Optimal Parameter Constraints on Dark Energy Models

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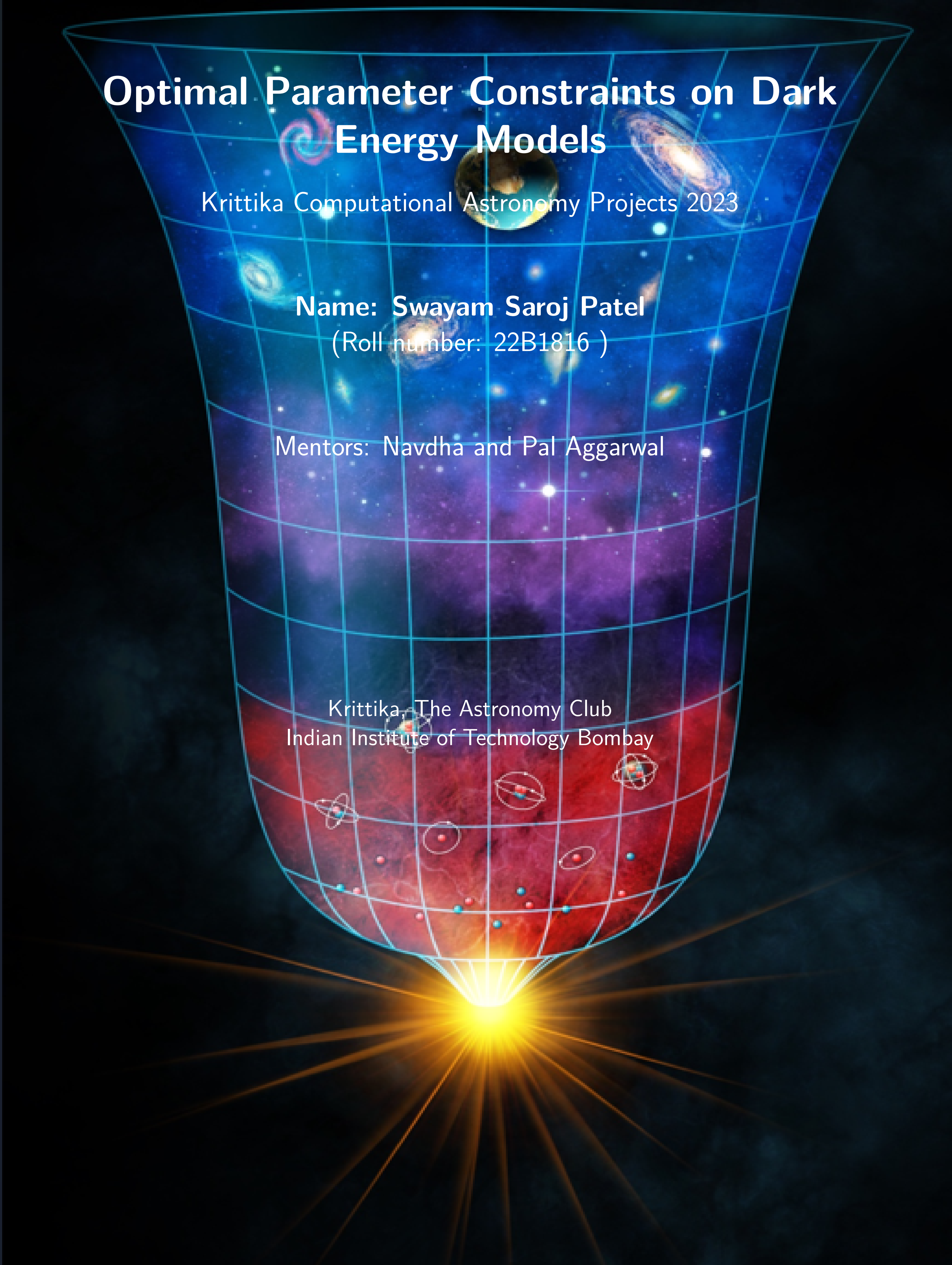


Contents

1 Introduction

For centuries, humans have been intrigued by the vast and dynamic nature of the universe. In modern astrophysics, it is necessary to dive deeply into the fundamental components that shape the cosmos to understand the cosmic fabric. This project aims to explore the puzzling realm of dark energy and alternative cosmological models to uncover the secrets of the universe.

The accelerated expansion of the universe is a phenomenon that is attributed to dark energy, which makes up a significant portion of cosmic energy. This expansion challenges our conventional understanding of the universe. The standard cosmological model, which includes a cosmological constant, is facing observational discrepancies that require the exploration of alternative models with different degrees of spatial curvature and dark energy dynamics.

Our project is focused on analyzing lower-redshift observations. We will be using a combination of Hubble parameter (H(z)) and Baryon Acoustic Oscillations (BAO) data to study cosmic mysteries. The primary objective is to determine optimal parameter constraints for four distinct cosmological models. Each of these models is characterized by different degrees of flatness and dark energy dynamics.

