

$$N_{ca} = \text{Capillary no.} = \frac{\text{viscous force}}{\text{surface tension force}} = \frac{\mu VL}{\sigma L} = \frac{\mu V}{\sigma}$$

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$$\text{Viscous force} = \tau A = \frac{\mu dV}{dy} A \propto \frac{\mu V^2 L^2}{L} \propto \mu VL$$

$$\text{Fraude no.} = \frac{V}{\sqrt{gL}}$$

$$\text{Weber no.} = \frac{\text{Inertia force}}{\text{surface tension force}}$$

$$(Fr No)^2 = \frac{\text{Inertia}}{\text{gravity}} = \frac{\rho V^2 L^2}{mg} = \frac{\rho V^2 L^2}{\rho L^3 g} = \frac{V^2}{Lg}$$

### Complex Potential and Complex Velocity

#### Complex Potential

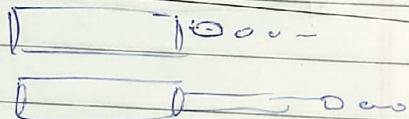
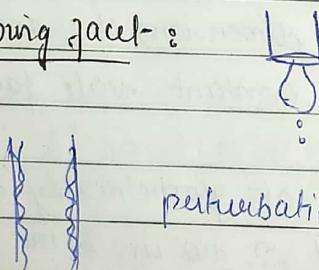
$$F(z) = \phi(x, y) + i\psi(x, y), \quad i^2 = -1$$

$$\frac{dF(z)}{dz} = \frac{\partial \phi}{\partial x} + i \frac{\partial \psi}{\partial x}, \quad \text{or} \quad \frac{\partial \psi}{\partial y} - i \frac{\partial \phi}{\partial y}$$

$$\frac{df(z)}{dz} = \lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z - z_0} = \frac{f(x+iy_0) - f(x_0+iy_0)}{x - x_0}$$

$$\frac{f(x_0+iy_0) - f(x_0+iy_0)}{i(y-y_0)}$$

Dripping faucet:



perturbation of particular wavelength amplified

Rayleigh instability

$$\text{Complex velocity } w(z) = F'(z) = \frac{\partial \psi}{\partial y} - i \frac{\partial \phi}{\partial y} = u(x, y) - i v(x, y)$$

$$\bar{w}w = (u-i v)(u+i v) = u^2 + v^2$$

(I)

$$F(z) = Cz$$

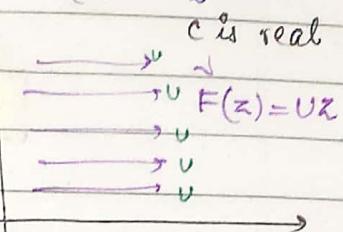
$$w(z) = \frac{dF}{dz} = C$$

$$\underline{u(x,y) - iv(x,y) = C}$$

Assume,  $C$  is real  $\Rightarrow v = 0$

(II)

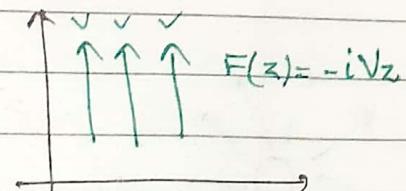
$$F(z) = Uz$$



(III)

$$F(z) = -icz$$

$$w(z) = u - iv \Rightarrow \begin{cases} v = c \\ u = 0 \end{cases}$$



not applicable for boundary layer flow

$$e^{i\theta} = \cos\theta + i\sin\theta$$

(IV)

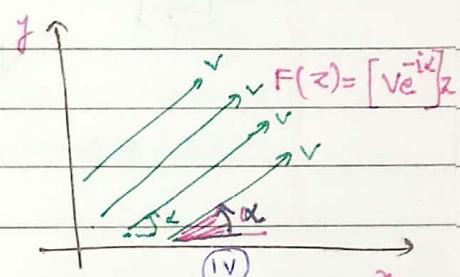
$$F(z) = (Ce^{-i\alpha})z$$

$$= C(\cos\alpha - i\sin\alpha)z$$

$$w(z) = C(\cancel{\cos\alpha} e^{-i\alpha}) = C(\cos\alpha - i\sin\alpha)$$

$$u = C\cos\alpha$$

$$v = C\sin\alpha$$



(V)

$$F(z) = Clm z$$

$$w(z) = \frac{C}{z} = \frac{C}{x+iy}$$

$$u = \cancel{\frac{Cx}{x^2+y^2}} = C(x-iy) = \frac{(x-iy)}{(x^2+y^2)}$$

$$v = \cancel{\frac{Cy}{x^2+y^2}} = -\frac{iy}{x^2+y^2}$$

$$\text{Take } z = x+iy$$

$$= r\cos\theta + ir\sin\theta$$

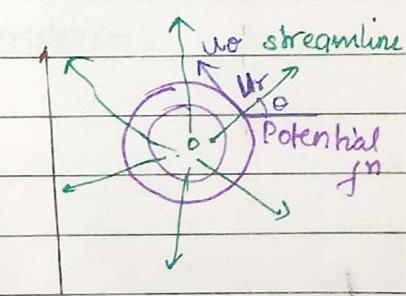
$$= re^{i\theta}$$

$$\ln z = \ln r + i\theta$$

$$\phi + i\psi = Clm r + iCo$$

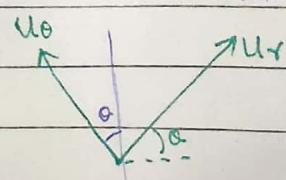
$$\phi = Clm r \quad \cancel{x \text{ constant } r}$$

$$\psi = Co \quad \cancel{x \text{ constant } \theta}$$



$$u = U_r \cos\theta - U_\theta \sin\theta$$

$$v = U_r \sin\theta + U_\theta \cos\theta$$



$$u - iv = (u_r \cos\theta - u_\theta \sin\theta) - i(u_r \sin\theta + u_\theta \cos\theta)$$

$$= u_r(\cos\theta - i\sin\theta) + iu_\theta(-\sin\theta + i\cos\theta)$$

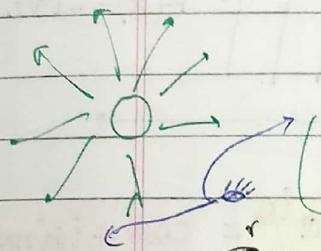
$$u - iv = (u_r - iu_\theta)e^{-i\theta}$$

(u\_r - iu\_\theta)

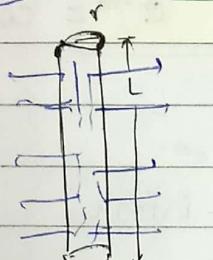
--- (1)

$$\omega = \frac{c}{r} = u - iv$$

$$\Rightarrow u - iv = \frac{ce^{-i\theta}}{r} \quad \text{--- (1)}$$



$$\text{Equating (1) \& (1)} \quad u_r - iu_\theta = \frac{c}{r}$$



$$Q = m^3/s$$

$$A = 2\pi rL$$

$$V_r = \frac{Q}{A} = \frac{Q}{2\pi rL}$$

$$\Rightarrow V_r = \frac{C}{r}$$

$$u_\theta = 0$$

$$V_r = \frac{C}{r}$$

(strength of source)

$m$ : volume flow rate from 1 to the  
source / sink per unit depth

✓ Volume flow :  $\int \hat{n} \cdot \vec{v} dA$

Any surface around origin

$$= \int_0^{2\pi} u_r (r d\theta L)$$

- Volume flow rate per unit depth  $\Rightarrow \frac{Q}{2\pi L}$

$$\begin{aligned} \Rightarrow \text{volume flow rate} &= \int_0^{2\pi} u_r r d\theta \\ &= \int_0^{2\pi} \frac{C}{r} r d\theta \end{aligned}$$

$$\boxed{\frac{m = Q}{L} = 2\pi C}$$

$$\boxed{C = \frac{m}{2\pi}}$$

$$\boxed{F(z) = \frac{m}{2\pi} \ln z}$$

source at  $z = 0$ ,  $F(z) = \frac{m}{2\pi} \ln z$

source at  $z = z_0$   $F(z) = \frac{m}{2\pi} \ln(z - z_0)$

(vi)  $F(z) = -iC \ln z$

$z = r e^{i\theta}$

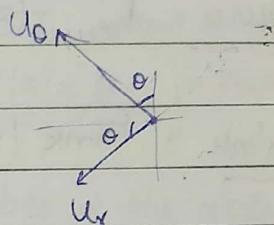
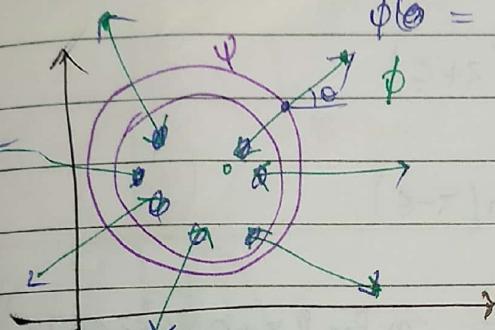
$\ln z = \ln r + i\theta$

