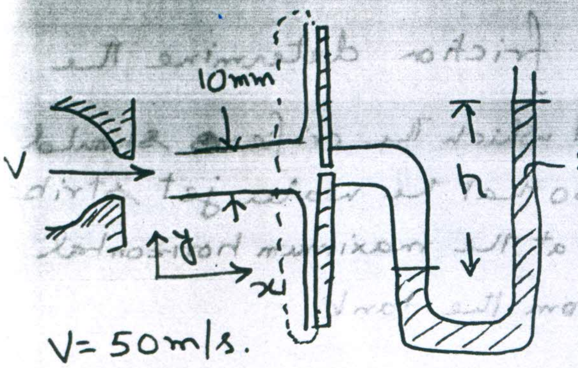
Bernoulli's Eq.



horizontal axisymmetric jet of air with 10 mm diameter.

Vertical disk of 200 mm dia.

Jet speed = 50 m/s at nozzle exit.

Calculate

i) the deflection, h of the manometer

ii) force exerted by the jet on the disk.

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{const.}$$

$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\, \vec{V}\cdot d\vec{A}$$

Apply Bernoulli eq^n bet^n jet exit and stagnation point.

$$\frac{p}{\rho} + \frac{V^2}{2} = \frac{p_0}{\rho} + 0. \Rightarrow p_0 - p = \frac{1}{2}\rho V^2$$

From hydrostatics $p_0 - p = SG\, \rho_{H_2O}\, g\, \Delta h$

$$\therefore \Delta h = \frac{\frac{1}{2}\rho V^2}{SG\, \rho_{H_2O}\, g} \qquad \rho_{AIR} = 1.23\ \text{kg/m}^3$$

$$\Delta h = \frac{1.23 \times (50)^2}{2 \times 1.75 \times 10^3 \times 9.81} = 0.0896\text{m}. \quad (1a)$$

From momentum,

$$R_x = u_1\{-\rho V A\} + u_2\{\rho V A\}$$

$$u_1 = V, \quad u_2 = 0.$$

$$R_x = -\rho V^2 A$$

$$R_x = -1.23\ \frac{\text{kg}}{\text{m}^3} \times (50)^2\ \frac{\text{m}^2}{\text{s}^2} \times \frac{\pi}{4}(0.01)^2\ \text{m}^2 = -0.242\ \text{N}$$

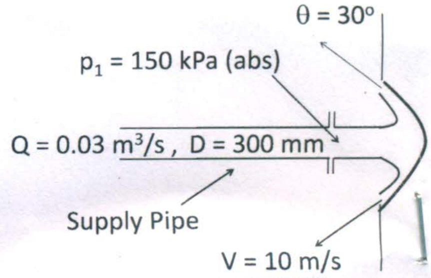
(to the left)

This is the force needed to hold the plate.

The 'force' of the jet on the plate is

$$K_x = -R_x = 0.242\ \text{N} \text{ (to right)}$$

A conical spray head is shown in the figure. The fluid is water and the exit stream is uniform along the entire inner surface of the spray head. Evaluate (a) the thickness of the water film along the spray head, when the radius of the spray head is 400 mm, and (b) the axial force exerted by the spray head on the connecting supply pipe.



At $r = 400\,mm$

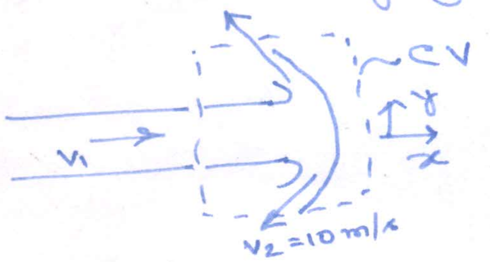
$$\cancel{F_{Sx}} + \cancel{F_{Bx}} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

$=0 \qquad\qquad =0\ (SS)$

Incompressible, uniform flow at each section.

gage pressure to be used to cancel P_{atm}.

Use continuity & x-component of momentum eq^n.



