Degrees of Freedom (f)

Degrees of freedom $f = V - E$

V = total no. of process variables

E = total no. of equations

✓ case 1 : $f = 0$ exactly specified Ideal case

$V = E$ solution may not be unique for nonlinear equations.

✓ case 2 : $f > 0$ Underspecified common case

$V > E$ How to make it exactly specified ?

✓ case 3 : $f < 0$ Overspecified Uncommon case

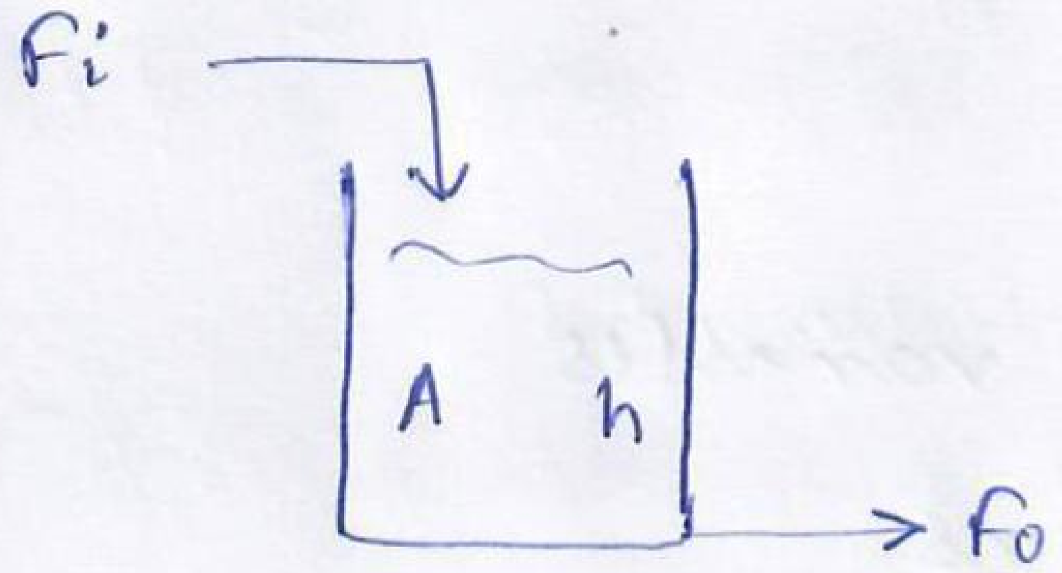
For underspecified system ; two ways to get $f = 0$.

(i) specify more no. of disturbance variables

(ii) Include more no. of equations (control eqns).

Degrees of freedom (contd...)

Ex. Liquid tank system (revisited).



Model: $A \frac{dh}{dt} = F_i - F_o$

$\therefore E = 1$

variables: F_i, h, F_o

$\therefore V = 3$

Degrees of freedom $f = V - E = 2$

How to make $f = 0$?

(i) Disturbance (LV) $\equiv F_i$ (Need to specify it through direct measurement)

$f = 2 - 1 = 1$

(ii) control obj: $h = h_{sp}$

CV	MV
h	F_o

Controller eqn:

