GATE 2023 EC 48

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Question: Let an input x[n] having discrete time Fourier transform $X(e^{j\omega}) = 1 - e^{-j\omega} + 2e^{-3j\omega}$ be passed through an LTI system. The frequency response of the LTI system is $H(e^{j\omega}) = 1 - \frac{1}{2}e^{-2j\omega}$. The output $y[n]$ of the system is

Solution:

Parameter	Value
$X(e^{j\omega})$	$1 - e^{-j\omega} + 2e^{-3j\omega}$
$H(e^{j\omega})$	$1 - \frac{1}{2}e^{-2j\omega}$
$Y(e^{j\omega})$	$X(e^{j\omega}) \cdot H(e^{j\omega})$
$y[n]$	?
$\delta[n]$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{j\omega n} d\omega$

TABLE I
PARAMETERS

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$y[n] = x[n] * h[n] \tag{2}$$

$$x(n) * h(n) \longleftrightarrow X(e^{j\omega}) \cdot H(e^{j\omega})$$

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega}) \tag{3}$$

$$Y(e^{j\omega}) = (1 - e^{-j\omega} + 2e^{-3j\omega}) \cdot \left(1 - \frac{1}{2}e^{-2j\omega}\right) \tag{4}$$

$$= (1 - e^{-j\omega} + \frac{5}{2}e^{-3j\omega} - \frac{1}{2}e^{-2j\omega} - e^{-5j\omega}) \tag{5}$$

$$y[n] = \mathcal{F}^{-1}\{Y(e^{j\omega})\} \tag{6}$$

$$y[n] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Y(e^{j\omega})e^{j\omega n} d\omega \tag{7}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left(1 - e^{-j\omega} + \frac{5}{2}e^{-3j\omega} - \frac{1}{2}e^{-2j\omega} - e^{-5j\omega}\right) e^{j\omega n} d\omega \tag{8}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{j\omega n} d\omega - \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{j\omega(n-1)} d\omega + \frac{1}{2\pi}\int_{-\infty}^{+\infty} +\frac{5}{2}e^{j\omega(n-3)} d\omega - \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{2}e^{j\omega(n-2)} d\omega - \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{j\omega(n-5)} d\omega \tag{9}$$

$$y[n] = \delta[n] - \delta[n-1] + \frac{5}{2}\delta[n-3] - \frac{1}{2}\delta[n-2] - \delta[n-5] \tag{10}$$

$$y[n] = \delta[n] - \delta[n-1] + 2.5\delta[n-3] - 0.5\delta[n-2] - \delta[n-5] \tag{11}$$

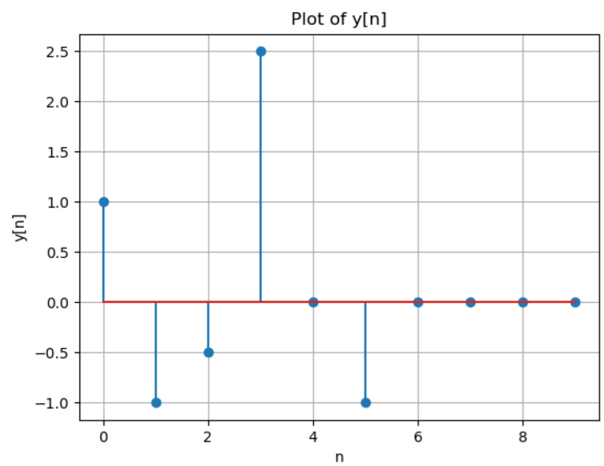

Fig. 1. $y(n)$ vs n