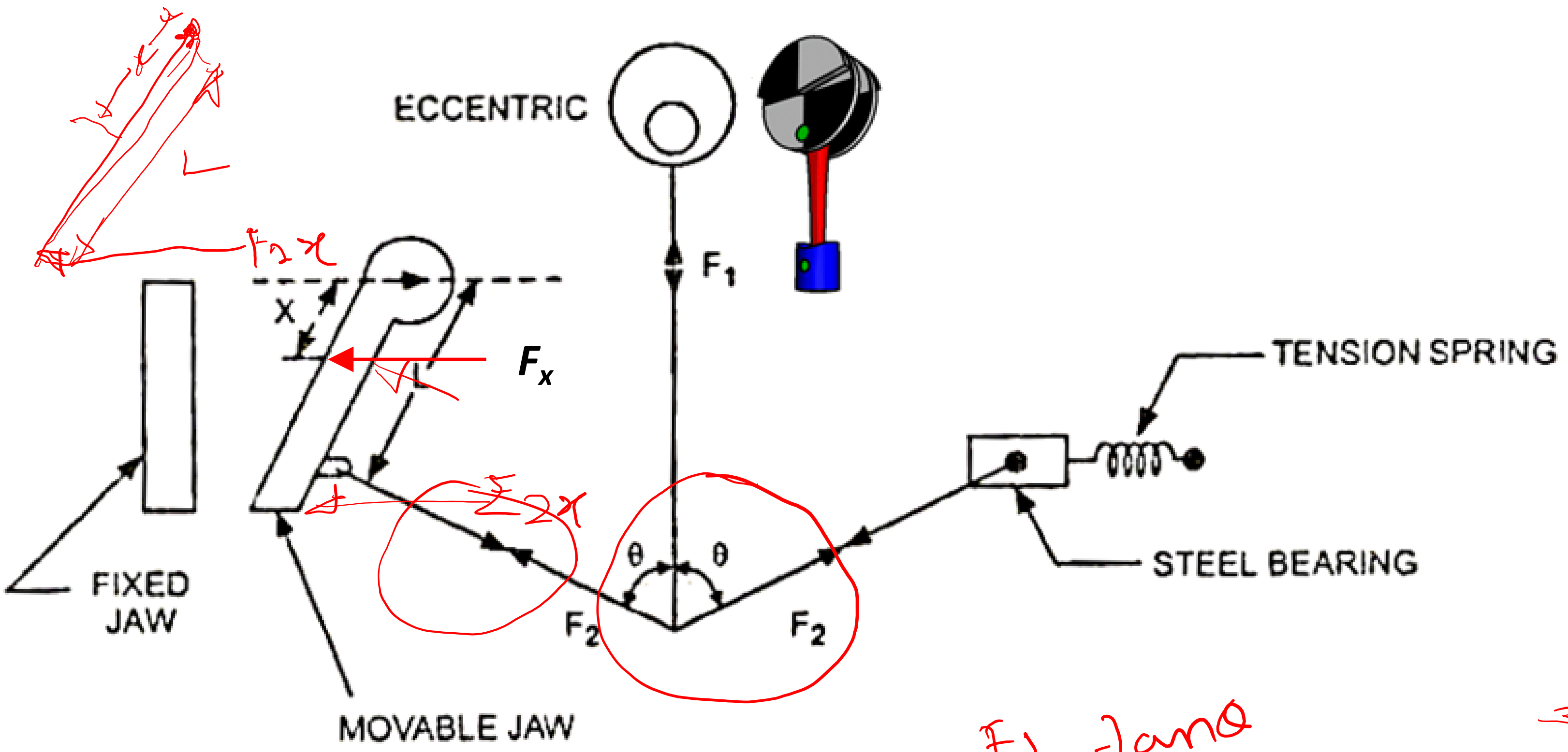




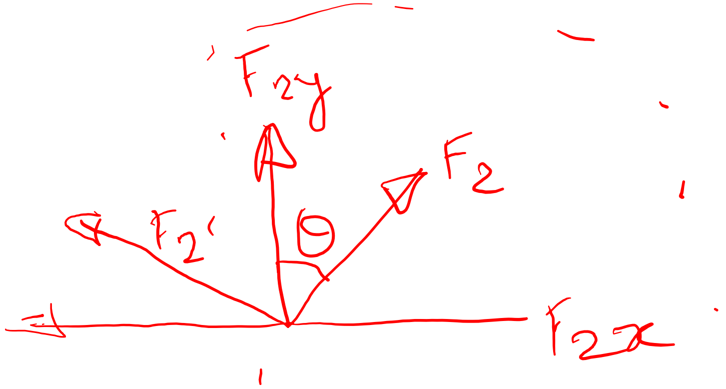
Overhead Eccentric (Single Toggle) Originally restricted to sampler sizes by structural limitations. Now in same size of Blake which it has tended to supersede, because overhead eccentric encourages feed and discharge, allowing higher speeds and capacity, but with higher wear and more attrition breakage and slightly lower energy efficiency.

Dodge Bottom pivot gives closer sized product than Blake,but Dodge is difficult to build in large sizes, and is prone to choking. Generally restricted to laboratory used.

Force on the moving jaw



$$\boxed{F_x = (F_1 L \tan\theta)/2x}$$



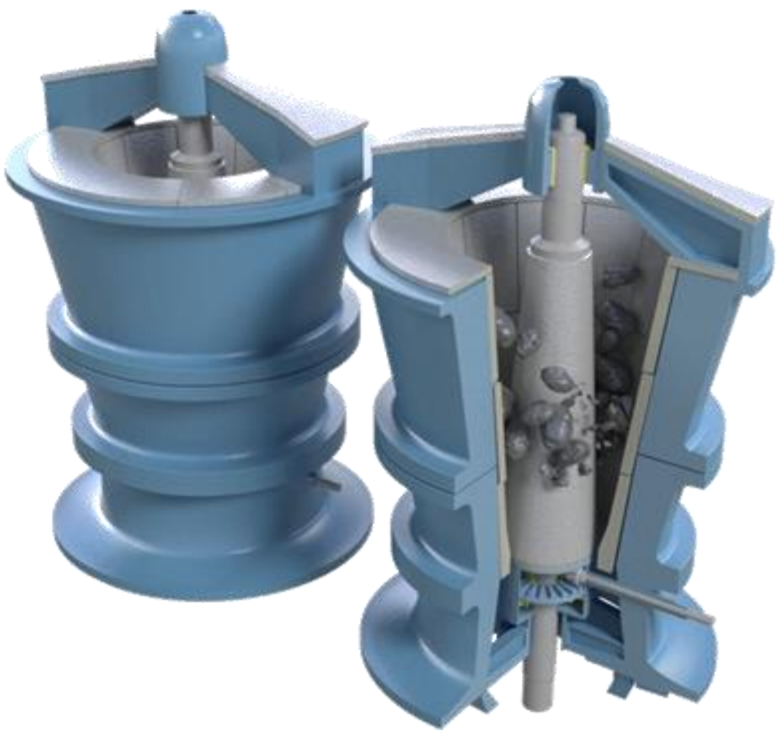
$F_{2x} = F_2 \sin\theta = \frac{F_1}{2}\tan\theta$

