Cumulative Review (Problems)

1. Evaluate $\int_1^\infty \frac{1}{x^p} dx$ converges if ______, otherwise it diverges.

2. Evaluate
$$\int \frac{-\csc\theta}{\csc\theta - \cot\theta} d\theta$$

3. Evaluate
$$\int \arcsin x \; dx$$

4. A sequence is bounded if it is ____.

5. Use the integral test to determine if the following series converges or diverges
$$\sum_{n=1}^{\infty} \frac{1}{4n^2+1}$$

6. Evaluate the limit
$$\lim_{x\to\infty} x\left(\frac{\pi}{2} - \arctan x\right)$$

7. Evaluate
$$\int \frac{2x+3}{x^3-2x^2+3x-6} dx$$

8. Evaluate
$$\int \frac{x+1}{\sqrt{3x^2+6x}} dx$$

9. How would you approach the following?
$$\int \frac{x}{x^2+1} dx$$

10. If f has n derivatives at center a, then the polynomial
$$P_n(x) = \_\_\_\_\_ + \cdots + \_\_\_\_\_$$
is called the nth degree Taylor polynomial for f at a.

11. Use the direct comparison test to determine if the following series converges or diverges
$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

12. Evaluate
$$\int \frac{x+5}{x^2+3x} dx$$

13. Evaluate
$$\int \frac{\sqrt{x^2-9}}{x} dx$$
$$\frac{\sqrt{x^2-9}}{3} - \arctan\frac{x}{3} + C$$

14. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.
$$\sum_{n=1}^{\infty} \frac{(\ln n)(x-3)^n}{n}$$

15. Evaluate
$$\int \frac{1}{\sqrt{1-4x-x^2}} dx$$

16. Evaluate
$$\int \sin(10x)\cos(3x) dx$$
by using one of the following identities:
$$\sin(mx)\sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$
$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$
$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

17. Evaluate
$$\int \frac{1}{\cos\theta - 1} d\theta$$

18. Determine the convergence or divergence of the series using the *integral test*

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

19. Evaluate

$$\int \tan^3 4x \, dx$$

20. Evaluate

$$\int_2^5 \frac{1}{\sqrt{x-2}} dx$$

21. Evaluate

$$\int e^{2x} x^2 \, dx$$

22. Evaluate

$$\int x\sqrt{5-4x^4} dx$$

using the reduction formula

$$\int \sqrt{a^2-x^2} dx = \frac{1}{2}\left(x\sqrt{a^2-x^1} + a^2 \arcsin \frac{x}{a}\right) + C$$

23. Find the exact value of the function

$\operatorname{arccsc}(-2)$

24. Evaluate

$$\int \sin^3 x \cos^4 x \, dx$$

25. Evaluate

$$\int \frac{1}{\sqrt{x^2-10x+21}} dx$$

26. Evaluate

$$\int_1^3 \ln 2x \, dx$$

27. :: Section 8.2 Evaluate

$$\int xe^{3x} dx$$

28. Evaluate

$$\int e^{2x} \cos x \, dx$$

29. Evaluate

$$\int \tan^2 2x \, dx$$

30. Differentiate

$$y = \frac{1}{2}\left[x\sqrt{4-x^4} + 4 \arcsin \frac{x}{2}\right]$$

31. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$$

32. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$$

33. Evaluate

$$\int x \sin x \, dx$$

34. Solve the differential equation

$$xy\frac{dy}{dx} = 1 - \ln x; quady(1) = 2$$

35. Use the limit comparison test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

36. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=0}^{\infty}(1.2)^n$$

37.

$$\int \frac{x^2+3}{x\sqrt{x^2-4}}dx$$

38. Evaluate

$$\int \frac{3}{2x^2-7x-4}dx$$

39. A geometric series is a series of the form

$$\sum_{n=0}^{\infty}\underline{\quad ? \quad}$$

40. Find the volume of the solid formed by revolving the region bounded by $y = e^{-2x}$ and the x-axis from $[0, \infty)$ about the x-axis.

