NCERT Physics 12.7 Q6

EE23BTECH11061 - SWATHI DEEPIKA*

Question: Obtain the resonance frequency of a series LCR circuit with $L = 2.0\,H$, $C = 32\,\mu F$, and $R = 10\,\Omega$. What is the Q-value of the circuit.

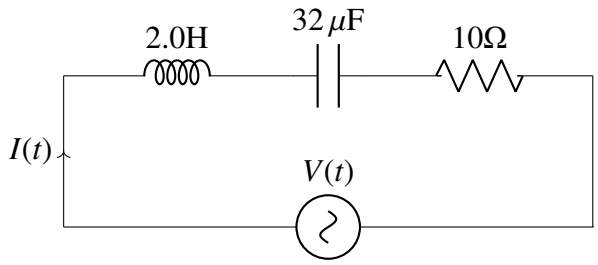

Fig. 1. LCR Circuit

Solution: In Figure Fig. 1 the following information is provided:

Symbol	Value	Description
L	2.0 H	Inductance
C	32 μF	Capacitance
R	10 Ω	Resistance
Q	$\frac{V_L}{V_R}$	Quality Factor
V_L	$sLI(s)$	Voltage across inductance
V_C	$RI(s)$	Voltage across capacitor

TABLE I
PARAMETERS

The equivalent s domain of the circuit is :

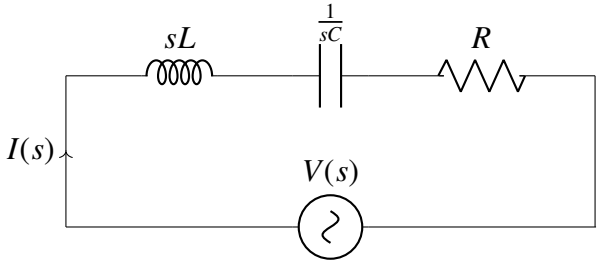

Fig. 2. LCR Circuit in s-domain

$$\frac{V(s)}{I(s)} = H(s) \tag{1}$$

$$\frac{V(s)}{I(s)} = R + sL + \frac{1}{sC} \tag{2}$$

At resonance,

$$Ls + \frac{1}{sC} = 0 \tag{3}$$

$$\implies s = j\frac{1}{\sqrt{LC}} \tag{4}$$

$$s = j\omega_0 \tag{5}$$

Comparing equations (4) and (5), we get

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{6}$$

$$I_s(t) = I_0 \sin(\omega t + \phi) \tag{7}$$

$$\frac{di}{dt} = I_0\omega\cos(\omega t + \phi). \tag{8}$$

$$V_s = L\frac{di}{dt} \tag{9}$$

$$= LI_0\omega\cos(\omega t + \phi). \tag{10}$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}. \tag{11}$$

$$V_s = LI_0\omega\left(\frac{e^{j(\omega t+\phi)} + e^{-j(\omega t+\phi)}}{2}\right). \tag{12}$$

$$V_s = j\omega LI_0\sin(\omega t + \phi). \tag{13}$$

$$V_s = j\omega LI_s. \tag{14}$$

$$V_s = sLI_s. \tag{15}$$

$$V_R = I_s \cdot R \tag{16}$$

From (15) and (16) ,

$$Q = \left(\frac{V_L}{V_R}\right)_{\omega_0} = \frac{|sLI(s)|}{|RI(s)|} \tag{17}$$

$$\implies Q = \frac{1}{\sqrt{LC}}\frac{L}{R} \tag{18}$$

$$\implies Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{19}$$

$$\frac{V(s)}{I(s)} = 10 + 2s + \frac{1}{32 \times 10^{-6} s} \tag{20}$$

$$H(s) = 10 + 2s + \frac{1}{32 \times 10^{-6} s} \tag{21}$$

From, (6) :

$$\omega_0 = \frac{1}{\sqrt{2\,\mathrm{H} \times 32\,\mu\mathrm{F}}} = 125\,\mathrm{rad/s} \tag{22}$$

The resonant frequency is 125 rad/s.

From, (19) :

$$Q = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}} = \frac{1}{40 \times 10^{-3}} = 25 \tag{23}$$

Therefore, the quality factor of the LCR circuit is 25.

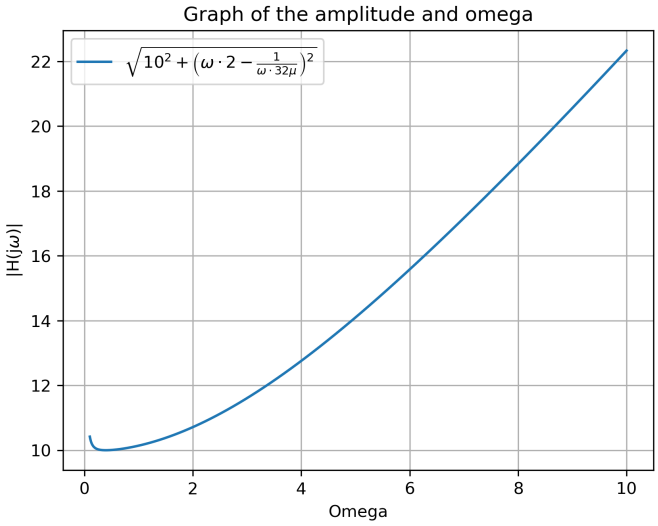

Fig. 3. Impedance vs ω

$$H(s) = R + sL + \frac{1}{sC} \tag{24}$$

$$\implies H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \tag{25}$$

$$\implies |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{26}$$