An Algorithmic Exploration of Mathematics with Examples

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1 Explaination

1.1 Introduction

In this paper there is an original idea, which can be summarized as the potential connection between algorithms (algorithms which implement artificial intelligence in the context of mathematics) and mathematics, articulated and some examples for it expressed as concrete python code. As a consequence of not able to verify the relevant literature, I do not know where this idea fits in mainstream knowlegde (mainstream mathematics and computer science).

1.2 Mathematical Tools and the Concept of Infinite Computer Resources

Without a doubt, there already exist software solutions that employ algorithms for mathematical problem-solving. The idea of connecting algorithms and mathematics may not be considered original, with well-known examples like Mathematica, Matlab, Sage, and others serving as powerful tools for exploring the field of mathematics and beyond. The software discussed in this paper, however, stands out as potentially more powerful than its counterparts, given a relaxation from one constraint: the need to run the very software we discuss.

When I mention not running the software, I am intuitively pointing towards the significance of the underlying code, which can be utilized differently. It's not just a relaxation from running the software; it's also a departure from the need for optimization. I confidently make the claim that a mathematics software can be capable of solving a multitude of mathematical problems, much more various than anyone thought before, provided a hypothetical object I call as "a computer enriched with infinitely many computational resources".

1.3 Exploring the Notion of Infinite Computing

I do not know why this idea is not discussed already popularly before so I did. Also not like that, it is an idea, which is too abstracted or isolated from what a programmer generally does. This is idea so brilliant, that it might be problematic even after addressing that ”may be the people in past might have already thought about it but rejected the importance and instead choose to spend time behind things which actually do work”, because the questioning of sanity. I lay out the idea in here along with the, precise details, because of which I think what I think. I am going to write a down a few examples to do that.

- **Chess:** A popular example can be what everyone already knows, that chess could be solved if we had enough time (zillions of years). We could even use the existing chess engines to feed its code in a commonly used i3 intel computer, which should keep functioning for a long time, possibly even after human civilization ends, or make our own code for that computer of-course confirming there is no bugs exist in that code. And then, as expected, chess would be solved (the engine saying checkmate in how many moves or if its a draw on perfect play).

- **File Compression:** Another example could be compressing a file, assuming the encoding as a programming language. Its not a typical method of compressing files but when systematically testing every conceivable code (for all valid python programs let's say) we will get strike by an eureka moment that what the program returns is the file itself we are given to compress. That moment, the code which generated that desired output, is the compressed file. Here is the code of the new compressing mechanism in python.

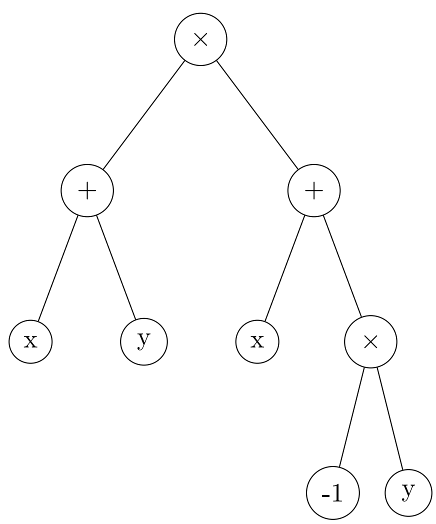
```
def compress(file_data):
    for file_size in range(1, 9999):
        for string in itertools.product([chr(i) for i
            in range(256)], repeat=file_size):
            string = "".join(string)
            code_string = f'def a():\n\t{string}\n\
                treturn A'
            try:
                exec(code_string)
                result = eval("a()")
                if isinstance(result, str) and result
                    == file_data:
                    return code_string
            except Exception as e:
                tmp = str(e)
    return file_data
```

