

NCERT Discrete 11.9.1 Q7

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Question: If a, b, c are in A.P.; b, c, d are in G.P and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P. prove that a, c, e are in G.P.

Solution:

Symbol	Value	Description
$x_1(n)$	a, b, c	A.P Sequence
$x_2(n)$	b, c, d	G.P Sequence
$x_3(n)$	$\left\{\frac{1}{c}, \frac{1}{d}, \frac{1}{e}\right\}$	A.P Sequence
$y(n)$	a, c, e	Sequence

TABLE I
PARAMETERS

$$b - a = c - b \quad (1)$$

$$2b = a + c \quad (2)$$

$$b = \frac{a + c}{2} \quad (3)$$

$$c^2 = b \times d \quad (4)$$

$$d = \frac{c}{b} \quad (5)$$

$$\frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d} \quad (6)$$

$$\frac{2}{d} = \frac{1}{c} + \frac{1}{e} \quad (7)$$

From (5),

$$\frac{2b}{c^2} = \frac{1}{c} + \frac{1}{e} \quad (8)$$

From (2),

$$\frac{a + c}{c^2} = \frac{1}{c} + \frac{1}{e} \quad (9)$$

$$\frac{a}{c^2} + \frac{1}{c} = \frac{1}{c} + \frac{1}{e} \quad (10)$$

$$a \times e = c^2 \quad (11)$$

$$(12)$$

So, a, c, e are in G.P

For $x_1(n)$,

$$x_1(n) = (x_1(0) + n(x_1(1) - x_1(0)))u(n) \quad (13)$$

Using Z-transform,

$$X_1(z) = \frac{x_1(0)}{1 - z^{-1}} + \frac{(x_1(1) - x_1(0))z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (14)$$

For $x_2(n)$,

$$x_2(n) = x_2(0) \left(\frac{x_2(1)}{x_2(0)} \right)^n u(n) \quad (15)$$

Using Z-transform,

$$X_2(z) = \frac{x_2(1)}{1 - \frac{x_2(1)}{x_2(0)}z^{-1}}, \quad |z| > \left| \frac{x_2(1)}{x_2(0)} \right| \quad (16)$$

For $x_3(n)$,

$$x_3(n) = (x_3(0) + n(x_3(1) - x_3(0)))u(n) \quad (17)$$

Using Z-transform,

$$X_3(z) = \frac{x_3(0)}{1 - z^{-1}} + (x_3(1) - x_3(0)) \frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (18)$$

(6) For $y(n)$,

$$y(n) = y(0) \left(\frac{y(1)}{y(0)} \right)^n u(n) \quad (19)$$

Using Z-transform,

$$Y(z) = \frac{y(1)}{1 - \frac{y(1)}{y(0)}z^{-1}}, \quad |z| > \left| \frac{y(1)}{y(0)} \right| \quad (20)$$