

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x \sqrt{x+2}$

8. $x \sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```
>>> 2*x/(1+x^2)
(2*((1+(x^2))^-1)*x)
>>> Sdx
```

THOUGHT PROCESS =

integrating $(2*((1+(x^2))^-1)*x)$ wrt x
 rewriting as $(2*((1+(x^2))^-1)*x)$

taking the constant outside of the integral

integrating $((1+(x^2))^-1)*x)$ wrt x

rewriting as $((1+(x^2))^-1)*x)$

rewriting as $((1+(x^2))^-1)*x)$

substituting $(1+(x^2))=y$

integrating $((2^-1)*((2+(-1*y))^-1))$ wrt y

rewriting as $((-2^-1)*((-2+y))^-1))$

taking the constant outside of the integral

integrating $((-2+y)^{-1})$ wrt y

rewriting as $((-2+y)^{-1})$

rewriting as $((-2+y)^{-1})$

the solution is $(\log(\text{abs}((-2+y))) * (1^-1))$

the solution is $((-2^-1)*(\log(\text{abs}((-2+(1+(x^2)))) * (1^-1)))$

the solution is $((-2^-1)*(\log(\text{abs}((-2+(1+(x^2)))) * (1^-1)))$

the solution is $(-1*\log(\text{abs}((-1+(x^2))))$

$(-1*\log(\text{abs}((-1+(x^2))))))$

>>>

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x \sqrt{x+2}$

8. $x \sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```
>>> (log(x))^2/x
((log(x)^2)*(x^-1))
>>> Sdx
```

THOUGHT PROCESS =

```
integrating ((log(x)^2)*(x^-1)) wrt x
rewriting as ((log(x)^2)*(x^-1))
rewriting as ((log(x)^2)*(x^-1))
substituting log(x)=y
integrating (y^2) wrt y
rewriting as (y^2)
rewriting as (y^2)
the solution is ((log(x)^3)*((3*1)^-1))
the solution is ((3^-1)*(log(x)^3))

((3^-1)*(log(x)^3))
>>>
```

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x \sqrt{x+2}$

8. $x \sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```
>>> sin(x)*sin(cos(x))
(sin(cos(x))*sin(x))
```

```
>>> Sdx
```

```
THOUGHT PROCESS =
```

```
integrating (sin(cos(x))*sin(x)) wrt x
rewriting as (sin(cos(x))*sin(x))
rewriting as (sin(cos(x))*sin(x))
```

```
substituting cos(x)=y
```

```
integrating (-1*sin(y)) wrt y
```

```
rewriting as sin(y)
```

```
rewriting as sin(y)
```

```
the solution is ((-1*cos(y))* (1^-1))
```

```
the solution is (-1* ((-1*cos(cos(x)))*(1^-1)))
```

```
the solution is cos(cos(x))
```

```
cos(cos(x))
```

```
>>>
```

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```
>>> (a*x+b)^(1/2)
((x*a)+b)^(2^-1))
>>> Sdx
```

THOUGHT PROCESS =

integrating $((x*a)+b)^(2^-1)$ wrt x
 rewriting as $((x*a)+b)^(2^-1)$
 rewriting as $((x*a)+b)^(2^-1)$

substituting $((x*a)+b)=y$

integrating $(y^(2^-1))*(a^-1)$ wrt y
 rewriting as $(y^(2^-1))*(a^-1)$

taking the constant outside of the integral

integrating $(y^(2^-1))$ wrt y

rewriting as $(y^(2^-1))$

rewriting as $(y^(2^-1))$

the solution is $((y^(3*(2^-1)))*(((3*(2^-1))*1)^{-1}))$

the solution is $((a^-1)*(((x*a)+b)^(3*(2^-1)))*(((3*(2^-1))*1)^{-1}))$

the solution is $(2*(3^-1)*(((x*a)+b)^(3*(2^-1)))* (a^-1))$

$(2*(3^-1)*(((x*a)+b)^(3*(2^-1)))* (a^-1))$

>>>

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```
>>> x*(x+2)^(1/2)
```

```
((2+x)^(2^-1))*x)
```

```
>>> Sdx
```

THOUGHT PROCESS =

integrating $((2+x)^(2^-1))*x$ wrt x

rewriting as $((2+x)^(2^-1))*x$

rewriting as $((2+x)^(2^-1))*x$

substituting $(2+x)=y$

integrating $((-2*(y^(2^-1)))+(y^(3*(2^-1))))$ wrt y

rewriting as $((-2*(y^(2^-1)))+(y^(3*(2^-1))))$

rewriting as $((-2*(y^(2^-1)))+(y^(3*(2^-1))))$

integrating over sums

integrating $(-2*(y^(2^-1)))$ wrt y

rewriting as $(-2*(y^(2^-1)))$

rewriting as $(-2*(y^(2^-1)))$

the solution is $((y^(3*(2^-1)))*(((3*(2^-1))*1)^{-1}))$

the solution is $(-2*((y^(3*(2^-1)))*(((3*(2^-1))*1)^{-1})))$

integrating over sums

integrating $(y^(3*(2^-1)))$ wrt y

rewriting as $(y^(3*(2^-1)))$

rewriting as $(y^(3*(2^-1)))$

the solution is $((y^(5*(2^-1)))*(((5*(2^-1))*1)^{-1}))$

the solution is $(((-2*((2+x)^(3*(2^-1)))*(((3*(2^-1))*1)^{-1}))-((2+x)^(5*(2^-1))*(((5*(2^-1))*1)^{-1})))$

the solution is $(((-4*(3^-1)*(2+x)^(3*(2^-1))))+(2*(5^-1)*(2+x)^(5*(2^-1))))$

((-4 * (3^-1) * ((2+x)^(3*(2^-1)))) + (2 * (5^-1) * ((2+x)^(5*(2^-1)))))
 >>>

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin(ax+b) \cos(ax+b)$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

>>> x*(1+2*x^2)^(1/2)
 (((1+(2*(x^2)))^(2^-1))*x)
 >>> Sdx

THOUGHT PROCESS =

integrating (((1+(2*(x^2)))^(2^-1))*x) wrt x
 rewriting as ((2^(2^-1))*(((2^-1)+(x^2))^(2^-1))*x)
 taking the constant outside of the integral
 integrating (((2^-1)+(x^2))^(2^-1))*x wrt x
 rewriting as (((2^-1)+(x^2))^(2^-1))*x
 rewriting as (((2^-1)+(x^2))^(2^-1))*x
 substituting ((2^-1)+(x^2))=y
 integrating ((2^-1)*(y^(2^-1))) wrt y
 rewriting as ((2^-1)*(y^(2^-1)))
 taking the constant outside of the integral
 integrating (y^(2^-1)) wrt y
 rewriting as (y^(2^-1))
 rewriting as (y^(2^-1))
 the solution is ((y^(3*(2^-1)))*(((3*(2^-1))*1)^-1))
 the solution is ((2^-1)*((((2^-1)+(x^2))^(3*(2^-1)))*(((3*(2^-1))*1)^-1)))
 the solution is ((2^-1)*((((2^-1)+(x^2))^(3*(2^-1)))*(((3*(2^-1))*1)^-1)))
 the solution is ((3^-1)*(2^(2^-1))*(((2^-1)+(x^2))^(3*(2^-1)))))
 ((3^-1)*(2^(2^-1))*(((2^-1)+(x^2))^(3*(2^-1)))))
 >>>

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```
>>> x/(x+4)^(1/2)
((4+x)^(-2^-1))*x
>>> Sdx
```

THOUGHT PROCESS =

integrating $((4+x)^{-2^-1}) * x$ wrt x

rewriting as $((4+x)^{-2^-1}) * x$

rewriting as $((4+x)^{-2^-1}) * x$

substituting $(4+x)=y$

integrating $((-4*(y^{-2^-1}))+ (y^{(2^-1)}))$ wrt y

rewriting as $((-4*(y^{-2^-1}))+ (y^{(2^-1)}))$

rewriting as $((-4*(y^{-2^-1}))+ (y^{(2^-1)}))$

integrating over sums

integrating $(-4*(y^{-2^-1}))$ wrt y

rewriting as $(-4*(y^{-2^-1}))$

rewriting as $(-4*(y^{-2^-1}))$

the solution is $((y^{(2^-1)})*(((2^-1)*1)^{-1}))$

the solution is $(-4*((y^{(2^-1)})*(((2^-1)*1)^{-1})))$

integrating over sums

integrating $(y^{(2^-1)})$ wrt y

rewriting as $(y^{(2^-1)})$

rewriting as $(y^{(2^-1)})$

the solution is $((y^{(3*(2^-1)}))*(((3*(2^-1)*1)^{-1}))$

the solution is $((-4*((4+x)^{(2^-1})*(((2^-1)*1)^{-1}))-((4+x)^{(3*(2^-1)})*(((3*(2^-1)*1)^{-1})))$

the solution is $((-8*((4+x)^{(2^-1)}))+(2*(3^-1)*(4+x)^{(3*(2^-1)}))))$

((-8*((4+x)^(2^(-1))))+(2*(3^(-1))*((4+x)^(3*(2^(-1))))))
 >>>

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin(ax+b) \cos(ax+b)$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x>0$

12. $\frac{-1}{(x^3-1)^{\frac{1}{3}}}x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x>0, m\neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1}x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

>>> $(x^3-1)^{(1/3)} * x^5$
 (((-1+(x^3))^(3^(-1))) * (x^5))
 >>> Sdx

THOUGHT PROCESS =

```

integrating (((-1+(x^3))^(3^(-1))) * (x^5)) wrt x
rewriting as (((-1+(x^3))^(3^(-1))) * (x^5))
rewriting as (((-1+(x^3))^(3^(-1))) * (x^5))
substituting (-1+(x^3))=y
integrating (((3^(-1))*(y^(3^(-1))))+((3^(-1))*(y^(4*(3^(-1)))))) wrt y
rewriting as (((3^(-1))*(y^(3^(-1))))+((3^(-1))*(y^(4*(3^(-1))))))
rewriting as (((3^(-1))*(y^(3^(-1))))+((3^(-1))*(y^(4*(3^(-1))))))
    integating over sums
    integrating ((3^(-1))*(y^(3^(-1)))) wrt y
    rewriting as ((3^(-1))*(y^(3^(-1))))
        taking the constant outside of the integral
        integrating (y^(3^(-1))) wrt y
        rewriting as (y^(3^(-1)))
        rewriting as (y^(3^(-1)))
        the solution is ((y^(4*(3^(-1))))*((((4*(3^(-1)))*1)^(-1)))
        the solution is ((3^(-1))*((y^(4*(3^(-1))))*((((4*(3^(-1)))*1)^(-1))
        integating over sums
        integrating ((3^(-1))*(y^(4*(3^(-1)))))) wrt y
        rewriting as ((3^(-1))*(y^(4*(3^(-1))))))
            taking the constant outside of the integral
            integrating (y^(4*(3^(-1)))) wrt y
            rewriting as (y^(4*(3^(-1))))
            rewriting as (y^(4*(3^(-1))))
```

the solution is $((y^{(7*(3^(-1)))}) * (((7*(3^(-1)))*1)^{-1}))$
 the solution is $((3^(-1))*((y^{(7*(3^(-1)))}) * (((7*(3^(-1)))*1)^{-1})))$
 the solution is $(((3^(-1))*((-1+(x^3))^{{(4*(3^(-1)))}}) * (((4*(3^(-1)))*1)^{-1}))) + ((3^(-1))*((-1+(x^3))^{{(7*(3^(-1)))}}) * (((7*(3^(-1)))*1)^{-1})))$
 the solution is $((((4^(-1))*((-1+(x^3))^{{(4*(3^(-1)))}})) + ((7^(-1))*((-1+(x^3))^{{(7*(3^(-1)))}})))$

$((((4^(-1))*((-1+(x^3))^{{(4*(3^(-1)))}})) + ((7^(-1))*((-1+(x^3))^{{(7*(3^(-1)))}}))))$
 >>>

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x>0$

12. $(x^3 - 1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x>0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```

>>> x^2/(2+3*x^3)^3
((2+(3*(x^3)))^(-3)*(x^2))
>>> Sdx
  
```

THOUGHT PROCESS =

integrating $((2+(3*(x^3)))^(-3)*(x^2))$ wrt x
 rewriting as $((27^(-1))*((2*(3^(-1)))+(x^3))^(-3)*(x^2))$
 taking the constant outside of the integral
 integrating $((2*(3^(-1)))+(x^3))^(-3)*(x^2)$ wrt x
 rewriting as $((((2*(3^(-1)))+(x^3))^(-3)*(x^2))$
 rewriting as $((((2*(3^(-1)))+(x^3))^(-3)*(x^2))$
 substituting $((2*(3^(-1)))+(x^3))=y$
 integrating $((3^(-1))* (y^(-3)))$ wrt y
 rewriting as $((3^(-1))* (y^(-3)))$
 taking the constant outside of the integral
 integrating $(y^(-3))$ wrt y
 rewriting as $(y^(-3))$
 rewriting as $(y^(-3))$
 the solution is $((y^(-2))*((-2*1)^{-1}))$
 the solution is $((3^(-1))*(((2*(3^(-1)))+(x^3))^(-2)*((-2*1)^{-1})))$
 the solution is $((3^(-1))*(((2*(3^(-1)))+(x^3))^(-2)*((-2*1)^{-1})))$

the solution is $((-162^{-1}) * (((2 * (3^{-1})) + (x^3))^{-2}))$

((-162^{-1}) * (((2 * (3^{-1})) + (x^3))^{-2}))
>>>

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3 - 1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

>>> 1 / (x * (log(x)^n))
((log(x)^(-1*n)) * (x^-1))
>>> Sdx
THOUGHT PROCESS =

integrating $((\log(x)^{-1*n}) * (x^{-1}))$ wrt x
rewriting as $((\log(x)^{-1*n}) * (x^{-1}))$
rewriting as $((\log(x)^{-1*n}) * (x^{-1}))$
substituting $\log(x)=y$
integrating (y^{-1*n}) wrt y
rewriting as (y^{-1*n})
rewriting as (y^{-1*n})
the solution is $((\log(x)^{(1+(-1*n))}) * (((1+(-1*n))*1)^{-1}))$
the solution is $(((1+(-1*n))^{-1}) * (\log(x)^{(1+(-1*n))}))$

((1+(-1*n))^-1) * (log(x)^(1+(-1*n)))
>>>

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```
>>> x/(9-4*x^2)
((9+(-4*(x^2)))^-1)*x)
>>> Sdx
```

THOUGHT PROCESS =

```
integrating (((9+(-4*(x^2)))^-1)*x) wrt x
rewriting as ((-4^-1)*((-2^-1)*((-3*(2^-1))+x)^-1)+(2^-1)*(((3*(2^-1))+x)^-1)))
taking the constant outside of the integral
integrating (((2^-1)*((-3*(2^-1))+x)^-1)+(2^-1)*(((3*(2^-1))+x)^-1)) wrt x
rewriting as (((2^-1)*((-3*(2^-1))+x)^-1)+(2^-1)*(((3*(2^-1))+x)^-1))
rewriting as (((2^-1)*((-3*(2^-1))+x)^-1)+(2^-1)*(((3*(2^-1))+x)^-1))
integrating over sums
integrating ((2^-1)*((-3*(2^-1))+x)^-1) wrt x
rewriting as ((2^-1)*((-3*(2^-1))+x)^-1)
taking the constant outside of the integral
integrating ((((-3*(2^-1))+x)^-1) wrt x
rewriting as ((((-3*(2^-1))+x)^-1)
rewriting as ((((-3*(2^-1))+x)^-1)
the solution is (log(abs((((-3*(2^-1))+x))))*(1^-1))
the solution is ((2^-1)*(log(abs((((-3*(2^-1))+x))))*(1^-1)))
integrating over sums
integrating ((2^-1)*(((3*(2^-1))+x)^-1)) wrt x
rewriting as ((2^-1)*(((3*(2^-1))+x)^-1))
taking the constant outside of the integral
integrating (((3*(2^-1))+x)^-1) wrt x
rewriting as (((3*(2^-1))+x)^-1)
```

```

rewriting as (((3*(2^-1))+x)^-1)
the solution is (log(abs(((3*(2^-1))+x)))*(1^-1))
the solution is ((2^-1)*(log(abs(((3*(2^-1))+x))))*(1^-1)))
the solution is (((2^-1)*(log(abs(((3*(2^-1))+x))))*(1^-1)))+((2^-
1)*(log(abs(((3*(2^-1))+x))))*(1^-1)))
the solution is ((((-8^-1)*log(abs(((3*(2^-1))+x))))+((-8^-
1)*log(abs(((3*(2^-1))+x))))))

((((-8^-1)*log(abs(((3*(2^-1))+x))))+((-8^-1)*log(abs(((3*(2^-1))+x))))))

>>>

```

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```

>>> e^(2*x+3)
(e^(3+(2*x)))
>>> Sdx

```

THOUGHT PROCESS =

```

integrating (e^(3+(2*x))) wrt x
rewriting as (e^(2*((3*(2^-1))+x)))
rewriting as (e^(3+(2*x)))
the solution is ((2^-1)*(e^(3+(2*x))))

```

```

((2^-1)*(e^(3+(2*x))))
>>>

```

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```
>>> x/e^(x^2)
((e^(-1*(x^2))) * x)
```

```
>>> Sdx
```

THOUGHT PROCESS =

integrating ((e^(-1*(x^2))) * x) wrt x
 rewriting as ((e^(-1*(x^2))) * x)

rewriting as ((e^(-1*(x^2))) * x)

substituting (-1*(x^2))=y

integrating ((-2^-1)*(e^(-1*y))) wrt y

rewriting as ((-2^-1)*(e^(-1*y)))

taking the constant outside of the integral

integrating (e^(-1*y)) wrt y

rewriting as (e^(-1*y))

rewriting as (e^(-1*y))

the solution is ((e^(-1*y)) * (-1^-1))

the solution is ((-2^-1)*((e^(-1*(-1*(x^2)))) * (-1^-1)))

the solution is ((2^-1)*(e^(x^2)))

```
((2^-1)*(e^(x^2)))
```

```
>>>
```

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$

2. $\frac{(\log x)^2}{x}$

3. $\frac{1}{x+x \log x}$

4. $\sin x \sin (\cos x)$

5. $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$

7. $x \sqrt{x+2}$

8. $x \sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\frac{1}{x-\sqrt{x}}$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

12. $(x^3-1)^{\frac{1}{3}} x^5$

13. $\frac{x^2}{(2+3x^3)^3}$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

15. $\frac{x}{9-4x^2}$

16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

```
>>> e^(arctan(x)) / (1+x^2)
((1+(x^2))^(-1) * (e^arctan(x)))
>>> Sdx
```