41. Derive the reduction formula

$$\int u^n \cos u \; du = u^n \sin u - n \int u^{n-1} \sin u \; du$$

42. Evaluate

$$\int \frac{x^3 e^{x^2}}{(x^2+1)^2}dx$$

43. Use trig substitution to evaluate

$$\int \frac{1}{\sqrt{4x^2+1}}dx$$

44. A geometric series with ratio r will:

   1. Diverge if ______.
   2. Converge to $S =$ ______ if ______.

45. Evaluate

$$\int x^2 \ln 3x \; dx$$

46. Use l'Hopital's rule to evaluate the limit

$$\lim_{x\to\frac{\pi}{3}} \frac{\cos(x)-\frac{1}{2}}{x-\frac{x}{3}}$$

47. Evaluate

$$\int_{13/2}^{13} \sqrt{169-x^2}dx$$

48. Compute $\frac{dy}{dx}$ for the function

$$y = \sinh^2 7x$$

49. Evaluate

$$\int \frac{1}{1+e^x}dx$$

50. Evaluate

$$\int \frac{4x^2}{x^2+9}dx$$

51. Find the function $y = f(t)$ passing through the point $(0, 15)$ with the first derivate

$$\frac{dy}{dt} = \frac{1}{4}t$$

52. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} n!(n-1)^n$$

53. Evaluate

$$\int \frac{3x-1}{x^2-5x+4}dx$$

54. Evaluate

$$\int_1^{e^2} \frac{\ln^2(x^3)}{x}dx$$

55. Evaluate

$$\int \sin 2x \cos 3x \ dx$$

by using one of the following identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$
$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$
$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

56. Evaluate the integral

$$\int \frac{2x}{x^2+6x+13} dx$$

57. Evaluate

$$\int \frac{1}{(1+25x^2)^{3/2}} dx$$

58. If $\sum_{n=1}^{\infty} b_n$ converges and a_n ______ b_n, then $\sum_{n=1}^{\infty} a_n$ also converges.

59. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

60. Determine if the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

61. Evaluate

$$\int -\operatorname{csch}^2 x \coth x \ dx$$

62. Write the first four terms of the sequence

$$a_{n+1} = 2a_n + 1, \ a_1 = 1$$

63. Evaluate

$$\int x^3 e^{2x} dx$$

64. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(5x)^n}{n^2}$$

65. How would you approach the following?

$$\int \frac{1}{x^2+1} dx$$

66. Evaluate Find the area of the region bounded by the curves:

$$y = \sin^2(\pi \cdot x); \ y = 0; \ x = 0; \ x = 1$$

67. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{-2, 5, 12, 19, \dots\}$$

68. A conservation organization releases 40 coyotes into a preserve. After 4 years, there are 70 coyotes in the preserve. The preserve has a carrying capacity of 175.

    1. Write a logistic function that models the population, $P(t)$, of coyotoes in the preserve.
    2. Use your answer from (a) to find $\lim_{t\to\infty} P(t)$

