Cumulative Review (Problems)

1. $$\int \arccos x \; dx$$

2. $$\lim_{x \to -\infty} 4 \sinh x$$

3. Given a suitable a_n and b_n to compare against, the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge if $\lim_{n\to\infty} \left(\frac{?}{?}\right) = L$ where L is both ______ and ______.

4. Evaluate
$$\int \frac{x^3 e^{x^2}}{(x^2+1)^2} dx$$

5. Find the limit of the sequence and state whether the sequence converges or diverges
$$\{1, {}^4\!/_3, {}^9\!/_7, {}^{16}\!/_{15}, {}^{25}\!/_{31}\}$$

6. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.
$$\sum_{n=1}^{\infty} \frac{(5x)^n}{n^2}$$

7. Use l'Hopital's rule to evaluate the limit
$$\lim_{x \to \frac{\pi}{3}} \frac{\cos(x) - \frac{1}{2}}{x - \frac{x}{3}}$$

8. A geometric series with ratio r will:
   1. Diverge if ______.
   2. Converge to $S =$ ______ if ______.

9. State the power series of the function $f(x) = \frac{1}{1+2x}$, centered at $c = 0$. What is the interval of convergence, and what are the first four terms?

10. Find the exact value of the function
$$\operatorname{arccsc}(-2)$$

11. Evaluate
$$\int \cos^2 \theta \sin 2\theta \; d\theta$$

12. $\int x e^{4x} dx$

13. If $\sum_{n=1}^{\infty} b_n$ diverges and a_n ________ b_n, then $\sum_{n=1}^{\infty} a_n$ also diverges.

14. $\int \frac{x+1}{(x^2+3)(x-1)} dx$

15. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3+5}$

16. Evaluate
$$\int_0^{\ln 2} \cosh x \; dx$$

17. Evaluate
$$\int_1^4 \frac{1}{(x-2)^{2/3}} dx$$

18. $\int \frac{1}{\sqrt{x^2+9}} dx$

19. Evaluate
$$\int \frac{4x+7}{(x+1)^2} dx$$

20. Evaluate
$$\int_0^1 \frac{x}{(x^2+1)^{3/2}} dx$$

21. How would you approach the following?
$$\int \frac{1}{x^2+1} dx$$

22. With $a_n = f(n)$, $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x) dx$ either both diverge if f is ______, ______, and ______ for $x \geq 1$.

23. Evaluate the integral

$$\int \cot^4 4x \ dx$$

using the reduction formula

$$\int \cot^m(u)du = -\frac{\cot^{m-1}(u)}{m-1} - \int \cot^{m-2}(u)du + C$$

24. Evaluate

$$\int \arcsin x \ dx$$

25. If $\sum_{n=1}^{\infty} b_n$ converges and a_n ________ b_n, then $\sum_{n=1}^{\infty} a_n$ also converges.

26. Evaluate

$$\int_0^1 \frac{1}{(x^2+1)^{3/2}} dx$$

27. Evaluate or simplify

$$\arccos\left(\cos\left(-\frac{\pi}{3}\right)\right)$$

28. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$$

29. Evaluate

$$\int_0^{\pi/4} x \sin 2x \ dx$$

30. Evaluate

$$\int_1^{\infty} \frac{1}{e^x} dx$$

31. Evaluate

$$\int \tan^2 2x \ dx$$

32. Derive the reduction formula

$$\int u^n \cos u \ du = u^n \sin u - n \int u^{n-1} \sin u \ du$$

33. $\int \frac{3}{\sqrt{10x-x^2}} dx$

34. A geometric series is a series of the form

$$\sum_{n=0}^{\infty} \underline{\quad ? \quad}$$

35. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} e^{-n}$$

36. Evaluate

$$\int e^{2x} x^2 \ dx$$

37. Write the nth-term formula for the following sequences

    1. $\{3, 7, 11, 15, \dots\}$
    2. $\{2, -1, \frac{1}{2}, -\frac{1}{4}, \dots\}$
    3. $\{1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \dots\}$

38. Use integration by parts to establish a reduction formula for the integral

$$\int x^n e^x dx$$

39. Evaluate

$$\tan\left(\arccos\left(\frac{1}{2}\right)\right)$$

40. Find the exact value of the function

$$\arcsin\left(-\frac{\sqrt{2}}{2}\right)$$

41. Evaluate

$$\int \frac{x+5}{x^2+3x} dx$$

42. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$$

43. Find the length of the curve

$$y = 3x^{\frac{3}{2}}; \text{ from } x = 0 \text{ to } x = \frac{5}{9}$$

44. Evaluate

$$\int xe^{3x} dx$$

45. Evaluate

$$\int_0^{\infty} \frac{4(1+\arctan x)}{1+x^2} dx$$

46. Find the function $y = f(t)$ passing through the point $(0, 15)$ with the first derivate

$$\frac{dy}{dt} = \frac{1}{4}t$$

47. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \frac{1}{2^n}$$

48. Evaluate the integral

$$\int \frac{2x}{x^2+6x+13} dx$$

49. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^n}$$

50. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} n!(x-1)^n$$

51. Find the derivative of y with respect to x:

$$y = 3\arcsin(4x^3)$$

52. $\sum_{n=1}^{\infty} \frac{n}{n+3}$

53. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

54. Evaluate

$$\int_1^2 \frac{1}{(x-1)^2} dx$$

55. Evaluate

$$\int x^2 e^{5x} dx$$

56. Evaluate

$$\int \sqrt{1-x^2} dx$$

57. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

58.