THOUGHT PROCESS =

```
integrating (((1+(x^2))^(-1) * (e^arctan(x))) wrt x
rewriting as (((1+(x^2))^(-1) * (e^arctan(x)))
rewriting as (((1+(x^2))^(-1) * (e^arctan(x)))
```

substituting arctan(x)=y

integrating (e^y) wrt y

rewriting as (e^y)

rewriting as (e^y)

the solution is ((e^arctan(x)) * (1^(-1)))

the solution is (e^arctan(x))

(e^arctan(x))

>>>

$$21. \tan^2(2x - 3)$$

$$22. \sec^2(7 - 4x)$$

$$23. \frac{\sin^{-1}x}{\sqrt{1-x^2}}$$

$$24. \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$$

$$25. \frac{1}{\cos^2 x (1 - \tan x)^2}$$

$$26. \frac{\cos \sqrt{x}}{\sqrt{x}}$$

$$27. \sqrt{\sin 2x} \cos 2x$$

$$28. \frac{\cos x}{\sqrt{1+\sin x}}$$

$$29. \cot x \log \sin x$$

$$30. \frac{\sin x}{1 + \cos x}$$

$$31. \frac{\sin x}{(1 + \cos x)^2}$$

$$32. \frac{1}{1 + \cot x}$$

$$33. \frac{1}{1 - \tan x}$$

$$34. \frac{\sqrt{\tan x}}{\sin x \cos x}$$

$$35. \frac{(1 + \log x)^2}{x}$$

$$36. \frac{(x+1)(x+\log x)^2}{x}$$

$$37. \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$$

```
>>> cos(x^(1/2))/x^(1/2)
(cos((x^(2^-1)))*(x^(-2^-1)))
>>> Sdx
```

THOUGHT PROCESS =

```
integrating (cos((x^(2^-1)))*(x^(-2^-1))) wrt x
rewriting as (cos((x^(2^-1)))*(x^(-2^-1)))
rewriting as (cos((x^(2^-1)))*(x^(-2^-1)))
substituting (x^(2^-1))=y
integrating (2*cos(y)) wrt y
rewriting as (2*cos(y))
taking the constant outside of the integral
integrating cos(y) wrt y
rewriting as cos(y)
rewriting as cos(y)
the solution is (sin(y)*(1^-1))
the solution is (2*(sin((x^(2^-1)))*(1^-1)))
the solution is (2*sin((x^(2^-1))))
```

```
(2*sin((x^(2^-1))))
```

```
>>>
```

$$21. \tan^2(2x - 3)$$

$$22. \sec^2(7 - 4x)$$

$$23. \frac{\sin^{-1}x}{\sqrt{1-x^2}}$$

$$24. \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$$

$$25. \frac{1}{\cos^2 x (1 - \tan x)^2}$$

$$26. \frac{\cos \sqrt{x}}{\sqrt{x}}$$

$$27. \sqrt{\sin 2x} \cos 2x$$

$$28. \frac{\cos x}{\sqrt{1+\sin x}}$$

$$29. \cot x \log \sin x$$

$$30. \frac{\sin x}{1 + \cos x}$$

$$31. \frac{\sin x}{(1 + \cos x)^2}$$

$$32. \frac{1}{1 + \cot x}$$

$$33. \frac{1}{1 - \tan x}$$

$$34. \frac{\sqrt{\tan x}}{\sin x \cos x}$$

$$35. \frac{(1 + \log x)^2}{x}$$

$$36. \frac{(x+1)(x+\log x)^2}{x}$$

$$37. \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$$

```
>>> cos(x) / (1+sin(x))^(1/2)
(cos(x) * ((1+sin(x)) ^ (-2^-1)))
```

```
>>> Sdx
```

```
THOUGHT PROCESS =
```

```
integrating (cos(x) * ((1+sin(x)) ^ (-2^-1))) wrt x
```

```
rewriting as (cos(x) * ((1+sin(x)) ^ (-2^-1)))
```

```
rewriting as (cos(x) * ((1+sin(x)) ^ (-2^-1)))
```

```
substituting (1+sin(x))=y
```

```
integrating (y^(-2^-1)) wrt y
```

```
rewriting as (y^(-2^-1))
```

```
rewriting as (y^(-2^-1))
```

```
the solution is (((1+sin(x)) ^ (2^-1)) * ((2^-1)*1)^-1))
```

```
the solution is (2*((1+sin(x)) ^ (2^-1)))
```

```
(2*((1+sin(x)) ^ (2^-1)))
```

```
>>>
```

$$21. \tan^2(2x - 3)$$

$$22. \sec^2(7 - 4x)$$

$$23. \frac{\sin^{-1}x}{\sqrt{1-x^2}}$$

$$24. \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$$

$$25. \frac{1}{\cos^2 x (1 - \tan x)^2}$$

$$26. \frac{\cos \sqrt{x}}{\sqrt{x}}$$

$$27. \sqrt{\sin 2x} \cos 2x$$

$$28. \frac{\cos x}{\sqrt{1+\sin x}}$$

$$29. \cot x \log \sin x$$

$$30. \frac{\sin x}{1+\cos x}$$

$$31. \frac{\sin x}{(1+\cos x)^2}$$

$$32. \frac{1}{1+\cot x}$$

$$33. \frac{1}{1-\tan x}$$

$$34. \frac{\sqrt{\tan x}}{\sin x \cos x}$$

$$35. \frac{(1+\log x)^2}{x}$$

$$36. \frac{(x+1)(x+\log x)^2}{x}$$

$$37. \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$$

```
>>> sin(x) / (1+cos(x))
(((1+cos(x))^-1)*sin(x))
>>> Sdx
THOUGHT PROCESS =
```

```
integrating (((1+cos(x))^-1)*sin(x)) wrt x
rewriting as (((1+cos(x))^-1)*sin(x))
rewriting as (((1+cos(x))^-1)*sin(x))
substituting (1+cos(x))=y
integrating (-1*(y^-1)) wrt y
rewriting as (-1*(y^-1))
taking the constant outside of the integral
integrating (y^-1) wrt y
rewriting as (y^-1)
rewriting as (y^-1)
the solution is (log(abs(y))*(1^-1))
the solution is (-1*(log(abs((1+cos(x))))*(1^-1)))
the solution is (-1*log(abs((1+cos(x)))))

(-1*log(abs((1+cos(x)))))
```

```
>>>
```

$$21. \tan^2(2x - 3)$$

$$22. \sec^2(7 - 4x)$$

$$23. \frac{\sin^{-1}x}{\sqrt{1-x^2}}$$

$$24. \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$$

$$25. \frac{1}{\cos^2 x (1 - \tan x)^2}$$

$$26. \frac{\cos \sqrt{x}}{\sqrt{x}}$$

$$27. \sqrt{\sin 2x} \cos 2x$$

$$28. \frac{\cos x}{\sqrt{1+\sin x}}$$

$$29. \cot x \log \sin x$$

$$30. \frac{\sin x}{1 + \cos x}$$

$$31. \frac{\sin x}{(1 + \cos x)^2}$$

$$32. \frac{1}{1 + \cot x}$$

$$33. \frac{1}{1 - \tan x}$$

$$34. \frac{\sqrt{\tan x}}{\sin x \cos x}$$

$$35. \frac{(1 + \log x)^2}{x}$$

$$36. \frac{(x+1)(x+\log x)^2}{x}$$

$$37. \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$$

```
>>> sin(x) / (1+cos(x))^2  
((1+cos(x))^-2)*sin(x))
```

```
>>> Sdx
```

```
THOUGHT PROCESS =
```

```
integrating (((1+cos(x))^-2)*sin(x)) wrt x
```

```
rewriting as (((1+cos(x))^-2)*sin(x))
```

```
rewriting as (((1+cos(x))^-2)*sin(x))
```

```
substituting (1+cos(x))=y
```

```
integrating (-1*(y^-2)) wrt y
```

```
rewriting as (y^-2)
```

```
rewriting as (y^-2)
```

```
taking the constant outside of the integral
```

```
integrating (y^-2) wrt y
```

```
rewriting as (y^-2)
```

```
rewriting as (y^-2)
```

```
the solution is ((y^-1)*((-1*1)^-1))
```

```
the solution is (-1*((1+cos(x))^-1)*((-1*1)^-1)))
```

```
the solution is ((1+cos(x))^-1)
```

```
((1+cos(x))^-1)
```

```
>>>
```

EXERCISE 7.3

Find the integrals of the functions in Exercises 1 to 22:

1. $\sin^2(2x+5)$

4. $\sin^3(2x+1)$

7. $\sin 4x \sin 8x$

10. $\sin^4 x$

13. $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$

16. $\tan^4 x$

19. $\frac{1}{\sin x \cos^3 x}$

22. $\frac{1}{\cos(x-a) \cos(x-b)}$

2. $\sin 3x \cos 4x$

5. $\sin^3 x \cos^3 x$

8. $\frac{1-\cos x}{1+\cos x}$

11. $\cos^4 2x$

14. $\frac{\cos x - \sin x}{1 + \sin 2x}$

17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

20. $\frac{\cos 2x}{(\cos x + \sin x)^2}$

3. $\cos 2x \cos 4x \cos 6x$

6. $\sin x \sin 2x \sin 3x$

9. $\frac{\cos x}{1 + \cos x}$

12. $\frac{\sin^2 x}{1 + \cos x}$

15. $\tan^3 2x \sec 2x$

18. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

21. $\sin^{-1}(\cos x)$

```
>>> sin(2*x+5)^2
(sin((5+(2*x)))^2)
>>> Sdx
```

THOUGHT PROCESS =

```
integrating (sin((5+(2*x)))^2) wrt x
rewriting as (sin((2*((5*(2^-1))+x)))^2)
rewriting as (4*(cos(((5*(2^-1))+x))^2)*(sin(((5*(2^-1))+x))^2))
rewriting as ((2^-1)+(-2^-1)*cos((10+(4*x)))))

integrating over sums
integrating (2^-1) wrt x
the solution is (x*(2^-1))

integrating over sums
integrating ((-2^-1)*cos((10+(4*x)))) wrt x
rewriting as ((-2^-1)*cos((4*((5*(2^-1))+x)))))

taking the constant outside of the integral
integrating cos((4*((5*(2^-1))+x))) wrt x
rewriting as cos((4*((5*(2^-1))+x)))
rewriting as cos((10+(4*x)))

the solution is (sin((4*((5*(2^-1))+x)))*(4^-1))

the solution is ((-2^-1)*(sin((4*((5*(2^-1))+x)))*(4^-1)))
the solution is (((-8^-1)*sin((10+(4*x))))+((2^-1)*x))

(((8^-1)*sin((10+(4*x))))+((2^-1)*x))
>>>
```

EXERCISE 7.3

Find the integrals of the functions in Exercises 1 to 22:

- | | | |
|-----------------------------------------------------------|-----------------------------------------------------|--------------------------------------------|
| 1. $\sin^2(2x + 5)$ | 2. $\sin 3x \cos 4x$ | 3. $\cos 2x \cos 4x \cos 6x$ |
| 4. $\sin^3(2x + 1)$ | 5. $\sin^3 x \cos^3 x$ | 6. $\sin x \sin 2x \sin 3x$ |
| 7. $\sin 4x \sin 8x$ | 8. $\frac{1 - \cos x}{1 + \cos x}$ | 9. $\frac{\cos x}{1 + \cos x}$ |
| 10. $\sin^4 x$ | 11. $\cos^4 2x$ | 12. $\frac{\sin^2 x}{1 + \cos x}$ |
| 13. $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$ | 14. $\frac{\cos x - \sin x}{1 + \sin 2x}$ | 15. $\tan^3 2x \sec 2x$ |
| 16. $\tan^4 x$ | 17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$ | 18. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$ |
| 19. $\frac{1}{\sin x \cos^3 x}$ | 20. $\frac{\cos 2x}{(\cos x + \sin x)^2}$ | 21. $\sin^{-1}(\cos x)$ |
| 22. $\frac{1}{\cos(x-a) \cos(x-b)}$ | | |

```
>>> sin(3*x)*cos(4*x)
(cos((4*x))*sin((3*x)))
```

```
>>> Sdx
```

THOUGHT PROCESS =

```

integrating (cos((4*x))*sin((3*x))) wrt x
rewriting as (cos((4*x))*sin((3*x)))
rewriting as (cos((4*x))*sin((3*x)))
rewriting as ((((-2^-1)*sin(x))+(2^-1)*sin((7*x)))) 
    integrating over sums
    integrating ((-2^-1)*sin(x)) wrt x
    rewriting as ((-2^-1)*sin(x))
        taking the constant outside of the integral
        integrating sin(x) wrt x
        rewriting as sin(x)
        rewriting as sin(x)
        the solution is ((-1*cos(x))*(1^-1))
    the solution is ((-2^-1)*((-1*cos(x))*(1^-1)))
    integrating over sums
    integrating ((2^-1)*sin((7*x))) wrt x
    rewriting as ((2^-1)*sin((7*x)))
        taking the constant outside of the integral
        integrating sin((7*x)) wrt x
        rewriting as sin((7*x))
        rewriting as sin((7*x))
        the solution is ((-1*cos((7*x)))*(7^-1))
    the solution is ((2^-1)*((-1*cos((7*x)))*(7^-1)))
    the solution is ((((-14^-1)*cos((7*x)))+(2^-1)*cos(x)))
        ((((-14^-1)*cos((7*x)))+(2^-1)*cos(x)))
>>>
```

EXERCISE 7.3

Find the integrals of the functions in Exercises 1 to 22:

- | | | |
|-----------------------------------------------------------|-----------------------------------------------------|--------------------------------------------|
| 1. $\sin^2(2x + 5)$ | 2. $\sin 3x \cos 4x$ | 3. $\cos 2x \cos 4x \cos 6x$ |
| 4. $\sin^3(2x + 1)$ | 5. $\sin^3 x \cos^3 x$ | 6. $\sin x \sin 2x \sin 3x$ |
| 7. $\sin 4x \sin 8x$ | 8. $\frac{1 - \cos x}{1 + \cos x}$ | 9. $\frac{\cos x}{1 + \cos x}$ |
| 10. $\sin^4 x$ | 11. $\cos^4 2x$ | 12. $\frac{\sin^2 x}{1 + \cos x}$ |
| 13. $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$ | 14. $\frac{\cos x - \sin x}{1 + \sin 2x}$ | 15. $\tan^3 2x \sec 2x$ |
| 16. $\tan^4 x$ | 17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$ | 18. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$ |
| 19. $\frac{1}{\sin x \cos^3 x}$ | 20. $\frac{\cos 2x}{(\cos x + \sin x)^2}$ | 21. $\sin^{-1}(\cos x)$ |
| 22. $\frac{1}{\cos(x-a) \cos(x-b)}$ | | |

```
>>> cos(2*x)*cos(4*x)*cos(6*x)
(cos((2*x))*cos((4*x))*cos((6*x)))
>>> Sdx
```

THOUGHT PROCESS =

```

integrating (cos((2*x))*cos((4*x))*cos((6*x))) wrt x
rewriting as (cos((2*x))*cos((4*x))*cos((6*x)))
rewriting as ((-
1*cos((4*x))*cos((6*x))* (sin(x)^2))+(cos((4*x))*cos((6*x))* (cos(x)^2)))
integrating over sums
integrating (-1*cos((4*x))*cos((6*x))* (sin(x)^2)) wrt x
rewriting as (-1*cos((4*x))*cos((6*x))* (sin(x)^2))
taking the constant outside of the integral
integrating (cos((4*x))*cos((6*x))* (sin(x)^2)) wrt x
rewriting as (cos((4*x))*cos((6*x))* (sin(x)^2))
rewriting as (cos((4*x))*cos((6*x))* (sin(x)^2))
rewriting as ((-8^-1)+((-8^-1)*cos((12*x)))+((-8^-
1)*cos((4*x)))+((-8^-1)*cos((8*x)))+((4^-1)*cos((10*x)))+((4^-
1)*cos((2*x))))
integrating over sums
integrating (-8^-1) wrt x
the solution is (x*(-8^-1))
integrating over sums
integrating ((-8^-1)*cos((12*x))) wrt x
rewriting as ((-8^-1)*cos((12*x)))
taking the constant outside of the integral
integrating cos((12*x)) wrt x
rewriting as cos((12*x))
rewriting as cos((12*x))
the solution is (sin((12*x))*(12^-1))
the solution is ((-8^-1)*(sin((12*x))*(12^-1)))

```

```

integrating over sums
integrating ((-8^-1)*cos((4*x))) wrt x
rewriting as ((-8^-1)*cos((4*x)))
    taking the constant outside of the integral
    integrating cos((4*x)) wrt x
    rewriting as cos((4*x))
    rewriting as cos((4*x))
    the solution is (sin((4*x))*(4^-1))
the solution is ((-8^-1)*(sin((4*x))*(4^-1)))
integrating over sums
integrating ((-8^-1)*cos((8*x))) wrt x
rewriting as ((-8^-1)*cos((8*x)))
    taking the constant outside of the integral
    integrating cos((8*x)) wrt x
    rewriting as cos((8*x))
    rewriting as cos((8*x))
    the solution is (sin((8*x))*(8^-1))
the solution is ((-8^-1)*(sin((8*x))*(8^-1)))
integrating over sums
integrating ((4^-1)*cos((10*x))) wrt x
rewriting as ((4^-1)*cos((10*x)))
    taking the constant outside of the integral
    integrating cos((10*x)) wrt x
    rewriting as cos((10*x))
    rewriting as cos((10*x))
    the solution is (sin((10*x))*(10^-1))
the solution is ((4^-1)*(sin((10*x))*(10^-1)))
integrating over sums
integrating ((4^-1)*cos((2*x))) wrt x
rewriting as ((4^-1)*cos((2*x)))
    taking the constant outside of the integral
    integrating cos((2*x)) wrt x
    rewriting as cos((2*x))
    rewriting as ((-1*(sin(x)^2))+(cos(x)^2))
        integrating over sums
        the solution is not found
        rewriting as cos((2*x))
        the solution is (sin((2*x))*(2^-1))
        the solution is ((4^-1)*(sin((2*x))*(2^-1)))
the solution is (((((x*(-8^-1))+((-8^-1)*(sin((12*x))*(12^-1))))+((-8^-1)*(sin((4*x))*(4^-1))))+((-8^-1)*(sin((8*x))*(8^-1))))+((4^-1)*(sin((10*x))*(10^-1))))+((4^-1)*(sin((2*x))*(2^-1))))
    the solution is (-1*(((((x*(-8^-1))+((-8^-1)*(sin((12*x))*(12^-1))))+((-8^-1)*(sin((4*x))*(4^-1))))+((-8^-1)*(sin((8*x))*(8^-1))))+((4^-1)*(sin((10*x))*(10^-1))))+((4^-1)*(sin((2*x))*(2^-1))))) )
integrating over sums
integrating (cos((4*x))*cos((6*x))*(cos(x)^2)) wrt x
rewriting as (cos((4*x))*cos((6*x))*(cos(x)^2))
rewriting as (cos((4*x))*cos((6*x))*(cos(x)^2))
rewriting as ((8^-1)+((4^-1)*cos((10*x)))+((4^-1)*cos((2*x)))+((8^-1)*cos((12*x)))+((8^-1)*cos((4*x)))+((8^-1)*cos((8*x)))+
    integrating over sums
    integrating (8^-1) wrt x
    the solution is (x*(8^-1))
integrating over sums