We will start our journey by studying the theoretical foundation of cosmology. This will include the Friedmann equations, distance measures, and an introduction to Monte Carlo Markov Chain (MCMC) analysis. In the following weeks, we will focus on the practical implementation of MCMC using the emcee library in Python. This is a useful tool for constraining cosmological parameters. [1]

2 Cosmological Models

The project outlines the exploration of parameter constraints for different cosmological models. These models provide alternative descriptions of the universe's evolution and key components. Here are the cosmological models considered in the project: [2]

Lambda Cold Dark Matter (ΛCDM) Model:
The ΛCDM model is the standard cosmological model in modern astrophysics. It posits a homogeneous and isotropic universe governed by the principles of general relativity. The model includes cold dark matter (CDM) as a non-relativistic form of matter and a cosmological constant (Λ) representing dark energy.

Theory:
Dark Matter (CDM): Cold dark matter consists of non-relativistic particles moving slowly compared to the speed of light. It plays a crucial role in structure formation, providing the gravitational scaffolding for the formation of galaxies and large-scale structures.

Dark Energy (Λ): The cosmological constant represents a form of dark energy responsible for the observed accelerated expansion of the universe. It introduces a repulsive force counteracting gravitational attraction

Flat ΛCDM: This assumes a flat spatial geometry, adhering to the standard cosmological model with a cosmological constant (Λ) driving accelerated expansion.

$$H(z) = H_0\sqrt{[\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}]}$$

Non-Flat ΛCDM: Unlike the flat ΛCDM model, this variant considers non-zero spatial curvature, allowing for a more flexible description of the universe's geometry.

$$H(z) = H_0\sqrt{[\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda,0}]}$$

XCDM (X for Dark Energy) Model:
Overview:
The XCDM model extends the LCDM model by introducing a dynamic component for dark energy instead of a cosmological constant. The letter "X" signifies this dynamic dark energy component, allowing for variations in its behavior.

Theory:
Dynamic Dark Energy (X): In the XCDM model, dark energy is represented by a dynamic field (X) with an equation of state w_X that may deviate from the constant value associated with Λ. This dynamic nature allows for a more flexible description of dark energy's influence on cosmic expansion.

Flat XCDM: Similar to the flat LCDM model but with a dynamic dark energy component denoted as X. This model explores scenarios where dark energy's equation of state may deviate from a cosmological constant.

$$H(z) = H_0\sqrt{\left[\Omega_{m,0}(1+z)^3 + \Omega_{X,0}(1+z)^{3(1+w_X)}\right]}$$

Non-Flat XCDM: Extending the exploration to non-flat geometries, this variant introduces the possibility of spatial curvature alongside a dynamic dark energy component.

$$H(z) = H_0\sqrt{\left[\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{X,0}(1+z)^{3(1+w_X)}\right]}$$

Φ-CDM Model:
The Φ-CDM model involves the consideration of additional parameters denoted by Φ(Phi), offering an alternative approach to describing the universe's evolution. This model may be explored in the project's extension phase, contingent on the progress made in the initial four weeks. Each of these models represents a distinct perspective on the interplay between dark energy, spatial curvature, and the overall dynamics of the cosmos. The project aims to constrain the parameters of these models using observational data, shedding light on which cosmological framework aligns most closely with the observed universe.

$$H(z) = H_0\sqrt{\left[\Omega_{m,0}(1+z)^3 + \Omega_{\phi,0}(1+z)^{3(1+w_\phi)}\right]}$$

3 Data

Table 1: Observational Data(H(z) and σ_H have units of $kms^{-1}Mpc^{-1}$)[3]

Redshift (z)	**Hubble Parameter (Hz)**	**Error(σ_H)**
0.07	69.0	19.6
0.09	69.0	12.0
0.12	68.6	26.2
0.17	83.0	8.0
0.179	75.0	4.0
0.199	75.0	5.0
0.20	72.9	29.6
0.27	77.0	14.0
0.28	88.8	36.6
0.352	83.0	14.0
0.3802	83.0	13.5
0.4	95.0	17.0
0.4004	77.0	10.2
0.4247	87.1	11.2
0.4497	92.8	12.9
0.47	89.0	50.0
0.4783	80.9	9.0
0.48	97.0	62.0
0.593	104.0	13.0
0.68	92.0	8.0
0.781	105.0	12.0
0.875	125.0	17.0
0.88	90.0	40.0
0.90	117.0	23.0
1.037	154.0	20.0
1.3	168.0	17.0
1.363	160.0	33.6
1.43	177.0	18.0
1.53	140.0	14.0
1.75	202.0	40.0
1.965	186.5	50.4

Table 2: BAO Data [4]

Redshift (z)	Measurement	Value	Reference
0.38	$D_M(r_{s,fid}/r_s)$	1512.39	Alam et al. (2017)
0.38	$H(z)(r_s/rs, fid)$	81.2087	Alam et al. (2017)
0.51	$D_M(r_{s,fid}/r_s)$	1975.22	Alam et al. (2017)
0.51	$H(z)(r_s/r_{s,fid})$	90.9029	Alam et al. (2017)
0.61	$D_M(r_{s,fid}/r_s)$	2306.68	Alam et al. (2017)
0.61	$H(z)(r_s/rs, fid)$	98.9647	Alam et al. (2017)
0.122	$D_V(r_{s,fid}/r_s)$	$539(17)$	Carter et al. (2018)
0.81	D_A/r_s	$10.75(43)$	DES Collaboration (2019)
1.52	$D_V(r_{s,fid)}/r_s$	$3843(147)$	Ata et al. (2018)
2.34	D_H/r_s	8.86	de Sainte Agathe et al. (2019)
2.34	D_M/r_s	37.41	de Sainte Agathe et al. (2019)

