Astronomical Instruments Competition (Indian Knowledge Systems)

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Executive Summary

This report provides comprehensive documentation of seven astronomical instruments constructed by our team, that are originally found at Jantar Mantar observatories. Each instrument represents a sophisticated application of spherical trigonometry, geometric principles and observational astronomy methods developed within the Indian astronomical tradition rooted in classical texts such as the *Sūrya Siddhānta* and *Brāhmasphuṭasiddhānta*.

The instruments covered in this report (in order) are:

1. Rāma Yantra (Cylindrical Altitude-Azimuth Instrument)
2. Dakṣiṇottara Bhitti Yantra (Meridian Wall Instrument)
3. Ṣaṣṭhāṁśa Yantra (60-Degree Arc Pinhole Instrument)
4. Jaya Prakāśa Yantra (Hemispherical Zodiac Instrument)
5. Nāḍī Valaya (Equatorial Sundial)
6. Digaṁśa Yantra (Azimuth Instrument)
7. Samrāṭa Yantra (Equinoctial Sundial)

Part I: Theoretical Foundations

Spherical Trigonometry in Indian Astronomy

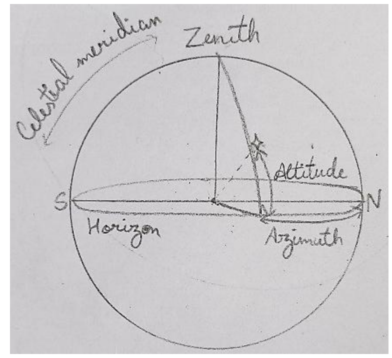
Historical Context

The *Sūrya Siddhānta*, dating to approximately the 5th century CE, described the following upon which all subsequent Indian astronomical calculations rest:

- **Spherical Earth:** Recognition of Earth's spherical shape
- **Trigonometric Functions:** Introduction of sine (*jyā*), cosine (*kojyā*), versine (*utkrama-jyā*), tangent, and secant functions
- **Spherical Geometry:** Methods for calculating positions of celestial bodies on the celestial sphere
- **Celestial Coordinates:** Techniques for converting between horizon and equatorial coordinate systems

Mathematical Framework

All instruments discussed in this report operate within the following coordinate systems:



1. Horizon-Zenith System (Altitude-Azimuth):

- **Azimuth (λ):** Angle measured eastward from north (0° to 360°)
- **Altitude (h):** Angle above the horizon (0° at horizon, 90° at zenith)
- **Zenith Distance (z):** Complementary angle to altitude, $z = 90° - h$

2. Equatorial System (Right Ascension-Declination):

- **Right Ascension (RA or α):** Angular distance east of the vernal equinox, measured along the celestial equator (0h to 24h)
- **Declination (δ):** Angular distance north or south of the celestial equator (-90° at south celestial pole to +90° at north celestial pole)
- **Hour Angle (H):** Angular distance measured westward from the meridian (0h to 24h)

3. Ecliptic System

- **Celestial Longitude (λ):** Angle along the ecliptic from the vernal equinox
- **Celestial Latitude (β):** Angle perpendicular to the ecliptic

Fundamental Transformation Equations

The conversion between horizon and equatorial coordinates is governed by the **spherical triangle formula**:

$$\sin(h) = \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(H)$$

Where:

- h = altitude
- φ = observer's latitude
- δ = declination
- H = hour angle

This relationship lies at the heart of all meridian-crossing observations and is explicitly or implicitly used by every instrument in this report.

As these instruments have been constructed for the geographical latitude of Roorkee (29°52'N ≈ 30°N), any object supposed to be inclined parallel to Earth's axis is inclined at 30° with the horizontal, and any object supposed to be inclined parallel to Earth's equator is inclined at 30° with the vertical (60° with the horizontal)

Part II: Individual Instruments

2.1 Rāma Yantra (Cylindrical Altitude-Azimuth Instrument)

2.1.1 Etymology and Designation

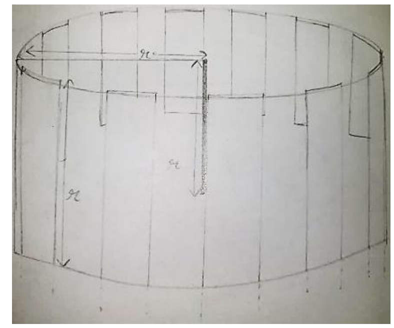
- *Rāma* = "the one who pleases", or a name of Bhagavān
- *Yantra* = "instrument"

The Rāma Yantra's name likely honors Jai Singh's ancestor Maharaja Ram Singh, as the instrument was special and of dual utility.

2.1.2 Functional Purpose

The Rāma Yantra measures a complete set of horizontal (altitude-azimuth) coordinates.

2.1.3 Design and Construction

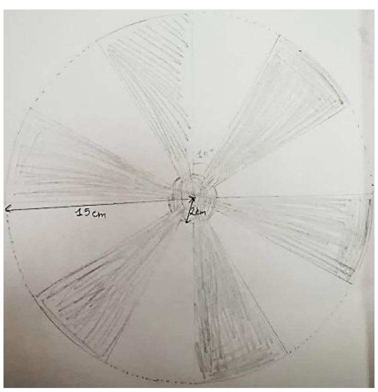


Structural Configuration: Twin Cylinders

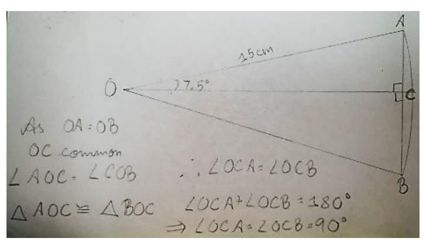
The Rāma Yantra has two identical but complementary structures built in a manner that only one of them is functional at a time. Each structure has a raised floor designed as 12 alternating sectors that end at a rectangular wall. The inner surfaces of the sectors and walls measure azimuth and altitude with the help of the shadow from a central pillar (gnomon). Radius of the sectors, and height of the wall and pillar above the floor are all equal.

Here, the construction of only one structure is given to avoid redundancy.

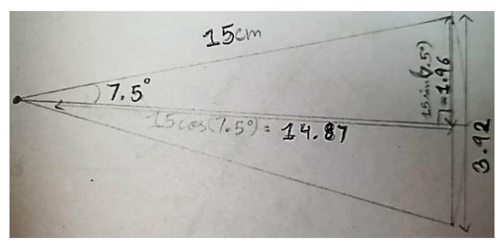
1. **Floor:**
    - A cardboard circle of radius 15 cm is divided into 24 equal sectors of 15° each
    - Leaving an inner circle of radius 2 cm intact, the large circle is cut such that alternate sectors remain attached to the centre and the immediately preceding and succeeding sectors are removed
    - The centre is pierced to later accommodate the central pillar.



Thought process to design the floor, actual structure has 12 sectors (shaded)



Dividing a sector into two congruent triangles

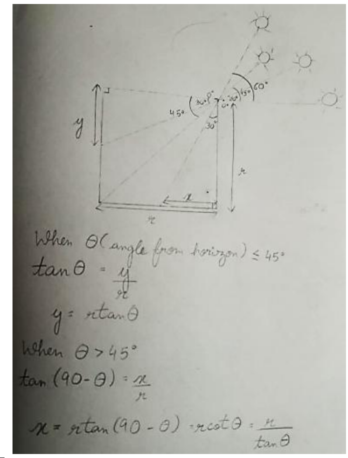


Length of edge obtained after cutting the arc

2. **Walls:**
    - 24 cardboard rectangles (12 for each structure) are cut with dimensions 20 cm x 3.92 cm
    - This is in accordance with the fact that the radially outward end of a sector is now 3.92 cm long, which will meet the surface of a wall

- After preparing the scale, the walls will be attached to the sectors such that 15 cm of vertical surface lies above the floor

3. **Scale:**
   Altitude
   - The trigonometric ideas in the picture on right side have been used to determine the height of the projection of shadow on the wall and the floor:
   - As it was decided to have a 1° resolution for both altitude and azimuth, a Python code was written to compute values for altitude in the range 0°-90° and export it in an Excel sheet:



Calculations for projection of shadow

```
import math
import pandas as pd
ang = 0 # Angle in degrees
r = 15  # Radius
data = []
for ang in range (0, 91):
    if ang <= 45:
        proj = math.tan(math.radians(ang))  # Projection component
        length = r*proj # Length of projection on surface
        data.append([ang, proj, length])
    else:
        proj = 1/math.tan(math.radians(ang))
        length = r*proj
        data.append([ang, proj, length])
df = pd.DataFrame(data, columns=['Angle in degrees', 'Projection Component',
'Projection in cm'])
df.to_excel('Projections.xlsx', index=False)
```

- This gave the following table:

Angle in degrees	Projection Component	Projection in cm	Projection in mm (rounded off to nearest 0.5 mm)
0	0	0	0
1	0.017455065	0.261825974	2.5
2	0.034920769	0.523811542	5
3	0.052407779	0.786116689	8
4	0.069926812	1.048902179	10.5
5	0.087488664	1.312329953	13
6	0.105104235	1.576563529	16
7	0.122784561	1.841768414	18.5
8	0.140540835	2.108112521	21
9	0.15838444	2.375766605	24
10	0.176326981	2.644904711	26.5
11	0.194380309	2.915704637	29
12	0.212556562	3.188348425	32
13	0.230868191	3.463022867	34.5
14	0.249328003	3.739920043	37.5
15	0.267949192	4.019237886	40
16	0.286745386	4.301180786	43
17	0.305730681	4.585960222	46
18	0.324919696	4.873795443	48.5
19	0.344327613	5.164914199	51.5
20	0.363970234	5.459553514	54.5
21	0.383864035	5.757960526	57.5
22	0.404026226	6.060393388	60.5
23	0.424474816	6.367122243	63.5

24	0.445228685	6.67843028	67
25	0.466307658	6.994614872	70
26	0.487732589	7.315988828	73
27	0.509525449	7.642881742	76.5
28	0.531709432	7.975641475	80
29	0.554309051	8.314635772	83
30	0.577350269	8.660254038	86.5
31	0.600860619	9.012909285	90
32	0.624869352	9.373040279	93.5
33	0.649407593	9.741113898	97.5
34	0.674508517	10.11762775	101
35	0.700207538	10.50311307	105
36	0.726542528	10.89813792	109
37	0.75355405	11.30331075	113
38	0.781285627	11.7192844	117
39	0.809784033	12.1467605	121.5
40	0.839099631	12.58649447	126
41	0.869286738	13.03930107	130.5
42	0.900404044	13.50606066	135
43	0.932515086	13.98772629	140
44	0.965688775	14.48533162	145
45	1	15	150
46	0.965688775	14.48533162	145
47	0.932515086	13.98772629	140
48	0.900404044	13.50606066	135
49	0.869286738	13.03930107	130.5
50	0.839099631	12.58649447	126
51	0.809784033	12.1467605	121.5
52	0.781285627	11.7192844	117
53	0.75355405	11.30331075	113
54	0.726542528	10.89813792	109
55	0.700207538	10.50311307	105
56	0.674508517	10.11762775	101
57	0.649407593	9.741113898	97.5
58	0.624869352	9.373040279	93.5
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67	0.424474816	6.367122243	63.5
68	0.404026226	6.060393388	60.5
69	0.383864035	5.757960526	57.5
70	0.363970234	5.459553514	54.5
71	0.344327613	5.164914199	51.5

72	0.324919696	4.873795443	48.5
73	0.305730681	4.585960222	46
74	0.286745386	4.301180786	43
75	0.267949192	4.019237886	40
76	0.249328003	3.739920043	37.5
77	0.230868191	3.463022867	34.5
78	0.212556562	3.188348425	32
79	0.194380309	2.915704637	29
80	0.176326981	2.644904711	26.5
81	0.15838444	2.375766605	24
82	0.140540835	2.108112521	21
83	0.122784561	1.841768414	18.5
84	0.105104235	1.576563529	16
85	0.087488664	1.312329953	13
86	0.069926812	1.048902179	10.5
87	0.052407779	0.786116689	8
88	0.034920769	0.523811542	5
89	0.017455065	0.261825974	2.5
90	6.12323E-17	9.18485E-16	0

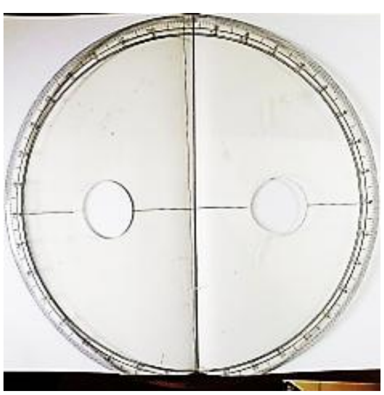
Note that the fourth column has been made by the user herself which contains values rounded off to nearest 0.5 mm due to the least count of the ruler available.

- In accordance with the table above, markings were made on all the 24 wall pieces covered in ivory paper
- For the altitude markings on the sectors, firstly shape in ivory paper was cut out for both the structures that exactly imitated the complicated floor design.
- Both pieces of paper were superimposed on each other such that the sectors of one fitted in the gaps of the other to give a complete circle
- A compass was used to draw circles with radius equal to each of the projection values

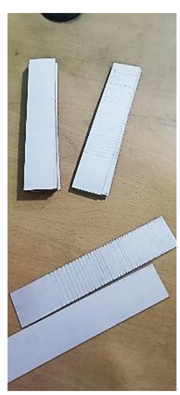
Note that it was possible to draw continuous circles despite the sectors having been chopped from the ends because of the largest projection value (14.5 cm) being smaller than the minimum radial distance after chopping (14.87 cm).

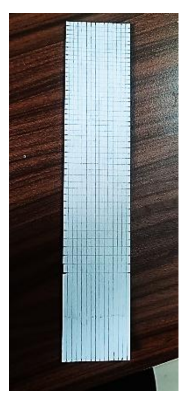
Azimuth

- A large protractor was used to mark 1° intervals clockwise on the (ivory paper) sectors in superimposed position
- A line was drawn for each division, connecting the mark with the centre and similarly the carboard walls were also marked by aligning them perpendicular to the sectors
- The respective paper pieces were then glued to the walls and the floor



Large protractor





4. **Assembly:**

- Each sector is glued perpendicularly to a wall piece using industry adhesive
- On a circular cardboard base of radius 16 cm, a circle of radius 15 cm is drawn and the combined wall+floor structure is carefully fixed on the boundary
- A thin cardboard ring of inner radius 14.8 cm and outer radius 15.3 cm is cut out and attached to the top of each structure, such that the walls are aligned in a perfect circle.
  This is necessary to bind those wall pieces that are bent at an angle
- A broomstick of length exactly 15 cm is slightly carved from one end (to obtain a fine point) and fixed at the centre of each structure using strong adhesive, acting as gnomon
- A thread is tied to the gnomon which can be used to operate for night sky objects
- Both the parts of the Rāma Yantra are glued on a wooden slab in a position such that their zero-azimuth marks point in the same direction

2.1.4 Dimensional Specifications

Parameter	**Specification**
Instrument Radius	15 cm
Instrument Height	20 cm
Primary Altitude Division	5°
Secondary Altitude Division	1°
Primary Azimuth Division	5°
Secondary Azimuth Division	1°
Minimum Resolution	1°
Average Error	0.54%

2.1.5 Operational Procedure

The life-size Rāma Yantra has empty spaces between two consecutive sectors to allow the observer to stand inside while recording the position. To ensure accuracy with convenience, a sector begins at the azimuth position where the just preceding sector of the complementary yantra ended.

- As the shadow/line of sight will meet the scale in a direction opposite to that of the celestial object, align the zero of the yantra with the South direction (instead of 0° North)
- Record the altitude and azimuth at the point where the tip and centre of the shadow falls, respectively
- Similar process for any night sky object, with the shadow replaced by line of sight

2.1.6 Data from Observations

Date	Time	Measured Value	Actual Value	Percentage Error	Average Error for a Set
16/12/2025	12:39 PM	**Alt** = 37° **Azi** = 188°	**Alt** = 36.5° **Azi** = 187.5°	**Alt** = 1.37% **Azi** = 0.27%	0.82%
18/12/2025	01:02 PM	**Alt** = 35.5° **Azi** = 192.5°	**Alt** = 35.5° **Azi** = 193.5°	**Alt** = 0.00% **Azi** = 0.52%	0.26%

2.1.7 Sources and References

[1][2][3]

2.2 Dakṣiṇottara Bhitti Yantra (Meridian Wall Instrument)

2.2.1 Etymology and Historical Context

The name *Dakṣiṇottara Bhitti Yantra* derives from Sanskrit:

- *Dakṣiṇa* = South
- *Uttara = North*
- *Bhitti* = Wall
- *Yantra* = Instrument

Literally, it is "the instrument that is a wall running from south to north." This was typically the **second instrument** constructed at an observatory, immediately after establishing the meridian line.

