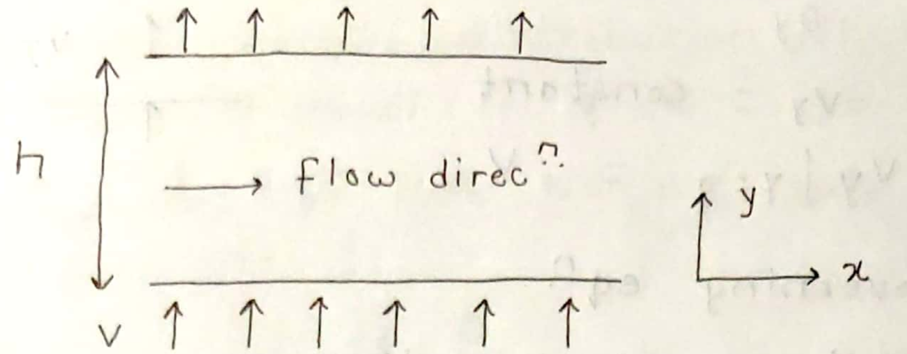
Momentum	Heat	Mass
∇v	∇T	∇C
$\tau_{yx} = -\mu \frac{dv}{dy}$	$q = -k \frac{dT}{dy}$	$N_A = -D_{AB} \frac{dC}{dy}$
Tensor (grad. of vector)	Vector	Vector

→ High viscosity oil spreads on solid surface. No slip condition not applicable.

- Fundamental Engg. Science
- Applications — Traditional (M^2, Heat, Mass)
  — New (Bio, Micro, Nano)
- Similarities / Dissimilarities — (M^2, Mass, Heat)
- Physical Insight
- Conservation / Flux
- Boundary Layers
- Similarity parameters
- Simultaneous heat, mass and momentum transfer
- Design eq^n / Numerical sol^n

3/01/19

Q1.



- Constant pressure gradient
- Two plates are porous and stationary.
- Steady, fully developed flow.
- No body forces.

Sol^n:

x component of the N-S eq^n

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x$$

$$\rho v_y \frac{\partial v_x}{\partial y} = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 v_x}{\partial y^2}$$

$$\rho v_y \frac{d v_x}{d y} = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 v_x}{\partial y^2}$$

A

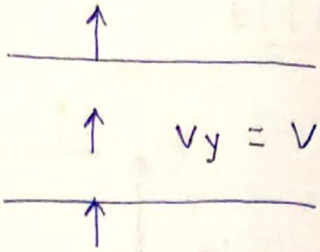
$\left\{\frac{\partial^2 v_x}{\partial z^2} = 0\right.$ only if the z dimension is very large compared to y $\}$

v_y can be obtained from eq^n of continuity.

$$\cancel{\frac{\partial v_x}{\partial x}} + \frac{\partial v_y}{\partial y} + \cancel{\frac{\partial v_z}{\partial z}} = 0$$

0 0

$$\Rightarrow \frac{\partial v_y}{\partial y} = 0$$



$$\Rightarrow v_y = \text{constant}$$

$$\Rightarrow v_y \big|_{y=0} = V$$

Governing eq^n.

$$V \frac{d v_x}{dy} = A + \mu \frac{d^2 v_x}{dy^2}$$

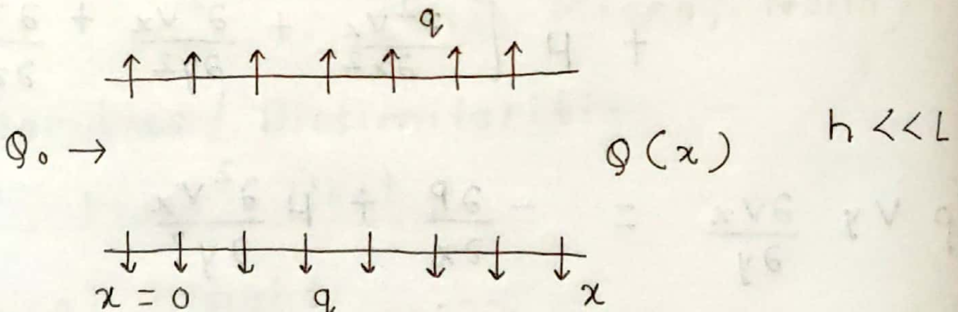
Boundary conditions :-

No slip $\Rightarrow$ $v_x = 0$ @ $y = 0$
$v_x = 0$ @ $y = h$

Sol^n :-

$$u_x = \frac{h}{\rho V}\left(\frac{\partial P}{\partial x}\right)\left[\frac{1 - \exp\left(\frac{\rho V y}{\mu}\right)}{1 - \exp\left(\frac{\rho V h}{\mu}\right)} - \frac{y}{h}\right]$$

Q2.



* Pressure gradient is constant only if cross-sectional area and ρ are constant and their is no suction.

- Pressure decreases as a function of x.
- Flow is dominated by viscous effects and is steady.
- Length is L, Depth is W and

separation is h. q has the unit m/s.

a) Find the pressure distribution, $P(x)$ in the flow channel if $q = 0$.

x component of the NS eq^n gets simplified to.

$$\mu \frac{d^2 v_x}{dy^2} - \frac{dP}{dx} = 0$$

{ steady state, fully-developed, body forces neglected }

The is no injection and suction.

$$\therefore \frac{dP}{dx} = \text{constant} = A$$

$$\mu \frac{d^2 v_x}{dy^2} = A$$

$$\mu \frac{dv_x}{dy} = Ay + c_1$$

$$\mu v_x = \frac{Ay^2}{2} + c_1 y + c_2$$

Boundary conditions :-

$y = 0 \quad v_x = 0$ } No slip

$y = h \quad v_x = 0$

$$\therefore v_x = \frac{1}{2\mu} A y^2 \left[1 - \frac{h}{y}\right]$$

$$= \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) y^2 \left[1 - \frac{h}{y}\right]$$

$$\langle v_x \rangle = \frac{1}{h}\int_0^h v_x \, dy = -\frac{1}{12\mu}\frac{\partial P}{\partial x} h^2$$

$$\therefore Q_0 = Wh \langle v_x \rangle = -\frac{1}{12\mu}\frac{\partial P}{\partial x} h^3 W,$$

Q_0 = Flow rate with $q = 0$.

$$\therefore \frac{dP}{dx} = -\frac{12\mu}{Wh^3} Q_0$$

Integrating and imposing the B.C. that
$P(x=0) = P_0$

$$\therefore P(x) = P_0 - \frac{12\mu Q_0}{Wh^3} x$$

b) Find the pressure distribution $P(x)$ in the channel for $q \neq 0$.

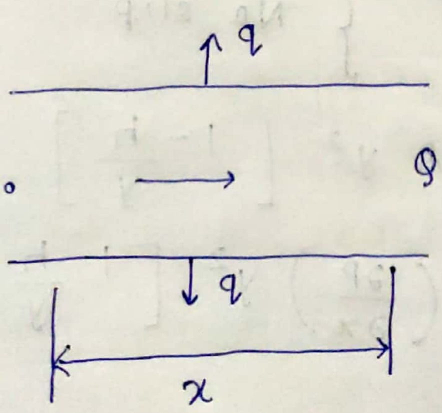
$$v_x = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) y^2 \left[1 - \frac{h}{y}\right]$$

$$Q_0 = Wh\langle v_x\rangle = -\frac{1}{12\mu}\frac{dP}{dx} h^3 W$$

$$\therefore \frac{dP}{dx} = -\frac{12\mu}{Wh^3} Q_0$$ → will change with x.

$Q(x)$ needs to be found out.

Applying conservation of mass



$$Q(x) = Q_0 - 2qWx$$

We have $$\frac{dP}{dx} = -\frac{12\mu}{Wh^3} Q(x)$$

$$\Rightarrow \frac{dP}{dx} = -\frac{12\mu Q(x)}{Wh^3} = -\frac{12\mu}{Wh^3}(Q_0 - 2qWx)$$

$$\Downarrow \quad p = P_0 \quad at \quad x = 0$$

$$P(x) = P_0 - \frac{12\mu x}{Wh^3}\left[Q_0 - \frac{2qWx}{2}\right]$$

c) Given that $P(x = L) = P_e$, find q.

d) Find the criterion necessary for viscous terms to dominate.

$$\frac{\rho\left(\overbrace{V_x \frac{\partial V_x}{\partial x}}^{I} + \overbrace{V_y \frac{\partial V_x}{\partial y}}^{II}\right)}{\mu\left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2}\right)} << 1 \quad - \quad (1)$$

$$\frac{\partial^2 V_x}{\partial x^2} << \frac{\partial^2 V_x}{\partial y^2} \Rightarrow \frac{\partial^2 V_x}{\partial x^2} \approx 0$$

$$\mu \frac{\partial^2 V_x}{\partial y^2} \approx \mu \frac{V_x}{h^2}$$

$$\frac{\partial V_x}{\partial x} \sim \frac{\partial}{\partial x}(Q/A) = \frac{d}{dx}(Q/A)$$

$$= \frac{1}{Wh}\frac{d}{dx}(Q_0 - 2qWx) = \frac{-2q}{h}$$

Ignoring the sign

$$\frac{\partial V_x}{\partial x} = \frac{2q}{h} \sim \frac{q}{h}$$

Term I becomes $V_x \frac{q}{h}$

Term II :

$v_y \approx q \qquad \frac{\partial v_x}{\partial y} \approx \frac{v_x}{h}$

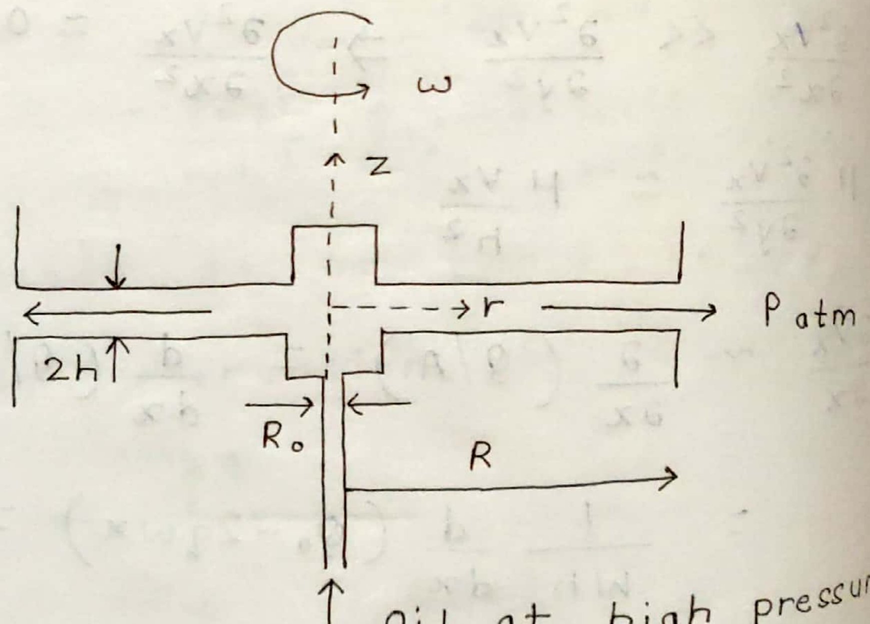
Eq^n ① becomes

$$\frac{\rho \left(v_x \frac{q}{h} + q \frac{v_x}{h} \right)}{\mu \, v_x / h^2} << 1$$

$$\frac{q \rho h}{\mu} << 1$$ q is the suction velocity

For our previous analysis to be valid, the reynolds no. based on suction velocity should be less than 1.