$F(z) = -iC \ln r + C_0 = C_0 - iC \ln r$

$\phi \circ = C_0$

$\psi = (-C \ln r)$

flow around vortex



$U_r = -U_r \cos \theta - U_\theta \sin \theta$

$V = -U_r \sin \theta - U_\theta \cos \theta$

$$U - iV = -U_r \cos \theta - U_\theta \sin \theta - i(U_r \sin \theta - U_\theta \cos \theta)$$

$$= -U_r \cos \theta + iU_\theta \sin \theta - iU_\theta \cos \theta - iU_r \sin \theta$$

$$= -U_r(\cos \theta - i \sin \theta) - iU_\theta(\cos \theta + i \sin \theta)$$

$$= -(U_r + iU_\theta) e^{-i\theta}$$

$$U - iV = F'(z) = -\frac{iC}{z} = -\frac{iCe^{-i\theta}}{r}$$

$$U_r + iU_\theta = \frac{iC}{r}$$

$U_\theta = \frac{C}{r}$

Circulation  $\oint \vec{v} \cdot d\vec{s} = \oint \vec{v} \cdot \vec{ds} = \int_0^{2\pi} U_\theta \cdot r d\theta = \int_0^{2\pi} C \cdot r d\theta = 2\pi C$

$\oint \vec{v} \cdot d\vec{s} = \int_0^{2\pi} (U_r \hat{r} + U_\theta \hat{\theta}) \cdot (dr \hat{r} + r d\theta \hat{\theta})$  — general term

$C = \frac{\Gamma}{2\pi}$

$\frac{m^3}{m \cdot s}$

$\Gamma > 0$  CCW

$F(z) = -i \frac{\Gamma}{2\pi} \ln z$

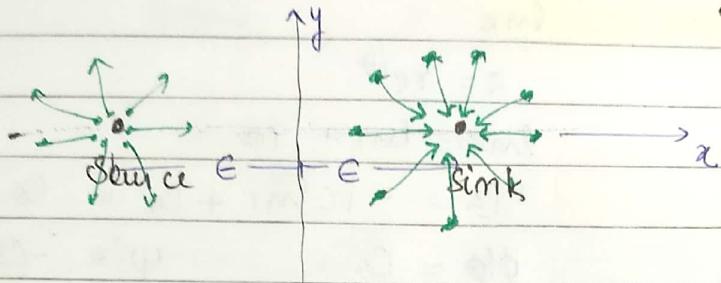
$\Gamma < 0$  CW

$F(z) = \frac{m}{2\pi} i \ln z$  Vortex at  $z=0$ ,  $F(z) = -i \frac{\sqrt{m}}{2\pi} \ln z$

When Vortex of strength  $i\sqrt{m}$  is located at  $z=z_0$

Vortex at  $z=z_0$ ,  $F(z) = -i \frac{\sqrt{m}}{2\pi} \ln(z-z_0)$

(vii)



✓ Source:  $F_{\text{source}}(z) = \frac{+m}{2\pi} \ln(z+E)$

✓ Sink:  $F_{\text{sink}}(z) = \frac{-m}{2\pi} \ln(z-E)$

$$F_{\text{source}}(z) + F_{\text{sink}}(z) = \frac{m}{2\pi} \ln \left( \frac{(z+E)(z-E)}{z^2} \right)$$

$$\begin{aligned} F(z) &= \frac{m}{2\pi} \ln \left( \frac{1+E/z}{1-E/z} \right) \rightarrow \left( \frac{1+E}{z} \right) \left( \frac{1+E+\dots}{z} \right) \\ &= \frac{m}{2\pi} \ln \left[ \left( 1+\frac{E}{z} \right) \left( 1-\frac{E}{z} \right)^{-1} \right] \quad \left( \frac{1+2E}{z} \right) + \frac{Cz^{10}}{z^2} \\ &= \frac{m}{2\pi} \ln \left( 1 + \frac{2E}{z} \right) \quad (\text{Considering } E \rightarrow 0) \end{aligned}$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

$$F(z) = \frac{m}{2\pi} \ln \left( 1 + \frac{2E}{z} \right) = \frac{m}{2\pi} \left[ \frac{2E}{z} + \dots \right]$$

$$F(z) = \frac{mE}{z} + \mu = \frac{\mu}{z}$$

↳ of the form  $\frac{c}{z} = f(z)$

$$F(z) = \frac{\mu}{ze^{i\theta}} = \frac{\mu e^{-i\theta}}{z}$$

$$\psi = -\frac{U \sin \theta}{r}$$

$$\psi = -\frac{Uy}{(x^2+y^2)}$$

$$\psi(x^2+y^2) = -Uy$$

$$\boxed{x^2+y^2 + \frac{Uy}{\psi} = 0}$$

$$x^2 + \left(y + \frac{U}{2\psi}\right)^2 = \frac{U^2}{4\psi^2}$$

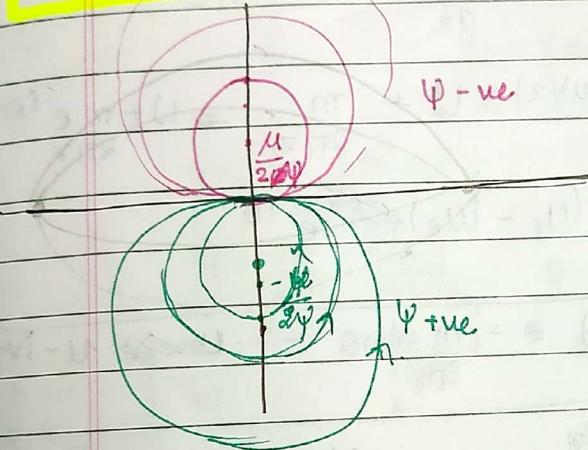
$$\phi = \frac{U \cos \theta}{r}$$

$$\phi = \frac{Ux}{(x^2+y^2)}$$

$$\Rightarrow x^2+y^2 - \frac{Ux}{\phi} = 0$$

$$\Rightarrow \left[x^2 + y - \frac{U}{2\phi}\right]^2 = \frac{U^2}{4\phi^2}$$

streamline is that line along which ~~stress~~  $\psi$  is constant.



DOUBLET when  $\epsilon \rightarrow 0$

$$F(z) = \frac{\mu}{z} \quad ] - \text{elemental flow}$$

Centres of the circle of radius  $(\frac{\mu}{2\psi})$  are located at

$y = \pm \frac{\mu}{2\psi}$  when  $\psi > 0 \Rightarrow$  lower half  
 $\psi < 0 \Rightarrow$  upper half

$$w(z) = \frac{dF}{dz} = -\frac{\mu}{z^2} = -\frac{\mu e^{-iz}}{r^2}$$

$$(U_r - i\omega U_\theta) e^{-iz} = -\frac{\mu e^{-iz}}{r^2} \cdot e^{-iz}$$

$$(U_r - i\omega U_\theta) = -\frac{\mu}{r^2} e^{(cos\theta - i\sin\theta)}$$

$$U_r = -\frac{\mu \cos \theta}{r^2}$$

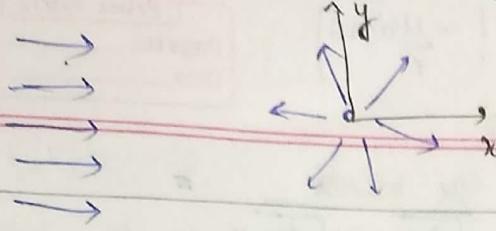
$$U_\theta = -\frac{\mu \sin \theta}{r^2}$$

When centre is  $z_0$

$$F(z) = \frac{\mu}{(z-z_0)}$$

o Cooling tower - <sup>some</sup> water evaporates to make remaining <sup>cool</sup>

## RANKINE HALF BODY



Source + uniform flow

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$$F(z) = U_\infty z + \frac{m}{2\pi} \ln z, \quad (\text{uniform flow})$$

$$\phi + i\psi = F(z) \rightarrow F(z) = (U_\infty z \cos\theta + U_\infty r \sin\theta) + \frac{m}{2\pi} \ln(r e^{i\theta})$$

$$\phi: U_\infty z \cos\theta + \frac{m}{2\pi} \ln r$$

$$\psi: U_\infty r \sin\theta + \frac{m}{2\pi}$$

$$\phi = U_\infty z \cos\theta + iU_\infty r \sin\theta + \frac{m}{2\pi} \ln r + i\theta$$

$$= [U_\infty z \cos\theta + \frac{m}{2\pi} \ln r] + i[U_\infty r \sin\theta + \frac{m}{2\pi}]$$

$$w(z) = \frac{df}{dz} = (U_r - iU_\theta) e^{-i\theta}$$

$$w(z) = U + \frac{m}{2\pi z} = U + \frac{m}{2\pi r} e^{-i\theta}$$

$$U + \frac{m}{2\pi r} e^{-i\theta} = (U_r - iU_\theta) e^{-i\theta}$$

$$(U e^{i\theta} + \frac{m}{2\pi r}) e^{-i\theta}$$

$$= (U_r - iU_\theta) e^{-i\theta}$$

$$(U + \frac{m \cos\theta}{2\pi r}) + i \frac{m \sin\theta}{2\pi r} = \text{constant } U - iv$$

$$U = U + \frac{m}{2\pi r} \cdot \frac{x}{x^2 + y^2}$$

$$V = \frac{m}{2\pi} \frac{y}{x^2 + y^2}$$

$$U_r = U e^{i\theta}$$

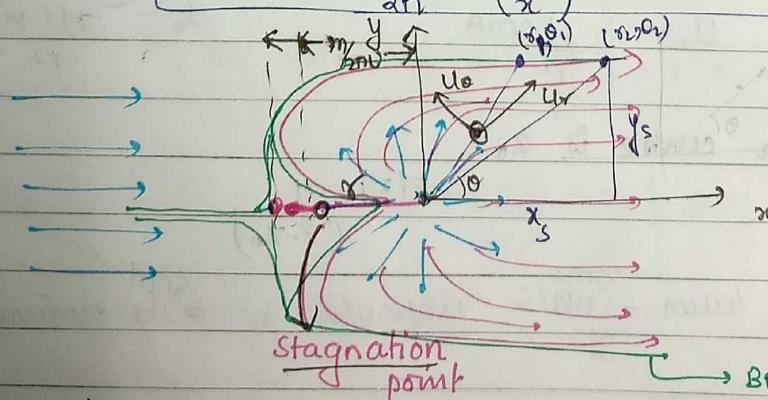
$$U_\theta = .$$

$$\left. \begin{aligned} U_r &= U \cos\theta + \frac{m}{2\pi r} \\ U_\theta &= -U \sin\theta \end{aligned} \right\}$$

$$\psi = U_y + \frac{m}{2\pi} \frac{y}{x^2 + y^2}$$

$$\psi = U \cdot y + \frac{m}{2\pi} \tan^{-1}\left(\frac{y}{x}\right)$$

$$\phi = U x + \frac{m}{4\pi} \ln(x^2 + y^2)$$



Stagnant point:  $U_r = U_\theta = 0$  (by def<sup>n</sup>)

$$U \cos\theta + \frac{m}{2\pi r} = 0$$

$$U \sin\theta = 0$$

$$\psi = \frac{m}{2}$$

We want to know where is this point located?

