Mechanical Operations
Group-2

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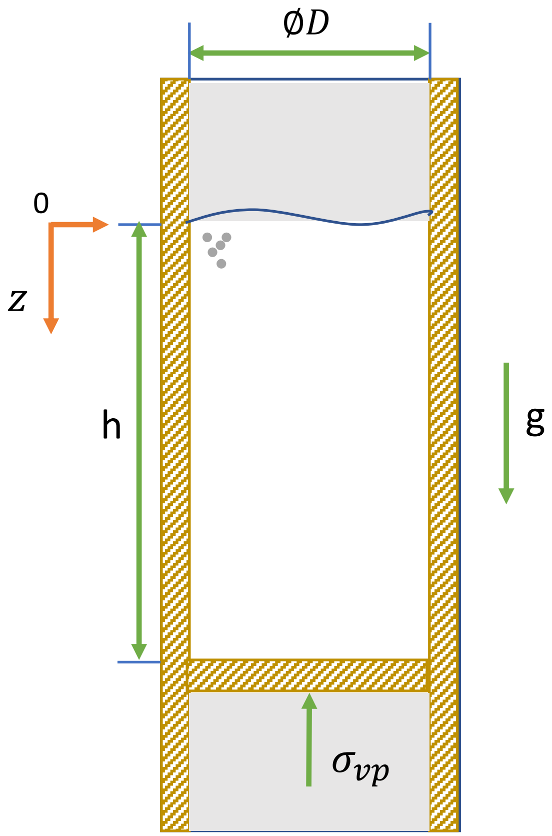
Problem - 1

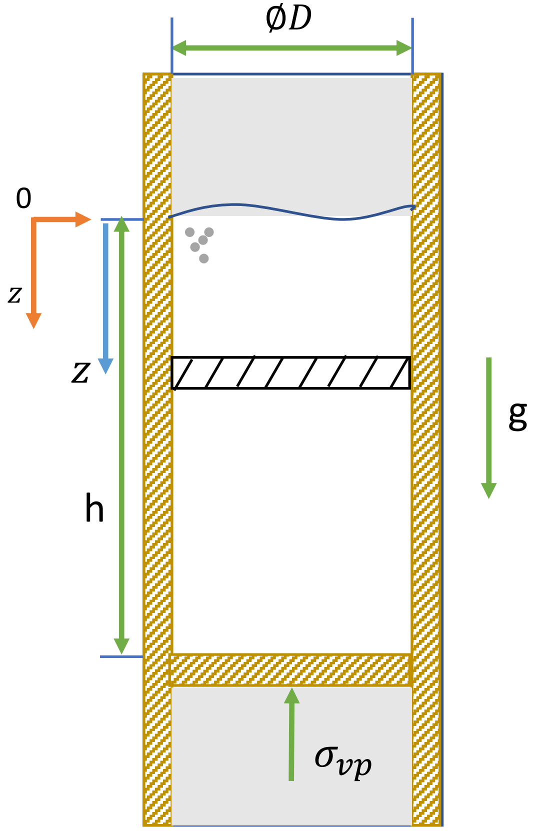
Consider a case where a column of bulk solid should be pushed upwards in a vertical channel, the wall shear stress on bulk solids then start acting downwards. Find out the expression for stress as a function of depth.

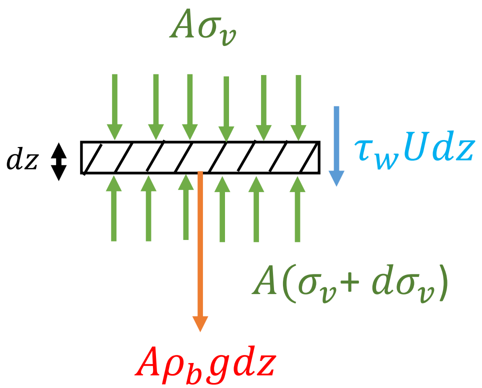


Motivation of the problem

- To find vertical stress at the bottom of the plug.
- The vertical stress which must be transferred through the piston to push the plug upwards.
- To decide whether it is practical to push a plug of bulk solid with a large height-to-diameter ratio upward through parallel channels such as tubes.

