Assignment 1
Probability and Random Variables

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I. Problem

Find the MGF for X ˜N(μ, σ^2) .

II. Solution

The Moment generating function for a Normal distribution is given as $M_X(s) = E[e^{sX}]$.

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} e^{sx}\, dx \tag{1}$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-x-(-\mu+\sigma^2)^2}{2\sigma^2}} e^{\sigma s + \frac{\sigma^2 s^2}{2}}\, dx \tag{2}$$

$$= e^{\sigma s + \frac{\sigma^2 s^2}{2}} \times E_{x}\tilde{~}N(\mu + s\sigma^2, \sigma^2)[1] \tag{3}$$

$$= e^{\sigma s + \frac{\sigma^2 s^2}{2}} \tag{4}$$

For the obtained expression, the MGF (0) =1. The same result is also obtained using the python code.

```
0th moment :
 [1. 1. 1.]

6th moment :
 [5.20609375e+02 9.13256836e+00 4.26392850e+06]

9th moment :
 55265909588.26437

12th moment :
 [1.53284936e+14 1.63654317e+02 8.83474172e+03 5.17842143e+04]
```

Python code link

https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment