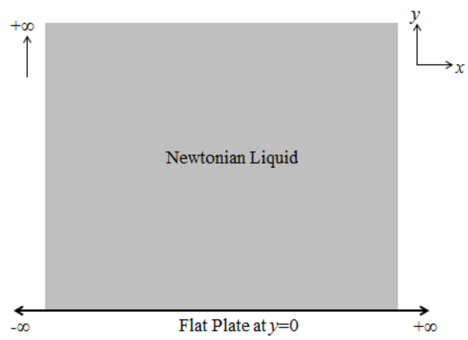
Assignment:1 (TP)

Consider an unsteady state momentum transfer event in semi-infinite domain, wherein a flat plate of infinite length is kept at $y=0$, and the domain $0<y$ contains a Newtonian liquid at rest, as shown in figure below. At time $t=0$, a constant stress τ_0, has been imposed on flat plate in $+x$ direction, which is maintained for $t \geq 0$. Assume 1D momentum transport in $+y$ direction. The constitutive relation for Newtonian liquid is given by: $\tau_{yx} = -\mu \partial V_x / \partial y$, where τ_{yx} is shear stress in $+x$ direction, working on any plane having normal in $+y$ direction. Viscosity of liquid= μ, Density of liquid= ρ.



a) Write down the momentum transport governing equation in terms of V_x.

b) Express the equation obtained in (a) in terms of shear stress, τ_{yx}. (Hint: differentiate the equation obtained in (a) w.r.t y and multiply both side by $-\mu$.)

c) Write down boundary and initial condition.

d) Solve the governing equation obtained in (b) in order to get the time-dependent stress profile, $\tau_{yx}(y,t)$. You can express your answer in terms of error function.

You may need the following, error function is given by: $erf(z) = \left(2/\sqrt{\pi}\right)\int_0^z \exp(-t^2)\,dt$, gamma function is given by: $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\,dt$, and $\Gamma(1/2) = \sqrt{\pi}$.

(Learning: The final expression for stress obtained in (d) can be integrated for getting the velocity profile across the fluid).

e) Comment on velocity of the bottom plate? (Will you maintain the bottom plate at constant velocity in order to achieve a constant shear stress at bottom plate.)