Q.



oil at high pressure P_0

- Upper plate is rotating at an angular velocity ω.
- Bottom plate is stationary.
- Pressure varies from R_0 to R, $P = P_0$ upto R_0. Convective and body force terms can be neglected.

Sol^n :

Velocity has two components v_r and v_θ. v_z can be neglected as no significant motion in z direction.

i) Start with the continuity eq^n to get the functional form of v_r.

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \underbrace{\cancel{\frac{\partial \rho}{\partial t}}}_{0} + \frac{1}{r}\frac{\partial}{\partial \theta}\underbrace{(\rho \cancel{v_\theta})}_{0} + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) = 0$$

$$\Rightarrow \quad r v_r = \text{constant}$$

$$\Rightarrow \quad v_r = \frac{c}{r}$$

$$c = f(z)$$

$$\Rightarrow \quad v_r = \frac{f(z)}{r} \quad \Rightarrow \quad v_r = v_r(r, z)$$

ii) Show that $v_\theta = \frac{\omega r}{2}\left(1+\frac{z}{h}\right)$ satisfies the θ component of NS eq^n.

L.H.S of the θ component of NS eq^n is zero as convective terms are neglected.

$$\frac{\partial P}{\partial \theta} = 0 \quad \{\text{angular symmetry}\}$$

$$\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} = 0 \quad \{\quad ,, \quad\}$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{\partial^2 v_\theta}{\partial z^2} = 0$$

iii) Write the r component of N.S. eq^n to show that $r\frac{\partial P}{\partial r}$ is a constant.

L.H.S. = 0

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) = 0$$

as $r v_r$ is a f^n of z.

$$-\frac{\partial P}{\partial r} = -\frac{\partial^2 v_r}{\partial z^2}\mu$$

or $$-\frac{\partial P}{\partial r} + \mu\frac{\partial^2 v_r}{\partial z^2} = 0$$

$$r\frac{\partial P}{\partial r} = r\mu\frac{\partial^2 v_r}{\partial z^2} = \mu\frac{\partial^2 (\mu v_r)}{\partial z^2}$$

$$\underbrace{r\frac{\partial P}{\partial r}}_{f(r)} = \underbrace{\mu\frac{\partial^2 (r v_r)}{\partial z^2}}_{f(z)} = K$$

$P \neq f(\theta)$

$P \neq f(z)$

$$r\frac{\partial P}{\partial r} = K \Rightarrow \frac{dP}{dr} = \frac{K}{r}$$

$$P = K \ln r + c$$

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At $r = R$, $p = p_{atm}$

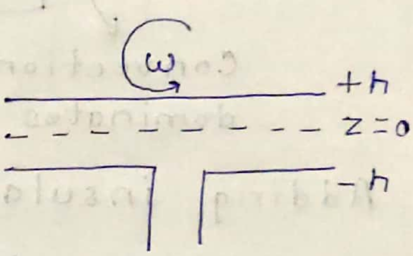
$$\therefore \quad p - p_{atm} = K \ln \frac{r}{R}$$

$$\Rightarrow \quad K = \frac{p_0 - p_{atm}}{\ln (r_0/R)} \qquad \left\{ \begin{array}{l} r = r_0 \\ p = p_0 \end{array} \right\}$$

Find the volume flowrate (leakage rate) and the ~~axial~~ vertical load that can be supported.

$$\frac{\partial^2 (r v_r)}{\partial z^2} = \frac{K}{\mu}$$

$$r v_r = \frac{K z^2}{2\mu} + C_1 z + C_2$$



B.C : @ $z = \pm h$ $\quad v_r = 0$

$$v_r = \frac{K}{2\mu r} (z^2 - h^2)$$

$$Q = \int_{-h}^{h} v_r \, 2\pi r \, dz = \int_{-h}^{h} \frac{K}{\cancel{2}\mu} \cdot \cancel{2}\pi (z^2 - h^2) \, dz$$

$$\Rightarrow Q = \frac{\pi K}{\mu} \left[\frac{z^3}{3} - z h^2 \right]_{-h}^{h}$$

$$= \frac{\pi K}{\mu} \left[\frac{h^3}{3} - h^3 - \left\{ -\frac{h^3}{3} + h^3 \right\} \right.$$

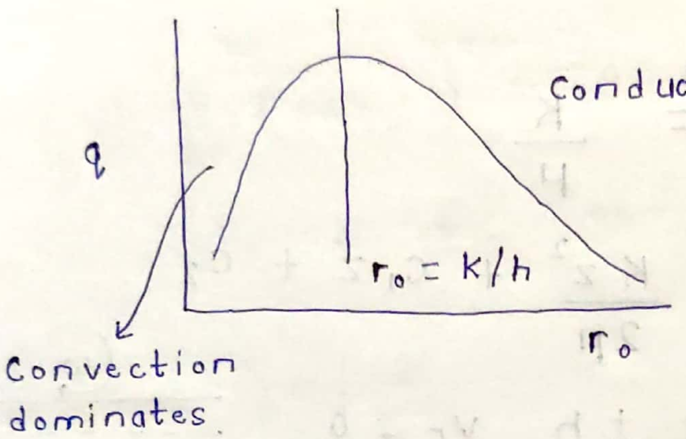
$$= -\frac{\pi K}{\mu} \frac{4}{3} h^3$$

$$Q = \frac{p_0 - p_{atm}}{\ln R/R_0} \frac{4}{3} \frac{\pi h^3}{\mu}$$

$$\text{Load} = \pi R_0^2 (P_0 - P_{atm}) + \int_{R_0}^{R} (P - P_{atm}) 2\pi r dr$$

$$= \frac{\pi (P_0 - P_{atm})}{2 \ln R/R_0} (R^2 - R_0^2)$$

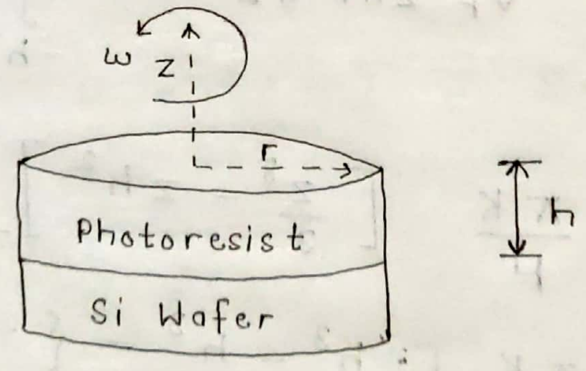
* Critical thickness of insulation not applicable for planar surface.



Adding insulation ;

Conduction ↓ Convection ↑

- Predicting film thickness during spin coating



- Angular symmetry
- Constant ω
- No pressure gradient
- No body force
- ∵ film is thin, it can be assumed to be a rigid body, i.e., $v_\theta \neq f(z)$

Solution :

Continuity eq^n :

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0$$

Momentum eq^n in r direction gets reduced to -

$$\rho\left(v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{\partial^2 v_r}{\partial z^2}\right]$$

As the film gets thinner $\frac{\partial v_r}{\partial r}$ becomes smaller and can be neglected.

$$\Rightarrow v_r \frac{\partial v_r}{\partial r} \sim 0$$

v_z is very small and can be neglected when compared to v_θ

After simplification,

$$\mu \frac{\partial^2 v_r}{\partial z^2} + \rho \frac{v_\theta^2}{r} = 0$$

$$\Rightarrow \frac{\partial^2 v_r}{\partial z^2} = -\frac{\rho}{\mu}\frac{v_\theta^2}{r}$$

$$\Rightarrow \frac{\partial v_r}{\partial z} = -\frac{\rho}{\mu}\frac{v_\theta^2}{r} z + c_1$$

$$\Rightarrow v_r = -\frac{\rho}{2\mu}\frac{v_\theta^2}{r} z^2 + c_1 z + c_2$$

@ $z = 0$, $c_2 = 0$

@ $z = h$, $\frac{\partial v_r}{\partial z} = 0$

$$\Rightarrow c_1 = \frac{\rho}{\mu} \frac{v_\theta^2}{r} h$$

$$\Rightarrow v_r = -\frac{\rho}{2\mu} \frac{v_\theta^2}{r} z^2 + \frac{\rho}{\mu} \frac{v_\theta^2}{r} hz$$

$$v_r = \frac{\omega^2 r}{\nu} \left(hz - \frac{z^2}{2} \right) \quad ; \quad v_\theta =$$

v_z can be obtained from eq^n of continuity.

