Fly-By Project

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ABSTRACT

Our goal for this project was to construct a stable binary system, then introduce a fly-by mass and then plot the effects on the orbit. Through the use of Python, we were able to successfully create an accurate simulation of this situation. We utilized several of the techniques previously discussed in the course in order to establish parameters, set vectors, and receive an output of a plot illustrating the path of the binary system. We found that the fly-by mass exerted a notable force on the binary system.

1. INTRODUCTION

For this project, we were tasked with constructing a variation on a three-body problem. A three-body problem is a situation where there are three masses that all exert forces on each other and thus causing changes in position. This makes three-body systems more complex than a two-body system, as it consists of three bodies constantly interacting with each other. For our specific variation of this problem, we had to create a system where a star flew nearby to a binary system, exerting a force on the orbiting mass, which would pull it off course. This would affect the position of the entire system as the star serving as the central point of the binary system would also have a force exerted upon it.

Grainy

$$\frac{d^2 \vec{r}_i}{dt^2} = \sum_{j \neq i} \frac{G m_j}{|\vec{r}_j - \vec{r}_i|^3} (\vec{r}_j - \vec{r}_i) \quad (1)$$

Equation one proved to be fundamental to our development of a successful, stable three-body system. This equation was essential for finding the force that each of the planets experiences as a result of its interaction in the three-body system.

2. METHODOLOGY

The methodology for this project was pretty straight forward. Since we are dealing with three stellar bodies, it was imperative to have the change in acceleration due to the gravitational forces applied by the other two bodies. Before taking the forces into account, the initial conditions had to be determined and accurately defined:

- The masses of all the stars
- The distance between the orbiting stars
- The initial positioning of all 3 stars
- The period of one orbit
- The speeds of each star

Note, a good portion of the initial conditions use other initial conditions to define themselves. For example, the velocities use the period and the initial positioning of the stars. Much of these equations were pulled straight from the Principles of Astrophysics textbook by Keeton ((2014)).

Initially, after deciding the masses for the stars and the separation vector (2), we used the reduced mass formula (3) to rewrite the positions as r_1 and r_2 (4). (Eqs (4.1), (4.4), (4.6) in Keeton (2014)).

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 \quad (2)$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad (3)$$

$$\mathbf{r}_1 = \mathbf{R} - \frac{\mu}{m_1}\mathbf{r} \quad \text{and} \quad \mathbf{r}_2 = \mathbf{R} + \frac{\mu}{m_2}\mathbf{r} \quad (4)$$

As the programming phase of the project went underway, the variable changed form due to how python handle indexes and the group's programming preferences:

$m_1 = M_0$ $\quad r_1 = r_0$
$m_2 = M_1$ $\quad r_2 = r_1$
$m_3 = M_2$ $\quad r_3 = r_2$

The next unknown initial variables to obtain are the velocities for each planet. Since the velocity of the fly-by is an independent variable subject to cause change in the binary orbit - along with the mass and the distance of the fly-by star, M_2 and r_2 - the value of v_2 is up to our discretion. The values of the orbiting stars, however, need another key factor: the period of the orbit. Keeton's formula for period helps point us in the right direction (5) (Eqs (3.14) Keeton (2014)).

$$P^2 = \frac{4\pi^2}{GM} a^3 \quad (5)$$

What are a and M here in terms of your other variables?

Furthermore, we're only interested in the effect the fly-by has on the orbit of the binary. For the simplest case, circular orbits about the center of mass (CM) of the two orbiting stars eases the much needed velocities:

$$v_0 = \frac{2\pi r_0}{P} \quad (6)$$

$$v_1 = \frac{2\pi r_1}{P} \quad (7)$$

Once the velocities are set and the interactions between the three stars are in motion thanks to the change in velocities driven by equation one, the simulation is set and is now able to produce data in the form of graphs.

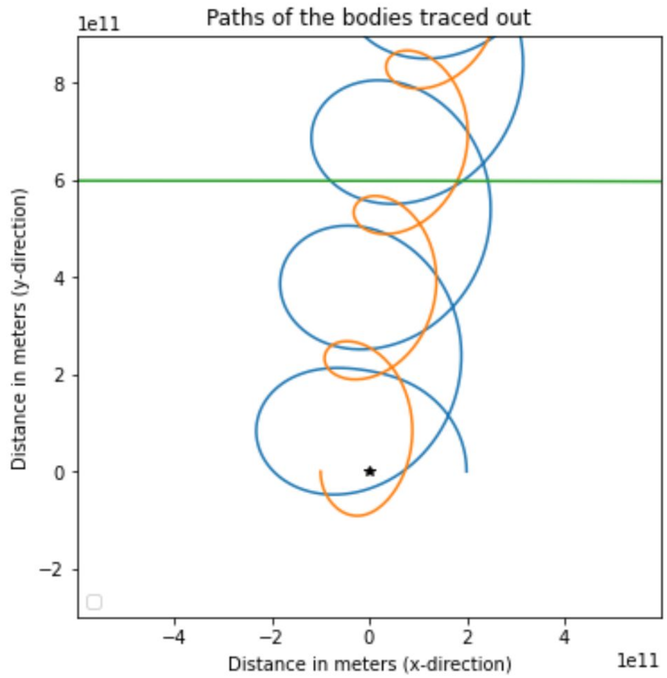
3. RESULTS (GROUP & SUB-ANALYSES) AND TAKEAWAYS

To quickly summarize the initial conditions we assigned to create our 'control' group simulation:

- $M_0 = 2M_\odot$
- $M_1 = 4M_\odot$
- $M_2 = 3M_\odot$
- $r_2 = 10AU$ (horizontal distance between fly-by and CM of binary)
- $r_{2y} = 4AU$ (vertical distance between fly-by and CM of binary)
- $v_2 = 5 \times v_0$

The result from the defined initial conditions is the following graph (on following page)

Did you test your code? As mentioned previously, there should be some validation



Note: The stars in the binary maintain their orbit despite the stationary positioning of their CM no longer remaining stationary.