4 Method

Chi-Minimization:
In cosmological parameter estimation, one common approach is to find the best-fit values for parameters by minimizing the chi-squared (χ^2) statistic. The chi-squared is a measure of the goodness of fit between a theoretical model and observational data. The theoretical model is used to predict the expected values of observables (e.g., Hubble parameter, BAO data) for a given set of cosmological parameters. The chi-squared statistic is then calculated by comparing the predicted values with the observed values, considering uncertainties in the observations.

$$\chi^2 = \Sigma((O_i - E_i)^2/\sigma_i^2)$$

where O_i is the observed value, $E_i$is the expected (model-predicted) value, and σ_i is the uncertainty associated with the observation.

Minimization:
The parameter values are adjusted iteratively to minimize the chi-squared statistic. This is often done using optimization algorithms (such as the Levenberg-Marquardt algorithm or the scipy.optimize.minimize function in Python) that systematically explore the parameter space to find the minimum χ^2 Best-Fit Parameters: The parameter values that result in the minimum χ^2 are considered the best-fit parameters for the given model.

Markov Chain Monte Carlo (MCMC) Methods:
Markov Chain Monte Carlo (MCMC) is a powerful statistical technique widely used in various fields, including physics, statistics, and machine learning. It is particularly valuable when dealing with complex probability distributions or high-dimensional parameter spaces. MCMC methods provide a systematic way to explore and sample from these distributions, allowing researchers to make inferences and estimate parameters in a principled manner.

The core idea behind MCMC is to construct a Markov chain whose equilibrium distribution is the target probability distribution of interest. The chain moves through the parameter space in a way that, over time, the samples it generates converge to a stationary distribution, which is the desired probability distribution. This convergence property ensures that the samples produced by the Markov chain are representative of the underlying distribution

MCMC allows researchers to obtain a representative sample from the posterior distribution of model parameters. This sample can then be used to estimate parameter values, calculate credible intervals, and make predictions about the underlying system.

In the context of the project, the emcee library in Python is used for MCMC analysis [5]. This library efficiently implements the Ensemble Sampler, a variant of MCMC, to explore the parameter space.

By combining chi-minimization and MCMC methods, the project aims to determine optimal parameter constraints for different cosmological models based on observational data. These methods allow for a robust statistical analysis that considers uncertainties and explores the entire parameter space.

5 Results

5.1 H(z) constraints

5.1.1 Flat ΛCDM

Table 3: **Flat ΛCDM**

Parameter	Optimal value obtained
H_O	$67.72^{+3.01}_{-3.10}$
Ω_{m0}	$0.33^{+0.06}_{-0.06}$

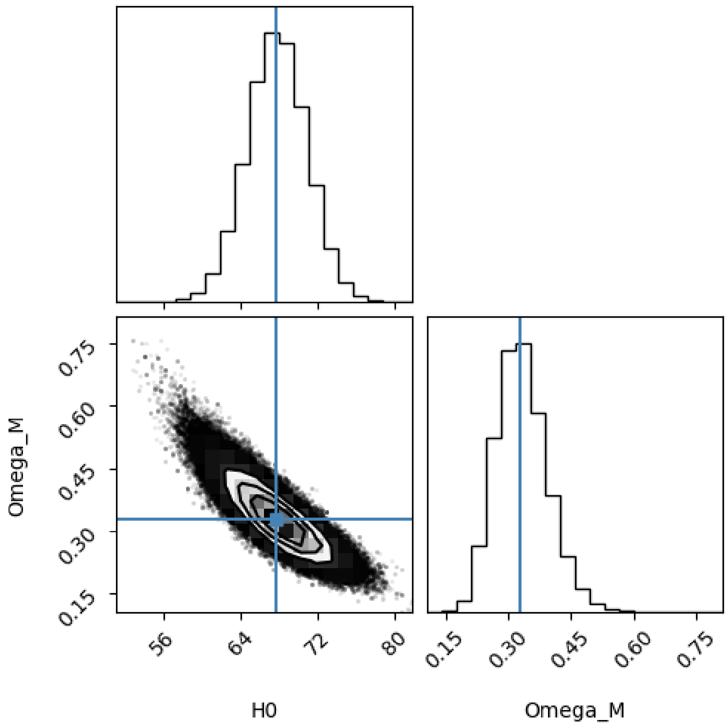


Figure 1: H(z) constraint MCMC(20000 steps) output plot

Qualitative Meaning:

- Hubble Constant(H_O):

  Represents the present-day expansion rate of the universe. And according to this model the universe is expanding at a rate of $67.72^{+3.01}_{-3.10}$ $kms^{-1}Mpc^{-1}$

- Density Parameters:

  – Matter Density Parameter($\Omega_{m,0}$):
    This parameter represents the fraction of the critical density composed of matter, both normal and dark matter. And according to this model $\Omega_{m,0}$ is $0.33^{+0.06}_{-0.06}$ that is suggesting about 33^{+6}_{-6} % of the universe's critical density is in the form of matter.