$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{\partial v_z}{\partial z} = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left\{ \frac{\omega^2 r^2}{\nu} \left(hz - \frac{z^2}{2} \right) \right\} + \frac{\partial v_z}{\partial z} = 0$$

$$\frac{1}{\cancel{r}} \times \frac{2\cancel{r}\omega^2}{\nu} \left(hz - \frac{z^2}{2} \right) + \frac{\partial v_z}{\partial z} = 0$$

$$\frac{\partial v_z}{\partial z} = \frac{2\omega^2}{\nu} \left(\frac{z^2}{2} - hz \right)$$

$$v_z = \frac{2\omega^2}{\nu} \left(\frac{z^3}{6} - \frac{hz^2}{2} \right) + c$$

@ $z = 0$, $v_z = 0$; $c = 0$

$$\therefore v_z = \frac{2\omega^2}{\nu} \left(\frac{z^3}{6} - \frac{hz^2}{2} \right)$$

$$v_z = \frac{\omega^2}{\nu}\left(\frac{z^3 - 3hz^2}{3}\right)$$

$$\frac{dz}{dt} = \frac{\omega^2}{\nu}\left(\frac{z^3 - 3hz^2}{3}\right)$$

$$\int_h^z \frac{dz}{z^3 - 3hz^2} = \int_0^t \frac{\omega^2}{3\nu}\, dt$$

$$\frac{dh}{dt} = \langle v_z \rangle \longrightarrow \text{Thickness avg.}$$

$$\langle v_z \rangle = \frac{1}{h}\int v_z\, dz$$

$$= \frac{1}{h}\int_0^h \left(\frac{z^3}{3} - hz^2\right) dz \cdot \frac{\omega^2}{\nu}$$

$$= \frac{1}{h}\left[\frac{z^4}{12} - \frac{hz^3}{3}\right]_0^h \cdot \frac{\omega^2}{\nu}$$

$$= \frac{1}{h}\left[\frac{h^4}{12} - \frac{h^4}{3}\right] = \frac{1}{h}\left[\frac{h^4 - 4h^4}{12}\right]$$

$$\langle v_z \rangle = -\frac{h^3}{4}\frac{\omega^2}{\nu}$$

$$\frac{dh}{dt} = -\frac{\omega^2}{4\nu} h^3$$

$$\int_{h_0}^h \frac{dh}{h^3} = \int_0^t -\frac{\omega^2}{4\nu}\, dt$$

$$-\frac{1}{2h^2}\Big|_{h_0}^h = -\frac{\omega^2}{4\nu} t$$

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$$-\frac{1}{2h^2} + \frac{1}{2h_0^2} = -\frac{\omega^2 t}{4\nu}$$

$$\Rightarrow \frac{1}{h_0^2} + \frac{\omega^2 t}{2\nu} = \frac{1}{h^2}$$

$$h = \left(\frac{1}{h_0^2} + \frac{\omega^2 t}{2\nu}\right)^{-1/2}$$

Q. Two parallel, plane circular discs of radius R lie one above the other a small distance apart. The space b/w them is filled with a fluid. The upper disc approaches the bottom at constant velocity V. At $r = R$, $p = p_0$. $p_r \neq f(z)$

Sol^n: Assumptions :-

1> R is very large compared to h $\Big\rangle$ $Re = \frac{hv}{\mu}$ is small

11> Leakage rate is small (small V_R) S.S.

111> $\frac{\partial P}{\partial z} \approx 0$

IV> V_z is very small compared to V_r

V> $\frac{\partial V_z}{\partial z}$ may not be small (as z is very small)

vi) $\frac{\partial}{\partial z}\left(\frac{\partial v_z}{\partial z}\right)$ can be appreciable.

vii) $\frac{\partial v_r}{\partial r}$ is small compared to $\frac{\partial v_r}{\partial z}$.

viii) $v_\theta = 0$, no θ dependence of v_r, v_z

Eq^n of continuity :-

$$\underbrace{\frac{\partial \rho}{\partial t}}_{0} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho \underbrace{v_\theta}_{0}) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0 \quad \text{— (A)}$$

Eq^n of motion in r direction –

$\underbrace{v_r}_{\text{appreciable}} \underbrace{\frac{\partial v_r}{\partial r}}_{\text{small}} \longrightarrow \therefore$ neglected

$\underbrace{v_z}_{\text{small}} \underbrace{\frac{\partial v_r}{\partial z}}_{\text{appreciable}} \longrightarrow \therefore$ neglected

$\frac{\partial^2 v_r}{\partial z^2}$ not neglected.

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{\partial^2 v_r}{\partial z^2}\right]$$

$$\Rightarrow 0 = -\frac{1}{\rho}\frac{dP}{dr} + \nu\left[\frac{1}{r}\cancel{\frac{\partial v_r}{\partial r}} - \cancel{\frac{v_r}{r^2}} + \cancel{\frac{\partial^2 v_r}{\partial r^2}} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

$$\Rightarrow -\frac{1}{\rho}\frac{dP}{dr} + \nu\frac{d^2 v_r}{dz^2} = 0 \quad - (B)$$

B.C. @ $z = 0$, $v_r = 0$
$z = h$, $v_r = 0$

$$v_r = \frac{1}{2\mu}\frac{dP}{dr}(z-h)z \quad - (C)$$

Substituting (C) in (A) and performing the necessary steps.

$z = 0$; $v_r = v_z = 0$

$z = h$; $v_r = 0$, $v_z = U$

$$U = -\frac{h^3}{12\mu}\frac{1}{r}\frac{d}{dr}\left(r\frac{dP}{dr}\right)$$

Integrating w.r.t. r

$$-\frac{12\mu U}{h^3}\frac{r^2}{2} = r\frac{dP}{dr} + C_1$$

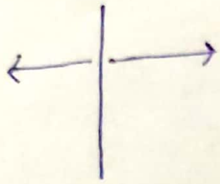
$$\Rightarrow P - P_0 = \frac{3\mu U}{h^3}(R^2 - r^2)$$

$$\text{Force} = \int_0^R 2\pi r\, dr\,(P - P_0) \Rightarrow \frac{3\pi\mu U R^4}{2h^3}$$

17.1.19

$$\rho C_p \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dot{q}$$

Adiabatic Surface



$$\frac{dT}{dx} = 0$$

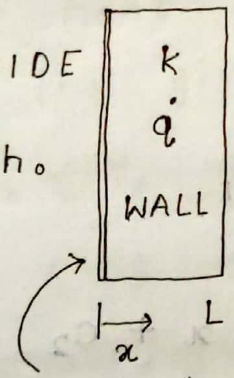
Insulated surface

$$-k \frac{dT}{dx}\Big|_{x=L} = 0$$

$$\frac{dT}{dx} = 0$$

Mathematically, AS and IS give rise to same boundary condⁿ.

Q.



strip heater

$T_i = 50°C$, $h_i = 20 \ W/m^2K$

$L = 200 \ mm$, $k = 4 \ W/mK$

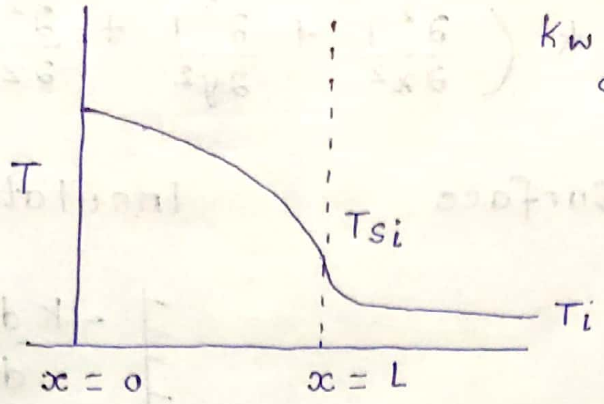
$\dot{q} = 1000 \ W/m^3$, $T_o = 25°C$

$h_o = 5 \ W/m^2 K$.

1. If no heat is lost to the outside, Draw the T-profile.

W | Air

$$k_W \frac{dT}{dx}\Big|_{0^-} = k_A \frac{dT}{dx}\Big|_{0^+}$$



II) Find T(0) and T(L) for condition I.

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{K} = 0$$

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{K}$$

$$\frac{dT}{dx} = -\frac{\dot{q}}{K}x + c_1$$

$$T(x) = -\frac{\dot{q}}{2K}x^2 + c_1 x + c_2$$

@ $x = 0$, $\frac{dT}{dx} = 0 \Rightarrow c_1 = 0$

$$-K\frac{dT}{dx}\Big|_{x=L} = h_i(T(L) - T_\infty)$$

$$+\frac{\dot{q}}{k}L = h_i\left(-\frac{\dot{q}}{2K}L^2 + c_2 - T_\infty\right)$$

$$c_2 = +\frac{\dot{q}}{h_i K}L + \frac{\dot{q}}{2K}L^2 + T_\infty$$

$$c_2 = 65\,^\circ C$$

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$$T(x) = -125x^2 + 65$$

$$T(L) = -125L^2 + 65$$

$$= -125 \times 0.2^2 + 65$$

$$T(L) = 60°C \qquad T(0) = 65°C$$

OR

$$\dot{q}\cancel{A}L = h_i \cancel{A}(T_L - T_i)$$

$$T_L = \frac{\dot{q}L}{h_i} + T_i = \frac{1000 \times 0.4}{20 \times 2} + 50$$

$$T_L = 60°C$$

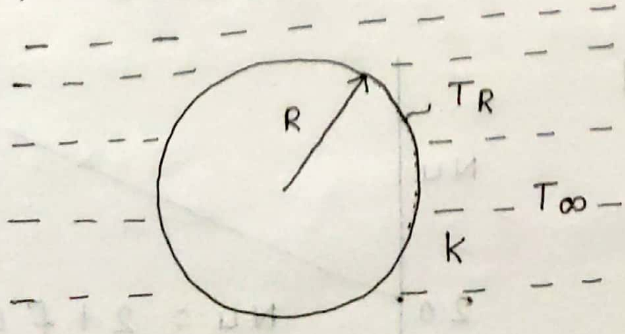
Amount of heat supplied by the strip heater for the given condition :-

$$q_{ST} = h_o(T_o - T_\infty)$$

$$= 5(65 - 25)$$

$$= 200 \ W/m^2$$

Heated Sphere in a large, motionless body of fluid, Obtain $T_{FLUID}(r)$.



$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

$$r^2 \frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

B.C. @ $r = R$ $\quad T = T_R$

$r \to \infty$ $\quad T = T_\infty$

$$\Rightarrow \frac{T(r) - T_\infty}{T_R - T_\infty} = \frac{R}{r}$$

Use this to obtain and expression for Nu.