```

```

integrating ((4^-1)*cos((10*x))) wrt x
rewriting as ((4^-1)*cos((10*x)))
    taking the constant outside of the integral
    integrating cos((10*x)) wrt x
    rewriting as cos((10*x))
    rewriting as cos((10*x))
    the solution is (sin((10*x))*(10^-1))
the solution is ((4^-1)*(sin((10*x))*(10^-1)))
integating over sums
integrating ((4^-1)*cos((2*x))) wrt x
rewriting as ((4^-1)*cos((2*x)))
    taking the constant outside of the integral
    integrating cos((2*x)) wrt x
    rewriting as cos((2*x))
    rewriting as ((-1*(sin(x)^2))+(cos(x)^2))
        integating over sums
        integrating (-1*(sin(x)^2)) wrt x
        rewriting as (-1*(sin(x)^2))
            taking the constant outside of the integral
            the solution is not found
            rewriting as (-1*(sin(x)^2))
            rewriting as ((-2^-1)+((2^-1)*cos((2*x))))
                integating over sums
                the solution is not found
                substituting sin(x)=y
                the solution is not found
                performing integration by parts f(x)=-1 and
g(x)=(sin(x)^2)
                the solution is not found
                performing integration by parts f(x)=(sin(x)^2) and g(x)=-1
the solution is not found
the solution is not found
rewriting as cos((2*x))
the solution is (sin((2*x))*(2^-1))
the solution is ((4^-1)*(sin((2*x))*(2^-1)))
integating over sums
integrating ((8^-1)*cos((12*x))) wrt x
rewriting as ((8^-1)*cos((12*x)))
    taking the constant outside of the integral
    integrating cos((12*x)) wrt x
    rewriting as cos((12*x))
    rewriting as cos((12*x))
    the solution is (sin((12*x))*(12^-1))
the solution is ((8^-1)*(sin((12*x))*(12^-1)))
integating over sums
integrating ((8^-1)*cos((4*x))) wrt x
rewriting as ((8^-1)*cos((4*x)))
    taking the constant outside of the integral
    integrating cos((4*x)) wrt x
    rewriting as cos((4*x))
    rewriting as cos((4*x))
    the solution is (sin((4*x))*(4^-1))
the solution is ((8^-1)*(sin((4*x))*(4^-1)))
integating over sums
integrating ((8^-1)*cos((8*x))) wrt x

```

```
rewriting as ((8^-1)*cos((8*x)))
taking the constant outside of the integral
integrating cos((8*x)) wrt x
rewriting as cos((8*x))
rewriting as cos((8*x))
the solution is (sin((8*x))*(8^-1))
the solution is ((8^-1)*(sin((8*x))*(8^-1)))
the solution is (((((x*(8^-1))+((4^-1)*(sin((10*x))*(10^-
1))))+((4^-1)*(sin((2*x))*(2^-1))))+((8^-1)*(sin((12*x))*(12^-
1))))+((8^-1)*(sin((4*x))*(4^-1))))+((8^-1)*(sin((8*x))*(8^-1))))
the solution is (((16^-1)*sin((4*x)))+((32^-1)*sin((8*x)))+((4^-
1)*x)+((48^-1)*sin((12*x))))+((16^-1)*sin((4*x)))+((32^-1)*sin((8*x)))+((4^-
1)*x)+((48^-1)*sin((12*x))))
```

EXERCISE 7.3

Find the integrals of the functions in Exercises 1 to 22:

$$1. \sin^2(2x + 5)$$

4. $\sin^3(2x + 1)$

7. $\sin 4x \sin 8x$

10. $\sin^4 x$

13. $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$

16. $\tan^4 x$

$$19. \frac{1}{\sin x \cos^3 x}$$

22. $\frac{1}{\cos(x-a)\cos(x-b)}$

```
>>> sin(2*x+1)^3  
(sin((1+(2*x)))^3)
```

>>> Sax
THOUGHT PROCESS =

THOUGHT PROCESS -

```

integrating (sin((1+(2*x)))^3) wrt x
rewriting as (sin((2*((2^(-1)+x)))^3)
rewriting as (8*(cos(((2^(-1)+x))^3)*(sin(((2^(-1)+x))^3))
rewriting as (((-4^(-1))*sin((3+(6*x))))+(3*(4^(-1))*sin((1+(2*x)))))

integating over sums
integrating ((-4^(-1))*sin((3+(6*x)))) wrt x
rewriting as ((-4^(-1))*sin((6*((2^(-1)+x)))))

taking the constant outside of the integral
integrating sin((6*((2^(-1)+x))) wrt x
rewriting as sin((6*((2^(-1)+x)))
rewriting as sin((3+(6*x)))

```

```

the solution is ((-1*cos((6*((2^(-1)+x))))*(6^(-1)))
the solution is ((-4^(-1))*((-1*cos((6*((2^(-1)+x))))*(6^(-1))))
integrating over sums
integrating (3*(4^(-1))*sin((1+(2*x)))) wrt x
rewriting as (3*(4^(-1))*sin((2*((2^(-1)+x)))))

taking the constant outside of the integral
integrating sin((2*((2^(-1)+x))) wrt x
rewriting as sin((2*((2^(-1)+x)))
rewriting as (2*cos(((2^(-1)+x)))*sin(((2^(-1)+x)))
rewriting as sin((1+(2*x)))

the solution is ((-1*cos((2*((2^(-1)+x))))*(2^(-1)))
the solution is ((3*(4^(-1)))*((-1*cos((2*((2^(-1)+x))))*(2^(-1)))
the solution is ((-3*(8^(-1))*cos((1+(2*x))))+((24^(-1))*cos((3+(6*x)))))

((-3*(8^(-1))*cos((1+(2*x))))+((24^(-1))*cos((3+(6*x)))))

>>>

```

EXERCISE 7.3

Find the integrals of the functions in Exercises 1 to 22:

- | | | |
|-----------------------------------------------------------|-----------------------------------------------------|--------------------------------------------|
| 1. $\sin^2(2x + 5)$ | 2. $\sin 3x \cos 4x$ | 3. $\cos 2x \cos 4x \cos 6x$ |
| 4. $\sin^3(2x + 1)$ | 5. $\sin^3 x \cos^3 x$ | 6. $\sin x \sin 2x \sin 3x$ |
| 7. $\sin 4x \sin 8x$ | 8. $\frac{1 - \cos x}{1 + \cos x}$ | 9. $\frac{\cos x}{1 + \cos x}$ |
| 10. $\sin^4 x$ | 11. $\cos^4 2x$ | 12. $\frac{\sin^2 x}{1 + \cos x}$ |
| 13. $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$ | 14. $\frac{\cos x - \sin x}{1 + \sin 2x}$ | 15. $\tan^3 2x \sec 2x$ |
| 16. $\tan^4 x$ | 17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$ | 18. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$ |
| 19. $\frac{1}{\sin x \cos^3 x}$ | 20. $\frac{\cos 2x}{(\cos x + \sin x)^2}$ | 21. $\sin^{-1}(\cos x)$ |
| 22. $\frac{1}{\cos(x-a) \cos(x-b)}$ | | |

```

>>> sin(x)^3*cos(x)^3
((cos(x)^3)*(sin(x)^3))
>>> Sdx

```

THOUGHT PROCESS =

```

integrating ((cos(x)^3)*(sin(x)^3)) wrt x
rewriting as ((cos(x)^3)*(sin(x)^3))
rewriting as ((cos(x)^3)*(sin(x)^3))
rewriting as (((-32^(-1))*sin((6*x)))+(3*(32^(-1))*sin((2*x)))))

integrating over sums
integrating ((-32^(-1))*sin((6*x))) wrt x
rewriting as ((-32^(-1))*sin((6*x)))
taking the constant outside of the integral
integrating sin((6*x)) wrt x
rewriting as sin((6*x))

```

```

rewriting as sin((6*x))
the solution is ((-1*cos((6*x)))*(6^-1))
the solution is ((-32^-1)*((-1*cos((6*x)))*(6^-1)))
integrating over sums
integrating (3*(32^-1)*sin((2*x))) wrt x
rewriting as (3*(32^-1)*sin((2*x)))
taking the constant outside of the integral
integrating sin((2*x)) wrt x
rewriting as sin((2*x))
rewriting as (2*cos(x)*sin(x))
rewriting as sin((2*x))
the solution is ((-1*cos((2*x)))*(2^-1))
the solution is ((3*(32^-1))*((-1*cos((2*x)))*(2^-1)))
the solution is ((-3*(64^-1)*cos((2*x)))+((192^-1)*cos((6*x))))
((-3*(64^-1)*cos((2*x)))+((192^-1)*cos((6*x))))
```

>>>

EXERCISE 7.3

Find the integrals of the functions in Exercises 1 to 22:

- | | | |
|-----------------------------------------------------------|-----------------------------------------------------|--------------------------------------------|
| 1. $\sin^2(2x + 5)$ | 2. $\sin 3x \cos 4x$ | 3. $\cos 2x \cos 4x \cos 6x$ |
| 4. $\sin^3(2x + 1)$ | 5. $\sin^3 x \cos^3 x$ | 6. $\sin x \sin 2x \sin 3x$ |
| 7. $\sin 4x \sin 8x$ | 8. $\frac{1 - \cos x}{1 + \cos x}$ | 9. $\frac{\cos x}{1 + \cos x}$ |
| 10. $\sin^4 x$ | 11. $\cos^4 2x$ | 12. $\frac{\sin^2 x}{1 + \cos x}$ |
| 13. $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$ | 14. $\frac{\cos x - \sin x}{1 + \sin 2x}$ | 15. $\tan^3 2x \sec 2x$ |
| 16. $\tan^4 x$ | 17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$ | 18. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$ |
| 19. $\frac{1}{\sin x \cos^3 x}$ | 20. $\frac{\cos 2x}{(\cos x + \sin x)^2}$ | 21. $\sin^{-1}(\cos x)$ |
| 22. $\frac{1}{\cos(x-a) \cos(x-b)}$ | | |

>>> sin(x)*sin(2*x)*sin(3*x)
 $(\sin((2*x))*\sin((3*x))*\sin(x))$
>>> Sdx

THOUGHT PROCESS =

```

integrating (sin((2*x))*sin((3*x))*sin(x)) wrt x
rewriting as (sin((2*x))*sin((3*x))*sin(x))
rewriting as (2*cos(x)*(sin(x)^2)*sin((3*x)))
rewriting as (((-4^-1)*sin((6*x)))+((4^-1)*sin((2*x)))+((4^-1)*sin((4*x))))
integrating over sums
integrating ((-4^-1)*sin((6*x))) wrt x
rewriting as ((-4^-1)*sin((6*x)))
taking the constant outside of the integral
integrating sin((6*x)) wrt x
```

```

rewriting as sin((6*x))
rewriting as sin((6*x))
the solution is ((-1*cos((6*x)))*(6^-1))
the solution is ((-4^-1)*((-1*cos((6*x)))*(6^-1)))
integating over sums
integrating ((4^-1)*sin((2*x))) wrt x
rewriting as ((4^-1)*sin((2*x)))
taking the constant outside of the integral
integrating sin((2*x)) wrt x
rewriting as sin((2*x))
rewriting as (2*cos(x)*sin(x))
rewriting as sin((2*x))
the solution is ((-1*cos((2*x)))*(2^-1))
the solution is ((4^-1)*((-1*cos((2*x)))*(2^-1)))
integating over sums
integrating ((4^-1)*sin((4*x))) wrt x
rewriting as ((4^-1)*sin((4*x)))
taking the constant outside of the integral
integrating sin((4*x)) wrt x
rewriting as sin((4*x))
rewriting as sin((4*x))
the solution is ((-1*cos((4*x)))*(4^-1))
the solution is ((4^-1)*((-1*cos((4*x)))*(4^-1)))
the solution is ((((-16^-1)*cos((4*x)))+((-8^-1)*cos((2*x)))+((24^-1)*cos((6*x))))+
((( -16^-1)*cos((4*x)))+((-8^-1)*cos((2*x)))+((24^-1)*cos((6*x)))))


```

EXERCISE 7.3

Find the integrals of the functions in Exercises 1 to 22:

1. $\sin^2(2x + 5)$
 2. $\sin 3x \cos 4x$
 3. $\cos 2x \cos 4x \cos 6x$
 4. $\sin^3(2x + 1)$
 5. $\sin^3 x \cos^3 x$
 6. $\sin x \sin 2x \sin 3x$
 - 7. $\sin 4x \sin 8x$**
 8. $\frac{1 - \cos x}{1 + \cos x}$
 9. $\frac{\cos x}{1 + \cos x}$
 10. $\sin^4 x$
 11. $\cos^4 2x$
 12. $\frac{\sin^2 x}{1 + \cos x}$
 13. $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$
 14. $\frac{\cos x - \sin x}{1 + \sin 2x}$
 15. $\tan^3 2x \sec 2x$
 16. $\tan^4 x$
 17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$
 18. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$
 19. $\frac{1}{\sin x \cos^3 x}$
 20. $\frac{\cos 2x}{(\cos x + \sin x)^2}$
 21. $\sin^{-1}(\cos x)$
 22. $\frac{1}{\cos(x-a)\cos(x-b)}$

```
>>> sin(4*x)*sin(8*x)
(sin((4*x))*sin((8*x)))
>>> Sdx
```

THOUGHT PROCESS =

```

integrating (sin((4*x))*sin((8*x))) wrt x
rewriting as (sin((4*x))*sin((8*x)))
rewriting as (sin((4*x))*sin((8*x)))
rewriting as (((-2^-1)*cos((12*x)))+((2^-1)*cos((-4*x)))))

integating over sums
integrating ((-2^-1)*cos((12*x))) wrt x
rewriting as ((-2^-1)*cos((12*x)))
taking the constant outside of the integral
integrating cos((12*x)) wrt x
rewriting as cos((12*x))
rewriting as cos((12*x))
the solution is (sin((12*x))*(12^-1))
the solution is ((-2^-1)*(sin((12*x))*(12^-1)))
integating over sums
integrating ((2^-1)*cos((-4*x))) wrt x
rewriting as ((2^-1)*cos((-4*x)))
taking the constant outside of the integral
integrating cos((-4*x)) wrt x
rewriting as cos((-4*x))
rewriting as cos((-4*x))
rewriting as cos((4*x))
the solution is (sin((-4*x))*(-4^-1))
the solution is ((2^-1)*(sin((-4*x))*(-4^-1)))
the solution is (((-24^-1)*sin((12*x)))+((-8^-1)*sin((-4*x)))))

((( -24^-1)*sin((12*x)))+((-8^-1)*sin((-4*x))))
```

>>>

EXERCISE 7.3

Find the integrals of the functions in Exercises 1 to 22:

1. $\sin^2(2x + 5)$ 2. $\sin 3x \cos 4x$ 3. $\cos 2x \cos 4x \cos 6x$

4. $\sin^3(2x + 1)$ 5. $\sin^3 x \cos^3 x$ 6. $\sin x \sin 2x \sin 3x$

7. $\sin 4x \sin 8x$ 8. $\frac{1 - \cos x}{1 + \cos x}$ 9. $\frac{\cos x}{1 + \cos x}$

10. $\sin^4 x$ 11. $\cos^4 2x$ 12. $\frac{\sin^2 x}{1 + \cos x}$

13. $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$ 14. $\frac{\cos x - \sin x}{1 + \sin 2x}$ 15. $\tan^3 2x \sec 2x$

16. $\tan^4 x$ 17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$ 18. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

19. $\frac{1}{\sin x \cos^3 x}$ 20. $\frac{\cos 2x}{(\cos x + \sin x)^2}$ 21. $\sin^{-1}(\cos x)$

22. $\frac{1}{\cos(x - a) \cos(x - b)}$

>>> sin(x)^4

(sin(x)^4)

>>> Sdx

THOUGHT PROCESS =

```

integrating (sin(x)^4) wrt x
rewriting as (sin(x)^4)
rewriting as (sin(x)^4)
rewriting as ((3*(8^-1))+((-2^-1)*cos((2*x)))+((8^-1)*cos((4*x))))
    integating over sums
integrating (3*(8^-1)) wrt x
the solution is (x*(3*(8^-1)))
integating over sums
integrating ((-2^-1)*cos((2*x))) wrt x
rewriting as ((-2^-1)*cos((2*x)))
    taking the constant outside of the integral
integrating cos((2*x)) wrt x
rewriting as cos((2*x))
rewriting as ((-1*(sin(x)^2))+(cos(x)^2))
    integating over sums
integrating (-1*(sin(x)^2)) wrt x
rewriting as (-1*(sin(x)^2))
    taking the constant outside of the integral
integrating (sin(x)^2) wrt x
rewriting as (sin(x)^2)
rewriting as (sin(x)^2)
rewriting as ((2^-1)+((-2^-1)*cos((2*x))))
    integating over sums
    the solution is not found
    substituting sin(x)=y
    the solution is not found
    the solution is not found
    rewriting as (-1*(sin(x)^2))
    rewriting as ((-2^-1)+((2^-1)*cos((2*x))))
        integating over sums
        integrating (-2^-1) wrt x
        the solution is (x*(-2^-1))
        integating over sums
        integrating ((2^-1)*cos((2*x))) wrt x
        rewriting as ((2^-1)*cos((2*x)))
            taking the constant outside of the integral
            the solution is not found
            rewriting as (((-2^-1)*(sin(x)^2))+((2^-1)*(cos(x)^2)))
                integating over sums
                the solution is not found
                rewriting as ((2^-1)*cos((2*x)))
                    substituting (2*x)=y
                    the solution is not found
                    performing integration by parts f(x)=(2^-1) and
g(x)=cos((2*x))
                    the solution is not found
                    performing integration by parts f(x)=cos((2*x)) and
g(x)=(2^-1)
                    the solution is not found
                    the solution is not found
                    substituting sin(x)=y
                    integrating (-1*((1+(-1*(y^2)))^(-2^-1))*(y^2)) wrt y
                    rewriting as (((-1+y)^(-2^-1))*((1+y)^(-2^-1))*(y^2))
                    rewriting as (((-1+y)^(-2^-1))*((1+y)^(-2^-1))*(y^2))
                    substituting (1+y)=z

