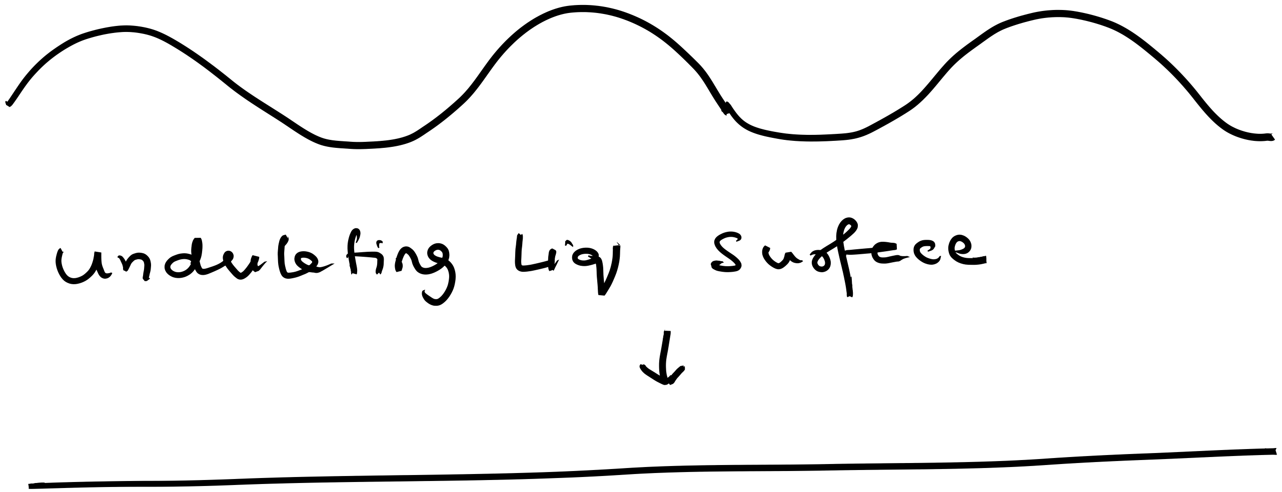
14.03.2022

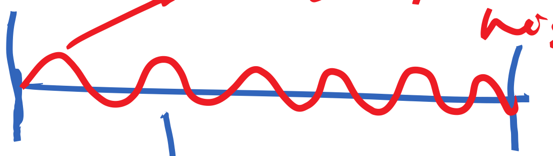
Lecture 25

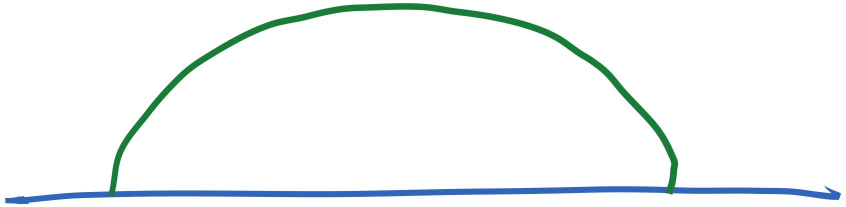


undulating Liq surface

↓

Flattens down due to Surface Tension





Drop of Liquid on a solid Surface.

→ Hemispherical shape due to Surface Tension.

Pressure across a non planar Liquid Surface (which might be in equilibrium).

$\boxed{\text{Laplace Pressure}}$

If you have a Flat Liquid Surface
At equilibrium

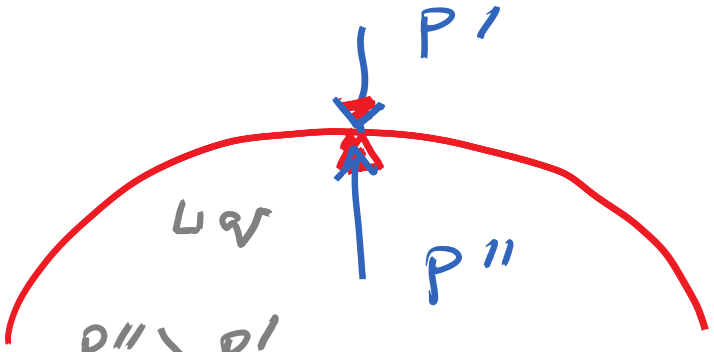
At the interface $P' = P''$

$(P_{Liq\ h=0} = P_{atm})$

For a Curved Liq Surface (at Eqlbm)