$$\int_1^{\infty} \frac{4}{(1+x^2)\arctan x} dx$$

59. $\int \frac{\cos^3 x}{\sqrt{\sin x}} dx$

60. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{-2, 5, 12, 19, \dots\}$$

61. Evaluate

$$\int_1^{e^2} \frac{\ln^2(x^3)}{x} dx$$

62. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

63. Evaluate

$$\int \sin(10x)\cos(3x)dx$$

by using one of the following identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$
$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$
$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

64. Evaluate

$$\int_{13/2}^{13} \sqrt{169-x^2}dx$$

65. Evaluate

$$\int \frac{2x+3}{x^3-2x^2+3x-6}dx$$

66. Determine the convergence or divergence of the series using the *limit comparison test*

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

67. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} 3(\pi)^{-n}$$

68. Compute $\frac{dy}{dx}$ for the function

$y = \sinh 7x$

69. The series $\sum_{n=1}^{\infty} |a_n|$ is ______ if $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} |a_n|$ ______.

70. $\frac{d}{dx}x^2 \arcsin 4x$

71. Find the length of the curve $y = 2x^{\frac{3}{2}}$ from $x = 0$ to $x = 9$.

72. Evaluate

$$\int \frac{-\csc\theta}{\csc\theta - \cot\theta}d\theta$$

73. Consider the repeating decimal $0.1313\cdots$; Convert this decimal to a fraction.

74. Evaluate

$$\int \frac{4x+1}{x^2+9}dx$$

75. $\int \sin^2(4x)dx$

76.

$$\int \frac{\sinh x}{1+\cosh x}dx$$

77. If f has n derivatives at center a, then the polynomial

$P_n(x) =$ ______ $+\cdots+$ ______

is called the nth degree Taylor polynomial for f at a.

78. Evaluate

$$\int \sqrt{9-4x^2}dx$$

79. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \left(1+\frac{1}{n}\right)^n$$

80. Find the equation of the line tangent to the curve

$$2x + \arctan y = y^2 - 1;\ \text{at the point } P(\frac{-\pi}{8}, -1)$$

81. $\int_1^4 \frac{2}{\sqrt{x-1}}dx$

82. Evaluate

$$\int_0^{\infty} \frac{1}{1+x^2}dx$$

83. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{n!}{3^n}$$

84. Determine if the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

85. Find the volume of the solid formed by revolving the region bounded by $y = e^{-2x}$ and the x-axis from $[0, \infty)$ about the x-axis.

86. Determine the convergence or divergence of the series using the *integral test*

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

87. Evaluate

$$\int_1^{\infty} \frac{1}{\sqrt{x+2}} dx$$

88. Compute $\frac{dy}{dx}$ for the function

$$y = \sinh^2 7x$$

89. A sequence is bounded ______ if there is a number M such that $a_n \geq M$ for all n.

90. Determine if the given function y is a solution of the differential equation y''. Assume that C is an arbitrary constant.

$$y = C_1 \sin 5t + C_2 \cos 5t; \quad y''(t) + 25y = 0$$

91. Evaluate

$$\int \frac{1}{4+9x^2} dx$$

92. $\int \frac{x+3}{2x^2-9x-5} dx$

93. Evaluate

$$\int_2^4 8x \ln x \; dx$$

94. Evaluate $\displaystyle\int \frac{1}{x^2+2x-3} dx$ via:

   1. Trigonometric substitution
   2. Partial fraction decomposition

95. Use the integral test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{1}{4n^2+1}$$

96. Verify the identity using the definitions of hyperbolic functions

$$\coth x = \frac{e^{2x}+1}{e^{2x}-1}$$

97. $\int \frac{\sqrt{x^2-4}}{x} dx$

98. $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$,

99. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = 2 + (-1)^n$$

100. $\lim_{x \to 0^+} \frac{1 - \ln x}{e^{1/x}}$

101. $\int_1^e x^2 \ln x \, dx$

102. Evaluate or simplify

$$\cos(2 \arccos x)$$

103. Evaluate

$$\int x^2 \ln 3x \; dx$$

104. Evaluate Find the volume of the solid formed when the region bounded by the curves

$$y = \cos \frac{x}{2}; y = \sin \frac{x}{2}; x = 0; x = \frac{\pi}{2}$$

is revolved about the x-axis.