From continuity

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4}{\pi} \times 0.03\,\frac{m^3}{s} \times \frac{1}{(0.3)^2\,m^2}$$

$V_1 = 0.424\ m/s.$

Assume velocity in jet sheet is constant at $V = 10\,m/s$.

$$\therefore\ Q = 2\pi R\, t\, V \Rightarrow t = \frac{Q}{2\pi R V} = \frac{1}{2\pi} \times 0.03 \times \frac{1}{0.4} \times \frac{1}{10} = 1.19\,mm$$

$$\boxed{t = 1.19\,mm}$$

From momentum,

$$R_x + p_{1g} A_1 = u_1\{-\rho Q\} + u_2\{+\rho Q\}.$$

$u_1 = V_1 \qquad u_2 = -V \sin\theta$

$$R_x + p_{1g} A_1 = -(V_1 + V\sin\theta)\rho Q.$$

$$R_x = -p_{1g} A_1 - (V_1 + V\sin\theta)\rho Q.$$

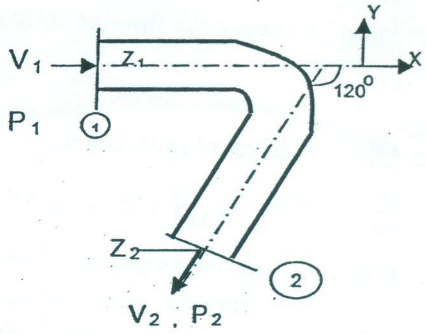
$$R_x = -(150-101)\times 10^3\,\frac{N}{m^2} \times \frac{\pi}{4}(0.3)^2\,m^2 - (0.424 + 10\sin 30)\,\frac{m}{s} \times 999\,\frac{kg}{m^3} \times 0.03\,\frac{m^3}{s}$$

$R_x = -3.63\,kN$

But R_x is the force on CV,

$\therefore$ Force on supply pipes $= -R_x = 3.63\,kN$ (to the right)

Q2. The diameter of a pipe bend is 30 cm at inlet and 15 cm at outlet and the flow is turned through 120^0 in a vertical plane. The axis at inlet is horizontal and the centre of the outlet section is 1.5 m below the centre of the inlet section. Total volume of water in the bend is 0.9 m^3. Neglecting friction, calculate the magnitude and direction of the force exerted in the bend by water flowing through it at 250 L/s and when the inlet pressure is 0.15 N/mm^2 (Note that the pressure at the inlet is absolute pressure and the outlet is not open to the atmosphere).



$A_1 = \frac{\pi}{4}(0.3)^2 = 0.07069 m^2$ $\quad$ $A_2 = \frac{\pi}{4}(0.15)^2 = 0.01767 m^2$

$V_1 = \frac{0.25}{0.07069} = 3.537 m/s$ $\quad$ $V_2 = \frac{0.25}{0.01767} = 14.148 m/s.$

$Z_1 = 1.5 m, \quad Z_2 = 0, \quad p_1 = 0.15 \times 10^6 Pa = 1.5 \times 10^5 Pa$

Applying Bernouli eq^n bet^n (1) & (2)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{1.5 \times 10^5}{9810} + \frac{(3.537)^2}{2 \times 9.81} + 1.5 = \frac{p_2}{\rho g} + \frac{(14.148)^2}{2 \times 9.81} + 0.$$

$p_2 = 70900 Pa$ (abs).

Apply momentum eq^n in the x-direction

$$p_{1g} A_1 + R_x + p_{2g} A_2 \cos 60 = -\rho Q V_1 + \rho Q(-V_2 \cos 60)$$

$$\underset{3464.3}{0.49 \times 10^5 \times 0.0707} + R_x - \underset{265}{3.0 \times 10^4 \times 0.01767 \cos 60} = -10^3 \times 0.25(3.537 + 0.5 \times \cancel{3.537}\ 14.148)$$

(−2601)

$R_x = \cancel{-45125.60 N}\ -5800 N$

Momentum eq^n in the y direction

$$0 - W + R_y + p_{2g} A_2 \sin 60 = \rho Q(-V_2 \sin 60)$$

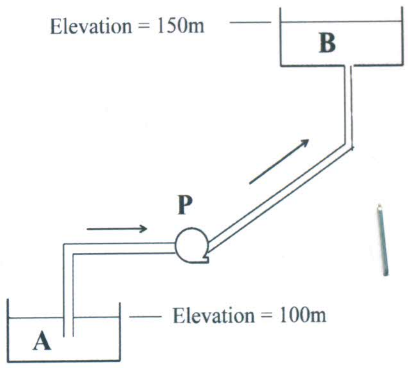
$$-0.9 \times 10^3 \times 9.81 + R_y - 3.0 \times 10^4 \times 0.01767 \times 0.866 = -10^3 \times 0.25 \times 14.148 \times 0.866$$

$$-8829 + R_y - 459 = -3063$$

$R_y = \cancel{+11433 N}\ 6225 N.$

These are the forces on the CV.

