Exam 3 Review (Problems)

1. Find the limit of the sequence and state whether the sequence converges or diverges

   $\{1, {}^4/_3, {}^9/_7, {}^{16}/_{15}, {}^{25}/_{31}\}$

2. Consider the repeating decimal $0.1313\cdots$; Convert this decimal to a fraction.

3. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

   $$\sum_{n=1}^{\infty} \frac{(5x)^n}{n^2}$$

4. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

   $$\sum_{n=1}^{\infty} \frac{(-1)^n 3}{2n+1}$$

5. Find the interval of convergence for the power series

   $$\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$$

6. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

   $$\sum_{n=1}^{\infty} 3(\pi)^{-n}$$

7. With $a_n = f(n)$, $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ either both diverge if f is ______, ______, and ______ for $x \geq 1$.

8. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

   $$1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \frac{1}{\sqrt[3]{25}} + \cdots$$

9. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

   $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$$

10. Determine if the series converges or diverges

    $$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

11. Write the first four terms of the sequence

    $a_{n+1} = 2a_n + 1, \; a_1 = 1$

12. If $\sum_{n=1}^{\infty} b_n$ diverges and a_n ________ b_n, then $\sum_{n=1}^{\infty} a_n$ also diverges.

13. If f has n derivatives at center a, then the polynomial

    $P_n(x) =$ ______ $+ \cdots +$ ______

    is called the nth degree Taylor polynomial for f at a.

14. Determine the convergence or divergence of the series using the *integral test*

    $$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

15. Find the limit of the sequence and state whether the sequence converges or diverges

    $$a_n = \frac{n}{n+1}$$

16. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

    $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

17. A sequence is bounded ______ if there is a number M such that $a_n \leq M$ for all n.

18. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$$

19. Use the direct comparison test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

20. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^2$$

21. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{n!}{3^n}$$

22. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = 2 + (-1)^n$$

23. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^n}$$

24. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{3, 6, 12, 24, 48, \dots\}$$

25. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$$

26. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \frac{n}{n+3}$$

27. A sequence is bounded if it is ______________________________.

28. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \left(1 + \frac{1}{n}\right)^n$$

29. The series $\sum_{n=1}^{\infty} (-1)^n$ and $\sum_{n=1}^{\infty} (-1)^{n+b}$ converge if the following two conditions are met:

    1. $\lim_{n\to\infty} a_n = \underline{?}$
    2. $a_{n+1} \underline{?}\, a_n$ for all n

30. Given a suitable a_n and b_n to compare against, the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge if $\lim_{n\to\infty} \left(\frac{\underline{?}}{\underline{?}}\right) = L$ where L is both ______ and ______.

31. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \frac{1}{2^n}$$

32. Given the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^3+1}$$

    1. Use the first 4 terms to approximate the sum; then explain what the maximum error is in this approximation.
    2. Determine how many terms are required to estimate the sum with an error of no more than 0.0001.

33. The series $\sum_{n=1}^{\infty} a_n$ is ______ if $\sum_{n=1}^{\infty} |a_n|$ converges.

34. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

35. If $\sum_{n=1}^{\infty} b_n$ converges and a_n _______ b_n, then $\sum_{n=1}^{\infty} a_n$ also converges.

36. A sequence is bounded ______ if there is a number M such that $a_n \geq M$ for all n.

37. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} n!(n-1)^n$$

38. A sequence is *monotonic* if all of its terms are entirely either:

    1. ______ (a_1 ______ $\cdots$ ______ a_n), or
    2. ______ (a_1 ______ $\cdots$ ______ a_n).

39. If you are given an alternating series:

    1. Check for ______ by applying a test on $\sum_{n=1}^{\infty} |a_n|$
    2. If the absolute value of the series ______, then test for ______ using the ______.

40. A geometric series is a series of the form

$$\sum_{n=0}^{\infty} \underline{\quad ? \quad}$$

41. Use the limit comparison test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

42. Use the integral test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{1}{4n^2+1}$$

43. Determine the convergence or divergence of the series using the *limit comparison test*

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

44. The series $\sum_{n=1}^{\infty} |a_n|$ is ______ if $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} |a_n|$ ______.

