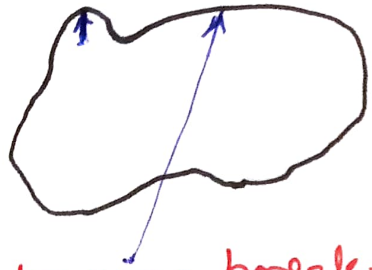
Droplet, squeezed away from spherical shape will have regions of higher and lower curvature



Resulting imbalance of Laplace pressure drives the liquid to a perfectly symmetric sphere.

Spontaneous break-up of liquid string into droplets.
(or gas string into bubbles)

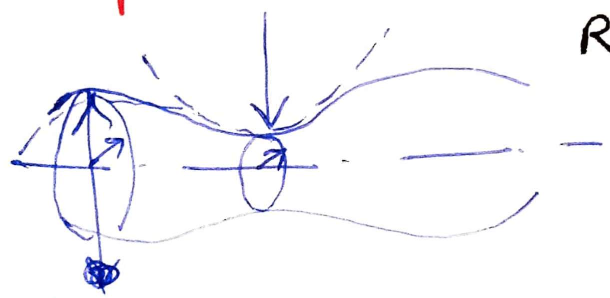
Cylinder of gas or liquid surrounded by another immiscible fluid

Principal radii of curvature

Axial Curvature = 0

Radial curvature defined by the radius of the cylinder

Small deformation of cylindrical shape due to thermal fluctuations.
(Sinusoidal modulations)



Radial Curvature (At the peaks) < Radial Curvature (At the trough)

Radial curvature (and corresponding Principal radius) dominates for large wavelength

$\Rightarrow$ (Laplace pressure)$_{peak}$ < (Laplace Pressure)$_{trough}$

$\Rightarrow$ Fluid driven from trough to peak
$\Rightarrow$ perturbation gets amplified.

For short wavelength, axial curvature dominates.
Axial curvature at trough is negative

(Laplace Pressure)$_{peak}$ > (Laplace Pressure)$_{trough}$

$\Rightarrow$ Fluid driven from peak to trough $\Rightarrow$ Decay of perturbation

Rayleigh Instability

Spontaneous break-up into droplets / bubbles

Perturbations with large wavelength gets amplified

Perturbation with short wavelength gets decayed.

Critical wavelength below which the fluctuations decay

↳ defines the size of droplet / bubble.

Critical wavelength = circumference of the jet for a stream of inviscid liquid falling through air.

The wavelength that provides fastest growth of perturbation

$\approx$ 1.43 times the circumference

$\approx$ Characteristic diameter of droplets.

When the immiscible fluid surrounding the gas or liquid string is inside a channel, viscous effect on the surrounding continuous ~~fo~~ trough slows down the fluid during movement due to collapsing

break-up

Dimensionless Number

Bond No. $= \dfrac{\rho g L^2}{\sigma}$

Compares gravitational force acting on the fluid with interfacial force $= \dfrac{(\rho L^3)g}{\left(\frac{\sigma}{L}\right)L^2}$

Bo No. $<< 1$
$\Rightarrow$ almost spherical droplet with L as diameter

Bo No. $>> 1$
$\Rightarrow$ flat puddle of liquid.

Capillary No. $= \dfrac{\mu v}{\sigma}$

or alternatively $\dfrac{\mu u d}{4 \sigma L}$

Compares viscous forces with the interfacial forces.

Viscous stress $\equiv \tau \equiv \mu \dfrac{v}{L}$

Interfacial Stress $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{\sigma L}{L^2}$

Ratio $= \dfrac{\mu V}{\sigma}$

For channel dimension (L), and droplet dimension (d), the ratio becomes $\dfrac{\mu V/L}{\left(\dfrac{\sigma \pi d}{\pi d^2/4}\right)} = \dfrac{\mu V d}{4 \sigma L}$

Weber No. $= \dfrac{\rho u^2 L}{\sigma}$

compares inertial and interfacial stresses

$= \dfrac{(\text{mass})(\text{acceleration})}{\text{interfacial force}}$

$= \dfrac{(\rho L^3)(u^2/L)}{\sigma L}$

Significance of Capillary Number

* In shear flow, large droplets are elongated, and undergo splitting into droplets due to Rayleigh Plateau instability.

* The fragmentation continues until the radius of the droplet is small enough such that the laplace pressure balances the shear stress (Ca No. ≈ 1).

* The order of magnitude of the droplet radius can be approximated by setting Ca No. $= 1$ $\Rightarrow$ $d \approx 4L\left(\frac{\gamma}{\mu u}\right)$

Significance of Weber Number

* A large value implies formation of elongated jet that breaks up into droplets far away from the nozzle

* A small value implies dripping of droplets without formation of pronounced jet.