5.1.2 Non-Flat ΛCDM

Table 4: **Non-Flat ΛCDM**

Parameter	Optimal value obtained
H_O	$66.80^{+3.78}_{-3.88}$
Ω_{m0}	$0.30^{+0.14}_{-0.15}$
$\Omega_{\Lambda 0}$	$0.60^{+0.26}_{-0.33}$

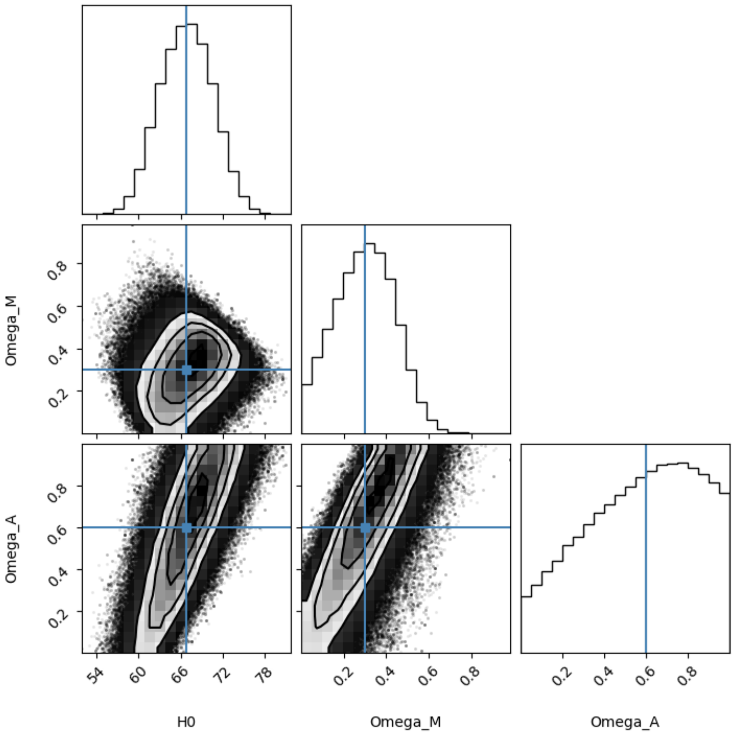


Figure 2: H(z) constraint MCMC(20000 steps) output plot

Qualitative Meaning:

- Hubble Constant(H_O):

  Represents the present-day expansion rate of the universe. And according to this model the universe is expanding at a rate of $66.80^{+3.78}_{-3.88}$ $kms^{-1}Mpc^{-1}$

- Density Parameters:

  – Matter Density Parameter($\Omega_{m,0}$):

    This parameter represents the fraction of the critical density composed of matter, both normal and dark matter. And according to this model $\Omega_{m,0}$ is $0.30^{+0.14}_{-0.15}$ that is suggesting about 30^{+14}_{-15} % of the universe's critical density is in the form of matter

  – Dark Matter Density Parameter ($\Omega_{\Lambda,0}$):

    This parameter represents the fraction of the critical density attributed to dark energy. This model suggests that the density of energy is $0.60^{+0.26}_{-0.33}$ that is 60^{+26}_{-33} %

5.1.3 Flat XCDM

Table 5: **Flat XCDM**

Parameter	Optimal value obtained
H_O	$69.23^{+6.55}_{-5.69}$
Ω_{m0}	$0.31^{+0.07}_{-0.07}$
ω_x	$-1.18^{+0.53}_{-0.51}$

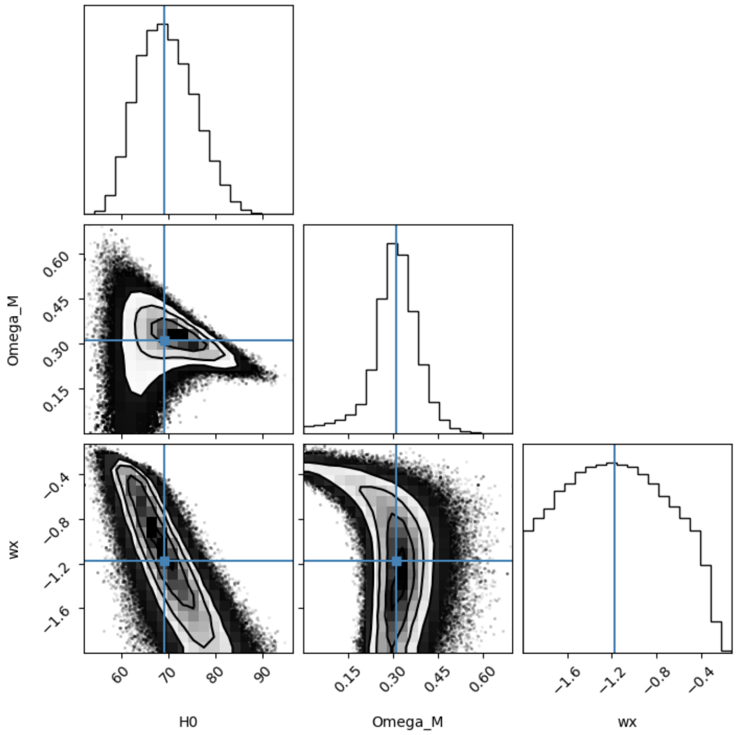


Figure 3: H(z) constraint MCMC(20000 steps) output plot

Qualitative Meaning:

- Hubble Constant(H_O):

  Represents the present-day expansion rate of the universe. And according to this model the universe is expanding at a rate of $69.23^{+6.55}_{-5.69}$ $kms^{-1}Mpc^{-1}$

- Density Parameters:

  – Matter Density Parameter($\Omega_{m,0}$):

    This parameter represents the fraction of the critical density composed of matter, both normal and dark matter. And according to this model $\Omega_{m,0}$ is $0.31^{+0.07}_{-0.07}$ that is suggesting about 31^{+7}_{-7} % of the universe's critical density is in the form of matter

- Equation of State Parameter(ω_x):

  It describes the pressure-density relationship of dark energy.The pressure-density relationship of dark energy is typically characterized by a negative pressure. And here the ratio is $-1.18^{+0.53}_{-0.51}$.

