Assignment 3
Linear Programming

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I. Problem

Minimise and Maximise $Z = x + 2y$ subject to $x + 2y \geq 100$; $2x - y \leq 0$; $2x + y \leq 200$; $x, y \leq 0$.

II. Solution

In order to obtain the maximum and minimum value we need to solve the system of inequalities by adding slack variables. The equations now become:

$$x + 2y - Z = 0 \tag{1}$$
$$x + 2y - S_1 = 100 \tag{2}$$
$$2x - y + S_2 = 0 \tag{3}$$
$$2x + y + S_3 = 200 \tag{4}$$

The simplex table can be formed as

$$\left(\begin{array}{cccccc} x & y & s_1 & s_2 & s_3 & b \\ \hline 1 & 2 & -1 & 0 & 0 & 50 \\ 2 & -1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 200 \\ \hline 1 & 2 & 0 & 0 & 0 & 0 \end{array}\right) \tag{5}$$

The pivot element is 2 as the minimum ratio 50 occurs for y as the entering variable. Now reducing the simplex matrix we get

$$\left(\begin{array}{cccccc} x & y & s_1 & s_2 & s_3 & b \\ \hline \frac{1}{2} & 1 & \frac{-1}{2} & 0 & 0 & 50 \\ \frac{5}{2} & 0 & \frac{-1}{2} & 1 & 0 & 50 \\ \frac{3}{2} & 0 & \frac{1}{2} & 0 & 1 & 150 \\ \hline 1 & 2 & 0 & 0 & 0 & 0 \end{array}\right) \tag{6}$$

This can be expressed in the form of matrix inequality for maximization and minimization respectively as:

$$\max_{\{x\}} \mathbf{c}^T \mathbf{x} \tag{7}$$
$$s.t \quad \mathbf{A}\mathbf{x} \leq \mathbf{b}; \mathbf{x} \geq 0 \tag{8}$$
$$\min_{\{x\}} \mathbf{c}^T \mathbf{x} \tag{9}$$
$$s.t \quad \mathbf{A}\mathbf{x} \geq \mathbf{b}; \mathbf{x} \geq 0 \tag{10}$$

where

$$\mathbf{c} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{11}$$
$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 2 & 1 \end{pmatrix} \tag{12}$$
$$\mathbf{b} = \begin{pmatrix} 100 \\ 0 \\ 200 \end{pmatrix} \tag{13}$$

Solving for Z by this reduction method we get

$$MaxZ = 400 \tag{14}$$
$$MinZ = 100 \tag{15}$$

This can be solved in Python which generates the result as shown in Fig 1.

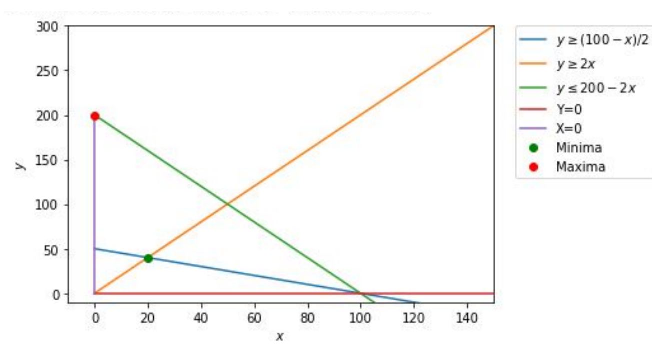

Fig. 1. Plot from python code with Maxima and Minima points

The following python code generates the maxima and minima values

Link : https://github.com/Swati-Mohanty/EE5600/blob/master/Assignment