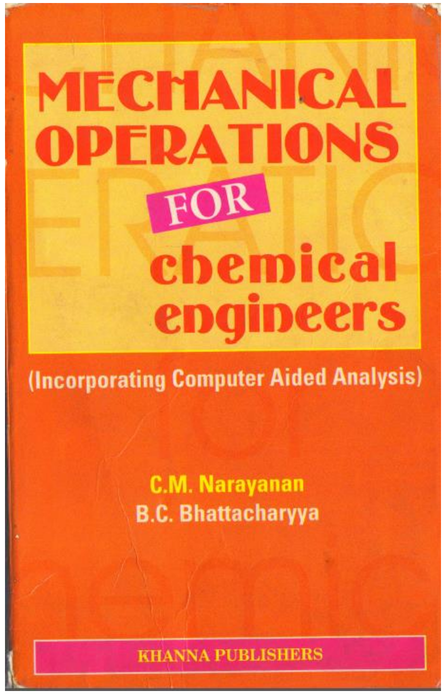
Course Distribution

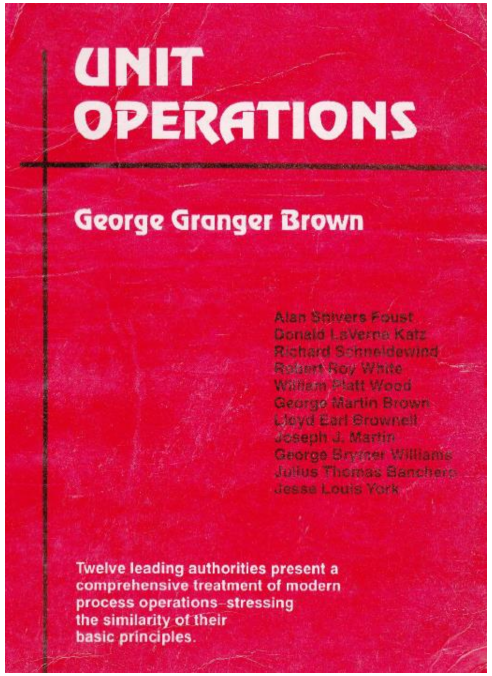
- Particle characterization (defining a particle/distribution)
  - Particle size and shape
  - Particle size investigation
    - Size measurement of fine particles
      - Defining a fine particles
      - Measurement
    - Sieving
  - Industrial screening equipment
- Size Reduction
  - Primary/Coarse size reduction
  - Intermediate size reduction
  - Fine size reduction
- Separation
  - Classification
  - Solid-Liquid separation
    - Dynamics of particle in liquid medium
    - Sedimentation
    - Flotation
  - Solid-Gas separation
    - Dynamics of particle in gas medium
    - Miscellaneous methods
    - Fluidization
- Storage, Handling and Transport of Solids
  - Storage of solids
  - Transport of solids

