

Cumulative Review (Problems)

1. Evaluate

$$\int x^2 \sin 2x \, dx$$

2. Find the volume of the solid formed by revolving the region bounded by $y = e^{-2x}$ and the x-axis from $[0, \infty)$ about the x-axis.

3. Evaluate

$$\int_1^2 \frac{1}{(x-1)^2} dx$$

4. Evaluate

$$\int \frac{1}{\sqrt{1-4x-x^2}} dx$$

5. Evaluate

$$\int_0^{\ln 2} \cosh x \, dx$$

6. Evaluate

$$\int_0^1 \frac{1}{(x^2+1)^{3/2}} dx$$

7.

$$\int_0^1 \frac{1}{\sqrt{16-x^2}} dx$$

8. Evaluate

$$\int \frac{1}{x^2-9} dx$$

9. Evaluate

$$\int \sin^4 \theta \, d\theta$$

10. Evaluate the expression without a calculator to a value or to show that the value does not exist. Simplify the answer to the extent possible

$$\sinh(2 \ln 5)$$

11.

$$\int_1^\infty \frac{4}{(1+x^2) \arctan x} dx$$

12. Evaluate

$$\int e^{2x} x^2 \, dx$$

13. Evaluate

$$\int \tan^3 4x \, dx$$

14. Evaluate

$$\int \sin 2x \cos 3x \, dx$$

by using one of the following identities:

$$\sin(mx) \sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$

$$\sin(mx) \cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$

$$\cos(mx) \cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

15. Find the general solution of the equation

$$y'(t) - \frac{y}{16} = -11$$

16. Evaluate $\int_1^\infty \frac{1}{x^p} dx$ converges if _____, otherwise it diverges.

17. Evaluate

$$\int \frac{x+4}{x^2+5x+6} dx$$

18. Derive the reduction formula

$$\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$$

19. :: Section 8.2 Evaluate

$$\int x e^{3x} dx$$

20. Evaluate the integral

$$\int \frac{2x}{x^2 + 6x + 13} dx$$

21.

$$\int \frac{1}{x\sqrt{9x^2 - 6}} dx$$

22. Evaluate

$$\int x^4 \sin 2x \, dx$$

23. Evaluate

$$\int_{-\infty}^e 23e^{-x} dx$$

24. Evaluate

$$\int x \sin x^2 \, dx$$

25. Evaluate

$$\int \sin^3 x \cos^4 x \, dx$$

26. Evaluate

$$\int \sqrt{25 - 4x^2} dx$$

27. Evaluate

$$\int \sqrt{9 - 4x^2} dx$$

28. Evaluate

$$\int \frac{x^3 + x - 3}{x^2 - 4} dx$$

29. Verify the identity using the definitions of hyperbolic functions

$$\coth x = \frac{e^{2x} + 1}{e^{2x} - 1}$$

30. Evaluate

$$\int \frac{1}{\cos \theta - 1} d\theta$$

31. Evaluate

$$\int_1^4 \frac{1}{(x-2)^{2/3}} dx$$

32. Evaluate

$$\int \sqrt{1 - x^2} dx$$

33. Evaluate

$$\int x^2 e^{5x} dx$$

34. Evaluate

$$\int_2^4 8x \ln x \, dx$$

35. Evaluate

$$\int \sin(6x) \sin(4x) dx$$

by using one of the following identities:

$$\sin(mx) \sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$

$$\sin(mx) \cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$

$$\cos(mx) \cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

36. Evaluate

$$\int x \sqrt{5 - 4x^4} dx$$

using the reduction formula

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left(x \sqrt{a^2 - x^2} + a^2 \arcsin \frac{x}{a} \right) + C$$

37.

$$\int \frac{\sinh x}{1 + \cosh x} dx$$

38. Evaluate

$$\int \frac{-\csc \theta}{\csc \theta - \cot \theta} d\theta$$

39. Evaluate

$$\int e^{2x} \cos x \, dx$$

40. Evaluate

$$\int \arcsin x \, dx$$

41. Find the length of the curve

$$y = 3x^{\frac{3}{2}}; \text{ from } x = 0 \text{ to } x = \frac{5}{9}$$

42. Evaluate

$$\int \sin^3 x \cos x \ln(\sin x) dx$$

using the reduction formula

$$\int x^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} (-1 + (n+1) \ln u) + C, n \neq -1$$

43. Evaluate

$$\int \cos^2 \theta \sin 2\theta \, d\theta$$

44. Evaluate

$$\int_0^{\pi/6} \ln(2 \sec x) dx$$

45. Evaluate

$$\tan \left(\arccos \left(\frac{1}{2} \right) \right)$$

46. Evaluate

$$\int_0^\infty \frac{4(1 + \arctan x)}{1 + x^2} dx$$

47. Evaluate

$$\int \frac{x+1}{\sqrt{3x^2+6x}} dx$$

48. Find the derivative of y with respect to x :

$$y = 3 \arcsin(4x^3)$$

49.

