Worcester Polytechnic Institute

RBE/ME 501 – ROBOT DYNAMICS
HOMEWORK 2
Spring 2024 – Instructor: L. Fichera

INSTRUCTIONS

- **This homework includes 5 preliminary programming questions and 3 problems (*)**
- **To solve the 3 problems, you will have to have Peter Corke's MATLAB Robotics toolbox installed on your system** (ver 10.4 or newer). See https://petercorke.com/toolboxes/robotics-toolbox/.
    - **Submit your solutions to the programming questions on MATLAB Grader:** https://grader.mathworks.com/courses/129496-rbe-501-robot-dynamics-spring-2024/assignments/361941-homework-2
    - **Submit your solutions to the other problems on Gradescope.**
      You will find separate entries to submit:
        - A zip file containing your MATLAB code
            - All source files must be professionally commented; ideally, anyone not familiar with the code should be able to understand what it does just from reading the comments;
            - All functions **must** include a help section;
            - Use I/O and/or figures to illustrate what your code does. Code that runs silently, that is, without generating any visible output, will not be graded;
        - A PDF file detailing your solution; the PDF **must** contain:
            - All necessary schematics/diagrams used to derive the solution;
            - Commentary to explain your thought process (for instance: "To solve the inverse kinematics, I apply method X"); if unsure how much level of detail is enough, err on the side of verbosity; no points will be awarded for just reporting the right solution;
            - All calculations (even though they are numerically performed in MATLAB);
    - Check your code before submitting it! **Code that runs with an error will not be graded.**
    - The PDF must be clearly legible. Plots and figures must be of professional quality. Figure DPI >= 300. Remember to add axis labels and units.
- **Soft deadline**: Friday 16-Feb-24 at 8:00 pm
- **Hard deadline**: Friday 23-Feb-24 at 8:00 pm

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Preliminary Programming Questions (25 points total – 5 points for each correct solution)

This section must be completed online at:
https://grader.mathworks.com/courses/129496-rbe-501-robot-dynamics-spring-2024/assignments/361941-homework-2

1. Converting exponential coordinates of rotation to rotation matrices

2. Converting twists to homogeneous transformation matrices

3. Product of Exponentials Formula

4. Adjoint Transformations

5. Product of Exponentials

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Problem 1 (25 points):

The 3-DoF manipulator in Figure 1 consists of a three degrees-of-freedom manipulator, with link lengths as follows: L1: 0.3 m, L2: 0.3 m, L3: 0.3 m. A model of this manipulator was pre-created for your convenience: open the zip archive with the starting MATLAB code provided on Canvas, then run the `hw2problem1.m` file.

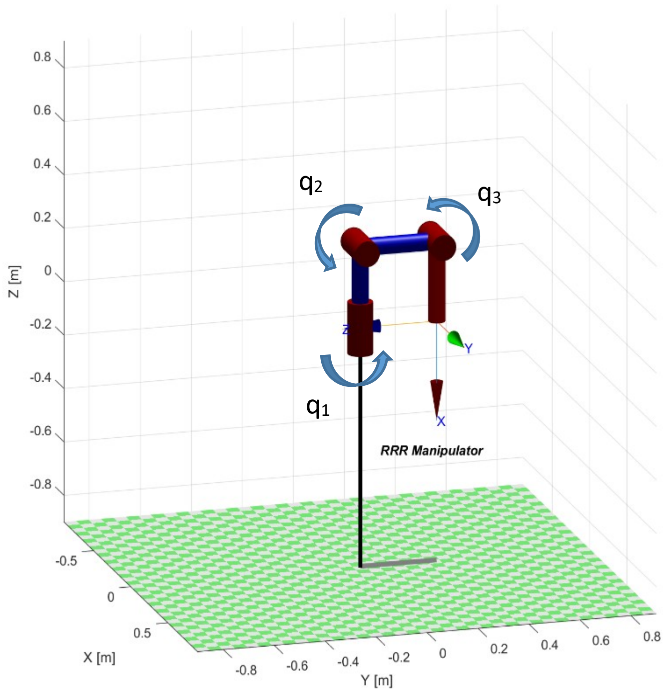

Figure 1: Three DoF manipulator.

