Practice Problems

State Space model based controller Design

(Book: Modern Control Engineering by K Ogata)

1. Consider the transfer function $\frac{y(s)}{u(s)} = \frac{10.4s^2+47s+160}{s^3+14s^2+56s+160}$ and derive state space realization in a) controllable canonical form b) observable canonical form.

2. Consider the transfer function of the system $y(s) = \frac{25s+5}{s^3+5s^2+25s+5}u(s)$. Derive a state space realization of the system where state transition matrix A = $\begin{bmatrix} -5 & -25 & -5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

3. Consider a state space model $\dot{X} = AX + BU$ where $X = [X_1\ X_2\ X_3]^T$ and characteristic polynomial is $|sI - A| = s^3 + a_1 s^2 + a_2 s + a_3$. Show that
$M^{-1}AM = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}$ where M is controllability matrix.

4. Consider for the above system W is given as $W = \begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Define $T = MW$ and show that $T^{-1}AT = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}$ and $T^{-1}B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

5. Show that the transform matrix T= MW transforming $X = TZ$ converts any state equation to controllable canonical form [M and W defined in earlier questions].

6. Consider the completely observable 3rd order system
$$\dot{X} = AX$$
$$Y = CX$$
Show that $N^T A (N^T)^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}$ where N is observability matrix.

7. Show that the transform matrix $Q = (WN^T)^{-1}$ transforming $X = TQ$ converts any state space model to observable canonical form [N and W defined in earlier questions].

8. Consider the state equation $\frac{dx_1}{dt} = x_1 + x_2\ and \frac{dx_2}{dt} = -4x_1 - 3x_2 + 2u$. Transform the state equation to controllable canonical form and observable canonical form.

9. Obtain state feedback controller gain for the following system by placing the regulator poles at [-3, -5]:
$$\frac{dx_1}{dt} = x_2\ ; \frac{dx_2}{dt} = -2x_1 - 3x_2 + 2u\ ; y = x_1$$

10. Consider the transfer function of a process $\frac{y(s)}{u(s)} = \frac{10}{(s+1)(s+2)(s+3)}$. Obtain state feedback controller gain matrix by placing the regulator poles at $\left[\left(-2 \pm j\sqrt{3}\right), -10\right]$.