NCERT Physics 12.7 Q6

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Question: Obtain the resonance frequency of a series LCR circuit with $L = 2.0\,H$, $C = 32\,\mu F$, and $R = 10\,\Omega$. What is the Q-value of the circuit.

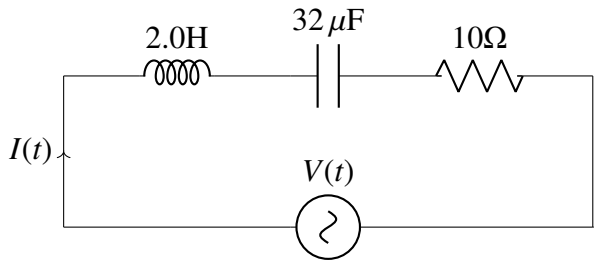

Fig. 1. LCR Circuit

Solution: In Figure Fig. 1 the following information is provided:

Symbol	Value	Description
L	2.0 H	Inductance
C	32 μF	Capacitance
R	10 Ω	Resistance
Q	$\frac{V_L}{V_R}$	Quality Factor
V_L	$sLI(s)$	Voltage across inductance
V_C	$RI(s)$	Voltage across capacitor

TABLE I
PARAMETERS

The equivalent s domain of the circuit is :

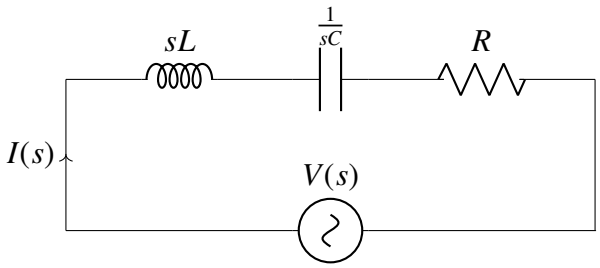

Fig. 2. LCR Circuit in s-domain

Now, the voltage transfer function ($\frac{V(s)}{I(s)} = H(s)$) is given by Ohm's Law in the Laplace domain:

$$\frac{V(s)}{I(s)} = R + sL + \frac{1}{sC} \tag{1}$$

Now, after substitution the equation is

$$\frac{V(s)}{I(s)} = 10 + 2s + \frac{1}{32 \times 10^{-6} s} \tag{2}$$

$$H(s) = 10 + 2s + \frac{1}{32 \times 10^{-6} s} \tag{3}$$

This is the voltage transfer function for the series LCR circuit in the Laplace domain.

At resonance, the circuit becomes purely resistive. The reactances of capacitor and inductor cancel out as follows:

$$Ls + \frac{1}{sC} = 0 \tag{4}$$

$$\implies s = j\frac{1}{\sqrt{LC}} \tag{5}$$

s can be expressed in terms of resonant frequency as

$$s = j\omega_0 \tag{6}$$

Comparing equations (5) and (6), we get

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{7}$$

Substituting values:

$$\omega_0 = \frac{1}{\sqrt{2\,\mathrm{H} \times 32\,\mu\mathrm{F}}} = 125\,\mathrm{rad/s} \tag{8}$$

The resonant frequency is 125 rad/s.

Quality Factor (Q) of an LCR circuit is defined as the ratio of voltage across inductor or capacitor to that across the resistor at resonance.

$$Q = \left(\frac{V_L}{V_R}\right)_{\omega_0} = \frac{|sLI(s)|}{|RI(s)|} \tag{9}$$

$$\implies Q = \frac{1}{\sqrt{LC}}\frac{L}{R} \tag{10}$$

$$\implies Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{11}$$

Substituting the values,

$$Q = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}} = \frac{1}{40 \times 10^{-3}} = 25 \tag{12}$$

Therefore, the quality factor of the LCR circuit is 25.

Plot the amplitude of the transfer function w.r.t omega :

$$H(s) = R + sL + \frac{1}{sC} \tag{13}$$

$$\implies H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \tag{14}$$

$$\implies |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{15}$$

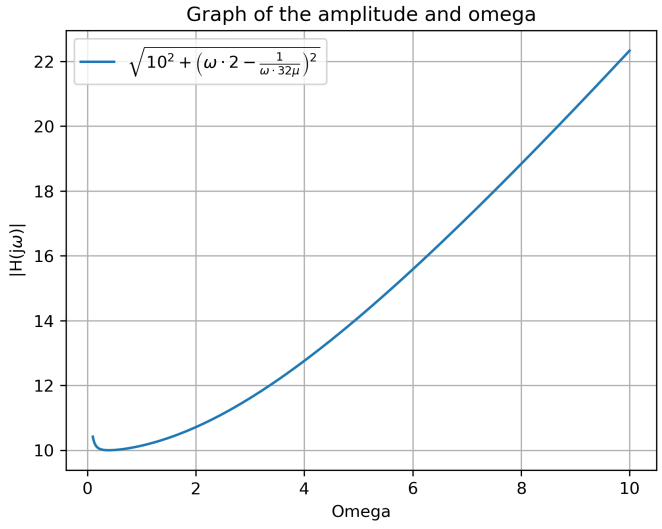

Fig. 3. Impedance vs ω