Solution





Where,

A - cross sectional area
ρ_b - bulk density
σ_v - vertical normal stress
U - perimeter
τ_w - shear stress

Applying force balance on the differential element,

$$\rho_b g A dz + \cancel{A}\sigma_v + \tau_w U dz = A(\cancel{\sigma_v} + d\sigma_v)$$

$$\frac{d\sigma_v}{dz} = \rho_b g + \frac{\tau_w U}{A}$$

Now,

Angle of wall friction ← $\boxed{\tan\varphi_x = \frac{\tau_w}{\sigma_h}}$ and $\boxed{K = \frac{\sigma_h}{\sigma_v}}$ → Lateral stress ratio

We assume ***K*** is constant throughout the material.
(In reality *K* varies through out the material)

Therefore, we get:

$$\frac{d\sigma_v}{dz} = \rho_b g + \frac{K\sigma_v \tan\varphi_x U}{A}$$

Application of Janssen's Equation

Integrating,

$$\int_{\sigma_{v0}}^{\sigma_v} \frac{d\sigma_v}{\rho_b g + \frac{K\sigma_v \tan\varphi_x U}{A}} = \int_0^z dz$$

$$\boxed{\therefore\ \sigma_v = \frac{-\rho_b g A}{K \tan\varphi_x U} + \left(\sigma_{v0} + \frac{\rho_b g A}{K \tan\varphi_x U}\right) e^{\frac{K \tan\varphi_x U z}{A}}}$$

Expression for vertical normal stress

If surcharge stress $\sigma_{v0} = 0$,

$$\sigma_v = \frac{\rho_b g A}{K \tan \varphi_x U} \left(e^{\frac{K \tan \varphi_x U z}{A}} - 1\right)$$

$$\frac{\rho_b g A}{K \tan \varphi_x U} \rightarrow \sigma_{vc}$$

Where,

$$U = \pi D$$

$$A = \frac{\pi D^2}{4}$$

For cylindrical plug

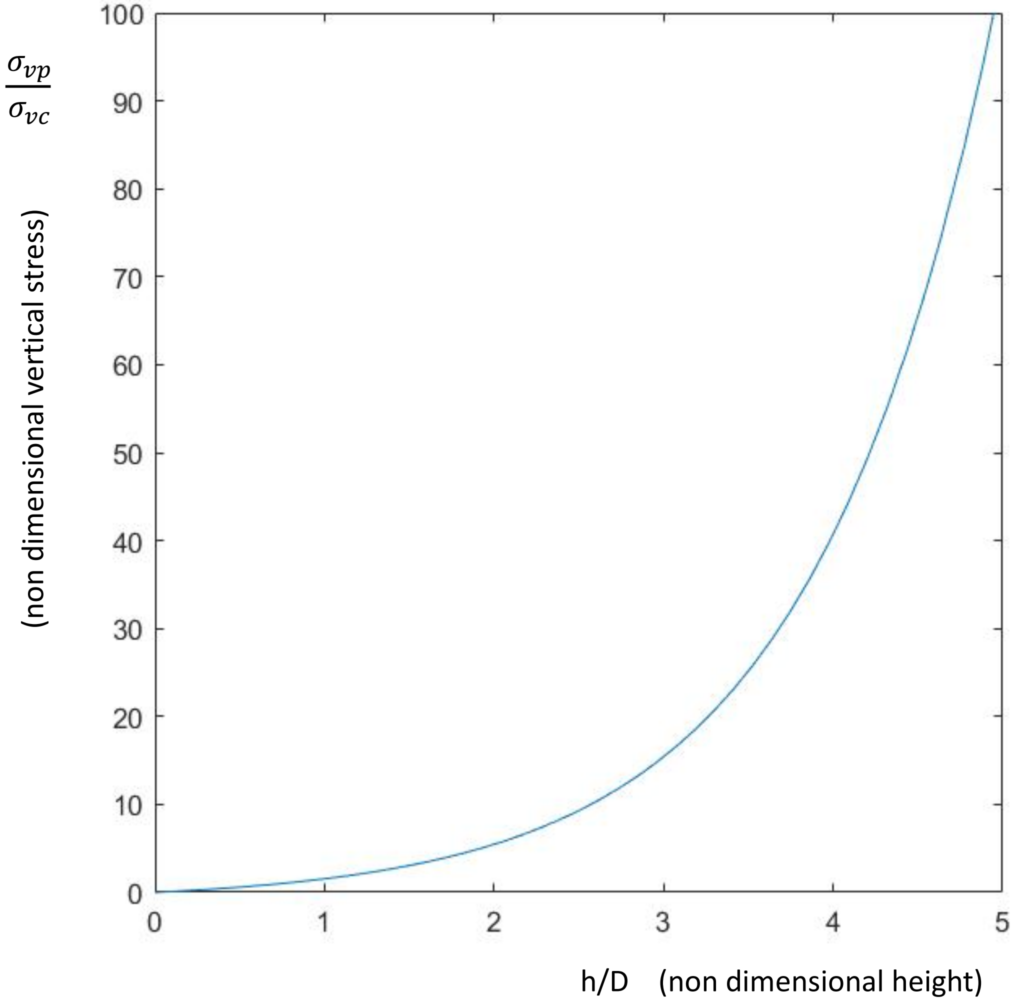
$$\boxed{\frac{\sigma_v}{\sigma_{vc}} = \left(e^{\frac{4K \tan \varphi_x z}{D}} - 1\right)}$$

