1. Why is this class?

   - Realizing process designs
   - Modeling vs. simulation.
   - Approach to designing a system
     1. building a small scale model as a PoC
     2. validation
     3. scaling.

this course => building & solving these models using analytical methods.

2. Curricula.

   1. PDEs
      - classification
      - boundary conditions, types
      - principle of linear superposition
   2. Adjoint operator & special ODEs
      - props (adj. op.)
      - props (eigenvalues, eigenfunctions)
   3. Separation of linear, homogeneous PDEs by separation of variables
      - cartesian, cylindrical, spherical

applicable when there are sources & sinks.

4. Solution of non-homogeneous PDEs using Green's function
5. Solution of PDEs by similarity solution
6. Solution of PDEs by (approximate) integral methods

3. Books

1. S. Pushparam - Math Techniques in ChemE.
2. A. Varma &

(cam)

3.

4. Partial Differential Equations

- One dep. var ⇒ ODE, else PDE
- Order of PDE = highest derivative

(order can also be w.r.t each indep. var.).

e.g. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \rightarrow 2$

- Linear PDEs
  - linear functions of the unknown & its derivatives
  - products of two terms, derivatives or unknowns

raised to exponents ≠ 1 ⇒ non-linear

- higher order derivatives are fine if the terms are linear.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad ✓$$

$$\frac{\partial u}{\partial t} = \underline{\underline{u \cdot \frac{\partial^2 u}{\partial x^2}}} \quad \times$$

~ "source of N.L."

$$\frac{\partial u}{\partial t} = \left(\underline{\underline{\frac{\partial^2 u}{\partial x^2}}}\right)^2 \quad \times$$

- Homogeneous, non-homogeneous PDEs:
  - if any of the terms in the PDEs do not contain the dependent var. ⇒ N.H.

$$\frac{\partial u}{\partial t} = \left(\frac{\partial^2 u}{\partial x^2}\right)^2 \rightarrow H.$$

$$\frac{\partial u}{\partial t} = \left(\frac{\partial^2 u}{\partial x^2}\right)^2 + 2x \Rightarrow N.H.$$

⑤ Boundary conditions

- Governing equation → satisfied at all points inside the control volume
- B.C.s → satisfied only at the boundaries of the c.v.

- Types of B.C.s

① Dirichlet B.C.

- value of the dep. var. (i.e. the unknown) is specified at the boundaries

e.g. $u = 0$ @ $t = 0$

② Neumann B.C.

- value of the derivative of the dep. var. (i.e. the unknown) is specified at the boundaries

e.g. $\frac{\partial u}{\partial t} = 0$ @ $t = 0$,

$\frac{\partial T}{\partial x} = 0$ @ $x = 1$

(⇒ boundary is insulated, so temp. gradient is 0).

③ Robin mixed B.C.

- unknown + derivative as an eq^n

e.g. $-k\frac{dT}{dx} = h(T - T_\infty)$

@ $x = 1$

④ Cauchy B.C.

- indep. var. & deriv. both present on the same boundary.

e.g. $T = T_0$ & $-k\frac{\partial T}{\partial x} = q_0$

@ $t = 0$.

⑤ Physical B.C.

- based on the structure/physics of the problem.

e.g. no slip conditions at pipe walls $\Rightarrow u = 0$