JC's Lecture Notes on Heat Transfer

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Preface

There are a large number of textbooks on heat transfer. All of them are better than this lecture notes in many ways and I do not claim this to be a Heat Transfer textbook. This is my lecture notes on heat transfer. I found these lecture notes to be useful for students and hence maintain it in a publishable form.

In India, many students write competitive exams like GATE after completing their degree. They need a concise source of knowledge that will enable them to solve problems quickly and effectively. This book also serves to this need.

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Dedication

This 'Lecture Notes' is dedicated to the young faces I see every year in my Heat Transfer course at IIT Kharagpur, with whom every year I pretend that we will be designing a heat exchanger some day.

Contents

Chapter 1

Convection

1.1 Introduction

If fluid flows over a solid body, heat transfer occurs between the flowing fluid and the solid. This situation is shown in Figure **??**. This mode of heat transfer is called *convective heat transfer*. The law of convective heat transfer is called *Newton's Law of Cooling*. For a body of uniform temperature (T_b), this law can be written as (Fig. **??**):

$$\dot{Q} = qA_b = h(T_b - T_\infty)A_b$$

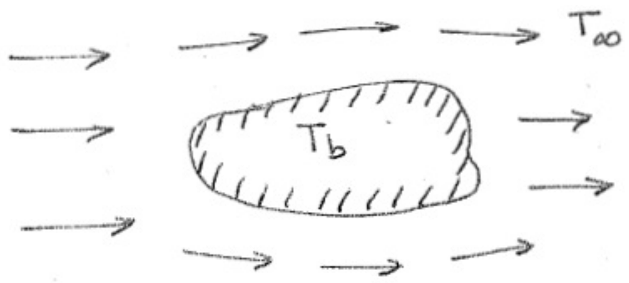


Figure 1.1: Newton's law of cooling for a body with uniform temperature

Here $\dot{Q}$ is the rate of heat transfer from the entire object, T_∞ is the temperature far away from the surface, A_b is the area of the body and h is called the 'heat transfer coefficient'. It can be noted that the heat transfer from the body is dependent on the flow conditions. If the fluid flows at a higher velocity, even for the same T_b and T_∞, the cooling is faster. This is accounted for because h is a function of the 'flow conditions' and fluid properties.

If the body has variable surface temperature, Newton's Law of cooling should be applied to a differential area Fig. ??:

$$d\dot{Q} = dq.dA = h_x(T - T_\infty).dA$$

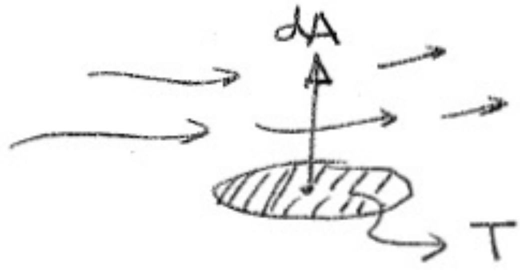


Figure 1.2: Newton's law of cooling for a differential area.

Again, $T_\infty =$ is the temperature of the fluid far away from the differential surface in the normal direction (see Figure ??). It can be seen that the heat flux by convection at a given location can be obtained if we know the local surface temperature (T), corresponding T_∞ and h_x. Again, h_x is dependent on the local conditions such as fluid velocity and physical properties. For most cases, T and T_∞ are known. Hence, the goal of convective heat transfer study is to estimate the local convective heat transfer coefficient, h_x.

It is clear that fluid flow plays a crucial role in convective heat transfer. Hence, convective heat transfer study is usually classified according to the nature of fluid flow across the solid body:

- External flow: Boundary layer grows indefinitely.
- Internal flow: Growing boundary layers from the surfaces merge to produces the so called fully developed flow.

The above classification is based on the nature of the boundary layer around the object. However, other details of the flow such as whether the flow is laminar or turbulent is also very important. The flow of fluid required for convection may be generated by equipments such as fan, blower or pump. Even in absence of such machineries, the density difference generated by the temperature gradient can generate sufficiently strong circulation. If the fluid flow is flow is generated using a pump, blower or fan, the corresponding convection phenomenon is called *forced convection*. If the flow is generated

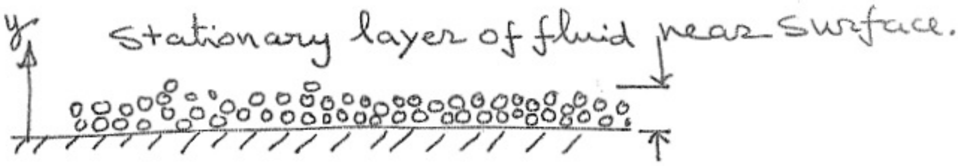


Figure 1.3: Heat transfer near a solid surface.

by density difference alone, it is called *free or natural convection.* Most of the convective heat transfer occurring in the nature are due to free convection.

Although convective heat transfer is a dominant mode of heat transfer for flowing fluid, conduction of heat through the fluid is also very important. In fact conductive heat transfer is the only mode of heat transfer if the fluid is stationary. Such a situation was discussed in Section. Since the fluid is stationary near a solid surface, conduction is the only mode of heat transfer near the solid surface even for a flowing fluid. Hence, although we potray conductive and convective as two distinct mode of heat transfer, convection cannot occur without conduction. Consider the solid surface and the nearby fluid molecules shown in Figure **??**. At steady state, the normal direction heat flux in solid and fluid phase can be written as:

$$q = -k_s \frac{dT}{dY}\bigg|_{y=0-} = -k_f \frac{dT}{dY}\bigg|_{y=0+} = h(T_w - T_\infty)$$

where k_f is the thermal conductivity of the fluid. Taking the last pair:

$$-k_f \frac{dT}{dY}\bigg|_{y=0} = h(T_w - T_\infty)$$

and using the following non-dimensional quantities:

$$\hat{T} = \frac{T - T_\infty}{T_W - T_\infty}$$

$$\hat{y}_t = \frac{y}{\delta_T}$$

we have:

$$\frac{d\hat{T}}{d\hat{y}_t}\bigg|_{\hat{y}_t=0} = \frac{h\delta_T}{k_f}$$

The quantity at the right hand side is a dimensionless quantity known as *Nusselt Number*, abbreviated as Nu. As the above equation suggests, Nusselt number is the dimensionless temperature gradient at the solid wall in the fluid phase. Note the similarity and difference between the Nusselt number and the Biot number introduced earlier. The other interpretation of the Nusselt number as a ratio of thermal transport coeeficient is obvious. The third interpretation of Nusselt number is "dimensionless heat transfer coefficient". In convection studies, we often seek a methodology to obtain the Nusselt number for a given flow.

1.2 Boundary layer theory

It is clear from the above analysis that we need to know the temperature gradient at the wall in the fluid phase to obtain the Nu or h. Hence, we will need to know the temperature profile near the wall. This region is called thermal boundary layer. Before we analyze the thermal boundary layer, we will re-visit the momentum boundary layer. We will seek to find the velocity profile at the boundary layer and obtain the gradient of velocity at the wall by differentiating the velocity profile near the wall and then apply the same strategy to obtain the temperature profile and its gradient at the wall.

1.2.1 Momentum Boundary Layer

The nature of the boundary layer is dependend on the flow geometry. We will consider laminar flow over a flat plate as shown in Figure **??** to demonstrate the procedure. It can be seen that this is external flow. For other geometry similar analysis may be carried out. However, for complicated geometry, the analysis becomes complicated and an empirical approach is often takesn. These points will be discussed in more detailed later.

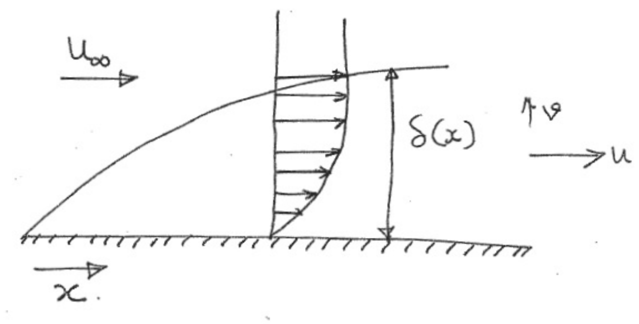


Figure 1.4: Velocity Boundary Layer

To obtain the velocity profile in the velocity boundary layer, we need to write down the continuity and momentum balance (Navier-Stokes) equations for this geometry. We will consider a steady 2D flow with constant properties and the equation of motion and continuity for this case is written as:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{P}.\frac{\partial P}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \quad (1.1a)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{P}\frac{\partial P}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] \quad (1.1b)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1.1c)$$

The equation given above can simplified considerably before attempting solution. For external flow over a flat plate, pressure shows negligible variation through out the domain and hence pressure gradient in both x and y direction can be neglected. Hence:

$$\frac{\partial P}{\partial x} \approx \frac{\partial P}{\partial y} \approx 0 \quad (1.2)$$

Additional simplification is also possible using an order of magnitude analysis shown below:

$$O\left(\frac{\partial^2 u}{\partial x^2}\right) \cong \frac{u_\infty}{l^2}$$

whereas,

$$O\left(\frac{\partial^2 u}{\partial y^2}\right) = \frac{u_\infty}{\delta^2}$$

Because $\delta << l$,

$$\frac{\partial^2 u}{\partial y^2} >> \frac{\partial^2 u}{\partial x^2} \quad (1.3)$$

Using Eqs **??** and **??**, eq. **??** becomes:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \quad (1.4a)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1.4b)$$

Please note that we do not need equation because we do not need to solve for P. Equation can be written as:

$$2u\frac{\partial u}{\partial x} - u\frac{\partial u}{\partial x} + \frac{\partial (uv)}{\partial y} - u\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Using Continuity equation:

$$2u\frac{\partial u}{\partial x}+\frac{\partial (uv)}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$$

or,

$$\frac{\partial u^2}{\partial x}+\frac{\partial (uv)}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$$

This form is amicable for solution by a method called *Boundary Integral method*. In boundary integral method, this equation is integrated over the height of the boundary layer:

$$\int_0^{\delta(x)}\frac{\partial u^2}{\partial x}.dy+\int_0^{\delta(x)}\frac{\partial (uv)}{\partial y}dy=\int_0^{\delta(x)}\nu\frac{\partial^2 u}{\partial y^2}.dy$$

or,

$$\int_0^{\delta(x)}\frac{\partial u^2}{\partial x}.dy+(uv)\Big|_{\delta(x)}-(uv)\Big|_0=\nu\left[\frac{\partial u}{\partial y}\Big|_{\delta(x)}-\frac{\partial u}{\partial y}\Big|_0\right]$$

Since velocity (both u and v) is zero at $y=0$ (No slip), and x direction velocity gradient is zero at the outer skin of the velocity boundary layer, the above equation becomes:

$$\int_0^{\partial(x)}\frac{\partial u^2}{\partial x}dy+u_\infty v_\infty=-\nu\frac{\partial u}{\partial y}\Big|_{y=0}$$

At this point, it is tempting to set $v_\infty=0$, but such a substitution will violate continuity equation. We need to substitute this quantity using the continuity equation. Using the similar boundary integral on the continuity equation:

$$\int_0^{\partial(x)}\frac{\partial u}{\partial x}dy+\int_0^{\partial(x)}\frac{\partial v}{\partial y}dy=0$$

or,

$$\int_0^{\partial(x)}\frac{\partial u}{\partial x}dy+v_\infty=0$$

or,

$$v_\infty=-\int_0^{\partial(x)}\frac{\partial u}{\partial x}.dy$$

Substituting v_∞ from above into the momentum equation:

$$\int_0^{\partial(x)}\frac{\partial u^2}{\partial x}dy-u_\infty\int_0^{\partial(x)}\frac{\partial u}{\partial x}dy=-v\frac{\partial u}{\partial y}\Big|_{y=0}$$

or,

$$\int_0^{\partial(x)} \frac{\partial}{\partial x}[u(u-u_\infty)]dy = -v\frac{\partial u}{\partial y}\bigg|_{y=0}$$

Pulling out the differentiation outside the integral will be useful. Since the limit is a function of x, we will have to do it using Leibnitz rule:

$$\frac{d}{dt}\int_{\alpha(t)}^{\beta(t)} f(x,t)dx = \int_{\alpha(t)}^{\beta(t)} \frac{\partial}{\partial t}f(x,t)dx + [f(\beta,t)\frac{d\beta}{dt} - f(\alpha,t)\frac{d\alpha}{dt}]$$

Because of the particular form of the function, both $f(\alpha,t)$ and $f(\beta,t)$ are zero. Hence, after application of Leibnitz rule, the momentum equation becomes:

$$\frac{d}{dx}\int_0^{\partial(x)} u(u-u_\infty)dy = -v\frac{\partial u}{\partial y}\bigg|_{y=0}$$

Non-dimensionalized momentum equation

Next we will non-dimensionalize this equation using:

$$\hat{u} = \frac{u}{u_\infty}$$

$$\hat{y} = \frac{y}{\delta(x)}$$

Transforming various quantities in terms of the scaled variables:

$$\frac{\partial u}{\partial y} = u_\infty\frac{\partial \hat{u}}{\partial y} = u_\infty\frac{\partial \hat{u}}{\partial \hat{y}}.\frac{\partial \hat{y}}{\partial y} = \frac{u_\infty}{\delta(x)}\frac{\partial \hat{u}}{\partial \hat{y}}$$

$$dy = d\hat{y}\delta(x)$$

$$@y = 0, \hat{y} = 0\text{and}@y = \delta(x), \hat{y} = 1.0$$

Using these transformations, the momentum equation becomes:

$$\frac{d}{dx}\int_0^{1.0} u_\infty.\hat{u}(u_\infty\hat{u} - u_\infty)\delta(x)d\hat{y} = -\nu\frac{u_\infty}{\delta(x)}\frac{d\hat{u}}{d\hat{y}}\bigg|_{\hat{y}=0}$$

Simplifying

$$\frac{d}{dx}\left[\delta(x)\int_0^{1.0} \hat{u}(\hat{u}-1)d\hat{y}\right] = -\frac{\nu}{u_\infty\delta(x)}\left[\frac{d\hat{u}}{d\hat{y}}\bigg|_{y\hat{=}0}\right]$$

At this point the reader will ask what have we achieved by using this dimensionless form? This dimensionless form allow us to exploit a special feature of physical systems known as scaling law or combination of variables. Instead of expressing the functionality of u as $u(x, y)$, we say that $\hat{u}(\hat{y})$ where $\hat{y}$ contains both x and y. This assumption may not be correct always but fortunately such 'scaling law' holds for many cases at least with good approximation. In this case, we are imposing the following restriction: at a fixed frction of the boundary layer thickness, the velocity is a certain fraction of the free stream velocity irrespective of the location on the flat plate. Many natural system (including our body) follow such feature. For example, our height is about four times of our forearm's length including palm.

Although we assumed $\hat{u}(\hat{y})$, the functional form of $\hat{u}$ cannot be solved readily. This impasse can be resolved by assuming a second degree polynomial

$$\hat{u} = a + b.\hat{y} + c.\hat{y}^2 + d\hat{y}^3$$

and estimating the coefficient using four conditions at the boundary, namely:

$$@\hat{y} = 0, \hat{u} = 0$$
$$@\hat{y} = 1.0, \hat{u} = 1.0.$$
$$@\hat{y} = 1.0, \frac{d\hat{u}}{d\hat{y}} = 0$$

The fourth boundary condition can be obtained by restricting the original equation of motion to the wall condition:

$$u.\frac{\partial u}{\partial x}\bigg|_{y=0} + v.\frac{\partial v}{\partial y}\bigg|_{y=0} = v\frac{\partial^2 u}{\partial y^2}\bigg|_{y=0}$$

which gives:

$$\frac{\partial u}{\partial y^2}\bigg|_{y=0} = 0$$

In terms of non-dimensional variables:

$$\frac{u_\infty}{[\delta(x)]^2}.\frac{d^2\hat{u}}{d\hat{y}^2}\bigg|_{\hat{y}=0} = 0$$

or,

$$@\hat{y} = 0, \frac{d^2\hat{u}}{d\hat{y}^2}\bigg|_{\hat{y}=0} = 0$$

We can evaluate the four constants in the polynomial by using these four conditions:

$$\hat{u} = \frac{3}{2}\hat{y} - \frac{1}{2}\hat{y}^3$$

Please note that the solution is in terms of the dimensionless variables which involves u_∞ and $\delta(x)$. While u_∞ is constant, the functional form of $\delta(x)$ needs to be known in order to complete the solution. The non-dimensional momentum equation will be used to obtain the explicit form of $\delta(x)$. Substituting in the solution from eq into eq :

$$\frac{d}{dx}\left[\delta(x)\int_0^{1.0}(1.5\hat{y}-0.5\hat{y}^3)(1.5\hat{y}-0.5\hat{y}^3-1)d\hat{y}\right]=-\frac{\nu}{u_\infty\delta(x)}.\frac{3}{2}$$

The definite integral can be evaluated readily and its value is $-39/280$. Substituting this value and simplifying:

$$\delta(x)\frac{d\delta(x)}{dx}=\frac{3\times 280\times\nu}{2\times 39\times u_\infty}$$

Solving for $\delta(x)$,

$$[\delta(x)]^2=21.538\frac{\nu}{u_\infty}x+c$$

At $x=0,\delta(x)=0 \implies c=0$
Finally,

$$\delta(x)=4.64\sqrt{\frac{\nu x}{u_\infty}}=4.64xRe_x^{-1/2}$$

Using this expression for $\delta(x)$, we can obtain the explicit expression of $u(x,y)$. Here Re_x is the local Reynold's number on the plate at a distance x.

Example 1.1. Air at 27°C 1 atm flows over a flat plate at a speed of 2 m/s. Calculate the hydrodynamic boundary layer thickness at a distance of 20 and 400 cm from the leading edge of the plate. Viscosity of air at 27°C is 1.85×10^{-5} kg/m s. Density of air under these condition is 1.177 kg/m^3

Solution: At a distance x =20 cm, the Reynolds number is:

$$\mathrm{Re}=\frac{xU_\infty\rho}{\mu}=2.54\times10^4$$

Hence, the boundary layer thickness will be:

$$\delta(x=20\mathrm{cm})=4.64\times20\times10^{-2}\frac{1}{\sqrt{2.54\times10^4}}\ \mathrm{m}=0.0058\ \mathrm{m}$$

At x = 400 cm, the Reynolds number is 5.08×10^5. Hence, the flow is turbulent in this region. The relation we developed cannot be used. □

1.2.2 Thermal Boundary Layer

In this section we will obtain the temperature distribution near a hot flat plate. The flow and geometry remains identical to the case discussed earlier, i.e., laminar flow over a flat plate with constant properties. Only difference is that the temperature of the plate is different from the temperature of the fluid supplied. We will assume that the temperature of the plate is higher than that of the fluid although the opposite may me treated in identical manner. The situation is shown in Figure ??.

It can be seen that a thermal boundary layer is established around the plate as shown in the figure. In this figure, the thermal boundary layer has been shown to be thicker than the velocity boundary layer. However, dipending the relative magnitude of the momentum and thermal diffusivity, the opposite is also possible. We will learn more about this in this section.

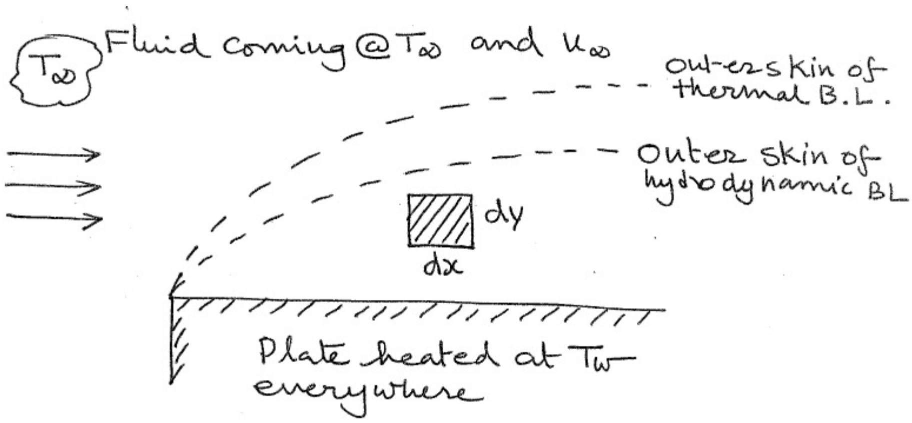


Figure 1.5: Thermal Boundary Layer

To analyzed the velocity distribution in the velocity boundary layer, we used the momentum and continuoity equation. Similarly, to analyze the temperature distribution in the therlmal boundary layer, we need the energy balance equation along with continuity and momentum balance equation. Next, we will derive the energy balance equation in the boundary layer. For viscous fluid, the energy balance equation involves one energy generation terms that is due to fluid friction (viscous dissipation) but in our analysis we will assume that such features are absent. We will also assume steady state.

Steady state energy balance in the thermal Boundary layer

A three dimensional elementary volume is shown in Figure. Since fluid can have energy in the form of internal energy as well as flow work, we need to write the enthalpy balance for this case. At steady state, the total enthalpy of the elementary volume remains constant and hence the energy balance translates into Total Enthalpy in = Total Enthalpy out. The specific enthaply (per unit volume) of a fluid can be written as:

$$h = \rho \hat{c_p}(T - T_0)$$

where $\hat{c_p}$ is the specific heat per unit mass (not mol) and $T_0 =$ is a suitable reference temperature which we can take as 0 K for our convenience. Hence, the enthalpy per unit volume of fluid is simply given by $\rho\hat{c_p}T$.

We will discuss another important aspect of the energy blance before writing the balance equation. As evident from the boundary layer flow discussed in the previous section, the velocity is strongest in the x direction. On the other hand, the velocity in the y direction is much smaller. Hence, in x direction, advection is the dominant mode of heat transfer while both advenction and conduction are important in y direction. The reader is also advised to recall that flux of any quantity across a plane is velocity times density at that plane. Keeping these things in mind, we can write the following terms:

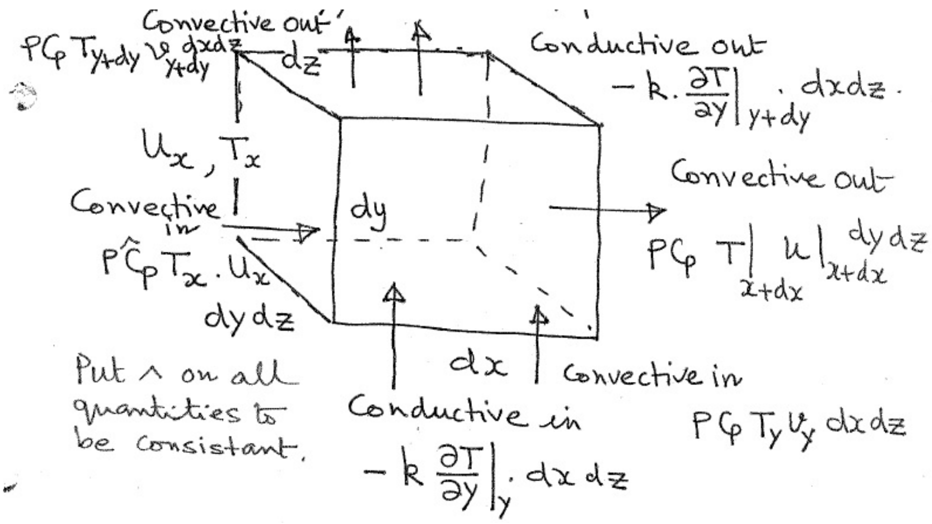


Figure 1.6: Energy balance in an elementary volume

$$\rho\hat{c_p}Tu\Big|_x dydz + \left[-k\frac{\partial T}{\partial y}\Big|_y + \rho\hat{c_p}Tv\Big|_y\right] dxdz = \rho\hat{c_p}Tu\Big|_{x+dx} dydz + \left[-k\frac{\partial T}{\partial y}\Big|_{y+dy} + \rho\hat{c_p}Tv\Big|_{y+dy}\right] dxdz$$

Simplification of this equation leads to:

$$\frac{\partial(Tu)}{\partial x} + \frac{\partial(Tv)}{\partial Y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{1.5}$$

An alternative form can be developed using the continuity equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial Y^2}$$

we get the usual form of the boundary layer energy equation.