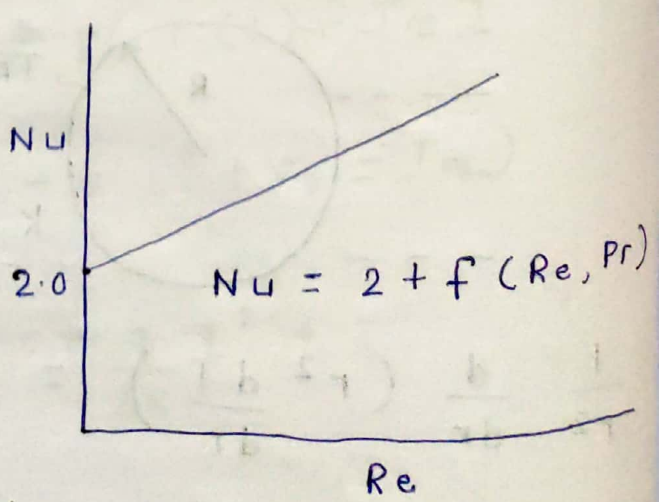
$$\frac{dT}{dr} = -\frac{R}{r^2}(T_R - T_\infty)$$

$$-kA \left.\frac{dT}{dr}\right|_{r=R} = hA(T_R - T_\infty)$$

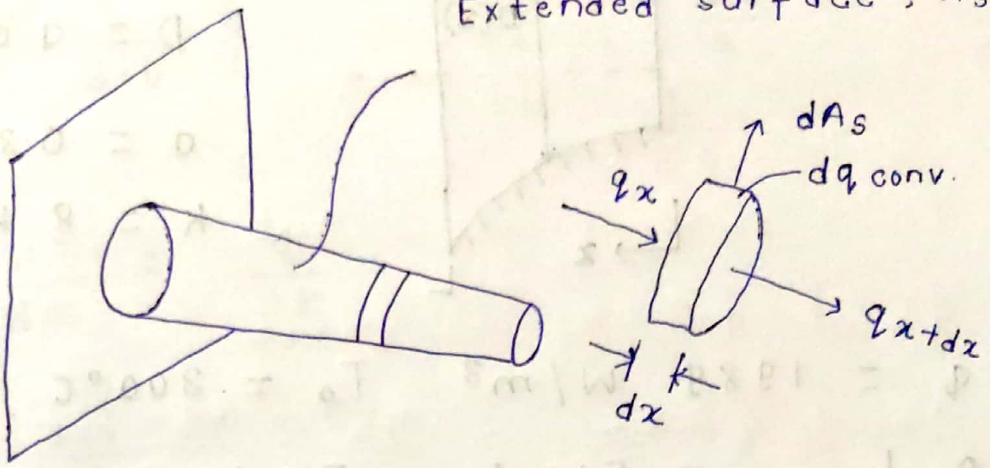
$$k\cancel{A}\frac{\cancel{R}}{R^{\cancel{2}}}\cancel{(T_R - T_\infty)} = h\cancel{A}\cancel{(T_R - T_\infty)}$$

$$\frac{hR}{k} = 1$$

$$\frac{hD}{k} = 2$$



General Conduction Analysis



$$q_x = q_{x+dx} + dq_{conv}$$

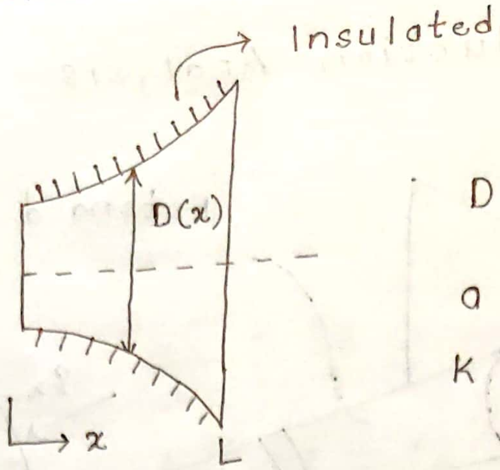
$$q_x = -kA_c\frac{dT}{dx} \quad , \quad dq_{conv.} = h\,dA_s\,(T-T_\infty)$$

$$\frac{d}{dx}\left(A_c\frac{dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}(T-T_\infty) = 0$$

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{A_s}{k}\frac{dA_s}{dx}\right)$$

$$(T-T_\infty) = 0$$

Q.



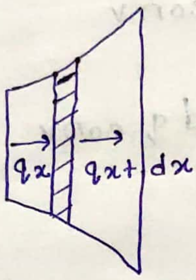
$D = a e^{1.8x/L}$

$a = 0.8\,m, \quad L = 1.8\,m$

$k = 8\ W/mK$

$\dot{q} = 1989\ W/m^3 \qquad T_0 = 300°C,$

$Q_x|_{x=0} = 500\ W \qquad$ Find T_L and $Q_x|_L$.



In + generation = out

$$-k \cancel{\frac{\pi a^2 e^{2\times1.8x/L}}{4}} \frac{dT}{dx} + 1989 \cdot \frac{\pi a^2 e^{2\times1.8x/L}}{4} \cdot dx$$

$$= \cancel{q_x} + \frac{dq_x}{dx} \cdot dx$$

$$\Rightarrow \frac{dq_x}{dx} = 1989 \frac{\pi a^2}{4} e^{2\times1.8x/L}$$

$$q_x = 1989 \frac{\pi a^2}{4} \times \frac{L}{3.6} e^{3.6x/L} + c_1$$

$c_1 \approx 0$ {In comparison to the other terms}

$$\therefore q_x = 1989 \frac{\pi a^2}{\ } \frac{L}{\ } e^{3.6x/L}$$

$$q_x = -KA\frac{dT}{dx}$$

$$\dot{q}\,\frac{\pi a^2}{4}\,\frac{L}{3.6}\,e^{3.6x/L} = -K\,\frac{\pi a^2}{4}\,e^{3.6x/L}\,\frac{dT}{dx}\cdot\frac{L}{3.6}$$

$$\Rightarrow -\frac{dT}{dx} = \frac{\dot{q}}{2K}$$

$$T = -\frac{\dot{q}}{2K}x + C_1$$

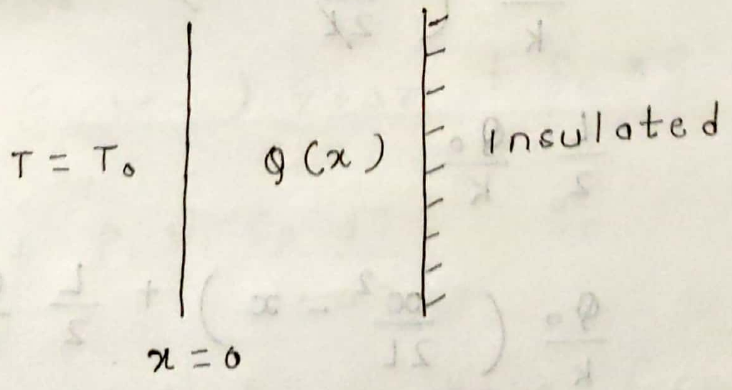
$$T|_0 = 300 = C_1$$

$$T(x) = -\frac{\dot{q}}{2K}x + 300$$

$$T|_{x=L} = 76.2\,^\circ C$$

$$q|_{x=L} = 18295\ W$$

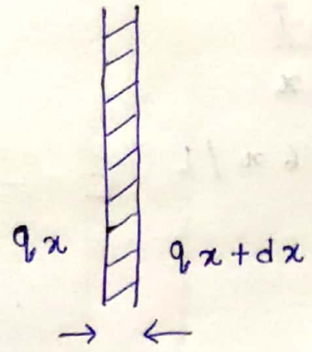
Q. $q(x) = q_0\left(1 - \frac{x}{L}\right)$ {heat generation}



a) Derive the DE for temp. profile.

b) BC ?

c) Solve DE to find temp. profile.



In + generation = out

$$q_x + \dot{q}\, A\, dx = q_{x+dx}$$

$$\cancel{q_x} + \dot{q}\, A\, dx = \cancel{q_x} + \frac{dq_x}{dx}\, dx$$

$$\dot{q}\, A\, \cancel{dx} = \frac{dq_x}{dx}\, \cancel{dx} \Rightarrow \frac{dq_x}{dx} = \dot{q}\, A$$

$$\frac{d}{dx}\left(-k \cancel{A} \frac{dT}{dx}\right) = q_0 \left(1 - \frac{x}{L}\right) \cancel{A}$$

$$\frac{d^2 T}{dx^2} = \frac{q_0}{k}\left(\frac{x}{L} - 1\right)$$

$$\frac{dT}{dx} = \frac{q_0}{k}\left(\frac{x^2}{2L} - x\right) + c_1$$

$$0 = \frac{q_0}{k}\left(\frac{L^{\cancel{2}}}{2\cancel{L}} - L\right) + c_1$$

$$c_1 = \frac{L}{2} \frac{q_0}{k}$$

$$\frac{dT}{dx} = \frac{q_0}{k}\left(\frac{x^2}{2L} - x\right) + \frac{L}{2}\frac{q_0}{k}$$

