UNIT OPERATIONS

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Twelve leading authorities present a comprehensive treatment of modern process operations–stressing the similarity of their basic principles.

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Preface

This textbook is the first to carry the title *Unit Operations*, but it is not the first to treat the subject.

Modern practice and equipment are emphasized as well as mathematical interpretations, as only by properly designed, constructed, and operated equipment can mathematical treatment yield useful results. The object is to build the student's knowledge and power progressively and continuously until he has a reasonably clear concept of how to approach the problems of design and operation of processing equipment. The unit operations are grouped according to similarities in action or in methods of calculation and presented in sequence according to increasing difficulty.

By grouping similar operations and using a common nomenclature in similar theoretical discussions, we find that the student makes more rapid progress, less effort is required to master nomenclature, and a better understanding is gained of the relationships among the different unit operations. The association and comparison of similar operations from different industrial processes is the essence of unit operations and the major factor in developing chemical, metallurgical, or process engineers capable of successfully designing new plants for conducting new processes. The full advantage of the study of unit operations can be realized only if the unit operations are themselves associated and compared so the engineer may more skillfully select the most suitable operation and equipment desired for each step in the process. The tendency of the specialist to treat each unit operation as a specialty having its own peculiar result, rationalization, and nomenclature is of questionable value in any sustained educational effort and is to be resisted by all means in an undergraduate curriculum.

The arrangement in order of increasing difficulty rather than in order of assumed importance continually presents new advanced intriguing problems to the student, maintains his interest, and encourages him to continue his own development beyond the limitations of the book. The treatment of those operations covering solids in Part I requires little more preparation than is ordinarily given in high school, whereas the treatment of mass transfer in Part IV is suitable for a postgraduate course and is presented with a critical attitude tending to develop the research point of view.

The inductive method is generally followed, relying upon observations from experience rather than upon deductive rationalizations. This method is a powerful tool of the practicing engineer and has been found most satisfactory for undergraduate students. However, kinetic explanations are not neglected and receive increasing emphasis in the last part on energy and mass transfer as an important means to a thorough understanding of the mechanisms involved.

Physics, calculus, and a beginning course in material and energy balances, or thermodynamics, are assumed as prerequisites to unit operations. Even with this background the student may be confused regarding dimensions and energy balances, and these subjects are treated rather fully. It is hoped that all chapters have received sufficiently extensive treatment to meet the requirements of any undergraduate curriculum so that the desired emphasis may be obtained by omission rather than addition. About 180 recitations should be required to cover the entire material in an adequate manner with undergraduate students, allowing 8 to 10 for the first five chapters and 50 to 60 each for Parts II, III, and IV. In a postgraduate course for students who have completed an undergraduate course in unit operations, this time could be reduced by one-third or one-half. With appropriate omissions the text has been used successfully for undergraduate courses of three quarters with a total of 117 class meetings and of two semesters with a total of 105 class meetings, as well as for a single-semester short course of 60 class meetings.

References to the literature are included for the purpose of attracting the student's attention to other sources of information as well as to acknowledge sources. An effort has been made to give credit for all material used, but so many workers have contributed so much that it is impossible to recognize the contributions of everyone. Indebtedness to previous texts and handbooks and to manufacturers of equipment is freely acknowledged. The specific help and suggestions of L. F. Stutzman and George Thodos, Associate Professors, and D. A. Dahlstrom, Assistant Professor of Chemical Engineering, at Northwestern University, F. Charles Moesel and Cedomir Sliepcevich, Assistant Professors of Chemical Engineering at The University of Michigan, Dr. Joseph Allerton, of Sayville, Long Island, and Verne C. Kennedy, Jr., of Chicago, and the frank criticisms of students who have used the material as mimeographed notes have been invaluable. Tolerance and your cooperation in helping to eliminate errors and suggest improvements as they may appear are requested.

THE AUTHORS

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(11) MASS TRANSFER

Principles of molecular diffusion and diffusion between phases. Concept of stage wise processes.

Distilation, vapour liquid equilibria, two component ideal mixtures. McCabe-Thiele, method, nonideal binary systems, enthalpy concentration diagrams, plate efficiencies, plate and packed columns. H. E. T. P. and H. T. U. Batch, azeotropic, extractive, steam and vacuum distilations.

Absorption, mechanism of absorption theories of absorption equipment, transfer coefficient and absorption with chemical reaction. Principles of absorption.

Extraction, equilibrium data use of trangular diagrams, cocurrent and countercurrent arrangements, continuous extraction and transfer coefficient. Principles of leaching and methods of calculations.

Humidification. vapour gas mixtures. humidity chart and equipment for humidification and dehumidification.Drying. principles of drying. types of dryers and calculation methods.

Principles of crystallisation and crystallisation equipment. Absorption and ion exchange.

Text Books :

McCabe and Smith, Unit Operations in Chemical Engineering,' McGraw-Hill (Asian Edition).

Treybal, R. E. Mass Transfer Operation. McGraw-Hill

CHAPTER 1

Introduction to the Unit Operations

In general there are two different approaches to the study of industrial processing. Each particular industry, such as the alcohol, petroleum, plastic, copper, or steel industry, including its characteristic operations, may be studied as a unit; or the different operations common to many industrial processes may be classified, each according to its function without regard to the industry using it, and each such operation studied as a unit operation. Thus heat transfer is a single or unit operation common to practically all industries, and knowledge of the principles of heat transfer is equally useful to an engineer in any industry requiring the transfer of heat.

As industrial processes have become more varied and technical, the fields open to the engineer have widened and it has become increasingly difficult, if not impossible, to cover the various industries in an adequate manner without limiting the students to a few closely related fields. By studying the unit operations themselves and their functions the engineer is trained to recognize these functions in new industrial processes; and by applying his knowledge and skill in the corresponding unit operations he is able to design, construct, and operate a plant for a new process with almost as much confidence as for a proved process. For these reasons the study of unit operations has proved to be the more efficient approach to the study of industrial processing.

Although the importance of these operations that are common to different industries was recognized as early as 1893 by Professor George Lunge,* the concept of unit operations was first crystallized by A. D. Little † in 1915.

> The arts of pulverizing, evaporating, filtering, distilling, and other operations constantly carried on in chemical works have been so thoroughly developed as to amount almost to special sciences.*
>
> Any chemical process, on whatever scale conducted, may be resolved into a coordinate series of what may be termed "Unit Operations," as pulverizing, drying, roasting, crystallizing, filtering, evaporating, electrolyzing, and so on. The number of these basic unit operations is not large and relatively few of them are involved in any particular process. The complexity of chemical engineering results from the variety of conditions as to temperature, pressure, etc., under which the unit operations must be carried out in different processes, and from the limitations as to materials of construction and design of apparatus imposed by the physical and chemical character of the reacting substances.†

A study of the unit operations is just as valuable to the operating engineer as to the designer, since all industrial operations, or plants, are composed physically of a series of unit operations in their proper sequence. The ability or capacity of a plant is no greater than that of its weakest unit. The operator analyzes his complex operations into units for individual improvement, and the designer synthesizes complex operations from a number of unit operations.

UNIT OPERATIONS CLASSIFIED

In this treatment the unit operations are classified or grouped according to their function and the phase

* Professor George Lunge of the Federal Polytechnic School of Zurich, in an address on the "Education of Industrial Chemists" presented at the Congress of Chemists at the Exposition in Chicago, 1893.

† Arthur D. Little as chairman of the Visiting Committee of the Department of Chemistry and Chemical Engineering of the Massachusetts Institute of Technology in a report to the President of the Institute in 1915.

or phases treated. A *phase* is a homogeneous and mechanically distinct or separable mass. Thus sand and water are two mechanically distinct masses, and each represents a separate phase; whether the sand is separate from or suspended in the water makes no difference. An oil phase floating upon water, or emulsified with the water, is a homogeneous mass mechanically distinct from the water whether or not it is continuous; and it is, therefore, a separate phase from the water phase. Similarly, a copper ore contains the mineral chalcopyrite as a separate solid phase from the surrounding gangue or rock, no matter how finely the mineral may be dispersed.

The phases present at any one time may be one or more solid phases, and one or more fluid phases. Sand and water represent one solid and one fluid phase, oil and water are two fluid phases, and the mineral and gangue are two (at least) solid phases. A mixture of solid salt, ice, water, and water vapor contains two solid and two fluid phases. Gases are fluids. Ordinarily there will exist only one gaseous phase.

The order of treatment begins with unit operations that treat solids alone, such as mechanical size separation, size reduction, and conveying of solids. These are followed by operations involving fluids. Since all fluids must be confined to store them or to direct their flow, a solid boundary phase is always involved, whether the solid particles are flowing through the fluid as in classification and flotation, or whether the fluid is flowing through a solid as in fluid transportation or filtration. The operations involving transfer of material from one phase to another are next treated by the method of equilibrium stages or contacts. These include leaching (solid to liquid), extraction (liquid to liquid), gas absorption and distillation (vapor to liquid), and adsorption (fluid to solid). Heat transfer and evaporation follow. Heat transfer deals with the rate of energy transfer and serves as a means of leading directly to the concept of rate of mass transfer as applied in crystallization, drying, absorption, distillation, and the more complicated operations involving catalysts and rates of reaction.

PRACTICAL OPERATIONS

In the study of unit operations, it must always be remembered that a unit operation is simply a unit of a more complex operating plant: a heat exchanger in a sugar plant, a crusher in a cement plant, a distillation column in a petroleum refinery, and that the important requirement in each case is a satisfactory workable overall operation. It makes no difference whether the result is obtained by exact mathematical calculation, by empirical approximation, or by a good guess based on the application of sound judgment, provided it is a satisfactory, workable, economical operation in its entirety.

The unit operations are the best available methods for classifying and formulating the combined experience of engineers as a guide to the operation and design of industrial plants. But these data, although of great help, are inadequate in themselves to insure successful operations. The successful engineer must develop sound judgment by his willingness to try, to recognize failures, and to keep on trying until he arrives at a satisfactory result. Seldom if ever does he have the opportunity to assemble either on paper or in physical form the ideal or perfect operation. Engineering operations require approximations and compromises. If made too nearly perfect, they may cost too much and last too long. Many plants become obsolete before they wear out.

All the information now available started with a single observation. As additional observations were made, the engineering mind began to draw conclusions which could be presented in the form of an empirical tabulation, such as the power required to operate crushing and grinding machines. Frequently these tabulated data could be presented in the form of a graph as a more satisfactory basis for extrapolating and interpolating the results. The next step was to derive an equation for the line representing the plotted data and to indicate means for estimating how the constants in the equation would be affected by different conditions. These equations might then be rationalized or sometimes "derived." However, the student and engineer should always keep in mind that these conclusions are drawn more or less soundly from a series of more or less reliable observations that have been empirically correlated; also, they should remember that the practical operator in the plant who may never have seen the equation or heard the term "unit operation" has probably made more observations himself than all those involved in deriving the equation. But it has taken the practical operator a much longer time to acquire his skill without understanding than it has the modern student of unit operations to acquire his comprehensive understanding.

FUNDAMENTAL CONCEPTS

Certain concepts or conclusions drawn from many observations are regarded as fundamental because, the more carefully the observations are made, the more closely do the data conform to the previous conclusion. Perhaps the most important of these to the engineer is the law of conservation of mass and energy.

Operations involving atomic energy have emphasized the concept that mass and energy are directly related. The quantity of energy equivalent to a unit of mass is so large, about 3×10^{16} ft-lb of energy per pound-mass, or the mass is so small, about 2.6×10^{-14} pound-mass per British thermal unit (Btu), that ordinary means of measurement are incapable of detecting any increase or decrease in mass accompanying a chemical process. In engineering operations, when nuclear changes are not involved, the mass of the products equals the mass of the reactants. This is in accord with engineering experience over many years and simplifies calculations, since material balances can then be made independently of energy balances.

The following four concepts are basic and form the foundation for the calculation of all operations. If nuclear changes are involved, the energy changes become so great that the first and second concepts are not independent and a combined energy and mass balance must be made.

1. The Material Balance

If matter may be neither created nor destroyed, the total mass for all materials entering an operation equals the total mass for all materials leaving that operation, except for any material that may be retained or accumulated in the operation. By the application of this principle, the yields of a chemical reaction or engineering operation are computed.

In continuous operations, material is usually not accumulated in the operation, and a material balance consists simply in charging (or debiting) the operation with all material entering and crediting the operation with all material leaving, in the same manner as used by any accountant. The result must be a balance. The accountant uses dollars as his unit, and the engineer uses pounds, tons, etc. In making a material balance, the engineer should not attempt to use units that may be created or destroyed during the process, such as units of volume or moles, or cubic feet, gallons, barrels, or molecules.

As long as the reaction is chemical and does not destroy or create atoms, it is proper and frequently very convenient to employ atoms as the basis for the material balance. The material balance may be made for the entire plant or for any part of it as a unit, depending upon the problem at hand. It is most conveniently made by adopting as a basis for calculation a fixed quantity of material which passes through the operation unchanged.

2. The Energy Balance

Similarly, an energy balance may be made around any plant or unit operation to determine the energy required to carry on the operation or to maintain the desired operating conditions. The principle is just as important as that of the material balance, and it is used in the same way. The important point to keep in mind is that all energy of all kinds must be included, although it may be converted to a single equivalent form such as Btu's, calories, or foot-pounds for the sake of addition. A balance cannot be made of heat or electrical energy alone, since all energy is convertible and all forms must be included in the balance.

3. The Ideal Contact

Whenever the materials being processed are in contact for any length of time under specified conditions, such as conditions of temperature, pressure, chemical composition, or electrical potential, they tend to approach a definite condition of equilibrium which is determined by the specified conditions. In many cases the rate of approach to these equilibrium conditions is so rapid or the length of time is sufficient that the equilibrium conditions are practically attained at each contact. Such a contact is known as an equilibrium or ideal contact. The calculation of the number of ideal contacts is an important step required in understanding those unit operations involving transfer of material from one phase to another, such as leaching, extraction, absorption, and distillation.

4. Rates of an Operation

In most operations equilibrium is not attained, either because of insufficient time or because it is not desired. As soon as equilibrium is attained no further change can take place and the process stops, but the engineer must keep the process going. For this reason rate operations, such as rate of energy transfer, rate of mass transfer, and rate of chemical reaction, are of the greatest importance and interest. In all such cases the rate and direction depend upon a difference in potential or driving force. The rate usually may be expressed as proportional to a potential drop divided by a resistance. An application of this principle to electrical energy is the familiar Ohm's law for steady or direct current.

$$I = \frac{E_1 - E_2}{R} = \frac{-\Delta E}{R}$$

where I = rate of electron transfer or current of electricity (coulombs/sec, or amp).

E = electrical potential, and ΔE is the increase in potential between points 1 and 2 (volts).

R = resistance (ohms).

In heat transfer under similar conditions for steady flow, the time rate of heat transfer from mass A in contact with mass B is

$$\frac{dQ}{dt} = \frac{T_A - T_B}{R} = \frac{-(T_B - T_A)}{R} = \frac{-\Delta T}{R}$$

where $\frac{dQ}{dt}$ = the instantaneous time rate of heat transfer or the quantity of heat transferred per unit of time from mass A to mass B.

T_A = temperature of mass A.

T_B = temperature of mass B.

$-\Delta T$ = the temperature drop.

R = resistance to heat transfer.

In solving rate problems as in heat transfer or mass transfer with this simple concept, the major difficulty is the evaluation of the resistance term. In practice, the values of the resistance term are generally computed from an empirical correlation of many determinations of transfer rates under different conditions.

The basic concept that rate depends directly upon a potential drop and inversely upon a resistance may be applied to any rate operation, although the rate may be expressed in different ways with particular coefficients for particular cases.

APPLICATION OF CONCEPTS

These principles, used singly or in combination, and the coordinated knowledge of the unit operations as presented in this textbook, the handbooks, and other technical literature constitute the science or theory of the unit operations. Practical engineering consists in applying the understanding of these operations and practical knowledge of the many types of equipment that may be employed to the design and operation of a commercial plant that will show not only a material balance but also a favorable dollar balance.

PART I

Solids

THIS section deals with those operations which treat material in the solid state only: screening, size reduction, and handling of solids. Before discussing these operations the properties of solids should be reviewed.

CHAPTER

2

Properties of Solids

AMONG the many properties of solids, those listed below are of particular significance in engineering operations.

Density, usually expressed by the symbol ρ, is defined as the mass per unit volume. The units are usually pounds per cubic foot, or grams per cubic centimeter.

Specific gravity is the ratio of the density of the material to the density of some reference substance, or ρ/ρ_{ref}. For solids and liquids the reference substance is usually water at 4° C. For most engineering work the specific gravity may be given the same numerical value as density in grams per cubic centimeter, but the specific gravity is a dimensionless ratio.

Bulk (or apparent) density ρ_b is the total mass per unit of total volume. For example, the true density of quartz is 2.65 grams/cc. But a quartz sand of 2.65 grams mass may occupy a total or bulk volume of 2 cc and have a bulk density ρ_b of 1.33 grams/cc. The bulk density is not an intrinsic characteristic of the material since it varies with the size distribution of the particles and their environment. The porosity of the solid itself and the material with which the pores, or voids, are filled also influence the bulk density. For a single nonporous particle the true density ρ equals the bulk density ρ_b.

Hardness[2] * of certain solids such as metals and plastics may be defined as resistance to indentation. The hardness of minerals is usually defined as resistance to scratching and is usually expressed in terms of Mohs' scale, which is based on a series of minerals of increasing hardness numbers as follows:

1	Talc	6	Feldspar
2	Gypsum	7	Quartz
3	Calcite	8	Topaz
4	Fluorite	9	Corundum, sapphire
5	Apatite	10	Diamond

Each mineral in the list will scratch all those of a lower number. A mineral of unknown hardness is rubbed against these test minerals, and its hardness is indicated by the softest material which just scratches it. The approximate hardnesses of some common materials are: dry finger nail, 2.5; copper

TABLE 1. SOME PROPERTIES OF SOLIDS

Material	Density,[1] ρ, lb/cu ft	Bulk Density, lb/cu ft	Mohs Hardness [1]	Crystalline Form [1]
Alumina, Al_2O_3	249		9.0	Hexagonal
Bauxite	159	Crushed 80	Soft	Amorphous
Barites, $BaSO_4$	280	Crushed 180	2.5–3.5	Orthorhombic
Calcite, $CaCO_3$	169	Crushed 90–96	3	Hexagonal
Gypsum, $CaSO_4 \cdot 2H_2O$	145	Crushed 80–100 Powdered 60–80	1.5–2.0	Monoclinic
Hematite, Fe_2O_3	306–330	Crushed 150	5.5–6.5	Triclinic
Pyrites, FeS_2	308–318		6.0–6.5	Cubic
Halite, NaCl	131–162	Crushed 80 Powdered 45	2.5	Cubic
Galena, PbS	460–480		2.5–2.7	Cubic
Quartz, SiO_2	165	Gravel 100–110	7	Hexagonal
Sphalerite, ZnS	255		3.5–4.0	Cubic
Beans, corn, flaxseed, wheat		45–48		
Cottonseed, oats		25–26		
Coal, bituminous	84	44–52	Soft	
Coke		23–34		
Petroleum coke		35–40	Soft	
Portland cement		100		

* Superior numbers refer to entries in the bibliographies. For this chapter, see p. 8.

penny, 3.0; tooth enamel, 5.0; penknife, 5.5; ordinary glass, 5.8.

Brittleness or friability refers to the ease with which a substance may be broken by impact. The hardness of a mineral is not a sure criterion of its brittleness. Horn, some plastics, and gypsum are soft and tough and are not easily broken by impact. Coal is soft and also friable. Friability is the inverse quality to toughness. *Toughness* is the property of metals and alloys called impact resistance.

The crystalline structure and crystal size influence the friability. The structure also determines the shape into which particles naturally break when subjected to a crushing operation. For example, galena, PbS, breaks into cubes; mica into plates; and magnetite into somewhat rounded grains. Those crystalline planes which are easily broken are termed *cleavage planes*. The quantity of work necessary to fracture a unit area of cleavage plane can be determined by experiment. When metals and alloys are stressed beyond their yield points, a similar cleavage takes place in the crystals; but the crystals do not break apart, they simply deform. Wood and asbestos are fibrous, do not possess cleavage planes, and do not crush readily, but must be torn or shredded.

Friction is the resistance to sliding of one material against another material. The coefficient of friction is the ratio of the force parallel to the surface of friction in the direction of motion required to maintain a constant velocity, to the force perpendicular to the surface of friction and normal to the direction of motion.

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PROBLEM

A copper tube, 1 in. I.D. and 2 ft long, is filled with steel balls of 1-in. diameter. The space between the balls is filled with water. The specific gravity of steel is 7.8. What is the bulk density of the contents of the tube?

CHAPTER

3

Screening

THE separation of materials on the basis of size is frequently important as a means of preparing a product for sale or for a subsequent operation. It is also a widely used means of analysis, either to control or gage the effectiveness of another operation, such as crushing or grinding, or to determine the value of a product for some specific application.

In the marketing of coal, for example, the size of the particles is the basis of its classification for sale. Certain equipment such as stokers require definite limits of size for successful operation. In the case of sand and gravel for concrete, on the other hand, only a properly blended series of sizes will insure the most dense packing, requiring the minimum of cement and securing the greatest strength and freedom from voids.

It has frequently been observed that the rate of a chemical reaction between a solid and a fluid is roughly proportional to the surface involved. Since the surface areas may be computed from a knowledge of the sizes of the particles, a sizing operation is of particular value in controlling the rates of reactions involving solids. The combustion of powdered coal illustrates the desirability of controlling the grinding operation to produce material of definite size limits in order to control the rate of combustion. Since the setting of Portland cement must take place within a specified time, it has been necessary to specify certain size limits. The hiding power of a paint pigment is indicated by size since it depends upon the projected area of the particles.

Screening is accomplished by passing the material over a surface provided with openings of the desired size. The equipment may take the form of stationary or moving bars, punched metal plate, or woven wire mesh. Screening consists in separating a mixture of various sizes of particles into two or more portions, each of which is more uniform in size of particle than is the original mixture.

Dry screening refers to the treatment of a material containing a natural amount of moisture or a material that has been dried before screening. Wet screening refers to an operation in which water is added to the material being treated for the purpose of washing the fine material through the screen.

The material that fails to pass through the screen is referred to as oversize or plus material, and that which passes through the screen openings is referred to as undersize or minus material. When more than one screen is used and more than two sizes are produced, the various fractions may be designated according to the openings employed in making the separations. For example, Table 2 shows three different ways of indicating sizes.

TABLE 2. THREE METHODS OF INDICATING SIZE FRACTIONS

First	Second	Third
Oversize $\frac{1}{4}$ in.	$+\frac{1}{4}$ in.	$+\frac{1}{4}$ in.
Through $\frac{1}{4}$ in. on $\frac{1}{8}$ in.	$-\frac{1}{4}+\frac{1}{8}$ in.	$\frac{1}{4}/\frac{1}{8}$ in.
Through $\frac{1}{8}$ in. on $\frac{1}{16}$ in.	$-\frac{1}{8}+\frac{1}{16}$ in.	$\frac{1}{8}/\frac{1}{16}$ in.
Undersize	$-\frac{1}{16}$ in.	$\frac{1}{16}/0$ in.

INDUSTRIAL SCREENING EQUIPMENT

Grizzlies are widely used for screening large sizes, particularly of 1 in. and over. They consist simply of a set of parallel bars separated by spacers at the ends. The bars may be laid horizontally or inclined longitudinally 20 to 50 degrees from the horizontal, depending upon the nature of the material treated.

The usual cross section of the bars is trapezoidal with the wide base upward to prevent clogging or wedging of the particles between the bars. Inverted railroad rails are frequently employed. Owing to the wear on the bars, they are frequently made of manganese steel.

Grizzlies are usually constructed about 3 to 4 ft wide with the bars from 8 to 10 ft in length. They are frequently used before material is sent to a crusher to remove the smaller particles from the feed to the crusher.

In some grizzlies, a cam arrangement causes a slight lengthwise reciprocal movement of alternate bars, permitting better flow of material through them and preventing clogging. Endless chains passing over sheaves may replace the bars, constituting

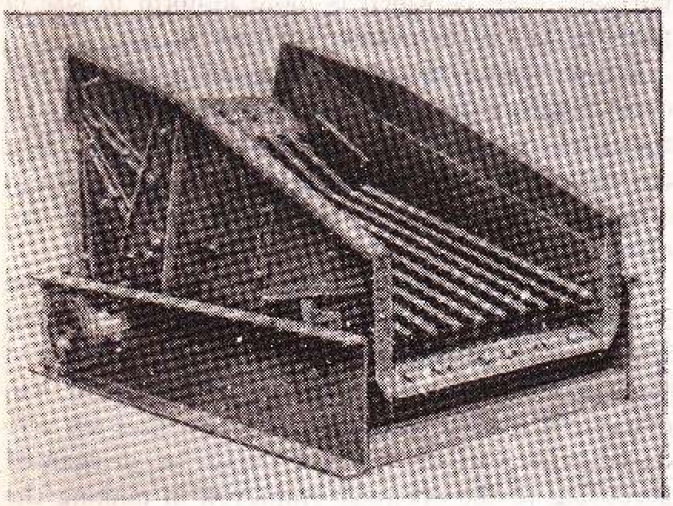

Fig. 1. Mechanically vibrated bar grizzly. The material enters at the top left and works its way downward to the right. The large or oversize particles are discharged over the lower right end, and the smaller particles pass through the slots between the bars into a hopper directly below. (*Nordberg Mfg. Co.*)

the chain grizzly. These more elaborate grizzlies are for somewhat sticky or clay-like material. Figure 1 shows a grizzly mounted on springs with the whole assembly vibrated mechanically.

