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IE-501

HW Assignment 1

Discussed with Dattaraj Salunkhe, Roll no: 22B1296.

Q3

→ An LP formulation is as follows:

·] Suppose on day t his investments are x_t then he must invest $\frac{x_t}{2}$ on $t+1$ and gets a return of $2x_t$ on $t+2$

·] Decision Variables:

C_t: Money in hand on day t

x_t: Money invested on day t

·] Objective Function:

$$\max C_6$$

·] Constraints:

$$C_1 = 100$$

$$C_{t+1} = C_t - x_t - x_{t-1} + 2x_{t-2} \quad \text{for } t \in \{1,2,3,4,5\}$$

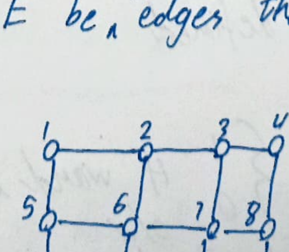
and $\{x_{t-1}, x_{t-2}\} =$ when $t-1 \le 0$ or $t-2 \le 0$

$$C_t \ge 0, \; x_t \ge 0 \; \forall t$$

Q1

→ I will first be defining the constituencies

c) Let the state be a graph $G(V,E)$ where V a set are all the wards as vertex and E be a set of edges that connect adjacent wards only, like for a 3×4:



So let $G_j'(V',E')$ be a subgraph of $G(V,E)$ ie $V' \subseteq V$ &.

And for $G_j'(V',E')$ to be a constituency the constraints are as foll

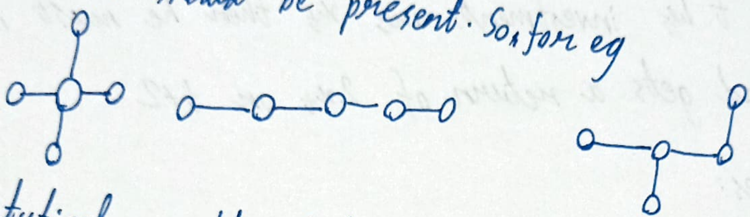
i) Each node needs to be connected. So,

$$\delta_i \geq 1 \quad \text{for } i \in V'$$

Here δ_i is degree of node i ie $\delta_i = \sum_{k \in V'} \alpha_{ik}$

& $\alpha_{ik} = \begin{cases} 1 & \text{if } \exists (i,k) \in E' \\ 0 & \text{ow} \end{cases}$

ii) Adjacents wards should be present. So, for eg



It is intutively visible that the chromatic numbers of these graphs are 2 ie

$$\chi(G_j') = 2$$

ie $X_{iu} = \begin{cases} 1 & \text{if vertex } i \text{ is coloured with } u \in U \\ 0 & \text{ow} \end{cases}$

$y_u = \begin{cases} 1 & \text{if color } u \in U \text{ used to color any vertex } v \\ 0 & \text{ow} \end{cases}$

& min y_u

st $X_{iu} + X_{ku} \leq 1 \quad \forall (i,k) \in E'$, $i,k \in V'$

$X_{iu} = \{0,1\}$

iii) Number of wards in it. So,

let $C_i = \begin{cases} 1 & \text{if } \cancel{\text{ward}}\ i \in V' \\ 0 & \text{ow} \end{cases}$

$\therefore \sum C_i = k$

Thus the constituencies are defined

a) Decesion Variables:

$x_{ij} = \begin{cases} 1 & \text{if ward } i \text{ is in } \cancel{\text{ward}} \text{ constituency } j \\ 0 & \text{o/w} \end{cases}$

$y_j = \begin{cases} 1 & \text{if A wins in const } j \\ 0 & \text{o/w} \end{cases}$

$w_i = \begin{cases} 1 & \text{if A wins in ward } i \\ 0 & \text{o/w} \end{cases}$

tive function:

$$Max \sum_{j}^{n} y_j$$

nstraints: $\sum_{j}^{n} x_{ij} \leq 1$ for each i {ie each ward in only one constiteuency}

$$\left[floor\left(\frac{k}{2}\right)+1\right] y_j \leq \sum_{i \in A} w_i \cdot x_{ij} \text{ for each } j$$

This constraint ensures if A win in y_j the it has to win atleast ~~int~~ $\frac{k}{2}$ $\left[floor\left(\frac{k}{2}\right)+1\right]$ ward to ensure majority

3).) So one of the pattern that can definitly bring the maximum in $n \times k$ are all the row ~~being~~ make to be a constituency each.

b) The only change will happen will be in the objective function which will be

$$Min \sum_{j}^{n} y_j$$

.) I think a stair like pattern of ward for constitency may lead to minimum majority win for A.

Q4

→ Let $f(w)$ be a function such that it gives the length of the word 'w' & let $g_l(w)$ be a function that gives the number of times letter l is there in the ~~work~~ word 'w'.

& & $x_w = \begin{cases} 1 & \text{if } w \in D \\ 0 & o/w \end{cases}$

Objective function:

$$Max \sum_{w \in D} f(w) \cdot x_w$$

$$st. \ \forall g_l(w) \cdot x_w < n_l$$

Q5

a) Decision Variables:

$$x_{ijt} = \begin{cases} 1 & \text{if student } s_i \text{ meets professor } p_j \text{ at time } t \\ 0 & o/w \end{cases}$$

Objective Functions:

$$\text{Max} \sum_{i}^{m} \sum_{j}^{n} \sum_{t}^{a} x_{ijt}$$

Constraints:

$x_{ijt} \leq 1$ if $t \in A_{pi}$ and $t \in A_{si}$ $\forall ijt$

~~{that is not~~

$x_{ijt} \leq 1$ if $j \in M_{si}$ and $i \in M_{pj}$ $\forall ijt$

$\sum x_{ijt} \leq 1$ $\forall i,t$

$\sum x_{ijt} \leq 1$ $\forall jt$

$x_{ijt} \in \{0,1\}$ $\forall ijt$

b) For this we only need to have another thing, the bigshot score of ~~proffesor~~ P_j, which is b_{pj}

∴ Modified Objective Fuction:

$$\text{Max} \sum\sum\sum b_{pj} \cdot x_{ijt}$$

c) A new element:

$$w_{ij} = \begin{cases} b_{pj} & \text{if } j \in M_{si} \text{ and } i \in M_{pj} \\ 1 & \text{ow} \end{cases}$$

So the new objecbive fuction is:

$$\text{Max} \sum\sum\sum w_{ij} \cdot x_{ijt}$$

Q2

$x_1, x_2, x_3 \rightarrow$ Amount betted on N winning a set

$y_1, y_2, y_3 \rightarrow$ Amount betted on J winning a set

$X \rightarrow$ Amount betted on N winning the match

$Y \rightarrow$ Amount betted on J winning the match.

Objective function:

$$\text{Max}\Big[\left(1+\tfrac{1}{\alpha}\right)(x_1+x_2+x_3)+\left(1+\tfrac{1}{\beta}\right)X+(1+\alpha)(y_1+y_2+y_3)+(1+\beta)Y - (x_1+x_2+x_3+y_1+y_2+y_3+X+Y)\Big]$$

Constraints:

$$\cancel{x_1+x_2+x_3+1} \quad \sum_{i=1}^{3} x_i+y_i+X+Y \leq B$$

$$\forall x_i, y_i, X, y \geq 0 \quad i \in [1,3]$$