69. Evaluate

$$\int_0^1 \frac{x}{(x^2+1)^{3/2}} dx$$

70. Evaluate

$$\int x^2 e^{5x} dx$$

71. Evaluate

$$\arccos\left(-\frac{\sqrt{3}}{2}\right)$$

72. Verify the identity using the definitions of hyperbolic functions

$$\coth x = \frac{e^{2x}+1}{e^{2x}-1}$$

73. Evaluate

$$\int \sqrt{9-4x^2}dx$$

74. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{n!}{3^n}$$

75. Compute $\frac{dy}{dx}$ for the function

$y = \ln \sinh 7x$

76. Evaluate

$$\tan\left(\arccos\left(\frac{1}{2}\right)\right)$$

77. Evaluate

$$\int \frac{1}{(25x^2+1)^2}dx$$

78. Find the derivative of y with respect to x:

$y = 3\arcsin(4x^3)$

79. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3}{2n+1}$$

80. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \frac{1}{2^n}$$

81. Evaluate

$$\int_0^{\pi/4} x \sin 2x \, dx$$

82. Compute $\frac{dy}{dx}$ for the function

$y = \sinh 7x$

83. Evaluate

$$\int \frac{4x+7}{(x+1)^2}dx$$

84. Find the general solution of the equation

$$y'(t) - \frac{y}{16} = -11$$

85. Evaluate

$$\int \frac{1}{x^2\sqrt{4-x^2}}dx$$

86. A sequence is *monotonic* if all of its terms are entirely either:

1. ______ (a_1 ______ $\cdots$ ______ a_n), or
2. ______ (a_1 ______ $\cdots$ ______ a_n).

87. Evaluate

$$\int \frac{e^x}{(e^{2x}+1)(e^x-1)}dx$$

88. Find the exact value of the function

$$\arcsin\left(-\frac{\sqrt{2}}{2}\right)$$

89. Evaluate

$$\int \frac{e^x}{(e^x+4)^{-4}}dx$$

90. With $a_n = f(n)$, $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ either both diverge if f is ______, ______, and ______ for $x \geq 1$.

91. Evaluate

$$\int \sin^3 x \, dx$$

92.

$$\int \arccos x \; dx$$

93. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} e^{-n}$$

94. Evaluate or simplify

$$\arccos\left(\cos\left(-\frac{\pi}{3}\right)\right)$$

95. If you are given an alternating series:

   1. Check for ______ by applying a test on $\sum_{n=1}^{\infty} |a_n|$
   2. If the absolute value of the series ______, then test for ______ using the ______.

96. Given a suitable a_n and b_n to compare against, the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge if $\lim_{n\to\infty}\left(\frac{?}{?}\right) = L$ where L is both ______ and ______.

97. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

98. Evaluate the following without use of a calculator

$\coth(\ln 6)$

99. Evaluate

$$\int \sin^3 x \cos x \ln(\sin x) dx$$

using the reduction formula

$$\int x^n \ln u \; du = \frac{u^{n+1}}{(n+1)^2}(-1 + (n+1)\ln u) + C, n \neq -1$$

100. Evaluate

$$\int \frac{\sin x}{\cos x + \cos^2 x} dx$$

101. Evaluate

$$\int \frac{8x^3 + 13x}{(x^2+2)^2} dx$$

102. Evaluate

$$\int_0^1 \frac{1}{(x^2+1)^{3/2}} dx$$

103.

$$\int_0^1 \frac{1}{\sqrt{16 - x^2}} dx$$

104. Evaluate $\int_0^1 \frac{1}{x^p} dx$ converges if ______, otherwise it diverges.

105.

$$\int \frac{1}{x\sqrt{9x^2 - 6}} dx$$

106. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \frac{n}{n+3}$$

107. Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$$

108. Evaluate

$$\int \sin(6x)\sin(4x) dx$$

by using one of the following identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$
$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$
$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

