INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Chemical Engineering

Mid-semester (Autumn) Examination 2018-2019

Subject: Advanced Mathematical Techniques in Chemical Engineering (CH61015)

Remarks:

1. This question paper contains two parts: **Part A** and **Part B**. Attempt both parts.
2. Write all the answers of a part together.
3. Unless otherwise stated, usual mathematical notations apply.
4. Time = 2 h; maximum marks = 60; total number of printed pages = 3.

Part A: Linear algebra

1. Prove that a set of vectors of the form $[x_1, x_2 \dots x_n]^T \in \mathbb{R}^n$, satisfying m equations (given below) is a subspace of $\mathbb{R}^n(\mathbb{R})$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0$$

$$\cdots$$

$$\cdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0$$

where $a_{ij} \in \mathbb{R}\ \forall\ i, j$.

...10 marks

2. For the following two systems of equations in unknowns x_i's and y_i's, determine whether the corresponding range spaces are identical.

System 1

$$x_1 + 3x_2 + 3x_3 + 2x_4 = a_1$$

$$2x_1 + 6x_2 + 9x_3 + 7x_4 = a_2$$

$$-x_1 - 3x_2 + 3x_3 + 4x_4 = a_3$$

System 2

$$y_1 + 2y_3 + 3y_4 = b_1$$
$$y_2 + 4y_3 + 5y_4 = b_2$$
$$3y_1 + y_2 + 10y_3 + 14y_4 = b_3$$

...10 marks

3. Let $\underline{\underline{A}} = \begin{bmatrix} 3 & -2 \\ 4 & -5 \end{bmatrix}$ be a matrix in $\mathbb{R}^{2\times 2}$ which defines a linear transformation $t_A : \mathbb{R}^2 \to \mathbb{R}^2$ by the rule $t_A(\underline{x}) = \underline{\underline{A}}\underline{x}\ \forall\ \underline{x} \in \mathbb{R}^2$. Find the matrix of t_A relative to the basis $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$ for $\mathbb{R}^2$.

...10 marks

Part B: Differential equations

4. Consider a Sturm-Liouville problem $Lu = \lambda r(x)u$ where L is the operator, λ is the eigenvalue, and $r(x)$ is the weight function. If λ_m and λ_n are distinct eigenvalues, prove that the corresponding eigenfunctions, u_m and u_n, are orthogonal functions with respect to weight function r.

...10 marks

5. Solve the following problem completely including all the derivations:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

subject to boundary conditions:

at $x = 0, \dfrac{\partial u}{\partial x} = 0$

at $x = 1, u = 0$

at $y = 0, \dfrac{\partial u}{\partial y} = -q_0$

at $y = 1, \dfrac{\partial u}{\partial y} + 5u = 0$

...10 marks

6. Solve the following transient heat conduction problem:

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

At $t = 0, T = T_0(1 + x)$

At $x = 0, \dfrac{\partial T}{\partial x} = 0$

At $x = L, -k\dfrac{\partial T}{\partial x} = h(T - T_\infty)$

Make the above equation suitably non-dimensional and completely solve the problem.

...10 marks

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