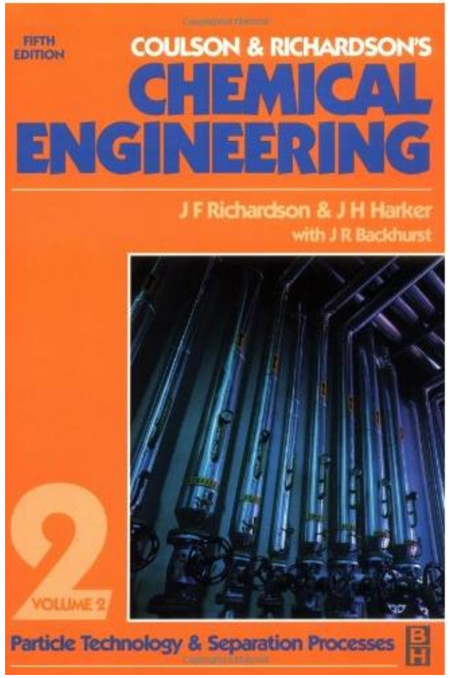
PARTICLE CHARACTERIZATION

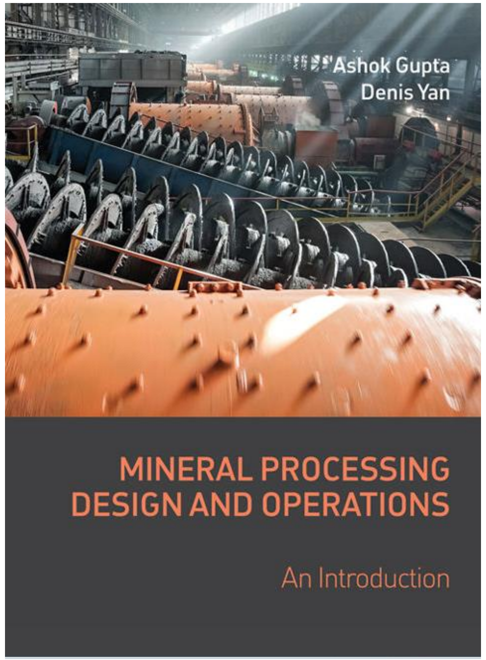
Resource

- **Books**









- **Relevant journal papers mentioned in the individual topics**

A liquid is a hen-pecked husband whereas a solid particle is a male chauvinist……

Particle Properties

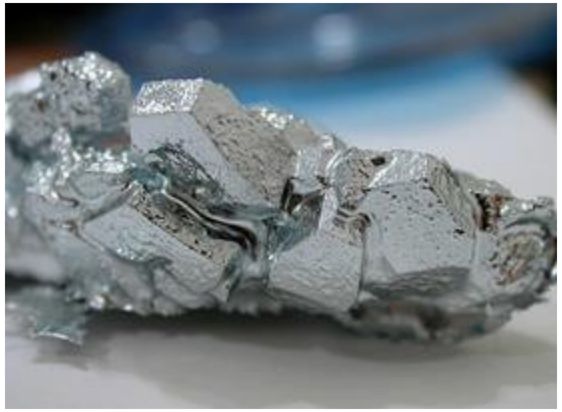
1. Physical

- Hardness, Softness
- Tenacity, Brittleness, Friability
- Structure, Fracture
- Friction
- Aggregation, Stickiness
- Colour, lustre (shine/gloss)
- Electro-conductivity
- Magnetic susceptibility

2. Physio-Chemical

- Surface Properties- Adhesion, Contact angle
- Change in magnetic properties by heat
- Change in properties on adding depressants, activators