1.4 Computationally Modelling Mathematics and what it Means

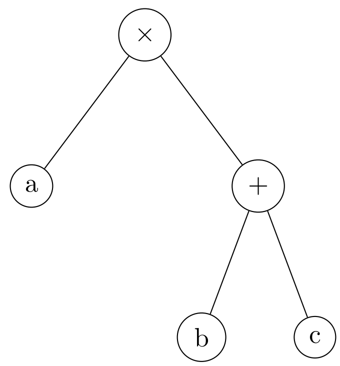
We have a very fast computer now, but a fast computer on its own, can do only a few things. Only after computationally modelling something, providing exactly the steps for a task, we could be able to utilize what that theoritical device could potentially provide us. I have developed a program in python, you will realize if you checked the last section, and in there is complete code on how to solve 15 example math questions. In this paper we will only give an outline on how that program works, rest to be understood by that code, which unfortunately effortfull but worth it . This code doing this naive but functioning attempt at computationally modelling some mathematics, motivates us to work more on this framework of infinite computing. Also when checking out the last section, you will be intrigued realizing that it is very possible to model if atleast high school mathematics. I even believe in future if enough work is done on this idea, examinations like JEE Main, can be completely computationally modelled, giving a remark on the very formal education system which organizes human to run the country, and also provide helpfulness and more hope to education. But that is only a dream.

1.5 Introduction by Some Simple Algebra

We will try to transform the equation $(x+y)\times(x-y)$ into $x^2 - y^2$ using what can be seen as simple steps, simple enough to be able to be coded. The first thing we will try to do is model $(x+y)\times(x-y)$ using a data structure. We use a tree in this case and represent it as follows:



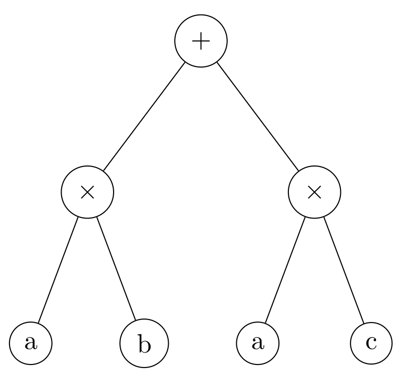
The creation of nodes and the tree is done according to the brackets in the equation. And also we can see that the attempt for creating a new function for subtraction (addition and multiplications are functions here) is avoided in order to reduce the model, by using addition multiplication and a -1. We will also be avoiding any negative numbers and completely rely on the -1. Then comes the question, how can we transform this equation, given we want to simplify it to $x^2 - y^2$? We wil using a concept called "formulas" to do that. The below shows the distributive formula's LHS.



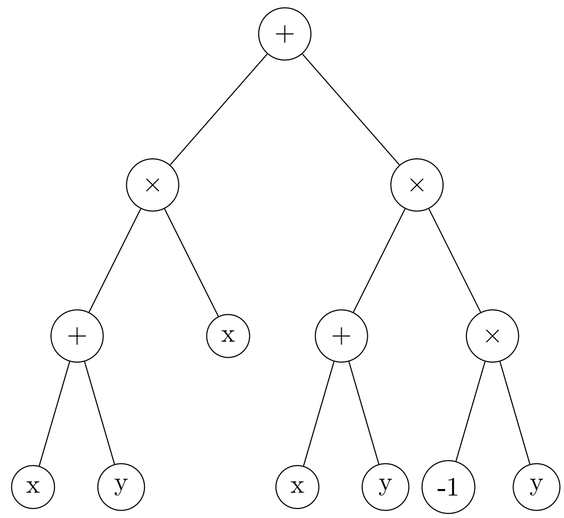
Let's apply the formula on the equation now. We usually need to test the formula at every position of the equation heirarchy but in this case it works at the root. The variables can take on any values, but the rest should be overlapping, in order to judge a formula is applicable.

$$a = x + y$$
$$b = x$$
$$c = -y$$

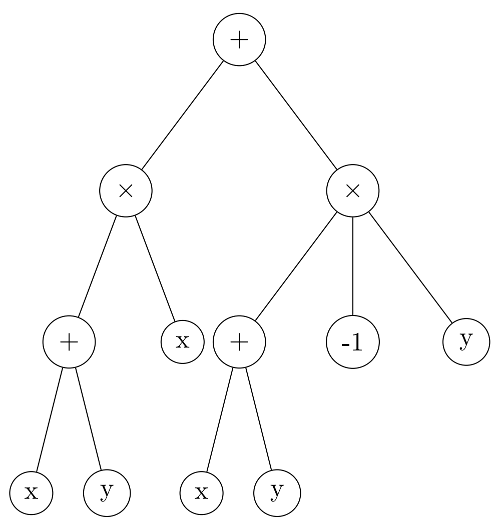
These will be the values we get on application. Now we will substitute the values to the RHS of the formula (and also draw the tree of RHS of the formula)