105.

$$\int \frac{x^2+3}{x\sqrt{x^2-4}} dx$$

106. Use trig substitution to evaluate

$$\int \frac{1}{\sqrt{4x^2+1}} dx$$

107. Evaluate

$$\int \cos^2\left(\frac{x}{c}\right)\,dx$$

108. Evaluate

$$\int \frac{2x-1}{4x^2-9}dx$$

109. Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n(x+1)^n}{2^n}$$

110. Evaluate Solve the differential equation

$$\frac{dy}{dx} = \tan^3 x \sec x; \quad y(\pi/3) = 0$$

111. Evaluate

$$\int x^2 \sin 2x \; dx$$

112. Find the 10th degree Maclaurin Polynomial for

$$f(x) = \cos x$$

And then compare the value of $\cos 0.5$ approximated by the polynomial with the value given by a calculator (≈ 0.878).

113. $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

114. Evaluate

$$\int x \sin x^2 \; dx$$

115. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^2$$

116. Evaluate

$$\int -\operatorname{csch}^2 x \coth x \; dx$$

117. $\int_0^{\frac{1}{3}} \frac{1}{1+9x^2} dx$

118. Evaluate

$$\int \frac{e^x}{(e^{2x}+1)(e^x-1)}dx$$

119. Evaluate

$$\int x\sqrt{5-4x^4}dx$$

using the reduction formula

$$\int \sqrt{a^2-x^2}dx = \frac{1}{2}\left(x\sqrt{a^2-x^1} + a^2 \arcsin\frac{x}{a}\right) + C$$

120. Find the Taylor polynomials of degrees 0, 1, and 2, of the function f centered at point a

$$f(x) = \cos x; a = \frac{\pi}{4}$$

121. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \frac{n}{n+3}$$

122. Evaluate

$$\int e^{2x} \cos x \; dx$$

123. Evaluate

$$\int \sin^3 x \cos x \ln(\sin x)dx$$

using the reduction formula

$$\int x^n \ln u \; du = \frac{u^{n+1}}{(n+1)^2}(-1+(n+1)\ln u) + C, n \neq -1$$

124. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(\ln n)(x-3)^n}{n}$$

125. The series $\sum_{n=1}^{\infty}(-1)^n$ and $\sum_{n=1}^{\infty}(-1)^{n+b}$ converge if the following two conditions are met:

1. $\lim_{n\to\infty} a_n = \underline{?}$
2. $a_{n+1}\,\underline{?}\,a_n$ for all n

126. Find the area of the surface generated when the given curve is revolved about the x-axis

$$y = \frac{x^3}{3} + \frac{1}{4x};\ \text{from } x = 1 \text{ to } x = 2$$

127.

$$\int_0^1 \frac{1}{\sqrt{16-x^2}}dx$$

128. Evaluate

$$\int \frac{cost}{\sin^2 t - 9\sin t + 18}dt$$

129. Evaluate

$$\int \frac{1}{(1+25x^2)^{3/2}}dx$$

130. Compute $\frac{dy}{dx}$ for the function

$$y = \ln\sinh 7x$$

131. A sequence is bounded if it is

____.

132. Evaluate

$$\int \frac{\sin x}{\cos x + \cos^2 x}dx$$

133. $\int \sec^4 x \tan x\, dx$

134. $\int \frac{x}{9+4x^2}dx$

135. Evaluate

$$\int x^4 \sin 2x\ dx$$

136. $\frac{d}{dx}\arctan e^x$

137. $\int \frac{\cos x}{\sqrt{25-\sin^2 x}}dx$

138. Evaluate $\int_0^1 \frac{1}{x^p}dx$ converges if ______, otherwise it diverges.

139. $\sum_{n=1}^{\infty}\frac{1}{n(n+3)}$

140.

$$\int \frac{1}{x\sqrt{9x^2-6}}dx$$

141. $\sum_{n=0}^{\infty}\frac{3^n}{n!}$

142. $\lim_{x\to 1}\frac{x^3-1}{x^2-1}$

143. $\int e^{3x}dx$

144. Evaluate

$$\int \sin^3(2x)dx$$

145. A sequence is *monotonic* if all of its terms are entirely either:

1. ______ $(a_1$ ______ $\cdots$ ______ $a_n)$, or
2. ______ $(a_1$ ______ $\cdots$ ______ $a_n)$.

