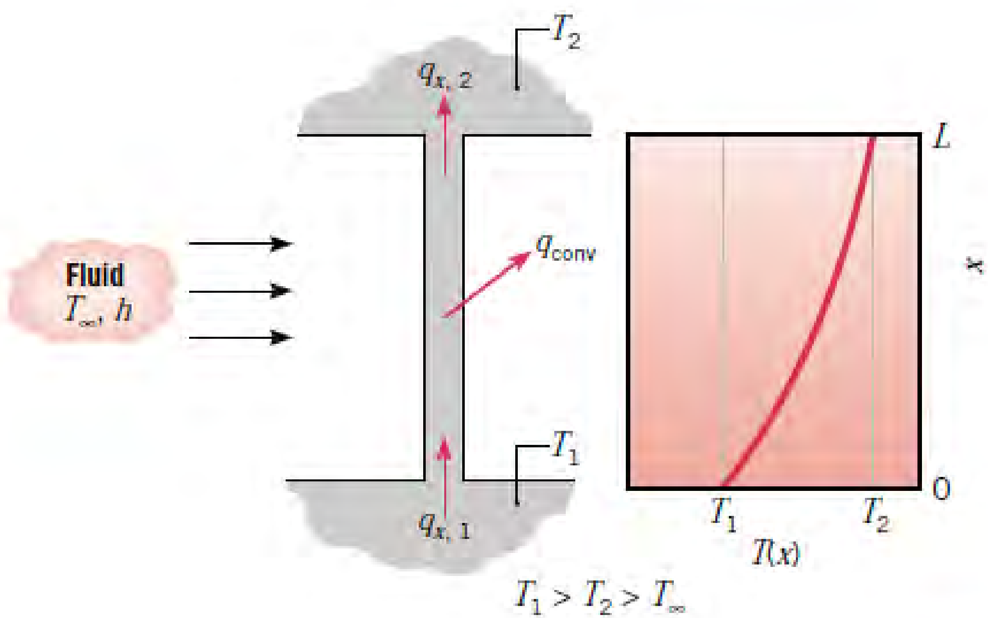
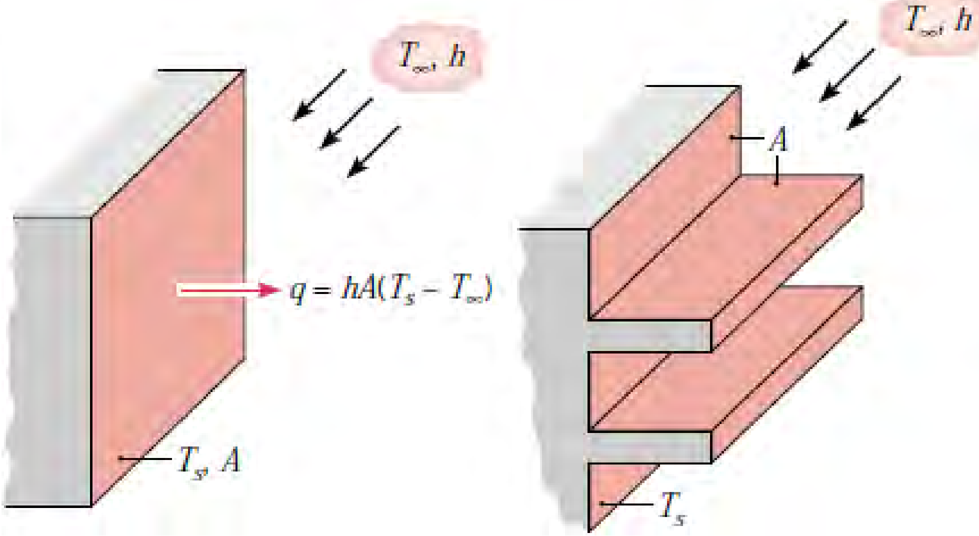
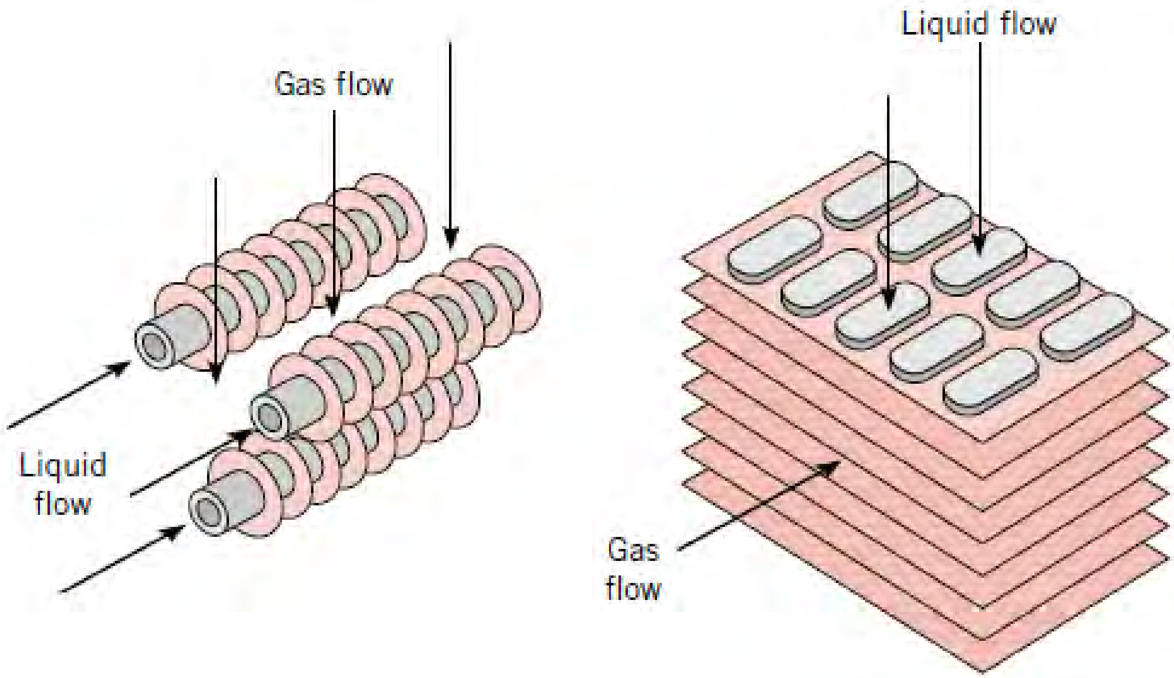
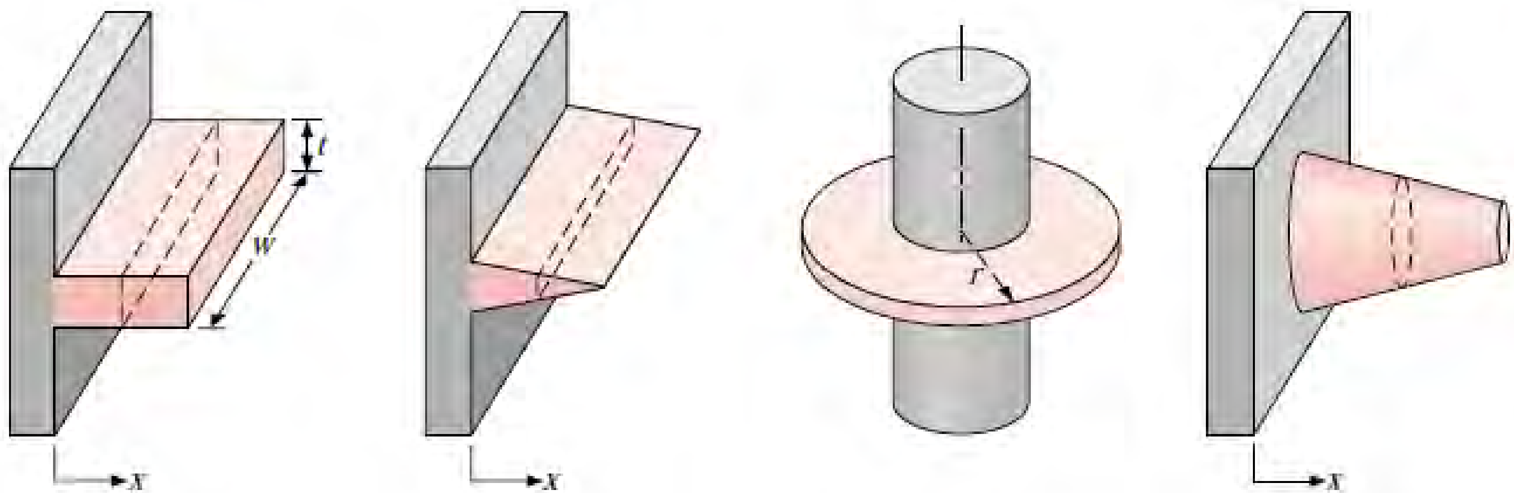
Heat Transfer from Extended Surfaces






Gas flow
Liquid flow
Liquid flow
Gas flow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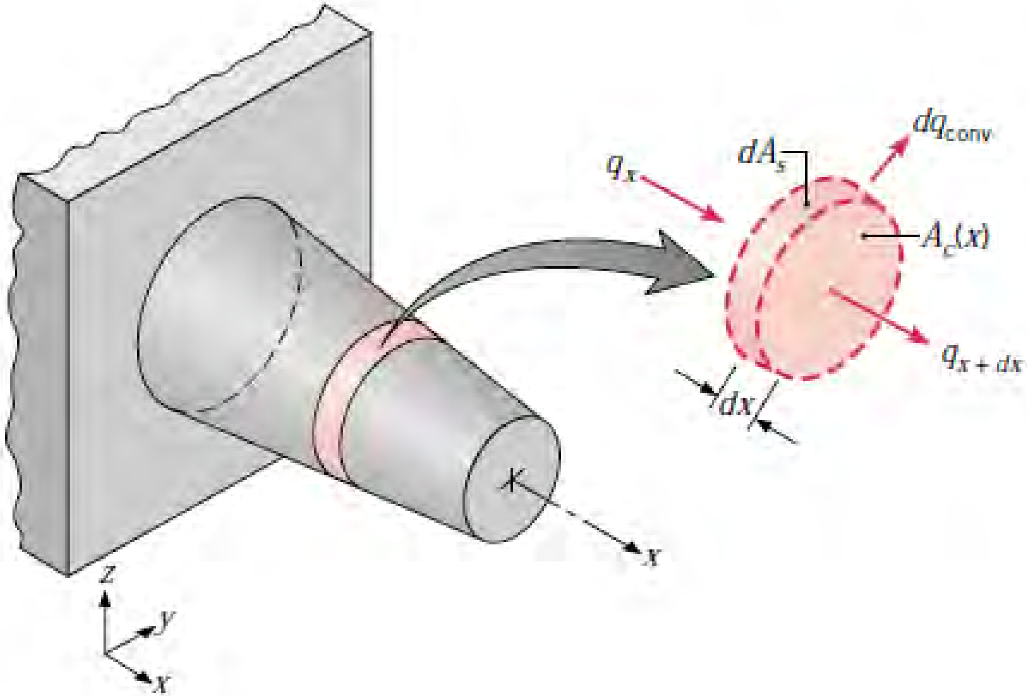


one-dimensional conditions

temperature is uniform across the fin thickness

thermal conductivity is constant

radiation from the surface is negligible

heat generation effects are absent

convection heat transfer coefficient is uniform over the surface

$$q_x = q_{x+dx} + dq_{conv}$$

$$q_x = -kA_c\frac{dT}{dx}$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}\,dx$$