```

```

the solution is not found
substituting (-1+y)=z
the solution is not found
the solution is not found
performing integration by parts f(x)=-1 and g(x)=(sin(x)^2)
integrating (sin(x)^2) wrt x
rewriting as (sin(x)^2)
rewriting as (sin(x)^2)
rewriting as ((2^-1)+((-2^-1)*cos((2*x)) ))
integrating over sums
the solution is not found
substituting sin(x)=y
the solution is not found
the solution is not found
performing integration by parts f(x)=(sin(x)^2) and g(x)=-1
integrating -1 wrt x
integrating ((2*cos(x)*sin(x))*(x^-1)) wrt x
rewriting as (-2*cos(x)*sin(x)*x)
taking the constant outside of the integral
the solution is not found
rewriting as (-2*cos(x)*sin(x)*x)
substituting cos(x)=y
the solution is not found
substituting cos(x)=y
the solution is not found
rewriting as cos((2*x))
the solution is (sin((2*x))*(2^-1))
the solution is ((-2^-1)*(sin((2*x))*(2^-1)))
integrating over sums
integrating ((8^-1)*cos((4*x))) wrt x
rewriting as ((8^-1)*cos((4*x)))
taking the constant outside of the integral
integrating cos((4*x)) wrt x
rewriting as cos((4*x))
rewriting as cos((4*x))
the solution is (sin((4*x))*(4^-1))
the solution is ((8^-1)*(sin((4*x))*(4^-1)))
the solution is (((-4^-1)*sin((2*x)))+((32^-1)*sin((4*x)))+(3*(8^-1)*x))

((( -4^-1)*sin((2*x)))+((32^-1)*sin((4*x)))+(3*(8^-1)*x))
>>>

```

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2-9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2+3x+2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2+1)(x-1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3-x^2-x+1}$

10. $\frac{2x-3}{(x^2-1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2-4)}$

12. $\frac{x^3+x+1}{x^2-1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4-1}$

```
>>> x / ((x+1) * (x+2))
((1+x)^-1) * ((2+x)^-1) * x
>>> Sdx
```

THOUGHT PROCESS =

```
integrating (((1+x)^-1) * ((2+x)^-1) * x) wrt x
rewriting as ((-1*((1+x)^-1)) + (2*((2+x)^-1)))
rewriting as ((-1*((1+x)^-1)) + (2*((2+x)^-1)))
  integrating over sums
  integrating (-1*((1+x)^-1)) wrt x
  rewriting as (-1*((1+x)^-1))
  taking the constant outside of the integral
  integrating ((1+x)^-1) wrt x
  rewriting as ((1+x)^-1)
  rewriting as ((1+x)^-1)
  the solution is (log(abs((1+x))) * (1^-1))
the solution is (-1 * (log(abs((1+x))) * (1^-1)))
  integrating over sums
  integrating (2*((2+x)^-1)) wrt x
  rewriting as (2*((2+x)^-1))
  taking the constant outside of the integral
  integrating ((2+x)^-1) wrt x
  rewriting as ((2+x)^-1)
  rewriting as ((2+x)^-1)
  the solution is (log(abs((2+x))) * (1^-1))
the solution is (2 * (log(abs((2+x))) * (1^-1)))
the solution is ((-1 * log(abs((1+x)))) + (2 * log(abs((2+x)))))

((-1 * log(abs((1+x)))) + (2 * log(abs((2+x)))))
```

>>>

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2-9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2+3x+2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2+1)(x-1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3-x^2-x+1}$

10. $\frac{2x-3}{(x^2-1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2-4)}$

12. $\frac{x^3+x+1}{x^2-1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4-1}$

```
>>> 1 / (x^2-9)
```

```
((-9+(x^2))^ -1)
```

```
>>> factor
```

```
(((-3+x)*(3+x))^ -1)
```

```
>>> Sdx
```

THOUGHT PROCESS =

integrating $(((-3+x)*(3+x))^ -1)$ wrt x

rewriting as $(((-6^{-1})*((3+x)^{-1})) + ((6^{-1})*((-3+x)^{-1})))$

rewriting as $(((-6^{-1})*((3+x)^{-1})) + ((6^{-1})*((-3+x)^{-1})))$

integrating over sums

integrating $((-6^{-1})*((3+x)^{-1}))$ wrt x

rewriting as $((-6^{-1})*((3+x)^{-1}))$

taking the constant outside of the integral

integrating $((3+x)^{-1})$ wrt x

rewriting as $((3+x)^{-1})$

rewriting as $((3+x)^{-1})$

the solution is $(\log(\text{abs}((3+x))) * (1^{-1}))$

the solution is $((-6^{-1}) * (\log(\text{abs}((3+x))) * (1^{-1})))$

integrating over sums

integrating $((6^{-1})*((-3+x)^{-1}))$ wrt x

rewriting as $((6^{-1})*((-3+x)^{-1}))$

taking the constant outside of the integral

integrating $((-3+x)^{-1})$ wrt x

rewriting as $((-3+x)^{-1})$

rewriting as $((-3+x)^{-1})$

the solution is $(\log(\text{abs}((-3+x))) * (1^{-1}))$

the solution is $((6^{-1}) * (\log(\text{abs}((-3+x))) * (1^{-1})))$

the solution is $(((-6^{-1}) * \log(\text{abs}((3+x)))) + ((6^{-1}) * \log(\text{abs}((-3+x)))))$

```
(((-6^{-1}) * log(abs((3+x)))) + ((6^{-1}) * log(abs((-3+x)))))
```

```
>>>
```

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2 - 9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2 + 3x + 2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2 + 1)(x-1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3 - x^2 - x + 1}$

10. $\frac{2x-3}{(x^2 - 1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2 - 4)}$

12. $\frac{x^3 + x + 1}{x^2 - 1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4 - 1}$

```
>>> 3*x/((x-1)*(x-2)*(x-3))
(3*(-1+x)^-1)*((-2+x)^-1)*((-3+x)^-1)*x
>>> Sdx
```

THOUGHT PROCESS =

integrating $(3*(-1+x)^{-1}) * ((-2+x)^{-1}) * ((-3+x)^{-1}) * x$ wrt x
 rewriting as $(3*(-2*((-2+x)^{-1})) + ((2^{-1}) * ((-1+x)^{-1})) + (3*(2^{-1}) * ((-3+x)^{-1})))$

taking the constant outside of the integral

integrating $((-2*((-2+x)^{-1})) + ((2^{-1}) * ((-1+x)^{-1})) + (3*(2^{-1}) * ((-3+x)^{-1})))$ wrt x

rewriting as $((-2*((-2+x)^{-1})) + ((2^{-1}) * ((-1+x)^{-1})) + (3*(2^{-1}) * ((-3+x)^{-1})))$

rewriting as $((-2*((-2+x)^{-1})) + ((2^{-1}) * ((-1+x)^{-1})) + (3*(2^{-1}) * ((-3+x)^{-1})))$

integrating over sums

integrating $(-2*((-2+x)^{-1}))$ wrt x

rewriting as $(-2*((-2+x)^{-1}))$

taking the constant outside of the integral

integrating $((-2+x)^{-1})$ wrt x

rewriting as $((-2+x)^{-1})$

rewriting as $((-2+x)^{-1})$

the solution is $(\log(\text{abs}((-2+x)))) * (1^{-1}))$

the solution is $(-2 * (\log(\text{abs}((-2+x)))) * (1^{-1})))$

integrating over sums

integrating $((2^{-1}) * ((-1+x)^{-1}))$ wrt x

rewriting as $((2^{-1}) * ((-1+x)^{-1}))$

taking the constant outside of the integral

integrating $((-1+x)^{-1})$ wrt x

rewriting as $((-1+x)^{-1})$

rewriting as $((-1+x)^{-1})$

the solution is $(\log(\text{abs}((-1+x)))) * (1^{-1}))$

the solution is $((2^{-1}) * (\log(\text{abs}((-1+x)))) * (1^{-1})))$

integrating over sums

integrating $(3*(2^{-1}) * ((-3+x)^{-1}))$ wrt x

rewriting as $(3*(2^{-1}) * ((-3+x)^{-1}))$

taking the constant outside of the integral
 integrating $((-3+x)^{-1})$ wrt x
 rewriting as $((-3+x)^{-1})$
 rewriting as $((-3+x)^{-1})$
 the solution is $(\log(\text{abs}((-3+x))) * (1^{-1}))$
 the solution is $((3 * (2^{-1})) * (\log(\text{abs}((-3+x))) * (1^{-1})))$
 the solution is $(((-2 * (\log(\text{abs}((-2+x))) * (1^{-1}))) + ((2^{^-1}) * (\log(\text{abs}((-1+x))) * (1^{-1})))) + ((3 * (2^{-1})) * (\log(\text{abs}((-3+x))) * (1^{-1}))))$
 the solution is $(((-6 * \log(\text{abs}((-2+x)))) + (3 * (2^{-1}) * \log(\text{abs}((-1+x)))) + (9 * (2^{-1}) * \log(\text{abs}((-3+x)))))$
 $(((-6 * \log(\text{abs}((-2+x)))) + (3 * (2^{-1}) * \log(\text{abs}((-1+x)))) + (9 * (2^{-1}) * \log(\text{abs}((-3+x)))))$
 >>>

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2 - 9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2 + 3x + 2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2 + 1)(x - 1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3 - x^2 - x + 1}$

10. $\frac{2x-3}{(x^2 - 1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2 - 4)}$

12. $\frac{x^3 + x + 1}{x^2 - 1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4 - 1}$

>>> $x / ((x-1) * (x-2) * (x-3))$

$(((-1+x)^{-1}) * ((-2+x)^{-1}) * ((-3+x)^{-1}) * x)$

>>> Sdx

THOUGHT PROCESS =

integrating $(((-1+x)^{-1}) * ((-2+x)^{-1}) * ((-3+x)^{-1}) * x)$ wrt x
 rewriting as $((-2 * ((-2+x)^{-1})) + ((2^{-1}) * ((-1+x)^{-1})) + (3 * (2^{-1}) * ((-3+x)^{-1})))$
 rewriting as $(((-2 * ((-2+x)^{-1})) + ((2^{-1}) * ((-1+x)^{-1})) + (3 * (2^{-1}) * ((-3+x)^{-1})))$
 integrating over sums
 integrating $(-2 * ((-2+x)^{-1}))$ wrt x
 rewriting as $(-2 * ((-2+x)^{-1}))$
 taking the constant outside of the integral
 integrating $((-2+x)^{-1})$ wrt x
 rewriting as $((-2+x)^{-1})$
 rewriting as $((-2+x)^{-1})$
 the solution is $(\log(\text{abs}((-2+x))) * (1^{-1}))$
 the solution is $(-2 * (\log(\text{abs}((-2+x))) * (1^{-1})))$
 integrating over sums
 integrating $((2^{-1}) * ((-1+x)^{-1}))$ wrt x

```

rewriting as ((2^-1) * ((-1+x)^-1))
taking the constant outside of the integral
integrating ((-1+x)^-1) wrt x
rewriting as ((-1+x)^-1)
rewriting as ((-1+x)^-1)
the solution is (log(abs((-1+x))) * (1^-1))
the solution is ((2^-1) * (log(abs((-1+x))) * (1^-1)))
integating over sums
integrating (3*(2^-1) * ((-3+x)^-1)) wrt x
rewriting as (3*(2^-1) * ((-3+x)^-1))
taking the constant outside of the integral
integrating ((-3+x)^-1) wrt x
rewriting as ((-3+x)^-1)
rewriting as ((-3+x)^-1)
the solution is (log(abs((-3+x))) * (1^-1))
the solution is ((3*(2^-1)) * (log(abs((-3+x))) * (1^-1)))
the solution is ((-2*log(abs((-2+x)))) + ((2^-1)*log(abs((-1+x)))) + (3*(2^-1)*log(abs((-3+x)))))
((-2*log(abs((-2+x)))) + ((2^-1)*log(abs((-1+x)))) + (3*(2^-1)*log(abs((-3+x)))))

>>>

```

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2 - 9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2 + 3x + 2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2 + 1)(x - 1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3 - x^2 - x + 1}$

10. $\frac{2x-3}{(x^2 - 1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2 - 4)}$

12. $\frac{x^3 + x + 1}{x^2 - 1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4 - 1}$

```

>>> 2*x/(x^2+3*x+2)
(2* ((2+(3*x)+(x^2))^-1) *x)
>>> Sdx

```

THOUGHT PROCESS =

```

integrating (2*((2+(3*x)+(x^2))^-1)*x) wrt x
rewriting as (2*((-1*((1+x)^-1)) + (2*((2+x)^-1))))
taking the constant outside of the integral
integrating ((-1*((1+x)^-1)) + (2*((2+x)^-1))) wrt x
rewriting as ((-1*((1+x)^-1)) + (2*((2+x)^-1)))
rewriting as ((-1*((1+x)^-1)) + (2*((2+x)^-1)))
integrating over sums
integrating (-1*((1+x)^-1)) wrt x
rewriting as (-1*((1+x)^-1))

```

taking the constant outside of the integral
 integrating $((1+x)^{-1})$ wrt x
 rewriting as $((1+x)^{-1})$
 rewriting as $((1+x)^{-1})$
 the solution is $(\log(\text{abs}((1+x))) * (1^{-1}))$
 the solution is $(-1 * (\log(\text{abs}((1+x))) * (1^{-1})))$
 integating over sums
 integrating $(2 * ((2+x)^{-1}))$ wrt x
 rewriting as $(2 * ((2+x)^{-1}))$
 taking the constant outside of the integral
 integrating $((2+x)^{-1})$ wrt x
 rewriting as $((2+x)^{-1})$
 rewriting as $((2+x)^{-1})$
 the solution is $(\log(\text{abs}((2+x))) * (1^{-1}))$
 the solution is $(2 * (\log(\text{abs}((2+x))) * (1^{-1})))$
 the solution is $((-1 * (\log(\text{abs}((1+x))) * (1^{-1}))) + (2 * (\log(\text{abs}((2+x))) * (1^{-1}))))$
 the solution is $((-2 * \log(\text{abs}((1+x)))) + (4 * \log(\text{abs}((2+x)))))$
 $((-2 * \log(\text{abs}((1+x)))) + (4 * \log(\text{abs}((2+x)))))$
>>>

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2 - 9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2 + 3x + 2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2 + 1)(x - 1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3 - x^2 - x + 1}$

10. $\frac{2x-3}{(x^2 - 1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2 - 4)}$

12. $\frac{x^3 + x + 1}{x^2 - 1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4 - 1}$

>>> $(1-x^2) / (x * (1-2*x))$

$((1+(-1*(x^2))) * ((1+(-2*x))^{-1}) * (x^{-1}))$

>>> Sdx

THOUGHT PROCESS =

integrating $((1+(-1*(x^2))) * ((1+(-2*x))^{-1}) * (x^{-1}))$ wrt x
 rewriting as $((2^{-1}) * ((-2 * ((-2^{-1})+x)^{-1}) + (2 * (x^{-1}))))$
 taking the constant outside of the integral
 integrating $((-2 * ((-2^{-1})+x)^{-1}) + (2 * (x^{-1})))$ wrt x
 rewriting as $((-2 * ((-2^{-1})+x)^{-1}) + (2 * (x^{-1})))$
 rewriting as $((-2 * ((-2^{-1})+x)^{-1}) + (2 * (x^{-1})))$
 integating over sums
 integrating $(-2 * ((-2^{-1})+x)^{-1})$ wrt x
 rewriting as $(-2 * ((-2^{-1})+x)^{-1})$
 taking the constant outside of the integral

```

integrating ((((-2^-1)+x)^-1) wrt x
rewriting as ((((-2^-1)+x)^-1)
rewriting as ((((-2^-1)+x)^-1)
the solution is (log(abs((((-2^-1)+x))))*(1^-1))
the solution is (-2*(log(abs((((-2^-1)+x))))*(1^-1)))
integating over sums
integrating (2*(x^-1)) wrt x
rewriting as (2*(x^-1))
taking the constant outside of the integral
integrating (x^-1) wrt x
rewriting as (x^-1)
rewriting as (x^-1)
the solution is (log(abs(x))* (1^-1))
the solution is (2*(log(abs(x))* (1^-1)))
the solution is ((-2*(log(abs((((-2^-1)+x))))*(1^-1)))+(2*(log(abs(x))* (1^-1))))
the solution is (log(abs(x))+(-1*log(abs((((-2^-1)+x))))))

(log(abs(x))+(-1*log(abs((((-2^-1)+x))))))
>>>

```

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

$$1. \frac{x}{(x+1)(x+2)}$$

$$2. \frac{1}{x^2 - 9}$$

$$3. \frac{3x-1}{(x-1)(x-2)(x-3)}$$

$$4. \frac{x}{(x-1)(x-2)(x-3)}$$

$$5. \frac{2x}{x^2 + 3x + 2}$$

$$6. \frac{1-x^2}{x(1-2x)}$$

$$7. \frac{x}{(x^2 + 1)(x - 1)}$$

$$8. \frac{x}{(x-1)^2 (x+2)}$$

$$9. \frac{3x+5}{x^3 - x^2 - x + 1}$$

$$10. \frac{2x-3}{(x^2 - 1)(2x+3)}$$

$$11. \frac{5x}{(x+1)(x^2 - 4)}$$

$$12. \frac{x^3 + x + 1}{x^2 - 1}$$

$$13. \frac{2}{(1-x)(1+x^2)}$$

$$14. \frac{3x-1}{(x+2)^2}$$

$$15. \frac{1}{x^4 - 1}$$

```

>>> x / ((x^2+1)*(x-1))
(((-1+x)^-1)*(1+(x^2))^-1)*x
>>> Sdx

```

THOUGHT PROCESS =

```

integrating ((((-1+x)^-1)*(1+(x^2))^-1)*x) wrt x
rewriting as ((((-2^-1)*(1+(x^2))^-1))+((2^-1)*((-1+x)^-1)))
rewriting as ((((-2^-1)*(1+(x^2))^-1))+((2^-1)*((-1+x)^-1)))
integating over sums
integrating ((-2^-1)*(1+(x^2))^-1) wrt x
rewriting as ((-2^-1)*(1+(x^2))^-1)
taking the constant outside of the integral
integrating ((1+(x^2))^-1) wrt x
rewriting as ((1+(x^2))^-1)
rewriting as ((1+(x^2))^-1)
the solution is (arctan((x*((1^(2^-1))^-1)))*((1^(2^-1))^-1))

```

```

the solution is (((-2^-1)*(arctan((x*((1^(2^-1))^(-1))))*((1^(2^-
1))^(-1))))
integrating over sums
integrating ((2^-1)*((-1+x)^(-1)) wrt x
rewriting as ((2^-1)*((-1+x)^(-1)))
taking the constant outside of the integral
integrating ((-1+x)^(-1) wrt x
rewriting as ((-1+x)^(-1)
rewriting as ((-1+x)^(-1)
the solution is (log(abs((-1+x)))*(1^-1))
the solution is ((2^-1)*(log(abs((-1+x)))*(1^-1)))
the solution is ((((-2^-1)*arctan(x))+((2^-1)*log(abs((-1+x))))))

((((-2^-1)*arctan(x))+((2^-1)*log(abs((-1+x)))))

>>>

```

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2 - 9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2 + 3x + 2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2 + 1)(x - 1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3 - x^2 - x + 1}$

