Hands-on Start to Mathematica

Text / documents

- Notebooks
- Cells
- Stylesheets

Means of starting

- Free-form linguistic Input

In[ ]:= integrate (1/(1-x^(3)))

Indefinite integrals:

Approximate form | Hide steps

$$\int \frac{1}{1-x^3}\,dx = \frac{1}{6}\left(\log(x^2+x+1) - 2\log(1-x) + 2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right) + \text{constant}$$

Possible intermediate steps:

Take the integral:

$$\int \frac{1}{1-x^3}\,dx$$

For the integrand $\frac{1}{1-x^3}$, use partial fractions:

$$= \int\left(\frac{x+2}{3(x^2+x+1)} - \frac{1}{3(x-1)}\right)dx$$

Integrate the sum term by term and factor out constants:

$$= \frac{1}{3}\int \frac{x+2}{x^2+x+1}\,dx - \frac{1}{3}\int \frac{1}{x-1}\,dx$$

Rewrite the integrand $\frac{x+2}{x^2+x+1}$ as $\frac{2x+1}{2(x^2+x+1)} + \frac{3}{2(x^2+x+1)}$:

$$= \frac{1}{3}\int\left(\frac{2x+1}{2(x^2+x+1)} + \frac{3}{2(x^2+x+1)}\right)dx - \frac{1}{3}\int \frac{1}{x-1}\,dx$$

Integrate the sum term by term and factor out constants:

Integrate the sum term by term and factor out constants:

$$= \frac{1}{6} \int \frac{2x+1}{x^2+x+1} \, dx + \frac{1}{2} \int \frac{1}{x^2+x+1} \, dx - \frac{1}{3} \int \frac{1}{x-1} \, dx$$

For the integrand $\frac{2x+1}{x^2+x+1}$, substitute $u = x^2 + x + 1$ and $du = (2x+1)\, dx$:

$$= \frac{1}{6} \int \frac{1}{u} \, du + \frac{1}{2} \int \frac{1}{x^2+x+1} \, dx - \frac{1}{3} \int \frac{1}{x-1} \, dx$$

The integral of $\frac{1}{u}$ is $\log(u)$:

$$= \frac{\log(u)}{6} + \frac{1}{2} \int \frac{1}{x^2+x+1} \, dx - \frac{1}{3} \int \frac{1}{x-1} \, dx$$

For the integrand $\frac{1}{x^2+x+1}$, complete the square:

$$= \frac{\log(u)}{6} + \frac{1}{2} \int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}} \, dx - \frac{1}{3} \int \frac{1}{x-1} \, dx$$

For the integrand $\frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}$, substitute $s = x + \frac{1}{2}$ and $ds = dx$:

$$= \frac{\log(u)}{6} + \frac{1}{2} \int \frac{1}{s^2 + \frac{3}{4}} \, ds - \frac{1}{3} \int \frac{1}{x-1} \, dx$$

Factor $\frac{3}{4}$ from the denominator:

$$= \frac{\log(u)}{6} + \frac{1}{2} \int \frac{4}{3\left(\frac{4s^2}{3} + 1\right)} \, ds - \frac{1}{3} \int \frac{1}{x-1} \, dx$$

Factor out constants:

$$= \frac{\log(u)}{6} + \frac{2}{3} \int \frac{1}{\frac{4s^2}{3} + 1} \, ds - \frac{1}{3} \int \frac{1}{x-1} \, dx$$

For the integrand $\frac{1}{\frac{4s^2}{3} + 1}$, substitute $p = \frac{2s}{\sqrt{3}}$ and $dp = \frac{2}{\sqrt{3}} ds$:

$$= \frac{\log(u)}{6} + \frac{1}{\sqrt{3}} \int \frac{1}{p^2+1} \, dp - \frac{1}{3} \int \frac{1}{x-1} \, dx$$

The integral of $\frac{1}{p^2+1}$ is $\tan^{-1}(p)$:

$$= \frac{\tan^{-1}(p)}{\sqrt{3}} + \frac{\log(u)}{6} - \frac{1}{3}\int \frac{1}{x-1}\,dx$$

For the integrand $\frac{1}{x-1}$, substitute $w = x-1$ and $dw = dx$:

$$= \frac{\tan^{-1}(p)}{\sqrt{3}} + \frac{\log(u)}{6} - \frac{1}{3}\int \frac{1}{w}\,dw$$

The integral of $\frac{1}{w}$ is $\log(w)$:

$$= \frac{\tan^{-1}(p)}{\sqrt{3}} + \frac{\log(u)}{6} - \frac{\log(w)}{3} + \text{constant}$$

Substitute back for $w = x-1$:

$$= \frac{\tan^{-1}(p)}{\sqrt{3}} + \frac{\log(u)}{6} - \frac{1}{3}\log(x-1) + \text{constant}$$

Substitute back for $p = \frac{2s}{\sqrt{3}}$:

$$= \frac{\tan^{-1}\left(\frac{2s}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(u)}{6} - \frac{1}{3}\log(x-1) + \text{constant}$$

Substitute back for $s = x + \frac{1}{2}$:

$$= \frac{\log(u)}{6} - \frac{1}{3}\log(x-1) + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \text{constant}$$

Substitute back for $u = x^2 + x + 1$:

$$= \frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log(x-1) + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \text{constant}$$

Factor the answer a different way:

$$= \frac{1}{6}\left(\log\left(x^2+x+1\right) - 2\log(x-1) + 2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right) + \text{constant}$$

