$\therefore \boxed{y_s = \frac{m}{2u}} \rightarrow$ Total width.

strength of source - vol. flow rate from/to the source/sink per unit depth.

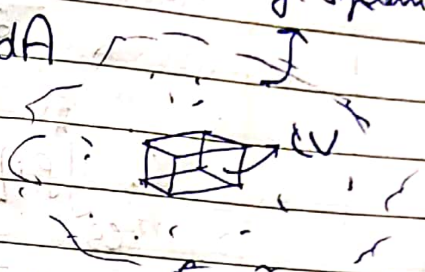
SDG

$\frac{\text{Extensive Property}}{m} =$ Intensive Property.

Consider a control vol. surrounded by a huge ~~surrounding~~ system having air :-

Total mass influx of fluid into CV $= \int_{CS} \rho V dA$

CS → control surfaces



~~Mass~~ Change in mass present in CV $= \cancel{\frac{d}{dt}(\rho \forall)}$

$$\frac{d}{dt}\int_{CV} \rho d\forall$$

$$\int_{CS} \rho V dA + \frac{d}{dt}\int_{CV} \rho d\forall = \text{accumulation of fluid mass in CV.}$$

(for N = m).

Now, for any extensive property N and its intensive property η we will have,

$$\therefore \underbrace{\int_{CV} \rho \frac{N}{m} d\forall}_{\text{extensive property per unit volume.}} + \int_{CS} \rho \frac{N}{m} V dA = \frac{dN}{dt}\Big|_{system}.$$

↳ brought from system to CV.

$$\boxed{\left.\frac{dN}{dt}\right|_{system} = \frac{\partial}{\partial t}\int_{CV} \rho\eta \, d\forall + \int_{CS} \rho\eta V \, dA}$$

η : intensive property

Since, system is very large so change in mass of the system is <u>zero</u>.

Now taking $N = m \therefore \eta = 1$

$$\therefore \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho V dA = 0.$$

In steady state, ρ does not depend on time

hence, $\displaystyle \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall \, \eta = 0$

$$\therefore \int_{CS} \rho\eta V dA = 0 \Rightarrow \int_{CS} \rho\eta V dA = 0$$

$$\Rightarrow \int_{CS} V dA = 0 \quad \text{(considering m as N so } \eta = 1\text{)}$$

Continuity eqn. in integral form

* Mass into the control surface is -ve

* Mass coming out of control surface is +ve, (because area vector points outward).

For $N = P$, $\eta = v$.

$$\therefore \left.\frac{dP}{dt}\right|_{syst.} = \frac{\partial}{\partial t}\int_{CV} \rho v \, d\forall + \int_{CS} \rho v^2 dA$$

surface force

body force

$F_S + F_B$

$\quad ||$

$$F_{syst} = \frac{\partial}{\partial t}\int_{CV} \rho v \, d\forall + \int_{CS} \rho v^2 dA$$: Eqn. of motion for a system of fluid flowing.

* v : velocity w.r.t. the control volume.

* This is the force on the control volume. It is the equal and opposite force applied on CV by ext. agent to maintain its properties.

For steady state ; $F_S + F_B = \underbrace{\int_{CS} \rho v (v dA)}_{\text{convective momentum}}$.

This arises due to ~~press. ptnd~~ adds the ~~press. energy term in Bernoulli's eqn.~~

* This eqn. of motion is a vector eqn.

1st law of Thermod. :-

$$\dot{Q} + \dot{W} = \left.\frac{dE}{dt}\right|_{syst.}$$

$$E_{syst.} = \int_{mass} e \, dm = \int_{\forall} e \rho \, d\forall$$

$$e = u + \frac{v^2}{2} + gz$$

u : internal energy ~~due~~ ~~to pressure~~

$\frac{v^2}{2}$: kinetic energy

gz : Potential energy

* Think about Bernoulli's eqn.

$$\vec{F} = \vec{F_S} + \vec{F_B} = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho\, dA + \int_{CS} \vec{V}\, \underbrace{\rho \vec{V}\cdot d\vec{A}}_{\text{const. since mass flow rate}}$$

Momentum eqn. for inertial control volume

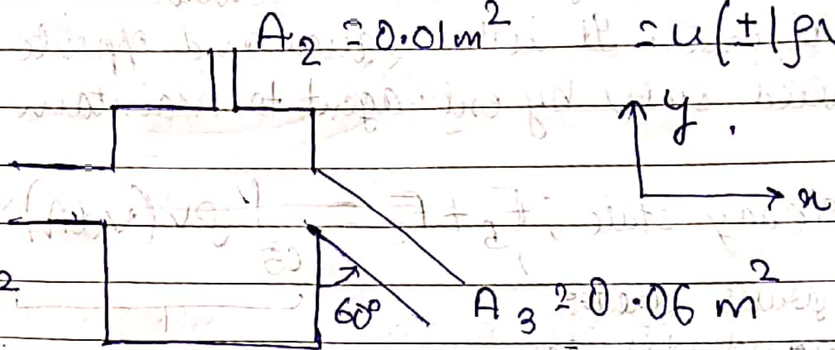
Steps to evaluate $\rho\vec{V}\cdot d\vec{A}$:-

$$\rho \vec{V}\cdot d\vec{A} = \rho|V(dA)|\cos\alpha = \pm|\rho V(dA)\cos\alpha|,$$

∴ Momentum entering in x-direction = $u\rho\vec{V}\cdot d\vec{A}$

$= u(\pm|\rho V(dA)\cos\alpha|)$

ex:



$\rho = 1050\ kg/m^3$, $v_1 = 4\hat{i}$ m/s,

$v_2 = -8\hat{j}$ m/s

Find $\vec{V_3}$ at steady state,

$$\int_{CS} \rho\vec{V}\cdot d\vec{A} = 0.$$

$$1050\left(4\hat{i}\cdot(-0.05)\hat{i} + (-8\hat{j})\cdot(0.01\hat{j}) + (V_{3x}\hat{i} + V_{3y}\hat{j})\cdot(0.03\sqrt{3}\hat{i} + 0.03\hat{j})\right)$$

$\Rightarrow$ $-0.2 - 0.08 + 0.03 V_{3x} \sqrt{3} - 0.03 V_{3y} = 0$

$V_{3x} = V_3 \sin 60°$, $V_{3y} = -V_3 \cos 60°$.

$$0.28 = 0.03 V_3 \cdot \frac{3}{2} + \frac{V_3 \, 0.03}{2}$$

$$= 0.06 V_3 \Rightarrow |V_3| = \frac{\cancel{28}\,14}{\cancel{6}\,3} = 4.67 \text{ m/s}.$$

$$\vec{V}_3 = \left(\frac{7}{\sqrt{3}}\hat{i} - \frac{7}{3}\hat{j}\right) \text{ m/s}.$$

$$= (4.04\hat{i} - 2.33\hat{j}) \text{ m/s}$$

ex: Find net efflux of momentum through above CV in SS.

