

Fundamentals, Balances, and Transport Processes

Beginning with simple atomic encounters from statistical mechanics, macroscopic balance equations and transport properties are derived

Main mixing and separation unit operations grouped according to their driving force

Unit operation	molecular / thermal	mechanical/ext. force	electro-magnetic
Mixing and aggregation, Combination, Control of segregation.	diffusion ¹⁾ dissolving ²⁾ extracting ^{2c)} desorption ^{2c)}	spraying ²⁾ aeration ²⁾ stirring ²⁾ active mixing ^{1, 2)} dosing ^{1, 2)}	electro-phoretic mixing ¹⁾ mixing with mag- netic beads ²⁾
Separation	thermodiffusion ¹⁾ pressure diffusion ¹⁾ counter-current diffusion ¹⁾ condensation ^{2a)} evaporation ^{2a)} crystallization ^{2a)} distillation/ rectification ^{2a)} drying ^{2b)} absorption ^{2c)} adsorption ^{2c)} extraction ^{2c)} ion exchange ^{2c)} membrane processes.	sedimentation ²⁾ cycloning ²⁾ centrifugation ²⁾ pressure diffusion ^{1, 2)} (ultracentrifuge) filtration osmosis gas permeation classification sorting	electro deposition ²⁾ magneto deposition ²⁾ electro filtration electro dialysis electro osmosis electrophoresis magneto-striction
superscripts for <i>employed phases:</i> 1) single phase 2) multiphase a) with own co-phase b) own + additional co-phase c) additional co-phase			

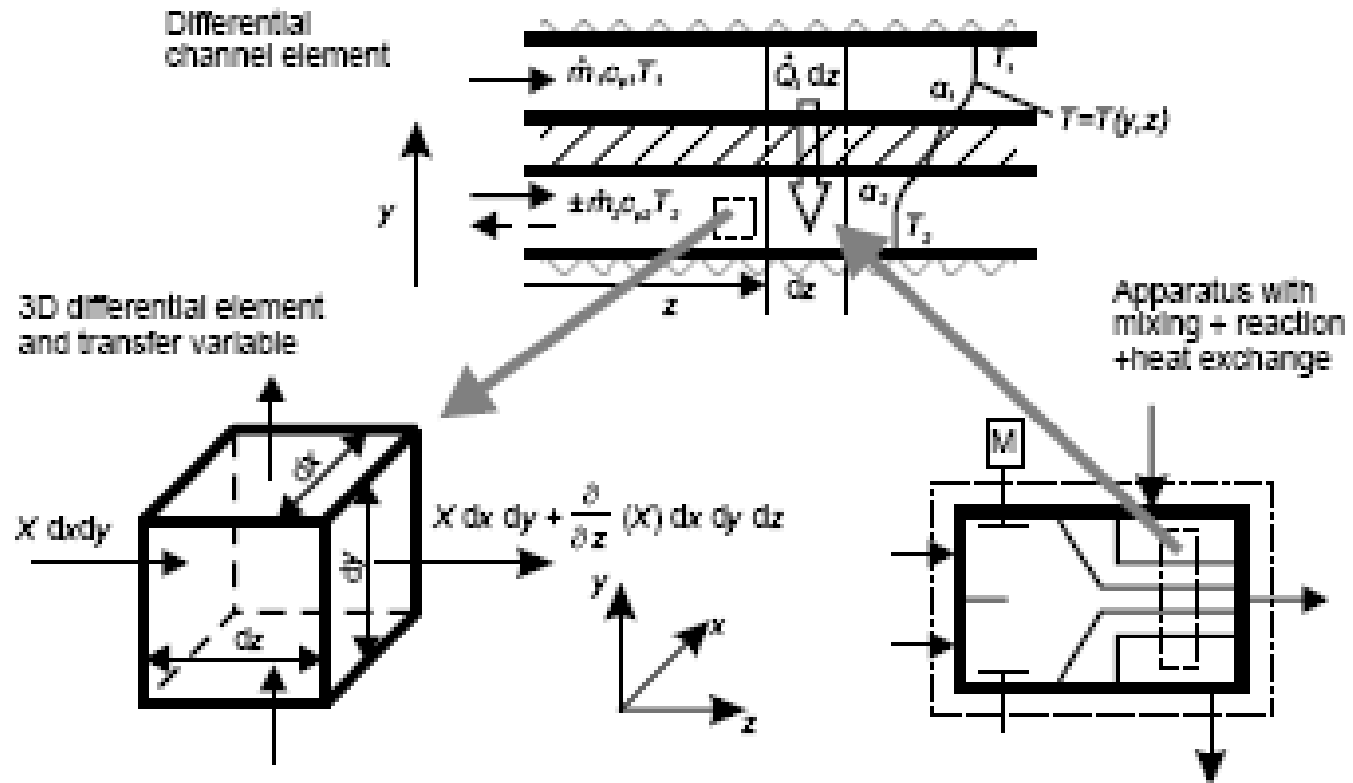
Balances and Transport Equations

Conservation and balance equations of mass, species, momentum, energy, as well as the definition of the entropy and its application.

The conservation laws of mass (continuity equations) and energy (first law of thermodynamics) can be described as

$$\left[\begin{array}{c} \text{System change} \\ \text{with time} \end{array} \right] = \left[\begin{array}{c} \text{Incoming} \\ \text{Flow} \end{array} \right] - \left[\begin{array}{c} \text{Outgoing} \\ \text{Flow} \end{array} \right] + \left[\begin{array}{c} \text{Source or} \\ \text{Sink} \end{array} \right]$$

Macroscopic balance equations



Overview of the various balancing volumes in process engineering : 3D differential element for general calculations, 1D differential element and complete equipment (active mixer) for process balances.

X stands for mass, species, momentum or energy.

The general balance equation with temporal and spatial derivatives of a system and a differential element with the volume V is written as (with X as the general balanced value):

$$V \cdot \frac{\partial X}{\partial t} = w \cdot \left[X \, dy \, dx - \left(X \, dy \, dx + \frac{\partial X}{\partial z} dz \, dy \, dx \right) \right]$$

In a steady process, the temporal derivative vanishes, $\partial/\partial t = 0$.

In systems with high velocities, the convective transport is dominant compared to conductive and diffusive fluxes.

For systems with dominant chemical reactions, only a change of substance needs to be considered.

Within the systems since no mass is generated or destroyed, hence, no sink or source appears in the balance equation.

For time dependent mass flow rates,

$$\frac{\partial m}{\partial t} = m_{in} - m_{out}$$

The momentum of a moving fluid can be expressed as the product of the mass and the flow velocity.

The integral of the momentum over the volume results in a net force of the fluid on the volume boundary or on the equipment.

In general, the momentum balance of a device can be written as

$$\sum J_{in} = \sum J_{out} - \sum J_{loss}$$

The momentum loss can be interpreted as the viscous momentum loss, which is expressed as pressure loss along the channel or device flow.

For fluid flow through an arbitrary channel:

$$\frac{\partial(mw)}{\partial t} = \left(\dot{mw} \right)_{in} - \left(\dot{mw} \right)_{out} + (pA_i)_{in} - (pA_i)_{out} + mg + F_z$$

Starting point for Navier-Stokes equation.

Additional forces in microfluidic applications result from surface effects

Similar to the mass, the energy itself is conserved according to the first law of thermodynamics for open systems.

$$\sum \dot{E}_{in} = \sum \dot{E}_{out} + \sum \dot{E}_{diss}$$

The energy dissipation takes into account that energy conversion from one form into another is accompanied by natural losses.

These losses are characterized by the entropy generation during a process according to the second law of thermodynamics.

In case of a channel with constant cross section, without chemical reactions and work consumed or produced, the entire energy of the fluid can be expressed according to the first law of thermodynamics

$$m e = \rho A \left(u + \frac{1}{2} w^2 + g z \right)$$

The enthalpy form of the energy balance can be written as

$$\rho \frac{dh}{dt} = \frac{dp}{dt} + \varepsilon - \operatorname{div} \vec{q}$$

The caloric equation of state gives the correlation between the internal energy or enthalpy and the temperature

$$d u = c_v d T \quad ; \quad d h = c_p d T$$

the energy equation can be rewritten as

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt} + \varepsilon - \text{div} (k \text{ grad } T)$$

k = thermal conductivity, (W/m K)

The solution of this equation gives the temperature distribution.

Isobaric process with no dissipation – usual form of energy equation

$$\rho c_p \frac{dT}{dt} = - \text{div} (k \text{ grad } T)$$

A chemical reaction within the system influences not only the species equation, but the reaction enthalpy Δh_R must also be considered in the energy balance due to the apparent heat consumption or release.

$$\Delta h_R = h_p - h_r$$

enthalpy of the reactants h_r and the products h_p

Dividing the transferred heat by the temperature, a new state variable, the entropy s (*state function*), is derived for further characterization of the states and processes.

$$ds \geq \frac{dq}{T} \quad \text{equal for rev. proc}$$

The dissipation function ε is the friction loss per volume and time unit.

Entropy always increases [$ds > 0$] by dissipation and irreversible processes, such as pressure loss or concentration homogenization by mixing.

The entropy production is a major indication of the efficiency of a process.

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Various control strategies are used for energy savings and entropy minimization.

For continuous flow systems, high pressure losses and unnecessary throttling should be avoided.

In mixing, high solution concentrations as well as high dilutions lead to high separation effort.

Heat transfer devices with high temperature gradients induce high entropy production.

Elementary transport processes and their description

Elementary transport processes and their description

The entire change of a state variable is described by the transport processes of conduction in the immobile phase (solids or resting fluids), convection in the fluid phase (gases and liquids), and by the generation or depletion in the control volume.

$$\begin{bmatrix} \text{Total Flow} \\ \text{Density} \end{bmatrix} = \begin{bmatrix} \text{Conduction over} \\ \text{the boundary} \end{bmatrix} + \begin{bmatrix} \text{Convection over} \\ \text{the boundary} \end{bmatrix} + \begin{bmatrix} \text{Source} \\ \text{or} \\ \text{Sink} \end{bmatrix}$$

Conductive transport is driven by a parameter gradient; convective transport is always accompanied by a volume flow rate with a certain mean velocity.

Transfer coefficients: D , $\nu = \mu/\rho$, $\alpha = k/\rho c_p$ (m^2/s)

The dynamic behavior of viscous fluids is governed by $Re = wd\rho/\mu$

Species transfer in convective flow, both Re and $Sc (= \mu/\rho D)$

Sc indicates the ratio between the momentum and species transfer.

For $Sc = O(1)$, as with gases, momentum transfer and species transfer are in the same order-of-magnitude.

Concentration gradients behave similar to velocity gradients.

For high Sc such as for liquids, concentration gradients continue much longer than velocity gradients, important for mixing issues.

Convective heat transfer – both Re and Pr

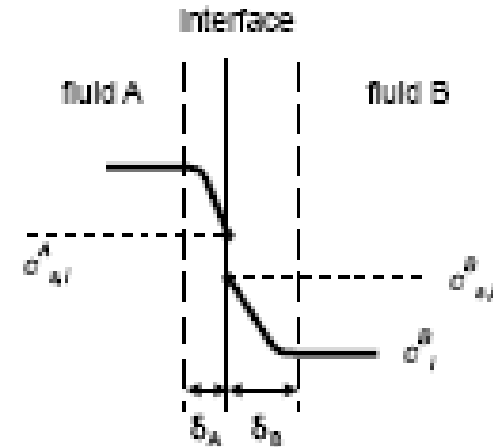
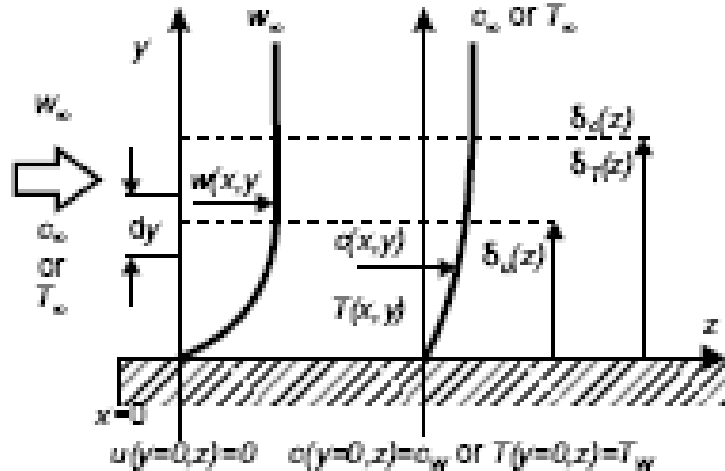
Low Pr numbers - oils or organic liquids.

