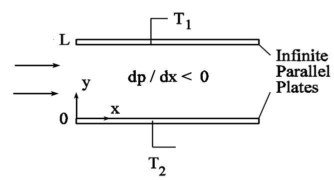
Indian Institute of Technology Kharagpur
Mid-Spring Semester 2018 – 2019

Date of Examination: 22.2.2019 Session: AN Duration 2 hrs Full Marks 30
Subject Number: CH30012 Subject: Transport Phenomena Department: Chemical Engineering
Specific charts, graph paper, log book etc., required: No
Specific Instructions: Assume and write any assumption and data that you feel are missing.

1. Consider a system of two concentric rotating cylinders. The two cylinders each rotate at a constant but different angular velocities (ω_i and ω_o) and you may neglect body forces.
   a) Determine the velocity profile $v_\theta(r)$ between the cylinders and the pressure distribution P(r).
   b) Determine a friction factor by calculating the force required to turn either of the two cylinders. Given that P = P_i at the inner cylinder (r = r_i) and for the outer cylinder one can express the force as (v_o is the linear velocity of the outer cylinder): **4+3=7**

$$F = \tau_{r\theta}A = C_f\left(\frac{1}{2}\rho v_o^2\right)A$$

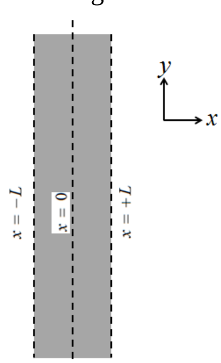
2. Consider the problem of steady, incompressible laminar flow between two stationary, infinite parallel plates maintained at different temperatures (neglect natural convection). Referred to as Poiseuille flow with heat transfer, this special case of parallel flow is one in which the x-velocity component is finite, but the y and z components are zero.



(i) What is the form of continuity equation for this case? In which way the flow is fully developed?
(ii) What forms do the x and y momentum equations take? What is the form of the velocity profile? Note that, unlike Couette flow, fluid motion is sustained by a finite pressure gradient (shown in the figure). How is this pressure gradient related to the maximum fluid velocity?
(iii) What is the appropriate form of energy equation, assuming viscous dissipation to be significant? Solve the equation for the temperature distribution and evaluate the heat flux at the upper (y=L) surface. **2+3+3=8**

3. Two large flat porous horizontal plates are separated by a relatively small distance L. The upper plate at y = L is at a temperature T_1 and the lower one at y = 0 is to be maintained at a lower temperature T_o. To reduce the amount of heat that must be removed from the lower plate, an ideal gas at T_o is blown upward through both the plates at a steady rate. Develop an expression for the temperature distribution and the amount of heat q_o that must be removed from the cold plate per unit area as a function of the fluid properties and gas flow rates. Use $\emptyset = \rho\, C_p\, v_y L / k$ where ρ is the density, c_p is the heat capacity, v_y is the upward velocity of air through the plates and k is the thermal conductivity of air to present the result in a compact form. **7**

4. A solid slab shown in the figure below, is initially kept at temperature T_0. At time $t = 0$, the temperature of the two boundaries situated at $x = \pm L$, is suddenly raised to T_s, and maintained there for all $t \geq 0$. Assume 1-D heat conduction in x-direction ($\pm y$ directions are infinite), thermal diffusivity of slab = α, answer the following:



a) Write down the governing equation along with initial (IC) and boundary condition (BC) for getting temperature profile across the slab: $T(x,t)$.
b) Consider: $\theta = \left(T_s - T\right)/\left(T_s - T\right)$, non-dimensionalise the governing equation along with IC and BC. Comment on your choice of characteristic time scale.
c) Solve the non-dimensionalised governing equation using *separation of variables* technique or otherwise. Report the final expression of θ in terms of $t, x/L, \alpha$.
d) Draw theta as a function of x/L for different $t = 0.1L^2/\alpha,\ L^2/\alpha, 5L^2/\alpha$, qualitatively.

EQUATION OF CONTINUITY (Cartesian, cylindrical and spherical coordinates)

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=0 \qquad \frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

$$\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^2 v_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\rho v_\theta\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(\rho v_\phi\right)=0$$

Stresses in cylindrical coordinates

$$\tau_{r\theta}=\mu[r\frac{\partial}{\partial r}(\frac{v_\theta}{r})+\frac{1}{r}\frac{\partial v_r}{\partial\theta}] \quad \tau_{\theta z}=\mu(\frac{\partial v_\theta}{\partial z}+\frac{1}{r}\frac{\partial v_z}{\partial\theta}) \quad \tau_{zr}=\mu(\frac{\partial v_r}{\partial z}+\frac{1}{r}\frac{\partial v_z}{\partial r})$$

EQUATION OF MOTION (Cartesian and Cylindrical coordinates)

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}+v_z\frac{\partial v_r}{\partial z}-\frac{{v_\theta}^2}{r}\right)$$
$$=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}-\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right]+\rho g_r$$

$$\boldsymbol{\rho}\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_r v_\theta}{r}\right)$$
$$=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right]+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$

<u>ENERGY EQUATION (all coordinate systems)</u>

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]+\mu\phi_v+\dot{Q}$$

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+v_z\frac{\partial T}{\partial z}\right)=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}\right]+\mu\phi_v+\dot{Q}$$

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial\phi}\right)=k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}\right]+\mu\phi_v+\dot{Q}$$

$$\mu\phi_v=\mu\left\{\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2\right\}$$

$$Sc=\frac{\mu}{\rho D_{AB}} \quad Sh=\frac{h_m l}{D_{AB}} \quad St=\frac{Nu}{Re*Pr} \quad Fo=\frac{\alpha t}{l^2} \quad Bi=hl/k_s \quad Le=\frac{\alpha}{D_{AB}} \quad C_f=\frac{\tau_w}{\frac{1}{2}\rho v^2} \quad C_D=\frac{F_D/A_P}{\frac{1}{2}\rho v^2}$$