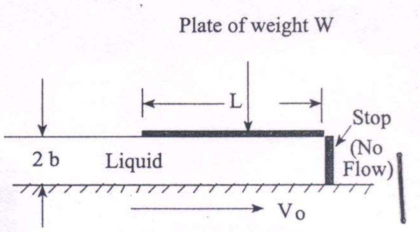
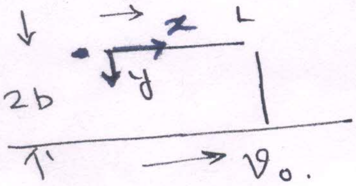
The lower plate of a lubricated thrust bearing moves to the right at velocity V_o. The stop at the right prevents fluid flow beyond that point. Find the weight W that can be supported by the fluid (of viscosity μ and of density ρ). Assume the plate to be wide so that the end effects can be neglected.



The effect of the stop is to provide an unfavorable pr. grad to the flow with max pr. at the stop ($x = L$).

x comp. of eqn of motion (after simplification)



$$0 = \mu \frac{d^2u}{dy^2} - \frac{dp}{dx}$$

$$\frac{du}{dy} = \frac{1}{\mu}\left(\frac{dp}{dx}\right) . y + c_1$$

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right) y^2 + c_1 y + c_2$$

BC. $u = 0$ at $y = 0 \Rightarrow$ $c_2 = 0$.

$u = v_o$ at $y = 2b \Rightarrow$ $v_o = \frac{1}{2\mu}\left(\frac{dp}{dx}\right) . 4b^2 + 2c_1 b$.

$$c_1 = \frac{v_o}{2b} - \frac{1}{2\mu}\left(\frac{dp}{dx}\right) \frac{\cancel{4b^2}}{\cancel{2b}}$$ 2b

$$\therefore u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)\left[y^2 - 2by\right] + \frac{v_o y}{2b}$$

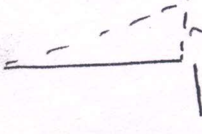
$$\text{Avg. } u, \; \langle u \rangle = \frac{1}{2b}\int_0^{2b} u\, dy = \frac{1}{2b}\left[\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\left\{\frac{y^3}{3} - \frac{2by^2}{2}\right\} + \frac{v_o}{4b} y^2\right]_0^{2b}$$

$$= \frac{1}{2b}\left[\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\left(\frac{8b^3}{3} - 4b^3\right) + v_o b\right].$$

As no flow $\langle u \rangle = 0$

$$-\frac{1}{\cancel{2}\mu}\left(\frac{dp}{dx}\right) . \frac{\cancel{4}\,2}{3} b^3 = -v_o b.$$

$$\left(\frac{dp}{dx}\right) = \frac{3\mu v_o}{2b^2}$$



$p_1 = \frac{3\mu v_o}{2b^2} . L$.

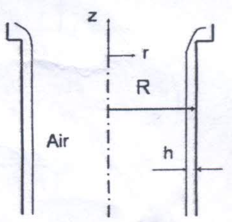
pr. at the stop.

$\therefore$ Av. pr. $= \frac{p_1 + 0}{2} = \frac{3\mu v_o L}{4b^2}$

$\therefore$ Load supported $=$ $p_{av} \times L \times 1$ ~ (unit width)

$= 3\mu v_o L^2$

A wetted wall column is used to measure mass transfer coefficients. A liquid of density ρ and viscosity μ flows down the inside of a tube of radius R, as shown in the figure. After an initial region, the flow becomes fully developed and the thickness of the liquid layer is constant and equal to h. Simplify the Navier-Stokes and continuity equations to obtain $v_z(r)$ for laminar flow in the film.

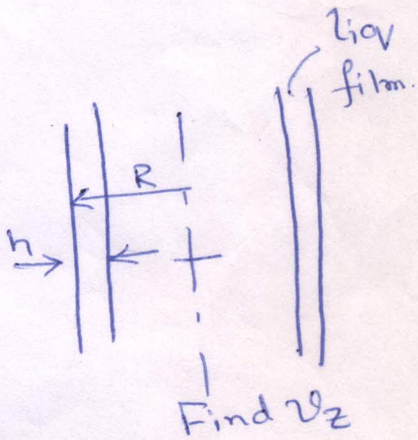


Incompressible flow (ρ const), fully developed $\frac{\partial}{\partial z} = 0$.

$\frac{\partial}{\partial \theta} = 0$ (Symmetry), $v_\theta = 0 = v_r$.

Cont. eqⁿ. $\Rightarrow \frac{\partial}{\partial r}(r v_r) = 0 \Rightarrow r v_r = \text{const}.$

As $v_r = 0$ at $r = 0$ $\therefore r v_r = 0$.



Navier Stoke's eqⁿ. (z comp).

$$\frac{d}{dr}\left(r \frac{dv_z}{dr} \right) = \frac{\rho g r}{\mu}$$

$$\frac{dv_z}{dr} = \frac{\rho g r}{2\mu} + \frac{c_1}{r}$$

$$v_z = \frac{\rho g r^2}{4\mu} + c_1 \ln r + c_2$$

At $r = R$, $v_z = 0$, at $r = R-h$ $\frac{dv_z}{dr} = 0$.

$$0 = \frac{\rho g (R-h)}{2\mu} + \frac{c_1}{R-h}$$

$$\therefore c_1 = -\frac{\rho g (R-h)^2}{2\mu}$$

$$\therefore v_z = \frac{\rho g r^2}{4\mu} - \frac{\rho g (R-h)^2}{2\mu} \ln r + c_2.$$

$$0 = \frac{\rho g R^2}{4\mu} - \frac{\rho g (R-h)^2}{2\mu} \ln R + c_2.$$

$$\therefore v_z = \frac{\rho g}{2\mu}\left[(R-h)^2 \ln \frac{R}{r} - \frac{(R^2 - r^2)}{2} \right]$$