Process Dynamics & Control

Process Dynamics & Control (CH 61016)

Class Teacher:	1. Prof Amar Nath Samanta

2. Prof Parag Deshpande

Class TA

Mr. Shubham Raj	Ms. Yashaswini Thakur

Mr. Nande Pradip Sugriv	Mr. Yagnesh Chainy

Class Timing

Wednesday	: 10 AM to 11 AM

Thursday	: 9 AM to 10 AM

Friday	: 11 AM to 1 PM (PD)

Attendance

Biometric method

Institute Deregistration Rule

Process Control Syllabus

1. Brief Review of process model and transfer function
2. Review of feedback control
3. Model based Control
   a) Direct Synthesis
   b) Internal Model Control

4. Control of time delay and inverse response processes

5. Traditional Advanced Control
   a) Feedforward Control
   b) Cascade Control
   c) Ratio Control
   d) Override Control

6. Multivariable Control
   a) Loop Interaction analysis
   b) Design of decentralized controllers
   c) Design of Decouplers

7. Model Predictive control
   a) Unconstrained DMC
   b) Quadratic DMC

8. Controller design using State-Space model
   a) State Feedback Controller
   b) Linear Quadratic Regulator
   c) State Observer

6. Nonlinear Control
   a) Feedback linearizing Control
   b) Backstepping Control
   c) Sliding mode control

Process Control

Books:

1.   Process Dynamics & Control   -- Seborg, Edgar, Mellichamp & Doyle

2.   Process Control   -- W Bequette

3.   Process Dynamics, Modelling and Control   – B A Ogunnaike & W H Ray

4.   Modern Control Engineering   -- K Ogata

Software: Knowledge of MATLAB & Simulink required for solving problems.

Benchmark Systems to be used for the course

- Tank Level System
  - 3 tank in series
  - Quadruple Tank
- Continuous Stirred Tank Heater
- Continuous Stirred Tank Reactor system
  - Isothermal Reactor (Van de Vusse reaction)
  - Non-isothermal Reactor
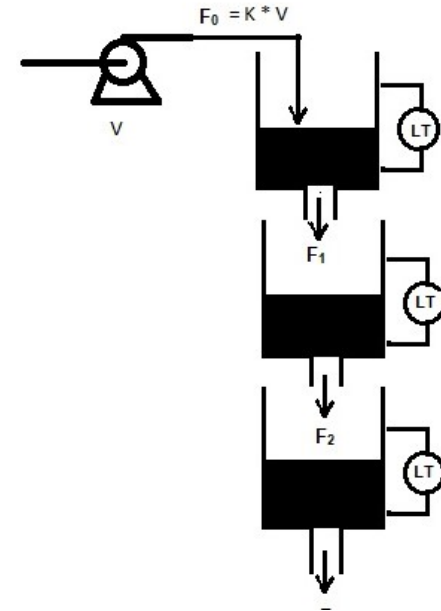 - Polymerization Reactor
- Distillation Column

MATLAB / SIMULINK model will be developed based on nonlinear model

Mathematical Model of tank level system

$$\frac{dh_1}{dt} = \frac{F_0}{A_1} - \frac{a_1}{A_1}\sqrt{2gh_1}$$

$$\frac{dh_2}{dt} = \frac{a_1}{A_2}\sqrt{2gh_1} - \frac{a_2}{A_2}\sqrt{2gh_2}$$

$$\frac{dh_3}{dt} = \frac{a_2}{A_3}\sqrt{2gh_2} - \frac{a_3}{A_3}\sqrt{2gh_3}$$

$F_0 = K * V$

Control objective:

Control the 3rd tank level by manipulating pump voltage V, where $F_0 = K\,V$

Add q_2 input flow on the tanks 2 and set q_2 as manipulated variable.

Which one is difficult to control?

