Assignment 10
Probability and Random Variables

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I. Problem

Let X_1 , X_2 , X_3... be i.i.d N(1,1) random variable. Let $S_n = (X_1)^2+(X_2)^2+(X_3)^2+....+(X_n)^2$ for n $\geq$ 1. Then $\lim_{n\to\infty} \frac{Var(S_n)}{n}$ is :

II. Solution

We know, If Z_i for $1 \leq i \leq$ n are independent standard normal random variables , $X = \sum_{i=1}^{n} Z_i^2$ has chi-sqaure distribution with n degree of freedom.The moment generating function for the same is given as:

$$= \int_{-\infty}^{\infty} e^{tx^2} f(x;1,1)\,dx \tag{1}$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} e^{tx^2}\,dx \tag{2}$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-1)^2}{2\times 1}} e^{\frac{tx^2}{2}}\,dx \tag{3}$$

$$= e^{\frac{t^2-t}{2}} \tag{4}$$

From moment generating function, the second moment is the variance(t=1).This gives $Var(X^2) = 1$. The given random variables are i.i.d, so $Var(S_n) = nVar(X^2)$.

$$\lim_{n\to\infty} \frac{Var(S_n)}{n} = n\frac{Var(X^2)}{n} \tag{5}$$

$$= Var(X^2) = 1 \tag{6}$$

Download latex code from here-

https://github.com/Swati−Mohanty/AI5002/blob/main/Assignment_10/codes/assignment10.tex