5.1.4 Non-Flat XCDM

Table 6: **Non-Flat XCDM**

Parameter	Optimal value obtained
H_O	$66.67^{+5.58}_{-4.37}$
Ω_{m0}	$0.22^{+0.09}_{-0.10}$
Ω_{K0}	$0.31^{+0.25}_{-0.21}$
ω_x	$-1.27^{+0.58}_{-0.50}$

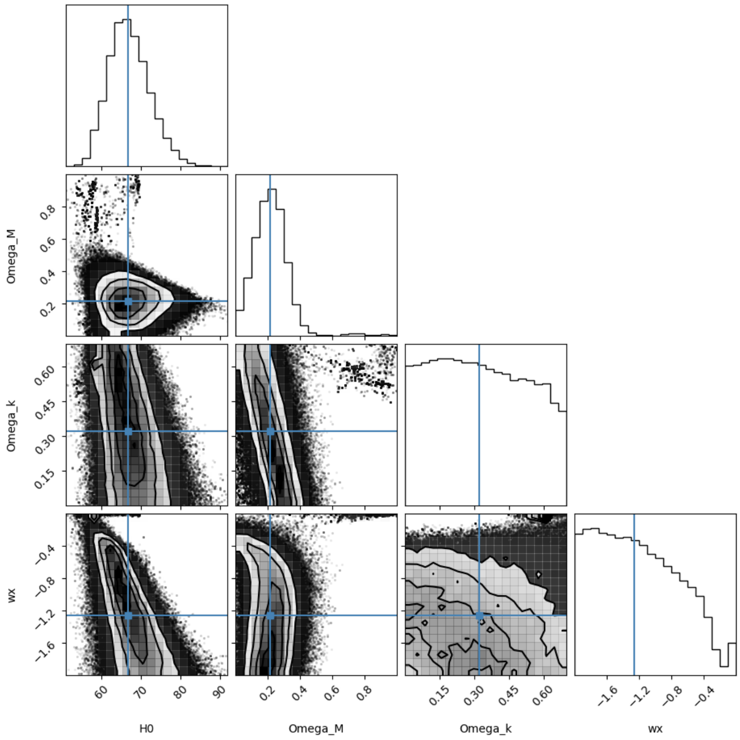


Figure 4: H(z) constraint MCMC(20000 steps) output plot

Qualitative meaning:

- Hubble Constant(H_O):

  Represents the present-day expansion rate of the universe. And according to this model the universe is expanding at a rate of $66.67^{+5.58}_{-4.37}\ kms^{-1}Mpc^{-1}$

- Density Parameters:

  – Matter Density Parameter($\Omega_{m,0}$):

    This parameter represents the fraction of the critical density composed of matter, both normal and dark matter. And according to this model $\Omega_{m,0}$ is $0.22^{+0.09}_{-0.10}$ that is suggesting about $0.22^{+0.09}_{-0.10}$ % of the universe's critical density is in the form of matter

  – Curvature Density Parameter($\Omega_{K,0}$):

    The parameter $\Omega_{K,0}$ represents the curvature density of the universe. In cosmology, the curvature of the universe is described by the spatial geometry of space-time. Here $\Omega_{K,0}$ is $0.31^{+0.25}_{-0.21}$ which indicates a closed universe.

- Equation of State Parameter(ω_x):

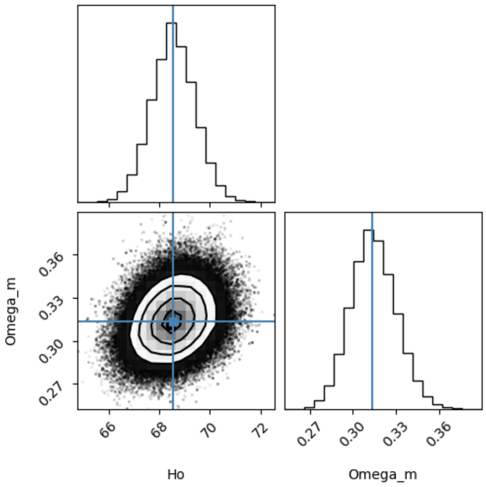
  It describes the pressure-density relationship of dark energy.The pressure-density relationship of dark energy is typically characterized by a negative pressure. And here the ratio is $-1.27^{+0.58}_{-0.50}$.

