

Assignment 4

Probability and Random Variables

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I. PROBLEM

Find the probability distribution of

- number of heads in two tosses of a coin.
- number of tails in the simultaneous tosses of three coins.
- number of heads in four tosses of a coin.

II. SOLUTION

Let Y denote the random variable tossing a coin. Considering a fair coin, the probability of getting a Head or Tail $P(Y) = 0.5 = p = 1-p$. In general, the probability of getting j Head/Tail in n tosses is given as:

$$P(Y = j) = {}^nC_j \times p^j (1-p)^{(n-j)} = {}^nC_j \times p^n \quad (1)$$

The binomial random variable for n tosses with p probability is: $X \sim B(n, p)$

(i) X = random variable of number of Heads. The probability distribution of getting exactly j Heads in 2 tosses of coin is given as: $X \sim B(2, 0.5) \implies n = 2; j \in (0, n)$ Using equation (1)

$$P(X = j) = {}^2C_j \times 0.5^2 \quad (2)$$

(ii) Let X = random variable of number of Tails. The probability distribution of getting exactly j Tails in 3 tosses of coin is given as: $X \sim B(3, 0.5) \implies n = 3; j \in (0, n)$ Using equation (1)

$$P(X = j) = {}^3C_j \times 0.5^3 \quad (3)$$

(iii) Let X = random variable of number of Heads. The probability distribution of getting exactly j Heads in 4 tosses of coin is given as: $X \sim B(4, 0.5) \implies n = 4; j \in (0, n)$ Using equation (1)

$$P(X = j) = {}^4C_j \times 0.5^4 \quad (4)$$

The probability distribution of X for n tosses is:

n	j	0	1	2	3	4
2	$P(X=j) = {}^2C_j 0.5^2$	0.25	0.5	0.25	0	0
3	$P(X=j) = {}^3C_j 0.5^3$	0.125	0.375	0.375	0.125	0
4	$P(X=j) = {}^4C_j 0.5^4$	0.0625	0.25	0.375	0.25	0.0625

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Bernoulli simulation
[0.2506, 0.5003, 0.2491]
[0.1258, 0.3806, 0.3705, 0.1231]
[0.0625, 0.2513, 0.3752, 0.2519, 0.0591]
Binomial simulation
[0.2459, 0.5013, 0.2528]
[0.123, 0.3801, 0.3752, 0.1217]
[0.0645, 0.2494, 0.3711, 0.2533, 0.0617]

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Figure 1: Simulation for tossing a fair coin

The probabilities were simulated using the python code.

Download python code from here

https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment_4/codes/cointoss.py

Download latex code from here-

https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment_4/codes/assignment4.tex