→ Non-dimensionalized equation

σ_{vc} is the maximum vertical stress below a stationary column of bulk solid only subjected to the force of gravity

The vertical stress σ_{vp} , at the bottom of the plug (i.e., at z = h) can be calculated. This is the vertical stress which must be transferred through the piston to push the plug upwards.

Results and Conclusion



required vertical stress, σvp, at the bottom of the plug as a function of height h (non-dimensional representation)

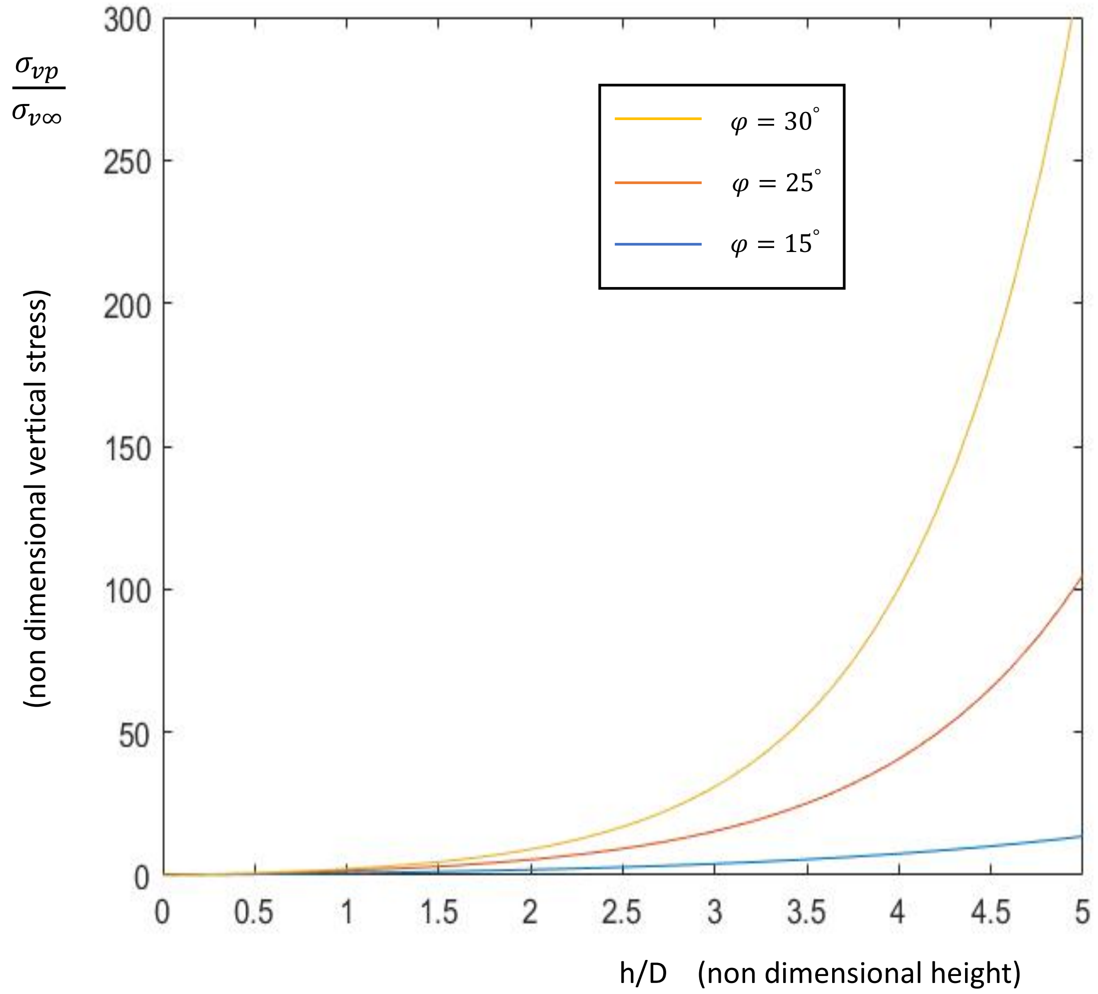
$$\sigma_{vp} = \sigma_{vc}\left(e^{\frac{4K \tan \varphi_x h}{D}} - 1\right)$$