5.2 H(z) + BAO constraints

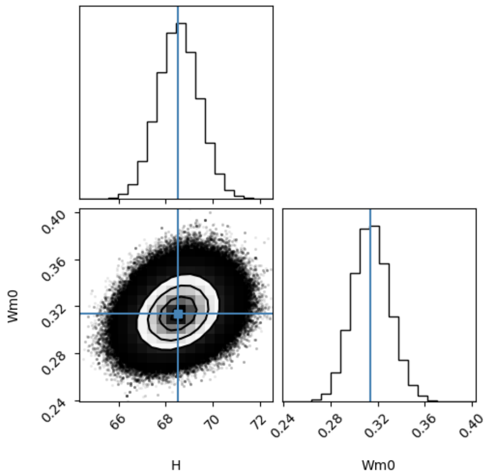
5.2.1 Flat ΛCDM

Table 7: **Flat ΛCDM**

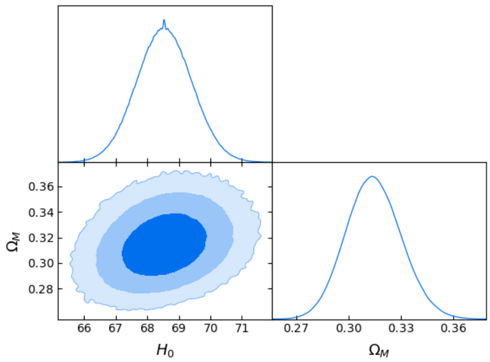
Parameter	Optimal value obtained
H_O	$68.5397^{+0.870}_{-0.863}$
Ω_{m0}	$0.3141^{+0.016}_{-0.015}$



(a) 20000 steps MCMC



(b) 20000 steps MCMC



(c) Contour plot

Figure 5: Contour Plots of Hz+BAO constraints of Flat ΛCDM

Qualitative Meaning:

- Hubble Constant(H_O):

  Represents the present-day expansion rate of the universe. And according to this model the universe is expanding at a rate of $68.5397^{+0.870}_{-0.863}\ kms^{-1}Mpc^{-1}$

- Density Parameters:

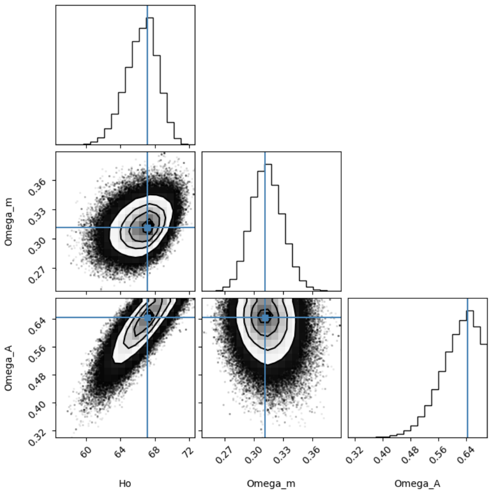
  – Matter Density Parameter($\Omega_{m,0}$):

    This parameter represents the fraction of the critical density composed of matter, both normal and dark matter. And according to this model $\Omega_{m,0}$ is $0.3141^{+0.016}_{-0.015}$ that is suggesting about $31.41^{+1.6}_{-1.5}$ % of the universe's critical density is in the form of matter.

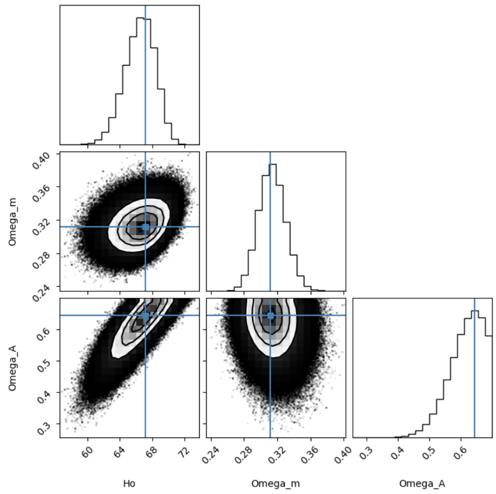
5.2.2 Non-Flat ΛCDM

Table 8: **Non-Flat ΛCDM**

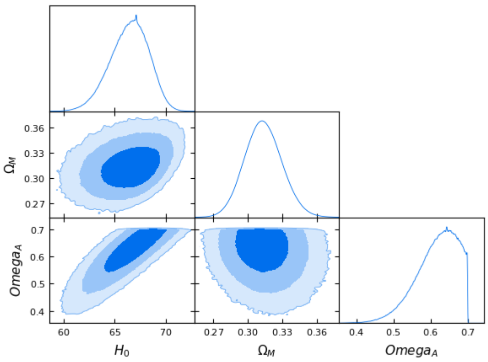
Parameter	Optimal value obtained
H_O	$66.5455^{+1.767}_{-2.023}$
Ω_{m0}	$0.3131^{+0.016}_{-0.016}$
$\Omega_{\Lambda 0}$	$0.6242^{+0.049}_{-0.064}$



(a) 20000 steps MCMC



(b) 20000 steps MCMC



(c) Contour plot

Figure 6: Contour Plots of Hz+BAO constraints of Non-Flat ΛCDM

Qualitative Meaning:

- Hubble Constant(H_O):

