Consider a system of elementary reactions in series of the type $A \rightarrow B \rightarrow C$. The kinetics of the system is given by the following equations.

$$\begin{aligned}
\frac{dC_A}{dt} &= -k_1 C_A \\
\frac{dC_B}{dt} &= k_1 C_A - k_2 C_B \\
\frac{dC_c}{dt} &= k_2 C_B
\end{aligned}$$

The reactions are carried out in a batch reactor with the respective initial concentrations as C_{A0}, C_{B0} and C_{C0}, respectively.

(a) By sequentially solving the above three equations, determine the expressions for the time evolutions of C_A, C_B and C_C. Determine the concentrations of the species as $t \rightarrow \infty$.

(b) Convert the above system of equations to a matrix equation and solve for the concentrations. Check if the solutions match the ones obtained from part (a).

(c) Determine the equilibrium solution(s) and check if they match the answers from part (a).

(d) Comment upon the stability of the solutions. Analyze your comment in reference to the magnitudes and signs of k_i's.

(e) Determine the solutions for the following cases.

(i) $k_1 = 2; k_2 = 1$

(ii) $k_1 = 1; k_2 = 2$

(iii) $k_1 = 1; k_2 = 1$

Sketch the time evolutions of the concentrations in each case.

(f) For each of the cases analyzed in part (d), determine the monotonicity of the solution functions and compare it against the graphical visualization. What can you learn about the concentration of B from the above?

(g) Determine the effect of initial conditions on the solutions for all the cases discussed in part (d).

(h) Sketch the phase portraits for all the above cases.