PROBLEMS

B–10–1. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 1 \quad 0]$$

Transform the system equations into (a) controllable canonical form and (b) observable canonical form.

B–10–2. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 1 \quad 1]$$

Transform the system equations into the observable canonical form.

B–10–3. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

By using the state-feedback control $u = -\mathbf{K}\mathbf{x}$, it is desired to have the closed-loop poles at $s = -2 \pm j4$, $s = -10$. Determine the state-feedback gain matrix **K**.

B–10–4. Solve Problem B–10–3 with MATLAB.

B–10–5. Consider the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

Show that this system cannot be stabilized by the state-feedback control $u = -\mathbf{K}\mathbf{x}$, whatever matrix **K** is chosen.

B–10–6. A regulator system has a plant

$$\frac{Y(s)}{U(s)} = \frac{10}{(s+1)(s+2)(s+3)}$$

Define state variables as

$$x_1 = y$$
$$x_2 = \dot{x}_1$$
$$x_3 = \dot{x}_2$$

By use of the state-feedback control $u = -\mathbf{K}\mathbf{x}$, it is desired to place the closed-loop poles at

$$s = -2 + j2\sqrt{3}, \qquad s = -2 - j2\sqrt{3}, \qquad s = -10$$

Determine the necessary state-feedback gain matrix **K**.

B–10–7. Solve Problem B–10–6 with MATLAB.

B–10–8. Consider the type 1 servo system shown in Figure 10–58. Matrices **A**, **B**, and **C** in Figure 10–58 are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5 & -6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 0 \quad 0]$$

Determine the feedback gain constants k_1, k_2, and k_3 such that the closed-loop poles are located at

$$s = -2 + j4, \qquad s = -2 - j4, \qquad s = -10$$

Obtain the unit-step response and plot the output $y(t)$-versus-t curve.

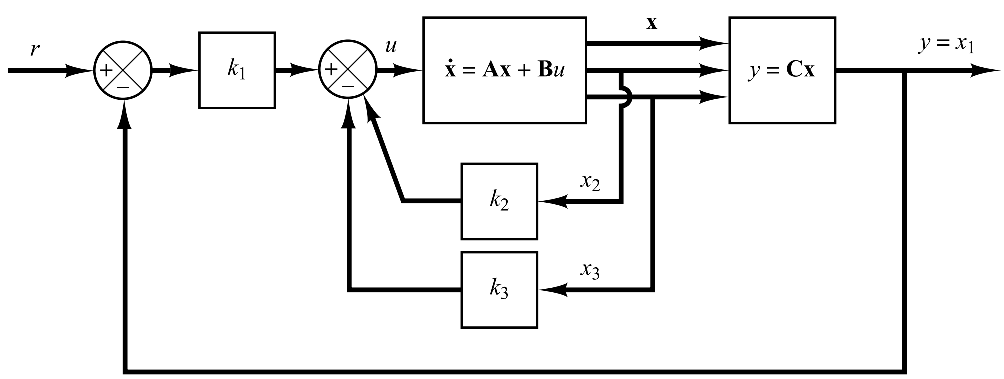

Figure 10–58
Type 1 servo system.

B–10–9. Consider the inverted-pendulum system shown in Figure 10–59. Assume that

$$M = 2 \text{ kg}, \qquad m = 0.5 \text{ kg}, \qquad l = 1 \text{ m}$$

Define state variables as

$$x_1 = \theta, \qquad x_2 = \dot{\theta}, \qquad x_3 = x, \qquad x_4 = \dot{x}$$

and output variables as

$$y_1 = \theta = x_1, \qquad y_2 = x = x_3$$

Derive the state-space equations for this system.

It is desired to have closed-loop poles at

$$s = -4 + j4, \qquad s = -4 - j4, \qquad s = -20, \qquad s = -20$$

Determine the state-feedback gain matrix **K**.

Using the state-feedback gain matrix **K** thus determined, examine the performance of the system by computer simulation. Write a MATLAB program to obtain the response of the system to an arbitrary initial condition. Obtain the response curves $x_1(t)$ versus t, $x_2(t)$ versus t, $x_3(t)$ versus t, and $x_4(t)$ versus t for the following set of initial condition:

$$x_1(0) = 0, \quad x_2(0) = 0, \quad x_3(0) = 0, \quad x_4(0) = 1 \text{ m/s}$$

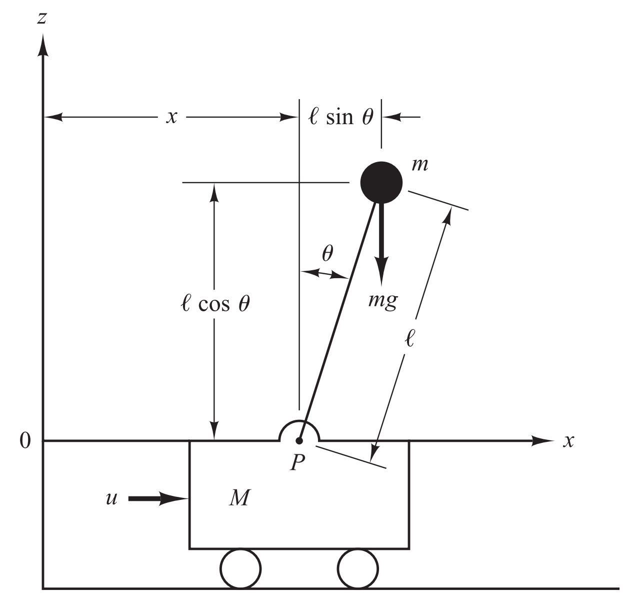

Figure 10–59
Inverted-pendulum system.