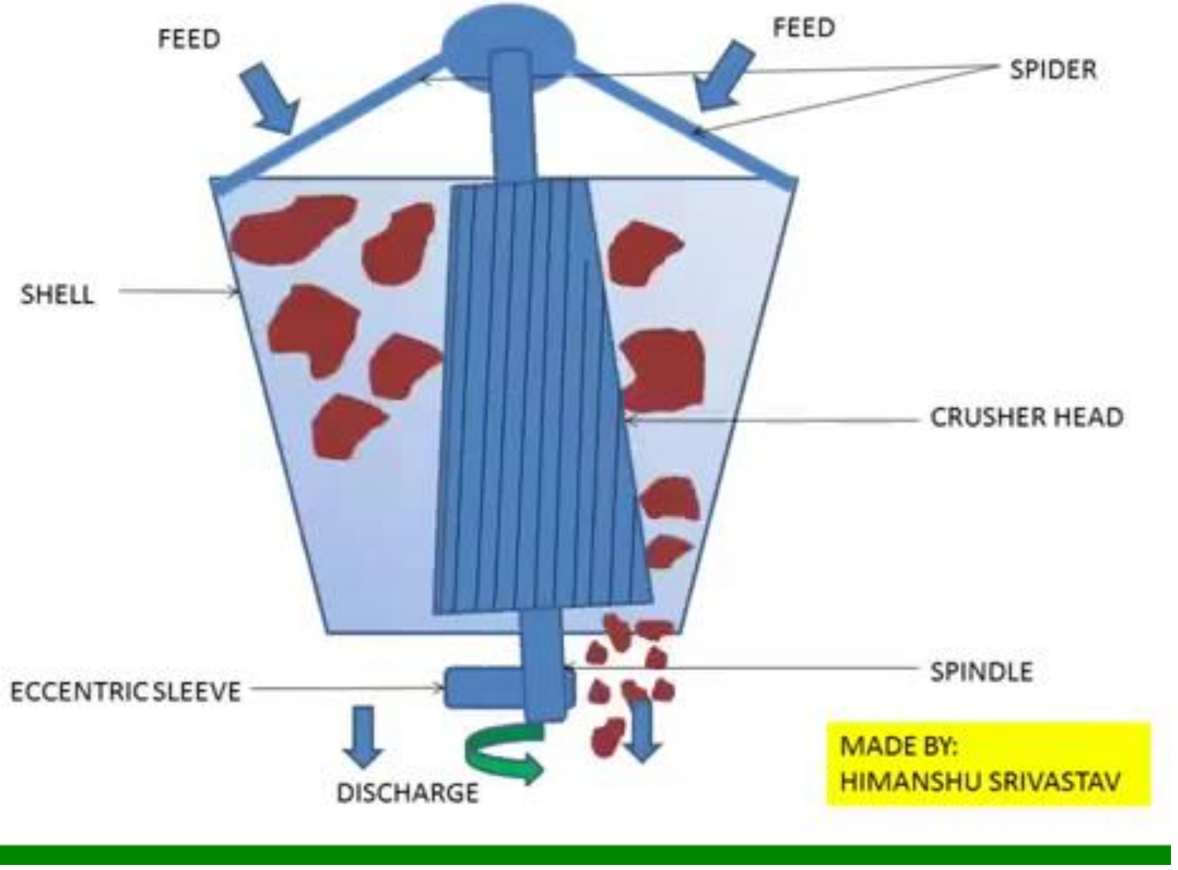
$F_{2y} = F_2 \cos\theta$

Force Balance on the movable jaw of a Double-toggle (Blake) Jaw Crusher

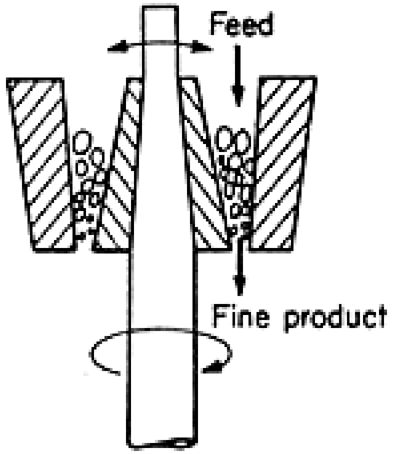
$F_{2x} \times L =$

Gyratory Crusher





It consists essentially of a fixed crushing surface around the axis of which gyrates a movable crushing surface which has the shape of a conical in the erect position. The material to be crushed is fed into the downward converging annular space between these two crushing surfaces. It may be looked upon as a jaw crusher wound around a vertical axis through the mid point of the swing jaw shaft.



Parts of a gyratory crusher -

1. **Plate**: made of cast steel and heavily ribbed as it forms the support of the driving seats.

2. **Spider**: made of cast iron in small machine and of hard steel in large machines, since it is only two armed and is supporting the breaking head, it must be heavy and strongly formed to prevent any wear.

3. **Spindle**: it carries the maximum load and is therefore built of great strength and capable of withstand continuous shock or load.

4. **Suspension bearing**: it is a split steel nut with a downward taper and fits the sleeve of the spindle. It rests on the wearing rings of the spider.

5. **Gear**: it is made mostly of cast steel.

6. **Counter shaft bearing**: it is made as a integral part of the bottom plate and is made extra long and capable of adjustment to take up wear on the gear and

Operation of Gyratory Crusher

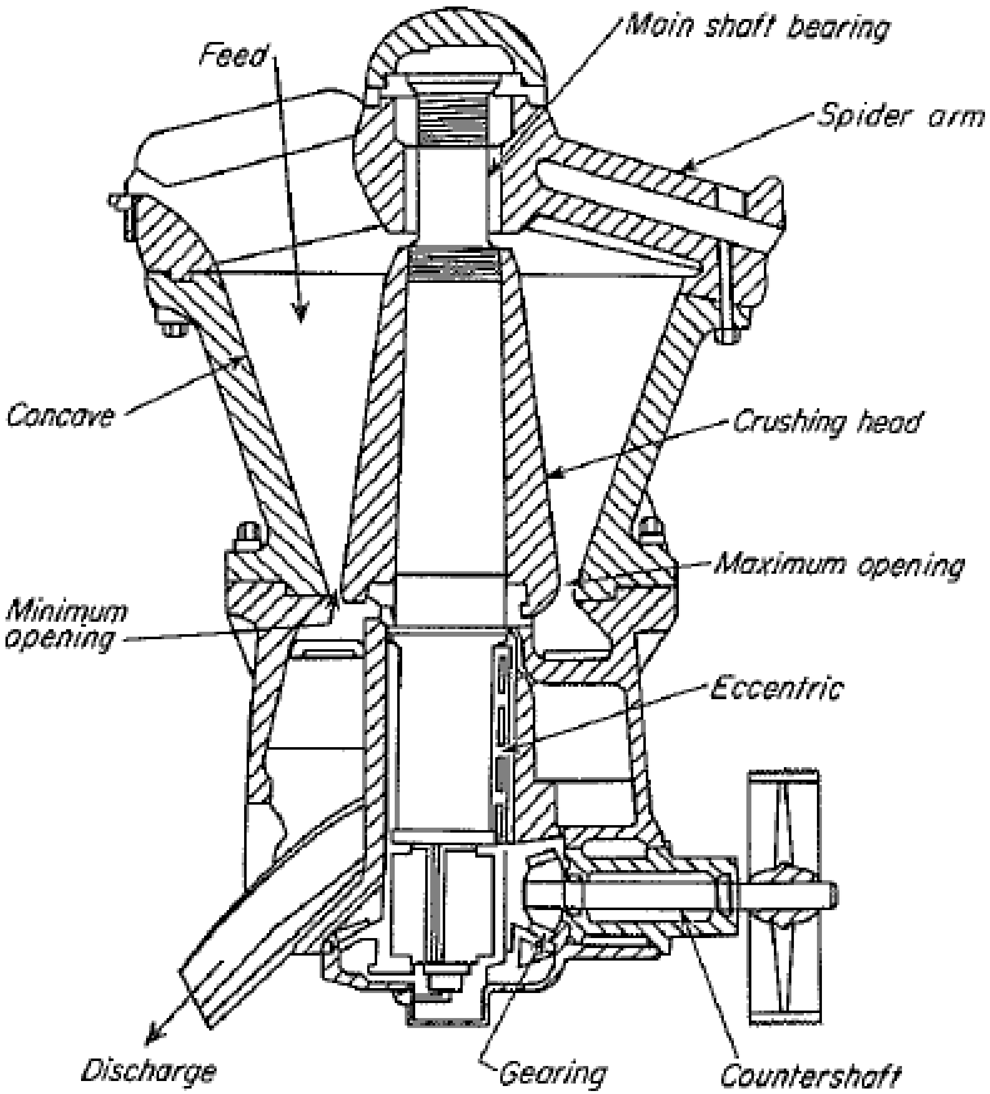
The crusher can be of short spindle, long spindle or fixed spider type. The capacity varies with different types of materials and operations. The capacity is expressed in terms of reduction tons.
Adjustments of gyratory crusher are a) width of discharge opening, b)throw and c) speed.

a) Width of the discharge opening is made by raising or lowering the spindle by means of adjustment nuts. In fixed spindle type the adjustment is made by changing the annular *. Range of adjustment is limited because of change in nip angle.

b) Throw is adjustable by changing the eccentric sleeves.

c) Reduction ratio varies between 3 and with an average of 6.

- ❖ Size of a gyratory crusher = gape x mantle diameter



Size distribution: The crushed product in gyratory crusher is finer. On an average 10% material is larger than open set whereas in jaw crusher its 25%.

Comparison of Jaw and Gyratory Crusher -

1. Cost: gyratory crushers cost 29 to 72% of that of jaw crusher per ton of feed i.e. for same hourly reduction capacity and same reduction ratio, same gape.

2. Installation:
a. cost proportional to weight of the machine
b. foundation – stronger in jaw crusher
c. housing more in gyratory
d. head room more in gyratory
e. vertical height more in gyratory

3. Feeding: easier in gyratory crusher than jaw crusher.

4. Efficiency: reduction per unit energy more in gyratory. Idle HP 40-50% in jaw and 30% in gyratory crusher.

5. Maintenance: higher cost in gyratory, labor cost several times more in gyratory

6. Feed: jaw crusher can take very hard feed, less choking, low angle of nip, so feed not thrown out.

7. Set: easily adjustable in jaw crusher.

8. Lubrication: higher lubricant quantity in gyratory but easily controllable.

Selection: The capacity of gyratory crusher for the same gape is several times more than in jaw crusher, so the selection is on the basis of capacity.

