

**IN SEMESTER ASSESSMENT (ISA) B. Tech. IV SEMESTER –MARCH 2021****TEST 1****UE19MA251 - LINEAR ALGEBRA AND ITS APPLICATIONS**

Time : 2 Hours

Answer All Questions

Max Marks: 60

1 a)	Use the method of Gaussian elimination to decide if the planes $y-4z=8$, $2x-3y+2z=1$, $5x-8y+7z=1$ have a common point of intersection in \mathbb{R}^3 . What happens if the right hand side of the third equation is changed to $-3/2$ instead of the present number 1? What are all the solutions in that case?	6
b)	Compute the inverse of $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ using Gauss Jordan method.	7
c)	Factorize $PA=LU$ for $A = \begin{pmatrix} 0 & 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 & 1 \\ 0 & 1 & 3 & 3 & 1 \end{pmatrix}$	7
2 a)	Find all independent vectors on the intersection of the two planes $x_1-x_3+x_4=0$ and $x_1-2x_2-3x_4=0$. These independent vectors are the null space of which matrix? What is the nullspace matrix?	6
b)	Let V be the set of all vectors of the form $\begin{pmatrix} a + 2b + 4c \\ b + 2c \\ -a + 3b + 6c \end{pmatrix}$ where a, b, c are arbitrary. (i) Find vectors u_1, u_2, u_3 such that $V = \text{span}(u_1, u_2, u_3)$. (ii) Is V a subspace of \mathbb{R}^3 ? (iii) Find a basis and the dimension of V .	7
c)	If the vectors $(1,1,2)$, $(1,2,4)$, $(2,4,8)$ span the column space of A , determine whether or not the vector $b=(2,3,5)$ is in $C(A)$. What value should replace the third component "5" in the vector b so that the system $Ax=b$ has infinitely many solutions? Express this new vector b as a linear combination of columns of A .	7

3 a)	<p>For each of the following linear transformations T, find a basis and the dimension of the range and kernel of T:</p> <p>(i) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (x+y, y-z)$</p> <p>(ii) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (x+2y, 2x-y)$</p>	6
b)	<p>If V is a subspace spanned by $(1,1,0,1)$ and $(0,0,1,0)$ find a basis for the orthogonal complement V^\perp. Find the projection matrix from b to V and also the vector in V closest to the vector $b=(0,1,0,-1)$ in V^\perp.</p>	7
c)	<p>Project $b=(1,0,0)$ onto the lines through $a_1=(-1,2,2)$, $a_2=(2,2,-1)$ and $a_3=(2,-1,2)$. Find the projection matrix P from b onto the column space of A. Also find the solution to $Ax=b$ and show that projection vector $p=b$.</p>	7