Data for Simulation in Matlab/Simulink

A_1, A_3 = 28 cm² A_2 = 32 cm² V^s = 3 V

**a_1 = 0.06725 cm² a_2 = 0.05683 cm²
a_3 = 0.07089 cm² K=3.14**

- Transfer Function model

Define : $x_1 = h_1 - h_1^s; \quad x_2 = h_2 - h_2^s; \quad x_3 = h_3 - h_3^s; \quad u = V - V^s$

Linearizing around steady state value h_1^s

$$\frac{dh_1}{dt} = \frac{K}{A_1} V - \frac{a_1}{A_1}\sqrt{2g}\left[\sqrt{h_1^s} + \frac{1}{2\sqrt{h_1^s}}(h_1 - h_1^s)\right]$$

Subtracting above from steady state equation,

$$\frac{dx_1}{dt} = \frac{K}{A_1} u - \frac{a_1\sqrt{g}}{A_1\sqrt{2h_1^s}} x_1 \quad \rightarrow \text{Transfer function } \frac{x_1(s)}{u(s)} = \frac{K_{p_1}}{\tau_{p_1}s+1}$$

Where, $K_{p_1} = \frac{K}{a_1}\sqrt{\frac{2h_1^s}{g}}$ and $\tau_{p_1} = \frac{A_1}{a_1}\sqrt{\frac{2h_1^s}{g}}$
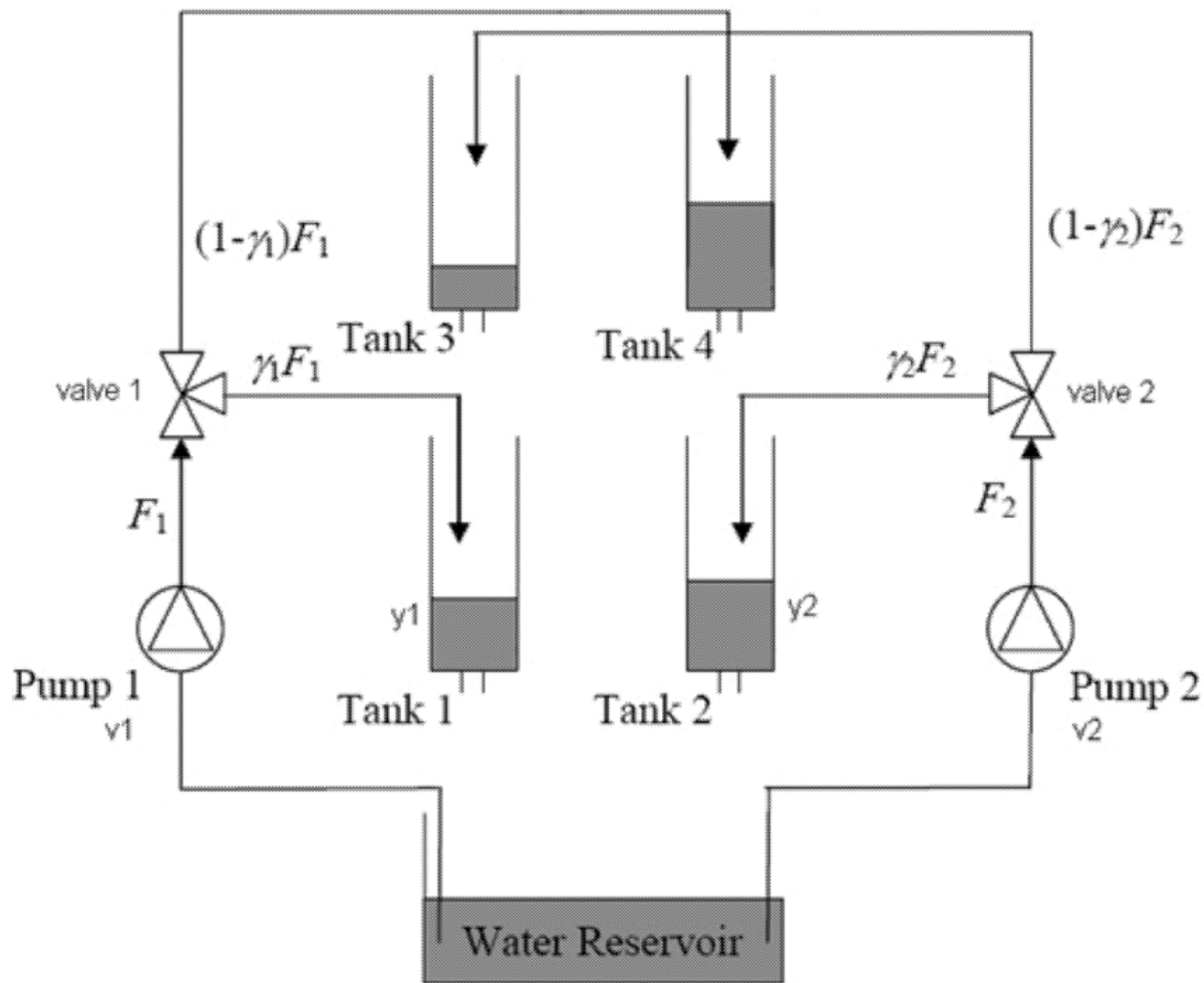
Similarly,

