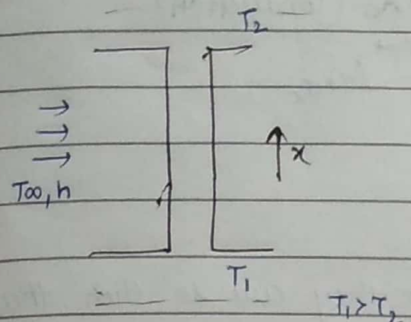
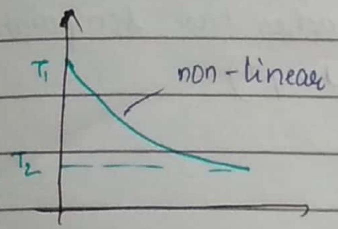
28.02

Heat Transfer from Extended Surfaces



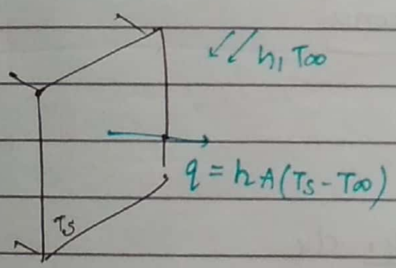
Without convection, a linear profile

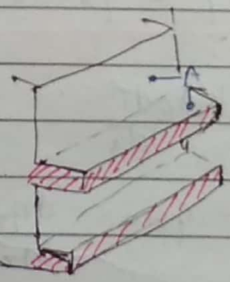


Why extended surfaces ?

– we want to maximise q in surfaces like that of AC (removal of heat - cooling)

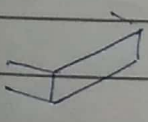
- increase h (not possible in case of fixed fluid)
- $(T_s - T_\infty)\uparrow$ → fixed; lowering T_∞ not possible everywhere
- increasing A



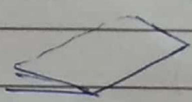


Extended surfaces to transfer heat called FINS

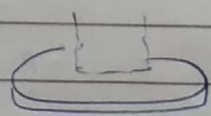
Types



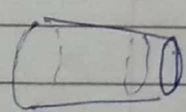
rectangular



triangular

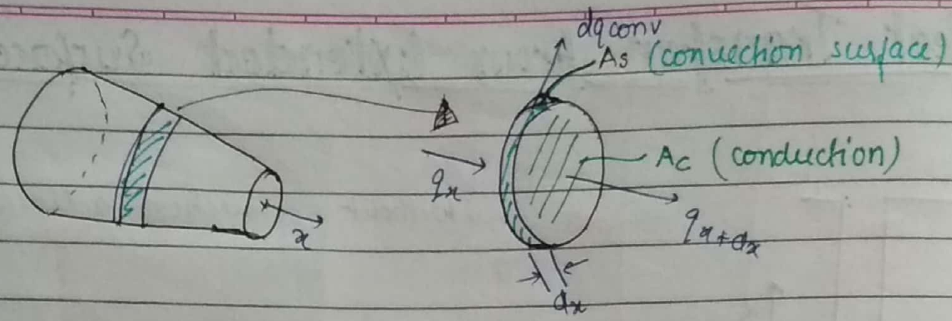


annular



Design considerations depends on several parameters :

→ ΔP changes with area (increases) , hence cost changes
→ space, ~~mater~~
→ material of construction



Assumptions:

→ one dimensional conditions (because they are so thin that it quickly reaches base temperature hence the whole substance is of same tempʳ)
→ tempʳ uniform across fin thickness.
→ k constant
→ negligible radiation loss
→ no heat generation
→ h uniform over the surface

Hence, $\boxed{q_x = q_{x+dx} + dq_{conv}}$

$$q_x = -kA_c\frac{dT}{dx}$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}\cdot dx$$

$$q_{x+dx} = -kA_c\frac{dT}{dx} - k\frac{d}{dx}\left(A_c\frac{dT}{dx}\right)\cdot dx$$

$$dq_{conv} = hA_s(T - T_\infty)$$

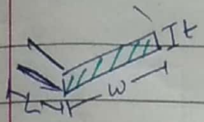
Hence, $\frac{d}{dx}\left(A_c\frac{dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}(T-T_\infty)$

$$\boxed{\frac{d^2T}{dx^2} + \frac{1}{A_c}\frac{dA_c}{dx}\cdot\frac{dT}{dx} - \frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}(T-T_\infty) = 0}$$

general form of energy eqⁿ from extended surface

A_c, A_s – may vary or remain uniform

Rectangular :



$P = 2w + 2t$ $\quad A_s = P \cdot x$ $\quad \frac{dA_s}{dx} = P$

$A_c = wt$

Uniform CS

$$\boxed{\frac{d^2T}{dx} - \frac{1}{A_c}\frac{h}{k}\left(\frac{dA_s}{dx}\right)(T - T_\infty) = 0}$$

P

Defining, $\theta(x) = T(x) - T_\infty$

$$\boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0}$$ $\quad m^2 = \frac{1}{A_c}\frac{hP}{k}$

2nd order D.E.

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary conditions:

I) At $x = 0$, $T = T_s$ $\therefore \theta = \theta_b$ (θ of base) $= T_b - T_\infty$

II)

A. Convection heat transfer : $h\theta(L) = -k\frac{d\theta}{dx}\Big|_{x=L}$

B. $L \to \infty$ $\quad \theta(L) = 0$ (infinite fin, the temp^r at tip is equal to that of fluid)

C. $\theta(L) = \theta_L$ (given)

D. $\frac{d\theta}{dx}\Big|_{x=L} = 0$ (adiabatic surface)

UNIFORM Cross sectional area

a) $hA_c(T(x=L) - T_\infty) = -A_c k\frac{dT}{dx}\Big|_{x=L}$

$$h\,\theta(L) = -k\frac{d\theta}{dx}\Big|_{x=L}$$

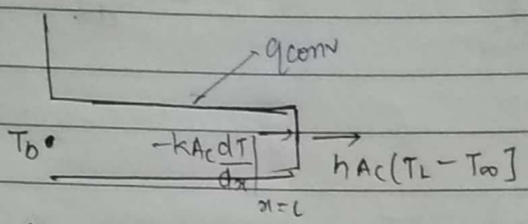
$$\theta_b = C_1 + C_2$$

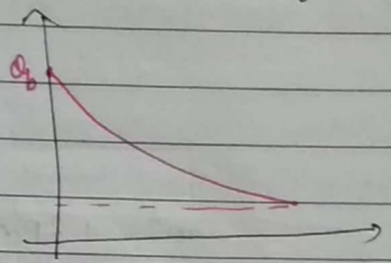
$$h(C_1 e^{mL} + C_2 e^{-mL}) = km(C_2 e^{-mL} - C_1 e^{mL})$$

$$\boxed{\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}}$$

$$\sinh x = \frac{e^{x} - e^{-x}}{2} \qquad \frac{d(\sinh u)}{dx} = \cosh u \frac{du}{dx}$$

