Spy Problem 4

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1 Problem

The spy after having his lunch date with Miss D'Dnominator and diffusing the bomb, is driving a 2009 Lemnis-Cate sports car, which after the brakes are applied, can decelerate at a rate given by $-4\sqrt{x+12}\ \frac{ft}{s^2}$, where v is the velocity of the car at any time t. He is driving at a speed of 60 $\frac{mi}{h}$(88 $\frac{ft}{s}$) on the Espionage Highway, in the remote republic of San DiBeeches. Suddenly, he sees a Llama in the road!! The automatic laser range determinator(or is it determinant) immediately alerts him that the llama is 200 ft in front of him. After a reaction time of 0.7 s, he steps on the brake and, the critical question is posed:

Will Our Hero Stop Before Hitting the Llama?
(Or Will llama burgers be the special of the day at the local eatery?)

If:

a) **The llama is hit, what is the speed of the car at the moment of the impact?**

b) **The llama is not hit, by what distance does the car stop ahead of the llama?**

2 Solution

2.1 Understanding the Problem

We see that the acceleration is a function in terms of the velocity at a certain given point of time. Since we know that acceleration is simply the derivative of the velocity function in terms of t, $a = -4\sqrt{v+12}$ is simply a first order, non-linear differential equation.

$$\frac{dv}{dt} = -4\sqrt{v+12}$$

2.2 Solving For Velocity Function

We note that the differential equation is autonomous since it is in the form $y' = f(t,y)$ and $f(t,y) = -4\sqrt{v+12}$ lacks the variable t.
To solve, we simply integrate both sides based on t:

$$\begin{gathered}\frac{dv}{dt} = -4\sqrt{v+12} \to \frac{\frac{dv}{dt}}{\sqrt{v+12}} = -4 \\ \to \int \frac{\frac{dv}{dt}}{\sqrt{v+12}} dt = -4dt \\ \to \int \frac{dv}{\sqrt{v+12}} = -4t + C \\ \to 2\sqrt{v+12} = -4t + C \\ \to v(t) = 4(t+C)^2 - 12\end{gathered}$$

We see that our derived function for velocity has a unknown constant. This expected since this is an autonomous equation. We are given the initial condition that at $t = 0$, $v = v_0 = 88\ \frac{ft}{s}$. We substitute the point $(0, 88)$ into our derived equation for velocity to determine the constant.

$$\begin{gathered}88 = 4(0+C)^2 - 12 \to 4(C)^2 = 100 \\ \to C = \pm 5\end{gathered}$$

To determine whether the constant is positive or negative, we think back to our given conditions. Can bound $0 \leq t \leq 5$ and $v \geq -12$. Any t less than 0 is impossible since time cannot be negative and any time larger than 5 seconds is trivial, since the car reaches a velocity of $-12\frac{ft}{s}$ at $t = 5$ so the velocity afterwards would always be constant since $a = 0$ for $v = -12$. Any v less than -12 is impossible for the car since the acceleration is 0 at $v = -12$. Only when $C = -5$ will $\frac{dv}{dt} = -4\sqrt{v + 12}$.
NOTE: Proceding, we will be solving the functions on the interval of $t : [0, 5]$.

$$v(t) = 4(5 - t)^2 - 12$$

2.3 Solving For the Position Function

Now that we have the velocity function. We can easily solve for the position function by taking the indefinite integral of the velocity.

$$\begin{gathered} d(t) = \int v(t)dt = \int 4(5 - t)^2 - 12\ dt \\ = \int 4t^2 - 40t + 100 - 12\ dt = \frac{4}{3}t^3 - 20t^2 + 88t \\ d(t) = \frac{4}{3}t^3 - 20t^2 + 88t \end{gathered}$$

We let the contant C that we produce through integration to be 0 since we assigned car's starting position to be the origin.

2.4 Will the Llama Die?

To determine if the Llama will die, we simply have to see if at the time the car comes to a complete stop(accounting for the reaction time), is the position greater than the position of the Llama(200 ft).
The time it takes for the car to come to a complete stop is:

$$\begin{gathered} 4(t - 5)^2 - 12 = 0 \\ t = 5 - \sqrt{3} \end{gathered}$$

Thus, the distance the car travels before it fully stops is $\frac{320}{3} + 8\sqrt(3)\ ft$. However, we also have to account for the distance traveled due to reaction time so the total stopping distance is $\frac{320}{3} + 8\sqrt(3) + 88 * 0.7 = 182.123\ ft$. Since $182.123 < 200$, the **Llama will not die**. Yay! But Captain Calculus won't be able to enjoy some llama burgers. D: In fact, there will be a distance of $200 - 182.123 = 17.8769... \approx 17.877\ ft$ between the car and the llama.