# GATE 2023 EC 48

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**Question:** Let an input x[n] having discrete time Fourier transform $X(e^{j\omega}) = 1-e^{-j\omega}+2e^{-3j\omega}$ be passed through an LTI system. The frequency response of the LTI system is $H(e^{j\omega}) = 1 - \frac{1}{2}e^{-2j\omega}$ . The output $y[n]$ of the system is

**Solution:**

| Parameter | Value |
|---|---|
| $X(e^{j\omega})$ | $1 - e^{-j\omega} + 2e^{-3j\omega}$ |
| $H(e^{j\omega})$ | $1 - \frac{1}{2}e^{-2j\omega}$ |
| $Y(e^{j\omega})$ | $X(e^{j\omega}) \cdot H(e^{j\omega})$ |
| $y[n]$ | ? |

TABLE I
PARAMETERS

$$y[n] = x[n] * h[n] \tag{1}$$

$$x(n) * h(n) \longleftrightarrow X(e^{j\omega}) \cdot H(e^{j\omega})$$

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega}) \tag{2}$$

$$Y(e^{j\omega}) = (1 - e^{-j\omega} + 2e^{-3j\omega}) \cdot \left(1 - \frac{1}{2}e^{-2j\omega}\right) \tag{3}$$

$$= (1 - e^{-j\omega} + \frac{5}{2}e^{-3j\omega} - \frac{1}{2}e^{-2j\omega} - e^{-5j\omega}) \tag{4}$$

$$y[n] = \mathcal{F}^{-1}\{Y(e^{j\omega})\}$$

$$y[n] = \delta[n] - \delta[n-1] + \frac{5}{2}\delta[n-3] - \frac{1}{2}\delta[n-2] - \delta[n-5] \tag{5}$$

$$y[n] = \delta[n] - \delta[n-1] + 2.5\delta[n-3] - 0.5\delta[n-2] - \delta[n-5] \tag{6}$$

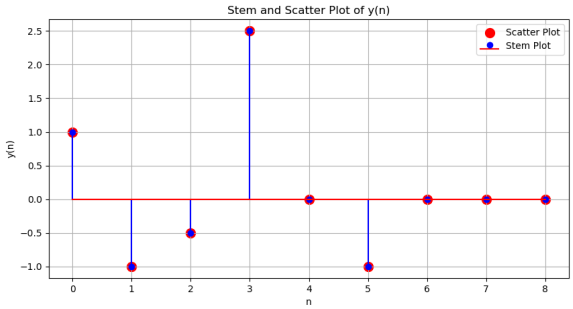


Fig. 1. $y(n)$ vs $n$