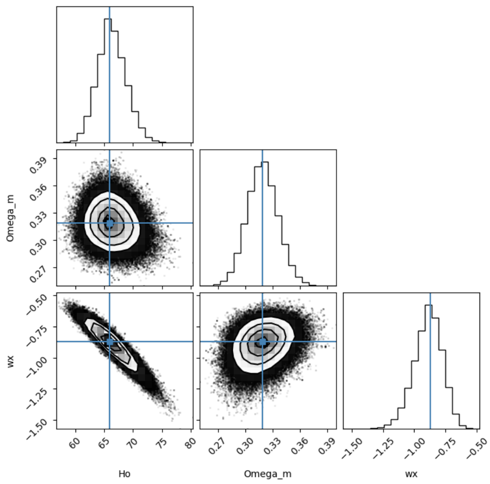
  Represents the present-day expansion rate of the universe. And according to this model the universe is expanding at a rate of $66.5455^{+1.767}_{-2.023}$ $kms^{-1}Mpc^{-1}$

- Density Parameters:
  - Matter Density Parameter($\Omega_{m,0}$):
    This parameter represents the fraction of the critical density composed of matter, both normal and dark matter. And according to this model $\Omega_{m,0}$ is $0.3131^{+0.016}_{-0.016}$ that is suggesting about $31.31^{+1.6}_{-1.6}$ % of the universe's critical density is in the form of matter
  - Dark Matter Density Parameter ($\Omega_{\Lambda,0}$):
    This parameter represents the fraction of the critical density attributed to dark energy. This model suggests that the density of energy is $0.6242^{+0.049}_{-0.064}$ that is $62.42^{+4.9}_{-6.4}$ %

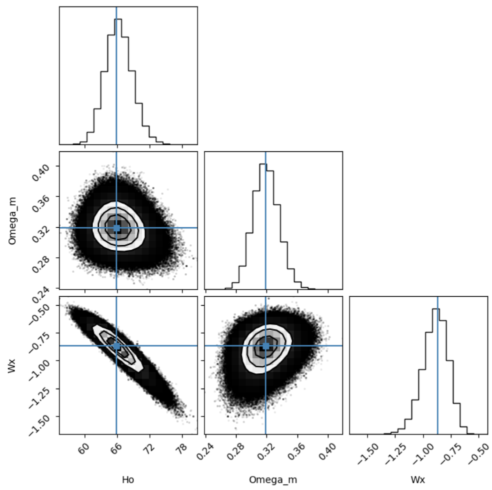
5.2.3 Flat XCDM

Table 9: **Flat XCDM**

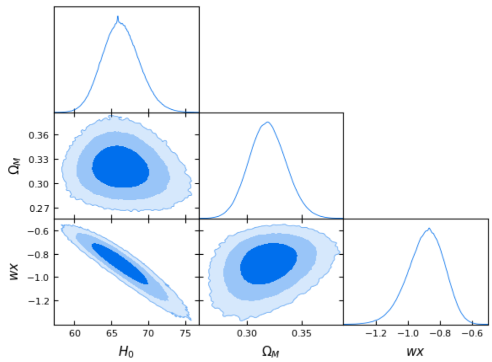
Parameter	Optimal value obtained
H_O	$66.2249^{+2.557}_{-2.392}$
Ω_{m0}	$0.3186^{+0.017}_{-0.016}$
ω_x	$-0.8837^{+0.106}_{-0.120}$



(a) 20000 steps MCMC



(b) 20000 steps MCMC



(c) Contour plot

Figure 7: Contour Plots of Hz+BAO constraints of Flat XCDM

Qualitative Meaning:

- Hubble Constant(H_O):

  Represents the present-day expansion rate of the universe. And according to this model the universe is expanding at a rate of $66.2249^{+2.557}_{-2.392}$ $kms^{-1}Mpc^{-1}$

- Density Parameters:

  – Matter Density Parameter($\Omega_{m,0}$):
    This parameter represents the fraction of the critical density composed of matter, both normal and dark matter. And according to this model $\Omega_{m,0}$ is $0.3186^{+0.017}_{-0.016}$ that is suggesting about $31.86^{+1.7}_{-1.6}$ % of the universe's critical density is in the form of matter

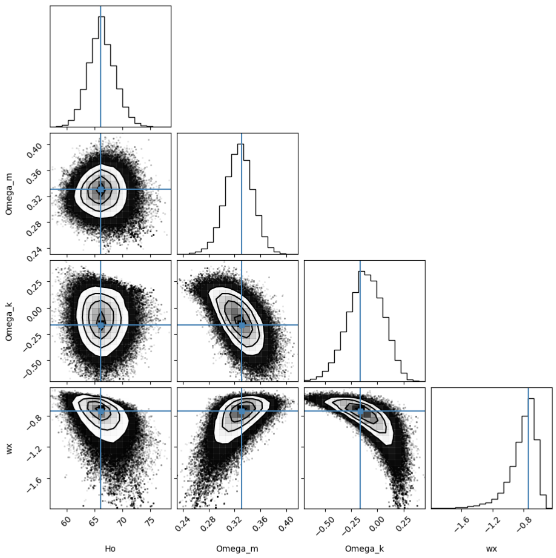
- Equation of State Parameter(ω_x):

  It describes the pressure-density relationship of dark energy.The pressure-density relationship of dark energy is typically characterized by a negative pressure. And here the ratio is $-0.8837^{+0.106}_{-0.120}$.