$$\theta = \pi, \rho \downarrow$$

$r$  becomes -

$$-U + \frac{m}{2\pi r} = 0$$

$$\delta = \frac{m}{2\pi r} \frac{m}{2\pi U}$$

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$$\Psi = Ur \sin \theta + \frac{m \theta}{2\pi}$$

$$\theta = \pi, \quad \boxed{\Psi = \frac{m}{2}}$$

$$\therefore \frac{m}{2} = Ur \sin \theta + \frac{m \pi}{2\pi}$$

Body streamline

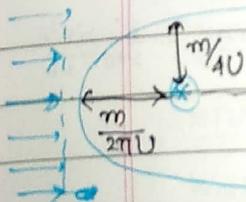
$$y_s = \frac{m}{2\pi U} (\pi - \theta) \sin \theta$$

$$\frac{m \sin \theta}{2\pi} = \frac{m}{2\pi}$$

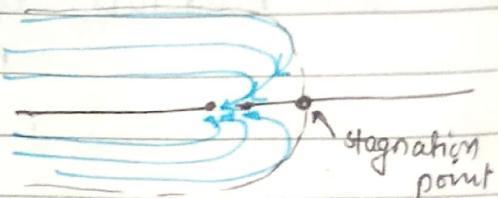
$$\frac{m \sin \theta}{2\pi} = \frac{m}{2\pi}$$

$$y_s = \frac{m}{2\pi U} (\pi - \theta)$$

$$x_s = \frac{m}{2\pi U} \frac{(\pi - \theta)}{\tan \theta}$$



e.g. uniform flow + sink



If there is upper flow only, strength will be diff.

$$y_s = y_s \sin \theta = \frac{m}{2\pi U} \sin \theta$$

$$x_s = y_s \cos \theta = \frac{m}{2\pi U} \frac{(\pi - \theta)}{\sin \theta} \cos \theta$$

To get an idea of how this envelope looks?

$$\theta = \pi/2 \quad (y_s \text{ at origin} = \frac{m}{4U})$$

$$\theta = 0 \quad (y_s \text{ at } \infty = \frac{m}{2U})$$

$U, P_\infty$

Bernoulli's eq<sup>n</sup>:

$$\frac{\rho V_s^2}{2} + P_s = \frac{\rho U^2}{2} + P_\infty$$

$$\bar{V}_s = \bar{W}_s = (U \cos \theta + \frac{m}{2\pi r}) - i U \sin \theta$$

$$V_s^2 = |V_s|^2 = W_s \bar{W}_s = \left( U \cos \theta + \frac{m}{2\pi r} \right)^2 + (U \sin \theta)^2 = U^2 + \left( \frac{m}{2\pi r} \right)^2 + \frac{m U \cos \theta}{\pi r}$$

$$\text{Let } \nu = \pi - \theta \quad \tau_s = \frac{m}{2\pi U} \frac{\nu}{\sin \nu}$$

$$\therefore V_s^2 = U^2 + \left( \frac{m}{2\pi U} \frac{\pi - \nu}{\sin \nu} \right)^2 - \frac{m U \cos \nu}{\pi \cdot \frac{\pi - \nu}{2\pi U} \frac{\nu}{\sin \nu}}$$

$$V_s^2 = U^2 + \frac{U^2 \sin^2 \nu}{\nu^2} - \frac{2 U^2 \cos \nu \sin \nu}{\nu}$$

$$P_s - P_\infty = \frac{\rho [U^2 - V_s^2]}{2} = \frac{\rho}{2} \left[ U^2 \frac{\sin^2 \nu}{\nu^2} - U^2 \frac{\sin^2 \nu}{\nu^2} \right] = \frac{\rho U^2}{2} \left[ \frac{\sin 2\nu}{\nu} - \frac{\sin^2 \nu}{\nu^2} \right]$$

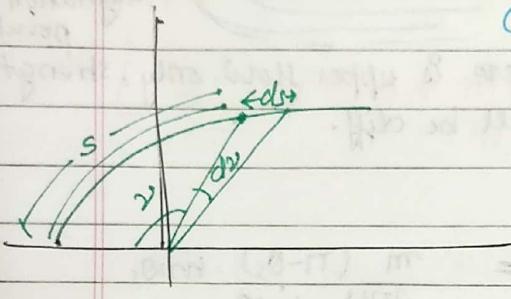
Introduce dimensionless pressure or pressure coefficient

$$\text{as } C_p = \frac{(P_s - P_\infty)}{\frac{1}{2} \rho U^2} = \frac{\sin 2\psi}{2} - \frac{\sin^2 \psi}{2}$$

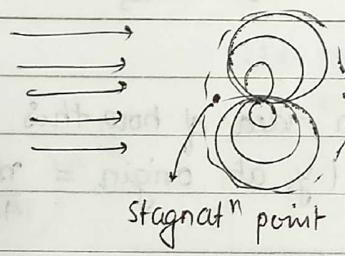
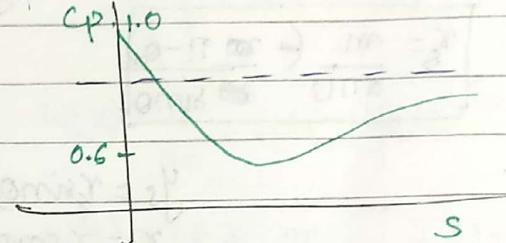
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Change in pressure head as  $\alpha$  of overall kinetic head  
from Bernoulli's eqn.

$$P_s - P_\infty = \left(1 - \frac{V_s L}{U^2}\right) \cdot \frac{\rho U^2}{2}$$



$$C_p = \frac{2 \sin 2\psi \cos \psi}{2} - \frac{1 \sin^2 \psi}{2}$$



$C_p$  is measured gradually

and as  $s \rightarrow \infty$ ,

$C_p \rightarrow 1$  because  $P_s \rightarrow P_\infty$  because  $V_s \rightarrow U$

$$s=0, \psi=0 \quad \frac{2 \sin 2\psi \cos \psi}{2} - \frac{1 \sin^2 \psi}{2} = 1$$

$$\int u ds = dm \quad \Rightarrow m = \int u ds$$

Energy through  
 $ds$  per unit time

$$dE_k = (\rho u ds) \frac{u^2}{2}$$

$$E_k = \int dE_k = \frac{1}{2} \int u^3 ds$$

$$\frac{E_k}{m} = \frac{\int u^3 ds}{\int u ds}$$

Circular  
pipe

maxm velocity

$$u = \left(2\bar{U}\right) \left[1 - \left(\frac{s}{R}\right)^2\right]$$

$$\bar{u} = \int u ds = \int_0^R u 2\pi r dr = \frac{R}{\pi R^2} \int_0^R u^3 dr$$

$$ds = 2\pi r dr$$

$$= \frac{4\bar{U}}{\pi R^2} \int_0^R (R - r)^3 dr = \frac{4\bar{U}}{\pi R^2} \left[ \frac{R^4}{4} - \frac{R^2}{2} \right]$$

$$\bar{U} = \frac{8\bar{U}}{3\pi R^2} R$$

$$\frac{1}{2} \int u^3 ds = \frac{(2\pi)(8\bar{U}^3)R}{2\bar{U}(\pi R^2)} \int_0^R \left(\frac{R-r}{R}\right)^3 dr$$

$$8\bar{U}^3 \int_0^R \left(\frac{R-r}{R}\right)^3 dr = 8\bar{U}^3 \int_0^1 (1-\frac{r}{R})^3 dr$$

$$= \frac{8\bar{V}^2}{R^2} \left[ \frac{\gamma^4}{4} - \frac{\gamma^{10}}{10R^6} - \frac{3\gamma^6}{6R^2} + \frac{3\gamma^8}{8R^4} \right]$$

$$= \frac{8\bar{V}^2}{R^2} \left[ \frac{1}{4} - \frac{1}{10} - \frac{1}{2} + \frac{3}{8} \right] R^4$$

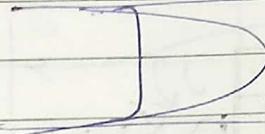
$$= \frac{V^2}{R^2} R^4$$

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$$= \frac{8V^2}{R^2} \int_0^R \left( R^2 - \frac{R^8}{8R^6} - \frac{3R^4}{4R^2} + \frac{3R^6}{6R^2} \right) dR = 8V^2 \cdot \left( \frac{1}{2} - \frac{1}{8} - \frac{3}{4} + \frac{1}{2} \right)$$

