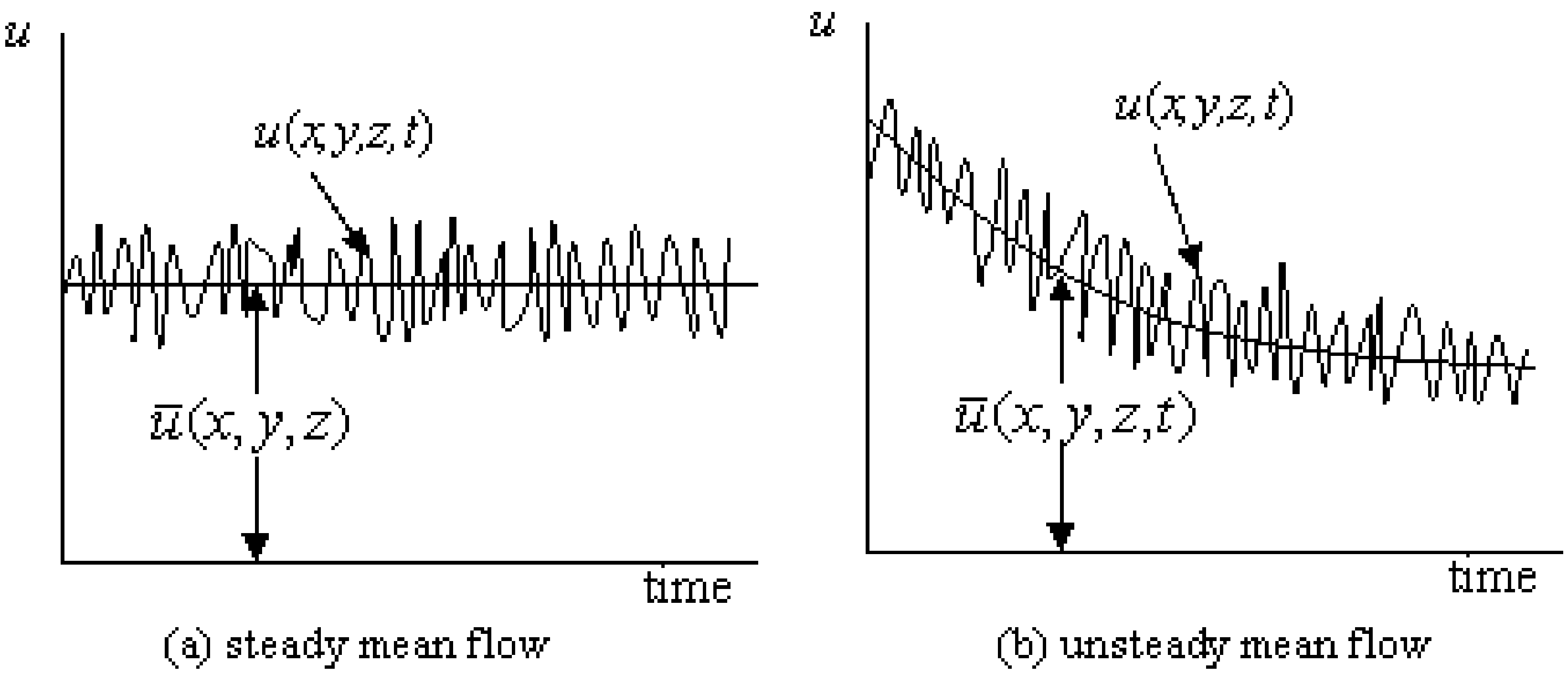
Turbulent velocity



Time dependence of velocity component in the x-direction in a turbulent flow

The derivation of the conservation laws for time-averaged flow begins with the transformation

$$u = \overline{u} + u', \qquad P = \overline{P} + P'$$

$$v = \overline{v} + v', \qquad T = \overline{T} + T'$$

$$w = \overline{w} + w'$$

where

$$\overline{u} = \frac{1}{\text{period}} \int_0^{\text{period}} u \, d(\text{time}) \qquad \int_0^{\text{period}} u' \, d(\text{time}) = 0$$

Definitions of time average property and fluctuating component are the foundation of a special kind of algebra that emerges in the process of substituting the $(\bar{\ }) + (\ )'$ decomposition into the mass, momentum, and energy equations and then time averaging these equations according to definition.

