Assignment 5
Probability and Random Variables

Swati Mohanty (EE20RESCH11007)

I. Problem

In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

II. Solution

P(win) = p = $\frac{1}{6}$ = 0.167
P(loss) = 1-p = $\frac{5}{6}$ = 0.833.

Let Y denote the random variable of winning the game in Nth trial
X_i denote the amount in each case and $X_i \in (+1, -1)$.
The possible cases are:
(i)Wins in first throw:

$$P(Y = 1) = p \tag{1}$$
$$X_1 = +1 \tag{2}$$

(ii)Wins in the second throw :

$$P(Y = 2) = (1 - p) \times p = p - p^2 \tag{3}$$
$$X_2 = \overline{X_1} + X_1 = -1 + 1 = 0 \tag{4}$$

(iii)Wins in the third throw :

$$P(Y = 3) = (1 - p) \times (1 - p) \times p = p(1 - p)^2 \tag{5}$$
$$X_3 = \overline{X_1} + \overline{X_1} + X_1 = -1 - 1 + 1 = -1 \tag{6}$$

(iv)Does not wins in any throw :

$$P(Y = 3) = (1 - p) \times (1 - p) \times (1 - p) = (1 - p)^3 \tag{7}$$
$$X_3 = \overline{X_1} + \overline{X_1} + X_1 = -1 - 1 - 1 = -3 \tag{8}$$

Net amount = Expectation = E[Y]

$$E[Y] = \sum_{n=1}^{3} P(Y = n)X_n \tag{9}$$
$$= (p \times 1) + 0 + (p(1 - p)^2 \times -1) + ((1 - p)^3 \times -3) \tag{10}$$
$$= 0.167 \times 1 + 0.107 \times -1 + 0.596 \times -3 \tag{11}$$
$$= -1.73 \tag{12}$$

The probabilities were simulated using the python code.

```
Simulated results :
-1.2642025634898237
Theoretical results
-1.6449999999999998
```

Figure 1: Simulation for tossing a fair coin

Download python code from here

https://github.com/Swati−Mohanty/AI5002/blob/main/Assignment_5/codes/die.py

Download latex code from here-

https://github.com/Swati−Mohanty/AI5002/blob/main/Assignment_5/codes/assignment5.tex