If hourly capacity is tons/gape in sq is
< 0.115 select jaw crusher
> 0.115 select gyratory crusher

Assume gape D=10”
So theoretical length of L= 1.7 D = 17”
L x D = 170 sq. inch
Jaw crusher capacity 10 TPH
So, ratio is 10/170 i.e. < 0.115 hence select jaw crusher.
A gyratory crusher of the same gape size has a capacity of 33 TPH
Ratio= $33/170 > 0.115$

**TPH=Tons Per Hour*

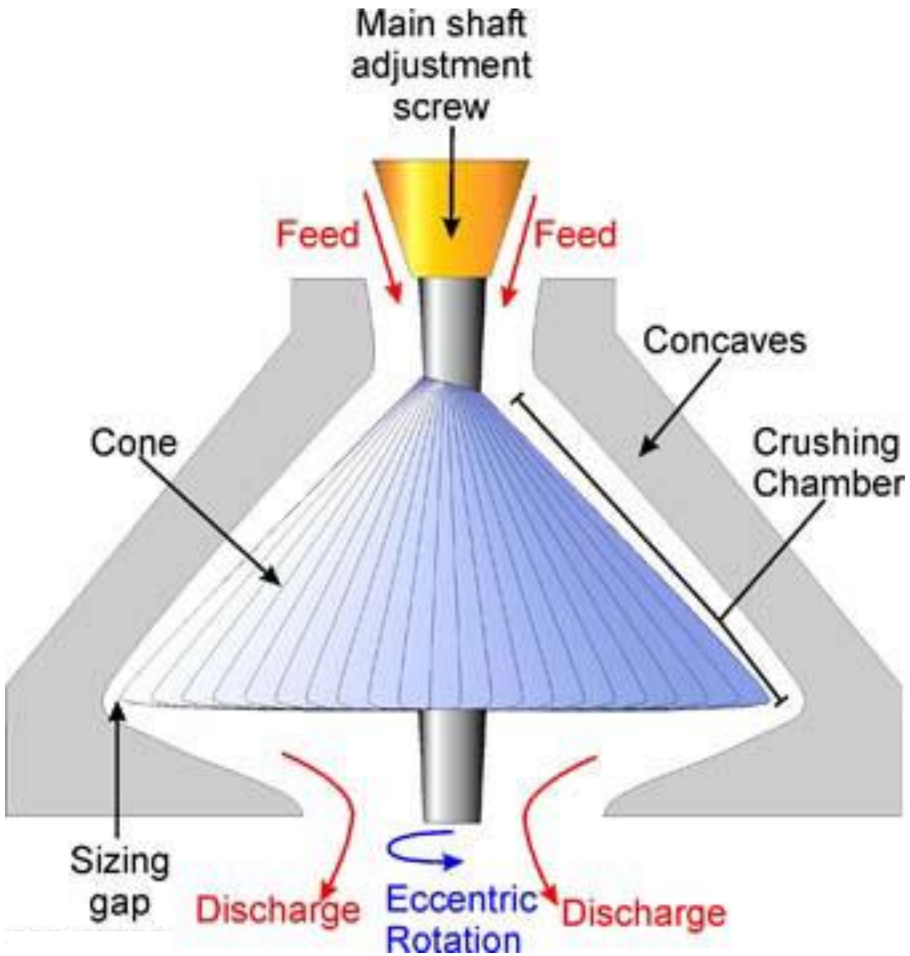
Cone Crusher

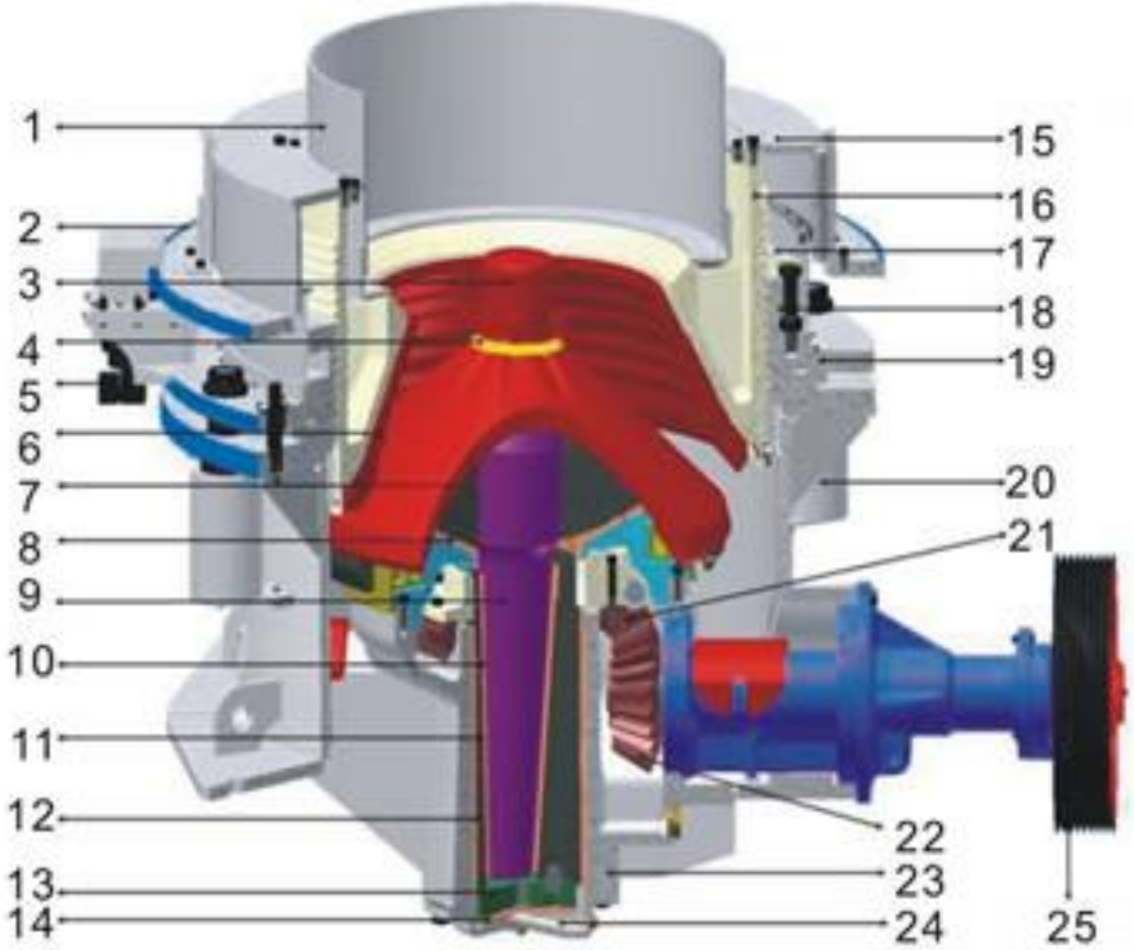
Cone crusher is an intermediate crusher, very similar in construction to the gyratory. The essential difference lies in the shape of crushing head which is very much flared. Another difference is that the spindle is supported on the bottom eccentric bearing. The head gyrates within a truncated cone which is flared outwards in the crushing chamber to allow more volume or area of crushing and more discharge area. The depth of cone is 1/3 rd of the diameter at the base. The throw of the head is 5-9 times the setting as against in gyratory. The large throw allows easy discharge of material. The speed of machine is kept high to crush the material at least twice during its passage. In cone crusher feed opening = 2 x gape.

Cone crushers are usually denoted by the diameters of the bottom of breaking head in inches or feet. Capacities range up to 150 TPH with ¼” closed set to 900 TPH with 2 ½” closed set. RPM of the mills varies between 435-700 and power up to 300 HP.

To prevent damage to the crushing surface due to tramp iron etc the bowl is usually held by spring which release when hard material falls inside the crusher.

