

# NCERT Discrete 11.5.9 Q20

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**Question:** If  $a, b, c$  are in A.P.;  $b, c, d$  are in G.P and  $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$  are in A.P. prove that  $a, c, e$  are in G.P.

**Solution:**

Symbol	Value	Description
$x_1(n)$	$\{a, b, c, \dots\}$	A.P Sequence
$x_2(n)$	$\{b, c, d, \dots\}$	G.P Sequence
$x_3(n)$	$\{\frac{1}{c}, \frac{1}{d}, \frac{1}{e}, \dots\}$	A.P Sequence
$y(n)$	$\{a, c, e, \dots\}$	Sequence

TABLE I  
PARAMETERS

$$b - a = c - b$$

$$2b = a + c$$

$$c^2 = b \times d$$

$$d = \frac{c^2}{b}$$

$$\frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d}$$

$$\frac{2}{d} = \frac{1}{c} + \frac{1}{e}$$

From (4),

$$\frac{2b}{c^2} = \frac{1}{c} + \frac{1}{e}$$

From (2),

$$\frac{a + c}{c^2} = \frac{1}{c} + \frac{1}{e}$$

$$\frac{a}{c^2} + \frac{1}{c} = \frac{1}{c} + \frac{1}{e}$$

$$a \times e = c^2$$

So,  $a, c, e$  are in G.P

For  $x_1(n)$ ,

$$x_1(n) = (x_1(0) + n(x_1(1) - x_1(0)))u(n) \quad (11)$$

Using Z-transform,

$$X_1(z) = \frac{x_1(0)}{1 - z^{-1}} + \frac{(x_1(1) - x_1(0))z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (12)$$

For  $x_2(n)$ ,

$$x_2(n) = x_2(0) \left( \frac{x_2(1)}{x_2(0)} \right)^n u(n) \quad (13)$$

Using Z-transform,

$$X_2(z) = \frac{x_2(1)}{1 - \frac{x_2(1)}{x_2(0)}z^{-1}}, \quad |z| > \left| \frac{x_2(1)}{x_2(0)} \right| \quad (14)$$

For  $x_3(n)$ ,

$$x_3(n) = (x_3(0) + n(x_3(1) - x_3(0)))u(n) \quad (15)$$

Using Z-transform,

$$X_3(z) = \frac{x_3(0)}{1 - z^{-1}} + (x_3(1) - x_3(0)) \frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (16)$$

For  $y(n)$ ,

$$y(n) = y(0) \left( \frac{y(1)}{y(0)} \right)^n u(n) \quad (17)$$

Using Z-transform,

$$Y(z) = \frac{y(1)}{1 - \frac{y(1)}{y(0)}z^{-1}}, \quad |z| > \left| \frac{y(1)}{y(0)} \right| \quad (18)$$