PDC CLASSTEST

NAME : SNEH LATA SINGH
ROLL NO: 181E33006

1.

a) order of system = 6 (order of system = 2×3 = 6)

Dynamical equation

$$Y_1 = \frac{dx_1}{dt}$$

$$Y_2 = \frac{dx_2}{dt}$$

$$Y_3 = \frac{dx_3}{dt}$$

$$\frac{dy_1}{dt} = -a(x_1 - x_2)$$

$$\frac{dy_2}{dt} = -b(x_2 - x_1) - b(x_2 - x_3)$$

$$\frac{dy_3}{dt} = -a(x_3 - x_2)$$

Dynamical variables : $x_1, x_2, x_3, Y_1, Y_2, Y_3$

b) Four equilibrium solutions

$$Ax = 0$$

$$\frac{d}{dt}\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} -a & a & 0 \\ b & -2b & b \\ 0 & a & -a \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -ax_1 + ax_2 = 0$$

$$x_1 = x_2 \quad (a \neq 0)$$

$$ax_2 - ax_3 = 0$$

$$x_2 = x_3 \quad (a \neq 0)$$

$$b(x_1 - 2x_2 + x_3) = 0$$

$$b \neq 0.$$

$x_1 = x_2 = x_2$

equilibrium solution

$x_1 = x_2 = x_3 =$ constant (or zero)

c)

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -a & a & 0 & 0 & 0 & 0 \\ b & -2b & b & 0 & 0 & 0 \\ 0 & a & -a & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

From Wolframalpha we get

$\lambda_1 = 0$ $\qquad V_1 = [1\ 1\ 1\ 0\ 0\ 0]^T$

$\lambda_2 = -i\sqrt{a}$ $\qquad V_2 = \left[\frac{-i}{\sqrt{a}}\ \ 0\ \ \frac{i}{\sqrt{a}}\ \ -1\ \ 0\ \ 1\right]^T$

$\lambda_3 = i\sqrt{a}$ $\qquad V_3 = \left[\frac{i}{\sqrt{a}}\ \ 0\ \ \frac{-i}{\sqrt{a}}\ \ -1\ \ 0\ \ 1\right]^T$

$\lambda_4 = -\sqrt{a-2b}$ $\qquad V_4 = \left[\frac{-1}{\sqrt{a-2b}}\ \ \frac{2b}{a\sqrt{-a-2b}}\ \ \frac{-1}{\sqrt{-a-2b}}\ \ 1\ \ \frac{-2b}{a}\ \ 1\right]^T$

$\lambda_5 = \sqrt{-a-2b}$

$$V_5 = \left[\frac{1}{\sqrt{-a-2b}}\ \ -\frac{2b}{a\sqrt{-a-2b}}\ \ \frac{1}{\sqrt{-a-2b}}\ \ 1\ \ \frac{-2b}{a}\ \ 1\right]^T$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \sum_{i=1}^{6} c_i e^{\lambda_i t} V_i$$

d) Let $a<0$, $b<0$

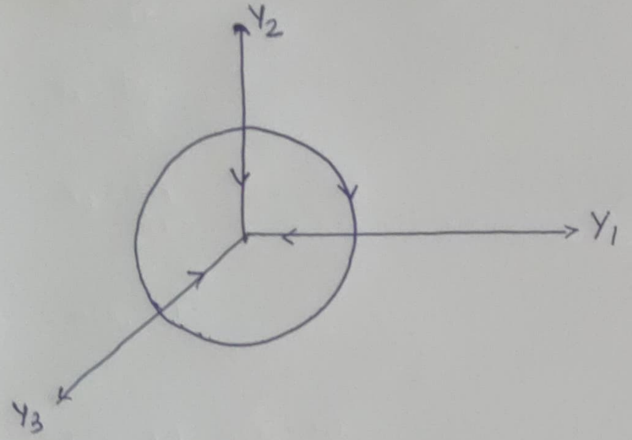
so all eigen values are imaginary.

Solution will be in the form of Real + j Imaginary.

so the phase potrait must be centre solutions.

Let $a>0$, $b>0$

All eigen values can be imaginary based on $a>b$.



2. Constant yeast decay rate = β.

Let growth parameter be = α.

$$\frac{dx}{dt} = \alpha x(1-x)$$

Since constant decay rate,

It should be subtracted from growing rate.

$$\frac{dx}{dt} = \alpha x(1-x) - \beta.$$

$$f(\alpha, x) = \alpha x - \alpha x^2 - \beta.$$

$$\frac{dx}{dt} = 0. \quad \text{(for equilibrium)}$$

$$-\alpha x^2 + \alpha x - \beta = 0$$

$$x^2 - x + \frac{\beta}{\alpha} = 0$$

$$x = \frac{1 \pm \sqrt{1 - \frac{4\beta}{\alpha}}}{2}$$

$x_e = f(\alpha)$

For diagram

$$\left.\frac{df}{dx}\right|_{x_e} = 0$$

$$\frac{\partial f}{\partial x} = \alpha - 2\alpha x$$

$$\left.\frac{df}{\partial x}\right|_{x_e} = \alpha(1) - 2\left(\frac{+1 \pm \sqrt{1-4\beta/\alpha}}{2}\right)\alpha$$

$$= \alpha\left[1 - \left(1 \pm \sqrt{1-4\beta/\alpha}\right)\right]$$

for λ stable

$$\left.\frac{\partial f}{\partial x}\right|_{x_e} < 0$$

α is always +ve

so $1-\left(1+\sqrt{1-4\beta/\alpha}\right) > 0$ is ~~uns~~ unstable

< 0 is stable

$$\sqrt{1-\frac{4\beta}{\alpha}} > 0 \quad \text{stable}$$

$$1 > \frac{4\beta}{\alpha}$$

$$\alpha > 4\beta$$