$\cosh x =$





$$q_f = q_b = -kA_c \frac{dT}{dx}\Big|_{x=0}$$

$$= -kA_c \frac{d\theta}{dx}\Big|_{x=0}$$

By Fourier's Law $\boxed{q_f = \sqrt{hPkA_c}\,\theta_b \dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}}$

Overall q will be given by integration of q all surfaces

$$q_f = \int_{A_f} h[T(x) - T_\infty]\, dA_s$$

$$q_f = \int_{A_f} h\theta(x)\, dA_s$$

$$\frac{\theta}{\theta_b} = \cos$$

b) $\frac{d\theta}{dx}\Big|_{x=L} = 0$

D) Infinite fin:

$$\frac{\theta}{\theta_b} = e^{-mx} \qquad m = \frac{hP}{kA_c}$$

$$q_f = M = \sqrt{hPkA_c}\,\theta_b$$

01.03

FIN PERFORMANCE :

Ratio of fin heat transfer ~~to~~ rate to the heat transfer rate that would've existed without fin

$$\boxed{\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b}}$$

$A_{c,b}$ - cross sectional area of base (no fin)

- If the value ε_f is not greater than 2, the use is not justified.

$$\boxed{R_{t,f} = \frac{\theta_b}{q_f}}$$

$$R_{t,b} = \frac{1}{hA_{c,b}}$$ Resistance of base

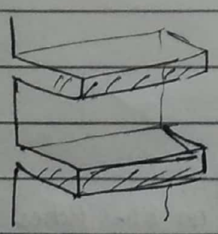
$$\boxed{\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}}$$

Efficiency :

$$\boxed{\eta_f = \frac{q_f}{q_{max}} = \frac{q_f}{hA_f\theta_b}} \qquad [\; 0 \le \eta_f < 1 \;]$$

To find q_{max}, we assume that fin reaches base temp^r without time lag, but ~~at~~ actually, this does not happen. There exists a finite temp^r difference (and a profile)

Overall surface efficiency :



$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{hA_t\theta_b}$$

$$A_t = NA_f + A_b$$

[A_b calculat^n difficult]

$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b$$

$$q_t = h[N\eta_f \cancel{h}A_f\cancel{\theta} + (A_t - NA_f)]\theta_b$$

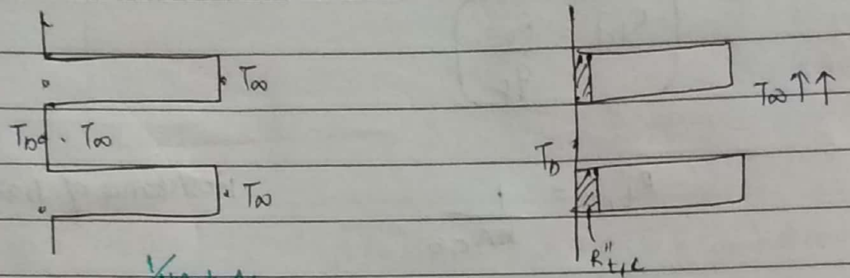
$$= hA_t\left[\frac{N\eta_f A_f}{A_t} + 1 - \frac{NA_f}{A_t}\right]\theta_b$$

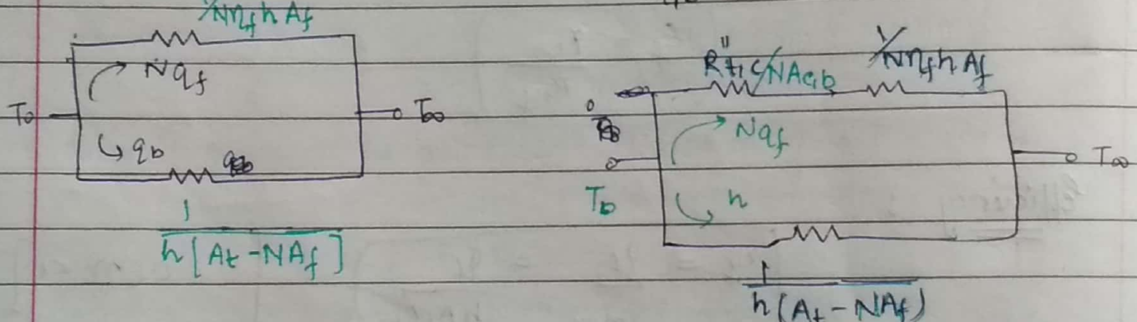
$$\boxed{q_t = hA_t\left[1 - \frac{NA_f}{A_t}(1-\eta_f)\right]\theta_b}$$

Overall surface eff.
$$\boxed{\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)}$$