$$\frac{d\vec{p}}{dt} = \int_{CS} \rho \vec{V} (\vec{V} \cdot d\vec{A})$$

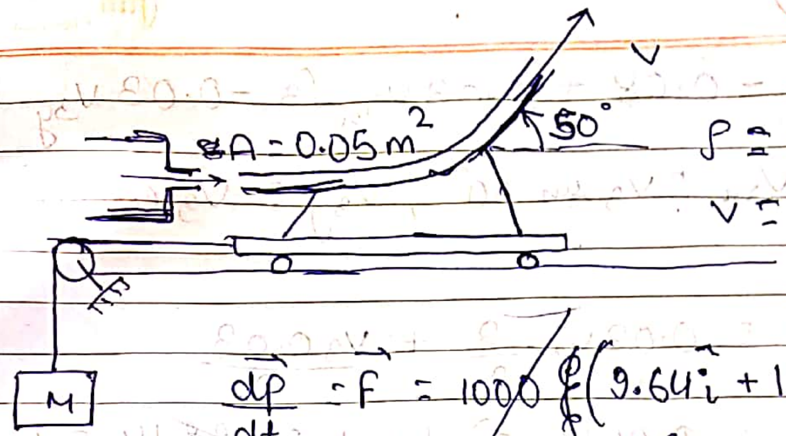
$$= 1050\left((4\hat{i})(-0.2) - (8\hat{j})(-0.08) + (4.04\hat{i} - 2.33\hat{j})(0.28)\right)$$

$$= 1050(-0.8\hat{i} + 0.64\hat{j} + 1.\cancel{8}1312\hat{i} - 0.6524\hat{j})$$

$$= 1050(0.3312\hat{i} - 0.0124\hat{j})$$

$$= (\overset{347.8}{\cancel{48.68}}\hat{i} - 13.02\hat{j}) \text{ kg m/s} \quad \text{Ans}.$$

Magnitude = $\overset{348.04 \text{ kg m/s}}{\cancel{51.36 \text{ m/s}}}$ Ans ex:



$\rho = 1000\ kg/m^3$

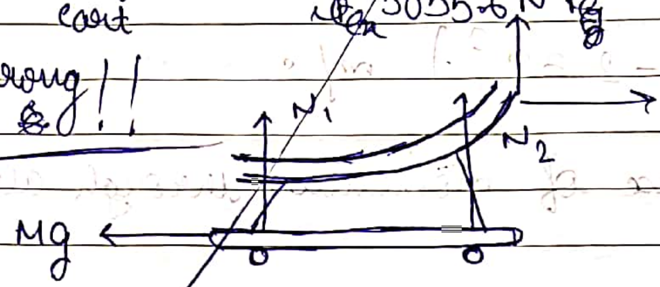
$V = 15 m/s$

$\frac{d\vec{p}}{dt} = \vec{F} = 1000\ (9.64\hat{i} + 11.49\hat{j})$

$\qquad\qquad f_x \qquad f_y \qquad (0.31 + 0.44)$

$F_{\text{acting on cart}} = 2988.4\hat{i} + 5055.6\hat{j}$

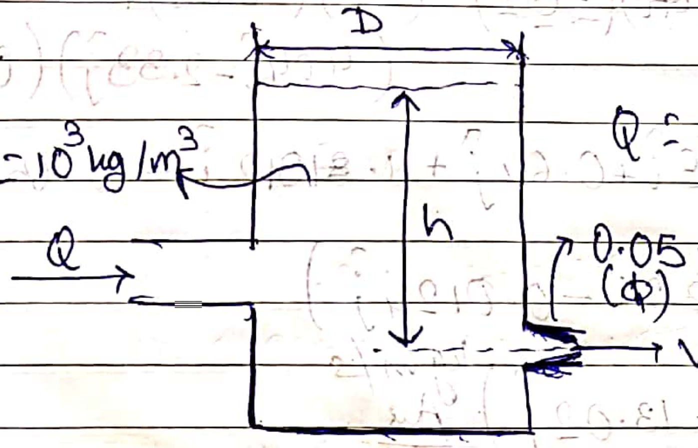
wrong!!



$M = 304.94\ kg$ (taking $g = 9.8 m/s^2$).

*. Take momentum of the water entering also.

ex:



$Q = 1 \times 10^{-2}\ m^3/s$

$V_{exit} = \sqrt{2gh}$

$t = 0,\ h = 2.743\ m$

Find $h(t)$ and steady state height of water in the tank.

Net ~~m~~ mass flow rate $= \frac{dm}{dt} = 10^3 (10^{-2} - \sqrt{2gh} \cdot (\frac{\pi (0.05)^2}{4}))$

$$\rho \pi D^2 \frac{dh}{dt} = 10^{-2} - \sqrt{2gh}\left(\frac{\pi(0.05)^2}{4}\right)$$

$$\int_{2.743}^{h} \frac{dh}{0.01 - 0.009\sqrt{h}} = \int_0^t \frac{dt}{A_{top}}$$

Put $h = u^2$

$dh = 2u\,du$

$$\int_{1.66}^{u^2} \frac{2u\,du}{0.01 - 0.009u} = \frac{t}{A_{top}}$$

* In Bernoulli's eqn, we either take absolute or gauge pressure because the diff in pressure matters in the end.

* When we consider force on a body due to a fluid, we consider a gauge pressure.

ex: $\vec{F_S} + \vec{F_B} = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho\, dA + \int_{CS} \rho \vec{V}(\vec{V}\cdot\vec{dA})$

$W = 44.48\,N$ — wt. of nozzle

P at ~~opening~~ entering $= 1.324 \times 10^5\ N/m^2$

P at leaving $= 10^5\ N/m^2$ (1 atm).

$\forall = 2.46 \times 10^{-3}\ m^3$

D at enter $= 0.076\ m$

D at leave $= 0.025\ m$.

$\vec{V_1} = -1.83\hat{j}\ m/s$, $\vec{V_2} = V_2 \cos 30^\circ\,\hat{i} - V_2 \sin 30^\circ\,\hat{j}$

$$F_{SX} + F_{BX} = \cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, dA}^{\,0} + \int \rho u (\vec{v}\cdot\vec{dA})$$

~~(because steady state)~~.

$$F_{SY} + F_{BY} = \cancel{\frac{\partial}{\partial t}\int_{CV} v\rho\, dA}^{\,0} + \int \rho v (\vec{v}\cdot\vec{dA})$$

(steady state)

~~$F_{SX} = P_{1g} A_1 \; P_{2g} A_2 \cos$~~ surface force on top $= P_{1g} A_1$

surface force at bottom $= P_{2g} A_2 (-\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j})(-\hat{j})$

$P_{2g} = 0$ (gauge press. at bottom).

$P_{gauge} = P_{absolute} - P_{atm}$.

$\therefore F_{SX} = 0$, $F_{BX} = 0$

$u_1(-\rho V A_1) + u_2(+\rho V A_2)$

$u_1 = 0$

$1.83 \times 0.076^2 = V_2 \times 0.0254^2 \Rightarrow V_2 = 16.4 \text{ m/s}$.

$R_X = 16.4 \times \cos 30^\circ \left(1000 \times 16.4 \times \pi \times \frac{0.0254^2}{4}\right)$

$= 117.8 \text{ N}$

→ force experienced on control volume.

