Assignment (Laplace Transform)

Answers/Hints:

1)

a) Function is not piecewise continuous since $\lim_{t\to 2\pm} f(t)$ do not exists.

b) Function is continuous everywhere.

c) Function has a jump discontinuity at $t=0$ and hence the function is piecewise continuous.

d) Function is continuous everywhere.

2)

Definition of Laplace transform and integration by parts give

$$L\{f(t)\}=\int_0^\infty e^{-st}te^{t^2}\sin\left(e^{t^2}\right)\mathrm{d}t=\frac{1}{2}\left[\cos(1)-sL\left\{\cos\left(e^{t^2}\right)\right\}\right]$$

Note that $L\left\{\cos\left(e^{t^2}\right)\right\}$ exists because $\cos\left(e^{t^2}\right)$ is continuous and is of exponential order. Hence $L\{f(t)\}$ exists.

3)

a) $L\left\{e^{-t}\cos^2 t\right\}=\dfrac{(s+1)^2+2}{(s+1)\left\{(s+1)^2+4\right\}}=\dfrac{s^2+2s+3}{(s+1)\left(s^2+2s+5\right)}$

b) $f(t)=e^{at}H(t-2)=e^{a(t-2)}e^{2a}H(t-2)=e^{-2(s-a)}\dfrac{1}{s-a}$

c) $L\{f(t)\}=\dfrac{1}{1-e^{-\pi s}}\displaystyle\int_0^\pi e^{-st}\sin t\,\mathrm{d}t=\dfrac{1+e^{-\pi s}}{\left(1-e^{-\pi s}\right)\left(s^2+1\right)}$

d) $L\{f(t)\}=e^{-as}\dfrac{1}{s^2}+a\dfrac{e^{-as}}{s}=\dfrac{e^{-as}}{s^2}[1+as]$

4)

(a) $\dfrac{e^{-s}+e^{-2s}-2e^{-3s}}{s}$

(b) $\dfrac{e^{-2s}}{s^2}$

(c) $\dfrac{e^{-\pi s}}{s^2+1}$

5)

a) $L^{-1}\left\{\dfrac{s+3}{\left(s^2+6s+13\right)^2}\right\}=L^{-1}\left\{\dfrac{s+3}{\left[\left(s+3\right)^2+4\right]^2}\right\}=e^{-3t}L^{-1}\left\{\dfrac{s}{\left(s^2+4\right)^2}\right\}=\dfrac{1}{4}te^{-3t}\sin 2t$

b) $\dfrac{1}{3}e^{t}+3te^{t}-\dfrac{1}{3}e^{-2t}$

c) $\dfrac{1}{\sqrt{2}}H(t-\pi)\sinh\left\{\sqrt{2}\left(t-\pi\right)\right\}$

d) $\dfrac{3}{2}e^{-t}+\dfrac{5}{4}te^{-t}-\dfrac{3}{2}\cos t+\dfrac{1}{4}\sin t-\dfrac{1}{4}t\sin t$

6)

a) $3(t-1)u(t-1)$

b) $u(t-3)\sin 2(t-3)$