The Pr numbers of air or water have the order of $O(1)$, permitting a heat transfer as fast as momentum transfer.

Coupled heat and mass or species transfer, such as convective condensation of aerosol droplets, are described by thermal diffusivity and diffusion coefficient. Both coefficients are combined to Lewis number $Le (=Sc/Pr) = \alpha/D$, ($\alpha=k/\rho c_p$)

A high Le number allows droplet generation from vapor cooling, while for low Le numbers, vapor will directly condense at the wall.

Two-film model for the fluid/fluid exchange of mass, species, energy



Species transport equations

$$J_i^A = \beta_i^A A (c_{i,bulk}^A - c_{i,interface}^A); \quad J_i^B = \beta_i^B A (c_{i,interface}^B - c_{i,bulk}^B); \quad \beta_i = D_{Ai} / \delta_i$$

For the absorption of a gas into a liquid Henry's law $p_{i,g} = H_i c_{ij}$

For the evaporation or condensation of a binary mixture (distillation, rectification), Raoult's law

$$p_i = p_t y_i = \pi_i x_i$$

Concentration difference at the interface of emulsions => Nernst distribution

Molecular velocities and macroscopic fluid properties

Macroscopic gas properties from kinetic theory and molecular behavior

Bird, Stewart, Lightfoot

Viscosity from the kinetic theory of gases : Chapman and Cowling

$$\mu = 0.499 \rho \lambda \overline{c_m} \quad \lambda = \text{mean free path}; \overline{c_m} = \text{mean molecular velocity}$$

$$\mu = \frac{M_m}{\sqrt{6} \pi \sigma^2} \sqrt{RT} \quad \mu \neq f(p), \mu \uparrow \sqrt{T}, \text{ unlike liquids where } \mu \downarrow \text{ with } \uparrow T$$

$$k = \frac{k}{\pi^{3/2} \sigma^2} \sqrt{RT} \quad D_{AB} = \frac{1}{3} \lambda \overline{c_m}$$

Modeling, Calculation Methods, and Simulation

1. Physical variables and dimensional analysis
2. Similarity and scaling laws
3. Order-of-magnitude analysis
4. Lumped element modeling
5. Numerical simulation and analytical modeling

Momentum Transfer

In single-phase flow, the fluid motion in microchannels is determined by wall friction, viscous forces and inertial forces.

Continuity Equation
$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{w})$$

Equation of motion
$$\frac{\partial(mw)}{\partial t} = \left(\dot{mw} \right)_{in} - \left(\dot{mw} \right)_{out} + (pA_i)_{in} - (pA_i)_{out} + mg + F_z$$

For one-dimensional differential channel elements with viscous flow, the momentum balance can be written as

$$\frac{\partial(\rho A w)}{\partial t} = -\frac{\partial(\rho w A w)}{\partial z} - \frac{\partial(p A)}{\partial z} + p \frac{\partial A}{\partial z} - \tau L_c - \rho A g$$

L_c is the perimeter of the differential channel element

With continuity equation, the above equation can be simplified for all space co-ordinates as (\vec{k} is the sum of external forces)

$$\rho \frac{D\vec{w}}{Dt} = \rho \left(\frac{\partial}{\partial t} + \vec{w} \cdot \text{div} \right) \vec{w} = -\text{grad } p + \mu \nabla^2 \vec{w} + \vec{k}$$

The 2nd term on LHS is also called the inertial term and describes convection. It is the only nonlinear term in the Navier-Stokes eqn.

The energy equation for fluid dynamics

The total energy conservation can be expressed as the sum of Mechanical and thermal parts.

Mechanical Energy

$$\frac{\rho}{2} \frac{D \vec{w}}{Dt} = \rho \vec{w} \cdot \vec{g} + \frac{\partial \rho}{\partial t} - \frac{D p}{Dt} - \Phi$$

Thermal Energy

$$\rho \frac{D h}{Dt} = - \operatorname{div} \vec{q} + \frac{D p}{Dt} + \Phi$$

The dissipation Φ of mechanical energy is accompanied by entropy production

- inner product of the viscous stress tensor and the velocity gradient tensor.

Relevant boundary conditions

- no wall slip, i.e. zero normal and parallel velocity at the wall (if no suction or blowing occurs, and for gases with $\text{Kn} < 0.01$),
- no temperature jump at the wall, i.e. the temperature and/or temperature gradients can be described at the wall.

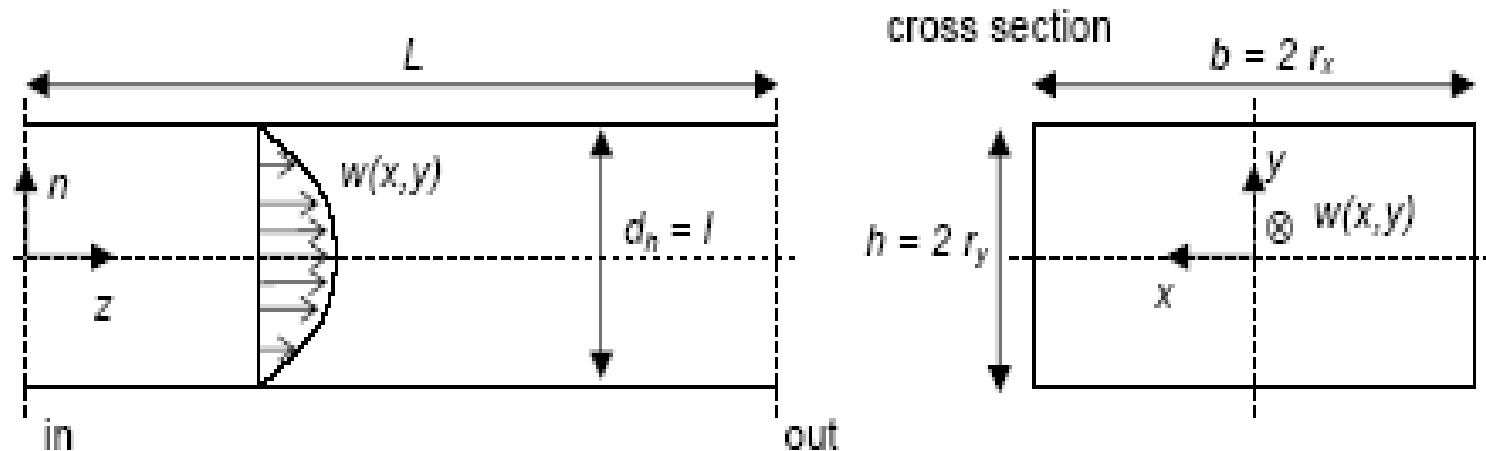
For constant density, non-viscous flow in all three directions, the momentum balance can be written as

$$\frac{\partial \vec{w}}{\partial t} = -(\vec{w} \text{ div}) \vec{w} - \frac{\text{grad } p}{\rho} + \vec{g} \quad \text{Euler equation}$$

Additional forces in microchannel flow may occur with surface effects in multiphase flow, such as bubbles and droplets.

Momentum Transport of Single-Phase Flow

Basic equations for long, small channels



Typical setup for a slender microchannel with rectangular cross section.

Length in flow directions is much larger than cross-sectional dimensions

dimensionless variables are introduced with reference values for slender channels with rectangular cross section (width b , depth h , length L , hydraulic diameter $d_h = 1$)

$$z^* = \frac{z}{l}; \quad y^* = \frac{y}{l}; \quad w^* = \frac{w}{w_r}; \quad v^* = \frac{v \text{Re}}{w_r}; \quad p^* = \frac{p - p_r}{\rho w_r^2};$$

$$T^* = \frac{T - T_r}{\Delta T}; \quad \eta^* = \frac{\eta}{\eta_r}; \quad \lambda^* = \frac{\lambda}{\lambda_r}; \quad \Phi^* = \frac{\Phi l^2 \text{Re}^2}{w_r^2}$$

The equations describing a steady and incompressible flow in long channels with variable viscosity and thermal conductivity are

the continuity equation

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0,$$

the z - momentum equation

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{\partial p}{\partial z^*} + \frac{\partial}{\partial y^*} \left[\eta^* \frac{\partial w^*}{\partial y^*} \right] + \frac{1}{\text{Re}^2} \frac{\partial}{\partial z^*} \left[\eta^* \frac{\partial w^*}{\partial z^*} \right],$$

The y – momentum equation

$$\frac{1}{\text{Re}^2} \left[v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} \right] = -\frac{\partial p}{\partial y^*} + \frac{1}{\text{Re}^2} \left[\frac{\partial}{\partial y^*} \left(\eta^* \frac{\partial v^*}{\partial y^*} \right) + \frac{\partial}{\partial z^*} \eta^* \frac{\partial v^*}{\partial z^*} \right]$$

Thermal energy equation

$$v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{1}{\text{Pr}} \frac{\partial}{\partial y^*} \left[\lambda^* \frac{\partial T^*}{\partial y^*} \right] + \frac{1}{\text{PrRe}^2} \frac{\partial}{\partial z^*} \left[\lambda^* \frac{\partial T^*}{\partial z^*} \right] + \frac{\text{Ec}}{\text{Re}^2} \Phi^*$$

Reynold's number = $\rho w_r l / \eta_r$; Prandtl number = ν / α

Eckert number = $w_r^2 / c_p \Delta T$

In flow regimes with high Re numbers, the terms with $1/\text{Re}^2$ are very small and can be ignored.

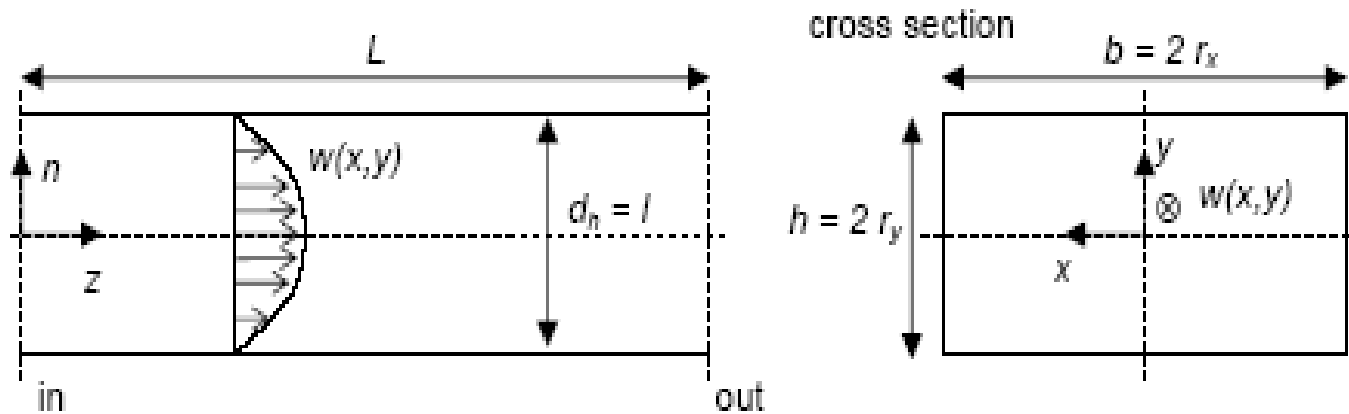
In microflows where the Re number is in the order of $O(1)$, the full Navier-Stokes equations must be considered and the influence of u and v velocities is not negligible.

