$u + \eta - \eta^2$

$2\eta - \eta^2$

$2-1$

Brinkmann — Br → will tell about viscous dissipation

$Br = 2$ → $q'' = 0$ → No heat flux → why?

↳ at $y = L$, $T = T_L$ → rate at which heat flows is in opp direction → which will cancel out.

{ friction between the layers is generating heat
∴ temp inside can be higher than ~~the~~ the boundaries (T_1, T_L) }

$$Br = \frac{\mu u^2}{k \Delta T}$$

$$Br = \frac{\mu \left(\frac{u}{L}\right) u}{\frac{k \Delta T}{L}}$$

Boundary layer

$y=0$

$F = \int_A \tau_0 \, dA$ — force exerted by the stream on the plate

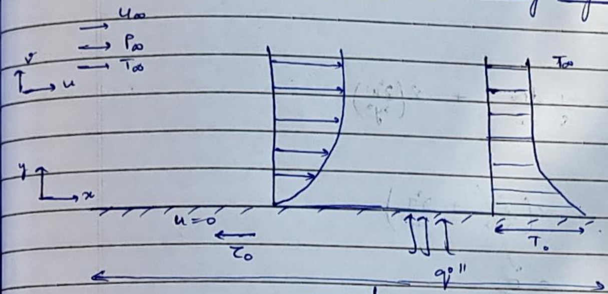
$q' = \int_A q_0'' \, dA$ → resistance to the transfer of heat from the plate to the stream

$$\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0} \qquad q_0'' = k \left.\frac{dT}{dy}\right|_{y=0}$$

→ $q = h(T_1 - T_2)$

$$h = \frac{-k\left(\frac{dT}{dy}\right)_{y=0}}{\Delta T}$$

Laminar Boundary layer



$T_0 > T_\infty$

Temperature & velocity profile close to BL?

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow v \sim \frac{u_\infty \delta}{L}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$u_\infty \times \frac{u_\infty}{L} \sim v\frac{u_\infty}{\delta}$, $\quad \frac{1}{\rho}\frac{P}{L}$, $\quad \cancel{\frac{u_\infty}{L^2}} \to 0$, $\quad \frac{u_\infty}{\delta^2}$

q'' → why is it called resistance to heat transfer

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

τ_0? q_0''? → aim

BC:- at $y = 0$, $u = 0$, $v = 0$, $T = T_0$

at $y \to \infty$, $u = u_\infty$, $v = 0$, $T = T_\infty$

at $y = 0$, $v \neq 0$ ↳ in case of boiling even if the wall is impermeable

$x \sim L$ — δ → thickness of HBL

$y \sim \delta, \delta_T$ — δ_T → thickness of TBL

$u \sim u_\infty$

$v \sim$?

$T \quad \Delta T$

$$\Rightarrow u_\infty \times \frac{u_\infty \delta}{L^2} \sim \frac{u_\infty \delta}{L} \times \frac{u_\infty \delta}{L \delta} \sim \nu\left(\frac{u_\infty \delta}{L L^2}\right) \sim \nu\left(\frac{u_\infty \delta}{L \delta^2}\right)$$

$$\frac{\partial p}{\partial y} = 0$$

$$\left[\frac{\partial p}{\partial x} = \frac{dp}{dx} = \frac{dP_\infty}{dx}\right]$$

→ $\neq 0$ in general

→ for our case $\frac{dP_\infty}{dx} = 0$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP_\infty}{dx} + \nu\left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$\rightarrow u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\cancel{\frac{\partial^2 T}{\partial x^2}} + \frac{\partial^2 T}{\partial y^2}\right)$$

≈ 0

$$\Rightarrow u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right)$$

$$\rightarrow \tau_0 = \mu\frac{\partial u}{\partial y} \sim \mu\frac{u_\infty}{\delta}$$

assumption

$$\Rightarrow u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\cancel{\frac{dP_\infty}{dx}}^{\,0} + \nu\frac{\partial^2 u}{\partial y^2}$$

at steady state there is a balance between inertia & viscous.