a. **Calculate each of the screw axes $\xi_i = (\omega_i, \nu_i)$ with respect to the space frame (5 points).**
   Show your calculations in the text box below (continues on the next page):

w1 = [0 0 1]^T p1 = [0 0 0] v1 = -[w1] p1 = [0 0 0]
S1 = [0 0 1 0 0 0]^T

W2 = [1 0 0]^T p2 = [0 0 L1] v2 = -[w2]
v2= [0 L1 0]
S2 = [1 0 0 0 L1 0]^T

w2 = [1 0 0]^T   p3 = [0 L2 L3]^T
v3 = -[w3] p3 = [0 L3 -L2]
S2 = [1 0 0 0 L3 -L2]^T

$\boldsymbol{\xi_1} = [0\ 0\ 1\ 0\ 0\ 0 \qquad]^{\boldsymbol{T}}$

$\boldsymbol{\xi_2} = [1\ 0\ 0\ \ 0\ L1\ 0 \qquad {}^{\boldsymbol{T}}$

$\boldsymbol{\xi_3} = [\quad 1\ 0\ 0\ 0\ L3\ \text{-}L2 \qquad {}^{\boldsymbol{T}}$

Once you have calculated the screw axes, complete line 38 of `hw2problem1.m` to hardcode the screw axes into the script.

b. Calculate the manipulator kinematics using the product of exponentials formula (5 points)
To complete this step, you will have to first calculate the homogeneous transformation matrix $\boldsymbol{M}$ for the home configuration. Calculate $\boldsymbol{M}$ and enter it in the text box below:

$$\boldsymbol{M} = \begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & L2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Once you have calculated the home configuration, type it into the Matlab script (line 44), then use this information to calculate the forward kinematics (line 62). You should be able to use the `fkine` function developed in the preliminary questions for this homework.

c. Calculate the space Jacobian of the manipulator (5 points)

Calculate the space Jacobian by adding code to line 96. You should be able to use the `jacob0` function developed in Section 1 of this homework.

d. Calculate the inverse kinematics for 30 different configurations using a numerical approach (10 points)

To solve this step, fill out the code on lines 150-151. These two lines of code should generate a vector `deltaQ` that should be applied to the vector of joint variables to move closer to the target pose. In your report, comment on what numerical method you chose to implement; take note of the configurations for which the inverse kinematics algorithm fails (if any), investigate the cause of the failure (ill-conditioned Jacobian? Unreachable configuration?), and attempt to find a different solution for these configurations.

Problem 2 (20 points total):

Repeat the same steps of problem 1 for the elbow manipulator shown below. Assume the same link lengths as in problem 1. A model of this manipulator was pre-created for your convenience: open the zip archive with the starting MATLAB code provided on Canvas, then run the `hw2problem2.m` file.

Note: unlike the prior problem, the starting code provided in *hw2problem2.m* is minimal. You are welcome to structure this script the way you want. Remember to comment your code adequately and to use I/O and/or figures to illustrate what your code does. Code that runs silently, that is, without generating any visible output, will not be graded.

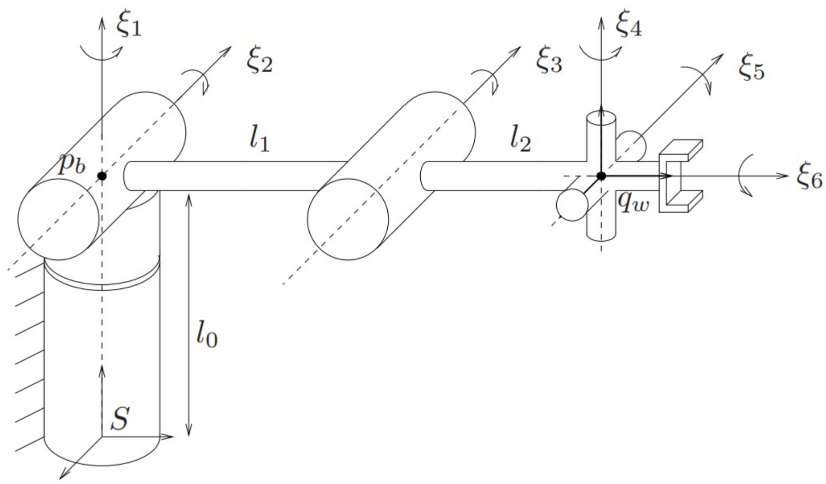

Figure 2: Elbow Manipulator

a. **Calculate each of the screw axes $\xi_i = (\omega_i, v_i)$ with respect to the space frame (5 points):**