$$\frac{dx_2}{dt} = \frac{a_1}{A_2}\sqrt{\frac{g}{2h_1^s}}\, x_1 - \frac{a_2}{A_2}\sqrt{\frac{g}{2h_2^s}}\, x_2 \text{ and Transfer function } \frac{x_2(s)}{x_1(s)} = \frac{K_{p_2}}{\tau_{p_2}s+1}$$

$$\frac{dx_3}{dt} = \frac{a_2}{A_3}\sqrt{\frac{g}{2h_2^s}}\, x_2 - \frac{a_3}{A_3}\sqrt{\frac{g}{2h_3^s}}\, x_3 \text{ and Transfer function } \frac{x_3(s)}{x_2(s)} = \frac{K_{p_3}}{\tau_{p_3}s+1}$$

Where, $K_{p_j} = \frac{a_{j-1}}{a_j}\sqrt{\frac{h_j^s}{h_{j-1}^s}}$ and $\tau_{p_j} = \frac{A_j}{a_j}\sqrt{\frac{2h_j^s}{g}}$ for j = 2,3

So, Process Transfer function $G(s) = \dfrac{x_3(s)}{u(s)} = \dfrac{K_{p_1}K_{p_2}K_{p_3}}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)(\tau_{p_3}s+1)}$

Quadruple Tank problem

Quadruple Tank problem

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2$$

$$\frac{dh_2}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1$$

Control Objective

Both the levels of tank1 and tank2 should be controlled by manipulating voltages to the pumps.

Data for simulation:

A_1, A_3 = 28 cm²

A_2, A_4 = 32 cm²

a_1, a_3 = 0.071 cm²

a_2, a_4 = 0.057 cm²

k_1, k_2 = 3.33, 3.35

v_1, v_2 = 3.0, 3.0

γ_1, γ_2 = 0.7, 0.6

Jacketed heated stirred tank
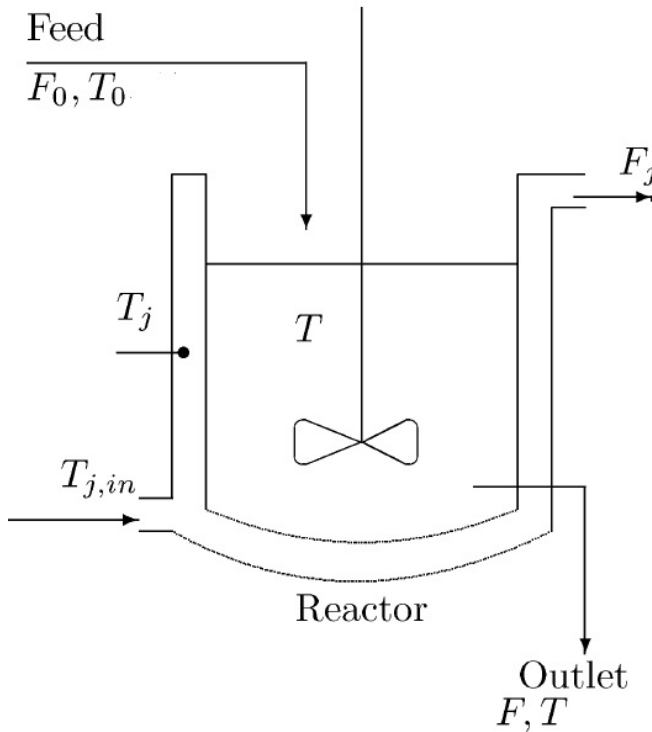
Assumptions:
- Constant hold-up in tank and jacket
- Constant heat capacities and densities
- Incompressible flow