146. Compute $\frac{dy}{dx}$ for the function:

$$y = \sinh^2 4x$$

147. Evaluate

$$\int \frac{8x^3+13x}{(x^2+2)^2}dx$$

148. Evaluate Find the area of the region bounded by the curves:

$$y = \sin^2(\pi\cdot x);\ y = 0;\ x = 0;\ x = 1$$

149. Differentiate

$$y = \frac{1}{2}\left[x\sqrt{4-x^4} + 4\arcsin\frac{x}{2}\right]$$

150. Use the direct comparison test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty}\frac{1}{n!}$$

151. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{2n+1}}{(2n+1)!}$$

152. Approximate $e^{-0.06}$ using the Taylor polynomial

$$P_2(x) = 1 - x + \frac{x^2}{2}$$

153. $\int xe^{x^2}\,dx$

154. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \frac{1}{\sqrt[3]{25}} + \cdots$$

155. Use the limit comparison test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

156. Evaluate

$$\int \frac{x+4}{x^2+5x+6}dx$$

157. Evaluate

$$\int \frac{x^3+x-3}{x^2-4}dx$$

158. Evaluate

$$\int \sin 2x \cos 3x \; dx$$

by using one of the following identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$
$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$
$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

159. $\int \frac{x^3+x+3}{x^2-x}dx$

160. Prove the reduction formula:

$$\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1}dx$$

161. Evaluate

$$\int \sin(6x)\sin(4x)dx$$

by using one of the following identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$
$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$
$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

162. A sequence is bounded ______ if there is a number M such that $a_n \leq M$ for all n.

163. Evaluate

$$\int \frac{e^x}{(e^x+4)^{-4}}dx$$

164. Evaluate

$$\int 2\cos^4 5x \; dx$$

165. Evaluate

$$\int \frac{1}{\sqrt{1-4x-x^2}}dx$$

166. How would you approach the following?

$$\int \frac{x}{x^2+1}dx$$

167. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3}{2n+1}$$

168. Evaluate

$$\int \frac{1}{(25x^2+1)^2}dx$$

169. Evaluate

$$\int \frac{\sqrt{x^2-9}}{x} dx$$

$$\frac{\sqrt{x^2-9}}{3} - \arctan\frac{x}{3} + C$$

170. Evaluate

$$\int \frac{1}{\sqrt{x^2-10x+21}} dx$$

171. Evaluate

$$\int \frac{3}{2x^2-7x-4} dx$$

172. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \frac{n}{n+1}$$

173. $\int x^2 \cos 3x \; dx$

174. Evaluate

$$\int \sin^3 x \; dx$$

175. $\int \frac{x}{9+4x^4} dx$

176. Evaluate

$$\int \sqrt{25-4x^2} dx$$

177. Evaluate

$$\int \sin^4 \theta \; d\theta$$

178. Evaluate $\int_1^\infty \frac{1}{x^p} dx$ converges if ______, otherwise it diverges.

179. Evaluate

$$\int \frac{x+1}{\sqrt{3x^2+6x}} dx$$

180. Evaluate

$$\int_{-\infty}^{e} 23e^{-x} dx$$

181. Find the area of the surface generated by revolving the curve $y = 2x^2$ from $x = 0$ to $x = 1$ about the y-axis.

182. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=0}^{\infty} (1.2)^n$$

183. Evaluate

$$\int x \sin x \; dx$$

184. Evaluate

$$\int_1^3 \ln 2x \; dx$$

185. Evaluate the following without use of a calculator

$\coth(\ln 6)$

186. A conservation organization releases 40 coyotes into a preserve. After 4 years, there are 70 coyotes in the preserve. The preserve has a carrying capacity of 175.

   1. Write a logistic function that models the population, $P(t)$, of coyotoes in the preserve.
   2. Use your answer from (a) to find $\lim_{t\to\infty} P(t)$

187. $\int \frac{1}{\sqrt{2-3x}} dx$

188. If you are given an alternating series:

   1. Check for ______ by applying a test on $\sum_{n=1}^{\infty} |a_n|$
   2. If the absolute value of the series ______, then test for ______ using the ______.

189. Evaluate

$$\int \frac{1}{x^2\sqrt{4-x^2}} dx$$

190. Evaluate the expression without a calculator to a value or to show that the value does not exist. Simplify the answer to the extent possible

$\sinh(2\ln 5)$

191. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$$

192. Evaluate

$$\int \frac{1}{1+e^x} dx$$

193. Find the Taylor polynomials of degrees 0, 1, and 2, of the function f centered at point a

$f(x) = \ln(1+8x)$

194. Find the general solution of the equation. Express the solution explicitly as a function of the independent variable