F_{BY}

R_Y

ex:

$F_{SY} = P_{1g}A_1 \times 10^{5} \times \ldots = 600.63\,N.$

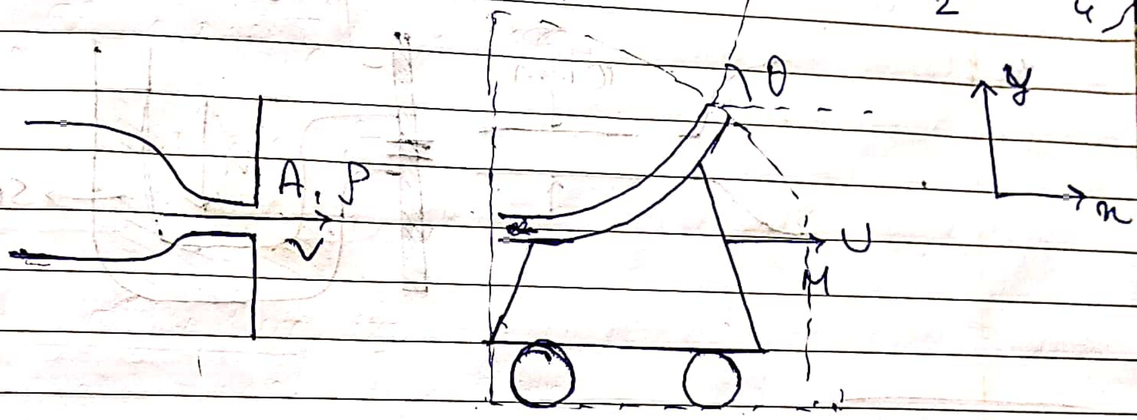
$F_{BY} = 10^3 \times 2.46 \times 10^{-3} \times 10 + 44.48 = 69.08$

~~= 669.71 N~~

~~R_Y = 669.71 N~~

$R_Y = 600.63 + 69.08 - 10^4\left(1.83 \times \frac{\pi \times 0.076^2}{4} \times \ldots + \frac{16.38}{2} \times \frac{\pi \times 0.054^2}{4}\right)$

ex:



velo. wrt control volume = $(V-u)\,\hat{i}$

$F_{SX} + F_{BX} = 0 \qquad R_x = -\left[-(V-u)\rho(V-u)A + (V-u)\cos\theta\, \rho(V-u)A\right]$

$= \rho A (V-u)^2 (1-\cos\theta)$

$= 2\rho A (V-u)^2 \sin^2\frac{\theta}{2}$ Ans

$F_{SY} = 0, \quad F_{BY} = -W\hat{j}$

$R_Y = -W - (V-u)\sin\theta\, \rho(V-u)A$

$= -(W + \rho A (V-u)^2 \sin\theta)$. Ans.

Power $= 2\rho A (V-u)^2\, u \sin^2\frac{\theta}{2}$ Ans $(R_x \cdot u)$.

$\frac{d(\text{Power})}{du} = 0$

SDG Bernoulli's Eqn :-

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = const.$$ (conservation of energy).

Valid for :-
→ Ideal fluid (non-viscous/ inviscid)
→ Incompressible flow.

} - Limitation.

* Flow of gases can be considered incompressible for very less flow rate.

→ valid when there is ~~no~~ no $\underbrace{\text{~~sudden~~ sharp change}}$ in area.
↓ causes an additional pressure drop.

→ Cannot be used across a pump / turbine (or something which adds / removes energy from the flowing fluid).

Looking from the perspective of 1st Law of Thermodi

$$\dot{Q} + \dot{W} = \frac{dE}{dt}, \quad E_{syst} = \int_{\text{mass of system}} e\,dm = \int_{\forall_{syst.}} e\,\rho\,d\forall$$

$$e = u + \frac{V^2}{2} + gz$$

↳ energy per unit mass.

$$\frac{dN}{dt} = \frac{\partial}{\partial t}\int_{CV} \eta\rho\,d\forall + \int_{CS} \eta\rho\vec{V}\cdot d\vec{A} \quad \text{where } \eta = \frac{N}{M}$$

Now, we use N as Total Energy.

$$\therefore \dot{Q} + \dot{W} = \frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall + \int_{CS} e\rho(\vec{V}\cdot d\vec{A})$$

work done on system → −ve
work done by system → +ve

system centric.

$$\dot{W} = \dot{W}_s + \dot{W}_{normal} + \dot{W}_{shear} + \dot{W}_{other}$$

$\dot{W}_s$ → by a pump/turbine

$\dot{W}_{other}$ → electric/magnetic field work.

$\dot{W}_{normal}$ → the press.-term missing here comes up from this (∵ press. does work while applied normal to a surface).

$$\dot{Q} + \cancel{\dot{W}_s}^{0} + \cancel{\dot{W}_{oth}}^{0} + \dot{W}_{normal} = \cancel{\frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall}^{0} + \int_{CS} e\rho\vec{V}\cdot d\vec{A}$$

shaft work

$$\dot{Q} = \dot{m}(u_2 - u_1) + \dot{m}\left(\frac{P_2}{\rho} - \frac{P_1}{\rho}\right) + \dot{m}g(z_2 - z_1) + \int_{A_2}\frac{V_2^2}{2}\rho V_2\, dA_2 - \int_{A_1}\frac{V_1^2}{2}\rho V_1\, dA_1$$

(assum. : steady state &, $W_{shaft} = 0$).

$\dot{W}_{shear}$ is considered 0 here, since it occurs on the surface where $V=0$, due to no slip

Considering avg. velocity :-

$$\int_{A_2}\frac{\rho V_2^2}{2} V\, dA = \alpha\int_{A}\frac{\bar{V}^2}{2}\rho V\, dA = \frac{\alpha\, \dot{m}\bar{V}^2}{2}$$

$$\alpha = \frac{\int_A \rho V^3 dA}{\dot{m}\bar{V}^2} \longrightarrow \text{KE coefficient}$$

while considering a pipe, $V_z = V_{max}\left(1-\left(\frac{r}{R}\right)^2\right)^2$

For laminar flow, $\alpha = 2$.

$$\frac{u}{U} = \left(\frac{y}{r}\right)^{1/n} = \left(1-\frac{r}{R}\right)^{1/n}$$

$U \rightarrow$ max. velo.

$$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)}$$

$$\frac{\dot{Q}}{\dot{m}} = u_2 - u_1 + \frac{P_2}{\rho} - \frac{P_1}{\rho} + g(z_2 - z_1) + \frac{\alpha_2\bar{V}_2^2}{2} - \frac{\alpha_1\bar{V}_1^2}{2}$$

$\dot{m} \rightarrow$ Head Loss.

$$\underbrace{\frac{P_1}{\rho} + \frac{\alpha_1\bar{V}_1^2}{2} + gz_1}_{\text{Mech. energy/mass at a flow cross-section}} = \left(\frac{P_2}{\rho} + \frac{\alpha_2\bar{V}_2^2}{2} + gz_2\right) + \underbrace{\left[(u_2 - u_1) - \frac{\dot{Q}}{\dot{m}}\right]}_{\text{Total Head loss } (h_{LT})}$$ SNG

h_{LT} = Total head loss $= h_L + h_{LM}$

$h_L \rightarrow$ major $\rightarrow$ losses occuring in straight horiz. pipe of uniform cross-section

$h_{LM} \rightarrow$ minor $\rightarrow$ the addnl. pressure drop caused due to a pump / measuring device in a pipe, the loss due to this is minor.

