Consider the case of a single linear spring of spring constant k with mass m attached to it such that the motion of the mass is confined only along the direction of the spring axis. The following equations govern the dynamics of the system.

Free undamped system

$$m\frac{d^2x}{dt^2} + kx = 0 \tag{1}$$

Free vibration with damping

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \tag{2}$$

Forced vibration without damping

$$m\frac{d^2x}{dt^2} + kx = F_0 \sin \omega t \tag{3}$$

Forced vibration with damping

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_0 \sin \omega t \tag{4}$$

x is the unstretched length of the spring and c is called the damping coefficient. $F_0 \sin \omega t$ in equations (3)-(4) is the external sinusoidal forced acting on the system.

(a) Solve equation (1) by converting the equation to a matrix equation. What is/are the equilibrium solution(s) of the system. Sketch the phase portrait. Comment upon the stability of the system.

(b) Repeat part (a) for equation (2). Examine the effect of the parameter c on the stability of the system to come up with the concept of *critical damping coefficient*. Examine the dynamics of the system for critical, lesser than critical, and greater than critical damping coefficient.

(c) Repeat parts (a) and (b) for equations (3) and (4). Examine the effect of F_0 and ω on the dynamics of the system.