Problem - 4

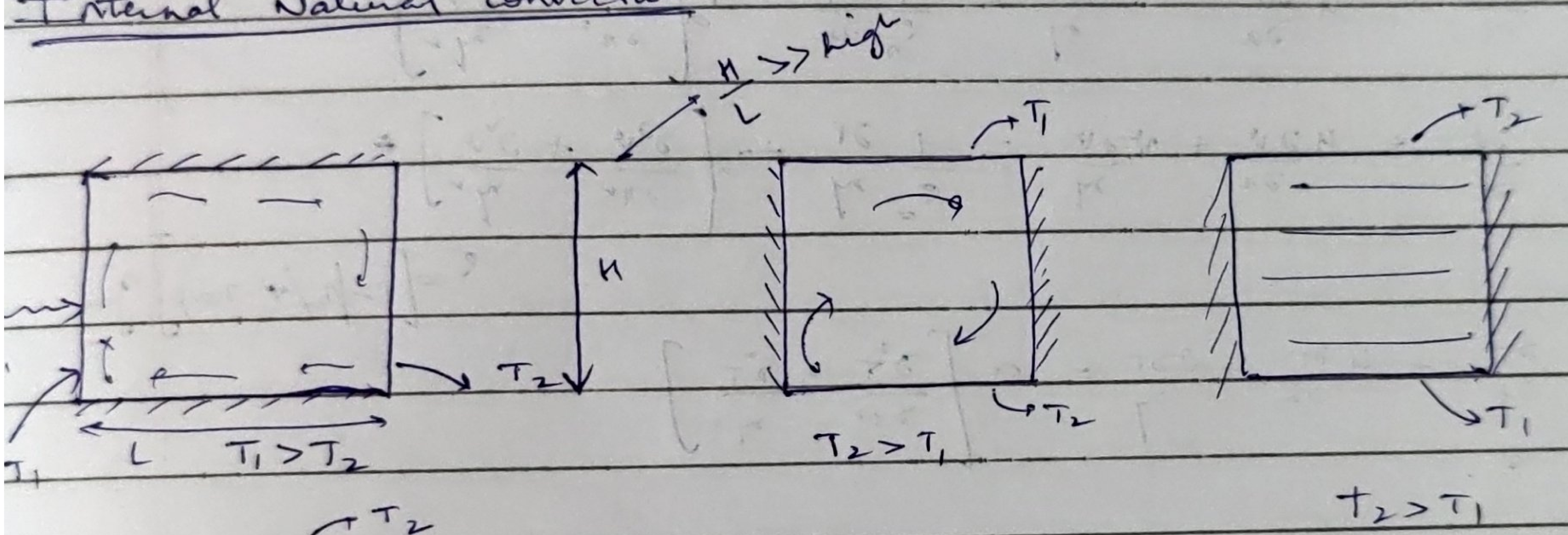
Similarity solution

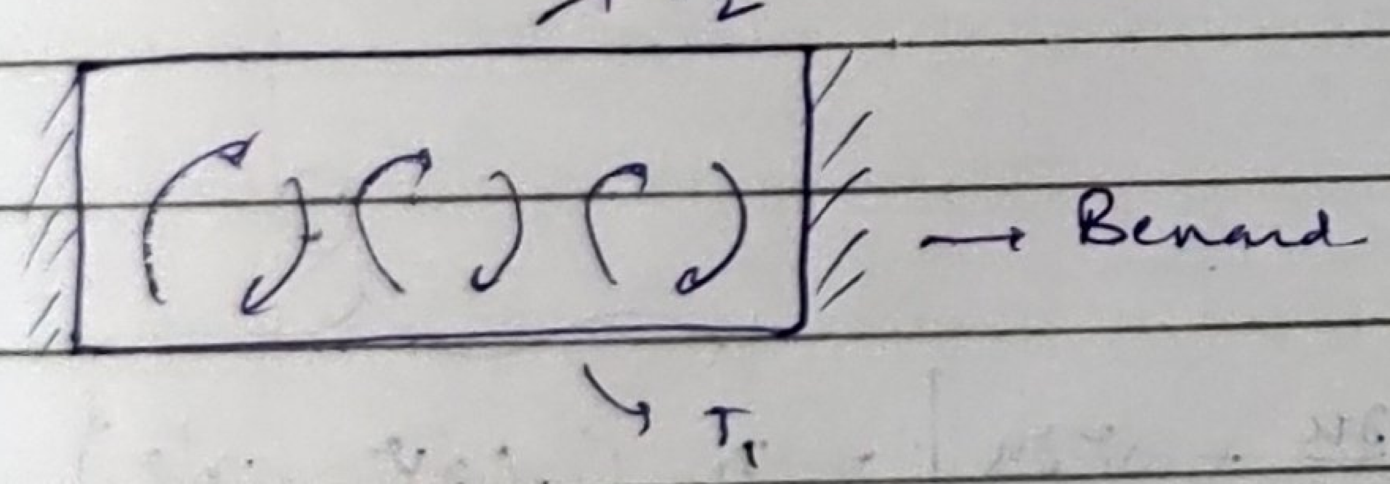
heat transfer results including the effect of turbulence