For straight horiz. pipe,

$$\frac{P_1}{\rho} + \cancel{\frac{\alpha_1 \bar{V}_1^2}{2}} + \cancel{g z_1} = \frac{P_2}{\rho} + \cancel{\frac{\alpha_2 \bar{V}_2^2}{2}} + \cancel{g z_2} + h_L + \underbrace{\cancel{h_{LM}}}_{\text{neglected}}$$

$$\Rightarrow \frac{P_1}{\rho} = \frac{P_2}{\rho} + h_L$$

$$\Rightarrow \boxed{h_L = \frac{P_1 - P_2}{\rho}} = \frac{\Delta P}{\rho}$$

* From Hagen-Poiseaulle eqn., whatever ΔP we get, its only for laminar flow

$$h_L = \frac{64}{Re} \cdot \frac{L}{D} \cdot \frac{\bar{V}^2}{2}$$ only for laminar flow

$$h_L = f \cdot \frac{L}{D} \cdot \frac{\bar{V}^2}{2}$$

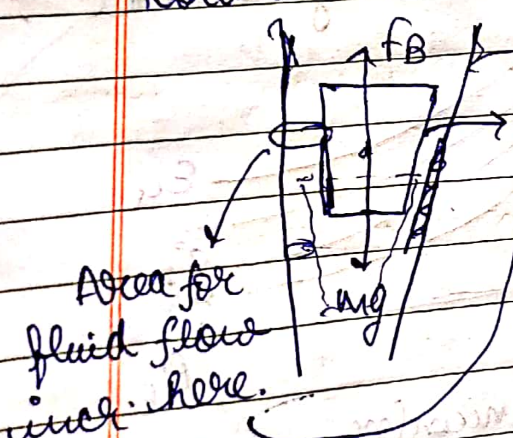
f → Friction factor

$\boxed{f = \frac{64}{Re} \text{ for laminar flow}}$

SNG

Flow meters :-
- → Venturimeter / orifice meter / V-element meter
- → Rotameter
- → Turbine meter
- → Hot wire anemometer.
- → Pitot tube (measures local velo. and not avg. velocity).

Rotameter :-



This slant is not very prominent in real.

∴ Apart from mg and f_B, we have a drag force acting there on float as well.

$$C_D = \frac{\text{Drag force} / \text{Projected Area}}{\frac{1}{2} \rho u^2}$$

Assum: this for a spherical body,

$$C_D = \frac{\frac{6\pi\mu r u}{\pi r^2}}{\frac{1}{2}\rho u^2} = \frac{3\pi\mu\rho u^3}{\pi} = \frac{24}{R_{EP}}$$

Particle Reynold's number.

SDG

* Pressure drop fn. in a straight horiz. pipe :-

$$\Delta P = f(\underbrace{\mu, \rho}_{\text{props.}}, \underbrace{L, D}_{\text{geometry}}, \underbrace{V}_{\text{operational param.}}, \underbrace{\varepsilon}_{\text{roughness.}})$$

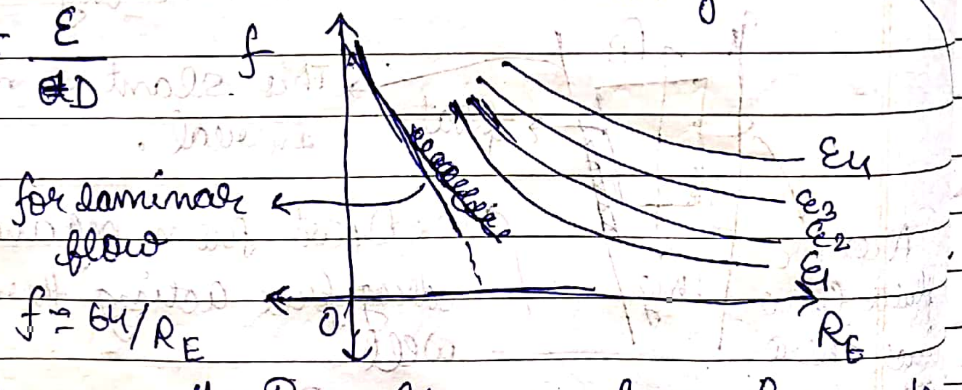
$\rho, V, D, \mu \longrightarrow$ composition of Reynold's no.

$$\frac{D_1 V_1 \rho_1}{\mu_1} = \frac{D_2 V_2 \rho_2}{\mu_2}$$ (2 diff. fluids in 2 diff. situations but exhibiting same kind of flow).

$\therefore \Delta P$ is a fn. of just Re and ε.

A graph of f (friction factor) v/s Re for diffn. values of ε - Moody Chart.

Roughness $= \frac{\varepsilon}{D}$



* Beyond a specific Reynold's number, f ceases to change with Re, we get a family of ‖le lines

Minor losses (h_{LM}) :-

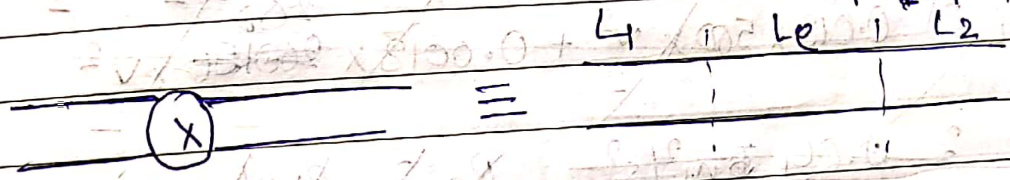
$h_{LM} = \frac{K\bar{V}^2}{2}$ → loss co-efficient

or

$h_{LM} = f \cdot \frac{L_e}{D} \cdot \frac{\bar{V}^2}{2}$ → equivalent length

the additional length brought about in a str. horiz. pipe accounting for the pressure drop due to valve or a turbine. (this length has same press. drop as that across the pump).

→ can also arise due to bends in pipes.



* Types of valves :-

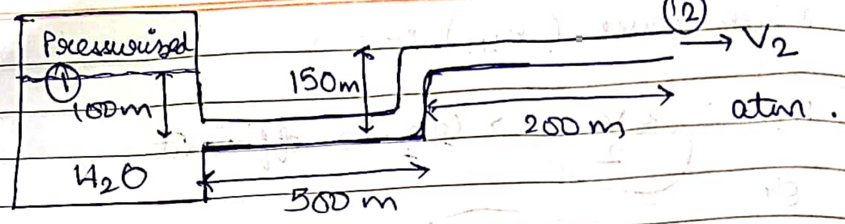
a) gate (valve is either completely closed or open).

b) globe → this has ~~less~~ much more pressure drop across than gate valve.

* In cases involving sudden change (like expansion or compress.), K values are preferred to proceed with L_e.

* ~~In cases~~

ex: (1)



$Q = 0.14\ m^3/s$, $D = 0.2\ m$, $K = 0.5$

$\mu = 1.3\times10^{-3}\ Ns/m^2$, $\frac{\varepsilon}{D} = 0.0013$, $\frac{L_e}{D} = 12$

Find gauge press.;

$V = \frac{0.14\times4}{\pi\times(0.2)^2} = 4.46\ m/s$

$\cancel{\frac{P_1}{\rho} = \frac{P_{atm}}{\rho} + g(150) + h_{LT}}$

Apply Bernoulli's eqn between (1) and (2).

~~h_{LT} → minor~~ $\cancel{h_{LM} = 0.013 \times \frac{12.4}{0.02} \times \frac{V^2}{2} \times 2 = 0.06\ m^2/s^2}$

$\cancel{h_L = 0.013\times500\times\frac{V^2}{2} + 0.0013\times200\times\frac{V^2}{2}}$

$\cancel{= 0.041\ m^2/s^2}$ $\cancel{Re = \frac{\rho V d}{\mu} = \frac{1000\times4.46\times0.2}{1.3\times10^{-3}}}$

$\cancel{P_1 = 1000(101.325 + 1500 + 9.1)}$

Flow is highly turbulent hence $\alpha_1 = \alpha_2 = 1$.

