

Assignment 4

Probability and Random Variables

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I. PROBLEM

Find the probability distribution of

- (i) number of heads in two tosses of a coin.
- (ii) number of tails in the simultaneous tosses of three coins.
- (iii) number of heads in four tosses of a coin.

II. SOLUTION

Let Y denote the event of tossing a coin. Considering a fair coin, the probability of getting a Head or Tail $P(Y) = 0.5$

We have i coin tosses, with probability p of Heads and (1-p) of Tails. We conduct the trials independently.

In general, the probability of getting of j Head/Tail is given as:

$$P(Y = j) = {}^nC_j \times p^j(1 - p)^{(n-j)} \quad (1)$$

Consider a random variable X where X = Number of successes. Suppose we have n trials. We write $X \sim B(n, p)$

- (i) Let X denote the random variable of number of Heads. The probability distribution of getting exactly j Heads in 2 tosses of coin is given as:

$X \sim B(2, 0.5)$

Using equation

$$P(X = j) = {}^2C_j \times 0.5^j(1 - 0.5)^{(2-j)} \quad (2)$$

We get the pdf as below:

$$P(X = 0) = {}^2C_0 \times 0.5^0(1 - 0.5)^{(2-0)} = 0.25 \quad (3)$$

$$P(X = 1) = {}^2C_1 \times 0.5^1(1 - 0.5)^{(2-1)} = 0.5 \quad (4)$$

$$P(X = 2) = {}^2C_2 \times 0.5^2(1 - 0.5)^{(2-2)} = 0.25 \quad (5)$$

The distribution table is given as:

j	0	1	2
P(X=j)	0.25	0.5	0.25

- (ii) Let X denote the random variable of number of Tails. The probability distribution of getting exactly

j Tails in 3 tosses of coin is given as:

$X \sim B(3, 0.5)$

Using equation

$$P(X = j) = {}^3C_j \times 0.5^j(1 - 0.5)^{(3-j)} \quad (6)$$

We get the pdf as below:

$$P(X = 0) = {}^3C_0 \times 0.5^0(1 - 0.5)^{(3-0)} = 0.125 \quad (7)$$

$$P(X = 1) = {}^3C_1 \times 0.5^1(1 - 0.5)^{(3-1)} = 0.375 \quad (8)$$

$$P(X = 2) = {}^3C_2 \times 0.5^2(1 - 0.5)^{(3-2)} = 0.375 \quad (9)$$

$$P(X = 3) = {}^3C_3 \times 0.5^3(1 - 0.5)^{(3-3)} = 0.125 \quad (10)$$

The probability distribution of X is:

j	0	1	2	3
P(X=j)	0.125	0.375	0.375	0.125

- (iii) Let X denote the random variable of number of Heads. The probability distribution of getting exactly j Heads in 4 tosses of coin is given as:

$X \sim B(4, 0.5)$

Using equation

$$P(X = j) = {}^4C_j \times 0.5^j(1 - 0.5)^{(4-j)} \quad (11)$$

We get the pdf as below:

$$P(X = 0) = ({}^4C_0)0.5^0(1 - 0.5)^{(4-0)} = 0.0625 \quad (12)$$

$$P(X = 1) = {}^4C_1 \times 0.5^1(1 - 0.5)^{(4-1)} = 0.25 \quad (13)$$

$$P(X = 2) = {}^4C_2 \times 0.5^2(1 - 0.5)^{(4-2)} = 0.375 \quad (14)$$

$$P(X = 3) = {}^4C_3 \times 0.5^3(1 - 0.5)^{(4-4)} = 0.25 \quad (15)$$

$$P(X = 4) = {}^4C_4 \times 0.5^4(1 - 0.5)^{(4-4)} = 0.0625 \quad (16)$$

The probability distribution of X is:

j	0	1	2	3	4
P(X=j)	0.0625	0.25	0.375	0.25	0.0625

The probabilities were simulated using the python code.

Download python code from [here](#)

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↳ Bernoulli simulation
[0.2506, 0.5003, 0.2491]
[0.1258, 0.3806, 0.3705, 0.1231]
[0.0625, 0.2513, 0.3752, 0.2519, 0.0591]
Binomial simulation
[0.2459, 0.5013, 0.2528]
[0.123, 0.3801, 0.3752, 0.1217]
[0.0645, 0.2494, 0.3711, 0.2533, 0.0617]

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Figure 1: Simulation for tossing a fair coin

https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment_4/codes/cointoss.py

Download latex code from here-

https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment_4/codes/assignment4.tex