10. $\frac{2x-3}{(x^2 - 1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2 - 4)}$

12. $\frac{x^3 + x + 1}{x^2 - 1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4 - 1}$

```

>>> x / ((x-1)^2 * (x+2))
((( -1+x)^-2) * ((2+x)^-1) * x)
>>> Sdx
THOUGHT PROCESS =

```

```

integrating ((( -1+x)^-2) * ((2+x)^-1) * x) wrt x
rewriting as ((-2*(9^-1)*((2+x)^-1)) + ((3^-1)*((-1+x)^-2)) + (2*(9^-
1)*((-1+x)^-1)))
rewriting as ((-2*(9^-1)*((2+x)^-1)) + ((3^-1)*((-1+x)^-2)) + (2*(9^-
1)*((-1+x)^-1)))
integrating over sums
integrating (-2*(9^-1)*((2+x)^-1)) wrt x
rewriting as (-2*(9^-1)*((2+x)^-1))
taking the constant outside of the integral
integrating ((2+x)^-1) wrt x
rewriting as ((2+x)^-1)
rewriting as ((2+x)^-1)
the solution is (log(abs((2+x)))*(1^-1))
the solution is ((-2*(9^-1))*log(abs((2+x)))*(1^-1)))
integrating over sums
integrating ((3^-1)*((-1+x)^-2)) wrt x

```

```

rewriting as ((3^-1) * ((-1+x)^-2))
taking the constant outside of the integral
integrating ((-1+x)^-2) wrt x
rewriting as ((-1+x)^-2)
rewriting as ((-1+x)^-2)
the solution is (((-1+x)^-1) * ((-1*1)^-1))
the solution is ((3^-1) * (((-1+x)^-1) * ((-1*1)^-1)))
integating over sums
integrating (2*(9^-1) * ((-1+x)^-1)) wrt x
rewriting as (2*(9^-1) * ((-1+x)^-1))
taking the constant outside of the integral
integrating ((-1+x)^-1) wrt x
rewriting as ((-1+x)^-1)
rewriting as ((-1+x)^-1)
the solution is (log(abs((-1+x))) * (1^-1))
the solution is ((2*(9^-1)) * (log(abs((-1+x))) * (1^-1)))
the solution is ((((-3^-1) * ((-1+x)^-1)) + (-2*(9^-
1) * log(abs((2+x)))) + (2*(9^-1) * log(abs((-1+x))))))

((((-3^-1) * ((-1+x)^-1)) + (-2*(9^-1) * log(abs((2+x)))) + (2*(9^-
1) * log(abs((-1+x)))))

>>>

```

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2 - 9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2 + 3x + 2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2 + 1)(x - 1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3 - x^2 - x + 1}$

10. $\frac{2x-3}{(x^2 - 1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2 - 4)}$

12. $\frac{x^3 + x + 1}{x^2 - 1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4 - 1}$

```

>>> (2*x-3) / ((x^2-1)*(2*x+3))
((-3+(2*x))*((-1+(x^2))^-1)*((3+(2*x))^-1))
>>> Sdx

```

THOUGHT PROCESS =

```

integrating (((-3+(2*x))*((-1+(x^2))^-1)*((3+(2*x))^-1)) wrt x
rewriting as ((((-10^-1)*((-1+x)^-1))+(-12*(5^-1)*(((3*(2^-1))+x)^-
1))+((5*(2^-1)*((1+x)^-1))))
rewriting as ((((-10^-1)*((-1+x)^-1))+(-12*(5^-1)*(((3*(2^-1))+x)^-
1))+((5*(2^-1)*((1+x)^-1))))
integating over sums
integrating ((-10^-1)*((-1+x)^-1)) wrt x
rewriting as ((-10^-1)*((-1+x)^-1))
taking the constant outside of the integral

```

```

integrating ((-1+x)^-1) wrt x
rewriting as ((-1+x)^-1)
rewriting as ((-1+x)^-1)
the solution is (log(abs((-1+x)))* (1^-1))
the solution is ((-10^-1)*(log(abs((-1+x)))* (1^-1)))
integating over sums
integrating (-12*(5^-1)* (((3*(2^-1))+x)^-1)) wrt x
rewriting as (-12*(5^-1)* (((3*(2^-1))+x)^-1))
taking the constant outside of the integral
integrating (((3*(2^-1))+x)^-1) wrt x
rewriting as (((3*(2^-1))+x)^-1)
rewriting as (((3*(2^-1))+x)^-1)
the solution is (log(abs(((3*(2^-1))+x)))* (1^-1))
the solution is ((-12*(5^-1))* (log(abs(((3*(2^-1))+x)))* (1^-1)))
integating over sums
integrating (5*(2^-1)* ((1+x)^-1)) wrt x
rewriting as (5*(2^-1)* ((1+x)^-1))
taking the constant outside of the integral
integrating ((1+x)^-1) wrt x
rewriting as ((1+x)^-1)
rewriting as ((1+x)^-1)
the solution is (log(abs((1+x)))* (1^-1))
the solution is ((5*(2^-1))* (log(abs((1+x)))* (1^-1)))
the solution is ((((-10^-1)*log(abs((-1+x))))+(-12*(5^-1)*log(abs(((3*(2^-1))+x)))))+(5*(2^-1)*log(abs((1+x)))))

((((-10^-1)*log(abs((-1+x))))+(-12*(5^-1)*log(abs(((3*(2^-1))+x)))))+(5*(2^-1)*log(abs((1+x)))))

>>>

```

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2-9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2+3x+2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2+1)(x-1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3-x^2-x+1}$

10. $\frac{2x-3}{(x^2-1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2-4)}$

12. $\frac{x^3+x+1}{x^2-1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4-1}$

```

>>> 5*x/((x+1)*(x^2-4))
(5*(-4+(x^2))^-1)*((1+x)^-1)*x
>>> Sdx

```

THOUGHT PROCESS =

integrating (5*((-4+(x^2))^-1)*((1+x)^-1)*x) wrt x

```

rewriting as (5*(((-2^-1)*(2+x)^-1))+((3^-1)*(1+x)^-1))+((6^-
1)*((-2+x)^-1)))
taking the constant outside of the integral
integrating ((((-2^-1)*(2+x)^-1))+((3^-1)*(1+x)^-1))+((6^-1)*((-2+x)^-1))) wrt x
rewriting as ((((-2^-1)*(2+x)^-1))+((3^-1)*(1+x)^-1))+((6^-1)*((-2+x)^-1)))
rewriting as ((((-2^-1)*(2+x)^-1))+((3^-1)*(1+x)^-1))+((6^-1)*((-2+x)^-1)))
integating over sums
integrating ((-2^-1)*(2+x)^-1) wrt x
rewriting as ((2+x)^-1)
rewriting as ((2+x)^-1)
the solution is (log(abs((2+x)))*(1^-1))
the solution is ((-2^-1)*(log(abs((2+x)))*(1^-1)))
integating over sums
integrating ((3^-1)*(1+x)^-1) wrt x
rewriting as ((3^-1)*(1+x)^-1)
taking the constant outside of the integral
integrating ((1+x)^-1) wrt x
rewriting as ((1+x)^-1)
rewriting as ((1+x)^-1)
the solution is (log(abs((1+x)))*(1^-1))
the solution is ((3^-1)*(log(abs((1+x)))*(1^-1)))
integating over sums
integrating ((6^-1)*((-2+x)^-1)) wrt x
rewriting as ((6^-1)*((-2+x)^-1))
taking the constant outside of the integral
integrating ((-2+x)^-1) wrt x
rewriting as ((-2+x)^-1)
rewriting as ((-2+x)^-1)
the solution is (log(abs((-2+x)))*(1^-1))
the solution is ((6^-1)*(log(abs((-2+x)))*(1^-1)))
the solution is ((((-2^-1)*(log(abs((2+x)))*(1^-1)))+((3^-
1)*(log(abs((1+x)))*(1^-1))))+((6^-1)*(log(abs((-2+x)))*(1^-1))))
the solution is ((-5*(2^-1)*log(abs((2+x))))+(5*(3^-
1)*log(abs((1+x))))+(5*(6^-1)*log(abs((-2+x)))))

((-5*(2^-1)*log(abs((2+x))))+(5*(3^-1)*log(abs((1+x))))+(5*(6^-
1)*log(abs((-2+x)))))

>>>

```

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2 - 9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2 + 3x + 2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2 + 1)(x - 1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3 - x^2 - x + 1}$

10. $\frac{2x-3}{(x^2 - 1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2 - 4)}$

12. $\frac{x^3 + x + 1}{x^2 - 1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4 - 1}$

```
>>> 2 / ((1-x) * (1+x^2))
(2 * ((1+(-1*x))^-1) * ((1+(x^2))^-1))
>>> Sdx
```

THOUGHT PROCESS =

```
integrating (2 * ((1+(-1*x))^-1) * ((1+(x^2))^-1)) wrt x
rewriting as (-2 * (((-2^-1) * ((1+(x^2))^-1)) + ((2^-1) * ((-1+x)^-1))))
taking the constant outside of the integral
integrating (((-2^-1) * ((1+(x^2))^-1)) + ((2^-1) * ((-1+x)^-1))) wrt x
rewriting as (((-2^-1) * ((1+(x^2))^-1)) + ((2^-1) * ((-1+x)^-1)))
rewriting as (((-2^-1) * ((1+(x^2))^-1)) + ((2^-1) * ((-1+x)^-1)))
integating over sums
integrating ((-2^-1) * ((1+(x^2))^-1)) wrt x
rewriting as ((-2^-1) * ((1+(x^2))^-1))
taking the constant outside of the integral
integrating ((1+(x^2))^-1) wrt x
rewriting as ((1+(x^2))^-1)
rewriting as ((1+(x^2))^-1)
the solution is (arctan((x * ((1^(2^-1))^-1))) * ((1^(2^-1))^-1))
the solution is ((-2^-1) * (arctan((x * ((1^(2^-1))^-1))) * ((1^(2^-1))^-1)))
integating over sums
integrating ((2^-1) * ((-1+x)^-1)) wrt x
rewriting as ((2^-1) * ((-1+x)^-1))
taking the constant outside of the integral
integrating ((-1+x)^-1) wrt x
rewriting as ((-1+x)^-1)
rewriting as ((-1+x)^-1)
the solution is (log(abs((-1+x))) * (1^-1))
```

```

the solution is ((2^-1)*(log(abs((-1+x)))*(1^-1)))
the solution is (((-2^-1)*(arctan((x*(1^(2^-1))^(-1))))*((1^(2^(-1)))^(-1)))+((2^-1)*(log(abs((-1+x)))*(1^-1))))
the solution is (arctan(x)+(-1*log(abs((-1+x)))))

(arctan(x)+(-1*log(abs((-1+x)))))

>>>

```

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$

2. $\frac{1}{x^2 - 9}$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

5. $\frac{2x}{x^2 + 3x + 2}$

6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2 + 1)(x - 1)}$

8. $\frac{x}{(x-1)^2(x+2)}$

9. $\frac{3x+5}{x^3 - x^2 - x + 1}$

10. $\frac{2x-3}{(x^2 - 1)(2x+3)}$

11. $\frac{5x}{(x+1)(x^2 - 4)}$

12. $\frac{x^3 + x + 1}{x^2 - 1}$

13. $\frac{2}{(1-x)(1+x^2)}$

14. $\frac{3x-1}{(x+2)^2}$

15. $\frac{1}{x^4 - 1}$

```

>>> (3*x-1) / (x+2)^2
((-1+(3*x))*((2+x)^(-2)))
>>> Sdx

```

THOUGHT PROCESS =

```

integrating ((-1+(3*x))*((2+x)^(-2))) wrt x
rewriting as (3*((-7*(3^-1)*((2+x)^(-2)))+((2+x)^(-1))))
taking the constant outside of the integral
integrating ((-7*(3^-1)*((2+x)^(-2)))+((2+x)^(-1))) wrt x
rewriting as ((-7*(3^-1)*((2+x)^(-2)))+((2+x)^(-1)))
rewriting as ((-7*(3^-1)*((2+x)^(-2)))+((2+x)^(-1)))
integrating over sums
integrating (-7*(3^-1)*((2+x)^(-2))) wrt x
rewriting as (-7*(3^-1)*((2+x)^(-2)))
taking the constant outside of the integral
integrating ((2+x)^(-2)) wrt x
rewriting as ((2+x)^(-2))
rewriting as ((2+x)^(-2))
the solution is (((2+x)^(-1))*((-1*1)^(-1)))
the solution is ((-7*(3^-1))*(((2+x)^(-1))*((-1*1)^(-1))))
integrating over sums
integrating ((2+x)^(-1)) wrt x
rewriting as ((2+x)^(-1))

```

```

rewriting as ((2+x)^-1)
the solution is (log(abs((2+x)))* (1^-1))
the solution is (((-7*(3^-1))*(((2+x)^-1)*((-1*1)^-
1)))+(log(abs((2+x)))* (1^-1)))
the solution is ((3*log(abs((2+x))))+(7*((2+x)^-1)))

((3*log(abs((2+x))))+(7*((2+x)^-1)))
>>>

```

17. $\frac{\cos x}{(1-\sin x)(2-\sin x)}$ [Hint : Put $\sin x = t$]

Reprint 2025-26

INTEGRALS

18. $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$ **19.** $\frac{2x}{(x^2+1)(x^2+3)}$ **20.** $\frac{1}{x(x^4-1)}$

21. $\frac{1}{(e^x-1)}$ [Hint : Put $e^x = t$]

```

>>> cos(x)/((1-sin(x))*(2-sin(x)))
(cos(x)*((1+(-1*sin(x)))^-1)*((2+(-1*sin(x)))^-1))
>>> Sdx
THOUGHT PROCESS =

```

```

integrating (cos(x)*((1+(-1*sin(x)))^-1)*((2+(-1*sin(x)))^-1)) wrt x
rewriting as (cos(x)*((-1+sin(x))^ -1)*((-2+sin(x))^ -1))
rewriting as (cos(x)*((-1+sin(x))^ -1)*((-2+sin(x))^ -1))
substituting (-1+sin(x))=y
integrating (((-1+y)^ -1)*(y^ -1)) wrt y
rewriting as ((-1*(y^ -1))+((-1+y)^ -1))
rewriting as ((-1*(y^ -1))+((-1+y)^ -1))
integrating over sums
integrating (-1*(y^ -1)) wrt y
rewriting as (-1*(y^ -1))
taking the constant outside of the integral
integrating (y^ -1) wrt y
rewriting as (y^ -1)
rewriting as (y^ -1)
the solution is (log(abs(y))* (1^-1))
the solution is (-1*(log(abs(y))* (1^-1)))
integrating over sums
integrating ((-1+y)^ -1) wrt y
rewriting as ((-1+y)^ -1)
rewriting as ((-1+y)^ -1)
the solution is (log(abs((-1+y)))* (1^-1))
the solution is ((-1*(log(abs((-1+sin(x))))* (1^-1)))+(log(abs((-1+(-1+sin(x))))* (1^-1))))

```

```

the solution is (log(abs((-2+sin(x))))+(-1*log(abs((-1+sin(x))))))

(log(abs((-2+sin(x))))+(-1*log(abs((-1+sin(x))))))

>>>

```

17. $\frac{\cos x}{(1-\sin x)(2-\sin x)}$ [Hint : Put $\sin x = t$]

Reprint 2025-26

INTEGRALS

18. $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$ **19.** $\frac{2x}{(x^2+1)(x^2+3)}$ **20.** $\frac{1}{x(x^4-1)}$

21. $\frac{1}{(e^x-1)}$ [Hint : Put $e^x = t$]

```

>>> 2*x/((x^2+1)*(x^2+3))
(2*((1+(x^2))^-1)*((3+(x^2))^-1)*x)
>>> Sdx

```

THOUGHT PROCESS =

```

integrating (2*((1+(x^2))^-1)*((3+(x^2))^-1)*x) wrt x
rewriting as ((1+(x^2))^-1)
rewriting as ((1+(x^2))^-1)
the solution is arctan(x)

```

```

arctan(x)
>>>

```

17. $\frac{\cos x}{(1-\sin x)(2-\sin x)}$ [Hint : Put $\sin x = t$]

Reprint 2025-26

INTEGRALS

18. $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$ 19. $\frac{2x}{(x^2+1)(x^2+3)}$ 20. $\frac{1}{x(x^4-1)}$

21. $\frac{1}{(e^x-1)}$ [Hint : Put $e^x = t$]

```
>>> 1/(e^x-1)
((-1+(e^x))^-1)
```

>>> Sdx

THOUGHT PROCESS =

```
integrating ((-1+(e^x))^-1) wrt x
rewriting as ((-1+(e^x))^-1)
rewriting as ((-1+(e^x))^-1)
substituting (e^x)=y
integrating (((-1+y)^-1)*(y^-1)) wrt y
rewriting as ((-1*(y^-1))+((-1+y)^-1))
rewriting as ((-1*(y^-1))+((-1+y)^-1))
    integating over sums
    integrating (-1*(y^-1)) wrt y
    rewriting as (-1*(y^-1))
        taking the constant outside of the integral
        integrating (y^-1) wrt y
        rewriting as (y^-1)
        rewriting as (y^-1)
        the solution is (log(abs(y))*(1^-1))
the solution is (-1*(log(abs(y))*(1^-1)))
integating over sums
integrating ((-1+y)^-1) wrt y
rewriting as ((-1+y)^-1)
rewriting as ((-1+y)^-1)
the solution is (log(abs((-1+y)))*(1^-1))
the solution is ((-1*(log(abs((e^x)))*(1^-1)))+(log(abs((-1+(e^x)))*(1^-1)))
the solution is (log(abs((-1+(e^x))))+(-1*log(abs((e^x))))))
(log(abs((-1+(e^x))))+(-1*log(abs((e^x)))))
```

>>>

EXERCISE 7.6

Integrate the functions in Exercises 1 to 22.

