NCERT Physics 12.7 Q6

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Question: Obtain the resonance frequency of a series LCR circuit with $L = 2.0\,H$, $C = 32\,\mu F$, and $R = 10\,\Omega$. What is the Q-value of the circuit.

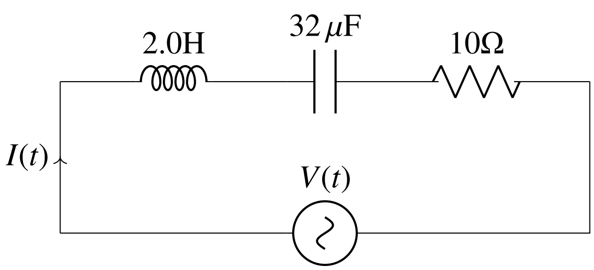

Fig. 1. LCR Circuit

Solution:

Symbol	Value	Description
L	2.0 H	Inductance
C	32 μF	Capacitance
R	10 Ω	Resistance
Q	$\frac{V_L}{V_R}$	Quality Factor
V_L	$sLI(s)$	Voltage across inductance
V_C	$RI(s)$	Voltage across capacitor

TABLE I
PARAMETERS

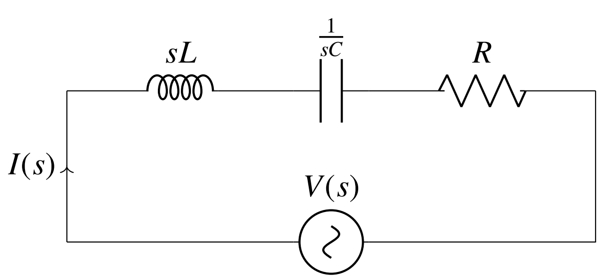

Fig. 2. LCR Circuit

1) Frequency Response of the Circuit
   From KVL:

$$V(t) = V_R + V_L + V_C \tag{1}$$

From Fig. 2,

$$V(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s) \tag{2}$$

$$= I(s)\left(R + Ls + \frac{1}{sC}\right) \tag{3}$$

$$\implies I(s) = \frac{V(s)}{\left(R + Ls + \frac{1}{sC}\right)} \tag{4}$$

At resonance,

$$Ls + \frac{1}{sC} = 0 \tag{5}$$

$$\implies s = j\frac{1}{\sqrt{LC}} \tag{6}$$

s in terms of angular resonance frequency as

$$s = j\omega_0 \tag{7}$$

From (6) and (7), we get

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{8}$$

2) Quality Factor
   a) voltage across inductor,

$$Q = \left(\frac{V_L}{V_R}\right)_{\omega_0} = \frac{|sLI(s)|}{|RI(s)|} \tag{9}$$

$$= \frac{1}{\sqrt{LC}}\frac{L}{R} \tag{10}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}} \tag{11}$$

   b) Using voltage across capacitor,

$$Q = \left(\frac{V_C}{V_R}\right)_{\omega_0} = \frac{\left|\frac{I(s)}{sC}\right|}{|RI(s)|} \tag{12}$$

$$= \frac{\sqrt{LC}}{RC} \tag{13}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}} \tag{14}$$

3) Plot of Impedance vs Angular Frequency

$$H(s) = \frac{V(s)}{I(s)} \tag{15}$$

Using (4),

$$H(s) = R + sL + \frac{1}{sC} \tag{16}$$

$$\implies H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \tag{17}$$

$$\implies |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{18}$$

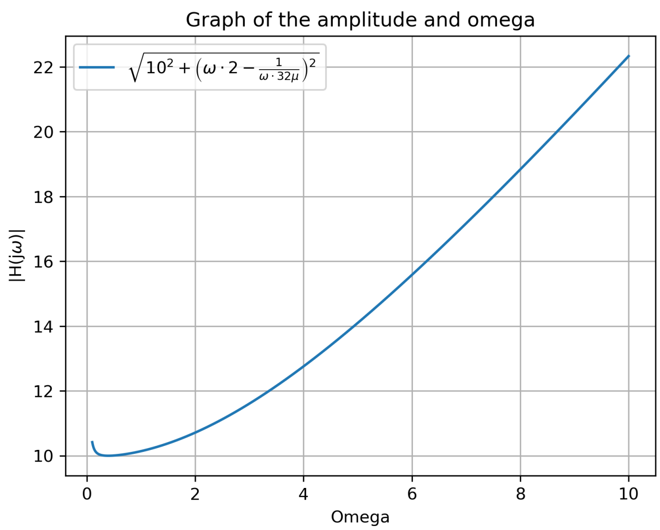

Fig. 3. Impedance vs ω (using values in Table I)

Substituting values,

$$\omega_0 = \frac{1}{\sqrt{(2.0)(32 \times 10^{-6})}} \tag{19}$$

$$\omega_0 = \frac{1}{\sqrt{64 \times 10^{-6}}} \tag{20}$$

$$\omega_0 = \frac{1}{8 \times 10^{-3}} \tag{21}$$

$$\omega_0 = 125 \text{ Hz} \tag{22}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{23}$$

$$Q = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}} \tag{24}$$

$$Q = \frac{100}{4} \tag{25}$$

$$Q = 25 \tag{26}$$