

NCERT Discrete 11.9.1 Q7

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Question: If a, b, c are in A.P.; b, c, d are in G.P and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P. prove that a, c, e are in G.P.

Solution: In the question, following information is provided:

Symbol	Remarks
$x(0)$	a
$x(1)$	b
$x(2)$	c
$x(3)$	d
$x(4)$	e

TABLE I
PARAMETERS

$$b - a = c - d \quad (1)$$

$$2b = a + c \quad (2)$$

$$b = \frac{a + c}{2} \quad (3)$$

$$c^2 = b \times d \quad (4)$$

$$d = \frac{c^2}{b} \quad (5)$$

$$\frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d} \quad (6)$$

$$\frac{2}{d} = \frac{1}{c} + \frac{1}{e} \quad (7)$$

$$\frac{2b}{c^2} = \frac{1}{c} + \frac{1}{e} \quad (8)$$

$$\frac{a + c}{c^2} = \frac{1}{c} + \frac{1}{e} \quad (9)$$

$$(a + c) \times e = (e + c) \times c \quad (10)$$

$$a \cdot e = c^2 \quad (11)$$

So, a, c, e are in G.P

For a, b, c in A.P,

$$x_1(n) = (a + n(b - a))u(n) \quad (12)$$

Using Z-transform,

$$X_1(z) = \frac{a}{1 - z^{-1}} + \frac{(b - a)z^{-1}}{(1 - z^{-1})^2} \quad (13)$$

For b, c, d in G.P,

$$x_2(n) = b \left(\frac{c}{b} \right)^n u(n) \quad (14)$$

Using Z-transform,

$$X_2(z) = \frac{c}{1 - \frac{c}{b}z^{-1}}, \quad |z| > \left| \frac{c}{b} \right| \quad (15)$$

For $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ in A.P,

$$x_3(n) = \left(\frac{1}{c} + n \left(\frac{1}{d} - \frac{1}{c} \right) \right) u(n) \quad (16)$$

Using Z-transform,

$$X_3(z) = \frac{1}{c(1 - z^{-1})} + \left(\frac{1}{d} - \frac{1}{c} \right) \frac{z^{-1}}{(1 - z^{-1})^2} \quad (17)$$

For a, c, e in G.P,

$$y(n) = a \left(\frac{c}{e} \right)^n u(n) \quad (18)$$

Using Z-transform,

$$Y(z) = \frac{c}{1 - \frac{c}{a}z^{-1}}, \quad |z| > \left| \frac{c}{a} \right| \quad (19)$$