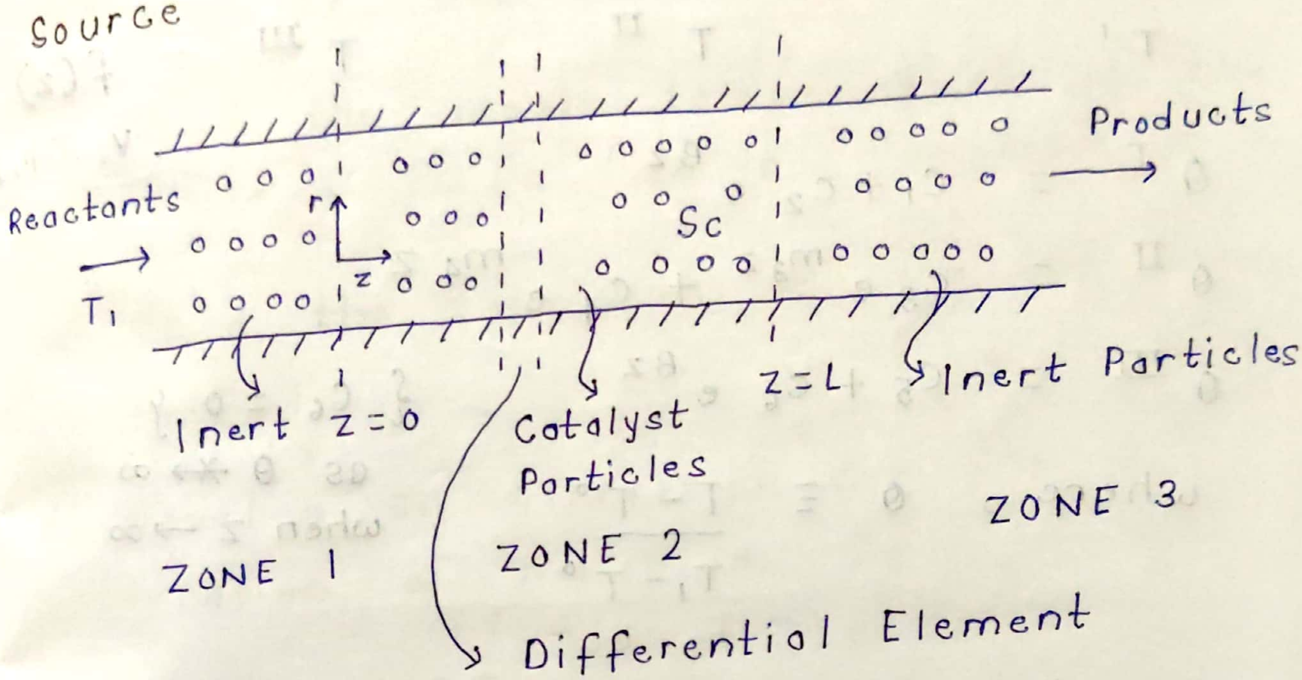
$$T(x) = \frac{q_0}{k}\left(\frac{x^3}{6L} - \frac{x^2}{2}\right) + \frac{L}{2}\frac{q_0}{k} x + c_2$$

$x = 0$, $T = T_0$, $c_2 = T_0$

$$T(x) = \frac{\rho_0}{K}\left(\frac{x^3}{6L} - \frac{x^2}{2}\right) + \frac{L}{2}\frac{\rho_0}{K}x + T_0$$

$$T(x) - T_0 = \frac{\rho_0}{2K}L^2\left(\frac{x}{L} - \frac{x^2}{L^2} + \frac{1}{3}\frac{x^3}{L^3}\right)$$

Heat Conduction with Chemical Heat Source



Volumetric Heat Generation = S_c

In + generation − out = 0

$$\pi R^2 q_z\big|_z + \pi R^2 \rho_1 v_1 C_p (T - T_0)\big|_z - \pi R^2 q_z\big|_{z+\Delta z}$$

$$- (\quad\checkmark\quad)_{z+\Delta z} + S_c \pi R^2 \Delta z = 0$$

$$\frac{dq_z}{dz} + \rho_1 v_1 C_p \frac{dT}{dz} = S_c$$

Assuming that we have finely grained catalyst particles.

For zone 1 and zone 3, $S_c = 0$

$$-k_z \frac{d^2T}{dz^2} + \rho_1 v_1 C_p \frac{dT}{dz} = S_c$$

ZONE -

I $\qquad$ II $\qquad$ III

B.C.S -

T^{I} $\qquad$ T^{II} $\qquad$ $T^{III} = f(z, v_1, C...)$

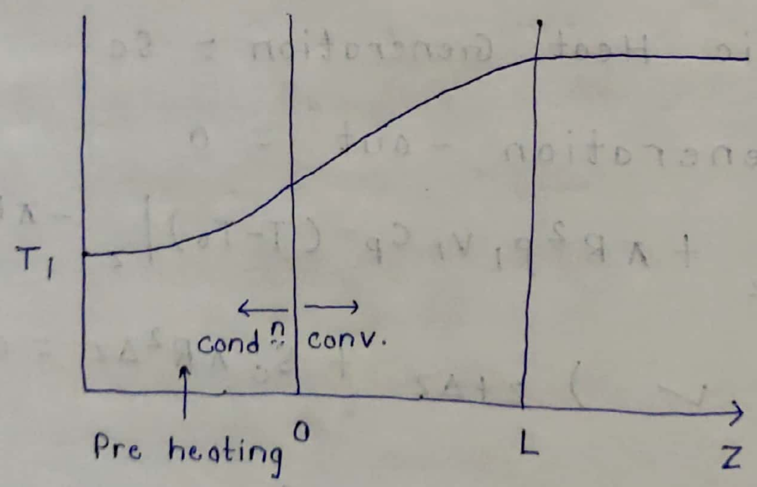
$$\theta^{I} = C_1 + C_2 e^{Bz}$$

$$\theta^{II} = C_3 e^{m_3 z} + C_4 e^{m_4 z}$$

$$\theta^{III} = C_5 + C_6 e^{Bz}$$

$\{ C_6 = 0 \}$ as $\theta \not\to \infty$ when $z \to \infty$

where, $\theta \equiv \frac{T - T^{\circ}}{T_1 - T^{\circ}}$



Validity of LC Method :-

Biot number should be small.

Alternatives $\to$ Analytical, Numerical, Graphical

Q. Length = L , Diameter = D

$q(x) = q_0 \sin(x/L)$, q_0 (W/m³)

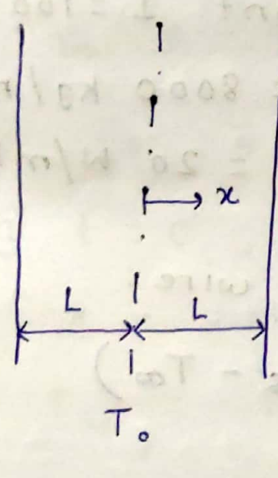
a) Obtain an expression for the local heat flux, $q''(x)$ and the total heat transfer from the fuel rod to water

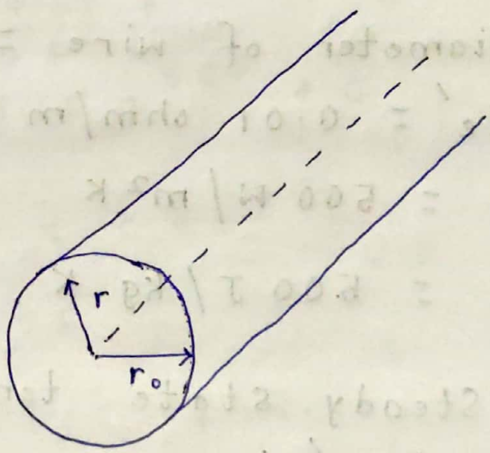
b) Find the mean temperature of the water, $T_m(x)$ as a function of x.

c) Find the variation of the rod temp. $T_s(x)$.

d) Develop an expression for the x location of maximum T_s.

HEISLER CHART



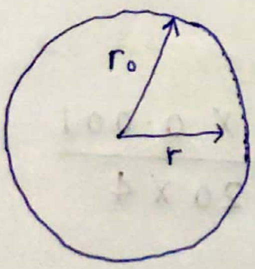


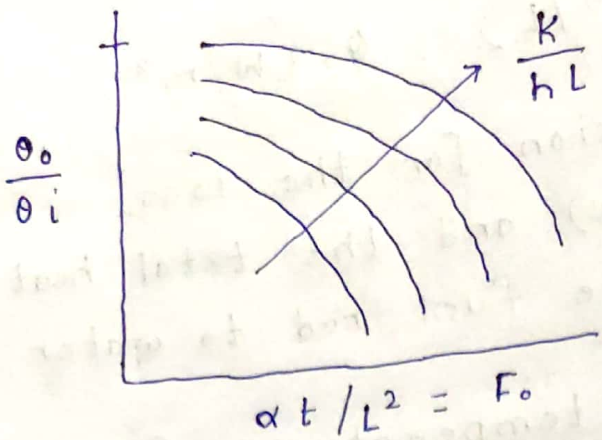
$\theta = T(x,t) - T_\infty$

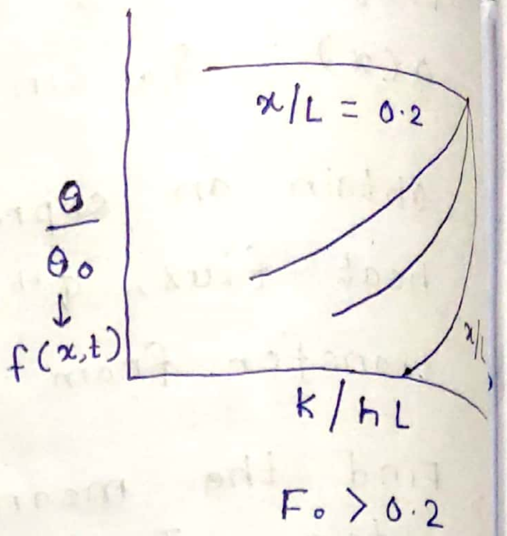
$\theta_i = T_i - T_\infty$

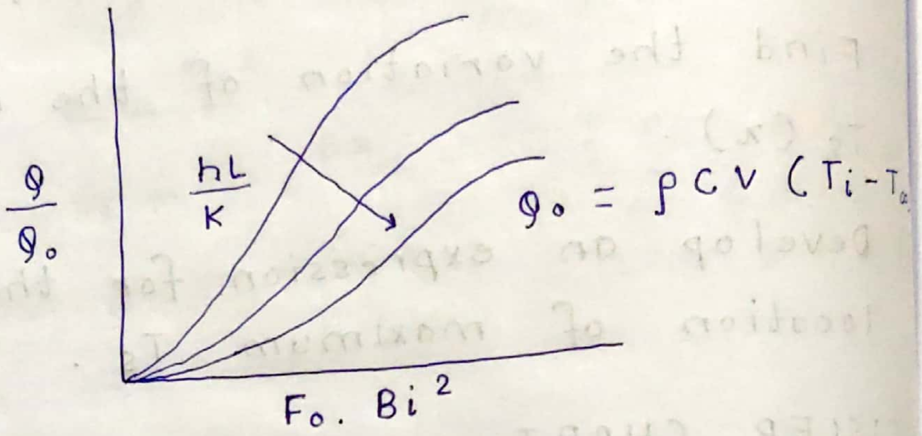
$T(r,t) - T_\infty$

$\theta_0 = T_0 - T_\infty$









Q. Diameter of wire $= 1\,mm$ $T_\infty = 25\,°C$

Current $I = 100\,A$

$R_e' = 0.01\ ohm/m$.

