Exam 3 Review (Problems)

1. Use the direct comparison test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

2. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

3. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} n!(x-1)^n$$

4. A geometric series with ratio r will:
   1. Diverge if ______.
   2. Converge to $S =$ ______ if ______.

5. Determine if the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

6. A sequence is bounded ______ if there is a number M such that $a_n \geq M$ for all n.

7. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$$

8. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} 3(\pi)^{-n}$$

9. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{-2, 5, 12, 19, \dots\}$$

10. Given a suitable a_n and b_n to compare against, the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge if $\lim_{n\to\infty} \left(\frac{?}{?}\right) = L$ where L is both ______ and ______.

11. The series $\sum_{n=1}^{\infty} |a_n|$ is ______ if $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} |a_n|$ ______.

12. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \frac{n}{n+3}$$

13. The series $\sum_{n=1}^{\infty} a_n$ is ______ if $\sum_{n=1}^{\infty} |a_n|$ converges.

14. A sequence is bounded ______ if there is a number M such that $a_n \leq M$ for all n.

15. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = 2 + (-1)^n$$

16. A geometric series is a series of the form

$$\sum_{n=0}^{\infty} \underline{\quad ? \quad}$$

17. If $\sum_{n=1}^{\infty} b_n$ diverges and a_n ________ b_n, then $\sum_{n=1}^{\infty} a_n$ also diverges.

18. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

19. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{n!}{3^n}$$

20. If f has n derivatives at center a, then the polynomial

$$P_n(x) = \underline{\hspace{2cm}} + \cdots + \underline{\hspace{2cm}}$$

is called the nth degree Taylor polynomial for f at a.

21. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} e^{-n}$$

22. Write the first four terms of the sequence

$$a_{n+1} = 2a_n + 1, \; a_1 = 1$$

23. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \frac{n}{n+1}$$

24. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \left(1 + \frac{1}{n}\right)^n$$

25. A sequence is *monotonic* if all of its terms are entirely either:

    1. ______ (a_1 ______ $\cdots$ ______ a_n), or
    2. ______ (a_1 ______ $\cdots$ ______ a_n).

26. Determine the convergence or divergence of the series using the *limit comparison test*

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

27. Given the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^3+1}$$

    1. Use the first 4 terms to approximate the sum; then explain what the maximum error is in this approximation.
    2. Determine how many terms are required to estimate the sum with an error of no more than 0.0001.

28. If $\sum_{n=1}^{\infty} b_n$ converges and a_n ________ b_n, then $\sum_{n=1}^{\infty} a_n$ also converges.

29. Use the limit comparison test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

30. Determine the convergence or divergence of the series using the *integral test*

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

31. Write the nth-term formula for the following sequences

    1. $\{3, 7, 11, 15, \dots\}$
    2. $\{2, -1, \frac{1}{2}, -\frac{1}{4}, \dots\}$
    3. $\{1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \dots\}$

32. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^2$$

33. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$$

34. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$$

35. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(5x)^n}{n^2}$$

36. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{3, 6, 12, 24, 48, \dots\}$$

37. Use the integral test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{1}{4n^2+1}$$

38. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

39. If you are given an alternating series:

    1. Check for ______ by applying a test on $\sum_{n=1}^{\infty} |a_n|$
    2. If the absolute value of the series ______, then test for ______ using the ______.

40. Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$$

41. Find the Taylor polynomials of degrees 0, 1, and 2, of the function f centered at point a

$$f(x) = \ln(1+8x)$$

42. Find the 10th degree Maclaurin Polynomial for

$$f(x) = \cos x$$

And then compare the value of $\cos 0.5$ approximated by the polynomial with the value given by a calculator (≈ 0.878).

43. Consider the repeating decimal $0.1313\cdots$; Convert this decimal to a fraction.

44. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \frac{1}{2^n}$$

45. A sequence is bounded if it is

________________________________.

46. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=0}^{\infty} (1.2)^n$$

47. Find the Taylor polynomials of degrees 0, 1, and 2, of the function f centered at point a

$$f(x) = \cos x; a = \frac{\pi}{4}$$

48. Find the limit of the sequence and state whether the sequence converges or diverges

$$\{1, {}^4\!/_3, {}^9\!/_7, {}^{16}\!/_{15}, {}^{25}\!/_{31}\}$$

49. The series $\sum_{n=1}^{\infty} (-1)^n$ and $\sum_{n=1}^{\infty} (-1)^{n+b}$ converge if the following two conditions are met:

    1. $\lim_{n\to\infty} a_n = \underline{?}$
    2. $a_{n+1} \,\underline{?}\, a_n$ for all n

50. Approximate $e^{-0.06}$ using the Taylor polynomial

$$P_2(x) = 1 - x + \frac{x^2}{2}$$

51. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$$

52. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \frac{1}{\sqrt[3]{25}} + \cdots$$

53. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3}{2n+1}$$

54. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(\ln n)(x-3)^n}{n}$$

55. With $a_n = f(n)$, $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ either both diverge if f is ______, ______, and ______ for $x \geq 1$.

56. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^n}$$

Exam 3 Review (Answers)

1. *(Section 9.4 and 9.5)*

   (D.C.T.) Comparing against $b_n = \sum_{n=1}^{\infty} \frac{1}{2^n}$ (convergent geometric series), because $a_n < b_n$ the series similarly converges.

2. *(Quiz 5)*

   By means of partial fraction decomposition and evaluation of a telescopic series, the series converges to the value $^{11}/_{18}$.

3. *(Exam 3 Studyguide)*

   (Ratio test) Because $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, the series diverges for all x.

4. *(Section 9.1–9.3)*

   1. Diverge if $|r| \geq 1$.
   2. Converge to $S = \dfrac{a}{1-r}$ if $0 < |r| < 1$.

5. *(Section 9.4 and 9.5)*

   (D.C.T.) Compared against $\frac{1}{n^2}$ (convergent p-series), because $a_n \leq b_n$ the series similarly converges.

6. *(Section 9.1–9.3)*

   below

7. *(Exam 3 Studyguide)*

   (Check for absolute convergence using Ratio Test) Because $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$, the series converges absolutely for all values of x.

8. *(Exam 3 Studyguide)*

   (Geometric series) The series has the ratio $r = {}^1/\pi$, which because $|r| < 0$ converges. The series converges to:

   $a_1 = {}^3/\pi;\ r = {}^1/\pi$

   $$S_n = \frac{{}^3/\pi}{1 - {}^1/\pi} \approx 1.4008$$

9. *(Quiz 5)*

   $$a_n = 7n - 9$$

10. *(Section 9.4 and 9.5)*

    $\underline{\frac{a_n}{b_n}}$; finite and positive.

11. *(Section 9.4 and 9.5)*

    conditionally convergent; diverges.

12. *(Quiz 5)*

    By the nth term test, $\lim_{n\to\infty} \frac{n}{n+3} = 1 \neq 0$, therefore the series diverges.

13. *(Section 9.4 and 9.5)*

    absolutely convergent

14. *(Section 9.1–9.3)*

    above

15. *(Section 9.1–9.3)*

    The function oscillates between the values 1 and 3, thus limit does not exist, and therefore the sequence diverges.

16. *(Section 9.1–9.3)*

    $$\sum_{n=0}^{\infty} ar^n$$

17. *(Section 9.4 and 9.5)*

    and $a_n \geq b_n$

18. *(Quiz 6)*

    Converges conditionally.

19. *(Exam 3 Studyguide)*

    (Ratio test) Because $\lim_{n\to\infty} \left|\frac{n+1}{3}\right| = \infty > 1$, the series diverges.

20. *(Section 10.1)*

    $$\underline{f(a)} + \cdots + \underline{\frac{f^{(n)}(a)}{n!}}$$

21. *(Section 9.1–9.3)*

    Converges, and the sum equals $\frac{e}{e-1}$.

22. *(Quiz 5)*

    $$\{1, 3, 7, 15\}$$

23. *(Section 9.1–9.3)*

    lim = 1, therefore the sequence converges.

24. *(Section 9.1–9.3)*

    $\lim = e$, therefore the sequence converges.

25. *(Section 9.1–9.3)*

    1. non-decreasing ($a_1 \leq \cdots \leq a_n$), or
    2. non-increasing ($a_1 \geq \cdots \geq a_n$).

26. *(Quiz 5)*

    By the limit comparison test and selection of the harmonic series/divergent p-series as the comparison series,

    $$\lim_{n\to\infty} \frac{n}{n^2+1} \cdot \frac{n}{1} = 1$$

    Because the limit exists and is both positive and finite, the original series behaves similarly to the comparison series, and thus diverges.