$$\frac{dV}{dt} = F_0 - F = 0$$

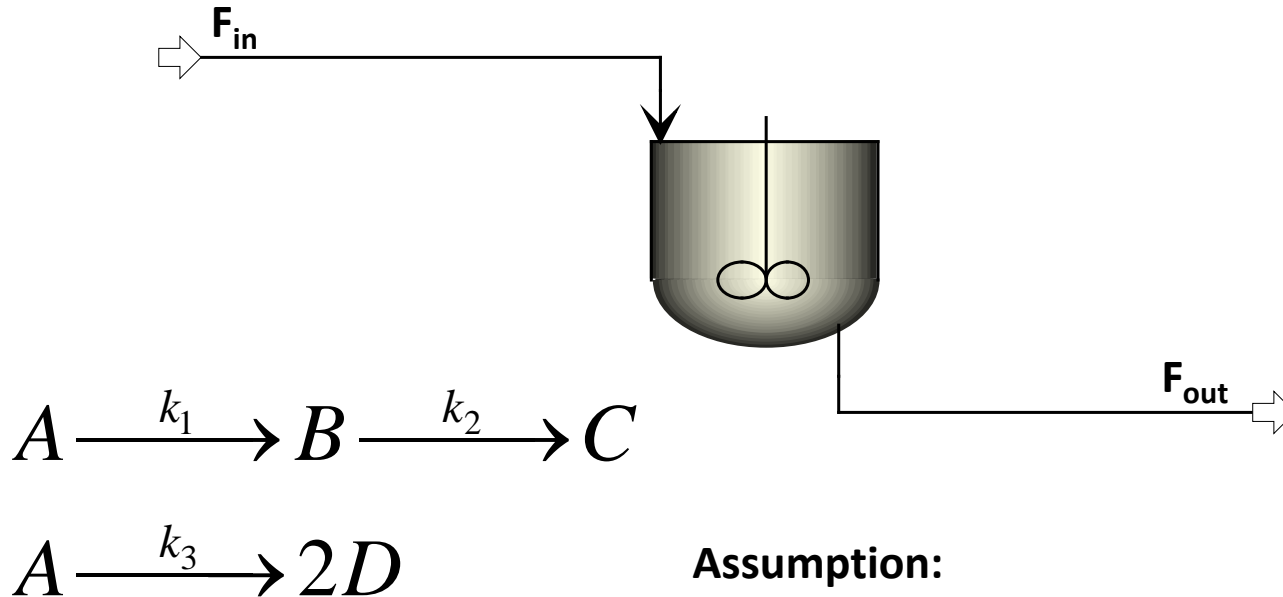$$\frac{dT}{dt} = \frac{F}{V}(T_0 - T) + \frac{UA}{\rho C_P V}(T_j - T)$$

$$\frac{dT_j}{dt} = \frac{F_j}{V_j}(T_{ji} - T_j) - \frac{UA}{\rho_j C_{Pj} V_j}(T_j - T)$$

Parameter and Steady State Values
F = 30 l/min; F_j^s = 50 l/min; T_0 = 15⁰C; T_{Jin} = 93⁰C; V = 300 l; V_j = 30 l;
ρC_p = 1 Kcal/K l; ρC_{pj} = 1.384 ; UA = 100 Kcal/min K; T^s = 60⁰C ; T_J = 73.5 ⁰C

Continuous Stirred Tank Reactor

F$_{in}$

F$_{out}$

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

$$A \xrightarrow{k_3} 2D$$

Assumption:

1. **Exit Condition = Reactor Condition**
2. **Isothermal Reaction**
3. **Constant volume, i.e, F$_{in}$ = F$_{out}$=F**
4. **Only reactant A in feed is consumed, i.e, other reactant is in large excess. No product in the feed**

