# LA Lecture 5

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## 1 Topics Covered

1. PA = LU (Section 2.7) (covered in notes of lecture 4)
2. Vector spaces and sub spaces (Section 3.1)

### 1.1 Vector Spaces

Examples of vectors spaces:
$R^2$ = all real valued 2-dimensional vectors (or say) all vectors with 2 components (or) the x-y plane
Similarly, $R^n$ = all column vectors with n components

$$\begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

is a vector in $R^2$ while

$$\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

is a vector in $R^3$.

**Vector space has to be closed under addition and multiplication.**

**Subspace of $R^2$** For a line to be subspace in $R^2$, the line must pass though origin else it won't satisfy the properties of a vector space.

**Various subspaces of $R^2$**

1. $R^2$ itself. (plane)
2. any line through
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
(L)
3. zero vector (Z)

**Various subspaces of $R^3$**

1. $R^3$ itself.

2. any line through
$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
(L)
3. any plane through
$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
4. zero vector (Z)

**How to create a subspace from a matrix?**
Let A =
$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}$$
All the linear combinations of columns of A form a subspace. Called as **column subspace**, C(A). This subspace is a plane passing through the origin. If the 2 columns of A were lying on the same line, then we would have got that line as subspace instead of plane. Since columns of A are vectors in $R^3$, the vectors in subspace obtained from it are vectors in $R^3$.
Let's say we have 5 vectors in $R^{10}$. What is the subspace formed by these vectors?
We won't get $R^5$ because these vectors have 10 components, not 5. We might get a 5-d flat thing passing through origin. If those 5 vectors lie on the same line, then we will only get a line passing through origin.