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PES UNIVERSITY

PES University, Bangalore
(Established under Karnataka Act No. 16 of 2013)

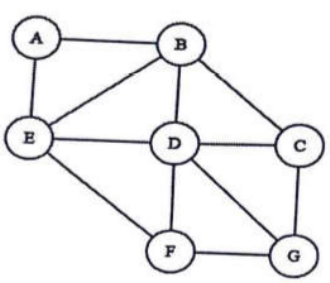
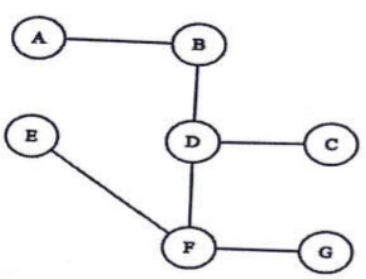
UE18CS323

DEC 2020: END SEMESTER ASSESSMENT (ESA) B.TECH. V SEMESTER

UE18CS323: GRAPH THEORY, APPLICATIONS AND COMBINATORICS

Time: 3 Hrs | Answer All Questions | Max Marks: 100

NOTE: **State any assumptions made.**

1.	a)	Define i. Distance between two vertices ii. Eccentricity of a vertex iii. Radius of a connected graph iv. Circumference of a graph v. Centre of a graph vi. Girth of a graph	6
	b)	Prove that every cut set in a connected graph G must contain atleast one branch of every spanning tree.	4
	c)	Which of the following graphs contain Euler circuit/Euler path/Neither? Justify your answer i. K_4 ii. K_5 iii. $K_{5,7}$ iv. $K_{3,3}$	4
	d)	Identify the fundamental cut sets of the graph G with respect to the spanning tree S  Graph G  Spanning Tree S	6
2.	a)	Find the chromatic polynomial of a connected graph with 3 vertices	8
	b)	Apply Welsh Powell algorithm and find the chromaticity of the graph G	2

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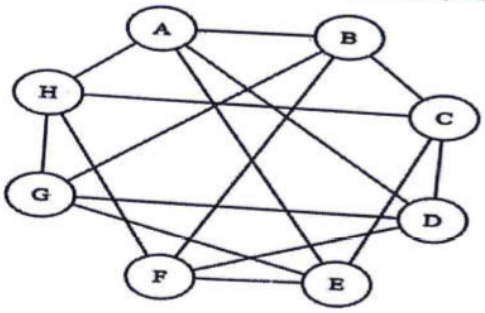
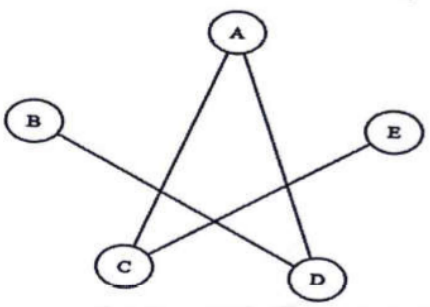


Graph G

c) Find the maximum independent set and thereby chromaticity of the graph



6

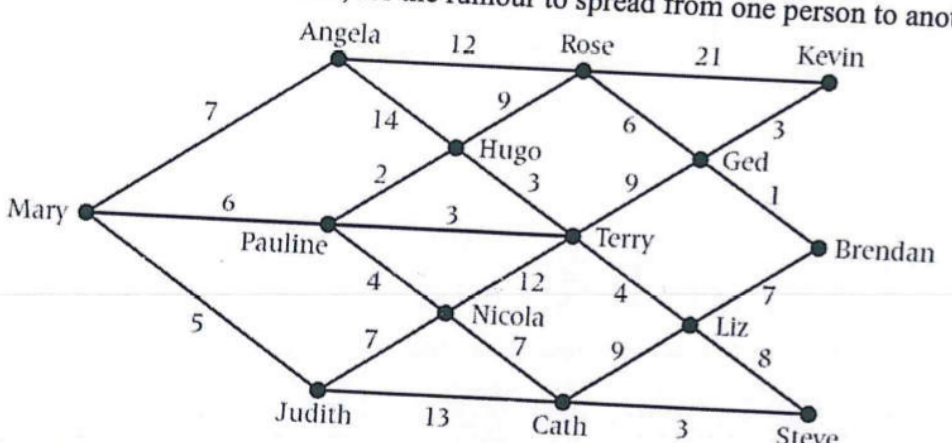
d) If a graph G has subgraphs G1 and G2 such that G1 ∪ G2 = G and G1 ∩ G2 = K_n for some positive integer n then prove that

