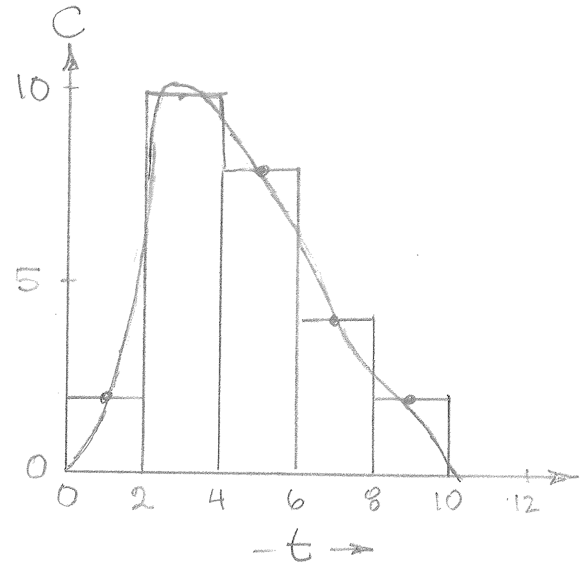
14.1 From experiment

t	t_{mean}	C
0-2	1	2
2-4	3	10
4-6	5	8
6-8	7	4
8-10	9	2
10-12	11	0

with linear interpolation



Find the variance & then N

From Eqs 13.1 and 13.3

$$\bar{t} = \frac{\Sigma tC}{\Sigma C} = \frac{1(2)+3(10)+5(8)+7(4)+9(2)}{2+10+8+4+2} = \frac{118}{26} = 4.538$$

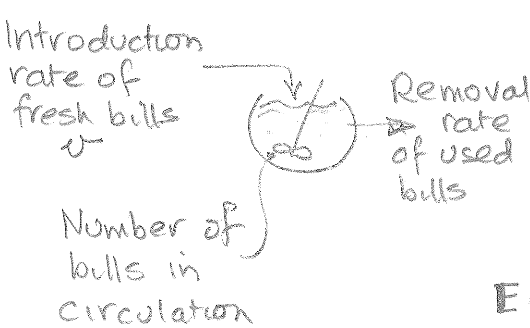
$$\sigma^2 = \frac{\Sigma t^2 C}{\Sigma C} - \bar{t}^2 = \frac{1^2(2)+3^2(10)+\cdots}{2+10+\cdots} - (4.538)^2 = 4.4038$$

From Eq 3

$$\frac{1}{N} = \sigma_\theta^2 = \frac{\sigma^2}{\bar{t}^2} = \frac{4.4038}{(4.538)^2} = 0.2138$$

$$\therefore N = \frac{1}{0.2138} = 4.68 \text{ tanks}$$

14.3 The analogy with mixed flow should be evident. All we need to do is to identify the various quantities. Referring to the sketch



$v = 10^9$ bills/yr

$V = 1.25 \times 10^9$ bills

$\bar{t} = \frac{V}{v} = 1.25$ yrs

n = no. of bills

n_0 = total no. of bills.

$$\mathbf{E}(t) = \frac{\text{fraction}}{\text{yr}} = \frac{n}{n_0}$$

14.3 a) (continued) How many new bills will be in circulation at any time t?

$$F(t) = 1 - e^{-t/\bar{t}} = 1 - e^{-t/1.25} \qquad \left.\begin{array}{ll} t=0 & F(t)=0 \\ t=\infty & F(t)=1 \end{array}\right\}$$

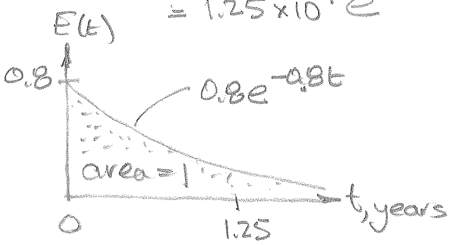
b) The number of bills in circulation which are over 21 yrs of age is given by the E(t) function. Thus

$$n = \int_{21}^{\infty} n_0 E(t)\,dt \quad \text{--- where} \quad n_0 = 1.25\times10^9$$

$$E(t) = \frac{1}{\bar{t}} e^{-t/\bar{t}} = 0.8e^{-0.8t}$$

$$\therefore n = \int_{21}^{\infty} (1.25\times10^9)(0.8e^{-0.8t})$$

$$= 1.25\times10^9 e^{-0.8t}\Big|_{21}^{\infty} = (-1.25\times10^9)\left[\frac{1}{e^{\infty}} - \frac{1}{e^{0.8(21)}}\right] = 63.2 \text{ bills}$$