A rough figure for the capacity of grizzlies is approximately 100 to 150 tons of material per square foot of area per 24 hr when the bars are spaced to give about 1 in. of clear opening.

Stationary screens are made of punched metal plate or woven wire mesh, usually set at an angle with the horizontal up to about 60 degrees. They are suitable for intermittent small-scale operations, such as screening sand, gravel, or coal by throwing the material against the screen. When large tonnage

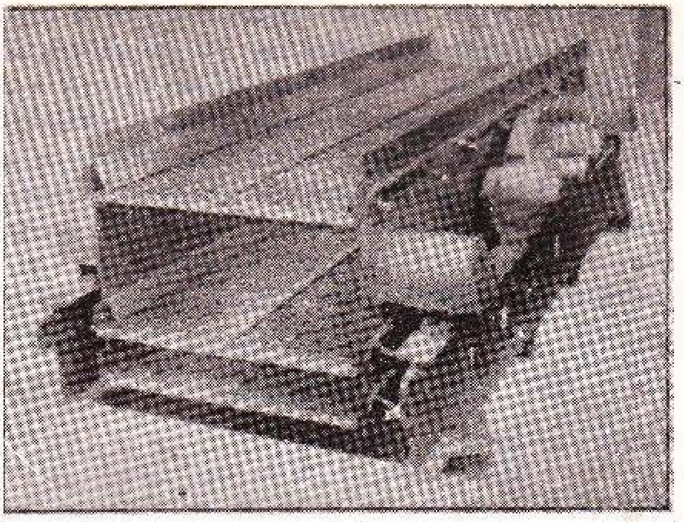

Fig. 2. Triple-decked mechanically vibrated screen. (*W. S. Tyler Co.*)

is to be handled stationary screens are usually abandoned in favor of the vibrating screens.

Vibrating screens are used where large tonnages are to be treated. The vibrating motion is imparted to the screen surface by means of cams, eccentric shafts, unbalanced flywheels, or electromagnetic means. A complete screen may have a single screening surface, or it may be double- or triple-decked, as indicated in Fig. 2. This screen is driven by an eccentric shaft, as shown in Fig. 3. The wire screens

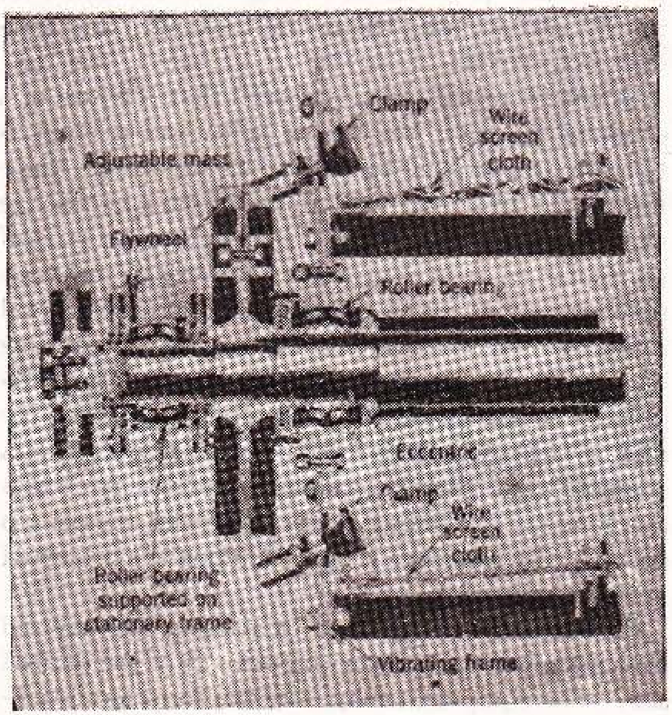

Fig. 3. Sectional drawing of the mechanism of a mechanically vibrated screen. (*W. S. Tyler Co.*)

are held in place under tension by the clamps and supported on the vibrating frame. The rotating shaft is supported on the stationary frame by the outer roller bearings which are fastened to the stationary frame. The eccentric carries the roller bearings which support the vibrating frame. The vibrating frame is positioned by springs, four of which are indicated under covers on each side of the screen shown in Fig. 2. The flywheel is mounted eccentrically on the shaft as a counterbalance to balance the vibrating frame, screens, and load of material being screened. An adjustable mass is provided for controlling the eccentricity of the flywheel. Rotation of the shaft gives the screens a circular motion in the vertical plane. As the screen passes the top of its cycle, the material is thrown clear of the screen surface. The material will be moved along the screen in either direction, depending upon the direction of rotation. Such screens are operated (Fig. 4) with a slope that may vary from the horizontal to about 45 degrees. These screens are operated with an amplitude (diameter of the described circle of motion) up to about ¼ in., depending on the size of the material, and with a frequency of vibration or rotation of about 1200 to 1800 per minute. They are made for heavy duty with openings above 1 in. and are widely used for dry screening of particles from 1 in. down to about 35 mesh * (0.0164 in.) at an angle of about 20 degrees. For wet screening the angle is reduced to about 5 or 10 degrees. Usually the feed enters the top of the screen, but sometimes it is found desirable to feed at the lower end of the screen and discharge at the top.

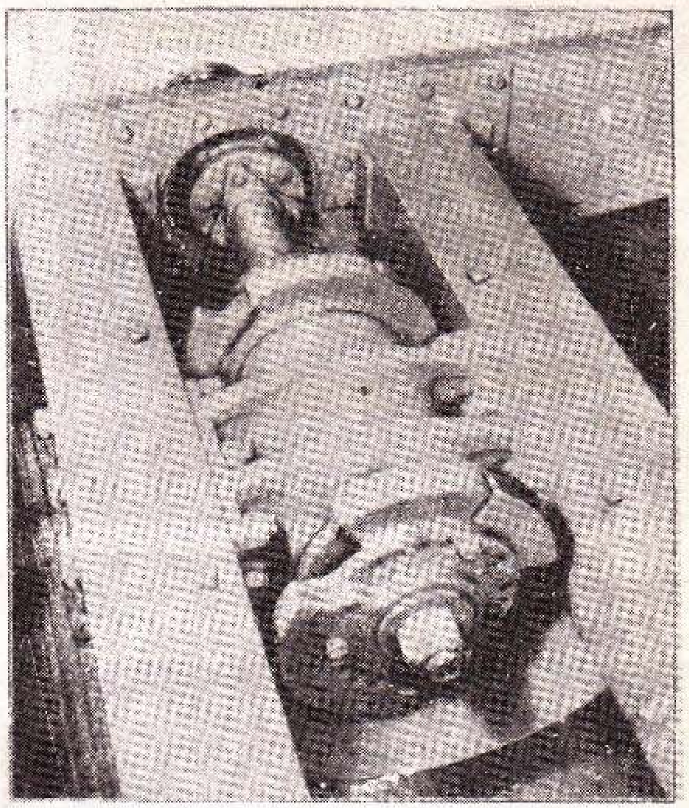

FIG. 5. Vibrating mechanism consisting of an adjustable unbalanced flywheel and shaft mounted rigidly on a vibrating screen frame, which is in turn supported on springs. (*Nordberg Mfg. Co.*)

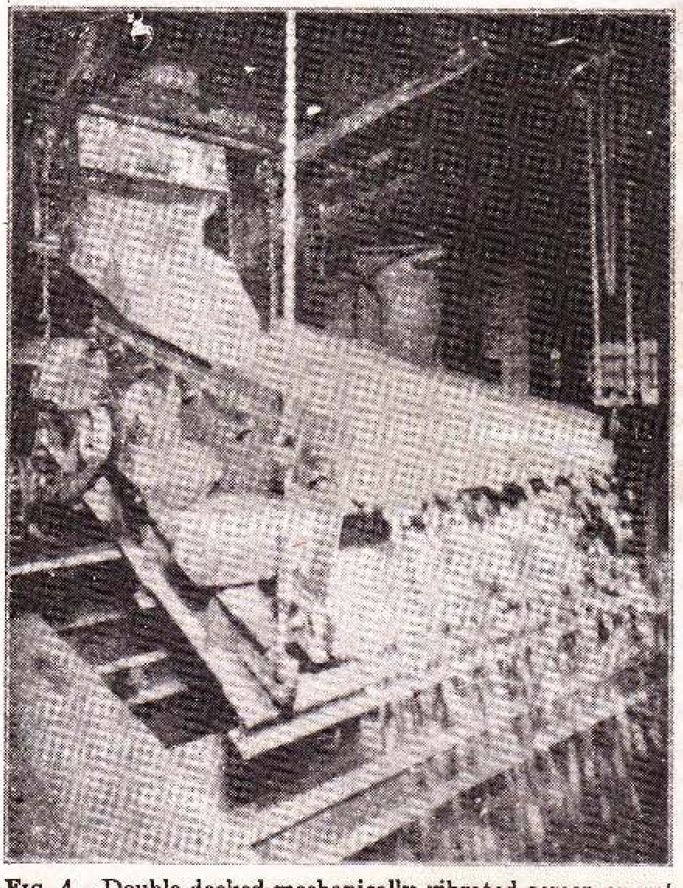

FIG. 4. Double-decked mechanically vibrated screen operating under test conditions. (*W. S. Tyler Co.*)

In another type of screen the vibrating frame is mounted on springs, and the belt-driven rotating shaft is mounted only on the vibrating frame. Vibration is caused by creating an unbalance on the rotating shaft by mounting an adjustable unbalanced flywheel (Fig. 5) to give the desired amplitude. Such a drive is mechanically simpler than the counterbalanced eccentric-shaft drive previously described,

* *Mesh* is a term stating the number of openings per linear inch of screen surface. The size of the opening depends on the size of the wire, but for screens of a standard series the mesh is a specific designation of the aperture, as stated in Table 4.

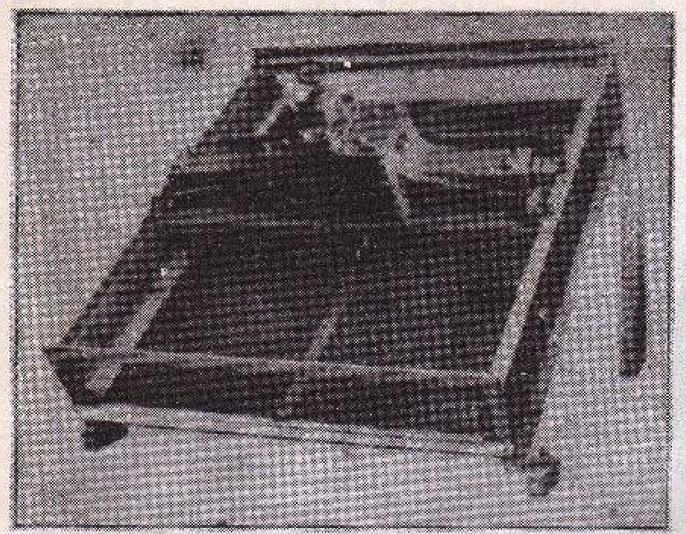

FIG. 6. Vibrating screen employing an electromagnetic vibrating unit attached to the center of the screen. (*W. S. Tyler Co.*)

but it is not capable of as rugged construction. If two unbalanced wheels are rotated in opposite directions at the same number of revolutions per minute (rpm), the vibration may be strictly normal to the plane of the screen.

An electromagnetic vibrator may be attached to the center of the screen, as shown in Fig. 6. The frame is rigid and the screen is vibrated by the solenoid whose core is fastened to the center of the screen. The core of the solenoid works against adjustable spring tension, as shown in Fig. 7. In this way the amplitude of vibration may be varied up to about ⅛ in. In other types the core is fastened to the frame supporting the screen and vibrates the frame and screen. With such an arrangement the solenoid may be mounted obliquely to the screen surface, giving the screen a motion at an angle to the normal. High amplitude in a screen vibrated

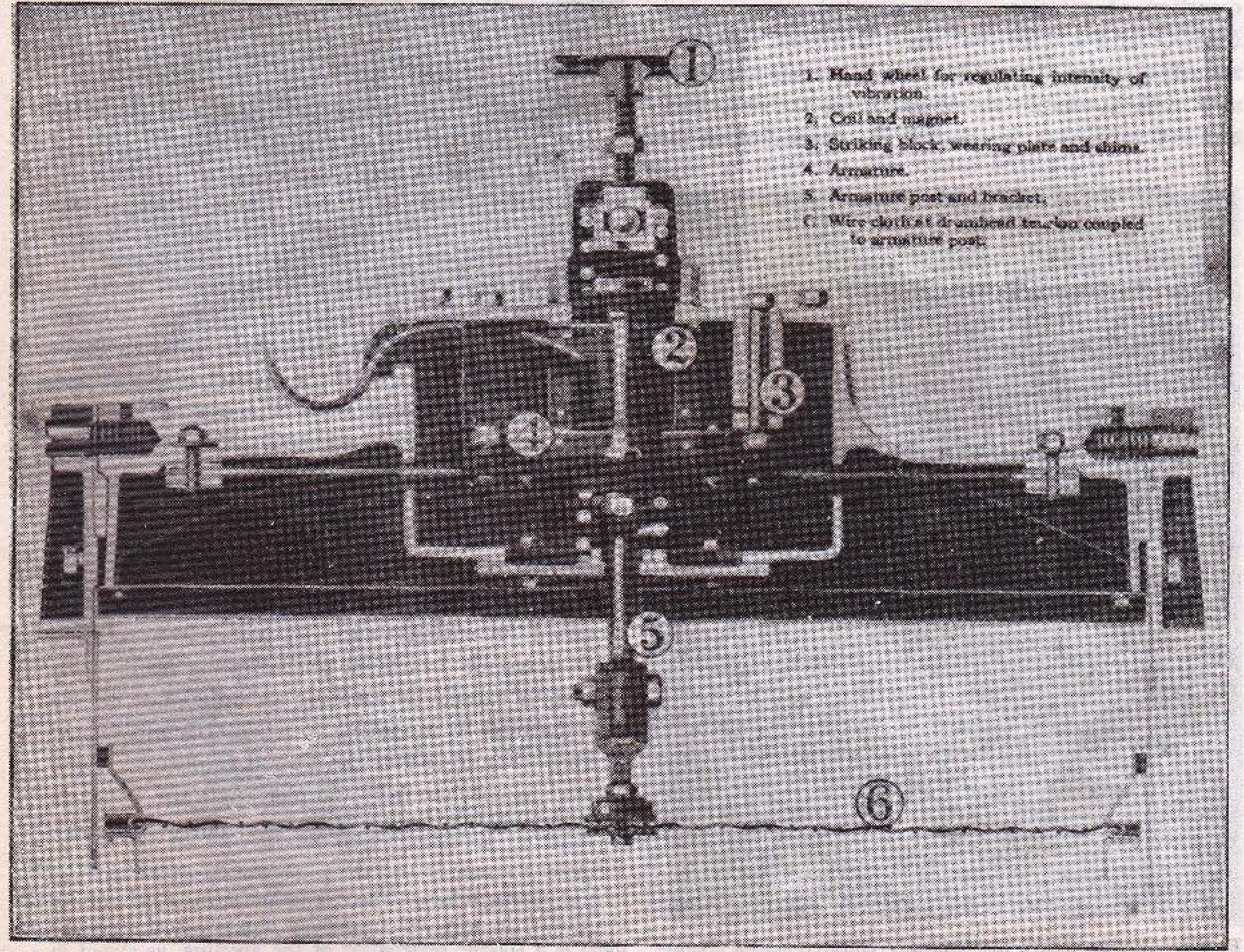

FIG. 7. Cross section of electromagnetic vibrator of Fig. 6. The handwheel, 1, adjusts the spring tension and limits the length of stroke caused by the electromagnet, 2, operating on the armature directly connected to the screen, 6. (*W. S. Tyler Co.*)

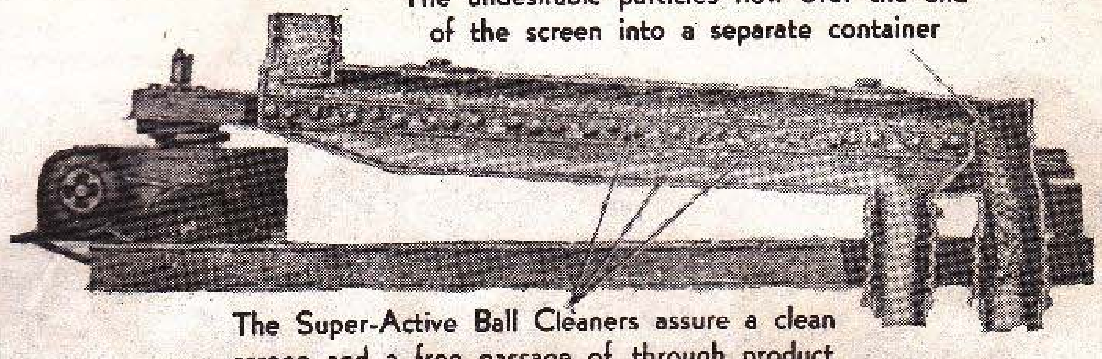

FIG. 8. Reciprocating screen with ball cleaners. The oversize flows over the end of the screen. The balls are confined in limited areas directly beneath the screen, and supported on a coarse wire screen. The balls bounce against this supporting screen and give additional vibration to the screen directly above. The desired fine material passes through the screen. (*J. H. Day Co.*)

in a fixed frame (Fig. 6) may cause fatigue failure of the screen near the clamps. If the screen is supported in a vibrating frame this limitation does not apply but a greater energy is required to overcome the inertia of the mass of the frame. The solenoid is usually caused to strike a block or anvil, thereby suddenly halting the upward motion of the screen and throwing the material clear of the screen. This is particularly desirable in the handling of sticky materials. These screens are normally used for material from about 8 mesh down to 100 mesh or finer and have been successful in wet screening. The frequency of vibration is determined by the frequency of the alternating current used and varies from about 900 to 7200 vibrations per minute. The lower frequency is used for coarser screening (8 mesh) and the higher frequencies for the finer screening.

The capacity of a vibrating screen varies widely with the character of the material treated from 2 tons/sq ft of surface per 24 hr for materials such as damp clay or powdered soap, up to 30 tons for dry material such as coke on screens of about 6 to 8 mesh.

Oscillating screens are characterized by a relatively low speed (300 or 400 oscillations per minute) in a plane essentially parallel to the screen. The riddle is a screen driven in an oscillating path by an eccentric or other mechanism attached to the sole support of the screen, usually a vertical bar extending from the top of the screen box. It is the cheapest form of screen on the market and is used for batch screening.

A sifter is a box-like container holding a number of screen cloths nested on top of one another and oscillated by an eccentric or counterweights in a nearly circular orbit. Many of these devices carry a coarse screen directly below the screening cloths on which rubber balls are confined to limited areas and are caused to bounce against the lower surface of the screen cloths as the device is oscillated.

Reciprocating screens (Fig. 8) are driven by an eccentric under the screen at the feed end. The motion varies from gyratory (about 2 in. in diameter) at the feed end to a reciprocating motion at the discharge end. These screens are usually inclined about 5 degrees, giving the screen a motion normal to the cloths of about $\frac{1}{10}$ in. Further vibration may be caused by including rubber balls as shown confined to local areas below the active screen surface. This type of screen is popular and widely used for screening dry chemicals down to about 300 mesh.

Trommels, or revolving screens, consist of a screen cylindrical or conical in form rotated about its axis.

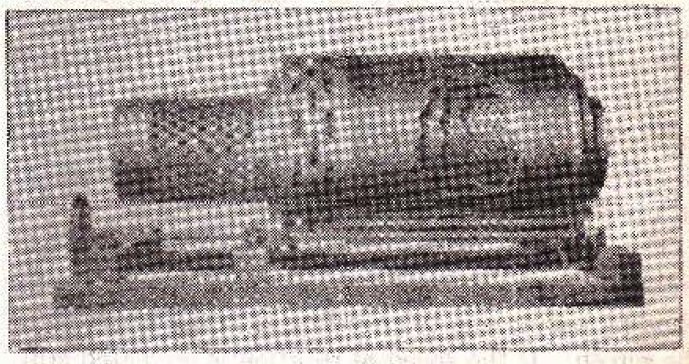

FIG. 9. Compound trommel. (*C. O. Bartlett and Snow Co.*)

Simple trommels may be arranged in series with the undersize of the first passing to the second trommel and the undersize of the second passing to the third, etc. Sometimes the trommels are built with screens of different sizes throughout their length, the feed

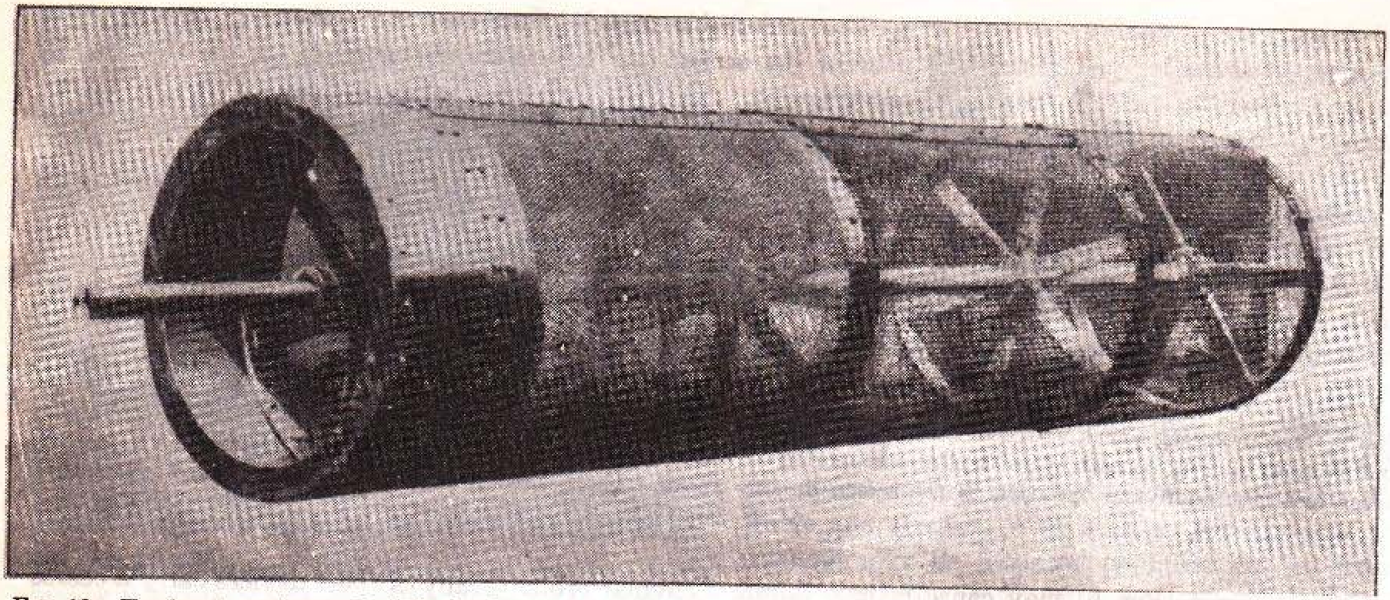

FIG. 10. Tandem-type trommel. The feed enters at the left end and passes over successively coarser screens as it works its way to discharge of the oversize at the far right. (*C. O. Bartlett and Snow Co.*)

entering at the end of the finest screen (Fig. 10). In this way it is possible to collect materials of different size ranges from a single trommel. But the operation is not so efficient as that of a series of simple trommels, or a compound trommel.

The compound trommel (Fig. 9) contains two or more concentric screening surfaces mounted on the same shaft. The coarser screening surface is the innermost, and the finest the outermost, with intermediate sizes arranged between the two limits. With provision made for the separate removal of the oversize from each screening surface, the undersize of each screen becomes the feed to the screen of the next smaller aperture.

Conical trommels have the shape of a truncated cone and are generally mounted with their axes horizontal.

Trommels are quite efficient for coarse sizes. The inclination of the trommel varies from about ¾ in. (for wet screening) to 3 in./ft of length, depending on the nature of the material to be processed.

The capacity of the trommel increases with increased speed of rotation up to a point where blinding occurs due to crowding of material through the screen. If the speed of rotation is increased still further to the *critical speed,* the material no longer cascades over the screen surface but is carried around by centrifugal force. The best operating speed is usually about 0.33 to 0.45 times the critical speed.

The critical speed of rotation of a trommel may be computed by equating the force of gravity tending to cause the particle to fall to the centrifugal force tending to carry the particle around.

$$mg = \frac{2mv^2}{D} \quad \text{or} \quad g = \frac{2v^2}{D}$$

where m = mass (lb).
g = acceleration of gravity (ft/sec^2).
v = velocity of particle, or of trommel, in circular path (fps).
D = diameter of trommel (ft).