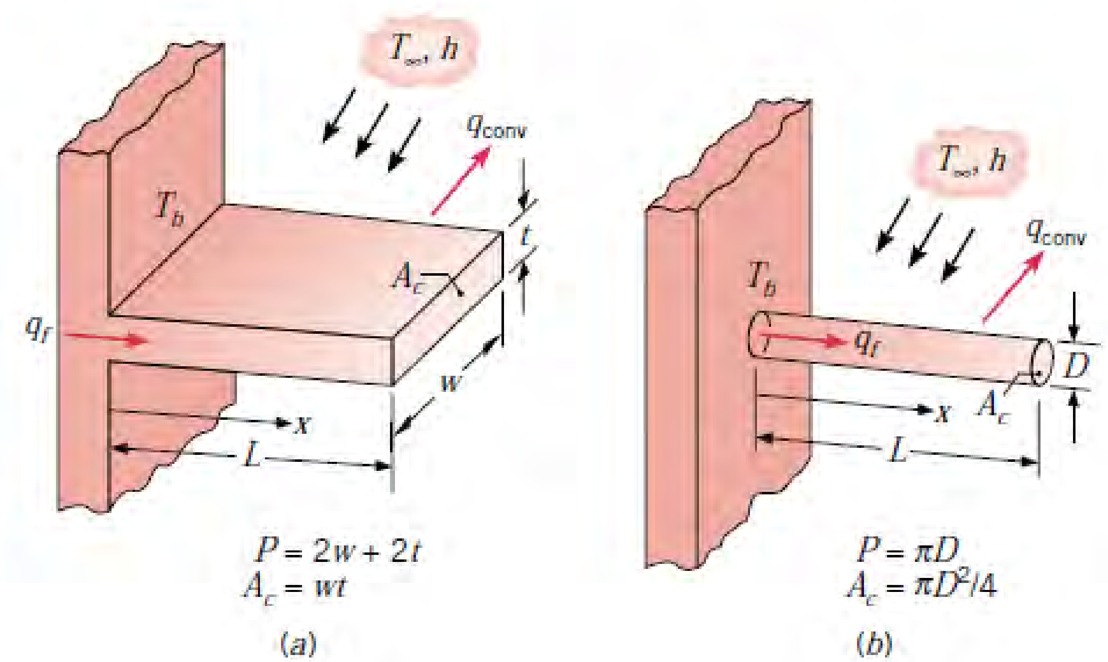
$$q_{x+dx} = -kA_c\frac{dT}{dx} - k\frac{d}{dx}\left(A_c\frac{dT}{dx}\right)dx$$

$$dq_{conv} = h\,dA_s(T - T_\infty)$$

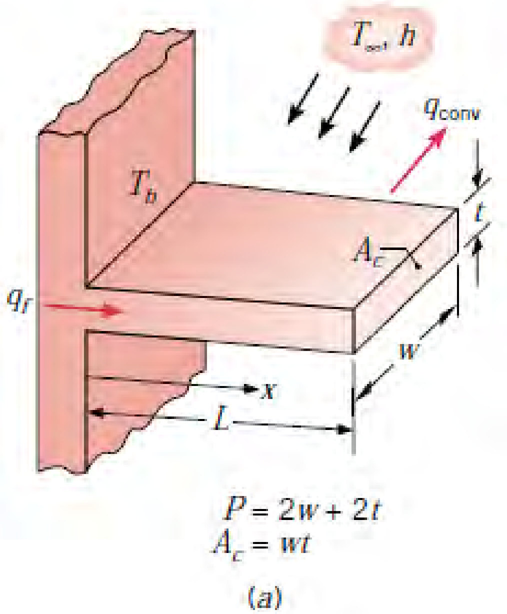
$$\frac{d}{dx}\left(A_c\frac{dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T - T_\infty) = 0$$

general form of the energy equation for an extended surface



$A_s = Px$ $\qquad$ $dA_s/dx = P$,



$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

$$\theta(x) \equiv T(x) - T_\infty$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$m^2 \equiv \frac{hP}{kA_c}$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta(0) = T_b - T_\infty \equiv \theta_b$$

Case	Tip Condition ($x = L$)
A	Convection heat transfer: $h\theta(L) = -k\, d\theta/dx\|_{x=L}$
B	Adiabatic $d\theta/dx\|_{x=L} = 0$
C	Prescribed temperature: $\theta(L) = \theta_L$
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$

$$hA_c[T(L) - T_\infty] = -kA_c \left.\frac{dT}{dx}\right|_{x=L}$$

$$h\theta(L) = -k \left.\frac{d\theta}{dx}\right|_{x=L}$$

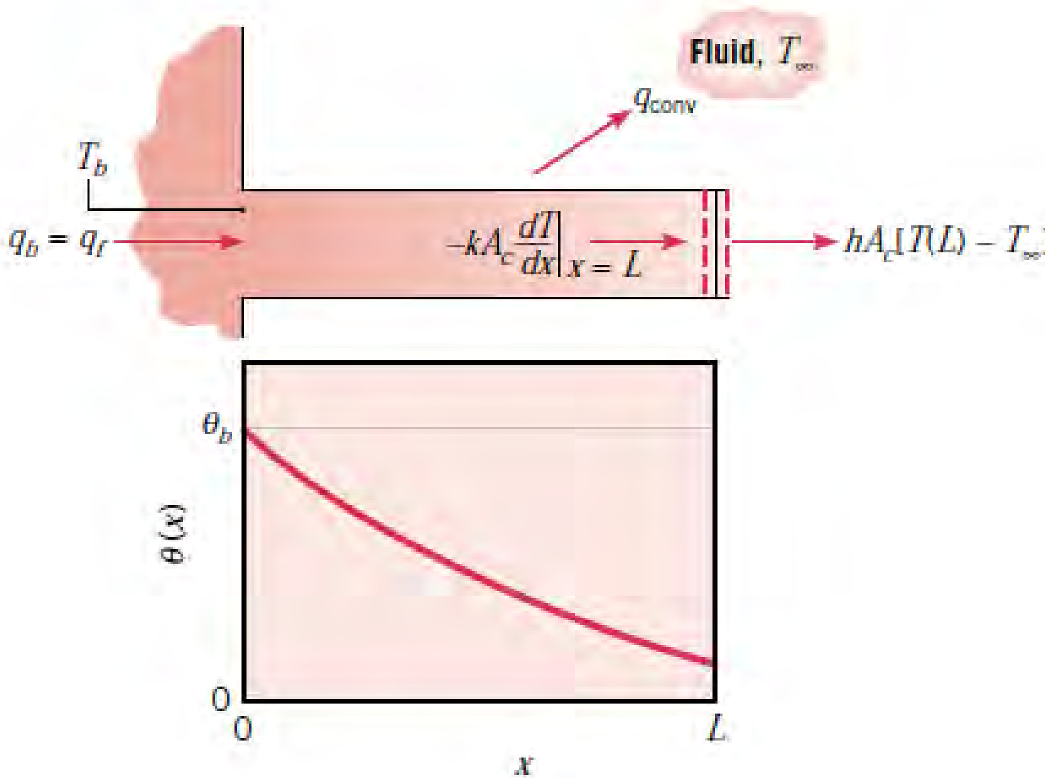
$$\theta_b = C_1 + C_2$$

$$h(C_1 e^{mL} + C_2 e^{-mL}) = km(C_2 e^{-mL} - C_1 e^{mL})$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L - x) + (h/mk) \sinh m(L - x)}{\cosh mL + (h/mk) \sinh mL}$$

$$\sinh x = \tfrac{1}{2}(e^x - e^{-x}) \qquad \cosh x = \tfrac{1}{2}(e^x + e^{-x}) \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$$

$$\frac{d}{dx}(\sinh u) = (\cosh u)\frac{du}{dx} \qquad \frac{d}{dx}(\cosh u) = (\sinh u)\frac{du}{dx} \qquad \frac{d}{dx}(\tanh u) = \left(\frac{1}{\cosh^2 u}\right)\frac{du}{dx}$$



$$q_f = q_b = -kA_c \frac{dT}{dx}\bigg|_{x=0} = -kA_c \frac{d\theta}{dx}\bigg|_{x=0}$$

$$\boxed{q_f = \sqrt{hPkA_c}\,\theta_b \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}}$$

$$q_f = \int_{A_f} h[T(x) - T_\infty]\, dA_s$$

$$q_f = \int_{A_f} h\theta(x)\, dA_s$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

$$C_1 e^{mL} - C_2 e^{-mL} = 0$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b \tanh mL$$

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -k d\theta/dx\|_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic $d\theta/dx\|_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

Fin Performance

- ratio of the fin heat transfer rate to the heat transfer rate that would exist without the fin