N: no. of fins

$$\boxed{R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}}$$



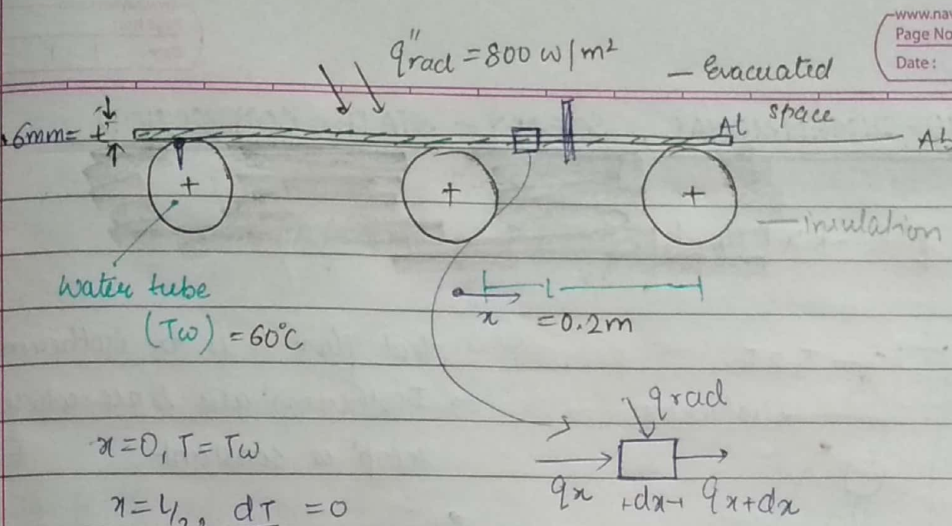


Q Copper tubing is joined to the absorber of flat plate solar radiator. Assume temp^r of ~~tube~~ plate directly above water $= T_w = 60°C$.
what is the maximum temp^r on the plate and the heat transfer rate per unit length of the tube?

$K_{Al} = 180\ W/m\cdot K$

here, extended surface is plate.
This problem can be simplified geometrically.
~~In b/w insulated materials,~~ Temp^r max^m in b/w tubes
because $\frac{dT}{dx} = 0$ at adiabatic surface.

Domain reduced



Assumption

① 1-D

$x=0,\ T=T_w$

$x=L/2,\ \dfrac{dT}{dx}=0$

unit width
$A = (1)\times t$

$$q_x = q_{rad} + q_{x+dx}$$

$$q_x = q_{rad} + q_x + \frac{d}{dx} q_x\, dx$$

$$\frac{q_x - q_{x+dx}}{dx} = q''_{rad} \qquad \frac{dq_x}{} = -q''_{rad}(1\times dx)$$

q_{x+dx}

$q_x + \dfrac{dq_x}{dx}$

$-\dfrac{dq_x}{dx}$

$$\Rightarrow \frac{d}{dx}\left(+kA_x\frac{dT}{dx}\right) = +q''_{rad}$$

$$\Rightarrow k(1\times t)\frac{d^2T}{dx^2} = q''_{rad}$$

$$\Rightarrow (kt)\,T = q''_{rad}\frac{x^2}{2} + C_1x + C_2 \qquad (kt)\left(\frac{dT}{dx}\right) = q''_{rad}x + C_1$$

$$(kt)\,T_w = C_2 \quad \text{—— ①}$$

$$(kt)(0) = q''_{rad}\frac{L}{2} + C_1$$

$$\therefore (kt)\,T = q''_{rad}\frac{x^2}{2} - q''_{rad}\frac{L}{2}x + (kT)\,T_w$$

$$\boxed{T = \frac{q''_{rad}L^2}{(kt)\,2}\left(\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)\right) + T_w}$$

$$= \frac{400}{27}\left(\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)\right) + 60^\circ C$$

C_2

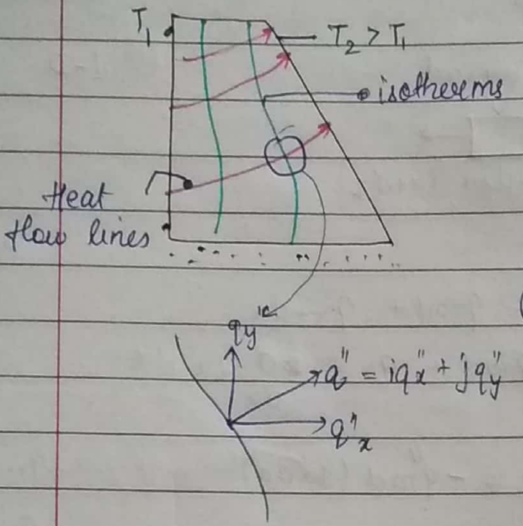
$\frac{1}{4}$

$T_{max} = \cancel{53.52}\,^\circ C \quad \underline{56.3\,^\circ C}$

14.03.18

TWO-DIMENSIONAL, STEADY-STATE CONDUCTION

$$q'' = -k\nabla T = -k\left(\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{z}\frac{\partial T}{\partial z}\right)$$



Heat flux is ⊥r to isotherms

→ Isotherms are lines where temp^r is constant

(I) write eq^n of heat flow

(II)

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0} \quad (1)$$

→ steady state

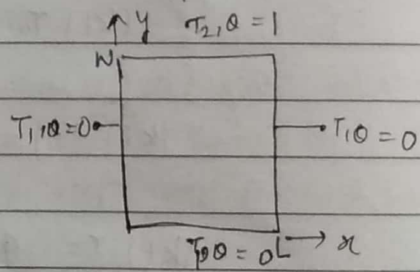
→ 2-D flow

→ no heat generation

→ Sol^n varies depending on ~~set~~^{n} of boundary conditions.

→ Advantage of analytical sol^n - we can get temp^r at any point

→ Numerical sol^n - we get approx sol^n at diff points



Solving (1)

$$\theta = \frac{T - T_1}{T_2 - T_1}$$

(1) is double diff w.r.t x & y, so we need 2 B.C. each for x & y

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$

$\theta(0, y) = 0$ $\quad$ $\theta(x, 0) = 0$

$\theta(L, y) = 0$ $\quad$ $\theta(x, W) = 1$

We assume $\theta(x, y) = X(x) \cdot Y(y)$ $\quad$ λ^2

$$\frac{1}{X}\frac{d^2 X}{dx^2} = \frac{-1}{Y}\frac{d^2 Y}{dy^2}$$

$$\frac{d^2X}{dx^2} + \lambda^2 X = 0$$

$$\frac{d^2Y}{dy^2} - \lambda^2 Y = 0$$

(λ is +ve constant)

$$\frac{d^2X}{dx^2} + \lambda^2 X = \frac{d^2Y}{dy^2} - \lambda^2 Y$$

$$X = C_1 \sin\lambda x + C_2 \cos\lambda x$$
$$Y = C_3 e^{\lambda y} + C_4 e^{-\lambda y}$$