When considering the energy equation, the viscous dissipation cannot be neglected in microchannels,

The ratio $\text{Ec}/\text{Re}^2 = \nu^2/(c_p \Delta T d^2)$ (ν = kinematic viscosity) does not incorporate velocity and is proportional to the inverse square of the hydraulic diameter.

Lecture 10

Basic equations for long, small channels



Typical setup for a slender microchannel with rectangular cross section.

z - momentum equation

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{\partial p}{\partial z^*} + \frac{\partial}{\partial y^*} \left[\eta^* \frac{\partial w^*}{\partial y^*} \right] + \frac{1}{\text{Re}^2} \frac{\partial}{\partial z^*} \left[\eta^* \frac{\partial w^*}{\partial z^*} \right],$$

The describing PDEs of the velocity w through a certain cross sectional area is comparable to the displacement of a thin membrane or the stress function in a bar under torsion. That means for the velocity

$$w(x, y) = \frac{\Delta P}{\eta L} \phi$$

with $\Delta p/L$ as pressure loss per length and ϕ as the torsion function.

A solution for the torsion function on a rectangular cross section can be found with a Fourier series:

$$\phi = \sum_{m,n=1}^{\infty} \frac{16(2r_x)^2(2r_y)^2}{mn\pi^4(n^2 4r_x^2 + m^2 4r_y^2)} \sin\left(\frac{m\pi x}{2r_x}\right) \sin\left(\frac{n\pi y}{2r_y}\right),$$

with $2r_x$ and $2r_y$ as the width and depth of the channel. The sum of the first 4 elements is sufficient for rectangular channels with an aspect ratio of approx. $a = 1$.

Higher order elements are negligible, but must be considered for flat channels.

The basic equations are valid for laminar flow with a typical Re number below the critical Re number at the transition from laminar to turbulent flow.

The critical Re numbers for internal flows are in the range of $Re_c = O(10^3)$.

circular pipe flow: $Re_c \approx 2300$ ($l = d$, diameter)

rectangular channel flow: $Re_c \approx 2300$ ($l = d_h$: hydraulic diameter $d_h = 4A/l_p = 2bh/(b+h)$)

plane channel flow: $Re_c \approx 2000$ ($l = h$, distance of the walls)

plane Couette flow: $Re_c \approx 1800$ ($l = b$, distance of the walls)

The governing equations can be simplified for long channel geometries with established mean values for the velocity and for constant fluid properties.

For different locations 1 (at the inlet) and 2 (at the outlet) of the channel, the basic equations can be written as:

continuity equation:

$$\rho w_2 A_2 = \rho w_1 A_1$$

momentum equation or mechanical energy equation, also called the Bernoulli equation

$$\frac{w_2^2}{2} + \frac{p_2}{\rho} + g y_2 = \frac{w_1^2}{2} + \frac{p_1}{\rho} + g y_1 + w_{t12} - \varphi_{12}$$

technical energy w_{t12} and the dissipation φ_{12}

The Bernoulli equation, is complemented by the technical energy w_{t12} and the dissipation. It represents the momentum balance and the mechanical energy balance.

W_{t12} : specific technical work, positive for added work (pump, actuator, etc.) and negative for produced work (turbine, opened valve, etc.) between location 1 and 2,

Thermal energy
$$e_2 = e_1 + q_{12} + \varphi_{12}$$

q_{12} : specific heat, positive for added heat (heating, light or electromagnetic waves, etc.) and negative for cooled fluid (heat loss to ambient, etc.) between locations 1 and 2

φ_{12} : specific viscous dissipation, simultaneously decreasing the mechanical energy and increasing the inner energy between location 1 and 2. The viscous dissipation is linked to the pressure loss Δp .

The pressure loss Δp_{12} in a slender channel is determined from the Bernoulli equation (neglecting technical work).

$$\Delta p_{12} = \rho \varphi_{12} = p_1 - p_2 + \frac{\rho}{2} (w_1^2 - w_2^2) + g (y_1 - y_2)$$

For a constant cross section and negligible gravitation forces, the viscous dissipation in a channel element correlates with the pressure loss:

$$\varphi_{12} = \frac{p_1 - p_2}{\rho}$$

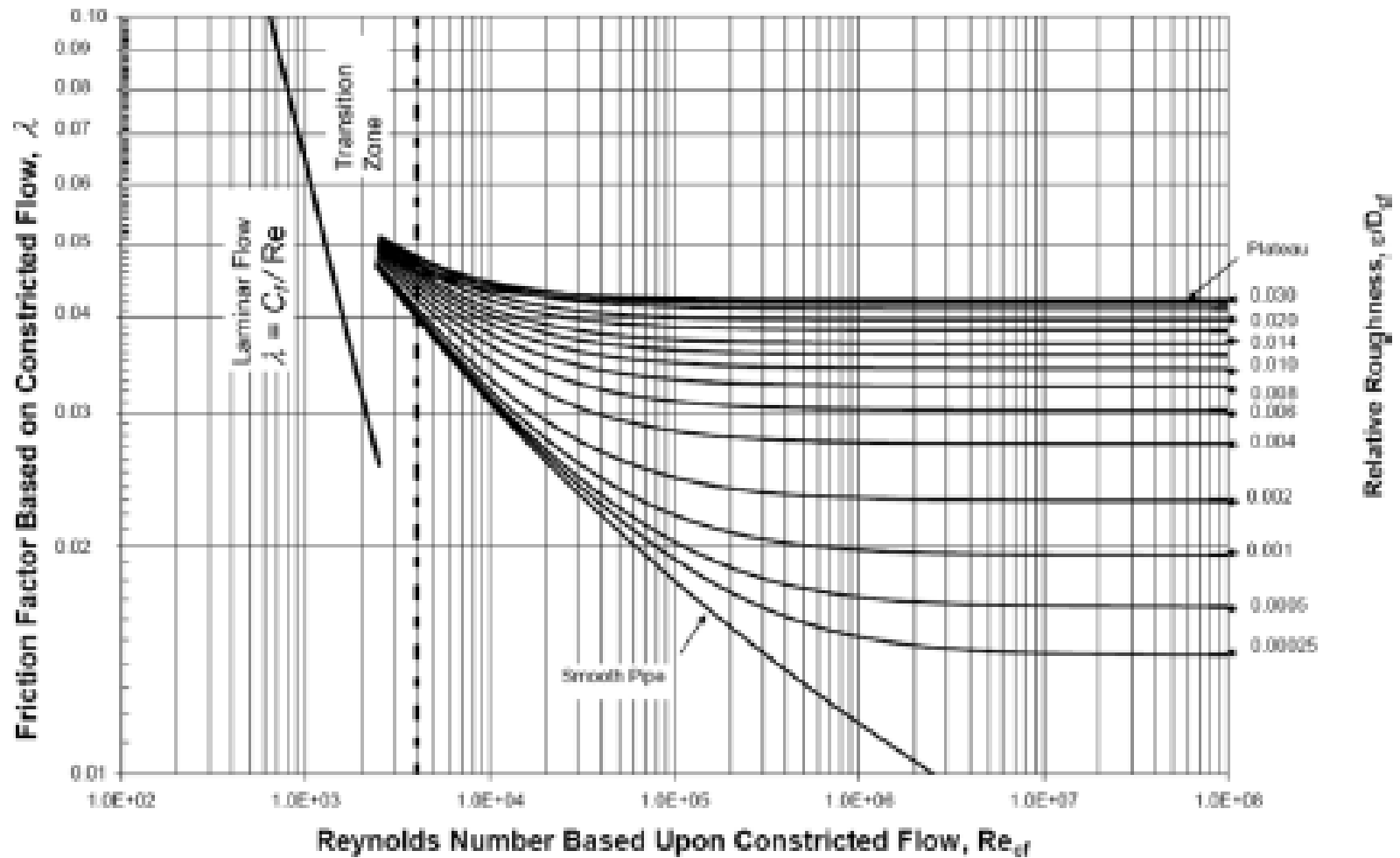
The pressure loss is approximated by the sum of individual losses consisting of fittings, bends, valves and straight pipes of length l_i

$$\Delta p = \sum_i \left(\lambda_i \frac{l_i}{d_{h,i}} + \zeta_i \right) \cdot \frac{\rho}{2} w_{ref,i}^2$$

The reference velocity $w_{ref,i}$ must be determined for each channel element i .

The channel friction factor λ of the straight pipe is determined by the flow regime and the cross section.

The pressure loss coefficient ζ_i is affected by flow internals, such as curves, bends, fittings, and other channel joints.



Flow friction factors for straight channels with a rough surface according to Moody

Channel friction factor coefficients C_f in fully developed flow through straight microchannels with different cross-sections.

Cross section, char. length	$C_f = \lambda \cdot \text{Re}$	w_{\max}/\bar{w}
circle, D	64	2.000
square, h	56.92	2.0962
rectangular, $h; b$ aspect ratio $\alpha_A = h/b$ see also Eq. 3.67	$96 [1 - 1.3553\alpha_A + 1.9467\alpha_A^2 - 1.7012\alpha_A^3 + 0.9564\alpha_A^4 - 0.2537\alpha_A^5]$	–
slab, $\alpha \rightarrow 0$	96	1.500
hexagon	60	–
60° trapezoid $h/b = 4.00$	55.66	2.181
2.00	55.22	2.162
1.00	56.60	2.119
0.50	62.77	1.969
0.25	72.20	1.766
KOH trapezoid $h/b = 1.00$	56.15	2.137

Convective Fluid Dynamics in Microchannels

The flow in microchannels is generally regarded as straight laminar flow. This is correct for straight channels with low flow velocity and, therefore, low Re numbers.

In straight channels, the flow remains laminar with straight streamlines below a Re number of 2300, although first flow disturbances with wavy streamlines may appear for even lower Re numbers.

Straight laminar flow changes when the fluid flows through curves, bends, or around obstacles.

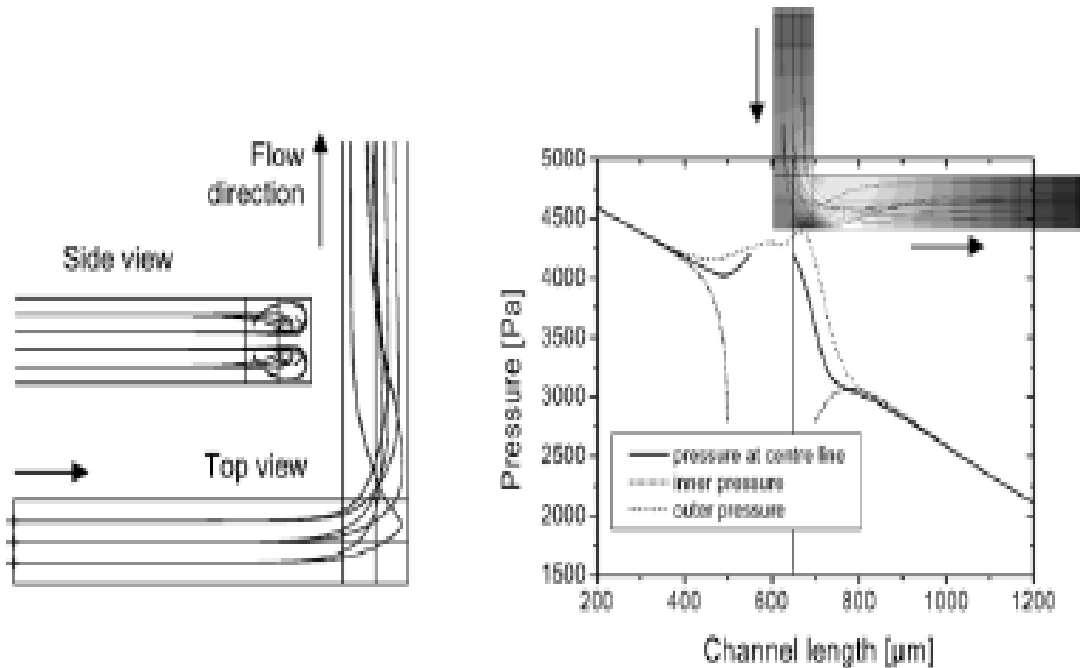
Centrifugal forces in bends push the fluid from the center of the channel, where the bulk fluid flows with high velocity, to the outward side.