* $P' > P''$
or
$P' < P''$

Fig 1



$P'' > P'$

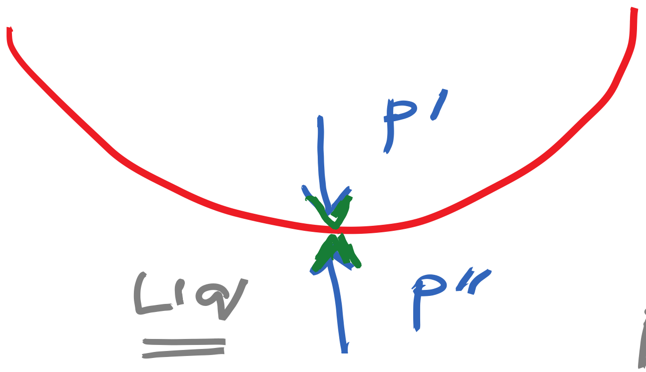
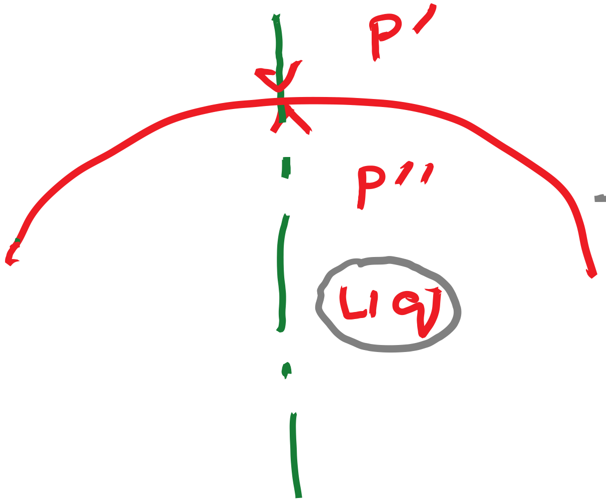