109. Evaluate

$$\int \frac{1}{x^2 - 9} dx$$

110. Evaluate

$$\int 2\cos^4 5x \ dx$$

111. Evaluate

$$\int_0^{\pi/4} \sin^3 4x \ dx$$

112. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^2$$

113. Evaluate

$$\int \frac{1}{4 + 9x^2} dx$$

114. Evaluate

$$\int \frac{2x - 1}{4x^2 - 9} dx$$

115. Evaluate

$$\int_0^{\infty} \frac{4(1 + \arctan x)}{1 + x^2} dx$$

116. Evaluate

$$\int x^2 \sin 2x \ dx$$

117. Evaluate

$$\int \sqrt{1 - x^2} dx$$

118. Evaluate

$$\int_2^4 8x \ln x \ dx$$

119. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^n}$$

120. Evaluate

$$\int_1^2 \frac{1}{(x-1)^2} dx$$

121. Evaluate

$$\int \tan^4 9t \ dt$$

122. Evaluate Solve the differential equation

$$\frac{dy}{dx} = \tan^3 x \sec x; \quad y(\pi/3) = 0$$

123. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$$

124. Evaluate

$$\int_1^{\infty} \frac{1}{e^x} dx$$

125. Evaluate

$$\int x \sin x^2 \ dx$$

126. Evaluate

$$\int_1^4 \frac{1}{(x-2)^{2/3}} dx$$

127. Evaluate

$$\int \frac{4x + 1}{x^2 + 9} dx$$

128. Consider the repeating decimal $0.1313\cdots$; Convert this decimal to a fraction.

129. Evaluate the integral

$$\int \cot^4 4x \ dx$$

using the reduction formula

$$\int \cot^m(u)du = -\frac{\cot^{m-1}(u)}{m-1} - \int \cot^{m-2}(u)du + C$$

130. Evaluate

$$\int \frac{1}{\sqrt{9x^2+4}}dx$$

131. Evaluate

$$\int \sin^4 \theta \ d\theta$$

132. Evaluate

$$\int \sin^3(2x)dx$$

133. Evaluate

$$\int \sqrt{25-4x^2}dx$$

134. Evaluate

$$\int_0^\infty \frac{1}{1+x^2}dx$$

135. Find the area of the surface generated when the given curve is revolved about the x-axis

$$y = \frac{x^3}{3} + \frac{1}{4x}; \text{ from } x = 1 \text{ to } x = 2$$

136. Evaluate

$$\int_0^{\ln 2} \cosh x \ dx$$

137. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} 3(\pi)^{-n}$$

138. Prove the reduction formula:

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$$

139.

$$\int \frac{\sinh x}{1+\cosh x}dx$$

140. Evaluate

$$\int x^4 \sin 2x \ dx$$

141. Evaluate the expression without a calculator to a value or to show that the value does not exist. Simplify the answer to the extent possible

$\sinh(2 \ln 5)$

142.

$$\lim_{x \to -\infty} 4 \sinh x$$

143. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \left(1 + \frac{1}{n}\right)^n$$

144. Determine the convergence or divergence of the series using the *limit comparison test*

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

145. Evaluate

$$\int \frac{9}{\sqrt{64-81x^2}}dx$$

146. The series $\sum_{n=1}^{\infty} a_n$ is ______ if $\sum_{n=1}^{\infty} |a_n|$ converges.

147. Find the length of the curve

$$y = 3x^{\frac{3}{2}}; \text{ from } x = 0 \text{ to } x = \frac{5}{9}$$

148.

$$\int_1^{\infty} \frac{4}{(1+x^2)\arctan x} dx$$

149. Find the general solution of the equation. Express the solution explicitly as a function of the independent variable

$e^{9t}y'(t) = -2$

150. The series $\sum_{n=1}^{\infty} |a_n|$ is ______ if $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} |a_n|$ ______.

151. Determine if the given function y is a solution of the differential equation y''. Assume that C is an arbitrary constant.