When N = number of revolutions of trommel per minute,

$$v = \frac{\pi DN}{60}$$

$$g = \frac{2}{D}\left(\frac{\pi DN}{60}\right)^2$$

$$N^2 = \frac{\overline{60}^2 gD}{2(\pi D)^2} = \frac{\overline{60}^2 g}{2\pi^2 D}$$

$$N = \sqrt{\frac{\overline{60}^2 g}{2\pi^2 D}}$$

At sea level $g = 32.17$ ft/sec^2 and $N = 76.65/\sqrt{D}$.

With compound trommels the speed of rotation is naturally governed by the diameter of the outside screen.

Trommels are usually about 3 to 4 ft in diameter, from 5 to 8 ft in length, and driven at 15 to 20 rpm with 2½- to 5-hp motors.

The trommel is best suited for material from ¼ to 2½ in. in size.

Reels are revolving screens driven at relatively high speed. They are used in the flour milling industry and for other light, dry, nonabrasive material. The screening surface consists of silk bolting cloth supported by wire mesh. Speed of rotation is above the critical speed for a trommel and is such as to throw the undersized particles outward through the bolting cloth by centrifugal force. The surface may be cleared by brushes inside the reel. Reels are generally 24 to 40 in. in diameter and 5 to 8 ft long, and they are rotated at speeds that may vary from 100 to 200 rpm.

The effectiveness of screens is based upon both the recovery in the product of the desired material in the feed and the exclusion or rejection from the product of the undesired material in the feed. For example, the specifications for hydraulic hydrated lime (ASTM C141-42) require that the product contain not more than 10 per cent by weight of material coarser than 200 mesh.

If x_P = mass fraction of desired material in product,
x_F = mass fraction of desired material in feed,
x_R = mass fraction of desired material in reject,
P = total mass of product,
F = total mass of feed,
R = total mass of reject,

$$\underline{\text{Recovery}} = \frac{x_P P}{x_F F}$$

Rejection (1 − Effectiveness of recovery of

$$\text{undesired material}) = 1 - \frac{(1 - x_P)P}{(1 - x_F)F}$$

Effectiveness (recovery × rejection)

$$= \frac{x_P P}{x_F F}\left(1 - \frac{(1 - x_P)P}{(1 - x_F)F}\right)$$

Weighing the entire feed and product is not practical, and it is desirable to express the effectiveness from the analyses of samples alone. A material balance around the screening operation gives

$$x_F F = x_P P + x_R R$$

$$F = P + R$$

Substituting for R

$$x_F F = x_P P + x_R F - x_R P$$

Collecting terms

$$F(x_F - x_R) = P(x_P - x_R)$$

$$\frac{P}{F} = \frac{(x_F - x_R)}{(x_P - x_R)}$$

Substituting for P/F

$$\underline{\text{Recovery}} = \frac{x_P(x_F - x_R)}{x_F(x_P - x_R)}$$

$$\underline{\text{Rejection}} = 1 - \frac{(1 - x_P)(x_F - x_R)}{(1 - x_F)(x_P - x_R)}$$

Effectiveness * (recovery × rejection)

$$= \frac{x_P(x_F - x_R)}{x_F(x_P - x_R)}\left[1 - \frac{(1 - x_P)(x_F - x_R)}{(1 - x_F)(x_P - x_R)}\right]$$

These equations permit the desired calculation of recovery, rejection, or effectiveness of any sizing operation from size analysis of the streams, without knowledge of the quantities.

Rapid feeding or too steep an angle of the screen gives insufficient time for complete separation of the fine and coarse material. Excessive dampness in the feed may cause cohesion of small particles to form larger masses, or the adhesion of small particles to large particles. Worn screens with enlarged apertures will pass more oversize material into the undersize fraction. Clogged screens (blinded) retain more undersize material in the oversize fraction. The effectiveness as defined above is a numerical expression for the effect of all these factors.

Capacity of screens and effectiveness are closely related. If a low efficiency or effectiveness may be tolerated, the screen may be operated at high capacity. The ability of the device to prevent blinding of the screen surface is probably the most important single factor determining capacity of the screen. In dry screening, the greater the amount of moisture or dampness in any particular material, the

* Other expressions for effectiveness are used, such as the recovery, or rejection, as defined above, or the product of recovery and enrichment.

$$\frac{x_P(x_F - x_R)}{x_F(x_P - x_R)}\left(\frac{x_P - x_F}{1 - x_F}\right)$$

All these expressions give different values, depending on whether the undersize is considered the reject or the product. The expression for effectiveness in the text gives the same value, regardless of whether the undersize is the product or reject.

lower is the capacity of the screen. Because of its greater surface area, finer material can tolerate a greater percentage of moisture. If the feed contains a high proportion of material of a size just slightly smaller than the size of the openings in the screen, called "near mesh," the capacity of the screen will be greatly reduced. For example, if the size of the openings is 1/8 in. and there is a large proportion of 3/32-in. grains in the material to be screened, the screen's capacity, for the same degree of effectiveness, will be much lower than if most of the undersize material is smaller than 1/32 in.

The ratio of the open area of the screen to the total area is an important factor in determining its capacity. Because of the direct dependence of screening capacity upon the area of the screen surface and upon the screen aperture, the capacity is usually expressed [1]* in terms of tons of feed per square foot of screen area per millimeter of screen aperture per 24 hr, as indicated in Table 3. For example, a vibrating screen having 6 sq ft of surface and an aperture of 2 mm may be expected to have an approximate capacity of $(5 \text{ to } 20)(6)(2) = 60 \text{ to } 240$ tons of ore per 24 hr.

TABLE 3. THE APPROXIMATE CAPACITY OF SCREENS FOR DENSE MATERIALS SUCH AS ORES

Type of Screen	Capacity Range, tons/sq ft area/mm aperture/24 hr
Grizzlies	1–6
Stationary screens	1–5
Vibrating screens	5–20
Shaking and oscillating screens *	2–8
Trommels	0.3–2

* If the oscillating screens are also vibrated by means of the rubber balls described, the capacities will be somewhat increased over those of the simple oscillating screens.

DETERMINING PARTICLE SIZE

The size of a particle may be expressed in different ways. If the particle is a sphere, the diameter, the projected area, the volume, or the surface of a particle may be the significant size. If the particle is a cube, the edge length, the projected area, the volume, or the surface may be the significant dimension indicating size.

Various methods are used for measurements of particle size. These depend on the size range, the physical properties, and the condition of dryness or wetness permissible. The following methods are used in laboratory and control work.

Microscope. For very small sizes of the order of a few microns (1 micron equals 0.001 mm), the sample may be placed under a microscope; the size may be determined by simple measurement of a photomicrograph of known magnification, or it may be determined directly by means of a filar micrometer. This device consists of a movable cross hair built into a standard microscope eyepiece. The movement of the cross hair is actuated by a calibrated micrometer screw. The cross hair is moved until it appears in contact with one edge of the particle, and a reading is taken on the micrometer; then the cross hair is moved to the opposite edge of the particle, and another reading is made. The difference in readings is a measure of the particle "diameter." This number divided by the optical magnification of the objective and eyepiece will give the true dimension in inches or other units. The microscopic method is frequently employed to measure particles of dust from the atmosphere and to evaluate the effectiveness of air filters.

Screening. Perhaps the simplest method for laboratory sizing consists in passing the material successively over a series of screens or sieves having progressively smaller openings. The size of a material which has passed through one screen and has been retained on a screen having openings of a smaller size is usually considered to be the arithmetic average of the two screen openings and is called the "average dimension" (or "average diameter") represented by the symbol D_{avg}.

Sedimentation. Sedimentation methods are based on the fact that small particles of a given material fall in a fluid at a rate that is proportional to their size. One method involves shaking a sample of the solid in water; after the mixture stands a definite length of time, portions are removed from different levels by means of a pipette. These portions are evaporated to dryness, and the residues weighed. Other modifications have been developed, such as having one balance pan suspended in the pulp of suspended solids and weighing at intervals as the particles settle on the pan.

Elutriation also depends on the velocity of settling. If the material is placed in a rising stream of fluid having a fixed upward velocity, particles whose normal falling velocity is less than the velocity of the fluid will be carried upward and out of the vessel.

* The bibliography for this chapter appears on p. 22.

If fractions obtained from a series of fluid velocities are collected and weighed, a complete size analysis may be obtained.

Centrifuging. Sedimentation is too slow for particles of diameter under ½ micron. Therefore a centrifugal force is substituted for the normal force of gravity when the size of very small particles is to be determined.

Other Methods. The coercive (magnetic) force of a paramagnetic material such as magnetite is directly proportional to its specific surface, regardless of its shape. This relationship has served as a means of determining the surface, or size, of such particles. The amount of light transmitted through a suspension of a definite quantity of the finely divided solid in kerosene in a tube of specified dimensions depends upon the projected area of the particles and is used as a method of determining particle size.[2] The surface size of quartz particles has been measured in research work by the rate of solution in solutions of hydrofluoric acid. It is assumed that the rate of solution in mass per unit of time is directly proportional to the surface area of quartz.

SCREEN ANALYSES

Screens are generally used for control and analytical work. They are constructed of wire mesh cloth, the diameters of the wire and the spacing of the wires being closely specified. These screens form the bottoms of metal pans about 8 in. in diameter and 2 in. high, whose sides are so fashioned that the bottom of one sieve nests snugly on the top of the next.

Screen Aperture and Screen Interval. The clear space between the individual wires of the screen is termed the screen aperture. Frequently the term *mesh* is applied to the number of apertures per linear inch; for example, a 10-mesh screen will have 10 openings per inch, and the aperture will be 0.1 in. minus the diameter of the wire. Mesh is therefore a nominal figure which does not permit accurate computation of the screen openings or aperture without knowledge of the wire sizes used by the manufacturer.

The screen interval is the relationship between the successive sizes of screen openings in a series. A simple arithmetic series might be used such that the screen openings are 10, 9, 8, 7, 6, 5, 4, 3, 2, and 1 in., for example. The weakness of such a system is that there is a large relative difference between the 1-in. and 2-in. sizes, but the 9-in. and 10-in. sizes are almost alike for practical purposes. All the material under 1 in. down to a micron would be in one fraction.

A more satisfactory series of screens is one in which the opening of each successive member varies from the next by a multiplier such as to give a series having openings of 8, 4, 2, 1, ½, and so forth. These sizes

TABLE 4. TYLER SCREENS[5]

Standard Interval $= \sqrt{2}$, Aperture, in.	Interval $= \sqrt[4]{2}$, for Closer Sizing			
	Aperture, in.	Aperture, mm	Mesh Number	Wire Diameter, in.
1.050	1.050	26.67	.....	0.148
	0.883	22.43	.....	0.135
0.742	0.742	18.85	.....	0.135
	0.624	15.85	.....	0.120
0.525	0.525	13.33	.....	0.105
	0.441	11.20		0.105
0.371	0.371	9.423	.....	0.092
	0.312	7.925	2½	0.088
0.263	0.263	6.680	3	0.070
	0.221	5.613	3½	0.065
0.185	0.185	4.699	4	0.065
	0.156	3.962	5	0.044
0.131	0.131	3.327	6	0.036
	0.110	2.794	7	0.0326
0.093	0.093	2.362	8	0.032
	0.078	1.981	9	0.033
0.065	0.065	1.651	10	0.035
	0.055	1.397	12	0.028
0.046	0.046	1.168	14	0.025
	0.0390	0.991	16	0.0235
0.0328	0.0328	0.833	20	0.0172
	0.0276	0.701	24	0.0141
0.0232	0.0232	0.589	28	0.0125
	0.0195	0.495	32	0.0118
0.0164	0.0164	0.417	35	0.0122
	0.0138	0.351	42	0.0100
0.0116	0.0116	0.295	48	0.0092
	0.0097	0.248	60	0.0070
0.0082	0.0082	0.208	65	0.0072
	0.0069	0.175	80	0.0056
0.0058	0.0058	0.147	100	0.0042
	0.0049	0.124	115	0.0038
0.0041	0.0041	0.104	150	0.0026
	0.0035	0.088	170	0.0024
0.0029	0.0029	0.074	200	0.0021
	0.0024	0.061	230	0.0016
0.0021	0.0021	0.053	270	0.0016
	0.0017	0.043	325	0.0014
0.0015	0.0015	0.038	400	0.0010

vary in a geometric progression, and the factor or screen interval is 2. If closer sizing is desired, an additional screen is inserted between each two screens of the previous series and the screen interval becomes $\sqrt{2}$. The standard screens used in the United States employ a screen interval in which the factor is $\sqrt{2}$ although $\sqrt[4]{2}$ is sometimes used for careful work and research.

The first commercial laboratory screens using this system were the *Tyler Standard screens.* This series of screens is based upon a 200-mesh screen with wire 0.0021 in. thick and with an opening of 0.0029 in. (0.0074 cm). The other sizes vary by a fixed ratio of $\sqrt{2}$. A supplementary set can be purchased for intermediate sizes so that the complete set varies by $\sqrt[4]{2}$.

The United States screens introduced by the National Bureau of Standards differ but slightly from the Tyler series, being based on a 1-mm opening (No. 18 mesh) and varying by $\sqrt[4]{2}$.

The British standard screens are similar but have wires of different gage.

Method of Making a Screen Analysis. In making a screen analysis, the individual screens comprising the entire series, varying for example by the ratio $\sqrt{2}$ from 3 mesh to 200 mesh, are cleaned with a brush and tapped free from any adhering particles. They are nested together with the coarsest or 3 mesh at the top and the finest or 200 mesh at the bottom. A bottom pan and top cover are put in place to complete the set. A weighed amount of material is placed upon the top screen, and the cover is replaced. The assembly may be supported and rotated by one hand and bumped against the other hand at intervals to set up a jarring action. After a period of time, the fines, −200 mesh, are removed from the bottom pan. The pan is replaced, and shaking is resumed to see whether any more fines are revealed. When no new material appears in the bottom pan, thus indicating for all practical purposes that the screening operation seems to have been completed, the sieves are disassembled and the individual fractions are weighed. The material, for example, which passed the 100-mesh screen but was retained on the 150-mesh screen is designated the 100/150 or −100 +150 fraction.

Since the shaking of the screens is a tedious process and open to error, mechanical screening is desirable. In one of these machines, the Ro-Tap, the screens are fastened into a vertical framework which is given an elliptical motion in a horizontal plane, a sharp tap or blow being given at the top of the screens for each revolution. Shaking is continued for 15 to 20 min. Other machines employ vibrators or other motions.

Factors which militate against accurate results are overloading of the screens, which may result in blinding which is the wedging of particles in the openings; or electrostatic forces causing small particles to adhere to one another or to large particles. A small amount of moisture may also cause adhesion or cohesion of particles.

Wet-and-dry screening is suitable for very precise screen analyses since it avoids the dangers of adhesion and cohesion. The weighed sample is placed in a beaker and pulped with a nonsolvent, frequently water, and then decanted over the finest screen in the series, for example, 200 mesh. More water is added; stirring and decantation are repeated until no fines are in suspension after stirring. Water from a wash bottle is played on the screen until the drip is clear. The water is decanted from the undersize fraction, and the material is dried. The oversize is also dried and put over the entire series of screens as usual. The new −200 fraction is weighed with the fraction obtained by wet screening. This procedure gives more accurate results since the chance of fine particles clinging to large ones is minimized.

Method of Reporting Screen Analyses. The customary manner of reporting screen analyses is shown in Table 5, in which the mass fractions retained on each of the screens are given.

TABLE 5. TYPICAL SCREEN ANALYSIS

Tyler Screen Mesh	Average Diameter of Particles, D_{avg}		Mass (or Weight) Fraction	Mass Fraction through Each Screen
	cm	in.		
− 8 + 10	0.2007	0.0791	0.03	1.0
− 10 + 14	0.1410	0.0555	0.14	0.97
− 14 + 20	0.1001	0.0394	0.25	0.83
− 20 + 28	0.0711	0.0280	0.20	0.58
− 28 + 35	0.0503	0.0198	0.14	0.38
− 35 + 48	0.0356	0.0140	0.09	0.24
− 48 + 65	0.0252	0.0099	0.06	0.15
− 65 +100	0.0178	0.0070	0.04	0.09
−100 +150	0.0126	0.00496	0.03	0.05
−150 +200	0.0089	0.0035	0.02	0.02
			Total 1.00	

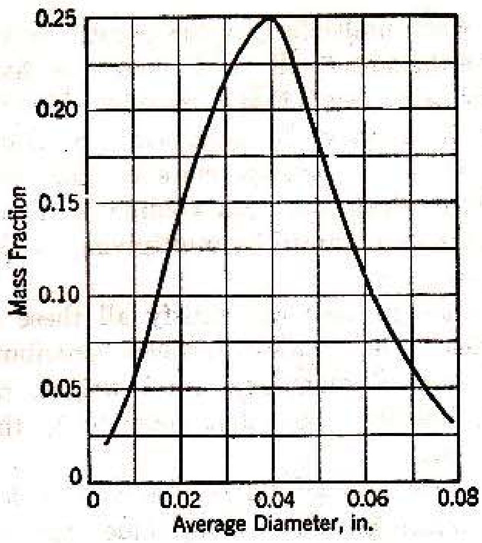

FIG. 11. Fractional plot of screen analysis of Table 5, showing the mass fraction (column 4) retained on screens in the series interval $\sqrt{2}$ as a function of the average particle diameter retained in each fraction (column 3).

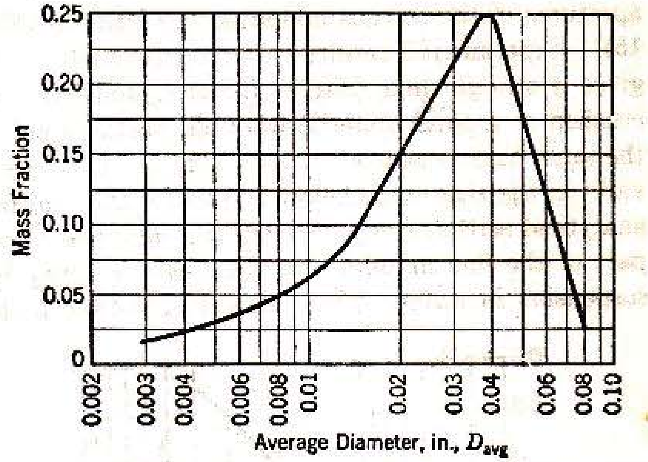

FIG. 13. Fractional plot of screen analysis of Table 5, showing the mass fraction retained on screens in the series interval $\sqrt{2}$ plotted against the logarithm of the average particle diameter retained in each fraction.

These data may be presented graphically by any one of several methods (Figs. 11 to 15). But most of the resulting curves are valuable primarily as pictures of the size distribution of the mixture. Such pictures tell a great deal to an experienced observer but are misleading unless the method of plotting and the materials comprising the mixture are not changed from curve to curve. *Fractional plots* of the mass fraction retained on each screen versus average screen aperture (Fig. 11), or *cumulative plots* of the mass fraction passing each screen versus particular screen aperture (Fig. 12), may be the basis for comparisons of different mixtures of the same materials, indicating changes with time or shipment. The fractional data (Fig. 11) give different curves for different screen intervals and are therefore specific to the particular screen series used as in Table 5. This limitation does not apply to plots of the cumulative data (Figs. 12 and 14) which give the same values regardless of screen intervals. The cumulative plot does not require the computation of average diameter but rather the addition of the fractions passing through the screens.

Ordinary rectangular coordinate plots crowd many points in the small size range into a narrow section of the curve. A better picture is obtained with the logarithm of the average of screen apertures (Figs. 13 and 14), as this spreads the points for the small particles along the dimension scale.

Still further use can be made of a plot of the logarithm of the mass fraction retained on each screen against the logarithm of the arithmetic average of the

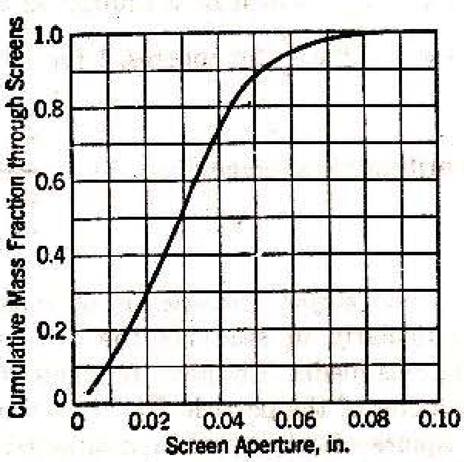

FIG. 12. Cumulative plot of screen analysis of Table 5, showing the mass fraction passing through screens (column 5) as a function of the screen aperture.

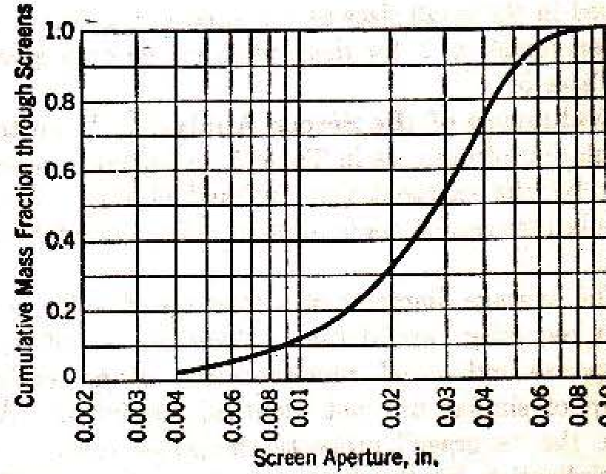

FIG. 14. Cumulative plot of screen analysis of Table 5, showing the mass fraction passing screens as a function of the logarithm of the screen aperture.

apertures of the screens bounding the fraction (Fig. 15). Experimental results indicate that such a plot gives a straight-line relation for the small sizes of crushed or ground material when all particles are of the same basic crystal structure. The straight line is valid in the size range below 200 mesh, the limiting analytical screen, except for natural deposits where part of the fine material has been carried away by suspension in water. Thus, an extrapolation of the

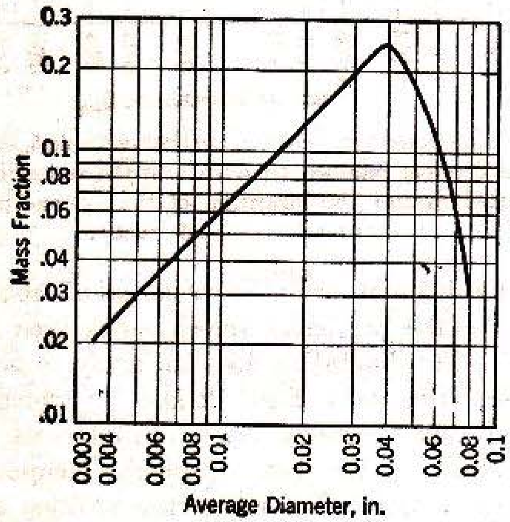

Fig. 15. Log-log fractional plot of screen analysis of Table 5, showing the mass fraction retained on each screen in the series interval $\sqrt{2}$ as a function of the average particle diameter retained in each fraction.

straight line (Fig. 15) will give approximate quantities of the material in each small size range. This extrapolation must be stopped when the total quantity through the 200-mesh screen is obtained by a cumulation of the small fractions. The extrapolation is valid only if the same screen size interval is maintained in the small sizes as was used in the standard screen range, $\sqrt{2}$ for these plots of the data given in Table 5.

Evaluation of the Screen Analysis. From the tabulation of data, as in Table 5, extended if necessary by the extrapolation outlined above, various calculations can be made to provide further information.

The average diameter of a mixture of solids is a term requiring careful use. "Average" signifies a composite individual representative of an entire group of similar but not identical specimens. As such, the "average" property should be capable of multiplication by the number of specimens in the entire group to give a total value for that property. Thus, strictly, *average diameter* is that diameter which, when multiplied by the number of particles, will give the sum of all the diameters in that group. The *average surface* is that surface by which the total surface area may be obtained. Similarly, the *average volume* or *average mass* is that volume or mass from which the total volume or mass of the group may be obtained by multiplying by the number of particles.

No single particle can satisfy all these average properties. The smallest particles contribute little to the sum of diameters, total weight, or total volume, but they contribute heavily to the total surface area.

Frequently average diameter is used to designate the composite particle having some other average property than the average diameter as defined above. Care must be exercised in interpreting average diameter since the term may be so defined as to be useful only for comparison of particles having some other average property. For example, if N_1, N_2, N_3, etc., are the number of particles and x_1, x_2, x_3, etc., are the mass fractions having diameters D_1, D_2, D_3, etc., respectively:

1. The *true arithmetic average diameter* is

$$\frac{N_1D_1 + N_2D_2 + \cdots + N_ND_N}{N_1 + N_2 + \cdots + N_N} = \frac{\Sigma(N_iD_i)}{\Sigma N_i}$$

Since

$$\Sigma N_i = \frac{M}{\rho}\left[\frac{x_1}{C_1D_1^3} + \frac{x_2}{C_2D_2^3} + \cdots + \frac{x_N}{C_ND_N^3}\right] = \frac{M}{\rho}\sum\frac{x}{CD^3}$$

where M = total mass of all particles.

