Assignment 4
Probability and Random Variables

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I. Problem

Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

II. Solution

Let Y denote the random variable tossing a coin. Considering a fair coin, the probability of getting a Head or Tail P(Y) = 0.5 =p=1-p
In general , the probability of getting of j Head/Tail in n tosses is given as:

$$P(Y = j) = {}^{n}C_j \times p^j (1-p)^{(n-j)} = {}^{n}C_j \times p^n \quad (1)$$

The binomial random variable for n tosses with p probability is: $X \sim B(n, p)$
(i) X = random variable of number of Heads. The probability distribution of getting exactly j Heads in 2 tosses of coin is given as: X $\sim B(2, 0.5) \implies n = 2; j \in (0, n)$Using equation (1)

$$P(X = j) = {}^{2}C_j \times 0.5^2 \quad (2)$$

We get the pdf as below:

$$P(X = 0) = {}^{2}C_0 \times 0.5^2 = 0.25 \quad (3)$$
$$P(X = 1) = {}^{2}C_1 \times 0.5^2 = 0.5 \quad (4)$$
$$P(X = 2) = {}^{2}C_2 \times 0.5^2 = 0.25 \quad (5)$$

(ii)Let X = random variable of number of Tails. The probability distribution of getting exactly j Tails in 3 tosses of coin is given as: X $\sim B(3, 0.5) \implies n = 3; j \in (0, n)$Using equation (1)

$$P(X = j) = {}^{3}C_j \times 0.5^3 \quad (6)$$

We get the pdf as below:

$$P(X = 0) = {}^{3}C_0 \times 0.5^0 (1-0.5)^{(3-0)} = 0.125 \quad (7)$$
$$P(X = 1) = {}^{3}C_1 \times 0.5^3 = 0.375 \quad (8)$$
$$P(X = 2) = {}^{3}C_2 \times 0.5^3 = 0.375 \quad (9)$$
$$P(X = 3) = {}^{3}C_3 \times 0.5^3 = 0.125 \quad (10)$$

(iii)Let X = random variable of number of Heads. The probability distribution of getting exactly j Heads in 4 tosses of coin is given as: X $\sim B(4, 0.5) \implies n = 4; j \in (0, n)$Using equation (1)

$$P(X = j) = {}^{4}C_j \times 0.5^4 \quad (11)$$

We get the pdf as below:

$$P(X = 0) = ({}^{4}C_0) 0.5^0 (1-0.5)^{(4-0)} = 0.0625 \quad (12)$$
$$P(X = 1) = {}^{4}C_1 \times 0.5^4 = 0.25 \quad (13)$$
$$P(X = 2) = {}^{4}C_2 \times 0.5^4 = 0.375 \quad (14)$$
$$P(X = 3) = {}^{4}C_3 \times 0.5^4 = 0.25 \quad (15)$$
$$P(X = 4) = {}^{4}C_4 \times 0.5^4 = 0.0625 \quad (16)$$

The probability distribution of X for n tosses is:

n	j	0	1	2	3	4
2	P(X=j)=$^{2}C_j 0.5^2$	0.25	0.5	0.25	0	0
3	P(X=j)=$^{3}C_j 0.5^3$	0.125	0.375	0.375	0.125	0
4	P(X=j)=$^{4}C_j 0.5^4$	0.0625	0.25	0.375	0.25	0.062

The probabilities were simulated using the python code.

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Bernoulli simulation
[0.2506, 0.5003, 0.2491]
[0.1258, 0.3806, 0.3705, 0.1231]
[0.0625, 0.2513, 0.3752, 0.2519, 0.0591]
Binomial simulation
[0.2459, 0.5013, 0.2528]
[0.123, 0.3801, 0.3752, 0.1217]
[0.0645, 0.2494, 0.3711, 0.2533, 0.0617]
```

Figure 1: Simulation for tossing a fair coin

Download python code from here

https://github.com/Swati−Mohanty/AI5002/blob/main/Assignment_4/codes/cointoss.py

Download latex code from here-

https://github.com/Swati−Mohanty/AI5002/blob/main/Assignment_4/codes/assignment4.tex