O(inertia) = O(viscous)

$$\frac{u_\infty^2}{L} \sim \nu\frac{u_\infty}{\delta^2}$$

$$\delta \sim \sqrt{\frac{\nu L}{u_\infty}} \Rightarrow \left(\frac{\delta}{L}\right)^2 \sim Re$$

$$Re^{-1/2} \sim \left(\frac{\delta}{L}\right)$$

$$\rightarrow \tau_0 \sim \mu\frac{u_\infty}{\delta}$$

$$\boxed{\tau_0 \sim \rho u_\infty^2 Re^{-1/2}}$$

friction factor

$$c_f \sim \frac{\tau_0}{\rho u_\infty^2}$$

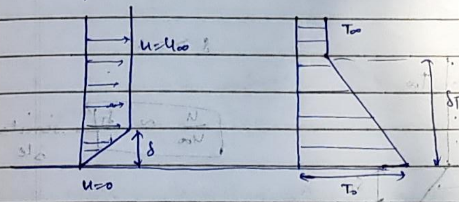
$$c_f \sim Re^{-1/2}$$

$$q'' \sim h\,\Delta T$$

$$h \sim \frac{k\frac{\Delta T}{\delta_T}}{\Delta T}$$

$$h \sim \frac{k}{\delta_T}$$

→ Thick TBL



$\delta_T = ?$

$\delta_T >>> \delta$ → assumptions

at the edge of TBL $u = u_\infty$ as $\boxed{\delta_T >> \delta}$

Impose the scaling at the edge of TBL

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{u_\infty}{L} \sim \frac{v}{\delta_T} \qquad v \sim \frac{u_\infty \delta_T}{L}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$$\underbrace{u_\infty\frac{\Delta T}{L} \sim v\frac{\Delta T}{\delta_T}}_{\text{convective heat transfer}} \sim \underbrace{\alpha\frac{\Delta T}{\delta_T^2}}_{\text{conductive heat transfer}}$$

at SS $\rightarrow u_\infty\frac{\Delta T}{L} \sim \alpha\frac{\Delta T}{\delta_T^2}$

$$\delta_T \sim \sqrt{\frac{\alpha L}{u_\infty}}$$

$\frac{\alpha}{u_\infty L} = Pe$ → Peclet Number

$$\boxed{\frac{\delta_T}{L} \sim Re^{-1/2} Pr^{-1/2}}$$

$Pe = Re\,Pr$

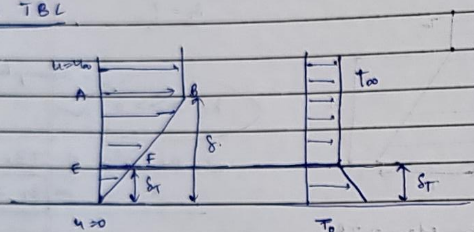
$h \sim \frac{k}{\delta_T} \sim \frac{k}{L} Re^{1/2} Pr^{1/2}$

$Nu = \frac{hL}{k} \sim Re^{1/2} Pr^{1/2}$

$\frac{\delta_T}{\delta} \sim Pr^{-1/2} >> 1$

$\boxed{Pr << 1} \rightarrow$ ✓ for liq metals.

Thin TBL



$\delta >> \delta_T$

$\frac{u}{u_\infty} \sim \frac{\delta_T}{\delta}$ → similar △le

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$ at the edge TBL

$\frac{u}{L} \sim \frac{v}{\delta_T}$

$v \sim \frac{u\delta_T}{L}$

$u \sim u_\infty \frac{\delta_T}{\delta}$

$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$

$u\frac{\Delta T}{L} \sim v\frac{\Delta T}{\delta_T} \quad \alpha\frac{\Delta T}{\delta_T^2}$

$\frac{u}{L} \sim \frac{\alpha}{\delta_T^2}$

$\delta_T^3 \sim \frac{\alpha L \delta}{u_\infty}$

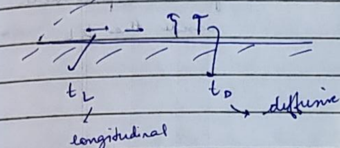
$\delta_T^3 \sim \frac{\alpha L \delta}{u_\infty}$

$\frac{\delta_T^3}{L^3} \sim \frac{\alpha L^2}{u_\infty} Re^{-1/2}$

$\frac{\delta_T}{L} \sim Pr^{-1} Re^{-1} Re^{-1/2}$

$\frac{\delta_T}{L} \simeq Pr^{-1/3} Re^{-1/2}$

Momentum BL	Thick TBL	~~Thermal BL~~ Thin TBL
$\frac{\delta}{L} \sim Re_L^{-1/2}$	$\delta_T \sim Re_L^{-1/2} Pr^{-1/2}$	$\frac{\delta_T}{L} \sim Re_L^{-1/2} Pr^{1/3}$
$\tau_0 \sim \rho u_\infty^2 Re_L^{-1/2}$	$h \sim \frac{k}{L} Re_L^{1/2} Pr^{1/2}$	$h \sim \frac{k}{L} Re_L^{1/2} Pr^{1/3}$.



momentum is transfered due to bulk motion & molecular diffusion
So at ss both the time scale must be same.