We get this 20¢ because we assume a continuous distribution of ages when in fact this is not so

---

14.5 For plug flow

$$k\bar{t} = \ln\frac{C_{A0}}{C_A} = \ln\frac{1000}{1} = 6.9078$$

For small deviation from plug flow, by the tanks in series model first calculate σ^2 from the tracer curve. From Fig 13.P13

$$\sigma^2 = \frac{a^2}{24} = \frac{4^2}{24} = \frac{2}{3}$$

$$\bar{t} = 10$$

From Eq 3

$$\frac{1}{N} = \sigma_\theta^2 = \frac{\sigma^2}{\bar{t}^2} = \frac{2/3}{(10)^2} = 0.67\times10^{-2} \quad \cdots \therefore N = 150 \text{ tanks}$$

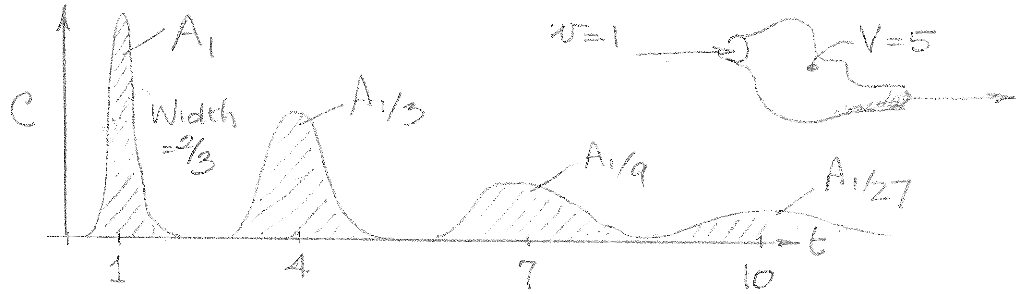
$$\therefore k\bar{t}_i = \frac{6.9078}{150} = 0.0461$$

For tanks in series

$$\frac{C_A}{C_{A0}} = \frac{1}{(1+k\bar{t}_i)^N} = \frac{1}{(1+0.0461)^{150}} = \frac{1}{863} = 0.00116$$

$$\therefore C_A = 1.16$$

14.7



First approximate each pulse by plug flow. Then from Fig 12·1

$$\frac{A_2}{A_1} = \frac{1}{3} = \frac{R}{R+1} \quad \cdots \quad \therefore R = \frac{1}{2}$$

$$\frac{V_{p1}}{(R+1)v} = 1 = \frac{V_{p1}}{(\frac{1}{2}+1)1} \quad \therefore V_{p1} = \frac{3}{2}$$

$$\frac{V_{p1}}{(R+1)v} + \frac{V_{p2}}{Rv} = 3 = \frac{3/2}{(1/2+1)1} + \frac{V_{p2}}{(1/2)1} \qquad \therefore V_{p2} = 1$$