Boundary integral method for the thermal boundary layer

Integrating equation **??** over the thickness of the thermal boundary layer $\delta_t(x)$:

$$\int_0^{\delta_t(x)} \frac{\partial(Tu)}{\partial x}dy + \int_0^{\delta_t(x)} \frac{\partial(Tv)}{\partial y}dy = \int_0^{\delta_t(x)} \alpha\frac{\partial^2 T}{\partial y^2}dy$$

simplifying:

$$\int_0^{\delta_t(x)} \frac{\partial(Tu)}{\partial x}dy + (Tv)\Big|_{y=\delta_t(x)} = -\alpha\frac{\partial T}{\partial Y}\Big|_{y=0}$$

The velocity at the outer skin of the *thermal boundary layer* may be obtained by integrating the continuity equation over the thickness of the *thermal boundary layer*:

$$\int_0^{\delta_t(x)} \frac{\partial u}{\partial x}dy + \int_0^{\delta_t(x)} \frac{\partial v}{\partial y}dy = 0$$

or,

$$\int_0^{\delta_t(x)} \frac{\partial u}{\partial x}dy + v\Big|_{\delta_t(x)} = 0$$

Substituting:

$$\int_0^{\delta_t(x)} \frac{\partial(Tu)}{\partial x}dy - T_\infty\int_0^{\delta_t(x)} \frac{\partial u}{\partial x}dy = -\alpha\frac{\partial T}{\partial y}\Big|_{y=0}$$

Simplifying:

$$\int_0^{\delta_t(x)} \frac{\partial}{\partial x}[u(T-T_\infty)]dy = -\alpha\frac{\partial T}{\partial y}\Big|_{y=0}$$

Bringing the differential outside the integral using Leibnitz rule:

$$\frac{d}{dx}\int_0^{\delta_t(x)} [u(T-T_\infty)]dy = -\alpha\frac{\partial T}{\partial y}\Big|_{y=0}$$

We will use the following transformations for non-dimensionalization:

$$\hat{T} = \frac{T - T_\infty}{T_w - T_\infty} \tag{1.6a}$$

$$\hat{y}_t = \frac{y}{\delta_t(x)} \tag{1.6b}$$

$$\hat{u} = \frac{u}{u_\infty} \tag{1.6c}$$

$$dy = \delta_t(x) d\hat{y}_t \tag{1.6d}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \hat{y}_t}\frac{d\hat{y}_t}{dy} = \frac{1}{\delta_t(x)}\frac{\partial}{\partial \hat{y}_t} \tag{1.6e}$$

Using the above transformations:

$$\frac{d}{dx}\int_0^1 \left[\hat{u}u_\infty \hat{T}(T_w - T_\infty)\delta_t(x)\right] d\hat{y}_t = -\alpha . \frac{1}{\delta_t(x)}\left(\frac{\partial}{\partial \hat{y}_t}\left[\hat{T}(T_w - T_\infty) + T_\infty\right]\right)_{\hat{y}_t=0}$$

Simplifying:

$$u_\infty \frac{d}{dx}\int_0^1 [\hat{u}\hat{T}.\delta_t(x)]d\hat{y}_t = -\frac{\alpha}{\delta_t(x)}\frac{\partial \hat{T}}{\partial \hat{y}_t}\bigg|_{\hat{y}_t=0}$$

With additional simplification:

$$\frac{d}{dx}\left[\delta_t(x)\int_0^1 \hat{u}\hat{T}d\hat{y}_t\right] = -\frac{\alpha}{u_\infty \delta_t(x)}\frac{\partial \hat{T}}{\partial \hat{y}_t}\bigg|_{\hat{y}_t=0} \tag{1.7}$$

Again, we shall assume scaling behaviour for dimensionless temperature profile and assume that a third degree polynomial can be used to approximate the function:

$$\hat{T} = a + b\hat{y}_t + c\hat{y}_t^2 + d\hat{y}_t^3$$

This function must satisfy the following conditions:

$$@\hat{y}_t = 0, \quad \hat{T} = 1$$

$$@\hat{y}_t = 1, \quad \hat{T} = 0$$

Because $u = v = 0$ at $y = 0$, from equation,

$$@\hat{y}_t = 0 \quad \frac{\partial^2 \hat{T}}{\partial \hat{y}_t^2} = 0$$

Additionally, because the temperature in the thermal boundary layer smoothly approaches the free stream temperature,

$$@\hat{y}_t = 1, \quad \frac{\partial \hat{T}}{\partial \hat{y}_t} = 0$$

The constants can be evaluated using these conditions and the final expression for $\hat{T}$ becomes:

$$\hat{T} = 1 - \frac{3}{2}\hat{y}_t + \frac{1}{2}\hat{y}_t^3 \tag{1.8}$$

substituting $\hat{T}$ from equation and $\hat{u}$ from equation into equation, we obtain:

$$\frac{d}{dx}\left[\delta_t(x)\int_0^1 \left(\frac{3}{2}\hat{y} - \frac{1}{2}\hat{y}^3\right)\left(1 - \frac{3}{2}\hat{y}_t + \frac{1}{2}\hat{y}_t^3\right)d\hat{y}_t\right] = -\frac{\alpha}{u_\infty \delta_t(x)}\left(-\frac{3}{2}\right)$$

Now, to evaluate the definite integral, we need a relation between $\hat{y}$ and $\hat{y}_t$ which can be written as:

$$\frac{\hat{y}}{\hat{y}_t} = \frac{\delta_t(x)}{\delta(x)}$$

Because of the close similarity of the momentum and energy equation, it is expected that the ratio of the two thickness is function of momentum diffusivity (ν) and thermal diffusivity (α) only. Denoting this function as $\psi(\nu, \alpha)$:

$$\frac{\hat{y}}{\hat{y}_t} = \frac{\delta_t(x)}{\delta(x)} \equiv \psi(\nu, \alpha)$$

we get:

$$\hat{y} = \psi \hat{y}_t$$

Substituting:

$$\frac{d}{dx}\left[\delta_t(x)\int_0^1 \left(\frac{3}{2}\psi\hat{y}_t - \frac{1}{2}\psi^3\hat{y}_t^3\right)\left(1 - \frac{3}{2}\hat{y}_t + \frac{1}{2}\hat{y}_t^3\right)d\hat{y}_t\right] = \frac{3\alpha}{2u_\infty \delta_t(x)}$$

Now the definite integral can be evaluated in terms of ψ:

$$\int_0^1 \left(\frac{3}{2}\psi\hat{y}_t - \frac{1}{2}\psi^3\hat{y}_t^3\right)\left(1 - \frac{3}{2}\hat{y}_t + \frac{1}{2}\hat{y}_t^3\right)d\hat{y}_t = \left[\frac{3}{20}\psi - \frac{3}{280}\psi^3\right] \equiv \zeta(\psi)$$

Substituting from above and with some rearranging:

$$2\delta_t(x)\frac{d\delta_t(x)}{dx} = \frac{3\alpha}{u_\infty \xi(\psi)}$$

Solution of this equation with the boundary condition $@x = 0, \delta_t(x) = 0$ gives:

$$\delta_t(x) = \sqrt{\frac{3\alpha x}{u_\infty \xi(\psi)}}$$

Now, we need to look back to check whether our solution contradicts our initial assumption that the ratio of the two boundary layer thickness is a function of two diffusiities or not. It can be seen from equation and equation, that this is indeed the case. Hence, our solution is consistent with our assumption. Now we need to obtain a closed form expression for ψ.

$$\begin{aligned}
\psi &\equiv \frac{\delta_t(x)}{\delta(x)} \\
&= \sqrt{\frac{3\alpha x}{u_\infty \xi(\psi)}} \Big/ 4.64\sqrt{\frac{\nu x}{u_\infty}} \\
&= \frac{\sqrt{3}}{4.64}\sqrt{\frac{\alpha}{\nu}}\frac{1}{\sqrt{\zeta(\psi)}} \\
&= 0.373\sqrt{\frac{\alpha}{\nu}}\left[\frac{3}{20}\psi - \frac{3}{280}\psi^3\right]^{-0.5}
\end{aligned}$$

Although we can solve for ψ in terms of α/ν, substantial simplification is possible if we restrict ourselves for a case where the thermal boundary layer is thinner than the hydrodynamic boundary layer. Mathematically,

$$0 < \psi \leq 1.0$$

In this range:

$$\frac{3}{20}\psi - \frac{3}{280}\psi^3 \approx \frac{3}{20}\psi$$

Using this simplification,

$$\psi = 0.373\sqrt{\frac{\alpha}{\nu}}\frac{1}{\sqrt{3\psi/20}}$$

Solving for ψ:

$$\psi = 0.975\sqrt[3]{\frac{\alpha}{\nu}} \approx \sqrt[3]{\frac{\alpha}{\nu}}$$

The quantity ν/α is one of the most important dimensionless number used in heat transfer an is known as Prandtl Number (Pr). In terms of Prandtl number, the thermal boundary layer thickness is given by:

$$\psi \equiv \frac{\delta_t(x)}{\delta(x)} = \frac{1}{\sqrt[3]{\mathrm{Pr}}}$$

Finally:

$$\delta_t(x) = \frac{\delta(x)}{\sqrt[3]{\mathrm{Pr}}} = \frac{4.64x}{\sqrt[3]{\mathrm{Pr}}\sqrt{Re_x}}$$

1.3 Correlations for external flow

With this explicit expression of the thermal boundary layer thickness, we complete the solution of the temperature profile in the thermal boundary layer. Now, let us see how we can use the obtained temperature profile to obtain an expression for the heat transfer coefficient:

$$\begin{aligned}
q_w &= -k\left.\frac{\partial T}{\partial y}\right|_{y=0} \\
&= -k\frac{(T_w - T_\infty)}{\delta_t}\left.\frac{\partial \hat{T}}{\partial \hat{y}_t}\right|_{\hat{y}_t=0} \\
&= \frac{3}{2}k\frac{(T_w - T_\infty)}{\delta_t} \\
&= \frac{3}{2}k\frac{(T_w - T_\infty)\sqrt[3]{\mathrm{Pr}}\sqrt{Re_x}}{4.64x} \\
&= 0.323\frac{k}{x}(T_w - T_\infty)\sqrt[3]{\mathrm{Pr}}\sqrt{Re_x}
\end{aligned}$$

Using Newton's law of cooling:

$$q_w = h_x(T_w - T_\infty)$$

Comparing:

$$\begin{aligned}
h_x &= 0.323\frac{k}{x}(Pr)^{\frac{1}{3}}.Re_x^{\frac{1}{2}} \\
\frac{h_x.x}{k} &= 0.323(Pr)^{\frac{1}{3}}.Re_x^{\frac{1}{2}} \\
Nu_x &= 0.323(Pr)^{\frac{1}{3}}Re_x^{\frac{1}{2}}
\end{aligned}$$

The above expression for Nusselt number can be used for obtaining the local heat transfer coefficient. However, the use of the above expression is limited to:

- Laminar flow (Re $<$ 1E5)
- Flat plate geometry
- Pr ≥ 1
- Local Nu
- Constant surface temperature

If any of the above conditions is violated, the expression for Nusselt number changes. Fresh analysis may be needed. For example, if Pr ≤ 0.001 (typical for liquid metals), we need to repeat part of the analysis for this modified condition. This is shown in Example . For more drastic changes such as change in geometry, the entire analysis has to be repeated.

Example 1.2. Obtain the Nusselt number correlation applicable for very low Prandtl number.

Solution: Another set of Pr range is $Pr \leq 0.001$ which happens for liquid metals. For this case we can safely assume that the hydrodynamic boundary layer is so thin that the entire thermal boundary layer experiences the free stream velocity. Mathematically:

$$v \approx 0; \quad u = u_\infty$$

For this case, the energy balance simplifies to:

$$u_\infty \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}$$

Using boundary integral:

$$\frac{u_\infty}{\alpha} \int_0^{\delta_t(x)} \frac{\partial T}{\partial x} dy = \int_0^{\delta_t(x)} \frac{\partial^2 T}{\partial y^2} dy$$

$$\frac{u_\infty}{\alpha} \int_0^{\delta_t(x)} \frac{\partial T}{\partial x} dy = -\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

Bringing the differential outside the integral is slightly trickier in this case. For this, we will first transform the variable T into $\hat{T}$. In terms of $\hat{T}$:

$$\frac{u_\infty}{\alpha} \int_0^{\delta_t(x)} \frac{\partial \hat{T}}{\partial x} dy = -\left.\frac{\partial \hat{T}}{\partial y}\right|_{y=0}$$

Now we can bring the derivative out with the understanding that the lower limit is constant and the function is zero at the upper limit.

$$\frac{u_\infty}{\alpha} \frac{d}{dx} \int_0^{\delta_t(x)} \hat{T} dy = -\left.\frac{\partial \hat{T}}{\partial y}\right|_{y=0}$$

In terms of scaled $\hat{y}_t$:

$$\frac{u_\infty \delta_t(x)}{\alpha} \frac{d}{dx} \left[\delta_t(x) \int_0^1 \hat{T} d\hat{y}_t \right] = -\left.\frac{\partial \hat{T}}{\partial \hat{y}_t}\right|_{\hat{y}_t=0}$$

Substituting the dimensionless temperature profile $\hat{T} = 1 - \frac{3}{2}\hat{y}_t + \frac{1}{2}\hat{y}_t^3$ into the above equation:

$$\frac{u_\infty \delta_t(x)}{\alpha} \frac{d}{dx} \left[\delta_t(x) \int_0^1 (1 - \frac{3}{2}\hat{y}_t + \frac{1}{2}\hat{y}_t^3) d\hat{y}_t \right] = \frac{3}{2}$$

Which simplifies to:

$$\frac{d\delta_t(x)^2}{dx} = \frac{8\alpha}{u_\infty}$$

Solving with the boundary condition $\delta_t(x) = 0 \quad @x = 0$:

$$\delta_t(x) = \sqrt{\frac{8\alpha x}{u_\infty}}$$

Now we can obtain the required Nusselt number correlation:

$$q_w = h.(T_w - T_\infty) = \frac{3}{2} k(T_w - T_\infty) \sqrt{\frac{u_\infty}{8\alpha x}}$$

which gives:

$$\frac{h.x}{k} = \frac{3}{2} \frac{1}{\sqrt{8}} x \sqrt{\frac{u_\infty}{\alpha x}} = 0.53 \sqrt{\frac{u_\infty x}{\nu} \frac{\nu}{\alpha}}$$

Hence the required correlation is:

$$Nu = 0.53 Re^{\frac{1}{2}} Pr^{\frac{1}{2}}$$

□

One of the major achievements of the above analysis is that it expresses Nusellt number (dimensionless heat transfer coefficient) as simple power law of Reynols number and Prandtle number. It has been observed that irrespective of the flow geometry and other features, this functionality holds. Only variation occurs in the exponents or the coefficient. For this reason, for all cases of forced convection, both internal and external, the following generalized form is used:

$$Nu = C Re^m Pr^n$$

It may be noted that while the exponents (m and n) and the constant (C) may be obtained theoretically for some cases, majority of the cases these are obtained experimentally.

One such experimental set-up is shown where the same flat plate geometry was used for obtaining the relation between the three vital dimensionless numbers of heat transfer. In this case, instead of constant surface temperature, constant surface heat flux condition is used (These two are the most

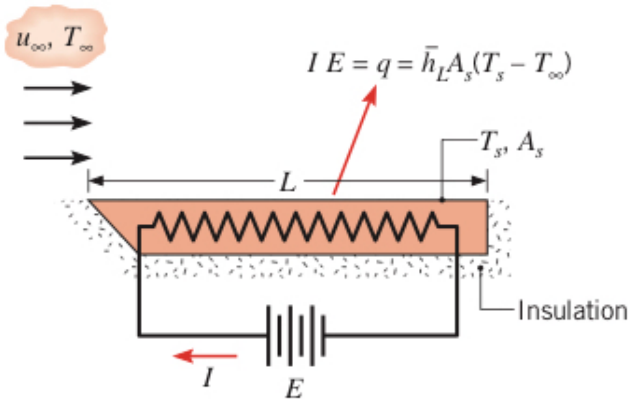


Figure 1.7: A typical set-up for determination of Nusselt number correlations

important conditions encountered in engineering). The constant surface heat flux is maintained by using an electric coil. If the same number of turns are maintained per unit area, the heat flux remains constant over the entire surface. In this case, the surface heat flux is a measurable quantity. Hence, if we could measure the difference between the local surface temperature and the free stream temperature (using a set of thermocouples, say), we can readily obtain the local heat transfer coeffiient (h_x)and the local Nu_x.

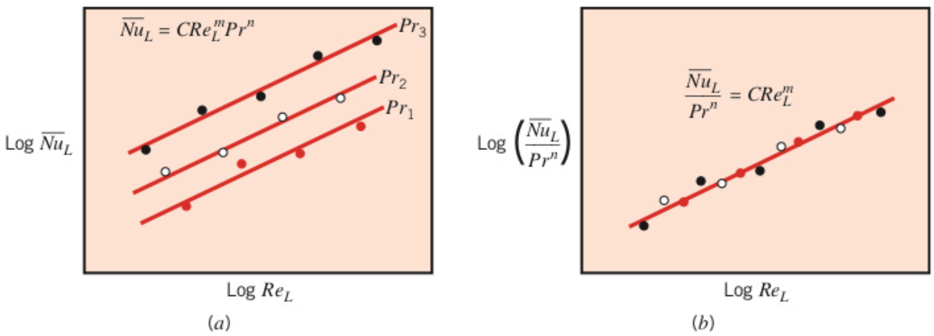


Figure 1.8: Typical sequence of data processing for evaluation of C, m and n in Nusselt number correlations.

Now it is possible to plot the local Nusselt number against the local Reynolds number and Prandtl number and see if the relation can be adequately represented by the power law relation discussed above and also evaluate the constant by fitting the data to the expression given in equation. First let us consider a situation where the same Pr is maintained (using the same fluid) but the Reynolds number is varied by changing the free stream velocity. The experimentally obtained Nu changes as the Re changes. The data may be plotted using a log-log plot as shown in Fig. If the power law functionality is valid it will show approximate straight line in the log-log plot. Fig shows several such plot obtained for various fluids and thus different Prandtl numbers. The slope of all such plots should be approximately the same and can be used to obtain the exponent m.

Once m is known, we set out to evaluate n. The exponent n is around 0.3 for most cases. Hence, we assume this value and $Nu_x/Pr^{0.3}$ is plotted against Re in log-log scale for data over all Pr number as shown in Fig. If our choice of n is correct, data for different Pr should collapses into a single straight line as shown. Usually, some variation is observed when $n = 0.3$ is used. Such variations are minimised by changing n slightly. The best fit line can be used to obtain C. The above procedure is also used for verification of the theoretically obtained correlations. The correlations for a few situations of importance for external forced convection is given in Table **??**

Some correlations are more elaborate that the simple ones given in Table **??**. One such example is for flow of a fluid over a bank of tubes as shown in Figure **??** The form of the correlation is similar to other correlations but the details are little involved. The rules for obtaining various quantities in the correlation are listed below:

$$\overline{Nu}_D = C_1 Re_{D,Max}^m Pr^{0.36} \left(\frac{Pr}{Pr_s} \right)^{1/4}$$

1. The value of the constants are dependent on the arrangement of tubes. Two arrangements are possible: align and staggered. These are shown in Figure **??**.

2. This correlation is for number of rows (N_L) greater than 20. For lesser number of rows, a correction factor is needed.

   $$\overline{Nu}_D\big|_{Corrected} = C_2 \overline{Nu}_D\big|_{Obtained}$$

   The constant C_2 is given in Table **??**.

3. All properties except Pr_s are evaluated at the mean temperature defined by

   $$T_m = (T_{in} + T_{out})/2$$

Table 1.1: External Forced Convection Correlations. Source: Fundamentals of Heat and Mass Transfer, Incropera et al., Seventh Edition

Condition	Correlation
Flat plate, laminar flow, constant wall temperature. Properties evaluated at $T_f = (T_w + T_\infty)/2$ $Pr \geq 0.6; \quad Re_x \leq 5 \times 10^5$	$Nu_x = 0.332 Re^{1/2} Pr^{1/3}$ $\overline{Nu}_L = 0.664 Re^{1/2} Pr^{1/3}$
Flat plate, laminar flow, constant wall heat flux. Properties evaluated at $T_f = (T_w + T_\infty)/2$ $Pr \geq 0.6; \quad Re_x \leq 5 \times 10^5$	$Nu_x = 0.453 Re^{1/2} Pr^{1/3}$ $\overline{Nu}_L = 0.680 Re^{1/2} Pr^{1/3}$
Flat plate, turbulent flow, constant wall temperature. Properties evaluated at $T_f = (T_w + T_\infty)/2$ $0.6 \leq Pr \leq 60; \quad 5 \times 10^5 \leq Re_x \leq 10^8$	$Nu_x = 0.0296 Re^{4/5} Pr^{1/3}$
Flat plate, turbulent flow, constant wall heat flux. Properties evaluated at $T_f = (T_w + T_\infty)/2$ $0.6 \leq Pr \leq 60; \quad 5 \times 10^5 \leq Re_x \leq 10^8$	$Nu_x = 0.0308 Re^{4/5} Pr^{1/3}$
Isolated solid sphere. All properties except μ_s (surface viscosity) are evaluated at T_∞. $1.0 \leq (\mu/\mu_s) \leq 3.2$ $0.71 \leq Pr \leq 380; \quad 3.5 \leq Re_D \leq 7.6 \times 10^4$	$\overline{Nu}_D = 2 + \phi(Re) Pr^{0.4} (\mu/\mu_s)^{0.25}$ $\phi(Re) = \left(0.4\sqrt{Re_D} + 0.06 Re^{0.66}\right)$
Cylinder in cross flow. All properties are evaluated at T_f. C and m are given in Table **??**. $Pr \geq 0.7$	$\overline{Nu}_D = C Re_D^m Pr^{1/3}$

Table 1.2: C and m for Nusselt number correlation for cylinder in cross flow. Source: Fundamentals of Heat and Mass Transfer, Incropera et al., Seventh Edition

Re_D	C	m
0.4 – 4	. 0.989	. 0.330
4 – 40	. 0. 911	0.385
40 – 4000.	. 0.683.	. 0.466.
4000 –40,000	. 0.192.	0.618.
40,000 – 400,000	0.027.	. 0.805.

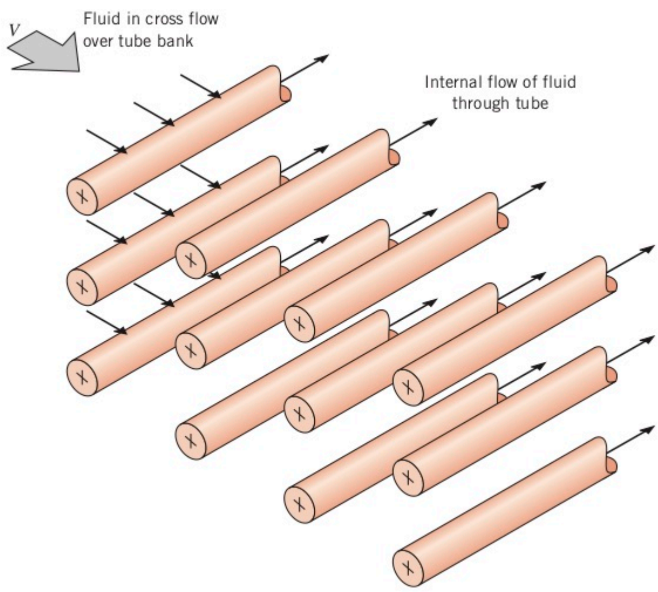


Figure 1.9: 3D view of flow of internal and external fluid through a bank of tubes.

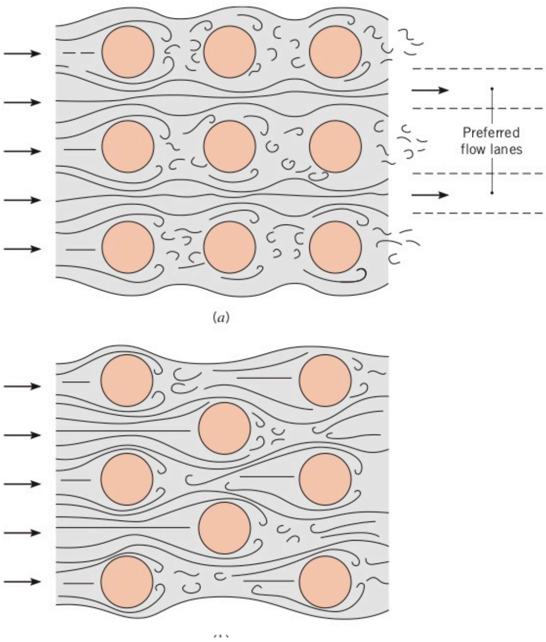


Figure 1.10: Bank of tubes: aligned and staggered arrangements

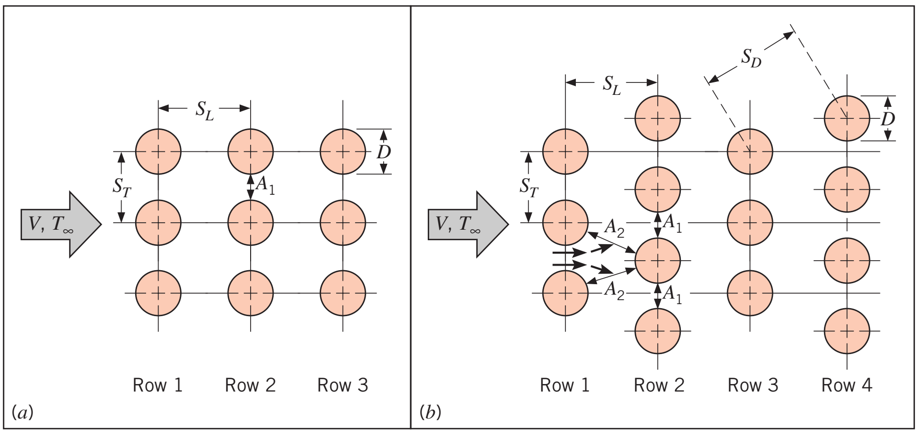


Figure 1.11: Bank of tubes: Details of the geometry

Table 1.3: Constants for the Nusselt number correlation for a bank of tubes. If $10^2 \leq Re \leq 10^3$, approximate as single (isolated) cylinder.

Configuration	$Re_{D,Max}$	C_1	m
Aligned	$10^1 - 10^2$	0.80	0.40
Staggered	$10^1 - 10^2$.	0.90	0.40
Aligned ($S_T/S_L > 0.7$)	$10^3 - 2 \times 10^5$	0.27	0.63
Staggered ($S_T/S_L < 2$)	$10^3 - 2 \times 10^5$	$0.35(S_T/S_L)^{0.2}$	0.60
Staggered ($S_T/S_L > 2$)	$10^3 - 2 \times 10^5$	0.40	0.60
Aligned	$2 \times 10^5 - 2 \times 10^6$	0.21	0.84
Staggered	$2 \times 10^5 - 2 \times 10^6$	0.22	0.84

Table 1.4: Correction factor C_2 for a bank of tubes $N_L < 20$. $Re_{D,Max} \geq 1000$

N_L	1	2	3	4	5	7	10	13	16
Aligned	0.70	0.80	0.86	0.90	0.92	0.95	0.97	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.92	0.95	0.97	0.98	0.99

4. C_1 and m are given in Table **??**.

5. $Re_{D,Max} = V_{max} D/\nu$

6. Range of Re and Pr: $0.7 \leq Pr \leq 500; \quad 1000 \leq Re_{D,Max} \leq 2 \times 10^6$

7. Aligned:
$$V_{Max} = \frac{S_T}{S_T - D} V$$

8. Staggered:
$$\begin{aligned} V_{Max} &= \frac{S_T}{S_T - D} V \quad \text{if } 2(S_D - D) > (S_T - D) \\ &= \frac{S_T}{2(S_D - D)} V \qquad \text{Otherwise} \end{aligned}$$

   Meaning of various quantities in this equation are provided in Figure **??**

1.4 Examples for external forced convection

Along with the four step problem solving methodology discussed at the very beginning of the course, the following five steps must be followed for all convection related problems.