. A pump delivers water from a tank A (water surface elevation = 100m) to tank B (water surface elevation = 150 m). The suction pipe is 50 m long (f=0.025) and 30 cm in diameter. The delivery pipe is 900 m long (f=0.02) and 20 cm in diameter. Evaluate a relation between the head developed by the pump (H_P) and the flow rate (Q), considering only major losses. Furthermore, from an independent experimental study, the head discharge relationship for the pump is correlated as $H_P = 80 - 7000\,Q^2$, where H_P is in meters and Q is in m^3/s. Based on the above information, calculate the discharge in the pipeline and the power delivered by the pump.



Suction Pipe

$D_1 = 0.3\ m$

$L_1 = 50\ m$

$f_1 = 0.025$

Delivery pipe.

$D_2 = 0.2\ m$

$L_2 = 900\ m$

$f_2 = 0.02$

Suction Pipe: Head Loss $= h_{L_1} = \dfrac{f_1 L_1 V_1^2}{2gD_1} = \dfrac{0.025 \times 50}{0.3} \dfrac{V_1^2}{2g}$

$h_i = 4.167 \dfrac{V_1^2}{2g}\ m.$

Delivery Pipe: Head Loss $= h_{L2} = \dfrac{f_2 L_2 V_2^2}{2gD_2} = \dfrac{0.02 \times 900}{0.2} \dfrac{V_2^2}{2g} = 90 \dfrac{V_2^2}{2g}\ m$

Total head loss $= 4.167 \dfrac{V_1^2}{2g} + 90 \dfrac{V_2^2}{2g}$

Continuity $V_1 (0.3)^2 = V_2 (0.2)^2 \Rightarrow V_1 = 0.444 V_2$

$\dfrac{V_1^2}{2g} = 0.1975 \dfrac{V_2^2}{2g}$

$\therefore H_L = 4.167 \dfrac{V_1^2}{2g} + 90 \dfrac{V_2^2}{2g} = 90.82 \dfrac{V_2^2}{2g}\ m.$

Static head $= 150 - 100 = 50\ m.$

$\therefore H_P$ = head delivered by pump

= Static head + friction head.

$$H_p = 50 + 90.82 \frac{V_2^2}{2g}$$

$$H_p = 50 + 90.82 \times \frac{Q^2}{\left[\frac{\pi}{4} \times (0.2)^2\right]^2} \times \frac{1}{2 \times 9.81}$$

$$\boxed{H_p = 50 + 4690\, Q^2}$$

$$h_p = 490 + 46008\, Q^2$$

By the given pump performance relation

$$H_p = 80 - 7000\, Q^2$$

$$\therefore \quad 80 - 7000\, Q^2 = 50 + 4690\, Q^2$$

$$\Rightarrow \quad Q = 0.0506 \frac{m^3}{s} \quad (50.6\ L/s).$$

$$H_p = 50 + 4690\, Q^2 = 50 + 4690\,(0.0506)^2$$

$$\boxed{H_p = 62.04\, m}$$

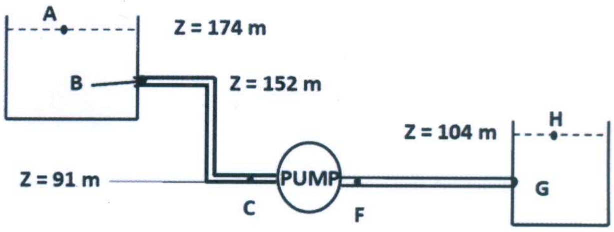
$\therefore$ Power delivered by the pump, $P = \rho g Q H_p$ (W)

