PROBLEM SET – 2

1) Solve the following initial value problems for $t > 0$ using Laplace transform method:

a) $\dfrac{d^2y}{dt^2} + y = 1;\ \ y(0) = y'(0) = 0$

b) $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} = \big(1 - H(t-1)\big);\ \ y(0) = 1, y'(0) = -1$

c) $\dfrac{d^2y}{dt^2} + y = f(t);\ \ y(0) = y'(0) = 0;$ where $f(t) = \begin{cases} \cos t, & 0 \le t \le \pi \\ 0, & t > \pi \end{cases}$

2) Solve the following boundary value problems using Laplace transform methods:

a) $\dfrac{d^2y}{dt^2} + y = \sin t, \quad y(0) = 1,\ y\left(\dfrac{\pi}{2}\right) = \pi,$

b) $\dfrac{d^2y}{dt^2} + 9\lambda = t, \quad y(0) = 1, \quad y'\left(\dfrac{\pi}{3}\right) = -1$

3) Solve the following differential equations using Laplace transform method:

a) $t\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} = -1;\ \ y(0) = 0$

b) $t\dfrac{d^2y}{dt^2} + (t+1)\dfrac{dy}{dt} + 2y = e^{-t};\ \ y(0) = 0$

4) Solve the following integral equations using Laplace transform method:

a) $y(t) = \sin t + 2\int_0^t y(u)\cos(t-u)\,du$

b) $\dfrac{dy(t)}{dt} + 3y(t) + 2\int_0^t y(u)\,du = t;\ \ y(0) = 1$