C is a constant depending on the shape of the particle by which D^3 is multiplied to obtain the volume; $\frac{\pi}{6}$ for spheres, 1 for cubes, etc.

$$\text{True arithmetic average diameter} = \frac{\sum\frac{x}{CD^2}}{\sum\frac{x}{CD^3}}$$

2. The *mean surface diameter* is of considerable value, particularly in studying the flow of fluids through porous media (Chapter 16) where it is used as the diameter of the particle D_p. It is that diameter the square of which, when multiplied by the number of particles and also by a suitable constant, R, depending upon the particle shape (π for spheres, 6 for cubes, etc.), gives the total surface of the aggre-

gate number of particles

$$B_1D_1^2N_1 + B_2D_2^2N_2 + \cdots = B(D_{sur})^2\Sigma N_i$$

The mean surface diameter (D_{sur}) is

$$(D_{sur}) = \sqrt{\frac{\Sigma N_iB_iD_i^2}{B\Sigma N_i}} = \sqrt{\frac{\sum \frac{x_iB_i}{C_iD_i}}{B\sum \frac{x_i}{C_iD_i^3}}}$$

3. Similarly the *mean volume diameter* or mean mass diameter equals

$$\sqrt[3]{\frac{\Sigma N_iC_iD_i^3}{C\Sigma N_i}} = \sqrt[3]{\frac{\Sigma x_i}{C\sum \frac{x_i}{C_iD_i^3}}}$$

One of the important properties of solids is the surface area. Since it is impractical to determine the number of particles in a mixture, the usual basis for evaluation of surface is a unit of mass. The *specific surface* or the *surface area per unit of mass* is an important property of solids which varies widely depending upon the condition of the surface as well as the particle size.

The *specific surface* could be computed easily if the particles were of known geometry, but they are of many different shapes and highly irregular. If the sphere is considered, its surface area is πD^2, where D is the diameter of the sphere. Its mass is $\rho\pi D^3/6$, where ρ is its density. The *specific surface of spherical particles* is area divided by mass or $6/\rho D$.

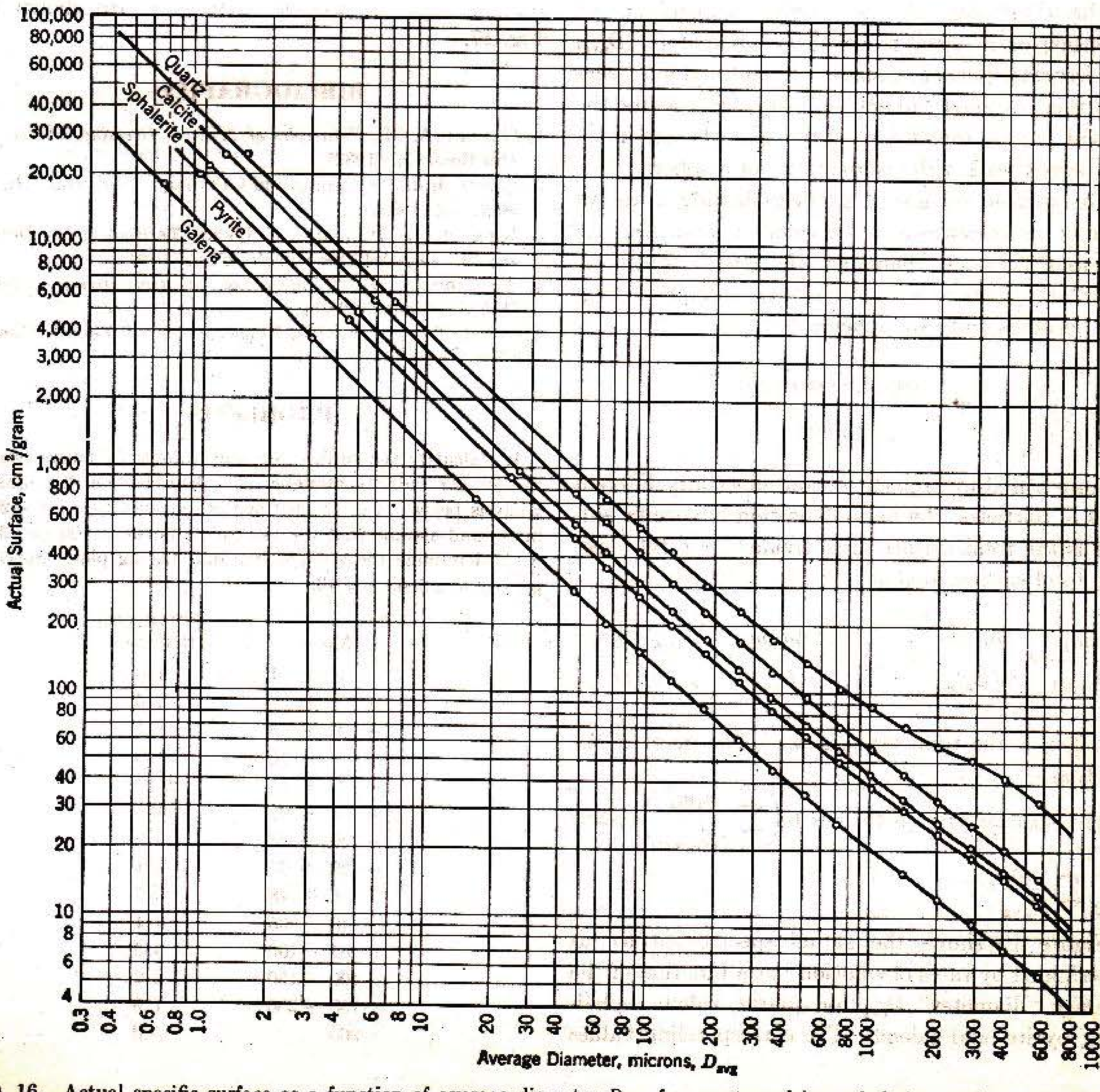

FIG. 16. Actual specific surface as a function of average diameter D_{avg} for quartz, calcite, sphalerite, pyrite, and galena.

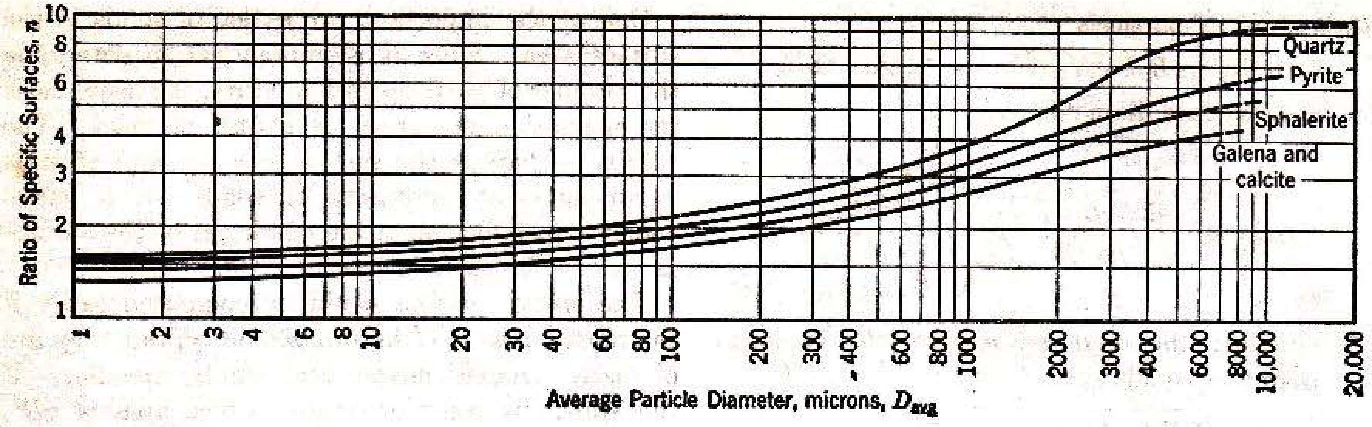

FIG. 17. Ratio n of specific surfaces [2] as a function of average diameter of particles D_{avg} for quartz, pyrite, sphalerite, calcite and galena.

The dimension of the particle controlling its retention on a screen is called its "diameter," D_{avg}. For irregular particles usually encountered in screening, this so-called "diameter," D_{avg}, is usually the second largest dimension of the particle and must not be confused with the diameter of a sphere.

The specific surface of particles having a known ratio of actual surface to the calculated surface of a sphere of the same "diameter" ($D_{avg} = D$ for sphere) is $6n/\rho D_{avg}$, where *n is the ratio of specific surfaces* and becomes unity for spheres.

$$n = \frac{\text{(Specific surface)}}{6/\rho D_{avg}}$$

Since a mixture of particles contains many different sizes of particles, the basic definition of specific surface is the total surface area divided by total mass. The total surface area is

$$\frac{6n_1m_1}{\rho(D_{avg})_1} + \frac{6n_2m_2}{\rho(D_{avg})_2} + \cdots + \frac{6n_Nm_N}{\rho(D_{avg})_N} = \frac{6}{\rho}\sum\frac{n_im_i}{(D_{avg})_i}$$

and Σm_i is the total mass, if m_i is the mass of the fraction i.

The specific surface then is $6\left(\sum\frac{n_im_i}{(D_{avg})_i}\right)/\rho\Sigma m_i$ or $\frac{6}{\rho}\sum\frac{nx}{D_{avg}}$.

Figure 16 shows the actual specific surface as determined by rates of solution [2] as a function of the average "diameter" D_{avg} for quartz, calcite, sphalerite, pyrite, and galena. The corresponding values for the ratio of specific surfaces n are shown in Fig. 17.

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PROBLEMS

1. Calculate the surface per unit volume in square centimeters per cubic centimeter of galena having the screen analysis below. Use the method of extrapolation for −200 mesh, and assume that the two points between 100 and 200 mesh determine the straight line in a log-log plot. Specific gravity of galena = 7.43.

Mesh	Percentage Retained
− 3 + 4	1.0
− 4 + 6	4.0
− 6 + 8	8.1
− 8 + 10	11.5
− 10 + 14	16.0
− 14 + 20	14.8
− 20 + 28	13.2
− 28 + 35	8.1
− 35 + 48	6.2
− 48 + 65	4.1
− 65 +100	3.6
−100 +150	2.2
−150 +200	1.9
−200	5.3

2. Calculate the specific surface in square centimeters per gram of pyrite having the screen analysis below. Specific gravity of pyrite = 5.0.

Mesh	Percentage Retained
− 3 + 4	0
− 4 + 6	4.0
− 6 + 8	7.2
− 8 + 10	12.0
− 10 + 14	17.6
− 14 + 20	15.4
− 20 + 28	12.0
− 28 + 35	10.0
− 35 + 48	7.2
− 48 + 65	6.0
− 65 +100	3.8
−100 +150	2.8
−150 +200	2.0

3. In the petroleum industry, gas oil is cracked in contact with solid catalysts to yield high-octane blending stocks, with a finely divided clay as the catalyst. The yields obtained are a function of the surface area of the catalyst. The catalyst has a density of 1.20 grams/cc and approximately the same specific surface ratio as quartz. A sample of this material was screened, and the material through the 200-mesh screen was further sized by air elutriation. From the resulting analysis given below, determine the specific surface (square centimeters per gram), the arithmetic average diameter and the mean surface diameter of the catalyst.

The specific surface so computed does not include the surface in the capillaries which may increase the total surface about 3000-fold.

Screened Fraction

Mesh	Mass Fraction
− 48 + 65	0.088
− 65 +100	0.178
−100 +150	0.293
−150 +200	0.194
−200	0.247

Elutriated Fraction

Size Limits, microns	Mass Fraction (of Original Sample)
80–60	0.113
60–40	0.078
40–20	0.042
20–0	0.014

4. Powdered coal with the screen analysis given below as "Feed" is fed to a vibrating 48-mesh screen in an attempt to remove the undesired fine material. When the screen was new the oversize and undersize analyses were as listed under columns headed "New." After 3 months' operation, the analyses are as headed "Old." What is the effectiveness of the screen (*a*) when new and (*b*) when old?

Screen Analyses—Mass Fractions

Mesh	Feed	Oversize New	Oversize Old	Undersize New	Undersize Old
− 3 + 4	0.010	0.012	0.014	.....	.....
− 4 + 6	0.022	0.027	0.031	.....	.....
− 6 + 8	0.063	0.078	0.088	.....	.....
− 8 + 10	0.081	0.100	0.112	.....	.....
− 10 + 14	0.102	0.126	0.142	.....	.....
− 14 + 20	0.165	0.204	0.229	.....	.....
− 20 + 28	0.131	0.162	0.182	.....	.....
− 28 + 35	0.101	0.125	0.104	.....	0.093
− 35 + 48	0.095	0.117	0.065	.....	0.171
− 48 + 65	0.070	0.029	0.025	0.246	0.186
− 65 +100	0.047	0.015	0.008	0.183	0.146
−100 +150	0.031	0.005	.....	0.141	0.111
−150 +200	0.020	.....	.....	0.105	0.071
−200	0.062	.....	.....	0.325	0.222

5. Table salt is being fed to a vibrating screen at the rate of 300 lb/hr. The desired product is the 48/65 mesh fraction. A 48- and a 65-mesh screen are therefore used (double deck), the feed being introduced on the 48-mesh screen, the product being discharged from the 65-mesh screen. During the operation it was observed that the average proportion of oversize:product:undersize was 2:1½:1.

(*a*) Calculate effectiveness of the screener.

(*b*) If screen dimensions were 2 ft by 4 ft, calculate the capacity of the 65-mesh screen on the basis of a perfectly functioning 48-mesh screen and also on the basis of the actual performance of the screen.

Screen Mesh	Feed, mass fraction	Oversize, mass fraction	Product, mass fraction	Undersize, mass fraction
− 10 + 14	0.000356	0.0008	......	....
− 14 + 20	0.00373	0.008	0.0005	0.00003
− 20 + 28	0.089	0.189	0.016	0.00012
− 28 + 35	0.186	0.389	0.039	0.0009
− 35 + 48	0.258	0.337	0.322	0.0036
− 48 + 65	0.281	0.066	0.526	0.344
− 65 +100	0.091	0.005	0.067	0.299
−100 +150	0.062	0.005	0.024	0.237
−150 +200	0.025	0.001	0.002	0.11

6. One ton per hour of dolomite is produced by crushing and then screening through a 14-mesh screen. According to the screen analysis below, calculate (*a*) the total load to crusher and (*b*) the effectiveness of the screen.

Tyler Mesh	Feed to Screen, %	Screen Undersize, Product, %	Screen Oversize, Circulating Load, %
4 on	14.3	....	20
8 on	20.0	....	28
14 on	20.0	0.0	28
28 on	28.5	40.0	24
48 on	8.6	30.0	0 through 28 mesh
100 on	5.7	20.0	
100 through	2.86	10.0	

7. The data below were obtained on the operation of a 6-mesh (square) hummer screen at the tipple of a coal mine. The screening was done to separate a very fine refuse from a fine coal stream so that it could be reprocessed. Calculate (*a*) the recovery and rejection of each size fraction and (*b*) the screen effectiveness.

Feed to Screen, 131 Tons/Hr (Approximately 5% Moisture)

Size	Sample Weight
+¼ in.	3825 grams
¼ × 6 mesh	1006
6 × 14	750
14 × 28	303
28 × 48	219
48 × 0	807

Overflow from Screen

Size	Sample Weight
+¼ in.	2905 grams
¼ × 6 mesh	767
6 × 14	405
14 × 28	117
28 × 48	68
48 × 0	278

Underflow from Screen, 9.8 Tons/Hr (Dry Solids)

Size	Sample, %
¼ × 6 mesh	11.3
6 × 8	7.8
8 × 14	6.9
14 × 28	8.6
28 × 48	3.[illegible]
48 × 0	62.1

CHAPTER 4

Size Reduction of Solids

In industries that process raw material in the solid state or use solid material in the processing of fluids, reduction in the size of the solid particles is frequently required. In the production of gypsum plaster, the raw gypsum rock is removed from the quarry in large blocks, sometimes 5 ft in diameter. It must be reduced to particles fine enough to pass through a 100-mesh screen in order to provide sufficient specific surface for hydration to take place rapidly. This means a reduction in size from 60 in. to 0.005 in. Pigments in paints must be very fine in order to give good coverage when applied to a surface.

Reduction in size involves the production of smaller mass units from larger mass units of the same material; it therefore follows that the operation must cause fracture to take place in the larger units. This fracturing or shattering of the larger mass units is accomplished by the application of pressure. All *true solid materials* are crystalline in nature; that is, the atoms in the individual crystals are arranged in definite repeating geometric patterns, and there are certain planes in the crystal along which shear takes place more readily. The pressure applied must be sufficient to cause failure by shear along these cleavage planes. If the shear along these planes results in deformation but not rupture, the deformation is called plastic deformation. The segments of the crystal slide along on each other like a pack of cards, the only result being a change in dimensions of the crystal. In order to bring about actual size reduction, it is necessary that the material be actually fractured and that shear movement, once started, results in complete separation of the segments between which the shear failure occurred.

From this, it might appear that the best method of causing rupture to take place in solid material would be the application of shearing loads. However, the orientation of crystals in solid matter is usually so irregular that the direct application of compressive loads is just as effective as shearing loads. All equipment for size reduction of solids uses compression, or shear, or both, as disrupting forces.

OBJECTIVES

The purpose of size reduction is not only to make "little ones out of big ones" when the effectiveness can be measured by the degree of fineness of the product, but also to produce a product of the desired size or size range. The size requirements for various products may vary widely, and hence different machines and procedures are employed. A size range entirely satisfactory for one purpose may be highly undesirable for another, even when the same substance is involved. Powdered coal is widely used for firing industrial furnaces, and lump coal is also fed into furnaces by mechanical stokers. But powdered coal could not be used in the stoker, and lump coal could not be used in the equipment designed for firing pulverized or powdered coal.

In many cases, it is necessary to use a product with rather narrow limits in size variation. It is usually impossible to accomplish this by size reduction only. Screening and classification by various means are required to secure the desired limitation in size range. The two unit operations of size reduction and size separation are further closely associated in that laboratory screen analyses are necessary to evaluate the effectiveness of a given size reduction operation as well as to furnish data for estimating the power or energy required.

Ores of metals consist of varying amounts of valuable minerals associated with undesired gangue minerals. The first step in processing ores for the recovery of metal values is the separation of the values from the gangue, since the ore as taken from the mine contains both types of minerals together in solid masses. Unless the valuable mineral exists in great enough concentration to permit the ore to be reduced to the metal without previous treatment, in which case the gangue is usually separated in the molten state, it is necessary to break up the ore mass mechanically, thus freeing the valuable minerals from the gangue. The minerals are then separated by gravity or flotation methods resulting in concentration of the valuable minerals.

The purposes of size reductions are therefore twofold: (1) To produce solids with desired size ranges or specific surfaces. (2) To break apart minerals or crystals of chemical compounds which are intimately associated in the solid state.

STAGES OF REDUCTION

For successful size reduction, it is necessary that every lump or particle must be broken by contact with other particles or by direct contact with the moving parts of the machine. As the breaking action proceeds, the number of particles increases, requiring more contacts per unit mass. Thus the capacity of a particular machine of fixed dimensions, as in tons per day, is much less for small sizes than for the larger sizes, since it is necessary for the smaller particles to remain in the machine for longer periods of time to sustain the required number of contacts. No device has been developed capable of automatically adjusting itself to the varying requirements of contact. In commercial operations, sufficient capacity in the intermediate and fine ranges of size reduction is obtained either by operating several similar units in parallel or, better, by employing machines which furnish greater numbers of contacts per unit of time.

Machines providing the required large number of contacts, particularly for smaller-size material, have been developed, primarily for the last stages of size reduction.

For commercial reduction in size of masses of solids 1 ft or more in diameter to 200-mesh size, usually at least three stages or steps are followed which are divided according to the types of machines best adapted to each stage. The three steps are:

1. Coarse size reduction: feeds from 2 to 96 in. or more.
2. Intermediate size reduction: feeds from 1 to 3 in.
3. Fine size reduction: feeds from 0.25 to 0.5 in.

OPERATING VARIABLES

The *moisture content* of solids to be reduced in size is important. If it is below 3 or 4 per cent by weight, no particular difficulties are encountered; indeed, it appears that the presence of this amount of moisture is of real benefit in size reduction if for no other reason than for dust control. When moisture content exceeds about 4 per cent, most materials become sticky or pasty with a tendency to clog the machine. This is particularly true in the coarse and intermediate stages.

A large excess of water (50 per cent or more) facilitates the operation by washing the feed into and the product out of the zone of action and by furnishing a means for transporting the solids about the plant as a suspension or slurry. *Wet grinding* is mostly confined to the fine stage of reduction.

The *reduction ratio* is the ratio of the average diameter of the feed to the average diameter of the product. Most machines in the coarser ranges of crushing have a reduction ratio from about 3 to 7. Fine grinders may have a reduction ratio as high as 100.

In *free crushing*, the crushed product with whatever fines have been formed is quickly removed after a relatively short sojourn in the crushing zone. The product may flow out by gravity, be blown out with compressed air, be washed out with water, or be thrown out by centrifugal force. This method of operation prevents the formation of an excessive amount of fines by limiting the number of contacts.

In *choke feeding* (the antithesis of free crushing), the crusher is equipped with a feed hopper and kept filled (or choked) so that it does not freely discharge the crushed product. This increases greatly the proportion of fines produced and decreases the capacity. In some instances choke feeding may result in economy of operation, eliminating one or more reducing stages because of the large quantity of fines produced.

Each stage in size reduction may, and frequently does, have a size-separating unit following it. If the oversize material is returned to the crusher, the operation is termed *closed circuit*. If no material is returned for recrushing, the operation is called *open circuit*. Closed-circuit operation is economical of crushing power, which at best is high, permits smaller units per given tonnage, and produces a material with greater uniformity of size.

Although the size of the feed is an important factor in the selection of a machine, other factors must be considered, such as hardness or structure of the material. From the standpoint of crushing, minerals with a Mohs hardness of 4 or less are classed as soft; others are considered hard. Machines for the coarse preliminary crushing of soft materials do not need to be so sturdily constructed or so elaborate in design as machines for breaking hard materials. In the finer size ranges, similar machines are used for both hard and soft materials.

Machines exerting a tearing action and called *disintegrators* are employed for reducing the size of fibrous materials such as wood and asbestos.

COARSE SIZE REDUCTION

Machines for the coarser stages of size reduction handle feed sizes from 3 to 4 in. and up. For hard materials, either jaw, gyratory, or disk crushers are used. For soft materials where the production of fines is to be limited, as in crushing coal for sale, such devices as hammer mills or toothed rolls are employed.

Coarse Crushers for Hard Materials

Jaw Crushers. Jaw crushers are represented by the Blake and Dodge types and operate by applying a crushing pressure.

The *Blake crusher* (Fig. 18) consists essentially of a cast-steel frame supporting one fixed and one movable jaw. The jaws are made of cast steel lined with a tough abrasion resistant metal, such as manganese steel. The movable jaw is pivoted at the top and operated by the eccentric, pitman, and toggles. The pitman is given a nearly vertical motion by the eccentric, and, since one of the toggles is mounted in rigid journals at one end of the crusher frame, the reciprocating motion of the pitman causes the other toggle to move the jaw back and forth. The jaw is held against the toggle by a tension link and spring. Crushing is accomplished only when the movable jaw moves toward the fixed jaw. This means an intermittent power requirement. In order to equalize this, one or two heavy flywheels are mounted on the main shaft of the crusher. The machine is driven by flat belts or V-belts.

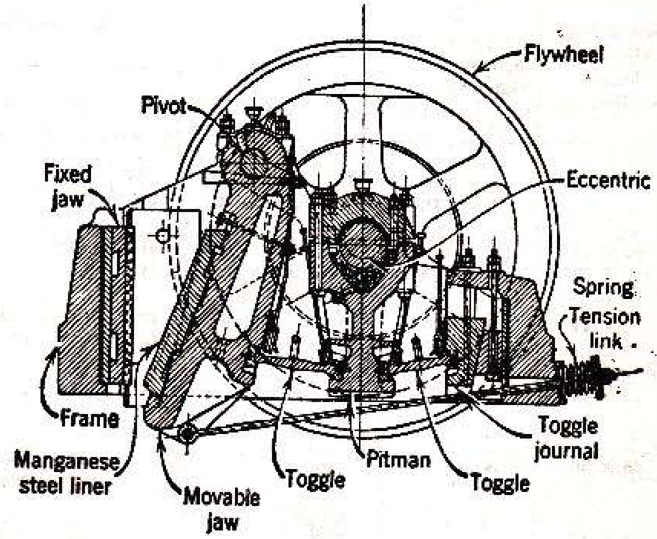


FIG. 18. Sectional drawing of Blake-type jaw crusher. (*Allis-Chalmers Mfg. Co.*)

The *Dodge crusher* (Fig. 19) is subject to uneven stresses inherent in its design and therefore is made only in small sizes. It differs from the Blake crusher in that the movable jaw is pivoted at the bottom and the width of the discharge opening remains practically constant, thereby yielding a more closely sized product. No toggles are required, the jaw being operated through the pitman by the eccentric. If only one size-reducing machine is being employed, the uniformity in size of product may be of advantage, but otherwise the machine is of limited use.