Continuous Stirred Tank Reactor

Constant volume : dV/dt = 0

Dynamic model

Data for the CSTR

$V = 1$ l

$F = 78$ l/h

$C_{Af} = 10$ mol/l

kl = 50 h-1

$k2 = 100$ h-1

$k3 = 10$ l.mol-1.h-1

- $\frac{dC_A}{dt} = \frac{F}{V}\left(C_{Af} - C_A\right) - k_1 C_A - k_3 C_A^2$

- $\frac{dC_B}{dt} = -\frac{F}{V} C_B + k_1 C_A - k_2 C_B$

- $\frac{dC_C}{dt} = -\frac{F}{V} C_C + k_2 C_B$

- $\frac{dC_D}{dt} = -\frac{F}{V} C_D + \frac{1}{2} k_3 C_A^2$
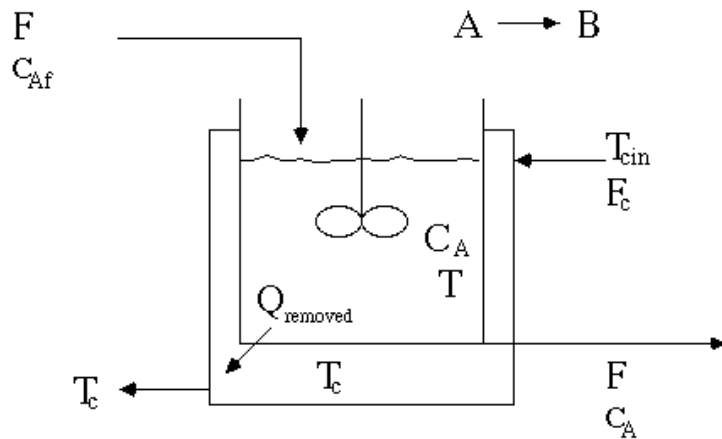
Since our objective is to control C_B, the reduced order model should be

- $\frac{dC_A}{dt} = \frac{F}{V}\left(C_{Af} - C_A\right) - k_1 C_A - k_3 C_A^2 = f_1(C_A, C_B, F)$

- $\frac{dC_B}{dt} = -\frac{F}{V} C_B + k_1 C_A - k_2 C_B = f_2(C_A, C_B, F)$
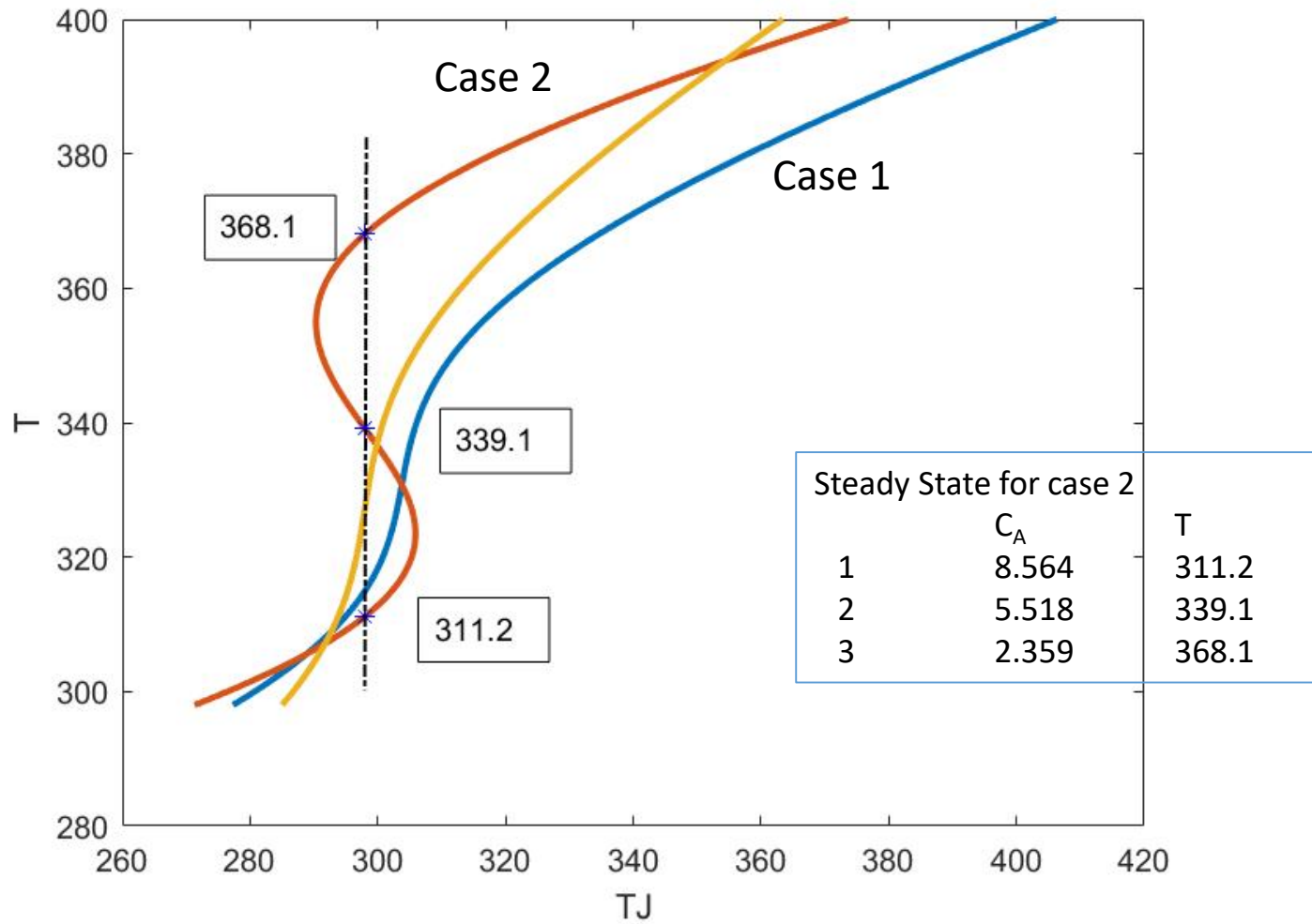
Steady State plot of Van-De-Vusse Reactor