$e^{9t}y'(t) = -2$

195. Evaluate

$$\int \frac{1}{x^2-9} dx$$

196. Find the general solution of the equation

$$y'(t) - \frac{y}{16} = -11$$

197. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

198. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

199. Evaluate

$$\int \frac{3x-1}{x^2-5x+4} dx$$

200. Evaluate

$$\int \frac{1}{\sqrt{9x^2+4}} dx$$

201. Evaluate

$$\arccos\left(-\frac{\sqrt{3}}{2}\right)$$

202. Evaluate

$$\int x^3 \cos 2x \; dx$$

203. Evaluate

$$\int_0^{\pi/4} \sin^3 4x \; dx$$

204. Evaluate

$$\int \tan^3 4x \; dx$$

205. Evaluate the limit

$$\lim_{x\to\infty} x\left(\frac{\pi}{2} - \arctan x\right)$$

206. Solve the differential equation

$$xy\frac{dy}{dx} = 1 - \ln x; quady(1) = 2$$

207. $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}$

208. Evaluate

$$\int_2^5 \frac{1}{\sqrt{x-2}} dx$$

209. Evaluate

$$\int \frac{4x^2}{x^2+9}dx$$

210. $\sum_{n=0}^{\infty}(1.2)^n$,

211. Evaluate

$$\int \frac{\ln x}{x^2}dx$$

212. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{3, 6, 12, 24, 48, \dots\}$$

213. Evaluate

$$\int \frac{1}{\cos\theta - 1}d\theta$$

214. How would you approach the following?

$$\int \frac{1}{\sqrt{x^2+1}}dx$$

215. Evaluate

$$\int_0^{\pi/6} \ln(2\sec x)dx$$

216. The series $\sum_{n=1}^{\infty} a_n$ is ______ if $\sum_{n=1}^{\infty} |a_n|$ converges.

217. Write the first four terms of the sequence

$$a_{n+1} = 2a_n + 1,\ a_1 = 1$$

218. Evaluate

$$\int x^3 e^{2x} dx$$

219. State the 3rd degree Taylor polynomial of the function $f(x) = \frac{1}{x^2}$, centered at $c = 2$.

220. Evaluate

$$\int \tan^4 9t\ dt$$

221. Evaluate

$$\int \frac{9}{\sqrt{64-81x^2}}dx$$

222. Given the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^3+1}$$

1. Use the first 4 terms to approximate the sum; then explain what the maximum error is in this approximation.
2. Determine how many terms are required to estimate the sum with an error of no more than 0.0001.

223. Evaluate

$$\int \sin^3 x \cos^4 x\ dx$$

Cumulative Review (Answers)

1. *(Exam 2)*

   $x \arccos x - \sqrt{1-x^2} + C$

3. *(Section 9.4 and 9.5)*
   $\frac{a_n}{b_n}$; finite and positive.

5. *(Section 9.1–9.3)*
   lim = 0, therefore the sequence converges.

6. *(Exam 3 Studyguide)*
   The series converges for $-1/5 \leq x \leq 1/5$.

8. *(Section 9.1–9.3)*
   1. Diverge if $|r| \geq 1$.
   2. Converge to $S = \dfrac{a}{1-r}$ if $0 < |r| < 1$.

9. *(Final Studyguide)*
   - $S_n = \sum_{n=0}^{\infty} (-1)^n 2^{n+1} x^n$
   - Converges for $\left(-\frac{1}{2}, \frac{1}{2}\right)$
   - $2 + 4x + 8x^2 - 16x^3 + \cdots$

   —

11. *(Quiz 3)*

   $-\dfrac{1}{2}\cos^4\theta + C$

12. *(Final Studyguide)*
   $\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x} + C$

13. *(Section 9.4 and 9.5)*
   and $a_n \geq b_n$

14. *(Final Studyguide)*
   $-\frac{1}{4}\ln(x^2+3) + \frac{1}{2\sqrt{3}}\arctan(\frac{x}{\sqrt{3}}) + \frac{1}{2}\ln|x-1|$

15. *(Final Studyguide)*
   Converges absolutely (root test).

16. *(Quiz 2)*

   $\dfrac{3}{4}$

18. *(Final Studyguide)*
   $\ln\left|\frac{\sqrt{x^2+9}+x}{3}\right| + C$

21. *(Exam 2 Studyguide)*
   Use the inverse tangent integration formula

22. *(Section 9.4 and 9.5)*
   positive, continuous, and decreasing for $x \geq 1$

23. *(Quiz 4)*

   $-\dfrac{1}{12}\cot^3(4x) + \dfrac{1}{4}\cot(4x) + x + C$

25. *(Section 9.4 and 9.5)*
   and $a_n \leq b_n$

28. *(Quiz 5)*
   Because $r = |3/4| < 1$ the series converges (geometric series), and converges to the value 3.

33. *(Final Studyguide)*
   $3\arcsin\left(\frac{x-5}{5}\right) + C$

34. *(Section 9.1–9.3)*

   $$\sum_{n=0}^{\infty} ar^n$$

35. *(Section 9.1–9.3)*
   Converges, and the sum equals $\frac{e}{e-1}$.