$$= 10^3 \times 9.81 \times 0.0506 \times 62.04 \text{ W}$$

$$= 9.81 \times 0.0506 \times 62.04 \text{ kW}.$$

$$\boxed{P = 3.8 \text{ kW}}$$

Water (kinematic viscosity = 1.0×10^{-6} m^2/s) is pumped from a reservoir at the rate of 1310 L/s and is being sent to another large tank. The path of water through the pipe is marked as BCFG with the pump being located between C and F. From B to C, the system consists of a square-edged entrance, 760 m of pipe, three gate valves, four 45° elbows (L_e/D = 20) and two 90° elbows. Gage pressure at C is 197 kPa. The system between F and G contains 760 m of pipe, two gate valves (L_e/D = 8) and four 90° elbows (L_e/D = 30). All the pipes are made of cast iron (ε = 0.26 mm) and of 508 mm diameter. Calculate the average velocity of water in the pipe, the gage pressure at F, the power input to the pump (of efficiency 80%) and the wall shear stress in section FG.



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i) Velocity $\bar{V} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 1310 \times \frac{1}{(0.508)^2} \times 10^{-3}$

$\bar{V} = 6.46$ m/s.

ii) To determine the pressure at F, use B' eqⁿ betⁿ F & ~~G~~ H

$$\left(\frac{P_F}{\rho} + \alpha\frac{V_F^2}{2} + gz_F\right) - \left(\frac{P_H}{\rho} + \alpha\frac{V_H^2}{2} + gz_H\right) = h_{LT}$$

$$h_{LT} = h_L + h_{LM} \Rightarrow h_L = f\frac{L}{D}\frac{V^2}{2}, \quad h_{LM} = \frac{V^2}{2}\sum f\frac{L_e}{D} + \frac{V^2}{2}K_{exit}$$

$V_H = 0, \quad P_H = P_{atm}, \quad \alpha = 1$

$$\frac{P_F}{\rho} = h_{LT} + g(z_H - z_F) - \frac{V_F^2}{2}.$$

$$\frac{P_F}{\rho} = h_{L_{FG}} + 2h_{Lm\,gate\ val.} + 4h_{Lm\,90^\circ} + h_{Lm\,exit} + g(z_H - z_F) - \frac{V^2}{2}$$

$$\frac{P_F}{\rho} = f\frac{L}{D}\frac{V^2}{2} + 2f\left(\frac{L_e}{D}\right)_{gv}\frac{V^2}{2} + 4f\left(\frac{L_e}{D}\right)_{90^\circ}\frac{V^2}{2} + K_{exit}\frac{V^2}{2} + g(z_H - z_F) - \frac{V^2}{2}$$

$(L_e/D)_{gv} = 8, \quad (L_e/D)_{90^\circ} = 30, \quad K = 1$

$Re = \frac{DV}{\nu} = \frac{0.508 \times 6.46}{1\times10^{-6}} = 3.28\times10^{6}, \quad \epsilon = 0.26\,mm, \quad \frac{\epsilon}{D} = 0.0005$

$\Rightarrow f = 0.017$

$$\therefore \frac{P_F}{\rho} = f\frac{\bar{V}^2}{2}\left[\frac{760}{0.508}\times 2\times 8 + 4\times 30\right] + g(z_H - z_F) - f\frac{V^2}{2}$$

$$P_F = \rho\left[1630 f \frac{V^2}{2} + g(z_H - z_F)\right].$$

$$= 999\left[\frac{1630}{2}\times 0.017\times(6.46)^2 + 9.81(104-91)\right]$$

$$P_F = 705 \text{ kPa (gage)}$$

iii) For fully dev. flow in a pipe $\tau = \frac{r}{2}\frac{dP}{dx}$

at the pipe centreline, $\tau = 0$.