1.2015

[78,1.266]

130

46

What is the Process Gain at these points ?
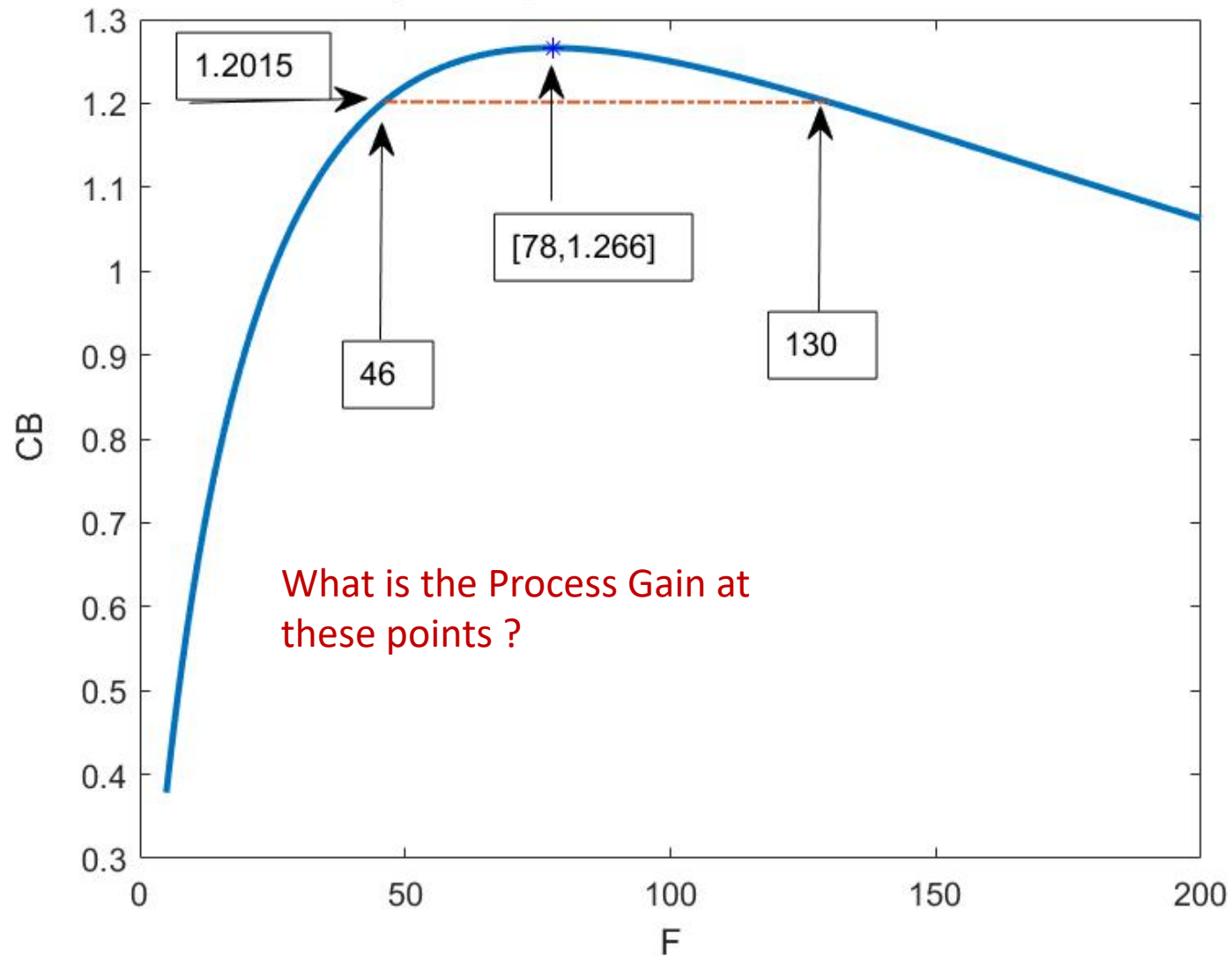
Non-Isothermal Jacketed CSTR