36. *(Quiz 3)*

   $\dfrac{1}{2}x^2e^{2x} - \dfrac{1}{2}xe^{2x} + \dfrac{1}{4}e^{2x} + C$

37. *(Section 9.1–9.3)*
   1. $a_n = 4n - 1$
   2. $a_n = (-1)^{n+1}2^{2-n}$
   3. $a_n = \frac{x^{n-1}}{(n-1)!}$

38. *(Quiz 3)*

$$x^n e^x - n \int x^{n-1} e^x dx$$

39. *(Quiz 1)*

$$\sqrt{3}$$

41. *(Exam 2)*

$$\frac{5}{3} \ln |x| - \frac{2}{3} \ln |x+3| + C$$

or

$$\frac{1}{3} \ln \left| \frac{x^5}{(x+3)^2} \right| + C$$

42. *(Exam 3 Studyguide)*

(Divergence test) lim = $^1/_3$ which $\neq 0$ and therefore diverges.

45. *(Exam 2)*

$$= 2\pi + \frac{\pi^2}{2}; \quad \therefore \text{converges}$$

47. *(Section 9.1–9.3)*

Converges, and the sum equals 2.

48. *(Quiz 1)*

$$\ln \left| x^2 + 6x + 13 \right| - 3 \arctan \frac{x+3}{2} + C$$

49. *(Exam 3 Studyguide)*

(Ratio Test) Because $\lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^n}} = 0 < 1$, the series converges.

50. *(Exam 3 Studyguide)*

(Ratio test) Because $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty$, the series diverges for all x.

52. *(Final Studyguide)*
Diverges (nth term test for divergence).

53. *(Quiz 6)*
Converges conditionally.

57. *(Final Studyguide)*
Converges conditionally.

58. *(Quiz 4)*

$4 \ln 2 \to$ Converges

59. *(Final Studyguide)*
$2(\sin x)^{\frac{1}{2}} + \frac{2}{5}(\sin x)^{\frac{5}{2}} + C$

60. *(Quiz 5)*

$$a_n = 7n - 9$$

61. *(Exam 2)*

24

62. *(Final Studyguide)*

Diverges (L.C.T. versus $\sum \frac{1}{n}$, divergent p-series).

66. *(Quiz 5)*

By the limit comparison test and selection of the harmonic series/divergent p-series as the comparison series,

$$\lim_{n\to\infty} \frac{n}{n^2+1} \cdot \frac{n}{1} = 1$$

Because the limit exists and is both positive and finite, the original series behaves similarly to the comparison series, and thus diverges.

67. *(Exam 3 Studyguide)*

(Geometric series) The series has the ratio $r = {}^1/_\pi$, which because $|r| < 0$ converges. The series converges to:

$a_1 = {}^3/_\pi;\ r = {}^1/_\pi$

$$S_n = \frac{^3/_\pi}{1 - {}^1/_\pi} \approx 1.4008$$

68. *(Quiz 2)*

$7 \cosh 7x$

69. *(Section 9.4 and 9.5)*

$\underline{\text{conditionally convergent}}$; $\underline{\text{diverges}}$.

70. *(Final Studyguide)*

$2x \arcsin(4x) + \frac{4x^2}{\sqrt{1-16x62}}$

71. *(Final Studyguide)*

$\frac{2}{27}\left(82^{\frac{3}{2}} - 1\right)$

72. *(Quiz 3)*

$\cot\theta + \csc\theta + C$

73. *(Section 9.1–9.3)*

$$0.1313\cdots = \frac{13}{99}$$

75. *(Final Studyguide)*

$\frac{1}{2}x - \frac{1}{16}\cos(8x) + C$

77. *(Section 10.1)*

$$\underline{f(a)} + \cdots + \underline{\frac{f^{(n)}(a)}{n!}}$$

79. *(Section 9.1–9.3)*

$\lim = e$, therefore the sequence converges.

81. *(Final Studyguide)*

$4\sqrt{3}$

83. *(Exam 3 Studyguide)*

(Ratio test) Because $\lim_{n\to\infty}\left|\frac{n+1}{3}\right| = \infty > 1$, the series diverges.

84. *(Section 9.4 and 9.5)*

(D.C.T.) Compared against $\frac{1}{n^2}$ (convergent p-series), because $a_n \leq b_n$ the series similarly converges.

86. *(Quiz 5)*

By the integral test, $\lim_{b\to x}\int_1^b \frac{x}{x^2+1}dx = \infty$ which is non-finite, therefore the series diverges.

87. *(Exam 2 Studyguide)*

$\infty \to$ diverges

89. *(Section 9.1–9.3)*

$\underline{\text{below}}$

90. *(Quiz 2)*

Yes (verified)

92. *(Final Studyguide)*

$-\frac{5}{22}\ln|2x+1| + \frac{8}{11}\ln|x-5| + C$

93. *(Quiz 3)*

$(64\ln 4 - 32) - (16\ln 2 - 8) \approx 53.6$

95. *(Section 9.4 and 9.5)*

(Integral test) Because $\int_1^\infty \frac{1}{4x^2+1}dx = \frac{\pi}{4} - \frac{1}{2}\arctan(2) \approx 0.2318$ (converges), the series also converges.

