Problem-1: Presently 90% of reactant A is converted into product by a second order reaction in a single mixed flow reactor. We propose to place a second reactor similar to the one being used in series with it.

(i) For the same treatment rate as that used presently, how will this addition of reactor affect the conversion of reactant ?

(ii) For the same 90% conversion, by how much can the treatment rate be increased in the latter case ?

Solution : Consider the case of a single mixed flow reactor/CSTR.

For CSTR, we have

$$\frac{\tau}{C_{Ao}} = \frac{X_A}{(-r_A)}$$

For second order reaction :

$$(-r_A) = k\,C_A^2 = k\,C_{Ao}^2\,(1-X_A)^2 \quad \text{for } \varepsilon_A = 0$$

$$\therefore \quad \frac{\tau}{C_{Ao}} = \frac{X_A}{k\,C_{Ao}^2\,(1-X_A)^2}$$

$$\tau\,k\,C_{Ao} = \frac{X_A}{(1-X_A)^2}$$

We have, $X_A = 0.90$

$$\tau\,k\,C_{Ao} = \frac{0.90}{(1-0.90)^2}$$

$$\therefore \quad \tau\,k\,C_{Ao} = 90$$

(i) Now, we will find the conversion that can be achieved using two equal-size CSTRs in series.

For CSTR-2 in series with the first one :

$$\frac{\tau_2}{C_o} = \frac{X_2 - X_1}{(-r)_2}$$

$(-r)_2$ is evaluated at X_2

$$(-r)_2 = k\,C_o^2\,(1 - X_2)^2 = k\,C_2^2$$

$$\therefore \quad \frac{\tau_2}{C_o} = \frac{X_2 - X_1}{k\,C_o^2\,(1 - X_2)^2}$$

$$\tau_2\,k\,C_o = \frac{X_2 - X_1}{(1 - X_2)^2}$$

For CSTR-1, we have

$$\tau_1\,k\,C_o = \frac{X_1}{(1 - X_1)^2} = 90, \quad X_1 = 0.90$$

$$\tau_1 = 90/k\,C_o$$

In two CSTRs in series $X_1 = 0.90$ is the intermediate conversion (i.e. conversion leaving CSTR-1) and X_2 is the overall conversion from the system as a whole.

For equal-size CSTRs in series,

$$\tau_1 = \tau_2 = \tau$$

$$\therefore \quad \tau_2 = \tau_1 = 90/k\,C_o \qquad \text{as } \tau k C_{Ao} = 90$$

$$\tau_2\, k\, C_o = \frac{X_2 - X_1}{(1 - X_2)^2}$$

$$\left(\frac{90}{k\,C_o}\right) k\,C_o = \frac{X_2 - X_1}{(1 - X_2)^2}$$

$$90 = \frac{X_2 - 0.90}{(1 - X_2)^2}$$

$$90\,(1 - X_2)^2 = X_2 - 0.90$$

$$90\,X_2^2 - 181\,X_2 + 90.90 = 0$$

$$\therefore \quad X_2 = 0.972 \;\ldots \text{(positive root)}$$

So using two equal-size CSTRs in series, 97.2% conversion of A is achieved.

$$\%\ \text{increase in conversion} = \left(\frac{0.972 - 0.90}{0.90}\right) \times 100 = 8$$

So addition of second mixed flow reactor/CSTR of equal size in series with the first one results in 8% increase in the conversion. **Ans.**

(ii) Now, we will consider the case of two equal-size CSTRs in series for obtaining 90% overall conversion.

$$\therefore \qquad X_2 = 0.90$$

For CSTR-1 :

$$\frac{\tau_1}{C_o} = \frac{X_1}{(-r)_1}$$

$$(-r)_1 = k\, C_o (1 - X_1)^2$$

$$\therefore \qquad \tau_1 = \frac{X_1}{k\, C_o\, (1 - X_1)^2}$$

Similarly, for CSTR-2 :

$$\frac{\tau_2}{C_o} = \frac{X_2 - X_1}{(-r)_2}$$

$$(-r)_2 = k\, C_o (1 - X_2)^2$$

$$\therefore \qquad \tau_2 = \frac{X_2 - X_1}{k\, C_o\, (1 - X_2)^2}$$

For equal-size CSTRs in series,

$$\tau_1 = \tau_2 \quad (= \tau)$$

$$\therefore \quad \frac{X_1}{k\,C_o\,(1-X_1)^2} = \frac{X_2 - X_1}{k\,C_o\,(1-X_2)^2}$$

$$\frac{X_1}{(1-X_1)^2} = \frac{X_2 - X_1}{(1-X_2)^2}$$

$$X_2 = 0.90$$

$$\frac{X_1}{(1-X_1)^2} = \frac{0.90 - X_1}{(1-0.90)^2}$$

$$X_1 = (90 - 100\,X_1)\,(1-X_1)^2$$

Solving by trial and error method, For X_1 = 0.75, 0.76, 0.765........
and found X_1= 0.7635, L.H.S = R.H.S

