Part 1: $R = R_{x,\theta} \cdot R_{y,\phi}$

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_{y,\phi} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ \sin\theta\sin\phi & \cos\theta & -\sin\theta\cos\phi \\ -\cos\theta\sin\phi & \sin\theta & \cos\theta\cos\phi \end{bmatrix}$$

$$\frac{\partial R}{\partial \phi} = \begin{bmatrix} -\sin\phi & 0 & \cos\phi \\ \sin\theta\cos\phi & 0 & +\sin\theta\sin\phi \\ -\cos\theta\cos\phi & 0 & -\cos\theta\sin\phi \end{bmatrix}$$

utting $\theta = \pi/2$, $\phi = \pi/2$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\boxed{4.10}$

$$H_1^0 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$v_1(t) = \begin{bmatrix} 3 \\ +1 \\ 0 \end{bmatrix}$$

position vector P_1^0 according to $H_1^0 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$$R_1^0 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$r = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

From R_1^0, $\dot{\imath}_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad S(\omega_z) = \begin{bmatrix} 0 & -1 & 0 \\ +1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

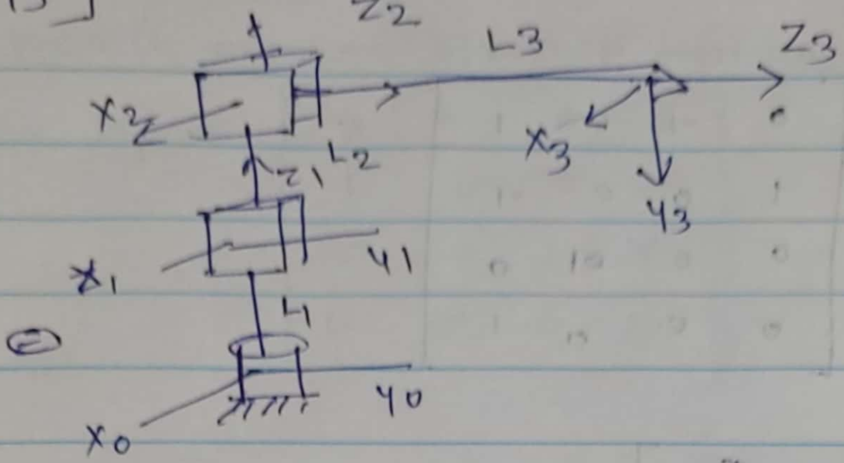
$$\dot{p}^0 = \omega \times r + v_1$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$$\dot{p}^0 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

$\dot{p}^0$ – velocity vector in 0th frame.

[4.15]



Calculating DH parameters:-

	θ_i	a_i	d_i	α_i
l_1	θ_1^*	0	l_1	0
l_2	0	0	l_2^*	-90
l_3	0	0	l_3^*	0

$$A_1 = \begin{bmatrix} C\theta_1^* & -S\theta_1^* & 0 & 0 \\ S\theta_1^* & C\theta_1^* & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & -0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} C\theta_1^* & -S\theta_1^* & 0 & 0 \\ S\theta_1^* & C\theta_1^* & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_1^* & 0 & +S\theta_1^* & 0 \\ S\theta_1^* & 0 & C\theta_1^* & 0 \\ 0 & -1 & 0 & l_2^* + L1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} C\theta_1^* & 0 & -S\theta_1^* & -l_3^* S\theta_1^* \\ S\theta_1^* & 0 & C\theta_1^* & l_3^* C\theta_1^* \\ 0 & -1 & 0 & l_2^* + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} \text{linear component} \\ \hline \text{Angular component} \end{bmatrix}$$

angular velocities

$$J\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad J\omega_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad J\omega_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

z axis of joint 1

Linear velocities

Joint 1.

$$J v_1 = z_0^0 \times (O_3^0 - O_0^0)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} -l_3^* S\theta_1^* \\ l_3^* C\theta_1^* \\ l_2^* + l_1^* \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -l_3^* S\theta_1^* \\ l_3^* C\theta_1^* \\ l_2^* + l_1^* \end{bmatrix}$$

$$= \begin{bmatrix} -l_3^* S\theta_1^* \\ -l_3^* C\theta_1^* \\ 0 \end{bmatrix}$$

$$J v_2 = z_1^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ - from matrix A_1

$$J_{V3} = z_2^0 = \begin{bmatrix} -s\theta_1 \\ c\theta_1 \\ 0 \end{bmatrix} \quad \text{-- from } T_2^0.$$

$$J = \begin{bmatrix} z_0^0 \times (o_3^0 - o_0^0) & z_1^0 & z_2^0 \\ z_0^0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_3^* s\theta_1^* & 0 & -s\theta_1^* \\ -l_3^* c\theta_1^* & 0 & c\theta_1^* \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

determinant

$$\det \begin{bmatrix} -c\theta_1^* l_3^* & 0 & -s\theta_1^* \\ -s\theta_1^* l_3^* & 0 & c\theta_1^* \\ 0 & 1 & 0 \end{bmatrix}$$

$$\det = c\theta_1^2 l_3^* + s\theta_1^2 l_3^* = (\sin\theta_1^2 + \cos^2\theta_1)\, l_3^* = l_3^*$$

~~here $l_3 \neq 0$, so when $\theta_1 = 0$, $c\theta_1^2$ -term will be 1, hence determinant $\neq 0$, at $\theta = 0$, there is no singularity.~~

$|J| = l_3^*$ & l_3 = link length can not be zero.

hence, there are no singularities.