

# NCERT Discrete 11.9.1 Q7

EE23BTECH11061 - SWATHI DEEPIKA\*

**Question:** If  $a, b, c$  are in A.P.;  $b, c, d$  are in G.P and  $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$  are in A.P. prove that  $a, c, e$  are in G.P.

**Solution:** In the question, following information is provided:

$$b - a = c - d \quad (1)$$

$$2b = a + c \quad (2)$$

$$b = \frac{a + c}{2} \quad (3)$$

$$c^2 = b \times d \quad (4)$$

$$d = \frac{c^2}{b} \quad (5)$$

$$\frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d} \quad (6)$$

$$\frac{2}{d} = \frac{1}{c} + \frac{1}{e} \quad (7)$$

$$\frac{2b}{c^2} = \frac{1}{c} + \frac{1}{e} \quad (8)$$

$$\frac{a + c}{c^2} = \frac{1}{c} + \frac{1}{e} \quad (9)$$

$$(a + c) \times e = (e + c) \times c \quad (10)$$

$$a \cdot e = c^2 \quad (11)$$

So,  $a, c, e$  are in G.P

For  $a, b, c$  in A.P,

$$x_1(n) = (a + n(b - a))u(n) \quad (12)$$

Using Z-transform,

$$X_1(z) = \frac{a}{1 - z^{-1}} + \frac{(b - a)z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (13)$$

For  $b, c, d$  in G.P,

$$x_2(n) = b \left( \frac{c}{b} \right)^n u(n) \quad (14)$$

Using Z-transform,

$$X_2(z) = \frac{c}{1 - \frac{c}{b}z^{-1}}, \quad |z| > \left| \frac{c}{b} \right| \quad (15)$$

For  $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$  in A.P,

$$x_3(n) = \left( \frac{1}{c} + n \left( \frac{1}{d} - \frac{1}{c} \right) \right) u(n) \quad (16)$$

Using Z-transform,

Symbol	Remarks
$x_1(n)$	$a, b, c$ in A.P
$x_2(n)$	$b, c, d$ in G.P
$x_3(n)$	$\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$
$y(n)$	$a, c, e$ in G.P
$x_1(0)$	$a$
$x_1(1)$	$b$
$x_1(2)$	$c$
$x_2(2)$	$d$
$x_3(2)$	$\frac{1}{e}$

TABLE I  
PARAMETERS

$$X_3(z) = \frac{1}{c(1-z^{-1})} + \left(\frac{1}{d} - \frac{1}{c}\right) \frac{z^{-1}}{(1-z^{-1})^2}, \quad |z| > 1 \quad (17)$$

For  $a, c, e$  in G.P,

$$y(n) = a \left(\frac{c}{a}\right)^n u(n) \quad (18)$$

Using Z-transform,

$$Y(z) = \frac{c}{1 - \frac{c}{a}z^{-1}}, \quad |z| > \left|\frac{c}{a}\right| \quad (19)$$