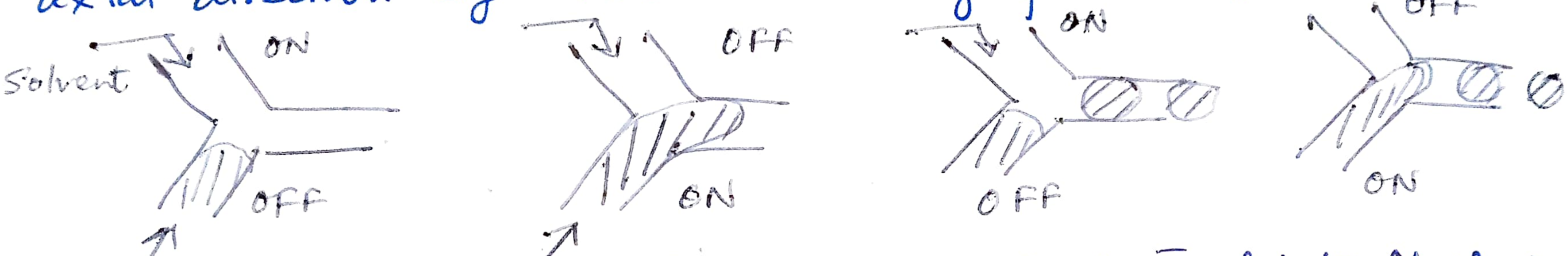
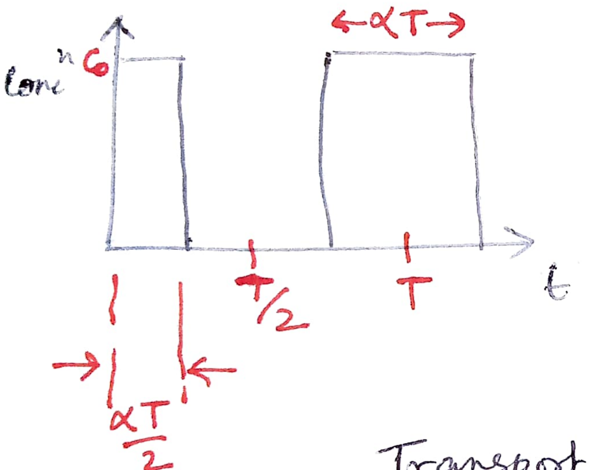
Mixing by Sequential Segmentation

Solvent and solute streams are broken up into segments along the axial direction by alternate switching of inlet flows





- Mean flow velocity $\bar{u}$ of both fluids in the mixing channel
- Switching time period $= T =$ Period of segmentation
- Characteristic mixing length $=$ Segment Length $L = \bar{u}\, T$

C_0 = initial conc^n of solute in the solute stream

α = Mixing Ratio

Transport Equation :
$$\frac{\partial c}{\partial t} + \bar{u}\frac{\partial c}{\partial x} = D^*\frac{\partial^2 c}{\partial x^2}$$

Here D^* = Dispersion Coefficient

At inlet $(x=0)$

$$c(0,t) = \begin{cases} C_0 & \text{for } 0 \le t \le \frac{\alpha T}{2} \\ 0 & \text{for } \frac{\alpha T}{2} < t \le T - \frac{\alpha T}{2} \\ C_0 & \text{for } T - \frac{\alpha T}{2} < t \le T \end{cases}$$

At $x \to \infty$

$$c(\infty, t) = \alpha\, C_0$$

Analytical Solution Available (Refer book on Micromixer)

SEQUENTIAL LAMINATION for Mixing

Seggregates the joined stream into two channels, and rejoins them in the next transformation stage
(Also referred as Split & Recombine Mixer)

Refer the picture in p/145 of the book on Micromixer

Consider time dependent striation thickness $w(t)$ with initial value $W(0) = W \sim$ the width of the microchannel.

The decrease of striation thickness follows $\frac{d \ln W(t)}{dt} = -\alpha(t)$

where $\alpha(t)$ is referred as stretching function, and is positive.

Basically this implies $W(t) = W e^{-\alpha t}$ i.e., two particles situated on neighboring trajectories diverge (on average) at an exponential rate, which is a property referred as "sensitivity to initial condition" applicable to ~~chator~~ chaotic advection.

Here, chaotic mixing is not caused by turbulence. Flow is of low Reynold's Number. Mixing is due to induced shear that causes additional interface including Taylor dispersion. However, the trajectories are chaotic that leads to stretching and folding.

