Kernel Functions

Exponential Kernel

`ExponentialKernel([α::Real=1]) <: MercerKernel`

The exponential kernel is given by the formula:

$$\kappa(\mathbf{x},\mathbf{y}) = \exp(-\alpha||\mathbf{x}-\mathbf{y}||) \qquad \alpha > 0$$

where α is a scaling parameter of the Euclidean distance. The exponential kernel, also known as the Laplacian kernel, is an isotropic Mercer kernel. The constructor is aliased by `LaplacianKernel`, so both names may be used:

```
ExponentialKernel()  # Default is Float64 with α = 1.0
LaplacianKernel(1)   # Integers will be converted to Float64
```

Squared Exponential Kernel

`SquaredExponentialKernel([α::Real=1]) <: MercerKernel`

The squared exponential kernel, or alternatively the Gaussian kernel, is identical to the exponential kernel except that the Euclidean distance is squared:

$$\kappa(\mathbf{x},\mathbf{y}) = \exp\left(-\alpha||\mathbf{x}-\mathbf{y}||^2\right) \qquad \alpha > 0$$

where α is a scaling parameter of the squared Euclidean distance. Just like the exponential kernel, the squared exponential kernel is an isotropic Mercer kernel. The squared exponential kernel is more commonly known as the radial basis kernel within machine learning communities. All aliases may be used in MLKernels.jl:

```
GaussianKernel()
RadialBasisKernel()
SquaredExponentialKernel()
```

Gamma Exponential Kernel

`GammaExponentialKernel([α::Real=1 [,γ::Real=1]]) <: MercerKernel`

The gamma exponential kernel is a generalization of the exponential and squared exponential kernels:

$$\kappa(\mathbf{x}, \mathbf{y}) = \exp(-\alpha||\mathbf{x} - \mathbf{y}||^{\gamma}) \qquad \alpha > 0, \ 0 < \gamma \leq 1$$

where α is a scaling parameter and γ is a shape parameter.

Rational-Quadratic Kernel

`RationalQuadraticKernel([α::Real=1 [,β::Real=1]]) <: MercerKernel`

The rational-quadratic kernel is given by:

$$\kappa(\mathbf{x}, \mathbf{y}) = \left(1 + \alpha||\mathbf{x}, \mathbf{y}||^{2}\right)^{-\beta} \qquad \alpha > 0, \ \beta > 0$$

where α is a scaling parameter and β is a shape parameter. This kernel can be seen as an infinite sum of Gaussian kernels. If one sets $\alpha = \alpha_0/\beta$, then taking the limit $\beta \to \infty$ results in the Gaussian kernel with scaling parameter α_0.

Gamma-Rational Kernel

`RationalQuadraticClass([α::Real [,β::Real [,γ::Real]]]) <: MercerKernel`

The gamma-rational kernel is a generalization of the rational-quadratic kernel with an additional shape parameter:

$$\kappa(\mathbf{x}, \mathbf{y}) = (1 + \alpha||\mathbf{x}, \mathbf{y}||^{\gamma})^{-\beta} \qquad \alpha > 0, \ \beta > 0, \ 0 < \gamma \leq 1$$

where α is a scaling parameter and β and γ are shape parameters.

Matern Kernel

`MaternKernel([ν::Real=1 [,θ::Real=1]]) <: MercerKernel`

The Matern kernel is a **Mercer** kernel [ras] given by:

$$\kappa(\mathbf{x}, \mathbf{y}) = \frac{1}{2^{\nu-1}\Gamma(\nu)} \left(\frac{2\sqrt{\nu}||\mathbf{x} - \mathbf{y}||}{\theta}\right)^{\nu} K_{\nu} \left(\frac{2\sqrt{\nu}||\mathbf{x} - \mathbf{y}||}{\theta}\right)$$

where Γ is the gamma function, K_{ν} is the modified Bessel function of the second kind, $\nu > 0$ and $\theta > 0$.

Linear Kernel

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`LinearKernel([a::Real=1 [,c::Real=1]]) <: MercerKernel`

The linear kernel is a **Mercer** kernel given by:

$$\kappa(\mathbf{x}, \mathbf{y}) = a\mathbf{x}^\intercal \mathbf{y} + c \qquad \alpha > 0,\ c \geq 0$$

Polynomial Kernel

`PolynomialKernel([a::Real=1 [,c::Real=1 [,d::Integer=3]]]) <: MercerKernel`

The polynomial kernel is a **Mercer** kernel given by:

$$\kappa(\mathbf{x}, \mathbf{y}) = (a\mathbf{x}^\intercal \mathbf{y} + c)^d \qquad \alpha > 0,\ c \geq 0,\ d \in \mathbb{Z}_+$$

Exponentiated Kernel

`ExponentiatedKernel([a::Real=1]) <: MercerKernel`

The exponentiated kernel is a **Mercer** kernel given by:

$$\kappa(\mathbf{x}, \mathbf{y}) = \exp(a\mathbf{x}^\intercal \mathbf{y}) \qquad a > 0$$

Periodic Kernel

`PeriodicKernel([α::Real=1 [,p::Real=π]]) <: MercerKernel`

The periodic kernel is given by:

$$\kappa(\mathbf{x}, \mathbf{y}) = \exp\left(-\alpha \sum_{i=1}^{n} \sin(p(x_i - y_i))^2\right) \qquad p > 0,\ \alpha > 0$$

where $\mathbf{x}$ and $\mathbf{y}$ are n dimensional vectors. The parameters p and α are scaling parameters for the periodicity and the magnitude, respectively. This kernel is useful when data has periodicity to it.

Sigmoid Kernel

`SigmoidKernel([a::Real=1 [,c::Real=1]]) <: Kernel`

The sigmoid kernel is given by:

$$\kappa(\mathbf{x}, \mathbf{y}) = \tanh(a\mathbf{x}^\intercal \mathbf{y} + c) \qquad \alpha > 0,\ c \geq 0$$

The sigmoid kernel is a not a true kernel, although it has been used in application.

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