Cone Crusher Video: https://www.youtube.com/watch?v=RGbNv88r5Wc



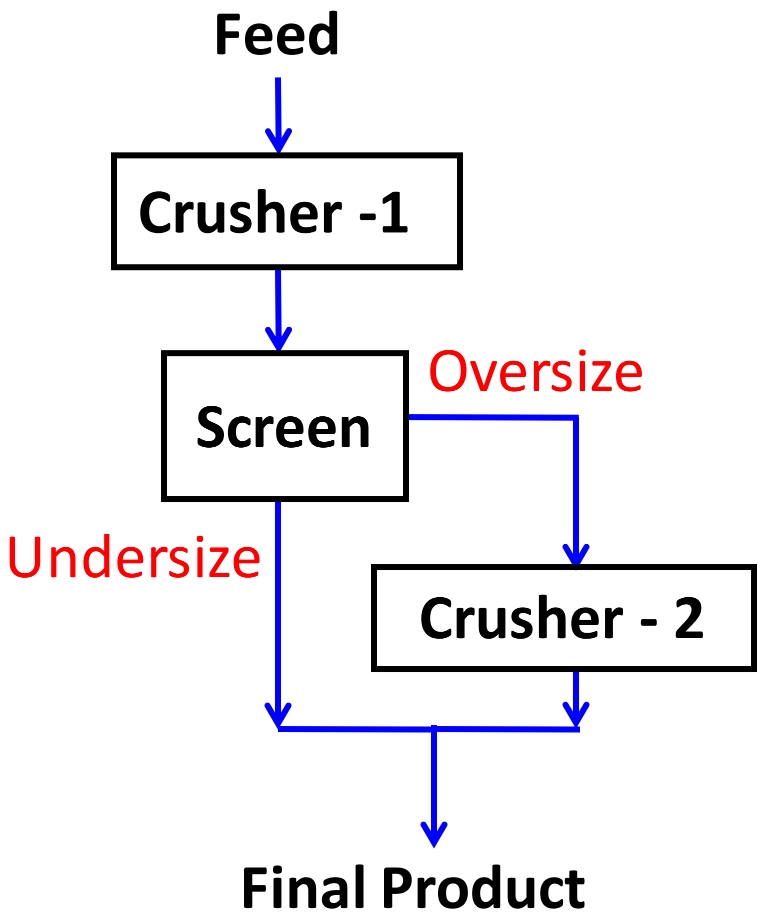


1. Feed Hopper
2. Drive Ring Pinion
3. Distribute Plate
4. Pressure Ring
5. Hydraulic Moto
6. Concave
7. Mantle
8. Socket Liner
9. Main Shaft
10. Eccentric Bushing
11. Eccentric
12. Main Frame Bushing
13. Upper Thrust Plate
14. Bottom Thrust Plate
15. Adjusting Cap
16. Adjusting sets
17. Locked Ring
18. Locked Cylinder
19. Supporting Sets
20. Tramp Release Cylinder
21. Gear
22. Pinion
23. Main Frame
24. Main Frame Bottom Cov
25. Pulley

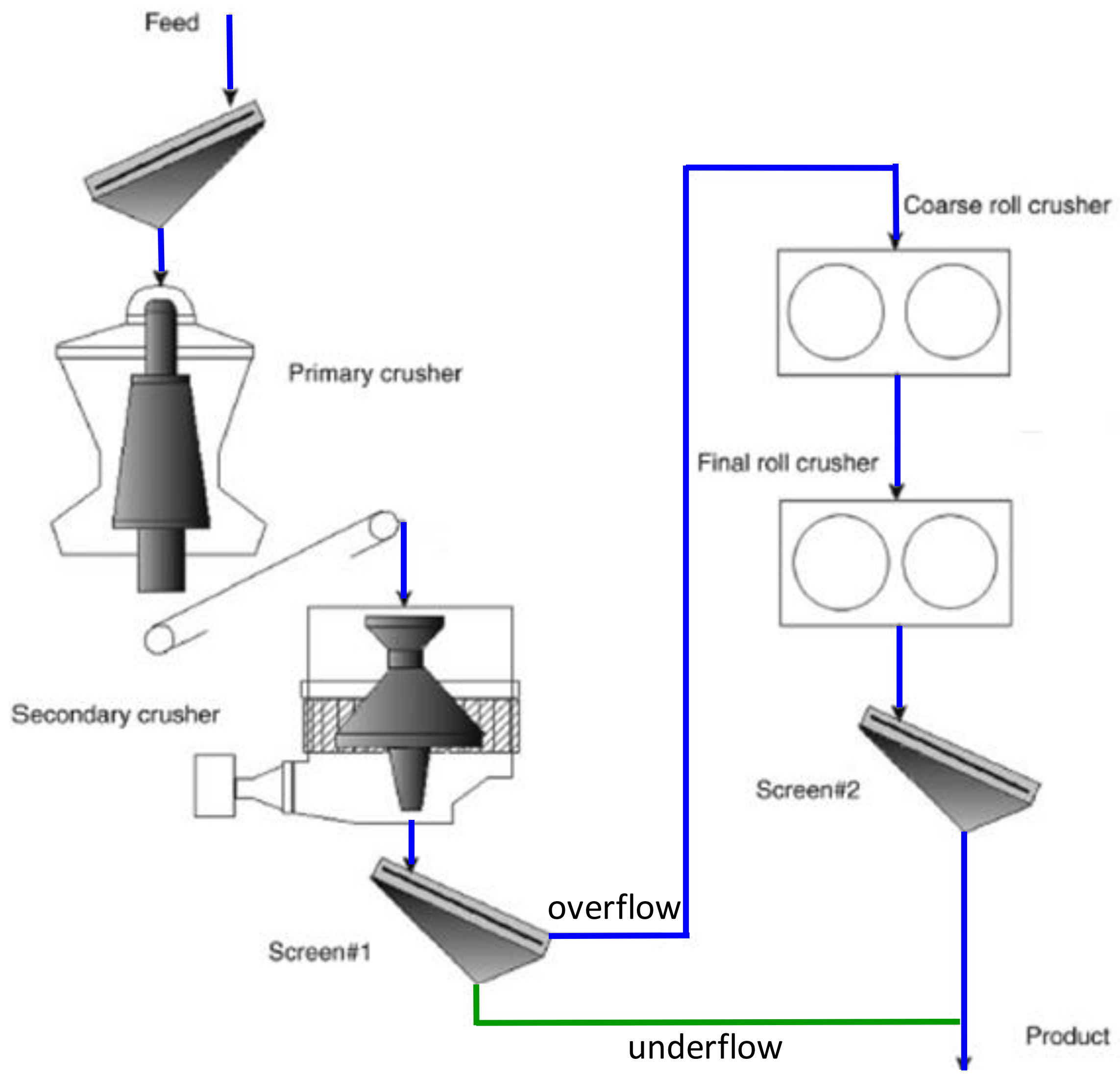
Open Circuit and Closed Circuit Grinding

Open Circuit crushing operation

- In open circuit crushing the material is fed to the crusher at a rate to produce the finished product in one pass through the crusher.
- The material to be ground must remain in the crusher until every particle has been reduced to a minimum size, considerable energy is thereby wasted in regrinding the material that's already of finished size.
- Open circuit crushing is particularly used when simplicity or layout may be the determining factor.



Open Circuit



Problem 5

Crushed product of a primary crusher having 57% of +1″ is sent to 1″ screen. The overflow of the screen is again crushed in a secondary crusher to –1″ size and sent along with underflow of the screen as final product.

(a) Draw the crushing circuit with all details.

(b) Calculate the rate of the material crushed in secondary crusher if the effectiveness of the screen is 80% based on oversize and undersize material

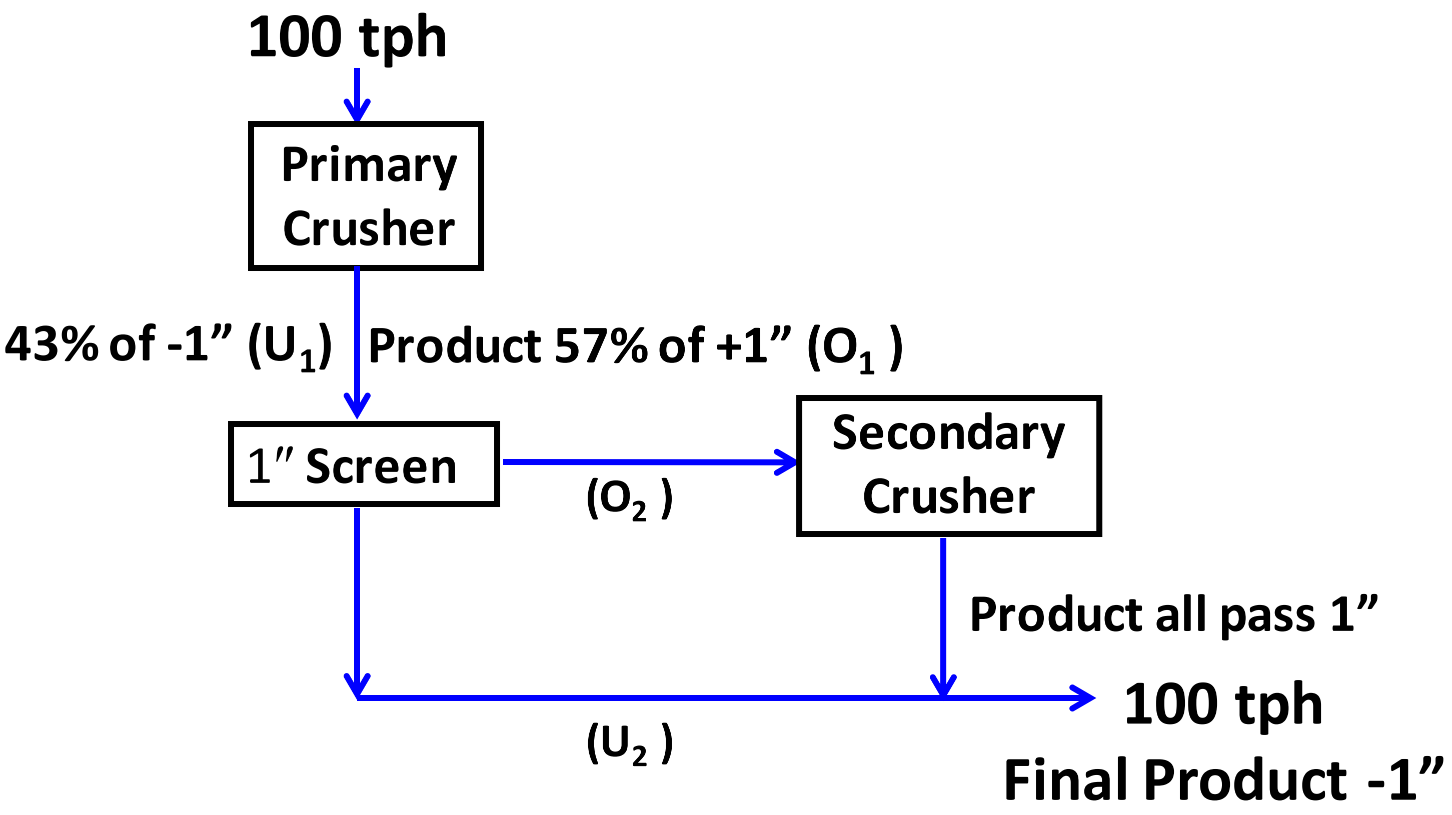
The feed rate to the primary crusher is 100 tons/hr.