1. Become immediately cognisant of the flow geometry
2. Specify the appropriate reference temperature and evaluate the pertinent fluid properties at that temperature.
3. Calculate the Reynolds number
4. Decide whether a local or surface average coefficient is required.
5. Select the appropriate correlation.

Example 1.3. (Source: Lienhard & Lienhard, Example 6.5) Air at 20°C and moving at 15 m/s is warmed by an isothermal steam heated plate at 110°C, 1/2 m × 1/2 m. Find the average rate of heat transfer. Calculate h, δ_t, δ at the trailing edge.

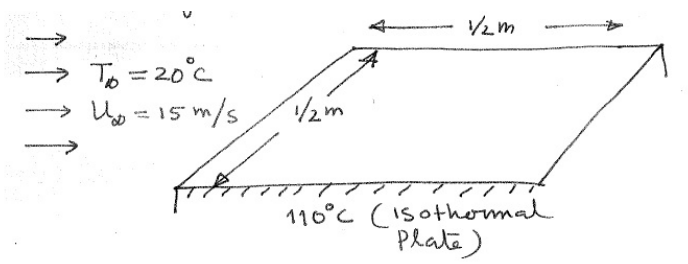


Figure 1.12: Schematic for example

Solution: This is for a flat plate geometry with constant surface temperature. The properties will be evaluated at $T_f = (110 + 20)/2 = 65°C$ At this temperature, $Pr = 0.707, \nu = 19.4 \times 10^{-6}$ and $Re = 386,000$ at the trailing edge. Hence, the flow is laminar since the transition Re for flat plate is 4×10^6.

With these restrictions, the correct expressions for the local heat transfer coefficient, thickness of the hydrodynamic boundary layer and the thermal boundary layer are:

$$\begin{aligned}
Nu(x) &= 0.332 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}} \\
\overline{Nu}_L &= 0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} \\
\delta(x) &= 4.64x(Re)^{-1/2} \\
\delta_t(x) &= \delta(x)(Pr^{-1/3}) \qquad (1.9)
\end{aligned}$$

Using appropriate values:

$$\begin{aligned}
&\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} = 367.8 \\
&\bar{h} = \frac{367.8 \times 0.0289}{0.5} = 21.26 \\
&h(x = L) = 0.5\bar{h} = 10.05 \\
&\delta(x = L) = \frac{4.64L}{\sqrt{Re}} = 3.96\text{mm} \\
&\delta_t(x = L) = \delta(x = L) Pr^{-\frac{1}{3}} = 4.44\text{mm}
\end{aligned}$$

□

Example 1.4. Discuss the variation of h with x and show that for isothermal flat plate heating $\bar{h} = 2h\big|_{x=L}$.

Solution: It can be seen that if properties are reasonably constant over the length of the plate,

$$h_x \propto \frac{1}{\sqrt{x}}$$

Hence, at the leading edge, $h \to \infty$ and it gradually decreases as x increases. Of course, it is not infinity at the leading edge. The reason our analysis leads to this unrealistic result *at the leading edge* is that the boundary layer approximations we made are not valid at the leading edge. For unit thickness of the plate:

$$\dot{Q} = \int_0^L h(x)\Delta T dx = \bar{h}\Delta T L$$

For isothermal plate, ΔT is constant. Hence,

$$\begin{aligned}
\bar{h} &= \frac{1}{L}\int_0^L \frac{0.332k}{x}\left(\frac{xu_\infty}{\nu}\right)^{\frac{1}{2}} Pr^{\frac{1}{3}}dx \\
&= \frac{1}{L}0.332k\left(\frac{u_\infty}{\nu}\right)^{\frac{1}{2}} Pr^{\frac{1}{3}}\int_0^L \frac{dx}{x^{\frac{1}{2}}} \\
&= \frac{1}{L}0.332k\left(\frac{u_\infty}{\nu}\right)^{\frac{1}{2}} Pr^{\frac{1}{3}}\left(2x^{\frac{1}{2}}\Big|_0^L\right) \\
&= \frac{2}{L}0.332k\left(\frac{u_\infty L}{\nu}\right)^{\frac{1}{2}} Pr^{\frac{1}{3}} \\
\overline{Nu_L} &= 0.664Re_L^{1/2}Pr^{\frac{1}{3}}
\end{aligned}$$

□

Example 1.5. Source: Lienhard & Lienhard, Problem 6.6. Air at 15°C flows at 1.8 m/s over a 0.6 m long heating panel. The panel is intended to supply 420 W/m^2 to the air, but the surface can sustain only about 105°C without being damaged. Is it safe? What is the average temperature of the plate?

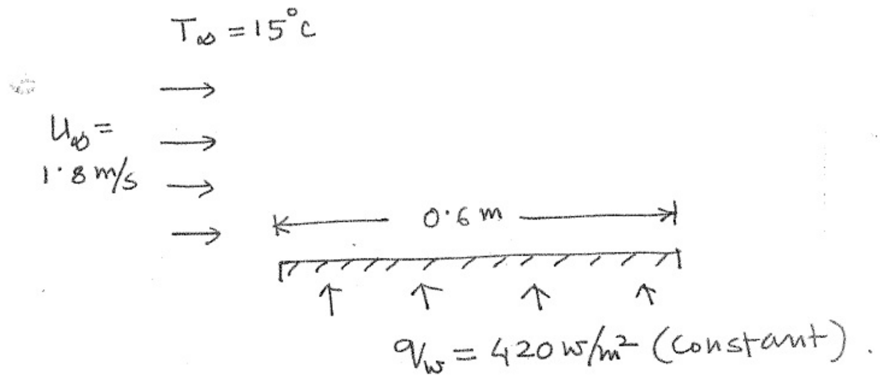


Figure 1.13: Schematic for example

Solution: This is the case of flat plate heat transfer with constant surface heat flux. According to the correlation $h(x)$ decreases with increase in x. Hence, the heat transfer coefficient will be least at the trailing edge. Since the heat flux is constant, the temperature gradient will be maximum at the trailing edge. Because the T_∞ of the inlet air is constant, this will lead to highest temperature of the plate at the trailing edge. Hence, we need to evaluate the local heat transfer coefficient at this location.

The first step for estimation of heat transfer coefficient is evaluation of propoerties. For this we need to know the film temperature. For the film temperature, we need to know the surface temperature which is unknown. Hence, we will have to guess a surface temperature. Take such a temperature so that you get the properties easily from table.

$$T_w = 2T_f - T_\infty = 2T_f - (15 + 273)$$

Let us start by taking $T_F = 300$K which gives $T_w = 312$ K. For such iterative problems it is easier to make a table such as this:

Table 1.5: Table for example

T_f (K)	T_w (K)	Re(L)	Pr	Nu	h	$\Delta(T)$	T_w
300	312	6.04×10^4	0.707	99	4.35	96.6	384
350	412	5.16×10^4	0.700	91	4.56	91.9	380

The correct correlation for this case is:

$$Nu = 0.453 Re^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{0.6 \times 1.8}{\nu}$$

The temperature quickly converges to $380K = 107°C$ which is slightly above the tolerable limit. The average temperature of the plate is defined by:

$$\begin{aligned}
\overline{\Delta T} &= \frac{1}{L}\int_0^L \Delta T(x)dx \\
&= \frac{1}{L}\int_0^L \frac{q_w}{h(x)}dx \\
&= \frac{q_w}{L}\int_0^L \frac{x/k}{Nu(x)}dx \\
&= \frac{q_w}{Lk}\int_0^L \frac{xdx}{0.453Re^{\frac{1}{2}}Pr^{\frac{1}{3}}} \\
&= \frac{q_w}{0.453LkPr^{\frac{1}{3}}}\int_0^L \frac{xdx}{x^{\frac{1}{2}}u_\infty^{\frac{1}{2}}/\nu^{\frac{1}{2}}} \\
&= \frac{q_w\nu^{\frac{1}{2}}}{0.453LkPr^{\frac{1}{3}}u_\infty^{\frac{1}{2}}}\int_0^L x^{\frac{1}{2}}dx \\
&= \frac{q_w\nu^{\frac{1}{2}}L^{\frac{3}{2}}\frac{2}{3}}{0.453LkPr^{\frac{1}{3}}u_\infty^{\frac{1}{2}}} \\
&= \frac{2}{3}\frac{q_w(L/k)}{0.453Re_L^{\frac{1}{2}}.Pr^{\frac{1}{3}}} \\
&= \frac{2}{3}\frac{q_w(L/k)}{Nu_L} \\
&= \frac{2}{3}\frac{q_w}{h_L} \\
&= \frac{2}{3}\Delta T_L
\end{aligned}$$

Therefore, the average temperature drop will be $0.667 \times (107-15)$°C = 61.36 °C. □

Example 1.6. Prob 7.42 (Incropera) A circular pipe of 25 mm outer diameter is placed in an airstream at 25°C and 1atms. The air moves at 15 m/s, while the outer surface is maintained at 100°C.Calculate the rate of heat loss/unit length of the pipe.

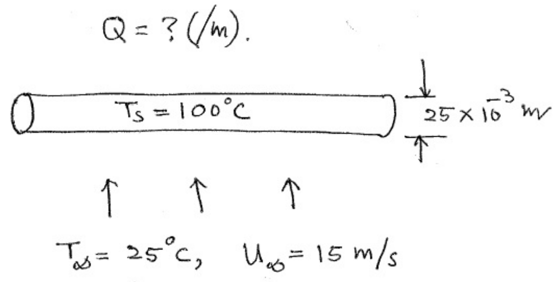


Figure 1.14: Schematic for example

Solution:

$$
\begin{aligned}
T_f &= \frac{25+100}{2} = 62.5°\text{C} = 335.5\text{K} \\
u_\infty &= 15m/s; \quad \nu = 19.4 \times 10^{-6}; \quad \Pr = 0.702 \\
Re &= \frac{25 \times 10^{-3} \times 15}{10.4 \times 10^{-6}} = 1.9 \times 10^4 \\
\overline{Nu_D} &= 0.193 Re^{0.618} Pr^{\frac{1}{3}} = 76.4 \\
\overline{h} &= \frac{76.4 \times k}{D} = \frac{76.4 \times 28.9 \times 10^{-3}}{25 \times 10^{-3}} = 88.3 \\
\dot{Q} &= 1 \times \pi \times 25 \times 10^{-3} \times 88.3 \times (100-25)\text{W/m} = 520\text{W/m}
\end{aligned}
$$

□

Example 1.7. Incropera 7.7 (Pg 443) A bank of tubes whose surface temperature is 70°C is available to heat air (15°C) flowing at 6 m/s. The tubes are arranged in staggered arrangement with geometries as shown in Figure. Determine the air side convection coefficient and rate of heat transfer.

Solution: First we have to find the value of V_{Max}. Since this is a staggered arrangement, the expression depends on the values of $2(S_D - D)$ and $S_T - D$. First let us evaluate the diagonal pitch S_D:

$$S_D = \left[S_L^2 - \left(\frac{S_T}{2} \right)^2 \right]^{\frac{1}{2}} = 37.7$$

Now, using $S_T = 31.3$ mm, $S_L = 34.3$ mm and $D = 16.4$ mm:

$$2(S_D - D) = 2(37.7 - 16.4) = 42.6$$

$$(S_T - D) = (31.3 - 16.4) = 14.9$$

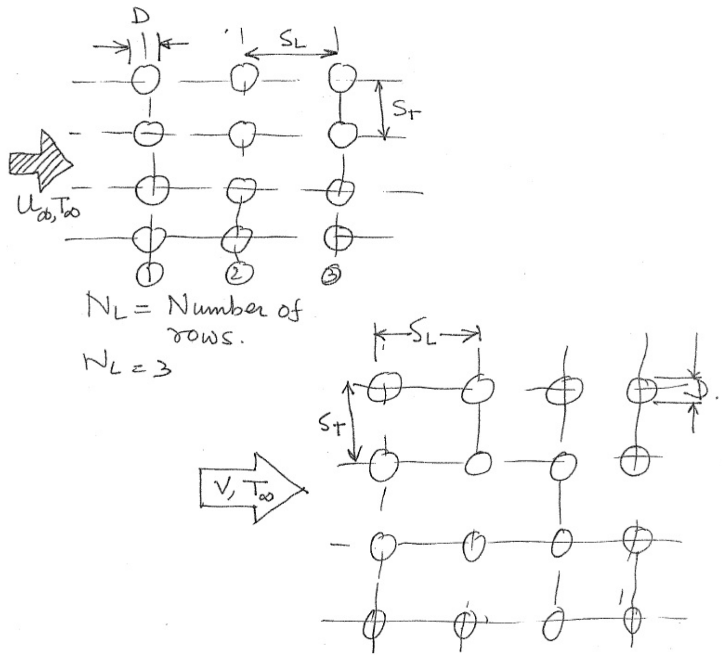


Figure 1.15: Schematic for example

Hence, V_{Max} occurs at A_1 which is given by:

$$V_{Max} = \frac{31.3}{31.3 - 16.4} \times 6\text{m/s} = 12.6m/s$$

Next, we will have to obtain the Reynolds number. For this we need to evaluate properties which is to be evaluated at the arithmetic mean of the inlet and outlet temperatures. However, the outlet temperature is not known. Let us assume that the outlet air reaches a temperature close to the surface temperature of the tubes and hence the properties will be evaluated at:

$$T_f = (15 + 70)/2°C \approx 42°C$$

Accordingly,

$$Re_{D,Max} = \frac{16.4 \times 10^{-3} \times 12.6}{17.4 \times 10^{-6}} = 11,875; \quad Pr = 0.705; \quad Pr_s = 0.701$$

Since $S_T/S_L = 0.912$ in this case,

$$C_1 = 0.35 \times (.912)^{\frac{1}{5}} = 0.3436$$
$$C_2(N_L < 20) = 0.95$$
$$N\bar{u}_D = 0.95 \times 0.34(11875)^{.6} \times \left(\frac{0.705}{0.701}\right)^{.25} = 90$$
$$\bar{h} = 90 \times \frac{0.0274}{0.0164} = 150W/m^2.K$$

Now, to obtain the outlet temperature, we need to write an energy balance. If N_T tubes are present in transverse direction, the total area of flow is $N_T S_T$. Hence the inlet mass flow rate for air is $\rho V N_T S_T$. Hence the total change in energy for the air between inlet and outlet is:

$$(T_o - T_i)\rho V N_T S_T c_{p,air}$$

This much energy was transferred to the air by heat transfer from a surface which is at T_s. The total area of heat transfer is $A = \pi D N$ where N is the total number of pipes in the bundle. Unit depth has been taken. Hence, the total amount of heat transfer is:

$$\pi D N_T N_L \bar{h} \Delta T_{lm}$$

where $\Delta T_l m$ is the logarithmic mean of the temperature difference between the inlet and the outlet. Its expression is given as:

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_o)}{ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} = \frac{T_o - T_i}{ln\left(\frac{T_s - T_i}{T_s - T_o}\right)}$$

Writing the energy balance:

$$\pi D N_T N_L \bar{h} \frac{T_o - T_i}{ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} = (T_o - T_i)\rho V N_T S_T c_{p,air}$$

Simplifying:

$$\frac{\pi D N_L \bar{h}}{\rho V S_T c_{p,air}} = ln\left(\frac{T_s - T_i}{T_s - T_o}\right)$$

Substituting appropriate values, we get $T_o = 50C$. Hence the previous calculation should be repeated for corrected value of T_f $\square$

1.5 Internal Forced Convection

In internal forced convection, we will discuss heat transfer between the surface of a tube and the fluid flowing inside the pipe. We shall take the following assumptions while discussing the internal forced convection:

1. Pipe is of constant cross section. Flow is steady. Mass flow rate is constant.
2. For fully developed flow the velocity profile is parabolic, given by:
$$u(r) = u_{max}\left(1 - (r/r_o)^2\right)$$
3. Hydrodynamic entry length for laminar flow: $\left(\frac{X_{fd,h}}{D}\right)_{lam} = 0.05Re_D$
4. Hydrodynamic entry length turbulent flow: $\left(\frac{X_{fd,h}}{D}\right)_{turb} > 10$

The development of velocity and temperature profile for a pipe are shown in Figures.

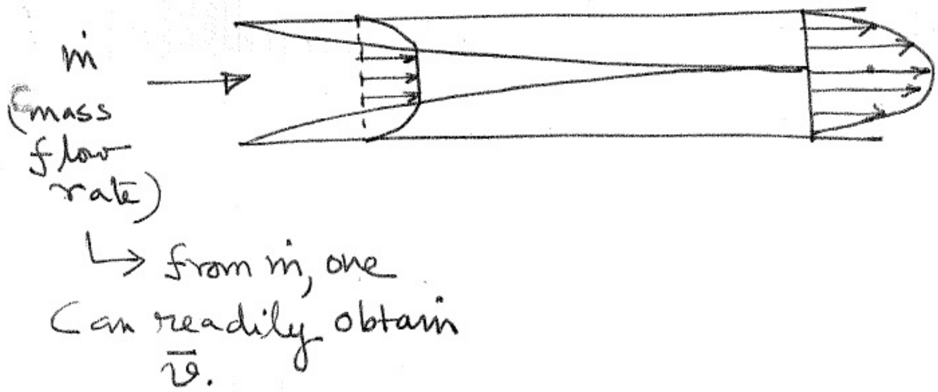


Figure 1.16: Development of velocity profile for pipe flow

First let us examine how we can apply Newton's law of cooling for the internal forced convection. In internal forced convection, the concept of T_∞ is not valid. Hence, we need another temperature to replace T_∞. The mean temperature at a given cross section is used instead of T_∞. The mean temperature is also called bulk temperature or the mixing cup temperature. We will denote it as T_m.

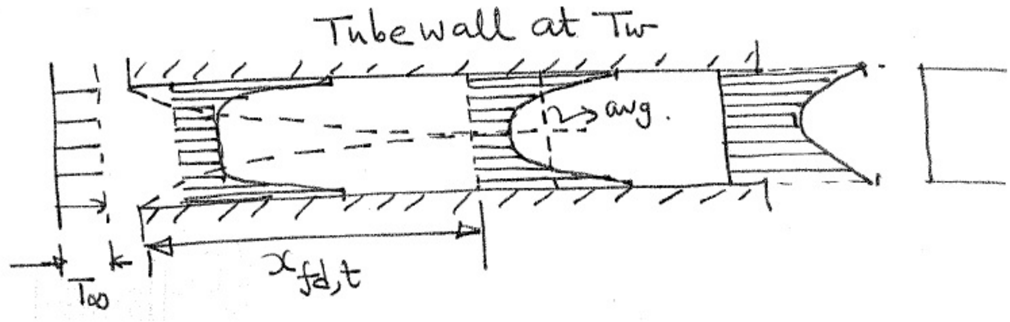


Figure 1.17: Evolution of temperature profile inside a pipe

Mean temperature

T_m is defined in terms of average enthalpy at a given cross section of pipe. If the specific enthalpy of the fluid at a radial location is denoted by $\hat{h}$, the total enthalpy passing through a given cross section of the pipe can be written as:

$$\int_0^{r_o} 2\pi r\rho u(r)\hat{c}_p(T(r,x) - T_{ref})dr$$

Since mass flow rate is constant, we can define a mean specific enthalpy for the cross section as:

$$\hat{h}_m = \frac{1}{\dot{m}}\int_0^{r_o} 2\pi r\rho u(r)\hat{c}_p(T(r,x) - T_{ref})dr$$

The average temperature at a cross section is defined using this average enthalpy:

$$\hat{h}_m = \hat{c}_p(T_m - T_{ref})$$

Setting $T_{ref} = 0$ and assuming $\hat{c}_p$ to be constant:

$$T_m(x) = \frac{2\pi\rho}{\dot{m}}\int_0^{r_o} ru(r)T(r,x)dr \tag{1.10}$$

and the Newton's law of cooling for internal flow can be written as:

$$q = h(T_s(x) - T_m(x)) \tag{1.11}$$

Fully developed condition

It should be noted that for heat transfer applications, the average temperature (T_m) keep increasing throughout the heat transfer length of the tube

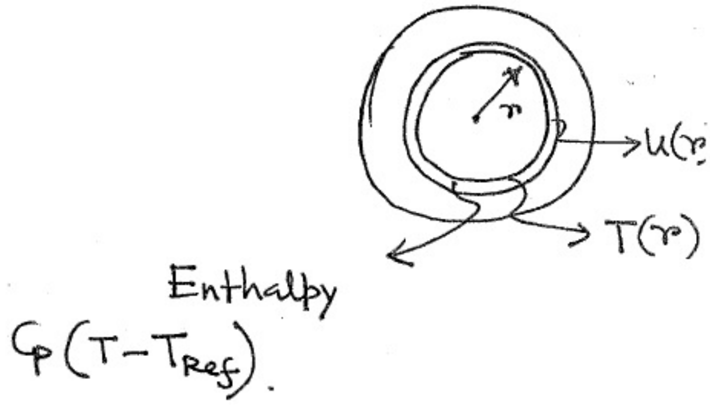


Figure 1.18: Mixing cup temperature

as shown in Fig. ($dT_m(x)/dx \neq 0$). Similarly, $dT(x,r)/dx$ also changes constantly over the entire heating length. Hence, for heat transfer, the condition for fully developed temperature profile cannot be written as:

$$\frac{d}{dx}T(x,r) = 0 \quad \text{or} \quad \frac{dT_m}{dx} = 0$$

However, the dimensionless temperature

$$\hat{T} = \frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}$$

reaches a constant value after a brief entry length for both constant surface temperature and constant surface heat flux conditions. Hence, for heat transfer, thermally fully developed condition is written as:

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right]_{fd,t} = 0 \tag{1.12}$$

The thermal entry length for laminar flow is given by:

$$\left(\frac{x_{fd,t}}{D}\right)_{lam} = 0.05 Re_D.Pr$$

For turbulent flow:

$$\left(\frac{x_{fd,t}}{D}\right)_{turb} \approx 10$$

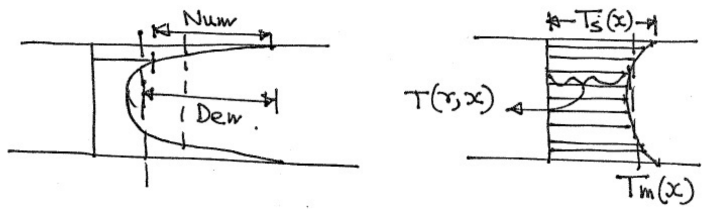


Figure 1.19: Schematic showing various quantities shown in equation

Constant h for thermally fully developed flow

A number of important deductions can be made from equation ??. Since $\hat{T}$ is independent of x, $\partial\hat{T}/\partial r$ is also independent of x. Mathematically,

$$\frac{\partial}{\partial r}\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right] \neq f(x)$$

or,

$$\frac{-1}{T_w(x)-T_m(x)}.\frac{\partial T(r,x)}{\partial r} \neq f(x)$$

Restricting for wall conditions, $r = r_o$:

$$\frac{-\partial T(r,x)/\partial r\Big|_{r=r_o}}{T_w(x)-T_m(x)} \neq f(x)$$

Using

$$q_w = -k\frac{\partial T(r,x)}{\partial r}\Big|_{r=r_o} = h(T_w(x)-T_m(x))$$

we obtain:

$$\frac{h}{k} \neq f(x) \tag{1.13}$$

Which implies that the Nusselt number is constant for thermally fully developed conditions, for both constant surface temperature and constant surface heat flux cases, provided properties are constant.

Fully developed flow: relation between temperature derivatives

For fully developed flow, constant surface heat flux, another useful relation can be obtained. Using Eq?? for fully developed constant heat flux case,

$$(T_w(x)-T_m(x)) = \frac{q}{h} = \text{constant}$$

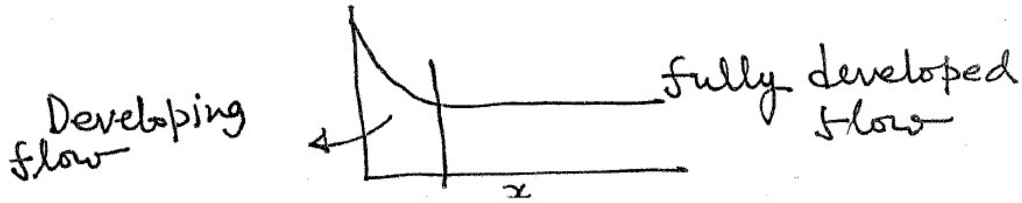


Figure 1.20: Variation of h over the length of tube

Differentiating wrt x:

$$\frac{dT_w(x)}{dx} = \frac{dT_m(x)}{dx} \tag{1.14}$$

Now, expanding **??**,

$$\left[\frac{\partial T_w(x)}{\partial x} - \frac{\partial T(r,x)}{\partial x}\right](T_w(x) - T_m(x)) - \left[\frac{\partial T_w(x)}{\partial x} - \frac{\partial T_m(x)}{\partial x}\right](T_w(x) - T(r,x)) = 0$$

Restricting for constant surface heat flux, using equation **??**and recognizing that $T_w(x) \neq T_m(x)$:

$$\frac{\partial T_w(x)}{\partial x} = \frac{\partial T(r,x)}{\partial x} \tag{1.15}$$

Combining equations **??** and **??**

$$\frac{\partial T_w(x)}{\partial x} = \frac{\partial T(r,x)}{\partial x} = \frac{dT_m(x)}{dx} \tag{1.16}$$

Energy balance for pipe flow

Additional equation for $dT_m(x)/dx$ can be obtained by writing an energy balance equation over a differential length of the tube as shown in Figure **??**:

$$\dot{m}\hat{c}_p T_m(x) + 2\pi r_o dx q_w = \dot{m}\hat{c}_p T_m(x + dx)$$

Simplifying:

$$\frac{dT_m(x)}{dx} = \frac{2\pi r_0 q_w}{\dot{m}\hat{c}_p} \tag{1.17}$$

Equation **??** is a generalized energy balance. It applies to constant surface temperature, constant surface heat flux or even for the entry region.