B–10–10. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}, \qquad \mathbf{C} = [1 \quad 0]$$

Design a full-order state observer. The desired observer poles are $s = -5$ and $s = -5$.

B–10–11. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -6 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \mathbf{C} = [1 \quad 0 \quad 0]$$

Design a full-order state observer, assuming that the desired poles for the observer are located at

$$s = -10, \qquad s = -10, \qquad s = -15$$

B–10–12. Consider the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1.244 & 0.3956 & -3.145 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1.244 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Given the set of desired poles for the observer to be

$$s = -5 + j5\sqrt{3}, \qquad s = -5 - j5\sqrt{3}, \qquad s = -10$$

design a full-order observer.

B–10–13. Consider the double integrator system defined by

$$\ddot{y} = u$$

If we choose the state variables as

$$x_1 = y$$
$$x_2 = \dot{y}$$

then the state-space representation for the system becomes as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

It is desired to design a regulator for this system. Using the pole-placement-with-observer approach, design an observer controller.

Choose the desired closed-loop poles for the pole-placement part to be

$$s = -0.7071 + j0.7071, \qquad s = -0.7071 - j0.7071$$

and assuming that we use a minimum-order observer, choose the desired observer pole at

$$s = -5$$

B–10–14. Consider the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 0 \quad 0]$$

Design a regulator system by the pole-placement-with-observer approach. Assume that the desired closed-loop poles for pole placement are located at

$$s = -1 + j, \qquad s = -1 - j, \qquad s = -5$$

The desired observer poles are located at

$$s = -6, \qquad s = -6, \qquad s = -6$$

Also, obtain the transfer function of the observer controller.

B–10–15. Using the pole-placement-with-observer approach, design observer controllers (one with a full-order observer and the other with a minimum-order observer) for the system shown in Figure 10–60. The desired closed-loop poles for the pole-placement part are

$$s = -1 + j2, \qquad s = -1 - j2, \qquad s = -5$$

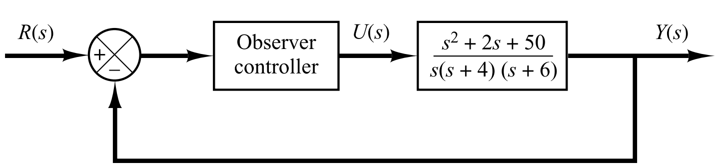

Figure 10–60
Control system with observer controller in the feedforward path.

The desired observer poles are

$$s = -10, \quad s = -10, \quad s = -10 \quad \text{for the full-order observer}$$

$$s = -10, \quad s = -10 \quad \text{for the minimum-order observer.}$$

Compare the unit-step responses of the designed systems. Compare also the bandwidths of both systems.

B–10–16. Using the pole-placement-with-observer approach, design the control systems shown in Figures 10–61(a) and (b). Assume that the desired closed-loop poles for the pole placement are located at

$$s = -2 + j2, \qquad s = -2 - j2$$

and the desired observer poles are located at

$$s = -8, \qquad s = -8$$

Obtain the transfer function of the observer controller. Compare the unit-step responses of both systems. [In System (b), determine the constant N so that the steady-state output $y(\infty)$ is unity when the input is a unit-step input.]

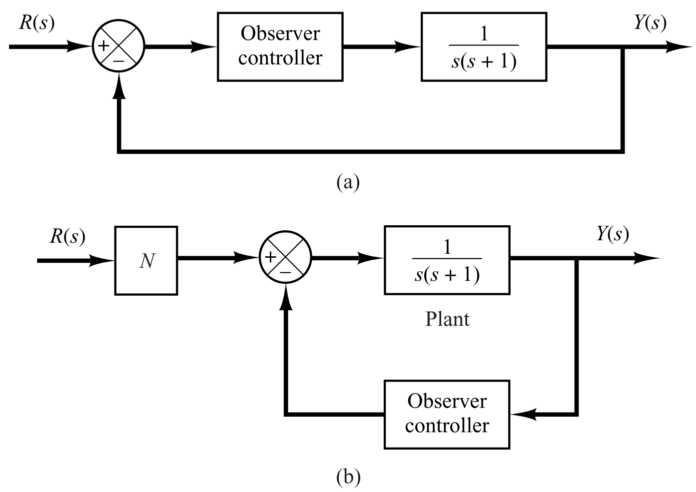

Figure 10–61
Control systems with observer controller: (a) observer controller in the feedforward path; (b) observer controller in the feedback path.

B–10–17. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -a \end{bmatrix}$$

$$a = \text{adjustable parameter} > 0$$