s1= [0 0 1 0 0 0]^T
w2= [- 1 0 0] v2=-[w2]p2 = [-1 0 0] x [0 0 0.3]
s2=[-1 0 0 0 -0.3 0]^T

w3= [-1 0 0] v3 = -[w3]p3 = [0 -0.3 0.3]^T
s3 =[-1 0 0 0 -0.3 0.3]^T

w4=[0 0 1]^T v4 = -[w4]p4 = [0.6 0 0]^T

w5 = [-1 0 0]^T v5=-[w5]p5 = [0 -0.3 0.6]^T

W6 = [0 1 0]^T v6= -[w6]p6 = [-0.3 0 0]

$\boldsymbol{\xi_1} = [0\ 0\ 1\ \ 0\ 0\ 0\ ]^{\boldsymbol{T}}$

$\boldsymbol{\xi_2} = [\ -1\ 0\ 0\ \ 0\ -0.3\ 0\ ]^{\boldsymbol{T}}$

$\boldsymbol{\xi_3} = [-1\ \ 0\ \ 0\ \ 0\ -0.3\ 0.3\ ]^{\boldsymbol{T}}$

$\boldsymbol{\xi_4} = [\ 0\ \ 0\ \ 1\ \ \ 0.6\ \ 0\ \ 0\ ]^{\boldsymbol{T}}$

$\boldsymbol{\xi_5} = [-1\ \ 0\ \ 0\ \ \ 0\ \ -0.3\ \ 0.6\ ]^{\boldsymbol{T}}$

$\boldsymbol{\xi_6} = [0\ \ 1\ \ 0\ \ -0.3\ \ 0\ \ 0\ ]^{\boldsymbol{T}}$

b. Calculate the home configuration (5 points):

$$\boldsymbol{M} = \begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.6 \\ 1 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 1 \end{matrix}$$

c. Calculate the space Jacobian of the manipulator (5 points)

Use the same procedure you used for the Jacobian in Problem 1. Submit your code to Gradescope.

d. Calculate the inverse kinematics for 30 different configurations using a numerical approach (10 points)

Use the same procedure you used for the Jacobian in Problem 1. Submit your code to Gradescope.

