External forced convection tutorial

Spring 2020

1 Problem set

Problem 1. Air at 27° C 1 atm flows over a flat plate at a speed of 2 m/s. Calculate the hydrodynamic boundary layer thickness at a distance of 20 and 400 cm from the leading edge of the plate. Viscosity of air at 27° C is 1.85×10^{-5} kg/m s. Density of air under these condition is 1.177 kg/m^3

Problem 2. Discuss the variation of h with x and show that for isothermal flat plate heating $\bar{h} = 2h\big|_{x=L}$.

Problem 3. Air at 20° C and moving at 15 m/s is warmed by an isothermal steam heated plate at 110° C. The dimension of the plate is 0.5 m $\times$ 0.5 m. Calculate h, δ_t, δ at the trailing edge. Calculate the total rate of heat transfer from the plate.

Problem 4. Air at 15° C flows at 1.8 m/s over a 0.6 m long heating panel. The panel is intended to supply 400 W/m^2 to the air, but the surface can sustain only about 105° C without being damaged. Is it safe? What is the average temperature of the plate?

Problem 5. A circular pipe of 25 mm outer diameter is placed in an crossflow airstream at 25° C and 1 atm. The air moves at 15 m/s, while the outer surface is maintained at 100° C. Calculate the rate of heat loss/unit length of the pipe.

Problem 6. A bank of tubes whose surface temperature is 70° C is available to heat air (15° C) flowing at 6 m/s. The tubes are arranged in staggered arrangement with geometries as shown in Figure (Figure will be supplied at class) Determine the air side convection coefficient.

2 External forced convection correlations

The correlations for a few situations of importance for external forced convection is given in Table 1. Some correlations are more elaborate that the simple ones given in Table 1. One such example is for flow of a fluid over a bank of tubes as shown in Figure 1. The form of the correlation is similar to other correlations but the details are little involved. The rules for obtaining various quantities in the correlation are listed below:

$$\overline{Nu}_D = C_1 Re_{D,Max}^m Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$$

1. The value of the constants are dependent on the arrangement of tubes. Two arrangements are possible: align and staggered. These are shown in Figure 2. The values of C_1 and m are given in Table 3. Applicable in $0.7 \leq Pr \leq 500$

2. This correlation is for number of rows (N_L) greater than 20 (See Figure 3 for clear understanding). For lesser number of rows, a correction factor is needed.

$$\overline{Nu}_D\Big|_{Corrected} = C_2 \overline{Nu}_D\Big|_{Obtained}$$

   The constant C_2 is given in Table 4.

3. All properties except Pr_s are evaluated at the mean temperature defined by

$$T_m = (T_{in} + T_{out})/2$$

4. $Re_{D,Max} = V_{max} D/\nu$
   For aligned arrangement:

$$V_{Max} = \frac{S_T}{S_T - D} V$$

   For staggered arrangement:

$$\begin{aligned} V_{Max} &= \frac{S_T}{S_T - D} V \quad \text{if } 2(S_D - D) > (S_T - D) \\ &= \frac{S_T}{2(S_D - D)} V \qquad \text{Otherwise} \end{aligned}$$

5. Meaning of various geometric quantities in this equation are provided in Figure 3

Table 1: External Forced Convection Correlations. Source: Fundamentals of Heat and Mass Transfer, Incropera et al., Seventh Edition

Condition	Correlation
Flat plate, laminar flow, constant wall temperature. Properties evaluated at $T_f = (T_w + T_\infty)/2$ $Pr \geq 0.6; \quad Re_x \leq 5 \times 10^5$	$Nu_x = 0.332 Re^{1/2} Pr^{1/3}$ $\overline{Nu}_L = 0.664 Re^{1/2} Pr^{1/3}$
Flat plate, laminar flow, constant wall heat flux. Properties evaluated at $T_f = (T_w + T_\infty)/2$ $Pr \geq 0.6; \quad Re_x \leq 5 \times 10^5$	$Nu_x = 0.453 Re^{1/2} Pr^{1/3}$ $\overline{Nu}_L = 0.680 Re^{1/2} Pr^{1/3}$
Flat plate, turbulent flow, constant wall temperature. Properties evaluated at $T_f = (T_w + T_\infty)/2$ $0.6 \leq Pr \leq 60; \quad 5 \times 10^5 \leq Re_x \leq 10^8$	$Nu_x = 0.0296 Re^{4/5} Pr^{1/3}$
Flat plate, turbulent flow, constant wall heat flux. Properties evaluated at $T_f = (T_w + T_\infty)/2$ $0.6 \leq Pr \leq 60; \quad 5 \times 10^5 \leq Re_x \leq 10^8$	$Nu_x = 0.0308 Re^{4/5} Pr^{1/3}$
Isolated solid sphere. All properties except μ_s (surface viscosity) are evaluated at T_∞. $1.0 \leq (\mu/\mu_s) \leq 3.2$ $0.71 \leq Pr \leq 380; \quad 3.5 \leq Re_D \leq 7.6\times10^4$	$\overline{Nu}_D = 2 + \phi(Re) Pr^{0.4} (\mu/\mu_s)^{0.25}$ $\phi(Re) = \left(0.4\sqrt{Re_D} + 0.06 Re^{0.66}\right)$
Cylinder in cross flow. All properties are evaluated at T_f. C and m are given in Table 2. $Pr \geq 0.7$	$\overline{Nu}_D = C Re_D^m Pr^{1/3}$