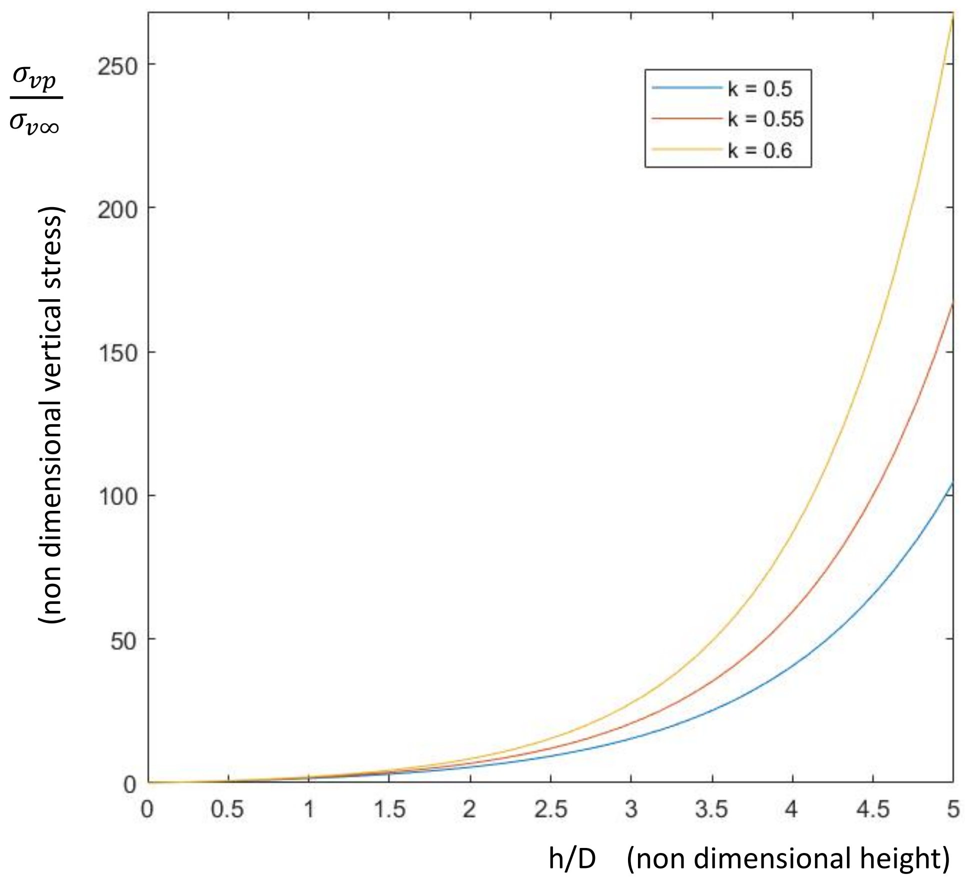
The dimensionless vertical stress, σ_{vp}/σ_{vc}, is plotted versus the non-dimensional height of the plug, h/D.
For h/D > 5 more than 100 times the vertical stress $\sigma_{v\infty}$, would be necessary.