So $V_{active,\,total} = \frac{3}{2} + 1 = 2.5$

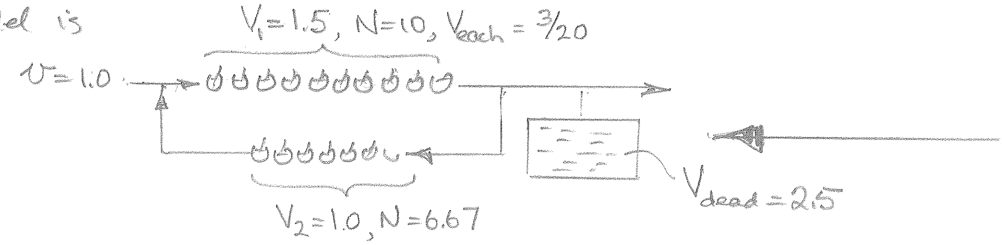
$\therefore V_{dead} = 5 - 2.5 = 2.5$

Now consider that the pulse output has width. Then from Fig 3

$$\frac{\Delta\theta}{\theta_{max}} = \frac{2/3}{1} = \frac{2}{\sqrt{N_1 - 1}} \quad \cdots \quad N_1 = 10 \text{ tanks}$$

and $\quad N_2 = 2/3\, N_1 = 2/3\,(10) = 6.67$ tanks

Thus our model is



Verify

$$\bar{t} = \frac{\Sigma tC}{\Sigma C} = \frac{1(A_1) + 4(A_1/3) + 7(A_1/9) + 10(A_1/27) + 13(A_1/81) + 16(A_1/243) + \cdots}{A_1 + A_1/3 + A_1/9 + A_1/27 + A_1/81 + A_1/243 + \cdots}$$

$$= \frac{1 + \frac{4}{3} + \frac{7}{9} + \frac{10}{27} + \frac{13}{81} + \frac{16}{243} + \frac{19}{729} + \cdots}{1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \frac{1}{729} + \cdots} = \frac{3.7339}{1.4993} = 2.49 \approx 2.5$$

the limit

This limit equals $\dfrac{V_1 + V_2}{v} = \dfrac{1.5 + 1.0}{v} = 2.5$

14.9 For N tanks in series

Fig 25 pg 292 shows that ... $$N = 1 + 4\left(\frac{\theta_{max}}{\Delta\theta}\right)^2$$

location of the maximum

width of the curve at 60% of its maximum height.

Using a ruler with mm scale and a keen eye we find from Fig P21

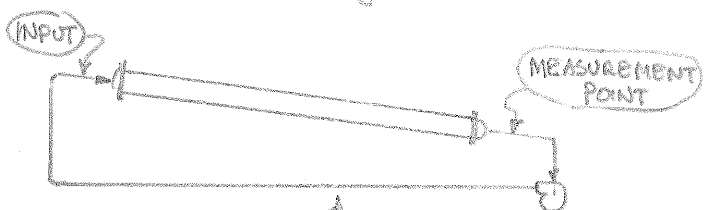
for 1st peak: $N_{1\,pass} = 1 + 4\left(\frac{16.2\text{ mm}}{3.2\text{ mm}}\right)^2 = 104$ tanks ... or N = 104 tanks/pass

for 2nd peak: $N_{2\,passes} = 1 + 4\left(\frac{32.6}{4.6}\right)^2 = 202$ tanks ... or N = 101 tanks/pass

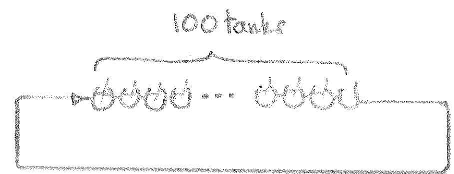
for 3rd peak: $N_{3\,passes} = 1 + 4\left(\frac{48.7}{5.6}\right)^2 = 304$ tanks ... or N = 101 tanks/pass

for 4th peak: $N_{4\,passes} = 1 + 4\left(\frac{65}{6.6}\right)^2 = 389$ tanks ... or N = 97 tanks/pass

These values average to $N \cong 100$ tanks/pass



Physical set up



Model

14.11 To find the non ideal characteristics of the experimental reactor determine the proper D/uL to use for the dispersion model, or the proper N value to use for the tanks-in-series model.

This is done in one of two ways — by matching the experimental tracer curve with the family of curves shown in Fig 13.9 (for the dispersion model) or with Fig 2 (for the tanks-in series model), or by calculating σ_θ^2, and from that D/uL or N.

