NCERT Physics 12.7 Q6

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Question: Obtain the resonance frequency of a series LCR circuit with $L = 2.0\,H$, $C = 32\,\mu F$, and $R = 10\,\Omega$. What is the Q-value of the circuit.

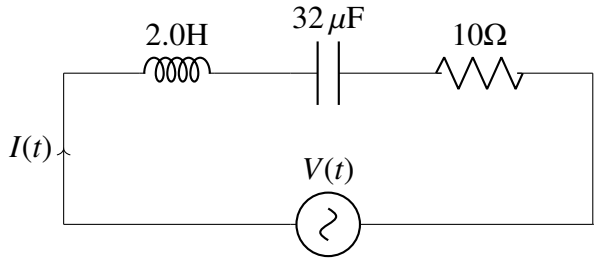

Fig. 1. LCR Circuit

Solution: In Figure Fig. 1 the following information is provided:

Symbol	Value	Description
L	2.0 H	Inductance
C	32 μF	Capacitance
R	10 Ω	Resistance
Q	$\frac{f_r}{\Delta f}$	Quality Factor

TABLE I
PARAMETERS

Series LCR Circuit Analysis

Now, the voltage transfer function ($\frac{V(s)}{I(s)} = H(s)$) is given by Ohm's Law in the Laplace domain:

$$\frac{V(s)}{I(s)} = R + sL + \frac{1}{sC} \tag{1}$$

Now, after substitution the equation is

$$\frac{V(s)}{I(s)} = 10 + 2s + \frac{1}{32 \times 10^{-6} s} \tag{2}$$

This is the voltage transfer function for the series LCR circuit in the Laplace domain.

Resonance Frequency (ω_0):

At resonance, the impedance is purely resistive, meaning the imaginary part of $Z(s)$ is zero.

$$\text{Im}\{Z(s)\} = \omega L - \frac{1}{\omega C} = 0 \tag{3}$$

Solving for ω_r:

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{4}$$

Substituting values:

$$\omega_0 = \frac{1}{\sqrt{2\,\text{H} \times 32\,\mu\text{F}}} = 125\,\text{rad/s} \tag{5}$$

The resonance frequency is 125 rad/s.

Quality Factor (Q) Calculation: The quality factor (Q) of a series RLC circuit is given by the formula:

$$Q = \frac{f_r}{\Delta f} \tag{6}$$

where f_r is the resonant frequency and Δf is the bandwidth of the circuit.

Resonant Frequency (f_r)

The resonant frequency (f_r) is related to the inductance (L) and capacitance (C) by the formula:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{7}$$

Bandwidth (Δf)

The bandwidth (Δf) is related to the resistance (R) and inductance (L) by:

$$\Delta f = \frac{R}{2\pi L} \tag{8}$$

Now, substitute these expressions into the formula for Q:

$$Q = \frac{f_r}{\Delta f} \tag{9}$$

$$Q = \frac{\frac{1}{2\pi\sqrt{LC}}}{\frac{R}{2\pi L}} \tag{10}$$

Simplify:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{11}$$

Substituting the values, we get :

$$Q = \frac{1}{10}\sqrt{\frac{2}{32\times10^{-6}}} = \frac{1}{40\times10^{-3}} = 25 \tag{12}$$

Therefore, the quality factor of the LCR circuit is 25.

The equivalent s domain of the circuit is :

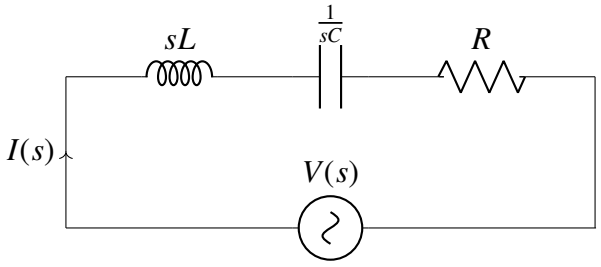

Fig. 2. LCR Circuit in s-domain