Scale Mixture of Rayleigh Distribution

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SCALE MIXTURE OF RAYLEIGH DISTRIBUTION

INTRODUCTION

A new model called "Scale Mixture of Rayleigh Distribution" is introduced, which is defined to be the quotient of two independent random variables. can be expressed as a quotient of scale mixture of Rayleigh and a particular Generalized Gamma distribution.

Rayleigh Distribution: It is a continuous and positive distribution named after Lord Rayleigh. It is the most popular model used for describing skewed positive data

Generalized Gamma Distribution: It is a continuous probability distribution with three parameters. It is a generalization of the two-parameter gamma distribution

Equations

- PDF of Rayleigh Distribution
  If a continuous RV X follows Rayleigh distribution with scale parameter $\sigma > 0$, then pdf of X $\sim$ R(σ) is:

$$f_X(x) = \frac{x}{\sigma} e^{\frac{-x^2}{2\sigma}}; x, \sigma > 0 \tag{1}$$

- PDF of GG Distribution
  If a RV Z follows three parameter GG distribution ,then pdf is:

$$f(z; a, d, p) = \frac{pa^d}{\Gamma(d/p)} e^{-(az)^p} z^{(d-1)}; a, d, p, z > 0 \tag{2}$$

It is denoted as Z $\sim$ GG(a,d,p).

Equations

- PDF of SMR Distribution
  An RV T follows SMR distribution with parameter $\sigma > 0$, and $q > 0$, if T can be expressed as he ratio of two independent RVs

$$T = \frac{X}{Y} \tag{3}$$

with X $\sim$ R(σ) and Y $\sim$ GG(1, q, 2)

With $q > 0$ and $\sigma > 0$,the pdf is:

$$f(t; \sigma, q) = \frac{qt}{2\sigma(t^2/(2\sigma) + 1)^{\frac{q}{2}+1}}; t > 0 \tag{4}$$

T is denoted as T $\sim$ SMR(q, σ).

PDF and CDF Plots

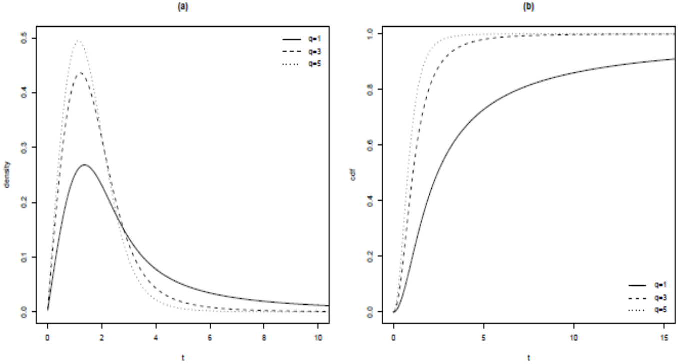

Figure: (a) pdf and (b) cdf in SMR model for $\sigma = 1$ and different values of q

Simulated PDF Plots

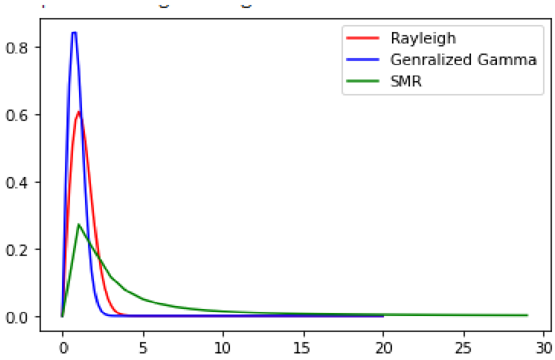

Figure: Python simulted pdf in SMR model for $\sigma = 1$ and q=1

The python code for the figure is
https://github.com/Swati-Mohanty/AI5002/blob/main/Project/codes/smr pdf.py

Simulated CDF Plots

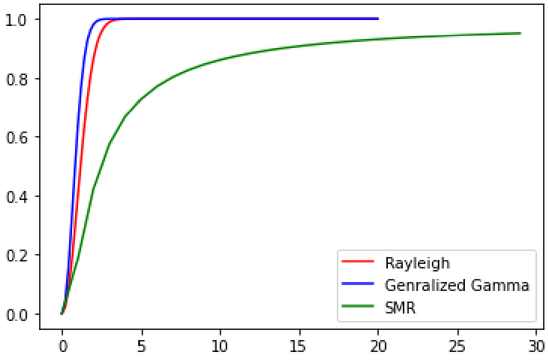

Figure: Python simulated cdf in SMR model for $\sigma = 1$ and q $= 1$

The python code for the figure is
https://github.com/Swati-Mohanty/AI5002/blob/main/Project/codes/smr_cdf.py

Simulation Study

The performance of ML estimates for finite sample size were studied to check if the estimators satisfy the desirable properties.