Table 2: C and m for Nusselt number correlation for cylinder in cross flow. Source: Fundamentals of Heat and Mass Transfer, Incropera et al., 7th Ed.

Re_D	C	m
0.4 – 4	. 0.989	. 0.330
4 – 40	. 0. 911	0.385
40 – 4000.	. 0.683.	. 0.466.
4000 –40,000	. 0.192.	0.618.
40,000 – 400,000	0.027.	. 0.805.

Table 3: Constants for the Nusselt number correlation for a bank of tubes. If $10^2 \leq Re \leq 10^3$, approximate as single (isolated) cylinder.

Configuration	$Re_{D,Max}$	C_1	m
Aligned	$10^1 - 10^2$	0.80	0.40
Staggered	$10^1 - 10^2$.	0.90	0.40
Aligned ($S_T/S_L > 0.7$)	$10^3 - 2 \times 10^5$	0.27	0.63
Staggered ($S_T/S_L < 2$)	$10^3 - 2 \times 10^5$	$0.35(S_T/S_L)^{0.2}$	0.60
Staggered ($S_T/S_L > 2$)	$10^3 - 2 \times 10^5$	0.40	0.60
Aligned	$2 \times 10^5 - 2 \times 10^6$	0.21	0.84
Staggered	$2 \times 10^5 - 2 \times 10^6$	0.22	0.84

Table 4: Correction factor C_2 for a bank of tubes $N_L < 20$. $Re_{D,Max} \geq 1000$

N_L	1	2	3	4	5	7	10	13	16
Aligned	0.70	0.80	0.86	0.90	0.92	0.95	0.97	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.92	0.95	0.97	0.98	0.99

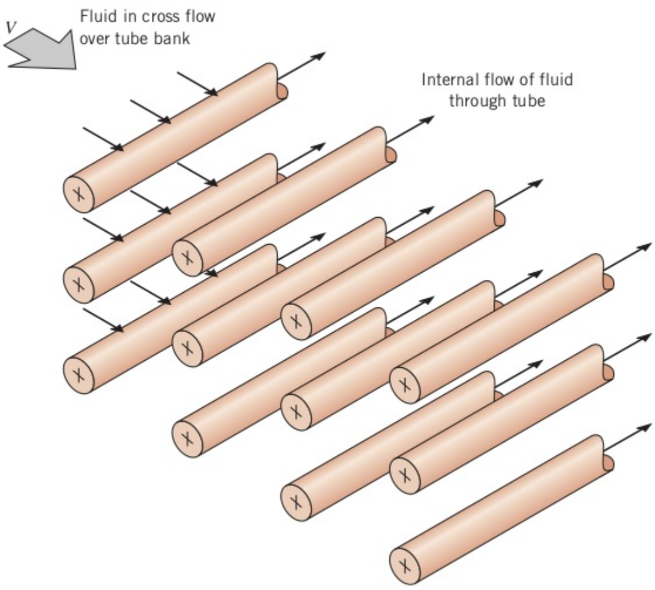

Figure 1: 3D view of flow of internal and external fluid through a bank of tubes.