At the wall, the fluid is forced either upwards or downwards, producing a symmetric, double vortex filling the entire channel.

This flow regime in curved channel elements is often called Dean flow.

$$\text{Dean Number} = \text{Re} (D / R_c)^{1/2}$$

The viscous wall friction acts against the centrifugal force and, therefore, dampens the vortex flow.



Left: Vortices in a 90° bend (L-mixer) with laminar vortex flow, $Re = 99$, $w^- = 0.85$ m/s; Right: Pressure distribution in the 90° bend at the center plane of the channels, $100 \times 100 \mu\text{m}^2$, $Re = 99$.

The investigated flow regimes are laminar with vortex formation. No onset of turbulence was observed in the bends.

In the inlet channel, a uniform pressure distribution in the cross section can be observed.

Due to the curvature of the bend the flow is altered into a new direction. At the outer side of the bend, the pressure is increased, comparable to the stagnation point of an impinging jet.

At the inner side of the bend, the pressure decreases directly at or shortly behind the sharp corner, which often results in recirculation, separation flow, or cavitation.

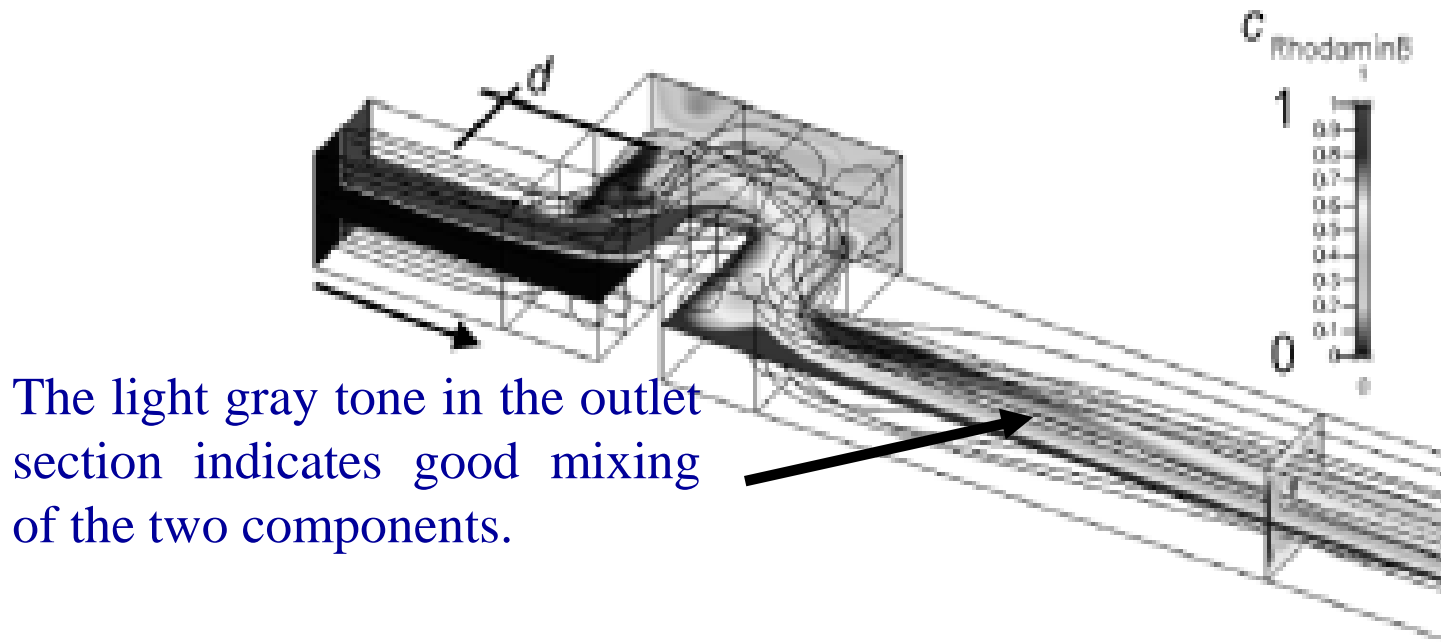
Approx. $100\ \mu m$ behind the bend, a uniform pressure establishes in the cross section.

At this point, the vortices are already dampened and straight laminar flow is established again, as shown by the streamlines in left side

The pressure loss in the bend results in vortex formation and is the basis for further calculation in mixing theory.

Mixing Elements

U-shaped 90° bend mixing element



The light gray tone in the outlet section indicates good mixing of the two components.

Streamlines and concentration profiles ($Sc = 3700$ for aqueous Rhodamin B solution) in a combined 90° bend mixer (U-mixer) with channel dimensions of $300 \times 300 \mu\text{m}$, offset d , mass flow rate of 250 g/h, ($Re = 270.7$)

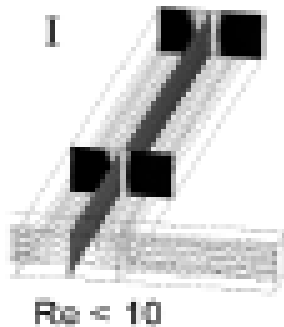
Fluid dynamics in T-junctions with symmetric inlet conditions

Dynamics of the mixing process in T-junctions are treated with symmetrical inlet conditions and 1 : 1 mixing of the reactants.

CFD simulations and visualizations

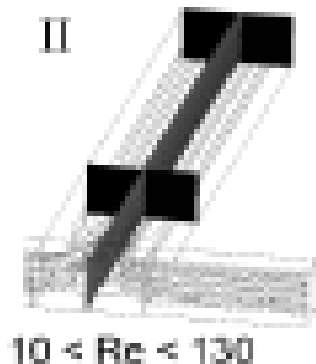
T-shaped micromixer (T600×300×300 which represents a mixing T with rectangular cross sections, a mixing channel width of 600 μm , two symmetric inlet channels with a width of 300 μm , and 300 μm overall depth)

Classification of the investigated flow regimes in symmetrical T-shaped micromixers



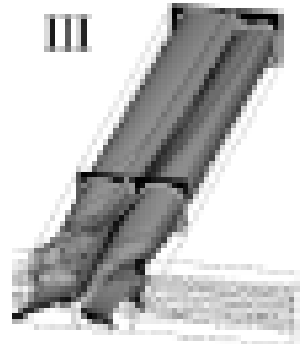
$Re < 10$, $Dn < 10$, regime I straight laminar flow, steady velocity profile with straight streamlines, diffusion dominates the mass transfer;

$$Dn = Re (D / R_c)^{1/2}$$



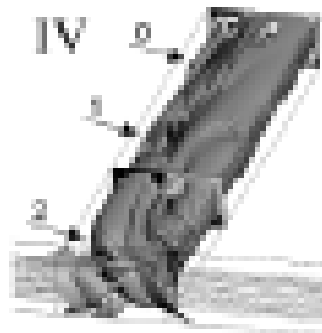
$10 < Re < 130$ (approx.), $Dn > 10$, regime II symmetric vortex flow, Dean flow with characteristic Dn number.

straight laminar flow is disturbed by the centrifugal forces created by an increasing Re number, and symmetric vortex flow establishes in the mixing channel



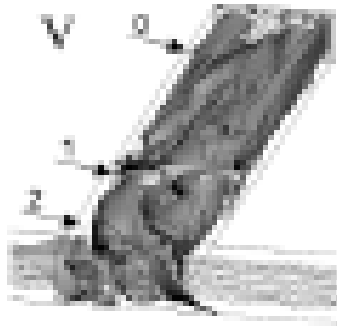
130 < Re < 240

130 < Re < 240 (approx.), regime III engulfment flow, breakup of the symmetry, fluid swaps to the opposite side. This leads to small lamellae, short diffusion lengths, and high mixing quality.



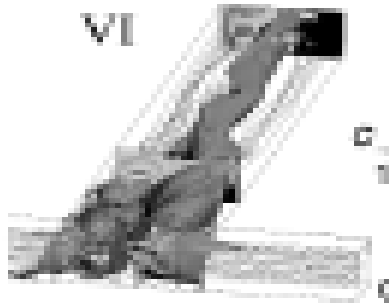
240 < Re < 400

240 < Re < 400 (approx.), regime IV regular, periodic pulsating flow, reproducible vortex break down



$400 < Re < 500$

$400 < Re < 500$ (approx.), regime V quasi-periodic pulsating flow, vortex breakdown, broad range of mixing quality;



$500 < Re$

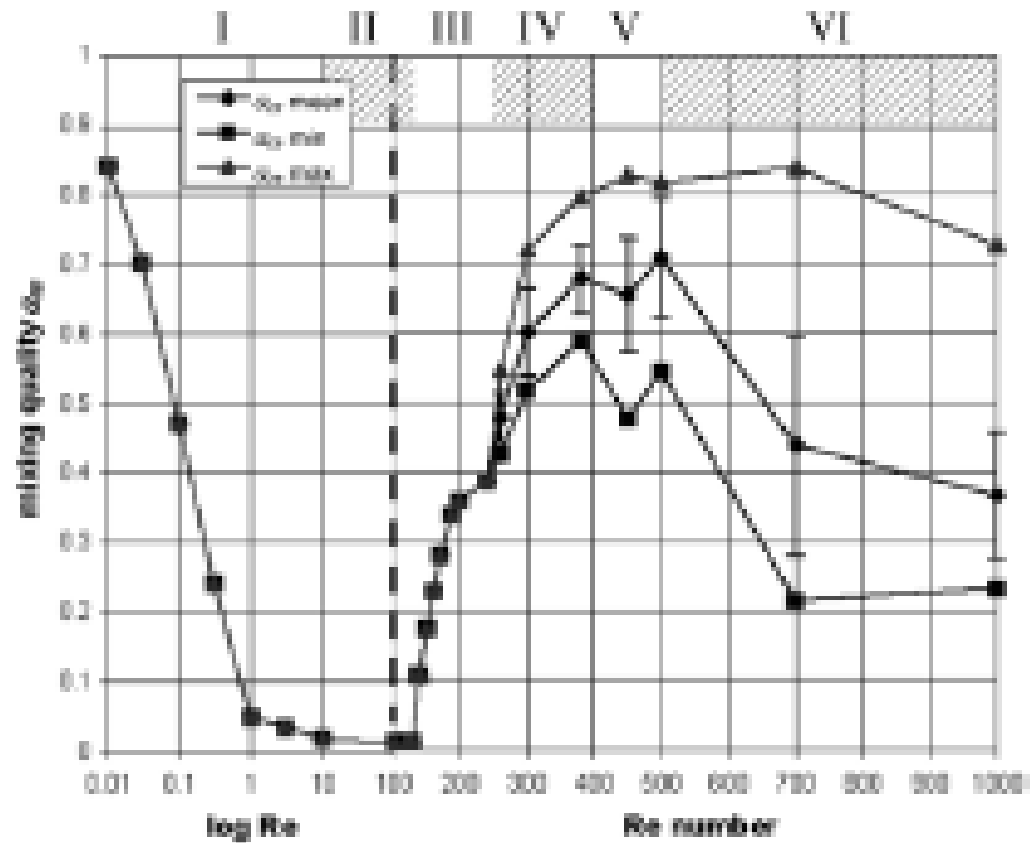
$500 < Re$ (approx.), regime VI chaotic pulsating flow, vortex breakdown, decreased mixing quality and single flushes of fluid swapping to the opposite side

Flow regimes with Re numbers higher than 1000 are avoided in T-shaped micromixers.