Aggregated particles



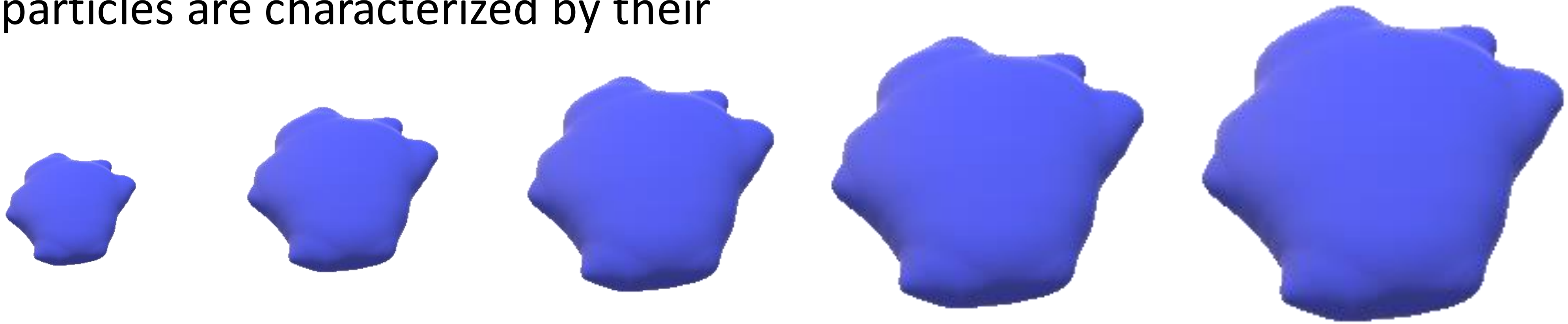
Soft mineral - Gallium

Different colours of minerals

Particle Properties

Individual solid particles are characterized by their

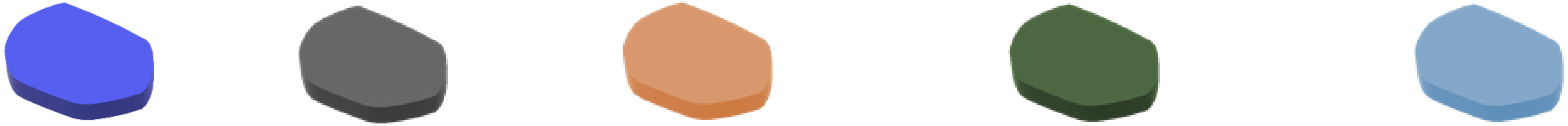
(1) Size



(2) Shape

Angular **Spongy** **Acicular** **Flakey** **Irregular**

(3) Density



Particle Properties

Individual solid particles are characterized by their

(1) Size

(2) Shape

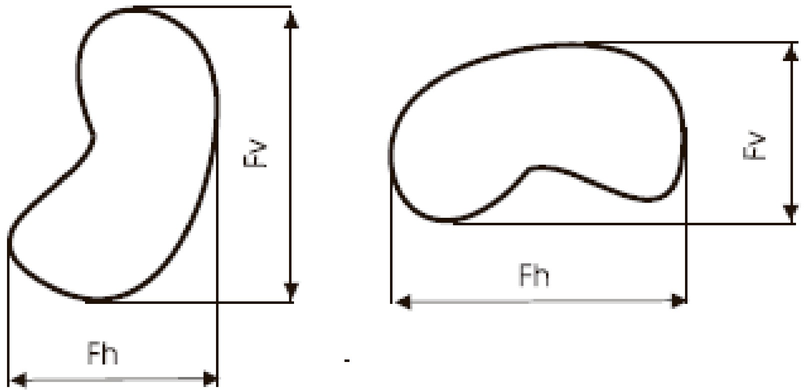
(3) Density

Note:

- Particles of homogeneous solids have the same density as the bulk material.
- Particles obtained by breaking a composite solid such as a metal bearing ore, have various densities, usually different from the density of the bulk material.
- Size and shape are easily specified for regular particles such as cubes and spheres but for irregular particles (such as mica flakes) the terms size and shape must be properly defined.

Particle Properties

How to define the size of a irregular particle ?



What else can be done to empirically define a particle size?

HISTORY

1. **Geoffrey Martin, 1923 :** Defined the size of an irregular particle as length of line bisecting the particle into two equal areas. The bisecting line is taken parallel to a fixed direction, irrespective of the orientation of the particle

2. **L. R. Feret, 1930 :** The Feret diameter or Feret's diameter is a measure of an object size along a specified direction. In general, it can be defined as the distance between the two parallel planes restricting the object perpendicular to that direction.

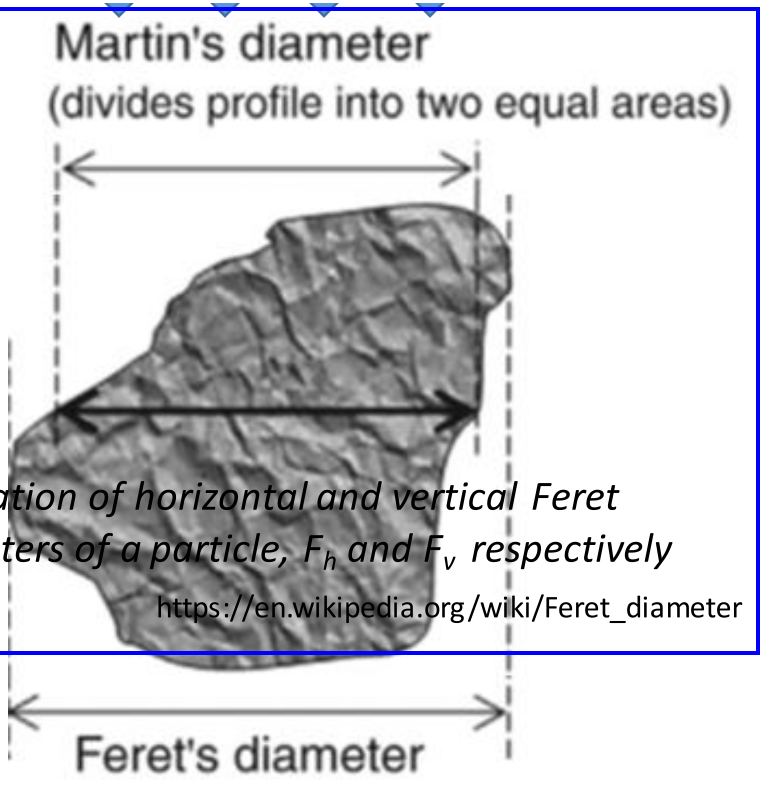


Illustration of horizontal and vertical Feret diameters of a particle, F_h and F_v respectively