$$F_o = F_{os} + K_c (h_{sp} - h)$$

$$\therefore f = 1 - 1 = 0$$

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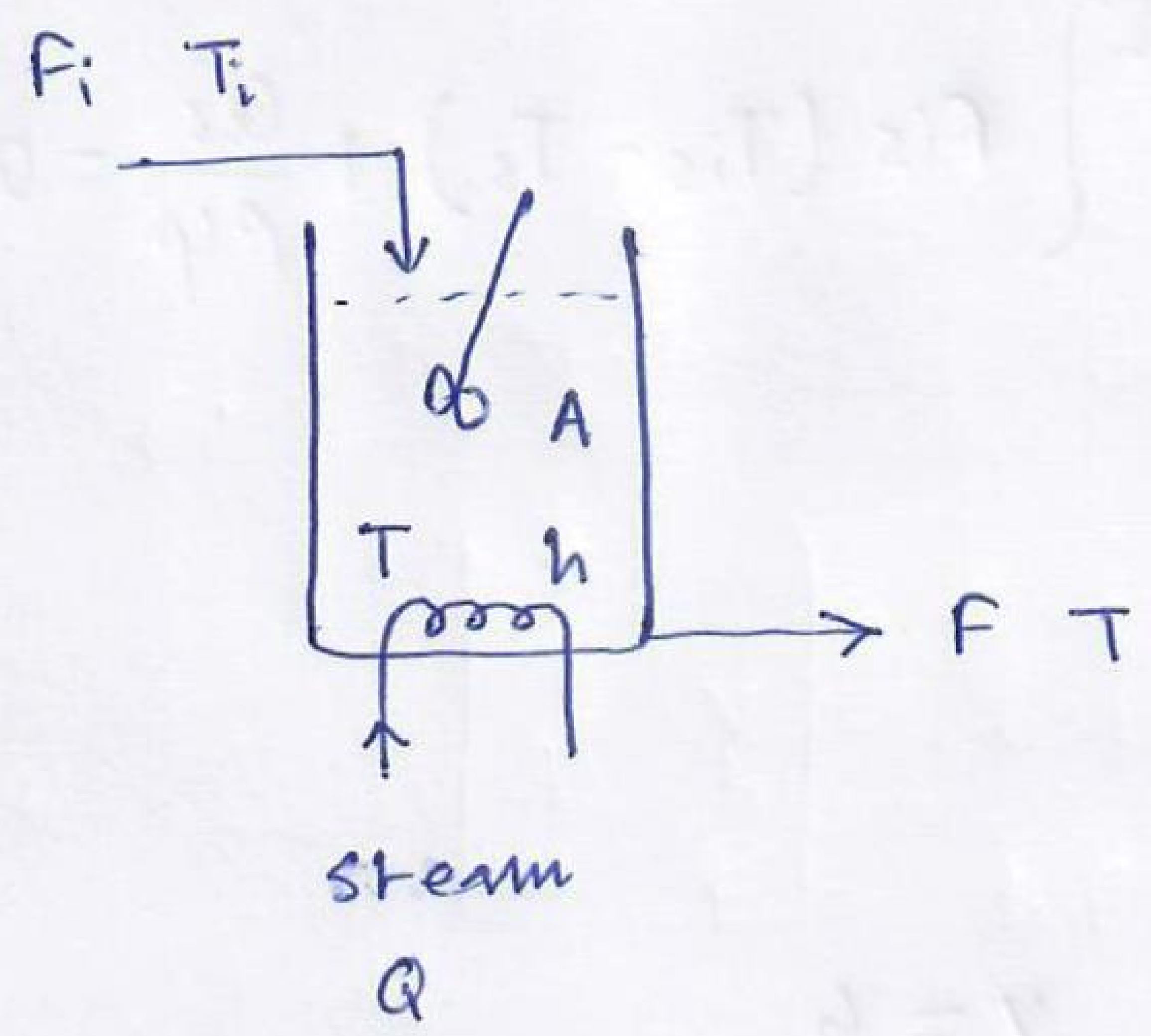
completely specified.

Stirred tank heater



$F \rightarrow$ volumetric flow rate

$Q \rightarrow$ Energy input per unit time

$\rho \rightarrow$ liq density

$C_p \rightarrow$ heat capacity

Assumptions:
1. Perfect mixing in the tank
2. ρ and C_p are constant
3. No heat loss (i.e., perfectly insulated).

✓ Total mass bal.

$$A \frac{dh}{dt} = F_i - F \quad \cdots (1)$$

✓ Energy bal. $\quad$ (energy $\equiv m C_p \, dT$).

Rate of accumulation = Rate of input − Rate of Output + Rate of energy supplied by steam

$$\frac{d}{dt}\left(\rho A h \, C_p (T - T_{ref})\right) = F_i \rho C_p (T_i - T_{ref}) - F \rho C_p (T - T_{ref}) + Q$$

$$A h \frac{dT}{dt} = F_i (T_i - T) + \frac{Q}{\rho C_p} \quad \cdots (2) \qquad T_{ref} = 0.$$

Dynamic model
$$\begin{cases} A \dfrac{dh}{dt} = F_i - F \\ Ah \dfrac{dT}{dt} = F_i (T_i - T) + \dfrac{Q}{\rho c_p} \end{cases}$$

Static model
$$\begin{cases} F_{is} - F_s = 0 \\ F_{is} (T_{is} - T_s) + \dfrac{Q_s}{\rho c_p} = 0 \end{cases}$$

Degrees of freedom

variables : F_i, T_i, F, T, Q, h $\quad \therefore \quad v = 6$

Parameters: A, ρ, c_p

Equations: eq ① and ② $\quad \therefore \quad E = 2$

$\therefore f = 6 - 2 = 4$ Underspecified

How to make $f = 0$?

(i) 2 disturbance variables : F_i, T_i (need to specify)

(ii) control objectives : $h = h_{sp}$

$T = T_{sp}$

CV	MV
h	F
T	Q

$$F = F_s + K_{cF} (h_{sp} - h)$$
$$Q = Q_s + K_{cQ} (T_{sp} - T)$$

$\therefore f = 4 - 2 - 2 = 0$

completely specified.