

On the properties of projectile motion and quadratic air resistance

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The equations of motion for normal projectile motion are derived and some properties of such motion are determined. One dimensional quadratic air resistance is studied and modeled using numerical and analytical methods and compared to experimental results, producing negligible discrepancies. Analytical approximations to two dimensional quadratic air resistance are made using a model which separates low, high and split angle trajectories. The equations of motion for these are derived, plotted and compared to numerical solutions to the original differential equation.

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