Assignment 4
Probability and Random Variables

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I. Problem

Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

II. Solution

Let Y denote the event of tossing a coin. Considering a fair coin, the probability of getting a Head or Tail P(Y) = 0.5
We have i coin tosses, with probability p of Heads and (1-p) of Tails. We conduct the trials independently.
In general , the probability of getting of j Head/Tail is given as:

$$P(Y = j) = \frac{n!}{j!(n-j)!}p^j(1-p)^{(n-j)} \tag{1}$$

Consider a random variable Y where Y = Number of successes.Suppose we have n trials. We write Y ˜B(n, p)
(i) Let Y1 denote the number of Heads. The probability distribution of getting exactly j Heads in 2 tosses of coin is given as:
Y1 ˜B(2, 0.5)
Using equation (1),

$$P(Y1 = 0) = \frac{2!}{0!(2-0)!}0.5^0(1-0.5)^{(2-0)} = 0.25 \tag{2}$$

$$P(Y1 = 1) = \frac{2!}{1!(2-1)!}0.5^1(1-0.5)^{(2-1)} = 0.5 \tag{3}$$

$$P(Y1 = 2) = \frac{2!}{2!(2-2)!}0.5^2(1-0.5)^{(2-2)} = 0.25 \tag{4}$$

The distribution table is given as:

j	0	1	2
P(Y1=1)	0.25	0.5	0.25

(ii)Let Y2 denote the number of Tails. The probability distribution of getting exactly j Tails in 3 tosses of coin is given as:
Y2 ˜B(3, 0.5)

$$P(Y2 = 0) = \frac{3!}{0!(3-0)!}0.5^0(1-0.5)^{(3-0)} = 0.125 \tag{5}$$

$$P(Y2 = 1) = \frac{3!}{1!(3-1)!}0.5^1(1-0.5)^{(3-1)} = 0.375 \tag{6}$$

$$P(Y2 = 2) = \frac{3!}{2!(3-2)!}0.5^2(1-0.5)^{(3-2)} = 0.375 \tag{7}$$

$$P(Y2 = 3) = \frac{3!}{3!(3-3)!}0.5^3(1-0.5)^{(3-3)} = 0.125 \tag{8}$$

The probability distribution of Y2 is:

j	0	1	2	3
P(Y2=j)	0.125	0.375	0.375	0.125

(iii)Let Y3 denote the number of Heads. The probability distribution of getting exactly j Heads in 4 tosses of coin is given as:
Y4 ˜B(4, 0.5)

$$P(Y3 = 0) = \frac{4!}{0!(4-0)!}0.5^0(1-0.5)^{(4-0)} = 0.0625 \tag{9}$$

$$P(Y3 = 1) = \frac{4!}{0!(4-1)!}0.5^1(1-0.5)^{(4-1)} = 0.25 \tag{10}$$

$$P(Y3 = 2) = \frac{4!}{2!(4-2)!}0.5^2(1-0.5)^{(4-2)} = 0.375 \tag{11}$$

$$P(Y3 = 3) = \frac{4!}{3!(4-3)!}0.5^3(1-0.5)^{(4-4)} = 0.25 \tag{12}$$

$$P(Y3 = 4) = \frac{4!}{0!(4-4)!}0.5^4(1-0.5)^{(4-4)} = 0.0625 \tag{13}$$

The probability distribution of Y3 is:

j	0	1	2	3	4
P(Y3=j)	0.0625	0.25	0.375	0.25	0.0625

The probabilities were simulated using the python code.

```
Bernoulli simulation
[0.2506, 0.5003, 0.2491]
[0.1258, 0.3806, 0.3705, 0.1231]
[0.0625, 0.2513, 0.3752, 0.2519, 0.0591]
Binomial simulation
[0.2459, 0.5013, 0.2528]
[0.123, 0.3801, 0.3752, 0.1217]
[0.0645, 0.2494, 0.3711, 0.2533, 0.0617]
```

Figure 1: Simulation for tossing a fair coin

Download python code from here

https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment_4/codes/cointoss.py

Download latex code from here-

https://github.com/Swati-Mohanty/AI5002/blob/main/Assignment_4/codes/assignment4.tex