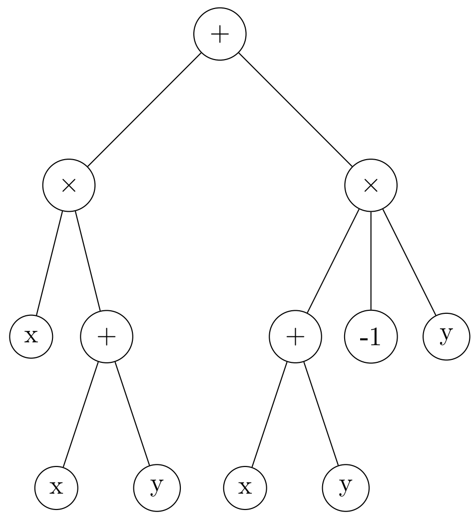
gets transformed into



So now we have with us $((x + y) \times x) + ((x + y) \times (-1 \times y))$
We will see now that there is an extra bracket despite mulitplication is commtative, so let's merge them. We merge any connected similar operation (function) which are commutative.



Done !
Now the next step would be that to apply the distributive formula again, but, the formula required is $(b + c) \times a$ and not $a \times (b + c)$
But we are not going to create a new formula to handle this situation, we instead, transform the equation in question in all possible ways (and try all formulas on every location), relying on the commutative property. One of the transformation will be



$x \times (x + y) + (x + y) \times (-1 \times y)$
Applying the distributive formula now finally
$((x \times x) + (x \times y)) + ((x + y) \times -1 \times y)$
Merging
$(x \times x + x \times y + ((x + y) \times -1 \times y))$
Now let's using the formula $a \times a = a^2$
$x^2 + x \times y + ((x + y) \times -1 \times y)$
Note that, we are applying the formulas not just in the root, but all position in the equation tree.
Again after some distribution, merging and squared formulas it would become
$x^2 + (x \times y) + (x \times y \times -1) + y^2$
Then the tricky part is programming this rebracketing which is possible to do.
One of the correct bracketing would be like
$x^2 + (-1 \times y^2) + ((x \times y) + ((x \times y) \times -1))$
Which will enable us to apply the formula $a + a \times (-1) = 0$ to get
$x^2 + (-1 \times y^2) + 0$
And finally after, $a + 0 = a$
$x^2 + (-1 \times y^2)$
Which would be the required transformation ! There is a need to define what's the right transformation as asked in the question in order to return an answer, and in here, it matches the simplification we wanted. Hence, this answer is correct.

1.6 Yet to be Written

Details for solving other than simple alegbra yet to written, but you can refer the code.

2 The List of The 15 Math Questions And the Code to Solve them

2.1 The Math Questions

1.
$$\frac{2}{5}(x-5)^{\frac{5}{2}} = \frac{10}{3}(x-5)^{\frac{3}{2}}$$

2.
$$\int \frac{dx}{\sin^2(x)\cos^2(x)} = \tan(x) - \cot(x)$$

3.
$$\int \frac{\sqrt{a^2-x^2}}{x^4}\,dx = \frac{(a^2-x^2)^{3/2}}{3a^2x^3}$$

4.
$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+b \end{vmatrix} = ab$$

5.
$$\begin{vmatrix} \frac{1}{a} & a & bc \\ \frac{1}{b} & b & ac \\ \frac{1}{c} & c & ab \end{vmatrix} = 0$$

6.
$$\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} = \frac{1}{2\sqrt{2}}$$

7.
$$\lim_{x\to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} = -\frac{1}{10}$$

8.
$$\lim_{x\to 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}} = -\frac{1}{3}$$

9.
$$\int \frac{dx}{1+e^x} = x - \ln(1+e^x)$$

10.
$$\int \arctan(x)\,dx = x\arctan(x) - \frac{1}{2}\ln(1+x^2)$$

11.
$$\int x^3\ln(x)\,dx = \frac{1}{4}x^4\ln(x) - \frac{1}{16}x^4$$

12.
$$\int (x^2+5)^3\,dx = \frac{x^7}{7} + 3x^4 + \frac{75x^2}{3} + 125x$$

13.
$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0$$

14.
$$\begin{vmatrix} x & 2 & 4 \\ 4 & 8 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

15.
$$i^{2015} + i^{2016} + i^{2017} + i^{2018} = 0$$

2.2 The Link of Code in Python Hosted on Github

```
https://github.com/SwastikMajumder/inf_math_ai/blob/
    main/main.py
```