Let us use the latter procedure. So first calculate $\bar{t}$ and σ^2 from the table of data with Eqs. 13.1 and 13.3. This gives

$\Sigma C = 213$ $\qquad \bar{t} = 2149/213 = 10.09$ min

$\Sigma tC = 2149$

$\Sigma t^2 C = 37685$ $\qquad \sigma^2 = 37685/213 - (10.09)^2 = 75.1168$

$$\therefore \sigma_\theta^2 = \frac{\sigma^2}{\bar{t}^2} = \frac{75.1168}{(10.09)^2} = 0.7378$$

Next determine the behavior in an ideal plug flow reactor

$k = 0.456\ \text{min}^{-1}$ $\quad k\bar{t} = (0.456)(10.09) = 4.6$

So for plug flow $\quad X_A = 1 - e^{-kt} = 1 - e^{-4.6} = 0.99$

Now we are ready to proceed with our problem

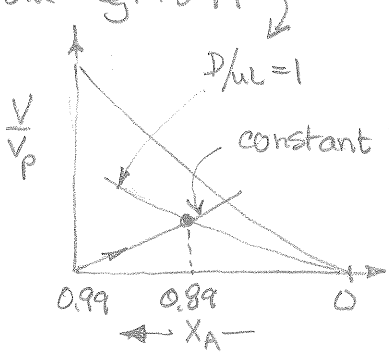
(a) Use the dispersion model

Here Eq. 13.15 relates σ_θ^2 with D/uL. So

$$\sigma_\theta^2 = 0.7378 = 2\left(\frac{D}{uL}\right) - 2\left(\frac{D}{uL}\right)^2\left[1 - e^{-uL/D}\right]$$

Solve by trial and error. This gives $D/uL = 1$

Then from Fig. 13.19



From this figure

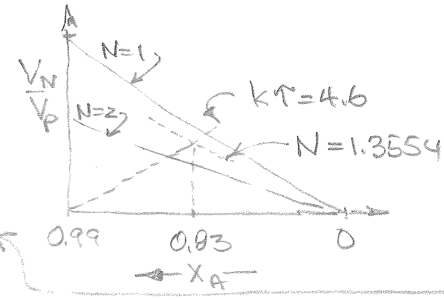
$X_{disp.} = 0.89$ ← a)

14.11 (continued) (b) Use the tanks-in-series model

From Eq. 3 we find

$$N = \frac{1}{\sigma_\theta^2} = \frac{1}{0.7388} = 1.3554 \text{ tanks}$$

So from Fig. 6.5 we find $X_{tanks} = 83\%$



b)

(c) Use the tracer data directly.

From Eq 11.13 $\quad \frac{C}{C_0} = \sum \left(\frac{C}{C_0}\right)_{batch} E\Delta t$

To find the E curve make the area under the C curve unity, or as shown in Example 11.1

$$E = \frac{C}{\text{area}} = \frac{C}{\sum C_i \Delta t} \quad \cdots \text{ or } \quad E\Delta t = \frac{C}{\sum C}$$

t, min	C	$e^{-0.456 t_i} C_i/213$
0	0	0
1	17	0.0506
3	38	0.0454
5	33	0.0158
7	26	0.0050
9	20	0.0015
11	16	0.0005
13	13	0.0002
15	10	0.0001
17	8	16×10^{-6}
19	6	5×10^{-6}
21	5	2×10^{-6}
23	4	1×10^{-6} ≈ 0
25–27	3	–
29–31	2	–
33–45	1	–
	$\sum C = 213$	$\sum = 0.1191$

Now follow the method of Example 11.4

$$\frac{C}{C_0} = \sum \left(\frac{C}{C_0}\right)_{batch} E\Delta t$$

$$= \sum e^{-kt_i} \frac{C}{\sum C}$$

$$= \sum_i e^{-0.456 t_i} \cdot \frac{C_i}{213}$$

now tabulate

$X_{\text{from curve}} = 0.88$ c)

(d) Which answer is most reliable

Naturally the direct use of the tracer curve gives the most reliable answer. In this problem the given RTD came from the dispersion model with $D/uL = 1$. Thus we'd expect that the answers to parts a) and c) should agree. They do.