2.2.2 Functional Purpose

The Dakṣiṇottara Bhitti Yantra serves to measure:

1. **Altitude (h)** of celestial objects when they cross the local meridian
2. **Zenith Distance (z)** of the Sun and stars at meridian transit

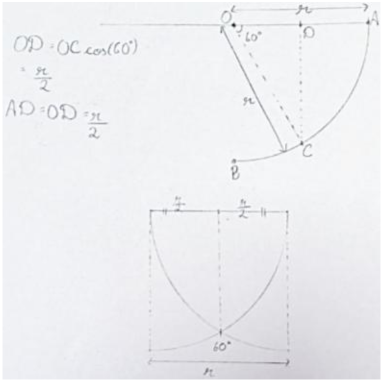
Additionally, this instrument provides **calibration reference** for other instruments at the observatory, ensuring their alignment is correct.

2.2.3 Design and Construction

Structural Components:

1. **Meridian Wall (Bhitti):**

- A vertical cardboard rectangle of 40cm x 30cm
- Flat surface with sufficient thickness (5 mm) to support shadow marking and measurement



2. Quadrant Scale (East-facing):

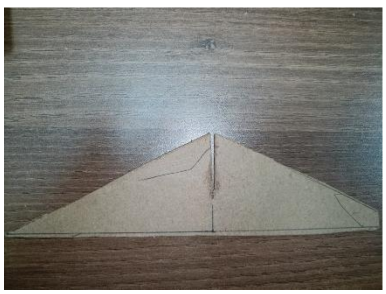
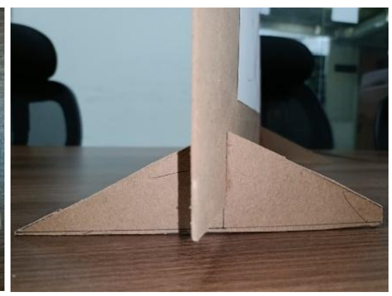
- Two quarter arcs of radius = 15 cm facing each other, intersecting each other at 60-degree mark
- Made on ivory sheet like a semicircle divided into half, and resulting arcs shifted such that one starts at the distance where the other ends (distance between the starting points = radius)
- Each arc divided into twelve 5-degree segments, further divided into 10 divisions each

3. Semi-circular Scale (West-facing):

- A large semicircular arc of radius 15 cm cut out of ivory sheet
- Divided into 0.5-degree angular segments, with distinct markers for every 5 and 10 degrees
- Center point of the arc at zenith

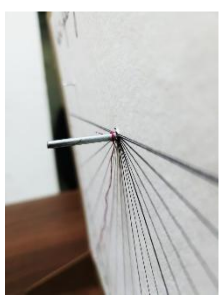
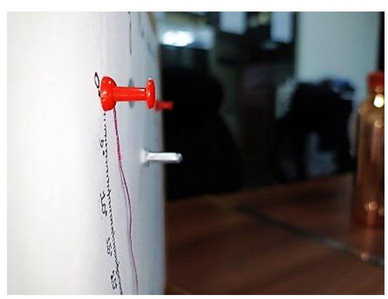
4. Supporting Stand:

- Two triangles 18-20 cm long, inserted at ~4.5 cm distance from the lower ends of the wall
- A slit of roughly 3.5 cm carved in the triangles as well as the wall for the supports to fit in

5. Shadow-Casting Elements (Gnomons):

- Two board pins at the upper ends of the quadrants, perpendicular to the wall
- A thin rod located at the center of semicircle, perpendicular to the wall
- A thread tied to it for, in the case of the Sun, extrapolating the shadow to meet the arc and, in the case of other celestial objects, physically generating the line of sight

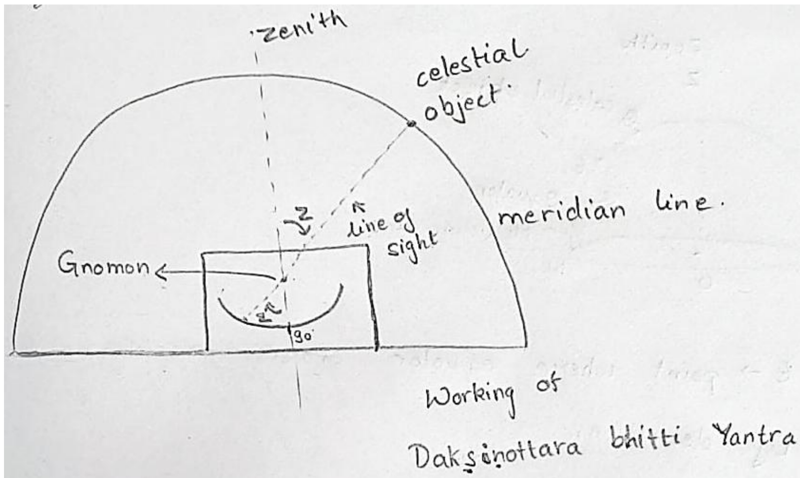



Parameter	**Specification**
Wall Height	30 cm
Wall Length	40 cm
Wall Thickness	0.5 cm
Arc Radius	15 cm

Minimum Resolution	0.5 degrees
Error	2.75%

2.2.4 Mathematical Principle: Altitude-Zenith Distance Relationship

The value read gives the altitude of the object. The mechanism behind this phenomenon is shown below:



When a celestial object crosses the meridian, its altitude directly corresponds to the shadow cast by the gnomon. The relationship is:

$$z = 90° - h$$

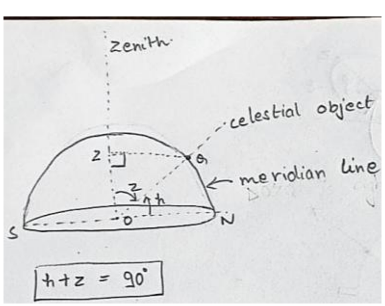
Where z is the zenith distance measured along the quadrant arc.

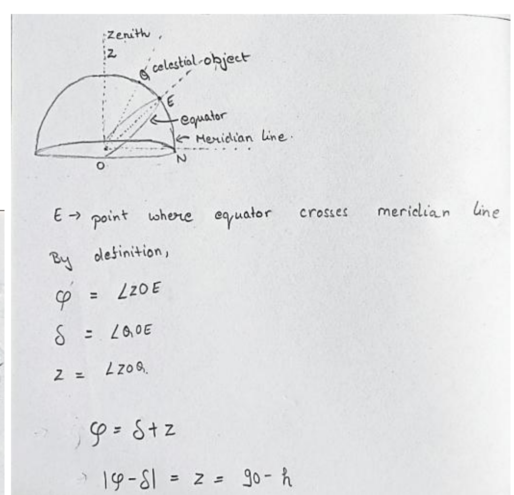
For the Sun on any given day, with declination δ and observer latitude φ:

$$|\varphi - \delta| = 90° - h_{\mathrm{Sun}}$$

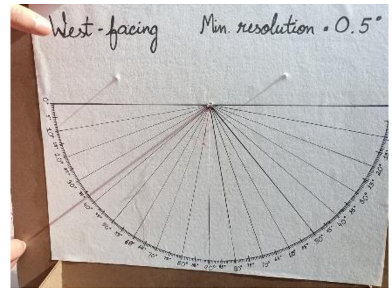
This simple yet elegant relationship means the Dakṣiṇottara Bhitti Yantra can determine position without requiring trigonometric calculations. The observer simply reads the arc at the point where the shadow/line of sight meets.

The formulae arrive from simple geometry applied to the horizon-zenith system. We consider the vertical plane passing through zenith, observer and meridian line. The diagrams given below here give a clear picture:





2.2.5 Operational Procedure



1. Ensure the wall is precisely oriented north-south (use astronomical observation or precise compass)
2. Observer's eye must be at the same level as the zenith point on the wall
3. Use the thread to extrapolate the shadow or line of sight, and record the value of altitude on scale

2.2.6 Data from Observations

Date	Time	Measured Value	Actual Value	Percentage Error
16/12/2025	11:58 AM	37°	36.5°	1.37%
16/12/2025	12:54 PM	38°	36°	5.56%
18/12/2025	12:13 PM	36°	37°	2.70%
18/12/2025	12:49 PM	36.5°	37°	1.35%

2.2.7 Sources and References

[4][5][6]

2.3 Ṣaṣṭhāṁśa Yantra (60-Degree Arc Pinhole Instrument)

2.3.1 Etymology and Designation

- *Ṣaṣṭha* = "sixth"
- *Aṁśa* = "part," "division," or "degree"
- *Yantra* = "instrument"

The name literally means "the sixth part instrument" or "60-degree instrument," referring to the 60° arc that forms its measurement scale.