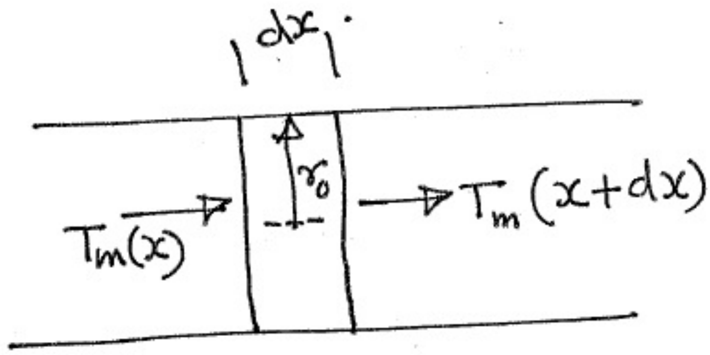


Figure 1.21: Pipe flow energy balace

Qualitative temperature profile for constant wall flux

For the constant surface heat flux case q_w is constant and hence from Eq **??**,

$$\frac{dT_m(x)}{dx} = \text{Constant}$$

provided $\hat{c}_p$ is constant. Because $h(x)$ is higher in the entry region, ΔT will be smaller in the entry region for constant q_w. When the flow becomes thermally fully developed, h reaches its constant (smaller) value so that ΔT also reaches a constant value (higher). This scenario is shown in Figure **??**.

Qualitative temperature profile for constant wall temperature

For constant wall temperature T_w is independent of x and it is advantageous to use $\Delta T(x) = T_w - T_m(x)$ instead of $T_m(x)$. Since,

$$\frac{d\Delta T(x)}{dx} = \frac{d}{dx}(T_w - T_m(x)) = -\frac{dT_m(x)}{dx}$$

we can re-write the energy balance for this case as:

$$\frac{d\Delta T}{dx} = -\frac{2\pi r_o}{\dot{m}\hat{c}_p} q_w = -\frac{2\pi r_o}{\dot{m}\hat{c}_p} h(x)\Delta T(x)$$

We have written $h(x)$ so that the entry region is also included. Integrating between the inlet and outlet:

$$\int_{\Delta T_i}^{\Delta T_o} \frac{d\Delta T}{\Delta T} = -\frac{2\pi r_o}{\dot{m}\hat{c}_p} \int_{x=0}^{x=L} h(x)dx$$

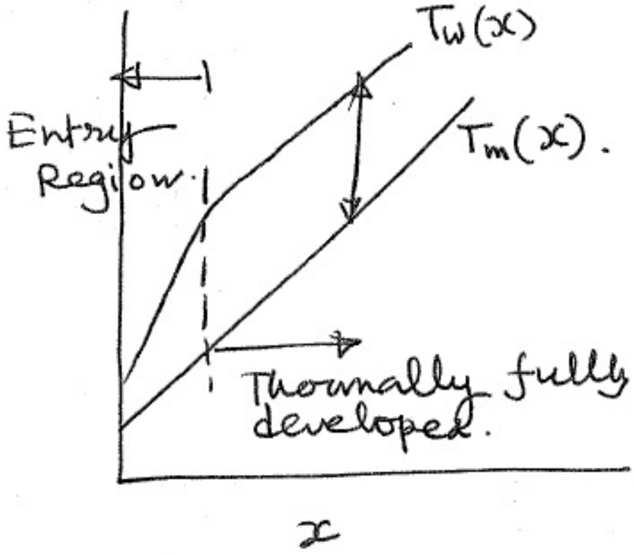


Figure 1.22: Length wise temperature variation for constant wall flux

$$\ln \frac{\Delta T_o}{\Delta T_i} = -\frac{2\pi r_o L \overline{h}_L}{\dot{m}\hat{c}_p}$$

$$\frac{\Delta T_o}{\Delta T_i} = \exp\left(-\frac{A_T \overline{h}_L}{\dot{m}\hat{c}_p}\right) \tag{1.18}$$

This expression shows that ΔT decays exponentially for the constant surface temperature case. This is shown in Figure. **??**

Rate of heat transfer for constant wall temperature

The total rate of heat transfer for a given length of tube is given by:

$$\begin{aligned} \dot{Q}_{tot} &= \dot{m}\hat{c}_p(T_{m,o} - T_{m,i}) \\ &= \dot{m}\hat{c}_p[(T_s - T_{m,i}) - (T_s - T_{m,o}) \\ &= \dot{m}\hat{c}_p(\Delta T_i - \Delta T_o) \end{aligned}$$

From equation **??**

$$\dot{m}\hat{c}_p = \frac{A_T \overline{h}_L}{ln(\Delta T_0/\Delta T_i)}$$

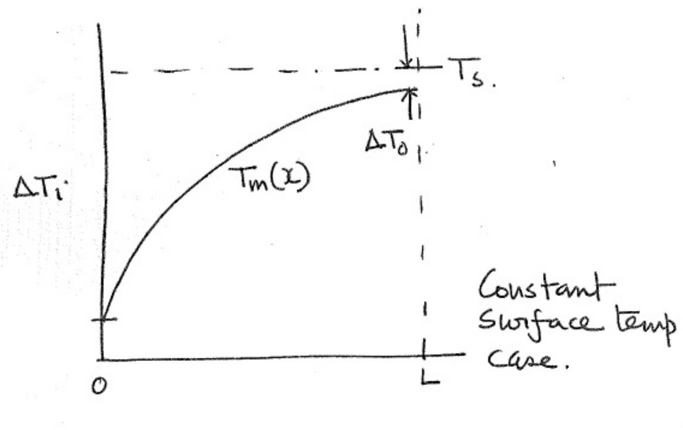


Figure 1.23: Lengthwise temperature variation for the constant surface temperature case

Substituting from above:

$$\dot{Q}_{tot} = A\bar{h_L}.\left(\frac{\Delta T_0 - \Delta T}{ln(\Delta T_0/\Delta T_i)}\right) = A.\bar{h_L}\Delta T_{lm} \tag{1.19}$$

The relationships developed in this section are given in Table ??

1.6 Correlation for internal flow

In the previous section, we have developed enough insight so that we can obtain the amount of heat transfer and temperature profiles for a given length of tube provided we know the heat transfer coefficient. In this section, we shall develop the analytical expression for heat transfer coefficient (Nusselt number correlation) for laminar, thermally and hydrodynamically fully developed flow. Again, we will write the energy balance for the elementary volume as shown in Figure ??.

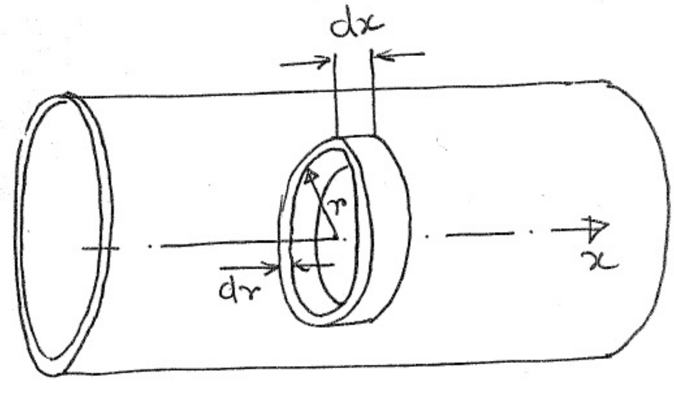


Figure 1.24: Elementary volume inside a pipe. Flow is thermally and hydrodynamically fully developed.

Table 1.6: Summary of improtant relations for internal forced convection

Condition	**Equations**
Hydrodynamic entry length, laminar flow	$\left(\frac{x_{fd,h}}{D}\right)_{lam} = 0.05Re_D$
Hydrodynamic entry lenght, turbulent flow	$\left(\frac{x_{fd,h}}{D}\right)_{turb} \approx 10$
Thermal entry length, laminar flow	$\left(\frac{x_{fd,t}}{D}\right)_{lam} = 0.05Re_D Pr$
Thermal entry length, turbulent flow	$\left(\frac{x_{fd,t}}{D}\right)_{turb} \approx 10$
Mean temperature or mixing cup temperature	$T_m(x) = \dfrac{2\pi\rho}{\dot{m}} \displaystyle\int_0^{r_o} ru(r)T(r,x)dr$
Thermally fully developed condition	$\dfrac{\partial}{\partial x}\left[\dfrac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right]_{fd,t} = 0$
For thermally fully developed condition	$h \neq f(x)$
Relation between temperature derivatives: fully developed, constant surface heat flux	$\dfrac{\partial T_s(x)}{\partial x} = \dfrac{\partial T(r,x)}{\partial x} = \dfrac{\partial T_m(x)}{\partial x}$
Energy balance for pipe flow	$\dfrac{dT_m(x)}{dx} = \dfrac{2\pi r_0 q_w}{\dot{m}\hat{c}_p}$
Length wise mean temperature variation, constant wall temperature	$\Delta T_o = \Delta T_i \exp\left(-\dfrac{A_T\bar{h}}{\dot{m}\hat{c}_p}\right)$
Total rate of heat transfer, constant wall temperature	$\dot{Q}_{tot} = A_T\bar{h}\Delta T_{lm}$

For laminar fully developed flow, no r velocity exists. Hence, no convection is possible in r direction. Heat transfer in r direction occur only through conduction. Hence, the x direction convective flow of energy is balanced by r direction conduction:

$$2\pi r dr u(r)\rho c_p[T_x - T_{x+dx}] + \left[-k.\frac{\partial T}{\partial r}.2\pi r dx\right]_r - \left[-k.\frac{\partial T}{\partial r}.2\pi r dx\right]_{r+dr} = 0$$

Simplifying:

$$\begin{aligned} -2\pi r u(r)\rho c_p \frac{\partial T}{\partial x} + 2\pi k \frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] &= 0 \\ u(r)\frac{\partial T}{\partial x} &= \frac{\alpha}{r}\frac{\partial}{\partial r}\left[r.\frac{\partial T}{\partial r}\right] \end{aligned}$$

Substituting the velocity profile

$$u_{max}\left(1-\frac{r^2}{r_o^2}\right)\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right]$$

Using $u_{max} = 2\bar{u}$, $\bar{u}\pi r_o^2\rho = \dot{m}$ and

$$\frac{\partial T}{\partial x} = \frac{\partial T_m}{\partial x}$$

$$\frac{2\dot{m}}{\pi r_o^2 \rho}\left(1-\frac{r^2}{r_o^2}\right)\frac{\partial T_m}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right]$$

Using energy balance:

$$2.\frac{\dot{m}}{\pi r_o^2 \rho}\left(1-\frac{r^2}{r_o^2}\right)\frac{2\pi r_o q_w}{\dot{m}c_p} = \frac{k}{\rho c_p}\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\right]$$

Simplifying:

$$\left(1-\frac{r^2}{r_o^2}\right)\frac{4q_w}{r_o k} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right]$$

If we restrict ourselves to constant wall heat flux case, this expression can be readily integrated:

$$\frac{\partial T}{\partial r} = \frac{4q_w}{k}\left(\frac{r}{2r_o} - \frac{r^3}{4r_o^3}\right) + \frac{C_1}{r}$$

Since the temperature gradient at $r = 0$ must be finite, $C_1 = 0$. Integrating once more:

$$T(r,x) = \frac{4q_w}{k}\left(\frac{r^2}{4r_o} - \frac{r^4}{16r_o^3}\right) + C_2$$

To evaluate C_2, we need a condition at $r = r_o$ which is given in terms of flux only. Hence, a little roundabout way is needed for evaluation of the constant C_2. Using the above expression for temperature profile into the expression for the mean temperature:

$$T_m = \frac{1}{\dot{m}} 2\pi\rho \int_0^{r_o} r \frac{2\dot{m}}{\pi r_o^2 \rho} \left(1 - \frac{r^2}{r_o^2}\right) \left[\frac{4q_w}{k}\left(\frac{r^2}{4r_o} - \frac{r^4}{16r_o^3}\right) + C_2\right] dr$$

which gives:

$$\begin{aligned} T_m &= \frac{4}{r_o^2}\left[\frac{4q_w}{r_o k}.\frac{7}{384} r_o^4 + C_2 \frac{r_o^2}{4}\right] \\ &= q_w.\frac{7}{24}.\frac{r_o}{k} + C_2 \end{aligned}$$

Using C_2 from above:

$$T(r,x) = \frac{4q_w}{k}\left(\frac{r^2}{4r_o} - \frac{r^4}{16r_o^3}\right) + T_m - \frac{7}{24}\frac{q_w r_o}{k}$$

and restricting to wall:

$$T_w(x) = \frac{4q_w}{k}\left(\frac{r_o}{4} - \frac{r_o}{16}\right) + T_m - \frac{7}{24}\frac{q_w r_o}{k}$$

we get,

$$T_w(x) - T_m = \frac{q_w}{k}\frac{11 r_o}{24} = \frac{q_w}{k}\frac{11D}{48}$$

Using Newton's law of cooling:

$$\frac{q_w}{h} = \frac{q_w}{k}\frac{11D}{48}$$

Finally:

$$\frac{h.D}{k} = 4.364$$

The above relation is applicable only for a thermally and hydrodynamically fully developed laminar flow with constant heat flux. For thermally and hydrodynamically fully developed laminar flow with constant surface temperature similar correlation can be obtained as shown in Table **??**. In the entry region, two different scenario are possible: in many cases, the fluid flows through the pipe for a reasonable length before entering the heating section. For these cases, the thermal profile develops in presence of a fully developed velocity profile. This case is called thermal entry length problem.

In another case, both flow and temperature profile develops together. This case is called combined entry length problem. The laminar flow correlations for the average Nusselt number for these cases for constant surface temperature are given in Table **??**. Please note that in these cases, product of Re, Pr and (D/x) appear in the correlation which is known as Greatz number:

$$Gz. = (D/x)Re_D Pr$$

For turbulent flow in smooth circular tube, the local Nusselt number is given by Dittus Boelter equation given in Table **??**. This equation is applicable to both constant surface temperature and constant surface heat flux. In this equation, $n = 0.4$ for heating $(T_s > T_m)$ and $n = 0.3$ for cooling $(T_s < T_m)$. The range of prameters for this equation are:

$$\begin{aligned} 0.6 \leq Pr &\leq 160 \\ Re &\geq 10,000 \\ L/D &> 10 \end{aligned}$$

For dittus boelter equation, properties are to be evaluated at $(T_{mi} + T_{mo})/2$.

1.7 Examples for internal forced convection

Example 1.8. Hot air flows with a mass rate of $\dot{m} = 0.05$ kg/s through an uninsulated sheet metal duct of diameter $D = 0.15$ m. The hot air enters the duct at 103 °C and after a distance of 5 m cools to 77 °C. The heat transfer coefficient of the outer surface is 6 W/m^2 K and the temperature of the outside air far from the pipe is 0 °C. Find:

- The rate of heat loss for the entire length of the pipe
- The heat flux and surface temperature at $x = 5$ m.

Solution: Since the inlet and outlet temperatures are given, the total heat transfer rate can be obtained as:

$$\dot{Q} = \dot{m}\hat{c}_p(T_{m,o} - T_{m,i})$$

We need the value of $\hat{c}_p$. The temperature for property evaluation for this case is:

$$\overline{T} = (103 + 77)/2\ °C = 90\ °C = 363\ K$$

Table 1.8: Heat Transfer Correlations for pipe flow

Condition	**Correlation**
Laminar, constant wall flux, thermally and hydrodynamically fully developed	$Nu_D = \frac{hD}{k} = 4.36$
Laminar, constant wall temperature,thermally and hydrodynamically fully developed	$Nu_D = \frac{hD}{k} = 3.66$
Laminar flow, constant surface temperature, thermal entry length problem. All Prandtl number.	$\overline{Nu}_D = 3.66 + \frac{0.068Gz}{1 + 0.04Gz^{2/3}}$
Laminar flow, constant surface temperature, combined entry length problem. $Pr \gtrapprox 5$	$\overline{Nu}_D = 3.66 + \frac{0.068Gz}{1 + 0.04Gz^{2/3}}$
Dittus Boelter equation	$Nu_D = 0.023Re^{0.8}Pr^n$

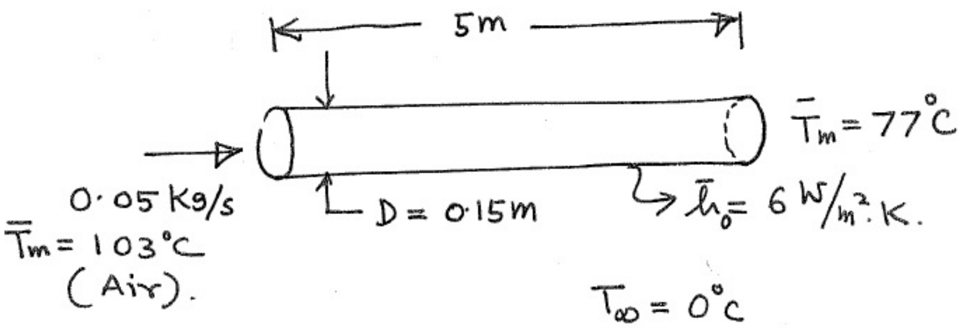


Figure 1.25: Schematic for example

Using value of $\hat{c}_p$ at this temperature:

$$\dot{Q} = 0.05 \times 1010 \times (130 - 70) = 1313\ W$$

To obtain the surface flux at $x = 5$ m, we can use the steady state flux equality:

$$q_L = h_{o,L}(T_{w,L} - T_\infty) = h_{i,L}(T_{m,L} - T_{w,L})$$

If $h_{i,L}$ can be obtained, we can solve the above equalities for $T_{w,L}$ and q_L. First let is obtain the Re:

$$Re = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.05}{\pi \times 0.15 \times 208 \times 10^{-7}} = 20,384$$

and $Pr = 0.7$. Since the flow is turbulent, the entry length is 1.5 m. Hence, at $x = 5$ m, the flow is both hydrodynamically and thermally fully developed. Using Dittus Boelter equation we get Nu= 57.9 and $h_{i,L} = 11.6$ W/m^{2}K. Now, using the second pair of the equality, we get, $T_{w,L} = 50.75$ and using the value of $T_{w,L}$, $q_L = 304.5$ W/m^2. □

Example 1.9. Engine oil is heated by flowing through a circular tube of diameter D= 50 mm and length L = 25 m. Before entering the 25 m heating section, the fluid passes through 1 m of unheated length. The surface of the tube is maintained at 150 °C. If the flow rate and inlet temperature of the oil are 0.5 kg/s and 20 °C, what is the outlet temperature $T_{m,o}$? What is the total rate of heat transfer for the entire tube?

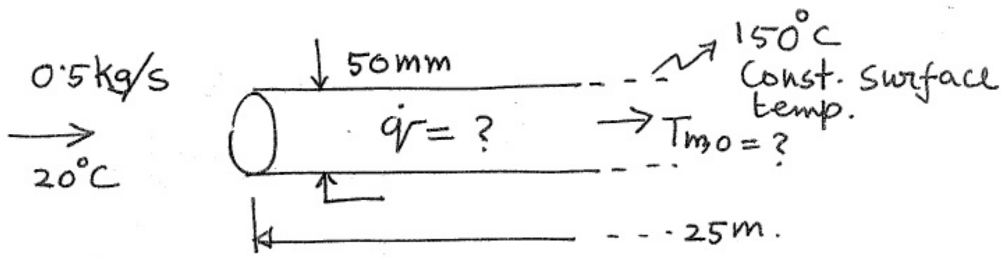


Figure 1.26: Schematic for example

Solution. We cannot proceed unless we evaluate various properties. Hence, we have to assume an outlet temperature. Let us assume $T_{m,o} = 140°C$ giving $\overline{T} = 80\ °C = 353\ K$. The values of various properties at this temperature are:

$\rho = 852$ kg/m^3, $\mu = 3.56 \times 10^{-2}$ Ns/m^2, Pr=546. Using these values:

$$Re = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.5}{\pi \times 50 \times 10^{-3} \times 3.56 \times 10^{-2}} = 358$$

For this laminar flow problem, the entry lengths are:

$$x_{fd,h} = 0.05DRe = 0.895 \text{ m}$$

$$x_{fd,t} = 0.05DRePr = 488 \text{ m}$$

Hence, the flow will be hydrodynamically developed while entering the heating length and the temperature profile will develop in presence of a fully developed velocity profile. Clearly, this is a thermal entry length problem. Hence we will require the value of the Greatz number:

$$Gz = 2 \times 10^{-3} \times 358 \times 546 = 390$$

Using appropriate correlation:

$$\overline{Nu}_D = 3.66 + \frac{0.068 \times 390}{1 + 0.04 \times 390^{2/3}}$$

Giving

$$\overline{Nu}_D = 11.11 \quad \text{and} \quad \overline{h} = 30.68$$

Using equation **??**, $T_{m,o} = 34$ °C. Hence, our initial assumption of outlet temperature was incorrect. Using 34 °C as next guess, we will see that the outlet temperature converge to 36 °C and the converged value of total heat transfer is $\dot{Q} = 15.7$ kW. □

Example 1.10. An annulus consists of the region between two concentric tubes having diameter of 4 cm and 5 cm. Ethylene glycol flows in this space at a velocity of 6.9 m/s. The entrance temperature is 20 °C and the exit temperature is 40 °C. Only the inner tube is the heating surface and it is maintained at 80 °C. Calculate the length of annulus necessary for the required heat transfer.

Solution. This is a case of internal flow through annulus. We will obtain the equivalent diameter and proceed as usual.

$$D_{eq} = \frac{4\pi(D_o^2 - D_i^2)/4}{\pi(D_i + D_o)} = (D_o - D_i) = 1 \text{ cm}$$

Using properties at $\overline{T} = 30$ °C:

$$Re = \frac{1 \times 10^{-2} \times 6.9 \times 1130}{15.5 \times 10^{-3}} = 5000$$

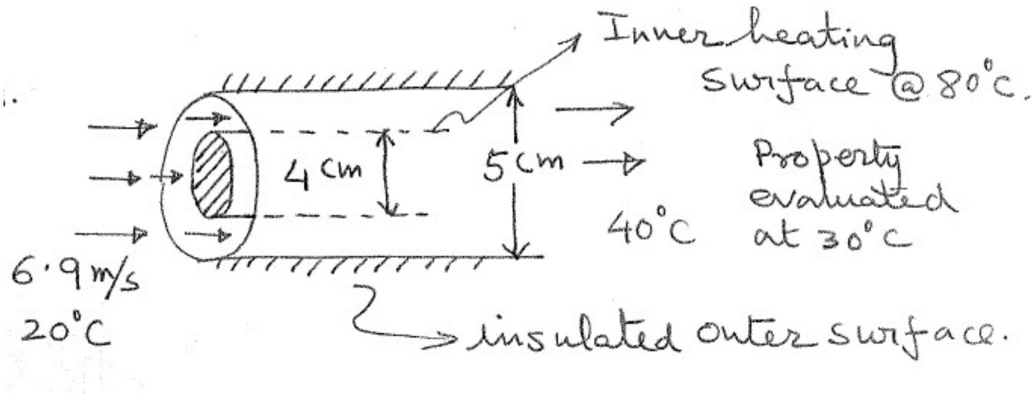


Figure 1.27: Schematic for example

and Pr = 151. Since this is a turbulent flow, we can use Dittus boelter equation which gives:

$$Nu = 0.023 \times (5 \times 10^3)^{0.8} \times 151^{1/3} = 111.47$$

or,

$$h = 2.82 \times 10^{-3} \text{ W/m}^2 \text{ K}$$

Using Eq **??**,

$$\frac{40}{60} = \exp\left(-\frac{\pi \times 4 \times 10^{-2} \times L \times 2.82 \times 10^3}{7.17 \times 2415}\right)$$

we get , L = 19.88m ≈ 20 m □

Example 1.11. Water at 0.4 Kg/s is to be cooled from 71° C to 32° C. Which would result in less pressure drop, to run the water through a tube of 12.5 mm diameter or through a tube of 15. mm diameter? Wall temperature of 12.5 mm diameter pipe is 4 °C and 15 mm diameter pipe is 27 °C.

Solution. For this case, properties to be evaluated at $\overline{T} = (71 + 32)/2 =$ 52 °C. At this temperature the values of various properties in SI unit are:

$$\hat{c}_p = 4.18 \times 10^{-3}, \quad k = 645 \times 10^{-3}, \quad \mu = 528 \times 10^{-6}, \quad Pr = 3.42$$

Using these values,

$$\dot{Q} = 6.52 \times 10^4 J$$

Now, we can obtain the Re as shown in Table **??**. It can be seen that for both cases, the flow is turbulent. Hence, the entrance length for hydrodynamic and thermal profile is same and is small as shown in Table. Hence, we can

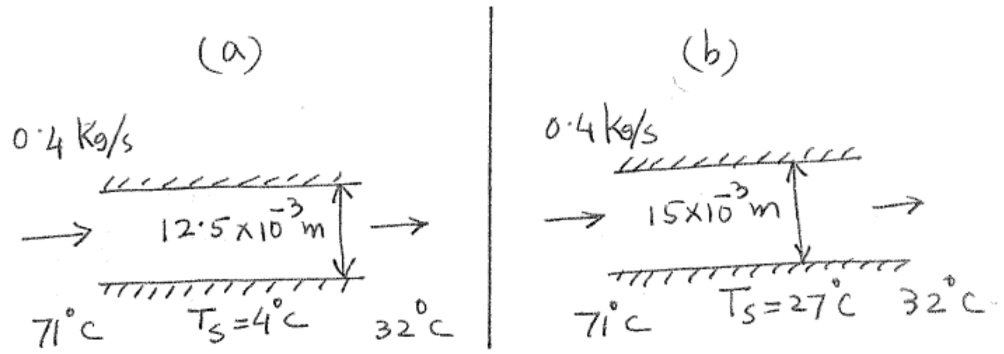


Figure 1.28: Schematic for example

Variable	Unit	Case-A	Case-B
D	m.	12.5×10^{-3}.	15×10^{-3}
$\overline{V}$.	m/s.	3.3	3.2
Re	–	7.72×10^4	6.43×10^4
$x_{fd,h}$	m	12.5×10^{-2}	15×10^{-2}
$x_{fd,t}$	m	12.5×10^{-2}	15×10^{-2}
Nu	–	281	243
h	$\mathrm{W/m^2}K$	$1.45{\times}10^4$.	$1.04{\times}10^4$
ΔT_{in}	°C	67	44
ΔT_{out}	°C	28	5
ΔT_{lm}	°C	44.7	18
L	m	2.56	3.39
f	–	4.84×10^-3	5.03×10^-3
ΔP	kPa	21	26

ignore this entrance length and use Dittus Boelter equation to obtain the Nu. Then we proceed as usual to get the required length of the pipe.