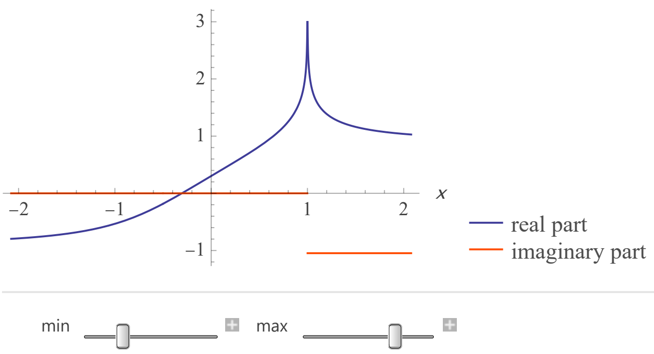
Which is equivalent for restricted x values to:

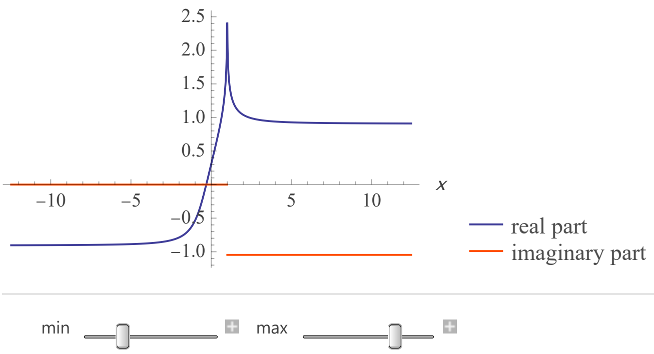
Answer:

$$= \frac{1}{6}\left(\log\left(x^2+x+1\right) - 2\log(1-x) + 2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right) + \text{constant}$$

Plots of the integral:

Complex-valued plots





Alternate forms of the integral:

$$\frac{1}{6}\left(\log\left(x^2+x+1\right) - 2\log(1-x)\right) + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \text{constant}$$

$$\frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log(1-x) + \frac{i\log\left(1-\frac{i(2x+1)}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{i\log\left(1+\frac{i(2x+1)}{\sqrt{3}}\right)}{2\sqrt{3}} + \text{constant}$$

Expanded form of the integral:

Step-by-step solution

$$\frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log(1-x) + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \text{constant}$$

Series expansion of the integral at x = 0:

$$\frac{\pi}{6\sqrt{3}} + x + \frac{x^4}{4} + \frac{x^7}{7} + O(x^9)$$

(Taylor series)

Big-O notation »

Series expansion of the integral at x = 1:

$$\frac{1}{18}\left(-6\log(1-x) + 2\sqrt{3}\,\pi + \log(27)\right) + \frac{x-1}{3} - \frac{1}{9}(x-1)^2 + \frac{1}{27}(x-1)^3 + O\left((x-1)^4\right)$$

(generalized Puiseux series)

Big-O notation »

WolframAlpha

In[∘]:=

integrate (1/(1-x^(3)))

```
Integrate[1/(1 - x^3), x]
```

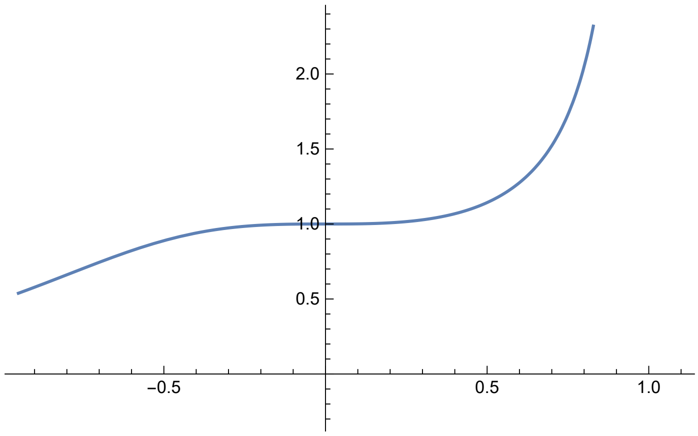
Out[∘]=

$$\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{1}{3}\text{Log}[1-x] + \frac{1}{6}\text{Log}\left[1+x+x^2\right]$$

In[∘]:=

```
Plot[1/(1 - x^3), {x, -0.95, 1.1}]
```

Out[∘]=



- Mathematica's Programming Language
  - 4 Basic Rules

0.0.1. Capital letters: function names

0.0.2. [] - anything we calculate

0.0.3. () - lists or ranges

0.0.4. Shift + Enter - run calculations (enter key carriage return)

In[∘]:=

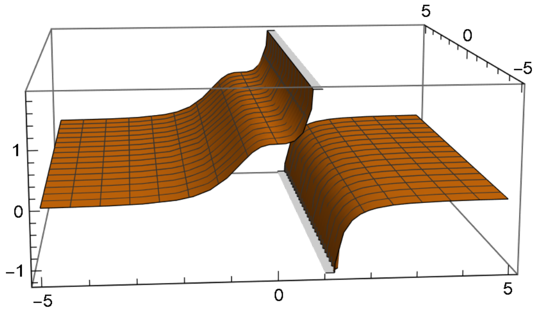
```
Integrate[log (x), x]
```

Out[∘]=

$$\frac{\log x^2}{2}$$

$$\text{Plot3D}\left[\frac{1}{1-x\hat{}3}, \{x, -5, 5\}, \{y, -5, 5\}\right]$$

Out[ ]=



Ctrl + Shift + (+) makes the 1/(1-x^3) look like above input.

■ Use of Palettes