$$\int \frac{x^2+3}{x\sqrt{x^2-4}} dx$$

50. Find the general solution of the equation. Express the solution explicitly as a function of the independent variable

$$e^{9t} y'(t) = -2$$

51. Evaluate

$$\int \frac{\cos t}{\sin^2 t - 9 \sin t + 18} dt$$

52. Evaluate

$$\int \tan^2 2x \, dx$$

53. Evaluate

$$\int_1^\infty \frac{1}{e^x} dx$$

54. A conservation organization releases 40 coyotes into a preserve. After 4 years, there are 70 coyotes in the preserve. The preserve has a carrying capacity of 175.

1. Write a logistic function that models the population, $P(t)$, of coyotes in the preserve.

2. Use your answer from (a) to find $\lim_{t \rightarrow \infty} P(t)$

55. Evaluate

$$\int \frac{\sin x}{\cos x + \cos^2 x} dx$$

56. Evaluate

$$\int \sin(10x) \cos(3x) dx$$

by using one of the following identities:

$$\sin(mx) \sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$

$$\sin(mx) \cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$

$$\cos(mx) \cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

57. Evaluate

$$\int_0^\infty \frac{1}{1+x^2} dx$$

58. Differentiate

$$y = \frac{1}{2} \left[x\sqrt{4-x^4} + 4 \arcsin \frac{x}{2} \right]$$

59. Evaluate

$$\int \frac{1}{4+9x^2} dx$$

60. Evaluate

$$\int x^2 \ln 3x \, dx$$

61. Evaluate

$$\int \cos^2 \left(\frac{x}{c} \right) dx$$

62. Evaluate the following without use of a calculator

$$\coth(\ln 6)$$

63. Evaluate

$$\int \sin^3(2x) dx$$

64. Evaluate

$$\int_2^5 \frac{1}{\sqrt{x-2}} dx$$

65. Evaluate

$$\int \frac{9}{\sqrt{64-81x^2}} dx$$

66. Evaluate

$$\int \frac{4x+7}{(x+1)^2} dx$$

67. Compute $\frac{dy}{dx}$ for the function

$$y = \ln \sinh 7x$$

68. Evaluate

$$\int_0^1 \frac{x}{(x^2+1)^{3/2}} dx$$

69. Evaluate

$$\int_{13/2}^{13} \sqrt{169-x^2} dx$$

70. Write the nth-term formula for the following sequences

1. $3, 7, 11, 15, \dots$

2. $2, -1, \frac{1}{2}, -\frac{1}{4}, \dots$

3. $1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \dots$

71. How would you approach the following?

$$\int \frac{x}{x^2+1} dx$$

72. Evaluate

$$\int x^3 \cos 2x \, dx$$

73. Find the area of the surface generated when the given curve is revolved about the x-axis

$$y = \frac{x^3}{3} + \frac{1}{4x}; \text{ from } x = 1 \text{ to } x = 2$$

74. Evaluate

$$\int \frac{e^x}{(e^x + 4)^{-4}} dx$$

75. Determine if the given function y is a solution of the differential equation y'' . Assume that C is an arbitrary constant.

$$y = C_1 \sin 5t + C_2 \cos 5t; \quad y''(t) + 25y = 0$$

76. Evaluate the limit

$$\lim_{x \rightarrow \infty} x \left(\frac{\pi}{2} - \arctan x \right)$$

77. Find the exact value of the function

$$\operatorname{arccsc}(-2)$$

78. Evaluate

$$\int_1^{\infty} \frac{1}{\sqrt{x+2}} dx$$

79. Evaluate

$$\int_1^{e^2} \frac{\ln^2(x^3)}{x} dx$$

80. Evaluate

$$\int -\operatorname{csch}^2 x \coth x \, dx$$

81. Evaluate

$$\int \frac{\ln x}{x^2} dx$$

82. Use l'Hopital's rule to evaluate the limit

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos(x) - \frac{1}{2}}{x - \frac{\pi}{3}}$$

83. Evaluate

$$\int x \sin x \, dx$$

84. Evaluate

$$\int \frac{3x - 1}{x^2 - 5x + 4} dx$$

85. Evaluate

$$\int_1^3 \ln 2x \, dx$$

86. Solve the differential equation

$$xy \frac{dy}{dx} = 1 - \ln x; \quad \text{quady}(1) = 2$$

87. Evaluate or simplify

$$\cos(2 \arccos x)$$

88. Evaluate

$$\int \frac{x + 5}{x^2 + 3x} dx$$

- 89.

