

**UE14CS311 - ADVANCED ALGORITHMS(AA) – Elective (RS)**

Time: 3 Hrs		Answer ALL Questions	Max Marks: 100
1.	a)	<p>Define the Asymptotic Growth bound <math>\Omega</math>. Decide if the following statements are TRUE/FALSE.</p> <p>i) <math>f(n) = \text{Big-O}(g(n))</math> and <math>g(n) = \text{Big-O}(h(n))</math> implies <math>f(n) = \text{Big-O}(h(n))</math></p> <p>ii) <math>f(n) = \omega(g(n))</math> and <math>g(n) = \omega(h(n))</math> does not imply <math>f(n) = \omega(h(n))</math></p> <p>iii) <math>2n^2 + \Theta(n) = \Theta(n^2)</math></p> <p>iv) <math>\lim_{n \rightarrow \infty} \{f(n)/g(n)\} = 0</math> if <math>f(n) = \text{Small-o}(g(n))</math></p>	06
	b)	<p>State the Master Theorem for solving Algorithm complexities, in the form of recurrences.</p> <p>Use either Substitution or Master method to find a closed form solution for</p> <p><math>T(n) = 2T(n^{1/2}) + \log_2 n</math></p>	06
	c)	Define Amortized Complexity.	03
	d)	What sequence of operations can cause the amortized complexity of Dynamic table with a strict load factor $\alpha \geq 0.5$ , to become $\Theta(n)$ . Outline the strategy and the corresponding potential function definition, to keep the amortized cost to 3.	05
2.	a)	Give formal definition of String matching problem and state the overlapping suffix lemma.	05
	b)	<p>Consider two binary trees T1 and T2. Both T1 and T2 have same number of nodes N. To decide, if node(keys) of T1 and T2 are mirror images of each other,</p> <p>i) Suggest a String Matching based scheme when <math>N \leq 1000</math>,</p> <p>ii) What are the pre-processing and subsequent matching involved?</p> <p>iii) Will your algorithm scale efficiently, if N is of the order of <math>10^{16}</math>?</p>	07
	c)	<p>State the formal properties of a Suffix Tree.</p> <p>Construct an algorithm for pattern matching using Suffix Arrays, with and without Longest Common Prefix(LCP) array, with respective complexities.</p>	08
3.	a)	Define Flow in a Flow Network.	05
	b)	Show the structure of a B-tree node with minimum and maximum load.	05
	c)	Show the Binomial Heap, initially empty, after 23 keys, sorted (in descending order), are inserted.	05
	d)	Assume a sequence of 1025 Insert operations, into an initially empty Fibonacci Heap, followed by an ExtractMin operation. What is the maximum degree of any node in the resulting Heap. Justify your answer.	05

P.T.O.

4.	a)	Let $A(x) = B(x) = x^{49} + x^{48} + \dots + x + 1$ . What is the Degree and Degree Bound of $C(x) = A(x) \cdot B(x)$ ? Compute $C_{49}$ and $C(x=1)$ .	05
	b)	Explain efficient polynomial multiplication in a clear block scheme. Suppose it is required to find $C(x) = A(x)/B(x)$ and compute point values for Quotient and the Remainder, repetitively for many values of $x$ , can it be done efficiently with the same/similar block scheme? What will be the resulting complexity?	07
	c)	State and prove Summation Lemma.	04
	d)	What is the result of a Bit-Reverse permute of the array: [ 0 , 9, 5, 13, 3, 11, 7, 15, 2, 10, 6, 14, 4, 12, 8, 16]	04
5.	a)	Derive a bound on the running time of EUCLID(a,b)	04
	b)	Compute the size of $Z_n^+$ and $Z_n^-$ when $n = 60$ and $n = 79$ .	04
	c)	Solve for $X$ if, for divisors $\{3, 5, 7\}$ , $X$ leaves respective remainders $\{2, 3, 2\}$ , using Chinese Remainder Theorem(CRT). How is CRT used in Digital Signature using RSA Cryptography?	08
	d)	Compute the approximate number of primes upto $2^{32}$ . Why is Rabin-Miller primality test, not a Las-Vegas randomized algorithm.?	04