Assignment 4
Probability and Random Variables

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I. Problem

Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

II. Solution

Let Y denote the random variable tossing a coin. Considering a fair coin, the probability of getting a Head or Tail P(Y) = 0.5 =p=1-p
In general , the probability of getting of j Head/Tail in n tosses is given as:

$$P(Y=j) = {}^{n}C_j \times p^j(1-p)^{(n-j)} = {}^{n}C_j \times p^n \tag{1}$$

The binomial random variable for n tosses with p probability is: X ˜B(n, p)
(i) X = random variable of number of Heads. The probability distribution of getting exactly j Heads in 2 tosses of coin is given as: X ˜B(2, 0.5)

$$\implies n = 2; j \in (0, n)$$

Using equation (1)

$$P(X=j) = {}^{2}C_j \times 0.5^2 \tag{2}$$

We get the pdf as below:

$$P(X=0) = {}^{2}C_0 \times 0.5^2 = 0.25 \tag{3}$$
$$P(X=1) = {}^{2}C_1 \times 0.5^2 = 0.5 \tag{4}$$
$$P(X=2) = {}^{2}C_2 \times 0.5^2 = 0.25 \tag{5}$$

(ii)Let X = random variable of number of Tails. The probability distribution of getting exactly j Tails in 3 tosses of coin is given as: X ˜B(3, 0.5)

$$\implies n = 3; j \in (0, n)$$

Using equation (1)

$$P(X=j) = {}^{3}C_j \times 0.5^3 \tag{6}$$

We get the pdf as below:

$$P(X=0) = {}^{3}C_0 \times 0.5^0(1-0.5)^{(3-0)} = 0.125 \tag{7}$$
$$P(X=1) = {}^{3}C_1 \times 0.5^3 = 0.375 \tag{8}$$
$$P(X=2) = {}^{3}C_2 \times 0.5^3 = 0.375 \tag{9}$$
$$P(X=3) = {}^{3}C_3 \times 0.5^3 = 0.125 \tag{10}$$

(iii)Let X = random variable of number of Heads. The probability distribution of getting exactly j Heads in 4 tosses of coin is given as: X ˜B(4, 0.5)

$$\implies n = 4; j \in (0, n)$$

Using equation (1)

$$P(X=j) = {}^{4}C_j \times 0.5^4 \tag{11}$$

We get the pdf as below:

$$P(X=0) = ({}^{4}C_0)0.5^0(1-0.5)^{(4-0)} = 0.0625 \tag{12}$$
$$P(X=1) = {}^{4}C_1 \times 0.5^4 = 0.25 \tag{13}$$
$$P(X=2) = {}^{4}C_2 \times 0.5^4 = 0.375 \tag{14}$$
$$P(X=3) = {}^{4}C_3 \times 0.5^4 = 0.25 \tag{15}$$
$$P(X=4) = {}^{4}C_4 \times 0.5^4 = 0.0625 \tag{16}$$

The probability distribution of X for j tosses is:

j	0	1	2	3	4
X ˜B(2,0.5)	0.25	0.5	0.25	0	0
X ˜B(3,0.5)	0.125	0.375	0.375	0.125	0
X ˜B(4,0.5)	0.0625	0.25	0.375	0.25	0.0625

The probabilities were simulated using the python code.

Download python code from here

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https://github.com/Swati−Mohanty/AI5002/blob/
    main/Assignment_4/codes/cointoss.py
```

Download latex code from here-

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https://github.com/Swati−Mohanty/AI5002/blob/
    main/Assignment_4/codes/assignment4.tex
```

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Bernoulli simulation
[0.2506, 0.5003, 0.2491]
[0.1258, 0.3806, 0.3705, 0.1231]
[0.0625, 0.2513, 0.3752, 0.2519, 0.0591]
Binomial simulation
[0.2459, 0.5013, 0.2528]
[0.123, 0.3801, 0.3752, 0.1217]
[0.0645, 0.2494, 0.3711, 0.2533, 0.0617]
```

Figure 1: Simulation for tossing a fair coin