$y = C_1 \sin 5t + C_2 \cos 5t; \quad y''(t) + 25y = 0$

152. Evaluate or simplify

$\cos(2 \arccos x)$

153. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$$

154. How would you approach the following?

$$\int \frac{1}{\sqrt{x^2+1}} dx$$

155. A sequence is bounded ______ if there is a number M such that $a_n \geq M$ for all n.

156. Evaluate

$$\int \cos^2\left(\frac{x}{c}\right) dx$$

157. Evaluate

$$\int_{-\infty}^{e} 23e^{-x} dx$$

158. Evaluate

$$\int \frac{x^3+x-3}{x^2-4} dx$$

159. Find the limit of the sequence and state whether the sequence converges or diverges

$\{1, {}^{4}/_{3}, {}^{9}/_{7}, {}^{16}/_{15}, {}^{25}/_{31}\}$

160. Evaluate

$$\int x^3 \cos 2x \; dx$$

161. Evaluate

$$\int \frac{cost}{\sin^2 t - 9\sin t + 18} dt$$

162. Evaluate

$$\int_0^{\pi/6} \ln(2 \sec x) dx$$

163. Use integration by parts to establish a reduction formula for the integral

$$\int x^n e^x dx$$

164. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \frac{1}{\sqrt[3]{25}} + \cdots$$

165. Evaluate

$$\int \frac{x+4}{x^2+5x+6} dx$$

166. Find the limit of the sequence and state whether the sequence converges or diverges

$a_n = 2 + (-1)^n$

167. The series $\sum_{n=1}^{\infty} (-1)^n$ and $\sum_{n=1}^{\infty} (-1)^{n+b}$ converge if the following two conditions are met:

1. $\lim_{n\to\infty} a_n = \underline{?}$

2. $a_{n+1} \underline{?} a_n$ for all n

168. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \frac{n}{n+1}$$

169. Given the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^3+1}$$

1. Use the first 4 terms to approximate the sum; then explain what the maximum error is in this approximation.
2. Determine how many terms are required to estimate the sum with an error of no more than 0.0001.

170. Evaluate $\displaystyle\int \frac{1}{x^2+2x-3}dx$ via:

1. Trigonometric substitution
2. Partial fraction decomposition

171. A sequence is bounded ______ if there is a number M such that $a_n \leq M$ for all n.

172. Evaluate

$$\int \frac{\ln x}{x^2}dx$$

173. Find the equation of the line tangent to the curve

$$2x + \arctan y = y^2 - 1; \text{ at the point } P(\frac{-\pi}{8}, -1)$$

174. Write the nth-term formula for the following sequences

1. $\{3, 7, 11, 15, \dots\}$
2. $\{2, -1, \frac{1}{2}, -\frac{1}{4}, \dots\}$
3. $\{1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \dots\}$

175. If $\sum_{n=1}^{\infty} b_n$ diverges and a_n ________ b_n, then $\sum_{n=1}^{\infty} a_n$ also diverges.

176. Compute $\frac{dy}{dx}$ for the function:

$$y = \sinh^2 4x$$

177. Evaluate Find the volume of the solid formed when the region bounded by the curves

$$y = \cos\frac{x}{2}; y = \sin\frac{x}{2}; x = 0; x = \frac{\pi}{2}$$

is revolved about the x-axis.

178. Evaluate

$$\int_1^{\infty} \frac{1}{\sqrt{x+2}}dx$$

179. Evaluate

$$\int \cos^2\theta \sin 2\theta \; d\theta$$

180. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

181. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{3, 6, 12, 24, 48, \dots\}$$

Cumulative Review (Answers)

2. *(Quiz 3)*

   $\cot\theta + \csc\theta + C$

4. *(Section 9.1–9.3)*

   bounded both above and below

5. *(Section 9.4)*

   (Integral test) Because $\int_1^\infty \frac{1}{4x^2+1}dx = \frac{\pi}{4} - \frac{1}{2}\arctan(2) \approx 0.2318$ (converges), the series also converges.

9. *(Exam 2 Studyguide)*

   Use u-substitution

10. *(Section 9.4)*

    $$\underline{f(a)} + \cdots + \underline{\frac{f^{(n)}(a)}{n!}}$$

11. *(Section 9.4)*

    (D.C.T.) Comparing against $b_n = \sum_{n=1}^\infty \frac{1}{2^n}$ (convergent geometric series), because $a_n < b_n$ the series similarly converges.