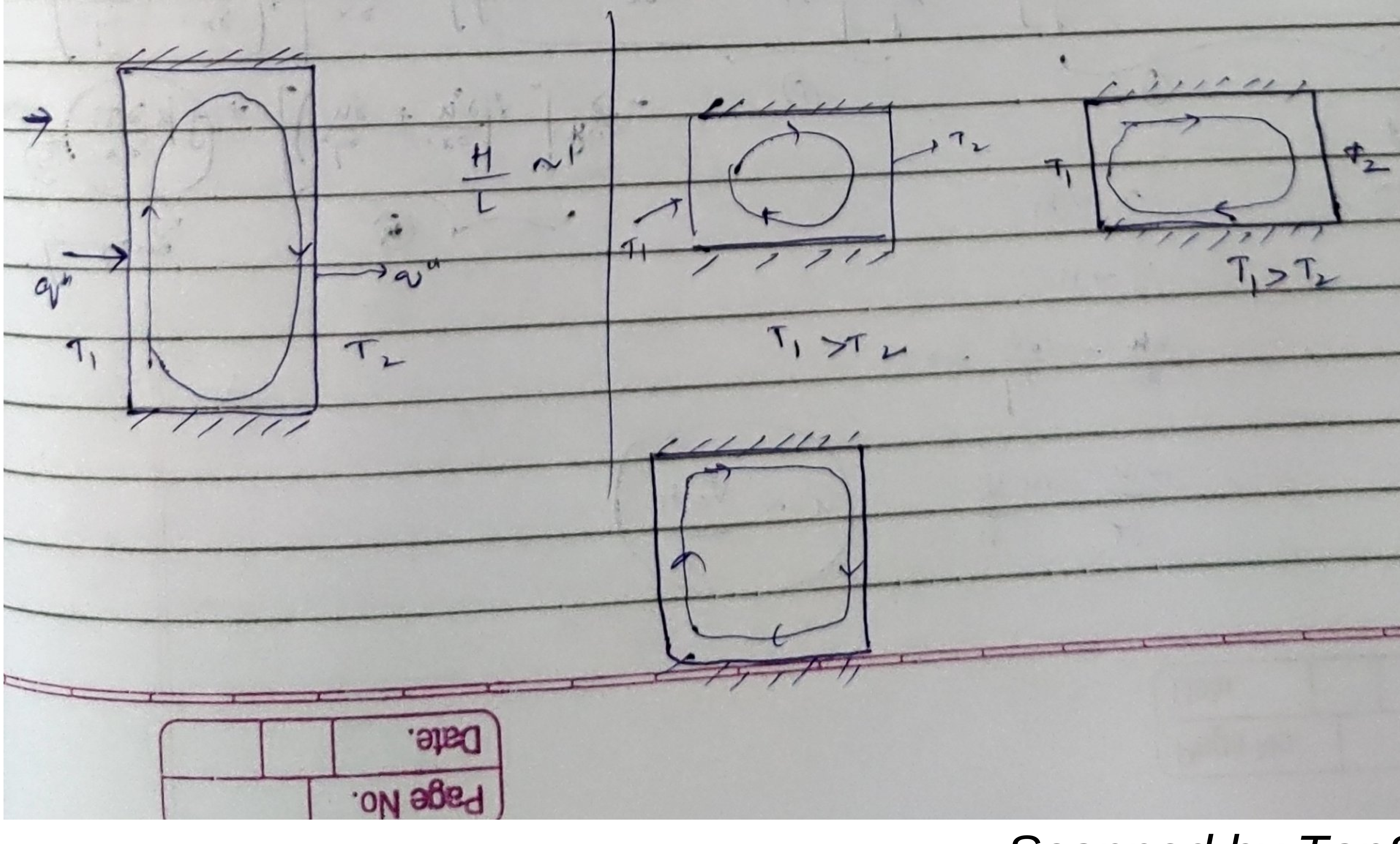
Inclined walls

→ Horizontal walls

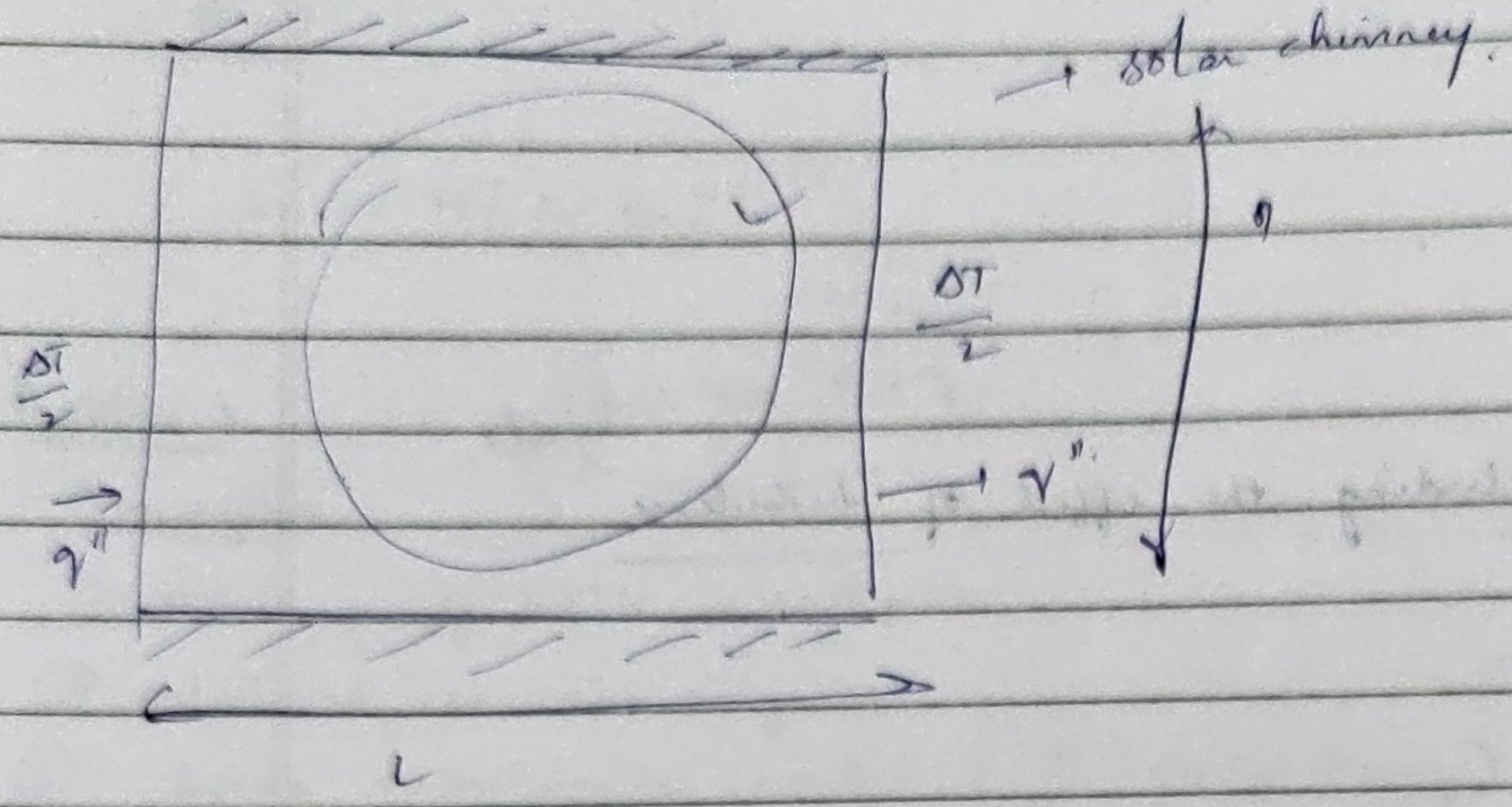
Internal Natural convection







* Internal natural convection



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_\infty}\frac{\partial P}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_\infty}\frac{\partial P}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] - \rho_\infty\left[1-\beta(T-T_\infty)\right]g$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right]$$

→ From momentum balance

$$\underbrace{\frac{\partial}{\partial