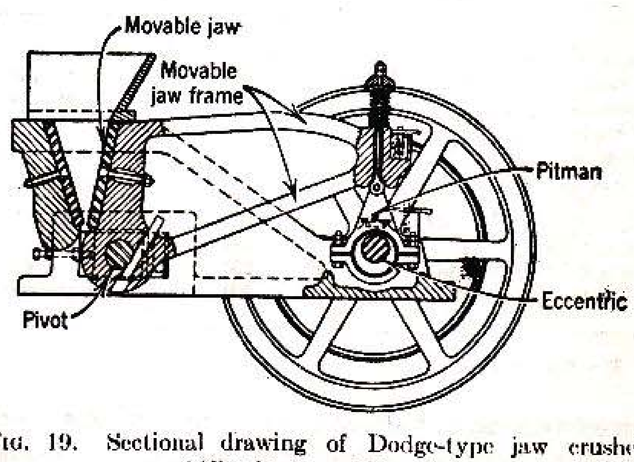


FIG. 19. Sectional drawing of Dodge-type jaw crusher. (*Allis-Chalmers Mfg. Co.*)

The power is applied through a long lever, and if the crusher becomes clogged enormous stresses are set up in the members which become excessive in ma-

TABLE 6. CAPACITIES OF BLAKE JAW CRUSHERS

(Allis-Chalmers Mfg. Co.)

Size of Feed Opening, Length × Gape, in.	Type of Jaw Plates *	Discharge Setting, in. 1½	2	2½	3	4	5	6	7	8	9	10	11	12	Rpm	Recommended Motor Horsepower	Crusher Weight, lb
15 × 10	A	7T †	11T	16T	20T	28T									235	15	10,000
	B	16	23	28	35	47											
24 × 15	A		22	28	35	48	60T								210	35	27,000
	B	25	34	43	52	69	86										
36 × 24	A				45	67	88	110T							210	75	70,000
	B				80	102	127	170									
42 × 40	A					90	103	130	155T	190T					190	125	140,000
	B					140	164	197	230	263							
48 × 36	A						120	155	187	225					190	150	145,000
	B						187	224	262	300							
48 × 42	A						120	155	187	225					190	150	160,000
	B						187	224	262	300							
60 × 48	A						150	210	240	265	300T				170	200	215,000
	B						262	314	368	420	472						
84 × 56	A									360	425	480T	565T	630T	90	200	422,000
	B									...	...	...	...	...			
84 × 60	A									360	425	480	565	630	90	250	430,000
	B									...	...	...	...	...			
84 × 66	A									420	480	510	570	630	90	250	460,000
	B									...	...	...	...	...			

* A = standard jaw plates (smooth).
B = "Nonchoking" jaw plates (corrugated).

† T = tons per hour.

TABLE 7. CAPACITIES OF DODGE CRUSHERS

(Allis-Chalmers Mfg. Co.)

Size of Feed Openings, Length × Gape, in.	Discharge Setting, in. ½	¾	1	1½	Rpm	Recommended Motor Horsepower	Crusher Weight, lb
6 × 4	¼T *	½T	1T		275	3	1,100
9 × 7		1	2	3T	235	6	3,250
12 × 8		1½	3	4	220	10	5,400
15 × 11		2	4	6	200	15	13,500

* T = tons per hour.

chines with gapes * above 11 in. The constant opening of the jaws at the discharge end gives the Dodge crusher an annoying tendency to clog which is absent in the Blake crusher.

* Gape is the greatest distance between the jaws or crushing surfaces.

There are many different designs of jaw crushers, some of which combine shear with compression. The Universal jaw crusher (Fig. 20) combines the principles of the Dodge and Blake crushers. It gives two crushing strokes per revolution because the pivot is above the bottom end of the jaw, causing the bottom of the jaw to move forward while the upper end of the jaw recedes.

Gyratory Crushers. Gyratory crushers were developed later to supply a machine with greater capacity. Actually, the crushing action of gyratories is similar to the action of jaw crushers in that the moving crushing element approaches to and recedes from a fixed crushing plate.

Figure 21 shows a *suspended-spindle type* of gyratory, consisting of an outer frame carrying an inverted conical surface known as "concaves" and an inner gyrating crushing head. The conical crushing head is supported on a spindle which hangs from a suitable bearing in the upper portion of the machine.

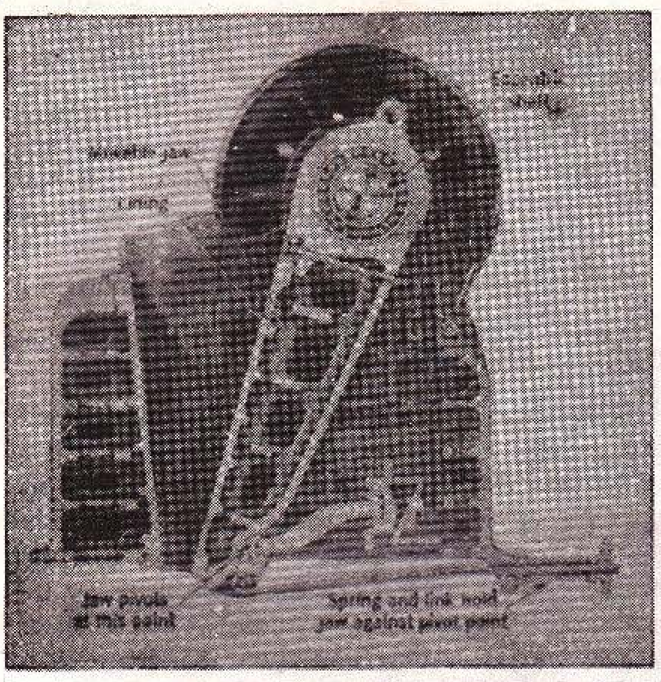


FIG. 20. Sectional drawing of Universal streamlined roller-bearing jaw crusher. (*Universal Engineering Corp.*)

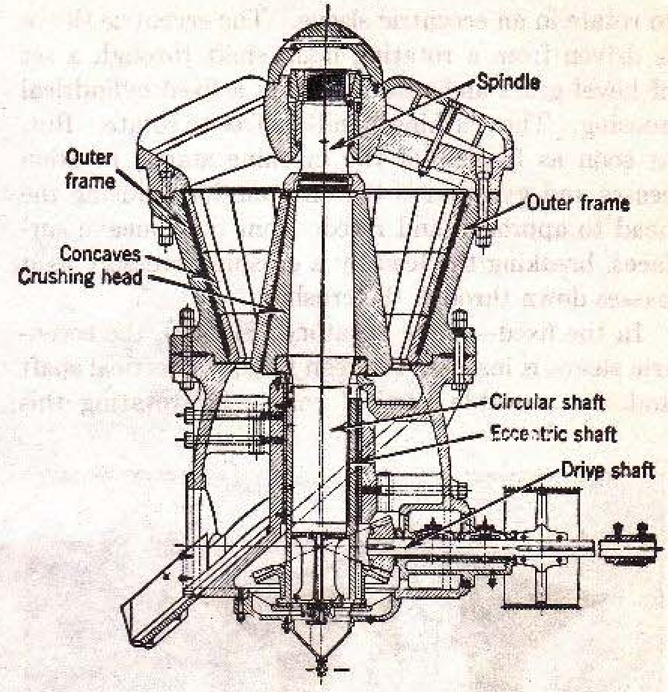


FIG. 21. Sectional drawing of gyratory crusher of suspended-spindle type. (*Allis-Chalmers Mfg. Co.*)

TABLE 8. CAPACITIES OF GYRATORY CRUSHERS

(Allis-Chalmers Mfg. Co.)

Size of Feed Opening, Gape × Length, in.	Finest Setting *		Coarsest Setting †		Driving Pulley, rpm	Recommended Motor, Horsepower	Crusher Weight, lb
	Size of Discharge Opening, in.	Capacity, tons/hr	Size of Discharge Opening, in.	Capacity, tons/hr			
2¼ × 10	⅜	½			700	3	700
8 × 34	1½	25	2½	47	450	15–25	20,000
10 × 40	1¾	39	3½	93	400	25–40	30,000
13 × 45	2	63	3½	128	375	50–75	45,000
16 × 56	3	120	4	176	350	60–100	62,000
20 × 68	3½	152	5	245	330	75–125	94,000
30 × 90	4	235	6½	450	325	125–175	169,000
36 × 126	5	365	6⅝	525	300	175–225	263,000
42 × 132	5½	475	6⅝	615	300	200–275	286,000
50 × 162	6	740	7¼	845	250	225–300	575,000
54 × 162	6¼	875	8	1050	250	225–300	630,000
60 × 174	6¼	990	10	1440	250	225–300	725,000
60 × 182	6½	1420	10½	1900	250	300–500	1,000,000

* Finest permitted for this size gyratory.
† Coarsest permitted for this size gyratory.

The lower end of the spindle is a circular shaft free to rotate in an eccentric sleeve. The eccentric sleeve is driven from a rotating main shaft through a set of bevel gears and rotates within a fixed cylindrical housing. The crushing spindle is free to rotate. But, as soon as feeding of the machine starts, rotation ceases and gyration is the only motion, causing the head to approach and recede from the concave surfaces, breaking the feed by a crushing pressure as it passes down through the crusher.

In the fixed-spindle gyratory (Fig. 22), the eccentric sleeve is inserted between the fixed vertical shaft and the movable vertical cone. By rotating this

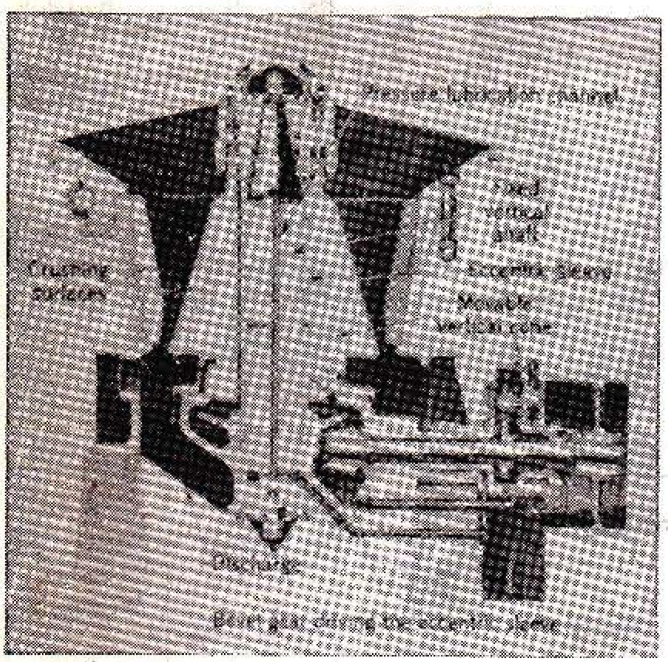


FIG. 22. Sectional drawing of Telsmith parallel pinch crusher. (*Smith Engineering Works.*)

eccentric sleeve the axis of the cone is given a cylindrical motion with a "parallel pinching" action on the material being crushed.

Gyratory crushers have large capacity because the action is continuous. The capacity is similar to that of a jaw crusher having the same gape and a length L equal to the perimeter of the gyratory. Since all the coarse crushers have greater capacities than the devices for the finer ranges of size reduction, a gyratory of sufficient size to handle the required size of feed may have an excessive capacity. Jaw crushers, therefore, are frequently used for the first coarse breaking operation, followed by gyratories.

Capacities of jaw and gyratory crushers with gapes of 4 in. to 2 ft may be approximated by the Taggart [5] * formula:

$$T = 0.6LS$$

where T = capacity (tons/hr).

L = length of feed opening (in jaw crushers, normal to gape; in gyratories, the perimeter of a circle whose diameter is the arithmetic average of the diameters of the two cones) (in.).

S = greatest width of discharge opening (in.).

Exercise. Compare the capacities as estimated by the Taggart formula with those given in Table 6.

The *power requirements* for jaw and gyratory crushers are about the same, but the gyratory load is somewhat more uniform since it is crushing continuously whereas the jaw crusher works intermittently.

In choosing between a jaw crusher or a gyratory crusher for a given installation, capacity is the criterion. If capacity requirements are small enough so that one jaw crusher is adequate, the jaw crusher is the usual choice because of its lower original cost and upkeep. If capacity requirements are large enough to keep a gyratory in continuous operation, the gyratory is usually preferred. Taggart [5] states an empirical rule that "if the hourly tonnage to be crushed divided by the square of the gape in inches is less than 0.115, use a jaw crusher; otherwise, a gyratory."

Coarse Crushers for Soft Materials

Such materials as coal, gypsum, some types of limestone, ice, fire clay and shales are less hard than 4 on the Mohs scale and do not require the heavy and expensive types of crushers needed for hard materials. Frequently, the size reduction desired for these soft materials excludes the very fine ranges, and most of the crushers designed for such materials produce a small amount of excessively fine material.

The *Bradford breaker* for coal (Fig. 23) combines the two features of breaking and screening. The periphery of the machine is a reinforced screen which allows the coal, when sufficiently reduced in size, to pass through. Breaking is accomplished by rotation of the cylinder. The coal is lifted on interior shelves and broken by falling and striking the coal below as the cylinder is rotated. Harder material such as slate and tramp iron are not broken and gradually pass out from the open end of the breaker as indicated.

* The bibliography for this chapter appears on p. 45.

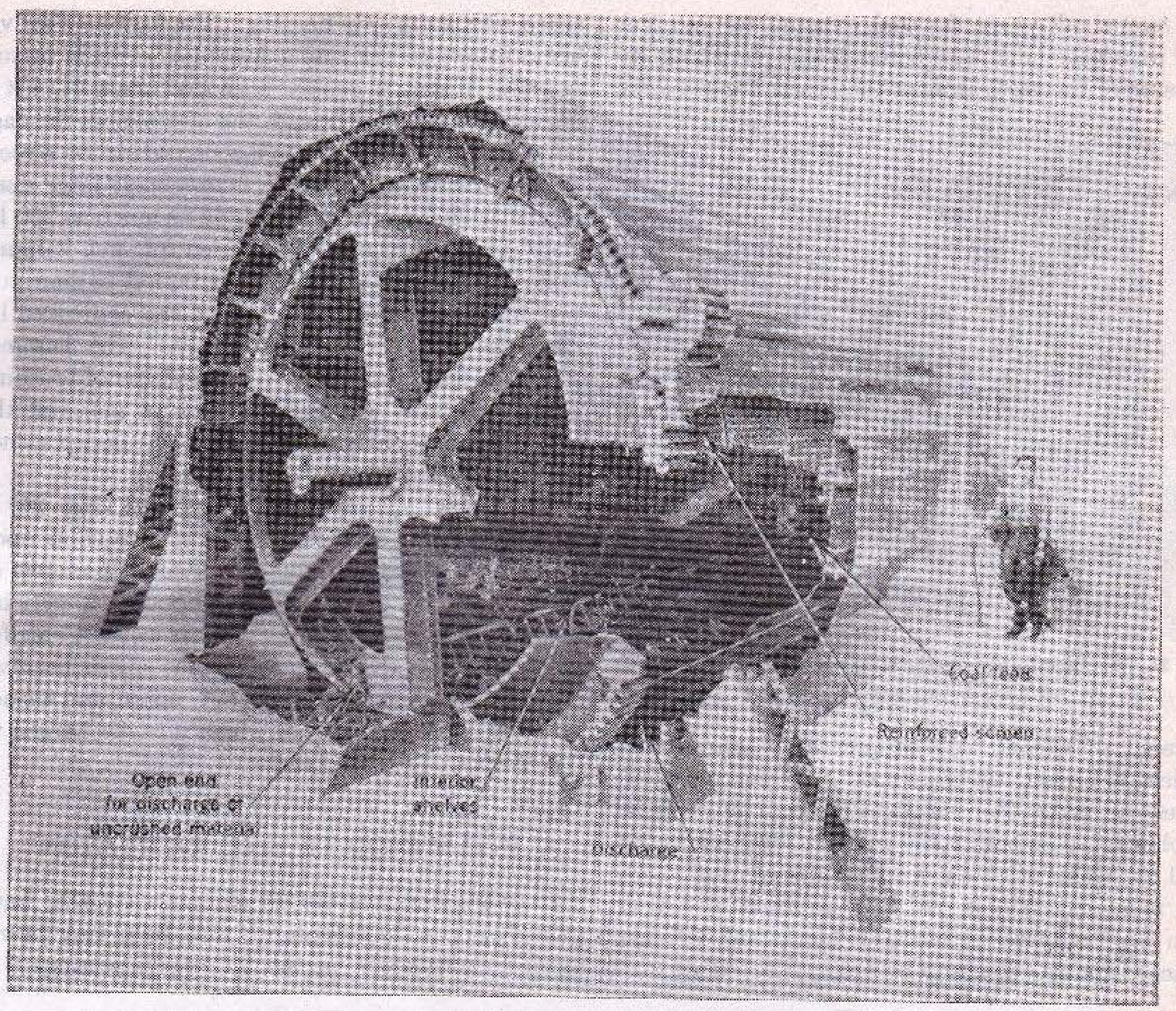


Fig. 23. Phantom drawing of Bradford breaker. Run-of-mine coal enters through the chute at the far end, is lifted, falls, and is broken by the impact, passing through perforations into the chute below; rock and refuse are plowed out as indicated in the foreground. (*Pennsylvania Crusher Co.*)

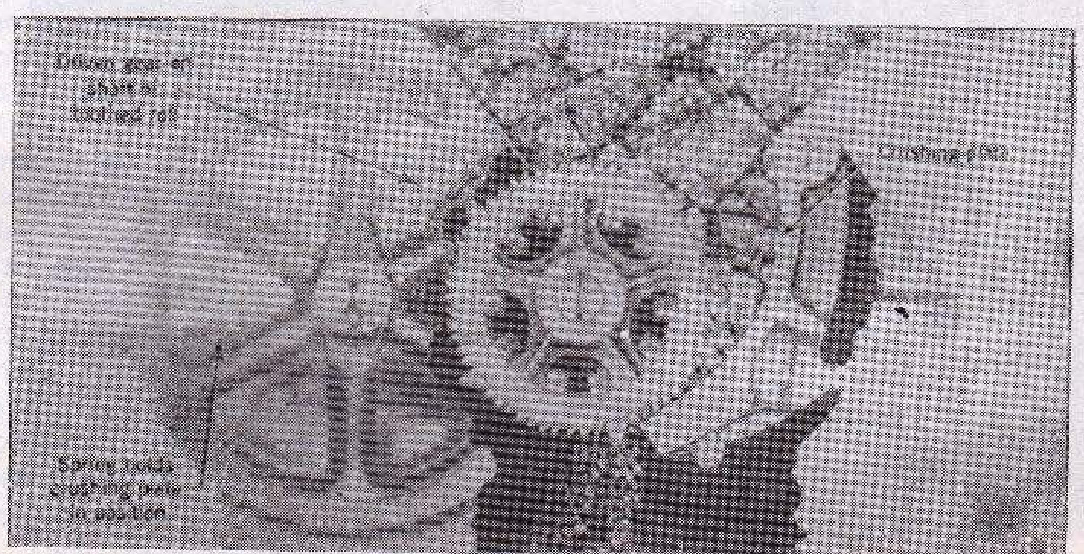


Fig. 24. Sectional drawing showing operation of toothed roll crusher. (*Link-Belt Co.*)

A *toothed roll crusher* for coal, gypsum, ice, or other soft materials (Fig. 24) accomplishes breaking by pressure of the teeth against the larger lumps of the material, disintegrating it in much the same manner as ice is broken up manually with an ice pick. Excessive production of fines is thus prevented. Knobbed and smooth rolls (Fig. 30) are also widely used for coarse crushing of soft materials.

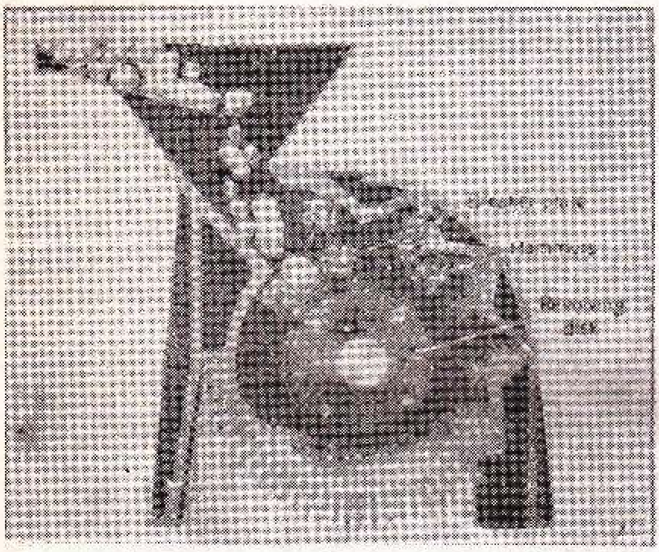


FIG. 25. Sectional drawing showing operation of a hammer mill. (*Allis-Chalmers Mfg. Co.*)

A *hammer mill* (Fig. 25) may be used for coal or even fibrous material. Heavy blocks of steel are attached by pins to a disk or disks revolving at high speed within a sturdy housing. The hammers deliver heavy blows to the feed material while it is in suspension, driving it against a breaker plate until it is fine enough to pass through the openings in the cage bars at the bottom of the mill constituting the screen. Some of these mills are built in extremely large sizes, the individual hammers weighing as much as 250 lb. Very sturdy housings are required for such hammer mills. The same type is also adapted to fine pulverizing, the size of the product being controlled by the sizes of the discharge screens. The hammer mill is probably the most versatile type of crushing device currently available. For wet material the cages or screens are replaced with corrugated grinding plates.

A so-called *squirrel-cage disintegrator* (Fig. 26) is useful in tearing apart fibrous material such as wood blocks and asbestos. The device consists of two or more concentric cages rotated in opposite directions. The feed is introduced into the inner cage. Centrifugal force drives the material into the spaces between the rotating cages where it is torn apart, and thence into the outer casing from which it is discharged to a conveyor or storage bin.