Solution:

As the product from the overflow from the screens are not fed back to the same crusher the entire crushing cycle is an open circuit operation

Solution (contd):

The crushing circuit for the problem is shown in the Figure



Weight of +1 material in feed to screen = 57% of 100 tons/hr = 57 tons/hr

Screen efficiency $\eta = \dfrac{\text{Weight of material present in feed}}{\text{Weight of overflow obtained from the screen}} = 0.8$

(Based of oversize material)

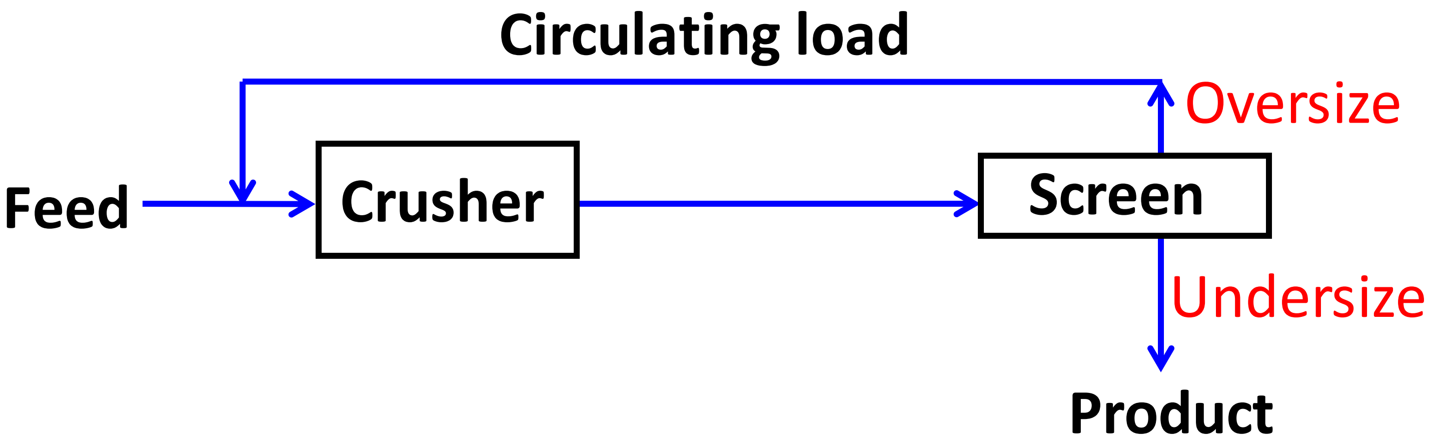
$\therefore$ Weight of overflow obtained from the screen $= \dfrac{57}{0.80} = 71.25 \text{ tons/hr } (O_2)$

Material crushed in secondary crusher = 71.25 tons/hr (O_2) (Ans)

Underflow from the screen = (100−71.25) = 28.75 tons/hr (U_2)

Close Circuit

- In Close circuit operation the new feed is fed to the crusher, crusher product is fed to the screen.
- The overflow from the screen is fed back to the same crusher.
- The quantity of the overflow material fed back to the crusher is called **circulating load**.



Problem 6

The details of crushing plant employing gyratory crusher are as follows:

2″ square screen is in closed circuit with crusher

New feed to crusher=100 tons/hr

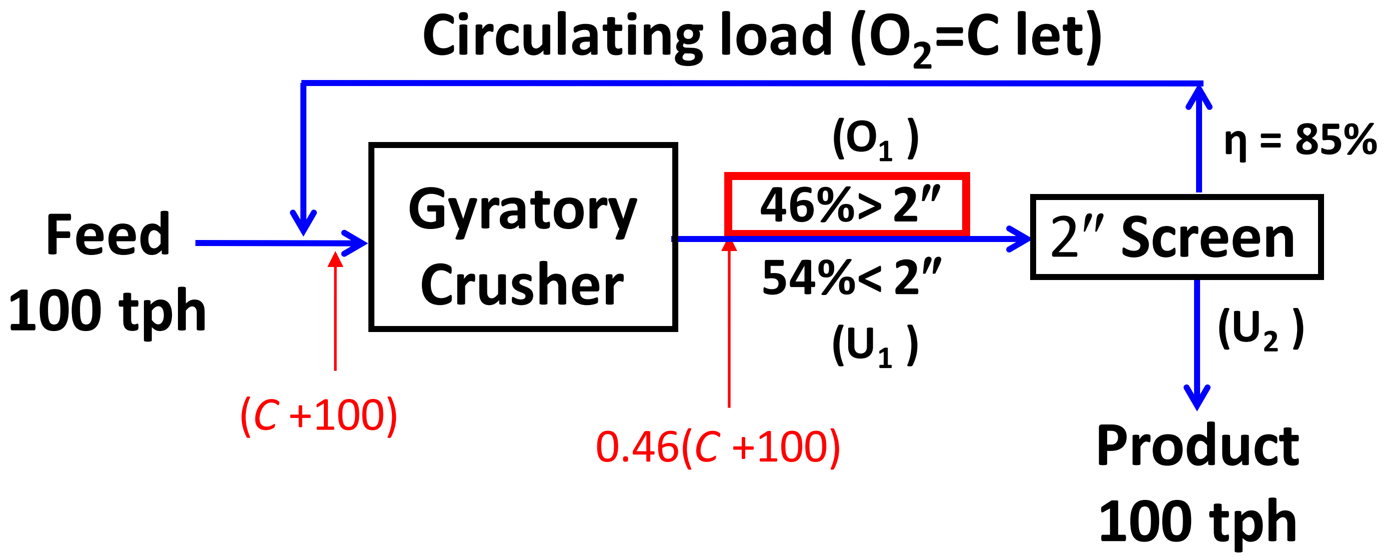
Crusher product contains 54% < 2″ and 46% > 2″ fed to screen

Screen efficiency=85% (based on oversize)

a. Draw the flow diagram and find circulating load.
b. Also find circulating load if the screen efficiency is based on undersize.

Solution:

a)



Let C tons/hr be the circulating load.

Load on the gyratory crusher = $C+100$ tons/hr

$+\,2''$ material in crusher product $=0.46(C+100)$ tons/hr

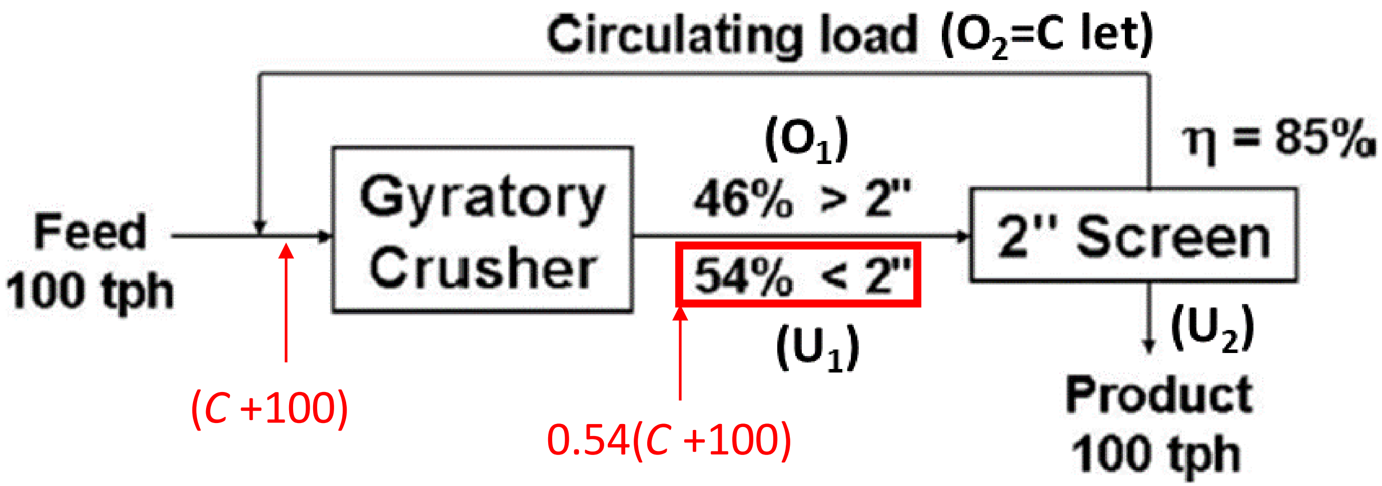
Screen efficiency

(Based of oversize material)

$$\eta = \frac{+2'' \text{ material in feed to the screen}}{+2'' \text{ fraction obtained from the screen}} = 0.85$$

$$\therefore \quad \frac{0.46(\mathrm{C}+100)}{\mathrm{C}} = 0.85 \quad \Rightarrow \quad \mathrm{C}=117.95 \text{ tons/hr}$$

