Indian Institute of Technology Kharagpur

Mid-Spring Semester 2017 – 18

Date of Examination: 27-02-2018 Session: AN Duration 2 hrs Full Marks 30
Subject Number: CH30012 Subject: Transport Phenomena
Department: Chemical Engineering
Specific charts, graph paper, log book etc., required: No
Specific Instructions: Assume and clearly write any assumption and data that you feel are missing.

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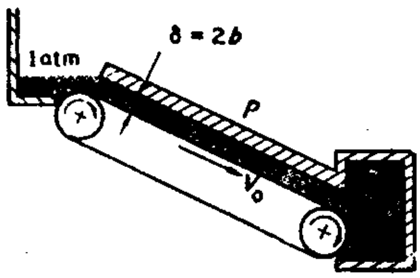
1. A belt of width W moves at velocity V_o as shown in the figure. A liquid fills the gap (of thickness $\delta = 2b$) between the belt and an upper plate. The upper plate is dragged down to the tank B with a velocity equal to V_o. After some time, the tank B gets filled up (without any leakage and thus stopping the flow) and steady state conditions prevail.

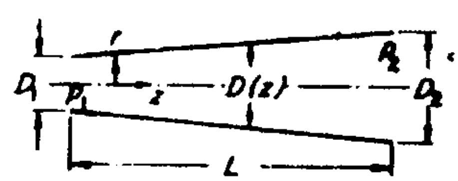


a) Obtain the velocity profile and the pressure gradient for this condition. Please take the origin of the coordinate system to be on the belt with the direction of the belt movement as z and the height of the film as y. The effect of gravity may not be neglected. θ is the angle of inclination with respect o the horizontal.

b) A weigh-bridge is designed based on the system just described. The object to be weighed is placed on the horizontal platform P and the belt is started at velocity V_o and steady state is reached. The film thickness δ is directly calibrated to give the load (W). Evaluate the relation between the load and the film thickness (δ) in terms of V_o, properties (μ and ρ) and system geometry (L, 2b, θ). **Marks 6+4 = 10**

2. Two hypothetical boundary layers can be represented as a) $\frac{u}{U} = \frac{y}{\delta}$ and b) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$. Calculate the momentum flux for the entire flow through the boundary layer for each profile. If the two profiles were subjected to the same pressure gradient condition, which would be most likely to separate first and why? **Marks 4+1 = 5**

3. Consider the flow of a fluid in the conical tube shown. The cone angle is small, i.e., $(D_2-D_1)/L \ll 1$ and a linear variation of D with z may be assumed. Both V_r and V_z are non-Zero, but V_r is small enough so that we can assume a quasi 1-D situation to obtain dp/dz in terms of the flow rate Q and the diameter D at any z. You may neglect the effect of body forces such that p = p(z) only. Integrate this expression to obtain the expression for $\frac{P_1-P_2}{L}$ in terms of Q, D_1, D_2, and the viscosity μ of the fluid.



Marks 6

4. A wind tunnel has a constant width of W = 305 mm but a flexible upper wall. The channel height is adjusted to generate a zero pressure gradient flow such that the free stream velocity remains constant within the tunnel. The boundary layers at the wall are represented by the 1/7 the turbulent velocity profile everywhere in the tunnel (kinematic viscosity of air is 1.45×10^{-5} m^2/s). At two locations in the channel (1 and 2) the following parameters are measured - Location 1 - H_1 = 305 mm, δ_1 = 12.2 mm, U_1 = 26.5 m/s, Location 2 at downstream - δ_1 = 16.6 mm. Evaluate the following

a) The height at location 2 (H_2).
b) The equivalent length of a flat plate to give δ_1 = 12.2 mm
c) Estimate the distance between locations 1 and 2. **Marks = 3x3 = 9**

Useful Relations

EQUATION OF CONTINUITY (Cartesian and Cylindrical coordinates)

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=0 \qquad \frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

EQUATION OF MOTION (Cartesian and Cylindrical coordinates)

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial z}-\frac{{v_\theta}^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]+\rho g_r$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$

INTEGRAL EQUATIONS

$$\frac{dN}{dt}\Big|_{system}=\frac{\partial}{\partial t}\int_{CV}\eta\,\rho\, d\forall+\int_{CS}\eta\,\rho\,\vec{V}.\vec{dA} \qquad \delta^*=\int_0^\delta\left(1-\frac{v_x}{U}\right)dy, \qquad \theta=\int_0^\delta\frac{v_x}{U}\left(1-\frac{v_x}{U}\right)dy$$

$$\frac{\tau_w}{\rho}=\frac{d}{dx}(U^2\theta)+\delta^* U\frac{dU}{dx} \qquad \delta=\frac{5.0x}{\sqrt{Re_x}}\ \ (laminar\ flow) \quad \delta=\frac{0.37}{(Re_x)^{\frac{1}{5}}}\ \ (turbulent\ flow) \quad \frac{\bar{v}_z}{U}=\left(\frac{y}{R}\right)^{\frac{1}{7}}$$

Laminar Flow: $C_f=\frac{0.664}{\sqrt{Re_x}}$ $C_D=\frac{1.328}{\sqrt{Re_L}}$ Turbulent Flow: $C_f=\frac{0.0594}{(Re_x)^{\frac{1}{5}}}$ $C_D=\frac{0.0742}{(Re_L)^{\frac{1}{5}}}$

For Mixed Flow, $C_{D\,Turb}=\frac{0.074}{(Re_L)^{\frac{1}{5}}}-\frac{1740}{Re_L}$, $10^5<Re<10^7$, $C_{D\,Turb}=\frac{0.455}{(\log Re_L)^{2.58}}-\frac{1610}{Re_L}$, $Re>10^7$

STOKES LAW $F=3\pi\mu V d$