The rules (theorems) of this algebra are

$$\overline{u+v} = \overline{u} + \overline{v}$$

$$\overline{\overline{u}u'} = 0$$

$$\overline{uv} = \overline{u}\,\overline{v} + \overline{u'v'}$$

$$\overline{u^2} = \overline{u}^2 + \overline{u'^2}$$

$$\overline{\frac{\partial u}{\partial x}} = \frac{\partial \overline{u}}{\partial x}$$

$$\frac{\partial \overline{u}}{\partial t} = 0$$

$$\overline{\frac{\partial u}{\partial t}} = 0$$

Mass conservation equation

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial u'}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial v'}{\partial y} + \frac{\partial \overline{w}}{\partial z} + \frac{\partial w'}{\partial z} = 0$$

Integrating this equation term by term over time and applying rules

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0$$

Boundary layer equations

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0 \qquad (1)$$

$$\overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} = -\frac{1}{\rho}\frac{d\overline{P}}{dx} + \nu\frac{\partial^2 \overline{u}}{\partial y^2} - \frac{\partial}{\partial y}\left(\overline{u'v'}\right) \qquad (2)$$

$$\overline{u}\frac{\partial \overline{T}}{\partial x} + \overline{v}\frac{\partial \overline{T}}{\partial y} = \alpha\frac{\partial^2 \overline{T}}{\partial y^2} - \frac{\partial}{\partial y}\left(\overline{v'T'}\right) \qquad (3)$$

$$\tau_{\text{app}} = \mu \frac{\partial \overline{u}}{\partial y} - \rho \overline{u'v'} = \rho(\nu + \epsilon_M) \frac{\partial \overline{u}}{\partial y} \qquad \text{apparent shear stress}$$

$$-q''_{\text{app}} = k \frac{\partial \overline{T}}{\partial y} - \rho c_P \overline{v'T'} = \rho c_P (\alpha + \epsilon_H) \frac{\partial \overline{T}}{\partial y} \qquad \text{apparent heat flux}$$

$$\overline{u} \frac{\partial \overline{u}}{\partial x} + \overline{v} \frac{\partial \overline{u}}{\partial y} = -\frac{1}{\rho} \frac{d\overline{P}}{dx} + \frac{\partial}{\partial y} \left[\left(\nu + \epsilon_M \right) \frac{\partial \overline{u}}{\partial y} \right]$$

$$\overline{u} \frac{\partial \overline{T}}{\partial x} + \overline{v} \frac{\partial \overline{T}}{\partial y} = \frac{\partial}{\partial y} \left[\left(\alpha + \epsilon_H \right) \frac{\partial \overline{T}}{\partial y} \right]$$

Summary of longitudinal velocity expressions for the inner region of a turbulent boundary layer

$u^+(y^+)$	Range	Reference
$u^+ = y^+$ $u^+ = 2.5 \ln y^+ + 5.5$	$0 < y^+ < 11.6$ $y^+ > 11.6$	Prandtl and Taylor [9]
$u^+ = y^+$ $u^+ = 5 \ln y^+ - 3.05$ $u^+ = 2.5 \ln y^+ + 5.5$	$0 < y^+ < 5$ $5 < y^+ < 30$ $y^+ > 30$	von Kármán [12]
$u^+ = 14.53 \tanh(y^+/14.53)$ $u^+ = 2.5 \ln y^+ + 5.5$	$0 < y^+ < 27.5$ $y^+ > 27.5$	Rannie [13]
$\dfrac{du^+}{dy^+} = \dfrac{2}{1 + \{1 + 4\kappa^2 y^{+2}[1 - \exp(-y^+/A^+)]^2\}^{1/2}}$ $\kappa = 0.4 \qquad A^+ = 26$	All y^+	van Driest [14]
$u^+ = 2.5 \ln(1 + 0.4y^+) + 7.8[1 - \exp(-y^+/11) - (y^+/11) \exp(-0.33y^+)]$	All y^+	Reichardt [15]
$\dfrac{du^+}{dy^+} = \dfrac{1}{1 + n^2 u^+ y^+[1 - \exp(-n^2 u^+ y^+)]}$ $n = 0.124$ $u^+ = 2.78 \ln y^+ + 3.8$	$0 < y^+ < 26$	Deissler [16]
$y^+ = u^+ + A[\exp Bu^+ - 1 - Bu^+ - \frac{1}{2}(Bu^+)^2 - \frac{1}{6}(Bu^+)^3 - \frac{1}{24}(Bu^+)^4]$ (last term in u^{+4} may be omitted)	All y^+ $A = 0.1108$ $B = 0.4$	Spalding [17]

