SRN

PES UNIVERSITY

PES University, Bangalore

(Established under Karnataka Act No. 16 of 2013)

UE14EC255

MAY 2016: END SEMESTER ASSESSMENT (ESA) B.Tech IV SEMESTER

UE14EC255 - PROBABILITY THEORY AND RANDOM PROCESSES

Time: 3 Hrs | Answer All Questions | Max Marks: 100

1	(a)	State the axioms of probability.	06
	(b)	Define the following: (i) Sample space (ii) Conditional probability (iii) Independent events (iv) Random variable.	08
	(c)	Show that for any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.	06
2	(a)	Define the cumulative distribution function (CDF) of a random variable X and discuss its properties.	08
	(b)	Consider a continuous random variable X, with probability density function (PDF) $f_X(x) = \begin{cases} c\,(4x - 2x^2), & 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$ Find the values of (i) c, and (ii) $P(X > 1)$.	06
	(c)	Consider a random variable X which has a uniform distribution between a and b, with the PDF $f_X(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise.} \end{cases}$ Derive the expressions for its (i) CDF, (ii) mean, and (iii) variance.	06
3	(a)	For a random variable X, show that $\text{var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$, where $\text{var}(X)$ denotes the variance of X and $\mathbb{E}(X)$ denotes the expectation of X.	04
	(b)	Let X and Y be two independent standard normal random variables. Find the joint distribution of $R = \sqrt{X^2 + Y^2}$ and $\theta = \tan^{-1}\left(\frac{Y}{X}\right)$. What can be said about the distribution of R and θ?	08
	(c)	Show that the MGF of a Gaussian random variable with mean μ and variance σ^2 is given by $M_X(t) = e^{\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)}$.	08
4	(a)	Show that the Poisson distribution $\text{Poi}(\lambda)$ is the limiting case of a Bernoulli distribution $\text{Ber}(n, p)$ with parameter $\lambda = np$.	06
	(b)	State and prove Markov's inequality.	06
	(c)	Define the covariance between random variables X and Y. Discuss its properties.	08
5	(a)	State and prove the central limit theorem.	08
	(b)	Explain (i) Random process, (ii) Strict-sense stationary (SSS) process, (ii) Wide-sense stationary (WSS) process, and (iv) Ergodic process.	08
	(c)	If $X(t, s)$ and $Y(t, s)$ are two independent, zero mean, WSS random processes with identical autocorrelations, then show that $Z(t, s) = aX(t, s) + bY(t, s)$ is also WSS.	04