2.3.2 Functional Purpose

The Ṣaṣṭhāṁśa Yantra is designed to measure three critical solar properties at meridian transit:

1. **Declination (δ):** The Sun's angular distance from the celestial equator
2. **Zenith Distance (z):** The Sun's angular distance from the observer's zenith at meridian crossing
3. **Solar Diameter (θ):** The angular size of the Sun's disk as observed from Earth

2.3.3 Design and Construction

Core Concept: Pinhole Projection

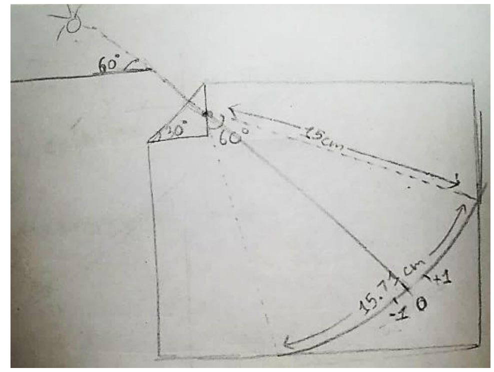
The Ṣaṣṭhāṁśa Yantra operates on the principle of **pinhole projection**, where sunlight passes through a small aperture and projects an inverted image onto a screen.

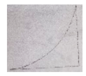
Structural Components:

1. **Measurement Scale (60° Arc):**

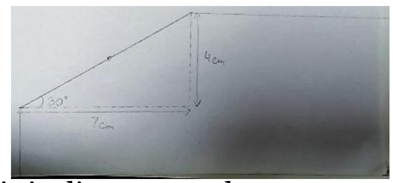
   - A paper strip 15.8 cm long and 10 cm wide is cut in accordance with the following calculation

     $$l = r\,\theta$$

     Where, l is arc length
     r is radius
     θ is the angle subtended at the arc in radians, 60° in this case
     So we have, $l = 15 \times \frac{\pi}{3} = 5\pi \approx 15.71$
   - It is to be arranged such that one end is parallel to the ground and the other touches the wall behind
   - Two curvilinear triangles  are made from paper to support the scale at its left and right edges
   - The centre of the scale is marked as 0°, with marks upwards upto 30°S and downwards upto 30°N

2. **Chamber Box:**

   - A cardboard box of length 16.5 cm, width 16.9 cm and height 17 cm is used to create the chamber
   - An inclined plane of 30° is slashed out at the top such that centre of this incline acts as the centre of the circle which the arc is a part of
   - The supports are glued such that they lie along the backwall of the chamber, and are at a distance of 5 cm from the centre
   - The scale is fixed on these supports

3. **Pinhole Camera:**

   - The top surface of the box is covered with paper and the centre of the inclined plane is pierced
   - The 30° angle is chosen so that the plane becomes perpendicular to the equator and direct sunlight can be received. This will lead to maximum light intensity and brighter image
   - Make sure that the hole is pierced with a needle and is not more than 1 mm wide
   - A cavity is made on the top above the scale, large enough for an observer to see through
   - The observation window is covered with black cloth, pasted at the edge of the incline

Note: Ensure that there is no opening in the box except the pinhole and the observation window. Leakage of light into the chamber from any other point can wash out the bright spot of sunlight.

2.3.4 Dimensional Specifications

Parameter	Specification
Box Length	16.5 cm
Box Width	16.9 cm
Box Height	17 cm
Inclination of Aperture Plane	30°
Aperture Diameter	~0.9 mm
Arc Radius (distance from aperture to scale)	15 cm

Scale Division	1°
Maximum Resolution	1°
Error	5.32%

2.3.5 Mathematical Principle

Relationship Between Altitude and Declination:

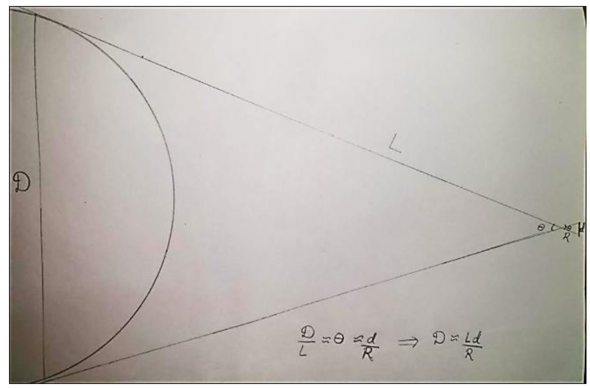
At meridian transit, the Sun's altitude is:

$$h = 90° - |\varphi - \delta|$$

Where ϕ is the observer's latitude and δ is the Sun's declination. Equivalently, the zenith distance is:

$$z = |\varphi - \delta|$$

Solar Diameter



When sunlight passes through a small circular aperture, it creates a circular image of the Sun on a screen at a distance R from the aperture. The relationship between the angular diameter θ (in radians) and the projected image diameter d is:

$$\theta = \frac{d}{R} \text{ (in radians)}$$

Using the relation in the picture, we can calculate the diameter of the Sun if the distance L between Earth and Sun is known:

$$D \approx \mathrm{L} \times \frac{\mathrm{d}}{R}$$

The standard value of Earth-Sun distance is 1 AU = 1.5×10^{11} meters. Thus,

$$D \sim 10^{12} \times d$$

Note: As it is not possible to do inside the chamber that we have built due to its size, the size of shadow and thus, the diameter of the Sun cannot be measured.

2.3.6 Operational Procedure

- Take the device out in sun around the time of solar noon. Its pinhole should face the South.
- Carefully lift the black cloth to look through the cavity on the top, ensuring that the pinhole is unobstructed and any other open spaces covered by the cloth.
- Record the reading on the scale.

2.3.7 Data from Observations

Date	Time	Measured Declination	Actual Declination	Percentage Error
18/12/2025	12:36 PM	-24.5°	-23.5°	4.26%
18/12/2025	01:18 PM	-25°	-23.5°	6.38%

2.3.8 Sources and References

[7][8]

2.4 Jaya Prakāśa Yantra (Hemispherical Zodiac Instrument)

2.4.1 Etymology and Significance

- *Jaya* = "victory, success", also the name of Maharaja Jai Singh
- *Prakāśa* = "light, illumination"
- *Yantra* = "instrument"

The name *Jaya Prakāśa Yantra* means "the instrument of Jai's light". According to tradition, this complex instrument was designed by Maharaja Jai Singh as his personal triumph in astronomical engineering—representing a "victory" in translating the celestial sphere into a functional terrestrial instrument.

2.4.2 Functional Purpose

The Jaya Prakāśa Yantra measures a complete set of celestial coordinates:

1. **Azimuth (λ):** Angle measured eastward from north (0° to 360°)
2. **Hour Angle (H):** Time-based angular measurement
3. **Zodiac sign (Rāśi):** Zodiac in which sun is currently present according to **Sāyana Sūrya**.
4. **Ascendent (Lagna):** Zodiac currently present at east point on the horizon (changes approximately every 2 hours)

Sāyana Sūrya (tropical system) refers to astronomical system where the ecliptic plane is divided into 12 zodiacs with reference to **vernal equinox** rather than being tied to a particular constellation (which changes slightly every year due to **precession of equinoxes** and completes one cycle in approximately 26,000 years).

2.4.3 Design and Construction

Structural Concept: Twin Hemispheres

The Jaya Prakāśa Yantra at Jaipur consists of **two complementary hemispherical bowls**, each working at alternate hours. The instrument was so designed to facilitate easy observations and maintenance of the instrument.

However, we are working on a small scale and hence do not need these sophistications. To make the instrument more readable, we use two hemispheres, one for Zodiac sign and Ascendent, and the other for hour angle or celestial longitudes. These hemispheres are:

- **Positioned:** Side by side, separated by 11 cm
- **Orientation:** Inverted position (with opening facing upward)
- **Structural Support:** Resting on a wooden platform with plastic legs (made from pipe)

Key Physical Components:

1. **Hemispherical Shells:**
    - We took a hollow spherical plastic ball and bisected it into two equal hemispherical parts.

- Diameter at rim: ~16.2 cm

2. **Celestial Coordinate Engravings:**
   - **Azimuth-Altitude System** on one hemisphere:
     - Center point at bottom of hemisphere = zenith
     - Rim = horizon (0° altitude)
     - For azimuth angle, a circular ring having degree marking (with maximum resolution of 1°) is attached. Equal azimuth line can be imagined till the shadow of cross-wire to calculate azimuth angle of the sun.
     - The circular ring was made by cutting a ring of ivory sheet having inner radius as 8.1 cm and outer one as 15 cm. Angles were marked using a protractor .
     - Altitude angle markings are omitted in our yantra due to small size and also because of its presence in other instruments.
   - **Hour lines** on complementary hemisphere:

     - Meridian line was drawn passing from zenith and north and south points of the horizon by placing a circle of 8.1 cm passing through these points, as shown in the figure on left.
     - A point 30° (Roorkee's latitude) below the south point on the horizon was marked as center on the meridian line. This corresponds to the celestial pole.
     - A circle with radius equivalent to maximum declination of sun (23.5°) was drawn. This corresponds to ecliptic circle, dividing the circle into 12 equal parts corresponding to 12 zodiacs of our consideration.
     - Hour lines: 15° apart (representing 24 hours) were drawn emanating from the center again by placing a circular cutout of 8.1 cm. These lines also correspond to celestial longitudes. The lines are to be drawn as shown in the figure on right.