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b}$$

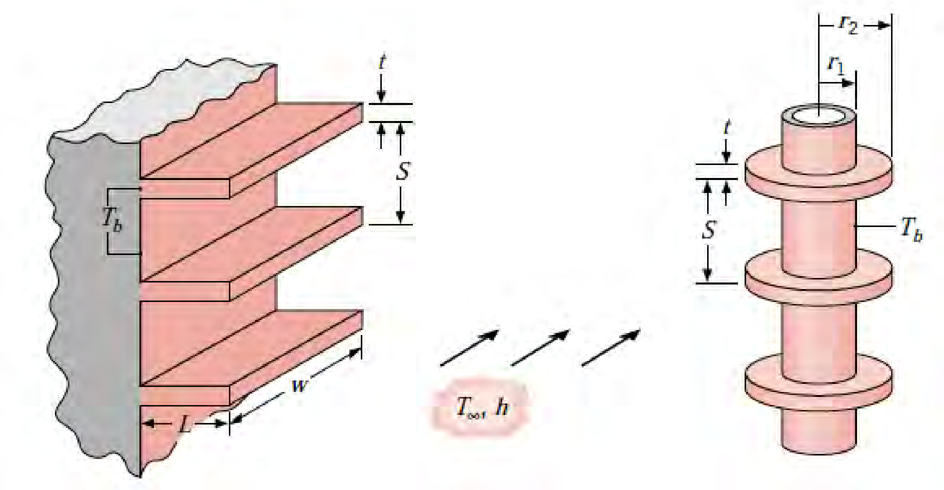
$$R_{t,f} = \frac{\theta_b}{q_f}$$

$$R_{t,b} = \frac{1}{hA_{c,b}}$$

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}$$

$$\eta_f \equiv \frac{q_f}{q_{\max}} = \frac{q_f}{hA_f\theta_b}$$

Overall surface efficiency



$$\boxed{\eta_o = \frac{q_t}{q_{\max}} = \frac{q_t}{hA_t\theta_b}}$$

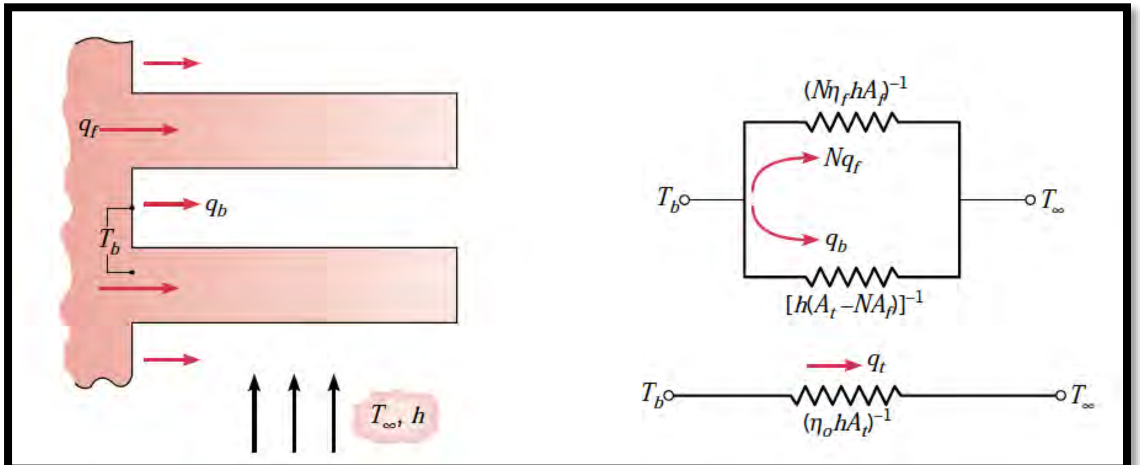
$$A_t = NA_f + A_b$$

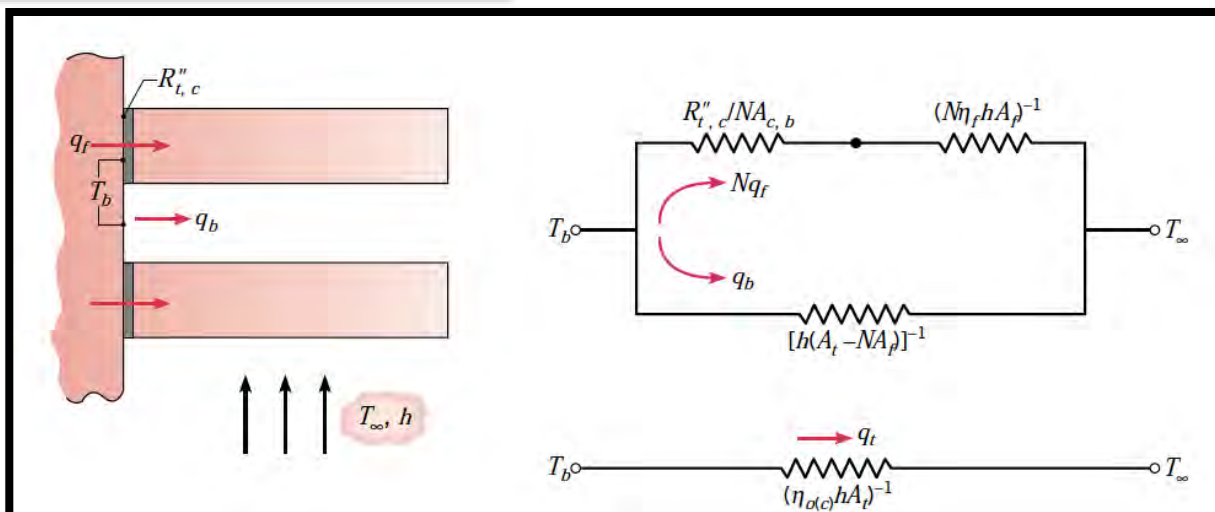
$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b$$

$$q_t = h[N\eta_f A_f + (A_t - NA_f)]\theta_b = hA_t\left[1 - \frac{NA_f}{A_t}(1 - \eta_f)\right]\theta_b$$

$$\eta_o = 1 - \frac{NA_f}{A_t}(1 - \eta_f)$$

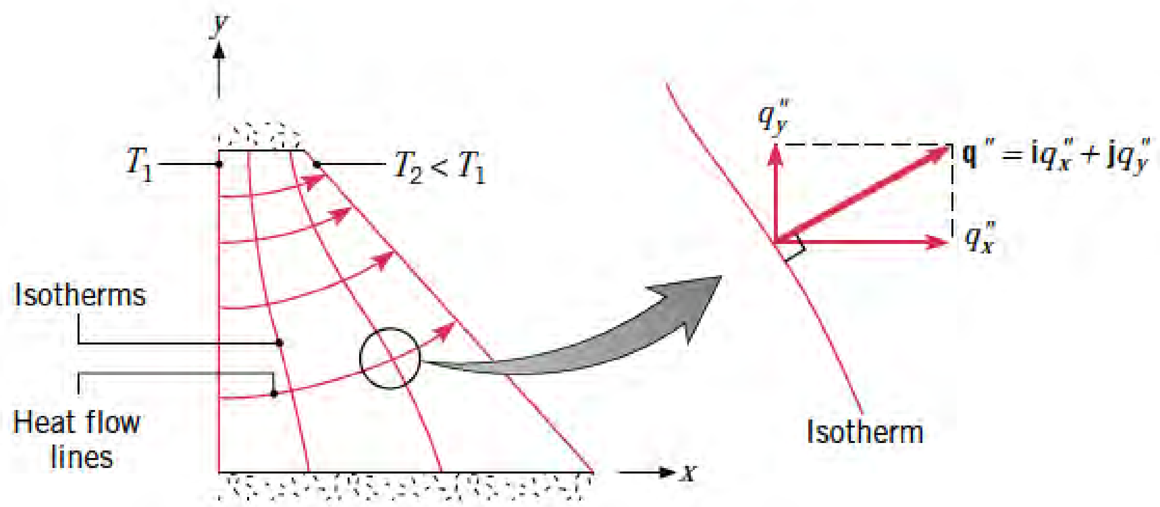
$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$$





Two-Dimensional, Steady-State Conduction

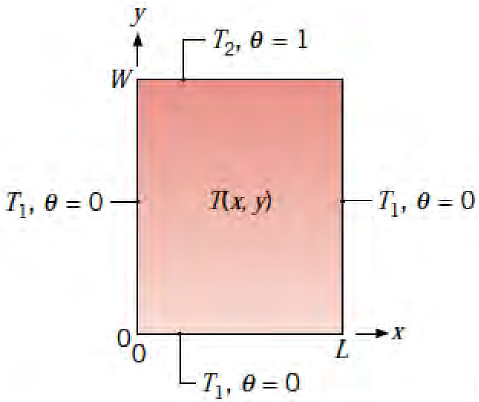
$$q'' = -k\nabla T = -k\left(\boldsymbol{i}\frac{\partial T}{\partial x} + \boldsymbol{j}\frac{\partial T}{\partial y} + \boldsymbol{k}\frac{\partial T}{\partial z}\right)$$



$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = 0$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

$$q_x'' = -k\frac{\partial T}{\partial x} \qquad q_y'' = -k\frac{\partial T}{\partial y} \qquad q_z'' = -k\frac{\partial T}{\partial z}$$



$$\theta \equiv \frac{T - T_1}{T_2 - T_1}$$

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$

$$\theta(0, y) = 0 \quad \text{and} \quad \theta(x, 0) = 0$$
$$\theta(L, y) = 0 \quad \text{and} \quad \theta(x, W) = 1$$

$$\theta(x, y) = X(x) \cdot Y(y)$$

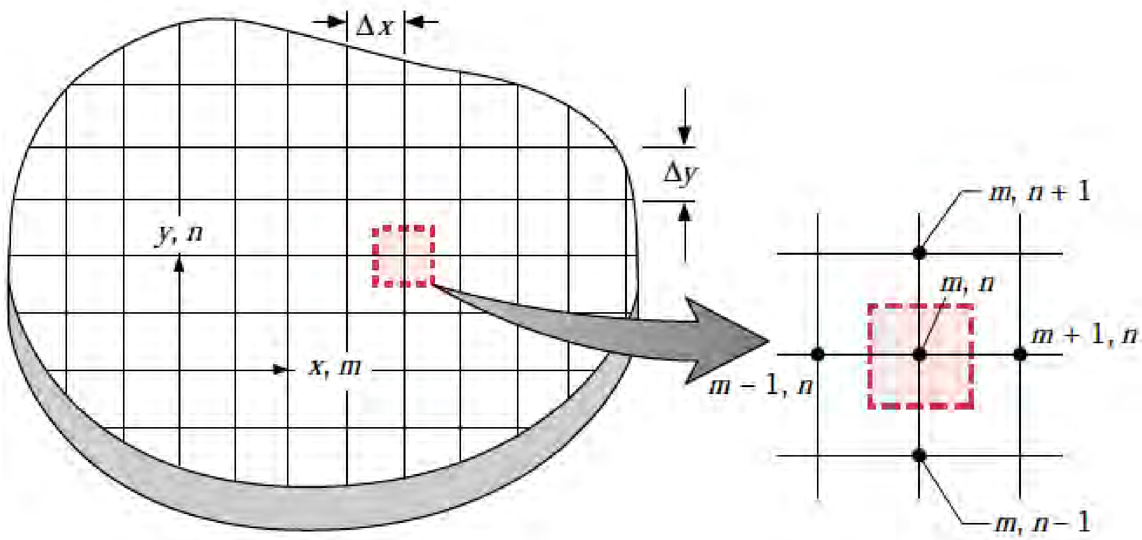
$$-\frac{1}{X}\frac{d^2 X}{dx^2} = \frac{1}{Y}\frac{d^2 Y}{dy^2}$$

$$\frac{d^2 X}{dx^2} + \lambda^2 X = 0$$
$$\frac{d^2 Y}{dy^2} - \lambda^2 Y = 0$$

$$X = C_1 \cos \lambda x + C_2 \sin \lambda x$$
$$Y = C_3 e^{-\lambda y} + C_4 e^{+\lambda y}$$

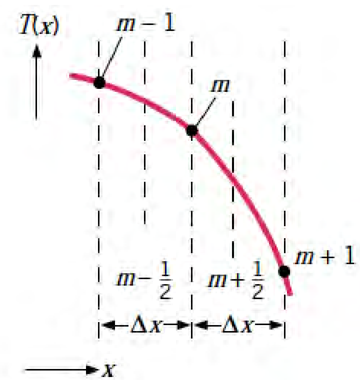
$$\theta = (C_1 \cos \lambda x + C_2 \sin \lambda x)(C_3 e^{-\lambda y} + C_4 e^{\lambda y})$$

Finite-Difference Equations



$$\left.\frac{\partial T}{\partial x}\right|_{m-1/2,n} = \frac{T_{m,n} - T_{m-1,\,n}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right|_{m+1/2,n} = \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$



$$\left.\frac{\partial^2 T}{\partial x^2}\right|_{m,n} \approx \frac{\partial T/\partial x|_{m+1/2,n} - \partial T/\partial x|_{m-1/2,n}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right|_{m+1/2,n} \approx \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

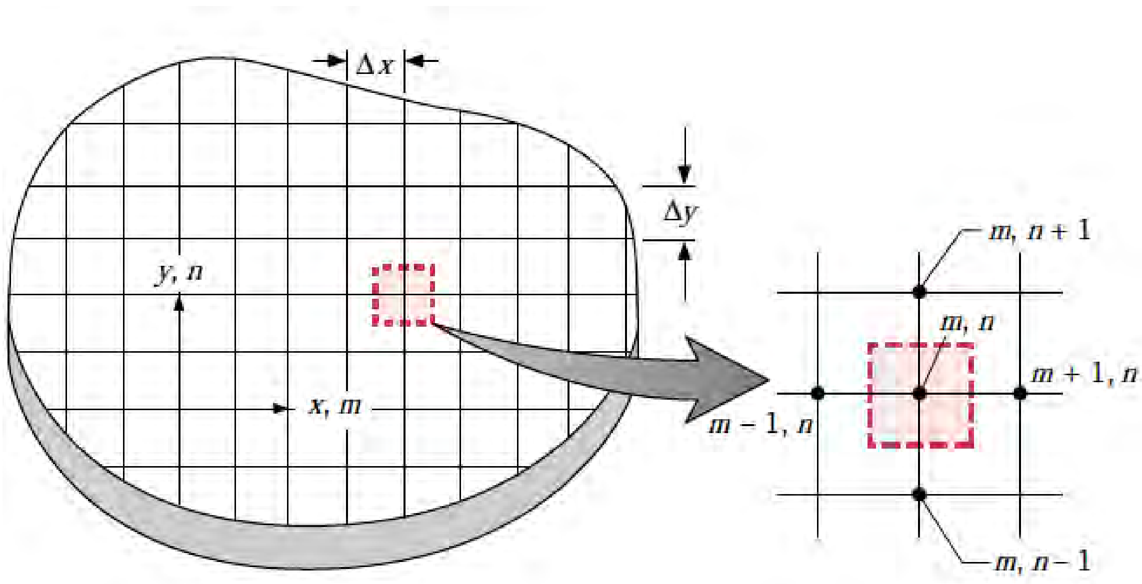
$$\left.\frac{\partial T}{\partial x}\right|_{m-1/2,n} \approx \frac{T_{m,n} - T_{m-1,n}}{\Delta x}$$

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_{m,n} \approx \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{(\Delta x)^2}$$