https://en.wikipedia.org/wiki/Feret_diameter

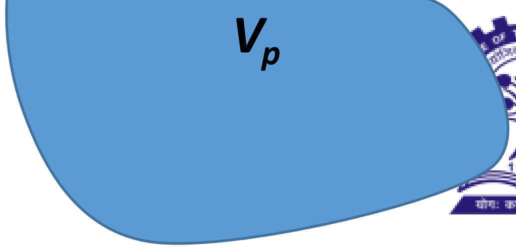
Mineral processing design and operations: an introduction. Elsevier, 2016.

Fails to define the actual size and shape of irregular particle

1. G Martin, CE Blyth, H Tongue - Transactions of the Ceramic Society, 1923

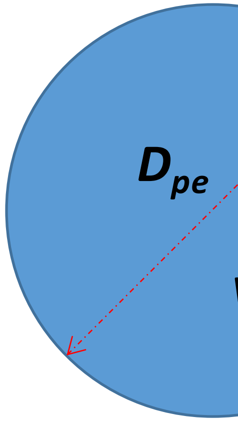
2. Feret, L. R. La grosseur des grains des matières pulvérulentes. 1930

Particle Properties



- Size of an irregular particle can be broadly defined as

The size of spherical particle having the same controlling characteristic as particle under consideration



What do we mean by *controlling characteristic* ?

Irregular particle

Equivalent
part

Depends on the system/process on which the particle is involved

- Example: Catalysis process ⟶ Surface area (d_s)

  Free Settling ⟶ Mass (d_v)

Definitions of derived diameter

1. Surface diameter, d_s: the diameter of sphere having same surface area as the particle
2. **Volume diameter, d_v: the diameter of sphere having same volume as the particle**
3. Drag diameter, d_d: the diameter of sphere having the same resistance to motion as the particle in a fluid of the same viscosity and at the same velocity
4. Projected diameter, d_p: the diameter of the sphere having the same projected area as the particle when viewed in a direction perpendicular to a plane of stability
5. Stokes diameter, d_{st} : $d_{st} = (d_v^3/d_d)^{1/2}$ the free falling velocity in the laminar region
6. **Sieve diameter, d_{avg}: the width of the minimum square aperture through which the particle will pass**
7. **Volume surface diameter (Sauter diameter) d_{vs} :** $d_{vs}= d_v^3/d_s^2$, the diameter of a sphere having same specific surface area as the particle under considertion

Particle shape

Different Shapes:

Spherical | Angular | Spongy | Acicular | Flakey | Irregular

Particle shape

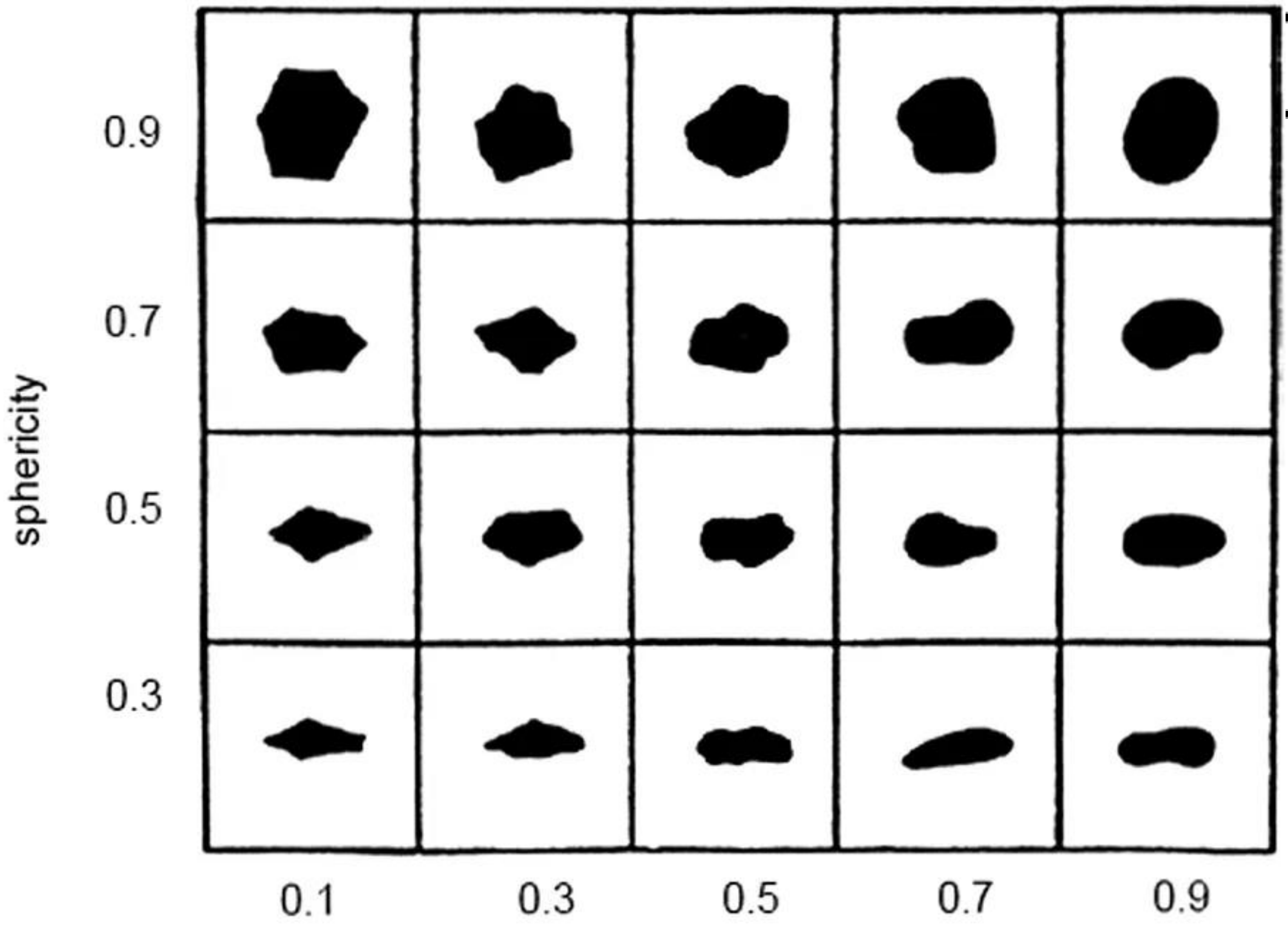
One of the method of defining shape is by using the term *sphericity* (ϕ_S). The sphericity is defined as the ratio of surface area of a sphere, having the same volume as that of the particles, to the actual surface area of the particle

$$\phi_s = \frac{\pi d_v^2}{S_p} \qquad .....(1)$$

Where, d_v is the volume diameter of the particle and S_p is the actual surface area of the irregular particle

Note: It defines only the particle shape and is independent of particle size.

Sphericity of different materials



$$\text{roundness} = \frac{\sum r_i / N}{r_{max}}$$

Sphericity, (ψ_s)
0.83
0.89
0.55
0.89
0.73
0.73
0.65
0.43
0.28
0.3 (average)
0.3 (average)

Visual chart for evaluating the roundness and sphericity of particle grains

Petroleum Production Engineering, 2nd Edition, 2017

Shape factor

The reciprocal of sphericity is called the *shape factor* or *surface shape factor* (λ_s)

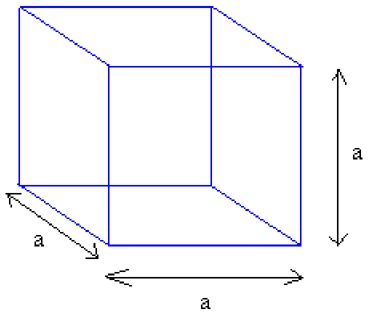
$$\lambda_s = \frac{1}{\phi_s} \qquad \text{.....(2)}$$

