NCERT Discrete 11.9.1 Q7

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Question: If a,b,c are in A.P.;b,c,d are in G.P and $\frac{1}{c}$, $\frac{1}{d}$, $\frac{1}{e}$ are in A.P. prove that a,c,e are in G.P.
Solution:

Symbol	Value	Description
$x_1(n)$	$\{a,b,c\}$	A.P Sequence
$x_2(n)$	$\{b,c,d\}$	G.P Sequence
$x_3(n)$	$\left\{\frac{1}{c},\frac{1}{d},\frac{1}{e}\right\}$	A.P Sequence
$y(n)$	$\{a,c,e\}$	Sequence

TABLE I
PARAMETERS

$$b-a=c-b \tag{1}$$
$$2b=a+c \tag{2}$$
$$b=\frac{a+c}{2} \tag{3}$$
$$c^2=b\times d \tag{4}$$
$$d=\frac{c^2}{b} \tag{5}$$
$$\frac{1}{d}-\frac{1}{c}=\frac{1}{e}-\frac{1}{d} \tag{6}$$
$$\frac{2}{d}=\frac{1}{c}+\frac{1}{e} \tag{7}$$

From (5),

$$\frac{2b}{c^2}=\frac{1}{c}+\frac{1}{e} \tag{8}$$

From (2),

$$\frac{a+c}{c^2}=\frac{1}{c}+\frac{1}{e} \tag{9}$$
$$\frac{a}{c^2}+\frac{1}{c}=\frac{1}{c}+\frac{1}{e} \tag{10}$$
$$a\times e=c^2 \tag{11}$$
$$\tag{12}$$

So, a,c,e are in G.P

For $x_1(n)$,

$$x_1(n)=(x_1(0)+n(x_1(1)-x_1(0)))u(n) \tag{13}$$

Using Z-transform,

$$X_1(z)=\frac{x_1(0)}{1-z^{-1}}+\frac{(x_1(1)-x_1(0))z^{-1}}{(1-z^{-1})^2},\quad |z|>1 \tag{14}$$

For $x_2(n)$,

$$x_2(n)=x_2(0)\left(\frac{x_2(1)}{x_2(0)}\right)^n u(n) \tag{15}$$

Using Z-transform,

$$X_2(z)=\frac{x_2(1)}{1-\frac{x_2(1)}{x_2(0)}z^{-1}},\quad |z|>\left|\frac{x_2(1)}{x_2(0)}\right| \tag{16}$$

For $x_3(n)$,

$$x_3(n)=(x_3(0)+n\,(x_3(0)-x_3(1)))\,u(n) \tag{17}$$

Using Z-transform,

$$X_3(z)=\frac{x_3(0)}{1-z^{-1}}+(x_3(1)-x_3(0))\,\frac{z^{-1}}{(1-z^{-1})^2},\quad |z|>1 \tag{18}$$

For $y(n)$,

$$y(n)=y(0)\left(\frac{y(1)}{y(0)}\right)^n u(n) \tag{19}$$

Using Z-transform,

$$Y(z)=\frac{y(1)}{1-\frac{y(1)}{y(0)}z^{-1}},\quad |z|>\left|\frac{y(1)}{y(0)}\right| \tag{20}$$