Thus it becomes clear that it is not practical to push a plug of bulk solid with a large height-to-diameter ratio upward through parallel channels such as tubes.

Plots were generated in MATLAB using fplot function

By varying K and φ , we get different plots

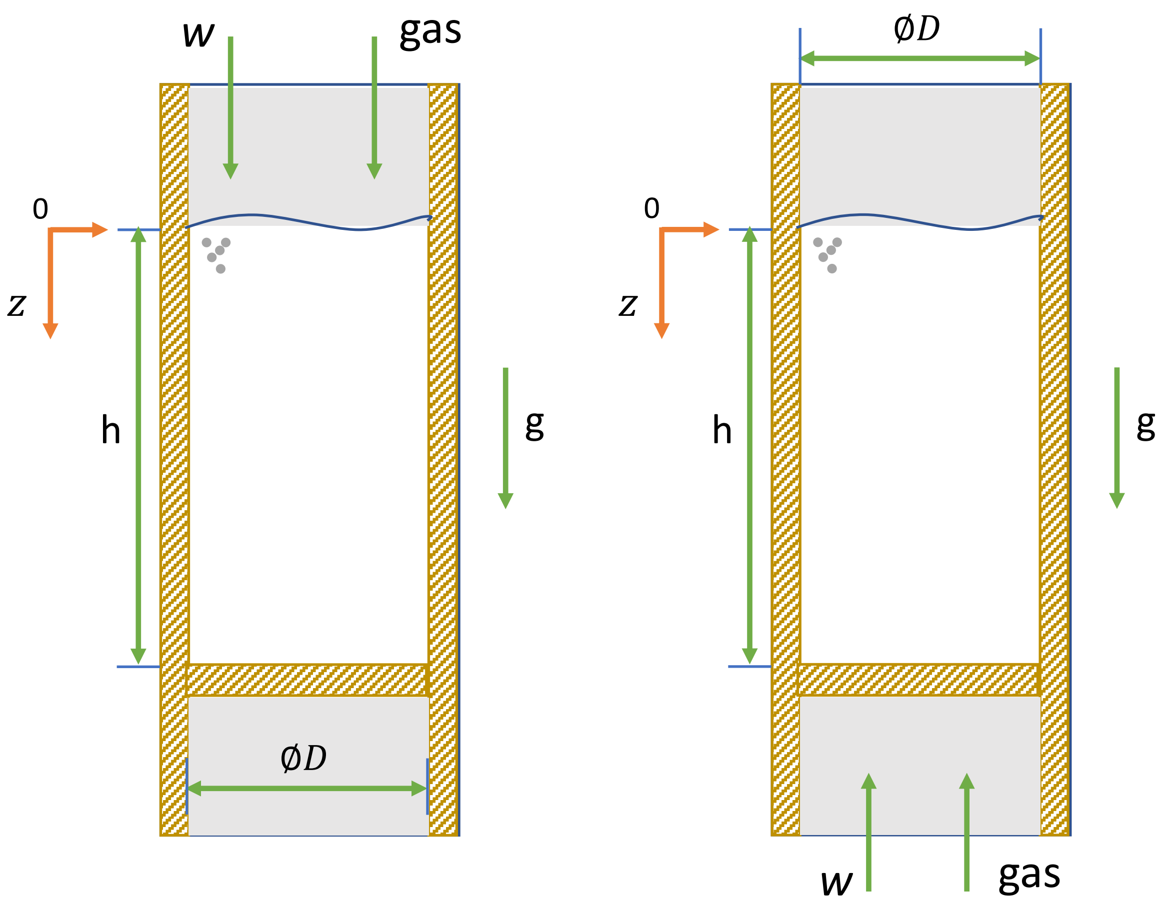




Plots were generated in MATLAB using fplot function

Problem - 2

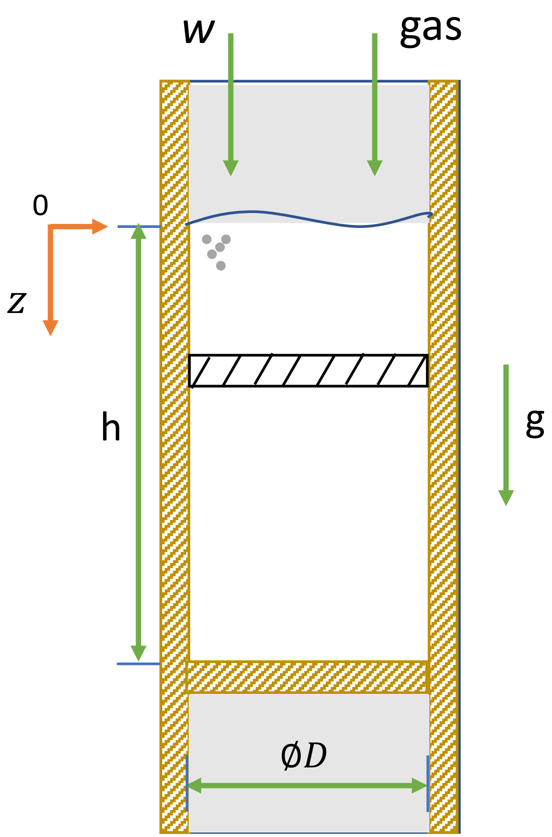
If a gas flows through a stationary packing of bulk solid, the stresses in the bulk solid are influenced by the pressure gradient. This effect can be taken into account for a setup as shown in figure below by adding pressure gradient, dp/dz.

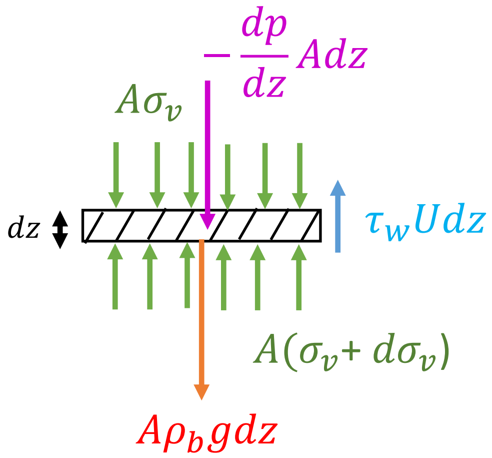


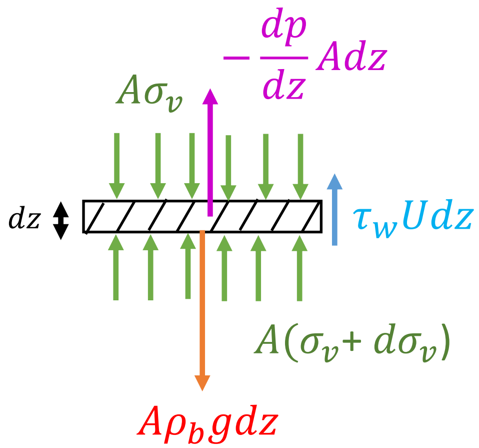
Motivation of the problem

- To understand the working of fluidized beds
- Determine onset of fluidization

Solution







Where,

A - cross sectional area

ρ_b - bulk density

σ_v - vertical normal stress

U - perimeter

τ_w - shear stress

$\frac{dp}{dz}$ - pressure gradient

Case 1: $\left(\rho_b g - \frac{dp}{dz}\right) > 0$ (gas flows downwards or net force is downwards when gas flows upwards)

When the gas is flowing downwards, the gas pressure p, decreases in z direction, i.e. the pressure gradient is dp/dz < 0. When gas flows upwards dp/dz > 0 , but resultant force acts downward. $\tau_w U dz$ (shear stress) acts upwards

$$\rho_b g A dz + A\sigma_v - \left(\frac{dp}{dz}\right) A dz = A(\sigma_v + d\sigma_v) + \tau_w U dz$$

Now,

Angle of wall friction ← $\boxed{\tan\varphi_x = \frac{\tau_w}{\sigma_h}}$ and $\boxed{K = \frac{\sigma_h}{\sigma_v}}$ → Lateral stress ratio

We assume **K** is constant throughout the material.
(In reality *K* varies through out the material)

$$\boxed{\frac{d\sigma_v}{dz} = \rho_b g - \left(\frac{dp}{dz}\right) - \frac{K\sigma_v \tan\varphi_x U}{A}}$$