Circulating load if the screen efficiency is based on oversize = 117.95 tons/hr (Ans (a)) b)



Now screen efficiency is defined based on undersize

−2" material in crusher product = 0.54(C +100) tons/hr

Screen efficiency

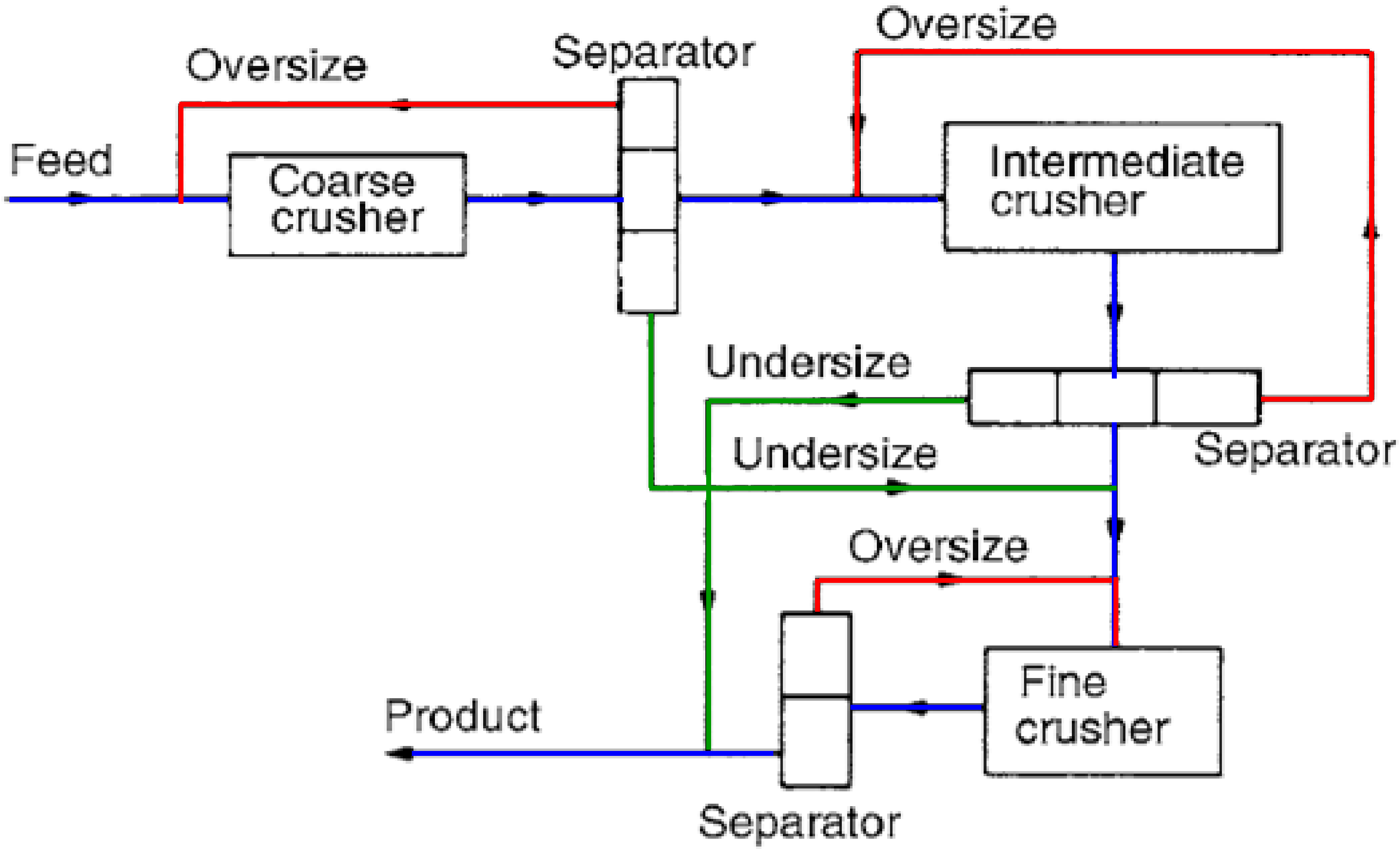
(Based of undersize material)

$$\eta = \frac{\text{-2}'' \text{ fraction obtained from the screen}}{\text{-2}'' \text{ material present in the feed to the screen}} = 0.85$$

$$\therefore \quad \frac{100}{0.54(\mathrm{C}+100)} = 0.85 \quad \Rightarrow \quad \mathrm{C}=117.86 \text{ tons/hr}$$

Circulating load if the screen efficiency is based on undersize = 117.86 tons/hr (Ans (b))

Flow diagram for closed circuit grinding system



Advantages of Close Circuit Operation-

1) increases the quantity of finished material and capacity
2) the size distribution is more uniform
3) power consumption decreases

The advantages of close circuit grinding arise from

i) marked increase in the near finished material in the feed,
ii) minimize the production of fines
iii) more rapid travel (resistance time is low), hence shorter time of pass,
iv) uniformity of size and

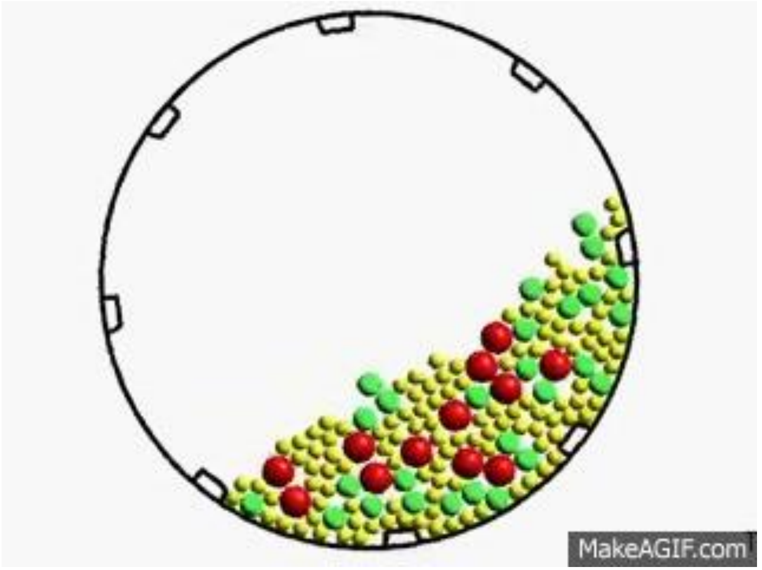
Tumbling Mills

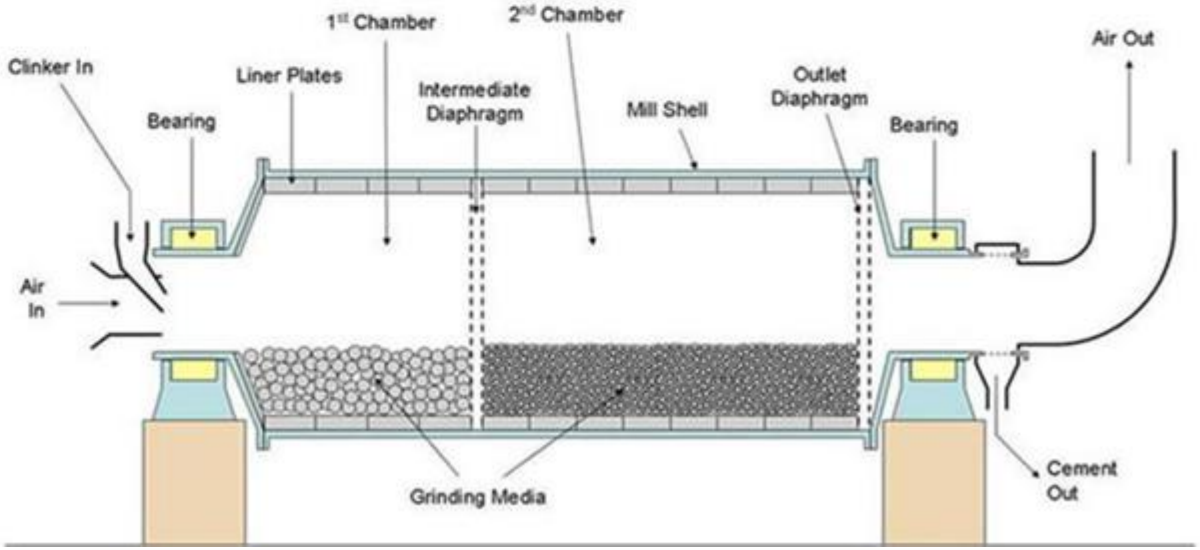
- In a tumbling mill the grinding of the material is affected by tumbling action of media which may be balls, rods or pebbles contained in a steel, rubber or stone lined steel shell.
- The **media used** in the charge generally describes a tubular mill. Thus, the medium could be **steel or cast iron balls** when the mill is designated as a ***ball mill***, or it could be steel rods where the mill is known as a ***rod mill***.
- The ball mill differs from the tube mill by its shortness in length and as a rule doesn't exceed the diameter.
- In conventional **ball mills** large balls are used on a coarse feed to produce comparatively coarse product.
- The **tube mills** are long compared to ball mills, L/D > 2.
- The compartment mill is a combination of the above types, consisting of a cylindrical shell divided into a number of sections by perforated partitions.
- Coarse grinding takes place at one end where coarse media is used and finishing is given at the other end with smaller balls.