Problem 1: Find the sphericity and Shape factor of a cubical particle of a side *a*

Volumetric diameter of the particle :

$$\cancel{V_p} = \frac{\pi}{6} d_v^3 \quad \Rightarrow \quad d_v = \left[\frac{6V_p}{\pi}\right]^{1/3} = \left[\frac{6a^3}{\pi}\right]^{1/3} = (1.24)a$$

a^3



Sphericity: $\phi_s = \frac{\pi d_v^2}{S_p} = \frac{\pi (1.24a)^2}{6a^2} = 0.806$

Shape factor : $\lambda_s = \frac{1}{\phi_s} = \frac{1}{0.806} = 1.24$

Specific surface ratio (n)

Ratio of the specific surface (surface per unit mass, s_p) of the particle to the specific surface of a spherical particle of the same diameter

$$s_{ps} = \frac{\pi d_{avg}^2}{\frac{\pi d_{avg}^3}{6} \times \rho_s} = \frac{6}{\rho_s d_{avg}}$$

$$n = \frac{\text{Specific surface area of the actual particle } (s_p)}{\text{Specific surface area of the spherical particle of same diameter } (s_{ps})}$$

$$n = \frac{s_p}{\left(\frac{6}{\rho_s d_{avg}}\right)} \quad \Rightarrow \quad s_p = \frac{6n}{\rho_s d_{avg}} \qquad .....(3)$$

Where, ρ_s is the density of the particle and the average size of the particle is d_{avg}

Relation with sphericity

From the definition of sphericity we have (from equation 1)

$$S_p = \frac{\pi d_v^2}{\phi_s}$$

Recall, S_p is the actual surface area of the irregular particle

So, specific surface area will be

$$s_p = \frac{S_p}{Mass\ of\ the\ particle}$$

$$= \frac{S_p}{Volume\ of\ the\ particle \times density} = \frac{\dfrac{\pi d_v^2}{\phi_s}}{\dfrac{\pi d_v^3}{6} \times \rho_s} = \frac{6}{d_v \rho_s \phi_s} \quad .....(4)$$

From equation 3 and 4

$$\frac{6n}{\rho_s d_{avg}} = \frac{6}{d_v \rho_s \phi_s}$$

$$n = \left(\frac{d_{avg}}{d_v}\right)\frac{1}{\phi_s} \quad .....(5)$$

...mineral and the most stable polymorph of calcium carbonate ($CaCO_3$)

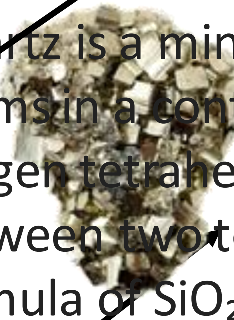
Specific surface ratio (n)

Special case

When, $d_{avg}=d_v$;