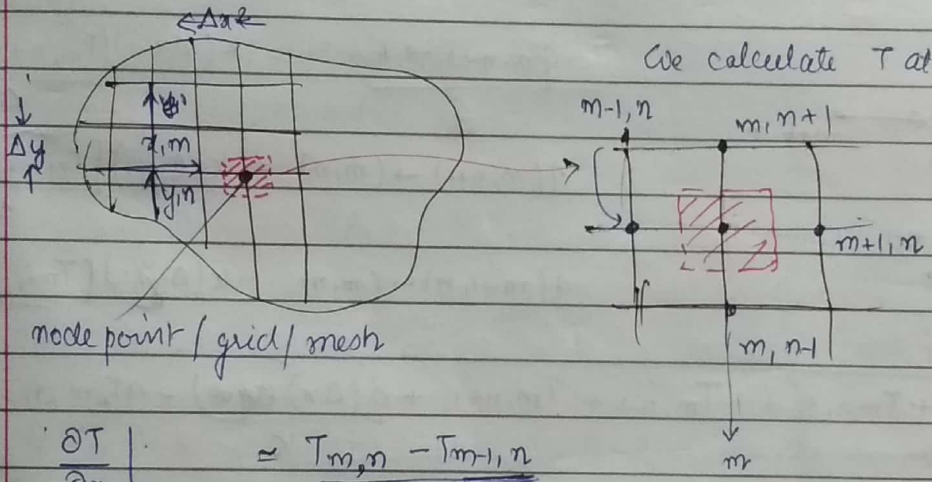
Discretization

Conversion of partial diff eq^n to set of algebraic eq^n (in x, y, z)

simplest

i) Finite difference
ii) Finite volume
iii) Finite element

(1) FINITE DIFFERENCE



We calculate T at nodal points (considering every thing is coming in)

$$T_{m,n} = T_{m-1,n} + \left.\frac{\partial T}{\partial x}\right|$$

$$\left.\frac{\partial T}{\partial x}\right|_{m-1/2,n} \simeq \frac{T_{m,n} - T_{m-1,n}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right|_{m+1/2,n} \simeq \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

$$\frac{\partial^2 T}{\partial x^2} \simeq \frac{\partial T/\partial x|_{m+1/2,n} - \partial T/\partial x|_{m-1/2,n}}{\Delta x}$$

$$\left[\left.\frac{\partial^2 T}{\partial x^2}\right|_{m,n} \simeq \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{(\Delta x)^2}\right]$$

Classical central differencing scheme

$$\left[\left.\frac{\partial^2 T}{\partial y^2}\right|_{m,n} \simeq \frac{T_{m,n+1} + T_{m,n-1} - 2T_{m,n}}{(\Delta y)^2}\right]$$

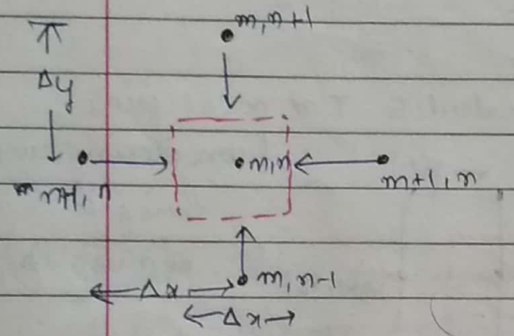
$\Delta x = \Delta y$ Putting in eq^n ① — $T_{m,n}$ is avg of Temp's at 4 points around m,n

$$\boxed{T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0} \quad (\times)$$

We get these for 100 × 50 divisions (if done so) and solved using Gauss Jordan, ... methods in MATLAB.

$E_{in} + E_g = 0$

$$\sum_{i=1}^{4} q_{(i) \to (m,n)} + \dot{q}(\Delta x \cdot \Delta y \cdot 1) = 0$$



$$q_{(m-1,n) \to (m,n)} = k(\Delta y \cdot 1)\left[\frac{T_{m-1,n} - T_{m,n}}{\Delta x}\right]$$

$$q_{(m,n-1) \to (m,n)} = k(\Delta x \cdot 1)\left[\frac{T_{m,n-1} - T_{m,n}}{\Delta y}\right]$$

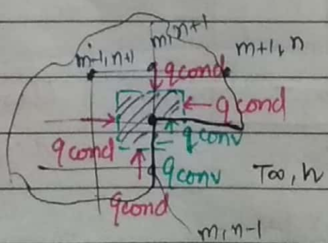
$$q_{(m,n+1) \to (m,n)} = k(\Delta x \cdot 1)\left[\frac{T_{m,n+1} - T_{m,n}}{\Delta y}\right]$$

$$q_{(m+1,n) \to (m,n)} = k(\Delta y \cdot 1)\left[\frac{T_{m+1,n} - T_{m,n}}{\Delta x}\right]$$

Add — $\boxed{\Delta x = \Delta y}$

$$\boxed{T_{m+1,n} + T_{m-1,n} + T_{m,n-1} + T_{m,n+1} + \frac{\dot{q}(\Delta x)^2}{k} - 4T_{m,n} = 0}$$

All the points are not enclosed as above.
Outer nodal points may encounter convection.
⇒ points at the boundary will require special attention.



$$q_{m-1,n \to m,n} = k(\Delta y \cdot 1)\left[\frac{T_{m-1,n} - T_{m,n}}{\Delta x}\right]$$

$$q_{m+1,n \to m,n} = k\left(\frac{\Delta y}{2} \cdot 1\right)\left[\frac{T_{m+1,n} - T_{m,n}}{\Delta x}\right]$$

Conduction occurs through half of area at 2 surfaces

$$q_{m,n-1 \to m,n} = k\left(\frac{\Delta x}{2} \cdot 1\right)\left[\frac{T_{m,n-1} - T_{m,n}}{\Delta y}\right]$$

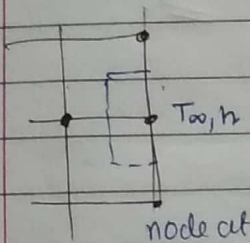
$$q_{m,n+1 \to m,n} = k(\Delta x \cdot 1)\left[\frac{T_{m,n+1} - T_{m,n}}{\Delta y}\right]$$

$$q_{\infty \to m,n} = h\left(\frac{\Delta x}{2}\cdot 1\right)(T_\infty - T_{m,n}) + h\left(\frac{\Delta y}{2}\cdot 1\right)(T_\infty - T_{m,n})$$