     - Zodiac inscriptions: 12 zodiac signs marked at their ecliptic positions (12 parts on the ecliptic circle)

3. **Zodiac signs and ascendent markings:**

   i. A circle of 8.1 cm was cut and angle markings were made on it with maximum resolution of 1° as shown in the figure on left.

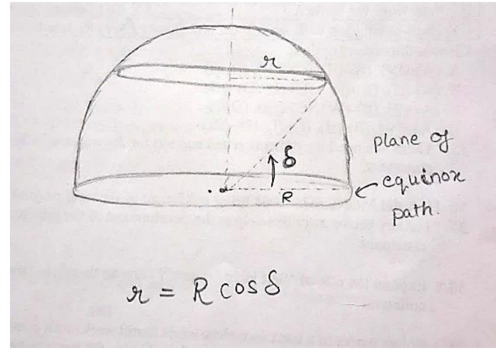
   ii. The circle was placed along the meridian line and points at angle 30°, 41.5°, 50°, 53.5°, 18.5°, 10°, 6.5° were marked on the inner surface of hemisphere at zenith distance points. (for explanation, see 2.3.5)

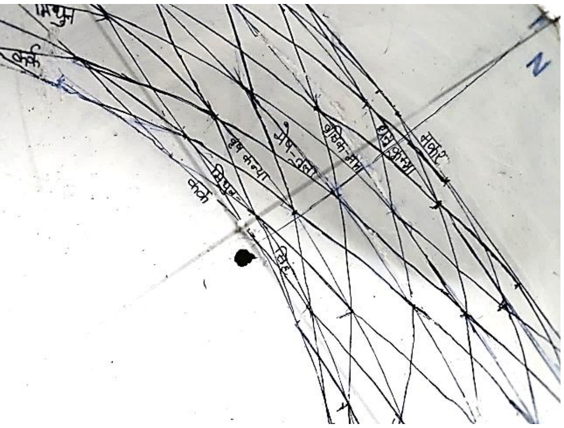
   iii. 4 circles with radius 8.2 cm, 7.9 cm, 7.6 cm and 7.4cm, respectively, were cut. The radius was obtained using the formula:

$$r = R \cos\delta$$

where r = required radius and R = radius of the hemisphere.

The formula is obtained from normal trigonometry as shown here:

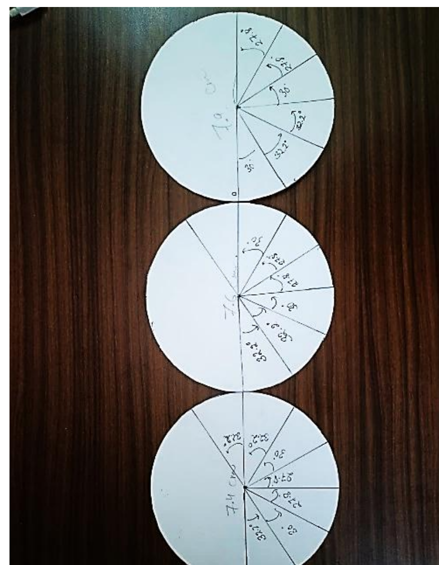




iv. For some particular declination angles of the sun (when it enters new zodiac), markings were made by placing the circle corresponding to that angle on the inner surface of hemisphere passing through the zenith distance point (marked in step ii), rising azimuth angle and setting azimuth angle of the sun on that day. (for data, see 2.3.5)

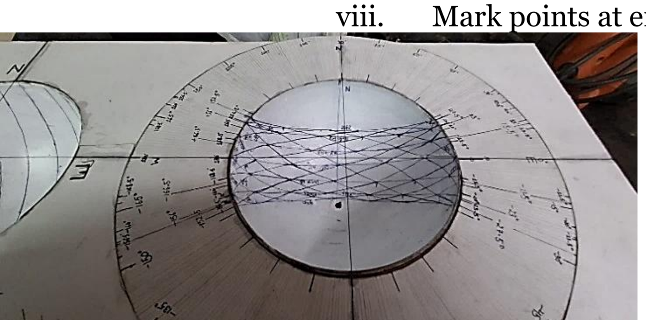
v. The lines so marked were labelled with the names of the zodiac sun is entering when it passes through them, as shown in the left.

vi. On the corresponding circle of each line, mark angles (given in section 2.3.5) for each zodiac in the order they appear, starting from the sign sun enters on that day as shown in the picture on right.



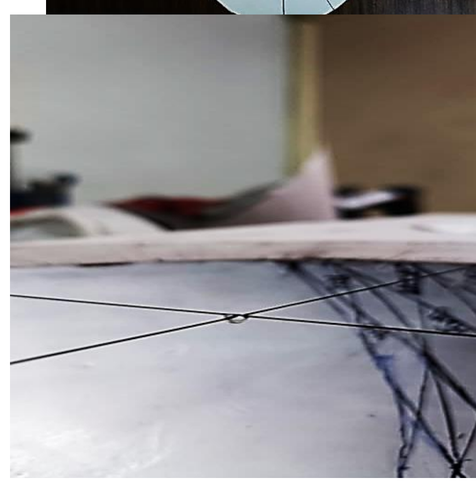
vii. Now place these circular cutouts on their corresponding lines, with the starting point of the angle of rising ascendant of that day coinciding with the west most point on the line.

viii. Mark points at end of every angle marking on the line (for explanation, see 2.3.5).



ix. Now join those points which mark the beginning of same zodiac on adjacent lines (made earlier) to make a long continuous line. Label the so formed line with the name of zodiac they correspond to as shown in the left.

4. **Cross-Wire System:**
   - Four metal wires stretched across the rim
   - North-South wire: Represents the meridian
   - East-West wire: Represents the equatorial line
   - The intersection point casts a sharp shadow for calculation



2.4.4 Dimensional Specification

Parameter	**Specification**
Hemisphere Diameter	16.2 cm
Scale Division Increment	15° (primary), 1° (secondary)
Least Count	1°

2.4.5 Mathematical Foundations: Spherical Geometry

The Celestial Sphere Representation:

The Jaya Prakāśa represents the **celestial sphere inverted**. It represents mirror image of the sky as it appears from Earth.

The shadow of sun corresponds to its position in the sky whose map is marked on the inverted bowl and can be easily read as shown in the below figure.

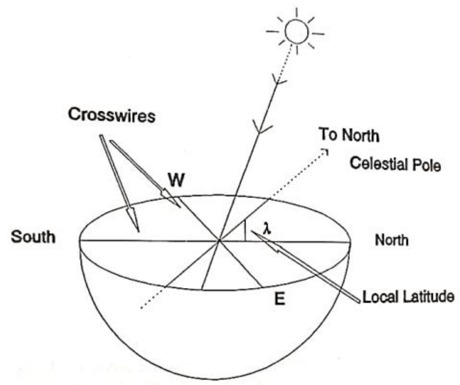


Fig. 4-1 The Principle of Jaya Prakāśa and Kapāla Yantras.

Source : "Maharaja Sawai Jai Singh and his Astronomy" by Virendra Nath Sharma.

Zenith-Horizon Relationship:

- **Zenith (Z):** Located at the lowest point (bottom center) of the hemispherical bowl
- **Horizon (H):** Traced along the rim of the bowl

Sun's path across the year and corresponding zodiac:

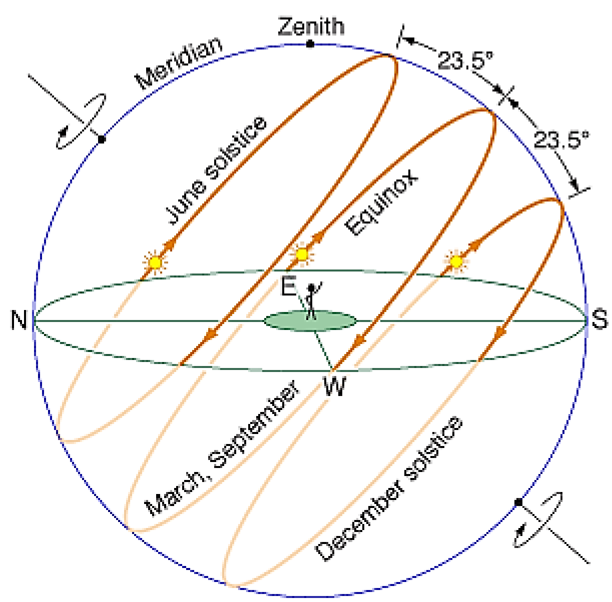
According to tropical system, the day of the year (Gregorian calendar) determines the zodiac sun is present in as per the following table (since in this system zodiacs are fixed with respect to vernal equinox which falls on 21 March every year):

Zodiac Sign	Dates (approximate)
Aries (Meṣa)	March 21 – April 19
Taurus (Vṛṣabha)	April 20 – May 20
Gemini (Mithuna)	May 21 – June 21
Cancer (Karka)	June 22 – July 22
Leo (Siṃha)	July 23 – August 22
Virgo (Kanyā)	August 23 – September 22

Libra (Tulā)	September 23 – October 23
Scorpio (Vṛścika)	October 24 – November 21
Sagittarius (Dhanu)	November 22 – December 20
Capricorn (Makara)	December 21 – January 19
Aquarius (Kumbha)	January 20 – February 18
Pisces (Mīna)	February 19 – March 20

The exact dates can vary slightly by a day or so each year. This is due to the Earth's orbit and the leap year cycle.