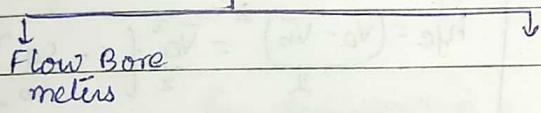
$$\alpha \frac{\vec{v}^2}{2} = \vec{N}^2$$

$\therefore$   $\boxed{\alpha = 2}$  Parabolic  
 $\boxed{\alpha = 1}$  turbulent



$\alpha$ : correction factor

## Flow Metering



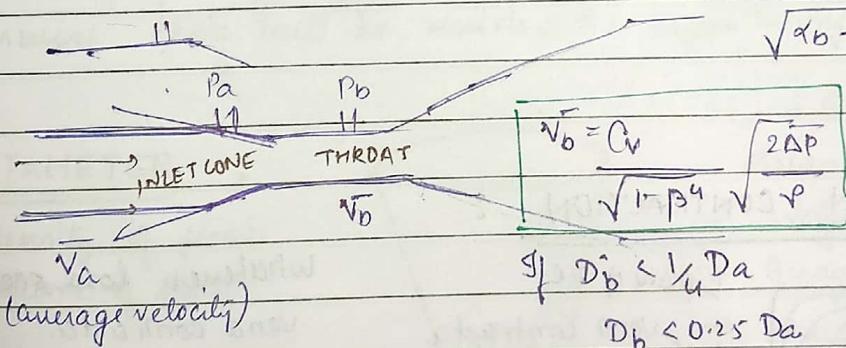
## VENTURI METER

Kinetic head converted  
pressure head.

$$\text{Energy Balance : } d_b \bar{v}_b^2 - 2 \bar{v}_a \bar{v}_a^2 = 2(P_a - P_b)$$

$$\text{Continuity eqn : } V_a^2 \pi r_a^2 = V_b^2 \pi r_b^2$$

Kinetic head converted to pressure head.



Cv: Venturi coefficient

$$0.97 < C_v < 0.99$$

$f_C$  is brought considering losses, all kinetic head not converted to pressure head)

If  $C_v$  is given, we can tell how much energy is lost]

- angle of discharge cone has to be small so that boundary layer doesn't separate from surface. (b/w  $5^\circ$  and  $15^\circ$ )
  - upstream cone shorter than downstream cone

Flange used to join pipes

## ORIFICEMETER

$$C_v = 0.6 \text{ (Huge energy loss)}$$

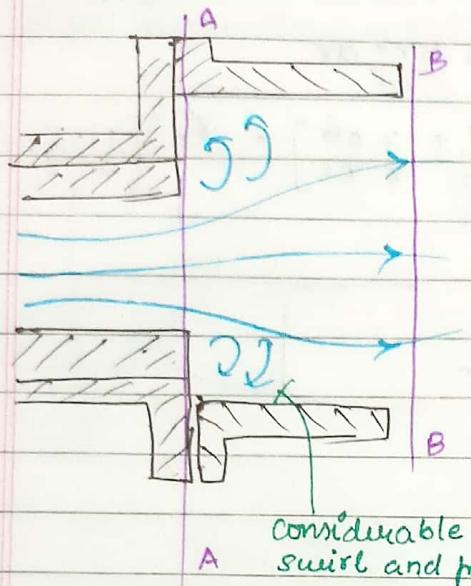
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## SUDDEN EXPANSION OF CROSS- SECTION :



1. Fluid stream separates from the wall and continues as a jet
2. At further downstream, jet expands to fill the entire cross-section

For the control volume AARB M<sup>2</sup> eqn.

$$(P_a S_b - P_b S_b) = \dot{m} [\bar{V}_b - \bar{V}_a]$$

A Considerable swirl and friction

energy eqn:

$$\frac{P_a - P_b}{\rho} = \frac{\bar{V}_b^2 - \bar{V}_a^2}{2} + h_{fe}$$

$$h_{fe} = \frac{(\bar{V}_a - \bar{V}_b)^2}{2} = \frac{\bar{V}_a^2}{2} \left[ 1 - \frac{S_a}{S_b} \right]^2 = \frac{\bar{V}_a^2}{2} K_e$$

↓  
frictional loss coeff.

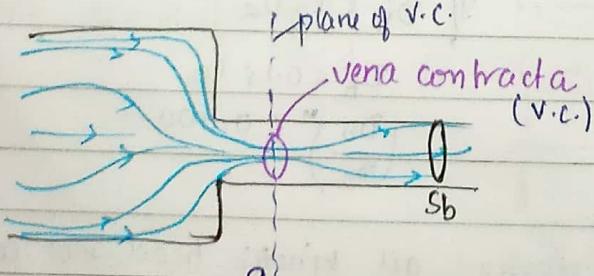
K<sub>e</sub>: Expansion loss coefficient

dry loss =  $\frac{(\text{velocity at upstream})^2}{2}$  (coefficient)

$$\frac{\sqrt{2}(V_0 - \bar{V}_a)}{2} = h_{fe}$$

$$\therefore i (\bar{V}_b - \bar{V}_a) = V^2$$

## SUDDEN CONTRACTION



Whatever loss occurs is after vena contracta.

The jet first expands contracts then expands.

Minm Area of c.s at v.c.

$$h_{fc} = K_c \frac{\bar{V}_b^2}{2}$$

$$\text{Vena contracta area} = 0.6 S_b$$

$$\text{Velocity at vena contracta} = \frac{\bar{V}_b}{0.6}$$

$$\text{Kinetic energy ratio} = \frac{\bar{V}_{eff}^2}{\bar{V}_b^2} = \frac{A_f}{A_i} \left( \frac{1}{0.6} \right)^2 = 2.8$$

Frictional energy loss as the stream expands beyond venturi contracts

$$K_c = \left[ 1 - \frac{0.6 S_b}{S_a} \right]^2 = 0.16$$

$$K_c \frac{V_a^2}{2} = K$$

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$$h_{fe} = K_c \frac{V_a^2}{2}$$

29  
8  
224

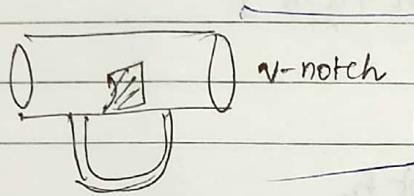
$$h_{fe} = K_c \cdot \frac{\bar{V}_b^2}{(0.6)^2} \cdot \frac{1}{2} = \frac{\bar{V}_b^2}{2} \times 0.16 \times 2.8 = 0.44 \frac{\bar{V}_b^2}{2}$$

$$K_c = 0.44$$

For laminar flow,  $K_c < 0.1$  and  $h_{fe}$  is neglected

$$\text{For turbulent } K_c = 0.4 \left[ 1 - \frac{S_b}{S_a} \right]$$

$$\frac{S_b}{S_a} \rightarrow 0 \quad K_c = 0.4$$



Orifice

$$U_o = C_0 \sqrt{\frac{2(P_a - P_b)}{\rho}} \quad (\text{in m/s})$$

$U_o$  - velocity through orifice

$$\beta = \frac{d}{D_{\text{dia}}}$$

$$C_0 \text{ depends on } Re_o = \frac{D_o U_o \rho}{\mu} = \frac{4 \dot{m}}{\pi D_o \mu}$$

$$Re_o > 30,000 \Rightarrow C_0 = 0.61$$

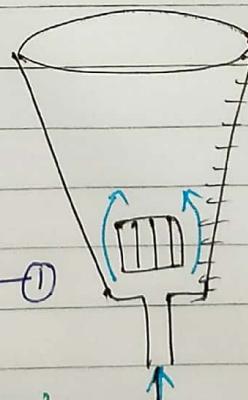
$\beta < 0.25$  - can be neglected

Availability of straight pipe both upstream or downstream is imp. otherwise there will be swirls, B.L before entry of the fluid.

### ROTAMETER :

$\rho_f$ : density of float

$V_f$ : volume of float



$$At \text{ eqm} \quad [ V_f \rho_f g - V_f \rho_g = F_{\text{drag}} ] \quad \text{--- (1)}$$

Balance of 3 forces

- Weight of float ↓
- Buoyancy force on float ↑
- Drag force on float ↑

$$F_D = A_f C_D \frac{\rho u^2}{2}$$

$$F_{\text{drag}} = G \pi r \mu u$$

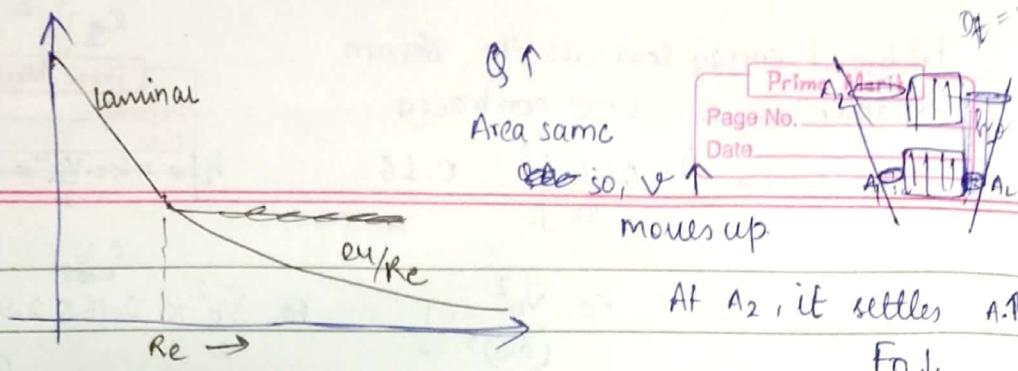
$$= C_D \pi r^2 \rho u^2 \frac{\rho}{2}$$

$C_D$ : drag coeff.  $A_f$ : Projected area

$u$ : velocity of fluid around float

$$C_D = \frac{12 \mu}{r u \rho} = \frac{24}{Re}$$

(for non-laminar)



$$D_t = D_f + ah \uparrow$$

$$q = \frac{u\eta}{4} (D_t^2 - D_f^2) = \frac{u\eta}{4} [(D_f + ah)^2 - D_f^2]$$

only \$f\_D\$ changes in eqn ①

$$= \frac{u\eta}{4} [2ah + a^2 h^2]$$

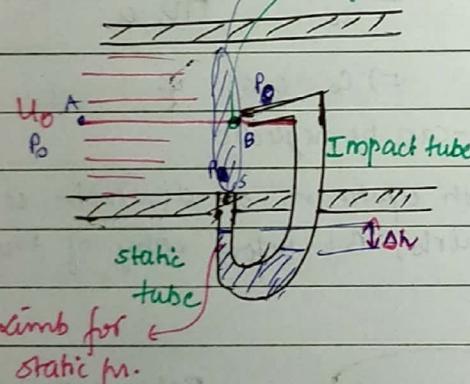
$$q \propto h^2$$

\$\therefore\$ Flow is a linear fm of height \$h\$.