x}\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right]}_{1} - \underbrace{\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]}_{2} = \underbrace{\frac{\partial}{\partial x}\left[\nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\right]}_{3} - \underbrace{\frac{\partial}{\partial y}\left[\nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right]}_{4} + \underbrace{g\beta\frac{\partial T}{\partial x}}_{5 \text{ buoyancy}}$$

(benard convection)

$x \sim \delta_T$

$y \sim H$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\sim \frac{u}{\delta_T} \sim \frac{v_c}{H}$$

$$u \sim \frac{v_c\,\delta_T}{H}$$

$$\sim \frac{1}{\delta_T}\underbrace{\left[\frac{v_c}{t} \sim u\frac{v_c}{\delta_T} \sim v_c\frac{v_c}{H}\right]}_{①} \quad \sim \frac{1}{H}\underbrace{\left[\frac{u}{t} \sim \frac{u^2}{\delta_T} \sim \frac{v_c u}{H}\right]}_{②}$$

$$\sim \frac{1}{\delta_T}\left[\nu\left(\frac{v_c}{\delta_T^2} \sim \frac{v_c}{H^2}\right)\right]_{③} \quad \sim \frac{1}{H}\left[\nu\left(\frac{u}{\delta_T^2} \sim \frac{u}{H^2}\right)\right]_{④}$$

$$\underbrace{\sim g\beta\frac{\Delta T}{\delta_T}}_{⑤}$$

$$\Rightarrow \sim \left[\cancel{\frac{v_c}{\delta_T t}} \sim \frac{v_c^2}{H\delta_T} \sim \frac{v_c^2}{H\delta_T}\right] \sim \left[\cancel{\frac{v_c\delta_T}{H^2 t}} \sim \frac{v_c^2\delta_T}{H^2}\right]$$

