Assignment 3
Linear Programming

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I. Problem

Minimise and Maximise $Z = x + 2y$ subject to $x + 2y \geq 100$; $2x - y \leq 0$; $2x + y \leq 200$; $x, y \leq 0$.

II. Solution

In order to obtain the maximum and minimum value we need to solve the system of inequalities by adding slack variables. The equations now become:

$$x + 2y - Z = 0 \tag{1}$$
$$x + 2y - S_1 = 100 \tag{2}$$
$$2x - y + S_2 = 0 \tag{3}$$
$$2x + y + S_3 = 200 \tag{4}$$

The simplex table can be formed as

$$\left(\begin{array}{cccccc} x & y & s_1 & s_2 & s_3 & b \\ \hline 1 & 2 & -1 & 0 & 0 & 50 \\ 2 & -1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 200 \\ \hline 1 & 2 & 0 & 0 & 0 & 0 \end{array}\right) \tag{5}$$

The pivot element is 2 as the minimum ratio 50 occurs for y as the entering variable. Now reducing the simplex matrix we get

$$\left(\begin{array}{cccccc} x & y & s_1 & s_2 & s_3 & b \\ \hline \frac{1}{2} & 1 & \frac{-1}{2} & 0 & 0 & 50 \\ \frac{5}{2} & 0 & \frac{-1}{2} & 1 & 0 & 50 \\ \frac{3}{2} & 0 & \frac{1}{2} & 0 & 1 & 150 \\ \hline 1 & 2 & 0 & 0 & 0 & 0 \end{array}\right) \tag{6}$$

This can be expressed in the form of matrix inequality for maximization and minimization respectively as:

$$\max_{\{x\}} \mathbf{c}^T\mathbf{x} \tag{7}$$
$$s.t \quad \mathbf{Ax} \leq \mathbf{b}; \mathbf{x} \geq 0 \tag{8}$$
$$\min_{\{x\}} \mathbf{c}^T\mathbf{x} \tag{9}$$
$$s.t \quad \mathbf{Ax} \geq \mathbf{b}; \mathbf{x} \geq 0 \tag{10}$$

where

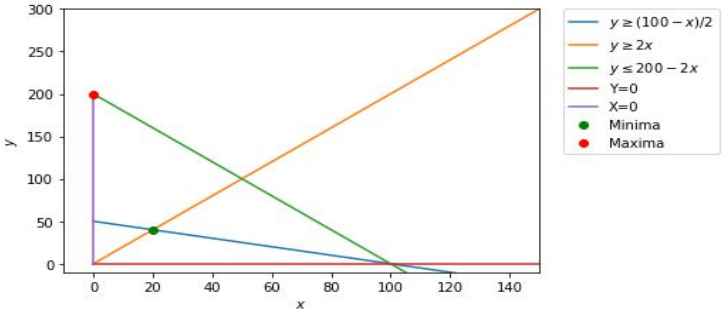
$$\mathbf{c} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{11}$$
$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 2 & 1 \end{pmatrix} \tag{12}$$
$$\mathbf{b} = \begin{pmatrix} 100 \\ 0 \\ 200 \end{pmatrix} \tag{13}$$

Solving for Z by this reduction method we get

$$MaxZ = 400 \tag{14}$$
$$MinZ = 100 \tag{15}$$

This can be solved in Python which generates the result as below:

```
Maximum=  [400.]
Minimum=  [100.]
<matplotlib.legend.Legend at 0x7fa35e61d6a0>
```



The following python code generates the maxima and minima values
Link : https://github.com/Swati-Mohanty/EE5600/blob/master/Assignment