Add all 4 q cond + 2 q conv

$$T_{m-1,n} + T_{m,n+1} + \tfrac{1}{2}(T_{m+1,n} + T_{m,n-1}$$

Assumptⁿ that : ~~point~~ avg temp^r throughout the cell is the ~~same~~ temp^r at the centroid (m,n) — valid for small grids

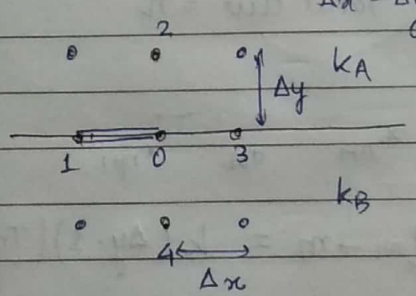


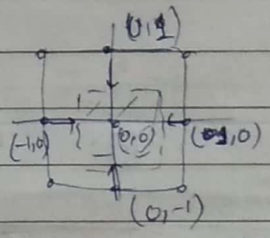
node at a plane surface with convection

$$Ax = B$$

x ↓ nodal point temp^r ; B ↓ RHS (constants)

Q Consider the nodal point 0 located on the boundary b/w materials of thermal conductivity K_A and K_B. Derive the finite difference equation, assuming no internal generation





$q_{(-1,0)} \to q_{(0,0)} =$ ~~T~~ $k_A + k\cdot$

$$q_{2\to 0} = k_A(\Delta x\cdot 1)\frac{(T_2 - T_0)}{\Delta y} \qquad q_{4\to 0} = k_B(\Delta x\cdot 1)\frac{(T_4 - T_0)}{\Delta y}$$

$$q_{1\to 0} = (k_A + k_B)\left(\frac{\Delta y}{2}\right)\left(\frac{T_1 - T_0}{\Delta x}\right) \qquad q_{3\to 0} = (k_A + k_B)\frac{\Delta y}{2}\frac{(T_3 - T_0)}{\Delta x}$$

Ans: $-T_0 + \frac{T_1}{4} + \frac{k_A}{2(k_A+k_B)} T_2 + \frac{T_3}{4} + \frac{k_B}{2(k_A+k_B)} T_4 = 0$

$\Delta x = \Delta y$

$$k_A(T_2 - T_0) + k_B(T_4 - T_0) + \frac{(k_A+k_B)}{2}(T_1 - T_0) + \frac{(k_A+k_B)}{2}(T_3 - T_0)$$

$$\frac{2k_A T_2}{k_A+k_B} + \frac{2k_B}{(k_A+k_B)} T_4 - \frac{\cancel{2(k_A+k_B)} T_0}{\cancel{(k_A+k_B)}} - T_0 - T_0 + T_1 + T_3 = 0$$

$$\frac{k_A}{2(k_A+k_B)} T_2 + \frac{k_B}{2(k_A+k_B)} T_4 - T_0 + \frac{T_1}{4} + \frac{T_3}{4} = 0$$

Ans

If $k_A = k_B$,

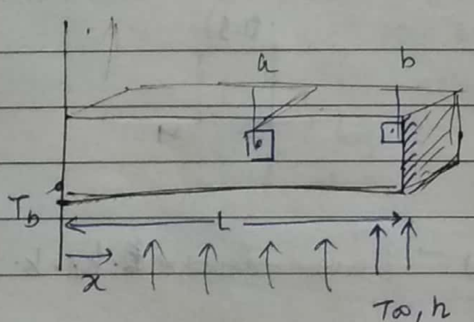
$$T_2 + T_4 + T_1 + T_3 = 4T_0$$

$$T_0 = \frac{T_1 + T_2 + T_3 + T_4}{4}$$

Q. Consider a 1-D fin of uniform cross-sectional area insulated at its tip ($x = L$). The temp^r at the base of the fin is T_b and of the adjoining fluid is T_∞ as well as the heat transfer coefficient h and K are known. Derive the finite

a) difference eq^n for any interior node m

b) difference eq^n for any node m located at the tip

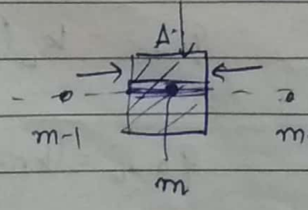


Total div $= n$

x_{m-1} m x_{m+1}

$$q_{m-1 \to m} = k(\Delta y \cdot 1)\frac{(T_{m-1} - T_m)}{\Delta x}$$

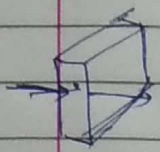
q



$$\frac{d^2T}{dx^2} - \frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}(T-T_\infty) = 0$$

We do not know whether it is cylinder / rectangle

$$\frac{d^2T}{dx^2}$$



$$q_x = q_{x+dx} + q_{conv}$$

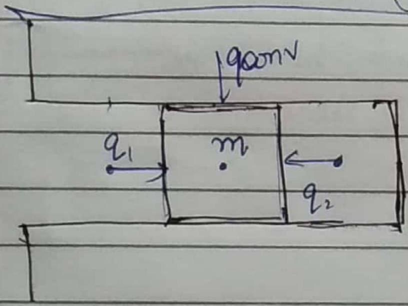
$$\cancel{-kA_c\frac{dT}{dx}} = -kA_c\left.\frac{dT}{dx}\right|_{x+dx} \div$$

$$q_x = q_x + \frac{dq_x}{dx}\cdot dx + hA_s(T-T_\infty)$$

$$\Rightarrow \frac{dq_x}{dx} + hA_s(T-T_\infty) = 0$$

$$-k\frac{dT}{dx} =$$

$$\frac{d}{dx}\left(A_c\frac{dT}{dx}\right.$$



a)