97. *(Final Studyguide)*

$\sqrt{x^2-4} - 2\,\mathrm{arcsec}\left(\frac{x}{2}\right) + C$

98. *(Final Studyguide)*

Converges (geometric series, $|r| < 1$) to 3.

99. *(Section 9.1–9.3)*

The function oscillates between the values 1 and 3, thus limit does not exist, and therefore the sequence diverges.

100. *(Final Studyguide)*

0

101. *(Final Studyguide)*

$\frac{1}{9}(2e^3 + 1)$

106. *(Quiz 4)*

$$\frac{1}{2}\ln\left|\sqrt{4x^2+1} + 2x\right| + C$$

109. *(Quiz 6)*

(-3,1)

112. *(Section 10.1)*

$$P_{10}(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$$
$$P_{10}(0.5) \approx 0.873$$

113. *(Final Studyguide)*
Diverges (root test).

115. *(Section 9.1–9.3)*
Diverges

116. *(Quiz 2)*

$$\frac{\coth^2 x}{2} + C$$

117. *(Final Studyguide)*
$\frac{\pi}{12}$

118. *(Exam 2 Studyguide)*

$$\frac{1}{2}\ln|e^x - 1| - \frac{1}{4}\ln|e^{2x}+1| - \frac{1}{2}\arctan(e^x) + C$$

120. *(Section 10.1)*

$$P_0(x) = \frac{\sqrt{2}}{2}$$
$$P_1(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$$
$$P_2(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$$

121. *(Quiz 5)*
By the nth term test, $\lim_{n\to\infty} \frac{n}{n+3} = 1 \neq 0$, therefore the series diverges.

124. *(Exam 3 Studyguide)*
(Check for absolute convergence using Root test; check end points using A.S.T. and L.C.T.) The series converges for $2 \le x < 4$.

125. *(Section 9.4 and 9.5)*
   1. $\lim_{n\to\infty} a_n = \underline{0}$
   2. $a_{n+1} \underline{\leq} a_n$ for all n

128. *(Exam 2)*

$$\frac{1}{3}\ln|\sin t - 6| - \frac{1}{3}|\sin t - 3| + C$$

131. *(Section 9.1–9.3)*
<u>bounded both above and below</u>

133. *(Final Studyguide)*
$\frac{1}{4}\sec^4 x + C$

134. *(Final Studyguide)*
$\frac{1}{8}\ln(9 + 4x^2) + C$

136. *(Final Studyguide)*
$\frac{e^x}{1+x^{2x}}$

137. *(Final Studyguide)*
$\arcsin\left(\frac{\sin x}{5}\right) + C$

139. *(Final Studyguide)*
Converges (telescoping series) to $\frac{11}{18}$.

141. *(Final Studyguide)*
Converges (ratio test).

142. *(Final Studyguide)*
$\frac{3}{2}$

143. *(Final Studyguide)*
$\frac{1}{3}e^{3x} + C$

144. *(Exam 2 Studyguide)*

$$-\frac{1}{2}\left(\cos(2x) - \frac{1}{3}\cos^3(2x)\right) + C$$

145. *(Section 9.1–9.3)*
   1. <u>non-decreasing</u> ($a_1 \underline{\leq} \cdots \underline{\leq} a_n$), or
   2. <u>non-increasing</u> ($a_1 \underline{\geq} \cdots \underline{\geq} a_n$).

146. *(Quiz 2)*

$$8\sinh(4x)\cosh(4x)$$

147. *(Quiz 4)*

$$4\ln(x^2+2) + \frac{3}{2(x^2+2)} + C$$

149. *(Quiz 1)*

$$\sqrt{4-x^2}$$

150. *(Section 9.4 and 9.5)*

(D.C.T.) Comparing against $b_n = \sum_{n=1}^{\infty} \frac{1}{2^n}$ (convergent geometric series), because $a_n < b_n$ the series similarly converges.

151. *(Exam 3 Studyguide)*

(Check for absolute convergence using Ratio Test) Because $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$, the series converges absolutely for all values of x.

152. *(Section 10.1)*

$$e^{-0.06} \approx 0.9418$$

153. *(Final Studyguide)*

$\frac{1}{2}e^{x^2} + C$

154. *(Quiz 6)*

(P-series) Because $p = 2/3 < 1$ the series diverges.

155. *(Section 9.4 and 9.5)*

(L.C.T.) Comparing against $\frac{1}{n^{3/2}}$ (convergent p-series), because $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$, which is finite and positive, the series similarly converges.

