NCERT Physics 12.7 Q6

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Question: Obtain the resonance frequency of a series LCR circuit with $L = 2.0\,H$, $C = 32\,\mu F$, and $R = 10\,\Omega$. What is the Q-value of the circuit.

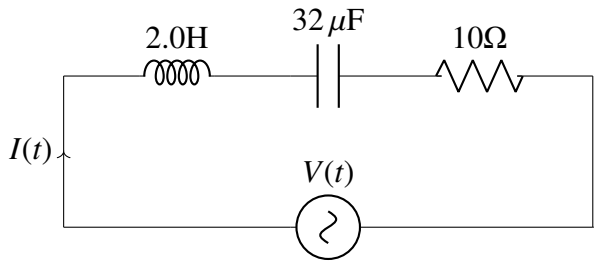

Fig. 1. LCR Circuit

Solution:

Symbol	Value	Description
L	2.0 H	Inductance
C	32 μF	Capacitance
R	10 Ω	Resistance
Q	$\frac{V_L}{V_R}$	Quality Factor
V_L	$sLI(s)$	Voltage across inductance
V_C	$RI(s)$	Voltage across capacitor
ω_0	$\frac{1}{\sqrt{LC}}$	Resonany frequency

TABLE I
PARAMETERS

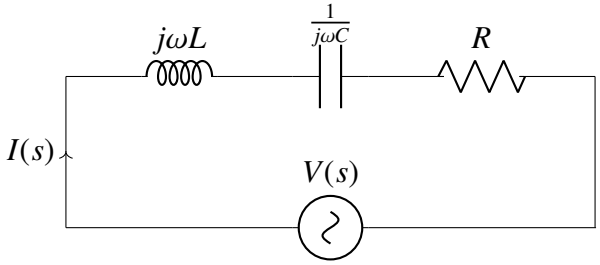

Fig. 2. LCR Circuit

1) Frequency Response of the Circuit

From Fig. 2,

$$V(s) = RI(s) + j\omega LI(s) + \frac{1}{j\omega C}I(s) \quad (1)$$

$$= I(s)\left(R + Lj\omega + \frac{1}{j\omega C}\right) \quad (2)$$

$$\implies I(s) = \frac{V(s)}{\left(R + Lj\omega + \frac{1}{j\omega C}\right)} \quad (3)$$

At resonance,

$$Lj\omega + \frac{1}{j\omega C} = 0 \quad (4)$$

$$\implies Lj\omega = \frac{-1}{j\omega C} \implies \omega^2 = \frac{1}{LC} \quad (5)$$

$$\omega = \frac{1}{\sqrt{LC}} \quad (6)$$

At resonance, Resonant frequency$(\omega_0) = \frac{1}{\sqrt{LC}}$

2) Quality Factor

a) voltage across inductor,

$$Q = \left(\frac{V_L}{V_R}\right)_{\omega_0} = \frac{|j\omega_0 LI(s)|}{|RI(s)|} \quad (7)$$

$$= \frac{1}{\sqrt{LC}}\frac{L}{R} \quad (8)$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}} \quad (9)$$

b) Using voltage across capacitor,

$$Q = \left(\frac{V_C}{V_R}\right)_{\omega_0} = \frac{\left|\frac{I(s)}{j\omega_0 C}\right|}{|RI(s)|} \quad (10)$$

$$= \frac{\sqrt{LC}}{RC} \quad (11)$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}} \quad (12)$$

3) Plot of Impedance vs Angular Frequency

$$H(s) = \frac{V(s)}{I(s)} \quad (13)$$

Using (3),

$$H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \tag{14}$$

$$\implies |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{15}$$

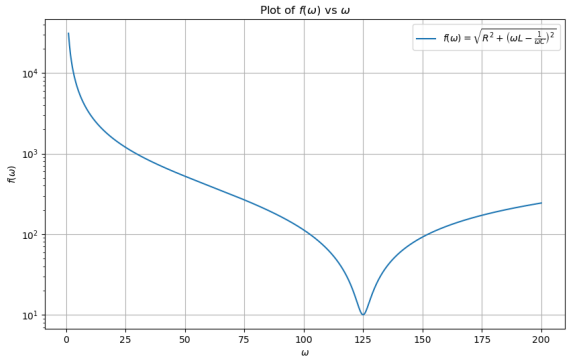

Fig. 3. Impedance vs ω (using values in Table I)

Substituting values,

$$\omega_0 = \frac{1}{\sqrt{(2.0)(32 \times 10^{-6})}} \tag{16}$$

$$\omega_0 = \frac{1}{\sqrt{64 \times 10^{-6}}} \tag{17}$$

$$\omega_0 = \frac{1}{8 \times 10^{-3}} \tag{18}$$

$$\omega_0 = 125 \text{ Hz} \tag{19}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{20}$$

$$Q = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}} \tag{21}$$

$$Q = \frac{100}{4} \tag{22}$$

$$Q = 25 \tag{23}$$