Compulsory Assignment
Probability and Random Variables

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I. Problem

Let, $X_1 \sim Bin(n_1, p)$ and $X_2 \sim Bin(n_2, q)$, independently. Find the PMF of $X_1 - X_2$.

II. Solution

Given, $X_1 \sim Bin(n_1, p)$ and $X_2 \sim Bin(n_2, q)$, independently.
$\therefore n_2 - X_2 \sim Bin(n_2, p)$
By additive/ reproductive property of binomial,
$X_1 + n_2 - X_2 \sim Bin(n_1 + n_2, p)$
Let, $D = X_1 - X_2$.

$$P(D = d) = P(X_1 - X_2 = d) \tag{1}$$
$$= P(X_1 - X_2 + n_2 = d + n_2) \tag{2}$$
$$= \binom{n_1 + n_2}{n_2 + d} p^{n_2+d} q^{n_1-d}, d = -n_2 \text{ to } n_1 \tag{3}$$

III. Problem

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

IV. Solution

Q3.7)Let X_1 denotes the number of heads and X_2 denotes the number of tails that occur when a coin is tossed 6 times.
We get, $X_1 \sim Bin(n = 6, p)$
and $X_2 \sim Bin(n = 6, 1 - p = q)$.
$\therefore n - X_2 \sim Bin(6, p)$.
By reproductive property,

$$X_1 + n - X_2 \sim Bin(6 + 6, p) \tag{4}$$

$X = X_1 - X_2$. Using (3),

$$P(X = x) = \binom{6 + 6}{6 + x} p^{6+x} q^{6-x}, x = -6 \text{ to } 6 \tag{5}$$

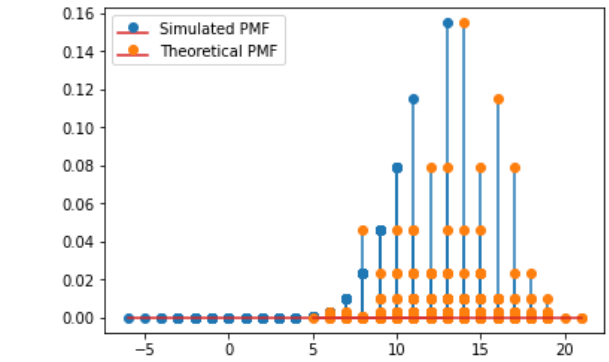

Figure 1: Probability Mass Function

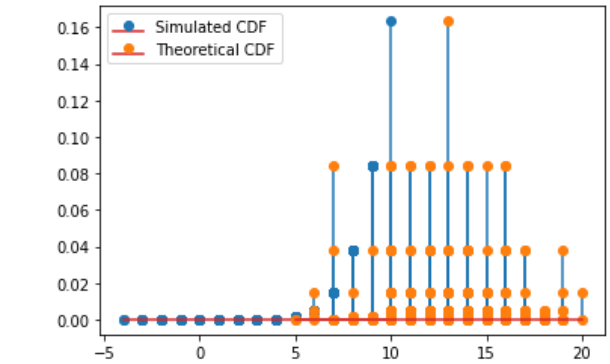

Figure 2: CDF

Suppose, the coin is unbiased. Then, $p = q = \frac{1}{2}$.

$$\therefore P(X = x) = \binom{12}{6 + x} \frac{1}{2}^{12}, x = -6 \text{ to } 6 \tag{6}$$

Download python code from here

https://github.com/Swati−Mohanty/AI5002/blob/main/Compulsory Assignment/codes/comp.py

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https://github.com/Swati−Mohanty/AI5002/blob/main/Compulsory Assignment/codes/compulsory.tex