In orifice meter, flow rate \$\propto \sqrt{\Delta P}\$ reading

### PITOT TUBE

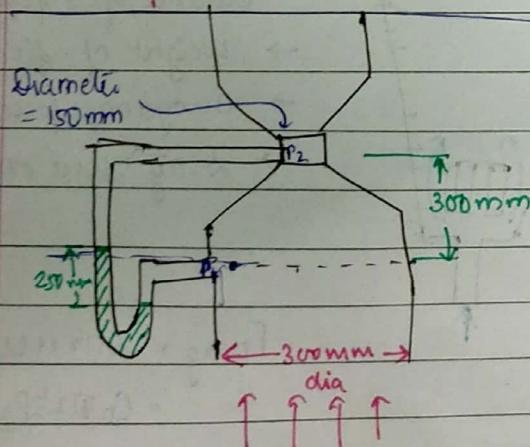
pressure is same across this c.s as no vertical velocity  
stagnant point



$$\frac{U_0^2}{2} + \frac{P_0}{\rho} = 0 + \frac{P_s}{\rho}$$

$$U_0 = \sqrt{2(P_s - P_0)} \quad \Delta P = \Delta h \cdot g$$

same pressure drop in this configuration.



All kinetic head converted to pressure head \$\rightarrow\$ measured by manometer.

oil (\$s\_{oil} = 0.9\$)

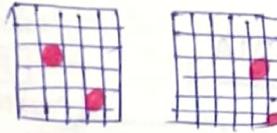
Particle image velocimetry

- measures local velocity

cross-correlation fm

1. Calculate \$(P\_1 - P\_2)\$

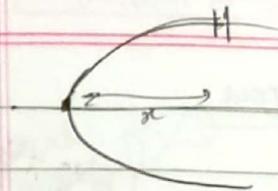
2. calculate flow rate (also referred as discharge rate) of oil



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such that we require

98% of pressure



$$C_v = 0.98$$

$$P_1 + \rho g(2.0) \\ - \rho g(0.2) = 0$$

$$- \rho g(0.3) = P_2$$

$$\Delta P = 250 h \rho_m g = 250 \times 10^3 \times 13.6 \times 10^3 \times 9.81 = 33.354 \text{ kPa}$$

$$V_2 = \frac{C_v}{\sqrt{1-p_2}} \sqrt{\frac{2\Delta P}{\rho}}$$

$$P = \frac{150}{300} = \frac{1}{2}$$

$$= \frac{0.98}{\sqrt{1-\frac{1}{2}}} \sqrt{\frac{2 \times 33.354 \times 10^3}{0.9 \times 10^3}}$$

$$P_2 + \frac{300}{1000} \times \rho_{air} g_{100} \times 9.81 + \frac{250}{1000} \times 13600 \times 9.81 = P_1 + \frac{250}{1000} \times 9000 \times 10$$

$$P_2 - P_1 = -33795.45 \text{ Pa}$$

$$\Delta P = 33795.45 \text{ Pa}$$

Assuming  $C_v = 1$

$$gz_1 + \frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

$$A_1 V_1 = A_2 V_2$$

$$\pi \left(\frac{300}{4}\right)^2 V_1 = \pi \left(\frac{150}{4}\right)^2 V_2$$

$$\therefore \frac{34 \times 10^3}{0.9 \times 10^3} = \frac{16 V_1^2 - V_1^2}{2} + g(0.3)$$

$$V_2 = 4V_1$$

$$V_1^2 = 4.644$$

$$V_1 = 2.155 \text{ m/s}$$

$$V_2 = 8.62 \text{ m/s}$$

$$Q = 0.152 \text{ m}^3/\text{s}$$

$$\text{Considering } C_v, \quad V_2 = 8.798 \text{ m/s} \\ Q = 0.155 \text{ m}^3/\text{s}$$

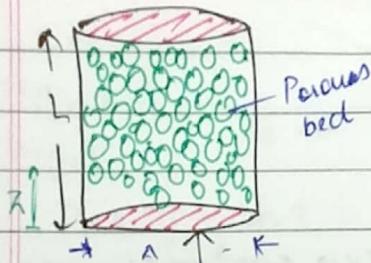
can't be used because there is hydrostatic head.

# FLOW THROUGH POROUS MEDIA

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$$\epsilon = \text{void fraction} = \frac{\text{Volume of void or pore}}{\text{Total volume}}$$



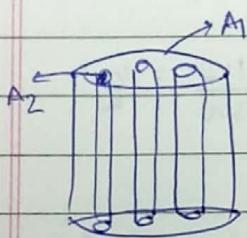
$$\epsilon = \frac{\text{Volume of void or pore}}{\text{Total volume}}$$

$$1 - \epsilon = \frac{\text{volume of solid}}{\text{total volume}}$$

$$\epsilon v_i = v_s$$

$$\frac{Q}{A} = \text{Darcy velocity} = \text{superficial velocity } v_s$$

$$= -k \frac{\partial p}{\mu \partial z}$$



$$\frac{Q}{E A} = \text{Interstitial velocity } v_i$$

$$= \frac{k}{\mu} \frac{\Delta p}{L}$$

property of medium

$k$ : permeability  $(m^2)$   
(Darcy)

$$\text{Total volume} = A_1 L$$

$$\frac{A_2 L}{A_1 L} = \epsilon$$

$$\text{Void "} = A_2 L$$

$$\delta \Omega [A_2 = A_1 \epsilon]$$

$\mu$ : viscosity of the fluid  
property of fluid

- Darcy velocity need not be in only one direction

We can have interstitial velocity in  $x, y, z$

$$\bar{v}_i = -k \frac{\partial p}{\mu \partial x} \hat{i} - k \frac{\partial p}{\mu \partial y} \hat{j}$$

• ' $k$ ' can be tensor

$$\begin{bmatrix} k_{xx} & . & . \\ . & k_{yy} & . \\ . & . & k_{zz} \end{bmatrix}$$

generally, we take  
diagonal elements

For isotropic medium,  $k_{xx} = k_{yy} = k_{zz}$

- When 2 fluids flow simultaneously,  $k$  becomes relative permeability

$$k_{oil} = -k \frac{k_{oil}}{k_{oil}} \frac{\partial p}{\partial z}$$

$$v_{water} = -\frac{k k_{water}}{\mu_{water}} \frac{\partial p}{\partial z}$$

$$D_h = \frac{4 A_i}{W_p} = \frac{4 A_i L}{W_p L} = \frac{4 \text{Flow volume}}{\text{Wetted surface area}}$$

$$= 4 \frac{E \{ \text{Bed volume} \}}{\{ \text{no. of particles} \}} \frac{\text{total vol of porous medium}}{\{ \text{surface area of particle} \}}$$

$$\{ \text{no. of particles} \} \frac{\{ \text{surface area of particle} \}}{\{ \text{particle} \}}$$

No. of particles =  $\frac{(\text{Bed volume})}{(\text{volume per particle})} (\text{Fract}^n \text{ of solid in bed})$

Fraction of solids in bed =  $(1-\epsilon)$

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Bed saddle

$$D_h = \frac{4\epsilon}{(1-\epsilon)} \frac{(\text{Bed volume})}{(\text{Bed volume})(\text{surface area per particle})} \frac{(\text{volume per particle})}{}$$

Coulson & Richardson  
Richardson  
McCabe & Smith

$$D_h = 4 \frac{\epsilon}{1-\epsilon} \left( \frac{\text{volume}}{\text{surface area}} \right) \text{particle}$$

$$\frac{4\pi R_s^3}{3} \frac{4\pi R_p^3}{3}$$

$$= 4 \left( \frac{\epsilon}{1-\epsilon} \right) \left( \frac{D}{6} \right) - \text{if spherical}$$

• If not spherical, we use sphericity factor multiplied by D

$\Phi_s$ : Sphericity : Surface volume ratio for a sphere of diameter  $D_p$  / Surface volume ratio for the particle whose nominal size is  $D_p$ .