Next, we need to evaluate pressure drop using:

$$\Delta P = 4f(L/D)\frac{1}{2}\rho\overline{V}^2; \quad f = 0.046\mathrm{Re}^{-0.2}$$

These values are shown in Table **??** and it can be seen that the narrower tube actually gives less pressure drop!

□

1.8 Exercise

Exercise 1.1. Obtain Nusselt number correlation by using dimensional analysis for an isothermal flat plate and laminar flow.

Exercise 1.2. Consider a flat plate subject to parallel flow (both top and bottom) characterised by U= 5 m/s, T= 20 °C. (a) Determine the convection heat transfer coefficient at L = 0.5 m and 2 m respectively from the leading edge. Determine the thickness of the thermal and hydrodynamic boundary layer at L= 0.5 m.

Exercise 1.3. A long, cylindrical, electrical heating element of diameter D = 10 mm is installed in a duct for which air moves in cross flow over the heater at a temperature and velocity of 27 °C and10 m/s, respectively. Estimate the steady-state surface temperature when, per unit length of the heater, electrical energy is being dissipated at a rate of 1000 W/m.

Exercise 1.4. A pre-heater involves the use of condensing steam at 100 °C on the inside of a bank of tubes to heat air that enters at 1 atm and 25 °C. The air moves at 5 m/s in cross flow over the tubes. Each tube is 1 m long and has an outside diameter of 10 mm. The bank consists of 196 tubes in a square, aligned array for which $S_T = S_L = 15$ mm. What is the total rate of heat transfer to the air?

Exercise 1.5. Copper spheres of 20 mm diameter are quenched by being dropped into a tank of water that is maintained at 280 K. The spheres may be assumed to reach the terminal velocity (2 m/s) on impact with the water surface and to drop freely through the water. What is the approximate depth of the water needed to cool the spheres from an initial temperature of 360 K to 320 K? Assume no spatial variation inside the sphere.

Exercise 1.6. A copper wire of diameter 2 mm carries a current "density" (really, a current per unit area) of 1000 A/cm^2. It is insulated with a material that has a thermal conductivity of 10^{-3} W/cm K. The electrical resistance of copper is 2×10^{-6} Ω cm. Determine the required air velocity if we want to maintain the outer surface of the insulation at 25 °C with 20 °C ambient air.

Exercise 1.7. Make a list of all dimensionless numbers you learnt in this course so far. Make a table for their name and expressions in terms of standard symbols. Add a column to the table with their physical significance. Memorize the expression for the following numbers: Re, Nu, Pr, Sh, Sc, Pe, Bi, Fo. Try to reason out the physical significance from the expressions. Note the difference between Bi and Nu.

Exercise 1.8. Heated air ($\dot{m}$=0.01 kg/s) required for a food-drying process is generated by passing ambient air at 20 °C through a 5 m long (D = 50 mm) tube. The tube is housed in a steam condenser. Saturated steam at atmospheric pressure is used in the condenser. Determine the outlet temperature of the air.

Exercise 1.9. Air at $\dot{m}$=0.0005 kg/s and 27 oC enters a rectangular duct (1 m long, the cross section is 8 mm by 10 mm) on which a uniform surface heat flux of 600 W/m2 is maintained. What is the temperature of the duct surface at the outlet? Hint: You may need to look for a correlation in the transition region.

Exercise 1.10. Water flows (0.2 kg/s) through a Teflon tube (k=0.35 W/m.K) of inner and outer radii equal to 10 and 13 mm respectively. A thin electrical heating tape wrapped around the outer surface of the tube generates a uniform heat flux of 2000 W/m2, while a convection current is established in the outside air due to natural convection (ho= 25 W/m2 K). The ambient air temperature is 300 K. What is the temperature of the outer surface at a location where the mixing cup temperature of water is 290 K? The electrical heating tape may be considered to be of negligible thickness and does not offer any thermal resistance. Hint: Use the series resistance concept developed in section 10.2.3 of Middleman. (8.67)

Exercise 1.11. Air at 20 °C is hydrodynamically fully developed as it flows in a 1 cm I.D. pipe. The average velocity is 0.7 m/s. If it enters a section where the pipe wall is maintained at 60 °C, what is the temperature 0.15 m farther downstream?

Chapter 2

Natural Convection, Boiling and Condensation

The heat transfer correlations developed in the previous chapter works for forced convection of fluid without phase change. If the heat transfer occur due to natural convection or if boiling of a fluid or condensation of a vapour are involved, the nature of the correlation changes substantially. This is because different physic is at play for these cases. For example, isobaric thermal expansion coefficient (β) is important for natural convection and the latent heat of evaporation or condensation (λ) are important for heat transfer with phase change. These variables were absent for forced convective heat transfer without phase change. Because of these new variables, the dimensionless groups and their functionality changes. Hence, we need to obtain the new dimensionless groups involved and their functional dependance. We will start by looking at natural convection.

2.1 Natural/Free Convection

In natural convection, no fan or blower is used for fluid movement. Rather fluid moves due to buoyancy force created by density gradient. In some situations, however, flow is not initiated even in presence of density gradient. One such situation is shown in Fig.

Like forced convection, natural convection is classification into internal and external forced convection. Additionally, natural convection is also classified into free boundary flow (no surface) and flow in presence of solid boundaries. We will consider only external natural convection in presence of a solid boundary in this book.

To analyse external free convection, we can write the Naviar-Stokes equa-

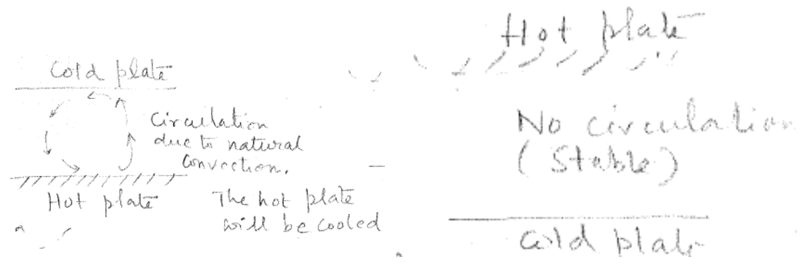


Figure 2.1: Stable and unstable configuration for natural convection

tion and solve the boundary layer problem and achieve an expression for the local Nusselt number. However, we will have to include the body force term in that equation. The solution of the momentum and energy equations are available in the text and the solutions are presented in Figure. We will refer to these figures for solving problems whenever required but in this chapter, we will avoid this detailed derivation and rather develop the correlation using dimensional analysis. As usual, experimental data can be used for determination of the constant and exponents.

For dimensional analysis, we need to know the variables that affects the convective heat transfer coefficient (h). We can use our previous understanding to this end and write all physical parameters that appeared in external forced convection correlation except the free stream velocity which is not present here. Although fluid moves, the velocity of fluid is not an independent variable and is determined by the isobaric expansion coefficient β, gravitatioal constant g and the temperature difference $\Delta\theta$. Since T is the symbol representing the time dimension in dimensional analysis, we will use the alternate notation for temperature as θ. This deviation will apply only to this section. Including the additional variables in place of u_∞, we can write:

$$h = h(l, k_f, c_p, \mu, \rho, g, \beta, \Delta\theta)$$

The number of variables is 9 and the number of dimensions is 4 (MLTΘ). Hence, we will get 5 dimensionless groups. Choosing

$$\begin{aligned} l &\Rrightarrow L \mapsto L \\ \rho &\Rrightarrow M.L^{-3} \mapsto M \\ \mu &\Rrightarrow \frac{M}{LT} \mapsto T \\ \Delta\theta &\Rrightarrow \Theta \mapsto \Theta \end{aligned}$$

to represent the four dimensions, we get:

$$\begin{aligned} L &\mapsto l \\ M &\mapsto \rho l^3 \\ T &\mapsto \rho l^2/\mu \\ \Theta &\mapsto \Delta\theta \end{aligned}$$

Now the remaining five quantities provides the five dimensionless groups:

$$\begin{aligned}
h &\Rrightarrow MT^{-3}\Theta^{-1} && \implies \pi_1 = \frac{hT^3\Theta}{M} = \frac{h\rho^2 l^3 \Delta\theta}{\mu^3} \\
c_p &\Rrightarrow L^2\Theta^{-1}T^{-2} && \implies \pi_2 = \frac{c_p\Theta T^2}{L^2} = \frac{c_p\Delta\theta\rho^2 l^2}{\mu^2} \\
\beta &\Rrightarrow \Theta^{-1} && \implies \pi_3 = \beta\Theta = \beta\Delta\theta \\
g &\Rrightarrow LT^{-2} && \implies \pi_4 = \frac{gT^2}{L} = \frac{g\rho^2 l^3}{\mu^2} \\
k &\Rrightarrow MLT^{-3}\Theta^{-1} && \implies \pi_5 = \frac{kT^3\Theta}{ML} = \frac{k\rho^2 l^2 \Delta\theta}{\mu^3}
\end{aligned}$$

Now, we have too many dimensionless groups for evaluation of constants and visualization of data. Hence, we will combine multiple dimensonless groups into one. We note that π_1/π_5 is our Nusselt number and π_2/π_5 is our Prandtl number. Hence, we will use Nu and Pr instead of π_1, π_2 and π_5. We will also combine π_3 and π_4 to a dimensionless group called Grashof number (Gr) given by:

$$Gr_l = \pi_3 \times \pi_4 = \frac{g\beta\Delta\theta\rho^2 l^3}{\mu^2}$$

Hence, the correlations for natural convection can be written as:

$$Nu = f(Pr, Gr)$$

It can be seen that the form of correlation is again similar to those of forced convection with the exception that the Reynolds number is replaced by the Grashof number. The physical significance of Grashoff number is also similar to Reynolds number: it is the ratio of buoyancy force to viscous force.

Like all other flows, natural convective currents also show transition from laminar to turbulent. This occurs when the buoyant force is more than the viscous force. For vertical plate, this transition occurs at a critical value of Rayleigh Number.

$$Ra_x = Gr_x.Pr = \frac{g\beta\Delta\theta x^3}{\nu\alpha} \approx 10^9$$

Similar to the previous cases, we will have to obtain the correct correlation depending on the geometry, nature of flow (Laminar or turbulent) etc. Table **??** shows a few such correlations.

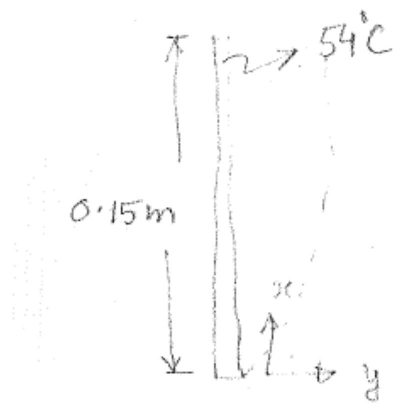
Table 2.1: External Natural Convection Correlations. Source: Fundamentals of Heat and Mass Transfer, Incropera et al., Seventh Edition

Condition	Correlation
Vertical plate, Laminar flow, Local Nu. All Pr. Property evaluated at T_f	$Nu_x = (Gr_x/4)^{1/4} \dfrac{0.75\sqrt{Pr}}{\left(0.609 + 1.221\sqrt{Pr} + 1.238Pr\right)^{1/4}}$
Vertical plate, Turbulent flow. Property evaluated at T_f	$\overline{Nu} = 0.10Ra^{1/3}$
Horizontal cylinder, $Ra_D < 10^{12}$ Property evaluated at T_f.	$\sqrt{\overline{Nu}} = 0.60 + \dfrac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}$
Sphere. Property evaluated at T_f. $Ra_D < 10^{11}$; $Pr > 0.7$	$\overline{Nu} = 2 + \dfrac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$

Example 2.1. A number of thin plates are to be cooled by vertically suspending them in a water bath at 20 ^{o}C. If the plates are initially at 54 ^{o}C and are 0.15 m long, what minimum spacing would prevent interference between their free convection boundary layers?

Solution: The thickness of the boundary layer at any x is dependent on the temperature of the plate. Higher the plate temperature, higher the thickness. The thickness also increases with x. Hence, we need to obtain the boundary layer thicknesses at the trailing edge of the plate at the initial time. At initial time $T_s = 54°C$ and $T_\infty = 20°C$. Hence, $T_f = 37°C$. At this temperature:

$$Pr = 4.62; \quad \beta = 362 \times 10^{-6}; \quad \mu = 695 \times 10^{-6}; \quad \rho = 999$$



Using these values:

$$Gr_{l=0.15} = \frac{9.81 \times 362 \times 10^{-6} \times 34 \times (0.15)^3 \times 999^2}{695^2 \times 10^{-12}} = 8.42 \times 10^8$$

$$Ra = Gr \times Pr = 3.9 \times 10^9$$

Hence the flow is laminar. Now we need to look at the velocity profile: We find that $\eta \approx 6$. which gives:

$$\frac{y}{x}\left(\frac{Gr_x}{4}\right)^{\frac{1}{4}} = 6$$

Solving for y at $x = 0.15$:

$$y = 0.00747\text{m}$$

Hence, the minimum gap that can be maintained is 15 mm. □

2.2 Heat transfer with phase change

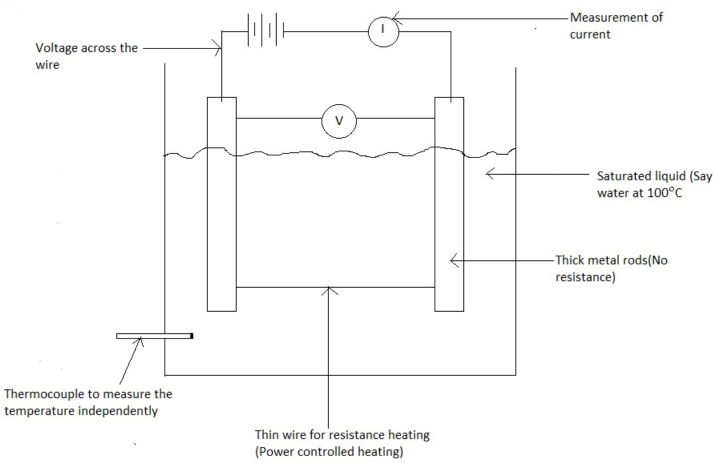
In chemical industry, we frequently need to boil a liquid or condensate a vapour. For example, in a binary distillation, an almost pure liquid is boiled at the bottom of the column and almost pure vapour is condensed at the top of the column. Boiling and condensation are also integral part of an evaporator where a solution is concentrated by boiling off the liquid. In many cases, these two operations are also used as efficient cooling or heating technique. A hot plate is quickly quenched by spraying water on the plate. The water evaporates quickly and takes away a great amount of latent heat to cool the plate. Such cooling techniques are frequently used in iron and steel industry.

Similarly, condensing saturated steam is one of the most efficient heating technique. In this section we will discuss these two operations. Despite their apparent simplicity, quantification of these two processes are more difficult than those considered before. We will start by looking at the details of the boiling process.

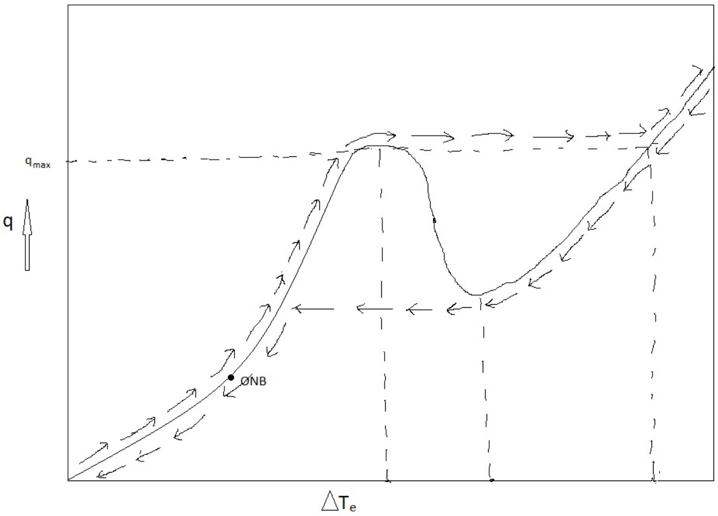
2.2.1 Boiling

The boiling of a liquid can occur under various conditions. If no circulation aid is used and a pool of liquid boils in presence of a heater surface, this is called pool boiling. If a circulating aid is used, this is called forced circulation boiling. Usually, the bulk of the boiling liquid reaches the saturation temperature while boiling. However, in some cases, only the liquid near the heating surface reaches the boiling point while the bulk remains at a subcooled state. This is called subcooled boiling. In this chapter we will discuss only pool boiling of a saturated liquid.

One of the early experimental work on saturated pool boiling was conducted by Professor Shiro Nukiyama from Tohoko Imperial University, Japan in 1934. The original paper was written in Japanese and later translated to english in 1966 in Int. J. of Heat and Mass Transfer. The set-up for this experiment is shown in Figure. The power input to the wire can be changed independently and the temperature of the wire get adjusted accordingly. This is called power controlled heating. In original experiment, Nukiyama used a nichrome wire whose melting point is 1400 ^{o}C.



The temperature of the wire is dependent on the heat transfer coefficient in a similar way as the constant surface heat flux case. It is measured by measuring the resistance of the wire. Usually, the temperature is expressed as excess temperature ($\Delta T_e = T_s - T_{sat}$). If we plot ΔT_e versus q (heat flux), we get a curve shown in Figure.



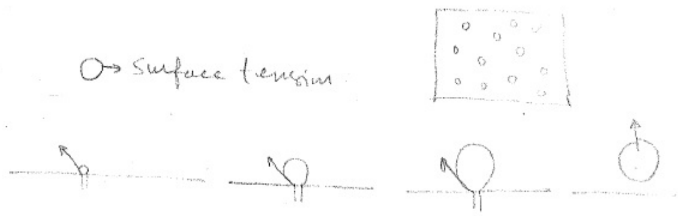
Nakiyama's boiling curve and hysteresis loop

If you keep the fluid at its saturation temperature (water at 100°C under 1 atm), it will never boil because you cannot transfer the latent heat (λ_{lv}) to it. Will it boil if you keep it at 101°C? It will boil very slowly. Usually, if $\Delta T_e <$ 5°C the heat supplied is carried by the fluid through natural convection and boiling takes place from the wire surface at a slow rate. This region is called free convection boiling. Here $q = h\Delta T_e$ and h is given by natural convection correlation. Close look at these correlation reveals that in laminar region $h \propto \Delta T_e^{1/4}$ and in turbulent region $h \propto \Delta T_e^{1/3}$. Accordingly, $q \propto \Delta T_e^{5/4}$ and $h \propto \Delta T_e^{4/3}$ respectively.

If we maintain an excess temperature higher than about 5°C, nucleate boiling starts. Usually, nucleate boiling occurs until an excess temperature of 30°C and is divided into three sub zones. In first sub zone, marked as AB in figure, individual bubbles appear on the surface, grows and goes up. Motion of bubble initiate a strong fluid circulation near the bubble and increases the heat flux considerably.

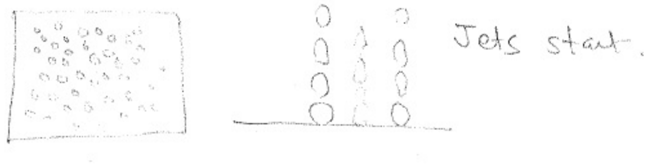
In the second sub zone of nucleate boiling (Marked as BP), the number of nucleation site increases. The vapour bubbles form so rapidly that they rise like a plume and interfere with each other. In this region both the heat transfer coefficient and heat flux keep increasing and reaches its maximum value.

In the third sub zone the vapor fraction on the surface gradually increases so that the surface remains only partially wet by the liquid. In this sub region,



the heat transfer coefficient decreases but the total heat flux increases because the small decrease in heat flux is overcompensated by the large increase in ΔT_e. After a certain excess temperature, the reduction in h becomes substantial and the heat flux start decreasing after it reaches the maximum heat flux at point C. Point C is considered as the end of nucleate pool boiling regime.

Beyond point C, the surface remains partially covered by vapour. This region is called the transition boiling or partial film boiling region. At a given region on the surface, the condition oscillates between nucleate and film boiling. The heat flux continues to decrease with increasing temperature gradient since more and more portion of the surface is covered by vapour. (In original Nukiyama's experiment, where a platinum wire was used, the reduced heat flux increases the temperature of the wire substantially and melts the wire. However, with another maerial, this part can be recovered) The heat flux keep decreasing with ΔT_e until a stable vapor film is established on the surface. This point is called Leidenfrost point. At this point, the heat flux reaches a local minimum. This is the end of the tansition boiling.



Beyond Leidenfrost point, the surface is covered by a blanket of vapor and the heat is transferred to the liquid through this vapor film primarily by conduction over the thin vapour film and radiation. This region is called film boiling.

Nucleate pool boiling correlation

This is the regime in which most of the boiling apparatus operate. Because it gives high heat flux at low temperature drops. The aim is to operate near the maximum heat flux zone but without exceeding it. In this case, the heat flux is high because growing bubbles add a strong circulation of liquid near the surface while detaching. This is similar to forced circulations. Hence one would expect:

$$\bar{N}u_L = CRe_L^m.Pr^n$$

Hence, we have to find out various variables related to this form. First let

us get the length scale of flow which is the bubble diameter. The bubble diameter is determined by balancing the buoyant and surface tension force:

$$\pi\alpha D_b\sigma = \frac{\pi D_b^3}{6}(\rho_l - \rho_v)g$$

Which gives the required length scale as:

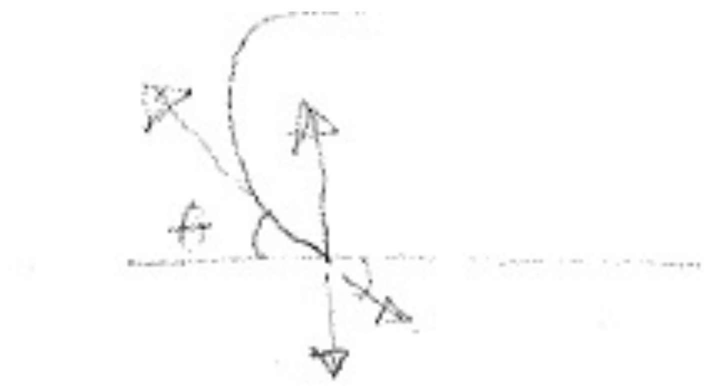
$$D_b \propto \sqrt{\frac{\sigma}{(\rho_l - \rho_v)g}}$$

The bubble Reynolds number is obtained in the following way:

$$Re_b = \frac{G_b D_b}{\mu_l}$$

From energy balance:

$$q'' = \left[nf\frac{\pi D^3}{6}\rho_v\right]\lambda$$



where n is the number of bubble per unit area and f is the frequency of bubble dislodgement. It can be noted that the quantity in the square bracket is actually the mass velocity of bubbles (G_b). Hence, the bubble Reynolds number can be written as:

$$Re_b \propto \frac{q''}{\lambda\mu_l}\sqrt{\frac{\sigma}{(\rho_l-\rho_v)g}}$$

Now, writing the correlation:

$$\frac{h}{k_l}\sqrt{\frac{\sigma}{(\rho_l-\rho_v)g}} = C\left[\sqrt{\frac{\sigma}{(\rho_l-\rho_v)g}}\frac{q''}{\mu_l\lambda}\right]^m\left[\frac{c_p\mu_l}{k_l}\right]^n$$

Substituting $h = q''/\Delta T$ and simplifying:

$$\begin{aligned}
\frac{h}{k_l}\sqrt{\frac{\sigma}{\Delta\rho g}} &= C\left[\sqrt{\frac{\sigma}{\Delta\rho g}}\frac{q''}{\lambda\mu_l}\right]^m Pr^n \\
&= C\left[\sqrt{\frac{\sigma}{\Delta\rho g}}\frac{h\Delta T}{\lambda\mu_l}\right]^m Pr^n \\
&= C\left[\frac{h}{k_l}\sqrt{\frac{\sigma}{\Delta\rho g}}\frac{k_l\Delta T}{\lambda\mu_l}\right]^m Pr^n \\
&= C\left[\frac{k_l\Delta T}{\lambda\mu_l}\right]^{m'} Pr^{n'} \\
&= C\left[\frac{c_p\Delta T}{\lambda}\right]^{m'} Pr^{n''}
\end{aligned}$$