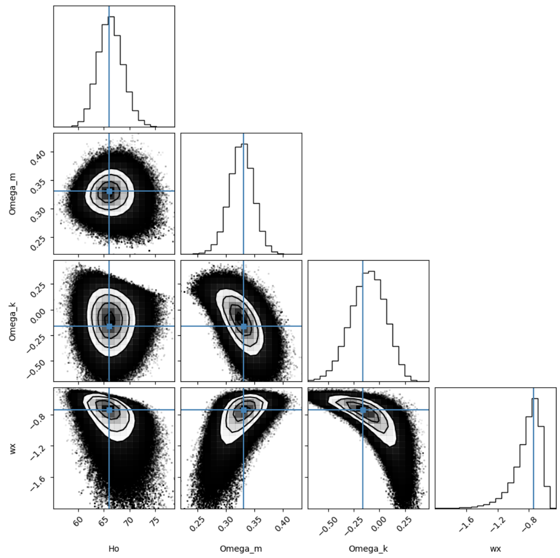
5.2.4 Non-Flat XCDM

Table 10: **Non-Flat XCDM**

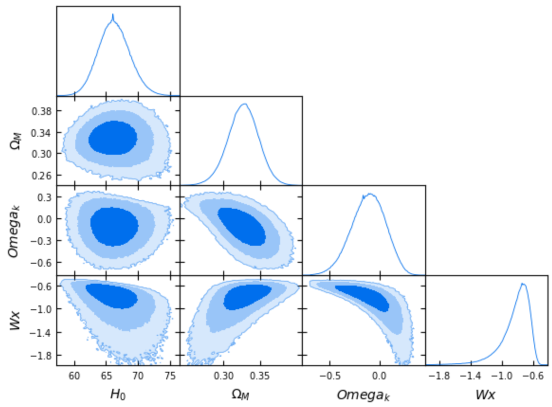
Parameter	Optimal value obtained
H_O	$66.2150^{+2.488}_{-2.346}$
Ω_{m0}	$0.3271^{+0.020}_{-0.021}$
Ω_{K0}	$-0.1071^{+0.1650}_{-0.1770}$
ω_x	$-0.7829^{+0.119}_{-0.119}$



(a) 20000 steps MCMC



(b) 20000 steps MCMC



(c) Contour plot

Figure 8: Contour Plots of Hz+BAO constraints of Non-Flat XCDM

Qualitative meaning:

- Hubble Constant(H_O):

  Represents the present-day expansion rate of the universe. And according to this model the universe is expanding at a rate of $66.2150^{+2.488}_{-2.346}\ kms^{-1}Mpc^{-1}$

- Density Parameters:

  – Matter Density Parameter($\Omega_{m,0}$):

This parameter represents the fraction of the critical density composed of matter, both normal and dark matter. And according to this model $\Omega_{m,0}$ is $0.3271^{+0.020}_{-0.021}$ that is suggesting about $32.71^{+2.0}_{-2.1}$ % of the universe's critical density is in the form of matter

– Curvature Density Parameter($\Omega_{K,0}$):

The parameter $\Omega_{K,0}$ represents the curvature density of the universe. In cosmology, the curvature of the universe is described by the spatial geometry of space-time. Here $\Omega_{K,0}$ is $-0.1071^{+0.1650}_{-0.1770}$ which indicates an open universe.

- Equation of State Parameter(ω_x):

It describes the pressure-density relationship of dark energy.The pressure-density relationship of dark energy is typically characterized by a negative pressure. And here the ratio is $-0.7829^{+0.119}_{-0.119}$.

6 Conclusion

This project immersed us in the captivating realm of cosmology, exploring alternative models to the standard ΛCDM and XCDM cosmology to comprehend the accelerated expansion of the universe. The primary focus was on constraining the parameters of these models using observational data, specifically the Hubble parameter (H(z)) and Baryon Acoustic Oscillations (BAO) data.

The initial weeks provided a theoretical foundation, covering essential cosmological concepts such as the Friedmann equations, distance measures, and an introduction to Monte Carlo Markov Chain (MCMC) analysis. This aimed to familiarize us with the theoretical framework that underpins subsequent data analysis.

The practical implementation phase involved utilizing the emcee library in Python for MCMC analysis. We applied this technique to constrain parameters in both flat and non-flat ΛCDM models using H(z) data. Additionally, we explored the concept of minimizing the chi-squared value to find optimal parameters, demonstrating the application of the Scipy.optimize.minimize function.

We extended our analysis by incorporating BAO data along with H(z) data to further constrain the parameters for various cosmological models. The inclusion of BAO data allowed us to refine our understanding and obtain more accurate parameter estimates. The MCMC technique was again employed to determine confidence intervals and quantify uncertainties based on 1-σ (68 percentile).

The journey through these phases illuminated the challenges and intricacies of cosmological parameter estimation, emphasizing the importance of combining different observational datasets for robust constraints. We engaged with various references, covering theoretical aspects, data sources, and statistical methods. Looking ahead, there is a potential extension to the project, exploring additional datasets such as higher-Redshift gamma-ray burst, H II starburst galaxy, and quasar data. Additionally, the ϕ-CDM model could be considered for analysis, depending on the progress made and the outcomes achieved.

In summary, this project equipped us with valuable skills in Python programming, data analysis, and the application of statistical techniques in the context of cosmology. It provided a hands-on experience in constraining cosmological parameters, fostering a deeper understanding of the complexities involved in unraveling the mysteries of the universe.

References

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