$$P(G, \lambda) = \frac{(P(G1,\lambda) * P(G2,\lambda)}{\lambda^n}$$

Where $\lambda^n = \lambda(\lambda\text{-}1)(\lambda\text{-}2).....(\lambda\text{-}n+1)$

4

3. a) Every day Mary thinks of a rumour to spread on her network of friends. The rumour originates from Mary and is then spread from one person to another. The following network shows the route through which the rumour spreads. The number on each edge represents the time in minutes, for the rumour to spread from one person to another.



8

i) Find the fastest time taken for the rumour to reach person.

ii) List the route through which Brendan first hears the rumour.

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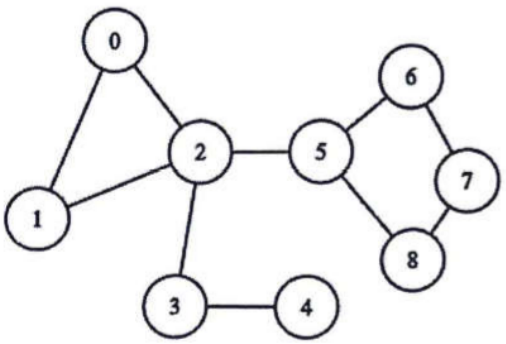
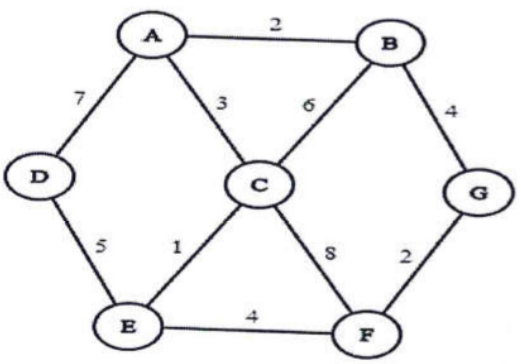
	b)	Apply Tarjan's Algorithm and identify articulation points in the graph 	6
	c)	Apply Reverse Delete Algorithm and find the Minimal Spanning Tree for the given graph 	6
4.	a)	A software company is visiting university campus to recruit students proficient in either C, Python or C++. 40 students are proficient in C, 60 students in Python and 50 in C++. 25 students are proficient in C and Python, 30 in Python and C++. 35 in C and C++. There are 10 students who are proficient in all three. How many students are eligible for the recruitment process?	6
	b)	Find the rook polynomial of a 3X3 board using expansion formula	4
	c)	Find the number of integer solutions of the equation $X_1 + X_2 + X_3 = 20$ such that $2 \le X_1 \le 5$, $4 \le X_2 \le 7$, $-2 \le X_3 \le 9$	6
	d)	Find the exponential generating function for the sequence i) 0, 1, 2a, $3a^2$, $4a^3$,..... ii) 0, 0, 1, 1, 1, 1,.....	4
5.	a)	Find the recurrence relation, initial condition for the sequence and solve it. i) 2, 16, 128, 1024, ... ii) 0, 2, 6, 12, 20, 30,.....	6
	b)	Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 3^n$, for $n \ge 1$ given that $a_0 = 2$	4

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	c)	Solve the recurrence relation $a_n + a_{n-1} - 6\,a_{n-2} = 0$, for $n \geq 2$ given that $a_0 = -1$, $a_1 = 8$	3
	d)	Find the generating function for the recurrence relation and solve it $a_{n+1} - a_n = n^2$, $n \geq 0$ and $a_0 = 1$	7