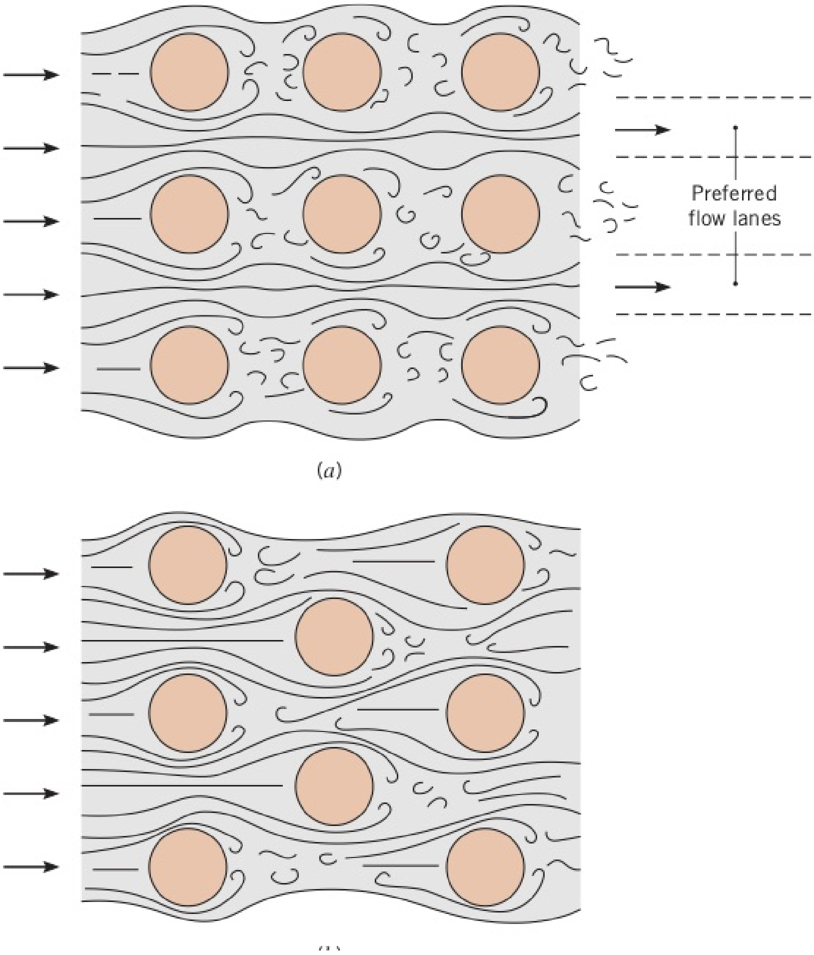

Figure 2: Bank of tubes: aligned and staggered arrangements

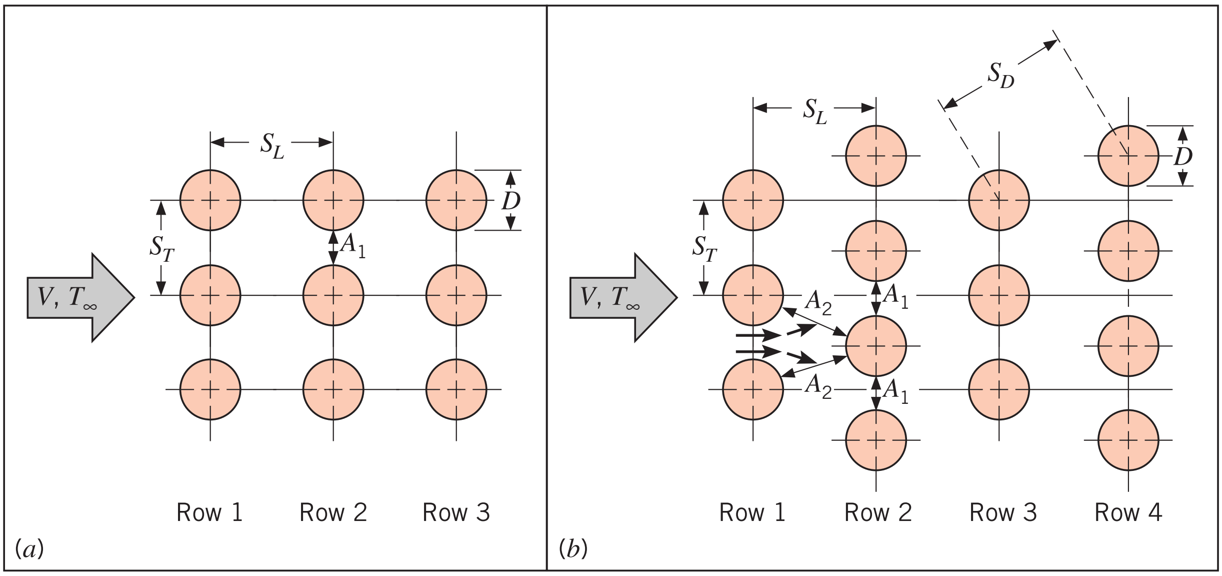

Figure 3: Bank of tubes: the geometry.

T (K)	ρ (kg/m^3)	c_p (kJ/kg·K)	$\mu \cdot 10^7$ (N·s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m·K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Air, $\mathcal{M}$ = 28.97 kg/kmol							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683
600	0.5804	1.051	305.8	52.69	46.9	76.9	0.685
650	0.5356	1.063	322.5	60.21	49.7	87.3	0.690
700	0.4975	1.075	338.8	68.10	52.4	98.0	0.695
750	0.4643	1.087	354.6	76.37	54.9	109	0.702
800	0.4354	1.099	369.8	84.93	57.3	120	0.709
850	0.4097	1.110	384.3	93.80	59.6	131	0.716
900	0.3868	1.121	398.1	102.9	62.0	143	0.720
950	0.3666	1.131	411.3	112.2	64.3	155	0.723
1000	0.3482	1.141	424.4	121.9	66.7	168	0.726
1100	0.3166	1.159	449.0	141.8	71.5	195	0.728

Figure 4: Properties of air