NCERT Discrete 11.5.9 Q20

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Question: If a,b,c are in A.P.;b,c,d are in G.P and $\frac{1}{c}$, $\frac{1}{d}$, $\frac{1}{e}$ are in A.P. prove that a,c,e are in G.P.
Solution:

Symbol	Value	Description	Z-Transform
$x_1(n)$	$\{a, b, c...\}$	A.P Sequence	$X_1(z)$
$x_2(n)$	$\{b, c, d...\}$	G.P Sequence	$X_2(z)$
$x_3(n)$	$\left\{\frac{1}{c}, \frac{1}{d}, \frac{1}{e}...\right\}$	A.P Sequence	$X_3(z)$
$y(n)$	$\{a, c, e...\}$	Sequence	$Y(z)$

TABLE I
PARAMETERS

$$b - a = c - b \tag{1}$$
$$2b = a + c \tag{2}$$

$$c^2 = b \times d \tag{3}$$

$$d = \frac{c^2}{b} \tag{4}$$

$$\frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d} \tag{5}$$
$$\frac{2}{d} = \frac{1}{c} + \frac{1}{e} \tag{6}$$

From (4),

$$\frac{2b}{c^2} = \frac{1}{c} + \frac{1}{e} \tag{7}$$

From (2),

$$\frac{a + c}{c^2} = \frac{1}{c} + \frac{1}{e} \tag{8}$$
$$\frac{a}{c^2} + \frac{1}{c} = \frac{1}{c} + \frac{1}{e} \tag{9}$$
$$a \times e = c^2 \tag{10}$$
$$y(1)^2 = y(0) \times y(2) \tag{11}$$

So, $y(0)$,$y(1)$,$y(2)$ are in G.P

1) For $y(n)$:

$$y(n) = y(0)\left(\frac{y(1)}{y(0)}\right)^n u(n) \tag{12}$$

$$y(n) \longleftrightarrow Y(z)$$

$$Y(z) = \frac{y(1)}{1 - \frac{y(1)}{y(0)}z^{-1}}, \quad |z| > \left|\frac{y(1)}{y(0)}\right| \tag{13}$$

2) For $x_1(n)$:

$$x_1(n) = (x_1(0) + n(x_1(1) - x_1(0)))u(n) \tag{14}$$

$$x_1(n) \longleftrightarrow X_1(z)$$

$$X_1(z) = \frac{x_1(0)}{1 - z^{-1}} + \frac{(x_1(1) - x_1(0))z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{15}$$

3) For $x_2(n)$:

$$x_2(n) = x_2(0)\left(\frac{x_2(1)}{x_2(0)}\right)^n u(n) \tag{16}$$

$$x_2(n) \longleftrightarrow X_2(z)$$

$$X_2(z) = \frac{x_2(1)}{1 - \frac{x_2(1)}{x_2(0)}z^{-1}}, \quad |z| > \left|\frac{x_2(1)}{x_2(0)}\right| \tag{17}$$

4) For $x_3(n)$:

$$x_3(n) = (x_3(0) + n\,(x_3(0) - x_3(1)))\,u(n) \tag{18}$$

$$x_3(n) \longleftrightarrow X_3(z)$$

$$X_3(z) = \frac{x_3(0)}{1 - z^{-1}} + (x_3(1) - x_3(0))\,\frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{19}$$