NCERT Discrete 11.9.1 Q7

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Question: If a,b,c are in A.P.;b,c,d are in G.P and $\frac{1}{c}$, $\frac{1}{d}$, $\frac{1}{e}$ are in A.P. prove that a,c,e are in G.P.
Solution: In the question, following information is provided:

Symbol	Remarks
$x_1(n)$	a,b,c in A.P
$x_2(n)$	b,c,d in G.P
$x_3(n)$	$\frac{1}{c},\frac{1}{d},\frac{1}{e}$
$y(n)$	a,c,e in G.P
$x_1(0)$	a
$x_1(1)$	b
$x_1(2)$	c
$x_2(2)$	d
$x_3(2)$	$\frac{1}{e}$

TABLE I
PARAMETERS

$$b-a=c-d \tag{1}$$
$$2b=a+c \tag{2}$$
$$b=\frac{a+c}{2} \tag{3}$$
$$c^2=b\times d \tag{4}$$
$$d=\frac{c^2}{b} \tag{5}$$
$$\frac{1}{d}-\frac{1}{c}=\frac{1}{e}-\frac{1}{d} \tag{6}$$
$$\frac{2}{d}=\frac{1}{c}+\frac{1}{e} \tag{7}$$
$$\frac{2b}{c^2}=\frac{1}{c}+\frac{1}{e} \tag{8}$$
$$\frac{a+c}{c^2}=\frac{1}{c}+\frac{1}{e} \tag{9}$$
$$(a+c)\times e=(e+c)\times c \tag{10}$$
$$a\cdot e=c^2 \tag{11}$$

So, a,c,e are in G.P

For a,b,c in A.P,

$$x_1(n)=(a+n(b-a))u(n) \tag{12}$$

Using Z-transform,

$$X_1(z)=\frac{a}{1-z^{-1}}+\frac{(b-a)z^{-1}}{(1-z^{-1})^2} \tag{13}$$

For b,c,d in G.P,

$$x_2(n)=b\left(\frac{c}{b}\right)^n u(n) \tag{14}$$

Using Z-transform,

$$X_2(z)=\frac{c}{1-\frac{c}{b}z^{-1}},\quad |z|>\left|\frac{c}{b}\right| \tag{15}$$

For $\frac{1}{c}$, $\frac{1}{d}$, $\frac{1}{e}$ in A.P,

$$x_3(n)=\left(\frac{1}{c}+n\left(\frac{1}{d}-\frac{1}{c}\right)\right)u(n) \tag{16}$$

Using Z-transform,

$$X_3(z) = \frac{1}{c(1-z^{-1})} + \left(\frac{1}{d} - \frac{1}{c}\right)\frac{z^{-1}}{(1-z^{-1})^2} \qquad (17)$$

For a,c,e in G.P,

$$y(n) = a\left(\frac{c}{a}\right)^n u(n) \qquad (18)$$

Using Z-transform,

$$Y(z) = \frac{c}{1-\frac{c}{a}z^{-1}}, \quad |z| > \left|\frac{c}{a}\right| \qquad (19)$$