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PES UNIVERSITY

UE18CS311

In Semester Assessment (ESA) B. Tech. 5th SEMESTER – Aug - Dec-2020

UE18CS311 - Advanced Algorithm

Time: 3 Hrs. Answer All Questions Max Marks: 100

You can bring one or more hand written notes. No printed or photocopied matter is allowed.

Answer precisely and briefly.

1	a	I) Consider the following recurrence relation.
		T(n) = 5 if n <= 2
		= T(n-1)+ n otherwise
		Find the closed form solution for T(n) for n >= 2.
		ii) Express recurrence and indicate the asymptotic complexity for the following cases. Give reasons.
		- addition x + x is const time
		- addition x + x is proportional to the number of bits in x – Assume that x also has n bits.
		algo g(n)
		if n = 1 then
		return 1
		else
		x ← g(n-1)
		return x+x
			2 + 2 + 2

	b	I) When will the amortized time complexity and asymptotic time complexity will be different for an algorithm? ii) What happens if we allocate a cost of 2 for each push operation of a dynamic table? Iii) Amortized cost of push, pop and multipop for n operations is O(n). What happens if we also introduce multipush?	2+2+2
	c	I) What is a possible potential function Φ(D) for the binary counter supporting increment operator? ii) What is the $\Phi(D_0)$? Iii) Show that $\Phi(D_i)$ is always positive non zero. iv) If the value in binary counter b_i is 0, how many of the k bits have been reset in the ith operation?	4
	d	Knowing that Hamiltonian cycle problem is in NP, Show that TSP is also in NP. Indicate the steps clearly.	4
2	a	Pattern : AABA Text: AABAACAADAABA Solve string matching using Rabin Karp algorithm. Can we solve using radix d = 10, and code for A B C D as 0 1 2 3 respectively? Hint: show the hash value for pattern and each sequence and indicate no match, match spurious and match exactly.	4
	b	I) The pattern P[1 .. m] has distinct characters. What is the least # of states required in the automaton for matching? Why? ii) x, y, and z are strings such that x is a suffix of z and y is a suffix of z. Which of the following does this imply? Why? a) x and y are always same b) x is a suffix of y if |x| > |y| c) y is a suffix of x if |x| > |y| d) z is a substring of both x and y	2+4

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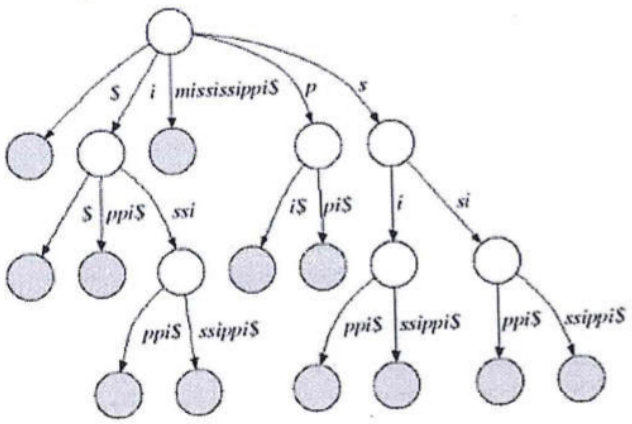
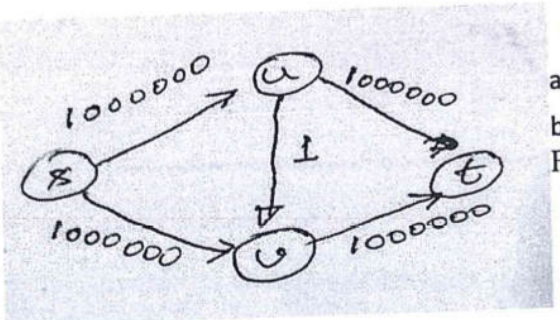
		You may give a counter example to disprove.	
	c	Given a suffix tree for the string T$ = "mississippi$", answer the following questions.  I) What is the number of leaves with respect to the length of T? ii) longest suffix will have the max # of nodes in the path. State true or false. Why? Iii) What is the longest repeated substring in T? How do you find it? iv) There is a path between node X (not the root) and node Y. This path will definitely exist in some other part of the tree. State true or false. Give an example from this tree.	1 + 1 + 2 + 2
	d	How do you find the following in a suffix tree? I) # of times a string P occurs in a string T in a suffix tree of T, ii) prefix P of a string T in suffix tree of the string is found	4
3	a	What will happen to the max flow i) if the capacity of each edge is increased by x units ii) if the capacity of each edge is doubled. iii) if an edge is removed Give your reasons.	6
	b	 For this flow graph, a) what will be the maxflow? b) What will be the number of augmentation if Ford-Fulkerson method is applied?	2+2
	c	I) What is the degree bound of the product of two polynomials A(x) and B(x) each of which is degree bound n? ii) To achieve the required degree bound, how should we alter A(x) and B(x)? iii) What is the more efficient complexity of multiplying two polynomials given in Co-efficient representation	1+1+2 +2