$n=\lambda_s$

Otherwise,



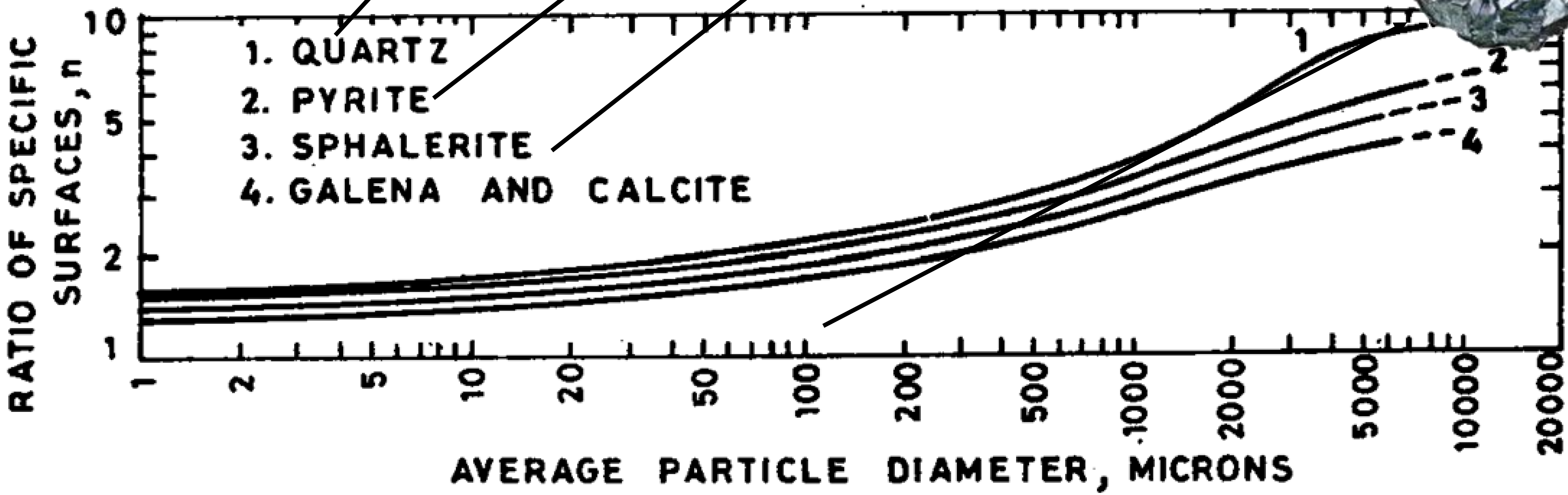
Quartz is a mineral composed of silicon and oxygen atoms in a continuous framework of SiO_4 silicon–oxygen tetrahedra, with each oxygen being shared between two tetrahedra, giving an overall chemical formula of SiO_2.

The mineral pyrite ... gold ... iron sulfide ... chemical formula FeS_2

Sphalerite is a mineral that is the chief ore of zinc [(Zn,Fe)S]

Natu... form... sulfi...

It must be noted that unlike sphericity, specific surface ratio n is a function of particle size (d_{avg})



Volume shape factor (λ_v)

- Sometimes used to calculate the volume of an irregular particle

- Volume of an irregular particle is assumed to be proportional to the cube of its diameter (which is true for a spherical particle)

$$V_p \,\alpha\, d_{avg}^3 \qquad \text{Or} \qquad V_p = \lambda_v d_{avg}^3 \qquad .....(6)$$

- The constant of proportionality λ_v is called the *volume shape factor*

- λ_v is actually not constant (though designated as shape factor) but a function of the particle size.

- In number of cases the variation of λ_v with particle size is not very large and an average value is considered for calculation.

Mixed Particle Sizes

- In real industrial process, we normally deal with mixtures of particles of different sizes.
- The mixture can be separated into a number of fractions with each fraction consisting of particles of given size d_{avgi}
- Let m_i be the total mass of the ith fraction.
- The specific surface of the ith fraction can be calculated using equation 3 as

$$s_i = \frac{6n_i}{\rho_s d_{avg_i}} \quad .....(7)$$

- The surface area of the ith fraction

$$S_i = m_i s_i = \frac{6n_i m_i}{\rho_s d_{avg_i}} \quad .....(8)$$

Mixed Particle Sizes

The specific surface area of the mixture (s_m)

$$s_m = \frac{Total\ surface\ area\ of\ all\ fractions}{Total\ mass\ of\ mixture}$$

$$= \frac{\frac{6n_1m_1}{\rho_s d_{avg_1}} + \frac{6n_2m_2}{\rho_s d_{avg_2}} + \dots\dots\dots\dots}{m_1 + m_2 + \dots\dots\dots} = \frac{\frac{6n_1m_1}{\rho_s d_{avg_1}} + \frac{6n_2m_2}{\rho_s d_{avg_2}} + \dots\dots\dots\dots}{M}$$

x_1

$$= \frac{6n_1x_1}{\rho_s d_{avg_1}} + \frac{6n_2x_2}{\rho_s d_{avg_2}} + \dots\dots\dots\dots$$

Where, x_i is the mass fraction of the ith fraction in the mixture= (m_i/M) and M is the total mass of the system

$$\boxed{s_m = \left(\frac{6}{\rho_s}\right)\sum \frac{n_i x_i}{d_{avg_i}}}$$

Recall n is the *specific surface ratio*

Mixed Particle Sizes

- Number of particles in the i[th] sample (N_i) :

$$N_i = \frac{Mass\ of\ the\ sample}{Mass\ of\ a\ single\ particle} = \frac{m_i}{\rho_s V_{pi}} = \frac{m_i}{\rho_s[\lambda_{vi} d_{avgi}^3]}$$

- Total number of particles per unit mass of the mixture :

$$\overline{N} = \frac{\sum N_i}{M} = \frac{1}{\rho_s} \sum \frac{x_i}{[\lambda_{vi} d_{avgi}^3]} \quad .....(9)$$

Note: If the distribution of the particle (x_i vs d_{avgi}) is known the specific surface area and the number of particle per unit mass of the mixture can be calculated.