High Re numbers are unsuitable due to high flow velocities producing intolerably high pressure loss.

The mixing quality is the standardized concentration field variance σ_c and is often used to characterize the state of mixing

$$\alpha_m = 1 - \sqrt{\frac{\sigma_c^2}{\sigma_{c,\max}^2}}$$



Development of the mixing quality α_m ; for $0.01 < Re < 100$ on a logarithmic scale and for $100 < Re < 1000$ on a linear scale

A T-shaped micromixer with typical channel dimensions from $50 < d_h < 600$ μm exhibits the most suitable mixing characteristics in the range of $200 < \text{Re} < 700$ due to the moderate flow velocities, ranging from $1 < w^- < 50$ m/s for gases and $0.1 < w^- < 10$ m/s for liquids

Heat Transfer and Micro Heat Exchangers

Heat Transfer Fundamentals: The energy balance

For a control volume:

$$\sum \dot{E}_{in} - \sum \dot{E}_{out} \pm \sum \dot{E}_{loss} = 0$$

First law of thermodynamics with a dissipation term Φ for closed systems

$$dE_{sys} = dU + dQ + dW + \Phi$$

For open systems, (P_t is the technical power).

$$d\dot{E}_{sys} = d\dot{H} + d\dot{Q} + dP_t + \dot{\Phi}$$

The dissipation Φ takes into account that the energy conversion from one form into another is accompanied by natural losses.

For a process device with mass flow rate m , heat flux q over the boundary, technical work W_t , or mechanical power P , and chemical reaction, the energy equation is written as

$$\dot{Q} = \left(\dot{m} (u_2 - u_1) \right) + \left(\dot{m} \left(\frac{p_2}{\rho} - \frac{p_1}{\rho} \right) \right) + \left(\dot{m} g (z_2 - z_1) \right) + m \left(\frac{\alpha_2 \overline{V_2^2}}{2} - \frac{\alpha_1 \overline{V_1^2}}{2} \right) + \dot{E}_q + P$$

The temporal energy change in a system consists of

- energy flowing in and out/dissipation,
- pressure,
- gravity,
- KE,
- energy produced inside the system (chemical reactions).
- technical work or mechanical power P .

The energy of the fluid flowing in a straight channel, without chemical reactions and technical work consumed or produced.

$$e = \left(u + \frac{w^2}{2} + g y \right)$$

With Fourier law $q = -\lambda \partial T / \partial x$ for conductive heat transfer perpendicular to the channel axis, the energy equation can be written as

$$\begin{aligned} \frac{\partial}{\partial t} \left(\rho \left(u + \frac{w^2}{2} \right) \right) = & - \frac{\partial}{\partial z} \left(\rho w \left(u + \frac{w^2}{2} \right) \right) + \rho g y - \\ & - \frac{\partial}{\partial z} (p w) - \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial z} (\tau w) + \dot{w}_t, \end{aligned}$$

Bernoulli equation for the energy balance in channel flow and can be simplified to suit each process.

For open systems and flow processes, the inner energy is replaced by the enthalpy $h = u + p/\rho$.

For flow processes, the total enthalpy $ht = u + p/\rho + w^2/2 + g y$ is applied together with the kinetic and potential energy part.

With the energy dissipation (ε) from shear stress and velocity gradient, the enthalpy form of the energy equation can be written as

$$\rho \frac{dh}{dt} = \varepsilon - \nabla \cdot \dot{q}$$

With the caloric equation of state, the correlation between the inner energy u or enthalpy h , and the temperature ($du = c_v dT$ and $dh = c_p dT$) the energy equation can be rewritten as

$$\rho c_p \frac{dT}{dt} = \varepsilon + \text{div}(\lambda \text{grad} T)$$

Solving this equation gives the temperature distribution for the actual process.

Heat conduction in small systems

Fourier's law of heat transfer describes the correlation between the steady heat flow and the driving temperature difference.

$$\dot{Q} = -\lambda A(r) \frac{dT}{dr}$$

The integration over the coordinate r leads, for constant heat conductivity λ , to

$$T_1 - T_2 = \dot{Q} \frac{1}{\lambda} \int_{r_1}^{r_2} \frac{1}{A(r)} dr$$

The integrated area divided by the thermal conductivity is often called the thermal resistance R_{th} in analogy to electrical resistance.

Thermal resistance of the simple geometrical elements plate, cylinder, and sphere.

plate or slab	cylindrical element	spherical element
$A = \text{const.}$	$A = 2r\pi L$	$A = 4r^2\pi$
$R_{th} = \frac{1}{\lambda} \frac{r_2 - r_1}{A}$	$R_{th} = \frac{1}{\lambda} \frac{\ln(r_2/r_1)}{2\pi L}$	$R_{th} = \frac{1}{\lambda} \frac{(1/r_1 - 1/r_2)}{4\pi}$

The heat conductivity in microsystems is influenced by the [microstructure of the material](#). Grain boundaries and crystal lattices form additional resistances to heat transfer.

In regular crystals, the heat transfer coefficient is dependent on the crystal orientation, and the Fourier equation of heat transfer must be expanded to the tensor notation

$$\dot{q} = -\lambda \frac{dT}{dx} \rightarrow \vec{\dot{q}} = -\Lambda \text{ grad } T$$

with the heat conductivity tensor

$$\Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{23} & \lambda_{33} \end{pmatrix}.$$

The solution of the three-dimensional heat conduction is often only possible with numerical methods.

The time dependent second Fourier law is derived from a differential element with the balance of the heat capacity and the heat conduction

$$\rho A(r) c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial r} \left(\lambda A(r) \frac{\partial T}{\partial r} \right).$$

Constant material properties, such as heat capacity c_p and heat conductivity λ , and mathematical simplification for different geometrical bodies ($n = 0$ for a plate, $n = 1$ for a cylinder, and $n = 2$ for a sphere) lead to,

$$\frac{\partial T}{\partial t} = a \left(\frac{\partial^2 T}{\partial r^2} + \frac{n}{r} \frac{\partial T}{\partial r} \right)$$

with the heat diffusivity $\alpha = \lambda / \rho c_p$.

The transient temperature development in a semi-infinite body, is given in one-dimensional form by



$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2}; \quad t \geq 0, x \geq 0.$$

$$T(x, t)$$

$$T(x, 0) = T_i$$

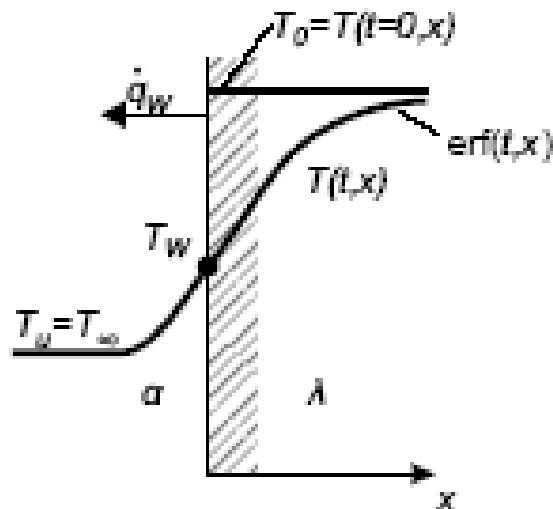
$$T(x=0, t > 0) = T_S$$

$$T(x \rightarrow \infty, t > 0) = T_i$$

The transient temperature development in a semi-infinite body, is given in one-dimensional form by

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2}; \quad t \geq 0, \quad x \geq 0.$$

The introduction of the dimensionless temperature θ leads to



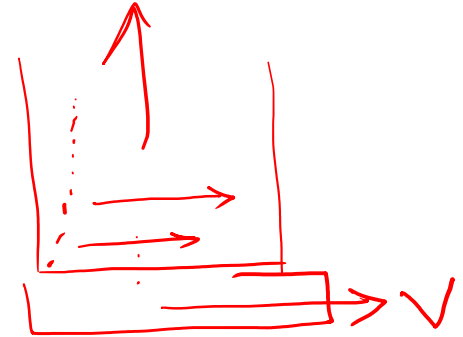
$$\frac{\partial \theta}{\partial t} = a \frac{\partial^2 \theta}{\partial x^2} \quad \text{with} \quad \theta = \frac{T - T_0}{T_w - T_0},$$

describing the temperature development in a solid body with defined wall temperature T_w

Temperature distribution in a semi-infinite body during cooling

With constant wall temperature $T_w = \text{const.}$, the dimensionless temperature is determined by the error function

$$\frac{T - T_0}{T_w - T_0} = \text{erf}(x^*) \quad \text{with} \quad x^* = \frac{x}{2\sqrt{at}}$$



The wall heat flux can be calculated from this equation.

For a constant α at the wall, the solution is derived with the help of two dimensionless numbers, the Fourier number (a = thermal diffusivity) and the Biot number (α = heat transfer coefficient)

$$Fo = \frac{a t}{x^2}$$

$$Bi = \frac{\alpha x}{\lambda_s}$$

The temperature development during cooling of the body is given by

$$\theta_C = \frac{T - T_\infty}{T_0 - T_\infty} = \operatorname{erf}(x^*) - e^{\text{Fo} \text{Bi}^2 + \text{Bi}} \operatorname{erfc}(\sqrt{\text{Fo}} \text{Bi} + x^*)$$

and for heating

$$\theta_H = 1 - \theta_C.$$

$$t_c = a t \left(\frac{\alpha}{\lambda} \right)^2 = \text{Fo} \cdot \text{Bi}^2$$

The characteristic time for heating or cooling is drawn from the combination of the Fo and Bi numbers and does not depend on the length (semi-infinite body).

Miniaturization will not influence the temperature development and the heat flux for a semi-infinite body.

For small bodies (high Fo and low Bi number), the temperature distribution can be approximated by asymptotic solutions.

The temperature inside a small body is only a function of time

$$\theta = \frac{T - T_{\infty}}{T_0 - T_{\infty}} = \exp\left(-\frac{\alpha A t}{m c_p}\right) \quad \text{for } Fo > 0.3 \text{ and } Bi < 0.2,$$

More exact for the three basic geometries (plate $n = 0$, cylinder $n = 1$, sphere $n = 2$):

$$\theta = \exp(-(n+1) Bi Fo)$$

Convective heat transfer in microchannels

The total heat transfer in microstructured devices consists of heat conduction through the walls and convective heat transfer from the wall into the fluid in microchannels.

For straight laminar flow, the dimensionless heat transfer coefficient, the Nu number, is constant.

For constant wall heat flux, $Nu_q = 4.3$, for constant wall temperature $Nu_T = 3.66$.

In a wide gap or narrow slit, the Nu number is 7.54 ($q = \text{const.}$) and 8.24 ($T = \text{const.}$) for double-sided heat transfer,

and 4.86 ($q = \text{const.}$) and 5.39 ($T = \text{const.}$) for single-sided heat transfer.

For smaller channels, the heat transfer coefficient increases due to the constant Nu number.