12. *(Exam 2)*

    $$\frac{5}{3}\ln|x| - \frac{2}{3}\ln|x+3| + C$$

    or

    $$\frac{1}{3}\ln\left|\frac{x^5}{(x+3)^2}\right| + C$$

14. *(Exam 3 Studyguide)*

    (Check for absolute convergence using Root test; check end points using A.S.T. and L.C.T.) The series converges for $2 \le x < 4$.

18. *(Quiz 5)*

    By the integral test, $\lim_{b\to x}\int_1^b \frac{x}{x^2+1}dx = \infty$ which is non-finite, therefore the series diverges.

20. *(Exam 2 Studyguide)*

    $2\sqrt{3} \to$ converges

21. *(Quiz 3)*

    $$\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

25. *(Exam 2)*

    $$\ln\left|\frac{x-5+\sqrt{(x-5)^2-4}}{2}\right| + C$$

26. *(Exam 2)*

    ≈ 2.68

30. *(Quiz 1)*

    $\sqrt{4-x^2}$

31. *(Exam 3 Studyguide)*

    (Divergence test) lim $= 1/3$ which $\neq 0$ and therefore diverges.

32. *(Exam 3 Studyguide)*

    (Check for absolute convergence using Ratio Test) Because $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$, the series converges absolutely for all values of x.

35. *(Section 9.4)*

    (L.C.T.) Comparing against $\frac{1}{n^{3/2}}$ (convergent p-series), because $\lim_{n\to\infty}\frac{a_n}{b_n} = 1$, which is finite and positive, the series similarly converges.

36. *(Quiz 5)*

    Because $r = |1.2| > 1$ the series diverges (geometric series).

39. *(Section 9.1–9.3)*

    $$\sum_{n=0}^{\infty} ar^n$$

43. *(Quiz 4)*

$$\frac{1}{2}\ln\left|\sqrt{4x^2+1}+2x\right|+C$$

44. *(Section 9.1–9.3)*

    1. Diverge if $|r| \geq 1$.
    2. Converge to $S = \dfrac{a}{1-r}$ if $0 < |r| < 1$.

52. *(Exam 3 Studyguide)*

    (Ratio test) Because $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, the series diverges for all x.

54. *(Exam 2)*

    24

56. *(Quiz 1)*

$$\ln\left|x^2+6x+13\right| - 3\arctan\frac{x+3}{2}+C$$

58. *(Section 9.4)*

    and $a_n \leq b_n$

59. *(Quiz 6)*

    Converges conditionally.

60. *(Section 9.4)*

    (D.C.T.) Compared against $\frac{1}{n^2}$ (convergent p-series), because $a_n \leq b_n$ the series similarly converges.

61. *(Quiz 2)*

$$\frac{\coth^2 x}{2}+C$$

62. *(Quiz 5)*

    $\{1,3,7,15\}$

63. *(Exam 2 Studyguide)*

$$\frac{1}{2}x^3e^{2x}-\frac{3}{4}x^2e^{2x}+\frac{3}{4}xe^{2x}-\frac{3}{8}e^{2x}+C$$

64. *(Exam 3 Studyguide)*

    The series converges for $-1/5 \leq x \leq 1/5$.

65. *(Exam 2 Studyguide)*

    Use the inverse tangent integration formula

67. *(Quiz 5)*

    $a_n = 7n-9$

71. *(Quiz 1)*

$$\frac{5\pi}{6}$$

74. *(Exam 3 Studyguide)*

    (Ratio test) Because $\lim_{n\to\infty}\left|\frac{n+1}{3}\right| = \infty > 1$, the series diverges.