$>>$

$$\to \sim \left[\frac{v_c}{\delta_T t} \sim \underbrace{\frac{v_c^2}{H\delta_T} \sim \frac{v_c^2}{H\delta_T}}\right] \times \frac{t\,\delta_T}{t} \sim \left[\cancel{\frac{v_c\delta_T}{H^2 t}} \sim \cancel{\frac{v_c^2\delta_T}{H^2}} \sim \underbrace{\cancel{\frac{v_c^2\delta_T}{H^3}}}\right]$$

inertia term

$>>>$

$$\sim \left[\frac{\nu v_c}{\delta_T^3} \sim \cancel{\frac{\nu v_c}{H^2\delta_T}}\right] \sim \left[\cancel{\frac{\nu v_c}{\delta_T H^2}} \sim \cancel{\frac{\nu v_c\delta_T}{H^4}}\right] \sim g\beta\frac{\Delta T}{\delta_T}$$

viscous

$$\Rightarrow \sim \left[\sim 1 \sim \underbrace{\frac{v_c t}{H} \sim \frac{v_c t}{H}}\right],$$

will dominate at initial time phase?

during the growing phase these terms will be small. ~~can be~~ (being t is small)

$$\Rightarrow \underbrace{\sim \frac{\partial^2 v}{\partial x\,\partial t}}_{\text{Inertia}} \quad \underbrace{\sim \nu\frac{\partial^3 v}{\partial x^3}}_{\text{viscous / friction}} \quad \underbrace{\sim g\beta\frac{\partial T}{\partial x}}_{\text{Buoyancy}}$$

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$$\sim \frac{v_c}{\delta_T t} \qquad \sim \nu\frac{v_c}{\delta_T^3} \qquad \sim g\beta\frac{\Delta T}{\delta_T}.$$

$\frac{N_A}{r} - N_A \int r c dr = \phi$

$\frac{d}{dt}(\dot{m}) = -D\frac{\partial c}{\partial x}$

Buoyancy → driving force

$\frac{\text{Inertia}}{\text{friction}} = \frac{v_c/\delta_T t}{\nu \frac{v_c}{\delta_T^{3}}}$

$\sim \frac{\delta_T^2}{\nu t} = \frac{\alpha t}{\nu t} = \frac{1}{Pr}$

$\rightarrow \sim \frac{1}{Pr} \qquad \sim 1 \qquad \sim \frac{g\beta \Delta T \delta_T^2}{\nu v_c}$

Viscous ~ Buoyancy.

$\frac{g\beta \Delta T \delta_T^2}{\nu v_c} \sim 1$ $\qquad \delta_T - (\alpha t)^{1/2}$

$v_c \sim \frac{g\beta\Delta T(\alpha t)}{\nu}$

$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$ (thermal inertia)

$\sim \frac{\Delta T}{t} \sim \left[\frac{v_c \Delta T}{H} \right] \sim \left[\frac{\alpha \Delta T}{\delta_T^2} \right]$

thermal inertia with time it will decrease

at $t = t_f$

convection ~ conduction

$\delta_T \sim (\alpha t)^{1/2}$

$\delta_{T_f} \sim (\alpha t_f)^{1/2}$

final

$v_c \frac{\Delta T}{H} \sim \alpha \frac{\Delta T}{\delta_T^2}$

$v_c \sim \frac{\alpha H}{\delta_f^2}$

$\frac{g\beta\Delta T(\alpha t_f)}{\nu} \sim \frac{\alpha H}{(\delta_{T_f})^2} \longrightarrow \alpha t_f$

$t_f^2 \sim \frac{H\nu}{g\beta\Delta T\alpha}$

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$\delta_{T,f} \sim (\alpha t_f)^{1/2}$

$(\delta_{T_f})^4 \sim (\alpha t_f)^2$

$\sim \alpha^2 \left(\frac{H\nu}{g\beta\Delta T\alpha}\right) \times \frac{H^3}{H^3}$

$\sim H^4 \cdot \frac{1}{\frac{g\beta\Delta T H^3}{\alpha\nu}} \sim \frac{H^4}{Ra_H}$