$$q_{m-1\to m} = k(A_c)\frac{(T_{m-1}-T_m)}{\Delta x}$$

$$q_{m+1\to m} = k(A_c)\frac{(T_{m+1}-T_m)}{\Delta x}$$

$$q_{conv} = -hP\cdot\Delta x\,(T_m - T_\infty)$$

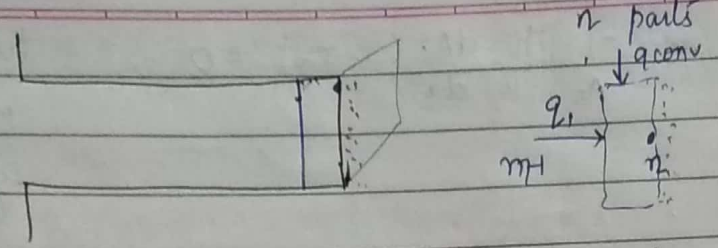
$$\Sigma q = 0$$

$$\Rightarrow kA_c\frac{(T_{m-1}-T_m)}{\Delta x} + kA_c\frac{(T_{m+1}-T_m)}{\Delta x} - hP\Delta x(T_m-T_\infty) = 0$$

$$T_{m-1} + T_{m+1} - 2T_m - \frac{hP(\Delta x)^2}{kA_c}(T_m - T_\infty) = 0$$

$$\Rightarrow \boxed{T_{m-1} + T_{m+1} + \frac{hP(\Delta x)^2}{kA_c}T_\infty + \left[-2 - \frac{hP(\Delta x)^2}{kA_c}\right]T_m = 0}$$

b)



$$q_{n-1 \to n} = kA_c \frac{(T_{n-1} - T_n)}{\Delta x}$$

$$q_{conv} = \frac{hP\Delta x}{2}(T_\infty - T_n)$$

$$kA_c \frac{(T_{n-1} - T_n)}{\Delta x} + \frac{hP\Delta x}{2}(T_\infty - T_n) = 0$$

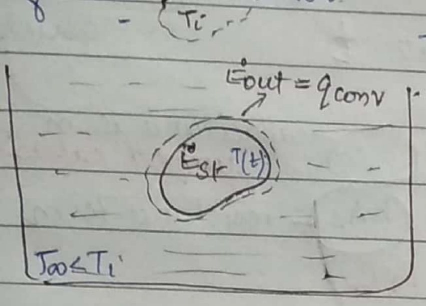
$$T_{n-1} - T_n + \frac{hP(\Delta x)^2}{kA_c 2}(T_\infty - T_n) = 0$$

$$T_{n-1} + \frac{hP(\Delta x)^2 T_\infty}{2kA_c} - \left[1 + \frac{hP(\Delta x)^2}{2kA_c}\right] T_n = 0$$

21.03.18

Transient Conduction

- unsteady state conduction



Two ways of heat transfer:
- from the core to the surface (conduction)
- from the surface to liquid (convection)

One of the simplifying assumptions:
- no temp^r gradient inside the solid, uniform temp^r at any time
- negligible resistance of solid / high thermal conductivity
  (practically not possible always but in case of small particles with very high k)

This approx can be valid for bulk even when we see cases where heat transfer Resistance inside solid is very small than outside R.

$\Rightarrow$ time scale of conduction << time scale of convection.

$$-\dot{E}_{out} = \dot{E}_{st} \qquad [\dot{E}_{in} = 0 = \dot{E}_g]$$

$$-hA_s(T-T_\infty) = mC_p \frac{\partial T}{\partial t}$$

$$\boxed{-hA_s(T-T_\infty) = \rho V C \frac{dT}{dt}}$$

Let $\theta = T - T_\infty$

Hence, $$\frac{\rho V C}{hA_s}\frac{d\theta}{dt} = -\theta$$

Integrating $$\frac{\rho V C}{hA_s}\int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = -\int_0^t dt$$ [$\theta_i = T_i - T_\infty$ max^m temp^r diff possible]

$$\frac{\rho V C}{hA_s} \ln \frac{\theta_i}{\theta} = t$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[\left(-\frac{hA_s}{\rho V C}\right)t\right]$$

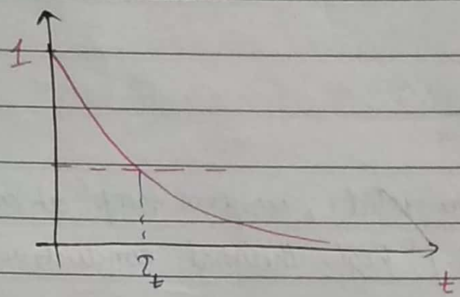
a thermophysical quantity

⇒ We can calculate T at any time 't'

⇒ If T is given, we can calculate t

Time constant $\tau_t = \dfrac{\rho V C}{hA_s}$ — capacitance term / lumped capacitance; resistance term

$= R_t C_t$



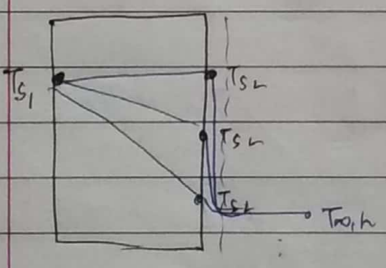
This analysis of taking no temp^r grad is called lumped capacitance analysis

$$Q = \int_0^t q\,dt = \int_0^t hA_s\theta\,dt$$

$$= \cancel{hA_s}\int \rho V C\theta_i\left[\exp\left(\frac{-t}{\tau_t}\right)\right]dt$$

$$\boxed{Q = \cancel{hA}(\rho V c)\theta_i\left[1 - e^{-t/\tau_t}\right]}$$

This is not valid in case of wall or if so, when?