With decreasing channel dimensions, the transfer area and the mass flow through the channel are also decreased, hence, the transported heat is limited by these conditions.

To maintain a high heat transfer coefficient with a high transport rate, an optimum channel dimension must be found - fabrication process.

At the entrance of a channel or behind channel elements, such as channel junctions, expansions or contractions, the disturbed flow enhances the radial transport in the channel.

This results in increased pressure loss as well as increased heat or mass transfer.

Dimensionless channel length X^* , starting at the entrance,

$$X^* = \frac{L}{d_h} \text{Pe} = \frac{L}{d_h} \text{Re} \cdot \text{Pr}$$

Pe is the heat transfer Peclet number.

The mean Nu number in the entrance flow Nu_{me} is calculated with the mean Nu number in straight channel flow, Nu_m according to the following correlation

$$Nu_{me} = \frac{Nu_m}{\tanh \left(2.432 Pr^{1/6} X^{* 1/6} \right)}.$$

This equation is valid for the entire channel length X^* and $Pr > 0.1$

Beyond a certain length, the velocity and temperature profiles do not alter - fully-developed flow

In turbulent flow (for $Re > Re_{crit} = 2300$ in channel flow) the pressure loss is proportional to the square mean velocity and the heat transfer can be calculated according to Gnielinski

$$Nu = \frac{\xi/8 (Re - 1000) Pr}{1 + 12.7 \sqrt{\xi/8} (Pr^{2/3} - 1)} \left(1 + \left(\frac{d_h}{l} \right)^{2/3} \right) K_{Pr}$$

$$\text{with } \xi = (1.8 \log_{10}(Re) - 1.5)^{-2} \text{ and } K_{Pr} = (Pr_{fluid}/Pr_{wall})^{0.11}$$

This correlation is valid for $0.5 < Pr < 2000$, $2300 < Re < 5 \cdot 10^6$ and $1 < L/d_h < \infty$.

Turbulent flow does not often occur in microchannels, however, the manifolds or inlet and outlet headers may produce turbulent conditions.

Surface roughness effect on heat transfer is less significant than on momentum transfer

Rarefied gases with slip boundary conditions

The previous equations are valid above the continuum limit (Knudsen number = mean free path/characteristic length).

$$\underline{\mathbf{Kn} < 10^{-2}}$$

The continuum and thermodynamic equilibrium assumptions are appropriate and flow situations can be described by conventional no-slip boundary conditions.

$$10^{-2} < \mathbf{Kn} < 10^{-1}$$

Slip Flow Regime

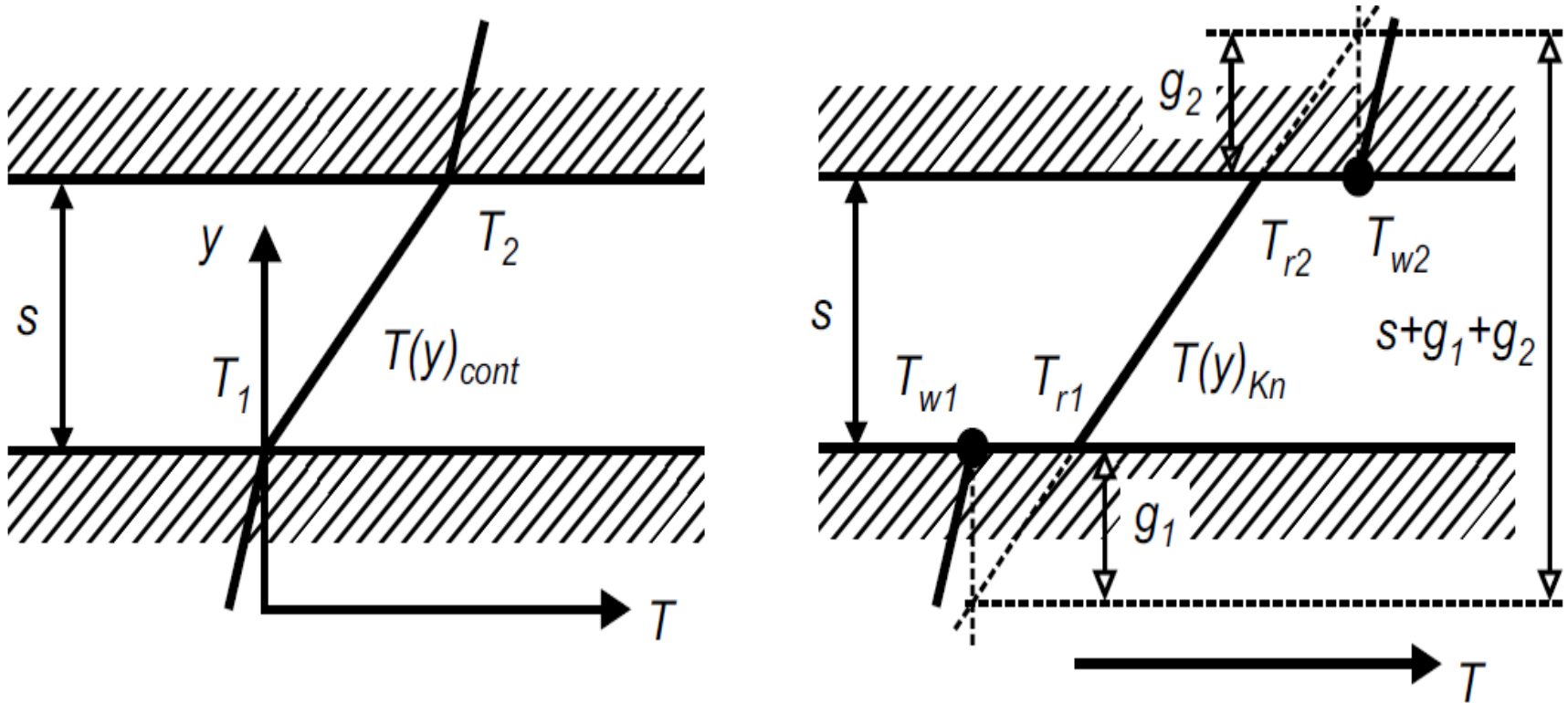
Navier-Stokes equations remain valid provided tangential slip-velocity and temperature-jump boundary conditions are implemented at the walls of the flow domain

For rarefied gases, the transfer processes at the wall differ due to the low number of gas particles.

The **reduced flow resistance** is produced by the slip velocity at the wall, which results from the molecular motion and insufficient momentum transfer between the wall and bulk fluid.

The molecular motion in the gas influences energy transfer into the fluid. A temperature jump occurs at the wall, which **increases the heat transfer resistance**.

Temperature gradient in a gap



Left: linear development for dense gases ($\text{Kn} < 0.01$);

Right: temperature jump in rarefied gases ($\text{Kn} > 0.01$).

Accommodation Coefficient, β

When a particle impacts with a surface, energy is transferred in the form of heat and stress, which leads to two main types of accommodation coefficients - thermal and transverse momentum.

The thermal accommodation coefficient is the fraction of heat transferred between the surface and the molecule. “The ratio of the average energy actually transferred between a surface and impinging gas molecules scattered by the surface, to the average energy which would theoretically be transferred if the impinging molecules reached complete thermal equilibrium with the surface.”

Transverse Momentum Accommodation Coefficients (TMAC)

The TMAC is the fraction of the momentum normal to the wall that is transferred to the wall in terms of stress. This stress is commonly known as pressure. By creating a pressure on the wall some of the vertical momentum is lost.

For rarefied gas flow ($0.01 < \text{Kn} < 0.1$), the boundary condition of the gas velocity at the wall is described by, (slip length ζ ,)

$$w(x=0) = \zeta \left(\frac{\partial w}{\partial x} \right)_{y=0}$$

The slip length ζ , can be calculated with the accommodation coefficient β and the mean free path Λ of the molecules. β describes the efficiency of the momentum and energy transfer from molecules to the wall and vice versa.

$$\zeta \approx \frac{2-\beta}{\beta} \Lambda \quad \beta = 2, \text{ from Kinetic theory for continuum regime}$$

Experimental data for β can be found in the literature.

The temperature jump at the wall is described in a similar way with the temperature jump coefficient g

$$T(x=0) = T_r = T_W + g \left(\frac{\partial T}{\partial x} \right)_{x=0}$$

The temperature jump coefficient g can be determined via kinetic theory from the thermal accommodation coefficient γ , a material parameter f , and the mean free path Λ

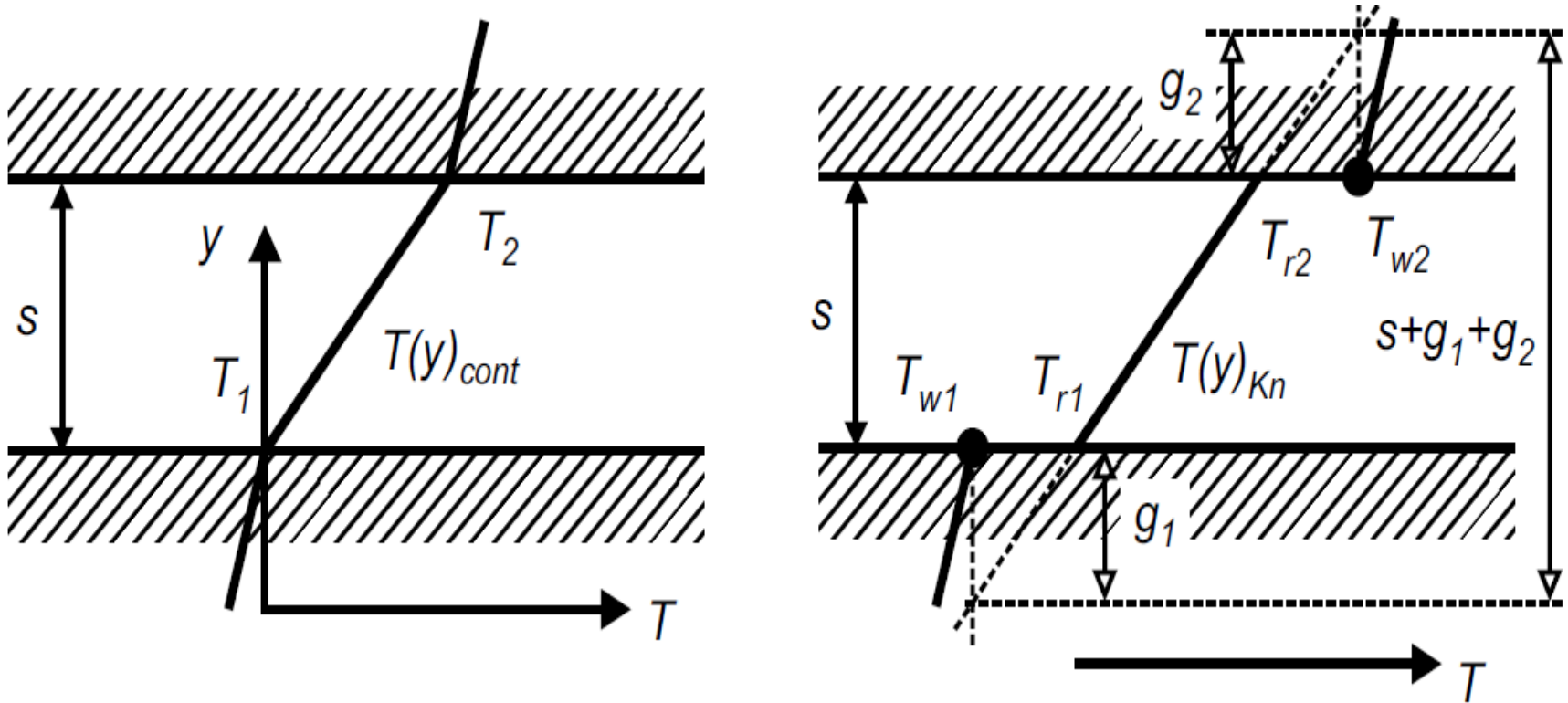
$$g = \frac{2 - \gamma}{\gamma} \frac{15}{8} f \Lambda$$

The material parameter f is calculated from the linearized Boltzmann transport equation and is given by

$$f = \frac{16}{15} \frac{\lambda}{\eta c_v} \frac{1}{\kappa + 1} = \frac{16}{15} \frac{1}{\text{Pr}} \frac{\kappa}{\kappa + 1}$$

λ = thermal conductivity, η = viscosity, κ = isentropic exponent = c_p/c_v

Temperature gradient in a gap



left: linear development for dense gases ($Kn < 0.01$); right: temperature jump in rarefied gases ($Kn > 0.01$).

