Exam 3 Review (Problems)

1. Write the nth-term formula for the following sequences

   1. $\{3, 7, 11, 15, \dots\}$
   2. $\{2, -1, \frac{1}{2}, -\frac{1}{4}, \dots\}$
   3. $\{1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \dots\}$

2. Find the limit of the sequence and state whether the sequence converges or diverges

   $$a_n = \frac{n}{n+1}$$

3. Find the limit of the sequence and state whether the sequence converges or diverges

   $$a_n = 2 + (-1)^n$$

4. Find the limit of the sequence and state whether the sequence converges or diverges

   $$a_n = \left(1 + \frac{1}{n}\right)^n$$

5. Find the limit of the sequence and state whether the sequence converges or diverges

   $$\{1, {}^4/_3, {}^9/_7, {}^{16}/_{15}, {}^{25}/_{31}\}$$

6. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

   $$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$$

7. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

   $$\sum_{n=0}^{\infty} (1.2)^n$$

8. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

   $$\sum_{n=1}^{\infty} \frac{n}{n+3}$$

9. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

   $$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

10. Determine the convergence or divergence of the series using the *integral test*

    $$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

11. Determine the convergence or divergence of the series using the *limit comparison test*

    $$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

12. Find the explicit nth term formula for the following sequence

    $$\{a_n\} = \{-2, 5, 12, 19, \dots\}$$

13. Find the explicit nth term formula for the following sequence

    $$\{a_n\} = \{3, 6, 12, 24, 48, \dots\}$$

14. Write the first four terms of the sequence

    $$a_{n+1} = 2a_n + 1, \; a_1 = 1$$

Exam 3 Review (Answers)

1. *(Section 9.1–9.3)*

    1. $a_n = 4n - 1$
    2. $a_n = (-1)^{n+1} 2^{2-n}$
    3. $a_n = \frac{x^{n-1}}{(n-1)!}$

2. *(Section 9.1–9.3)*

    lim = 1, therefore the sequence converges.

3. *(Section 9.1–9.3)*

    The function oscillates between the values 1 and 3, thus limit does not exist, and therefore the sequence diverges.

4. *(Section 9.1–9.3)*

    lim = e, therefore the sequence converges.

5. *(Section 9.1–9.3)*

    lim = 0, therefore the sequence converges.

6. *(Quiz 5)*

    Because $r = |3/4| < 1$ the series converges (geometric series), and converges to the value 3.

7. *(Quiz 5)*

    Because $r = |1.2| > 1$ the series diverges (geometric series).

8. *(Quiz 5)*

    By the nth term test, $\lim_{n\to\infty} \frac{n}{n+3} = 1 \neq 0$, therefore the series diverges.

9. *(Quiz 5)*

    By means of partial fraction decomposition and evaluation of a telescopic series, the series converges to the value $11/18$.

10. *(Quiz 5)*

    By the integral test, $\lim_{b\to x} \int_1^b \frac{x}{x^2+1} dx = \infty$ which is non-finite, therefore the series diverges.

11. *(Quiz 5)*

    By the limit comparison test and selection of the harmonic series/divergent p-series as the comparison series,

    $$\lim_{n\to\infty} \frac{n}{n^2+1} \cdot \frac{n}{1} = 1$$

    Because the limit exists and is both positive and finite, the original series behaves similarly to the comparison series, and thus diverges.

12. *(Quiz 5)*

    $$a_n = 7n - 9$$

13. *(Quiz 5)*

    $$a_n = 3(2)^{n-1}$$

14. *(Quiz 5)*

    $$\{1, 3, 7, 15\}$$