159. *(Final Studyguide)*

$\frac{1}{2}x^2 + x - 3\ln|x| + 5\ln|x-1| + C$

162. *(Section 9.1–9.3)*

<u>above</u>

163. *(Exam 2)*

$$-\frac{1}{3(e^x+4)^3}dx$$

164. *(Exam 2)*

$$\frac{3}{4}x + \frac{1}{10}\sin 10x \frac{1}{80}\sin 10x + C$$

166. *(Exam 2 Studyguide)*

Use u-substitution

167. *(Exam 3 Studyguide)*

1. (Check for absolute convergence using L.C.T.) Comparing against $\sum_{n=1}^{\infty} \frac{1}{n}$ (divergent p-series):

   $$\lim_{n\to\infty} \left|\frac{3}{2n+1} \cdot \frac{n}{1}\right| = \frac{3}{2}$$

   Which is both finite and positive, and so diverges similarly. We need to check for conditional convergence.

2. (A.S.T.)

   (a) $\lim_{n\to\infty} \frac{(-1)^n 3}{2n+1} = 0$ ✓

   (b) $a_{n+1} < a_n$ ✓

168. *(Exam 2)*

$$\frac{1}{10}\arctan 5x + \frac{x}{50x^2+2} + C$$

170. *(Exam 2)*

$$\ln\left|\frac{x-5+\sqrt{(x-5)^2-4}}{2}\right| + C$$

172. *(Section 9.1–9.3)*

lim = 1, therefore the sequence converges.

173. *(Final Studyguide)*

$\frac{1}{3}x^2\sin(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + C$

175. *(Final Studyguide)*

$\frac{1}{12}\arctan\left(\frac{2x^2}{3}\right) + C$

180. *(Exam 2)*

$$= \infty; \quad \therefore \text{diverges}$$

181. *(Final Studyguide)*

$\frac{\pi}{24}\left(17^{\frac{3}{2}} - 1\right)$

182. *(Quiz 5)*

Because $r = |1.2| > 1$ the series diverges (geometric series).

184. *(Exam 2)*

≈ 2.68

185. *(Quiz 2)*

$\frac{37}{35}$

187. *(Final Studyguide)*
$-\frac{2}{3}\sqrt{2-3x}+C$

188. *(Section 9.4 and 9.5)*
    1. absolute convergence
    2. diverges; conditional convergence; alternating series test.

191. *(Exam 3 Studyguide)*
(L.C.T.) Comparing against $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ (convergent p-series),

$$\lim_{n\to\infty}\frac{1}{n\sqrt{n+1}}\cdot\frac{n\sqrt{n}}{1}=1$$

Which is both finite and positive, and so converges similarly.

193. *(Section 10.1)*

$P_0(x)=0$
$P_1(x)=8x$
$P_2(x)=8x-32x^2$

196. *(Quiz 2)*

$y=Ce^{x/16}+176$

197. *(Quiz 6)*
(Integral test) Because lim $=\infty$ the series diverges.

198. *(Quiz 5)*
By means of partial fraction decomposition and evaluation of a telescopic series, the series converges to the value $^{11}/_{18}$.

201. *(Quiz 1)*

$\frac{5\pi}{6}$

203. *(Quiz 3)*

$\frac{1}{3}$

207. *(Final Studyguide)*
e^{ab}

208. *(Exam 2 Studyguide)*

$2\sqrt{3}\to$ converges

210. *(Final Studyguide)*
Diverges (geometric series, $|r|>=1$).

212. *(Quiz 5)*

$a_n=3(2)^{n-1}$

214. *(Exam 2 Studyguide)*
Use trig-substitution

216. *(Section 9.4 and 9.5)*
absolutely convergent

217. *(Quiz 5)*

$\{1,3,7,15\}$

218. *(Exam 2 Studyguide)*

$$\frac{1}{2}x^3e^{2x}-\frac{3}{4}x^2e^{2x}+\frac{3}{4}xe^{2x}-\frac{3}{8}e^{2x}+C$$

219. *(Final Studyguide)*
$\frac{1}{4}-\frac{1}{4}(x-2)+\frac{3}{16}(x-2)^2-\frac{1}{8}(x-2)^3$

220. *(Exam 2)*

$$\frac{\tan^3 9t}{27} - \frac{\tan 9t}{9} + t + C$$

221. *(Quiz 2)*

$$\arcsin \frac{9x}{8} + C$$

222. *(Exam 3 Studyguide)*

   1. $S_4 = -\frac{1}{3} + \frac{1}{17} - \frac{1}{55} + \frac{1}{129} \approx -0.2849$, and by alternating series remainder theorem, error $= a_5 = \left|\frac{1}{251}\right|$.
   2. $$a_{n+1} = \left|\frac{(-1)^n}{2n^3+1}\right| \leq 0.0001$$
      $$16.099 \leq n$$
      $$n = 17$$