INTERMEDIATE SIZE REDUCTION

Cone crushers, developed since the 1920's, have gained such wide acceptance that they may be regarded as standard in the intermediate range. A

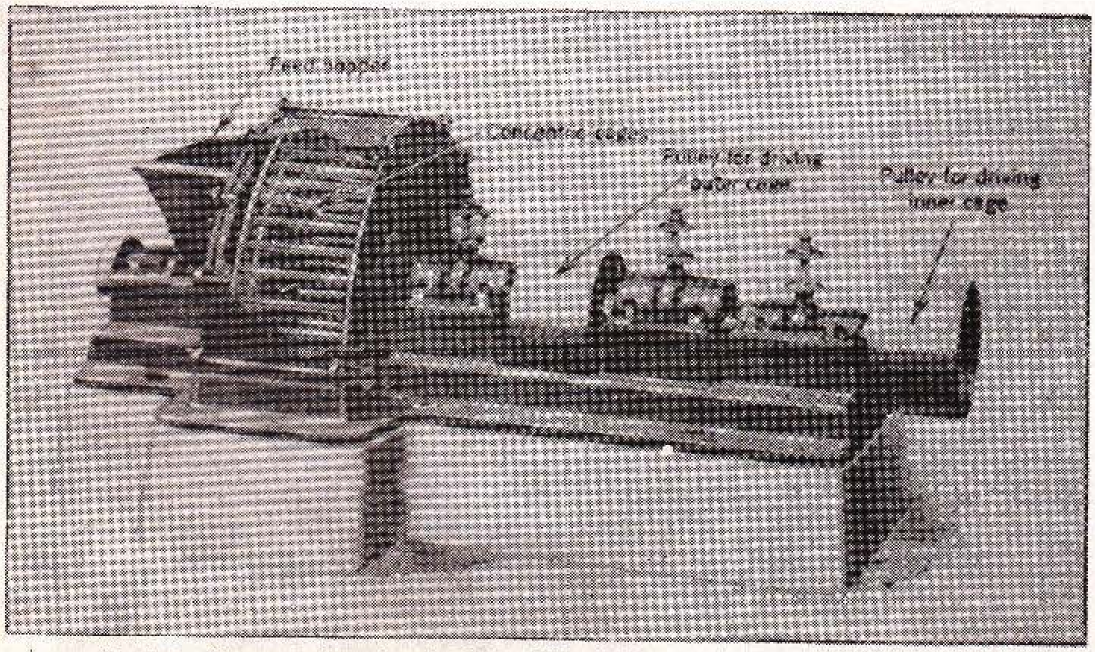


FIG. 26. Cutaway view of squirrel-cage disintegrator. (*C. O. Bartlett and Snow Co.*)

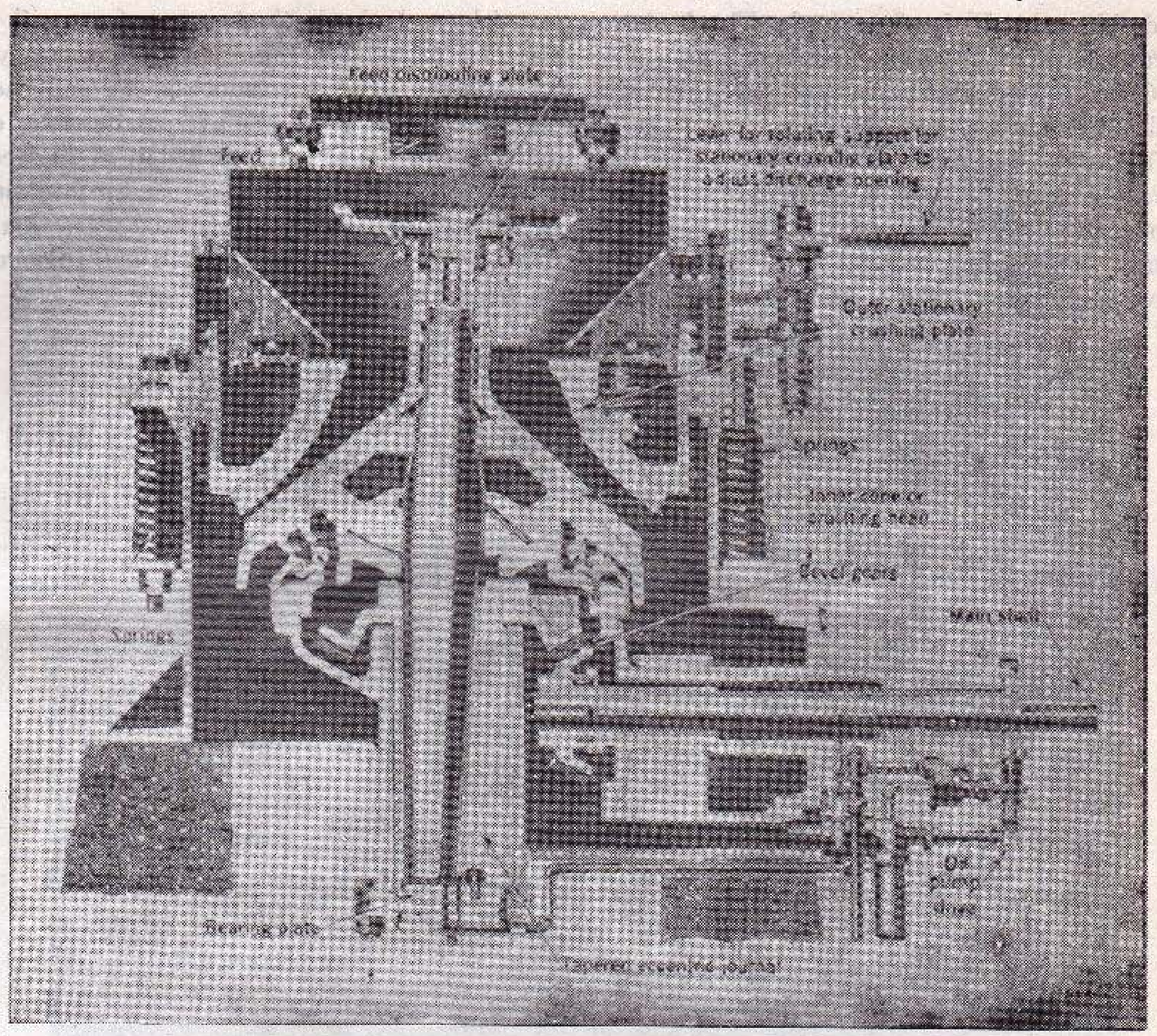


FIG. 27. Sectional drawing of cone crusher. (*Nordberg Mfg. Co.*)

standard cone crusher is shown in Figs. 27 and 28. The drive is similar to that of the gyratory crusher. The inner cone or "crushing head" is supported by the tapered eccentric journal which is rotated by the bevel gears driven by the main shaft. The entire weight of the crushing head and spindle is supported on a bearing plate supplied with oil under pressure. The operation is quite similar to that of the gyratory crusher, but there are two important points of difference. The outer stationary crushing plate flares outward to provide an increasing area of discharge so that the machine can quickly clear itself of the reduced product. This stationary crushing plate is held in position by a nest of heavy helical tension springs so that when tramp iron or other uncrushable objects enter the crushing zone the plate is lifted, preventing fracture of the plate and injury to the machine. These cone crushers are available in two sizes, the standard (Fig. 27) for coarser feed, and a

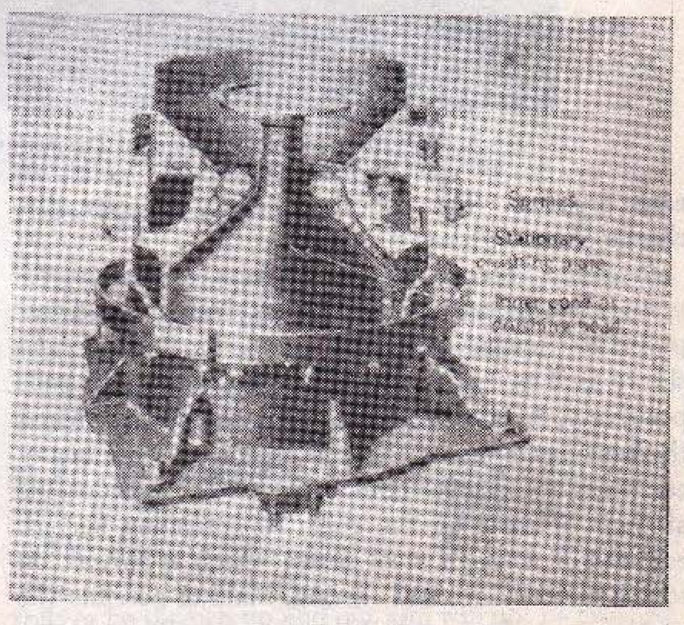


FIG. 28. Cutaway view showing action of cone crusher. (*Nordberg Mfg. Co.*)

so-called "short head" for finer feed. The feed to cone crushers must be dry and rather uniformly sized. Cone crushers give best results when operating in closed circuit with screens.

The Telsmith Gyrasphere, Fig. 29, is a variation of the cone crusher. The crushing head is spherical in contour, and the crushing plate is held in position by springs under compression instead of tension. The drive and oiling system is similar to that of the vented by a device in the bearing of one roll which gives it a limited lateral motion simultaneously with the rotation. The size reduction accomplished by rolls is relatively small, the average diameter of the product being about one-fourth that of the feed.

Cone crushers are replacing rolls for intermediate size reduction of ores because their reduction ratio is two or three times that of rolls and they require less maintenance.

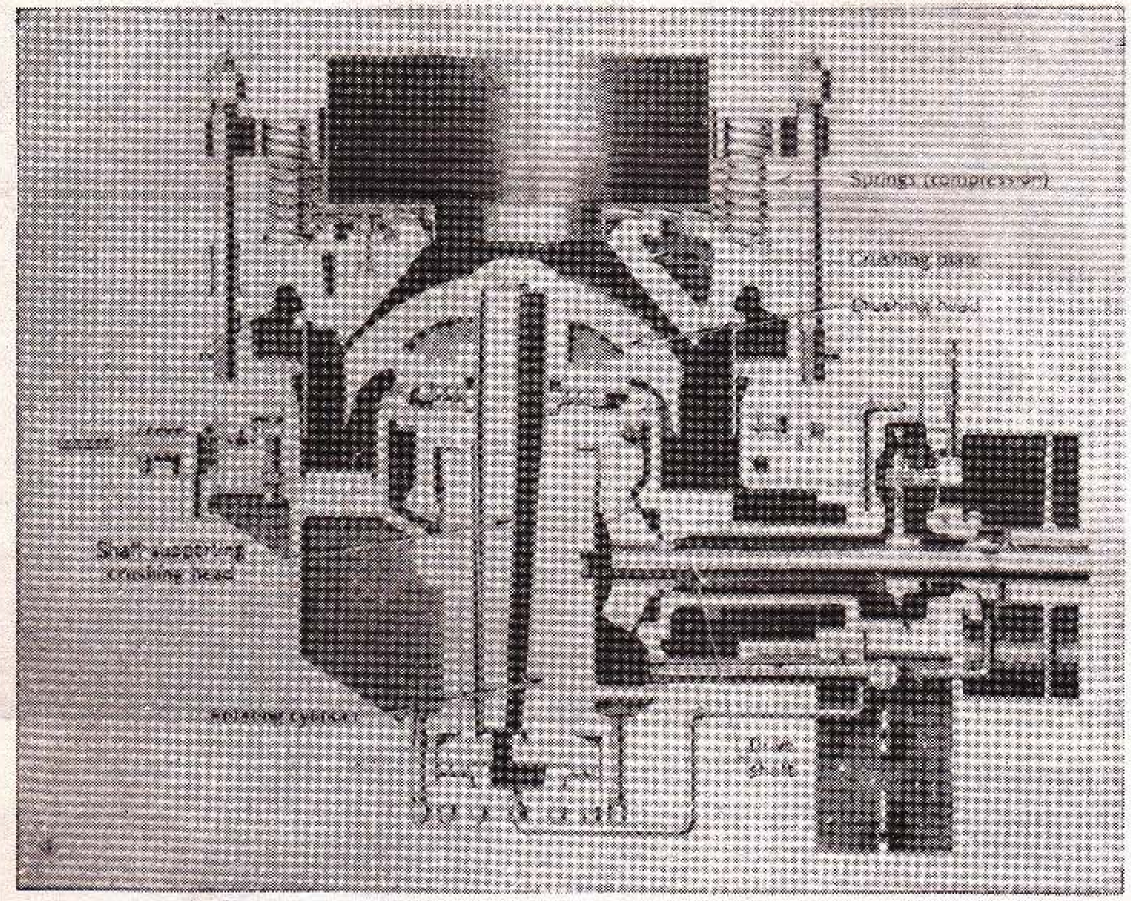


FIG. 29. Sectional drawing of Telsmith Gyrasphere. (*Smith Engineering Works.*)

cone crusher. The spherical head facilitates discharge of the crushed product.

Crushing rolls consist of two heavy cylinders revolving toward each other, the feed being nipped and pulled downward through the rolls by friction. As shown in Fig. 30, modern crushers drive both rolls positively, breakage being prevented by mounting the bearings of one of the rolls against nests of heavy compression springs. Since there is a considerable amount of wear on the rolls, the crushing surface consists of a tough steel sleeve which is shrunk on to the main cylindrical casting, making possible the replacement of worn crushing surfaces. The wearing of grooves in the surface of the rolls is largely pre-

The diameter and spacing of rolls may be varied over rather wide ranges, allowing considerable variations in size of feed and product. This flexibility is a favorable characteristic of crushing rolls, which, combined with the low initial cost, has encouraged the wide adoption of rolls for moderate size reduction of all sizes. The proper diameter and spacing of the rolls, the capacity in tons per hour, and the required horsepower for crushing rolls may be computed as follows.

The coefficient of friction of the mineral against the steel surfaces of the rolls incorporated with a relationship between the dimension of the material to be crushed and the diameter of the rolls determines

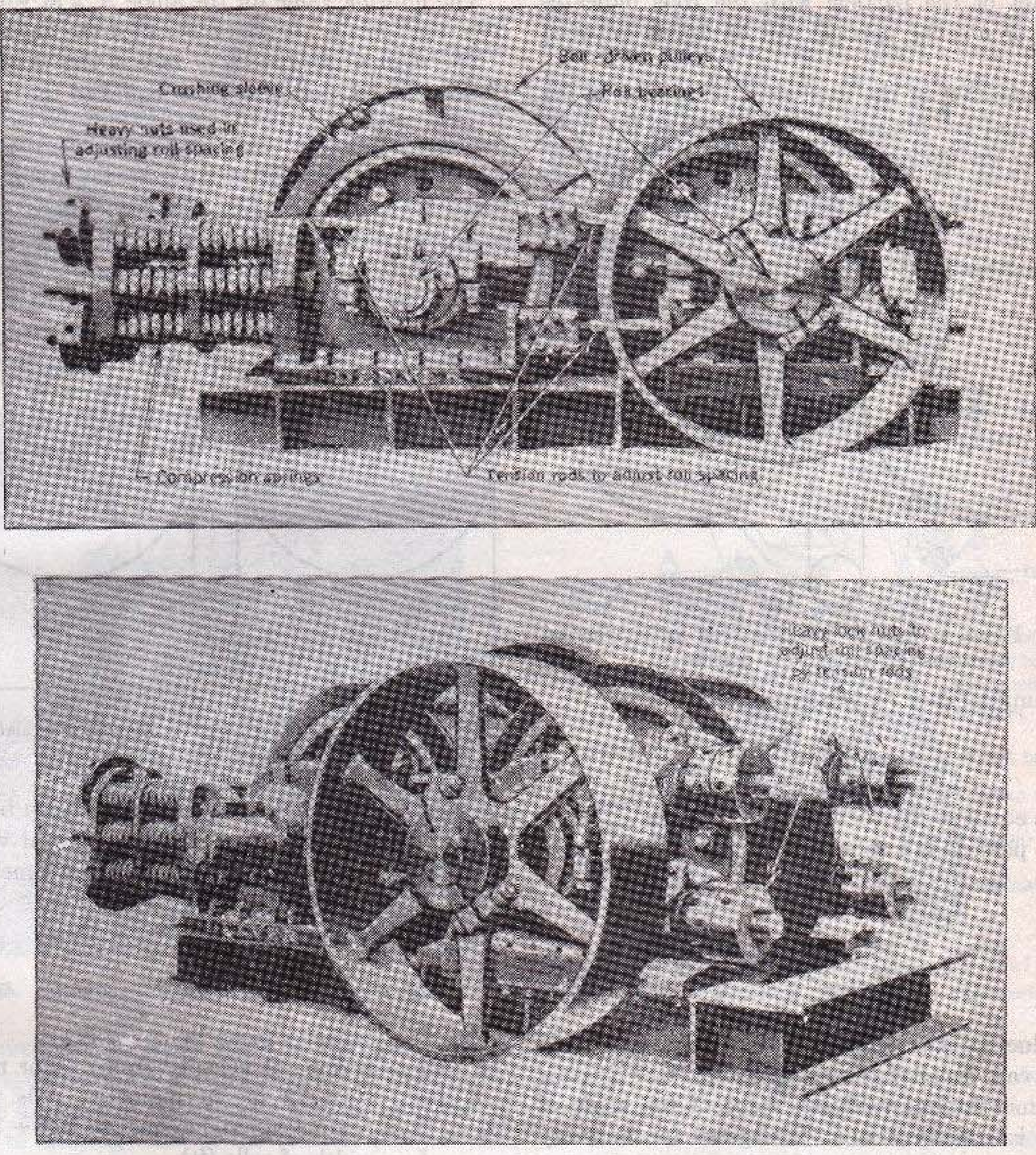


FIG. 30. Crushing rolls. (*Denver Equipment Co.*)

whether or not a particle will be drawn into the rolls and crushed. Figure 31 is a line diagram showing the outline of a spherical particle in position to be crushed between a pair of rolls. The vectors F_T and F_N represent the forces acting on the particle at the point of contact with the roll and may be represented by the resultant force F_R:

A_n = angle of nip (the value for angle A in Fig. 31 corresponding to F_R being horizontal).
D_r = diameter of the rolls.
D_f = diameter of the feed particle.
D_p = maximum dimension of the product (minimum distance between rolls).
F_T = tangential force on the particle.
F_N = normal force on the particle.
F_R = resultant of F_T and F_N.

If F_R is at a negative angle (pointing downward) with the horizontal, as shown in Fig. 31, the particle will be drawn between the rolls. If F_R is at a positive angle with the horizontal, the particle will ride on the rolls or be thrown up and out and will not be crushed. The angle A between the two tangents at the points of contact of the particle with the rolls indicates whether or not the particle will be drawn between the rolls.

The definition of the coefficient of friction is the ratio of the force tangent to the surface to the force normal to the surface. In Fig. 31, this is F_T/F_N.

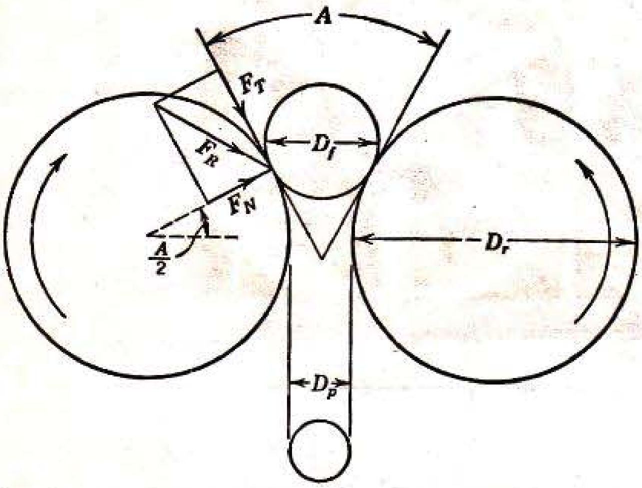


FIG. 31. Forces exerted by crushing rolls for a spherical particle in position to be crushed.

In the limiting case F_R is horizontal and

$$\tan\left(\frac{A}{2}\right) = \frac{F_T}{F_N}$$

which is equal to the coefficient of friction.

If the particle is a sphere,

$$\cos\frac{A}{2} = \frac{\dfrac{D_r}{2} + \dfrac{D_p}{2}}{\dfrac{D_r}{2} + \dfrac{D_f}{2}} = \frac{D_r + D_p}{D_r + D_f}$$

The value for the angle A corresponding to this limiting case is called the angle of nip, A_n.

For smooth steel rolls the value of the angle of nip A_n is usually about 32 degrees for ordinary rocks. In industrial operations general practice is to determine the theoretical minimum roll diameter D_r, add 1 in. to allow for wear, and select the next larger industrial roll.

If the rolls are operating on a slab of steel (or a particle of similar shape) as indicated in Fig. 32,

$$\cos\frac{A}{2} = \frac{\overline{ae}}{\overline{ac}} = \frac{\dfrac{D_r}{2} + \dfrac{D_p}{2}}{\dfrac{D_r}{2} + bc} = \frac{D_r + D_p}{D_r + \dfrac{D_f}{\cos(A/2)}}$$

$$D_r\left(1 - \cos\frac{A}{2}\right) = D_f - D_p$$

The limiting value for the angle $A/2$ at which the resulting force is horizontal is called the *angle of bite*

The *theoretical capacity of rolls* is the weight of a ribbon of feed having a width equal to the width of

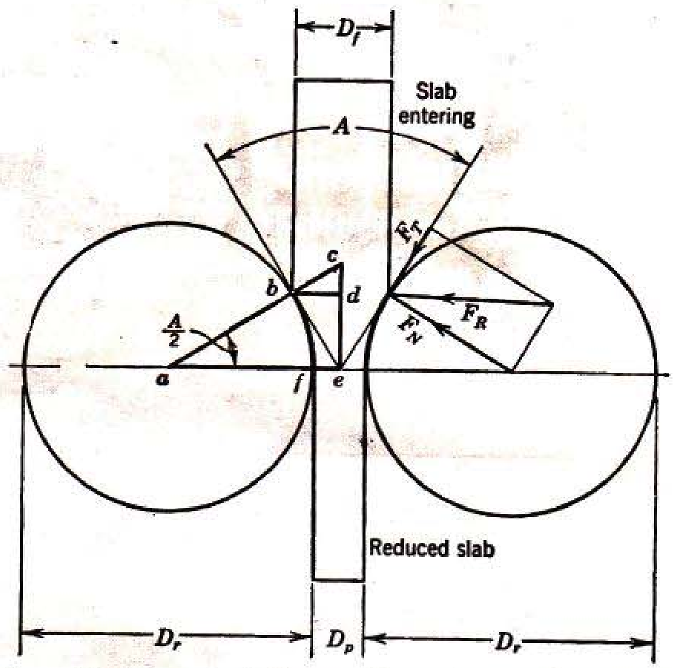


FIG. 32. Forces exerted by crushing rolls on a slab at the approximate angle of bite.

the rolls, a thickness equal to the distance between the rolls, and a length equal to peripheral velocity of the rolls in linear units per interval of time. This may be expressed in tons per hour:

$$T = \frac{60vLD_p\rho}{2000}$$

where T = capacity (tons/hr).
v = peripheral velocity (fpm). For rolls up to 72 in. in diameter, v is usually approximately equal to $300 + 84D_r$.
L = width of rolls (ft).
D_p = distance between rolls (ft).
ρ = density of material (lb/cu ft).

The *actual capacity* is usually from 0.10 to 0.30 of the theoretical.

With the increasing use of cone crushers for intermediate size reduction of ores, the application of rolls in this field is being limited to the size range between cone crushers and fine grinders.

Gravity stamps. The oldest method for size reduction of solids is undoubtedly a husky human being swinging a heavy hammer. When man began to devise mechanical methods for industrial operations, he naturally thought of a rock-crushing device involving a weight to be lifted and dropped on the material to be broken. For this reason the gravity stamp is the oldest recorded method for size reduction in the intermediate and fine size ranges. Gravity stamps are still used to a considerable extent because of the ease of construction in the field, especially for crushing gold ores when the gold is to be amalgamated with mercury, in spite of the fact that capacity is low and the costs are relatively high.

Figure 33 is a modern type of stamp mill. The stamps are vertical shafts raised by cams operating under collars fastened to the upper part of the shafts.

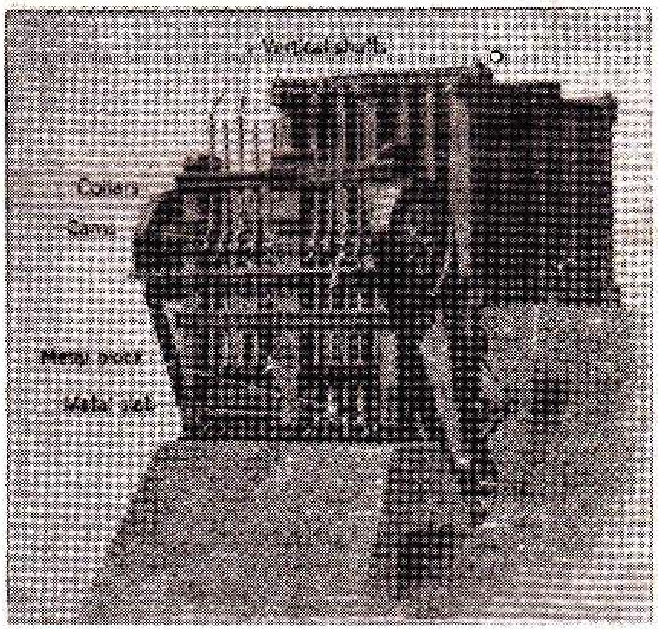


FIG. 33. Gravity stamp mill. (*Allis-Chalmers Mfg. Co.*)

The lower end of each shaft is equipped with a heavy cylindrical metal block which strikes on a stationary hard metal slab. Since a stamp mill has no means of clearing itself of the crushed product, the operation is usually carried out on suspensions of solids in water, which pass slowly through the crushing zone.

The reduction ratios in stamps may be as high as 150, making them one of the most flexible types of machines for size reduction.

FINE SIZE REDUCTION

Size reduction in the finer ranges has usually been termed fine grinding. This is due to the fact that most of the older devices for reduction in this range consisted of two main parts, a stationary surface and a surface rubbed against the stationary surface. The upper and nether millstones used for grinding flour from grain are typical. Such a machine causes disintegration mainly by the application of shear loads. Most recent devices in fine size reduction, such as ball mills, depend more on impact than on shearing forces. The division of the operations of size reduction into crushing and grinding is no longer descriptive of the operations used in coarse size reduction, as distinct from fine size reduction.

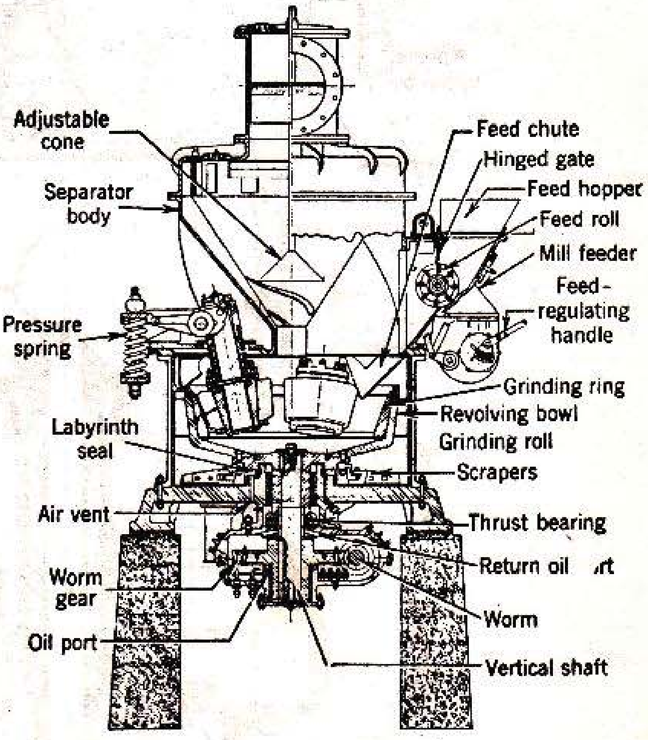


FIG. 34. Cutaway and sectional diagram of bowl mill with air classifier or separator. (*Combustion Engineering Co.*)

In the transition from the old-style shear-grinding devices to the widespread application of ball mills and rod mills, several machines appeared in which the material is reduced in size between rollers, or heavy balls, rolling against a crushing ring. In the Chilean mill, the horizontal axes of the rolls are usually stationary, and the flat pan carrying the crushing ring revolves. The bowl mill (Fig. 34) may be regarded as its modern development.