$\delta_{T_f} \sim H(Ra_H)^{-1/4}$

⇒ viscous ~ inertia

$\frac{\nu v_c}{(\delta_v)^3} \sim \frac{v_c}{(\delta_v)t}$

$(\delta_v)^2 \sim (\nu t)$

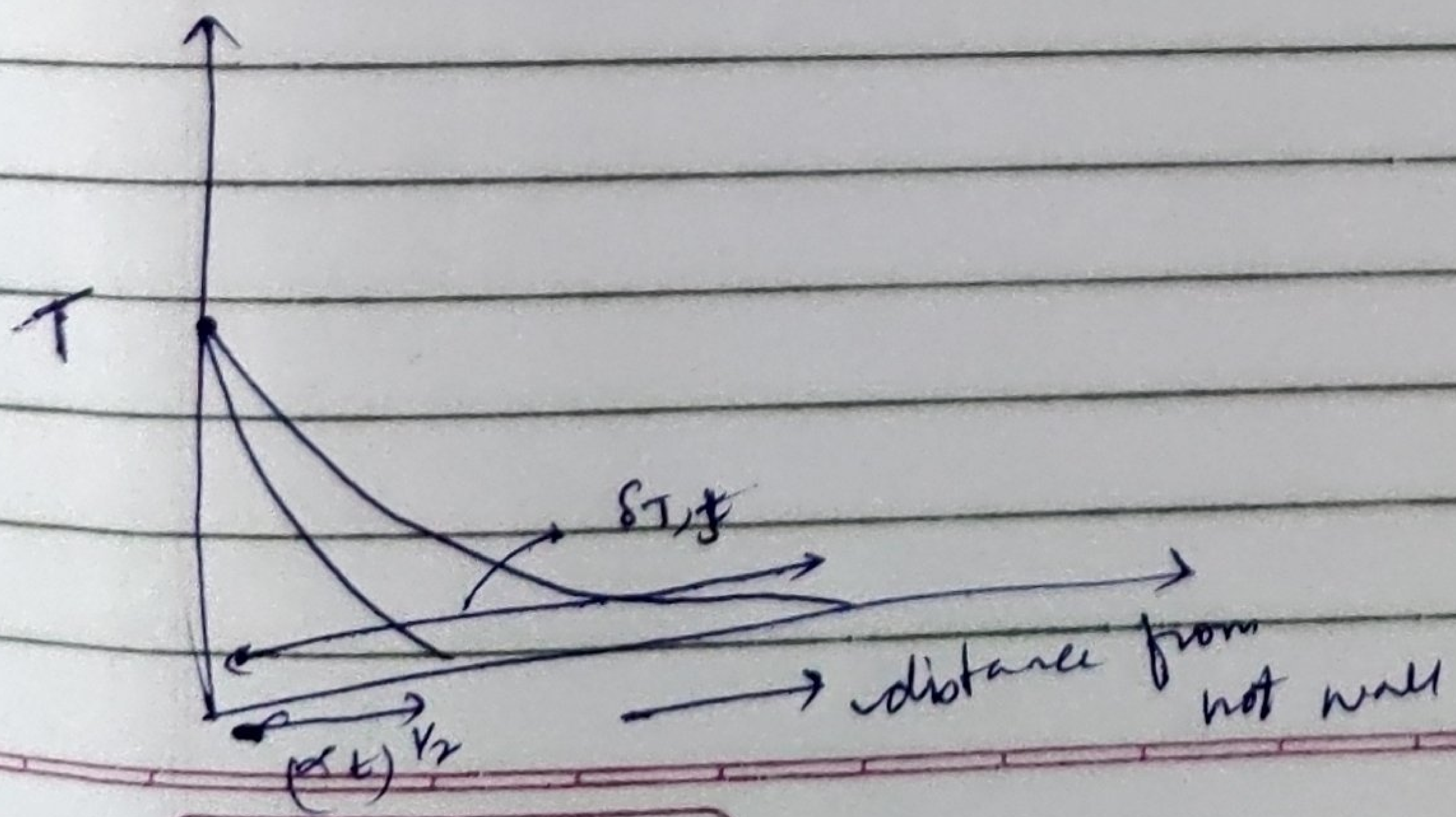
→ $\delta_{v,f} \sim (\nu t_f)^{1/2}$

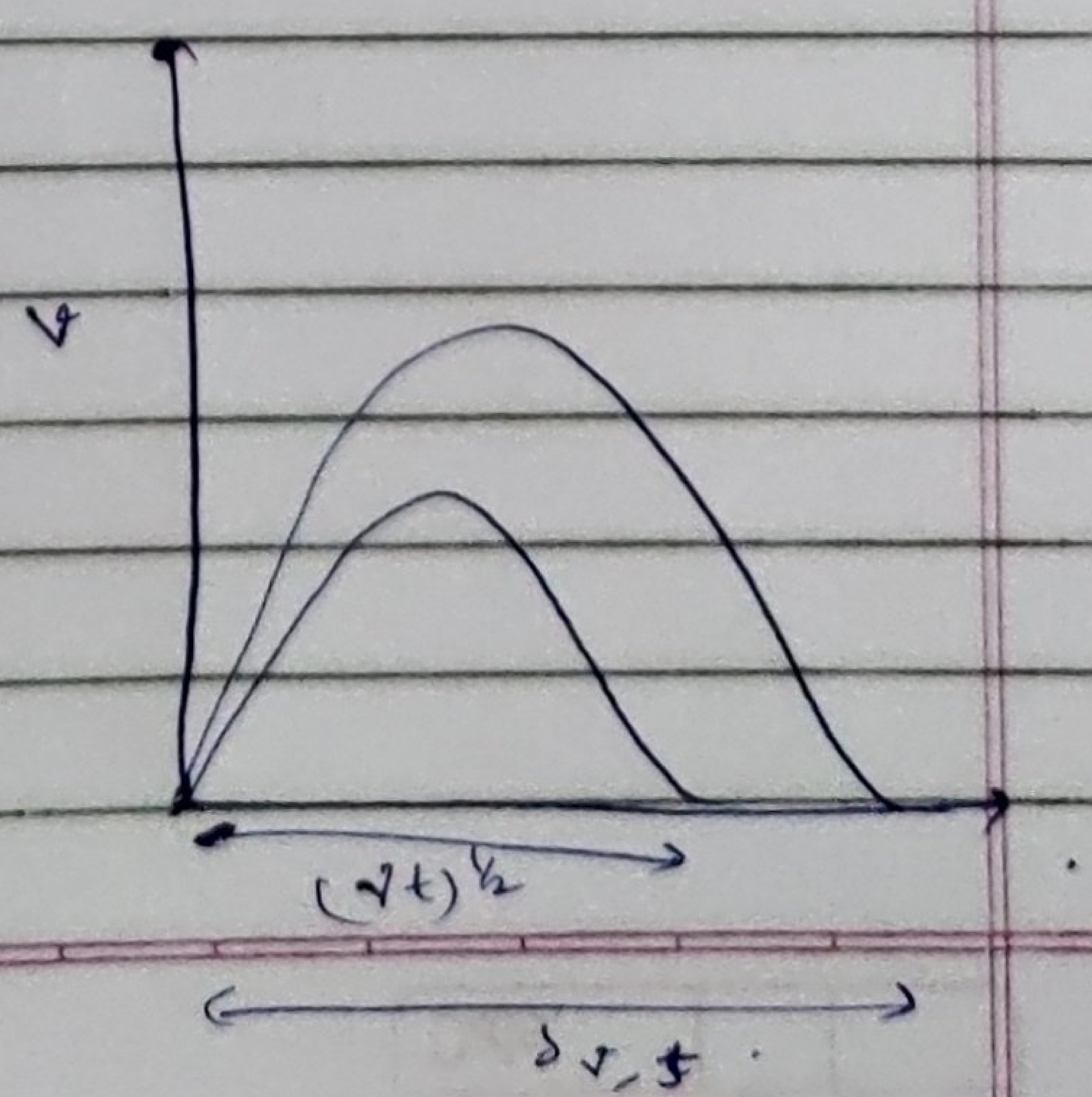
$(\delta_{v,f})^2 \sim \frac{\nu}{\alpha}(\alpha t_f)$