A → B

$$\frac{dC_A}{dt} = \frac{F}{V}\left(C_{A_f} - C_A\right) - r$$

$$\frac{dT}{dt} = \frac{F}{V}\left(T_f - T\right) + \left(\frac{-\Delta H}{\rho C_p}\right)r - \frac{UA}{V\rho C_p}(T - T_C)$$

$$r = k_0 e^{-\frac{E}{RT}}C_A$$

parameter	case 1	case 2	case 3
F/V, hr-1	1	1	1
k_0, hr-1	14,825*3600	9,703*3600	18,194*3600
$(-\Delta H)$, kcal/kgmol	5215	5960	8195
E, kcal/kgmol	11,843	11,843	11,843
ρc_p, kcal/(m^3 ^oC)	500	500	500
T_f, ^oC	25	25	25
C_{Af}, kgmol/m3	10	10	10
UA/V, kcal/(m^3 ^oC hr)	250	150	750
T_C, ^oC	25	25	25

Variation of Reactor Temp with Jacket Temperature

Case 2

Case 1

368.1

339.1

311.2

Steady State for case 2		
	C_A	T
1	8.564	311.2
2	5.518	339.1
3	2.359	368.1

Styrene Polymerization Reactor

x_1 Initiator Conc
x_2 Monomer Conc
x_3 Reactor Temp
x_4 Jacket Temp.

$$\dot{x}_1 = \frac{(F_i\,C_{ia} - F_o\,x_1)}{V} - k_d\,x_1$$

$$\dot{x}_2 = \frac{(F_m\,C_{ma} - F_o\,x_2)}{V} - k_p\,x_2\,\mathcal{R}$$

$$\dot{x}_3 = \frac{F_o\,(T_a - x_3)}{V} - \frac{\Delta H}{\rho\,C_p}\,k_p\,x_2\,\mathcal{R} - \frac{U A}{\rho\,C_p\,V}\,(x_3 - x_4)$$

$$\dot{x}_4 = \frac{F_j(T_{j,in} - x_4)}{V_j} + \frac{U A}{\rho_j\,C_{pj}\,V_j}\,(x_3 - x_4)$$