Application of Janssen's Equation

Assuming a constant pressure gradient and no surcharge stress, we get:

$$\int_0^{\sigma_v} \frac{d\sigma_v}{\rho_b g - \left(\frac{dp}{dz}\right) - \frac{K\sigma_v \tan\varphi_x U}{A}} = \int_0^{z} dz$$

$$\boxed{\therefore \quad \sigma_v = \frac{\left(\rho_b g - \frac{dp}{dz}\right) A}{K \tan\varphi_x U}\left[1 - e^{\frac{-K \tan\varphi_x U z}{A}}\right]}$$

Expression for vertical normal stress

Where,

$\left(\rho_b g - \frac{dp}{dz}\right) > 0$

Now, $\sigma_{v\infty} = \sigma_v(z \to \infty)$

$$\therefore \quad \sigma_{v\infty} = \frac{\left(\rho_b g - \frac{dp}{dz}\right) A}{K \tan \varphi_x U}$$

The final value of the vertical stress, σv∞, which is the maximum vertical stress attained below a bulk solid column of infinite height

Where,

$\left(\rho_b g - \frac{dp}{dz}\right) > 0$

Case 2: $\left(\rho_b g - \frac{dp}{dz}\right) < 0$ (gas flows upwards and before the onset of fluidization)

At $g\rho_b$ = dp/dz the vertical stress is zero throughout the complete plug, because the gravity force equals the force resulting from the pressure gradient of the upward gas flow. If the pressure gradient exceeds the gravity force only slightly, the resultant force on the bulk solid acts upwards.

Here $\tau_w = 0$ as there is no shear stress

Therefore we obtain the equation as

$$\rho_b g A dz + \cancel{A\sigma_v} - \left(\frac{dp}{dz}\right) A dz = A(\cancel{\sigma_v} + d\sigma_v)$$

Integrating the previous equation with our previous assumptions, we get:

$$\sigma_v = \left(\rho_b g - \frac{dp}{dz}\right) z$$

Where,

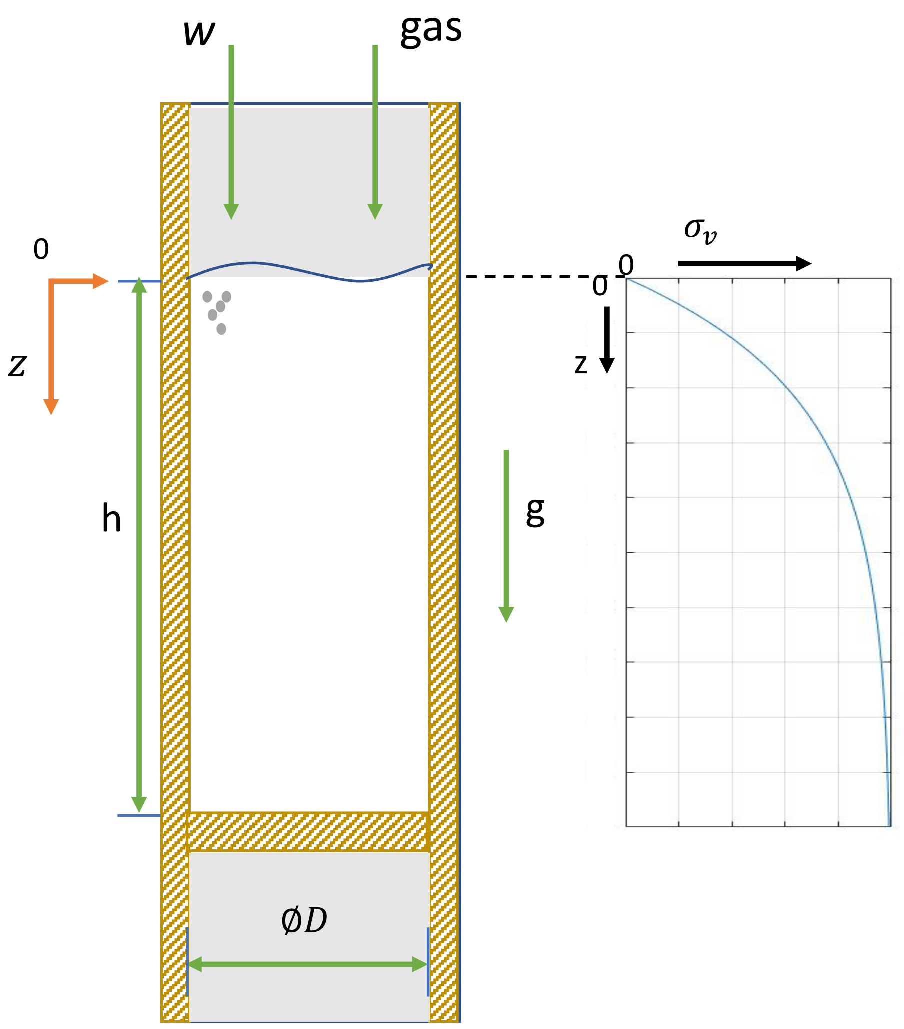
$$\left(\rho_b g - \frac{dp}{dz}\right) < 0$$