$$\text{Frictional loss} = \frac{\Delta P}{P} = \frac{g}{P} \frac{\rho_w \cdot L}{\tau_w}$$

$$Re = \frac{D_h \nu_i \rho}{\mu} = \frac{2DV_i \rho (1-\epsilon)}{3\mu (1-\epsilon)}$$

$$= \frac{2D \nu_s \rho}{3\mu (1-\epsilon)}$$

$$(N_{Re})_{PM} = \frac{d \nu_s \rho}{(1-\epsilon) \mu}$$

Porous medium

$$\alpha = 6 \Rightarrow \frac{1}{\alpha} = \frac{D_p}{6}$$

$$\frac{1}{\alpha} = \frac{D_p}{6} \quad \frac{1}{\alpha} = \frac{D_p}{6}$$

$$\pi r^2 p - (\pi r^2)(p + dp) = 2(2\pi r dL)$$

$$\frac{2\sum dp}{r} = 0$$

$$f = \frac{16}{Re} \quad \text{McCabe Smith}$$

Fanning Fanning, Moody, Darcy friction factor

f<sub>FM</sub>

Ergun equation

$$f_{FM} = 1.75 + \frac{150}{(N_{Re})_{PM}}$$

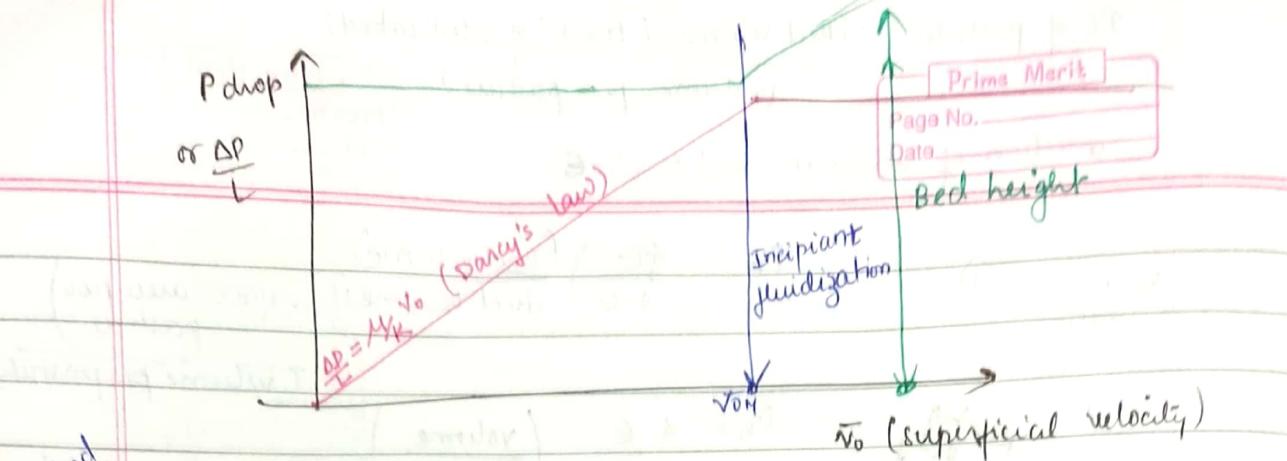
v<sub>o</sub>: superficial velocity

$$\frac{\Delta P}{L} = \frac{150 v_o^2 \mu}{\phi_s^2 D_p^2} \frac{(1-\epsilon)^2}{\epsilon^3} + \frac{1.75 f_{FM}^{-2} \rho g}{\phi_s D_p} \frac{(1-\epsilon)}{\epsilon^3}$$

① D<sub>p</sub>: nominal size of particle

$$y = k_1 x + k_2 x^2$$

(D<sup>L</sup>)



When we go on increasing  $v$ , pores' eq<sup>m</sup> disturbed.

- BUBBLING occurs.

*Fluidized catalytic cracking unit catalyst*

Churning

$v_{0M}$  - velocity

beyond which  
no more fluidization occurs

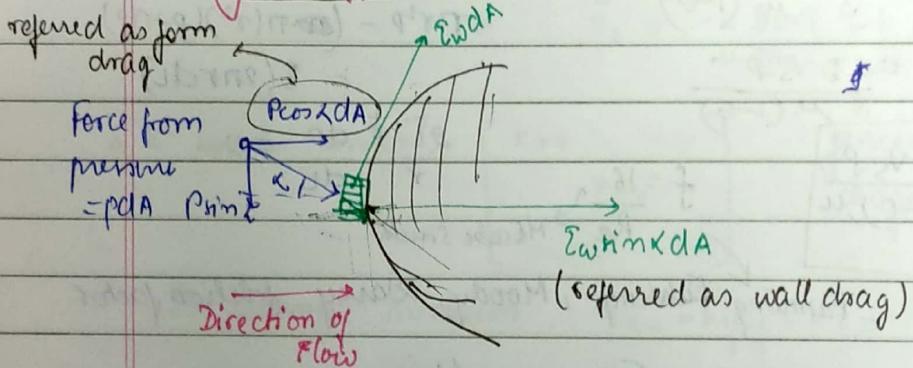
$$\frac{F_D}{F_B} \propto \frac{v^2}{w}$$

(Pressure drop across the bed)  $\Delta A = \text{wt. of bed per unit area}$   
of cross section (allowing for buoyancy force)

$$= (1-\epsilon) (\rho_p - \rho) L g - \text{① weight of solid}$$

08.11.17

## Drag Coefficient-



For flow past immersed object, the force in the direction of flow exerted by the fluid on the solid is called "drag".

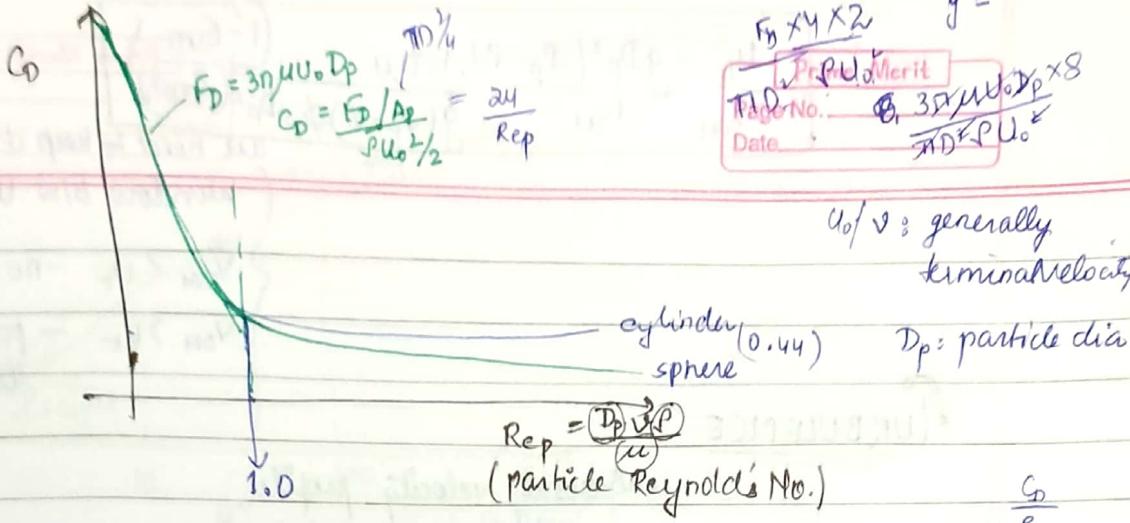
$$\text{Drag coefficient } (C_d) = \frac{\text{Total drag / Projected area}}{\frac{\rho U_0^2}{2}}$$

$\bullet U_0: \text{velocity of particle}$

$$f = \frac{\sum w}{\frac{\rho U_0^2}{2}}$$

Projected area is the area obtained by projecting the object on a plane  $\perp$  to the direction of flow.

sphere — circle



other particles do not interfere

### Free settling:

$$\frac{du}{dt} = F_e - F_b - F_D$$

$\uparrow mg$        $\uparrow m \nu g$        $\uparrow C_D \frac{\rho U^2}{2} A_p$

$$\frac{du}{dt} = g \frac{(\rho_p - \rho)}{\rho_p} - \frac{C_D \rho U^2 A_p}{2m}$$

If centrifuge is used?

assume, at terminal velocity,  $\frac{du}{dt} = 0$

$$U_t = \sqrt{\frac{2g(\rho_p - \rho)m}{A_p \rho_p C_D \rho}}$$

If we write  $m = \frac{1}{6} \pi D_p^3 \rho_p$

$$A_p = \frac{\pi D_p^2}{4}, \quad Re_p \leq 1 \Rightarrow C_D = \frac{24}{Re_p}$$

$$Re_p \gg 1 \Rightarrow C_D = \text{const} = 0.44$$

$$U_t = \sqrt{\frac{g D_p^2 (\rho_p - \rho)}{18 \mu}}$$

$$U_t = 1.75 \sqrt{\frac{g D_p (\rho_p - \rho)}{\rho}}$$

### Hindered settling

$$U_s = U_t (e)^n$$

$e$ : vol. f' of solid

( $0.6 < e < 2.5$ )  
Stokes region  
 $Re_p \uparrow$

- In Ergun eqn, when the size of particles is very small, then the

$$\boxed{Re_p < 1 \approx g (\rho_p - \rho) e_m^3 \frac{\phi_s^2 D_p^2}{15 \mu (1 - e_m)^2}}$$

$$\frac{U_t}{V_{OM}} = \frac{g D_p^2 (\rho_p - \rho)}{18 \mu} \cdot \frac{150 \mu}{g (\rho_p - \rho) \phi s D_p^2 (G_m^3)}$$

$(1 - G_m)$   
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 $(G_m^3)$   
Data

we need to keep it as high as possible.