The sun does not rise exactly in the east every day, rather it oscillates around east (where it rises on equinoxes) to rise at the northernmost point on summer solstice and southernmost point on winter solstice as shown in the figure below:



Source : physics.weber.edu

The plane of path of the sun over a day is however, parallel to **equatorial plane**.

On equinoxes sun follows the path of celestial equator which is marked by Meṣa- Tulā line in our instrument.

The celestial equator passes through three points: East and West on the Horizon and 30° (Roorkee's latitude) below the zenith towards the southern end on the meridian line.

The path of the sun on any day can be traced by finding three points viz. rising and setting azimuth and its declination on that day.

Using the below formula, zenith distance can be found:

$$|\varphi - \delta| = z$$

Given below is the data for dates at which the zodiac of sun changes, and the construction of these paths is already explained.

Month	Declination (°)	Sunrise Azimuth (° from E +ve towards N)	Sunset Azimuth (° from W +ve towards N)	Zodiac Sign (Tropical)
Jan 20	~ −20.0°	~-22.9	~-22.9	**Kumbha**
Feb 19	~ −11.5°	~-13.3	~-13.3	**Mīna**
Mar 21	~ 0°	~0	~0	**Meṣa**
Apr 20	~ +11.5°	~13.3	~13.3	**Vṛṣabha**
May 21	~ +20.0°	~22.9	~22.9	**Mithuna**
Jun 22	~ +23.5°	~27.3	~27.3	**Karka**
Jul 23	~ +20.0°	~22.9	~22.9	**Siṃha**
Aug 23	~ +11.5°	~13.3	~13.3	**Kanyā**
Sep 23	~ +0°	~0	~0	**Tulā**
Oct 24	~ −11.5°	~-13.3	~-13.3	**Vṛścika**
Nov 22	~ −20.0°	~-22.9	~-22.9	**Dhanu**
Dec 21	~ −23.5°	~-27.3	~-27.3	**Makara**

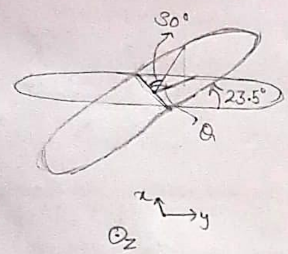
The values here are approximated for simplicity. The data is observational and directly taken from internet.

Ascendant:

Due to rotation of earth about its own axis, it seems to an observer on earth that the ecliptic plane is rotating (completing one cycle in 24 hours) causing the 12 zodiacs to change their position continuously in the celestial sphere.

Due to this every zodiac appears on the east point on the horizon for approximately 2 hours and is called the ascendant for that time and place.

However, since the ecliptic plane is tilted at an angle (23.5°) to celestial equator and the rotation is occurring along the celestial equator, the duration of each ascendant is not constant. The projection of each zodiac angle on the celestial equator is calculated as shown below to get the exact duration of each ascendant.



Along x axis, length

$= r\cos 30°$

Along y axis, length

$= r \sin 30° \cos 23.5°$

$$\therefore \quad \tan\theta = \frac{r \sin 30° \cos 23.5°}{r \cos 30°} = \tan 30° \cos 23.5°$$

Similarly do the above steps for other angles

Zodiac Sign	**Angles**
Aries (Meṣa)	27.8°
Taurus (Vṛṣabha)	30°
Gemini (Mithuna)	32.2°
Cancer (Karka)	32.2°
Leo (Siṃha)	30°
Virgo (Kanyā)	27.8°
Libra (Tulā)	27.8°
Scorpio (Vṛścika)	30°
Sagittarius (Dhanu)	32.2°
Capricorn (Makara)	32.2°

Aquarius (Kumbha)	30°
Pisces (Mīna)	27.8°

The zodiac sun is present in rises along with the sun on east point on the horizon and hence is the rising ascendant of that day.

The days, on which sun enters new zodiac, experience the complete duration of rising zodiac only after sunrise and hence taken as reference in our yantra for marking ascendants on these days.

2.4.6 Operational Procedure

Align the Yantra along the correct cardinal directions as mentioned on it.

Phase 1: For zodiac and ascendent

1. At any time of the day locate the shadow of the cross wire
2. The line of zodiac the sun has last crossed in the year represents one of the two possible zodiacs of the day (to find the exact zodiac, one need to know the next line that sun would reach in the year)
3. The line of the ascendant the sun has last crossed in the day represents the current ascendent of the day
4. For better demonstration, see the attached video

Phase 2: Hour of the day

1. At any time of the day locate the shadow of the cross wire
2. The line of hour the sun has last crossed in the day represents the current hour or the celestial longitude sun has recently crossed

2.4.7 Observation

Date	Time	Zodiac Observed	Lagna Observed	Actual Zodiac	Actual Lagna
18/12/2025	03:42 PM	Dhanu	Vṛṣabha	Dhanu	Vṛṣabha

2.4.8 Sources and References

[9][10][2][8]

2.5 Nāḍī Valaya (Equatorial Sundial)

2.5.1 Etymology and Historical Context

- *Nāḍī* = a unit of time ≅ 24 minutes
- *Valaya* = "circle", "ring"
- *Yantra* = "instrument"

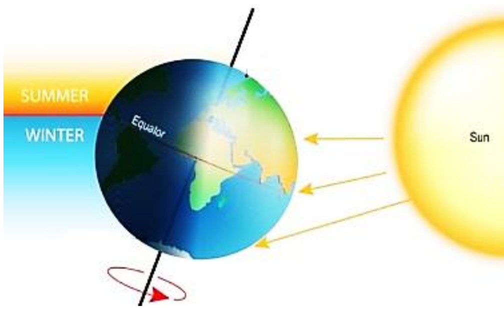
The name means "the time-ring instrument." The Nāḍī Valaya is an equatorial sundial, which means that it is designed in such a manner that the dial plates are parallel to Earth's equator and the shadow of the gnomon covers equal area in equal time intervals.

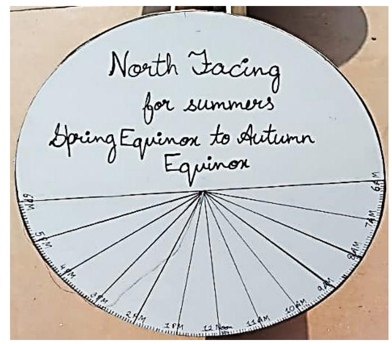
2.5.2 Functional Purpose

The Nāḍī Valaya measures:

1. **Local Solar Time**
2. **Hour Angle (H)**
3. **Noon Time**

For northern hemisphere, the Sun travels towards the south of equator due to the Earth's tilt away from the Sun during winter. This starts at the autumn equinox and ends at the spring equinox. Similarly, from spring equinox to autumn equinox, the Sun stays north of the equator. The plates are consequently constructed in a manner that only one of the plates works during one season.





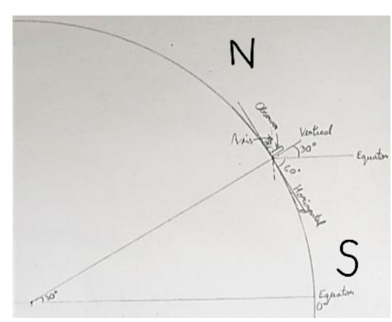
https://www.almanac.com/reason-seasons

The plates are parallel to the equator. The Sun stays on one side of the equator for half of the year, and on the other side of the equator for the other half of the year. Thus, only one of the plates is illuminated at a time.