The vertical stress is zero at the upper surface and decreases linearly downwards, i.e., a tensile stress acts in the bulk solid which is maximum at the bottom

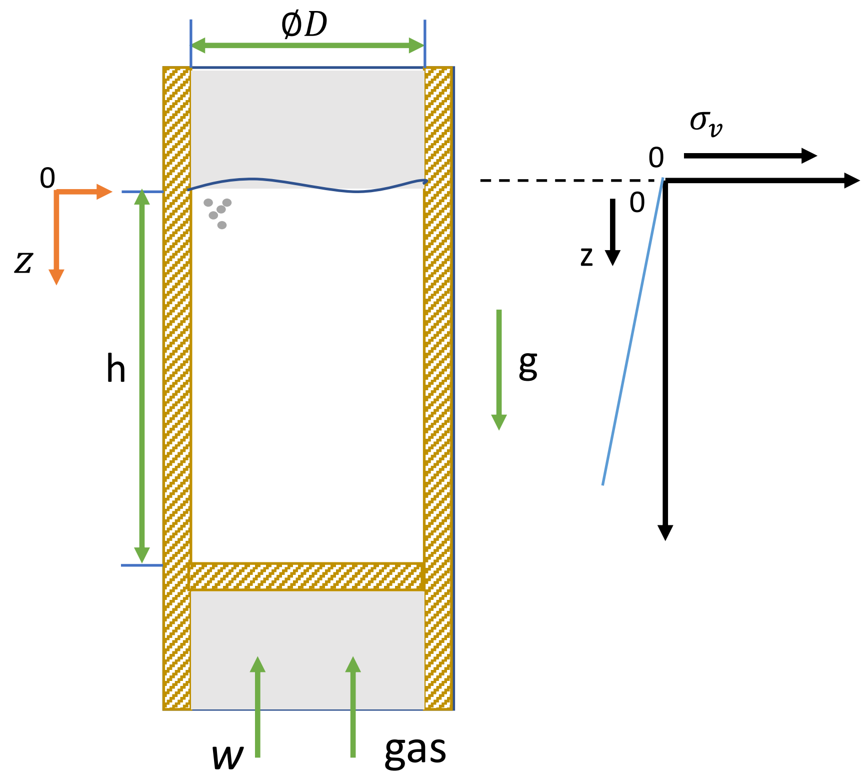
Assumption:

For the calculation of vertical stress it is assumed that in the presence of vertical tensile stresses the (horizontal) wall normal stress and, thus, the shear stress transferred between the bulk solid and the confining walls, is zero, i.e., adhesion and possible residual stresses in the horizontal direction which can result from a preceding consolidation and remain even after the material is relieved from the consolidation stress are excluded.

Results and Conclusion



Distributions of vertical stress and (absolute) gas pressure in a stationary particle bed (fixed bed) subjected to vertical gas flow (gas velocity: w)



upward gas flow where gpb < dp/dz (before the onset of fluidization)

- Thus, a negative vertical stress (tensile stress) is acting within the material.
- In this situation a loosely packed, cohesionless bulk solid starts to dilate, transforms into a fluidized bed, and due to the increased porosity the pressure drop decreases so that again gρb = dp/dz is fulfilled (onset of fluidization).
- A cohesive bulk solid can still be in the state of a fixed bed at considerable pressure gradients dp/dz > gρ_b even before the onset of fluidization.
- The reason for this is the ability of a cohesive bulk solid to transfer tensile stresses due to its tensile strength, which is dependent on the actual state of consolidation.