At any time t,

$$\frac{kA}{L}(T_{s_1} - T_{s_2}) = hA(T_{s_2} - T_\infty)$$

$$\frac{\Delta T_{inside}}{\Delta T_{outside}} = \frac{T_{s_1} - T_{s_2}}{T_{s_2} - T_\infty} = \frac{1/kA}{1/hA}$$ Resistance (conduct^n) / Resistance (convect^n)

$$= \frac{hL}{k} = Bi$$

Biot number

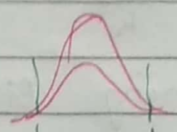
Nusselt no, k was for fluid - measure ~~of~~ ~~heat~~ cond^n ~~by~~ to ~~hea~~ convect^n for

But, Biot no says ~~wha~~ ratio of resistance inside to resistance outside

If $Bi << 1$ $\Rightarrow$ $R_{conv} >> R_{cond}$, hence, ~~conduct~~ ^{n} uniform temp^{r} inside ~~temp~~ solid
(<0.1)

$\rightarrow$ we can apply lumped parameter analysis

If $Bi >> 1$ $\Rightarrow$ $R_{conv} << R_{cond}$

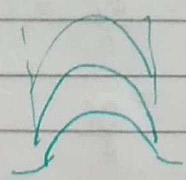


(~~similar~~ diff grad inside & outside

If $Bi \approx 1$ $R_{cond} \approx R_{conv}$

there will be temp^{r} gradient inside solid

similar gradient inside & outside always



L_c : characteristic length for any geometry

$= \dfrac{\text{Volume}}{\text{surface area across which it is flowing}}$

~~sphere cylinder~~ : $L_c = \dfrac{\frac{4}{3}\pi R^3}{4\pi R^2} = \dfrac{R}{3}$

Cylinder : $L_c = \dfrac{\pi R^2 L}{2\pi R L} = \dfrac{R}{2}$

Biot — thermal diffusivity

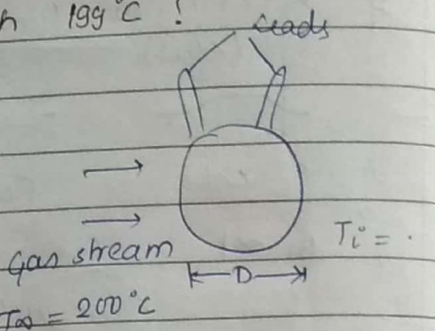
$$\frac{hA_s t}{\rho V c} = \frac{h\,t}{\rho L_c \cdot C} = \left(\frac{hL_c}{k}\right) \cdot \left(\frac{k}{\rho C}\right) \cdot \frac{t}{L_c^2}$$

$\dfrac{\alpha \cdot t}{L_c^2}$ — Fourier no.

$$\frac{hA_s t}{\rho V c} = Bi \cdot Fo$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-Bi \cdot Fo)$$

Q A thermocouple junction which may be approximated as a sphere, is ~~used~~ to be used for temp^{r} measurement in a gas stream. The convection coefficient b/w the junct^{n} surface and the gas is $h = 400\ W/m^2.K$ and the junct^{n} thermophysical properties are $k = 20\ W/m.K$, $c = 400\ J/kg.K$ and $\rho = 8500\ kg/m^3$. Determine the junct^{n} diameter needed for the thermocouple to have a time constant of 1 s. If the junct^{n} is at 25°C and is placed in a gas stream that is at 200°C, how long will it take for the junct^n to reach 199°C ?



$Bi = \frac{h l_c}{k}$

Assuming $Bi \ll 1$

$$\frac{\theta_i}{\theta} = \exp(-Bi \cdot Fo)$$

$$\tau_t = \frac{\rho V c}{h A_s} = \frac{\rho L_c \cdot c}{h}$$

$$L_c = \frac{1s \cdot 400}{8500 \cdot 400}$$

For sphere, $\frac{R}{3} = 1.1765 \times 10^{-4}$

$$D = 7.06 \times 10^{-4}\ m$$

$Bi = 2.35 \times 10^{-3}$

$$\frac{\theta_i}{\theta} = e^{-t/\tau}$$

$$\frac{25 - 200}{199 - 200} = e^{-t/1}$$

$$175 = e^{-t}$$

22.03

Spatial Effects

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$T(x,0) = T_i$ [substance at one temp' initially]

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$

$$\left.-k\frac{\partial T}{\partial x}\right|_{x=L} = h[T(L,t) - T_\infty]$$

$$T = T(x, t, T_i, T_\infty, L, k, \alpha, h)$$

thermal, physical } thermophysical parameters

Dimensionless : we can still solve the problem without knowing geometry.

$$\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} \qquad 0 \le \theta^* \le 1$$

$$x^* = \frac{x}{L}$$

$$t^* = \frac{\alpha t}{L^2} = F_0$$

$$\frac{\partial^2 \theta}{\partial x^{*2}} = \frac{\partial \theta^*}{\partial F_0}$$

$$\theta^*(x^*, 0) = 1 \qquad \text{dimension-invariant}$$

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=0} = 0$$

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=1} = -Bi\, \theta^*(1, t^*)$$

$$\theta^* = f(x^*, F_0, Bi)$$

Such sol^{n}s are complex to derive, so we use numerical sol^{n}s.
→ finite difference method.

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$$

$$t = p\Delta t$$
integer

$$\left.\frac{\partial T}{\partial t}\right|_{m,n} \approx \frac{T^{p+1}_{m,n} - T^{p}_{m,n}}{\Delta t} \quad \Big] \text{ discretization}$$

$$\frac{1}{\alpha}\frac{T^{p+1}_{m,n} - T^{p}_{m,n}}{\Delta t} = \frac{T^p_{m+1,n} + T^p_{m-1,n} - 2T^p_{m,n}}{(\Delta x)^2} + \frac{T^p_{m,n+1} + T^p_{m,n-1} - 2T^p_{m,n}}{(\Delta y)^2}$$

$\Delta x = \Delta y.$

$$T^{p+1}_{m,n} = F_0\left(T^p_{m+1,n} + T^p_{m-1,n} + T^p_{m,n+1} + T^p_{m,n-1}\right) + (1-4F_0)T^p_{m,n}$$

$$F_0 = \frac{\alpha \Delta t}{(\Delta x)^2}$$

RHS is of previous known time. — explicit formulation
- oscillatory nature (not stable)
Any small divergence will lead to oscillation.

When the prev time step coeff is either 0 or +ve, then the formulation is stable.