SEQUENTIAL LAMINATION ... Contd.

In the local coordinate system of the striation

$$\frac{\partial c^*}{\partial t^*} = \frac{\partial^2 c^*}{\partial x^{*2}} \quad \text{where} \quad x^* = \frac{x}{w(t)}$$

$$t^* = \int_0^t \frac{D}{w(t')^2}\, dt' \equiv$$ Dimensionless time to diffuse into all possible striations formed till time t.

Solution of above equation gives penetration distance δ in (x, t) space.

Mixing is complete when $\delta_x = W e^{-\alpha t}$

Reference: Ottino and Wiggins "Introduction: Mixing in microfluidics", Phil. Trans. R. Soc. Lond. A Vol. 362, Page 923-935 (2004)

CHARACTERIZATION OF MIXING

Dankwert's Seggregation Intensity in a cross-section

$$\boxed{I = \frac{\sigma^2}{\sigma_{max}^2}}$$

~~where~~ based on mean square deviation of the concentration profile for component 'i' in a cross-section

$$\sigma^2 = \int_A (c_i - \bar{c}_i)^2 \, dA$$

$$\sigma_{max}^2 = A \, \bar{c}_i \, (c_{i,max} - \bar{c}_i)$$

$\boxed{\text{Mixing Quality}}$ = Normalized Seggregation Intensity

$$\equiv \alpha_m = 1 - \sqrt{\frac{\sigma^2}{\sigma_{max}^2}}$$

$\alpha_m = 0$ means completely seggregated

$\alpha_m = 1$ means completely mixed

When the flow velocity varies over the cross-section, the velocity-weighted mixing quality $\alpha_v = 1 - \sqrt{\frac{\sigma_v^2}{\sigma_{v_{max}}^2}}$

where $\sigma_v^2 = \frac{1}{A_m \bar{v}} \int_{A_m} \left(c - \frac{\int_{A_m} c \, v \, dA}{\int_{A_m} v \, dA} \right) v \, dA$

RELEVANT DIMENSIONLESS NUMBERS

Fourier No. $= \dfrac{D\, t_{diff}}{L_{mixing}^2}$

For mixing in two streams flowing side by side in a simple ~~microchannel~~ microchannel of width w, and length L_{mixer}

$t_{res} = t_{diff} \Rightarrow \boxed{\dfrac{L_{mixer}}{U} \approx \dfrac{F_o L_{mixing}^2}{D} = F_o \dfrac{w^2}{D}}$

Peclet no. $= \dfrac{\text{Advective mass transport}}{\text{Diffusive mass transport}} = \dfrac{\bar{u}\, L_{characteristic}}{D}$

$\Rightarrow \dfrac{L_{mixer}}{W} = F_o \dfrac{\bar{u} W}{D} = F_o Pe_w$

For microfluidic channel, $100 < Pe < 10,000$

so that advective transport dominates over diffusive transport.

Also, $0.1 < F_o$ No. < 1.0

$$10 < \frac{L_{mixer}}{W} < 10,000$$

(Mixing channel becomes unacceptably long. This underscores the need for split and recombine n-pairs of solute solvent stream such that $L_{mixing} = \dfrac{W}{n}$

$\Rightarrow \dfrac{L_{mixer}}{W} = \dfrac{1}{n^2} F_o Pe_w$, and the length is reduced.

RELEVANT DIMENSIONLESS NUMBERS .... Contd.

When there is reaction in microchannel given by $\frac{dC_j}{dt} \propto r = k C_j^m$

Upon integration, the characteristic time

$$t_R \text{ for a reaction } \propto \frac{1}{k C_j^{m-1}}$$

Damköhler No. for design of microreactor is defined as

$$\left(\frac{t_{res}}{t_R} \right)$$

Euler No. for energy dissipation due to vortex formation in mixer

$$Eu = \frac{\Delta P}{\rho \bar{v}^2}$$

Mixing Effectiveness $(ME)_I = \frac{1}{Eu} \frac{d_h}{l_m} \approx \frac{\text{Revenue}}{\text{Effort}}$

Here, $l_m = \bar{v}\, t_{res}$ d_h = Hydraulic dia

shorter the l_m, bigger is the revenue, Larger the d_h greater the revenue

Larger the Eu, greater is the effort.

$$(Me)_{II} = \frac{Re}{Eu} \frac{d_h}{l_m}$$