Fig 2

$P'' < P'$

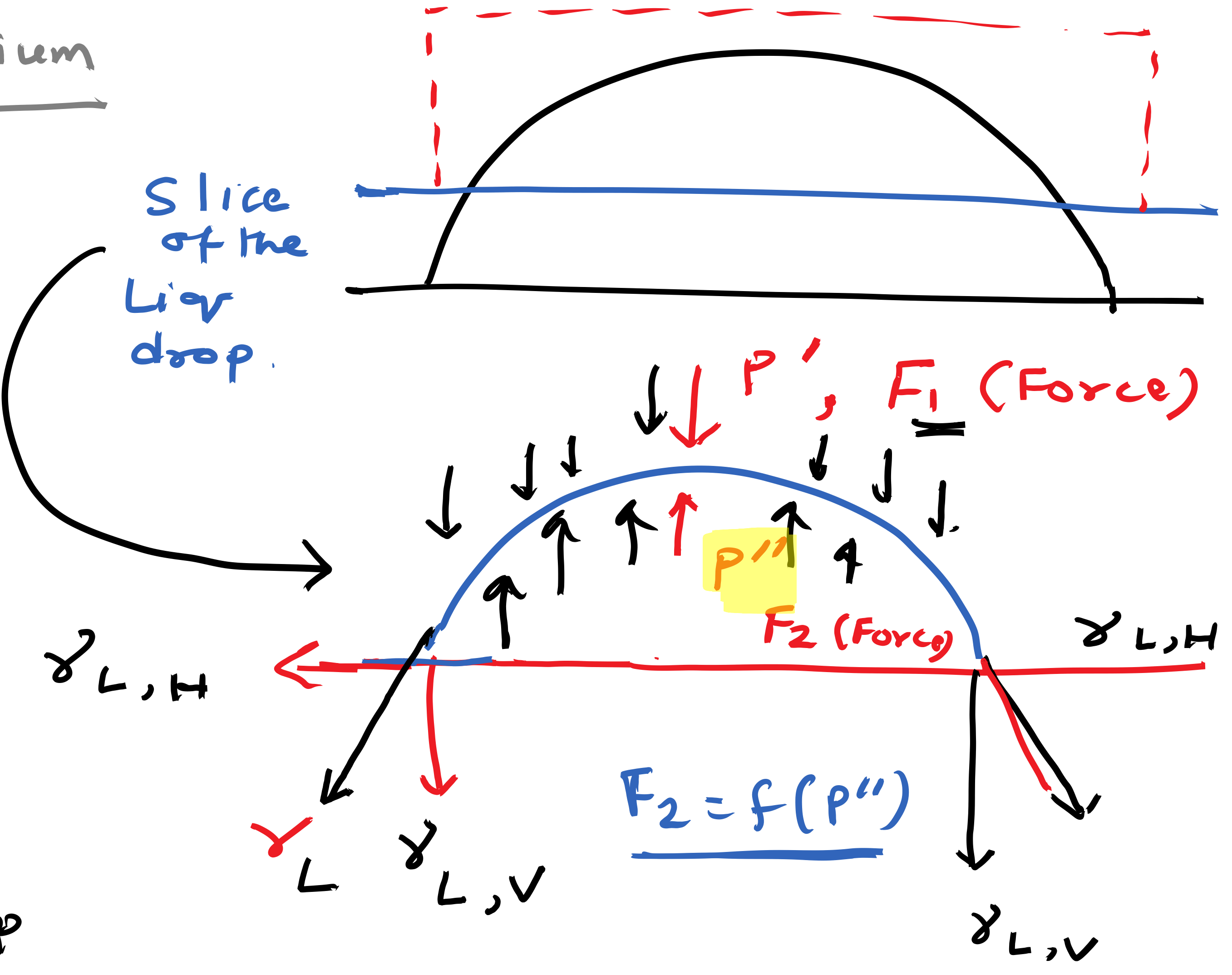
$P' \neq P''$

$\Delta P = P' - P'' =$ Laplace Pressure
(Pressure Jump)



$P'' > P'$

→ Equilibrium



Since the surface is at Eqllbm,

Net Force vertically Down

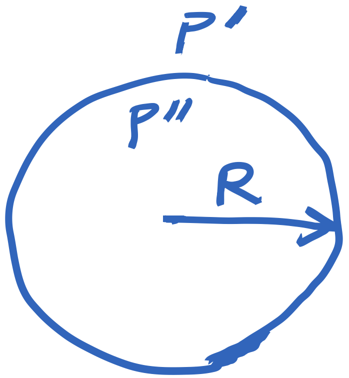
= Net Force Vertically up

$$F_1 + \gamma_{L,V} = F_2$$

Since $\gamma_{L,V} \neq 0$, $F_2 > F_1 \Rightarrow P'' > P'$

Spherical Liq. Drop.

$\Delta P \cdot 4\pi R^2 dR = 8\pi R dR \cdot \gamma$ $\boxed{\Delta P = \frac{2\gamma}{R}}$



We do some work (Supply some Energy) so that the drop dia changes from R to R+dR.