$$F_o = F_i + F_m + F_s.$$

Initiator F_i

Monomer F_m

Solvent F_s

T_j

T

$T_{j,in}$

Reactor
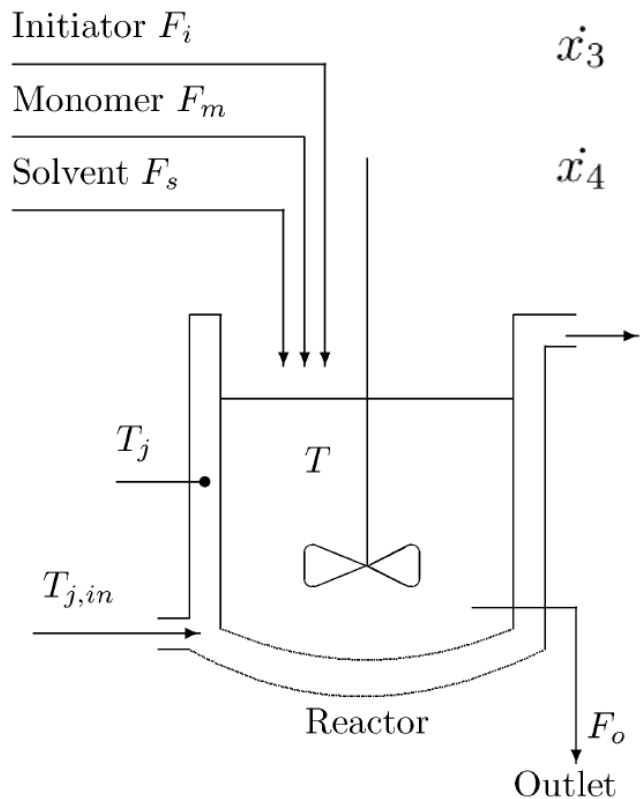
F_o

Outlet

The chain concentration of growing polymer is equal to

$$\mathcal{R} = (2\,f\,k_d\,x_1/k_t)^{0.5}$$
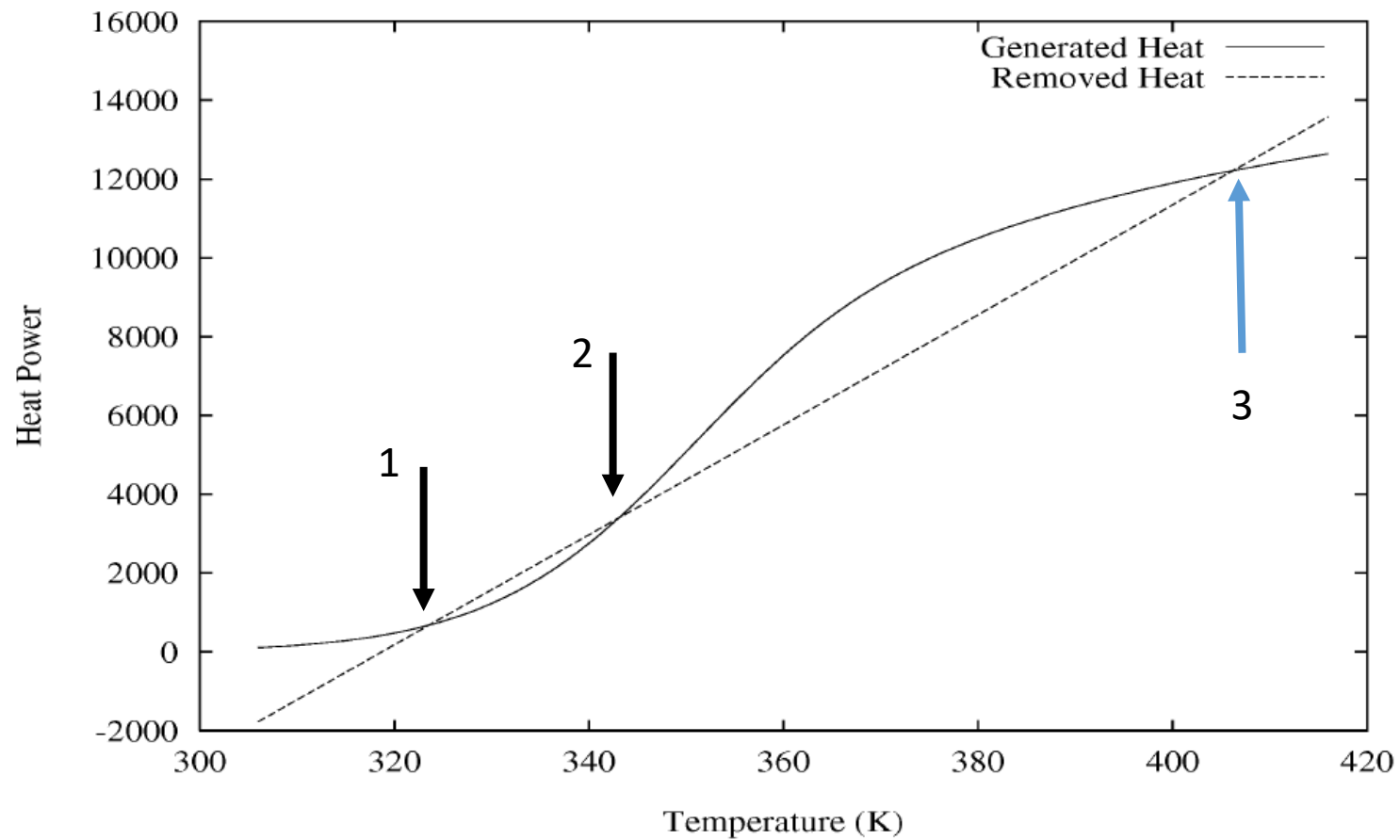
The dissociation, propagation and termination
Rate constants follow Arrhenius law :

$$k_i = k_{i0}\,e^{-\frac{E_i}{RT}} \quad \text{for I =d, p or t}$$

Steady State Heat Power vs Reactor Temperature

Phase Portrait

Legend:
- T=305K
- T=305K
- T=325K
- T=340K
- T=340K
- T=340K
- T=345K
- T=346K
- T=349K
- T=350K
- T=350K
- T=342,6K
- T=343K

Axis labels: Temperature (K), Monomer Conversion

Annotations: 1, 2 (unstable), 3