

On the properties of projectile motion and quadratic air resistance

Simon Halvdansson

February 22, 2016

The functions for projectile motion in a three dimensional space are derived. Some properties of such motion are defined. Numerical approximations and analytical solutions to the effects of one dimensional quadratic air resistance are made and compared to experimental results with small error. Analytical approximations to two dimensional quadratic air resistance are made using a model which separates low, high and split angle trajectories. The equations of motion for these are derived and plotted and compared to numerical integration of the original differential equation.

1	Introduction	4
2	Trajectory	4
2.1	Acceleration	5
2.2	Velocity	5
2.3	Position	5
2.4	Parabola	5
3	Properties of a projectile parabola	5
3.1	Time to highest point	5
3.2	Highest point	6
3.3	Range of projectile	6
3.4	Optimal angle for maximum range	6
3.5	Velocity at a given time	6
3.6	Angle of impact	6
4	Choosing initial values to match result values	6
4.1	Delimitations	6
4.2	Specific projectile range	7
4.3	Hitting a specific coordinate	7
4.3.1	Predetermined speed	7
4.3.2	Predetermined angle	7
5	Longest parabola starting at height	8
5.1	Expression for range	8
5.2	Algorithm	8
6	Computational approach to air resistance	9
6.1	Change at every time interval	9
6.2	Algorithm	10
7	Analytical solution to one-dimensional air resistance	10
7.1	Derivation	10
7.1.1	Position as a function of time	10
7.1.2	Determining the duration of the fall	11
7.1.3	Terminal velocity	11
7.2	Plots	11
7.2.1	Examples	11
7.2.2	Analysis	12
8	Analytical approximations to two-dimensional air resistance	12
8.1	Introduction	12
8.2	Low Angle Trajectory approximation	13
8.2.1	Differential equations of motion	13
8.2.2	Velocity	13

8.2.3	Position	13
8.2.4	Equations of motion	14
8.3	High Angle Trajectory approximation	14
8.3.1	Differential equations of motion	14
8.3.2	Ascending velocity	14
8.3.3	Ascending position	14
8.3.4	Descending velocity	15
8.3.5	Descending position	15
8.3.6	Equations of motion	16
8.4	Split Angle Trajectory approximation	16
8.4.1	Differential equations of motion	16
8.4.2	Equations of motion	16
8.5	Plots	16
8.5.1	Low Angle Trajectory	16
8.5.2	High Angle Trajectory	17
8.5.3	Split Angle Trajectory	17
9	Experiment	17
9.1	Setup	17
9.2	Results	18
9.3	Analysis	18
10	Conclusions	18
	Further reading	18