NCERT Discrete 11.9.1 Q7

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Question: If a,b,c are in A.P.;b,c,d are in G.P and $\frac{1}{c}$, $\frac{1}{d}$, $\frac{1}{e}$ are in A.P. prove that a,c,e are in G.P.
Solution:

Symbol	**Remarks**
$x_1(n)$	A.P Sequence
$x_2(n)$	G.P Sequence
$x_3(n)$	A.P Sequence
$y(n)$	Sequence of a, c, e
$x_1(0)$	a
$x_1(1)$	b
$x_1(2)$	c
$x_2(0)$	b
$x_2(1)$	c
$x_2(2)$	d
$x_3(0)$	$\frac{1}{c}$
$x_3(1)$	$\frac{1}{d}$
$x_3(2)$	$\frac{1}{e}$
$y(0)$	a
$y(1)$	c
$y(2)$	e

TABLE I
Parameters

$$x_1(1) - x_1(0) = x_1(2) - x_1(1) \tag{1}$$
$$2x_1(1) = x_1(0) + x_1(2) \tag{2}$$

$$x_1(1) = \frac{x_1(0) + x_1(2)}{2} \tag{3}$$

$$x_2(1)^2 = x_2(0) \times x_2(2) \tag{4}$$

$$x_2(2) = \frac{x_2(1)^2}{x_2(0)} \tag{5}$$

$$x_3(1) - x_3(0) = x_3(2) - x_3(1) \tag{6}$$
$$2x_3(1) = x_3(0) + x_3(2) \tag{7}$$

From (5),

$$2\frac{x_2(0)}{x_2(1)^2} = x_3(0) + x_3(2) \tag{8}$$

From (2),

$$\frac{x_1(0) + x_1(2)}{x_1(2)^2} = x_3(0) + x_3(2) \tag{9}$$
$$\frac{x_1(0)}{x_1(2)^2} + \frac{1}{x_1(2)} = x_3(0) + x_3(2) \tag{10}$$
$$\frac{x_1(0)}{x_3(2)} = x_2(2)^2 \tag{11}$$
$$y(0) \cdot y(2) = y(1)^2 \tag{12}$$

So, $y(0)$,$y(1)$,$y(2)$ are in G.P

For $x_1(n)$,

$$x_1(n) = (x_1(0) + n(x_1(1) - x_1(0)))u(n) \tag{13}$$

Using Z-transform,

$$X_1(z) = \frac{x_1(0)}{1 - z^{-1}} + \frac{(x_1(1) - x_1(0))z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{14}$$

For $x_2(n)$,

$$x_2(n) = x_2(0)\left(\frac{x_2(1)}{x_2(0)}\right)^n u(n) \tag{15}$$

Using Z-transform,

$$X_2(z) = \frac{x_2(1)}{1 - \frac{x_2(1)}{x_2(0)}z^{-1}}, \quad |z| > \left|\frac{x_2(1)}{x_2(0)}\right| \tag{16}$$

For $x_3(n)$,

$$x_3(n) = (x_3(0) + n\,(x_3(0) - x_3(1)))\,u(n) \tag{17}$$

Using Z-transform,

$$X_3(z) = \frac{x_3(0)}{1 - z^{-1}} + (x_3(1) - x_3(0)) \frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{18}$$

For $y(n)$,

$$y(n) = y(0) \left(\frac{y(1)}{y(0)} \right)^n u(n) \tag{19}$$

Using Z-transform,

$$Y(z) = \frac{y(1)}{1 - \frac{y(1)}{y(0)} z^{-1}}, \quad |z| > \left| \frac{y(1)}{y(0)} \right| \tag{20}$$