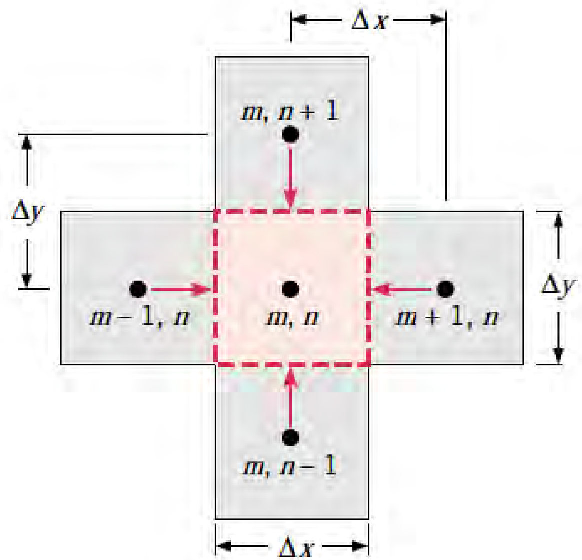
$$\left.\frac{\partial^2 T}{\partial y^2}\right|_{m,n} \approx \frac{\partial T/\partial y|_{m,n+1/2} - \partial T/\partial y|_{m,n-1/2}}{\Delta y}$$

$$\approx \frac{T_{m,n+1} + T_{m,n-1} - 2T_{m,n}}{(\Delta y)^2}$$

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0$$



$$\dot{E}_{\text{in}} + \dot{E}_g = 0$$



$$\sum_{i=1}^{4} q_{(i) \to (m,n)} + \dot{q}(\Delta x \cdot \Delta y \cdot 1) = 0$$

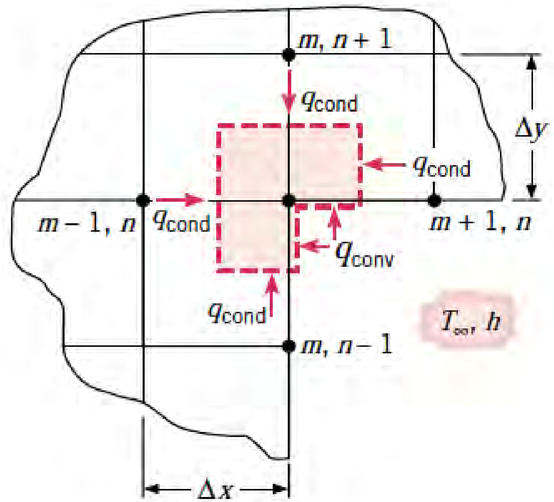
$$q_{(m-1,n) \to (m,n)} = k(\Delta y \cdot 1) \frac{T_{m-1,n} - T_{m,n}}{\Delta x}$$

$$q_{(m+1,n) \to (m,n)} = k(\Delta y \cdot 1) \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

$$q_{(m,n+1) \to (m,n)} = k(\Delta x \cdot 1) \frac{T_{m,n+1} - T_{m,n}}{\Delta y}$$

$$q_{(m,n-1) \to (m,n)} = k(\Delta x \cdot 1) \frac{T_{m,n-1} - T_{m,n}}{\Delta y}$$

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} + \frac{\dot{q}(\Delta x)^2}{k} - 4T_{m,n} = 0$$

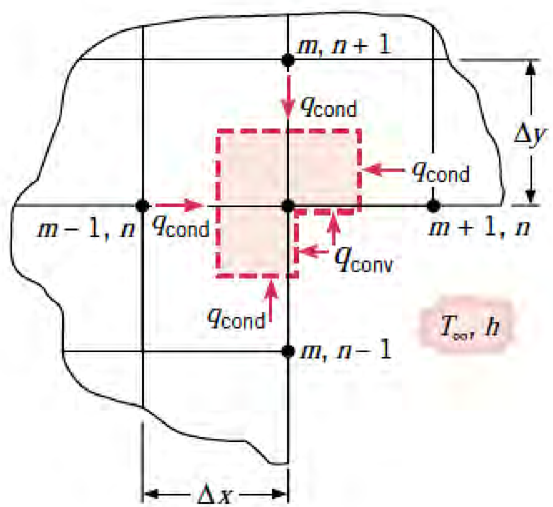


$$q_{(m-1,n)\to(m,n)} = k(\Delta y \cdot 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x}$$

$$q_{(m,n+1)\to(m,n)} = k(\Delta x \cdot 1)\frac{T_{m,n+1} - T_{m,n}}{\Delta y}$$

$$q_{(m+1,n)\to(m,n)} = k\left(\frac{\Delta y}{2} \cdot 1\right)\frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

$$q_{(m,n-1)\to(m,n)} = k\left(\frac{\Delta x}{2} \cdot 1\right)\frac{T_{m,n-1} - T_{m,n}}{\Delta y}$$



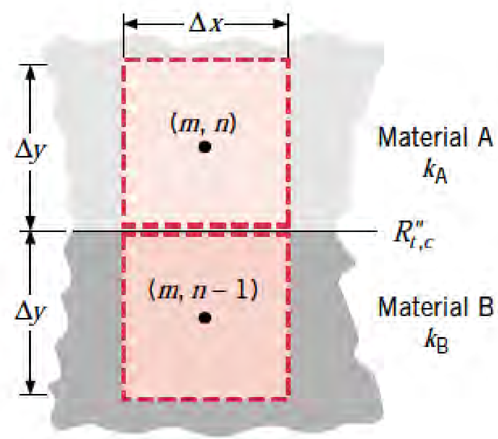
$$q_{(\infty)\to(m,n)} = h\left(\frac{\Delta x}{2}\cdot 1\right)(T_\infty - T_{m,n}) + h\left(\frac{\Delta y}{2}\cdot 1\right)(T_\infty - T_{m,n})$$

$$T_{m-1,n} + T_{m,n+1} + \frac{1}{2}(T_{m+1,n} + T_{m,n-1}) + \frac{h\Delta x}{k}T_\infty - \left(3 + \frac{h\Delta x}{k}\right)T_{m,n} = 0$$

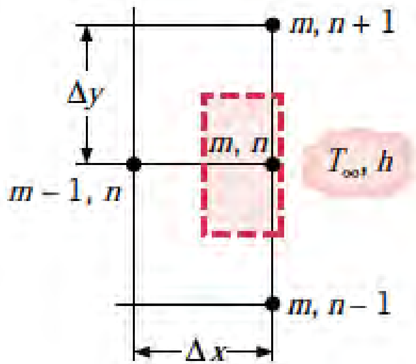
$$q_{(m-1,n)\to(m,n)} = \frac{T_{m-1,n} - T_{m,n}}{R_{t,\text{cond}}} = \frac{T_{m-1,n} - T_{m,n}}{\Delta x/k\,(\Delta y\cdot 1)}$$

$$q_{(m,n)\to(m,n-1)} = \frac{T_{m,n} - T_{m,n-1}}{R_{\text{tot}}}$$

$$q_{(\infty)\to(m,n)} = \frac{T_\infty - T_{m,n}}{R_{t,\text{conv}}} = \frac{T_\infty - T_{m,n}}{\{h[(\Delta x/2)\cdot 1 + (\Delta y/2)\cdot 1]\}^{-1}}$$

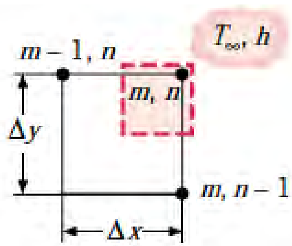


$$R_{\text{tot}} = \frac{\Delta y/2}{k_A(\Delta x \cdot 1)} + \frac{R''_{t,c}}{\Delta x \cdot 1} + \frac{\Delta y/2}{k_B(\Delta x \cdot 1)}$$



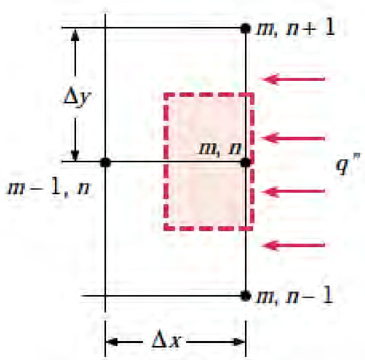
Node at a plane surface with convection

$$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2h\Delta x}{k}T_\infty - 2\left(\frac{h\Delta x}{k} + 2\right)T_{m,n} = 0$$



Node at an external corner with convection

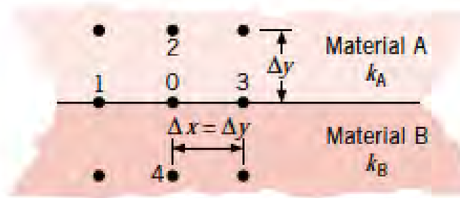
$$(T_{m,n-1} + T_{m-1,n}) + 2\frac{h\Delta x}{k}T_\infty - 2\left(\frac{h\Delta x}{k} + 1\right)T_{m,n} = 0$$



Node at a plane surface with uniform heat flux

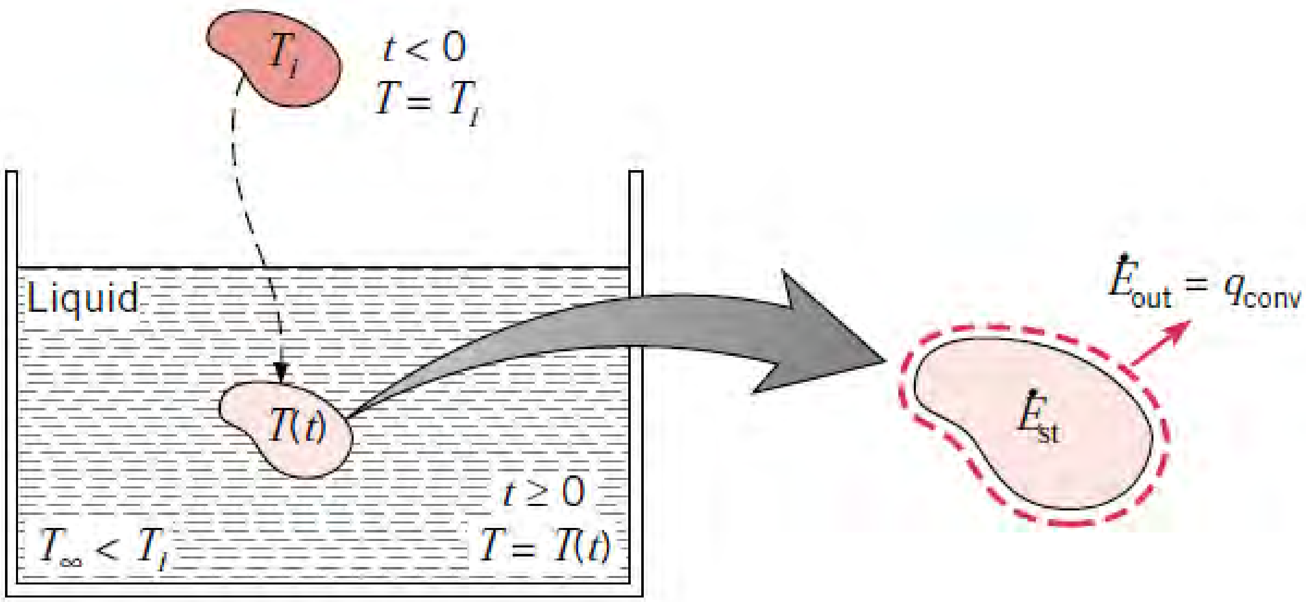
$$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2q''\Delta x}{k} - 4T_{m,n} = 0$$

Consider the nodal point 0 located on the boundary between materials of thermal conductivity k_A and k_B.



Derive the finite-difference equation, assuming no internal generation.

