NCERT Physics 12.7 Q6

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Question: Obtain the resonant frequency of a series LCR circuit with $L = 2.0\,H$, $C = 32\,\mu F$, and $R = 10\,\Omega$. What is the Q-value of the circuit.

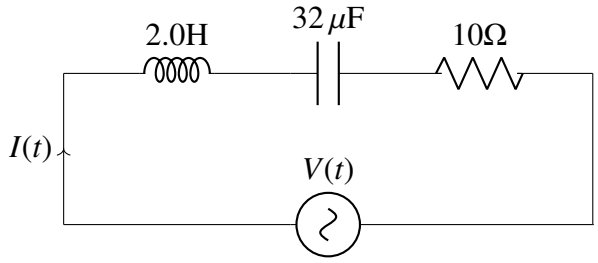

Fig. 1. LCR Circuit

Solution: In Figure Fig. 1 the following information is provided:

Symbol	Value	Description
L	2.0 H	Inductance
C	32 μF	Capacitance
R	10 Ω	Resistance
Q	$\frac{\omega_r}{\Delta\omega}$	Quality Factor

TABLE I
PARAMETERS

Series LCR Circuit Analysis

Now, the voltage transfer function ($\frac{V(s)}{I(s)} = H(s)$) is given by Ohm's Law in the Laplace domain:

$$\frac{V(s)}{I(s)} = R + sL + \frac{1}{sC} \tag{1}$$

Now, after substitution the equation is

$$\frac{V(s)}{I(s)} = 10 + 2s + \frac{1}{32 \times 10^{-6} s} \tag{2}$$

This is the voltage transfer function for the series LCR circuit in the Laplace domain.

Resonant Frequency (ω_0):

At resonance, the impedance is purely resistive, meaning the imaginary part of $Z(s)$ is zero.

$$\text{Im}\{Z(s)\} = \omega L - \frac{1}{\omega C} = 0 \tag{3}$$

Solving for ω_0:

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{4}$$

Substituting values:

$$\omega_0 = \frac{1}{\sqrt{2\,\text{H} \times 32\,\mu\text{F}}} = 125\,\text{rad/s} \tag{5}$$

Quality Factor (Q) Calculation:

$$\frac{\omega_r}{\Delta\omega} = \frac{\omega_0 L}{R} \tag{6}$$

$$Q = \frac{\omega_0 L}{R} = \frac{125\,\text{rad/s} \times 2\,\text{H}}{10\,\Omega} = 25 \tag{7}$$

Therefore, the quality factor of the LCR circuit is 25.

The equivalent s domain of the circuit is :

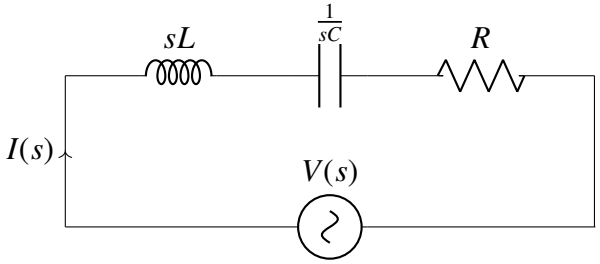

Fig. 2. LCR Circuit in s-domain