76. *(Quiz 1)*

    $\sqrt{3}$

77. *(Exam 2)*

$$\frac{1}{10}\arctan 5x+\frac{x}{50x^2+2}+C$$

79. *(Exam 3 Studyguide)*

    1. (Check for absolute convergence using L.C.T.) Comparing against $\sum_{n=1}^{\infty}\frac{1}{n}$ (divergent p-series):

       $$\lim_{n\to\infty}\left|\frac{3}{2n+1}\cdot\frac{n}{1}\right| = \frac{3}{2}$$

       Which is both finite and positive, and so diverges similarly. We need to check for conditional convergence.
    2. (A.S.T.)
       (a) $\displaystyle\lim_{n\to\infty}\frac{(-1)^n 3}{2n+1} = 0$ ✓
       (b) $a_{n+1} < a_n$ ✓

80. *(Section 9.1–9.3)*

Converges, and the sum equals 2.

82. *(Quiz 2)*

$7\cosh 7x$

84. *(Quiz 2)*

$y = Ce^{x/16} + 176$

86. *(Section 9.1–9.3)*

1. $\underline{\text{non-decreasing}}$ $(a_1 \leqq \cdots \leqq a_n)$, or
2. $\underline{\text{non-increasing}}$ $(a_1 \geqq \cdots \geqq a_n)$.

87. *(Exam 2 Studyguide)*

$$\frac{1}{2}\ln|e^x-1|-\frac{1}{4}\ln|e^{2x}+1|-\frac{1}{2}\arctan(e^x)+C$$

89. *(Exam 2)*

$$-\frac{1}{3(e^x+4)^3}dx$$

90. *(Section 9.4)*

$\underline{\text{positive}}$, $\underline{\text{continuous}}$, and $\underline{\text{decreasing}}$ for $x \geq 1$

92. *(Exam 2)*

$x \arccos x - \sqrt{1-x^2} + C$

93. *(Section 9.1–9.3)*

Converges, and the sum equals $\frac{e}{e-1}$.

95. *(Section 9.4)*

1. $\underline{\text{absolute convergence}}$
2. $\underline{\text{diverges}}$; $\underline{\text{conditional convergence}}$; $\underline{\text{alternating series test}}$.

96. *(Section 9.4)*

$\underline{\frac{a_n}{b_n}}$; $\underline{\text{finite}}$ and $\underline{\text{positive}}$.

97. *(Quiz 5)*

By means of partial fraction decomposition and evaluation of a telescopic series, the series converges to the value $^{11}/_{18}$.

98. *(Quiz 2)*

$$\frac{37}{35}$$

101. *(Quiz 4)*

$$4\ln(x^2+2)+\frac{3}{2(x^2+2)}+C$$

106. *(Quiz 5)*

By the nth term test, $\lim_{n\to\infty}\frac{n}{n+3} = 1 \neq 0$, therefore the series diverges.

107. *(Quiz 6)*

(-3,1)

110. *(Exam 2)*

$$\frac{3}{4}x+\frac{1}{10}\sin 10x\frac{1}{80}\sin 10x + C$$

111. *(Quiz 3)*

$$\frac{1}{3}$$

112. *(Section 9.1–9.3)*

Diverges

115. *(Exam 2)*

$$= 2\pi + \frac{\pi^2}{2}; \quad \therefore \text{converges}$$

118. *(Quiz 3)*

$(64\ln 4 - 32) - (16\ln 2 - 8) \approx 53.6$

119. *(Exam 3 Studyguide)*

(Ratio Test) Because $\lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^n}} = 0 < 1$, the series converges.

121. *(Exam 2)*

$$\frac{\tan^3 9t}{27} - \frac{\tan 9t}{9} + t + C$$

123. *(Exam 3 Studyguide)*

(L.C.T.) Comparing against $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ (convergent p-series),

$$\lim_{n\to\infty} \frac{1}{n\sqrt{n+1}} \cdot \frac{n\sqrt{n}}{1} = 1$$

Which is both finite and positive, and so converges similarly.

