Solving Differential Equations in Mathematica

- First Order Differential Equation
- Second Order Differential Equation
- Boundary Conditions

So, let's explore:

First Order Differential Equation:

```
In[•]:= DSolve[y'[x] == 2 x + 5, y[x], x]
```

$$\text{Out}[\bullet]= \left\{\left\{y[x] \to 5\,x + x^2 + c_1\right\}\right\}$$

```
In[•]:= DSolve[y'[x] == (2 y[x] + 5)/(x + 2), y[x], x]
```

$$\text{Out}[\bullet]= \left\{\left\{y[x] \to -\frac{5}{2} + (2 + x)^2\, c_1\right\}\right\}$$

```
In[•]:= DSolve[y'[x] == (2 x + y[x])/(y[x] - x), y[x], x]
```

$$\text{Out}[\bullet]= \left\{\left\{y[x] \to x - \sqrt{e^{2 c_1} + 3 x^2}\right\}, \left\{y[x] \to x + \sqrt{e^{2 c_1} + 3 x^2}\right\}\right\}$$

Second Differential Equation:

```
In[•]:= DSolve[y''[x] + 2 y'[x] - 8 y[x] == 6 Cos[x], y[x], x]
```

$$\text{Out}[\bullet]= \left\{\left\{y[x] \to e^{-4x} c_1 + e^{2x} c_2 - \frac{6}{85}(9\,\text{Cos}[x] - 2\,\text{Sin}[x])\right\}\right\}$$

```
In[•]:= {{y[x] → e^(-4 x) c1 + e^(2 x) c2 - 6/85 (9 Cos[x] - 2 Sin[x])}}[[1, 1, 2]]
```

$$\text{Out}[\bullet]= e^{-4x} c_1 + e^{2x} c_2 - \frac{6}{85}(9\,\text{Cos}[x] - 2\,\text{Sin}[x])$$

```
In[•]:= {{y[x] → e^(-4 x) c1 + e^(2 x) c2 - 6/85 (9 Cos[x] - 2 Sin[x])}}[[1, 1, 2]]
```

$$\text{Out}[\bullet]= e^{-4x} c_1 + e^{2x} c_2 - \frac{6}{85}(9\,\text{Cos}[x] - 2\,\text{Sin}[x])$$

```
In[•]:= DSolve[y''[x] + 2 y'[x] - 8 y[x] == 0, y[x], x]
```

$$\text{Out}[\bullet]= \left\{\left\{y[x] \to e^{-4x} c_1 + e^{2x} c_2\right\}\right\}$$

```
In[•]:= Simplify[{{y[x] → e^(-4 x) c1 + e^(2 x) c2}}]
```

$$\text{Out}[\bullet]= \left\{\left\{y[x] \to e^{-4x}\left(c_1 + e^{6x} c_2\right)\right\}\right\}$$

In[ ]:=
```
DSolve[{y'[t] == k (y[t] - T), y[0] == T1}, y[t], t]
```

DSolve: "List encountered within \!\({*SuperscriptBox[\\\"y\\\", \\\",\\\",MultilineFunction->None][t] == {k\\\\\\ \((\(-*FractionBox[\(1 + Sin[≪1≫]\), SqrtBox[\(Plus[≪2≫]\)]]\) + y[t])\), k\\\\\\ \((*FractionBox[\(Cos[t]\), SqrtBox[\(Plus[≪2≫]\)]] + y[t])\)}}\). There should be no lists on either side of the equations."

Out[ ]=
$$\text{DSolve}\left[\left\{y'[t] == \left\{k\left(-\frac{1+\text{Sin}[t]}{\sqrt{\text{Cos}[t]^2+(1+\text{Sin}[t])^2}}+y[t]\right), k\left(\frac{\text{Cos}[t]}{\sqrt{\text{Cos}[t]^2+(1+\text{Sin}[t])^2}}+y[t]\right)\right\}, y[0] == T1\right\}, y[t], t\right]$$

DSolve: There are fewer dependent variables than equations, so the system is overdetermined.

DSolve: "The function y[t] was specified without dependence on all the independent variables. Each function must depend on all the independent variables."

In[ ]:=
```
Dsolve[
  {y'''[x] + 2 y''[x] - 8 y[x] == 7,
   y'[2] == 4,
   y[1] == 0},
  y[x], x] // Simplify
```

Out[ ]=
$$\text{Dsolve}\left[\left\{7+8\,y[x] == 2\,y''[x]+y^{(3)}[x], y'[2] == 4, y[1] == 0\right\}, y[x], x\right]$$