Transient Conduction



$$-\dot{E}_{out} = \dot{E}_{st}$$

$$-hA_s(T - T_\infty) = \rho V c \frac{dT}{dt}$$

$$\theta \equiv T - T_\infty$$

$$\frac{\rho V c}{hA_s}\frac{d\theta}{dt} = -\theta$$

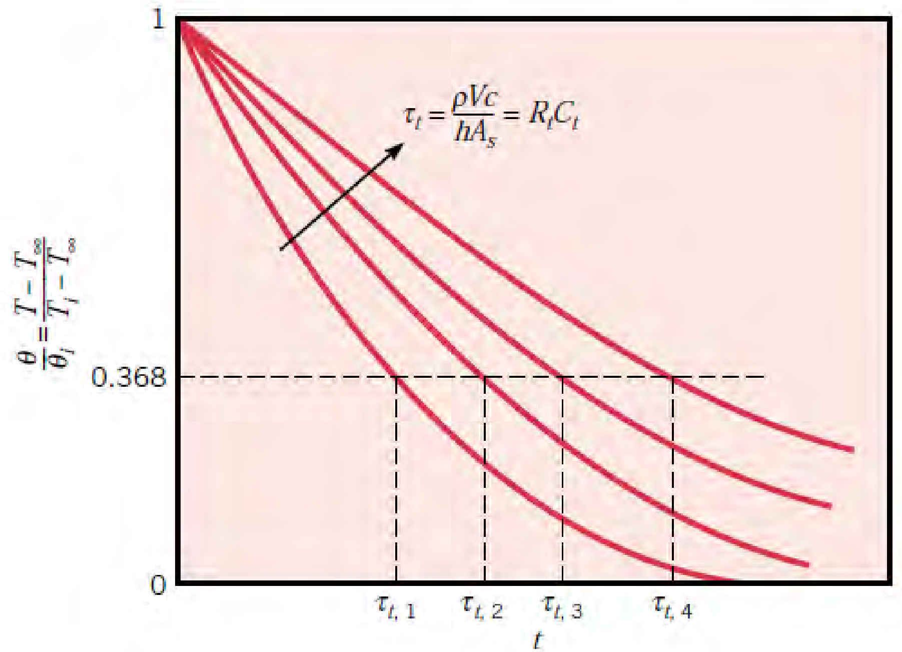
$$\frac{\rho V c}{h A_s} \int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = -\int_0^t dt$$

$$\theta_i \equiv T_i - T_\infty$$

$$\frac{\rho V c}{h A_s} \ln \frac{\theta_i}{\theta} = t$$

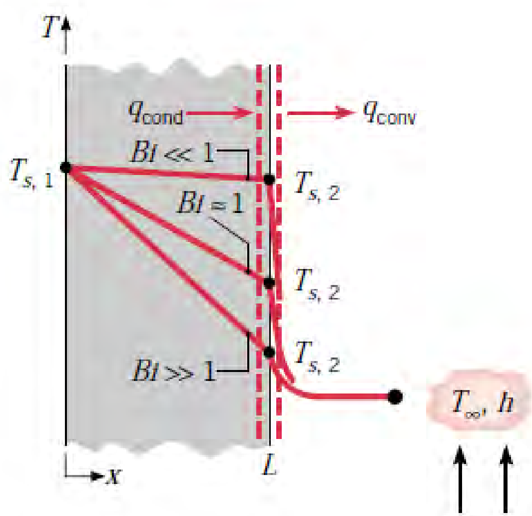
$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{h A_s}{\rho V c}\right) t\right]$$

$$\tau_t = \left(\frac{1}{h A_s}\right)(\rho V c) = R_t C_t$$



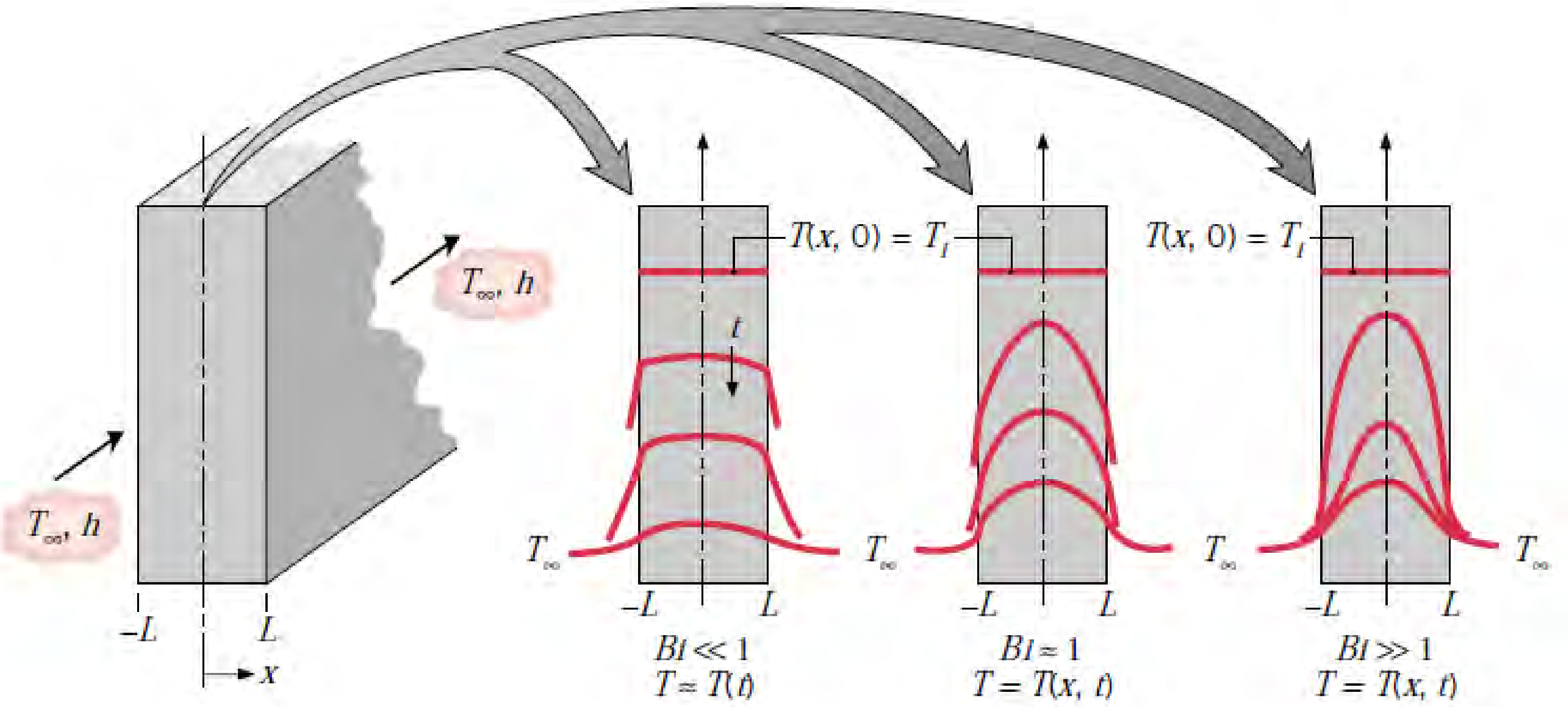
$$Q = \int_0^t q\, dt = hA_s \int_0^t \theta\, dt$$

$$Q = (\rho V c)\theta_i \left[1 - \exp\left(-\frac{t}{\tau_t} \right) \right]$$



$$\frac{kA}{L}(T_{s,1} - T_{s,2}) = hA(T_{s,2} - T_\infty)$$

$$\frac{T_{s,1} - T_{s,2}}{T_{s,2} - T_\infty} = \frac{(L/kA)}{(1/hA)} = \frac{R_{\text{cond}}}{R_{\text{conv}}} = \frac{hL}{k} \equiv Bi$$



$$\boxed{Bi = \frac{hL_c}{k} < 0.1}$$

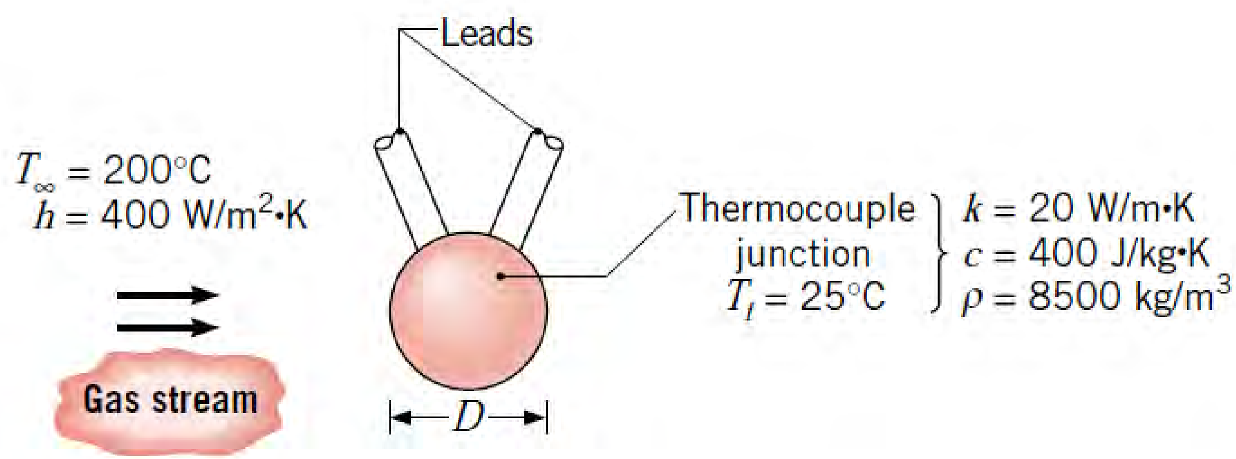
$$L_c \equiv V/A_s$$

$$\frac{hA_s t}{\rho V c} = \frac{ht}{\rho c L_c} = \frac{hL_c}{k}\frac{k}{\rho c}\frac{t}{L_c^2} = \frac{hL_c}{k}\frac{\alpha t}{L_c^2}$$

$$\frac{hA_s t}{\rho V c} = Bi \cdot Fo \qquad \boxed{Fo \equiv \frac{\alpha t}{L_c^2}}$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-Bi \cdot Fo)$$

A thermocouple junction, which may be approximated as a sphere, is to be used for temperature measurement in a gas stream. The convection coefficient between the junction surface and the gas is $h = 400\ \mathrm{W/m^2 \cdot K}$, and the junction thermophysical properties are $k = 20\ \mathrm{W/m \cdot K}$, $c = 400\ \mathrm{J/kg \cdot K}$, and $\rho = 8500\ \mathrm{kg/m^3}$. Determine the junction diameter needed for the thermocouple to have a time constant of 1 s. If the junction is at 25°C and is placed in a gas stream that is at 200°C, how long will it take for the junction to reach 199°C?



$$\tau_t = \frac{1}{h\pi D^2} \times \frac{\rho\pi D^3}{6} c$$

$$D = \frac{6h\tau_t}{\rho c} = \frac{6 \times 400\ \mathrm{W/m^2 \cdot K} \times 1\ \mathrm{s}}{8500\ \mathrm{kg/m^3} \times 400\ \mathrm{J/kg \cdot K}} = 7.06 \times 10^{-4}\ \mathrm{m}$$

$$Bi = \frac{h(r_o/3)}{k} = \frac{400\ \mathrm{W/m^2 \cdot K} \times 3.53 \times 10^{-4}\ \mathrm{m}}{3 \times 20\ \mathrm{W/m \cdot K}} = 2.35 \times 10^{-3}$$

$$t = \frac{\rho(\pi D^3/6)\,c}{h(\pi D^2)} \ln\frac{T_i - T_\infty}{T - T_\infty} = \frac{\rho D c}{6h} \ln\frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{8500\ \mathrm{kg/m^3} \times 7.06 \times 10^{-4}\ \mathrm{m} \times 400\ \mathrm{J/kg \cdot K}}{6 \times 400\ \mathrm{W/m^2 \cdot K}} \ln\frac{25 - 200}{199 - 200}$$

$$t = 5.2\ \mathrm{s} \approx 5\tau_t$$

Spatial Effects

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$T(x, 0) = T_i$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h[T(L, t) - T_\infty]$$

$$T = T(x, t, T_i, T_\infty, L, k, \alpha, h)$$

$$\theta^* \equiv \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}$$

$$x^* \equiv \frac{x}{L}$$

$$t^* \equiv \frac{\alpha t}{L^2} \equiv Fo$$

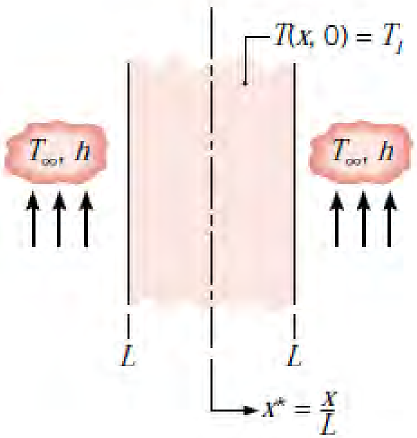
$$\frac{\partial^2 \theta^*}{\partial x^{*2}} = \frac{\partial \theta^*}{\partial Fo}$$

