INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

End-Spring Semester Examination, 2015-2016

Subject : Advanced Mathematical Techniques in Chemical Engineering

Subject No.: CH 61015

Date: *Time*: 2 Hrs *Full Marks*: 50

Instructions : Answer **all Questions**. Assume any missing data suitably. Closed notes/books

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1. Consider a problem, Lu=f, where, $L=3x\frac{d^2}{dx^2}+5\frac{d^2}{dx^2}+3$ with set of boundary conditions B such as at x=0, $\frac{du}{dx}=0$ and at x=1, $\frac{du}{dx}-4u=0$. Find out adjoint operator L* with associated boundary condition set B*. (10)

2.Working in a spherical coordinate system an engineer has landed into the following equation:

$x^2\frac{d^2y}{dx^2}-8x\frac{dy}{dx}+y=0$ **with BC at x=0, y=finite and x=1, y=1**

Find the solution of above equation. (6)

3. Consider PDE, $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ **with conditions: at t=0, u=1; at x=0, u=0 and at x=1,** $\frac{\partial u}{\partial x}+u=2$. **Divide this problem into sub-problems such that each sub-problem is well posed.** (10)

4. A transient chemical engineering system is described as,

$$\frac{dx_1}{dt}=x_1^2-ax_1x_2-x_1 \text{ and } \frac{dx_2}{dt}=bx_2^2+x_1x_2-2x_2$$

where, a and b are two positive process parameters. Find the possible steady states. Check stability of each steady state. (10)

3. Identify the following equations homogeneous, non-homogeneous, linear, non-linear: (5)

(i) $u\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$; (ii) $u\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+5x$; (iii) $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+5ux$

(iv) $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+5u$; (4)

4. Consider an enzyme catalyzed fermentation process is governed by Monod kinetics is occurring in a CSTR with volume V, flow rate q. Feed concentration of the reactant S is C_f. The reaction occurs isothermally. The reaction can be considered as an irreversible reaction as S→P, where rate equation is given as $-r_A=\frac{k_1c}{(1+k_2c)^2}$. (10)

Show that two parameters of this system are $Da=k_1V/q$ **and** $\sigma=k_2c_f$. **Find the condition for existence of unique steady state. If D=3, show that maximum value of** σ **is 8.**

———*END & Best Wishes*———