Assignment 4
Probability and Random Variables

Swati Mohanty (EE20RESCH11007)

I. Problem

Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

II. Solution

Let Y denote the event of tossing a coin. Considering a fair coin, the probability of getting a Head or Tail P(Y) = 0.5
We have i coin tosses, with probability p of Heads and (1-p) of Tails. We conduct the trials independently.
In general , the probability of getting of j Head/Tail is given as:

$$P(Y = j) = \frac{n!}{j!(n-j)!} p^j (1-p)^{(n-j)} \tag{1}$$

Consider a random variable X where X = Number of successes.Suppose we have n trials. We write X ˜B(n, p)
(i) Let X denote the random variable of number of Heads. The probability distribution of getting exactly j Heads in 2 tosses of coin is given as:
X ˜B(2, 0.5)
Using equation

$$P(X = j) = \frac{2!}{j!(2-j)!} 0.5^j (1-0.5)^{(2-j)} \tag{2}$$

We get the pdf as below:

$$P(X = 0) = \frac{2!}{0!(2-0)!} 0.5^0 (1-0.5)^{(2-0)} = 0.25 \tag{3}$$

$$P(X = 1) = \frac{2!}{1!(2-1)!} 0.5^1 (1-0.5)^{(2-1)} = 0.5 \tag{4}$$

$$P(X = 2) = \frac{2!}{2!(2-2)!} 0.5^2 (1-0.5)^{(2-2)} = 0.25 \tag{5}$$

The distribution table is given as:

j	0	1	2
P(X=j)	0.25	0.5	0.25

(ii)Let X denote the random variable of number of Tails. The probability distribution of getting exactly j Tails in 3 tosses of coin is given as:
X ˜B(3, 0.5)
Using equation

$$P(X = j) = \frac{3!}{j!(3-j)!} 0.5^j (1-0.5)^{(3-j)} \tag{6}$$

We get the pdf as below:

$$P(X = 0) = \frac{3!}{0!(3-0)!} 0.5^0 (1-0.5)^{(3-0)} = 0.125 \tag{7}$$

$$P(X = 1) = \frac{3!}{1!(3-1)!} 0.5^1 (1-0.5)^{(3-1)} = 0.375 \tag{8}$$

$$P(X = 2) = \frac{3!}{2!(3-2)!} 0.5^2 (1-0.5)^{(3-2)} = 0.375 \tag{9}$$

$$P(X = 3) = \frac{3!}{3!(3-3)!} 0.5^3 (1-0.5)^{(3-3)} = 0.125 \tag{10}$$

The probability distribution of X is:

j	0	1	2	3
P(X=j)	0.125	0.375	0.375	0.125

(iii)Let X denote the random variable of number of Heads. The probability distribution of getting exactly j Heads in 4 tosses of coin is given as:
X ˜B(4, 0.5)
Using equation

$$P(X = j) = \frac{4!}{j!(4-j)!} 0.5^j (1-0.5)^{(4-j)} \tag{11}$$

We get the pdf as below:

$$P(Y3 = 0) = \frac{4!}{0!(4-0)!} 0.5^0 (1-0.5)^{(4-0)} = 0.0625 \tag{12}$$

$$P(X = 1) = \frac{4!}{0!(4-1)!} 0.5^1 (1-0.5)^{(4-1)} = 0.25 \tag{13}$$

$$P(X = 2) = \frac{4!}{2!(4-2)!} 0.5^2 (1-0.5)^{(4-2)} = 0.375 \tag{14}$$

$$P(X = 3) = \frac{4!}{3!(4-3)!} 0.5^3 (1-0.5)^{(4-4)} = 0.25 \tag{15}$$

$$P(X = 4) = \frac{4!}{0!(4-4)!} 0.5^4 (1-0.5)^{(4-4)} = 0.0625 \tag{16}$$

The probability distribution of X is:

j	0	1	2	3	4
P(X=j)	0.0625	0.25	0.375	0.25	0.0625

The probabilities were simulated using the python code.

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Bernoulli simulation
[0.2506, 0.5003, 0.2491]
[0.1258, 0.3806, 0.3705, 0.1231]
[0.0625, 0.2513, 0.3752, 0.2519, 0.0591]
Binomial simulation
[0.2459, 0.5013, 0.2528]
[0.123, 0.3801, 0.3752, 0.1217]
[0.0645, 0.2494, 0.3711, 0.2533, 0.0617]
```

Figure 1: Simulation for tossing a fair coin

Download python code from here

https://github.com/Swati−Mohanty/AI5002/blob/main/Assignment_4/codes/cointoss.py

Download latex code from here-

https://github.com/Swati−Mohanty/AI5002/blob/main/Assignment_4/codes/assignment4.tex