$$\theta^*(x^*, 0) = 1$$

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=0} = 0$$

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=1} = -Bi\,\theta^*(1, t^*)$$

$$\theta^* = f(x^*, Fo, Bi)$$



$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo) \cos(\zeta_n x^*)$$

$$C_n = \frac{4 \sin \zeta_n}{2\zeta_n + \sin(2\zeta_n)}$$

$$\zeta_n \tan \zeta_n = Bi$$

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$$

$$t = p\,\Delta t$$

$$\left.\frac{\partial T}{\partial t}\right|_{m,\,n} \approx \frac{T_{m,\,n}^{p+1} - T_{m,\,n}^{p}}{\Delta t}$$

$$\frac{1}{\alpha}\frac{T_{m,\,n}^{p+1} - T_{m,\,n}^{p}}{\Delta t} = \frac{T_{m+1,\,n}^{p} + T_{m-1,\,n}^{p} - 2T_{m,\,n}^{p}}{(\Delta x)^2} + \frac{T_{m,\,n+1}^{p} + T_{m,\,n-1}^{p} - 2T_{m,\,n}^{p}}{(\Delta y)^2}$$

$$T_{m,\,n}^{p+1} = Fo(T_{m+1,\,n}^{p} + T_{m-1,\,n}^{p} + T_{m,\,n+1}^{p} + T_{m,\,n-1}^{p}) + (1 - 4Fo)T_{m,\,n}^{p}$$

$$Fo = \frac{\alpha\,\Delta t}{(\Delta x)^2}$$

$$T_m^{p+1} = Fo(T_{m+1}^p + T_{m-1}^p) + (1 - 2Fo)\,T_m^p$$

$$(1 - 2Fo) \geq 0$$

$$Fo \leq \frac{1}{2}$$

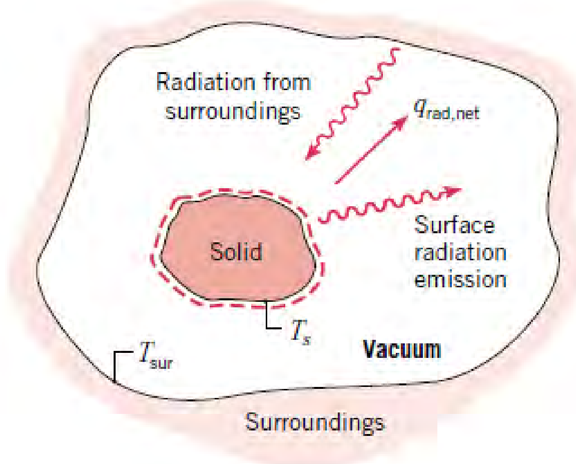
$$(1 - 4Fo) \geq 0$$

$$Fo \leq \frac{1}{4}$$

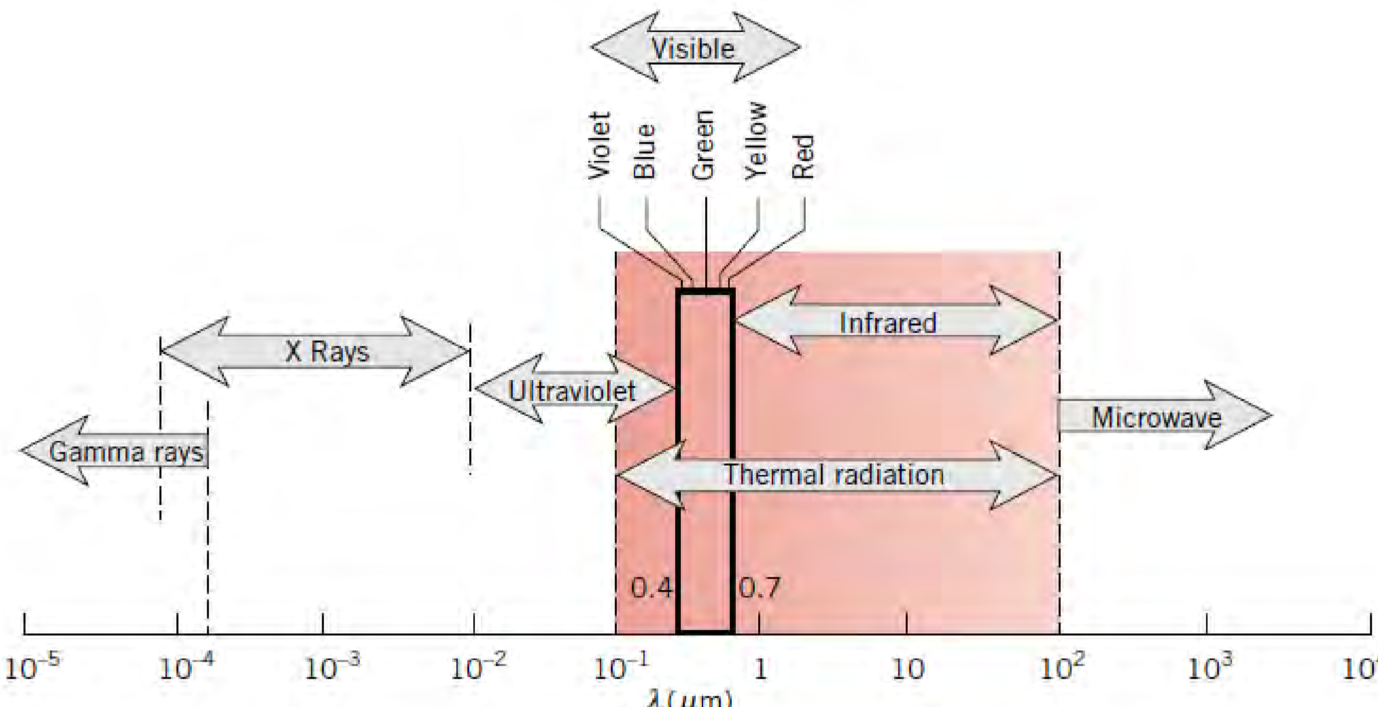
$$\frac{1}{\alpha}\frac{T_{m,n}^{p+1}-T_{m,n}^{p}}{\Delta t}=\frac{T_{m+1,n}^{p+1}+T_{m-1,n}^{p+1}-2T_{m,n}^{p+1}}{(\Delta x)^2}+\frac{T_{m,n+1}^{p+1}+T_{m,n-1}^{p+1}-2T_{m,n}^{p+1}}{(\Delta y)^2}$$

$$(1+4Fo)\,T_{m,n}^{p+1}-Fo\left(T_{m+1,n}^{p+1}+T_{m-1,n}^{p+1}+T_{m,n+1}^{p+1}+T_{m,n-1}^{p+1}\right)=T_{m,n}^{p}$$

Radiation



Radiation from surroundings
$q_{rad,net}$
Solid
Surface radiation emission
T_s
T_{sur}
Vacuum
Surroundings



Visible
Violet
Blue
Green
Yellow
Red
Infrared
X Rays
Ultraviolet
Microwave
Gamma rays
Thermal radiation
0.4
0.7
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
10^{2}
10^{3}
10^{4}
$\lambda(\mu m)$

$$c = \lambda \nu$$

where

$c =$ speed of light
$\lambda =$ wavelength
$\nu =$ frequency

$$E = h\nu$$

$$h = 6.625 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$E_b = \sigma T^4$$

energy radiated per unit time
and per unit area by the ideal radiator

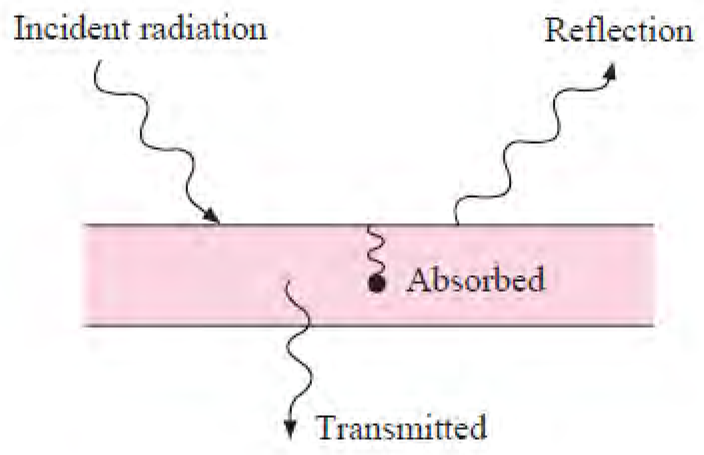
$$\sigma = 5.669 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$$

Stefan-Boltzmann constant

b for blackbody radiation

Emissive power of a blackbody

RADIATION PROPERTIES

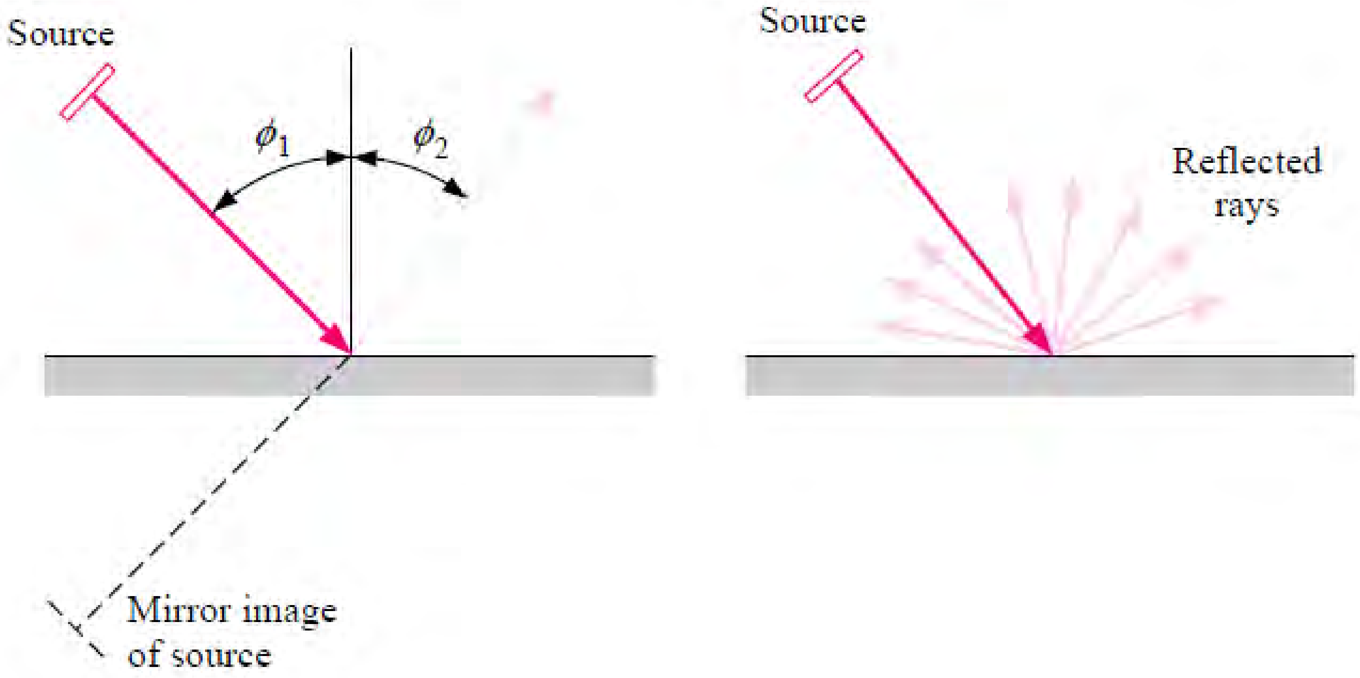


$$\rho + \alpha + \tau = 1$$

reflectivity

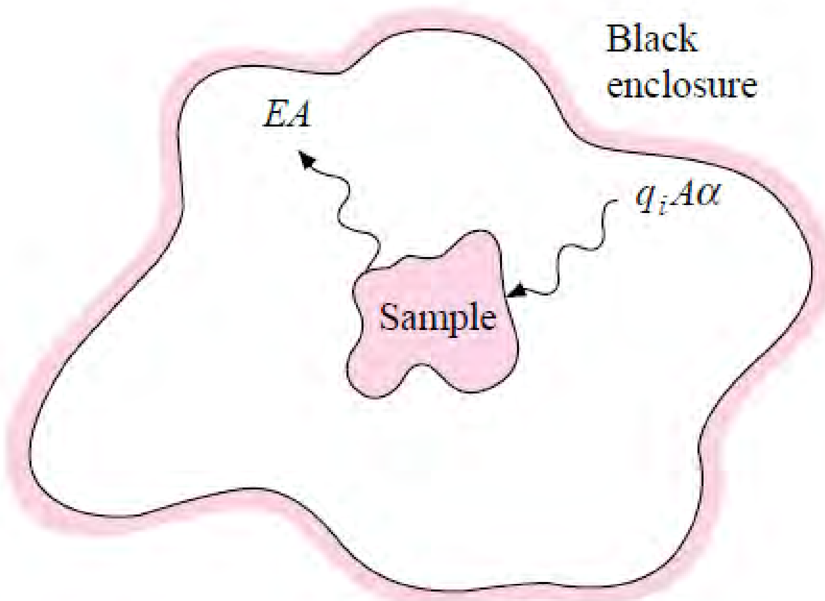
absorptivity

transmissivity



Ordinarily, a rough surface exhibits diffuse behaviour better than a highly polished surface