2.5.3 Design and Construction

Structural Components:

1. **Equatorial Plates (Two):**
    - Two circular cardboard plates of radius 15 cm, one to face North and the other South
    - Plates covered with ivory paper, lower semi-circle used for marking time from 6 AM to 6 PM
    - The semi-circle is divided into 12 equal sectors, further divided into 15 equal parts of 1° each
    - Note that the direction of time will reverse for each plate as both face different directions
2. **Inclination Angle:**
    - The inclination angle of the plates and gnomons are determined in accordance with the calculations shown in the following diagram
    - As the plates have to be parallel to the equator, they are tilted at an angle of 60° from the horizontal for Roorkee, backwards for North and forwards for South
    - The gnomon is perpendicular to the plate, consequently at an angle of 30° from the horizontal



3. **Support Structure:**

   - Two cardboard parallelograms of length 60 cm and height 20 cm with a separation of 5 cm between them to support the plates at inclination of 60°
   - Two cardboard rectangles glued together to form a leg; three such legs to stabilize the structure
   - Slits carved out for each slab in legs, similar slits in the slabs for interlocking and later gluing



4. **Gnomon:**

   - A hole pierced through the centre of each plate and a rod inserted (length 3 cm, thickness 2 mm)
   - The rod is perpendicular to the plate so that it receives sunlight uniformly throughout the day
   - A thread is tied to the gnomons so that the shadow can be extrapolated to meet the scale

2.5.4 Dimensional Specifications

Parameter	**Specification**
Plate Radius	15 cm
Inclination Angle (from horizontal)	60°
Actual Gnomon Length	~2.5 cm
Gnomon Thickness	2 mm
Primary Division	15° (≡ 1 hour)
Secondary Division	1° (≡ 4 minutes)
Least Count (Time)	1° (4 minutes)

2.5.5 Mathematical Principle: Equatorial Coordinate Alignment

Key Concept: Axis Parallel to Earth's Rotation

The crucial feature of the Nāḍī Valaya is that the gnomon's axis is oriented **parallel to Earth's rotational axis** (from south celestial pole to north celestial pole). This ensures:

1. The shadow rotates at a constant angular velocity: **15° per hour**
2. Time can be directly read from the angular position of the shadow
3. No complex trigonometric calculations are needed

Angular Velocity of Gnomon Shadow:

As the Earth rotates at an angular velocity of:

$$\omega = \frac{360°}{24 \text{ hours}} = 15° \text{ per hour}$$

The shadow of a gnomon parallel to the rotation axis also rotates at this same rate:

$$\text{Angle of shadow} = 15° \times t_{\text{hours past noon}}$$

Therefore,

when shadow crosses 12 noon $t_{\text{local solar time post noon}} = \frac{\text{shadow angle}}{15°} \times 60 \text{ minutes}$

before crossing 12 noon $t_{\text{local solar time before noon}} = \frac{\text{shadow angle}}{15°} \times 60 \text{ minutes}$

2.5.6 Operational Procedure

- Align the device along the north-south meridian such that the plate tilted forward and downward faces south
- Use the plate depending on the day of the year (South facing from autumn equinox to spring equinox and North facing from spring equinox to autumn equinox)
- Extend the taut thread so that it lies exactly at the centre of the shadow of the gnomon when seen from an eye level between the gnomon and the markings
- Record the reading where the thread meets the scale

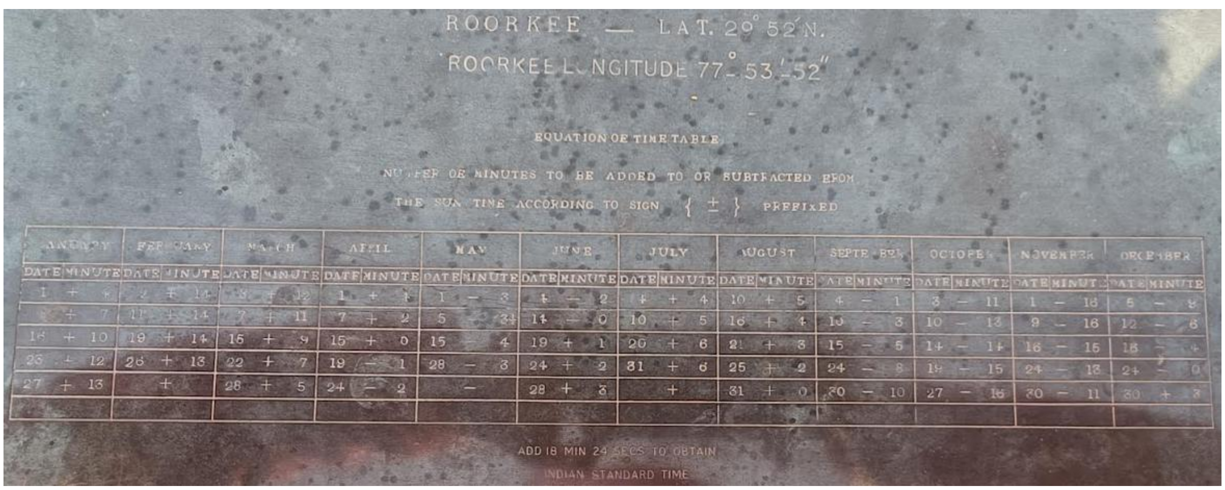
Equation of Time Tracking:

Unlike mean solar time (based on a fictitious uniformly-moving sun), the actual solar time (local apparent time) varies throughout the year due to:

1. **Earth's elliptical orbit:** Earth moves faster at perihelion (early January) than at aphelion (early July)
2. **Ecliptic obliquity:** The angle between Earth's rotational axis and its orbital plane affects the rate of solar time change

By observing solar noon times throughout the year (comparing gnomon shadow crossing time with a clock), the **equation of time** can be empirically determined:

$$\text{Equation of Time} = \text{Mean Solar Time} - \text{Apparent Solar Time}$$



Equation of time for Roorkee, taken from the sundial plaque at Thomason Building

2.5.7 Data from Observations

Date	Actual Time	Measured Time	Error
16/12/2025	11:48 AM	11:42 AM	-6 minutes
16/12/2025	12:59 PM	12:50 PM	-9 minutes

18/12/2025	12:52 PM	12:43 PM	-9 minutes

2.5.8 Sources and References

[11][12][13][8]

2.6 Digaṁśa Yantra (Azimuth Instrument)

2.6.1 Etymology and Designation

- *Dik/Dig* = "direction"
- *Aṁśa* = "part, angle, or division"
- *Yantra* = "instrument"

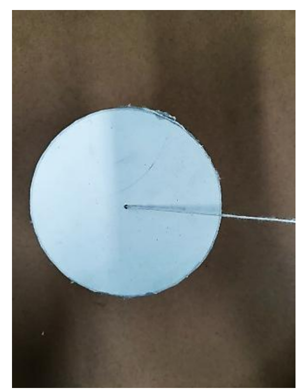
The name literally means "the direction-angle measurement instrument." The Digaṁśa Yantra measures **azimuth (λ)**, the compass bearing of a celestial object as measured from the north point of the horizon.

2.6.2 Functional Purpose

Primary functions:

1. **Azimuth Measurement:** Determines the compass direction of the Sun, Moon, and stars
2. **Earth Rotation Tracking:** Measures the degree of Earth rotation since the moment of solar noon (hour angle)

2.6.3 Design and Construction



Structural Components:

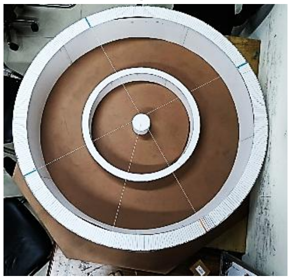
1. **Central Pillar:**

- A pipe of radius 4.5 cm is fixed on a large carboard base, acting as the central pillar
- Its top is covered with a circle cut out from ivory sheet of appropriate size
- A hole is pierced at the centre of the circle

2. **Inner Wall:**

- Cylindrical wall of inner radius = 27 cm, outer radius = 30 cm, height = 10 cm
- A ring of matching radii made from carboard (semi-circles are cut out and attached due to size constraints of carboard sheet) to cover the top and act as scale
- The vertical surfaces of the wall are made using ivory paper; many pieces are glued together and fixed to create the circular boundaries
- The surface of the ring covered with ivory paper, and fixed on the top of the wall using adhesive

3. **Outer Wall:**

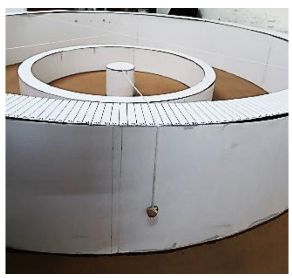


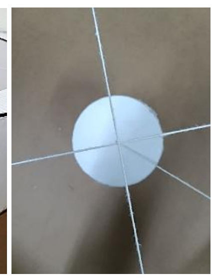
- Cylindrical wall of inner radius = 54 cm, outer radius = 60 cm, height = 20 cm
- A corresponding ring is made by cutting 4 equal parts from carboard and attaching them
- The ring's up-side surface is covered with ivory paper and marked with 360-degree divisions
- For this, the circumference is divided into equal parts and arc length for on division approximated as a straight line

- North is marked with orange and other cardinal directions measured with blue
- The ring is attached to the top of the outer wall. **Note that** all the structures are concentric. Circles of corresponding radii were etched in the base to help erecting the walls.