Hence $1-4F_o \geq 0$

$$\boxed{F_o \leq 1/4}$$

α fixed for system, $\therefore$ we need to adjust Δt & $(\Delta x)^2$

1-D

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$$

$$T_m^{p+1} = F_o\left(T_{m+1}^p + T_{m-1}^p\right) + (1-2F_o)T_m^p$$

$$\boxed{F_o \leq 1/2}$$

Implicit formulation

Or

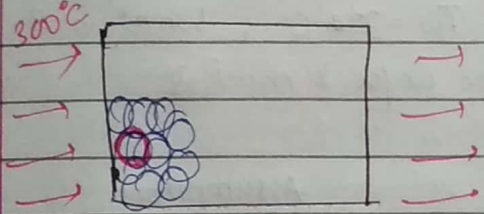
$$\frac{1}{\alpha}\frac{T_{m,n}^{p+1} - T_{m,n}^p}{\Delta t} = \frac{T_{m+1,n}^{p+1} + T_{m-1,n}^{p+1} - 2T_{m,n}^{p+1}}{(\Delta x)^2} + \frac{T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1} - 2T_{m,n}^{p+1}}{(\Delta y)^2}$$

$$(1+4F_o)T_{m,n}^{p+1} - F_o\left(T_{m+1,n}^{p+1} + T_{m-1,n}^{p+1} + T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1}\right) = T_{m,n}^p$$

→ doesn't require any stability cond^n
→ sol^n is consistent because values calculated at all points at one time.

Q. Thermal energy storage systems commonly involve a packed bed of solid spheres through which a hot gas flows if the system is beigng charged or a cold gas if it is being discharged. In a charging process, heat transfer from the hot gas increases thermal energy of the colder spheres. During discharge, stored energy decreases & heat flows from warm to cold gas.

Consider a p.b of 75 mm dia Al spheres ($\rho = 2700\ kg/m^3$, $c = 950\ J/kg \cdot K$, $k = 240\ W/m \cdot K$) and a charging process for which gas enters at $T = 300°C$. If T_i of spheres $= 25°C$ and $h = 75\ W/m^2 \cdot K$. How long does it take a sphere near the inlet to accumulate 90% of the max^m possible thermal energy. What is the corresp. temp^r at the centre of the sphere?



$h = 75$

neglect spatial effect

$T_i = 25°C$, $T_\infty = 300°C$.

$\boxed{\text{Show}}$

$Bi = \frac{h L_c}{k}$, $L_c = R/3$

$= \frac{75 \times D}{6 \times k}$

$= 3.9 \times 10^{-3} << 1$

$Bi < 0.1$

⇒ Lumped capacitance analysis is valid

$Q = (\rho V c)\theta_i$ $\quad \tau = \frac{\rho V c}{h A_s}$

$\theta = \theta_i \cdot e^{-t/\tau}$

$Q = \int_0^t q\, dt = \int_0^t h A_s \theta\, dt$

$= h A_s \theta_i \int_0^t e^{-t/\tau} dt$

$Q = h A_s \theta_i [1 - e^{-t/\tau}]$

$Q_{max} = h A_s \theta_i$

$Q = 0.90\ Q_{max}$

$\cancel{h A_s \theta_i}[1 - e^{-t/\tau}] = 0.90\ \cancel{h A_s \theta_i}$

$0.1 = e^{-t/\tau}$

$-\ln(0.1) = \frac{t}{\tau}$

$\frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$

$\tau = \frac{\rho R \cdot c}{3h}$

$= 427.5$

$t = 984.4$ sec.

$= 16.4'$ min

$\theta = \theta_i e^{-t/\tau}$

$\Rightarrow \theta = (T_i - T_\infty)\, e^{-\frac{984.4}{427.5}} = (25 - 300)\, e^{-\frac{984.4}{427.5}}$

$= -27.5$

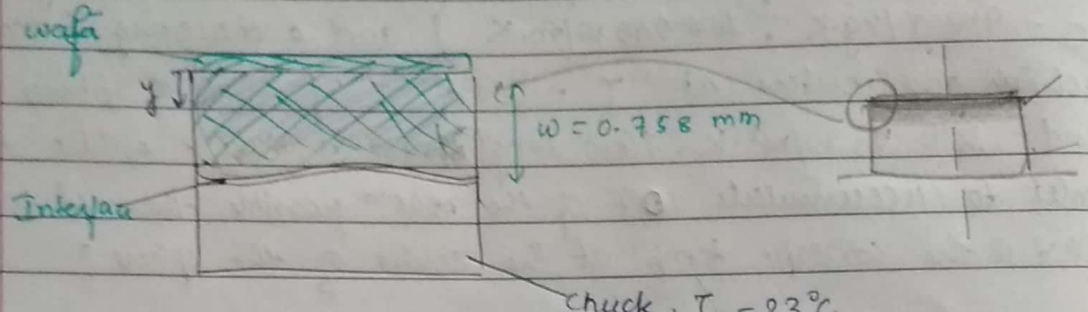
$\boxed{T = 272.5}$

28.03

Q

$T_w(0) = 100°c$

$\rho = 2900\ kg/m^3$, $c = 875\ J/kg.K$, $k = 177\ W/mK$



There exists R_{cont}. After 15 sec, $T_w = 33°C$. What is the thermal contact resistance b/w the wafer & chuck? $[m^2K/W]$

L_c wafer $= \dfrac{V}{A_s} =$

Assumptions

(i) Consider the chuck is at uniform temp^r at 23°C.

(ii) No conduction/radiation loss from wafer

(iii) ~~No~~ Uniform temp^r of wafer throughout

$Bi = \cancel{\frac{hL}{k}}$

Hence, no convective heat transfer from the system. Only through conduction.

wafer

$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

$$\rho C V \frac{dT}{dt} = 0 - \left[\cancel{k A_s} \frac{dT}{dy}\right] A_s \frac{(T - T_c)}{R_{t,c}}$$

$$\frac{\rho C w}{k} \frac{dT}{dt} = -\frac{dT}{dy} \frac{(T - T_c)}{R_{t,c}}$$

$$\frac{\rho C w R_t}{k} \ln(T - T_c) \Big|_{T_{w0}}^{T_{wt}} = -\int_0^t dt$$

$$\frac{\rho C w (R_t)}{k} \ln\left(\frac{33 - 23}{100 - 23}\right) = -15$$

$$R = \frac{15 \times k}{\ln\left(\frac{77}{10}\right) \cdot \rho C w}$$

$w \times A$

$q = \frac{\Delta T}{R}$

$w = \frac{k}{R}$

$q'' =$