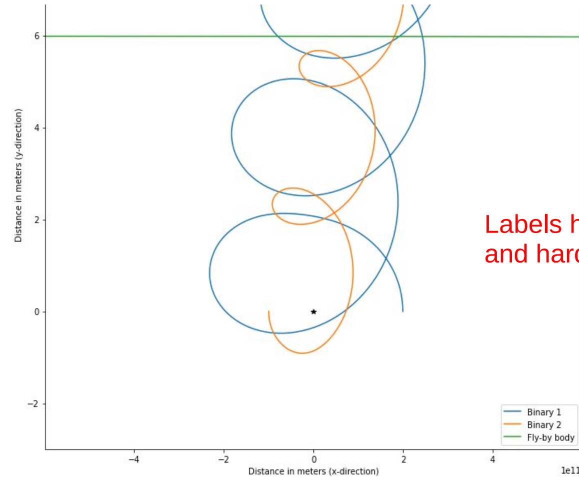
For our sub-analyses, we tampered with the mass and the velocity of the fly-by.

➔ Change in mass:

Increasing the mass of the fly-by from 3 $M_\odot$ to 120 $M_\odot$ drastically increases the force applied by the fly-by on the binary orbit. Keeping everything else constant, the fly-by did not simply change the momentum of the CM. It gravitationally sling-shot it at really velocities hurtling through space.

➔ Change in velocity:

Increasing the velocity of the fly-by resulted in the more fascinating results. As the velocity of the fly-by increases, the force it applies on the binary does not change. But what is force? Force can be defined as change in momentum (impulse) over time. To ensure that the force is maximized, one could say that the time it takes for changing the momentum should be ample. On the contrary, having a fly-by with a high velocity counteracts this by taking away the amount of time the change in momentum has. Furthermore, the change in momentum is dependent on the mass of the fly-by and the distance it flies by from. The quicker the incoming mass switches its relative direction of travel to outgoing mass, the less impulse the binary orbit is going to experience. Let's look at the graphs of the binary as the velocity is increased 10 fold, 100 fold, and 1000 fold.



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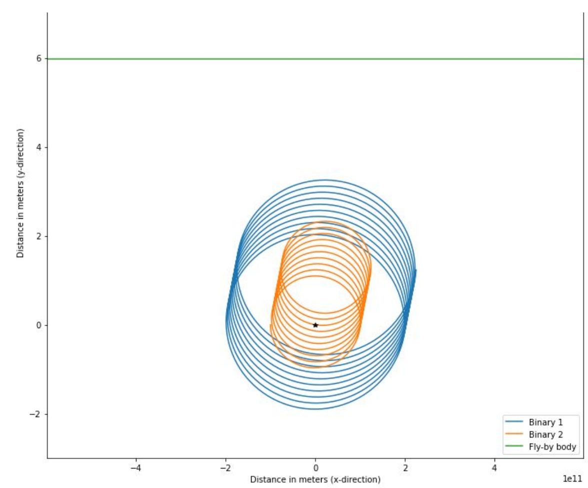

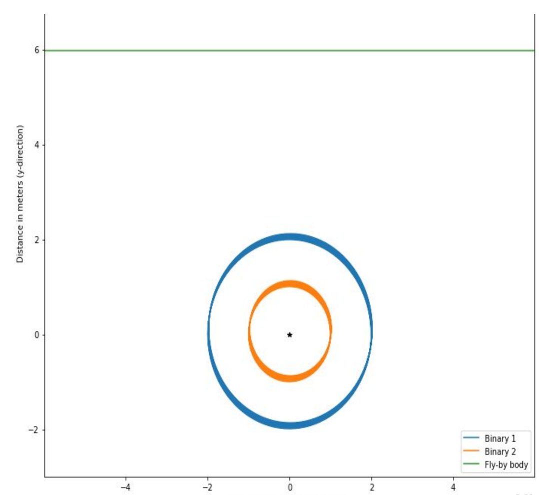

Note: Increasing the reducing the change in momentum of the CM of the binary orbit. This lack of impulse causes the binary to be virtually unaffected when the velocity is 1000 fold.

CONCLUSIONS

Overall, we found that we were able to create a fairly accurate representation or a three-body system featuring a fly-by. The general predictions that we initially made were found to be accurate, however I don't believe that any of us could have predicted the extent to which the position of the binary system would change. We found that the essential parameters (velocity, initial position and mass) could greatly influence the way that the fictional binary system was affected and therefore create vastly different graphs. By manipulating these parameters we could show the importance of each of them throughout the entirety of the interaction.

that's why we do the simulations!

REFERENCES

[2014] Keeton, C. (2014). Principles of astrophysics: Using gravity and stellar physics to explore the cosmos. New York: Springer.

Can you think if any physical scenarios to which this analysis might apply?