The Raymond roller mill (Fig. 35) consists of rollers suspended on balanced journals from a rapidly rotating spider mounted on the upper end of the main shaft. The revolving rolls exert pressure on a stationary confining ring by centrifugal force. A plow mounted on the apron or sleeve revolves with the shaft to throw the material into the crushing zone. This mill is usually provided with a sizing feature whereby the material cannot leave the machine until it is fine enough to pass through a screen of given mesh or be lifted by a stream of air of constant velocity. The so-called whizzer consists of vertical vanes rotating rapidly in a horizontal plane to knock oversize particles out of the rising stream.

Ball mills are horizontal rotating cylindrical or conical steel chambers, approximately half full of steel or iron balls, or flint stones. The size reduction is accomplished by the impact of these balls as they fall back after being lifted by the rotating chamber. The length of the cylinder is usually about equal to the diameter. Most ball mills are continuous in operation, feed entering at one end and discharging through the opposite end or through the periphery. They may be operated either wet or dry.

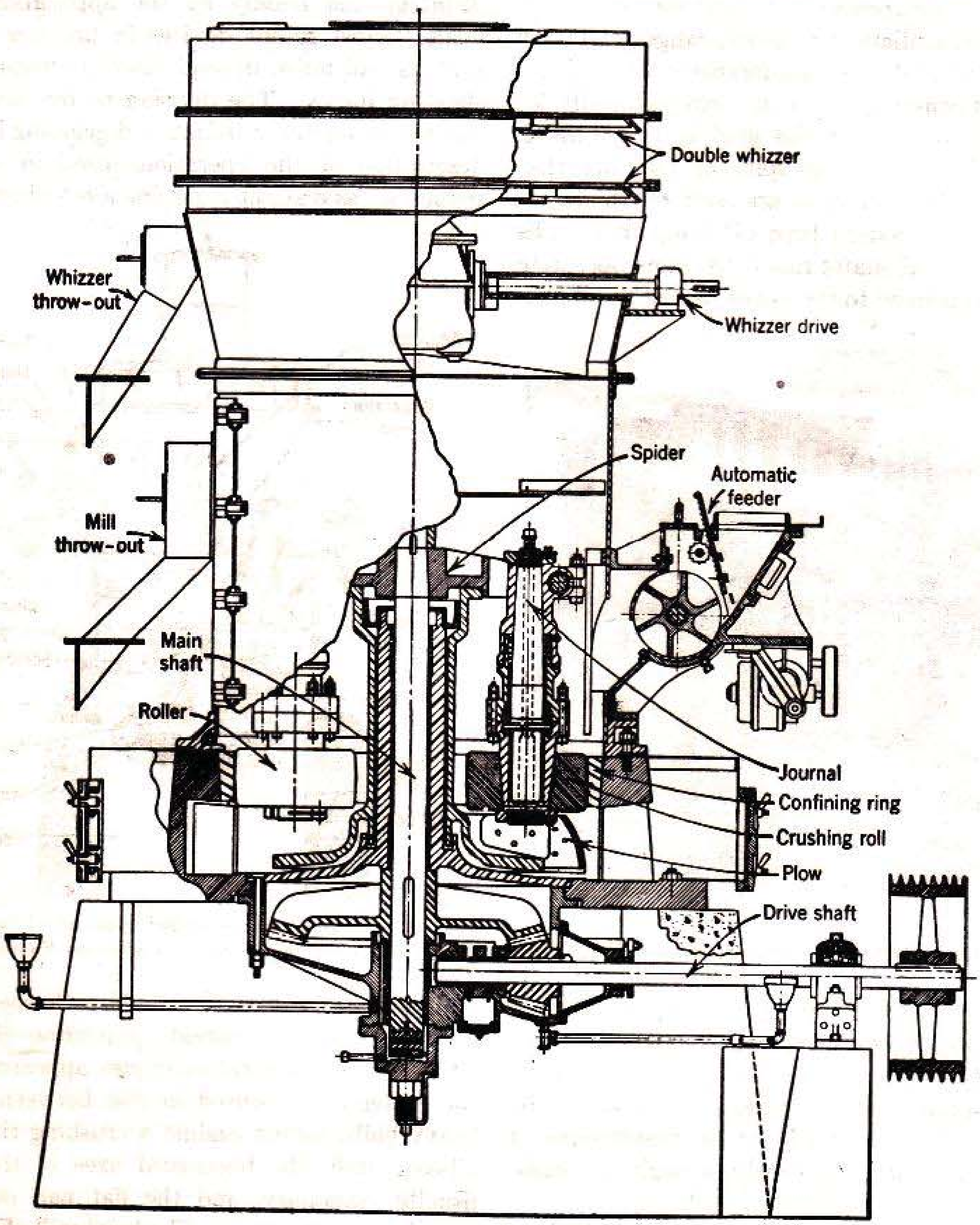


Fig. 35. Cutaway and sectional diagram of Raymond roller mill with air classifier or separator. (*Combustion Engineering Co.*)

In cylindrical ball mills the product may be discharged by overflow through a hollow trunnion (Fig. 36). The smaller particles are suspended and carried out by the circulating fluid, such as air or water.

The Hardinge mill (Fig. 37) is typical of cylindro-conical ball mills. The larger balls and larger par-

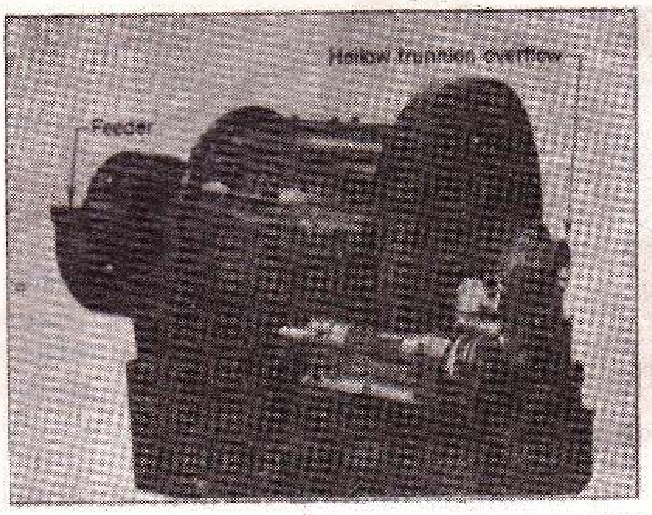


Fig. 36. Ball mill showing feeder and hollow trunnion. (*Allis-Chalmers Mfg. Co.*)

ticles of feed are supposed to segregate to a certain extent in the cylindrical portion of the mill with the greatest diameter. Whether or not this supposition is true, there is a definite relationship between size of particles and size of balls required for effective size reduction. In any case the lifting effect on the balls is greatest at the greatest diameter, and the larger balls will be most effective in size reduction at this point.

In "grate mills" the product passes out through the openings in a vertical grate or diaphragm (Fig. 38). In the trunnion mill, the product may be raised by radial plates or scoops on the outside of the grate (Fig. 39), pushed away from the grate by helical

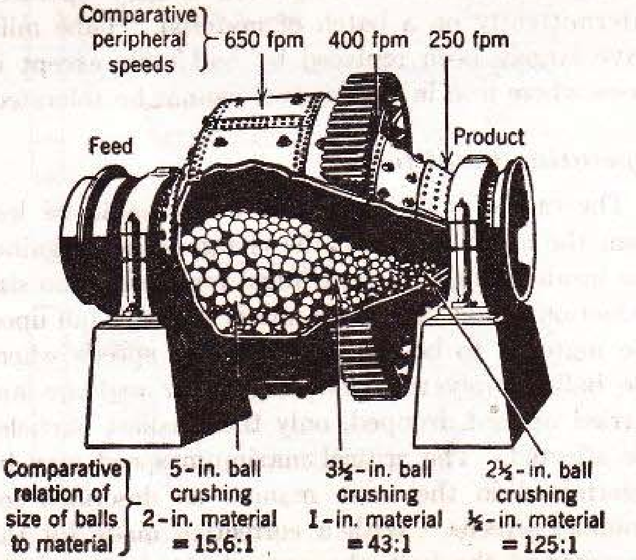


Fig. 37. Cutaway diagram indicating idealized operation of conical ball mill. (*Hardinge Co.*)

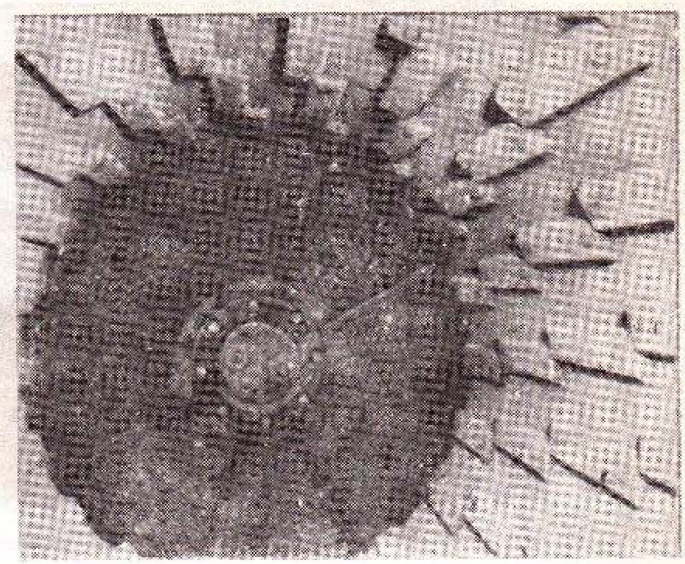


Fig. 38. Interior view of empty ball mill showing grate and rolled steel liners. (*Allis-Chalmers Mfg. Co.*)

vanes on the inner periphery of the cylinder, and discharged from the hollow trunnion by which the mill is supported. If the mill is supported by peripheral tires riding on rollers (Fig. 40), the material simply flows out through the grate and through the open end of the mill.

Compound ball mills consist of two to four cylindrical compartments separated by grates. Each successive compartment is of smaller diameter and contains balls of smaller sizes for finer grinding.

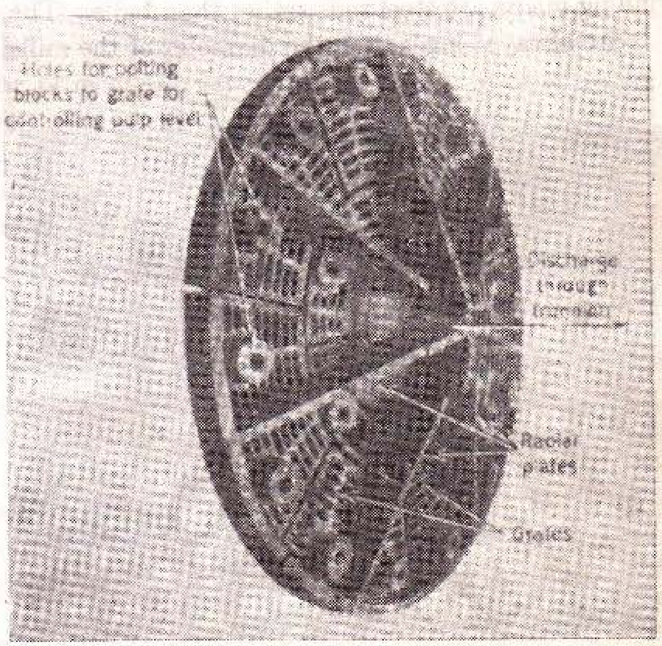


Fig. 39. Outside view of grate showing radial plates which raise the product and cause it to be discharged through the hollow trunnion. (*Allis-Chalmers Mfg. Co.*)

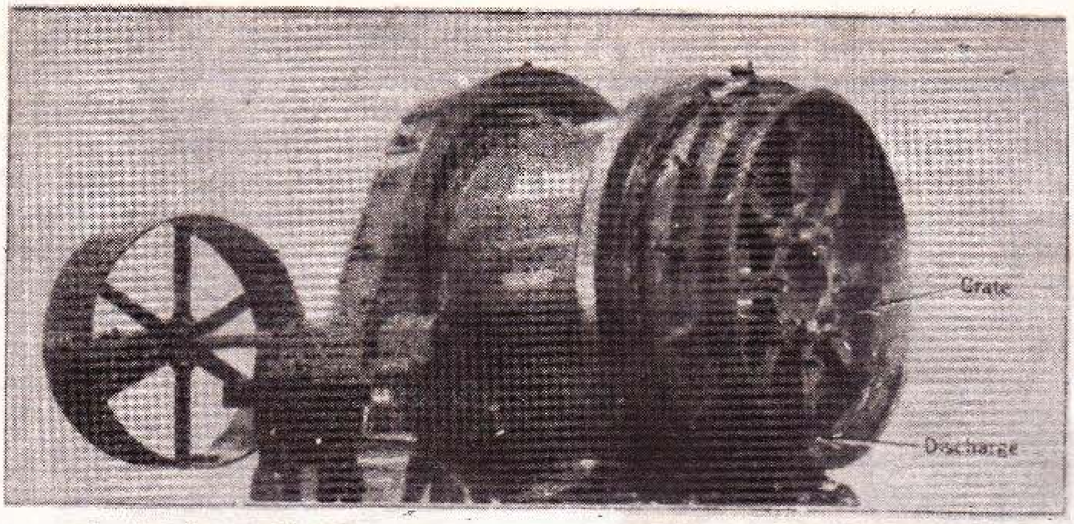


FIG. 40. Grate mill with open-end discharge. (*The Mine and Smelter Supply Co.*)

Such a mill is essentially a series of mills operating continuously.

The liners of ball mills are replaceable and usually made from alloy steel. Other materials such as rubber, cast iron, ceramic, and rock products are sometimes used. The wear on liners is usually from 0.1 to 0.5 lb/ton of product. The balls introduced into the mill vary from 1 to 6 in. in diameter, and the wear is from 1 to 3 lb/ton of product. It is customary to compensate for ball wear by introducing one or more full-sized balls to the mill at least once a day.

Rod mills are similar to ball mills except that the grinding media are steel rods rather than balls. The rods are always longer than the diameter of the mills and therefore lie in the mill parallel to the axis. The impact of the rods is received mainly by the larger particles, causing preferential reduction on the coarsest particles and giving a more closely sized product. Rod mills are more expensive to operate than ball mills, but their use is indicated when a small proportion of fines is desired in the product. Figure 41 shows the inside of a typical rod mill and indicates the wear and replacement of the rods by their different diameters. When the rods become badly worn they must be removed before they bend or break; if they become shorter than the diameter of the mill they may become wedged in such a position as to be held against the lining.

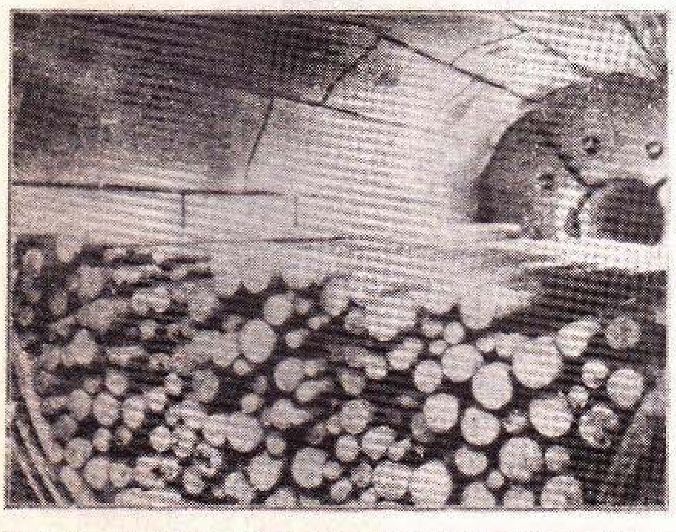


FIG. 41. Interior view of rod mill showing rods in various states of wear from service. (*Allis-Chalmers Mfg. Co.*)

Tube mill is a term used to identify a long cylindrical mill (usually about 22 ft long) utilizing pebbles of flint and ceramic linings and usually operated intermittently on a batch of material. Tube mills have largely been replaced by ball mills except in cases where iron in the product cannot be tolerated.

Operating Conditions

The rate of rotation of ball mills should be less than the speed at which the charge is held against the inside surface by centrifugal force, since no size reduction would take place unless the balls fall upon the material to be crushed. At low speeds where the balls simply roll over each other and are not carried up and dropped, only the smallest particles are affected. The critical maximum speed may be determined in the same manner as described for trommel screens. With a correction made for the diameter of the ball, the critical rate at sea level may be ascertained from the expression

$$N = \frac{76.65}{\sqrt{D - d}}$$

where N = revolutions per minute.
D = diameter of the mill (ft).
d = diameter of the balls (ft).

At low speeds where the contents are simply tumbled or rolled over, the power required to drive the mill varies directly with the speed of rotation. At higher speeds slippage occurs between the contents and the lining, and power requirements increase more slowly with speed of rotation.

Increasing the load (balls and material) in a ball mill will increase the power requirements until the maximum value is reached, after which the power requirement decreases with increasing load as the center of gravity of the load approaches the axis of rotation. For wet grinding the maximum power is required when the weight fraction of solids in the feed is about 0.60 to 0.75. The load may be increased by increasing the weight of balls introduced into the mill, by operating on material (wet pulp) of higher density, or by operating at a higher pulp level. The pulp level or quantity of material being ground in the mill is a major factor in the operation of the mill.

In the simple overflow type of continuous ball mill (no diaphragm), the feed enters at one end and the product flows out through the hollow trunnion

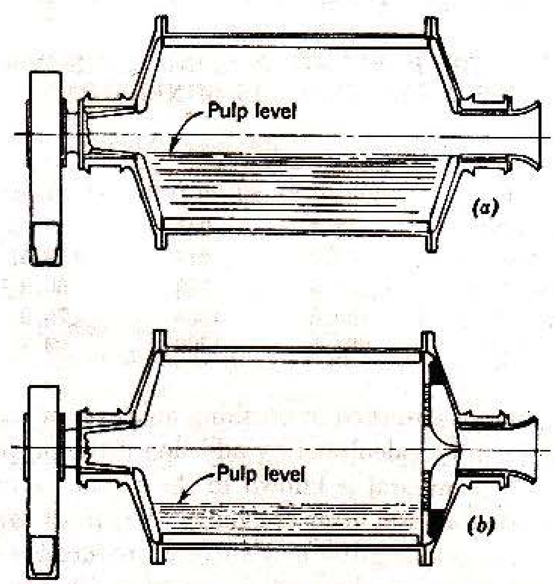


FIG. 42. (a) Sectional diagram of overflow ball mill. (b) Sectional diagram of ball mill equipped with diaphragm or grate allowing lower pulp levels. (*Allis-Chalmers Mfg. Co.*)

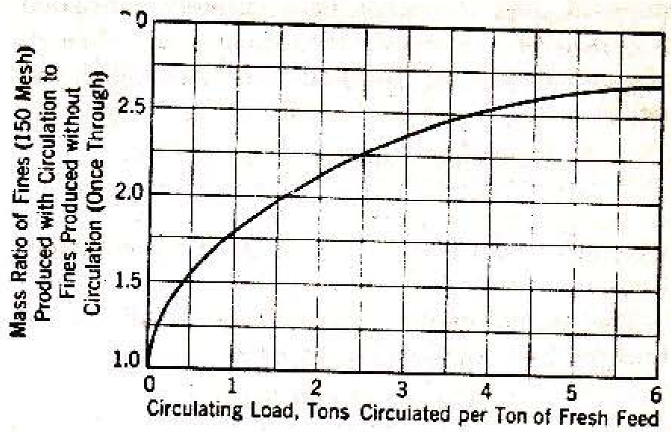


FIG. 43. Relation between circulating load and production of fines in a ball mill being operated in closed circuit.[6]

at the other end, as shown in Fig. 42*a*. The relatively fixed or constant pulp level provided by such a mill means that the effectiveness of grinding can be controlled only by the size and quantity of balls or the rate of feed. With the use of diaphragms the pulp level may be independently controlled at any desired level by making the diaphragm or grate solid for the desired distance from the periphery (Fig. 42*b*).

Lower pulp levels result in greater freedom of movement of the balls with consequent improvement in effectiveness of grinding. In a simple overflow type of mill the balls lose kinetic energy when falling into the dense pulp, and the contact forces between balls under the surface of the pulp is decreased. Mills with diaphragms or grates blocked to maintain the proper pulp level are reported to deliver 25 per cent more product of the correct size range with an increased power requirement of only 20 per cent. Low levels of pulp and decreased time in the mill result also in a decrease of overgrinding.

Closed circuit operation (see diagram accompanying example, p. 44) is usually necessary in ball mill operation since these mills do not have a sizing action on their product. A sizing device, such as a "classifier," is placed in series with the ball mill, and the oversize material from the sizing operation is returned to the mill for further size reduction. In such operations, the circulating load may be the major part of the feed. The present trend is to use high circulating loads. The approximate relationship between the production of fines and circulating load is shown in Fig. 43.

The *capacity of ball mills* depends very largely on the reduction ratio as well as on the hardness of the material, and it cannot be accurately calculated. A reasonably conservative estimate of the capacity of a cylindroconical (Hardinge type) ball mill in tons per 24 hr is

$$\frac{\text{Maximum diameter} \times \text{Length (ft)}}{C_1}$$

where C_1 varies from 6 to 3 for most normal operations.

The normal capacity of cylindrical ball mills in tons per 24 hr may be estimated as

$$\frac{\text{Volume of mill (cu ft)}}{C}$$

where C usually varies from about 1 to 2.

ENERGY REQUIREMENTS

Although most of the power required for driving crushers and grinders is used in overcoming mechanical friction, the actual energy used in size reduction is an important consideration and theoretically is proportional to the new surface produced, as there is no change in the material except size and the

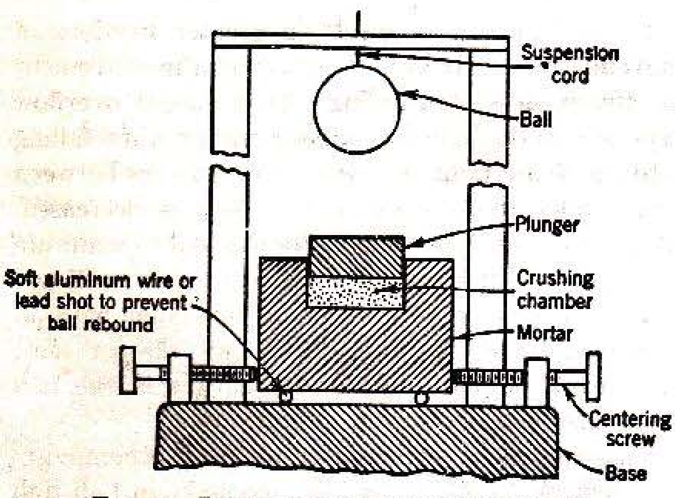


FIG. 44. Diagram of a drop weight crusher.[3]

creation of new surface. This principle was first recognized by Rittinger.[4] Rittinger's law was first confirmed beyond doubt * by the U. S. Bureau of Mines.[3] A drop weight crusher (Fig. 44) was used for accurate determination of the energy expended in crushing, and a rate of solution method for accurate determination of the surface of the particles. The results of their measurements on quartz (SiO_2),

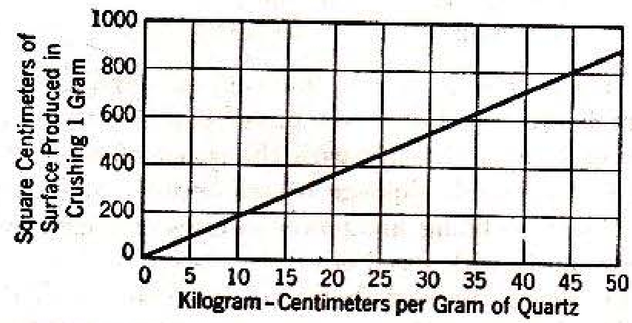


FIG. 45. Relation of energy input to surface produced in crushing quartz with a drop weight crusher.[3]

plotted in Fig. 45, show a constant energy requirement of 1 kg-cm for each 17.56 sq cm of new surface produced for this quartz, or, as usually expressed, 17.56 sq cm of new surface produced by the application of 1 kg-cm of mechanical energy.

Rittinger's number designates the new surface produced per unit of mechanical energy absorbed by the material being crushed. The values vary for different materials, depending on the elastic constants and their relation to the ultimate strength and on the manner or rate of application of the crushing force. A few values of Rittinger's number as determined by a drop weight crusher are given in Table 9.