The emissive power of a body: the energy emitted by the body per unit area and per unit time.



$$EA = q_i A\alpha$$

$$E_b A = q_i A(1)$$

$$\frac{E}{E_b} = \alpha \qquad \epsilon = \frac{E}{E_b}$$

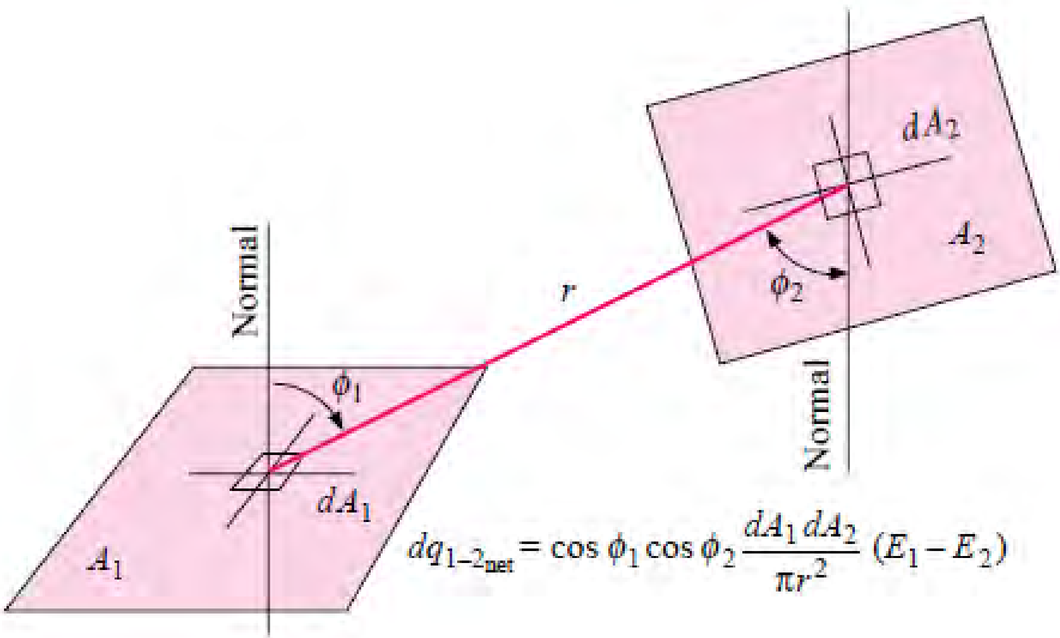
$$\boxed{\epsilon = \alpha}$$

Gray Body

- A gray body is defined such that the monochromatic emissivity of the body is independent of wavelength.
- The monochromatic emissivity is defined as the ratio of the monochromatic emissive power of the body to the monochromatic emissive power of a blackbody at the same wavelength and temperature.

$$\epsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}$$

RADIATION SHAPE/VIEW FACTOR



$F_{1-2} =$ fraction of energy leaving surface 1 that reaches surface 2

$F_{2-1} =$ fraction of energy leaving surface 2 that reaches surface 1

$F_{i-j} =$ fraction of energy leaving surface i that reaches surface j

Other names for the radiation shape factor are *view factor, angle factor,* and *configuration factor*. The energy leaving surface 1 and arriving at surface 2 is

$$E_{b1} A_1 F_{12}$$

and the energy leaving surface 2 and arriving at surface 1 is

$$E_{b2} A_2 F_{21}$$

Since the surfaces are black, all the incident radiation will be absorbed, and the net energy exchange is

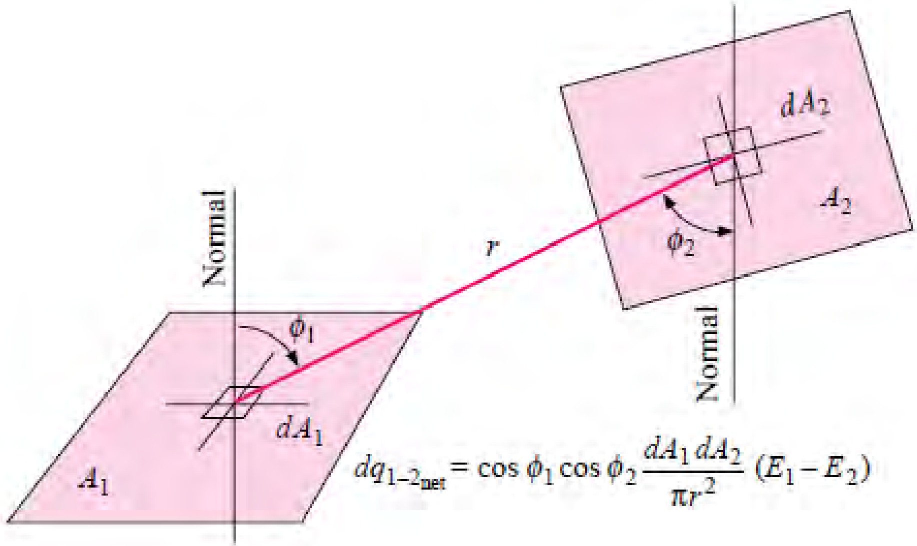
$$E_{b1} A_1 F_{12} - E_{b2} A_2 F_{21} = Q_{1-2}$$

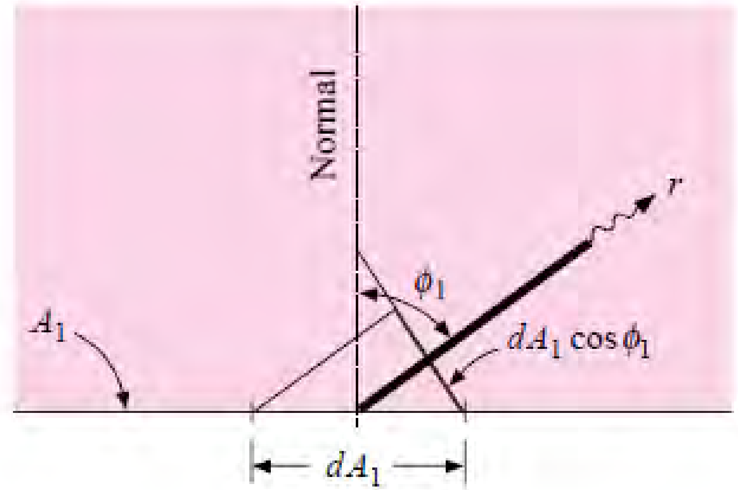
If both surfaces are at the same temperature, there can be no heat exchange, that is, $Q_{1-2} = 0$. Also, for $T_1 = T_2$

$$E_{b1} = E_{b2}$$

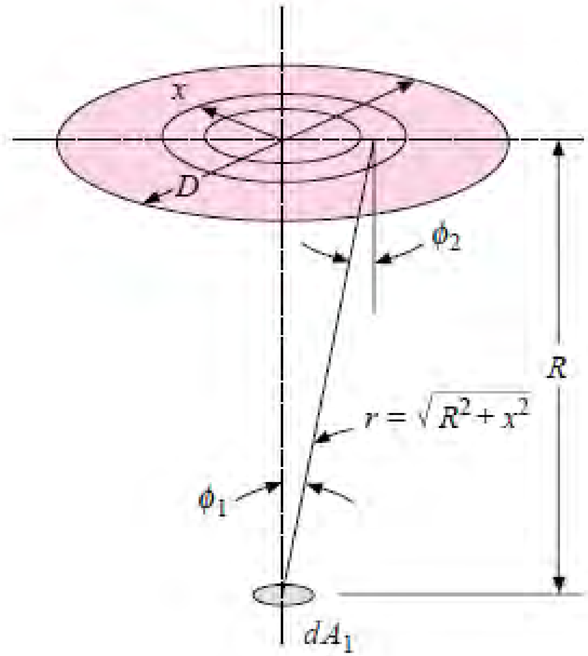
The net heat exchange is therefore

$$Q_{1-2} = A_1 F_{12} (E_{b1} - E_{b2}) = A_2 F_{21} (E_{b1} - E_{b2})$$





$$q_{\mathrm{net}_{1-2}} = (E_{b1} - E_{b2}) \int_{A_2} \int_{A_1} \cos\phi_1 \cos\phi_2 \frac{dA_1 dA_2}{\pi r^2}$$



$$dA_2 = 2\pi x \, dx$$

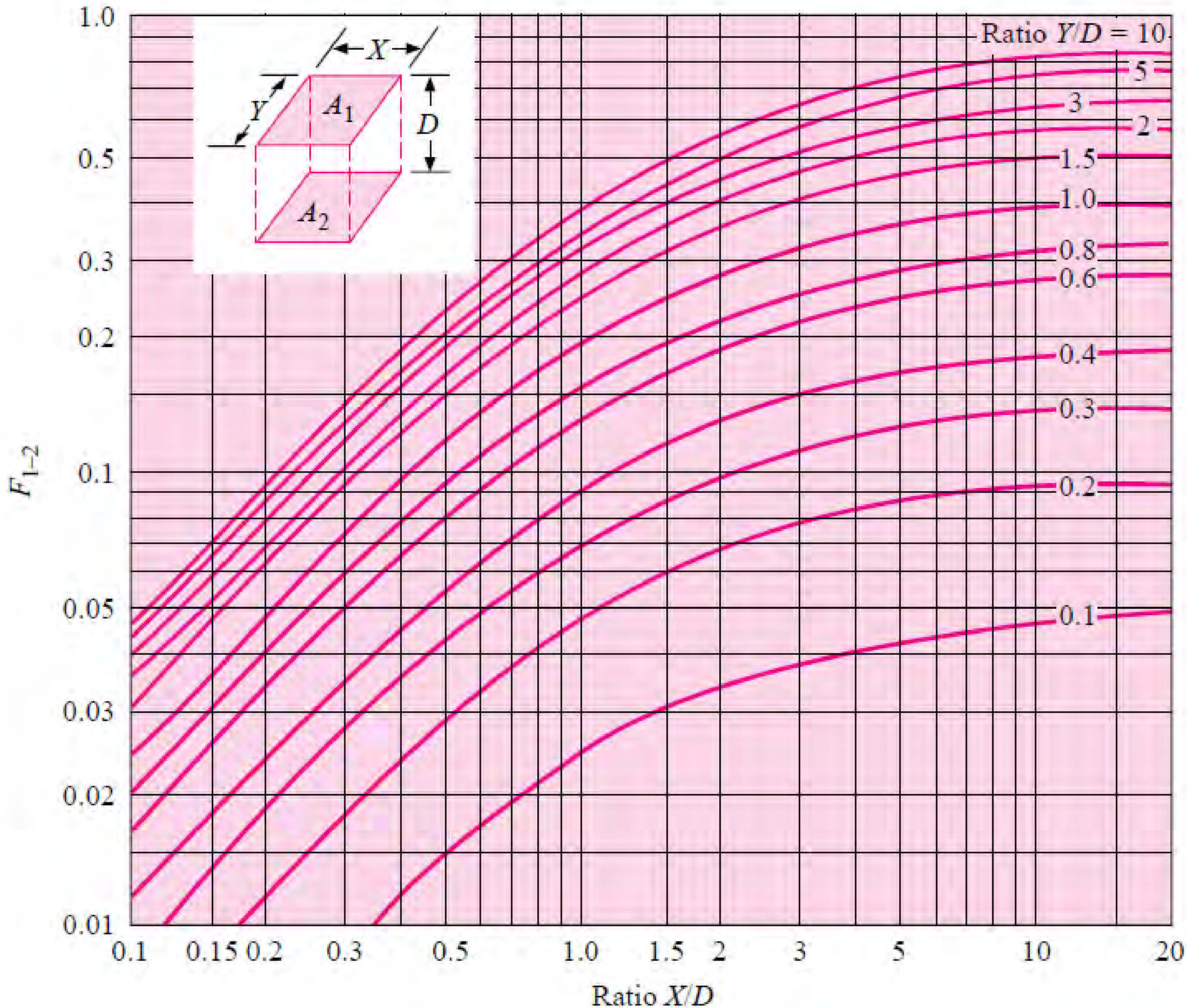
$$dA_1 \, F_{dA_1-A_2} = dA_1 \int_{A_2} \cos^2 \phi_1 \, \frac{2\pi x \, dx}{\pi r^2}$$