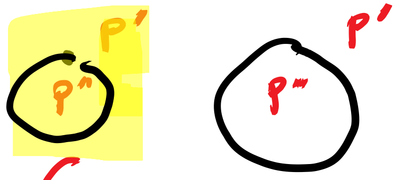
Necessary work done

$$P\,dV = (p' - p'')\,dV = \underline{\Delta P \cdot 4\pi R^2 dR}$$

$$V = \frac{4}{3}\pi R^3 \qquad \underline{dV = 4\pi R^2 dR}$$

Consequence of the work done/ Energy Supplied →

↳ Radius goes up. The Surface Area goes up → Total Interfacial Energy goes



$p'' > p'$

ΔP is more

$A = 4\pi R^2$

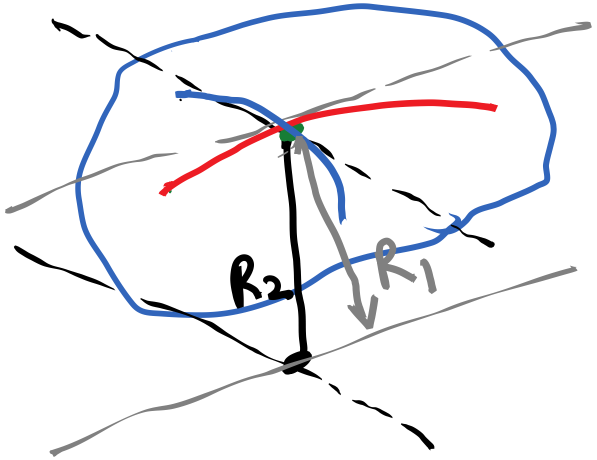
Total Energy at the surface $\gamma . A$.

$dA = 8\pi R dR$

Change in Energy at the Surface $\gamma . dA$

$= \underline{\underline{8\pi R dR}} \cdot \gamma$

$\boxed{\text{Lower is R, Higher is Laplace Pressure}}$



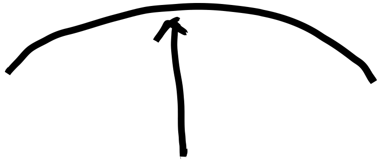
Arbitrarily Curved Surface

Any Point on a curved surface

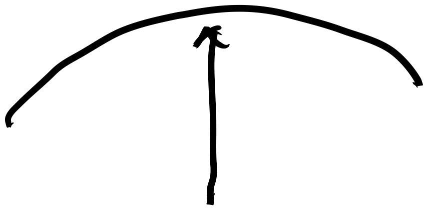
Define the Point $(x, y, z) \rightarrow$ 3 Coordinates reqd.

How do you define the Local Curvature at any point. (How many Parameters are required).

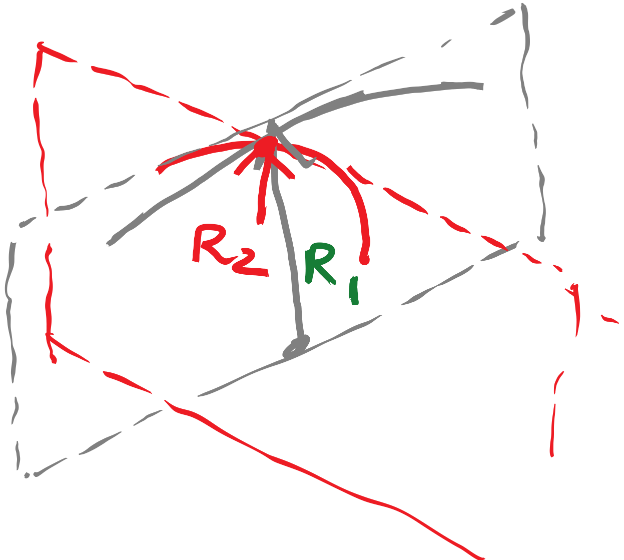
Any Point $\rightarrow$ Complete Curvature is obtained if two orthogonal curvatures are known.



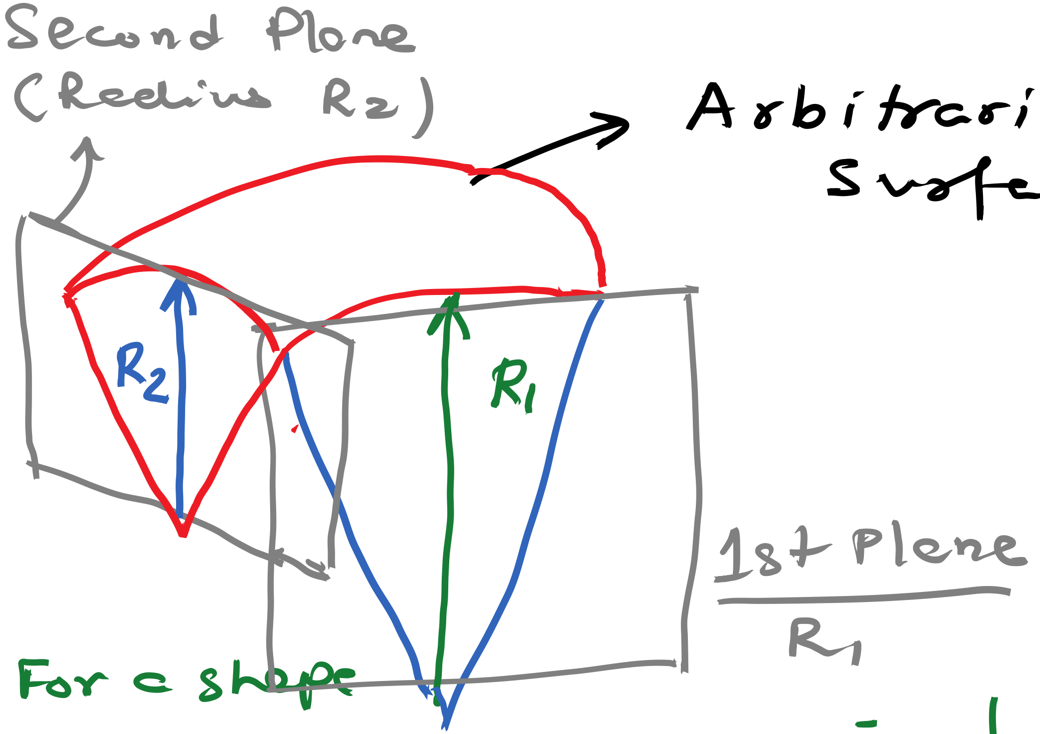
Curvature on the Sheet



Curvature orthogonal to the sheet

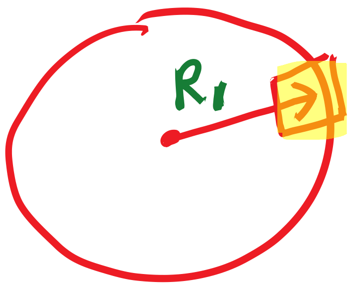


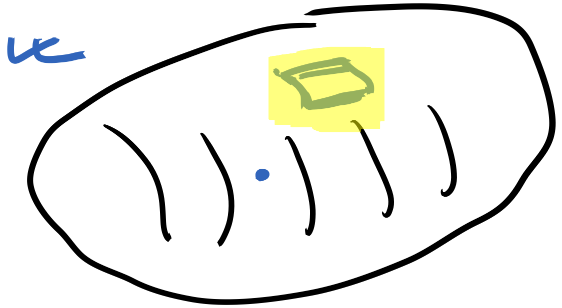
* If the curvature at two different planes orthogonal to each other is known at any point, then the Local curvature at that point is known.



For a shape

At all points radius is R_1, → Then it is a sphere



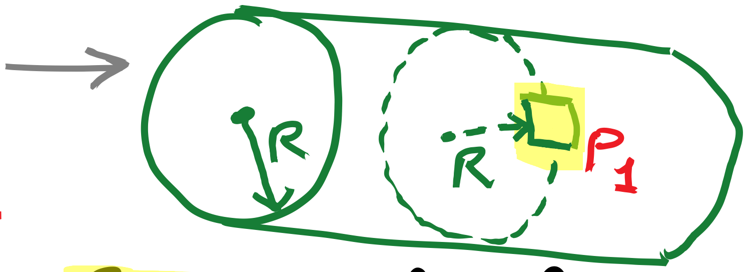


Local Curvature changes from point to point.

Cylinder

Point P_1 →

Curvature at the Radius at the Point



Radius is R in θ direction

Radius is ∞ in Z direction