GATE 2023 EC 48

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Question: Let an input x[n] having discrete time Fourier transform $X(e^{j\omega}) = 1 - e^{-j\omega} + 2e^{-3j\omega}$ be passed through an LTI system. The frequency response of the LTI system is $H(e^{j\omega}) = 1 - \frac{1}{2}e^{-2j\omega}$. The output $y[n]$ of the system is
Solution:

Parameter	Value
$X(e^{j\omega})$	$1 - e^{-j\omega} + 2e^{-3j\omega}$
$H(e^{j\omega})$	$1 - \frac{1}{2}e^{-2j\omega}$

TABLE I
PARAMETERS

Given the convolution equation:

$$y[n] = x[n] * h[n] \tag{1}$$

And the corresponding frequency domain representation:

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega}) \tag{2}$$

Given:

$$Y(e^{j\omega}) = (1 - e^{-j\omega} + 2e^{-3j\omega}) \cdot \left(1 - \frac{1}{2}e^{-2j\omega}\right) \tag{3}$$

Expanding:

$$Y(e^{j\omega}) = 1 - e^{-j\omega} + 2.5e^{-3j\omega} - 0.5e^{-j2\omega} - e^{-j5\omega} \tag{4}$$

Taking the inverse discrete-time Fourier transform (IDTFT):

$$y[n] = \delta[n] - \delta[n-1] - 0.5\delta[n-2] + 2.5\delta[n-3] - \delta[n-5] \tag{5}$$