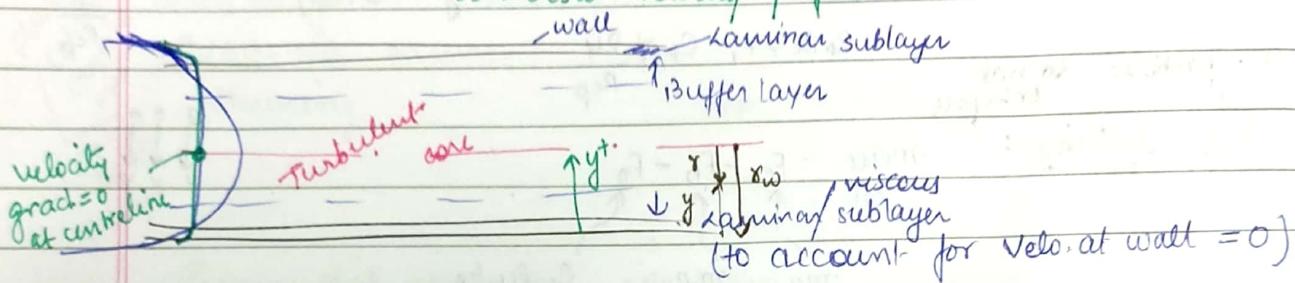
Window b/w  $U_t$  &  $V_{OM}$

$V_{OM} < U_t$  - no fluidization

$V_{OM} > U_t$  - particles will flow with fluid

## TURBULENCE

universal velocity profile



Dimensionless variables:

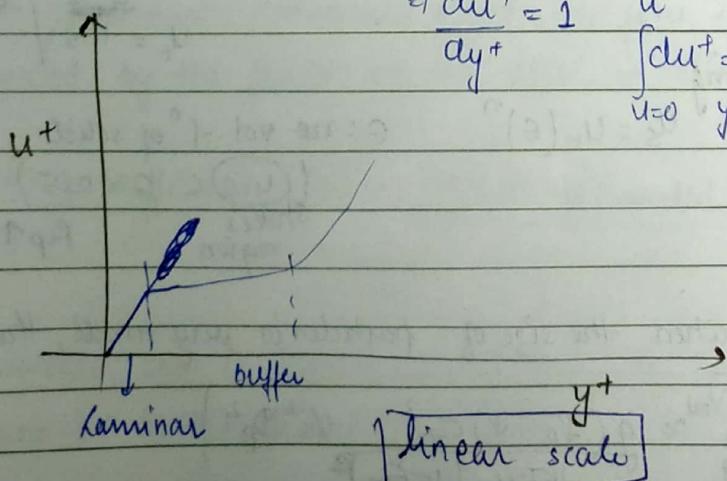
shear velocity ( $u_s$ )      • Friction velocity =  $v^* = \sqrt{\frac{f}{2}} = \sqrt{\frac{\tau_w}{\mu}}$  → has dimension

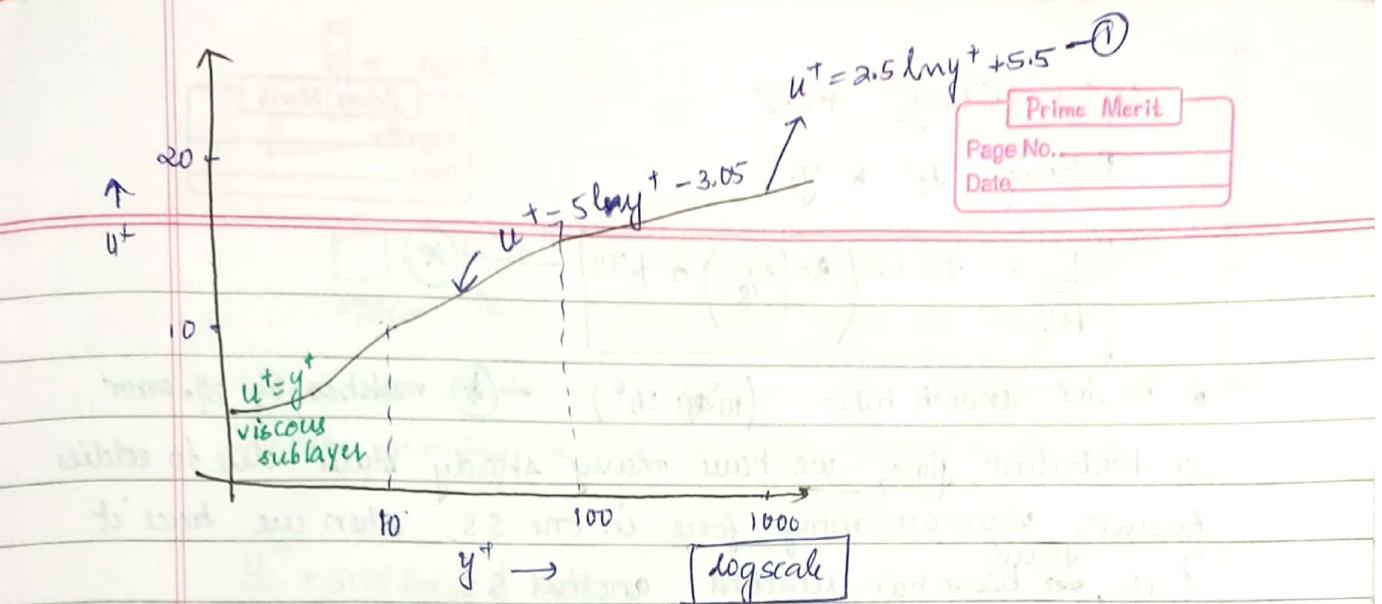
$$\rightarrow \left\{ \begin{array}{l} u^+ = \frac{u}{u^*} \quad \text{- dimensionless velocity} \\ y^+ = \frac{y u^* \rho}{\mu} \quad \text{- distance} \\ y: \text{distance from wall of tube } (\tau_w - r) \\ \tau_w = \tau + y \Rightarrow dr = -dy \end{array} \right.$$

For viscous sublayer,  $-\frac{dy}{dr} = \frac{\tau_w}{\mu}$

$$\frac{du}{dy} = \frac{\tau_w}{\mu}$$

$$\Rightarrow \frac{du^+}{dy^+} = 1 \quad u^+ \quad y^+ \\ \int du^+ \quad \int dy^+ \\ u=0 \quad y=0 \quad u^+ = y^+$$





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$y^+$	0 → 5	viscous sublayer
	5 → 30	buffer layer
	30 → centre of pipe	turbulent core

At the centre of the pipe  $u_c^+ = 2.5 \ln y_c^+ + 5.5 \quad \text{--- (1)}$

$$\begin{aligned} u_c^+ &= \frac{U_{\max}}{u^*} \\ y_c^+ &= \frac{r_w u}{\mu/\rho} \end{aligned}$$

To find  $u^+$  at any  $y^+$

subtracting (1) from (1)

$$u^+ - u_c^+ = 2.5 \ln \frac{y^+}{y_c^+}$$

$$\begin{aligned} \bar{V} &= \frac{1}{\pi r_w^2} \int_0^{r_w} u(2\pi r) dr \\ &= \frac{2}{\pi r_w^2} \int_0^{r_w} u(r_w - y) dy \end{aligned}$$

$$\begin{aligned} r &= r_w - y \\ dr &= -dy \\ y &= 0 \Rightarrow y = r_w \\ r &= r_w \Rightarrow y = 0 \end{aligned}$$

$u$  is taken  
as the  
turbulent  
part  
dominating

$$= 5 \left( \frac{\mu}{\rho} \right)^2 \int_0^{y_c^+} \left( 0.4 u_c^+ + \ln \frac{y^+}{y_c^+} \right) (y_c^+ - y^+) dy^+$$

$$\frac{\bar{V}}{U_{\max}} = u_c^+ - 3.75$$

[laminar  $\bar{V} = \frac{1}{2} U_{\max}$ ]

$$\frac{\bar{V}}{U_{\max}} = \frac{1}{1 + 3.75 \sqrt{f_2}}$$

$$y_c^+ = \frac{r_w \bar{V}}{\left( \frac{\mu}{\rho} \right) \sqrt{f_2}} = \frac{D \bar{V}}{2 \left( \frac{\mu}{\rho} \right) \sqrt{f_2}} = \frac{Re}{2} \sqrt{f_2}$$

$$u_c^+ = \frac{1}{\sqrt{f_2}} + 3.75$$

$$U_c^+ = 2.5 \ln y_c^+ + 5.5$$

Putting  $U_c^+ \propto y_c^+$

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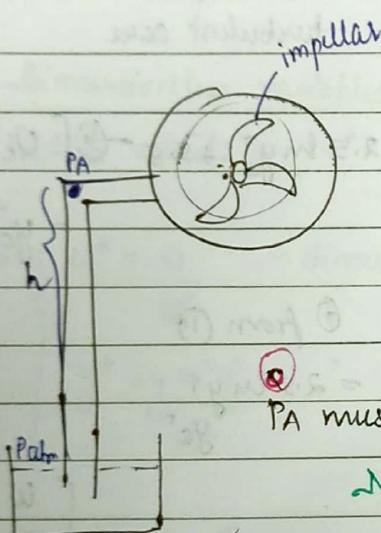
$$\frac{1}{\sqrt{f_{L_2}}} = 2.5 \ln \left( Re \sqrt{f_{L_2}} \right) + 10.75 \quad (*)$$

- $Re$  for smooth tubes ( $10^4$  to  $10^6$ )  $\rightarrow (*)$  matches by 2% error

For turbulent flow, we have many steady states due to eddies present. When we apply force in one S.S., when we trace it back, <sup>fluid</sup> same has reached another S.S.