Usually, the correlation for boiling is written in terms of the heat flux. Hence, further manipulation of this expression leads to:

$$
\begin{aligned}
h &= Ck_l\sqrt{\frac{\Delta\rho g}{\sigma}}\left(\frac{c_p\Delta T}{\lambda}\right)^{m'} Pr^{n''} \\
q'' = h\Delta T &= Ck_l\sqrt{\frac{\Delta\rho g}{\sigma}}\Delta T\left(\frac{c_p\Delta T}{\lambda}\right)^{m'} Pr^{n''} \\
&= C\frac{k_l}{c_p\mu}c_p\mu\sqrt{\frac{\Delta\rho g}{\sigma}}\Delta T.\left(\frac{c_p\Delta T}{\lambda}\right)^{m'} Pr^{n''} \\
&= C\mu\lambda\left(\frac{c_p\Delta T}{\lambda}\right)\sqrt{\frac{\Delta\rho g}{\sigma}}\left(\frac{c_p\Delta T}{\lambda}\right)^{m'} Pr^{n'''} \\
&= C\mu\lambda\sqrt{\frac{\Delta\rho g}{\sigma}}\left(\frac{c_p\Delta T}{\lambda}\right)^{m'} Pr^{n'''}
\end{aligned}
$$

A slightly different form is practically used to predict the heat flux in the entire nucleate boiling region and is known as Rohsenow correlation which is given by:

$$
q'' = \mu_l\lambda\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{\frac{1}{2}}\left(\frac{c_p\Delta T}{C_{sf}\lambda Pr^n}\right)^3
$$

Along with this nucleate boiling heat flux, the maximum and minimum heat flux can also be obtained and is given by:

$$
q''_{max} = C\lambda\rho_v\left[\frac{\sigma g(\rho_l-\rho_v)}{\rho_v^2}\right]^{\frac{1}{4}}
$$

$$
q''_{min} = C\lambda\rho_v\left[\frac{\sigma g(\rho_l-\rho_v)}{(\rho_l+\rho_v)^2}\right]^{\frac{1}{4}}
$$

Example 2.2. A circular copper pan of 0.3 m diameter is submerged in saturated water with surface temperature 118°C by an electric heater. What should be the power supply to the heater so that the temperature is maintained? (Pressure = 1atms)

Solution: The power supply should be such that the heat flux under this condition is same as the power input. This is in the nucleate pool boiling region. In the correlation, the properties are evaluated at T_{sat}. According to table, the properties at this temperature are:

$$
\rho_l = 958 \quad \rho_v = 0.6 \quad c_{pl} = 4220 \quad \sigma = 58.9\times10^{-3} \quad \mu_l = 279\times10^{-6}
$$

Table 2.2: Correlations for boiling Source: Fundamentals of Heat and Mass Transfer, Incropera et al., Seventh Edition. Property evaluated at T_{sat}. Refer to the text for various constants such as C_{sf} and n

Condition	Correlation
Nucleate pool boiling (Rohsenow correlation)	$$q'' = \mu_l \lambda \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{\frac{1}{2}} \left(\frac{c_p \Delta T}{C_{sf} \lambda Pr^n}\right)^3$$
Maximum heat flux	$$q''_{max} = C \lambda \rho_v \left[\frac{\sigma g(\rho_l - \rho_v)}{\rho_v^2}\right]^{\frac{1}{4}}$$
Minimum heat flux	$$q''_{min} = C \lambda \rho_v \left[\frac{\sigma g(\rho_l - \rho_v)}{(\rho_l + \rho_v)^2}\right]^{\frac{1}{4}}$$

$$c_{pl} = 4.22 \quad \sigma = 58.9 \times 10^{-3} \quad \text{and} \quad \Delta T_e = 18°\text{C}$$

Using C_{sf} from Table, we get:

$$q'' = \mu_l \lambda \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{\frac{1}{2}} \left[\frac{c_{pl}.\Delta T_e}{C_{s,f}.\lambda Pr}\right]^3 = 837 \text{ kW/m}^2$$

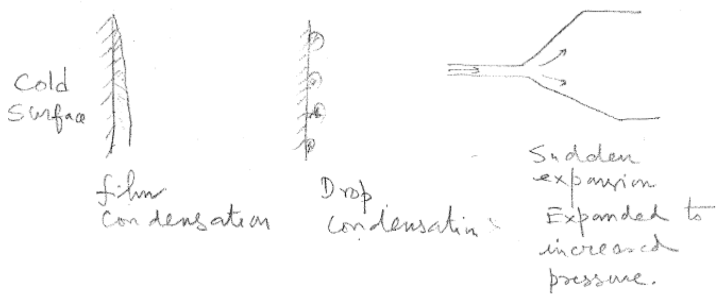
How far are you from critical heat flux?

$$q_{max} = C\lambda\rho_v \left[\frac{\sigma g(\rho_l - \rho_v)}{\rho_v^2}\right]^{\frac{1}{4}} = 1100 \text{ kW/m}^2$$

Hence we have some gap from the critical heat flux. □

2.2.2 Condensation

Normally in chemical industry, cooling surfaces are used for condensation. Drop/film formation depends on nature of surface tension. The heat transfer coefficient is of main importance and it is much higher (10 times usually) in dropwise condensation. Dropwise condensation is difficult to survive and hence designs are done for film condensation only. Here we shall obtain the film coefficient for such film type condensation.

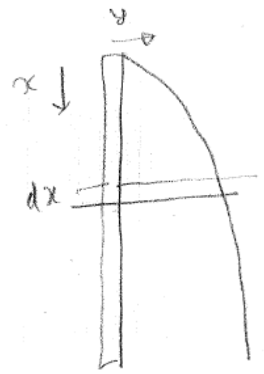


First we will look at the hydrodynamics of flow of a fluid on a vertical flat plate. Hence, the equation of motion can be written as:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_l}\frac{dp}{dx} + g_x + \nu_l\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

For fully developed flow, u is only a function of y and $v = 0$. Using these, the simplified equation of motion becomes:

$$0 = -\frac{1}{\rho_l}\frac{dp}{dx} + g_x + \nu_l\frac{\partial^2 u}{\partial y^2}$$



The pressure gradient may be assumed to be because of the gravity head:

$$\frac{dp}{dx} = \rho_v g$$

With this substitution and assuming shear stress at the free surface is zero, we can use the following boundary conditions:

$$\frac{\partial u}{\partial y}|_{y=\delta} = 0 \quad \text{and} \quad u|_{y=0} = 0$$

Using these BC, we can solve the velocity profile readily as:

$$u(y) = \frac{g\Delta\rho\delta^2}{\mu_l}\left(\frac{y}{\delta} - \frac{1}{2}\frac{y^2}{\delta^2}\right)$$

In this profile, δ is yet an unknown. This is obtained by using the total mass balance. We define:

$$\frac{\dot{m}}{b} = \int_0^\delta \rho_l u(y) dy \equiv \Gamma$$

we can obtain the following expression for Γ wh

$$\Gamma = \frac{g\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l} \tag{2.1}$$

Since Γ is known from the total mass flow rate, δ can be obtained.

Consider a vertical flat plate which is colder than the saturation temperature of the surrounding vapour. The vapour condenses on the plate and flows downward. More and more liquid mass accumulates on the surface as we go downward. We will use the same hydrodynamic equation obtained before but will consider Γ and δ to be a function of x.

$$\Gamma(x) = \frac{g\rho_l(\rho_l - \rho_v)[\delta(x)]^3}{3\mu_l} \tag{2.2}$$

We can also obtain an expression for this mass flow rate using energy balance:

$$-dq = -\lambda d\Gamma = q_s'' dx$$

q_s'' maybe obtained by assuming conductive transport in the liquid film and is given by:

$$q_s'' = -k_f \frac{(T_{sat} - T_s)}{\delta}$$

Substituting and simplifying:

$$\frac{d\Gamma}{dx} = \frac{k_f(T_{sat} - T_s)}{\delta\lambda} \tag{2.3}$$

From eq **??**:

$$\frac{d\Gamma}{dx} = \frac{g\rho_l(\rho_l - \rho_v)\delta^2}{\mu_l}\frac{d\delta}{dx}$$

Equating this with eq **??**:

$$\frac{d\delta}{dx}\frac{g\rho_l(\rho_l - \rho_v)\delta^2}{\mu_l} = \frac{k_f(T_{sat} - T_s)}{\delta\lambda}$$

Using the boundary condition a $x = 0$, $\delta = 0$

$$\delta(x) = \left[\frac{4k_f\mu_l(T_{sat} - T_s)x}{g\rho_l(\rho_l - \rho_v).\lambda}\right]^{\frac{1}{4}}$$

which gives,

$$h_x = \frac{k_f}{\delta} = \left[\frac{k_l^3 g\rho_l(\rho_l - \rho_v)\lambda}{4\mu_l(T_{sat} - T_s)x}\right]^{1/4}$$

The average coefficient can be obtained by integrating the above expression and is given by:

$$\overline{h_L} = \frac{1}{L}\int_0^L h_x dx = \frac{4}{3}h_L$$

The above analysis is known as Nusselt Equation. This is applicable for laminar flow. In these expressions, all properties to be evaluated at T_f except ρ_v and λ which are evaluated at T_{sat}. The above derivation is applicable for laminar film. For free surface flow, the Re is defined as:

$$Re_\delta = \frac{4\Gamma}{\mu_l} = \frac{4g\rho_l(\rho_l - \rho_v)[\delta(x)]^3}{3\mu_l^2}$$

Considering $\rho_l >> \rho_v$ and writing the Re at $x = L$:

$$Re_\delta = \frac{4g\rho_l^2\delta_L^3}{3\mu_l^2}$$

Since $h_L = k_l/\delta_L$ and $h_L = 3\overline{h_L}/4$:

$$\delta_L = \frac{4k_l}{3\overline{h_L}}$$

Substituting:

$$Re_\delta = \frac{256k_l^3 g\rho_l^2}{81\overline{h_L}^3 \mu_l^2}$$

Simplifying:

$$\frac{\overline{h_L}(\nu^2/g)^{1/3}}{k_l} = 1.47Re^{-1/3}$$

The expression for Re is also manipulated in Considering the mass flow and Reynolds number at the trailing edge and writing an overall energy balance:

$$\lambda\dot{m} = Q = \overline{h}_L \times (T_{sat} - T_s) \times L \times b$$

Substituting $\dot{m}$ from above into the expression for Re_δ:

$$Re_\delta = \frac{4\overline{h}_L \times (T_{sat} - T_s)L}{\mu_l\lambda}$$

Using the expression for $\overline{h}_L$ from above:

$$Re_\delta = \frac{4 \times 1.47Re^{-1/3}k_l \times (T_{sat} - T_s)L}{(\nu^2/g)^{1/3}\mu_l\lambda} = 3.78\left[\frac{k_l L(T_{sat} - T_s)}{(\nu^2/g)^{1/3}\mu_l\lambda}\right]^{3/4}$$

If $Re \geq 1800$, the film is turbulent and in between is the transition region. For turbulent film, the following correlation can be used:

$$Re_\delta = \left[\frac{0.069k_l L(T_{sat} - T_s)}{(\nu^2/g)^{1/3}\mu_l\lambda}Pr^{0.5} - 151Pr^{0.5} + 253\right]^{4/3}$$

$$\frac{\overline{h_L}(\nu_l^2/g)^{1/3}}{k_l} = \frac{Re_\delta}{8750 + 58Pr_l^{-0.5}(Re_\delta^{0.75} - 253)}$$

In transition region:

$$Re_\delta = \left[\frac{3.7k_l L(T_{sat} - T_s)}{(\nu^2/g)^{1/3}\mu_l\lambda}Pr^{0.5} + 4.8\right]^{0.82}$$

$$\frac{\overline{h_L}(\nu_l^2/g)^{1/3}}{k_l} = \frac{Re_\delta}{1.08Re_\delta^{1.22} - 5.2}$$

Table 2.3: Correlations for film condensation over vertical flat plate. Source: Fundamentals of Heat and Mass Transfer, Incropera et al., Seventh Edition

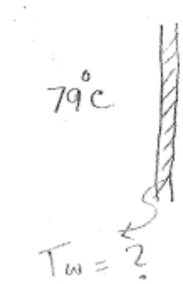
Condition	Correlation
Laminar: $Re_\delta < 30$: $Re_\delta = 3.78 \left[\frac{k_l L(T_{sat} - T_s)}{(\nu^2/g)^{1/3}\mu_l \lambda}\right]^{3/4}$	$Nu = \frac{\overline{h_L}(\nu^2/g)^{1/3}}{k_l} = 1.47 Re^{-1/3}$
Transition: $30 < Re_\delta < 1800$: $Re_\delta = \left[\frac{3.7 k_l L(T_{sat} - T_s)}{(\nu^2/g)^{1/3}\mu_l \lambda} Pr^{0.5} + 4.8\right]^{0.82}$	$\frac{\overline{h_L}(\nu_l^2/g)^{1/3}}{k_l} = \frac{Re_\delta}{1.08 Re_\delta^{1.22} - 5.2}$
Turbulent: $Re > 1800$ $Re_\delta = \left[\frac{0.069 k_l L(T_{sat} - T_s)}{(\nu^2/g)^{1/3}\mu_l \lambda} Pr^{0.5} - 151 Pr^{0.5} + 253\right]^{4/3}$	$\frac{\overline{h_L}(\nu_l^2/g)^{1/3}}{k_l} = \frac{Re_\delta}{8750 + 58 Pr_l^{-0.5}(Re_\delta^{0.75} - 253)}$

For vertical tube, the flat plate correlation may be used provided the tube curvature is much smaller than the film thickness. For horizontal tubes, the below correlation may be used.

$$\overline{h_D} = 0.729 \left(\frac{g\rho_l(\rho_l - \rho_v)k_l^3\lambda}{\mu_l(T_{sat} - T_s)D} \right)^{1/4}$$

Example 2.3. Shell and tube condenser with vertical 3/4″ 16 BWG Cu tubes has cholorobenzene condensing at atmospheric pressure in the shell. The tubes are 1.52 m long. Cooling water at 79 °C is flowing in the tube. The water side coefficient is 4540 W/m_2K. Determine the vapour si

1. What is the vapour side coeffiecient?
2. If the same condenser is used horizontally, what will be this coefficient? (*Cooling water has negligible change in temperature*)



Solution: The properties evaluated for this problem are:

$$\lambda = 324.9 \times 10^3 \quad L = 1.52 \quad h_i = 4540 \quad T_{sat,CB} = 131°C$$

Hence,

$$\bar{h_L} = 0.943 \left[\frac{g\rho_l^2 k_l^3 \lambda}{\mu_l.\Delta T.L} \right]^{\frac{1}{4}}$$

At $118°C$,

$$\mu_l = 0.28 \times 10^{-3} \quad k_l = 0.109 \quad \rho_l = 111.0$$

Using these values,

$$\bar{h_L} = 776 \text{W/m}_2 \text{ K}$$

Hence the overall heat transfer coefficient:

$$\frac{1}{U} = \frac{1}{4540} + \frac{1}{776} \implies U = 663$$

Hence, $\Delta T_i = 7.59$ and $T_W = 86.6$ The next guess would be:

$$T_w = \frac{(105+87)}{2} = 96°C$$

Using this value of T_w,

$$\mu_l = 0.29 \times 10^{-3} \quad \rho_l = 111.0 \quad k_l = 0.111$$

which gives

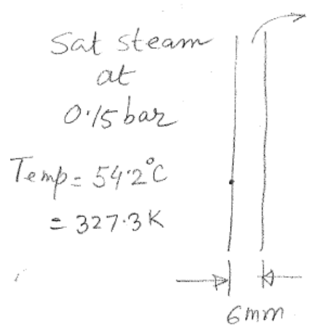
$$\bar{h_L} = 724 \text{ and } U = 624$$

giving

$$\Delta T_i = 7.15 \quad T_w = 91$$

Finally converges to $T_W = 91$ □

Example 2.4. Vertical condenser or 6mm diameter tubes, 1.52m long. Cooling water is flowing in the tube at $12°C$. The water side coefficient is 4540 W/m^2 °C. What is the vapour side coefficient?



Solution: This is an iterative solution. Let us take $T_w = 33$. Hence, $T_f = 316.6K$ and

$$\bar{h_L} = 0.729 \left(\frac{k_f^3 \rho_f^2 g \lambda}{\Delta T D_0 \mu_f} \right)^{\frac{1}{4}} = 3880$$

Hence,

$$\frac{1}{U} = \frac{1}{3880} + \frac{1}{4540} \implies U = 2092$$

Using this value,

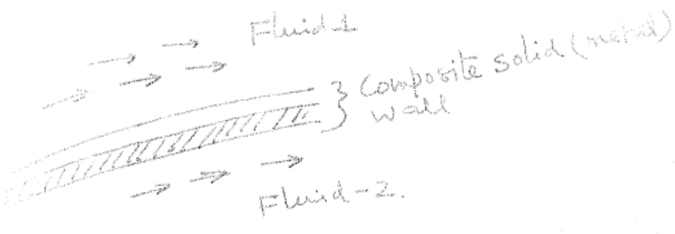
$$\Delta T_i = 19.4 \quad T_w = 31.44.$$

Iteration may be repeated but the value will converge to 31. □

Chapter 3

Heat Exchangers

In the chemical industry, we transfer heat from one fluid to another with a solid wall in between. This situation is shown in Fig. There are several configuration of equipment in which this can be achieved. One of those is double pipe heat exchanger and the other is shell and tube heat exchanger.



3.1 Overall heat transfer coefficent

The general problem for heat exchange between two fluids with a solid wall in between is that the temperature of the solid wall is not known. Only the bulk temperature of the fluid in either side of the solid wall is known. Of course, the heat transfer coefficient of either side is also known. Let us first examine a situation where the surface temperature of the solid is not known, but the bulk temperature (T_m for internal flow and T_∞ for the external flow) are known. The heat transfer coefficients are also known and the properties evaluated at some arbitrarily defined temperature. We will first show how we can use the available information to obtain the heat flux across the solid wall and then extend the procedure for cylindrical geometry. At steady state:

$$q = h_i(T_{mi} - T_1) = \frac{k_1}{t_1}(T_1 - T_i) = \frac{k_2}{t_2}(T_i - T_2) = h_o(T_2 - T_{mo})$$

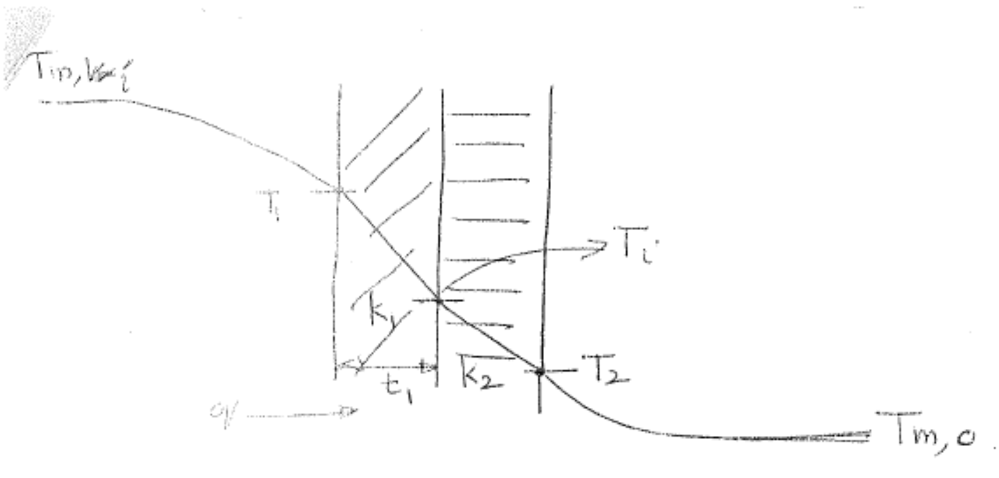


Figure 3.1: Figure 52

which implies:

$$T_{mi} - T_1 = \frac{q}{h_i}$$
$$T_1 - T_i = \frac{q}{k_1/t_1}$$
$$T_i - T_2 = \frac{q}{k_2/t_2}$$
$$T_2 - T_{mo} = \frac{q}{h_o}$$

Adding:

$$T_{mi} - T_{mo} = q\left[\frac{1}{h_i} + \frac{1}{h_o} + \frac{1}{k_i/t_i} + \frac{1}{k_2/t_2}\right]$$

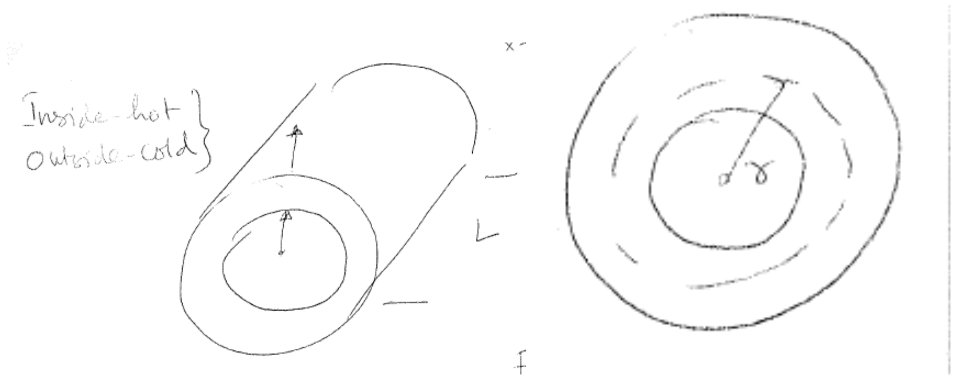
Rearranging:

$$q = \frac{1}{1/h_i + 1/h_o + t_1/k_1 + t_2/k_2}(T_{mi} - T_{mo}) \equiv U(T_{mi} - T_{mo})$$

Where U is called the overall heat transfer coefficient given by:

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + \frac{t_1}{k_1} + \frac{t_2}{k_2}$$

For radial system, since the area changes with r, we cannot equate the heat flux. However, the total amount of heat (Q) that passes through a cylindircal plane at any r remains constant at steady state in absence of heat generation.



Hence, in this case,

$$Q = h_i(T_{mi} - T_{wi})2\pi r_i L = h_o(T_{wo} - T_{mo})2\pi r_o L = -k\frac{dT_w}{dr}.2\pi r L$$

The last equality can be integrated:

$$\frac{Q}{2\pi k L}\int_{r_i}^{r_o}\frac{dr}{r} = -\int_{T_{wi}}^{T_{wo}} dT_w$$

$$\frac{Q\ln(r_o/r_i)}{2\pi k L} = (T_{wi} - T_{wo})$$

Following the similar procedure as above,

$$(T_{mi} - T_{mo}) = Q\left(\frac{1}{h_i r_i 2\pi L} + \frac{ln r_o/r_i}{k2\pi L} + \frac{1}{h_o r_o 2\pi L}\right)$$

or,

$$Q = \frac{1}{1/(h_i r_i 2\pi L) + ln(r_o/r_i)/(k2\pi L) + 1/(h_o r_o 2\pi L)}(T_{mi} - T_{mo})$$

Now, unlike previous case, here we have ambiguity in terms of area. Hence, the overall coefficient needs to be defined according to the area chosen. If we chose the inside area, we have an overall coefficient based on inside area and vice versa.

$$Q = U_o A_o(T_{mi} - T_{mo}) = U_i A_i(T_{mi} - T_{mo})$$

where

$$U_o A_o = U_i A_i = \frac{1}{1/(h_i r_i 2\pi L) + ln(r_o/r_i)/(k2\pi L) + 1/(h_o r_o 2\pi L)}$$

Using $A_o = 2\pi r_o L$ and $A_i = 2\pi r_i L$ we get:

$$\frac{1}{U_o r_o} = \frac{1}{h_o r_o} + \frac{\ln r_o/r_i}{k} + \frac{1}{h_i r_i}$$

and

$$\frac{1}{U_i r_i} = \frac{1}{h_i r_i} + \frac{\ln r_o/r_i}{k} + \frac{1}{h_o r_o}$$

respectively. If the tube wall thickness is negligible, $r_o/r_i \approx 1$. For this case,

$$\frac{1}{U_i} = \frac{1}{U_o} = \frac{1}{h_i} + \frac{1}{h_o}$$

The concept of overall coefficient can be used for estimation of wall temperature if the mean temperatures of the hot and cold fluids are known. For a differential cross section of a tube with thin wall:

$$dQ = h_i dA(T_{m,i} - T_w) = h_o dA(T_w - T_{m,o}) = U dA(T_{m,i} - T_{m,o})$$

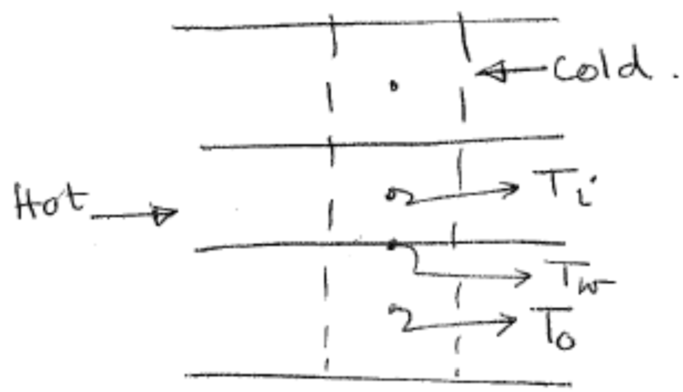
Denoting $(T_{m,i} - T_w) = \Delta T_i$, $(T_w - T_{m,o}) = \Delta T_o$ and $(T_{m,i} - T_{m,o}) = \Delta T_{ov}$ and noting that

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

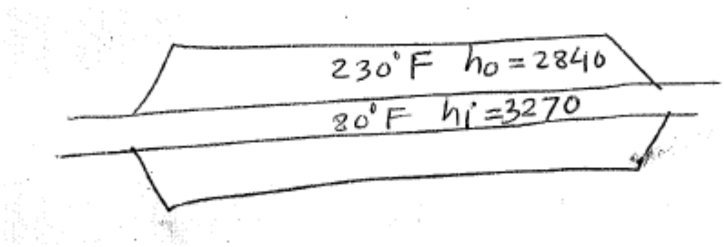
we get:

$$\Delta T_i = \frac{1/h_i}{1/h_i + 1/h_o}\Delta T_{ov} \quad \text{and} \quad \Delta T_o = \frac{1/h_o}{1/h_i + 1/h_o}\Delta T_{ov}$$

Normally, such estimation of wall temperature is needed for viscosity cor-



rections.