$\rho_{wire} = 8000\ kg/m^3$

$h = 500\ W/m^2K$.

$C = 500\ J/kg.K$ and $k = 20\ W/mK$.

i) Steady state temp. of wire :

$$I^2 R_e' \cancel{L} = h\pi D\cancel{L}\,(T_s - T_\infty)$$

$$T_s = T_\infty + \frac{I^2 R_e'}{h\pi D}$$

$$= 88.66\,°C$$

$$Bi = \frac{hD}{k \times 4} = \frac{500 \times 0.001}{20 \times 4} = \frac{0.0}{4}$$

$$= 6.25 \times 10^{-3}$$

$\therefore$ T is not a f^n of position

$$q_{in} - q_{out} + q_{gen} = q_{accu}$$

$q_{in} \to 0$

$$-h\pi D\cancel{L}(T_w - T_\infty) + I^2 R_e' \cancel{L} = \rho C_p \pi R^2 \cancel{L} \frac{\partial T_w}{\partial t}$$

ii) time required for the T to reach within 1°C of steady state temp.

$$\frac{dT_w}{dt} + \frac{h\cancel{\pi}\cancel{D} \times 4}{\rho C_p \cancel{\pi} D^{\cancel{2}}}(T_w - T_\infty) = \frac{I^2 R_e' \times 4}{\rho C_p \pi D^2}$$

$$\frac{dT_w}{dt} + \frac{4h}{\rho C_p D} T_w = \frac{4 I^2 R_e'}{\rho C_p \pi D^2} + \frac{4h T_\infty}{\rho C_p D}$$

$$\frac{dT_w}{dt} + 0.5\, T_w = 44.33$$

$$\Rightarrow T_w \cdot e^{0.5t} = \frac{44.33}{0.5} e^{0.5t} + C$$

$$T_w \cdot e^{0.5t} = 88.66\, e^{0.5t} + C$$

i.e., $T_w = 88.66 + c\, e^{-0.5t}$

@ $t = 0$, $T_w = 25$

$\therefore$ $c = 25 - 88.66 = 63.66$

$\therefore$ $T_w = 88.66 - 63.66\, e^{-0.5t}$

Given $T_w = 87.66$

$\Rightarrow$ $87.66 = 88.66 - 63.66\, e^{-0.5t}$

$\Rightarrow$ $1/63.66 = e^{-0.5t}$

which gives ; $t = 8.307$ s

Q. Small spheres (r_0 = 1.5 cm) of Coloured liquid has a density
$\rho_\infty = 900\ kg/m^3$ and $T_\infty = 325\ K$

$\rho_{sphere} = \rho_0 - \beta (T - T_\infty)$ $\rho_0 = 910$

$\beta = 2\ kg/m^3 K$

If the polymer liquid begins its ascent at a temp. of 350 K, how long before it begins to fall back to be reheated ?

$h = 20\ W/m^2 K$, C_p (sphere) = 400 J/

k (sphere) = 10 W/m K

Volume and area are constant.

Force of gravity = Force of buoyancy

$$\cancel{V} (\rho_0 - \beta (T - T_\infty)) \cancel{g} = \cancel{V} \times 900 \times \cancel{g}$$

$$910 - 2 (T - T_\infty) = 900$$

$$5 = T - T_\infty$$

$$T = T_\infty + 5 = 325 + 5 = 330$$

$$-h \cancel{\pi} \cancel{D^2} (T - T_\infty) = \rho\, C_p \frac{\cancel{4}}{3} \cancel{\pi} \frac{D^{\cancel{3}}}{\cancel{8}2} \frac{\partial T}{\partial t}$$

$$-h (T - T_\infty) = (\rho_0 - \beta (T - T_\infty)) C_p \frac{\partial T}{\partial t}$$

Incorrect

$$-\frac{6h}{D C_p} (T - T_\infty) = (\rho_0 - \beta (T - T_\infty)) \frac{\partial T}{\partial t}$$

$$\int_{T_0}^{T} dT \left(\frac{\rho_0 - \beta (T - T_\infty)}{T - T_\infty} \right) = \int_0^t - \frac{6h}{D C_p} dt$$

$$\int_{T_0}^{T} \left(\frac{\rho_0}{T - T_\infty} - \beta \right) dT = - \frac{6ht}{D C_p}$$

Incorrect

$$\ln (T - T_\infty) \rho_0 \Big|_{T_0}^{T} - \beta (T - T_0) = - \frac{6ht}{D C_p}$$

$$\ln \left(\frac{T - T_\infty}{-T_0 + T_\infty} \right) + \beta (T_0 - T) = - \frac{6ht}{D C_p}$$

$$\ln \left(\frac{330 - 325}{350 - 325} \right) + 2 (350 - 330) = - \frac{6 \times 20 t}{0.03 \times 400}$$

Governing equation -

$$-h (4\pi R^2)(T - T_\infty) = C_p V \frac{d}{dt} \left[T \left(\rho_0 - \beta (T - T_\infty) \right) \right]$$

$$-h (4\pi R^2)(T - T_\infty) = C_p V \frac{dT}{dt} \{ \rho_0 - 2T\beta + \beta T_\infty \}$$

$$-h (\cancel{4}\cancel{\pi}\cancel{R^2})(T - T_\infty) = C_p \cdot \frac{\cancel{4}}{3} \cancel{\pi} R^{\cancel{3}} \frac{dT}{dt} \{ \;,, \; \}$$

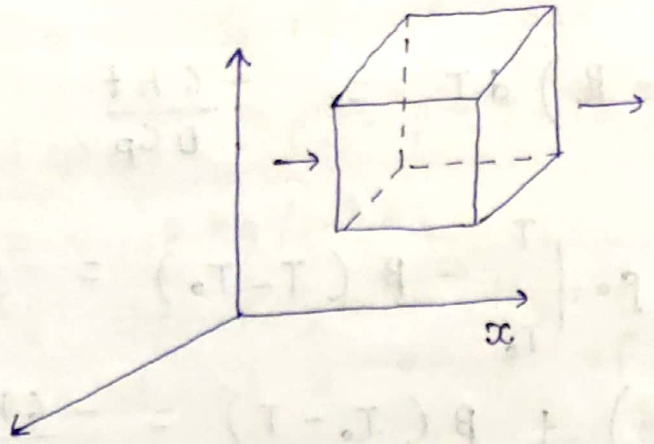
$$-20 (T - 325) = \frac{dT}{dt} (1560 - 4T)$$

$$\Rightarrow \frac{dT}{dt} = \frac{10 (325 - T)}{(1560 - 4T)}$$

$$\int_{350}^{330} \left(\frac{1560 - 4T}{325 - T} \right) dT = 10 \int_0^t dt$$

$$\Rightarrow t = 498 s \approx 500 s$$

Equation of Change for a non-isothermal System –



Conservation of energy –

Rate of accu. of IE and K.E. =

$$\left\{ \begin{matrix} \text{K.E. and} \\ \text{I.E. in} \end{matrix} \right\} - \left\{ \begin{matrix} \text{K.E. and} \\ \text{I.E. out} \end{matrix} \right\} + \begin{matrix} \text{viscous} \\ \text{Dissipation} \end{matrix}$$

$$\text{Accumulation} = \Delta x \Delta y \Delta z \frac{\partial}{\partial t} \left(\rho U + \frac{1}{2} \rho v^2 \right)$$

U = IE / mass of fluid

v = mag. of local fluid velocity

Rate of convection of IE and KE into the element :

x face :

$$\Delta y \Delta z \left\{ v_x \left(\rho U + \frac{1}{2} \rho v^2 \right) \Big|_x - v_x \left(\rho U + \frac{1}{2} \rho v^2 \right) \Big|_{x+\Delta x} \right.$$

Similary for y and z face.

– Conduction :

Net Rate of energy in by conduction

$$\Delta y \Delta z \left\{ q_x|_x - q_x|_{x+\Delta x} \right\} + \Delta x \Delta z \left\{ q_y|_y - q_y|_{y+\Delta y} \right\} + \Delta x \Delta y \left\{ q_z|_z - q_z|_{z+\Delta z} \right\}$$

Work done $\begin{cases} \rightarrow \text{Against Volumetric force} \\ \rightarrow \text{Against surface forces (viscous, pressure)} \end{cases}$

Work done by gravitational force :-

$$-\rho\,(\Delta x\, \Delta y\, \Delta z)\,(v_x g_x + v_y g_y + v_z g_z)$$

Work done against pressure :-

$$\Delta y \Delta z \left\{ P|_x - P|_{x+\Delta x} \right\} + \Delta x \Delta z \left\{ P|_y - P|_{y+\Delta y} \right\}$$

$$+ \Delta x \Delta y \left\{ P|_z - P|_{z+\Delta z} \right\}$$

Viscous Forces :-

$\tau_{xx} . v_x$	$\tau_{yx} . v_x$	$\tau_{zx} . v_x$
$\tau_{xy} . v_y$	$\tau_{yy} . v_y$	$\tau_{zy} . v_y$
$\tau_{xz} . v_z$	$\tau_{yz} . v_z$	$\tau_{zz} . v_z$
↓ Acting on area $\Delta y \Delta z$	↑ $\Delta x \Delta z$	↑ $\Delta x \Delta y$

Net Rate :-

$$\Delta y\, \Delta z \left\{ (\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz})|_{x+\Delta x} - (\tau_{xx} v_x + \ldots\ldots)|_x \right. + 2y + 2z$$

$$\Rightarrow \rho C \frac{DT}{dt} = \underset{\substack{\uparrow \\ \text{Heat} \\ \text{Conduction}}}{k \nabla^2 T} - \underset{\substack{\uparrow \\ \text{Expansion} \\ \text{Effects}}}{T \left(\frac{\partial P}{\partial T} \right) \nabla v} + \underset{\substack{\uparrow \\ \text{Viscous} \\ \text{Dissipation}}}{\mu \phi v}$$

where $\mu \phi v = 2\mu \left\{ \left(\frac{\partial v_x}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 \right\}$

$$+ \mu \left\{ \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2 + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)^2 + \right.$$