4. **Cross-Section and Marker:**

- A small pebble is tied to one end of a string and the other end tied to the centre of the central pillar
- The pebble hangs in air outside the outer wall; so the length of the string should be accordingly
- A thread is tied from North to South and another from East to West across the outer wall





Parameter	**Specification**
Outer Wall Outer Radius	60 cm
Outer Wall Inner Radius	54 cm
Inner Wall Outer Radius	30 cm
Inner Wall Inner Radius	27 cm
Central Pillar Radius	4.5 cm
Minimum Resolution	1 degree

2.6.4 Mathematical Principles

Azimuth:

Azimuth is defined as the angle measured eastward from the north point of the horizon:

- North (N): $\lambda = 0°$
- East (E): $\lambda = 90°$
- South (S): $\lambda = 180°$
- West (W): $\lambda = 270°$

The azimuth of a celestial object changes continuously as the object rises in the east, passes through the meridian and sets in the west.

Relationship to Hour Angle:

The azimuth relates to the hour angle (H) through the spherical triangle relationship:

$$\tan(\lambda) = \frac{\sin(H)}{\cos(\varphi)\tan(\delta) - \sin(\varphi)\cos(H)}$$

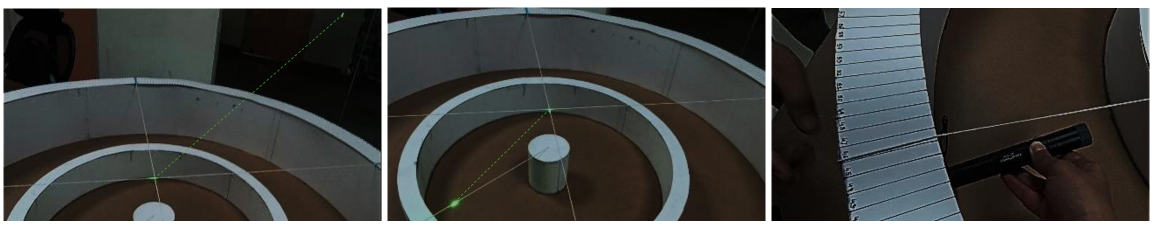
where:

- H = hour angle (0 h at meridian)
- φ = observer's latitude
- δ = declination

This equation shows that azimuth continuously changes as the hour angle increases (as Earth rotates).

2.6.5 Operational Procedure

1. Set the Yantra such that the North of the Yantra matches exactly the cardinal North.
2. The observer stands between two walls and aligns the celestial object with the crosswire
3. Another person standing outside the outer wall moves the pebble such that the string coincides with the observer's line of sight
4. The mark which is crossed over by the freely hanging pebble is recorded as the azimuth



Note: The string attached to the pebble must be taut.

Digaṁśa Yantra can be used in conjunction with Dakṣiṇottara Bhitti to give complete horizontal coordinates.

2.6.6 Sources and References

[14][15][2][8]

2.7 Samrāṭa Yantra (Equinoctial Sundial)

2.7.1 Etymology and Historical Significance

- *Samrāṭa* = "emperor," "supreme," or "paramount"
- *Yantra* = "instrument"

The name *Samrāṭa Yantra* translates to "the Emperor's Instrument" or "the Supreme Instrument," reflecting its position as the largest and most precise sundial

2.7.2 Functional Purpose

The Samrāṭa Yantra measures:

1. **Local Solar Time:** With accuracy of ±2 minutes
2. **Meridian Transit Time:** Precise moment when the Sun crosses the meridian

2.7.3 Design and Construction

1. **Gnomon:**
   - A right-angled triangular gnomon of base 25.5 cm, height 14.7 cm and thickness 0.85 cm is made such that the hypotenuse makes a 30° angle with the horizontal.
   - The gnomon is parallel to the Earth's axis and the meridian line.

2. **Quadrants:**
   - Two quadrants of radius 11.9 cm and thickness 1.7 cm are made
   - The quadrant scale has to be parallel to the equator, thus it will be a ring inclined at 60° with the horizontal, or perpendicular to the gnomon's hypotenuse.
   - To support the quadrants, supporting walls are made such that the back wall is appropriately higher than the front wall (due to the angle of the quadrant)
   - The quadrants, along with the walls, are attached to both sides of the gnomon such that the inner end meets the base tangentially to ground and the outer end points upwards.

3. **Scale:**
   - The apex of the gnomon points towards the North. During sunrise the gnomon will cast shadow in the west and vice versa. Thus, markings are made in the following manner.
   - A paper strip the size of the ring surface (37.4 cm long, 1.7 cm wide) is used. Left end is marked 6 AM, right end is marked 6 PM, and all the intermediate divisions are equal and uniform.
   - There are 30 equal divisions in an hour, each division marking the completion of 30 minutes is drawn as a line, and each line marking the completion of an hour is marked at the middle.
   - The paper strip is pasted on the quadrants such that the part to be covered by the gnomon is cut/torn out before pasting.

Note: As the gnomon covers 13 divisions, there is no shadow on the scale for about 26 minutes and time cannot be tracked.

2.7.4 Dimensional Specifications

Parameter	**Specification**
Gnomon Height	14.7 cm
Gnomon Base Length	25.5 cm
Gnomon Hypotenuse Angle (slope)	30°
Gnomon Thickness	0.85 cm
Quadrant Radius	11.9 cm
Quadrant Thickness	1.7 cm
Quadrant Ring Angle	60°
Scale Divisions (Primary)	15° (1 hour)
Scale Divisions (Secondary)	7.5° (30 minutes)
Scale Divisions (Tertiary)	0.5° (2 minutes)
Accuracy	±2 minutes

2.7.5 Mathematical Principle

The mathematics behind the Samrāṭa Yantra is same as that for the Nāḍī Valaya Yantra. Equinoctial sundial is another name for an equatorial sundial.

The gnomon hypotenuse is parallel to Earth's axis and the quadrants are parallel to Earth's equator. Thus, direct sunlight will be received and the shadow of the gnomon will travel with a uniform velocity, covering the same number of divisions in equal time intervals.

2.7.6 Operational Procedure

1. Align the Yantra in sunlight such that the hypotenuse apex points towards North.
2. Notice where the boundary of the shadow lies. Record it as the time.

The equation of time, as given for Nāḍī Valaya Yantra, should be used here as well.

2.7.7 Data from Observations

Date	Actual Time	Measured Time	Error
18/12/2025	12:08 PM	11:51 PM	-17 minutes
18/12/2025	12:41 PM	12:57 PM	+16 minutes
18/12/2025	12:44 PM	12:59 PM	+15 minutes
18/12/2025	01:30 PM	01:38 PM	+8 minutes

2.7.8 Sources and References

[16][17][2]

Secondary Resources

- Sawai Jai Singh II. *Zij-i Muhammad Shahi* (18th century, Persian). Astronomical tables and observational procedures.
- UNESCO World Heritage Documentation. "The Jantar Mantar, Jaipur." UNESCO Centre for World Heritage Documentation. https://whc.unesco.org
- Jantar Mantar Historical Documents. Architectural plans and engineering specifications from the original construction records.
- 5 Senses Tours. "Surya Siddhanta: Ancient Astronomy & Trigonometry." Analysis of ancient Indian trigonometric foundations and historical priority of Indian mathematics.
- The Astronomical Observatories of Jai Singh II by Sharma, V. N. (Definitive technical monograph)

Part III: Conclusion and Endnote

In the manner discussed above, the seven instruments were made. We have tried our best to put forth all the details of the construction as well as the necessary mathematics and ideas behind the design and working of the instruments.

Along the process, the team has not only picked up knowledge of astronomy and refined its hands-on skills, but also developed an appreciation for the genius of Raja Sawai Jai Singh II and the excellent craftsmanship of the labour involved. Many instances of our struggle to create a particular feature left us in awe of how the rendering on large-scale could have been made possible.

We have tried to present detailed observational data for most of the instruments along with honest error analysis. Despite this, there is little to no real data available for a couple of instruments, specifically the Digaṁśa Yantra, due to many factors, some of them being:

- Lack of solar visibility
- Cloudy and/or foggy weather
- Narrow observation window
- Unavailability of safe open space for testing

We would like to offer our gratitude to the IKS Division for giving us this opportunity to learn about Indian astronomical instruments through this competition. Special thanks to our mentor — professor and Head of the Centre for IKS at IIT Roorkee — Dr. Anil Kumar Gourishetty, for encouraging and supporting us throughout our work. This task could not have been achieved without the facilities of Rethink! The Tinkering Lab, and the efforts of all the team members.