45. Write the nth-term formula for the following sequences

    1. $\{3, 7, 11, 15, \dots\}$
    2. $\{2, -1, \frac{1}{2}, -\frac{1}{4}, \dots\}$
    3. $\{1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \dots\}$

46. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

47. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$$

48. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=0}^{\infty} (1.2)^n$$

49. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(\ln n)(x-3)^n}{n}$$

50. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{-2, 5, 12, 19, \dots\}$$

51. Does the following series converge or diverge?
    If it converges, find the sum.

$$\sum_{n=0}^{\infty} e^{-n}$$

52. A geometric series with ratio r will:

    1. Diverge if ______.
    2. Converge to $S =$ ______ if ______.

Exam 3 Review (Answers)

1. *(Section 9.1–9.3)*

   lim = 0, therefore the sequence converges.

2. *(Section 9.1–9.3)*

   $$0.1313\cdots = \frac{13}{99}$$

3. *(Exam 3 Studyguide)*

   The series converges for $-1/5 \le x \le 1/5$.

4. *(Exam 3 Studyguide)*

   1. (Check for absolute convergence using L.C.T.) Comparing against $\sum_{n=1}^{\infty} \frac{1}{n}$ (divergent p-series):

      $$\lim_{n\to\infty} \left| \frac{3}{2n+1} \cdot \frac{n}{1} \right| = \frac{3}{2}$$

      Which is both finite and positive, and so diverges similarly. We need to check for conditional convergence.
   2. (A.S.T.)

      (a) $\lim_{n\to\infty} \dfrac{(-1)^n 3}{2n+1} = 0$ ✓

      (b) $a_{n+1} < a_n$ ✓

5. *(Quiz 6)*

   (-3,1)

6. *(Exam 3 Studyguide)*

   (Geometric series) The series has the ratio $r = 1/\pi$, which because $|r| < 0$ converges. The series converges to:

   $a_1 = 3/\pi;\ r = 1/\pi$

   $$S_n = \frac{3/\pi}{1 - 1/\pi} \approx 1.4008$$

7. *(Section 9.4)*

   positive, continuous, and decreasing for $x \ge 1$

8. *(Quiz 6)*

   (P-series) Because $p = 2/3 < 1$ the series diverges.

9. *(Exam 3 Studyguide)*

   (Check for absolute convergence using Ratio Test) Because $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$, the series converges absolutely for all values of x.

10. *(Section 9.4)*

    (D.C.T.) Compared against $\frac{1}{n^2}$ (convergent p-series), because $a_n \le b_n$ the series similarly converges.

11. *(Quiz 5)*

    $$\{1, 3, 7, 15\}$$

12. *(Section 9.4)*

    and $a_n \ge b_n$

13. *(Section 9.4)*

    $$\underline{f(a)} + \cdots + \underline{\frac{f^{(n)}(a)}{n!}}$$

14. *(Quiz 5)*

    By the integral test, $\lim_{b\to x} \int_1^b \frac{x}{x^2+1} dx = \infty$ which is non-finite, therefore the series diverges.

15. *(Section 9.1–9.3)*

    lim = 1, therefore the sequence converges.

16. *(Quiz 6)*

    Converges conditionally.

17. *(Section 9.1–9.3)*

    above

18. *(Exam 3 Studyguide)*

    (L.C.T.) Comparing against $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ (convergent p-series),

    $$\lim_{n\to\infty} \frac{1}{n\sqrt{n+1}} \cdot \frac{n\sqrt{n}}{1} = 1$$

    Which is both finite and positive, and so converges similarly.

19. *(Section 9.4)*

    (D.C.T.) Comparing against $b_n = \sum_{n=1}^{\infty} \frac{1}{2^n}$ (convergent geometric series), because $a_n < b_n$ the series similarly converges.

20. *(Section 9.1–9.3)*

    Diverges

21. *(Exam 3 Studyguide)*

    (Ratio test) Because $\lim_{n\to\infty}\left|\frac{n+1}{3}\right| = \infty > 1$, the series diverges.

22. *(Section 9.1–9.3)*

    The function oscillates between the values 1 and 3, thus limit does not exist, and therefore the sequence diverges.