$$\lim_{x \rightarrow -\infty} 4 \sinh x$$

90. Evaluate

$$\int \frac{2x - 1}{4x^2 - 9} dx$$

91. Use trig substitution to evaluate

$$\int \frac{1}{\sqrt{4x^2 + 1}} dx$$

92. Evaluate

$$\arccos\left(-\frac{\sqrt{3}}{2}\right)$$

93. Evaluate

$$\int \frac{1}{(25x^2 + 1)^2} dx$$

94. Evaluate

$$\int \frac{1}{(1 + 25x^2)^{3/2}} dx$$

95. Evaluate Find the area of the region bounded by the curves:

$$y = \sin^2(\pi \cdot x); y = 0; x = 0; x = 1$$

96. Evaluate

$$\int \sin^3 x \, dx$$

97. Evaluate

$$\int \frac{1}{\sqrt{x^2 - 10x + 21}} dx$$

98. Evaluate

$$\int \frac{2x + 3}{x^3 - 2x^2 + 3x - 6} dx$$

99. Compute $\frac{dy}{dx}$ for the function

$$y = \sinh 7x$$

100. Evaluate

$$\int \frac{e^x}{(e^{2x} + 1)(e^x - 1)} dx$$

101. Evaluate Find the volume of the solid formed when the region bounded by the curves

$$y = \cos \frac{x}{2}; y = \sin \frac{x}{2}; x = 0; x = \frac{\pi}{2}$$

is revolved about the x-axis.

102. Find the function $y = f(t)$ passing through the point $(0, 15)$ with the first derivate

$$\frac{dy}{dt} = \frac{1}{4}t$$

103. Evaluate

$$\int \frac{1}{x^2 \sqrt{4 - x^2}} dx$$

104. Evaluate $\int_0^1 \frac{1}{x^p} dx$ converges if _____, otherwise it diverges.

105. Evaluate

$$\int \frac{x^3 e^{x^2}}{(x^2 + 1)^2} dx$$

106. Evaluate

$$\int_0^{\pi/4} \sin^3 4x \, dx$$

107. Evaluate

$$\int \frac{3}{2x^2 - 7x - 4} dx$$

108. Evaluate or simplify

$$\arccos\left(\cos\left(-\frac{\pi}{3}\right)\right)$$

109.

$$\int \arccos x \, dx$$

110. Evaluate

$$\int \frac{1}{\sqrt{9x^2 + 4}} dx$$

111. Evaluate

$$\int x^3 e^{2x} dx$$

112. Use integration by parts to establish a reduction formula for the integral

$$\int x^n e^x dx$$

113. Prove the reduction formula:

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$$

114. Find the exact value of the function

$$\arcsin\left(-\frac{\sqrt{2}}{2}\right)$$

115. Evaluate the integral

$$\int \cot^4 4x \, dx$$

using the reduction formula

$$\int \cot^m(u) du = -\frac{\cot^{m-1}(u)}{m-1} - \int \cot^{m-2}(u) du + C$$

116. Evaluate $\int \frac{1}{x^2 + 2x - 3} dx$ via:

1. Trigonometric substitution
2. Partial fraction decomposition

117. Evaluate

$$\int \frac{4x+1}{x^2+9} dx$$

118. How would you approach the following?

$$\int \frac{1}{\sqrt{x^2+1}} dx$$

119. Evaluate

$$\int \frac{4x^2}{x^2+9} dx$$

120. Evaluate Solve the differential equation

$$\frac{dy}{dx} = \tan^3 x \sec x; \quad y(\pi/3) = 0$$

121. Evaluate

$$\int \frac{\sqrt{x^2-9}}{x} dx$$

$$\frac{\sqrt{x^2-9}}{3} - \arctan \frac{x}{3} + C$$

122. Evaluate

$$\int \frac{1}{1+e^x} dx$$

123. Find the equation of the line tangent to the curve

$$2x + \arctan y = y^2 - 1; \text{ at the point } P\left(\frac{-\pi}{8}, -1\right)$$

124. Evaluate

$$\int \frac{8x^3 + 13x}{(x^2 + 2)^2} dx$$

125. Evaluate

$$\int 2 \cos^4 5x \, dx$$

126. Compute $\frac{dy}{dx}$ for the function

$$y = \sinh^2 7x$$

127. How would you approach the following?

$$\int \frac{1}{x^2+1} dx$$

128. Evaluate

$$\int_0^{\pi/4} x \sin 2x \, dx$$

129. Evaluate

$$\int \tan^4 9t \, dt$$

130. Compute $\frac{dy}{dx}$ for the function:

$$y = \sinh^2 4x$$

Cumulative Review (Answers)