1. $x \sin x$

5. $x \log 2x$

9. $x \cos^{-1} x$

13. $\tan^{-1} x$

2. $x \sin 3x$

6. $x^2 \log x$

10. $(\sin^{-1} x)^2$

14. $x (\log x)^2$

3. $x^2 e^x$

7. $x \sin^{-1} x$

11. $\frac{x \cos^{-1} x}{\sqrt{1-x^2}}$

15. $(x^2 + 1) \log x$

4. $x \log x$

8. $x \tan^{-1} x$

12. $x \sec^2 x$

Reprint 2025-26

264 MATHEMATICS

16. $e^x (\sin x + \cos x)$ 17. $\frac{x e^x}{(1+x)^2}$ 18. $e^x \left(\frac{1+\sin x}{1+\cos x} \right)$

19. $e^x \left(\frac{1}{x} - \frac{1}{x^2} \right)$ 20. $\frac{(x-3) e^x}{(x-1)^3}$ 21. $e^{2x} \sin x$

22. $\sin^{-1} \left(\frac{2x}{1+x^2} \right)$

```
>>> e^x * (sin(x) + cos(x))  
((cos(x) + sin(x)) * (e^x))
```

```
>>> Sdx
```

```
THOUGHT PROCESS =
```

```
integrating ((cos(x) + sin(x)) * (e^x)) wrt x  
rewriting as ((cos(x) + sin(x)) * (e^x))  
rewriting as ((cos(x) * (e^x)) + ((e^x) * sin(x)))
```

```
integrating over sums
```

```
integrating (cos(x) * (e^x)) wrt x
```

```
rewriting as (cos(x) * (e^x))
```

```
rewriting as (cos(x) * (e^x))
```

```
substituting (e^x) = y
```

```
integrating cos(log(y)) wrt y
```

```
rewriting as cos(log(y))
```

```
rewriting as cos(log(y))
```

```
substituting log(y) = z
```

```
integrating (cos(z) * (e^z)) wrt z
```

```
rewriting as (cos(z) * (e^z))
```

```
rewriting as (cos(z) * (e^z))
```

```
substituting (e^z) = a
```

```
integrating cos(log(a)) wrt a
```

```
rewriting as cos(log(a))
```

```

rewriting as cos(log(a))
substituting log(a)=b
the solution is not found
the solution is not found
performing integration by parts f(x)=cos(z) and g(x)=(e^z)
integrating (e^z) wrt z
rewriting as (e^z)
rewriting as (e^z)
integrating ((-1*sin(z))*(e^z)*(1^-1)) wrt z
rewriting as (-1*(e^z)*sin(z))
taking the constant outside of the integral
the solution is not found
rewriting as (-1*(e^z)*sin(z))
substituting (e^z)=a
the solution is not found
the solution is not found
the solution is not found
performing integration by parts f(x)=(e^z) and g(x)=cos(z)
integrating cos(z) wrt z
rewriting as cos(z)
rewriting as cos(z)
integrating ((e^z)*(sin(z)*(1^-1))) wrt z
rewriting as ((e^z)*sin(z))
rewriting as ((e^z)*sin(z))
substituting (e^z)=a
the solution is not found
performing integration by parts f(x)=(e^z) and g(x)=sin(z)
the solution is not found
performing integration by parts f(x)=sin(z) and g(x)=(e^z)
the solution is not found
performing integration by parts f(x)=cos(x) and g(x)=(e^x)
integrating (e^x) wrt x
rewriting as (e^x)
rewriting as (e^x)
integrating ((-1*sin(x))*((e^x)*(1^-1))) wrt x
rewriting as (-1*(e^x)*sin(x))
taking the constant outside of the integral
integrating ((e^x)*sin(x)) wrt x
rewriting as ((e^x)*sin(x))
rewriting as ((e^x)*sin(x))
substituting (e^x)=y
integrating sin(log(y)) wrt y
rewriting as sin(log(y))
rewriting as sin(log(y))
substituting log(y)=z
the solution is not found
the solution is not found
performing integration by parts f(x)=(e^x) and g(x)=sin(x)
integrating sin(x) wrt x
rewriting as sin(x)
rewriting as sin(x)
integrating ((e^x)*((-1*cos(x))*(1^-1))) wrt x
rewriting as (-1*cos(x)*(e^x))
taking the constant outside of the integral

```

```

the solution is not found
rewriting as (-1*cos(x)*(e^x))
substituting (e^x)=y
the solution is not found
the solution is not found
performing integration by parts f(x)=sin(x) and g(x)=(e^x)
integrating (e^x) wrt x
rewriting as (e^x)
rewriting as (e^x)
the solution is ((sin(x)*((e^x)*(1^-
1)))+(int((cos(x)*(e^x)),x)^*-1))
the solution is ((sin(x)*((e^x)*(1^-
1)))+(int((cos(x)*(e^x)),x)^*-1))
the solution is (((cos(x)*((e^x)*(1^-1)))+((-1*((sin(x)*((e^x)*(1^-
1)))+(int((cos(x)*(e^x)),x)^*-1))))^*-1))
the solution is (((2^-1)*cos(x)*(e^x))+((2^-1)*(e^x)*sin(x)))
integating over sums
integrating ((e^x)*sin(x)) wrt x
rewriting as ((e^x)*sin(x))
rewriting as ((e^x)*sin(x))
substituting (e^x)=y
integrating sin(log(y)) wrt y
rewriting as sin(log(y))
rewriting as sin(log(y))
substituting log(y)=z
integrating ((e^z)*sin(z)) wrt z
rewriting as ((e^z)*sin(z))
rewriting as ((e^z)*sin(z))
substituting (e^z)=a
integrating sin(log(a)) wrt a
rewriting as sin(log(a))
rewriting as sin(log(a))
substituting log(a)=b
the solution is not found
the solution is not found
performing integration by parts f(x)=(e^z) and g(x)=sin(z)
integrating sin(z) wrt z
rewriting as sin(z)
rewriting as sin(z)
integrating ((e^z)*((-1*cos(z))*(1^-1))) wrt z
rewriting as (-1*cos(z)*(e^z))
taking the constant outside of the integral
the solution is not found
rewriting as (-1*cos(z)*(e^z))
substituting (e^z)=a
the solution is not found
the solution is not found
performing integration by parts f(x)=sin(z) and g(x)=(e^z)
integrating (e^z) wrt z
rewriting as (e^z)
rewriting as (e^z)
integrating (cos(z)*((e^z)*(1^-1))) wrt z
rewriting as (cos(z)*(e^z))
rewriting as (cos(z)*(e^z))
substituting (e^z)=a
the solution is not found

```

```

    performing integration by parts f(x)=cos(z) and g(x)=(e^z)
    the solution is not found
    performing integration by parts f(x)=(e^z) and g(x)=cos(z)
    the solution is not found
    the solution is not found
    the solution is not found
    the solution is not found
the solution is not found
performing integration by parts f(x)=(e^x) and g(x)=sin(x)
integrating sin(x) wrt x
rewriting as sin(x)
rewriting as sin(x)
integrating ((e^x)*((-1*cos(x))*(1^-1))) wrt x
rewriting as (-1*cos(x)*(e^x))
    taking the constant outside of the integral
    integrating (cos(x)*(e^x)) wrt x
    rewriting as (cos(x)*(e^x))
    rewriting as (cos(x)*(e^x))
    substituting (e^x)=y
    integrating cos(log(y)) wrt y
    rewriting as cos(log(y))
    rewriting as cos(log(y))
    substituting log(y)=z
    the solution is not found
the solution is not found
performing integration by parts f(x)=cos(x) and g(x)=(e^x)
integrating (e^x) wrt x
rewriting as (e^x)
rewriting as (e^x)
integrating ((-1*sin(x))*((e^x)*(1^-1))) wrt x
rewriting as (-1*(e^x)*sin(x))
    taking the constant outside of the integral
    the solution is not found
    rewriting as (-1*(e^x)*sin(x))
    substituting (e^x)=y
    the solution is not found
the solution is not found
performing integration by parts f(x)=(e^x) and g(x)=cos(x)
integrating cos(x) wrt x
rewriting as cos(x)
rewriting as cos(x)
the solution is (((e^x)*(sin(x)*(1^-
1)))+(int(((e^x)*sin(x)),x)^*-1))
    the solution is (((e^x)*(sin(x)*(1^-
1)))+(int(((e^x)*sin(x)),x)^*-1))
    the solution is (((e^x)*((-1*cos(x)*(1^-1)))+((-1*
(((e^x)*(sin(x)*(1^-1)))+(int(((e^x)*sin(x)),x)^*-1)))*-1)))
    the solution is ((((-2^-1)*cos(x)*(e^x))+((2^-1)*(e^x)*sin(x))))
the solution is ((e^x)*sin(x))

((e^x)*sin(x))
>>>

```

Therefore, the given function is a solution of the given differential equation.

EXERCISE 9.2

In each of the Exercises 1 to 10 verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

1. $y = e^x + 1$: $y'' - y' = 0$
2. $y = x^2 + 2x + C$: $y' - 2x - 2 = 0$
3. $y = \cos x + C$: $y' + \sin x = 0$
4. $y = \sqrt{1+x^2}$: $y' = \frac{xy}{1+x^2}$
5. $y = Ax$: $xy' = y (x \neq 0)$
6. $y = x \sin x$: $xy' = y + x\sqrt{x^2 - y^2} (x \neq 0 \text{ and } x > y \text{ or } x < -y)$
7. $xy = \log y + C$: $y' = \frac{y^2}{1-xy} (xy \neq 1)$
8. $y - \cos y = x$: $(y \sin y + \cos y + x) y' = y$
9. $x + y = \tan^{-1} y$: $y^2 y' + y^2 + 1 = 0$
10. $y = \sqrt{a^2 - x^2} x \in (-a, a)$: $x + y \frac{dy}{dx} = 0 (y \neq 0)$

```
>>> dif(y)/dif(x)-2*x-2=0
((-1*(-2+(-2*x)+(dif(y)*(dif(x)^-1))))=0)
>>> difsolve
solving the differential equation ((-1*(-2+(-2*x)+(dif(y)*(dif(x)^-1))))=0)
rewritting as ((dif(y)+(-2*dif(x))+(-2*dif(x)*x))=0)
rearranged as ((dif(y)+(-2*dif(x))+(-2*dif(x)*x))=0)
integrating 1 wrt y
the solution is (y^1)
integrating -2 wrt x
the solution is (x^-2)
integrating (-2*x) wrt x
rewriting as (-2*x)
taking the constant outside of the integral
integrating x wrt x
rewriting as x
rewriting as x
the solution is ((x^2)*(2^-1))
the solution is (-2*((x^2)*(2^-1)))
the solution is ((((-1*(x^2))+(-2*x)+y+c1)=0)
((((-1*(x^2))+(-2*x)+y+c1)=0)
>>>
```

Therefore, the given function is a solution of the given differential equation.

EXERCISE 9.2

In each of the Exercises 1 to 10 verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

1. $y = e^x + 1$: $y'' - y' = 0$

2. $y = x^2 + 2x + C$: $y' - 2x - 2 = 0$

3. $y = \cos x + C$: $y' + \sin x = 0$

4. $y = \sqrt{1+x^2}$: $y' = \frac{xy}{1+x^2}$

5. $y = Ax$: $xy' = y (x \neq 0)$

6. $y = x \sin x$: $xy' = y + x \sqrt{x^2 - y^2} (x \neq 0 \text{ and } x > y \text{ or } x < -y)$

7. $xy = \log y + C$: $y' = \frac{y^2}{1-xy} (xy \neq 1)$

8. $y - \cos y = x$: $(y \sin y + \cos y + x) y' = y$

9. $x + y = \tan^{-1} y$: $y^2 y' + y^2 + 1 = 0$

10. $y = \sqrt{a^2 - x^2} x \in (-a, a)$: $x + y \frac{dy}{dx} = 0 (y \neq 0)$

```
>>> dif(y)/dif(x) = x*y/(1+x^2)
((( -1*dif(y)*(dif(x)^-1)) + ((1+(x^2))^ -1)*x*y))=0
>>> difsolve
solving the differential equation ((( -1*dif(y)*(dif(x)^-
1)) + ((1+(x^2))^ -1)*x*y))=0
rewriting as ((( -1*dif(y)) + (-1*dif(y)*(x^2)) + (dif(x)*x*y))=0
rearranged as not found
the solution is not found
solving the differential equation ((( -1*dif(y)*(dif(x)^-
1)) + ((1+(x^2))^ -1)*x*y))=0
homogenous, substituting z=y/x
    solving the differential equation ((( -1*dif(y)*(dif(x)^-1)*x) + (-
1*y) + (((1+(x^2))^ -1)*(x^2)*y))=0
    rewriting as ((( -1*dif(x)*y) + (-1*dif(y)*(x^3)) + (-1*dif(y)*x))=0
    rearranged as not found
    the solution is not found
homogenous, substituting z=x/y
    solving the differential equation ((( -1*dif(y)*((dif(x)*y)+(dif(y)*x))^ -1)) + (((1+((x^2)*(y^2)))^ -1)*(y^2)*x))=0
    rewriting as ((( -1*dif(y)) + (dif(x)*(y^3)*x))=0
    rearranged as ((( -1*dif(x)*x) + (dif(y)*(y^-3)))=0
        integrating (-1*x) wrt x
        rewriting as (-1*x)
            taking the constant outside of the integral
            integrating x wrt x
            rewriting as x
            rewriting as x
            the solution is ((x^2)*(2^-1))
        the solution is (-1*((x^2)*(2^-1)))
        integrating (y^-3) wrt y
        rewriting as (y^-3)
        rewriting as (y^-3)
```

```

the solution is ((y^-2)*((-2*x)^-1))
the solution is (((-2+(-2*(x^2)*(y^2)))+(4*(y^2)*c1))=0)
((-2+(-2*((x/y)^2)*(y^2)))+(4*(y^2)*c1))=0)
>>>

```

Therefore, the given function is a solution of the given differential equation.

EXERCISE 9.2

In each of the Exercises 1 to 10 verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

1. $y = e^x + 1$

: $y'' - y' = 0$

2. $y = x^2 + 2x + C$

: $y' - 2x - 2 = 0$

3. $y = \cos x + C$

: $y' + \sin x = 0$

4. $y = \sqrt{1+x^2}$

: $y' = \frac{xy}{1+x^2}$

5. $y = Ax$

: $xy' = y (x \neq 0)$

6. $y = x \sin x$

: $xy' = y + x \sqrt{x^2 - y^2} \quad (x \neq 0 \text{ and } x > y \text{ or } x < -y)$

7. $xy = \log y + C$

: $y' = \frac{y^2}{1-xy} \quad (xy \neq 1)$

8. $y - \cos y = x$

: $(y \sin y + \cos y + x) y' = y$

9. $x + y = \tan^{-1} y$

: $y^2 y' + y^2 + 1 = 0$

10. $y = \sqrt{a^2 - x^2} \quad x \in (-a, a) : \quad x + y \frac{dy}{dx} = 0 \quad (y \neq 0)$

```

>>> x*diff(y)/diff(x)=y
((( -1*diff(y)*(diff(x)^-1)*x)+y)=0)
>>> dsolve
solving the differential equation ((( -1*diff(y)*(diff(x)^-1)*x)+y)=0)
rewriting as ((( -1*diff(y)*x)+(diff(x)*y))=0)
rearranged as ((( -1*diff(x)*(x^-1))+(diff(y)*(y^-1)))=0)
integrating (-1*(x^-1)) wrt x
rewriting as (-1*(x^-1))
taking the constant outside of the integral
integrating (x^-1) wrt x
rewriting as (x^-1)
rewriting as (x^-1)
the solution is (log(abs(x))*(1^-1))
the solution is (-1*(log(abs(x))*(1^-1)))
integrating (y^-1) wrt y
rewriting as (y^-1)
rewriting as (y^-1)
the solution is (log(abs(y))*(1^-1))
the solution is ((( -1*abs(x))+(abs(y)*(e^c2)))=0)
((( -1*abs(x))+(abs(y)*(e^c2)))=0)
>>>

```

Therefore, the given function is a solution of the given differential equation.

EXERCISE 9.2

In each of the Exercises 1 to 10 verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

1. $y = e^x + 1$: $y'' - y' = 0$

2. $y = x^2 + 2x + C$: $y' - 2x - 2 = 0$

3. $y = \cos x + C$: $y' + \sin x = 0$

4. $y = \sqrt{1+x^2}$: $y' = \frac{xy}{1+x^2}$

5. $y = Ax$: $xy' = y (x \neq 0)$

6. $y = x \sin x$: $xy' = y + x\sqrt{x^2 - y^2} (x \neq 0 \text{ and } x > y \text{ or } x < -y)$

7. $xy = \log y + C$: $y' = \frac{y^2}{1-xy} (xy \neq 1)$

8. $y - \cos y = x$: $(y \sin y + \cos y + x) y' = y$

9. $x + y = \tan^{-1} y$: $y^2 y' + y^2 + 1 = 0$

10. $y = \sqrt{a^2 - x^2} x \in (-a, a)$: $x + y \frac{dy}{dx} = 0 (y \neq 0)$

```
>>> x+y*diff(y)/diff(x) = 0
((-1*((diff(y)*(diff(x)^-1)*y)+x))=0)
>>> difsolve
solving the differential equation ((-1*((diff(y)*(diff(x)^-1)*y)+x))=0)
rewriting as (((diff(x)*x)+(diff(y)*y))=0)
rearranged as (((diff(x)*x)+(diff(y)*y))=0)
integrating x wrt x
rewriting as x
rewriting as x
the solution is ((x^2)*(2^-1))
integrating y wrt y
rewriting as y
rewriting as y
the solution is ((y^2)*(2^-1))
the solution is (((2*(x^2))+(2*(y^2))+(4*c1))=0)
(((2*(x^2))+(2*(y^2))+(4*c1))=0)
>>>
```

EXERCISE 9.3

For each of the differential equations in Exercises 1 to 10, find the general solution:

$$1. \frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$2. \frac{dy}{dx} = \sqrt{4 - y^2} \quad (-2 < y < 2)$$

Reprint 2025-26

DIFFERENTIAL EQUATIONS 311

$$3. \frac{dy}{dx} + y = 1 \quad (y \neq 1)$$

$$4. \sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

$$5. (e^x + e^{-x}) \, dy - (e^x - e^{-x}) \, dx = 0$$

$$6. \frac{dy}{dx} = (1+x^2)(1+y^2)$$

$$7. y \log y \, dx - x \, dy = 0$$

$$8. x^5 \frac{dy}{dx} = -y^5$$

$$9. \frac{dy}{dx} = \sin^{-1} x$$

$$10. e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$$

```

>>> dif(y)/dif(x) + y = 1
((1+(-1*((dif(y)*(dif(x)^-1))+y)))=0)
>>> difsolve
solving the differential equation ((1+(-1*((dif(y)*(dif(x)^-
1))+y)))=0)
rewriting as ((dif(x)+(-1*dif(y))+(-1*dif(x)*y))=0)
rearranged as (((-1*dif(x))+(dif(y)*((1+(-1*y))^-
1)))=0)
integrating -1 wrt x
the solution is (x^-1)
integrating ((1+(-1*y))^-1) wrt y
rewriting as (-1*((-1+y)^-1))
taking the constant outside of the integral
integrating ((-1+y)^-1) wrt y
rewriting as ((-1+y)^-1)
rewriting as ((-1+y)^-1)
the solution is (log(abs((-1+y)))*(1^-1))
the solution is (-1*(log(abs((-1+y)))*(1^-1)))
the solution is (((-1*abs((-1+y)))+((e^(-1*x))*(e^c2)))=0)
(((1*abs((-1+y)))+((e^(-1*x))*(e^c2)))=0)
>>>

```

EXERCISE 9.4

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve each of them.

