

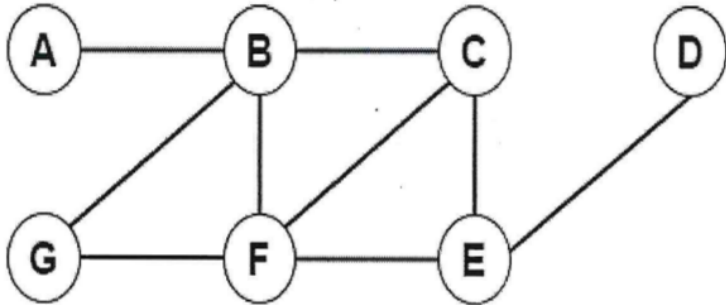
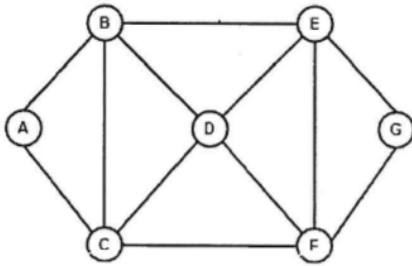
**DECEMBER 2021: END SEMESTER ASSESSMENT (ESA) B TECH
V SEMESTER**

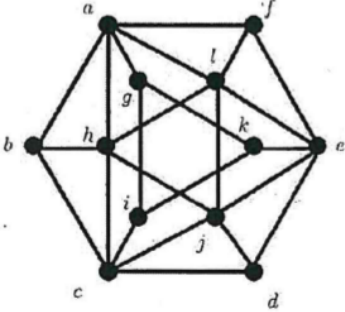

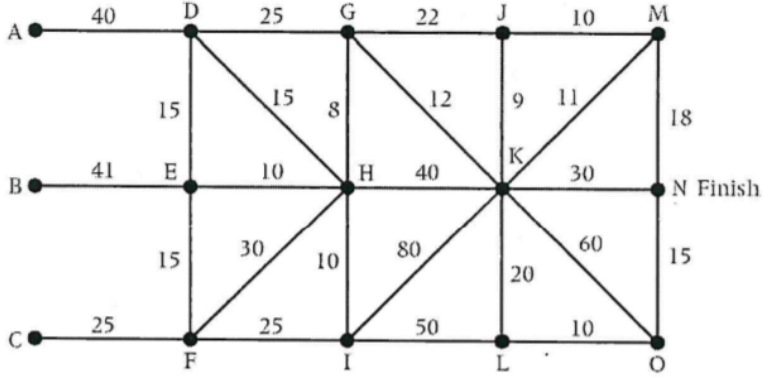
UE18/19CS323 – Graph Theory, Applications and Combinatorics

Time: 3 Hrs

Answer All Questions

Max Marks: 100

1	<p>a) With respect to the given Graph G, define and find:</p> <ul style="list-style-type: none"> i) Distance between F and D ii) Eccentricity of Vertex F iii) Diameter of the Graph G iv) Circumference of the Graph G <div style="text-align: center; margin-top: 20px;">  </div>	8
	<p>b) For a graph with n vertices and m edges, if δ is the minimum and Δ is the maximum of the degrees of vertices show that</p> $\delta \leq \frac{2m}{n} \leq \Delta$	4
	<p>c) Prove that every circuit has even number of edges in common with any cut set.</p>	4
	<p>d) Does the graph below have a Euler circuit? If so find one using Fleury's Algorithm.</p> <div style="text-align: center; margin-top: 20px;">  </div>	4