$\left(\frac{\delta_{v,t}}{\delta_{T,f}}\right)^2 \sim Pr$

$\boxed{\delta_{v,t} \sim Pr^{1/2} H Ra^{-1/4}}$

(final jet thickness)





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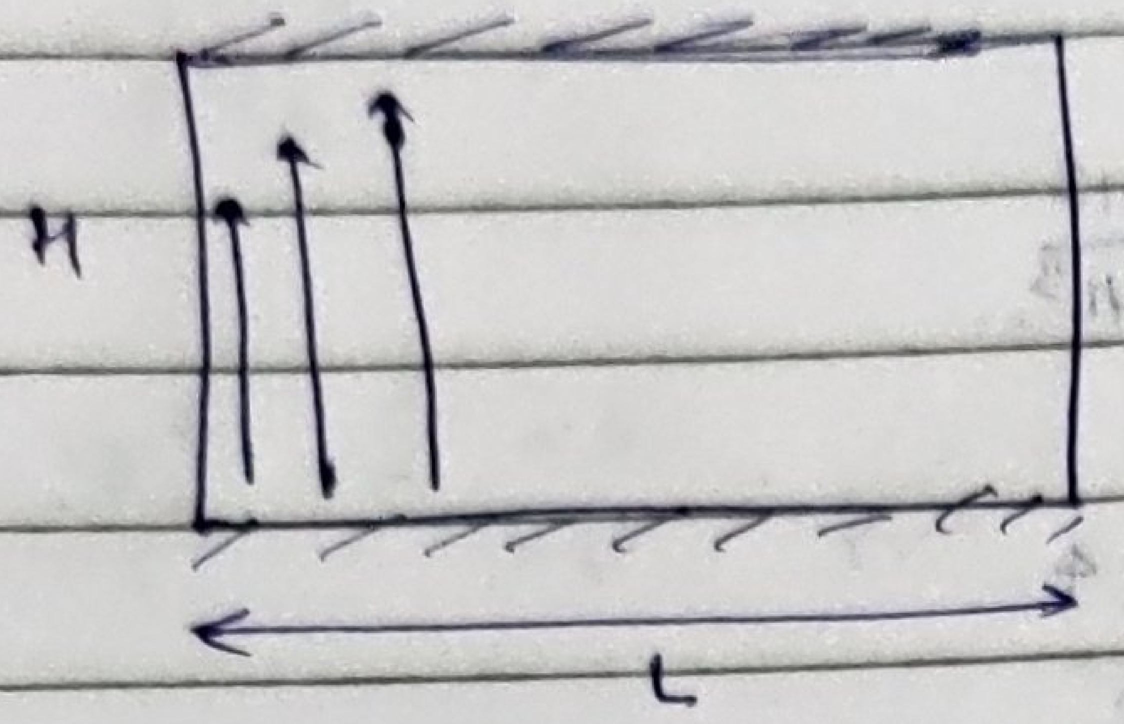
~~① distinct vel~~

① distinct vertical jet moving upward



$L < \delta_{T,j}$

$L > H\,Ra_H^{-1/4}$

$\frac{H}{L} < Ra_H^{1/4}$

② distinct layer of fluid

$\delta_{v,j} < L$

$(Pr)^{1/2}\, H\, Ra_H^{-1/4} < L$

$\frac{H}{L} < Pr^{-1/2}\, Ra_H^{1/4}$