Example 3.1. Toluene is being condensed at 230 $^\circ F$ on outiside of a tube through which cooling water is flowing with inlet 40 $^\circ F$ and outlet 120 $^\circ F$. What is the average wall temperature?

Solution: We can take the arithmetic mean temperature of in and out as $T_{m,i}$. Hence, $T_c = 80\ ^\circ F$. Since a vapour is being condensed, $T_{m,o} = 230\ ^\circ F$. Hence, $\Delta T_{ov} = 150\ ^\circ F$. Using the above formulae:

$$\Delta T_i = 150 \frac{1/3270}{1/3270 + 1/2840} = 70^\circ F$$

which gives:

$$T_w = 80 + 70 = 150^\circ F$$

□

3.2 Double pipe heat exchanger

In this section we will analyse the operation of the simplest kind of heat exchanger called the double pipe heat exchanger as shown in Figure. The

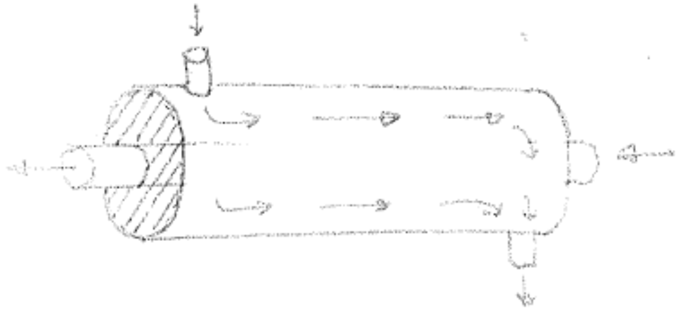


Figure 3.2: Figure 50

operation of a double pipe exchanger may be co-current or counter current as shown in Figure. In these two operating conditions, the evolution of the fluid temperatures are shown in Figure. It is important to note the sign of the derivative of various quantities in these two differneti configurations. For co-current flow:

$$\frac{dT_h}{dL} < 0; \quad \frac{dT_c}{dL} > 0$$

and for counter current flow:

$$\frac{dT_h}{dL} > 0; \quad \frac{dT_c}{dL} > 0$$

In analysing this exchanger, we will take the length of the exchanger (L) as independent variable and the total heat transfer Q as dependent variable. Since the diameters of the tubes are fixed, length and area (A) are linearly related. Hence, either of L or A may be considered as independent variable. Inside an exchanger, the inside and outside coefficients may change mainly

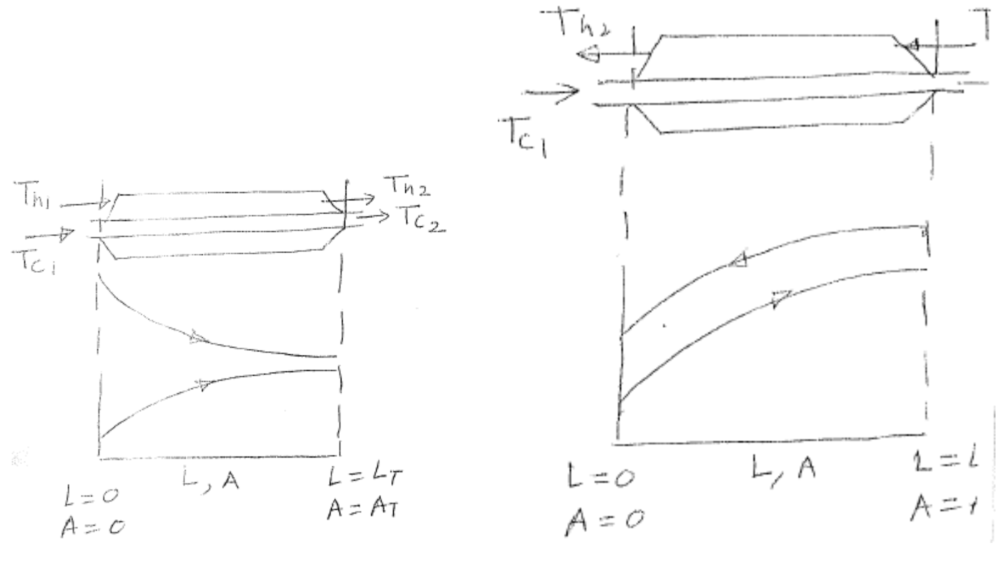


Figure 3.3: Figure 56

due to change in temperatures of the fluid and associated property variation. In this simple analysis we will assume that we can use an average value of the overall heat transfer coefficient and hence consider it to be constant along the length of the exchanger.

Now, we will obtain the total rate of heat exchange between the fluids in the exchanger as a function of the total length of exchanger L_T (or area A_T), terminal temperatures (inlet and outlet temperatures of both the fluids) and overall heat transfer coefficient. In the derivation of overall heat transfer coefficient, we considered a small section of pipe (ΔL) over which $T_{m,i}$ and $T_{m,o}$ may be considered uniform. However, for a heat exchanger, the mean temperature changes continuously along the length of the exchanger. Hence, the above analysis applies only over a differential area of the exchanger. Considering an exchanger with thin wall:

$$dQ = UdA(T_h - T_c)$$

where T_h and T_c are the mean temperature of the hot and cold fluid at any given location respectively. Now, consider a double pipe heat exchanger un-

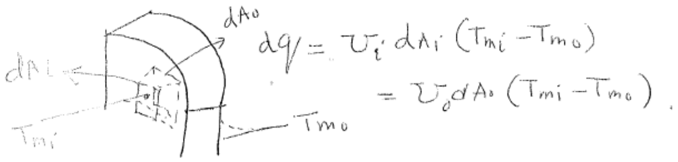
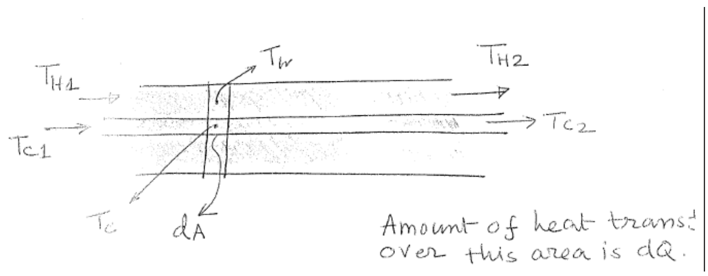


Figure 3.4: Figure 55

der co-current flow. The mean temperature at any location in the exchanger is denoted by T_h an T_c respectively. Denoting the mass flow rate of hot and cold fluid as $\dot{m}_h$ and $\dot{m}_c$ respectively, we can write an energy balance over a differential length as:

$$dQ = \dot{m}_c\hat{c}_{p,c}dT_c = -\dot{m}_h\hat{c}_{p,h}dT_h$$

The negative sign indicates that dT_c and dT_h has opposite sign for a co-current flow. The above equations are combined in the following way to



obtain the so called 'design equation' for heat exchanger: From eq, we can write:

$$dT_h = \frac{-dQ}{\dot{m}_h\hat{c}_{p,h}} \quad \text{and} \quad dT_c = \frac{dQ}{\dot{m}_c\hat{c}_{p,c}}$$

Subtracting:

$$d(T_h - T_c) = -dQ\left(\frac{1}{\dot{m}_h\hat{c}_{p,h}} + \frac{1}{\dot{m}_c\hat{c}_{p,c}}\right)$$

Using overall energy balance

$$Q = \dot{m}_h\hat{c}_{p,h}(T_{h1} - T_{h2}) = \dot{m}_c\hat{c}_{p,c}(T_{c2} - T_{c1})$$

to substitute the $\dot{m}\hat{c}_p$ terms:

$$d(T_h - T_c) = -dQ\left(\frac{T_{c2} - T_{c1}}{Q} + \frac{T_{h1} - T_{h2}}{Q}\right)$$

Using eq,

$$d(T_h - T_c) = -UdA(T_h - T_C)\left(\frac{T_{c2} - T_{c1} + T_{h1} - T_{h2}}{Q}\right)$$

Rearranging:

$$\frac{d(T_h - T_c)}{T_h - T_c} = \frac{U}{Q}dA[\Delta T_2 - \Delta T_1]$$

where $\Delta T_2 = (T_{h2} - T_{c2})$ and $\Delta T_1 = (T_{h1} - T_{c1})$. Integrating from left side of exchanger where

$$T_h - T_c = \Delta T_1; \quad A = 0$$

to the right side of the exchanger where

$$T_h - T_c = \Delta T_2; \quad A = A_T$$

and assuming U, c_p, Q to be constant:

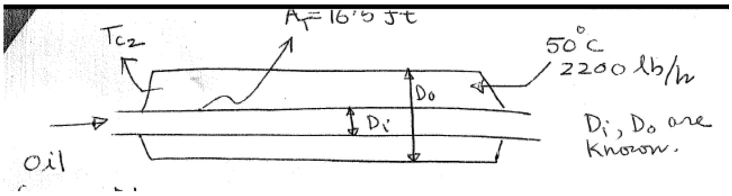
$$\ln\frac{\Delta T_2}{\Delta T_1} = \frac{U}{Q}A_T[\Delta T_2 - \Delta T_1]$$

The final form of the design equation is:

$$Q = UA_T\frac{\Delta T_2 - \Delta T_1}{ln\Delta T_2/\Delta T_1} = UA_T(LMTD)$$

Although the expression above is derived for co-current (or parallel) flow, the same expression is obtained for countercurrent flow.

Example 3.2. CCl_4 flowing at 19,000 kg/h is to be cooled from 85 °C to 40 °C using 13,500 Kg/h of cold water available at 20 ° C using a double pipe heat exchanger. The CCl_4 flows outside the central tube which has a film coefficient of 1700 W/m^2 °C. Inside film coefficient is 11000 W/m^2 °C. Calculate the area of a counter current exchanger. Data: c_{p,CCl_4} = 0.8 kJ/kg K; c_{p,H_2O} =4.2 kJ/kg K.



Solution. Using overall energy balance to obtain the outlet temperature of the cold fluid ($T_{c,2}$):

$$Q = (14000/3600)\times 0.837\times 10^3\times(85-40) = (13500/3600)\times 4.2\times 10^3\times(T_{c,2}-20)$$

gives $Q = 1.46 \times 10^5$ W and $T_{c2} = 32.6$ °C. Using this value:

$$LMTD = \frac{52.4 - 20}{\ln(52.4/20)} = 33.6.$$

Now, the overall coefficient can be obtained assuming thin metal wall:

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} \implies U = 1472$$

Using these values in the design equation, we get:

$$A \approx 4m^2$$

□

3.3 Shell and tube exchanger

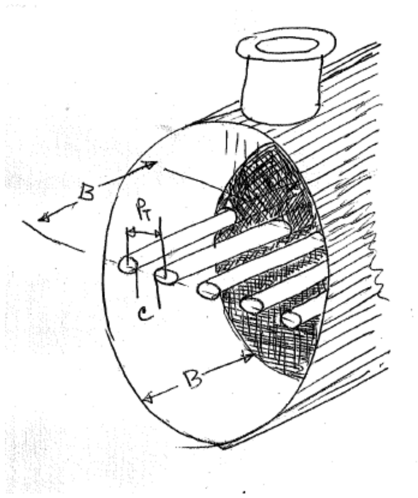
When area becomes more, many bends prevent use of double pipe exchanger and shell and tube exchanger is used instead. Its construction is shown in U is given by its usual formulae. In many cases, dirt resistance needs to be included. h_i is given by usual formulae (e.g. Dittus Boelter equation for fully developed turbulent flow). For h_o a correlation specially designed for the shell side of a shell and tube exchanger is available. This correlation considers the hybrid flow consisting of crossflow and logitudinal flow. The correlation also needs to consider the flow through the clearance between the baffles and the tubes. For shell and tube exchanger, usually, the subscript 1 means inlet and 2 means outlet.

3.3.1 The shell side heat transfer coefficient

First, we need to calculate the Reynold's number for flow. The Re is given by:

$$Re = \frac{D_{eq}\dot{m}}{\mu A_f}$$

where A_f is the cross sectional area through which flow occurs. Since the flow is between longitudinal and cross flow, we calculate the D_{eq} by assuming a pure longitudinal flow and A_f by assuming pure cross flow. Now, for cross flow, the area varies and becomes maximum at the mid plane. The flow area available at the mid plain is considered as A_f. According to Figure,



$$A_f = N_t \times (P_T - d_o) \times B$$

where B is the baffle Spacing, P_T is the tube pitch, d_o is the tube dia and N_t is the number of tubes in the row given by: $N_t = D_s/P_T$. The equivalent diameter may be obtained using the usual formulae:

$$D_{eq} = \frac{4 \times \text{free area}}{\text{wetted perimeter}}$$

For square pitch:

$$D_{eq} = \frac{4 \times (P_T^2 - \pi d_o^2/4)}{\pi d_o}$$

for triangular pitch:

$$D_{eq} = \frac{4 \times (0.5P_T \times 0.86P_T - 0.5\pi d_o^2/4)}{0.5\pi d_o}$$

Example 3.3. Water enters (12 kg/s) the tubes of a small single-pass counter flow shell and tube heat exchanger at 20 °C. On the shell side, 7 kg/s of water enters at 60 °C. Calculate the exit temperatures if the area of the exchanger is 52 m^2. The exchanger has the following configuration: 10 inch ID shell with 74 no 1 inch OD tubes laid on 1.25 inch square pitch. Baffles are 24 inch apart with 25% cut. The inside film coefficient is 1913 W/m^2 K. 1m^2 = 10.76 ft^2.

Solution: The heat load:

$$Q = 12 \times 4200 \times (T_{t,out} - 20) = 12 \times 4200 \times (60 - T_{s,out})$$

$$N_t = \frac{10}{1.25} \approx 8$$

$$A_f = 8 \times (1.25 - 1) \times 24 \text{ sq. in} = 0.3 \text{ ft}^2 = 0.031\text{m}^2$$

$$D_{eq} = \frac{4 \times (1.25^2 - \pi \times 1^2/4)}{\pi \times 1} \text{ in} = 0.0824 \text{ ft} = .02511\text{m}$$

$$Re = \frac{0.02511 \times 7}{577 \times 10^{-6} \times 0.031} = 9783$$

Using the given correlation, $J_H = 55$. Under these conditions:

$$Pr = 3.77$$

and using the expression for J_H:

$$Nu = 55\sqrt[3]{3.77} = 85$$

Using $k_f = 640 \times 10^{-3}$, giving: h_o =2191. Using h_i = 1913, U = 1021. Therefore, the second equation becomes:

$$Q = 1021 \times 52 \times (LMTD)$$

These two equations can be solved easily using iterative calculation and the exit temperatures are:

$$T_{t,out} = 37\ ^\circ\text{C and} T_{s,out} = 31\ ^\circ\text{C}$$

□

It can be seen that the exchanger is very long. To avoid such long tube bundle, we can increase the shell diameter to incorporate more number of tubes. But this will reduce the flow per tube and the tube side heat transfer coefficient will be reduced substantially. For example, in the above example, if we increase the number of tubes by two, the tube side heat transfer coefficient becomes *** and hence the overall heat transfer coefficient reduces by **. Hence, we do not really decrease the area required. Hence, we bend the tubes and make them pass the shell more than once. In this way, we pack more heat exchange area in a smaller length and at the same time keep the turbulent flow in the tubes. Similar things can be done to shells as well. The construction of an one-two exchange is shown in Figure. It can be seen that a combination of counter current and co-current flow occurs. As evident

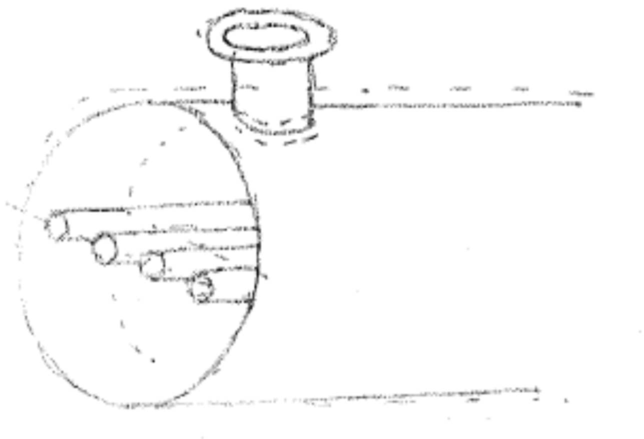


Figure 3.5: Figure 51

from the discussion and Figure, the shell and tube exchanger experience a combination of co-current and counter current flow. Hence, the design equation $Q = UA(LMTD)$ requires a correction factor in ΔT. The calculation procedure is given in the following:

- Use the corrected design equation $Q = U.A.(LMTD) \times F_T$

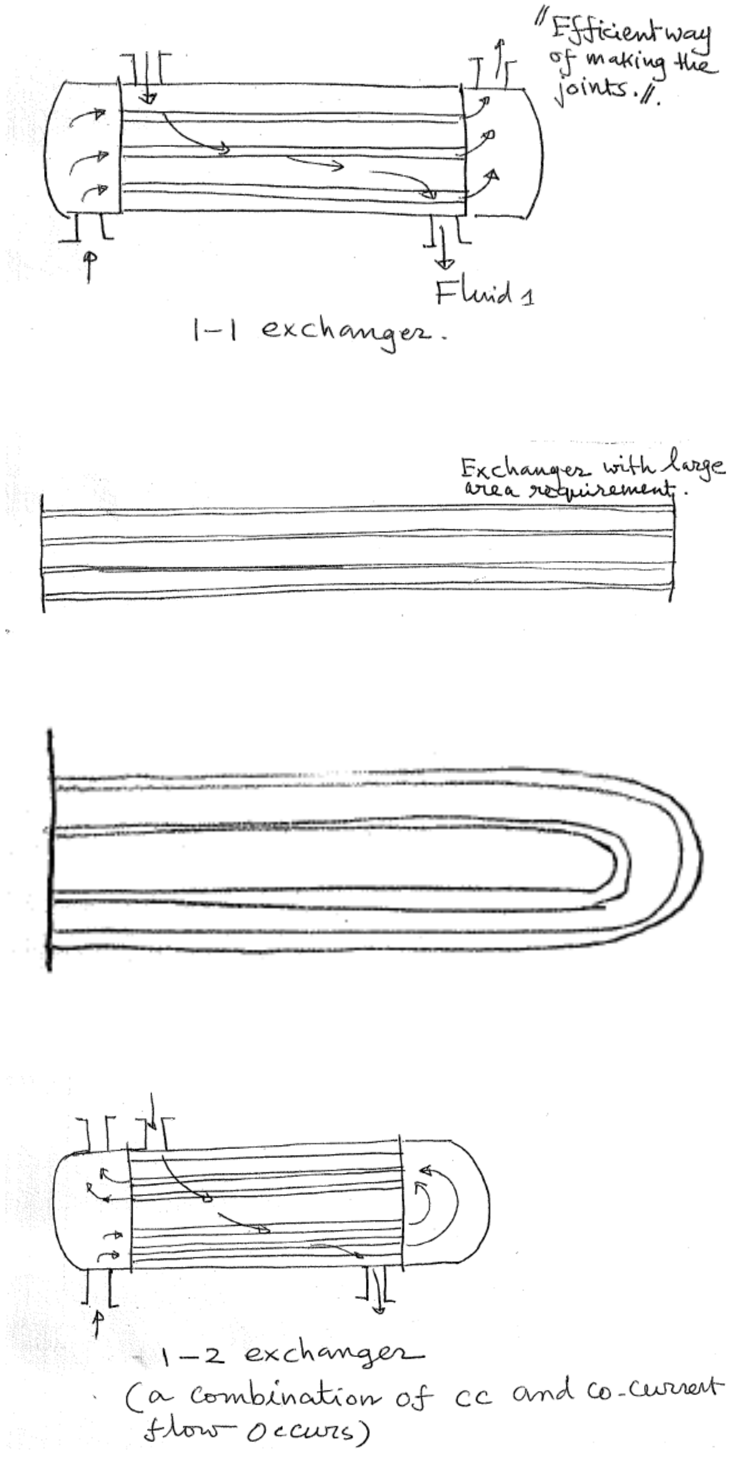
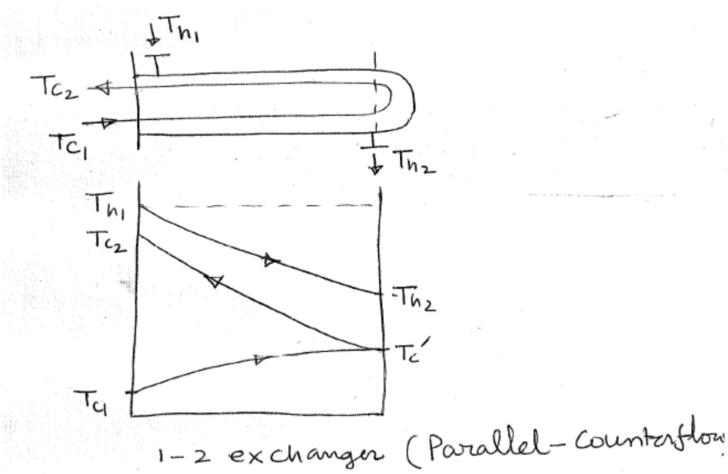


Figure 3.6: 1-2 Exchanger



- Assume counter current flow of the streams (as if in a double pipe exchanger)
- Calculate LMTD based on above.
- Obtain a correction factor F_T for the exchanger. For the correction factor, look up the correction factor chart applicable for the given exchanger. This will depend on the number of shell side passes and number of tube side passes as well as the nozzle arrangements. Usually, the nozzle arrangement is drawn on the correction factor chart.

Example 3.4. 43800 lb/hr of a 42 °API Kerosene leaves the bottom of a distillation column at 390 °F and will be cooled to 200 °F by 149000 lb/hr of 34 °API mid continent crude from storage at 100 °F. The shell side fluid is kerosene. The tube side coefficient is 121 Btu/hr ft^2 °F and the tubes are clean with negligible metal wall resistance. The following tube bundle configuration is prescribed: 21 inch ID shell with 158 no 1 inch OD tubes laid on 1.25 inch square pitch. Baffles are 5 inch apart with 25% cut. Determine the length of an 1-2 exchanger. *Data:* $\hat{c_p}$ of kerosene is 0.605 Btu/lb °F; $\hat{c_p}$ of crude is 0.490 Btu/lb °F. Viscosity of kerosene = 0.97 lb/ft-hr, thermal conductivity of kerosene is 0.0765 Btu/hr ft^2 (°F/ft). Viscosity correction can be neglected. The tube outside area is 0.344 ft^2/ft.