To evaluate the power input to the fluid

$$\frac{W_{pump}}{\dot{M}} = \left(\frac{P}{\rho} + \frac{V^2}{2} + gz\right)_{discharge} - \left(\frac{P}{\rho} + \frac{V^2}{2} + gz\right)_{suction}$$

$$W_p = \left(\frac{P_F}{\rho} - \frac{P_c}{\rho}\right)\rho A V = \left(\frac{P_F}{\rho} - \frac{P_c}{\rho}\right)Q$$

$$W_p = (705-197)\times 10^3 \times 1310\times 10^{-3} = 6.65\times 10^5 \frac{N.m}{s}.$$

$$\therefore \text{Actual power input to the pump} = \frac{W_p}{Eff.} = 8.32\times 10^5 \text{ W}$$

$$= 832 \text{ kW}.$$

iv) $$\tau_w = \frac{R}{2}\frac{dP}{dx}.$$

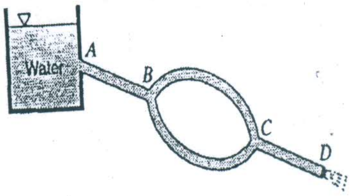
from F to G $\frac{\Delta P}{\rho} = f\frac{L}{D}\frac{V^2}{2}$

$$\therefore \frac{dP}{dx} = \frac{\Delta P}{L} = \rho\frac{f}{D}V^2 = 999\times\frac{0.017}{0.508}\times\frac{1}{2}(6.46)^2$$

$$\therefore \frac{dp}{dx} = 698 \frac{N}{m^2\ m}$$

$$\tau_w = \frac{R}{2}\frac{dP}{dx} = \frac{0.254}{2}\times 698 = 88.6\ N/m^2$$

Q9. In the given figure, pipe AB is 600 m long, of 180 mm diameter, with f = 0.035; pipe BC (upper) is 500 m long, of 120 mm diameter, with f = 0.025; pipe BC (lower) is 400 m long, of 160 mm diameter, with f = 0.030. The elevations are: reservoir water surface = 150 m, A = 100m, B = 60m, C = 50 m, D = 20 m. Neglecting velocity heads and minor losses (a) compute the flow in each pipe and (b) determine the pressures at B and C. Comment on the practicality of this system.



(Marks = 3+3+2)

The pressures at points B and C have to be equal or in other words ΔP in the two paths must be the same.

$$\Delta P_{upper} = \Delta P_{Lower}$$

$$0.025(4167)\frac{v_U^2}{2g} = 0.03(2500)\frac{v_L^2}{2g}$$

$$\Rightarrow v_u = 0.849\,v_L$$

Continuity $Q = 0.0254V_1 = 0.01131(0.849v_L) + 0.0201\,v_L = 0.0804\,v_2$

$$\Rightarrow v_1 = 1.167\,v_L, \quad v_2 = 0.369\,v_L$$

B' eqn (neglecting velocity heads) from water surface to D (neglecting minor losses)

$$0 + 150 = 0 + 20 + h_{L1} + h_{LL} + h_{L2}$$

(The head loss in this part [ΔP] is given by h_{LL}, only one branch needs to be taken into acc.)

$$130 = f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} + f_L\frac{L_L}{D_L}\frac{v_L^2}{2g} + f_2\frac{L_2}{D_2}\frac{V_2^2}{2g}$$

$$130 = 0.035\underset{600/0.18}{(3333)}(1.167V_L^2)/2g + 0.03\underset{400/0.16}{(2500)}V_L^2/2g + 0.02(2813)(0.369V_L^2)/2g$$

$V_L = 3.25$ m/s.

$V_L = 3.25$ m/s $\quad Q_L = 0.0653$ m^3/s.

$V_u = 2.76$ m/s $\quad Q_u = 0.0312$ m^3/s

$V_1 = 3.79$ m/s $\quad Q_1 = 0.0965$ m^3/s.

$V_2 = 1.2$ m/s $\quad Q_2 = 0.0965$ m^3/s.

b) B eqn water surface B: $\quad 150 = \frac{p_B}{\rho g} + 60 + h_{LAB}$.

$$\frac{p_B}{\rho g} = 90 - \frac{0.035(3333)(3.79)^2}{2g} = 4.48\text{ m}, \quad p_B = 4.48\times 9.81\times 10^3 = 43.9\text{ kPa}.$$

B eqn C to D $\quad \frac{p_C}{\rho g} + 50 = 20 + (h_L)_{CD}$.

$\frac{p_C}{\rho g} = -25.9$ m IMPOSSIBLE

The head loss in the parallel pipes is so large that it brings the pressure below zero absolute pressure.

$\therefore$ The system will not function.