1. $(x^2 + xy) dy = (x^2 + y^2) dx$

2. $y' = \frac{x+y}{x}$

3. $(x-y) dy - (x+y) dx = 0$

4. $(x^2 - y^2) dx + 2xy dy = 0$

5. $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$

6. $x dy - y dx = \sqrt{x^2 + y^2} dx$

7. $\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$

8. $x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$

9. $y dx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0$

10. $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$

```

>>> (x^2 + x*y)*dif(y) = (x^2 + y^2)*dif(x)
((( -1*((x*y)+(x^2))*dif(y)) + (((x^2)+(y^2))*dif(x)))=0)
>>> difsolve
solving the differential equation (((-
1*((x*y)+(x^2))*dif(y)) + (((x^2)+(y^2))*dif(x)))=0)
rewriting as ((( -1*((x*y)+(x^2))*dif(y)) + (((x^2)+(y^2))*dif(x)))=0)
rearranged as not found
the solution is not found
solving the differential equation (((-
1*((x*y)+(x^2))*dif(y)) + (((x^2)+(y^2))*dif(x)))=0)
homogenous, substituting z=y/x
    solving the differential equation ((( -1*dif(x)*(x^2)*y) + (-
1*dif(y)*(x^3)) + (-1*dif(y)*(x^3)*y) + (dif(x)*(x^2)))=0)
        rewriting as ((( -1*dif(x)*(x^2)*y) + (-1*dif(y)*(x^3)) + (-
1*dif(y)*(x^3)*y) + (dif(x)*(x^2)))=0)
        rearranged as ((( -1*dif(x)*(x^-1)) + ((1+y)*dif(y)*( (1+(-1*y)) ^-
1)))=0)
            integrating (-1*(x^-1)) wrt x
            rewriting as (-1*(x^-1))
                taking the constant outside of the integral
                integrating (x^-1) wrt x
                rewriting as (x^-1)
                rewriting as (x^-1)
                the solution is (log(abs(x))*(1^-1))
            the solution is (-1*(log(abs(x))*(1^-1)))
            integrating ((1+(-1*y))^-1) wrt y
            rewriting as (-1*(-1+y)^-1)
                taking the constant outside of the integral
                integrating ((-1+y)^-1) wrt y
                rewriting as ((-1+y)^-1)
                rewriting as ((-1+y)^-1)
                the solution is (log(abs((-1+y)))*(1^-1))
            the solution is (-1*(log(abs((-1+y)))*(1^-1)))

```

```

integrating (((1+(-1*y))^(-1)*y) wrt y
rewriting as (-1*((-1+y)^(-1)*y))
taking the constant outside of the integral
integrating (((-1+y)^(-1)*y) wrt y
rewriting as (((-1+y)^(-1)*y)
rewriting as (((-1+y)^(-1)*y)
substituting (-1+y)=z
integrating (1+(z^(-1))) wrt z
rewriting as (1+(z^(-1)))
rewriting as (1+(z^(-1)))
integating over sums
integrating 1 wrt z
the solution is (z*1)
integating over sums
integrating (z^(-1)) wrt z
rewriting as (z^(-1))
rewriting as (z^(-1))
the solution is (log(abs(z))*(1^(-1)))
the solution is ((-1+y)+(log(abs((-1+y)))*(1^(-1))))
the solution is ((-1+y)+(log(abs((-1+y)))*(1^(-1))))
the solution is (-1*((-1+y)+(log(abs((-1+y)))*(1^(-1)))))
the solution is ((((-1*abs((-1+y))*abs(x))+(((e^1)*(e^(-1*log(abs((-1+y)))))*(e^(-1*y)))*(e^c1))=0)
((((-1*abs((-1+(y/x)))*abs(x))+(((e^1)*(e^(-1*log(abs((-1+(y/x)))))*(e^(-1*(y/x))))*(e^c1))=0)
>>>

```

EXERCISE 9.4

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve each of them.

1. $(x^2 + xy) dy = (x^2 + y^2) dx$

2. $y' = \frac{x+y}{x}$

3. $(x - y) dy - (x + y) dx = 0$

4. $(x^2 - y^2) dx + 2xy dy = 0$

5. $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$

6. $x dy - y dx = \sqrt{x^2 + y^2} dx$

7. $\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$

8. $x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$

9. $y dx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0$

10. $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$

```

>>> dif(y)/dif(x) = (x+y)/x
((((-1*dif(y)*(dif(x)^(-1)))+((x+y)*(x^(-1))))=0)
>>> difsolve
solving the differential equation ((((-1*dif(y)*(dif(x)^(-1)))+((x+y)*(x^(-1))))=0)
rewritting as ((((-1*dif(y)*x)+(dif(x)*x)+(dif(x)*y))=0)
rearranged as not found

```

the solution is not found
 solving the differential equation $(((-1*\text{dif}(y)*(\text{dif}(x)^{-1})+((x+y)*(x^{-1})))=0)$
 homogenous, substituting $z=y/x$
 solving the differential equation $((1+(-1*\text{dif}(y)*(\text{dif}(x)^{-1})*x))=0)$
 rewriting as $((\text{dif}(x)+(-1*\text{dif}(y)*x))=0)$
 rearranged as $((\text{dif}(y)+(-1*\text{dif}(x)*(x^{-1})))=0)$
 integrating 1 wrt y
 the solution is (y^1)
 integrating $(-1*(x^{-1}))$ wrt x
 rewriting as $(-1*(x^{-1}))$
 taking the constant outside of the integral
 integrating (x^{-1}) wrt x
 rewriting as (x^{-1})
 rewriting as (x^{-1})
 the solution is $(\log(\text{abs}(x))*(1^{-1}))$
 the solution is $(-1*(\log(\text{abs}(x))*(1^{-1})))$
 the solution is $(((-1*\text{abs}(x))+((e^y)*(e^{c2})))=0)$
 $(((-1*\text{abs}(x))+((e^{(y/x)})*(e^{c2})))=0)$
 >>>

EXERCISE 9.4

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve each of them.

1. $(x^2 + xy) dy = (x^2 + y^2) dx$

2. $y' = \frac{x+y}{x}$

3. $(x - y) dy - (x + y) dx = 0$

4. $(x^2 - y^2) dx + 2xy dy = 0$

5. $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$

6. $x dy - y dx = \sqrt{x^2 + y^2} dx$

7. $\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$

8. $x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$

9. $y dx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0$

10. $\left(\frac{x}{1+e^y} \right) dx + e^y \left(1 - \frac{x}{y} \right) dy = 0$

>>> $(x-y)*\text{dif}(y) - (x+y)*\text{dif}(x) = 0$
 $((-1*((-1*(x+y)*\text{dif}(x))+((-1*y)+x)*\text{dif}(y)))=0)$
 >>> `difsolve`
 solving the differential equation $((-1*((-1*(x+y)*\text{dif}(x))+((-1*y)+x)*\text{dif}(y)))=0)$
 rewriting as $(((-1*((-1*y)+x)*\text{dif}(y))+((x+y)*\text{dif}(x)))=0)$
 rearranged as not found
 the solution is not found
 solving the differential equation $((-1*((-1*(x+y)*\text{dif}(x))+((-1*y)+x)*\text{dif}(y)))=0)$
 homogenous, substituting $z=y/x$

```

solving the differential equation (((-
1*diff(y)*(x^2))+(diff(x)*(y^2)*x)+(diff(x)*x)+(diff(y)*(x^2)*y))=0)
rewriting as (((-
1*diff(y)*(x^2))+(diff(x)*(y^2)*x)+(diff(x)*x)+(diff(y)*(x^2)*y))=0)
rearranged as ((((-1*diff(x)*(x^-1))+(1+(-1*y))*diff(y)*((1+(y^2))^-1)))=0)
integrating (-1*(x^-1)) wrt x
rewriting as (-1*(x^-1))
taking the constant outside of the integral
integrating (x^-1) wrt x
rewriting as (x^-1)
rewriting as (x^-1)
the solution is (log(abs(x))*(1^-1))
the solution is (-1*(log(abs(x))*(1^-1)))
integrating (-1*((1+(y^2))^-1)*y) wrt y
rewriting as (-1*((1+(y^2))^-1)*y)
taking the constant outside of the integral
integrating (((1+(y^2))^-1)*y) wrt y
rewriting as (((1+(y^2))^-1)*y)
rewriting as (((1+(y^2))^-1)*y)
substituting (1+(y^2))=z
integrating ((2^-1)*((2+(-1*z))^-1)) wrt z
rewriting as ((-2^-1)*((-2+z)^-1))
taking the constant outside of the integral
integrating ((-2+z)^-1) wrt z
rewriting as ((-2+z)^-1)
rewriting as ((-2+z)^-1)
the solution is (log(abs((-2+z)))*(1^-1))
the solution is ((-2^-1)*(log(abs((-2+(1+(y^2)))))*(1^-1)))
the solution is ((-2^-1)*(log(abs((-2+(1+(y^2)))))*(1^-1)))
the solution is (-1*((-2^-1)*(log(abs((-2+(1+(y^2)))))*(1^-1))))
integrating ((1+(y^2))^-1) wrt y
rewriting as ((1+(y^2))^-1)
rewriting as ((1+(y^2))^-1)
the solution is (arctan((y*((1^(2^-1))^-1)))*((1^(2^-1))^-1))
the solution is ((((-1*abs(x))+((abs((-1+(y^2))))^(2^-1)))*((e^arctan(y))*(e^c3))))=0)
((((-1*abs(x))+((abs((-1+((y/x)^2))))^(2^-1)))*((e^arctan((y/x)))*(e^c3))))=0)
>>>

```

EXERCISE 9.4

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve each of them.

1. $(x^2 + xy) dy = (x^2 + y^2) dx$

2. $y' = \frac{x+y}{x}$

3. $(x-y) dy - (x+y) dx = 0$

4. $(x^2 - y^2) dx + 2xy dy = 0$

5. $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$

6. $x dy - y dx = \sqrt{x^2 + y^2} dx$

7. $\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$

8. $x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$

9. $y dx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0$

10. $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$

```

>>> (x^2 - y^2)*dif(x) + 2*x*y * dif(y) = 0
((-1*((2*dif(y)*x*y)+((((-1*(y^2))+(x^2))*dif(x))))=0)
>>> difsolve
solving the differential equation (((-1*((2*dif(y)*x*y)+(((-1*(y^2))+(x^2))*dif(x))))=0)
rewriting as ((((-1*((-1*(y^2))+(x^2))*dif(x))+(-2*dif(y)*x*y))=0)
rearranged as not found
the solution is not found
solving the differential equation (((-1*((2*dif(y)*x*y)+(((-1*(y^2))+(x^2))*dif(x))))=0)
homogenous, substituting z=y/x
    solving the differential equation ((((-1*dif(x)*(x^2))+(-1*dif(x)*(x^2)*(y^2))+(-2*dif(y)*(x^3)*y))=0)
    rewriting as ((((-1*dif(x)*(x^2))+(-1*dif(x)*(x^2)*(y^2))+(-2*dif(y)*(x^3)*y))=0)
    rearranged as (((((2^-1)*dif(x)*(x^-1))+(dif(y)*((1+(y^2))^-1)*y))=0)
    integrating (((2^-1)*(x^-1)) wrt x
    rewriting as (((2^-1)*(x^-1))
        taking the constant outside of the integral
        integrating (x^-1) wrt x
        rewriting as (x^-1)
        rewriting as (x^-1)
        the solution is (log(abs(x))*(1^-1))
        the solution is ((2^-1)*(log(abs(x))*(1^-1)))
        integrating (((1+(y^2))^-1)*y) wrt y
        rewriting as (((1+(y^2))^-1)*y)
        rewriting as (((1+(y^2))^-1)*y)
        substituting (1+(y^2))=z
        integrating (((2^-1)*((2+(-1*z))^-1)) wrt z
        rewriting as (((-2^-1)*((-2+z)^-1))
            taking the constant outside of the integral
            integrating (((-2+z)^-1)) wrt z
  
```

```

rewriting as ((-2+z)^-1)
rewriting as ((-2+z)^-1)
the solution is (log(abs((-2+z)))* (1^-1))
the solution is ((-2^-1)*(log(abs((-2+(1+(y^2)))))* (1^-1)))
the solution is ((-2^-1)*(log(abs((-2+(1+(y^2)))))* (1^-1)))
the solution is ((-1+((abs((-1+(y^2)))^(-2^-1)))*(abs(x)^(2^-
1))* (e^c4)))=0)
((-1+((abs((-1+((y/x)^2)))^(-2^-1)))*(abs(x)^(2^-1))* (e^c4)))=0)
>>>

```

EXERCISE 9.4

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve each of them.

1. $(x^2 + xy) dy = (x^2 + y^2) dx$

2. $y' = \frac{x+y}{x}$

3. $(x - y) dy - (x + y) dx = 0$

4. $(x^2 - y^2) dx + 2xy dy = 0$

5. $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$

6. $x dy - y dx = \sqrt{x^2 + y^2} dx$

7. $\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$

8. $x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$

9. $y dx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0$

10. $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$

```

>>> x^2 * dif(y)/dif(x) = x^2 - 2*y^2 + x*y
((-1*dif(y)*(dif(x)^-1)*(x^2)) + (-2*(y^2)) + (x*y) + (x^2))=0)
>>> difsolve
solving the differential equation (((-1*dif(y)*(dif(x)^-1)*(x^2)) + (-
2*(y^2)) + (x*y) + (x^2))=0)
rewritting as (((-1*dif(y)*(x^2)) + (-
2*dif(x)*(y^2)) + (dif(x)*(x^2)) + (dif(x)*x*y))=0)
rearranged as not found
the solution is not found
solving the differential equation (((-1*dif(y)*(dif(x)^-1)*(x^2)) + (-
2*(y^2)) + (x*y) + (x^2))=0)
homogenous, substituting z=y/x
solving the differential equation (((-1*dif(y)*(dif(x)^-
1)*(x^3)) + (-2*(x^2)*(y^2)) + (x^2))=0)
rewritting as (((-1*dif(y)*(x^3)) + (-
2*dif(x)*(x^2)*(y^2)) + (dif(x)*(x^2)))=0)
rearranged as (((-1*dif(x)*(x^-1)) + (dif(y)*((1+(-2*(y^2)))^-
1)))=0)
integrating (-1*(x^-1)) wrt x
rewriting as (-1*(x^-1))
taking the constant outside of the integral
integrating (x^-1) wrt x
rewriting as (x^-1)
rewriting as (x^-1)

```

```

the solution is (log(abs(x)) * (1^-1))
the solution is (-1 * (log(abs(x)) * (1^-1)))
integrating ((1+(-2 * (y^2)))^-1) wrt y
rewriting as ((-2^-1) * ((-2 * (8^-(-2^-1))) * (((4^-1) * (8^(2^-1))) + y)^-1)) + (2 * (8^-(-2^-1)) * ((((-4^-1) * (8^(2^-1))) + y)^-1)))
    taking the constant outside of the integral
    integrating ((-2 * (8^-(-2^-1))) * (((4^-1) * (8^(2^-1))) + y)^-1) + (2 * (8^-(-2^-1)) * ((((-4^-1) * (8^(2^-1))) + y)^-1))) wrt y
        rewriting as ((-2 * (8^-(-2^-1))) * (((4^-1) * (8^(2^-1))) + y)^-1) + (2 * (8^-(-2^-1)) * ((((-4^-1) * (8^(2^-1))) + y)^-1)))
            rewriting as ((-2 * (8^-(-2^-1))) * (((4^-1) * (8^(2^-1))) + y)^-1) + (2 * (8^-(-2^-1)) * ((((-4^-1) * (8^(2^-1))) + y)^-1)))
                integating over sums
                integrating (-2 * (8^-(-2^-1)) * (((4^-1) * (8^(2^-1))) + y)^-1))
wrt y
    rewriting as (-2 * (8^-(-2^-1)) * (((4^-1) * (8^(2^-1))) + y)^-1)
        taking the constant outside of the integral
        integrating (((4^-1) * (8^(2^-1))) + y)^-1 wrt y
        rewriting as (((4^-1) * (8^(2^-1))) + y)^-1
        rewriting as (((4^-1) * (8^(2^-1))) + y)^-1
        the solution is (log(abs(((4^-1) * (8^(2^-1))) + y))) * (1^-1)
        the solution is ((-2 * (8^-(-2^-1))) * (log(abs(((4^-1) * (8^(2^-1))) + y))) * (1^-1)))
            integating over sums
            integrating (2 * (8^-(-2^-1)) * ((((-4^-1) * (8^(2^-1))) + y)^-1))
wrt y
    rewriting as (2 * (8^-(-2^-1)) * ((((-4^-1) * (8^(2^-1))) + y)^-1))
        taking the constant outside of the integral
        integrating ((((-4^-1) * (8^(2^-1))) + y)^-1) wrt y
        rewriting as ((((-4^-1) * (8^(2^-1))) + y)^-1)
        rewriting as ((((-4^-1) * (8^(2^-1))) + y)^-1)
        the solution is (log(abs((((-4^-1) * (8^(2^-1))) + y))) * (1^-1))
    the solution is ((2 * (8^-(-2^-1))) * (log(abs((((-4^-1) * (8^(2^-1))) + y))) * (1^-1)))
        the solution is ((((-2 * (8^-(-2^-1))) * (log(abs(((4^-1) * (8^(2^-1))) + y))) * (1^-1)) + ((2 * (8^-(-2^-1))) * (log(abs((((-4^-1) * (8^(2^-1))) + y))) * (1^-1))))
            the solution is ((-2^-1) * ((((-2 * (8^-(-2^-1))) * (log(abs(((4^-1) * (8^(2^-1))) + y))) * (1^-1)) + ((2 * (8^-(-2^-1))) * (log(abs((((-4^-1) * (8^(2^-1))) + y))) * (1^-1))))))
            the solution is ((((-1 * abs(x)) + ((e^(4^-1) * ((-4 * log(abs(((4^-1) * (8^(2^-1))) + y))) * (1^-1)) + ((2 * (8^-(-2^-1))) * (log(abs(((4^-1) * (8^(2^-1))) + y))) * (1^-1)))) * (8^-(-2^-1))) + (4 * log(abs(((4^-1) * (4 * y) + (8^(2^-1))))))) * (8^-(-2^-1))) * (e^c5))) = 0
            ((((-1 * abs(x)) + ((e^(4^-1) * ((-4 * log(abs(((4^-1) * ((-4 * (y/x)) + (8^(2^-1)))) * (8^-(-2^-1)))) + (4 * log(abs(((4^-1) * ((4 * (y/x)) + (8^(2^-1)))) * (8^-(-2^-1))))))) * (e^c5))) = 0
>>>

```

EXERCISE 12.1

Evaluate the following limits in Exercises 1 to 22.

