NCERT Physics 12.7 Q6

EE23BTECH11212 - SWATHI DEEPIKA*

Question: Obtain the resonant frequency of a series LCR circuit with $L = 2.0\,H$, $C = 32\,\mu F$, and $R = 10\,\Omega$. What is the Q-value of the circuit.

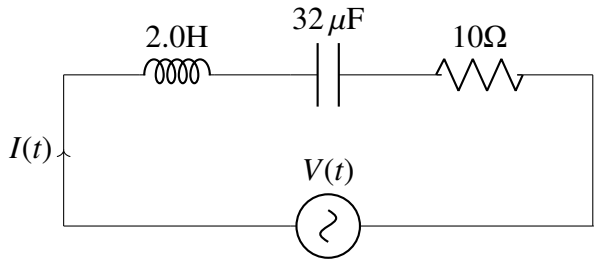

Fig. 1. LCR Circuit

Solution: In Figure Fig. 1 the following information is provided:

Symbol	Value	Description
L	2.0 H	Inductance
C	32 μF	Capacitance
R	10 Ω	Resistance
Q	$\frac{1}{R}\sqrt{\frac{L}{C}}$	Quality Factor

TABLE I
PARAMETERS

Series LCR Circuit Analysis

The resonant frequency (f_r) of a series LCR circuit can be calculated using the formula:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{1}$$

where L is the inductance, C is the capacitance, and π is a mathematical constant.

Given $L = 2.0\,\text{H}$ and C= $32\,\mu\text{F}$ ($32 \times 10^{-6}\,\text{F}$), the resonant frequency is calculated as follows:

$$f_r = \frac{1}{2\pi\sqrt{(2.0\,\text{H})(32 \times 10^{-6}\,\text{F})}} \approx \frac{1}{2\pi \times 0.008} \approx 20\,\text{Hz} \tag{2}$$

Next, the quality factor (Q) of the circuit can be calculated using the formula:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{3}$$

Given $R = 10$, $L = 2.0\,\text{H}$, and $C = 32\,\mu\text{F}$, the Q-value is found as:

$$Q = \frac{1}{10}\sqrt{\frac{2.0\,\text{H}}{32 \times 10^{-6}\,\text{F}}} \approx \frac{1}{10} \times \frac{1}{0.004} \approx 25 \tag{4}$$

Now, the voltage transfer function ($\frac{V(s)}{I(s)} = H(s)$) is given by Ohm's Law in the Laplace domain:

$$\frac{V(s)}{I(s)} = R + sL + \frac{1}{sC} \tag{5}$$

Now, after substitution the equation is

$$\frac{V(s)}{I(s)} = 10 + 2s + \frac{1}{32 \times 10^{-6}s} \tag{6}$$

This is the voltage transfer function for the series LCR circuit in the Laplace domain.

The equivalent s domain of the circuit is :

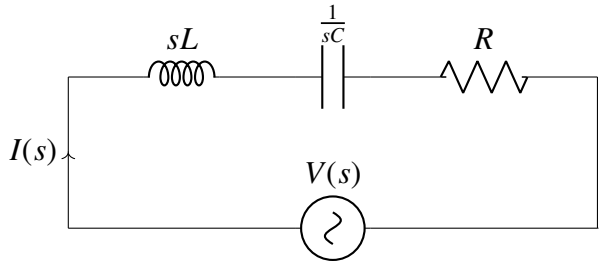

Fig. 2. LCR Circuit in s-domain