$Re = \frac{1000\times4.46\times0.2}{1.3\times10^{-3}} = 6.86\times10^5$

For finding f, we consider the Moody chart, which comes out to be 0.021.

$$\frac{P_1}{1000} + 0 + 0 = 0 + \frac{4.46^2}{2} + 10(50) + h_{LT}$$

$$h_{LT} = h_L + h_{LM}$$

$$h_L = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} = 0.021\left(\frac{500+150+200+2L_e}{0.2}\right)\times\frac{4.46^2}{2}$$

h_{LM}

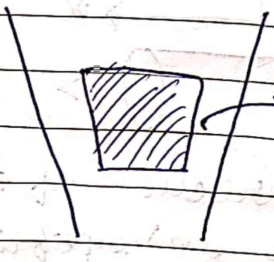
SNG

$$h_{LM} = \frac{0.5 \times 4.46^2}{2}$$

m/s

$= 0.06\ m^2/s^2$

SNG



$\rightarrow$ float (V_f, ρ_f)

Force balance of float :-

$$V_f \rho_f g - V_f \underbrace{\rho_L}_{\text{density of liquid}} g = \underbrace{A_f}_{\text{projected area of float}} \underbrace{C_D}_{\text{drag coefficient}} \rho \cdot \frac{u_{max}^2}{2}$$

$$\text{Flow rate} = q = u_{max} \cdot \frac{\pi}{4} \left(\underbrace{D_t^2}_{\text{diam of tube}} - \underbrace{D_f^2}_{\text{diam of float}} \right)$$

Q-R

height of float

$$= u_{max} \cdot \frac{\pi}{4} \left((D_f + a\underbrace{h}_{\text{height of float}})^2 - D_f^2 \right)$$

$a \rightarrow$ slope of float from top.

$$= u_{max} \cdot \frac{\pi}{4} \left(2 D_f a h + \underbrace{a^2 h^2}_{\text{neglected since slope is negligible}} \right)$$

$$q = \frac{\pi u_{max} D_f a h}{2}$$

Flow through Porous Media :-

Superficial velo. $= \dfrac{Q}{A}$ $Q \rightarrow$ flow rate.

Porosity $= \phi = \dfrac{\text{void volume}}{\text{total volume}} = \dfrac{A_{open}}{A}$

(void fraction)

$A_{open} \rightarrow$ actual area available for flow.

Interstitial velo. $= \dfrac{Q}{A_{open}}$

$=$ superficial velo. $/ \phi$

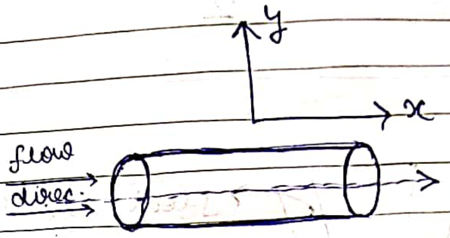
$\boxed{\phi < 1}$

Darcy's Law :-

$$V_{superficial} = -\frac{K}{\mu} \cdot \frac{\partial P}{\partial x}$$

→ $V_{superficial}$: volumetric flux

→ K : permeability (unit : m^2)



Pressure decr. in direction of flow hence, a -ve sign in $V_{superficial}$.

ex:

$Q = \frac{0.001}{60} m^3/s$

$\phi = 0.2, \quad K = 10^{-12} m^2.$

$$V_{superficial} = \frac{0.001}{60 \times \frac{\pi}{4}(1^2)} = 2.12 \times 10^{-5} \; m/s.$$

$$V_{interstitial} = \frac{2.12 \times 10^{-5}}{0.2} = 1.06 \times 10^{-4} \; m/s.$$

$$\frac{\partial P}{\partial x} = -\frac{2.12 \times 10^{-5} \times 10^{-3}}{10^{-12}} = -2.12 \times 10^{4} \; Pa/m$$

$$\Delta P = -2.12 \times 10^{4} \times 10 = -0.2 \; MPa \; Ans.$$

Hydraulic diameter (D_H) of porous medium :-

$$D_{H(PM)} = 4 \cdot \frac{\text{wetted area}}{\text{wetted perimeter}}$$

$$\text{wetted area} = \frac{\text{Total void volume}}{L}$$

$$\text{wetted perimeter} = \frac{\text{Total surface area of particles}}{L}$$

$D_{H(PM)} = \dfrac{4\varepsilon V}{\text{Total no. of particles} \times \text{surface area of 1 particle}}$ → void ratio

Total no. of particles $= \dfrac{\text{Total vol. of particles}}{\text{vol. of 1 particle}}$

$= \dfrac{(1-\varepsilon)V}{\text{vol. of 1 particle}}$

$$= \frac{4\varepsilon V}{(1-\varepsilon)\cancel{V} \times \dfrac{\text{surface area of 1 particle}}{\text{vol. of 1 particle}}}$$

Assum., particle is spherical :-

$$= \frac{4\varepsilon}{(1-\varepsilon)\dfrac{4\pi r^2}{\frac{4}{3}\pi r^3}} = \frac{4\varepsilon r}{3(1-\varepsilon)} = \frac{2\varepsilon D}{3(1-\varepsilon)}$$

For non-spherical particle :-

$$= \frac{4\varepsilon}{(1-\varepsilon)\dfrac{6}{\phi_s D_p}}$$

sphericity (ϕ_s); D_p → equ. diameter (diam. of sphere having same volume as particle).

sphericity $(\phi_s) = \dfrac{\text{surface area of sphere with diameter } D_p}{\text{surface area of actual particle}}$.

$$\boxed{D_{H(PM)} = \frac{2\varepsilon \phi_s D_p}{3(1-\varepsilon)}}$$

$Re_{(PM)}$ = Reynold's no. for porous media

$$= D_{H(PM)} \frac{V_{superficial}}{\varepsilon} \cdot \frac{\rho}{\mu}$$

$\cancel{\frac{Re(PM)}{D_H(PM)}}$ $Re(PM) = \frac{2}{3} \cdot \frac{\cancel{\varepsilon}\phi_s D_p}{1-\varepsilon} \times \frac{V_{superficial}}{\cancel{\varepsilon}} \times \frac{\rho}{\mu}$

$$= \frac{2\phi_s D_p V_{superf.} \rho}{3(1-\varepsilon)\mu}.$$

Friction factor $= f = \frac{150}{Re(PM)}$ → Kozeny Carman eqn.

(for lower Reynold's numbers).

$f = 1.75$ → stabilizes to this value for higher Reynold's number

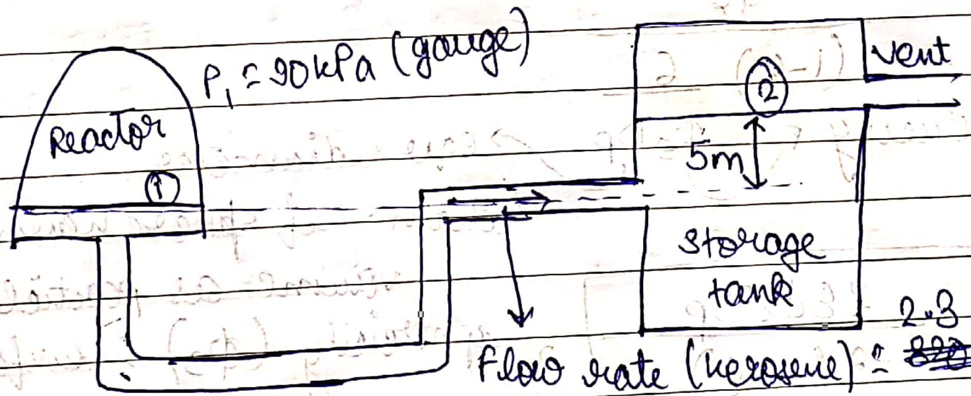
↳ Blake - Plummer eqn.