For flow processes, the ratio of the temperature jump coefficient and the characteristic length is important and can be derived from $g = \frac{2-\gamma}{\gamma} \frac{15}{8} f \Lambda$ as

$$\frac{g}{l_{char}} = \frac{2-\gamma}{\gamma} \frac{15}{8} f \text{ Kn} \quad \Lambda - \text{mean free path}$$

For monatomic gases with $\kappa = 5/3$, $\text{Pr} = 2/3$ ($\rightarrow f = 1$), and complete accommodation $\gamma=1$, the length ratio reduces to

$$\frac{g}{l_{char}} = \frac{15}{8} \text{ Kn}$$

The linear temperature distribution in a gap (of size s) is determined with the jump coefficient g and the starting equation $T(x=0) = T_r = T_W + g \left(\frac{\partial T}{\partial x} \right)_{x=0}$ as

$$\frac{T(y) - T_2}{T_1 - T_2} = \left(1 - \frac{y + g_1}{s + g_1 + g_2} \right)$$

The temperature jump coefficient g can be regarded as an additional distance of the gap.

For $0.1 < \text{Kn} < 10$, monatomic gases, and complete accommodation, the temperature gradient is only a function of the Kn number.

$$\frac{T(y/s) - T_2}{T_1 - T_2} = \left(1 - \frac{y/s + (15/8) \text{Kn}}{1 + (15/4) \text{Kn}} \right)$$

The derivation of the temperature profile and comparison of the coefficients yields the heat flux for $0.1 < \text{Kn} < 10$, which can be expressed as the ratio to the continuum heat flux.

$$\frac{\dot{q}}{\dot{q}_{cont}} = \frac{1}{1 + (15/4) \text{Kn}}$$

For $Kn > 10$ (free molecular flow), the ratio of the heat fluxes in a gap are also dependent on the Kn number.

$$\frac{\dot{q}_{FM}}{\dot{q}_{cont}} = \frac{4}{15 Kn}$$

The distance between the plates plays no role in this regime; the molecules only hit the walls and not each other. The unsteady heat transfer of rarefied gases can be treated in the same manner.

Assuming a round capillary with the outer radius r_A and constant wall heat flux q , the heat transfer of rarefied gas flow (for $\text{Kn} < 0.1$) is expressed with the dimensionless heat transfer coefficient, the Nu number

$$\text{Nu}_q = \frac{\dot{q}}{(T_W - \bar{T})} \frac{r_A}{\lambda} = \frac{\alpha r_A}{\lambda} = 24 \left(11 - 6 \Delta \bar{w} + (\Delta \bar{w})^2 + 24 \frac{g}{r_A} \right)$$

The dimensionless slip velocity Δw is determined with the slip length from the Eq. for slip length

$$\zeta \approx \frac{2 - \beta}{\beta} \Lambda$$

$$\Delta w = \frac{w(r_A)}{\bar{w}} = \left(1 + \frac{r_A}{4 \zeta} \right)^{-1}.$$

For non-circular cross sections, the half of the hydraulic diameter d_h can be taken for r_A .

The Nu number for constant wall temperature is approx. 5 % higher than for constant wall heat flux.

A numerical study with the Monte-Carlo method indicated that the slip flow model correctly represents convective heat transfer between continuum and molecular flow.

The influence of the axial heat conduction must be considered, however, the viscous heat dissipation, expansion cooling can be neglected.

Microfluidic Networks for Heat Exchange

Transport processes in micro process engineering are governed by two different mechanisms: the **conductive** and the **convective** transfer of a species or energy.

The fluid flow in microchannels is often regarded to be laminar with dominant conductive transport.

On structured surfaces and in bent and curved flow, secondary transversal flow components are introduced into the straight laminar flow, which enhance the transport processes.

Thus, two strategies enhance the overall transport in pressure-driven flow in passive devices: the implementation of small diameter channels with a short diffusion length and the creation of secondary flow structures perpendicular to the main flow direction.

The typically low flow rates in single microstructured elements can be enlarged by internal and external numbering-up or equal-up of the desired effects.

The internal numbering-up is limited by the available space and the uniform distribution of the fluids. A relative maldistribution of the liquids of approx. 5 % will lead to a corresponding decrease in the heat transfer efficiency.

Status-quo of microfluidic networks for device cooling

Goal: to provide high thermal performance with low pressure loss.

Cooling of electronic equipment is often accomplished in long, straight channels, which produces relatively low heat transfer coefficients.

With a cost effective manufacturing and proper system integration, a heat dissipation rate of up to 10 MW/m^2 (1 kW/cm^2) appears possible for single phase liquid flow.

The setup of different branching levels to spread fluid over an area and collect it again can be managed with the Constructal Theory of Bejan

Constructal theory of Bejan

The **constructal design** approach begins with the smallest elements on zero level and connects these with those on the next higher level.

This approach is inverse to the **fractal description of branched systems**, where an element is repeatedly miniaturized till almost infinitely small structures.

In nature, systems have a finite smallest size, and therefore, follow the constructal approach.

Optimum size of channel elements and the corresponding area covered

Murray's law

The Physiological Principle of Minimum Work: The Vascular System and the Cost of Blood Volume, 1926, PNAS

In every segment of vessel, flow is achieved with the least possible biological work.

Two energy terms contribute to the cost of maintaining blood flow: (a) the energy required to overcome viscous drag in a fluid obeying Poiseuille's law, and (b) the energy metabolically required to maintain the volume of blood and vessel tissue involved in the flow.

The larger the radius, the smaller is the power, P_f , required for flow, but the larger is the power, P_m , required for metabolic maintenance of the blood and vessel wall tissue.

The vessel can be neither too large nor too small if the total power, $P_t = P_f + P_m$, is to be minimized.

Murray's law

The optimum flow distribution and cross sections of the channels on different branching levels are influenced by - Murray's law, applicable, for example, in the branching of blood vessels or plant capillaries.

If the sum of the inner radii to the power of three on each branching level is constant, the channel network will need minimal power consumption or exhibit minimal pressure loss for a given flow rate.

For channels with circular cross section, the diameter of the highest and largest level element d_n to the power of 3 is equal to the sum of the diameters d_z of the next level elements to the power of 3.

$$d_n^3 = \sum_i d_{z,i}^3$$

The level with the largest elements has the notation “n” due to n branching levels of the system. The zero level of the system is the smallest level following the notation of the constructal design method.

For a symmetrical bifurcation, the above equation gives the following correlation for the diameters on various branching levels z

$$d_n^3 = 2^z d_z^3 \quad or \quad d_z / d_n = 2^{-z/3}$$

This correlation serves for an equal wall shear stress in the channels on each branching level z

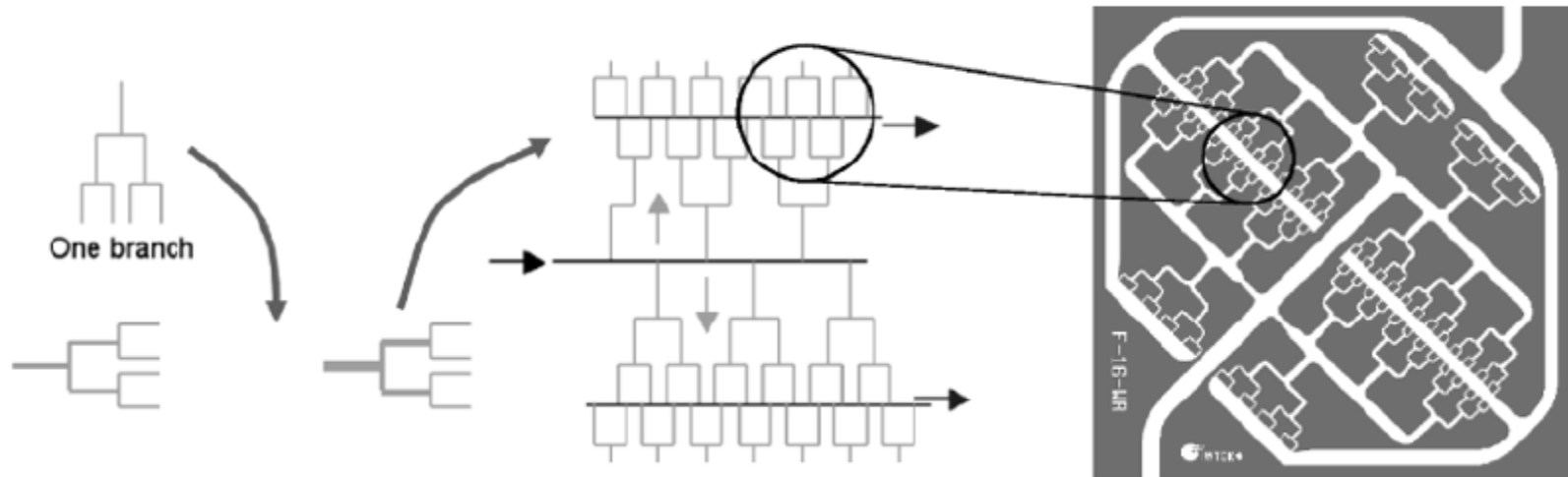
Since the original derivation of Murray's law, it has been noted that the application of other optimization principles (not just that of minimum work) result in the final expression: minimizing the total mass of the network, minimizing volume for a constant pressure drop and flow rate, keeping the shear stress constant in all channels, or minimizing flow resistance for a constant volume.

Combined channel elements

Low pressure loss and a low mean driving temperature difference ΔT are essential for an optimum operation and suitable heat exchanger performance.

Both effects lead to appropriate geometrical optimization of dendritic channel networks with minimal entropy generation, presented under the concept of “Bejan’s constructal theory”

Heat exchanger channel network



Setup of a two level heat exchanger network from branched channels (left to right)

Figure shows the combination of the branched elements from zero level to the desired covered area. The channel dimensions and the covered area are determined by the actual situation

The channel cross sections are designed according to Murray's law - the relation between the channel cross sections in different branched and connected levels.

The optimization of a channel network to a minimal pressure loss shows that the cube of the diameters of a parent channel should equal the sum of the cubes of the daughter channel diameters

This law can be derived from laminar flow in a branched circular tube system, but is also present in biological systems, such as plants and mammals.

To maintain a homogeneous fluid flow and temperature distribution, the emphasis must be focused on proper fluid distribution.

Micro Heat Exchanger Devices

The balance equations for cooling and heating, as well as the transfer correlation, are valid independently from the length scale.

Two approaches: LMTD and ε – NTU method

Typical overall heat transfer coefficients of microstructured heat exchangers range from 2.6 kW/m² K for gas/liquid flow to 26 kW/m² K for liquid/liquid flow.

Conventional plate heat exchangers exhibit very good heat transfer characteristics, ranging from 0.2 to 2.5 kW/m² K, under optimum conditions up to 5 kW/m² K for gas/liquid and liquid/liquid flow, respectively.