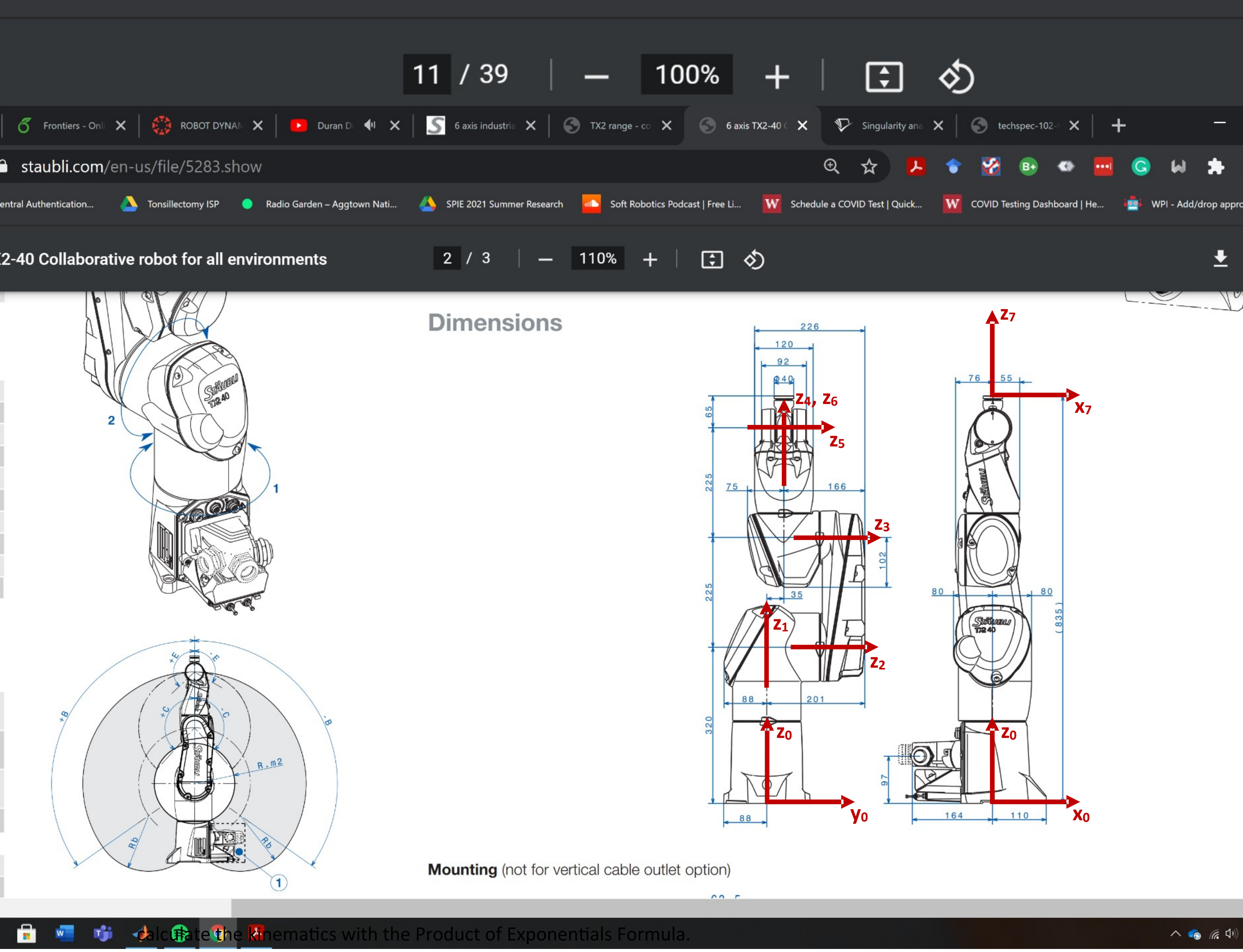

Figure 5: Figure for Problem 4

s1=[0 0 1 0 0 0]^T
w2=[0 1 0]^T v2 = -[w2]p2 = -[0 1 0]x [0.32, 0 0]
= [-0.32 0 0]^T
s2 = [0 1 0 -0.32 0 0]
w3=[0 1 0]^T v3=-[w3]p3 = -[0 1 0]x[0 0 0.545]
s3=[0 1 0   -0.545 0 0]

w4=[0 0 1]^T v4= -[w4]p4 = -[0 0 1]x[-0.035 0 0]
S4=[0 0 1 0.035 0 0]^T

w5=[0 1 0]^T v5=-[w5]p5 = -[0 1 0] x [0 0 0.77]
S5=[0 1 0 -0.77 0 0]^T

w6 = [0 0 1]^T v6 = -[w6]p6 =-[0 0 1]x[0 -0.035 0]
S6 =[0 0 1 0.035 0 0]

$$\boldsymbol{\xi_1} = [\quad 0\ 0\ 1\ \ 0\ 0\ 0 \quad]^{\boldsymbol{T}}$$

$$\boldsymbol{\xi_2} = [\quad 0\ \ 1\ \ 0\ \ -0.32\ \ \ 0\ \ \ 0 \quad {}^{\boldsymbol{T}}$$

$$\boldsymbol{\xi_3} = [\quad 0\ 1\ 0\ \ -0.545\ \ 0\ 0 \quad {}^{\boldsymbol{T}}$$

$$\boldsymbol{\xi_4} = [\quad 0\ \ 0\ \ 1\ \ 0.035\ \ 0\ \ 0 \quad {}^{\boldsymbol{T}}$$

$$\boldsymbol{\xi_5} = [\quad 0\ 1\ 0\ \ \ -0.77\ \ 0\ \ \ 0 \quad {}^{\boldsymbol{T}}$$

$$\boldsymbol{\xi_6} = [\quad 0\ 0\ 1\ \ 0.035\ \ 0\ \ 0 \quad {}^{\boldsymbol{T}}$$

b. Calculate the home configuration (10 points):

$$\boldsymbol{M} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.035 \\ 0 & 0 & 1 & 0.835 \\ 0 & 0 & 0 & 1 \end{matrix}$$

c. Calculate the space Jacobian of the manipulator (5 points)

Use the same procedure you used for the Jacobian in Problem 1. Submit your code to Gradescope.

d. Calculate the inverse kinematics for 30 different configurations using a numerical approach (10 points)

Use the same procedure you used for the Jacobian in Problem 1. Submit your code to Gradescope.