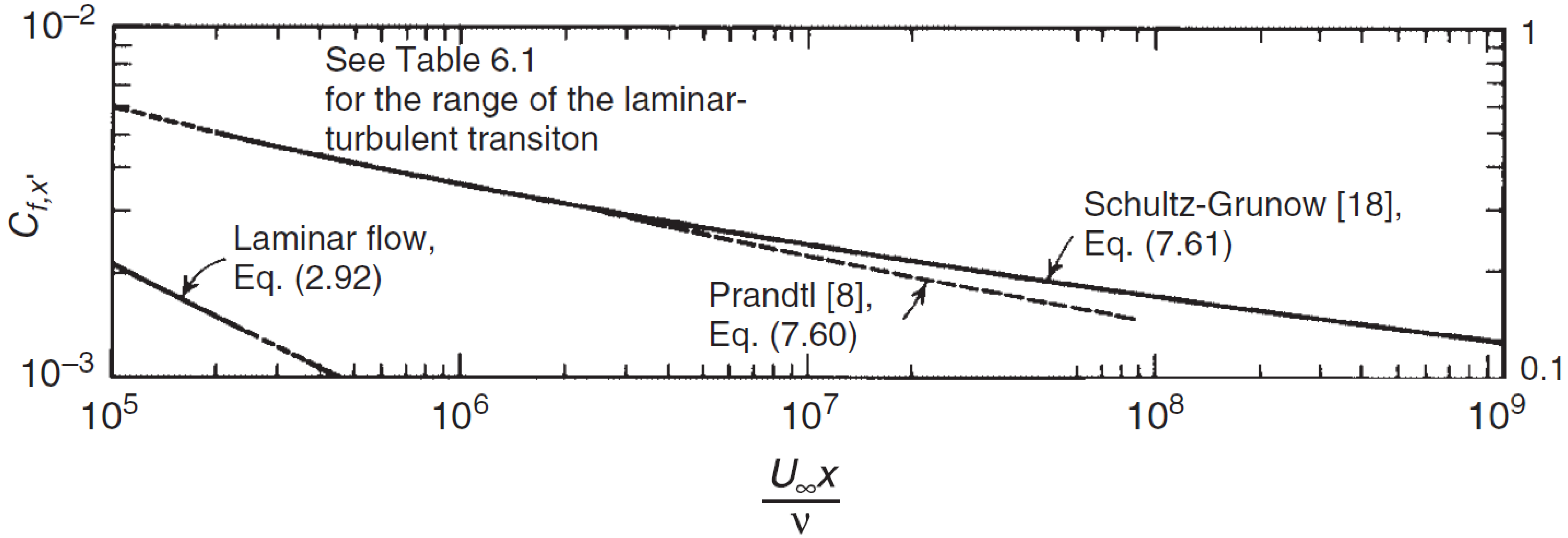
$$\frac{h}{\rho c_P U_\infty} = \frac{\frac{1}{2}C_{f,x}}{\mathrm{Pr}_t + \left(\frac{1}{2}C_{f,x}\right)^{1/2}\left[\mathrm{Pr}\, y^+_{\mathrm{CSL}} - B\,\mathrm{Pr}_t - (\mathrm{Pr}_t/\kappa)\ln\, y^+_{\mathrm{CSL}}\right]}$$