Design issues for exchange equipment

While microstructured heat exchangers deliver high transfer rates, they also have drawbacks, application limits, and factors to be considered during design and operation.

- Axial heat conduction in the relatively thick walls
- Equal distribution on a large number of channels
- Fouling or blocking of single passages or complete parts

SUMMARY

Microstructured equipment and related transport processes promise successful application in various fields, where high transfer rates, intelligent incorporation of microstructures in macro devices, and new process routes are needed.

High gradients and high specific surface in devices with various construction materials lead to fast equilibrium state.

The characteristic dimensions of microstructured internals are in the range of boundary layers, where high gradients enforce the transfer processes.

Cascading of effects and integration of various elements enhance and guide the entire process. Opportunities in the area of process intensification/new technologies.

Microfluidic network for flow, heat and mass transfer

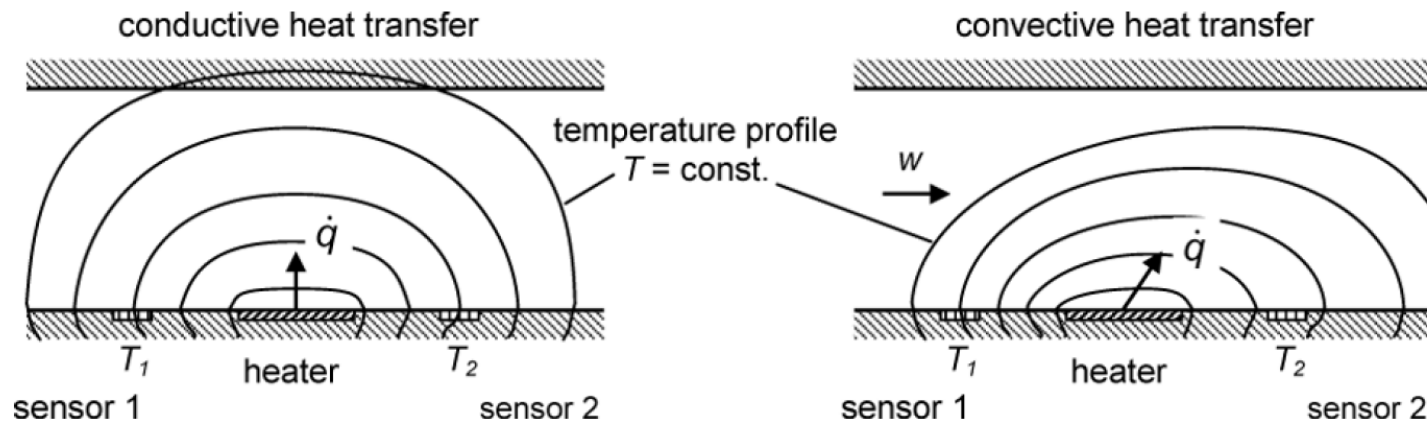
Example



Convective cooling for flow measurement

An interesting application of convective heat transfer is **flow velocity sensing** within a microchannel or on a surface.

A **thermal flow sensor** consists mainly of the heating element and temperature sensors, which measure the temperature of the heater and the fluid before and after the heater



Method A – heating of the fluid

Method B – Cooling of the heater

Setup of a thermoelectric flow sensor, in a channel consisting of a heater and two temperature sensors up- and downstream of the heater

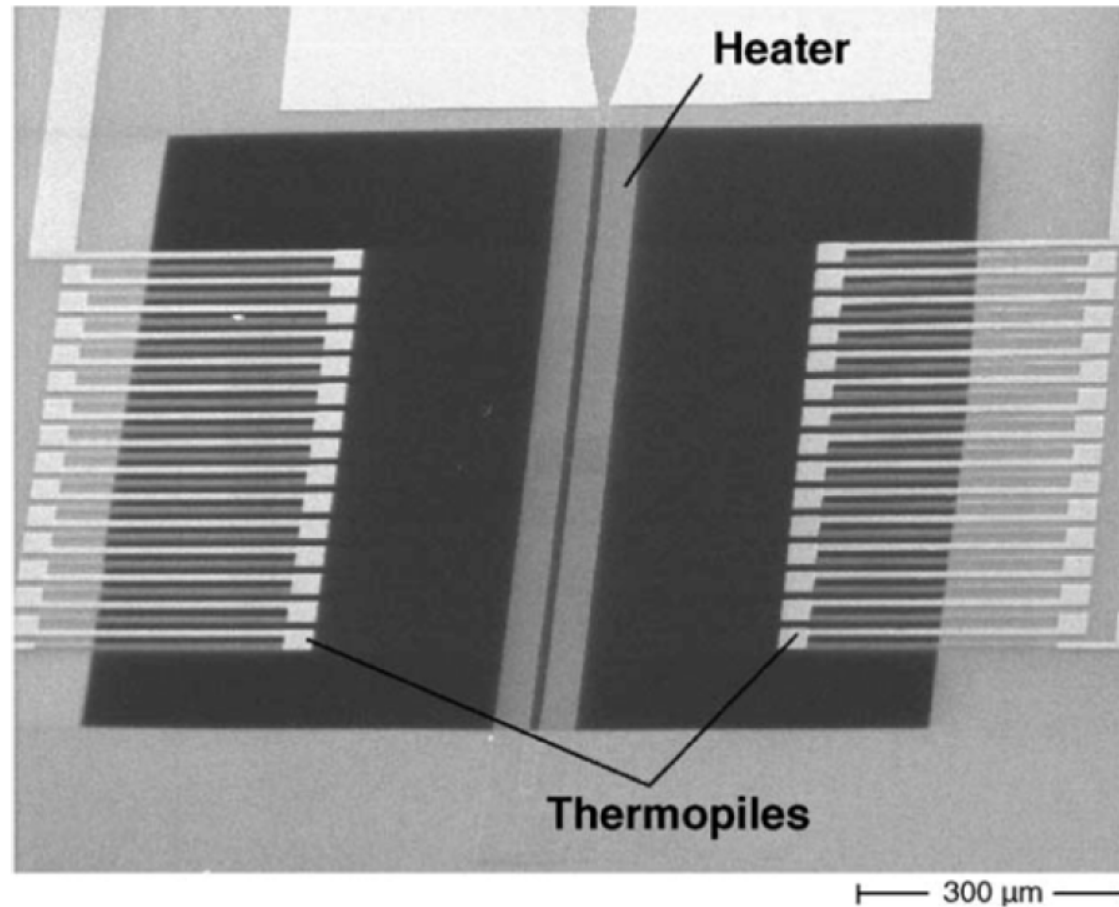
The complete sensor arrangement involves three complex domains: fluid flow, heat transfer, and thermoelectric energy and signal conversion.

The flow-induced heat transfer (convection) and the parallel heat conduction in the sensor wall/substrate determine the thermal signal domain, which is measured as temperature.

The basic sensor setup can be classified into two different techniques or methods. In method A, heating of the fluid imposes a temperature profile in the channel or near the wall, which is measured at certain positions by thermocouples.

In second method (B), fluid flow cools the heater, where temperature or electrical power is measured.

Local temperatures of a heated area- direction-sensitive velocimetry



Thermoelectric flow sensor consisting of an electrical heater on a thin membrane with parallel meandering thermoelectric sensors, two thermopiles with 15 thermo-elements

The dissipated electrical power in the heater spreads into the flowing fluid and the wall or substrate.

At steady-state, the energy balance of the heater/sensor expresses the electrical heating power P on one side and the dissipated heat into the substrate \dot{Q}_S and into the fluid \dot{Q}_f on the other side

$$P = \dot{Q}_S + \dot{Q}_f = G_S (T_H - T_S) + G_f (T_H - T_f)$$

where T_H , T_S , and T_f are the temperature of the heater, the substrate, and the bulk fluid, respectively, G_S is related to the solid thermal conductivity and G_f is the convective heat transfer coefficient..

The flow measurement relies on the variation of G_f with the flow and can be measured, from the temperature difference resulting from the applied electrical power of the heater.

If the sensor measures the mean temperature of the fluid, the energy dissipation into fluid is determined by

$$\dot{Q}_f = \dot{m}c_p (T_S - T_f)$$

If the heat dissipation into the substrate is \dot{Q}_S

$$P = \dot{Q}_S + \rho_f c_p A_C (T_S - T_f) \bar{w}$$

The fabricated sensor must be calibrated

Convective cooling of an electrical heater and measuring its temperature according to 2nd method avoids errors, such as measuring the heated fluid temperature or the unknown heat dissipation into the substrate. For this reason, many sensors are located on a thin membrane or fabricated on polymers or other thermal insulators, which minimizes parasitic heat losses and increases the sensor accuracy.

Convective heat transfer into the fluid $\dot{q}_f = \frac{\dot{Q}_f}{A_H} = \alpha_w (T_H - T_f)$

The heat transfer from cooling the heater is similar to cooling a plate on one side by surface flow

$$\text{Nu} = \frac{\alpha z}{\lambda_f} = 0.332 \text{Pr}^{1/3} \text{Re}^{1/2}$$

For the entire length of the heater – (equations are valid for laminar flow)

$$\text{Nu}_L = \frac{\alpha l_H}{\lambda_f} = 0.664 \text{Pr}^{1/3} \text{Re}^{1/2}$$

Both equations are valid for laminar flow.

Correlation between dissipated power, flow velocity and measured temperature difference is thus

$$\frac{\dot{Q}_f}{A_H} = \frac{\text{Nu } \lambda_f}{l_H} (T_H - T_f) = 0.664 \lambda_f \text{Pr}^{1/3} \left(\frac{\bar{w}}{l_H v} \right)^{1/2} (T_H - T_f)$$

Arranging the above equations to correlate the electrical heating power with the flow velocity gives the following equation.

$$P = G_S (T_H - T_S) + 0.664 \lambda_f A_H \text{Pr}^{1/3} \left(\frac{\bar{w}}{l_H v} \right)^{1/2} (T_H - T_f)$$

A detailed analysis of the correlation requires many parameters, such as heat conductivity of the substrate, fluid properties, correct geometry of the sensor.

For practical applications, the correlation between electrical power, measured temperature difference, and flow velocity can be reduced to (laminar flow)

$$P = \left(C_1 + C_2 w^{1/2} \right) (T_H - T_f)$$

Both coefficients can be determined by calibration measurements

The application of heat transfer measurements is a widespread and generally accepted method for determining the flow rate in channels.

Single channel element calculation

- design rules and correlations for high performance heat exchangers and microchannel networks

Heat transfer in laminar, channel flow, fully developed and constant heat flux

$$Nu = \frac{hD}{\lambda} = 4.3$$

At the entrance or after a bend, the straight laminar flow is disturbed, an additional pressure loss occurs, and transverse flow components enhance the transport process in the channel. The enhancement is given by

$$Nu_{me} = \frac{Nu_{fd}}{\tanh \left(2.432 Pr^{1/6} X^{* 1/6} \right)}$$

With the dimensionless entrance length X^* given by

$$X^* = \frac{L}{d_h} Pe = \frac{L}{d_h} Re \cdot Pr$$

To determine the optimum channel length x , the above equation is used to get

$$\frac{x}{d_h} = \text{Re} \cdot \left(0.411 \cdot \text{arctanh} \left(\frac{\text{Nu}_m}{\text{Nu}_{me}} \right) \right)^6$$

To yield a heat transfer enhancement of 30 % ($\text{Nu}_{me}/\text{Nu}_m = 1.3$), the entrance length divided by the hydraulic diameter should be less than 0.005Re ,

for a heat transfer enhancement of 10 %, the length should not exceed 0.06Re .

For longer channels, the pressure loss increases without additional benefit to the heat transfer.