Solution: The heat load:

$$\begin{aligned} Q &= 43800 \times 0.605 \times (390 - 200) = 5.03 \times 10^6 \text{ Btu/hr} \\ &= 149000 \times 0.490 \times (T_{c2} - 100) \implies T_{c2} = 170°\text{F} \end{aligned}$$

Assuming pure countercurrent flow:

$$LMTD = \frac{220 - 100}{\ln(220/100)} = 152 \text{ °F}$$

Now, we need to obtain the correction factor:

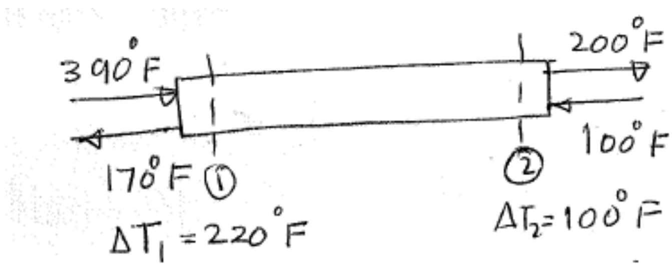


Figure 3.7: Pure counter-current flow

$$R = \frac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}} = \frac{390 - 200}{170 - 100} = 2.71$$

$$P = \frac{T_{t,out} - T_{t,in}}{T_{s,in} - T_{t,in}} = \frac{170 - 100}{390 - 100} = 0.241$$

From the chart,

$$F_T = 0.92$$

and

$$\Delta T = (LMTD)F_T = 152 \times 0.92 = 140°\text{F}$$

Now, we need to obtain the value of the overall heat transfer coefficient U. Since the inside coefficient is given and the metal wall and dirt resistance is negligible, we need to calculate the value of h_o to complete the calculation. Let us obtain the required quantities:

$$N_t = \frac{21}{1.25} \approx 17$$

$$A_f = 17 \times (1.25 - 1) \times 5 \text{ sq. in} = 0.147 \text{ ft}^2$$

$$D_{eq} = \frac{4 \times (1.25^2 - \pi \times 1^2/4)}{\pi \times 1} \text{ in} = 0.0824 \text{ ft}$$

$$Re = \frac{0.0824 \times 43800}{0.97 \times 0.147} = 25311$$

Using the given correlation, $J_H = 90$. Under these conditions:

$$Pr = \frac{0.605 \times 0.97}{0.0765} = 7.67$$

and using the expression for J_H:

$$Nu = 90\sqrt[3]{7.67} = 178$$

giving: h_o =165 Btu/hr ft^2 °F Therefore, the overall heat transfer coefficient will be:

$$U = (165^{-1} + 121^{-1})^{-1} = 70 \text{ Btu/hr ft}^2 \text{ °F}$$

The required heat exchange area therefore is:

$$A_T = \frac{5.03 \times 10^6}{70 \times 140} = 513 \text{ ft}^2$$

The required length would be:

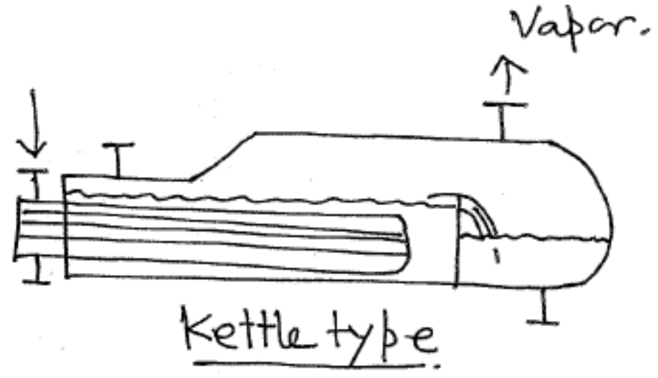
$$L_T = \frac{513}{158 \times 0.344} = 9.44 \text{ ft}$$

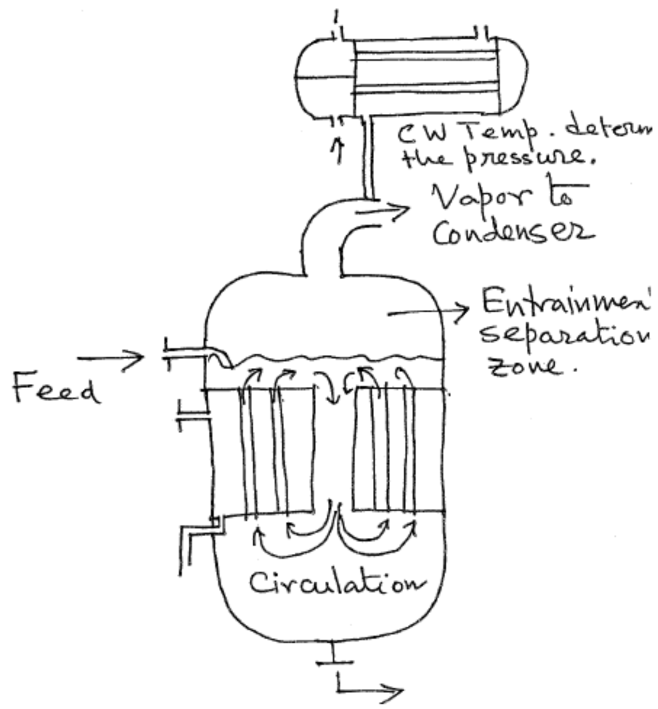
□

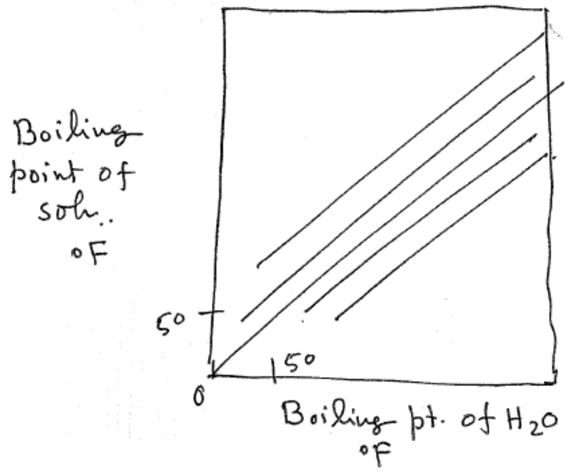
3.4 Evaporator

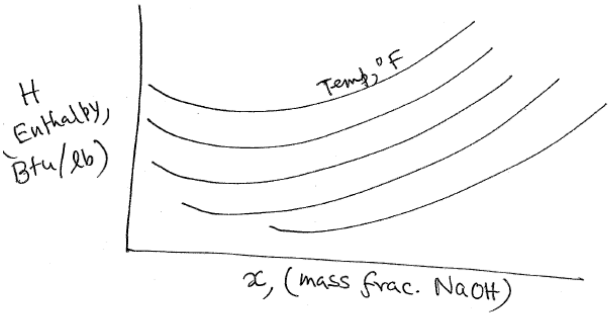
Evaporators are vaporizing exchangers. Any unfired heat exchanger in which one fluid undergoes vaporisation is called a vapourizer. If the vapour formed is steam, it is called evaporator. If it forms vapour using the bottom of a distillation column, it is called a reboiler, irrespective of whether the vapour is steam or not. In the bottom of a distillation column, a kettle type reboiler is seen whereas calandria type evaporators are typically used for concentrating solutions.

For a given system, our primary objective is to obtain the heat transfer area required. Along with that, we will also required to obtain the capacity of the evaporator (kg H_2O evaporated per hour) and economy (kg H_2O evaporated per kg of steam fed). In evaporator, we usually boil a concentrate solution. T_{sat} or boiling point of this solution is needed. This quantity is supplied by an empirical chart called *Duhring's Plot*. This plot is system specific. For the system NaOH-H_2O, the Duhring's Plot is shown below: We


Vapor.
Kettle type


CW Temp. determ the pressure.
Vapor to Condenser
Entrainment separation zone.
Feed
Circulation


Boiling point of soln. °F
50
0
150
Boiling pt. of H_2O °F


H Enthalpy, Btu/lb)
Temp, °F
x, (mass frac. NaOH)

will also need the enthalpy of the solution as a function of concentration. This is supplied by the enthalpy concentration diagram.

Now, we will start the analysis of the evaporator by writing the energy balance: Writing the heat transfer across the bundle:

$$UA\Delta T = \lambda_s \dot{m}_s$$

where $\lambda_s = h_{v,st} - h_{l,st}$. In this equation U, ΔT and λ_s may be obtained independently. Note that this ΔT is <u>not</u> LMTD. ΔT is the difference between the temperature of saturated steam used in the steam chest and that of the (saturated) boiling solution. The temperature of the boiling solution can be obtained from the D*ü*hring's Plot. Usually sensible heat exchange is negligible in an evaporator and hence only the latent heat is considered. In an evaporator problem, usually, the solution concentrations are known and the steam requirement and area required is asked. Hence, in the above equation, both A and $\dot{m}_s$ remains as unknowns. Therefore, we need another independent equation. This additional equation is obtained by writing the enthalpy balance around the entire evaporator:

$$\lambda_s \dot{m}_s + \dot{m}_f h_f = \dot{m}_c h_c + \dot{m}_v h_v$$

Various enthalpies and latent heats can be substituted in the above equation using the enthalpy concentration diagram and steam table. The mass flow rates of feed, vapour and products are obtained by writing the mass balance:

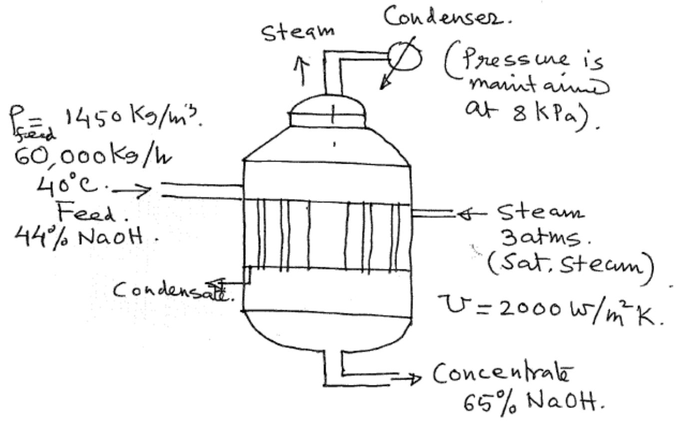
$$\dot{m}_f = \dot{m}_c + \dot{m}_v$$

$$\dot{m}_f x_f = \dot{m}_c x_c$$

These equations together form a closed set which is solved for $\dot{m}_s$, A and $\dot{m}_v$.

Example 3.5. 60,000 kg/h of 44% NaOH at 40 °C is to be concentrated to 65%. Saturated steam at 3 atm absolute pressure is available. An evaporator supplier can provide an evaporator which has an overall heat transfer coefficient of 2000 W/m^{2}K and operates at 8 kPa absolute pressure. Obtain the required heat transfer area. Calculate the steam requirement, heat transfer rate and area.

Solution. First, let us obtain all required properties:
Saturated steam at 3 atm: $T_{sat} = 407$ K, $\lambda_s = 2724$ kJ/kg.
At the pressure of the evaporator (8 kPa): $T_{sat} = 315$ K $= 107$ °F, $h_v = 2576$ kJ/kg.
The boiling point of the concentrated solution (Duhring's plot): 213 °F =



373 K. Using the enthalpy concentration diagram (1Btu/lb= 2.326 kJ/kg):
h_f (44% NaOH at 40 °C): 261 kJ/kg
h_c (65% NaOH at 213 °F): 756 kJ/kg
Now, from the mass balances:

$$\begin{aligned} 60000 &= \dot{m}_c + \dot{m}_v \\ 60000 \times 0.44 &= \dot{m}_c \times 0.65 \end{aligned}$$

we get $\dot{m}_c$ = 40,615 kg/h and $\dot{m}_v$ = 19,385 kg/h. Using these values in the overall energy balance we get $\dot{m}_s = 23854$ kg/h. Now, we can use the design equation and noting that $\Delta T = 34$ K, we get $A = 265$ m^2. The area required is very high. A large evaporator will be required. □

3.4.1 Negligible BPE

In many systems, such as milk, the boiling point elevation (0.3 °F) is negligible. For such systems, all streams have properties close to that of water. With this approximation, the enthalpy balance can be written as:

$$\begin{aligned} \lambda_s \dot{m}_s &= \dot{m}_c h_c + \dot{m}_v h_v - \dot{m}_f h_f \\ &= (\dot{m}_f - \dot{m}_v) h_c + \dot{m}_v h_v - \dot{m}_f h_f \\ &= \dot{m}_f (h_c - h_f) + \dot{m}_v (h_v - h_c) \\ &= \dot{m}_f c_{pf} (T_c - T_f) + \dot{m}_v \lambda_v \end{aligned}$$

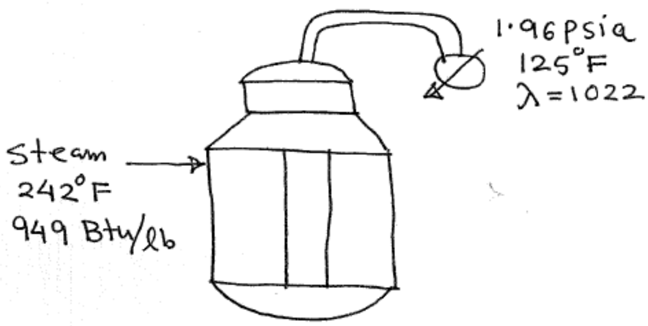
Example 3.6. It is desired to concentrate 50000 lb of a solution at 100 °F and 10% solid to a product containing 50% solid. Steam is available at 26.7 psia and the vapor space of the evaporator is operated at 2 psia.

Assume negligible BPE, specific heat=1.0, condensate leaves at saturation. The overall heat transfer coefficients is 125 Btu/hr ft^2 °F. Calculate the steam required and heating surface required.

Solution: Using mass balance:

$$50000 = \dot{m}_c + \dot{m}_v$$
$$50000 \times 0.1 = \dot{m}_c \times 0.5$$

we get $\dot{m}_c = 10,000$ and $\dot{m}_v = 40,000$. Properties of the steam @ 26.7 psia:



$T_{sat} = 242$ °F and $\lambda = 949$ Btu/lb.
@ 2 psia: $T_{sat} = 125$ °F and $\lambda = 1022$ Btu/lb.
Hence, the enthalpy balance becomes:

$$949 \times \dot{m}_s = 50000 \times 1 \times (125 - 100) + 40000 \times 1022$$

gives $\dot{m}_s = 44394$. The steam economy is 0.9. □

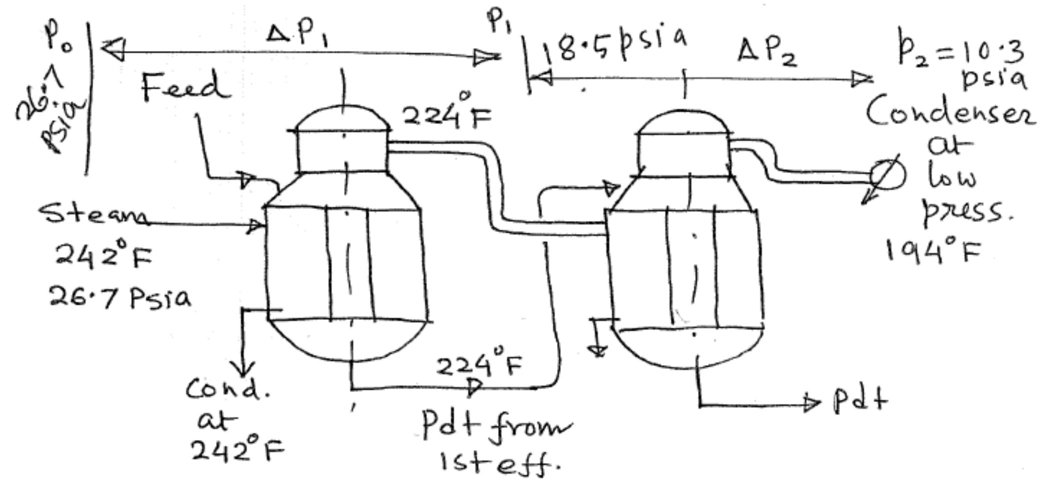
It can be seen that the calculations are much simpler if the BPE can be neglected. We will use this approximation for multiple effect evaporator discussed next.

3.4.2 Multiple effect evaporator

In many cases, the vapour generated in one evaporator is used as heating steam for another evaporator. In most cases, the feed/product streams are also connected as shown in Figure. This is called a multiple effect evaporator.

The first effect shown in Figure is not any different from the evaporator discussed before. It is also nothing extraordinary to feed the product stream to another evaporator to concentrate it further. However, the ingenuity exists in using the vapour generated in the first evaporator in the second instead of fresh steam. Since the vapor and the concentrate leaving the first evaporator are at the same temperature (in absence of BPE), higher vacuum must be applied in the vapor space of the second evaporator so that the temperature of the concentrate stream becomes lower than the vapor stream from the first evaporator and heat transfer becomes possible .

For example, consider the double effect evaporator shown in Figure **. In this case, the first effect operates at 18.5 psia (T_{sat} = 226 °F) using a steam at 26.5 psia (T_{sat} = 242 °F). Neglecting BPE, a good 16 °F temperature drop is maintained across the heating surface. The steam at 226 °F and 18.5 psia is fed to the next evaporator which is maintained at 10.3 psia. Because of lower pressure in the vapor space, a part of feed evporates by absorbing heat from its own body until it reaches the saturation temperature corresponding to 10.3 psia (194 °F). Then additional evaporation occurs by absorbing heat from the steam chest which is at 226 °F. In the second effect, the temperature drop across heating surface is 32 °F.



The feed order shown in the above example is called forward feed. The concentrated liquor flows from high pressure effects to lower pressure effect and hence no pump is required to transfer the liquor. But it has a disadvantage that the thicker liquor is fed to the coldest effect. At lower temperature, the heat transfer coefficient drops mainly due to viscosity effect and hence this type of feed is not suitable for all cases. For such cases, backward feed as shown in Figure may be used. However, backward feed will require a pump since the liquor must flow from low pressure effect to a high pressure effect. There are other feed arrangements such as mixed feed and parallel feed as shown in Figure.

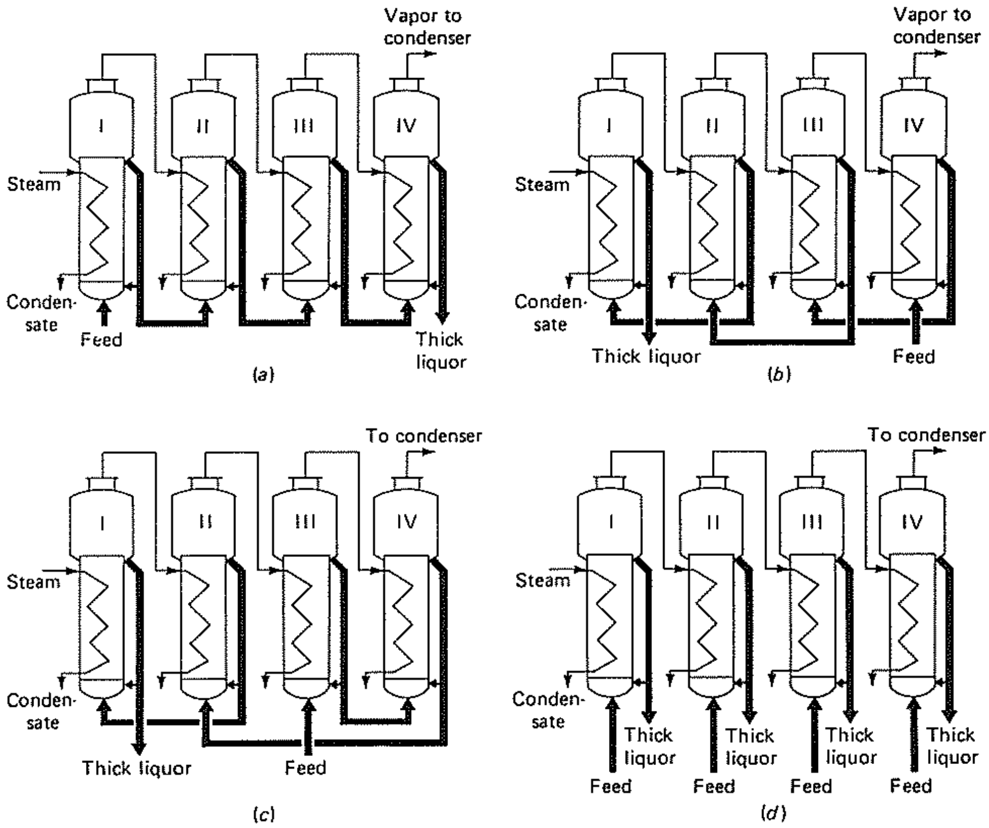


FIGURE 16.10
Patterns of liquor flow in multiple-effect evaporators: (*a*) forward feed; (*b*) backward feed; (*c*) mixed feed; (*d*) parallel feed. (▬▬) Liquor streams. (——) Steam and vapor condensate streams.

Next we will demonstrate a method to calculate the capacity, economy and area required for multiple effect evaporators. We will assume that all effects has the same area. This is usually the case because for multiple units of the same size, the cost is significantly less. We will also assume that the boiling point elevation is negligible.

In absence of BPE, most of the thermal energy goes for vapour generation. The latent heat absorbed by the vapour in the previous effect is transferred to the next effect.

$$\dot{m}_{s,1}\lambda_{s,1} = U_1 A \Delta T_1 = \dot{m}_{s,2}\lambda_{s,2} = U_2 A \Delta T_2$$

or,

$$U_i \Delta T_i = \text{constant}$$

or,

$$\Delta T_i \propto \frac{1}{U_i}$$

Defining $\Delta T_{ov} = \sum \Delta T_i$,

$$\frac{\Delta T_i}{\Delta T_{ov}} = \frac{1/U_i}{\sum 1/U_i}$$

Because the concentration and temperature of fluid is very different, U_i changes substantially from effect to effect. But nevertheless, the U_i can be obtained using the methods used in this course. The values of U_is are then used to obtain the values of ΔT_i in various effects using the above relation.

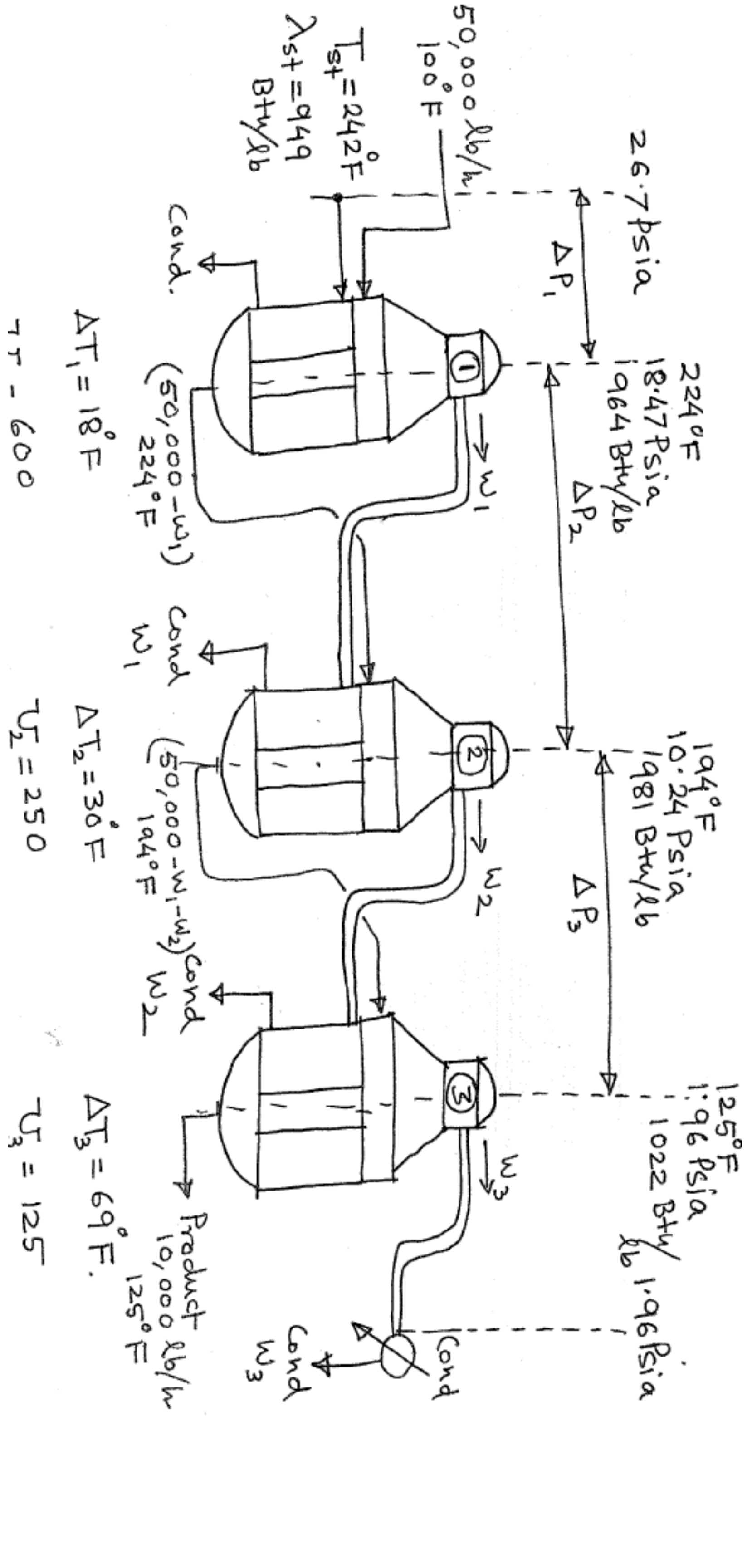
Example 3.7. It is desired to concentrate 50000 lb of a solution at 100 °F and 10% solid to a product containing 50% solid. Steam is available at 26 psia and the last effect of a triple effect evaporator is operated at 2 psia. Assume negligible BPE, specific heat=1.0 in all effects, condensate leaves at saturation. The overall heat transfer coefficients for the three effects are $U_1 = 600$, $U_2 = 250$ and $U_3 = 125$ Btu/hr ft^2 °F. The evaporator is operated at forward feed. Calculate the heating surface required and steam economy.

Solution: The terminal conditions are same as that of the previous example and the conditions are marked in Figure. It can be seen that the overall temperature drop is $\Delta T_{ov} = 117$ ° F. Using eq:

$$\Delta T_1 = 14 \text{ °F}; \quad \Delta T_2 = 34 \text{ °F}; \quad \Delta T_3 = 69 \text{ °F};$$

From the ΔT, we can obtain the temperature, pressure and enthalpies of various streams at different effects. These values are marked in the figure. Writing mass balnce in the overall envelope we get $\dot{m}_c$ =10000 lb/hr and

$$w_1 + w_2 + w_3 = 40000$$


50,000 lb/h
100°F
$T_{st} = 242°F$
$\lambda_{st} = 949$ Btu/lb
26.7 psia
ΔP_1
Cond.
1
224°F
18.47 Psia
964 Btu/lb
ΔP_2
w_1
$(50,000-w_1)$ 224°F
$\Delta T_1 = 18°F$
$U_1 = 600$
Cond w_1
2
194°F
10.24 Psia
981 Btu/lb
ΔP_3
w_2
$(50,000-w_1-w_2)$ 194°F
$\Delta T_2 = 30°F$
$U_2 = 250$
Cond w_2
3
125°F
1.96 Psia
1022 Btu/lb
1.96 Psia
w_3
$\Delta T_3 = 69°F.$
$U_3 = 125$
Product 10,000 lb/h 125°F
Cond
Cond w_3

Now, writing enthalpy balances for all three effects:

$$\begin{aligned}
\dot{m}_s \times 949 &= 50000 \times 1 \times (224 - 100) + w_1 \times 964 \\
w_1 \times 964 &= (50000 - w_1) \times 1 \times (194 - 224) + w_2 \times 981 \\
w_2 \times 981 &= (50000 - w_1 + w_2) \times 1 \times (125 - 194) + w_3 \times 1022
\end{aligned}$$

These four equations are solved for four unknowns: $w_1, w_2, w_3, \dot{m}_s$ and the solution is:

$$w_1 = 12,341; \quad w_2 = 13,280; \quad w_3 = 14,380; \quad \dot{m}_s = 19,069$$

Now, using the design equation:

$$A_1 = \frac{\dot{m}_s \lambda_s}{U_1 \Delta T_1} = \frac{19,069 \times 949}{600 \times 18} = 1675 \text{ ft}^2$$

$$A_2 = \frac{W_1 \lambda_2}{U_2 \Delta T_2} = \frac{12,341 \times 964}{250 \times 30} = 1586 \text{ ft}^2$$

$$A_3 = \frac{W_2 \lambda_2}{U_3 \Delta T_2} = \frac{13,280 \times 981}{125 \times 69} = 1510 \text{ ft}^2$$

Economy of the evaporator:

$$\frac{\text{kg H}_2\text{O evaporated}}{\text{kg steam used}} = \frac{40,000}{19,069} \approx 2.1$$

Capacity of heat exchange for this case is Q = 4.3 $\times 10^7$ Btu/h. □

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