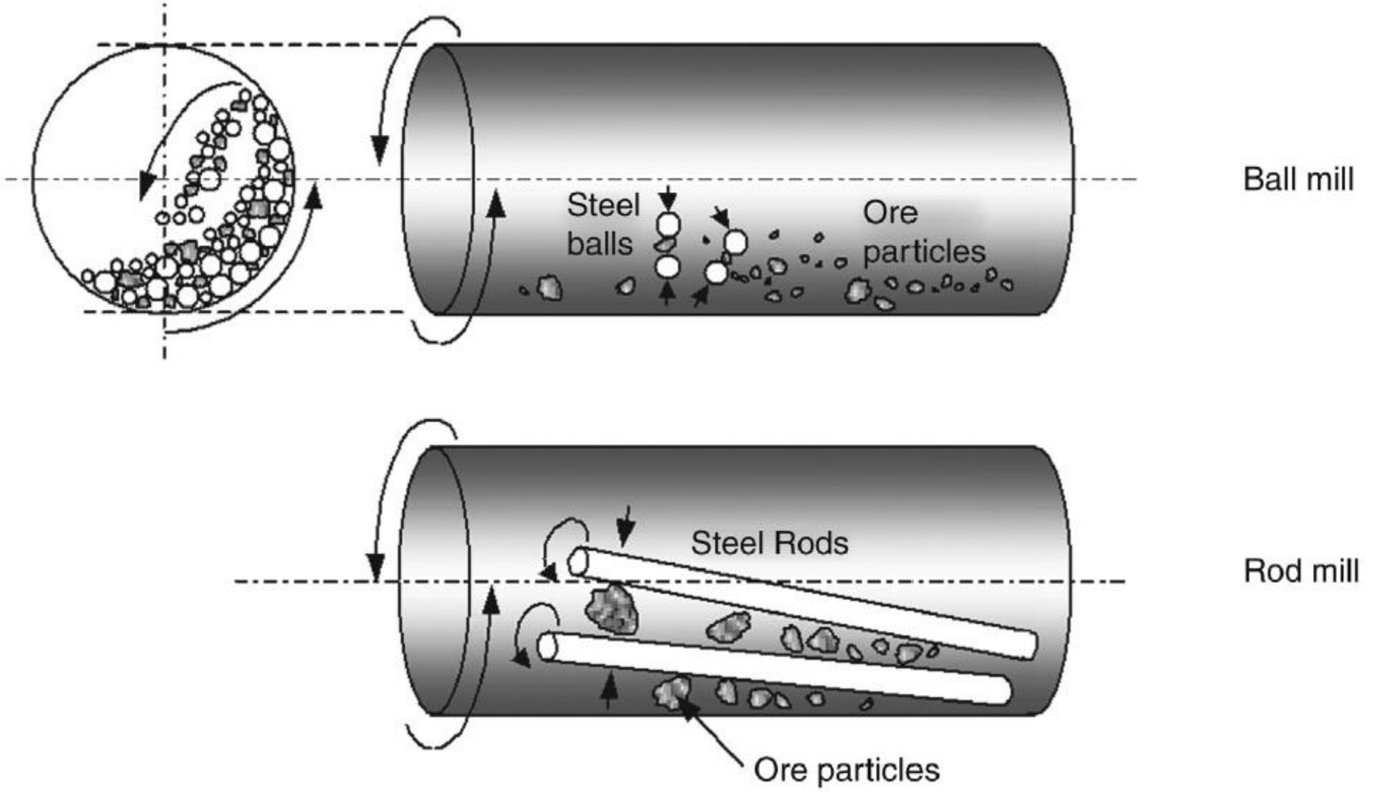
- Due to rotation of the mills the bigger balls tend to collect at the larger diameter portion towards the feed end and the small balls at the discharge end where the diameter is relatively small. It is thus counterpart of compartmental mills.

- Another type is **rod mill** which delivers a more uniform and granular product. The pebble is a fluid with ceramic pebbles as the grinding media and may be lined with ceramic or any other non-metallic liner.

- **Autogenous mills** are one of the latest developments and are gradually coming into use in different industries where the specific gravity of the material is high and contamination with the grinding media is not desired. In principle the balls are replaced by large lumps of the ore itself which act as media.