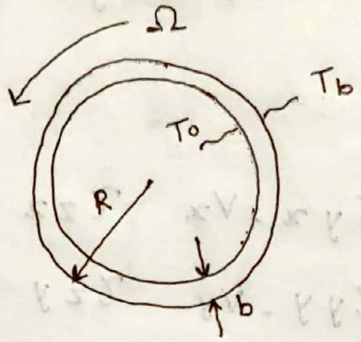
$$\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2 \Bigg]$$

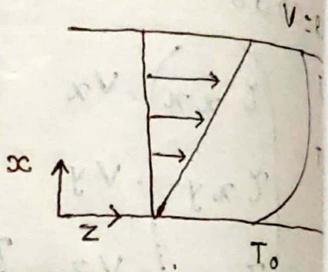
$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}$$

Energy Equation (in all coordinate system)

$$\underbrace{\rho C_p\left(\frac{DT}{Dt}\right)}_{\text{Convective}} = \underbrace{k\nabla^2 T}_{\text{Conductive}} + \underbrace{\mu\phi_v}_{\text{Viscous Dissipation}} + \underbrace{\dot{Q}}_{\text{Heat generat}}$$

Q





Thin gap : parallel plate approximation
no pressure gradient.

$$V_z = R\Omega\frac{x}{b}$$

Energy Eq^n :-

$$\rho C_p\left(\underset{0}{\cancel{\frac{\partial T}{\partial t}}} + \underset{0}{\cancel{v_x}}\frac{\partial T}{\partial x} + \underset{0}{\cancel{v_y}}\frac{\partial T}{\partial y} + v_z\underset{0}{\cancel{\frac{\partial T}{\partial z}}}\right) =$$

$$k\left(\frac{\partial^2 T}{\partial x^2} + \underset{0}{\cancel{\frac{\partial^2 T}{\partial y^2}}} + \underset{0}{\cancel{\frac{\partial^2 T}{\partial z^2}}}\right) + 2\mu$$

$$\left\{\left(\underset{0}{\cancel{\frac{\partial v_x}{\partial x}}}\right)^2 + \left(\underset{0}{\cancel{\frac{\partial v_y}{\partial y}}}\right)^2 + \left(\underset{0}{\cancel{\frac{\partial v_z}{\partial z}}}\right)^2\right\} + \mu$$

$$\left\{\left(\underset{0}{\cancel{\frac{\partial v_x}{\partial y}}} + \underset{0}{\cancel{\frac{\partial v_y}{\partial x}}}\right)^2 + \left(\underset{0}{\cancel{\frac{\partial v_x}{\partial z}}} + \frac{\partial v_z}{\partial x}\right)^2 +\right.$$

$$\left.\left(\underset{0}{\cancel{\frac{\partial v_y}{\partial z}}} + \underset{0}{\cancel{\frac{\partial v_z}{\partial y}}}\right)^2\right\} + \underset{0}{\cancel{\dot{Q}}} = 0$$

$$\Rightarrow k \frac{\partial^2 T}{\partial x^2} + \mu \left(\frac{d v_z}{dx} \right)^2 = 0$$

$$v_z = R\Omega \frac{x}{b} \Rightarrow \frac{dv_z}{dx} = \frac{R\Omega}{b}$$

$$\Rightarrow K \frac{d^2 T}{dx^2} + \mu \frac{R^2\Omega^2}{b^2} = 0$$

$$\frac{d^2 T}{dx^2} = - \frac{\mu}{K} \frac{R^2 \Omega^2}{b^2}$$

$$\frac{dT}{dx} = - \frac{\mu}{K} \frac{R^2\Omega^2}{b^2} x + c_1$$

$$T(x) = - \frac{\mu}{K} \frac{R^2\Omega^2}{2b^2} x^2 + c_1 x + c_2$$

@ $x = 0$ $T = T_o \Rightarrow c_2 = T_o$

@ $x = b$ $T = T_b$

$$T_b = - \frac{\mu}{K} \frac{R^2\Omega^2}{2\cancel{b^2}} \times \cancel{b^2} + c_1 b + T_o$$

$$\Rightarrow c_1 = \frac{T_b}{b} + \frac{\mu}{K} \frac{R^2\Omega^2}{2b} - \frac{T_o}{b}$$

$$T(x) = - \frac{\mu}{K} \frac{R^2\Omega^2}{2b^2} x^2 + \left(\frac{T_b - T_o}{b} \right) x$$

$$+ \frac{\mu}{K} \frac{R^2\Omega^2}{2b} x + T_o$$

$$\frac{T - T_o}{T_b - T_o} = \left(\frac{x}{b} \right) + \frac{1}{2} \frac{\mu R^2 \Omega^2}{k (T_b - T_o)} \left[\left(\frac{x}{b} \right) - \left(\frac{x}{b} \right)^2 \right]$$

$\frac{\mu v^2}{k(T_b - T_0)}$ gives $\frac{\text{Viscous}}{\text{Conductive HT}}$

↳ Brinkmann Number

$$\frac{1}{T_b - T_0} \frac{dT}{dx} = \frac{1}{b} + \frac{1}{2} \frac{\mu v^2}{k(T_b - T_0)} \left[\frac{1}{b} - 2\frac{x}{b} \right]$$

For maxima,

$$0 = \frac{1}{b} + \frac{\alpha}{b} - \frac{2\alpha x}{b^2}$$

$$2\alpha \frac{x}{b^{\cancel{2}}} = \frac{(\alpha+1)}{\cancel{b}}$$

$$x = \left(\frac{\alpha+1}{2\alpha}\right) b$$

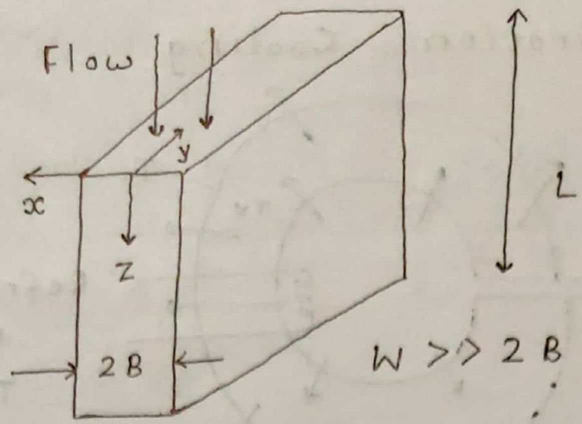
$$0 < \frac{\alpha+1}{2\alpha} < 1 \Rightarrow 0 < \alpha+1 < 2\alpha$$

$1 < \alpha$ where α is $\frac{\text{Brinkm}}{2}$

$$\frac{Br}{2} > 1 \Rightarrow Br > 2$$

$\therefore$ Brinkmann Number > 2

Q.



Find $T = f(x)$. Fully developed flow

Energy Eq^n gives :-

$$0 = k\frac{d^2T}{dx^2} + \mu\left(\frac{\partial v_z}{\partial x}\right)^2 \quad - \text{①}$$

$T = T_0$ at $x = \pm B$

$v_z = f(x) \longrightarrow$ N.S.

$$v_z = \frac{(P_0 - P_L)B^2}{2\mu L}\left[1 - \left(\frac{x}{B}\right)^2\right]$$

$$\frac{\partial v_z}{\partial x} = \frac{(P_0 - P_L)\cancel{B^2}}{\cancel{2}\mu L} \times \left(-\frac{\cancel{2}x}{\cancel{B^2}}\right)$$

$$\frac{\partial v_z}{\partial x} = -x\left(\frac{P_0 - P_L}{\mu L}\right) \quad - \text{②}$$

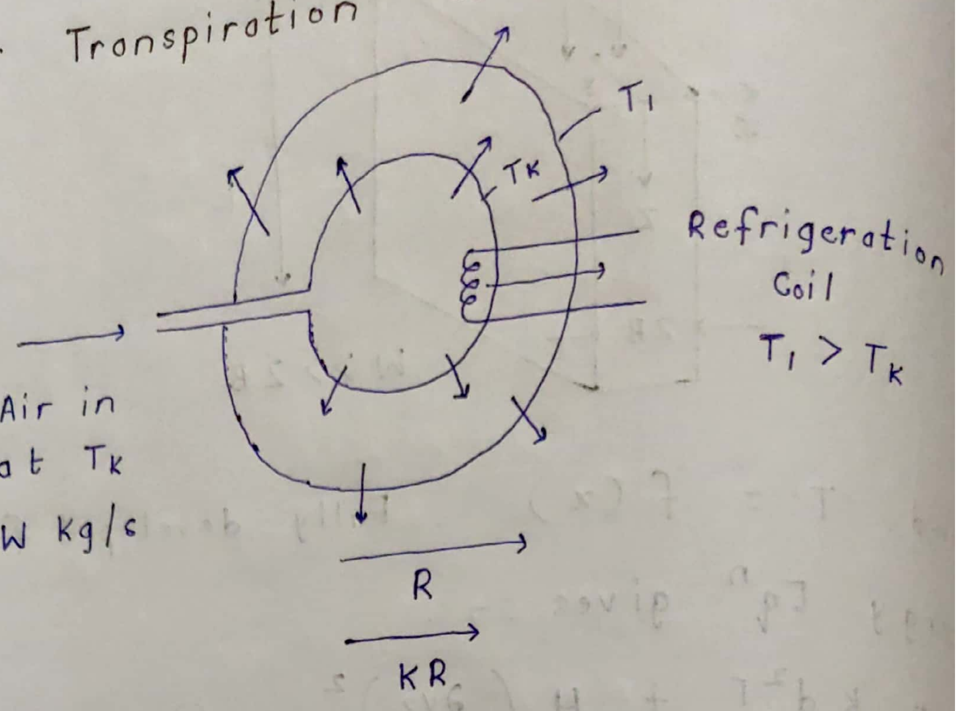
Substituting ② in ① gives.

$$\frac{d^2T}{dx^2} = -\frac{\mu}{k}\left(\frac{P_0 - P_L}{\mu L}\right)^2 x^2$$

which on solving and putting the B.C.s give

$$T(x) = T_0 + \frac{\mu}{k}\left(\frac{P_0 - P_L}{\mu L}\right)^2 \cdot \frac{B^4}{12}\left(1 - \frac{x^4}{B^4}\right)$$

Q. Transpiration Cooling



From continuity ;

$$4\pi r^2 v_r \rho = \text{constant} = W$$

$$\Rightarrow v_r = \frac{W}{4\pi r^2 \rho} \qquad \{ r\uparrow \quad v_r \downarrow \}$$

Energy eq^n in spherical coordinates get reduced to

$$\rho C_p v_r \frac{\partial T}{\partial r} = \frac{k}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

- No viscous dissipation as flow is slow
- No heat generation.