TABLE 9. DROP WEIGHT RITTINGER'S NUMBER FOR A FEW COMMON MINERALS[3]

Mineral	Rittinger's Number		
	sq in./ft-lb	sq cm/ft-lb	sq cm/kg-cm
Quartz (SiO_2)	37.7	243	17.56
Pyrite (FeS_2)	48.7	314	22.57
Sphalerite (ZnS)	121.0	780	56.2
Calcite ($CaCO_3$)	163.3	1053	75.9
Galena (PbS)	201.5	1300	93.8

The energy absorbed in crushing mixtures of these minerals can be calculated by addition if the proportion of each mineral is known in the various screen fractions before and after crushing. The most rapid means of estimating the new surface produced is by the use of screen analyses as discussed in Chapter 3. Other methods, such as the rate of solution, are more precise but more difficult to execute.

* The principle known as Kick's law, that "the energy required to produce analogous changes of configuration of geometrically similar bodies varies as the volumes or masses of these bodies," was at one time erroneously applied in the theory of crushing. It led to the false conclusion that the energy required in crushing was proportional to the decrease in volume or mass of the particles. This principle is now recognized as applicable only to plastic deformation of particles within the elastic limit and not to crushing.

The mechanical energy supplied to the crusher is always greater than that indicated by Rittinger's number, as friction losses and inertia effects in the equipment require more energy than the actual production of new surface. Also, fracture is accomplished, not by static loading, but by exceeding the minimum rate of loading or deformation. Even brittle substances adjust themselves to slowly applied loads, and fracture does not occur the instant the load is applied but only when the rate of loading exceeds a certain minimum.

The total energy supplied to the crusher, therefore, depends upon the rate of load application, which differs with the type of machine and conditions of operation. Table 10 gives values for the new surface produced per unit of energy supplied to the material being crushed in a laboratory ball mill operated at the same speed but with varying weights of similar balls in the machine while grinding equal weights of quartz.

TABLE 10. EXPERIMENTAL VALUES OF NEW SURFACE PRODUCED PER UNIT OF ENERGY FOR QUARTZ

Calculated by subtracting the energy required to drive the mill containing balls but no material from the total energy required to drive the operating mill for the same length of time.

Total Weight of Balls in Ball Mill, lb	sq in./ft-lb	sq cm/ft-lb	sq cm/kg-cm
36	5.6	36	2.6
71	10.1	65	4.6
142	12.7	82	5.9
178	14.6	94	6.8
249	12.1	78	5.6
Drop weight method	37.6	243	17.56

The new surface produced per unit of energy supplied to the material being crushed in a ball mill is much less than for the drop weight crusher. This may be explained by the high percentage of ineffective blows and other losses in the ball mill. The important practical point is the variation in effectiveness of size reduction with the total weight of balls charged, showing a maximum value at about 175 lb of balls in this particular mill.

Values of the Rittinger number as determined in the drop weight crusher represent maximum effectiveness in size reduction and may be used in calculating the *crushing effectiveness* for any such operation. In the ball mill with 178 lb of balls, the crushing effectiveness is 94/243 = 0.387. In this manner the performance of various machines, and variations in the same machine, can be compared.

The overall energy effectiveness (or efficiency) of a crusher is always much less than the crushing effectiveness, as the latter does not include the mechanical losses such as friction and inertia. The capacity of ball mills cannot be accurately calculated because of the effects of variables such as the relative grindability of the material and the range in size reduction. An approximate idea of the capacity and power requirements of ball mills, both cylindrical and conical, may be gained by reference to Table 11.

TABLE 11. CAPACITY AND POWER REQUIREMENTS OF BALL MILLS

Size, ft, diameter × length	Approximate Ball Load, lb	Approximate Rpm	Approximate Average Capacity, tons/24 hr: ½ in. to 48 mesh	½ in. to 65 mesh	¼ in. to 100 mesh	Motor Horsepower
3 × 2	1,000	35	[illegible]	9	5	6-8
3 × 4	2,000	35	24	18	10	12-15
4 × 4	3,300	30	42	30	20	20-25
5 × 4	5,000	29	80	55	30	30-40
5 × 6	7,500	29	120	85	50	40-50
6 × 3	6,000	25	125	85	50	50-60
6 × 5	10,000	25	210	150	90	75-100
6 × 6	12,000	25	250	175	100	90-120
6 × 12	24,000	25	500	340	200	150-200
7 × 6	21,600	23	500	350	200	110-160
8 × 6	28,000	22	620	450	260	150-225
10 × 9	74,000	17	1500	1100	650	550-600
Cylindroconical Mills						
2 × 3¼	600	40	4	3	2	2
3 × 3½	1,100	35	12	10	9	5-8
3 × 2	2,000	35	17	15	13	10
5 × 3	9,500	28	100	80	60	40-50
6 × 3	15,000	24	180	120	90	60-75
7 × 4	27,000	23	300	220	150	125
8 × 4	38,000	21	480	350	270	175-200
12 × 6	110,000	16	1800	1400	1000	700-800

Illustrative Example. A ball mill operating in closed circuit with a 100-mesh screen gives the screen analyses below. The ratio of the oversize to the undersize (product) stream is 1.0705 when 200 tons of galena are handled per day.

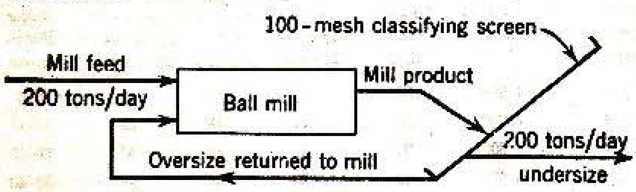
The ball mill requires 15.0 hp when running empty (with the balls but without galena) and 20.0 hp when delivering 200 tons per day of galena. Find:

1. The effectiveness of crushing based on drop weight crushing as 1.00.
2. The overall energy efficiency.

5. The classifying screen effectiveness.

Mesh	Mill Feed, weight %	Oversize from Screen, weight %	Undersize from Screen, weight %
− 4 + 6	1.0	0	0
− 6 + 8	1.2	0	0
− 8 + 10	2.3	0	0
− 10 + 14	3.5	0	0
− 14 + 20	7.1	0	0
− 20 + 28	15.4	0	0
− 28 + 35	18.5	13.67	0
− 35 + 48	17.2	32.09	0
− 48 + 65	15.6	27.12	0
− 65 +100	10.4	20.70	2.32
−100 +150	6.5	4.35	14.12
−150 +200	1.3	2.07	13.54
−200	0	0	70.02
	100.00	100.00	100.00

Solution.



Analysis. The Rittinger number measures the minimum energy required to form new surface. If the new surface created per unit time is calculated, then the minimum energy required for the formation can be calculated.

In order to evaluate the new surface of the product, the minus 200-mesh fractions may be evaluated by the straight-line plot, such as Fig. 15. This method is valid only for the product of a mechanical crushing device and not for the classified product. Therefore the size distribution of the mill product must be computed and extrapolated for the mass fractions retained below 200 mesh. The sum of these fractions must equal the minus 200-mesh fraction.

The fractions of mill product below 200 mesh are then converted to fractions of the undersize stream. The surfaces of the fractions are calculated either from the actual specific surfaces in Fig. 16 or from the relationship (p. 22):

$$\text{Total surface} = \frac{6}{\rho} \sum \frac{n_i m_i}{(D_{avg})_i}$$

n is evaluated from the data of Fig. 17.

Theoretical effectiveness of ball mill

$$= \frac{\text{Minimum power required to create new surface}}{\text{Power increase due to charge}}$$

Overall energy effectiveness of ball mill

$$= \frac{\text{Minimum energy required to create new surface}}{\text{Total energy used}}$$

Effectiveness of classifying screen

$$= \frac{x_P(x_F - x_R)}{x_F(x_P - x_R)} \left[1 - \frac{(1 - x_P)(x_F - x_R)}{(1 - x_F)(x_P - x_R)} \right]$$

Mesh	Calculated Size Distribution: Mass % in Oversize	Mass % in Undersize	Mass in Mill Product per 100 lb of Oversize, Oversize + (Undersize/1.0705), lb	Size Distribution, Mass % in Mill Product
− 28 + 35	13.67	0	13.67	7.07
− 35 + 48	32.09	0	32.09	16.60
− 48 + 65	27.12	0	27.12	14.02
− 65 +100	20.70	2.32	22.86	11.82
−100 +150	4.35	14.12	17.52	9.07
−150 +200	2.07	13.54	14.72	7.62
−200	0	70.02	65.35	33.80
	100.00	100.00	193.33	100.00

Mass % in mill product

$$= \frac{\text{Mass \% oversize} + \dfrac{\text{Mass \% undersize}}{1.0705}}{193.33}$$

The distribution of material smaller than 200 mesh in mill product from an extrapolation on a plot of

$$\log (\text{mass retained, per cent}) \text{ versus } \log (D_{avg})$$

is given below.

Average Diameter (D_{avg}), microns	Mass % in Mill Product	Mass % in Undersize	n	$\dfrac{mn}{D_{avg}}$
63	6.98	14.46	1.65	0.378
45	5.90	12.22	1.60	0.434
31.8	5.00	10.34	1.53	0.497
22.5	4.24	8.78	1.50	0.585
15.9	3.58	7.41	1.45	0.676
11.2	3.03	6.28	1.42	0.795
7.92	2.68	5.58	1.40	0.985
5.59	2.19	4.54	1.375	1.117
3.84	0.20	0.41	1.35	0.144
	33.80	70.02		$5.611 = \sum \dfrac{mn}{D_{avg}}$

Surface area for this fraction of the undersize stream

$$= \frac{5.611 \times 10^4 \times 6}{7.43} = 45{,}250 \text{ sq cm/70.02 grams}$$

Screen Mesh	Mass % in Undersize	Specific Surface, sq cm/gram	Actual Surface, sq cm
− 65 +100	2.32	85.8	199.1
−100 +150	14.12	115.4	1,629.4
−150 +200	13.54	155.1	2,100.0
−200	70.02		45,250.0
	100.00		49,180 sq cm surface area/100 grams of undersize

FEED SURFACE CALCULATIONS

Screen Mesh	Mass % in Feed	Specific Surface, sq cm/gram	Actual Surface, sq cm/100 grams
− 4 + 6	1.0	7.6	7.6
− 6 + 8	1.2	9.9	11.9
− 8 + 10	2.3	12.5	28.8
− 10 + 14	3.5	16.4	57.4
− 14 + 20	7.1	21.1	149.8
− 20 + 28	15.4	26.9	414.5
− 28 + 35	18.5	35.7	660.5
− 35 + 48	17.2	47.2	811.8
− 48 + 65	15.6	63.0	982.8
− 65 +100	10.4	85.8	892.3
−100 +150	6.5	115.4	750.1
−150 +200	1.3	155.1	201.6
			4,968.8 sq cm total surface/100 grams of feed

$$\text{New surface created} = (49{,}180 - 4968)$$

$$= 44{,}212 \text{ sq cm/100 grams of feed}$$

$$\text{Theoretical effectiveness} = \frac{(44{,}212)(9072)(200)}{(6.452)(1.98)(10^6)(24)(201.5)(5)}$$

$$= \frac{1.265 \text{ hp}}{5 \text{ hp}} = 0.253$$

$$\text{Overall energy effectiveness} = \frac{1.265}{20} = 0.0633$$

SCREEN EFFECTIVENESS CALCULATIONS

$$x_P = 1 - 0.0232 = 0.9768$$

$$x_F = 0.0907 + 0.0762 + 0.3380 = 0.5049$$

$$x_R = 0.0642$$

Screen effectiveness

$$= \frac{(0.9768)(0.4407)}{(0.5049)(0.9126)}\left[1 - \frac{(0.0232)(0.4407)}{(0.4951)(0.9126)}\right] = 0.914$$

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PROBLEMS

1. A short-head cone crusher is available for crushing 2 tons of pyrites per hour. On similar materials, the overall energy efficiency has been found to be 3.15 per cent. The raw feed is to be crushed by a jaw crusher, whose product constitutes the feed to the cone crusher. The cone crusher operates in closed circuit with a 14-mesh screen. The cone crusher product and recycle stream analyses are given below. On the basis of calculations and of any assumptions which you may find necessary, select a Dodge-type crusher that will do the job.

The surface ratio (n) may be considered to be 6.5 above 3 mesh. The full load energy requirement for the short-head cone crusher is 5 hp. The recycle ratio (recycle stream/product stream) is 1.

Product Mesh	Product Mass %	Recycle Stream Mesh	Recycle Stream Mass %
−14 + 20	29.8	− 3 + 4	3.8
−20 + 28	30.2	− 4 + 6	10.0
−28 + 35	25.0	− 6 + 8	19.6
−35 + 48	9.6	− 8 +10	26.0
−48 + 65	3.8	−10 +14	36.6
−65 +100	1.6	−14 +20	4.6
	100.0		100.0

2. A ball mill, operated in a closed circuit with a classifier, is used to grind calcite after it has had preliminary crushing in jaw crushers. Screen analyses of the various streams are given below.

The ball mill feed (25 tons/hr) is estimated to have a specific surface of 292 sq cm/gram. When the ball mill is operated with a recycle of 75 tons/hr, 75 kw are required to drive the ball mill. Determine the efficiency of the ball mill.

Tyler Screen Mesh	Ball Mill Feed, mass % retained	Recycle, Classifier Sands, mass % retained	Product, Classifier Overflow, mass % retained
0.525 in. − 0.371 in.	4.7		
0.371 in. − 3 mesh	20.1	6.3	
− 3 mesh + 4 mesh	17.9	7.0	
− 4 + 6	12.1	8.2	
− 6 + 8	8.6	9.3	
− 8 + 10	5.5	3.0	
− 10 + 14	4.7	15.4	
− 14 + 20	2.7	16.9	
− 20 + 28	3.5	20.7	
− 28 + 35	2.9	3.4	4.2
− 35 + 48	1.9	2.8	12.7
− 48 + 65	2.0	1.4	19.3
− 65 +100	1.7	1.2	13.7
−100 +150	1.7	0.8	11.7
−150 +200	1.5	0.6	9.8
−200	8.5	3.0	28.6
Totals	100.0	100.0	100.0

3. A cement plant is grinding 10 tons/hr of a hard rock (specific gravity, 3.8) in a high-speed disk grinder operating in a closed circuit with a 65-mesh screen. Regular checks upon the possibility of oversize particles passing through the screen show that all material in the undersize stream from the screen will pass through a 35-mesh screen.

Drop weight laboratory tests upon the material being crushed indicate that the absorption of 1 ft-lb of energy will result in creation of 110 sq cm of new surface and that the specific surface ratios are identical with those of sphalerite.

(*a*) If the energy efficiency of the grinder is 18 per cent and the known streams have the analyses given below, what is the horsepower required by the grinder?

(*b*) What is the effectiveness of the screen?

Mesh	Raw Feed to Grinder, mass fraction	Discharge from Grinder, mass fraction	Oversize from Screen, mass fraction
− 3 + 4	0.05		
− 4 + 6	0.10		
− 6 + 8	0.20		
− 8 + 10	0.30		
− 10 + 14	0.20	0.04	0.05
− 14 + 20	0.10	0.08	0.10
− 20 + 28	0.05	0.16	0.20
− 28 + 35		0.24	0.30
− 35 + 48		0.17	0.2025
− 48 + 65		0.10	0.0975
− 65 +100		0.08	0.05
−100 +150		0.06	
−150 +200		0.04	
−200 +270		0.02	
−270 +400		0.01	

4. A feed of 150 tons/day of pyrites must be comminuted from the material size given below as feed (the product from a controlling screen) to the size range given below as product (the feed to a reduction process). A ball mill is to be used. It will be loaded with balls to operate at a crushing effectiveness of about 32 per cent.

(*a*) What size cylindrical mill should be selected?

(*b*) What size motor will be needed to drive it?

(*c*) What is the overall energy efficiency?

Mesh	Feed, mass fraction	Product, mass fraction
− 3 + 4	0.036	
− 4 + 6	0.192	
− 6 + 8	0.365	
− 8 + 10	0.284	0.010
− 10 + 14	0.123	0.072
− 14 + 20		0.228
− 20 + 28		0.295
− 28 + 35		0.170
− 35 + 48		0.098
− 48 + 65		0.072
− 65 +100		0.046
−100 +150		0.009
−150 +200		0.002

5. Quartz from the mine is sent over a grizzly with 8-in. spacing and then to a Blake standard jaw crusher with a 40-in. by 42-in. feed opening and a 6-in. discharge setting. The crusher operates at 190 rpm and handles 130 tons/hr of feed. Screen analyses of the feed and product are given below.

(*a*) What are the theoretical power requirements?

(*b*) What size motor is recommended? Why?

(In the size range indicated the average surface ratio may be assumed to be 8.0)

Feed: Screen Aperture, in.		Feed: Mass Fraction	Product: Screen Aperture, in.		Product: Mass Fraction
−34	+28.6	0.181	−6	+4.23	0.123
−28.6	+24.0	0.343	−4.23	+2.99	0.248
−24.0	+20.3	0.220	−2.99	+2.11	0.167
−20.3	+17.0	0.165	−2.11	+1.49	0.105
−17.0	+14.3	0.054	−1.49	+1.05	0.068
−14.3	+12.0	0.037	−1.05	+0.81	0.051
			−0.81	+0.57	0.046
			−0.57	+0.403	0.039
			−0.403	+0.285	0.033
			−0.285	+0.201	0.028
			−0.201	+0.142	0.025
			−0.142	+0.100	0.023
			−0.100	+0.0707	0.018
			−0.0707	+0.0500	0.016
			−0.0500	+0.0353	0.010

6. In an attempt to evaluate the efficiency of a 24-in. by 15-in. Blake jaw crusher, a set of coarse analytical screens was constructed from welded steel rods. The standard Tyler $\sqrt{2}$ relationship between screen apertures was maintained in this series of large screens.

Calcite was fed to the crusher at the rate of 60 tons/hr. The discharge setting of the jaws was 5 in. The crusher was driven by a 35-hp motor. Screen analyses of the feed and the product are given in the table below.

(*a*) Calculate efficiency of the crusher, assuming the motor was operating at an average of ⅕ its rating.

(*b*) How many tons per hour of galena could be fed to the crusher and reduced over the same size range with the same power?

(*c*) What is the capacity according to Taggart's formula?

Aperture of Screen, in.	Feed, mass fraction on screen	Product, mass fraction on screen	Specific Surface Ratio n for Average Diameter of Material on Screen
22.6	0.0	0.0	
16.75	0.15	0.0	10.0
11.85	0.35	0.0	9.7
8.40	0.25	0.0	9.5
5.93	0.15	0.0	9.0
4.20	0.10	0.05	8.6
2.97	0.0	0.20	8.0
2.10	0.0	0.45	7.2
1.48	0.0	0.25	6.6
1.05	0.0	0.05	6.2

7. A grinder is to be used to reduce a siliceous ore of the feed size shown below. Laboratory tests on similar equipment indicate that the product size given below will be satisfactory, and that the grinder is approximately 8 per cent efficient in converting input energy into size reduction as evidenced by an increase in surface.

It is estimated that a crusher to handle 10 short tons/hr will cost about $4000. If the crusher operates on a 24-hr basis for 300 days/yr, it is estimated that maintenance costs, overhead, and ordinary replacement costs will be about 50 per cent of power costs. Electric power costs 2 cents/kwhr.

If this machine depreciates on a straight-line basis and its life is estimated at 10 yr, what is the processing cost per ton of ore?

Tyler Mesh	Feed, mass fraction	Product, mass fraction
− 6 + 8	0.143	
− 8 + 10	0.211	
− 10 + 14	0.230	
− 14 + 20	0.186	0.098
− 20 + 28	0.120	0.234
− 28 + 35	0.076	0.277
− 35 + 48	0.034	0.149
− 48 + 65		0.101
− 65 +100		0.068
−100 +150		0.044
−150 +200		0.029

8. A roll crusher is to be used to crush medium hard quartz (specific gravity, 2.65). The product from the crusher is to be fed to a number of rod mills (6 ft by 12 ft) at the rate of 8 tons/hr to each mill. The power consumption of each rod mill is 160 hp, with an overall energy effectiveness or efficiency of 2.0 per cent.

The rod mills operate in closed circuit with a 48-mesh screen. The ratio of recycle to product is 1:1.

If the surface ratio n for quartz is 10 for all sizes above 3 mesh, determine the setting (distance between the rolls) in the roll crusher.

Classifier Product

Mesh	Mass Fraction
− 35 + 48	0.05
− 48 + 65	0.80
− 65 +100	0.10
−100 +150	0.05

Recycle Stream

Mesh	Mass Fraction
−20 +28	0.05
−28 +35	0.10
−35 +48	0.80
−48 +65	0.05

9. Five tons of a hard rock (specific gravity, 3.8) are fed every hour to a cone crusher in closed circuit with a 48-mesh screen. Regular checks upon the possibility of oversize particles passing through the screen show that all material in the undersize stream from the screen will pass through a 28-mesh screen.

Drop weight laboratory tests upon the material being crushed indicate that the absorption of 1 ft-lb of energy will result in the creation of 110 sq cm of new surface. These tests also indicate that the surface area ratios for the material are identical with those of sphalerite.

If the energy efficiency of the grinder is 18 per cent and the known streams have the analyses given below, what is the horsepower required by the grinder? What is the effectiveness of the screen?

Mesh	Raw Feed to Grinder, mass fraction	Discharge from Grinder, mass fraction	Oversize from Screen, mass fraction
− 3 + 4	0.10		
− 4 + 6	0.20		
− 6 + 8	0.40		
− 8 + 10	0.20		
− 10 + 14	0.10	0.02	0.03
− 14 + 20		0.04	0.06
− 20 + 28		0.06	0.09
− 28 + 35		0.25	0.35
− 35 + 48		0.30	0.30
− 48 + 65		0.20	0.08
− 65 +100		0.06	0.05
−100 +150		0.04	0.04
−150 +200		0.03	

10. Quartz goes through two successive grinders on the same shaft which draws a total of 20 hp. The feed averages 2 in. in diameter and has a surface ratio n of 10. The grinders running empty require 2 hp. Their capacity is 3 tons/hr. The analyses of their products are given below.

(*a*) Calculate the horsepower used in each grinder.

(*b*) Calculate the efficiency of the grinders if Rittinger's number (new surface produced per unit of energy) is 37.6 sq in./ft-lb.

Primary Grinder

Mesh	%
− 4 + 8	20
− 8 + 14	30
−14 + 28	30
−28 + 48	15
−48 +100	5

Final Grinder

Mesh	%
− 28 + 48	10
− 48 +100	20
−100 +200	30
−200 mesh + 0.001 in.	30
− 0.001 in. + 0.0003 in.	10

11. A Hardinge mill is grinding cement clinker (specific gravity, 2.2) in an open circuit at the rate of 20 tons/hr. All grinder product must pass 48 mesh, and none is to be wasted. A total of 375 hp is required by the mill, with 5 hp

needed to run the mill empty. Following is a size analysis of the feed and product.

Mesh	Feed, mass %	Product, mass %	n
− ½ in. + ⅜ in.	25.0	....	7.2
− ⅜ in. + 3	27.3	....	6.1
− 3 + 8	19.2	....	4.2
− 8 + 20	19.6	....	3.1
− 20 + 48	8.9	....	2.9
− 48 + 65	....	2.1	2.7
− 65 +100	....	8.2	2.3
−100 +200	....	21.9	1.9
−200 mesh + 0.001 in.	....	67.8	1.6

A closed circuit system is suggested as a means of reducing power costs. A laboratory test indicates the following results could be obtained with a closed circuit unit using the above feed and screening the grinder product on a 48-mesh screen (1 kwhr = 1.341 hp-hr).

Mesh	Screen Feed, mass %	Oversize, mass %	Undersize, mass %
− 3 + 8	4.1	9.1	
− 8 + 20	13.2	29.3	
− 20 + 48	14.1	31.4	Nothing
− 48 + 65	20.2	23.6	coarser
− 65 +100	24.3	6.6	than
−100 +200	16.2	....	48 mesh
−200 mesh + 0.001 in.	7.9	....	

If electricity costs 1 cent/kwhr, how much in power costs could be saved per year by a closed circuit if 480 tons of ground clinker are produced daily, 365 days a year?

What is the energy consumption per hour if the same feed rate (20 tons/hr) is used in the closed circuit?

12. The double-roll toothed crusher of a coal corporation yields a product of the indicated screen analysis when crushing 239 tons/hr using 38 hp.

(*a*) If the surface ratio n is 2, what is Rittinger's number for this particular installation?

(*b*) From this calculation and any data you can find in the handbooks, what is your conclusion as to the efficiency of this crusher?

Screen Size	Feed, mass %	Product, mass %
+2¾ in.	16.88	.....
2¾ × 2	34.62	.....
2 × 1¾	10.82	.....
1¾ × 1½	14.80	0.39
1½ × 1¼	11.12	3.24
1¼ × 1	5.77	12.10
1 × ¾	1.75	16.26
¾ × ⅝	1.44	18.84
⅝ × ⅜	1.04	21.14
⅜ × ¼	0.42	8.02
¼ × 3/16	0.21	5.43
3/16 × 0	1.13	14.10
3/16 × 6 mesh	....	2.83
6 × 8	....	2.42
8 × 14	....	3.35
14 × 28	....	2.09
28 × 48	....	1.28
48 × 0	....	2.13