Problem 2. Size distribution data of a sample of sand of specific gravity 2.65 is shown in the table below. Calculate the specific surface area of the sample. Assume the volume equivalent diameter of the particle is same as the average diameter of the particle in each sample. Sphericity and density of sand is given as 0.9 and 2.65 respectively.

Size of each fraction (d_{avgi}, cm)	Wt % (w_i)
0.0853	7
0.0726	10.4
0.0511	14.2
0.0359	13.6
0.0253	9.2
0.0182	8.1
0.0128	8.2
0.0089	5.1
0.0037	24.2

Solution:

Size of each fraction (d_{avgi}, cm)	Wt % (w_i)	Weight fraction (x_i)	x_i/d_{avgi}
0.0853	7	0.07	0.820633
0.0726	10.4	0.104	1.432507
0.0511	14.2	0.142	2.778865
0.0359	13.6	0.136	3.788301
0.0253	9.2	0.092	3.636364
0.0182	8.1	0.081	4.450549
0.0128	8.2	0.082	6.40625
0.0089	5.1	0.051	5.730337
0.0037	24.2	0.242	65.40541
			$\sum$ 94.44921

The specific surface area of the mixture (s_m) is given as

$$s_m = \left(\frac{6}{\rho_s}\right)\sum \frac{n_i x_i}{d_{avg_i}}$$

recall n is the *specific surface ratio*

The relation between *specific surface ratio* with the *shpericity* is given as

$$n = \left(\frac{d_{avg}}{d_v}\right)\frac{1}{\phi_s}$$

volume equivalent diameter

In the problem statement it is given that $d_{avg} = d_v$

So, $n = \dfrac{1}{\phi_s}$ and *specific surface ratio* will be $s_m = \left(\dfrac{6}{\phi_s \rho_s}\right)\sum \dfrac{x_i}{d_{avg_i}}$

The *specific surface ratio* of the sand is $= \left(\dfrac{6}{0.9\times 2.65}\right)\times 94.44921 = 237.60\ \dfrac{cm^2}{gm}$

The average particle size for a mixture of particles may be defined in several ways.

1) arithmetic mean : $$D_A = \frac{\sum N_i d_i}{\sum N_i}$$

2) Length mean : $$D_L = \frac{\sum N_i d_i^2}{\sum N_i d_i}$$

3) Surface mean : $$D_S = \frac{\sum N_i d_i^3}{\sum N_i d_i^2}$$

4) Volume mean : $$D_v = \frac{\sum N_i d_i^4}{\sum N_i d_i^3}$$

5) Square root mean: $$D_{SR} = \left[\frac{\sum N_i d_i^2}{\sum N_i}\right]^{1/2}$$

6) Cube root mean: $$D_{CR} = \left[\frac{\sum N_i d_i^3}{\sum N_i}\right]^{1/3}$$

7) Harmonic mean: $$D_H = \left[\frac{\sum N_i}{\sum (N_i / d_i)}\right]$$

Problem 3: Calculate the statistical average diameters

Number of particles, N_i	*Particle size, d_i (microns)*
155,000	2.0
25,600	5.0
6,200	10.0
1,750	20.0
660	30.0
156	40.0
100	50.0
87	60.0

D_A = 3.0 microns

D_L = 7.0 microns

D_S = 21.0 microns

D_V = 36.4 microns

D_{SR} = 4.6 microns

D_{CR} = 7.6 microns

D_H = 2.3 microns

What we have learned?

- Different particle properties
- Quantification techniques for particle size, and shape
- Estimation of particle properties in a mixture of particles
- Statistical techniques for finding the average diameter of the particle in a mixture