If bulk motion is dominating $\rightarrow \frac{u_\infty}{t_L} \sim$ convective time scale $\frac{u_\infty^2}{L}$ ✓

$t_L \sim \frac{L}{u_\infty}$

diffusive time scale $\rightarrow \frac{u_\infty}{t_D} \sim \nu \frac{u_\infty}{\delta^2}$

$t_D \sim \frac{\delta^2}{\nu}$ ✓

$t_D = t_L$ → at ss.

$\frac{\delta^2}{\nu} \sim \frac{L}{u_\infty}$

$\delta \sim \sqrt{\frac{L\nu}{u_\infty}}$ ✓

$\boxed{\frac{\delta}{L} \sim Re^{-1/2}}$

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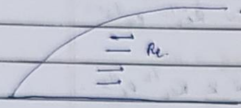


Significance of Re inside the BL

$$\boxed{\frac{\delta}{L} \sim Re^{-1/2}}$$

$$Re^{1/2} \sim \left(\frac{L}{\delta}\right)$$ → geometric parameter inside the BL

→ $\frac{\delta}{L} \sim Re_L^{-1/2}$

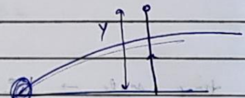
$\delta \sim L^{1/2}$

$\delta \sim x^{1/2}$

→ at $x=0$ $\frac{d\delta}{dx}$ → not defined → limitation

Integral solutions

$Y > \max(\delta, \delta_T)$



$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP_\infty}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\int_0^Y \frac{\partial (u^2)}{\partial x}dx + \int_0^Y \frac{\partial (uv)}{\partial y}dy = -\int_0^Y \frac{1}{\rho}\frac{dP_\infty}{dx}dy + \int_0^Y \nu\frac{\partial^2 u}{\partial y^2}dy$$

$$\frac{d}{dx}\int_0^Y \cdots = -\frac{1}{\rho}Y\frac{dP_\infty}{dx} + \frac{du_\infty}{dx}\int_0^Y u\,dy + \nu\frac{\partial u}{\partial y}\Big|_0$$

(Wall shear stress)

Similarity solutions

$$\frac{u}{u_\infty} = f'(\eta)$$

$\eta \propto y$

$\eta = k(x)\,y$

$$\eta = \frac{y}{\sqrt{Re_x}}$$

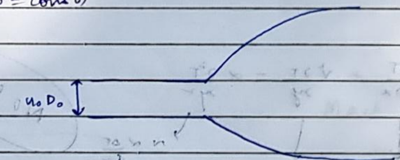
Pg 45 → Adrian

→

Jet problem

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\cancel{\frac{dP_\infty}{dx}} + \nu\frac{\partial^2 u}{\partial y^2}$$

$\sim \frac{U^2}{x}$; $\cancel{\frac{dP_\infty}{dx}} \to 0$ (P_∞ = const)



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$\sim \frac{U}{x}$; $\sim \frac{V}{D}$

$$V \sim \frac{DU}{x}$$

$$\frac{U^2}{x} \sim \frac{\nu U}{D^2}$$

$$\boxed{D^2 U \sim \nu x}$$

$$\int_{-\infty}^{\infty}\frac{\partial (u^2)}{\partial x}dy + \int_{-\infty}^{\infty}\frac{\partial uv}{\partial y}dy = \int_{-\infty}^{\infty}\nu\frac{\partial^2 u}{\partial y^2}dy$$

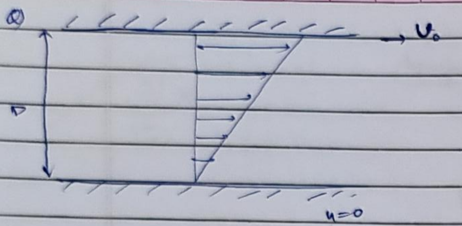
$$\int_{-\infty}^{\infty}\frac{\partial u^2}{\partial x}dy + \cancel{[uv]_{-\infty}^{\infty}} = \cancel{\nu\left[\frac{\partial u}{\partial y}\right]_{-\infty}^{\infty}}$$