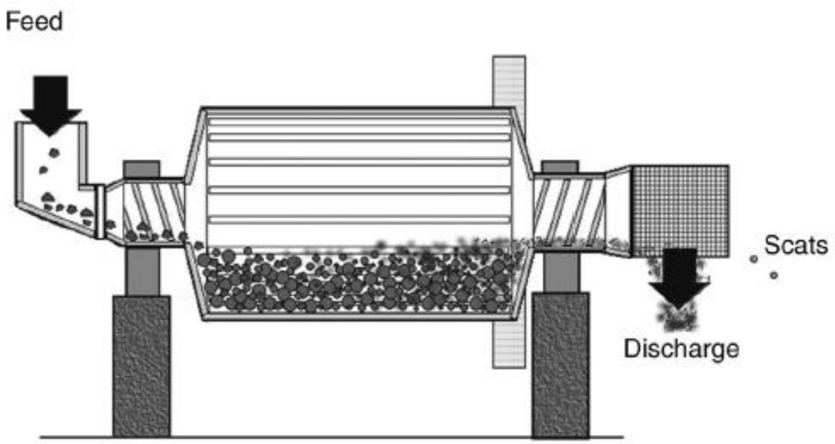




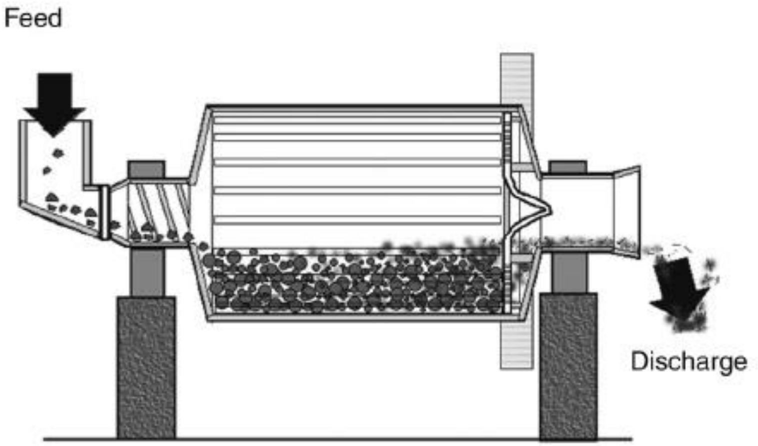


Mechanism of Crushing in Tubular Mills

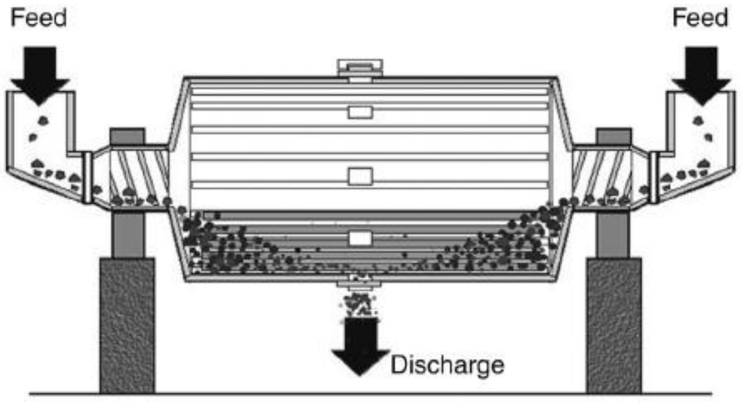
Tubular Mill Types based on discharge



A: Overflow discarge mill



B: Diaphragm or grate discharge mill



C: Centre-periphery discharge mill

Basic design parameters of the tubular mills

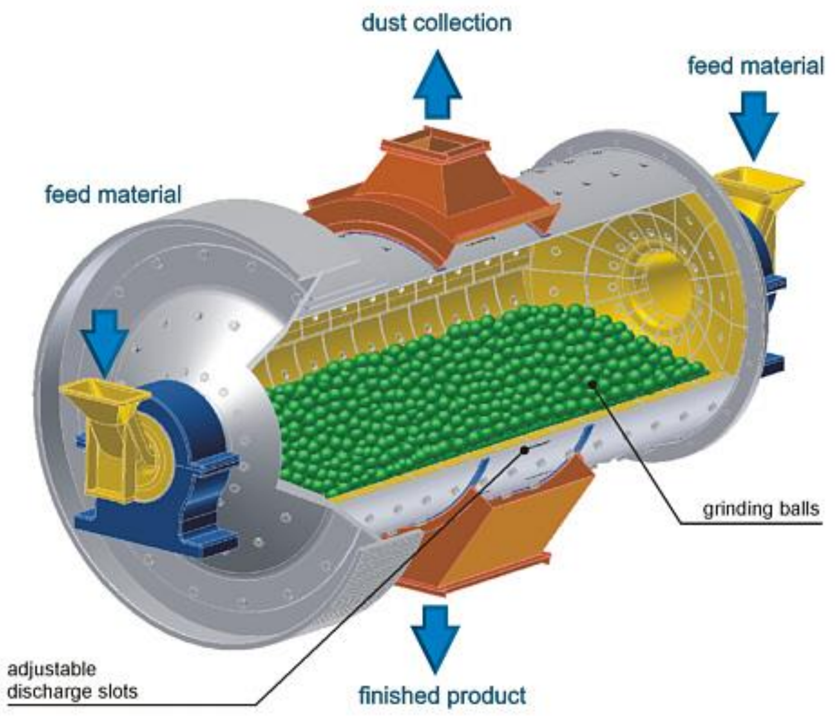
1. **Size:** diameter × length,
2. **Feed system:** one hopper feed, diameter 40–100 cm at 30° to 60° entry angle and top of feed hopper at least 1.5 m above the center line of the mill for ease of entry of feed,
3. **Feeder:** single or double helical scoop feeder or a spout feeder,
4. **Discharge system:** one exit unit, about 5–110 cm lower than the center line for overflow mills.

Tumbling mill characteristics

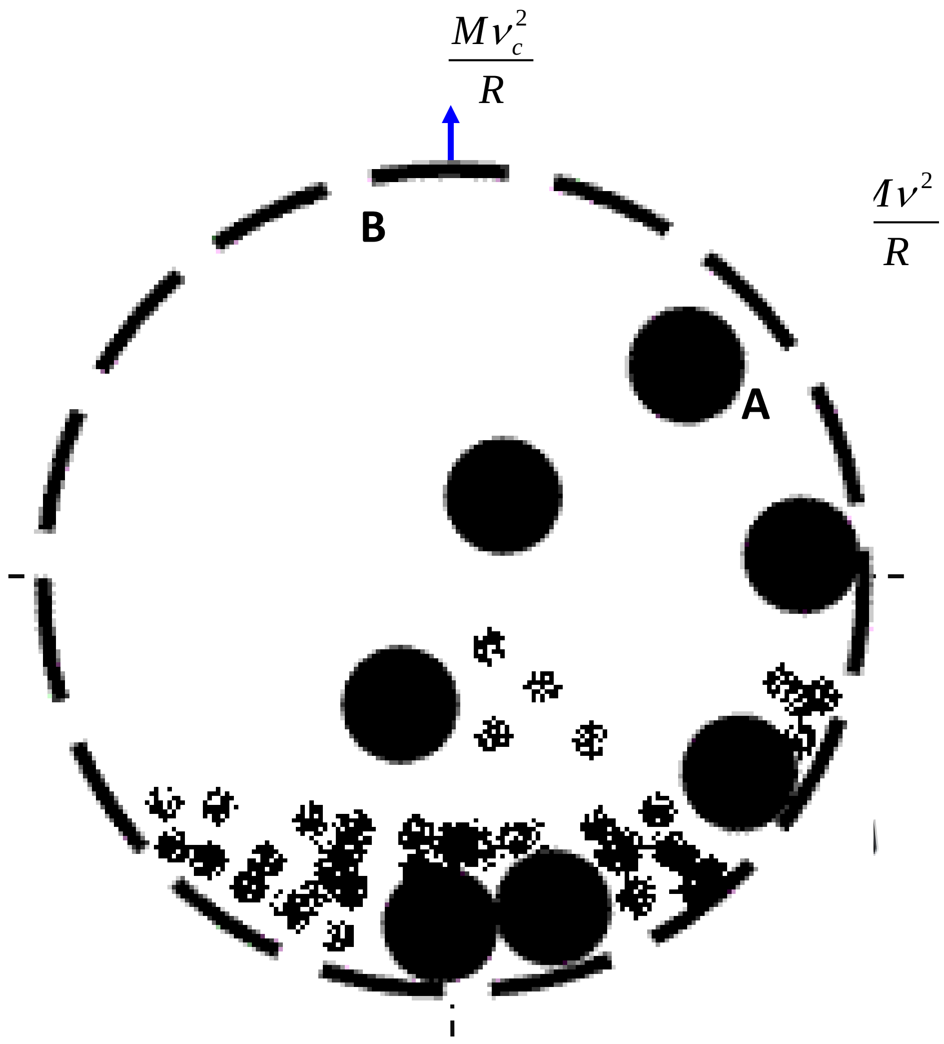
Parameter	Rod Mill	Ball Mill	Autogenous Mill
Length/diameter ratio*	1.4–1.8	0.5–3.5	0.25 to 0.5:1
Feed size	2.5 cm max	-1.9 cm	Course ore
		-1.25 cm to 0.9 cm	Normal ore
Reduction ratio	15:1 to 20:1	20: 1 to 200:1	

*Multicompartment mills: length/diameter ≈ 5.

Ball Mills


dust collection
feed material
feed material
grinding balls
adjustable
discharge slots
finished product

Mill Rotation and Critical Speed



If we consider ball at position A in equilibrium

$$Mg\cos\theta = \frac{Mv^2}{(R-r)}$$

M = mass of the ball
v = linear velocity (m/s) of the ball
R, *r* = radii of the mill and ball, respectively

At the critical speed (v_c) of the ball mill $\theta = 0°$ $\cos\theta = 1$

$$g = \frac{v_c^2}{(R-r)} = \frac{4\pi^2(R-r)^2\omega_c^2}{(R-r)\times 3600}$$

$$Where\ v = 2\pi(R-r)\omega/60 \text{ rpm}$$

$$\omega_c = \frac{42.3}{\sqrt{(D-d)}} \quad rpm \qquad ......(8)$$

Problem 4

In a ball mill of 2000 mm diameter, 100 mm diameter steel balls are being used for grinding at a speed of 15 rpm. At what speed will the mill have to run if the 100 mm balls are replaced with 50 mm balls, all the other conditions remaining same?

Solution:

Given

Diameter of the ball mill = 2000mm = 2 m
Diameter of the balls = 100 mm = 0.1 m

Grinding at a speed of 15 rpm

When

Diameter of the balls = 50 mm = 0.05 m

Find the speed of ball mill

Solution (contd.):

Critical speed of the ball mill

$$\omega_c = \frac{42.3}{\sqrt{(2-0.1)}} = 30.69\ rpm$$

As the mill is running at 15 rpm, the percent of critical speed the mill is operated

$$= \frac{15}{30.69} \times 100 = 48.9\ \%$$

Critical speed of the ball mill if the balls are of 50 mm diameter

$$\omega_c = \frac{42.3}{\sqrt{(2-0.05)}} = 30.26\ rpm$$

Operating speed of the ball mill= $30.26 \times \frac{48.9}{100} = 14.79\ \ rpm$ (Ans)

Wet vs. Dry Grinding

There are two types of grinding practices i.e. “wet” and “dry” grinding. The selection of the process is mainly governed by type of material and use of material. If water changes physical or chemical properties of the material, dry grinding is a must. Conversely, if the finished material is to be used as a suspension in water and classified by hydraulic method, then we prefer wet grinding.

Wet grinding has got the following advantages:

1. Lower power consumption per ton of product (15-20%)
2. Higher capacity per cubic foot of mill volume
3. Elimination of dust problem
4. Use of simple handling methods such as pumps, pipes *etc*
5. Wet screening or hydraulic classification can be used for closer product control.

The following two points are in favor of dry grinding:

1. lower wear rate of mill liners and grinding media compared to wet grinding
2. Higher percentage of fines in mill product. In many cases, this is desirable e.g. cement, talcum powder, pulverized coal *etc.*