$\mathrm{Pr}_t \cong 0.9, \qquad \kappa \cong 0.41, \qquad y^+_{\mathrm{CSL}} \cong 13.2$ taking B ~= 5.1

$$\mathrm{St}_x = \frac{\frac{1}{2}C_{f,x}}{0.9 + \left(\frac{1}{2}C_{f,x}\right)^{1/2}(13.2\mathrm{Pr} - 10.25)}$$

Local Stanton number expression produced by von K´arm´an s three-region velocity profile

$$\mathrm{St}_x = \frac{\frac{1}{2}C_{f,x}}{1 + 5\left(\frac{1}{2}C_{f,x}\right)^{1/2}\left\{\mathrm{Pr} - 1 + \ln\left[1 + \frac{5}{6}(\mathrm{Pr} - 1)\right]\right\}}$$



See Table 6.1
for the range of the laminar-
turbulent transiton
Schultz-Grunow [18],
Eq. (7.61)
Laminar flow,
Eq. (2.92)
Prandtl [8],
Eq. (7.60)
$C_{f,x'}$
10^{-2}
10^{-3}
1
0.1
10^5
10^6
10^7
10^8
10^9
$\frac{U_\infty x}{\nu}$

Problem 1

By holding and rubbing the ball in his hand, the pitcher warms the leather cover of the baseball to 30°C. The outside air temperature is 20°C and the ball diameter is 7 cm. The pitcher throws the ball at 50 miles/h (22.35 m/s) to the catcher, who is stationed 18.5m away.

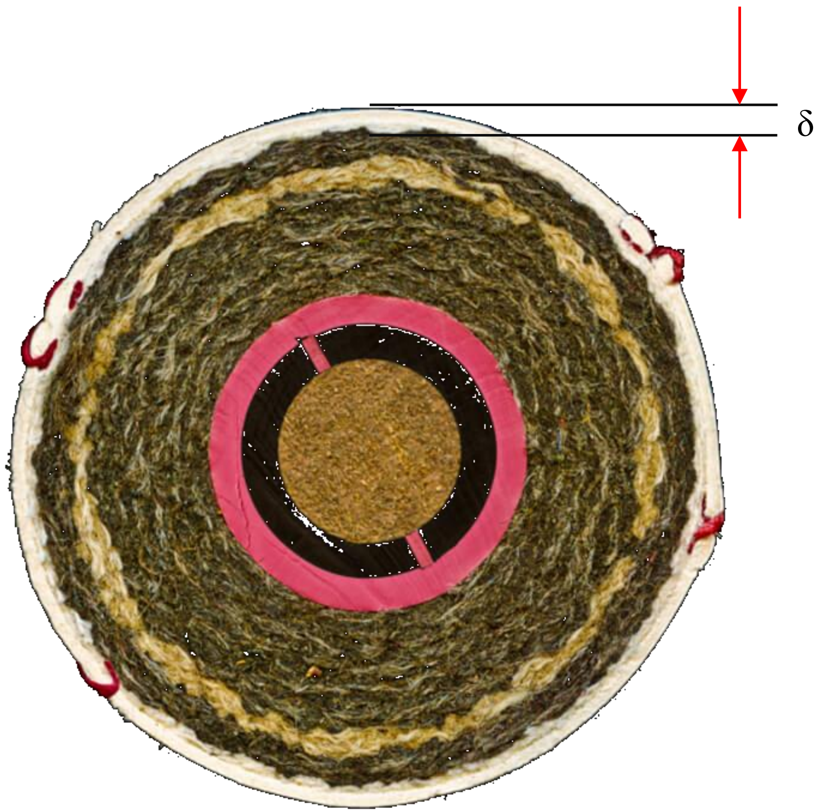
(a) Assume that the ball surface temperature remains constant, and calculate the heat transferred from the ball to the surrounding air during the throw.

(b) Calculate the temperature drop experienced by the leather cover to account for the heat transfer calculated in part (a). Assume that the thickness of the layer of leather that experiences the air cooling effect is comparable to the conduction penetration depth $\delta \sim (\alpha t)^{1/2}$, where α is the thermal diffusivity of leather. Validate the correctness of the constant surface temperature assumption made in part (a).

