13.1 Evaluate the intensity of dispersion for the middle section.

$$\bar{t}_2 = 1.66\,s$$

Width at 61% = 0.70

∴ standard deviation, $\sigma_2 = 0.35$ (from Fig 4)

$$\sigma_2^2 = 0.1225$$

$$\Delta\bar{t} = 1.66 - 0.068 = 0.98 \text{ (from Fig P1)}$$

$$\Delta(\sigma^2) = 0.1225 - 0.0196 = 0.1029$$

$$\therefore \frac{D}{uL} = \frac{\Delta(\sigma^2)}{\Delta(\bar{t})^2} = \frac{0.1029}{2(0.98)^2} = 0.0536$$

$$\frac{D}{ud_p} = \frac{D}{uL}\left(\frac{L}{d_p}\right) = (0.0536)\left(\frac{10.37 - 5.08}{0.028}\right) = 10.1$$

From the blast furnace experiment $\frac{D}{ud_p} = 10.1$

From the correlation for smaller solids, Fig 17 : $\frac{D}{ud_p} = 0.5$

Note: These results are very different. It could be because of the severe bypassing in the blast furnace, caused by the segregation of the solids and the severe channeling of the gas

---

13.3 From experiment:

t, sec	$\bar{t}$, sec	C
0-20	10	0
20-25	22.5	60
25-30	27.5	210
30-35	32.5	170
35-40	37.5	75
40-45	42.5	35
45-50	47.5	10
50-55	52.5	5
55-70	62.5	0

Calculate D/uL:

$$\Sigma C = 565$$

$$\Sigma tC = 17687.5$$

$$\Sigma t^2 C = 573781.25$$

$$\bar{t} = \frac{\Sigma tC}{\Sigma C} = 31.31 \text{ sec}$$

$$\sigma^2 = \frac{\Sigma t^2 C}{\Sigma C} - (\bar{t})^2 = 35.52 \text{ sec}^2$$

$$\frac{D}{uL} = \frac{\sigma^2}{2(\bar{t})^2} = 0.018$$

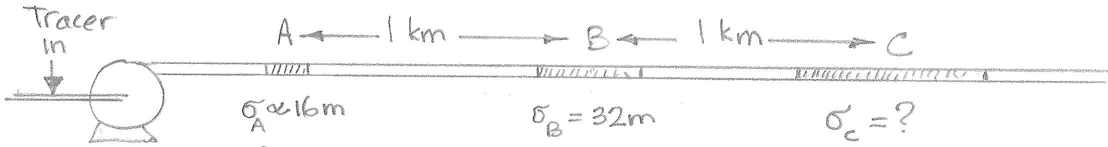
Now calculate the volume of the baffles

From mat. balance

$$\left.\begin{aligned} V &= (1.21\times10^2)(\pi\times1.75^2) = 1164\,cm^3 \\ v &= 1300\,cm^3/min = 21.67\,cm^3/s \end{aligned}\right\} \bar{t} = \frac{1164}{21.67} = 53.72 \text{ sec}$$

$$\therefore \text{Volume of baffles} = \frac{53.72 - 31.31}{53.72}(1164) = 486\,cm^3$$

13.5



$\sigma_A \propto 16m$ $\qquad$ $\sigma_B = 32m$ $\qquad$ $\sigma_C = ?$

$\sigma_A^2 \propto 256$ $\qquad$ $\sigma_B^2 \propto 1024$ $\qquad$ $\sigma_C^2 \propto ?$

Let us solve this with the dispersion model. We could also do this with the tanks-in-series model.

Then from Eq 8 $\quad \sigma^2 = 2(DL/u^3)$

or $\sigma^2 \propto L$ ... or $(\text{spread})^2 \propto L$

So $\quad \sigma_C^2 - \sigma_B^2 = \sigma_B^2 - \sigma_A^2$

or $\quad \sigma_C^2 - 1024 = 1024 - 256$

or $\sigma_C^2 = 1792$ $\qquad$ or Width $= k\,\sigma_C = 42.3\,m$

13.7 First find the σ^2 for this flow

$$Re = \frac{du\rho}{\mu} = (0.255)(1.1)/0.9\times10^{-6} = 3.12\times10^5$$

From Fig 15: $\quad D/ud_t = 0.22$

From Eq 8: $\quad \dfrac{D}{uL} = \left(\dfrac{D}{ud_t}\right)\left(\dfrac{d_t}{L}\right) = 0.22\left(\dfrac{0.255}{1000\,000}\right) = 5.61\times10^{-8}$

$$\frac{\sigma^2}{\bar{t}^2} = \sigma_\theta^2 = 2\left(\frac{D}{uL}\right) = 2\,(5.61\times10^{-8}) = 11.22\times10^{-8}$$

$$\therefore \sigma = (11.22\times10^{-8})^{1/2}\,\frac{1000\,000}{1.1} = 304.5 \text{ sec}$$

