

NCERT Discrete 11.5.9 Q20

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Question: If a, b, c are in A.P.; b, c, d are in G.P and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P. prove that a, c, e are in G.P.

Solution:

Symbol	Value	Description
$x_1(n)$	$\{a, b, c, \dots\}$	A.P Sequence
$x_2(n)$	$\{b, c, d, \dots\}$	G.P Sequence
$x_3(n)$	$\{\frac{1}{c}, \frac{1}{d}, \frac{1}{e}, \dots\}$	A.P Sequence
$y(n)$	$\{a, c, e, \dots\}$	Sequence

TABLE I
PARAMETERS

$$b - a = c - b \quad (1)$$

$$2b = a + c \quad (2)$$

$$c^2 = b \times d \quad (3)$$

$$d = \frac{c^2}{b} \quad (4)$$

$$\frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d} \quad (5)$$

$$\frac{2}{d} = \frac{1}{c} + \frac{1}{e} \quad (6)$$

From (4),

$$\frac{2b}{c^2} = \frac{1}{c} + \frac{1}{e} \quad (7)$$

From (2),

$$\frac{a+c}{c^2} = \frac{1}{c} + \frac{1}{e} \quad (8)$$

$$\frac{a}{c^2} + \frac{1}{c} = \frac{1}{c} + \frac{1}{e} \quad (9)$$

$$a \times e = c^2 \quad (10)$$

$$y(1)^2 = y(0) \times y(2) \quad (11)$$

So, $y(0), y(1), y(2)$ are in G.P

1) For $y(n)$:

$$y(n) = y(0) \left(\frac{y(1)}{y(0)} \right)^n u(n) \quad (12)$$

Using Z-transform:

$$Y(z) = \frac{y(1)}{1 - \frac{y(1)}{y(0)}z^{-1}}, \quad |z| > \left| \frac{y(1)}{y(0)} \right| \quad (13)$$

2) For $x_1(n)$:

$$x_1(n) = (x_1(0) + n(x_1(1) - x_1(0)))u(n) \quad (14)$$

Using Z-transform:

$$X_1(z) = \frac{x_1(0)}{1 - z^{-1}} + \frac{(x_1(1) - x_1(0))z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (15)$$

3) For $x_2(n)$:

$$x_2(n) = x_2(0) \left(\frac{x_2(1)}{x_2(0)} \right)^n u(n) \quad (16)$$

Using Z-transform:

$$X_2(z) = \frac{x_2(1)}{1 - \frac{x_2(1)}{x_2(0)}z^{-1}}, \quad |z| > \left| \frac{x_2(1)}{x_2(0)} \right| \quad (17)$$

4) For $x_3(n)$:

$$x_3(n) = (x_3(0) + n(x_3(0) - x_3(1)))u(n) \quad (18)$$

Using Z-transform:

$$X_3(z) = \frac{x_3(0)}{1 - z^{-1}} + (x_3(1) - x_3(0)) \frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (19)$$