. thin flat plate is installed in a water tunnel as a splitter. The plate is 0.3 m long and 1 m wide. The freestream speed is 2 m/s. Laminar boundary layer forms on both sides of the plate. The boundary layer velocity profile is approximated by

$$\frac{v_x}{U} = 2\left[\frac{y}{\delta}\right] - \left[\frac{y}{\delta}\right]^2$$

Obtain an expression for δ. Determine the total viscous drag force on the plate assuming the pressure drag is negligible. (kinematic viscosity of water is $1.0 \times 10^{-6}$ $m^2/s$).

Thin flat plate in water ~~channel~~ tunnel.

A. Check $Re_L = \frac{UL}{\nu} = \frac{2\times 0.3}{10^{-6}} = 6\times 10^5$, so laminar

$$\frac{u}{U} = 2\eta - \eta^2.$$

MI eqn $\tau_w = \rho U^2 \frac{d\delta}{dx}\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\eta.$

or, $\frac{U}{\delta}\,\mu \left.\frac{d(u/U)}{d(y/\delta)}\right|_{y/\delta=0} = \rho U^2 \frac{d\delta}{dx}\int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\,d\eta.$

$$= \rho U^2 \frac{d\delta}{dx}\int_0^1 (2\eta - 4\eta^2 + 2\eta^3 - \eta^2 + 2\eta^3 - \eta^4)\,d\eta.$$

or, $\frac{2\mu U}{\delta} = \rho U^2 \frac{d\delta}{dx}\left[\int_0^1 (2\eta - 5\eta^2 + 4\eta^3 - \eta^4)\,d\eta\right]$

or, $\frac{2\mu}{\delta} = \rho U \frac{d\delta}{dx}\left[\eta^2 - \frac{5}{3}\eta^3 + \eta^4 - \frac{1}{5}\eta^5\right]_0^1$

or, $\frac{2\mu}{\delta} = \rho U \frac{d\delta}{dx}\left[1 - \frac{5}{3} + 1 - \frac{1}{5}\right] = \frac{2}{15}\rho U \frac{d\delta}{dx}.$

or, $\frac{2\mu}{\delta} = \rho U \frac{d\delta}{dx}[$

$$\delta\, d\delta = 15\frac{\mu}{\rho U}\cdot dx$$

$$\delta^2 = 30\frac{\mu x}{\rho U} + c.$$

$\delta = 0$ at $x=0 \Rightarrow c = 0.$

$$\boxed{\frac{\delta}{x} = \frac{5.48}{\sqrt{\rho U x/\mu}} = \frac{5.48}{\sqrt{Re_x}}} \leftarrow$$

$$F_D = 2\int_0^L \tau_w\, b\,dx = 2\int_0^L \frac{2\mu U}{\delta}\, b\,dx = \frac{4}{5.48}\, b\mu U\sqrt{\frac{\rho U}{\mu}}\cdot\int_0^L \frac{dx}{x^{1/2}}.$$

$$= \frac{8}{5.48}\, b\mu U\sqrt{\frac{\rho U}{\mu}}\cdot L^{1/2}$$

$$= \frac{8}{5.48}\, b\times\mu U\sqrt{\frac{\rho U L}{\mu}}$$

$$= \frac{8}{5.48}\times 1\,m\times 10^{-3}\,\frac{kg}{m\,s}\cdot 2\,\frac{m}{s}\sqrt{\frac{10^3\times 2\times 0.3}{10^{-3}}\,\frac{kg}{m^3}\cdot\frac{m}{s}\,m\frac{m\,s}{kg}}$$

$$\boxed{F_D = 2.26\,N.} \leftarrow$$