1. $\lim_{x \rightarrow 3} x + 3$

2. $\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right)$

3. $\lim_{r \rightarrow 1} \pi r^2$

4. $\lim_{x \rightarrow 4} \frac{4x+3}{x-2}$

5. $\lim_{x \rightarrow -1} \frac{x^{10} + x^5 + 1}{x - 1}$

6. $\lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x}$

7. $\lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$

8. $\lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

9. $\lim_{x \rightarrow 0} \frac{ax+b}{cx+1}$

10. $\lim_{z \rightarrow 1} \frac{\frac{1}{z^3} - 1}{\frac{1}{z^6} - 1}$

11. $\lim_{x \rightarrow 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}, a + b + c \neq 0$

12. $\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$

13. $\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$

14. $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$

```

>>> x+3
(3+x)
>>> limit 3
limit x->3 (3+x) = 6
>>> pi * x^2
((x^2)*pi)
>>> limit 1
limit x->1 ((x^2)*pi) = pi
>>> (4*x+3)/(x-2)
((3+(4*x))*((-2+x)^{-1}))
>>> limit 4
limit x->4 ((3+(4*x))*((-2+x)^{-1})) = (19*(2^{-1}))
>>> ((x+ 1)^5 - 1)/x
((-1+((1+x)^5))*(x^{-1}))
>>> limit 0
limit x->0 ((-1+((1+x)^5))*(x^{-1})) = 5
>>> (3*x^2 - x - 10)/(x^2-4)
((-10+(-1*x)+(3*(x^2)))*((-4+(x^2))^ {-1}))
>>> limit 2
limit x->2 ((-10+(-1*x)+(3*(x^2)))*((-4+(x^2))^ {-1})) = (11*(4^{-1}))
limit x->0 (((x*a)+b)*((1+(x*c))^ {-1})) = None
>>> sin(a*x)/(b*x)
((x^{-1})*(b^{-1})*sin((x*a)))
>>> limit 0
limit x->0 ((x^{-1})*(b^{-1})*sin((x*a))) = ((b^{-1})*a)
>>> sin(a*x)/sin(b*x)
((sin((x*b))^ {-1})*sin((x*a)))
>>> limit 0
limit x->0 ((sin((x*b))^ {-1})*sin((x*a))) = ((b^{-1})*a)
  
```

15. $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

16. $\lim_{x \rightarrow 0} \frac{\cos x}{\pi - x}$

17. $\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$

18. $\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$

19. $\lim_{x \rightarrow 0} x \sec x$

20. $\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx} \quad a, b, a+b \neq 0,$ 21. $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$

22. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

```
>>> cos(x) / (pi - x)
(cos(x) * (((-1*x)+pi)^-1))
>>> limit 0
limit x->0 (cos(x) * (((-1*x)+pi)^-1)) = (pi^-1)
>>> (cos(2*x)-1) / (cos(x) - 1)
((-1+cos((2*x)))*((-1+cos(x))^-1))
>>> limit 0
limit x->0 ((-1+cos((2*x)))*((-1+cos(x))^-1)) = 4
>>> x*sec(x)
(sec(x)*x)
>>> trig0
((1*(cos(x)^-1))*x)
>>> limit 0
limit x->0 ((1*(cos(x)^-1))*x) = 0
```

9. Find the derivative of

(i) $2x - \frac{3}{4}$

(ii) $(5x^3 + 3x - 1)(x - 1)$

(iii) $x^{-3}(5 + 3x)$

(iv) $x^5(3 - 6x^{-9})$

(v) $x^{-4}(3 - 4x^{-5})$

(vi) $\frac{2}{x+1} - \frac{x^2}{3x-1}$

10. Find the derivative of $\cos x$ from first principle.

11. Find the derivative of the following functions:

(i) $\sin x \cos x$

(ii) $\sec x$

(iii) $5\sec x + 4\cos x$

(iv) $\operatorname{cosec} x$

(v) $3\cot x + 5\operatorname{cosec} x$

(vi) $5\sin x - 6\cos x + 7$

(vii) $2\tan x - 7\sec x$

```
>>> 2*x - 3/4
```

```
((-3*(4^-1))+(2*x))
```

```
>>> d/dx
```

```
2
```

```
>>> (5*x^3 + 3*x - 1)*(x - 1)
```

```
((-1+(3*x)+(5*(x^3)))*(-1+x))
```

```
>>> d/dx
```

```
(-1+(3*x)+(5*(x^3)) + ((-1+x)*(3+(15*(x^2)))))
```

```
>>> expand
```

```

(-4+(-15*(x^2))+(20*(x^3))+(6*x))
>>> x^(-3) * (5 + 3*x)
((5+(3*x))* (x^-3))
>>> d/dx
((-3*(5+(3*x))* (x^-4))+(3*(x^-3)))
>>> expand
((-15*(x^-4))+(-6*(x^-3)))
>>> x^5*(3 - 6*x^(-9))
((3+(-6*(x^-9)))* (x^5))
>>> d/dx
((5*(3+(-6*(x^-9)))* (x^4))+(54*(x^-5)))
>>> expand
((15*(x^4))+(24*(x^-5)))
>>> x^(-4) * (3 - 4*x^(-5))
((3+(-4*(x^-5)))* (x^-4))
>>> d/dx
((-4*(3+(-4*(x^-5)))* (x^-5))+(20*(x^-10)))
>>> expand
((-12*(x^-5))+(36*(x^-10)))
>>> 2/(x+1) - x^2/(3*x-1)
((-1*((-1+(3*x))^-1)*(x^2))+(2*((1+x)^-1)))
>>> d/dx
((-2*((-1+(3*x))^-1)*x)+(-2*((1+x)^-2))+(3*((-1+(3*x))^-2)*(x^2)))
>>> expand
((-2*((-1+(3*x))^-1)*x)+(-2*((1+x)^-2))+(3*((-1+(3*x))^-2)*(x^2)))
>>> sin(x)*cos(x)
(cos(x)*sin(x))
>>> d/dx
((-1*(sin(x)^2))+(cos(x)^2))
>>> fraction
(1+(-2*(sin(x)^2)))
>>> sec(x)
sec(x)
>>> trig0
(1*(cos(x)^-1))
>>> d/dx
((cos(x)^-2)*sin(x))
>>> 5*sec(x) + 4*cos(x)
((4*cos(x))+(5*sec(x)))
>>> trig0
((4*cos(x))+(5*(1*(cos(x)^-1))))
>>> d/dx
((-4*sin(x))+(5*(cos(x)^-2)*sin(x)))
>>> cosec(x)
cosec(x)
>>> trig0
(1*(sin(x)^-1))
>>> d/dx
(-1*cos(x)*(sin(x)^-2))
>>> 3*cot(x) + 5*cosec(x)
((3*cot(x))+(5*cosec(x)))
>>> trig0
((3*(cos(x)*(sin(x)^-1)))+(5*(1*(sin(x)^-1))))
>>> d/dx
(-3+(-3*(cos(x)^2)*(sin(x)^-2))+(-5*cos(x)*(sin(x)^-2)))
>>> 5*sin(x) - 6 * cos(x) + 7

```

```

(7+(-6*cos(x))+(5*sin(x)))
>>> d/dx
((5*cos(x))+(6*sin(x)))
>>> 2*tan(x)-7*sec(x)
((-7*sec(x))+(2*tan(x)))
>>> trig0
((-7*(1*(cos(x)^-1)))+(2*(sin(x)/cos(x))))
>>> d/dx
(2+(-7*(cos(x)^-2)*sin(x))+(2*(cos(x)^-2)*(sin(x)^2)))

```

4. For what values of x : $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$?

Example 25 Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that

$$CD - AB = O.$$

```

>>> [[x,0-5,0-1]]*[[1,0,2],[0,2,1],[2,0,3]]*[[x],[4],[1]]=0
([[(-48+(2*x)+((-2+x)*x))]]=0)
>>> factor
([[((-6+x)*(8+x))]]=0)
>>> mat
((((-6+x)=0) | ((8+x)=0)))
>>> [[2,5],[3,8]]*[[a,b],[c,d]]-[[2,0-1],[3,4]]*[[5,2],[7,4]]=0
([[(-3+(2*a)+(5*c)),((2*b)+(5*d))], [(-43+(3*a)+(8*c)),(-
22+(3*b)+(8*d))]]=0)
>>> mat
(((110+b)=0) & ((-44+d)=0) & ((-77+c)=0) & ((191+a)=0))

```

4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(i) (\csc \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$(ii) \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \csc \theta$$

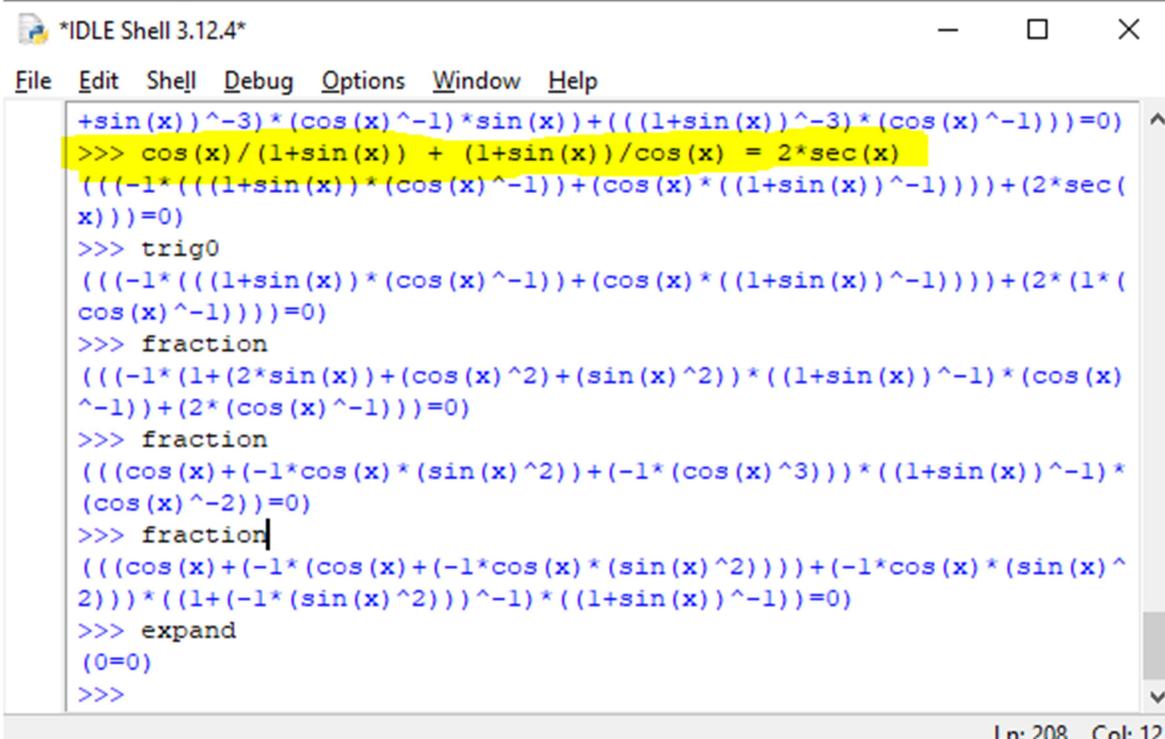
[Hint : Write the expression in terms of $\sin \theta$ and $\cos \theta$]

$$(iv) \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A} \quad [\text{Hint : Simplify LHS and RHS separately}]$$

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \csc A + \cot A, \text{ using the identity } \csc^2 A = 1 + \cot^2 A.$$

$$(vi) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A \quad (vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$(viii) (\sin A + \csc A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$



```
*IDLE Shell 3.12.4*
File Edit Shell Debug Options Window Help
+sin(x))^-3)*(cos(x)^-1)*sin(x))+((l+sin(x))^-3)*(cos(x)^-1)))=0
>>> cos(x)/(l+sin(x)) + (l+sin(x))/cos(x) = 2*sec(x)
((( -1*(((l+sin(x))*cos(x)^-1))+(cos(x)*((l+sin(x))^ -1))))+(2*sec(x)))=0)
>>> trig0
((( -1*(((l+sin(x))*cos(x)^-1))+(cos(x)*((l+sin(x))^ -1))))+(2*(1*(cos(x)^-1))))=0)
>>> fraction
((( -1*(1+(2*sin(x))+(cos(x)^2)+(sin(x)^2))*((l+sin(x))^ -1)*cos(x)^-1))+(2*(cos(x)^-1)))=0)
>>> fraction
(((cos(x)+(-1*cos(x)*(sin(x)^2))+(-1*(cos(x)^3)))*((l+sin(x))^ -1)*cos(x)^-2))=0)
>>> fraction
(((cos(x)+(-1*(cos(x)+(-1*cos(x)*(sin(x)^2)))))+(-1*cos(x)*(sin(x)^2)))*((1+(-1*(sin(x)^2)))^ -1)*((l+sin(x))^ -1))=0)
>>> expand
(0=0)
>>>
```

Ln: 208 Col: 12

4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(i) (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$(ii) \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

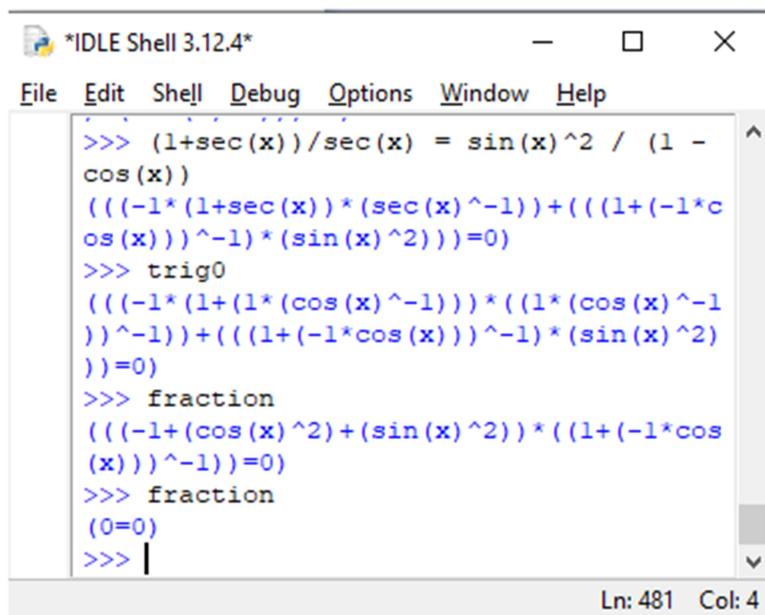
[Hint : Write the expression in terms of $\sin \theta$ and $\cos \theta$]

$$(iv) \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A} \quad [\text{Hint : Simplify LHS and RHS separately}]$$

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A, \text{ using the identity } \operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$(vi) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A \quad (vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$(viii) (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$



The screenshot shows an IDLE Shell window titled "IDLE Shell 3.12.4*". The menu bar includes File, Edit, Shell, Debug, Options, Window, and Help. The code input area contains the following Python code:

```

>>> (1+sec(x))/sec(x) = sin(x)^2 / (1 - cos(x))
((( -1 * (1+sec(x)) * (sec(x)^-1)) + (( (1+(-1*cos(x)))^-1) * (sin(x)^2)))=0)
>>> trig0
((( -1 * (1+(1*(cos(x)^-1))) * ((1*(cos(x)^-1))^ -1)) + (((1+(-1*cos(x)))^-1) * (sin(x)^2)))=0)
>>> fraction
((( -1+(cos(x)^2)+(sin(x)^2)) * ((1+(-1*cos(x)))^-1))=0)
>>> fraction
(0=0)
>>>

```

The status bar at the bottom right indicates "Ln: 481 Col: 4".

4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(i) (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$(ii) \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

[Hint : Write the expression in terms of $\sin \theta$ and $\cos \theta$]

$$(iv) \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A} \quad [\text{Hint : Simplify LHS and RHS separately}]$$

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A, \text{ using the identity } \operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$(vi) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A \quad (vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$(viii) (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

```

*IDLE Shell 3.12.4*
File Edit Shell Debug Options Window Help
>>> (cos(x) - sin(x) + 1)/(cos(x) + sin(x) - 1) = cosec(x) + cot(x)
((cosec(x)+cot(x)+(-1*(1+cos(x)+(-1*sin(x))))*((-1+cos(x)+sin(x))^-1)))=0
>>> trig0
(((1*(sin(x)^-1))+(cos(x)*(sin(x)^-1))+(-1*(1+cos(x)+(-1*sin(x))))*((-1+cos(x)+sin(x))^-1)))=0
>>> fraction
((((-1*sin(x))+((cos(x)^2)*sin(x))+(sin(x)^3))*((-1+cos(x)+sin(x))^-1)*(sin(x)^-2))=0
>>> fraction
((((-1*sin(x))+((1+(-1*(sin(x)^2)))*sin(x))+(sin(x)^3))*((-1+cos(x)+sin(x))^-1)*(sin(x)^-2))=0
>>> expand
(0=0)
>>>
Ln: 491 Col: 4

```

```

chat = i am your father
chat = you are mary
chat = am i a boy
yes
chat = what is my daughter's name
i
mary
chat = what is your father's name
you
chat = i am john
chat = what is your father's name
john
you
chat =

```

n5.py - D:\physicshuman\n5.py (3.12.4)

File Edit Format Run Options Window Help

```

import mathai
import parser
from base import *
command =\
"""
move 200
turn 135 pi/2+x
move 400
move -300
turn -90 -pi/2
box"""
box = []
linear = [[tree_form("d_0"), tree_form("d_1"), tree_form("d_250")]]
turn2 = parser.take_input("pi/2")
x = tree_form("d_0")
y = tree_form("d_-250")
prevturn = None
for item in command.split("\n"):
    tmp = item.split(" ")
    n = None
    if len(item.split(" "))>2:
        n = int(item.split(" ")[1])
    if tmp == "move":
        n = "d "+ str(n)
        turn = turn % 360
        m = turn // 90
        k = turn % 90
        slope = None
        if m == 0:
            x += tree_form(n)*parser.take_input("cos((k)*pi/180)")
            y += tree_form(n)*parser.take_input("sin((k)*pi/180)")
            slope = parser.take_input("tan((k)*pi/180)")
        elif m == 1:
            x += tree_form(n)*parser.take_input("~-sin((k)*pi/180)")
            y += tree_form(n)*parser.take_input("~-cos((k)*pi/180)")
            slope = parser.take_input("~-1/tan((k)*pi/180)")
        elif m == 2:
            x += tree_form(n)*parser.take_input("~-cos((k)*pi/180)")
            y += tree_form(n)*parser.take_input("~-sin((k)*pi/180)")
            slope = parser.take_input("~-tan((k)*pi/180)")
        elif m == 3:
            x += tree_form(n)*parser.take_input("~-1")
            y += tree_form(n)*parser.take_input("~-1")
            slope = None
    if turn2 != prevturn:
        if turn2 == "pi/2+x":
            mode("blue")
        else:
            mode("black")
        move(200)
        turn135 = turn + 135
        turn = turn % 360
        move(400)
        move(-300)
        turn = turn % 360
        turn(-90)
        box()
    prevturn = turn2
    turn2 = parser.take_input("pi/2")
    Ln: 17 Col: 15

```

