

# Assignment 4

## Probability and Random Variables

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### I. PROBLEM

Find the probability distribution of

- number of heads in two tosses of a coin.
- number of tails in the simultaneous tosses of three coins.
- number of heads in four tosses of a coin.

### II. SOLUTION

Let  $X_i$  denote the event of tossing a coin  $i$  times. Considering a fair coin, the probability of getting a Head or Tail  $P(X) = 0.5$

(i)  $Y_1$  = Occurance of Head in the event of tossing a coin twice =  $X_1 + X_2$

Possible outcomes = {HH, HT, TH, TT}

Probability of each outcome =  $0.5 \times 0.5 = 0.25$

No of Heads	Sample space	Probability
0	TT	0.25
1	HT, TH	0.5
2	HH	0.25

Hence the probability distribution of no of heads in two tosses of coin is:

Y1	0	1	2
P(Y1)	0.25	0.5	0.25

(ii) Let the event be defined as

$Y_2$  = Number of tails in the simultaneous tosses of three coins =  $X_1 + X_2 + X_3$

Outcomes = {HHH, HHT, HTH, HTT, THH, TTH, TTT}

Probability of each outcome =  $0.5 \times 0.5 \times 0.5 = 0.125 = \frac{1}{8}$

Similarly the probability distribution of  $Y_2$  is:

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

(iii) Let the event be defined as

$Y_3$  = Number of heads in four tosses of a coin =  $X_1 + X_2 + X_3 + X_4$

Outcomes = {HHHH, HHHT, HHTH, HHTT, HTHH,

HTHH, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHH, TTTH, TTTT}

Probability of each outcome =  $0.5 \times 0.5 \times 0.5 \times 0.5 = 0.625 = \frac{1}{16}$

In similar manner the probability distribution of  $Y_3$  is:

X	0	1	2	3	4
P(X)	1/16	1/4	3/8	1/4	1/16

The probabilities were simulated using the python code.

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Bernoulli simulation
[0.2506, 0.5003, 0.2491]
[0.1258, 0.3806, 0.3705, 0.1231]
[0.0625, 0.2513, 0.3752, 0.2519, 0.0591]
Binomial simulation
[0.2459, 0.5013, 0.2528]
[0.123, 0.3801, 0.3752, 0.1217]
[0.0645, 0.2494, 0.3711, 0.2533, 0.0617]
```

Figure 1: Simulation for tossing a fair coin

**Download python code from here**

[https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment\\_4/codes/cointoss.py](https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment_4/codes/cointoss.py)

**Download latex code from here-**

[https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment\\_4/codes/assignment4.tex](https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment_4/codes/assignment4.tex)