Ergun eqn. :- $f = \frac{150}{Re(PM)} + 1.75.$

$$\frac{\Delta P}{L} \cdot \frac{\phi_s d_p}{\rho \bar{V}_0^2} \cdot \frac{\varepsilon^3}{1-\varepsilon} = \frac{150\mu(1-\varepsilon)}{\phi_s d_p \bar{V}_0 \rho} + 1.75$$

SDG

ex: (2)



Flow rate (kerosene) = 2.3 m³/min ~~820 kg/m³~~

$P_2 = 0$ kPa (gauge) $D = 0.15$ m

$\rho_{kerosene} = 820$ kg/m³.

$$\frac{P_1}{\rho} + \cancel{\frac{V_1^2}{2}}^{0} + gh_1 = \cancel{\frac{P_2}{\rho}}^{0} + \cancel{\frac{V_2^2}{2}}^{\simeq 0} + gh_2 + h_L + h_{LM}$$

$$\Rightarrow \frac{90 \times 10^3}{820} + 10(-5) = h_{LT} = 59.76 \text{ J/kg}$$

$h_{LT} \rightarrow h_L + h_{LM}$

$\bar{V} = \frac{2.3\times4}{\pi\times0.15^2\times60} = 2.17\,m/s.$

$Re = \frac{820\times2.17\times0.15}{\mu}$ $\qquad \frac{\mu}{\rho} = 1.05\times10^{-6}\,m^2/s$

$= \frac{2.17\times0.15}{1.05\times10^{-6}} = 0.31\times10^{6}.$

$f = \frac{64}{Re} = 0.00021$

$K_{in} = 0.2,\; K_{exit} = 1.0,\; \frac{L_{eq}}{D} = 12$

$L_{eq} = 12\times0.15 = 1.8\,m$

$$h_{LT} = \underbrace{f\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2}}_{\text{major loss in whole pipe}} + \underbrace{h_{L_{inlet}}}_{\text{minor loss (inlet)}} + \underbrace{h_{L_{exit}}}_{\text{minor loss (exit)}} + \underbrace{3h_{L_{elbow}}}_{\text{minor loss (due to elbows)}}$$

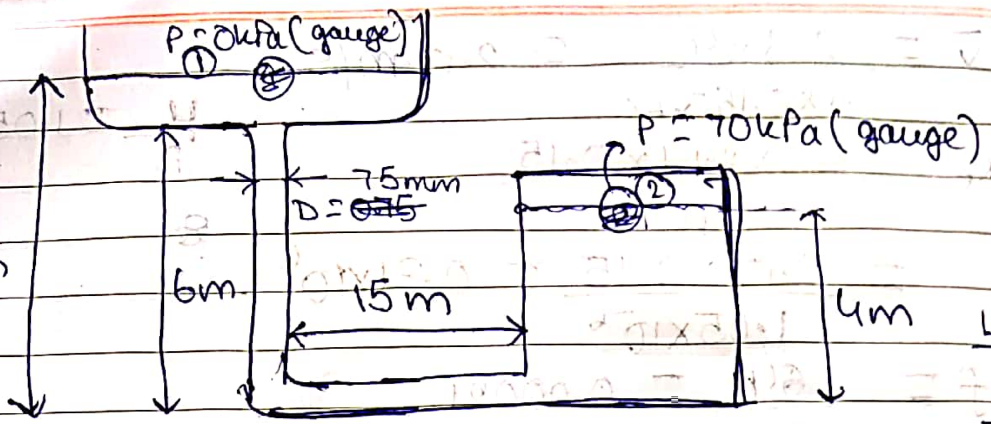
$$\cancel{59.76 = (0.00021)\frac{L}{0.15} + \frac{0.2\times2.17^2}{0.15} + \frac{1.0\times2.17^2}{0.15} + 3\times0.00021\times\frac{1.8}{0.15}\times\frac{2.17^2}{0.15}}$$

$$59.76 = \frac{2.17^2}{2}\left(0.00021\times\frac{L}{0.15} + \frac{0.2}{0.15} + \frac{1}{0.15} + 3\times0.00021\times12\right)$$

$$25.38 = L(1.4\times10^{-3}) + 8 + 7.56\times10^{-3}$$

mastërmind

ex : (3)



$\frac{L_e}{D} = 12$

$\Rightarrow L_e = 0.9$

$\frac{\varepsilon}{D} = 0.002$ (Fully rough region of Moody diagram).

$\rho = 999\ kg/m^3$, ν (kinematic viscosity) $= 1.1 \times 10^{-6}\ m^2/s$ (μ/ρ).

$$\cancel{\frac{P_1}{\rho}}^{0} + \cancel{\frac{\bar{V}_1^2}{2}} + g z_1 = \frac{P_2}{\rho} + \cancel{\frac{\bar{V}_2^2}{2}} + g z_2 + h_{LT}$$ (when flow from ① to ②)

$$10 \times 7 = \frac{70 \times 10^3}{999} + 10 \times 4 + h_{LT}$$

$h_{LT} = -40.1\ J/kg$ which is not true

$\therefore$ Flow is from ② to ①

$\therefore h_{LT} = +40.1\ J/kg$

$$40.1 = f \times \left(\frac{21 + L_e}{0.075}\right) \times \frac{\bar{V}^2}{2}$$

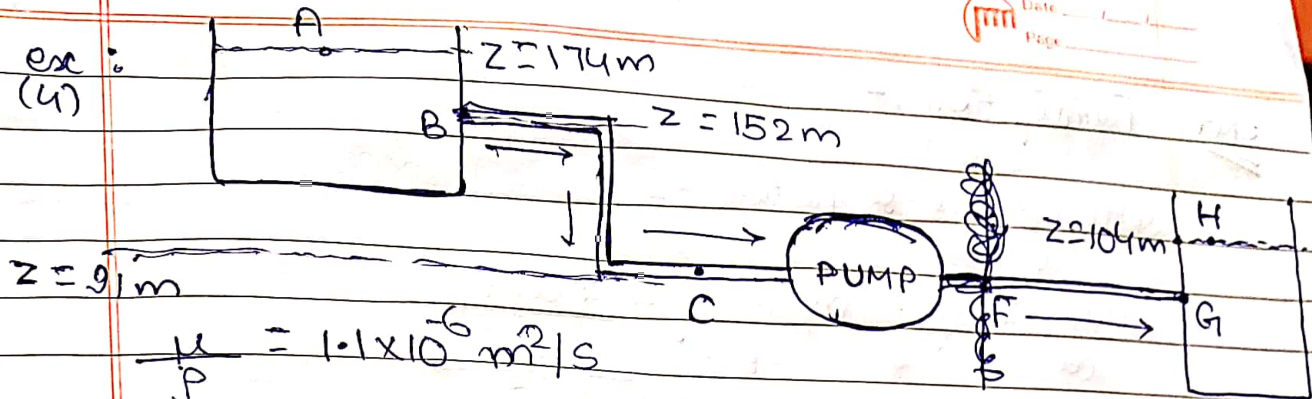
$$\cancel{= \frac{64 \times 1.1 \times 10^{-6}}{0.075 \times V} \times \frac{21.9}{0.075} \times \frac{V^2}{2}}$$

$$= \underbrace{0.023}_{\text{obtained from Moody diagram}} \left(\frac{21 + L_e}{0.075}\right) \frac{\bar{V}^2}{2}$$

$\Rightarrow \bar{V} = 3.46\ m/s$

$$Q = 3.46 \times \pi \times \frac{(0.075)^2}{4} = 0.015\ m^3/s.$$

ex: (4)



$\frac{\mu}{\rho} = 1.1 \times 10^{-6}\ m^2/s$

$\varepsilon = 0.26$ mm

$Q = 1310\ L/s = 1.31\ m^3/s$ $\quad D = 0.508$ m.