Data: $\alpha_{leather} = 0.001\ cm^2/s$; $c_{leather} = 1.5$ kJ/kg.K ; $\rho_{leather} = 860\ kg/m^3$;
at 20°C
$k_{air} = 0.025$ W/mK; $\nu_{air} = 0.15\ cm^2/s$; $Pr_{air} = 0.72$

Churchill and Bernstein correlation, for all Re_D and Pr>0.2

$$\overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1+\left(0.4/Pr\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$



Baseball ball cut section

Problem 2

1. Water flows with the velocity $U_\infty = 0.2$ m/s parallel to a plane wall. The following calculations refer to the position $x = 6$ m measured downstream from the leading edge. The water properties can be evaluated at 20◦C.

(a) A probe is to be inserted in the viscous sublayer to the position represented by $y^+ = 2.7$. Calculate the actual spacing y (mm) between the probe and the wall.

(b) Calculate the boundary layer thickness δ, and compare this value with the estimate based on the assumption that the length x is covered by turbulent boundary layer flow.

(c) Calculate the heat transfer coefficient averaged over the length x.

Problem 3

To examine the wall shear stress on a flat plate, maintained at a constant temperature of 30 °C ($T_w = 30°C$). An air stream of uniform velocity ($u_\infty = 10\ m/s$) and temperature ($T_\infty = 10°C$) at a distance from the wing is blown over it. Flow is assumed to be turbulent from $x = 0$. At distance $x\ (= 1\ m)$ from the plate leading edge in the direction of the flow, the wall shear stress τ_w equal to $0.23\ N\ m^{-2}$ is measured. In the same location, a wire of diameter $1\ \mu m$ is placed at 0.19 mm above the wall to measure the fluid mean temperature, which is found to be equal to $25°C$.

(a) Find the wall heat flux at the location of the measurement.

(b) Compare with the Colburn correlation and find the % difference relative to the direct measurement.

(c) Now the temperature measuring wire is replaced by a thermocouple. The thermocouple sensor has a spherical-shaped diameter $d \approx 50$ µm and is placed at 4.5 mm above the wall. What is the expected temperature?

(d) What is the sensor size-induced uncertainty in the mean temperature measurement at this distance from the wall?

(e) The uniform velocity of air is now changed 50 m/s. Do you expect any change in the measured temperature? (For both measured temperature, initially by the wire located 0.19 mm above the wall and then by the thermocouple located 4.5 mm above the wall) Explain your answer with proper justification.

Assume the wall co-ordinte is valid till $y^+ < 200$

Physical properties of air:
$\rho = 1.13\ kg\ m^{-3}$; $\nu = 16.7 \times 10^{-6}\ m^2\ s^{-1}$; $k = 0.026\ W\ m^{-1}\ K^{-1}$; $Pr = 0.7$; $C_p = 10^3\ J\ kg^{-1}\ K^{-1}$

The empirical constants are given as $Pr_t \approx 0.9$; $\kappa \approx 0.41$ and $y^+_{CSL} \approx 13.2$ for the above mention conditions

Problem 4

A flat sheet (tabular) iceberg drifts over the ocean as it is driven by the wind that blows over the top. The temperature of the surrounding seawater is 10°C, and the relative velocity between it and the iceberg is 10 cm/s. The length of the iceberg in the direction of drift is L. Calculate the corresponding wind velocity when the atmospheric air temperature is 40°C.

Problem 5

Air flows with velocity 3.24 m/s over the top surface of the flat iceberg discussed in Problem 7.14. The air temperature outside the boundary layer is 40°C, and the ice surface temperature is 0°C. The length of the iceberg in the direction of air flow is $L = 100$ m. The ice latent heat of melting is $h_{sf} = 333.4$ kJ/kg.

Calculate the L-averaged heat flux deposited by the air flow into the upper surface of the iceberg (model this surface as flat). Calculate, in millimeters per hour, the rate of melting caused by this heat flux, that is, the erosion (thinning) of the ice slab.