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		PV representation? iv) In the tree of input vectors to the recursive calls of recursive-FFT for degree bound n, what will be the difference between the coefficient indices of the of he left and right leaves of a node?	
	d	If we have n distinct points in PV representation and we convert to the CR representation, will the degree of the polynomial be n – 1 if we ignore the terms with 0 coefficient? Why?	2+2
4	a	Answer the following questions based on a subgroup generated by an element of $Z^{\bullet}_{n}$. I) What will be the number of elements in subgroup <1>? How is this related to n? ii) What will be the relationship between the number of elements in any subgroup and the number of elements in Z? Iii) How many elements will the biggest subgroup have with respect to the size of Z? iv) if n is even, how many elements will <2> have?	2+1+1 +1
	b	Find the smallest positive number which when divided by 5 give a remainder 1 and which when divided by 7 gives a remainder 3. How many such numbers less than or equal 1000 exist?	4+2
	c	Solve the equation 15 * x = 25 (mod 35). How many solutions would this equation have? Find all the solutions.	5

	d	int what(int x, int y, int n)
		{
		int res = 1;
		while (y > 0)
		{
		if (y & 1)
		res = res*x % n;
		y = y>>1;
		x = x*x % n;
		}
		return res;
		}
		Find the output for x = 3; y = 14; n = 5
		What does this program with respect to its parameters?
			4

5	a	Specify the recurrence relation for the matrix chain multiplication problem where p[i-1]*p[i] gives the dimension of the ith matrix. dp[i, j] is the cost of multiplying matrices in chain from position i till position j both inclusive. Min is obtained over all values of k, i <= k < j. Express dp as a recurrence. Also state the base condition.	4

	b	Algo longest_common_subsequence (X, Y)
		m ← length(X)
		n ← length(Y)
		// C is a table holding the iterative solution
		int C[__][___]; // question i
		// initialization code : question ii
		for i ← 1 to m do
		for j ← 1 to n do
		if $x_i = y_j$
		C[i, j] ← C[i - 1, j - 1] + 1
		else
		if C[i -1, j] ≥ C[i, j -1]
		C[i, j] ← C[i - 1, j] + 1
		else
		C[i, j] ← C[i, j - 1]
			1 + 2 + 1 +2

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```
return _____   // question iii
```

Answer the following questions precisely.

I) what is the dimension of the table C?

ii) What initialization is required? Write the code.

Iii) What do we return for just the final # of matches?

iv) There is one bug. spot the bug in the code. Give reason and correct it.

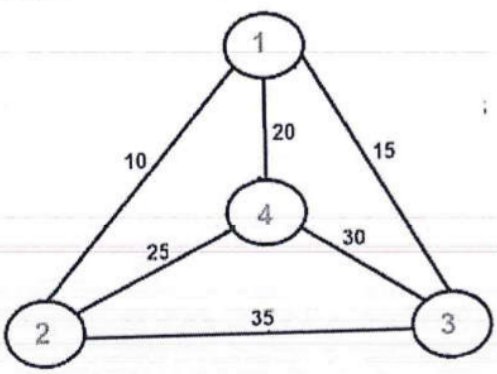
c

Answer the following questions with respect to Approximate Traveling salesman problem. 2+2+ 2

I) What is triangle inequality?

ii) The min cost spanning tree is traversed using pre-order. Why is the cost of the tree is half the cost of full walk?

Iii) Find the solution for the given graph starting from node 1.



d

This algorithm tries to find a in the array. The array contains equal number of a and b and no other element. 1 + 1 + 2

```
Algo find_a(int x[1 .. n])
begin
  repeat
    Randomly select one element out of n elements.
  until 'a' is found
end
```

I) What is the probability of finding a in one iteration?

ii) What is the probability of finding a in this algorithm?

Iii) Chance of succeeding in the ith iteration is given by i * (pf)^(i-1) * (ps) where is pf : probability of failure and ps : probability of success. Show that for large values of n, what is the expectation?