$\cancel{[uv]} \to 0$; $\cancel{\nu[\partial u/\partial y]} \to 0$ → Beyond BL

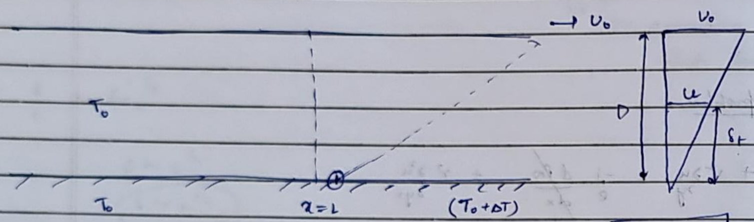
$$\frac{d}{dx}\int_{-\infty}^{\infty}u^2 dy = 0$$

$\int_{-\infty}^{\infty}u^2 dy$ = const → use order of magnitude analysis

$U^2 D = u_0^2 D_0$ → (2)



$$\boxed{u = \frac{U_0}{D} y}$$



$$\boxed{\frac{u}{U_0} \approx \frac{\delta_T}{D}}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

at the edge of TBL

$$u\frac{\Delta T}{x} \qquad \frac{u\delta_T}{x}\frac{\Delta T}{\delta_T} \qquad \sim \alpha\frac{\Delta T}{\delta_T^2}$$

NO HBL

$$u \sim \frac{\delta_T U_0}{D}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{u}{x} \sim \frac{v}{\delta_T}$$

$$v \sim \frac{u\delta_T}{x}$$

$$\frac{u\Delta T}{x} \sim \alpha\frac{\Delta T}{\delta_T^2}$$

$$\delta_T^2 \sim \frac{\alpha x}{u}$$

$$\delta_T^2 \sim \frac{\alpha x D}{\delta_T U_0}$$

$$\delta_T^3 \sim \frac{\alpha x D}{U_0}$$

$$\boxed{\delta_T \sim \left(\frac{\alpha x D}{U_0}\right)^{1/3}}$$

$$\Rightarrow \frac{T(x,y) - T_0}{\Delta T} = 1 - \frac{y}{\delta_T}$$

$$\frac{d}{dx}\int_0^{\delta_T} u(T_0 - T)\,dy = \cancel{\frac{dT_\infty}{dx}}^{\,0}\int_0^{\delta_T} u\,dy + \alpha\left(\frac{\partial T}{\partial y}\right)_{y=0}.$$

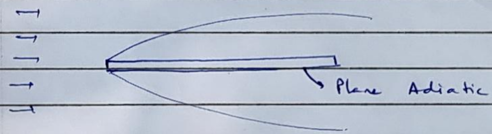
$$\frac{d}{dx}\int_0^{\delta_T} \frac{U_0}{D} y\left(\frac{y}{\delta_T} - 1\right)\Delta T\,dy = \qquad \delta_T = \left(\frac{9\alpha x}{4U_0}\right)^{1/2}$$

$$D = \left(\frac{9\alpha x}{4U_0}\right)^{1/2} \quad \text{(iv)}$$

$$x = \frac{D^2 4 U_0}{9\alpha}$$

Problem-3

Mach number $= \frac{u}{c}$ (sound)



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\cancel{\frac{dp_\infty}{dx}}^{\,0} + \nu\frac{\partial^2 u}{\partial y^2}$$

$u = 0,\ y = 0$

$u = u_1,\ y \to \infty.$

Can we solve this using blasius soln?

v = const

u can use blasius soln.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2$$

$T = T_1,\ y \to \infty$

$\frac{dT}{dy} = 0,\ y = 0$

$$\theta_a(\eta) = \frac{T - T_1}{u_1^2/2c_p} \rightarrow \text{similarity parameter.}$$

$$\theta_a'' + \frac{Pr}{2} f\theta_a' + 2Pr(f'')^2 = 0.$$

$\cancel{\theta_a(0) = \int_0^x \exp}$ $\theta_a(0) \sim \sqrt{Pr}$. L

Sound velocity → $a = \sqrt{\gamma R T}$

$T_w = T_1 \left[1 + 0.167\, Ma^2 \right]_{Ma = 0.8} = 302.178$