27. *(Exam 3 Studyguide)*

    1. $S_4 = -\frac{1}{3} + \frac{1}{17} - \frac{1}{55} + \frac{1}{129} \approx -0.2849$, and by alternating series remainder theorem, error $= a_5 = \left|\frac{1}{251}\right|$.
    2. $$a_{n+1} = \left|\frac{(-1)^n}{2n^3+1}\right| \leq 0.0001$$

       $16.099 \leq n$

       $n = 17$

28. *(Section 9.4 and 9.5)*

    and $a_n \leq b_n$

29. *(Section 9.4 and 9.5)*

    (L.C.T.) Comparing against $\frac{1}{n^{3/2}}$ (convergent p-series), because $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$, which is finite and positive, the series similarly converges.

30. *(Quiz 5)*

    By the integral test, $\lim_{b\to x} \int_1^b \frac{x}{x^2+1} dx = \infty$ which is non-finite, therefore the series diverges.

31. *(Section 9.1–9.3)*

    1. $a_n = 4n - 1$
    2. $a_n = (-1)^{n+1} 2^{2-n}$
    3. $a_n = \frac{x^{n-1}}{(n-1)!}$

32. *(Section 9.1–9.3)*

    Diverges

33. *(Quiz 5)*

    Because $r = |3/4| < 1$ the series converges (geometric series), and converges to the value 3.

34. *(Exam 3 Studyguide)*

    (L.C.T.) Comparing against $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ (convergent p-series),

    $$\lim_{n\to\infty} \frac{1}{n\sqrt{n+1}} \cdot \frac{n\sqrt{n}}{1} = 1$$

    Which is both finite and positive, and so converges similarly.

35. *(Exam 3 Studyguide)*

    The series converges for $-1/5 \leq x \leq 1/5$.

36. *(Quiz 5)*

    $a_n = 3(2)^{n-1}$

37. *(Section 9.4 and 9.5)*

    (Integral test) Because $\int_1^{\infty} \frac{1}{4x^2+1} dx = \frac{\pi}{4} - \frac{1}{2}\arctan(2) \approx 0.2318$ (converges), the series also converges.

38. *(Quiz 6)*

    (Integral test) Because $\lim = \infty$ the series diverges.

39. *(Section 9.4 and 9.5)*

    1. absolute convergence
    2. diverges; conditional convergence; alternating series test.

40. *(Quiz 6)*

    (-3,1)

41. *(Section 10.1)*

    $P_0(x) = 0$

    $P_1(x) = 8x$

    $P_2(x) = 8x - 32x^2$

42. *(Section 10.1)*

    $$P_{10}(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$$

    $P_{10}(0.5) \approx 0.873$

43. *(Section 9.1–9.3)*

$$0.1313\cdots = \frac{13}{99}$$

44. *(Section 9.1–9.3)*

Converges, and the sum equals 2.

45. *(Section 9.1–9.3)*

bounded both above and below

46. *(Quiz 5)*

Because $r = |1.2| > 1$ the series diverges (geometric series).

47. *(Section 10.1)*

$$P_0(x) = \frac{\sqrt{2}}{2}$$
$$P_1(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$$
$$P_2(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$$

48. *(Section 9.1–9.3)*

$\lim = 0$, therefore the sequence converges.

49. *(Section 9.4 and 9.5)*

1. $\lim_{n\to\infty} a_n = \underline{0}$
2. $a_{n+1} \underline{\leq} a_n$ for all n

50. *(Section 10.1)*

$$e^{-0.06} \approx 0.9418$$

51. *(Exam 3 Studyguide)*

(Divergence test) $\lim = 1/3$ which $\neq 0$ and therefore diverges.

52. *(Quiz 6)*

(P-series) Because $p = 2/3 < 1$ the series diverges.

53. *(Exam 3 Studyguide)*

1. (Check for absolute convergence using L.C.T.) Comparing against $\sum_{n=1}^{\infty} \frac{1}{n}$ (divergent p-series):

$$\lim_{n\to\infty} \left|\frac{3}{2n+1} \cdot \frac{n}{1}\right| = \frac{3}{2}$$

Which is both finite and positive, and so diverges similarly. We need to check for conditional convergence.

2. (A.S.T.)
   (a) $\lim_{n\to\infty} \frac{(-1)^n 3}{2n+1} = 0$ ✓
   (b) $a_{n+1} < a_n$ ✓

54. *(Exam 3 Studyguide)*

(Check for absolute convergence using Root test; check end points using A.S.T. and L.C.T.) The series converges for $2 \leq x < 4$.

55. *(Section 9.4 and 9.5)*

positive, continuous, and decreasing for $x \geq 1$

56. *(Exam 3 Studyguide)*

(Ratio Test) Because $\lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^n}} = 0 < 1$, the series converges.