JULY 2021: END SEMESTER ASSESSMENT (ESA) B TECH IV SEMESTER

UE17/18/19MA251 - Linear Algebra and Its Applications

Time: 3 Hrs	Answer All Questions	Max Marks: 100

1	a)	Determine the values of a and b for which the system of equations x+y+az=2b, x+3y+(2+2a)z=7b, 3x+y +(3+3a)z=11b will have (i) trivial solution (ii) unique non-trivial solution (iii) no solution (iv) infinity of solutions.	7
	b)	Factor A=LU and A=LDU for $A=\begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{pmatrix}$	6
	c)	Use Gauss-Jordan elimination on [A:I] to solve $AA^{-1}=I$ $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	7
2	a)	Let $A=\begin{pmatrix} 4 & -2 & 0 & 2 \\ -2 & 0 & -5 & -2 \\ 0 & 3 & 1 & 3 \end{pmatrix}$ (a) If column space of A is a subspace of R^k, find k? (b) Find a non-zero vector in nullspace of A. Also find l of R^l such that nullspace of A is a subspace of R^l.	7
	b)	Find a basis and the dimension of the subspaces $V=\{(a,b,c,d)/b-2c+d=0\}$, $W=\{(a,b,c,d)/a=d,b=2c\}$ and $V\bigcap W$ in R^4.	6
	c)	Find a matrix A that has V as its row space, and a matrix B that has V as its nullspace, if V is the subspace spanned by $\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix},\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix},\begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix}\right\}$	7
3	a)	What matrix P projects every point in R^3 onto the line of intersection of the planes x+y+z=0 and x-z=0? Find the nullspace matrix of P. What do the column space and row space of matrix P represent?	7

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	b)	Find the matrix of the linear transformation $T: R^3 \to R^2$ defined by $T(x,y,z)=(x+y, 2z-x)$ with respect to (i) the standard basis (1,0,0),(0,1,0),(0,0,1) and (ii) the basis (1,0,-1),(1,1,1), (1,0,0)	6
	c)	Find $\|E\|^2 = \|Ax-b\|^2$ and solve the normal equations $A^T A\hat{x} = A^T b$. Find the solution $\hat{x}$ and the projection $p = A\hat{x}$. (Use Least squares method) Given $A=\begin{pmatrix}1 & 0\\ 0 & 1\\ 1 & 1\end{pmatrix}, x=\begin{pmatrix}u\\ v\end{pmatrix}, b=\begin{pmatrix}1\\ 3\\ 4\end{pmatrix}$	7
4	a)	Let W = { (a, b, b) / a , b are real } and let v = (3, 2, 6), (i) Find an orthonormal basis for W. (ii) Find the projection of v onto W , say v_1 (iii) Decompose v into a sum of two vectors $v_1 + v_2$ where v_2 is projection of v onto $W^{\perp}$.	7
	b)	Check if the following symmetric matrix $A=\begin{pmatrix}7 & -4 & 4\\ -4 & 5 & 0\\ 4 & 0 & 9\end{pmatrix}$ is orthogonally diagonalizable. If so, orthogonally diagonalize it as $A = S\Lambda S^{-1} = Q\Lambda Q^{-1} = Q\Lambda Q^T$ where Q is an orthogonal matrix.	6
	c)	Use the Gram – Schmidt process to find a set of orthonormal vectors from the independent vectors $a_1 = (1, -1, 0)$, $a_2 = (0, 1, -1)$ and $a_3 = (1, 0, -1)$. How many non-zero orthonormal vectors are obtained?	7
5	a)	Write the quadratic form of the matrix $\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}$ and express it as a sum of squares using A=LDU factorization.	7
	b)	Test if A^TA is positive definite or positive semi-definite given $A=\begin{pmatrix}1 & 1\\ 1 & 2\\ 2 & 1\end{pmatrix}$	4
	c)	Find SVD of the matrix $A=\begin{pmatrix}1 & 1\\ 0 & 1\\ -1 & 1\end{pmatrix}$	9

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