$\bar{V} = \dfrac{1.31}{\frac{\pi \times (0.508)^2}{4}} = 6.46$ m/s. $\quad f = 0.017$ (moody chart).

Betwn. A and C :-

$\frac{P_A}{\rho}^{0} + \frac{\bar{V}_A^2}{2} + g(174) = \frac{P_C}{\rho} + \frac{\bar{V}_C^2}{2} + g(91) + h_{LT}$

$P_C = (g(83) - h_{LT})\rho$

$h_{LT} = h_{L(760m)} +$

Betwn. F and G :-

$\frac{P_F}{\rho} + \frac{\bar{V}_F^2}{2} + g(174) = \frac{P_H}{\rho}^{0} + \frac{\bar{V}_H^2}{2} + g(104) + h_{LT}$

$\frac{P_F}{\rho} = h_{LT} - g(70)$

$h_{LT} = h_{L(760m)} + h_{\text{valve + bends}}$

$= \frac{0.017 \times 760}{0.508} \times \frac{6.46^2}{2} + \frac{6.46^2}{2} \times 0.017 ($

SNG Fluidization :-

settling velo. of particle :-

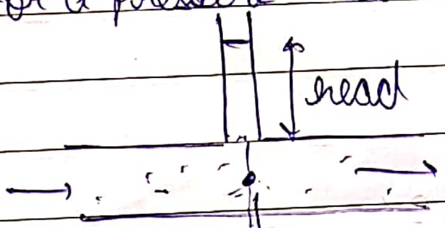
$$m\frac{du}{dt} = mg - m\left(\frac{\rho}{\rho_p}\right)g - \underbrace{\frac{C_D u^2 \rho A_p}{2}}_{\text{drag force}}$$

At the point, $\frac{du}{dt} = 0$; particle has now achieved terminal velocity

$\frac{u_t}{\bar{V}_{OM}} > 100$, for particles to not get carried away on slightest change in velo. above $\bar{V}_{OM}$

Cavitation / NPSH (net positive suction head) / Priming :-

For a pressure head :-

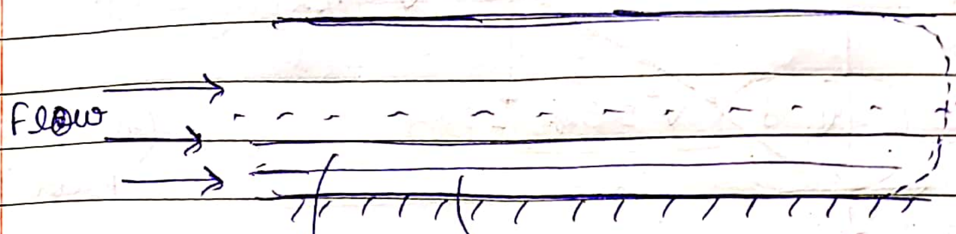


press. at this point is denoted by height of column which is called head.

Priming : act of filling the pump with ~~water~~ liquid before pumping liquid from it.

Scanned by CamScanner

Turbulent flow and universal velocity distribution :-



Sort of velo. profile for turbulent flow.

viscous sub-layer
(within this velocity profile is ~~linear~~ laminar)

Transition/
Buffer Layer
(layer betw. the laminar and turbulent eddy currents region)

*Most of the K.E. of eddies exist ~~between~~ in the buffer layers ~~as~~ and the outer portion of turbulent core.

*velo. gradient at center line is zero.

Dimensionless velocity :-

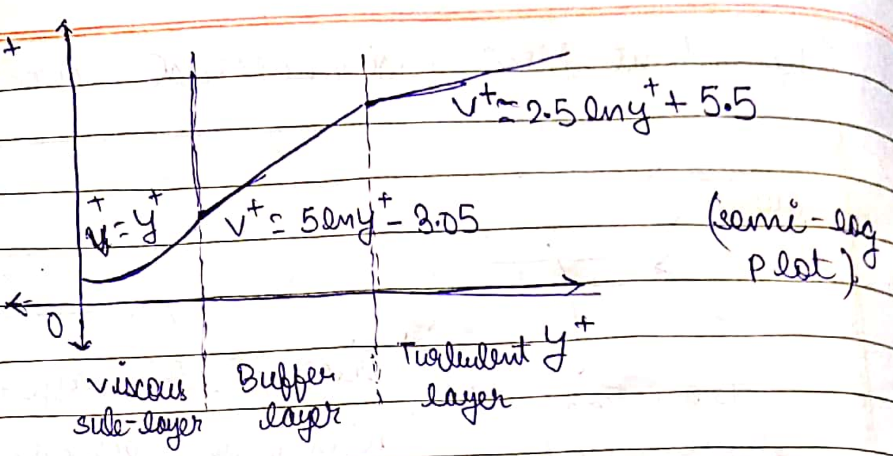
$V^+ = \frac{V}{V^*}$ → velo. at any point

V^* → friction velocity $= \bar{V}\sqrt{\frac{f}{2}}$

$\bar{V}$ → avg. velocity

f → friction factor

$V^* = \sqrt{\frac{\tau_w}{\rho}}$



v_c^+ (at centre) $= 2.5\ln y_c^+ + 5.5$ ($\because$ it is in turbulent region).

$$y_c^+ = \frac{r_w v^* \rho}{\mu}$$

$$v^+ - v_c^+ = 2.5\ln\left(\frac{y^+}{y_c^+}\right)$$

$$\Rightarrow v^+ = v_c^+ + 2.5\ln\left(\frac{y^+}{y_c^+}\right)$$

$$\bar{v} = \frac{1}{\pi r_w^2}\int_{0=y}^{r_w=y} v(2\pi(r_w-y))(dy)$$

$$= \frac{5\left(\frac{\mu}{\rho}\right)^2}{r_w^2 v^*}\int_0^{y_c^+}\left[0.4u_c^+ + \ln\left(\frac{y^+}{y_c^+}\right)\right]\left[y_c^+ - y^+\right]dy^+$$

$$\Rightarrow \frac{\bar{v}}{v^*} = v_c^+ - 3.75 = \frac{v_{max}}{v^*} - 3.75$$

$$\Rightarrow \frac{\bar{v}}{v^*} = \frac{v_{max}}{\bar{v}\sqrt{\frac{f}{2}}} - 3.75$$

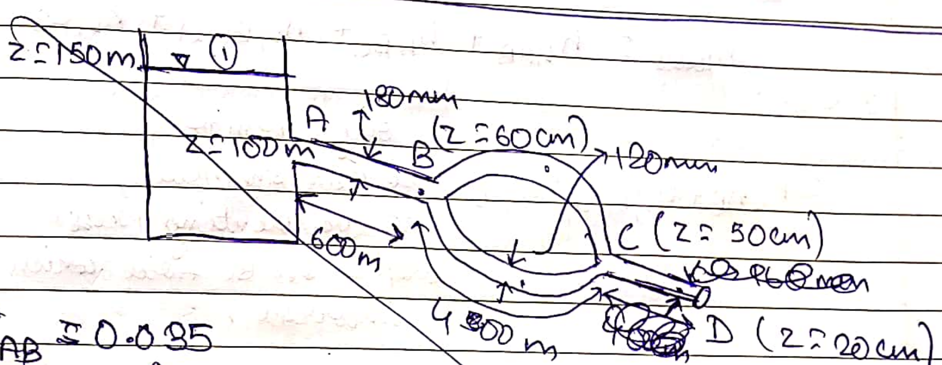
$$\Rightarrow \frac{\bar{v}}{v_{max}} = \frac{1}{1 + 3.75\sqrt{\frac{f}{2}}}$$

$$y_c^+ = \frac{r_w \bar{V}\sqrt{f/2}}{(\mu/\rho)} = \frac{\overset{\text{diameter}}{D}\bar{V}}{2}\frac{\sqrt{f/2}}{\mu/\rho} = \frac{Re}{2}\sqrt{\frac{f}{2}}$$

$$u_c^+ = \frac{1}{\sqrt{f/2}} + 3.75 \quad , \quad u_c^+ = 2.5\ln y_c^+ + 5.5$$

$$\frac{1}{\sqrt{f/2}} = 2.5\ln\left(Re\sqrt{\frac{f}{8}}\right) + 1.75.$$

SDG



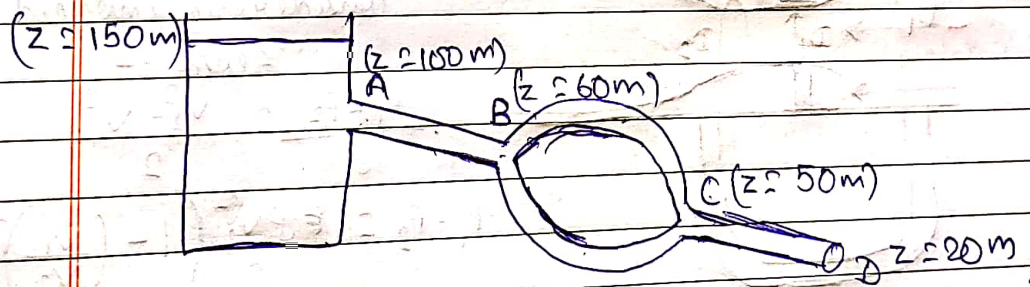
$f_{AB} = 0.035$

$f_{BC} = 0.025$

$f_{CD} = 0.030$

Betwn. point 1 and A :-

$$0 + \frac{v_1^2}{2} + g(150) = P_{A,gauge} + \frac{V_{AB}^2}{2} + g(100)$$



AB :-	BC (up) :-	BC (down) :-
L = 600m	L = 500 m	L = 400m
ϕ = 180mm	ϕ = 120mm	ϕ = 160mm
f = 0.035	f = 0.025	f = 0.030.

CD :-

$f = 0.02, \quad \frac{L_{CD}}{D_{CD}} = 2813.$

$$P_{A,gauge} + \frac{1}{2}V_A^2 + g(100)$$

$$= P_{B,gauge} + \frac{1}{2}V_B^2 + g(60)$$

$$V_{AB} \times \frac{\pi (0.18^2)}{4} = V_{B(up)} \times \frac{\pi (0.12)^2}{4} + V_{B(low)} \times \frac{\pi (0.16)^2}{4}$$

$$= V_{CD} \times \frac{\pi D_{CD}^2}{4} \quad \ldots \; (1)$$

Bernoulli between ~~..~~ surface and D :-

→ neglecting velocity heads.

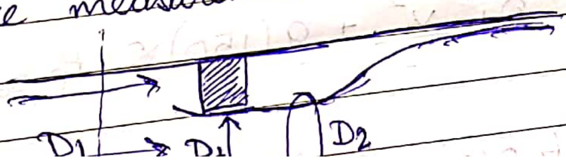
$$\cancel{P_1}^{0} + \frac{1}{\cancel{2}} \cancel{V_1^2}^{0} + gZ_1 = \cancel{P_D}^{0} + \frac{1}{\cancel{2}} \cancel{V_D^2} + gZ_D + h_{Losses}$$

$$h_{Losses} = h_{L,AB} + \underbrace{h_{L,BC(u)} + h_{L,BC(l)}}_{\text{only 1 term to be used otherwise, we are counting press. drop across BC twice which is incorrect.}} + h_{L,CD}$$

$$= h_{L,AB} + h_{L,BC(u)} + h_{L,CD}$$

$V_{AB} = 3.79 \, m/s$, $V_{CD} = 1.2 \, m/s$, $V_{B(u)}$

Pressure measurement :-



considering inviscid flow :-

$$P_1 - P_2 = \frac{V_2^2 - V_1^2}{\ldots}$$

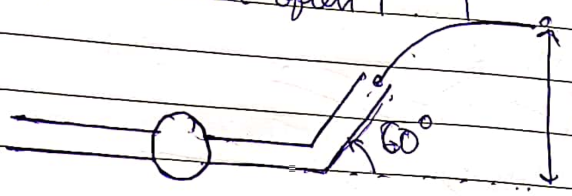
Empirical discharge coeff. :-

$$C = \frac{\text{Actual mass flow rate}}{\text{Theoretical mass flow rate}}$$

Venturimeter :-
a) costly
b) accurate
c) recovers press. very easily.
d) co-eff. is most often 1.

orifice meter :-
a) easy to manuf. (hence, cheapest).
b) not accurate (just rough value)

exc :



Let velo. of water jet be v.

$\therefore (v\sin 60^\circ)^2 = 2 \times 10 \times 10$

$\Rightarrow v^2 = 200 \times \frac{4}{3}$

$\Rightarrow v = 16.33$ m/s.

$L_{hose} = 30$ m

$D = 8$cm

$\varepsilon = 0.0008$ m

$\frac{\varepsilon}{D} = \frac{0.0008}{8\times10^{-2}} = 0.01$

$f = 0.038$ (from Moody chart).

Energy added ~~Major loss~~ (due to loss) $= 0.038 \times \frac{30}{8\times10^{-2}} \times \frac{16.33^2}{2}$

$= ~~2000~~ ~~2000 J/kg~~ 1900.02$ J/kg.

Total head = energy due to loss + velo. head

$= 1900.02 + \frac{16.33^2}{2} = 2033.35$ J/kg. Ans.

d = 2.5cm, $V_{hose} = \frac{16.33 \times \cancel{\pi}(2.5)^2}{\cancel{\pi} \times 8^2} = 1.59$ m/s.

Total head by pump now $= 0.038 \times \frac{30}{8\times10^{-2}} \times \frac{1.59^2}{2} + \frac{1.59^2}{2}$

$= 19.28$ J/kg.

Yes, head considerably reduced,
so, he has studied fluid mechanics.