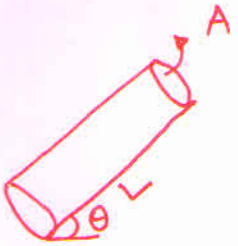

ρ_g = Actual gas density

ρ_s = " solid density

ϵ = void fraction (fraction of air volumewise)

ρ_{gd} = disperse gas phase density

ρ_{sd} = disperse solid phase density

$$\boxed{\rho_{gd} = \frac{M}{V_{total}} \;\&\; \rho_g = \frac{M}{V\epsilon} \Rightarrow \rho_{gd} = \rho_g\epsilon} \;\&\; \rho_{sd} = \rho_s(1-\epsilon)$$

$$(\Delta P)A - \underbrace{F_{gw}}_{\text{friction between pipe-wall \& gas}} - \underbrace{F_{gp}}_{\text{friction (between Particle \& gas)}} - AL\epsilon\rho_g \sin\theta - AL(1-\epsilon)\rho_s g\sin\theta = \frac{\rho_g A\epsilon\,\overline{U}_g^{\,2}}{2} + \frac{\rho_s A(1-\epsilon)\overline{U}_p^{\,2}}{2} \quad \text{—(1)}$$

$\overline{U}_g$ = Actual Avg. gas phase velocity

$\overline{U}_p$ = Actual Avg solid " "

$\overline{U}_{gz}$ = gas phase superficial velocity

$\overline{U}_{pz}$ = solid phase superficial velocity

$$\overline{U}_g = \frac{Q_g}{(A\epsilon)}$$

only fraction of cross section is available

$$\overline{U}_{gz} = \frac{Q_g}{A}$$

Q_g → volumetric flow rate of gas.

$$\boxed{\frac{\overline{U}_{gz}}{\epsilon} = \bar{u}_g} \qquad \boxed{\frac{\overline{U}_{pz}}{(1-\epsilon)} = \bar{u}_p}$$

calculation of F_{gw}

$$F_{gw} = (\Delta P_{gw})A$$

$$\frac{\Delta P_{gw}}{\frac{1}{2}\rho_{gd}\overline{U}_{gz}^{\,2}\frac{L}{D}} = f_{gw}$$

f_{gw} → friction factor

$$\boxed{Re = \frac{\rho_g\,\overline{U}_{gz}\,D}{\mu_g}}$$

for a given Re, we know f_{gw}

so $$\boxed{F_{gw} = \frac{1}{2}(f_{gw})\,\rho_{gd}\frac{L}{D}\,\overline{U}_{gz}^{\,2}\,A}$$

$$F_{gw} = \frac{1}{2}(f_{gw})\,\rho_g\epsilon\,\frac{L}{D}\,\overline{U}_{gz}^{\,2}\,A \quad \text{——(2)}$$

F_{gp} calculation (total Drag force between particles & gas)

Considering one-particle gas interaction.

d = particle diameter

$$\frac{F_{\text{Drag on one particle}}}{\frac{1}{2}\rho_g(\bar{U}_g-\bar{U}_p)^2\frac{\pi d^2}{4}} = C_D \rightarrow \text{drag co-efficient}$$

★ In dilute phase transport we ignore the friction between particle & wall as the contact time is really very small.

$$F_{gp} = \underbrace{(N)}_{\text{no of particles in the given volume}} \times F_{\text{on one particle}}$$

$$= \left(\frac{AL(1-\epsilon)}{\frac{4}{3}\pi\frac{d^3}{8}}\right)\left(C_D\frac{1}{2}\rho_g(\bar{U}_g-\bar{U}_p)^2\frac{\pi d^2}{4}\right)$$

$$= \frac{3}{4}C_D\frac{AL(1-\epsilon)}{d}\rho_g(\bar{U}_g-\bar{U}_p)^2 \quad \text{—— (3)}$$

from eqn (1), (2) & (3)

$$(\Delta P)\cancel{A} = \left(\frac{1}{2}f_{gw}\rho_g\epsilon\frac{L}{D}\bar{U}_{gz}^2\right)\cancel{A} + \frac{3}{4}C_D\frac{L}{d}(1-\epsilon)\rho_g(\bar{U}_g-\bar{U}_p)^2\cancel{A}$$

$$+\cancel{A}L\epsilon\rho_g g\sin\theta + \cancel{A}L(1-\epsilon)\rho_s g\sin\theta$$

$$+\frac{1}{2}\rho_g A\epsilon\frac{\bar{U}_g^2}{2} + \frac{1}{2}\rho_s A(1-\epsilon)\frac{\bar{U}_p^2}{2}$$

$\bar{U}_g = \bar{U}_{gz}/\epsilon$, $\bar{U}_p = \dfrac{\bar{U}_{pz}}{(1-\epsilon)}$

$$\Delta P = \frac{1}{2}f_{gw}\rho_g\epsilon\frac{L}{D}\bar{U}_{gz}^2 + \frac{3}{4}C_D\frac{L}{d}(1-\epsilon)\rho_g\left(\frac{\bar{U}_{gz}}{\epsilon}-\frac{\bar{U}_{pz}}{(1-\epsilon)}\right)^2$$

$$+L\epsilon\rho_g g\sin\theta + L(1-\epsilon)\rho_s g\sin\theta$$

$$+\frac{1}{2}\rho_g A\epsilon\frac{\bar{U}_g^2}{2} + \rho_s A(1-\epsilon)\frac{\bar{U}_p^2}{2}$$