Värmdö Gymnasium

On the properties of projectile motion and quadratic air resistance

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The equations of motion for normal projectile motion are derived. Some properties of such motion are defined. Numerical approximations and analytical solutions to the effects of one dimensional quadratic air resistance are made and compared to experimental results with small error. Analytical approximations to two dimensional quadratic air resistance are made using a model which separates low, high and split angle trajectories. The equations of motion for these are derived, plotted and compared to numerical solutions to the original differential equation.

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