Spy Problem 2

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1 Problem

Our friend the spy(who escaped from the diamon-smugglers in Problem 1) is on a secret mission in space. An encounter with an enemy agent leaves him with a mild concussion that causes him to forget where he is. Fortunately, he somehow remembers(from his highschool calculus class, no doubt), the formula for the height of a projectile:

$$h(t) = -\frac{1}{2}gt^2 + v_0t + h_0$$

where v_0 and h_0 are the initial velocity and height respectively and the values for g for various heavenly bodies. Therefore, to deduce his whereabouts, he throws a rock/ directly upwards(from ground level) and makes the two observations:

i) in 3 seconds, the rock, still rising, reaches a height of 108 feet

ii) the rock eventually reaches a maximum height of 121.5 feet

to enable him to determine the value of g for this location. **WHERE IS OUR HERO?**
(Note: You will need to know the different values of g at various places: 32 $\frac{ft}{sec^2}$ on Earth, 5.5 $\frac{ft}{sec^2}$ on the moon(of Earth), 12 $\frac{ft}{sec^2}$ on Mars, 28 $\frac{ft}{sec^2}$ on Venus, etc)

Writer's Note: The original height equation is $h(t) = \frac{1}{2}gt^2 + v_0t + h_0$ but I added in a $-$ in order to allow the numeric value of g to be positive since relatively, g and v_0 point in different directions.

2 Solution

We first approach the problem by determining the variables that we do not know. At first glance, we see that there are 4 different variables in the height equation: $v_0,\ g,\ h_0\ and\ t$. We immediately notice that we already know $h_0 = 0$ since Captain Calculus threw the object from the ground level. Now we have 3 variables left.

With only two conditions to help us, let us substitute in the values of the two conditions and see what we can do with them.
Condititon 1:
$h(3) = -(3)^2 * \frac{1}{2}g + 3 + v_0$
$\to 108 = -\frac{9}{2}g + 3v_0$
Condition 2:
$h(t_h) = -\frac{1}{2}g * (t_h)^2 + v * t_h$ where t_h is the amount of time it takes for the object to reach the highest point. Although we seemingly introduced a new unknown variable, we actually know what value t_h is in terms of g and v_0. We know that the velocity of the object is $0\frac{ft}{sec}$ at the very top of its path so $t_h = \frac{v_0}{g}$. Thus:
$\to 121.5 = -\frac{1}{2} * g * (\frac{v_0}{g})^2 + v_0 * \frac{v_0}{g}$
$\to 121.5 = \frac{v_0^2}{2g}$
We shall be expressing v_0 in terms of g so we can more easily use it later:
$v_0 = \sqrt{2 * 121.5 * g} = \sqrt{243g}$

Wow would you look at that! We have two equations with two unknowns! Now we can totally solve it:
$108 = -\frac{9}{2}g + 3v_0$
$v_0 = \sqrt{2 * 121.5 * g} = \sqrt{243g}$
$\to 108 = -\frac{9}{2}g + 3\sqrt{243g}$
$\to 108 + \frac{9}{2}g = 3\sqrt{243g}$
$\to 36 + \frac{3}{2}g = \sqrt{243g}$
$\to (36 + \frac{3}{2}g)^2 = (\sqrt{243g})^2$
$\to 36^2 + \frac{9}{4}g^2 + 36 * 3g = 243g$
$\to \frac{9}{4}g^2 - 135g + 1296 = 0$

$\rightarrow g = 12$ or 48

Although both 12 $\frac{ft}{sec^2}$ and 48 $\frac{ft}{sec^2}$ can be answers, only 12 $\frac{ft}{sec^2}$ is an answer since that is the only one that matches with the charts(we checked). Thus, Captain Calculus is on Mars with a gravity of 12 $\frac{ft}{sec^2}$.