

GATE 2023 EC 48

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Question: Let an input $x[n]$ having discrete time Fourier transform $X(e^{j\omega}) = 1 - e^{-j\omega} + 2e^{-3j\omega}$ be passed through an LTI system. The frequency response of the LTI system is $H(e^{j\omega}) = 1 - \frac{1}{2}e^{-2j\omega}$. The output $y[n]$ of the system is

Solution:

| Parameter | Value |
|------------------|--|
| $X(e^{j\omega})$ | $1 - e^{-j\omega} + 2e^{-3j\omega}$ |
| $H(e^{j\omega})$ | $1 - \frac{1}{2}e^{-2j\omega}$ |
| $Y(e^{j\omega})$ | $X(e^{j\omega}) \cdot H(e^{j\omega})$ |
| $y[n]$ | ? |
| $\delta[n]$ | $\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\omega n} d\omega$ |

TABLE I
PARAMETERS

$$y[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(e^{j\omega}) e^{j\omega n} d\omega \quad (5)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(1 - e^{-j\omega} + \frac{5}{2}e^{-3j\omega} - \frac{1}{2}e^{-2j\omega} - e^{-5j\omega} \right) e^{j\omega n} d\omega \quad (6)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\omega n} d\omega - \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\omega(n-1)} d\omega + \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{5}{2} e^{j\omega(n-3)} d\omega \quad (7)$$

$$- \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{2} e^{j\omega(n-2)} d\omega - \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\omega(n-5)} d\omega \quad (8)$$

$$y[n] = \delta[n] - \delta[n-1] + \frac{5}{2}\delta[n-3] - \frac{1}{2}\delta[n-2] - \delta[n-5] \quad (9)$$

$$y[n] = \delta[n] - \delta[n-1] + 2.5\delta[n-3] - 0.5\delta[n-2] - \delta[n-5] \quad (10)$$

$$y[n] = x[n] * h[n] \quad (1)$$

$$x(n) * h(n) \longleftrightarrow X(e^{j\omega}) \cdot H(e^{j\omega})$$

$$Y[e^{j\omega}] = X(e^{j\omega}) \cdot H(e^{j\omega}) \quad (2)$$

$$Y[e^{j\omega}] = (1 - e^{-j\omega} + 2e^{-3j\omega}) \cdot \left(1 - \frac{1}{2}e^{-2j\omega} \right) \quad (3)$$

$$= (1 - e^{-j\omega} + \frac{5}{2}e^{-3j\omega} - \frac{1}{2}e^{-2j\omega} - e^{-5j\omega}) \quad (4)$$

$$Y[e^{j\omega}] \longleftrightarrow y[n]$$