2	<p>a) What are isomorphic graphs? Are the two graphs shown below isomorphic? Map the vertices from G1 to corresponding vertices in G2.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> G1 G2 </div>	6
	<p>b) What is uniquely k-colorable Graph? Determine which of the following graphs are uniquely colorable with appropriate reasoning:</p> <ol style="list-style-type: none"> K_n C_n 	3
	<p>c) Consider a 4X4 sudoku problem.</p> <ol style="list-style-type: none"> If the 4X4 board is modeled as graph coloring problem, how many vertices and edges are there in the graph? Is the resulting graph k-regular? Will Welsh Powell algorithm give an optimal solution to the 4X4 sudoku problem? Justify your answer 	8
	<p>d) Let F be a forest with k components (trees). If n is the number of vertices, m is the number of edges in F, then prove that $n=m+k$</p>	3
3	<p>a) Three boys, John, Lee and Safraz, are to take part in a running race. They are each starting from a different point but they all must finish at the same point N. John starts from the point A, Lee from the point B and Safraz from the point C.</p> <p>The following diagram shows the network of streets that they may run along. The numbers on the arcs represent the time, in seconds, taken to run along each street.</p>  <p>Find the time taken for each of the three boys to complete the race. Write down the route that each boy should take.</p>	8

	b)	Google has 6 vacant positions to be filled. HR at Google has scrutinized applicants and shortlisted 6 of them to fill the vacant positions. Depending on the strengths of applicants, HR wants match the shortlisted applicants to job profiles. A1(applicant 1) is suitable for J2, A3 is suitable for J1 and J4, A4 is suitable for J3, A5 is suitable for J3 and J4, A6 is suitable for J6. Find if all shortlisted applicants can be matched to a suitable job profile. Find such a maximum match.	6																														
	c)	There are 5 jobs to be assigned to 4 machines. How can these jobs be assigned to machines so as to minimize the total cost? Solve using Hungarian method. <table border="1"><tr><td></td><td>J1</td><td>J2</td><td>J3</td><td>J4</td><td>J5</td></tr><tr><td>M1</td><td>9</td><td>11</td><td>15</td><td>10</td><td>11</td></tr><tr><td>M2</td><td>12</td><td>9</td><td>-</td><td>10</td><td>9</td></tr><tr><td>M3</td><td>-</td><td>11</td><td>14</td><td>11</td><td>7</td></tr><tr><td>M4</td><td>14</td><td>8</td><td>12</td><td>7</td><td>8</td></tr></table>		J1	J2	J3	J4	J5	M1	9	11	15	10	11	M2	12	9	-	10	9	M3	-	11	14	11	7	M4	14	8	12	7	8	6
	J1	J2	J3	J4	J5																												
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M3	-	11	14	11	7																												
M4	14	8	12	7	8																												
4	a)	Of 30 personal computers owned by faculty members in a certain university, 20 run on windows, 8 have 21 inch monitors, 25 have CD-ROM drives, 20 have atleast two of these features and six have all three. a. How many have none of the features? b. How many have exactly one feature? c. How many have two of the three features described?	6																														
	b)	Four persons P1, P2, P3, P4 who arrive late for a dinner party find that only one chair at each of five tables T1, T2, T3, T4 and T5 is vacant. P1 will not sit at T1 or T2, p2 will not sit at T2, P3 will not sit at T3 or T4 and P4 will not sit at T4 or T5. Find the number of ways they can occupy the vacant chairs.	6																														
	c)	In how many ways can we distribute 24 pencils to 4 children so that each child gets atleast 3 pencils but not more than eight. Solve using Generating functions	8																														
5	a)	Solve the recurrence relation $a_{n+1} = a_n + 2n + 3$ Given $n \geq 0$ and $a_0 = 1$	5																														
	b)	Solve the recurrence relation: $x_n = 2x_{n-1} + x_{n-2} - 2x_{n-3}$ $n \geq 3$ given $x_0 = 1, x_1 = 2, x_2 = 0$	5																														
	c)	Find the general solution to the recurrence relation $a_n - 7a_{n-2} + 10a_{n-4} = 0 \forall n \geq 4$	5																														
	d)	Solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n$ $n \geq 0$ Given $a_0 = 1, a_1 = 2, A = -2$ and $B = 1$	5																														