SRN

UE19CS311/ UE18CS311`
ADVANCED ALGORITHMS

PES University, Bangalore
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DECEMBER 2021: END SEMESTER ASSESSMENT (ESA) B.TECH
5th SEMESTER

UE19CS311/UE18CS311 - ADVANCED ALGORITHMS(AA)

Time: 3 Hrs	Answer All FIVE Questions	Max Marks: 100

1	a)	Define the set $\Omega(g(n))$ formally. Show that $T(n) = 4n^2 + 2n - 2$ belongs to the set **Big-O(n^3)**, from the definition of Big-O.	05
	b)	List the three methods of solving a Recurrence equation. Find the closed form solution for $T(n) = 3 T(n/4) + n \log n$, using Master Method.	05
	c)	Define Amortized complexity. Using **Aggregate Method**, find the amortized complexity of a Dynamic Table data structure, with only *"Insertion"* operation and a Load factor, $\alpha \geq \frac{1}{2}$.	05
	d)	Define the complexity classes NP, NP-Hard and NP-Complete, with an example each. How do you prove that a new problem is NP-Complete, by *"Polynomial Reduction"* technique ?	05
2	a)	Given a pattern $P[1 ...1000]$ = *"aa...aa"* and Text $T[1 ... 10000]$ = *"aa...aa"*, What are the values of the following: (Note : $\Sigma = \{ a \}$ and Ignore comparisons in pre-processing) i) Number of Shifts ii) Number of Valid Shifts iii) Number of comparisons done by Naive String Matcher iv) Number of comparisons done by Rabin-Karp v) Number of comparisons done by Boyer-Moore	05
	b)	Design an FSA and the corresponding transition table for matching the pattern P = *"PRESSESS"* . What is the bound on the complexity of FSA string matcher?	06
	c)	What is the strategy followed by KMP string matcher to improve on FSA stringmatcher? Show the KMP prefix function array for the pattern P = *"ababaaab"*	04
	d)	Draw the suffix tree for the string S = *"ukkonnen"*. Show the corresponding Suffix Array SA[S]. How is a Suffix Array used for string matching ?	05
3	a)	Define Flow and Capacity constraints in a flow network. Consider a flow network $G(V,E)$ with $V = \{ s, v1, v2, v3, v4, t\}$ and a directed edge set $E = \{(s, v1), (s, v2), (v1,v2), (v1,v3), (v3,v2), (v2,v4), (v4,v3), (v3,t), (v4,t) \}$. All the edges have initial **flow/capacity** value = **0 / 20**. Find the flow and capacity across the cuts $S = \{ s, v1, v2\}$ and $T = \{ v3, v4, t \}$.	05
	b)	Define Degree & Degree Bound of a Polynomial. Given $A(x) = x^{100} + 2$; $B(x) = x^{100} - 2$ and $C(x) = A(x) * B(x)$, Find C(x) and evaluate $C(x = i)$ where $i = \sqrt{-1}$; Verify $C(x = i)$ is equal to $A(x = i) * B(x = i)$;	05
	c)	Show the 8 complex roots of unity on the Unit circle. State and prove Halving Lemma.	04
	d)	What is the difference between DFT and FFT ? Outline the Divide and Conquer strategy used in arriving at Recursive DFT.	06

4	a)	Define Relative Prime numbers using GCD. Prove , if a, b, p $\in$ Z are Relative Prime then, the product (a x b) is also Relative Prime to p .	04
	b)	Verify that the set induced by $(Z^*_7, *_7)$, forms a Finite Abelian Group. What is the size of group formed by $(Z^*_{35}, *_{35})$?	05
	c)	Define Multiplicative Modular inverse as a Modular Linear equation. Solve the Modular linear equation $14x \equiv 40\ Mod\ 100$, by using the starter and repeater solutions.	05
	d)	What is Fermat's theorem ? Give the steps of an *efficient* Miller Rabin primality test for large numbers.	06

5	a)	What are the Key elements of a Dynamic Programming(DP) Problem ? For the matrix chain multiplication shown , find the optimal entry M[1, 3] , by the DP algorithm.	06

Matrix	A_1	A_2	A_3	A_4	A_5
Dimension	4 x 15	15 x 10	10 x 20	20 x 30	30 x 30

Show the DP recurrence & your compute steps, in filling the relevant M[i , j] entries.
(Note: Fill only what is needed to compute M[1 , 3])

	b)	Define a Randomized algorithm. What is the key difference in the nature of result produced by QuickSort (with Randomized pivot selection) and Miller-Rabin Primality Test ?	04
	c)	Assume your Software Startup venture, requires 5 programmers, from Day 1 , to be hired from the list of 50 candidates to be provided by a HR firm. The conditions are: – from Day 1, headcount in your Startup should be 5 engineers – The HR firm sends in one candidate per day, after the initial list of FIVE. – Test/Interview cost per candidate, is negligible – After every interview, you must hire & replace an existing engineer, if you find a better candidate. – Replacing an engineer,with a better candidate by hiring, means a hiring fees of Rs50K to the HR firm. What is the lowest and highest HR fees your Startup ends up paying to the HR firm? Find the Expected Value ,your Startup pays the HR firm , using Indicator Random variables.	05
	d)	Define an Approximation Scheme (AS) for an NP-Complete Optimization problem. Design and Prove a simple Poly Time 2-Approximate-Vertex-Cover algorithm.	05