128. *(Section 9.1–9.3)*

$$0.1313\cdots = \frac{13}{99}$$

129. *(Quiz 4)*

$$-\frac{1}{12}\cot^3(4x) + \frac{1}{4}\cot(4x) + x + C$$

132. *(Exam 2 Studyguide)*

$$-\frac{1}{2}\left(\cos(2x) - \frac{1}{3}\cos^3(2x)\right) + C$$

136. *(Quiz 2)*

$$\frac{3}{4}$$

137. *(Exam 3 Studyguide)*

(Geometric series) The series has the ratio $r = {}^1\!/\!_\pi$, which because $|r| < 0$ converges. The series converges to:

$a_1 = {}^3\!/\!_\pi;\ r = {}^1\!/\!_\pi$

$$S_n = \frac{{}^3\!/\!_\pi}{1 - {}^1\!/\!_\pi} \approx 1.4008$$

143. *(Section 9.1–9.3)*

$\lim = e$, therefore the sequence converges.

144. *(Quiz 5)*

By the limit comparison test and selection of the harmonic series/divergent p-series as the comparison series,

$$\lim_{n\to\infty} \frac{n}{n^2+1} \cdot \frac{n}{1} = 1$$

Because the limit exists and is both positive and finite, the original series behaves similarly to the comparison series, and thus diverges.

145. *(Quiz 2)*

$$\arcsin\frac{9x}{8} + C$$

146. *(Section 9.4)*

<u>absolutely convergent</u>

148. *(Quiz 4)*

$4\ln 2 \to$ Converges

150. *(Section 9.4)*

<u>conditionally convergent</u>; <u>diverges</u>.

151. *(Quiz 2)*

Yes (verified)

153. *(Quiz 5)*

Because $r = |{}^3\!/\!_4| < 1$ the series converges (geometric series), and converges to the value 3.

154. *(Exam 2 Studyguide)*

Use trig-substitution

155. *(Section 9.1–9.3)*

<u>below</u>

157. *(Exam 2)*

$= \infty;\quad \therefore$ diverges

159. *(Section 9.1–9.3)*

lim = 0, therefore the sequence converges.

161. *(Exam 2)*

$$\frac{1}{3}\ln|\sin t-6|-\frac{1}{3}|\sin t-3|+C$$

163. *(Quiz 3)*

$$x^n e^x - n\int x^{n-1}e^x dx$$

164. *(Quiz 6)*

(P-series) Because $p = {}^2/_3 < 1$ the series diverges.

166. *(Section 9.1–9.3)*

The function oscillates between the values 1 and 3, thus limit does not exist, and therefore the sequence diverges.

167. *(Section 9.4)*

   1. $\lim_{n\to\infty} a_n = \underline{0}$
   2. $a_{n+1} \leq a_n$ for all n

168. *(Section 9.1–9.3)*

lim = 1, therefore the sequence converges.

169. *(Exam 3 Studyguide)*

   1. $S_4 = -\frac{1}{3}+\frac{1}{17}-\frac{1}{55}+\frac{1}{129} \approx -0.2849$, and by alternating series remainder theorem, error $= a_5 = \left|\frac{1}{251}\right|$.
   2. $$a_{n+1} = \left|\frac{(-1)^n}{2n^3+1}\right| \leq 0.0001$$

      $16.099 \leq n$

      $n = 17$

171. *(Section 9.1–9.3)*

above

174. *(Section 9.1–9.3)*

   1. $a_n = 4n-1$
   2. $a_n = (-1)^{n+1}2^{2-n}$
   3. $a_n = \frac{x^{n-1}}{(n-1)!}$

175. *(Section 9.4)*

and $a_n \geq b_n$

176. *(Quiz 2)*

$$8\sinh(4x)\cosh(4x)$$

178. *(Exam 2 Studyguide)*

$$\infty \to \text{diverges}$$

179. *(Quiz 3)*

$$-\frac{1}{2}\cos^4\theta + C$$

180. *(Quiz 6)*

(Integral test) Because lim $= \infty$ the series diverges.

181. *(Quiz 5)*

$$a_n = 3(2)^{n-1}$$