5. (Quiz 2)

$$\frac{3}{4}$$

11. (Quiz 4)

$$4 \ln 2 \rightarrow \text{Converges}$$

12. (Quiz 3)

$$\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

15. (Quiz 2)

$$y = Ce^{x/16} + 176$$

20. (Quiz 1)

$$\ln \left| x^2 + 6x + 13 \right| - 3 \arctan \frac{x+3}{2} + C$$

23. (Exam 2)

$$= \infty; \quad \therefore \text{diverges}$$

34. (Quiz 3)

$$(64 \ln 4 - 32) - (16 \ln 2 - 8) \approx 53.6$$

38. (Quiz 3)

$$\cot \theta + \csc \theta + C$$

43. (Quiz 3)

$$-\frac{1}{2} \cos^4 \theta + C$$

45. (Quiz 1)

$$\sqrt{3}$$

46. (Exam 2)

$$= 2\pi + \frac{\pi^2}{2}; \quad \therefore \text{converges}$$

51. (Exam 2)

$$\frac{1}{3} \ln |\sin t - 6| - \frac{1}{3} |\sin t - 3| + C$$

58. (Quiz 1)

$$\sqrt{4 - x^2}$$

62. (Quiz 2)

$$\frac{37}{35}$$

63. (Exam 2 Studyguide)

$$-\frac{1}{2} \left(\cos(2x) - \frac{1}{3} \cos^3(2x) \right) + C$$

64. (Exam 2 Studyguide)

$$2\sqrt{3} \rightarrow \text{converges}$$

65. (Quiz 2)

$$\arcsin \frac{9x}{8} + C$$

70. (Section 9.1—9.3)

1. $a_n = 4n - 1$
2. $a_n = (-1)^{n+1} 2^{2-n}$
3. $a_n = \frac{x^{n-1}}{(n-1)!}$

71. (Exam 2 Studyguide)

Use u-substitution

74. (Exam 2)

$$-\frac{1}{3(e^x + 4)^3} dx$$

75. (Quiz 2)

Yes (verified)

78. (Exam 2 Studyguide)

$\infty \rightarrow$ diverges

79. (Exam 2)

24

80. (Quiz 2)

$$\frac{\coth^2 x}{2} + C$$

85. (Exam 2)

≈ 2.68

88. (Exam 2)

$$\frac{5}{3} \ln |x| - \frac{2}{3} \ln |x+3| + C$$

or

$$\frac{1}{3} \ln \left| \frac{x^5}{(x+3)^2} \right| + C$$

91. (Quiz 4)

$$\frac{1}{2} \ln \left| \sqrt{4x^2 + 1} + 2x \right| + C$$

92. (Quiz 1)

$$\frac{5\pi}{6}$$

93. (Exam 2)

$$\frac{1}{10} \arctan 5x + \frac{x}{50x^2 + 2} + C$$

97. (Exam 2)

$$\ln \left| \frac{x-5 + \sqrt{(x-5)^2 - 4}}{2} \right| + C$$

99. (Quiz 2)

$$7 \cosh 7x$$

100. (Exam 2 Studyguide)

$$\frac{1}{2} \ln |e^x - 1| - \frac{1}{4} \ln |e^{2x} + 1| - \frac{1}{2} \arctan(e^x) + C$$

106. (Quiz 3)

$$\frac{1}{3}$$

109. (Exam 2)

$$x \arccos x - \sqrt{1-x^2} + C$$

111. (Exam 2 Studyguide)

$$\frac{1}{2} x^3 e^{2x} - \frac{3}{4} x^2 e^{2x} + \frac{3}{4} x e^{2x} - \frac{3}{8} e^{2x} + C$$

112. (Quiz 3)

$$x^n e^x - n \int x^{n-1} e^x dx$$

115. (Quiz 4)

$$-\frac{1}{12} \cot^3(4x) + \frac{1}{4} \cot(4x) + x + C$$

118. (Exam 2 Studyguide)

Use trig-substitution

124. (Quiz 4)

$$4 \ln(x^2 + 2) + \frac{3}{2(x^2 + 2)} + C$$

125. (*Exam 2*)

$$\frac{3}{4}x + \frac{1}{10} \sin 10x \frac{1}{80} \sin 10x + C$$

127. (*Exam 2 Studyguide*)

Use the inverse tangent integration formula

129. (*Exam 2*)

$$\frac{\tan^3 9t}{27} - \frac{\tan 9t}{9} + t + C$$

130. (*Quiz 2*)

$$8 \sinh(4x) \cosh(4x)$$