Assignment (Laplace Transform)

1) Discuss the piecewise continuity of the following functions:

a) $f(t)=\dfrac{1}{t-2},\ t\neq 2$

b) $f(t)=\begin{cases} 2t, & t\leq 1 \\ 1+t^2, & t>1 \end{cases}$

c) $f(t)=\begin{cases} \dfrac{1-e^{-t}}{t}, & t\neq 0 \\ 0, & t=0 \end{cases}$

d) $f(t)=\begin{cases} t\sin\left(\frac{1}{t}\right), & t\neq 0 \\ 0, & t=0 \end{cases}$

2) Show that the function $f(t)=te^{t^2}\sin\left(e^{t^2}\right)$ possesses a Laplace transform.

3) Find Laplace transform of the following functions:

a) $e^{-t}\cos^2 t,\ \ t>0$

b) $f(t)=\begin{cases} 0, & 0\leq t<2 \\ e^{at}, & t\geq 2 \end{cases}$

c) $f(t)=|\sin t|,\ \ t>0$

d) $f(t)=tH(t-a),\ \ t>0$

4) Find Laplace transform of the following functions:

(a) $f(t)=u(t-1)+u(t-2)-2u(t-3)$

(b) $f(t)=\begin{cases} 0, & t<2 \\ t-2, & t\geq 2 \end{cases}$

(c) $f(t)=\sin(t-\pi)\,u(t-\pi)$

5) Find the inverse Laplace transform of the following functions:

a) $F(s)=\dfrac{s+3}{\left(s^2+6s+13\right)^2}$

b) $F(s)=\dfrac{4s+5}{(s-1)^2(s+2)}$

c) $F(s)=\dfrac{e^{-\pi s}}{s^2-2}$

d) $F(s)=\dfrac{2s^2+3}{(s+1)^2\left(s^2+1\right)^2}$

6) Find the inverse Laplace transform of the following functions

a) $F(s)=\dfrac{3e^{-s}}{s^2}$

b) $F(s)=\dfrac{2\,e^{-3s}}{s^2+4}$