Substituting v_r —

$$\cancel{\rho}\, C_p \frac{W}{4\pi \cancel{r^2} \cancel{\rho}} \frac{dT}{dr} = \frac{k}{\cancel{r^2}} \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$$

$$\frac{dT}{dr} = \frac{4\pi k}{W C_p} \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$$

$$\frac{dT}{dr} = \frac{4\pi k}{W C_p} \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$$

$$\frac{T - T_1}{T_K - T_1} = \frac{e^{-R_o/r} - e^{-R_o/R}}{e^{-R_o/KR} - e^{-R_o/R}}$$ { with transpira^n.

where, $R_o = \frac{W C_p}{4\pi k}$

For small R_o.

$$\Rightarrow \frac{T - T_1}{T_K - T_1} = \frac{\frac{1}{r} - \frac{1}{R}}{\frac{1}{KR} - \frac{1}{R}}$$ → no transpiration

For very small value of W and R very large, Nu = 2 can be obtained from the above temp. profile.

For stagnant fluid, equality of conduction and convective flux holds.

$$-4\pi (KR)^2 \left.\frac{dT}{dr}\right|_{r=R} k = h\,(4\pi K^2R^2)(T_K - T_1)$$

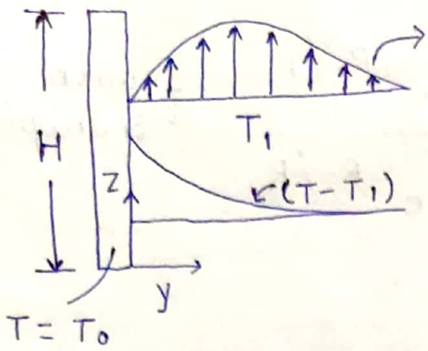
Nu = 2 gives the minimum possible value of h possible for a sphere.

$$Q = -4\pi (K^2R^2)\, q''_r \big|_{r=KR}$$

$$= 4\pi k\,(K^2R^2) \left.\frac{dT}{dr}\right|_{r=KR}$$

Efficiency $(\epsilon) = \frac{-Q_o + Q}{Q_o}$ { Q_o = w/o transpiration

Free Convection from a Vertical Plate :



$H >> \delta_t$

@ $y = 0$ $\quad v_z = 0$

as $y \to \infty$ $\quad v_z = 0$

$v_y = 0$

$z \to -\infty$ $\quad v_z = 0$

Continuity gives ;

$$\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

z Component of NS eq^n :-

$$\rho \left(v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = \mu \left(\frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g \beta (T-$$

$$\beta \equiv \frac{1}{V} \frac{\Delta V}{\Delta T} \Rightarrow \Delta V = \beta V \Delta T$$

Buoyant force $= \rho g \Delta V = \rho g \beta V \Delta T$

Energy Equation :-

$$\rho C_p \left(v_y \frac{\partial}{\partial y}(T-T_1) + v_z \frac{\partial}{\partial z}(T-T_1) \right)$$

$$= k \left(\frac{\partial^2}{\partial y^2}(T-T_1) + \frac{\partial^2}{\partial z^2}(T-T_1) \right)$$

$v_z >> v_y$ $\quad \to$ No heat generation

$\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial z}$ $\quad \to$ No viscous loss (slow flow)

↳ Because length scale in y is δ_t (thermal boundary layer) which is much greater than h.

z dir. : Convection prevalent
y dir. : Conduction ,,

Non - dimensionalising the governing eq^n:

$$\theta = \frac{T - T_1}{T_0 - T_1} \quad , \text{temp}$$

$$\phi_z = \left(\frac{\mu}{B \alpha H} \right)^{1/2} v_z \longrightarrow \text{Vertical velo.}$$

$$\phi_y = \left(\frac{\mu H}{\alpha^3 B} \right)^{1/4} v_y \longrightarrow \text{Horizontal velo.}$$

$$\alpha = \frac{k}{\rho C_p} \quad ; \quad B = \rho g \beta (T_0 - T_1)$$

$$\xi = z/H \quad , \quad \eta = \left(\frac{\beta}{\alpha \mu H} \right)^{1/4} y$$

$$\frac{\partial \phi_y}{\partial \eta} + \frac{\partial \phi_z}{\partial \xi} = 0 \quad \{ \text{continuity eq}^n \}$$

$$\frac{1}{Pr} \left(\phi_y \frac{\partial \phi_z}{\partial \eta} + \phi_z \frac{\partial \phi_z}{\partial \xi} \right) = \frac{\partial^2 \phi_z}{\partial \eta^2} + \theta$$

(Momentum eq^n)

$$\phi_y \frac{\partial \theta}{\partial \eta} + \phi_z \frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \eta^2} \quad (\text{Energy eq}^n)$$

For very small values of velocity L.H.S. of eq^n 2 can be neglected.

From engineering point of view, heat lost by plate is important.

$$\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0} \rightarrow \text{important}$$

is a f^n of z.

$$q''_{avg.} = \frac{1}{H}\int_0^H -k \left.\frac{\partial T}{\partial y}\right|_{y=0} dz = q''_0$$

$$q''_{avg.} = k\,(T_0 - T_1)\left(\frac{B}{\mu \alpha H}\right)^{1/4} \int_0^1 -\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0}$$

$$\frac{B}{\mu \alpha H} = \frac{\rho \beta g\,(T_0 - T_1)\,\rho C_p}{\mu k H}$$

$$= \frac{\rho^2 \beta g H^3 \Delta T}{\mu^2} \frac{C_p \mu}{k} \cdot \frac{1}{H^4}$$

$$= Gr\ Pr\ \frac{1}{H^4}$$

$$\therefore \left(\frac{B}{\mu \alpha H}\right)^{1/4} = \frac{1}{H}\,(Gr\ Pr)^{1/4}$$

$$\Rightarrow \theta = f\,(\eta, \xi, Pr)$$

$$\Rightarrow \left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0} = f\,(\xi, Pr)$$

$$\Rightarrow \int_0^1 \left.\left(\frac{\partial \theta}{\partial \eta}\right)\right|_{\eta=0} d\xi = f\,(Pr) = c \text{ (Let)}$$

This process is characterised by weak dependence on Pr.

Experimentally it is found

Pr	10	10^2	10^3	0.7
C	0.612	0.652	0.653	0.52

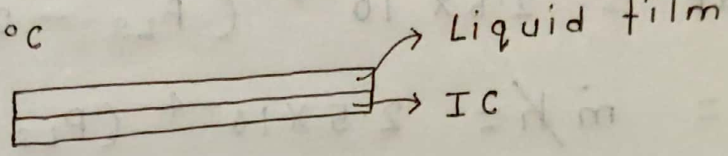
$$q''_{avg} = k(\Delta T)\frac{1}{H}(Gr.Pr)^{1/4} \underset{C}{\overset{\|}{f(Pr)}}$$

$$\simeq k(\Delta T)\frac{1}{H}(Gr.Pr)^{1/4}(0.6\ldots)$$

Q.

Atmosphere

$T_\infty = 35°C$



Relative humidity = 50%

$T_\infty = 35°C$

$k_c = 2.5 \times 10^{-4}$ mol / Pa.s.m^2

q'' from the chip = 10 W/cm^2

a) If the heat of vaporisation of the liquid is 35 kJ/mol, what is the flux of vapor from the surface of the liquid at steady state?

$\dot{m} \times 35 \times 10^3 = 10 \times 10^4$

$\dot{m} = 2.857$ mol/m^{2}s

b) The vapor pressure of the liquid can be represented by the following eq^n:-

$$\log_{10} P^{sat}(\text{mm Hg}) = 7.96681 - \frac{1668.21}{T + 228.0}$$

What is the temperature of the

surface of the liquid ?

$$R.H. = \frac{p_A}{p_{sat}} \times 100 = 0.5$$

$T = 35°C$

Using Antoinne's Equation,

$p_{sat} = 42.05$ mm Hg

$p_A = 21.025$ mm Hg $= \overset{2761}{\cancel{2803}}$ (Pa)

$\dot{m} = k_c \Delta P = 2.5 \times 10^{-4} (P_{LS} - 2761)$

$q'' = \dot{m} \cancel{h} = 2.5 \times 10^{-4} (P_{LS} - 2761) \cancel{h}$

$2.857 = 2.5 \times 10^{-4} (P_{LS} - 2761)$

$P_{LS} = 1.42 \times 10^{4}$ Pa

Liquid surface will be at saturation,

$$\therefore \log_{10} \left(\frac{1.42 \times 10^4}{1.013 \times 10^5 / 760} \right) = 7.96681 - \frac{1668.21}{T + 228}$$

$\therefore T = 53°C$

c) What is the temperature at the surface of the chip assuming a liquid film thickness of 100 μm and a 'k' for the liquid of 0.5 W/mK ?

$$-k \frac{\Delta T}{L} = q''$$

$$-0.5 \frac{(-T_s + 53)}{100 \times 10^{-6}} = 10 \times 10^4$$

$\Rightarrow \quad T_s - 53 = \frac{10}{0.5}$

$\Rightarrow \quad T_s = 73°C$

d) if heat flux is 10% more than present value, find T_s.

$$\frac{-0.5\,(53 - T_s)}{100 \times 10^{-6}} = 11 \times 10^4$$

$\Rightarrow \quad T_s - 53 = 22$

$\Rightarrow \quad T_s = 75°C$.