UE18EC201#2508#52

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PES University, Bangalore

(Established under Karnataka Act No. 16 of 2013)

DEC. 2020: END SEMESTER ASSESSMENT

UE19EC201: MATHEMATICS FOR ELECTRONICS ENGINEERS

Time: 180 Min. Answer all questions Max. Marks: 100

1	(a)	Examine the analyticity of the following function. Comment on its analyticity when $r=0$ and $n \leq 0$: $w = f(r,\theta) = r^n(\cos n\theta + i \sin n\theta)$.	06
	(b)	Find the Taylor's series expansion of (i) $f(z) = \frac{e^z}{1-z}$, around $z=0$, and (ii) $f(z) = \frac{1}{1-z}$ around $z=5$.	07
	(c)	Evaluate the following integral, where C is the circle $\|z\| = 3$. $$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\, dz.$$	07
2	(a)	A certain auditorium has 30 rows of seats. Row 1 has 11 seats, row 2 has 12 seats, row 3 has 13 seats and so on. Following this pattern, row 30 has 40 seats. A prize is to be given away by randomly selecting a row (with equal probability across rows) and then randomly selecting a seat in that row (with equal probability across seats in that row). Find (i) probability that seat 15 was selected, given that row 20 was selected, and (ii) probability that row 20 was selected, given that seat 15 was selected.	08
	(b)	State the mathematical properties of a cumulative distribution function.	06
	(c)	Suppose a manufactured product was found to contain an average of 2 defectives on inspection. Considering a Poisson distribution, find the probabilities of getting (i) no defectives, (ii) one defective, and (iii) three defectives.	06
3	(a)	Find the mean and variance of a uniformly distributed random variable $X \sim \mathcal{U}(a,b)$.	06
	(b)	Let X be a random variable, whose PDF is given by $f_X(x) = \frac{\frac{b}{\pi}}{b^2+x^2}$, for some positive constant b. Find the PDF of $Y = \frac{1}{X}$.	07
	(c)	Find the moment generating function of a Gaussian distributed random variable $X \sim \mathcal{N}(\mu, \sigma^2)$.	07

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4	(a)	Let X_1 and X_2 be independent and identically distributed standard uniform random variables, that is $X_1, X_2 \sim \mathcal{U}(0,1)$. Calculate the joint density function of $Y_1 = X_1 + X_2$, and $Y_2 = X_1 - X_2$.	07
	(b)	Let X and Y be independent Poisson random variables with parameters λ_1 and λ_2, respectively. Show that the conditional PMF of X given $X + Y = n$ is a binomial distribution with parameters $\left(n, \frac{\lambda_1}{\lambda_1+\lambda_2}\right)$.	07
	(c)	If X and Y are independent and identically distributed standard uniform random variables, that is $X \sim \mathcal{U}(0,1)$ and $Y \sim \mathcal{U}(0,1)$, calculate the PDF of $X + Y$.	06
5	(a)	Show that the random process $X(t) = A\sin(\omega_0 t + \theta)$ is stationary in wide-sense (WSS) and mean ergodic, given that θ is a uniformly distributed random variable, that is, $\theta \sim \mathcal{U}(0, 2\pi)$, and A and ω_0 are constants.	07
	(b)	State any five mathematical properties of a power density spectrum function.	05
	(c)	Find the mean value $\overline{Y}$, second moment (mean-squared) value $\overline{Y^2}$ and variance σ_Y^2 of the output $Y(t)$, given that the input $X(t) = 2\cos(2\pi t + \theta)$, where $\theta \sim \mathcal{U}(0, 2\pi)$, and the system function $$h(t) = \begin{cases} 5\delta(t) + 3, & 0 \le t \le 1, \\ 0 & \text{elsewhere.} \end{cases}$$	08