$\therefore$ the width at $1\sigma = (304.5)(1.1) = 335\,m$

From 5/95 to 95/5 Fig 12 shows that this includes

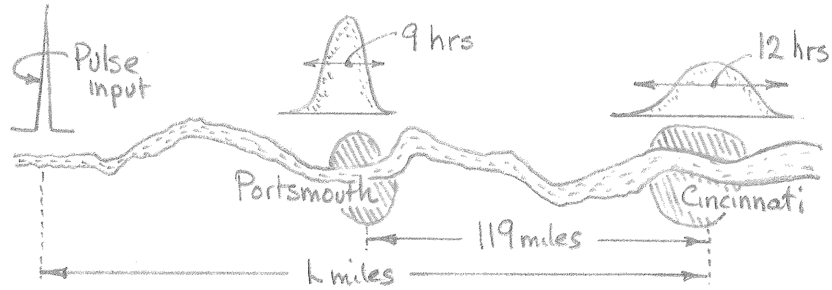
$(1.656\times2)\,\sigma$ of width

$\therefore$ the 5/95 to 95/5% width is

$$(335m)(1.655\times2) = 1105\,m$$

width of one σ^2

13.9 To start assume that a perfect pulse is injected. Then find how far upstream from Cincinnati this pulse was injected



According to any reasonable flow model (dispersion, tanks-in-series, or what have you)

$$\sigma^2_{\text{tracer curve}} \propto \begin{pmatrix}\text{distance from}\\ \text{point of injection}\end{pmatrix} \quad \text{--or--} \quad \begin{pmatrix}\text{spread}\\ \text{of curve}\end{pmatrix} \propto \sqrt{\begin{matrix}\text{distance downstream}\\ \text{from injection point}\end{matrix}}$$

∴ from the Cincinnati reading ... $12\text{ hrs} = kL^{1/2}$

and from the Portsmouth reading ... $9\text{ hrs} = k(L-119)^{1/2}$

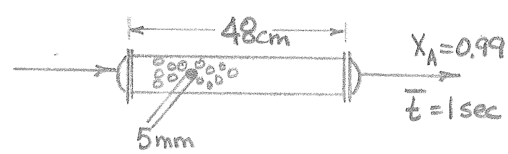
Dividing one by the other & solving for L gives $L = 272$ miles

Since the dumping may not be instantaneous any location where $L \leq 272$ miles is suspect or Ashland, Huntington, Pomeroy ⟵

Note: This solution assumes that different stretches of the Ohio river have the same flow & dispersion characteristics (a reasonable assumption), & that no suspect tributary joins the Ohio within 272 miles upstream from Cincinnati (a poor assumption — check a map for the location of Charleston W.Va. on the Kanawah.)

13.11 First calculate k assuming plug flow, then account for dispersion

a) Find k assuming plug flow



For a 1^{st} order reaction, $\varepsilon_A = 0$, we have

$$k\tau = \ln\frac{C_{A0}}{C_A} \quad \cdots \quad k_{plug} = \frac{1}{\tau}\ln\frac{C_{A0}}{C_A} = \frac{1}{(1\text{sec})}(4.6) = 4.6\text{sec}^{-1} \longleftarrow \text{a)}$$

13.11 (continued) b) Find k accounting for deviation from plug flow according to the dispersion model

From the curve of Fig 17 we find for $Re_p = 200$ & $\epsilon = 0.4$

$$\frac{D\epsilon}{ud_p} = 1 \quad \text{... thus} \quad \frac{D}{ud_p} = 2.5$$

The size ratio of real to plug flow reactor, or what is equivalent, the ratio of the corresponding rate constants is found either from Fig 19 or from Eq 22 if the k ratio or the L ratio is close to unity. Use Eq. 22. Then

$$\frac{L}{L_p} = \frac{k_{true}}{k_{measured}} = 1 + k\tau\left(\frac{D}{uL}\right) = 1 + k\tau\left(\frac{D}{ud_p}\right)\left(\frac{d_p}{L}\right)$$

$$= 1 + 4.6(2.5)\left(\frac{5\,mm}{480\,mm}\right) = 1.1198$$

$$\therefore k_{true} = (4.6\,sec^{-1})(1.1198) = 5.15\,sec^{-1} \longleftarrow \text{b)}$$

Note: Direct use of Eq. 19 pg 314 (trial & error unfortunately) would give a more accurate answer.
Also, are you sure that $k_{true} > k_{plug}$? Convince yourself.

c) Error in assuming plug flow $= \left(\frac{5.15 - 4.60}{5.15}\right) \times 100 = 11\%$ underestimate $\longleftarrow$ c)

13.13 For plug flow with $X_A = 0.999$, or $C_A = 1.000$ and $C_{A0} = 1000$

$$\frac{C_A}{C_{A0}} = e^{-k\bar{t}} \quad \text{.. or} \quad k\bar{t} = \ln\frac{1000}{1} = 6.9078 \quad \text{... or} \quad k = 0.69078$$

From Fig P13 $\quad \sigma^2 = \frac{4^2}{24} = \frac{2}{3}$

So from Eq 21

$$\frac{C_A}{C_{A0}} = \exp\left[(-k\bar{t}) + \frac{k^2\sigma^2}{2}\right] = \exp\left[-6.9078 + \frac{(0.69078)^2\frac{2}{3}}{2}\right]$$

or $\quad = 0.001172$

$$\therefore C_A = 1.17 \longleftarrow$$