$$r = (R^2 + x^2)^{1/2} \qquad \text{and} \qquad \cos \phi_1 = \frac{R}{(R^2 + x^2)^{1/2}}$$

$$dA_1 \, F_{dA_1-A_2} = dA_1 \int_0^{D/2} \frac{2R^2 x \, dx}{(R^2 + x^2)^2}$$

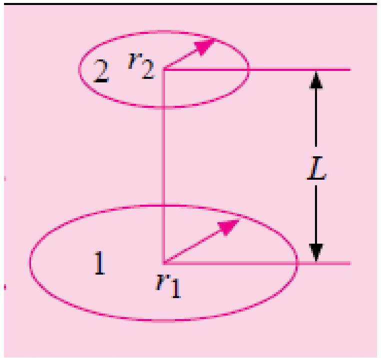
$$dA_1\ F_{dA_1-A_2} = -dA_1 \left(\frac{R^2}{R^2+x^2} \right) \Bigg]_0^{D/2} = dA_1 \frac{D^2}{4R^2+D^2}$$

$$F_{dA_1-A_2} = \frac{D^2}{4R^2+D^2}$$



Ratio Y/D = 10
5
3
2
1.5
1.0
0.8
0.6
0.4
0.3
0.2
0.1
X
Y
D
A_1
A_2
F_{1-2}
1.0
0.5
0.3
0.2
0.1
0.05
0.03
0.02
0.01
0.1
0.15
0.2
0.3
0.5
1.0
1.5
2
3
5
10
20
Ratio X/D

$$F_{1-2} = (2/\pi xy)\{\ln[(1+x^2)(1+y^2)/(1+x^2+y^2)]^{1/2} + x(1+y^2)^{1/2}\tan^{-1}[x/(1+y^2)^{1/2}] + y(1+x^2)^{1/2}\tan^{-1}[y/(1+x^2)^{1/2}] - x\tan^{-1}x - y\tan^{-1}y\}$$



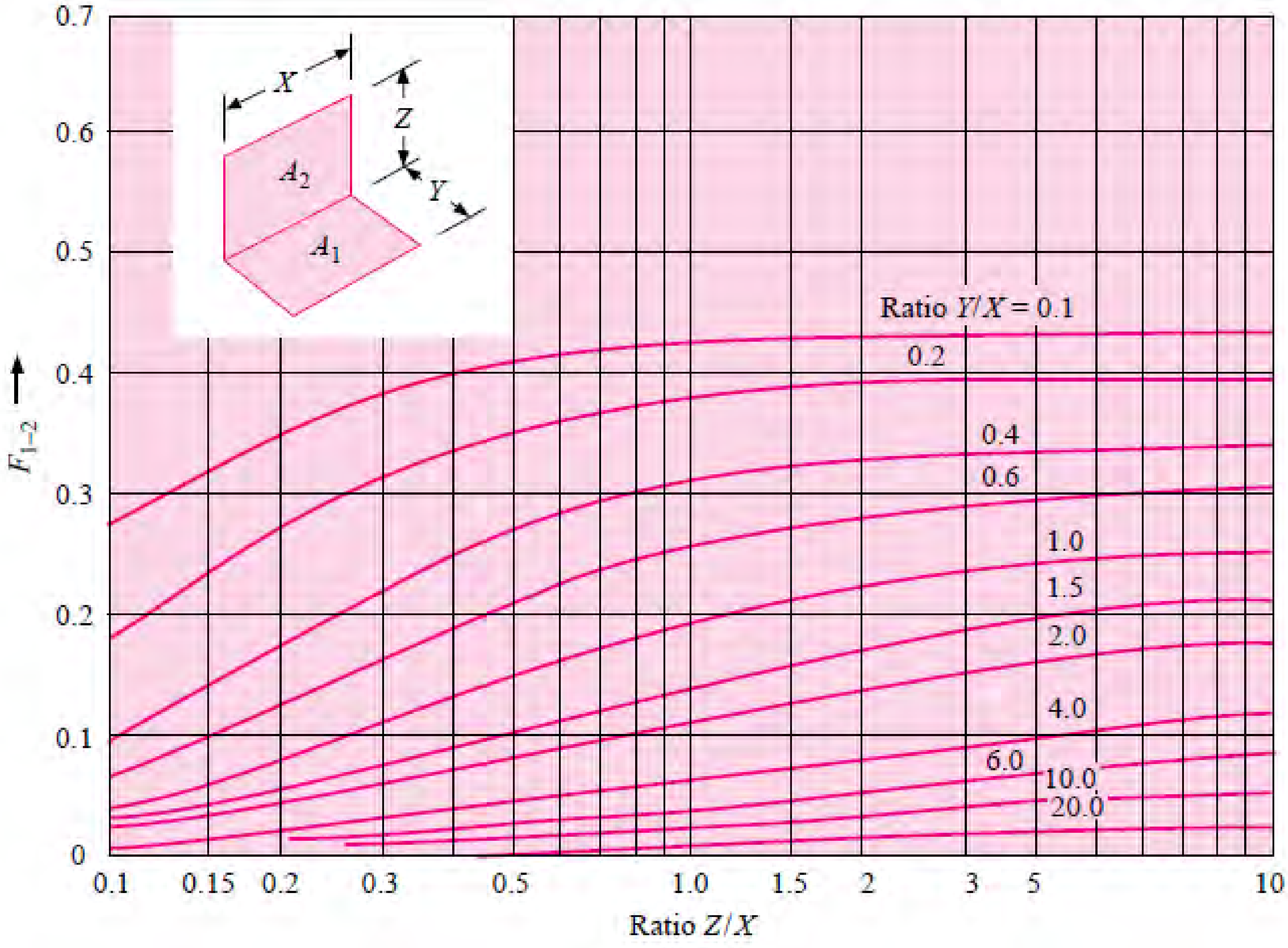
$$F_{1-2} = \{X - [X^2 - 4(R_2/R_1)^2]^{1/2}\}/2$$

Two parallel black plates 0.5 by 1.0 m are spaced 0.5 m apart. One plate is maintained at 1000°C and the other at 500°C. What is the net radiant heat exchange between the two plates?

$$\frac{Y}{D} = \frac{0.5}{0.5} = 1.0 \qquad \frac{X}{D} = \frac{1.0}{0.5} = 2.0$$

$$F_{12} = 0.285$$

$$q = A_1 F_{12}(E_{b1} - E_{b2}) = \sigma A_1 F_{12}(T_1^4 - T_2^4)$$
$$= (5.669 \times 10^{-8})(0.5)(0.285)(1273^4 - 773^4)$$
$$= 18.33 \text{ kW}$$



X
Z
Y
A_2
A_1
Ratio Y/X = 0.1
0.2
0.4
0.6
1.0
1.5
2.0
4.0
6.0
10.0
20.0
F_{1-2}
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.15
0.2
0.3
0.5
1.0
1.5
2
3
5
10
Ratio Z/X

$$F_{3-1,2} = F_{3-1} + F_{3-2}$$

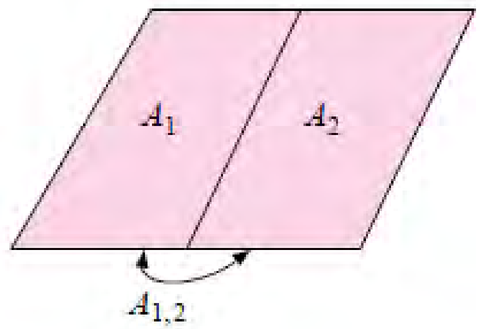
$$A_3 F_{3-1,2} = A_3 F_{3-1} + A_3 F_{3-2}$$

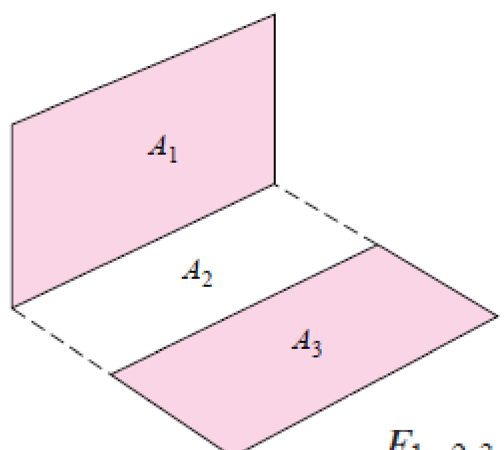
$$A_{1,2} F_{1,2-3} = A_1 F_{1-3} + A_2 F_{2-3}$$

$$A_3 F_{3-1,2} = A_{1,2} F_{1,2-3}$$

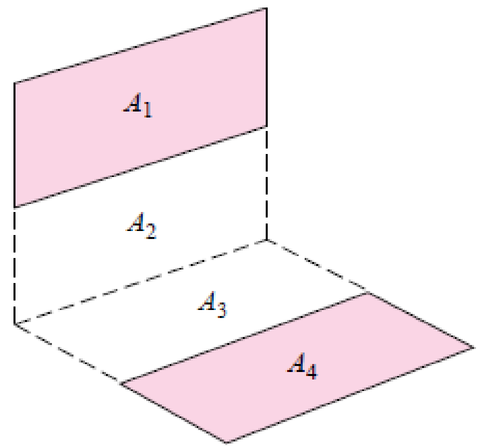
$$A_3 F_{3-1} = A_1 F_{1-3}$$

$$A_3 F_{3-2} = A_2 F_{2-3}$$





$$F_{1-2,3} = F_{1-2} + F_{1-3}$$



$$A_{1,2}F_{1,2-3,4} = A_1F_{1-3,4} + A_2F_{2-3,4}$$

$$A_1F_{1-3,4} = A_1F_{1-3} + A_1F_{1-4}$$

$$A_{1,2}F_{1,2-3} = A_1F_{1-3} + A_2F_{2-3}$$

$$A_{1,2}F_{1,2-3,4} = A_{1,2}F_{1,2-3} - A_2F_{2-3} + A_1F_{1-4} + A_2F_{2-3,4}$$

$$F_{1-4} = \frac{1}{A_1}(A_{1,2}F_{1,2-3,4} + A_2F_{2-3} - A_{1,2}F_{1,2-3} - A_2F_{2-3,4})$$