(\*)

## Centrifugal Pump



Process of putting in water into the pump - priming

$$P_A = \{P_{atm} - \rho g h\} < \text{Vapor pressure of liquid}$$

then, liquid will vapourise

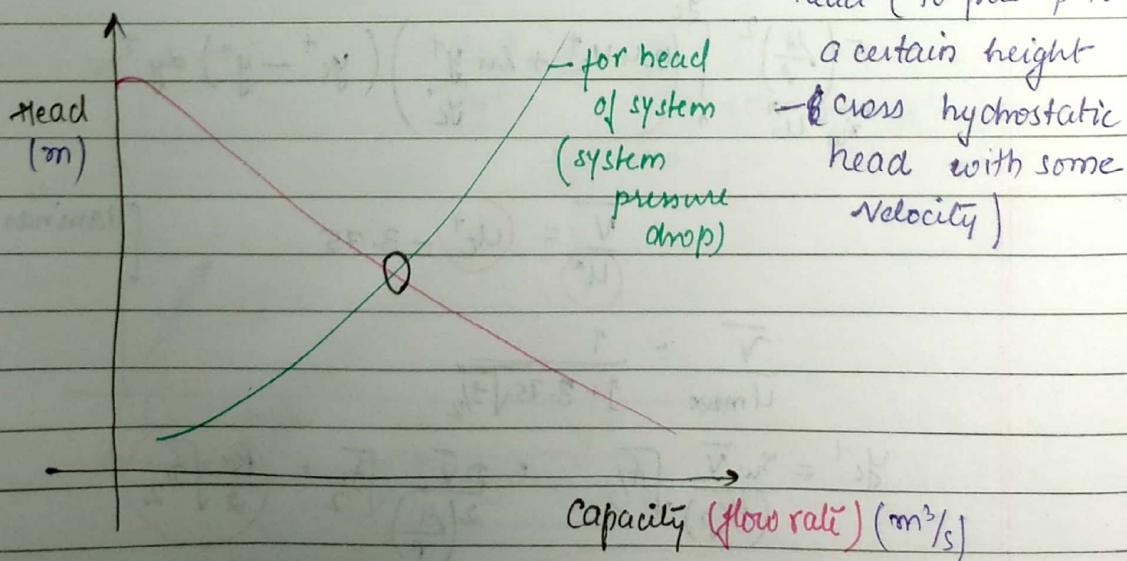
PA must be greater than vapour pressure

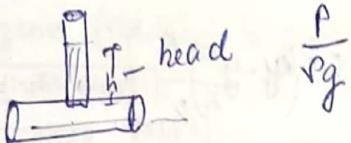
limitation on height-

$$h = 34 \text{ ft}, \rightarrow P_A = 0 \text{ (vacuum)}$$

Net Positive Suction Head (NPSH) is the difference b/w absolute stagnation pressure and vapour pressure

Before that condition (\*)





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—  They converge at one point,  
System Pump pump will work at this Q.

- when we want more pressure in downstream, ~~the~~ ~~the~~ ~~the~~ impeller simply rotates without throwing liquid.

$$\frac{u^+}{u_f} = 5.75 \log_{10} \left[ \frac{u_f y}{u_f p_c} \right] + 5.5$$

$$\bar{u} = \frac{Q}{\pi R^2} = \frac{1}{\pi R^2} \int_0^R u^+ 2\pi r dr$$

$$= \frac{2}{\pi R^2} \int_0^R u_f \left[ 5.75 \log_{10} \left( \frac{u_f y}{u_f p_c} \right) + 5.5 \right] r dr$$

$$= \frac{2 u_f}{R^2} \int_0^R \left[ 5.75 \log_{10} \left( \frac{u_f (r - r_w)}{u_f p_c} \right) + 5.5 \right] r dr$$

$$\frac{2 u_f}{R^2} \int_0^R \frac{\log_{10} \left( \frac{u_f (r - r_w)}{u_f p_c} \right) + 5.5}{r} dr$$

$$(r - r_w) = \frac{u}{u_f} 10^t$$

$$\frac{2 u_f}{R^2} \int_0^R \left[ 5.75 \cdot t + 5.5 \right] \left[ \frac{u}{u_f} 10^t + 20R \right] 10^t dr$$

$$= \frac{2}{R^2} \int_0^R \left[ 5.75 u + 10^t + \frac{20R}{u_f} \right] (R - r) dr$$

$$= \frac{2}{R^2} \int_0^R u_f \left[ 5.75 \log_{10} \left( \frac{u_f y}{u_f p_c} \right) + 5.5 \right] [R - y] dy$$

$$= \frac{2}{R^2} \int_0^R u_f \left[ 5.75 \log_{10} \left( \frac{u}{u_f p_c} \right) + 5.5 \right] dy$$

$$= \frac{2}{R^2} \int_0^R [2.5 \ln y + 5.5] [R - y] dy$$

$$\text{Solving} = \frac{2}{R^2} \int_0^R [2.5 y \ln y - 2.5 y^2] dy + 5.5 R \left[ \frac{Ry - y^2}{2} \right]_0^R$$

$$= \frac{2}{R^2} \int_0^R [2.5 y \ln y - 2.5 y^2] dy$$

$$= \frac{x^2 \ln x}{2} - \frac{x^3}{4}$$

$$\frac{\partial}{R^2} \left[ R \left( 2.5 \ln y - 1 \right) \right]_0^R + 5.5 \left( R y - y^2 \right)_0^R$$

$$-\frac{2}{R^2} \left[ 2.5 \left[ \frac{y^2 \ln y - y^2}{2} \right]_0^R \right]$$

$$= \frac{2}{R^2} \left[ R \left( 2.5 \ln R - 1 \right) + 5.5 \frac{R^2}{2} \right]$$

$$- \frac{2.5}{4R^2} [ 2R^2 \ln R - R^2 ]$$

$$= \frac{2.5 ( \ln R - 1 ) + 5.5}{2.5 \ln R + 1.75}$$

$$\bar{u} = \left[ 5.75 \log_{10} \left( \frac{R y_f}{\kappa} \right) + 1.75 \right] u_f$$

$$\frac{u}{u_f} = 5.75 \log_{10} \left[ \frac{y_f (R - y)}{u_f \kappa} \right] + 5.15$$

$$\boxed{\frac{u - \bar{u}}{u_f} = 5.75 \log_{10} \left( \frac{y}{R} \right) + 3.75} \quad \textcircled{1}$$

Kauman Prandtl equation for velocity distribution near hydrodynamically rough boundaries

$$\frac{u}{u_f} = 5.75 \log_{10} \left( \frac{y}{\kappa} \right) + 3.75$$

$k$ : height of roughness element.

$$u^+ - u_{ref}^+ = 5.75 \log_{10} \left[ \frac{y^+}{y_{ref}^+} \right]$$

$$\text{Find } \boxed{\frac{u - \bar{u}}{u_f} = \dots} \quad \text{same as } \textcircled{1}$$

The universal velocity profile with same gives same result irrespective of smooth / rough pipe.

Benzene ( $37.8^\circ\text{C}$ ) is pumped through a system at  $9.09 \text{ m}^3/\text{kg}$   
 The reservoir is at atmospheric pressure  
 The gage pressure at the end of discharge line is  $345 \text{ kN/m}^2$ . The discharge is  $10\text{ft}$   
 and the suction is  $4\text{ft}$  above the level of reservoir. The friction in  
 suction line is  $3.45 \text{ kPa}$ , and discharge line is  $37.9 \text{ kPa}$

Prime Merit

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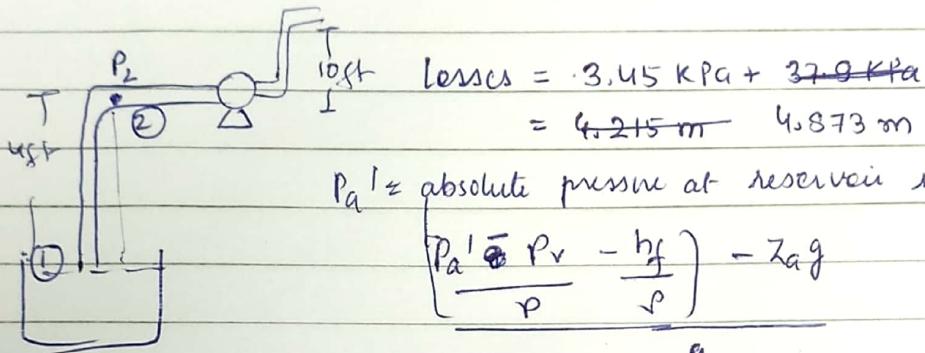
$$\rho_{\text{benzene}} = 865 \text{ kg/m}^3$$

$$P_{\text{vap}} = 26.2 \text{ kPa}$$

$$V_{\text{avg}} = 1.9 \text{ m/s}$$

Calculate developed head of pump  
 NPSH

If  $\eta = 60\%$ , calculate  $P_{\text{input}}^{\text{total}}$  power.



$$\dot{W}_{\text{pump}} = 0 + \frac{\dot{V}^2}{2g} + \dot{W} + g(0) - (10) - \frac{345}{\rho g} - \frac{\dot{V}^2}{2g} = h_L$$

$$\dot{W} = 49.507 - 4.873 = 48.76$$

$$\therefore (49.507 - 4.873) -$$

$$R_e =$$

$$8.85 - 0.4066 = \frac{43.414 \text{ m}}{7.22 \text{ m.}} = 2.35 \text{ m}$$

$$\dot{W}_{\text{in}} = 82.99 - 83.29 - 81.27 = 0 \text{ m} =$$

$$\text{Power to be supplied} = \dot{m} \dot{W}_{\text{in}}$$

$$= 630.038 \text{ kW}$$

$$\dot{m} = 2.18 \text{ kg/s}$$

$$\text{Power} = 1.738 \text{ kW}$$