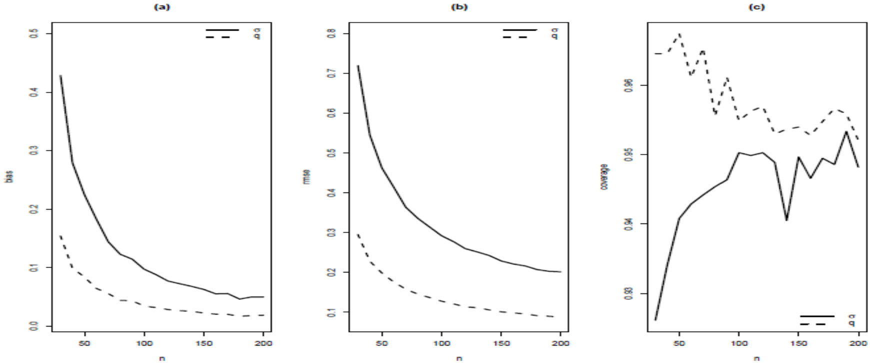

Figure: Graphics of (a) bias (b) RMSE and (c) coverage of simulator for $\sigma = 1, q = 1, n = 30...200$ in SMR model

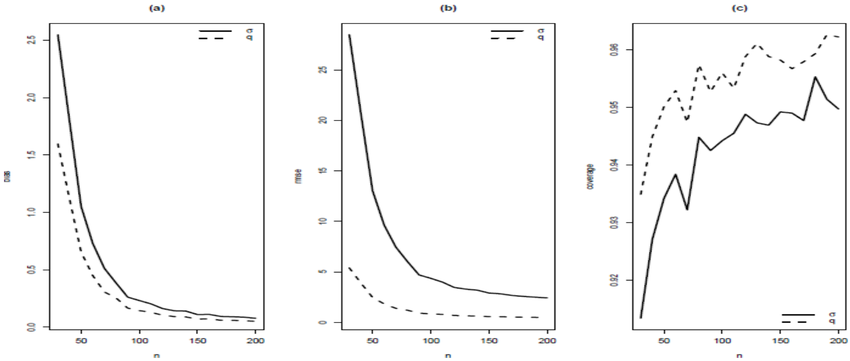

Figure: Graphics of (a) bias (b) RMSE and (c) coverage of simulator for $\sigma = 10, q = 1.5, n = 30...200$ in SMR model

INFERENCES

- As sample size increases, then bias and RMSE decreases. This suggests that the estimators are consistent.
- As sample size increases, the empirical coverage probability approaches to the nominal level (95%)

Application 1:Patients with Bladder cancer

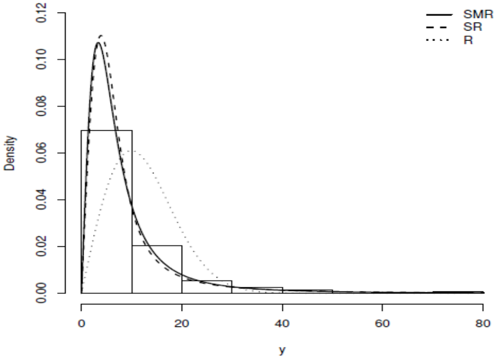

Figure: Density plot of patients with bladder cancer in the R, SR and SMR distribution

Statistical Values	
n	128
$\overline{T}$	9.366
S	10.508
$\sqrt{b_1}$	3.287
b_2	18.483
min(T)	0.08
max(T)	79.05

Table: Descriptive statistics

Application 1:Patients with Bladder cancer

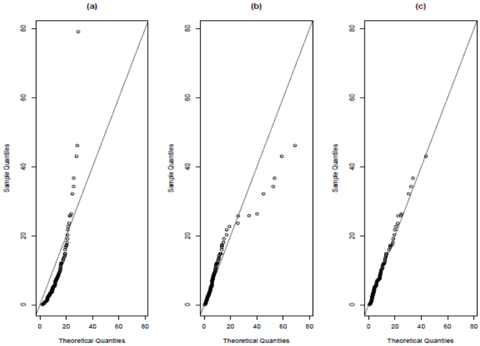

Figure: QQ plot of patients with bladder cancer in the (a)R, (b)SR and (c)SMR distribution

Application 2:Number of failures of an air conditioning system

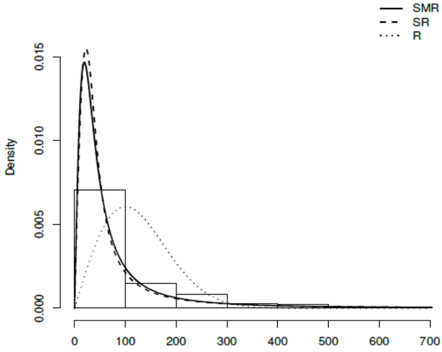

Figure: Density plot of number of failures of an air conditioning system in the R, SR and SMR distribution

Statistical Values	
n	188
$\overline{T}$	92.074
S	107.916
S	10.508
$\sqrt{b_1}$	2.139
b_2	8.023
min(T)	1
max(T)	603

Table: Descriptive statistics

Application 2:Number of failures of an air conditioning system

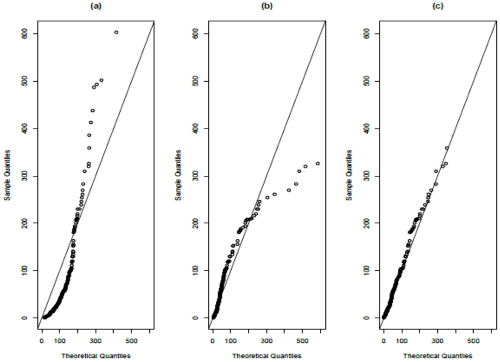

Figure: QQ plot of number of failures of an air conditioning system in the (a)R, (b)SR and (c)SMR distribution

CONCLUSION

- More flexible model as for its kurtosis coefficient and hazard function than the Rayleigh and slashed Rayleigh distribution.
- A simulation study is included, which suggests that the ML estimators are consistent even for moderate sample sizes
- QQ-plots show that our proposal provides a better fit than R and SR distributions, especially on the right tail of these data sets.

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