$$\tau_2 = \frac{X_2 - X_1}{k\,C_o\,(1 - X_2)^2}$$

$$\tau_2\,k\,C_o = \frac{0.90 - 0.7635}{(1 - 0.90)^2}$$

$$= 13.65$$

$$\tau_1 = \frac{X_1}{k\,C_o\,(1 - X_1)^2}$$

$$\tau_1\,k\,C_o = \frac{X_1}{(1 - X_1)^2}$$

$$= \frac{0.7635}{(1 - 0.7635)^2}$$

$$= 13.65$$

$$\tau_2 = \tau_1 = 13.65/k\,C_o$$

$$\tau_1 = \tau_2 = \tau \text{ for}$$

$$\tau_{N\ \text{reactors}} = N\,\tau_i$$

$$(\tau)_{N=2} = 2 \times 13.65/k\,C_o = 27.3/k\,C_o$$

Single CSTR with 90% conversion,

$$\tau k C_o = 90$$

$$(\tau)_{N=1} \quad 90/k C_o$$

$$\frac{(\tau)_{N=2}}{(\tau)_{N=1}} = \frac{(V/v)_{N=2}}{(V/v)_{N=1}}$$

$$V_{N=2} = 2 V_{N=1}$$

$$(v)_{N=2} = \frac{2 (V)_{N=1}}{(V)_{N=1}} \times \frac{(\tau)_{N=1}}{(\tau)_{N=2}} \times (v)_{N=1}$$

$$(v)_{N=2} = \frac{2 \times 90}{k C_o} \times \frac{1}{\left(\frac{27.3}{k C_o}\right)} \times (v)_{N=1}$$

$$(v)_{N=2} = \frac{2 \times 90}{27.3} \times (v)_{N=1}$$

$$= 6.59 (v)_{N=1}$$

$$\approx 6.6 (v)_{N=1}$$

So the treatment capacity will be increased by 6.6 times than that of the single reactor

- If we use parallel connection, $F_{A0} = F_{A01} + F_{A02}$ and $F_{A01} = F_{A01}$
- $F_{A0} = 2F_{A01}$, so treatment rate will be doubled.

Problem -2 Substance A reacts according to second-order kinetics. 95% conversion of A is achieved in a single flow reactor. We buy a second reactor identical to the first. For the same 95% conversion, by how much is the capacity increased if we operate these two reactors in parallel or in series ?

(i) The reactors are both plug flow.

(ii) The reactors are both mixed flow.

Solution : A → products ... second order reaction.

$-r_A = k\,C_A^2$ and $X_A = 0.95$

$$\frac{V}{F_{Ao}} = C_{Ao}\int_0^{X_A} \frac{dX}{(-r_A)} \quad \text{... performance equation for PFR.}$$

(i) For the two identical PFRs in series,

$$\frac{V_1}{F_o} = C_o\int_0^{X_1} \frac{dX}{(-r)}, \quad \frac{V_2}{F_o} = C_o\int_{X_1}^{X_2} \frac{dX}{-r}$$

$$\frac{V_1 + V_2}{F_o} = C_o\int_0^{X_1} \frac{dX}{(-r)} + C_o\int_{X_1}^{X_2} \frac{dX}{-r} = C_o\int_0^{X_2} \frac{dX}{(-r)}$$

$$V_1 = V_2 = V$$

$$\frac{2V}{F_o} = C_o\int_0^{X_2} \frac{dX}{(-r)} = C_o\int_0^{0.95} \frac{dX}{(-r)}$$

For one (single) PFR, $\dfrac{V}{F_{Ao}} = C_o \displaystyle\int_0^{0.95} \dfrac{dX}{(-r_A)}$

RHS of equations (1) and (2) remain the same (unchanged for one PFR as well as for two identical PFRs in series).

$$2V/F_o = V/F_{Ao}$$

$$F_o = 2\,F_{Ao}$$

$\therefore$ Feed rate can be doubled. ... **Ans.**

For plug flow reactors in parallel, τ should be identical for the conversion to be the same in each parallel reactor (parallel line).

$\therefore$ $$\tau_1 = \tau_2$$

For identical reactors :

$$V_1 = V_2$$

we have, $$F_o = (F_o)_1 + (F_o)_2$$

For reactors in parallel :

$$V/(F_o)_1 = V/(F_o)_2$$

$\therefore$ $$(F_o)_1 = (F_o)_2$$

$\therefore$ $$F_o = 2\,(F_o)_1$$

So the feed rate can be doubled. ... **Ans.**

(ii) For a single CSTR/mixed flow reactor :

$$\tau = \frac{C_{Ao} V}{F_{Ao}} = \frac{C_{Ao} X_A}{(-r_A)}$$

$$\tau k C_{Ao} = 380$$

$$\tau_{N=1} = 380/k\, C_o \quad (\text{as } C_o = C_{Ao})$$

$$(v)_{N=2} = \frac{(V)_{N=2}}{(V)_{N=1}} \times \frac{\tau_{N=1}}{\tau_{N=2}} \times (v)_{N=1}$$

$$(V)_{N=2} = 2\,(V)_{N=1}$$

$$(v)_{N=2} = \frac{2\,(V)_{N=1}}{(V)_{N=1}} \times \frac{380/k\, C_o}{78.4/k\, C_o} \times (v)_{N=1}$$

$$(v)_{N=2} = 9.694\,(v)_{N=1}$$

$$= 9.7\,(v)_{N=1}$$

$$(F_{Ao})_{N=2} = 9.7\,(F_{Ao})_{N=1}$$

Therefore, two identical CSTRs in series to achieve the overall conversion 95% the feed rate is 9.7 times of the feed rate of single CSTR.