23. *(Exam 3 Studyguide)*

    (Ratio Test) Because $\lim_{n\to\infty}\sqrt[n]{\frac{2^n}{n^n}} = 0 < 1$, the series converges.

24. *(Quiz 5)*

    $$a_n = 3(2)^{n-1}$$

25. *(Exam 3 Studyguide)*

    (Divergence test) $\lim = 1/3$ which $\neq 0$ and therefore diverges.

26. *(Quiz 5)*

    By the nth term test, $\lim_{n\to\infty}\frac{n}{n+3} = 1 \neq 0$, therefore the series diverges.

27. *(Section 9.1–9.3)*

    bounded both above and below

28. *(Section 9.1–9.3)*

    $\lim = e$, therefore the sequence converges.

29. *(Section 9.4)*

    1. $\lim_{n\to\infty} a_n = \underline{0}$
    2. $a_{n+1} \underline{\leq} a_n$ for all n

30. *(Section 9.4)*

    $\underline{\frac{a_n}{b_n}}$; finite and positive.

31. *(Section 9.1–9.3)*

    Converges, and the sum equals 2.

32. *(Exam 3 Studyguide)*

    1. $S_4 = -\frac{1}{3} + \frac{1}{17} - \frac{1}{55} + \frac{1}{129} \approx -0.2849$, and by alternating series remainder theorem, error $= a_5 = \left|\frac{1}{251}\right|$.
    2. $$a_{n+1} = \left|\frac{(-1)^n}{2n^3+1}\right| \leq 0.0001$$

       $16.099 \leq n$

       $n = 17$

33. *(Section 9.4)*

    absolutely convergent

34. *(Quiz 5)*

    By means of partial fraction decomposition and evaluation of a telescopic series, the series converges to the value $11/18$.

35. *(Section 9.4)*

    and $a_n \leq b_n$

36. *(Section 9.1–9.3)*

    below

37. *(Exam 3 Studyguide)*

    (Ratio test) Because $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, the series diverges for all x.

38. *(Section 9.1–9.3)*

    1. non-decreasing ($a_1 \underline{\leq} \cdots \underline{\leq} a_n$), or
    2. non-increasing ($a_1 \underline{\geq} \cdots \underline{\geq} a_n$).

39. *(Section 9.4)*

    1. absolute convergence
    2. diverges; conditional convergence; alternating series test.

40. *(Section 9.1–9.3)*

    $$\sum_{n=0}^{\infty} ar^n$$

41. *(Section 9.4)*

    (L.C.T.) Comparing against $\frac{1}{n^{3/2}}$ (convergent p-series), because $\lim_{n\to\infty}\frac{a_n}{b_n} = 1$, which is finite and positive, the series similarly converges.

42. *(Section 9.4)*

    (Integral test) Because $\int_1^{\infty}\frac{1}{4x^2+1}dx = \frac{\pi}{4} - \frac{1}{2}\arctan(2) \approx 0.2318$ (converges), the series also converges.

43. *(Quiz 5)*

    By the limit comparison test and selection of the harmonic series/divergent p-series as the comparison series,

    $$\lim_{n\to\infty}\frac{n}{n^2+1}\cdot\frac{n}{1} = 1$$

    Because the limit exists and is both positive and finite, the original series behaves similarly to the comparison series, and thus diverges.

44. *(Section 9.4)*

    conditionally convergent; diverges.

45. *(Section 9.1–9.3)*

    1. $a_n = 4n - 1$
    2. $a_n = (-1)^{n+1} 2^{2-n}$
    3. $a_n = \frac{x^{n-1}}{(n-1)!}$

46. *(Quiz 6)*

    (Integral test) Because lim $= \infty$ the series diverges.

47. *(Quiz 5)*

    Because $r = |3/4| < 1$ the series converges (geometric series), and converges to the value 3.

48. *(Quiz 5)*

    Because $r = |1.2| > 1$ the series diverges (geometric series).

49. *(Exam 3 Studyguide)*

    (Check for absolute convergence using Root test; check end points using A.S.T. and L.C.T.) The series converges for $2 \le x < 4$.

50. *(Quiz 5)*

    $a_n = 7n - 9$

51. *(Section 9.1–9.3)*

    Converges, and the sum equals $\frac{e}{e-1}$.

52. *(Section 9.1–9.3)*

    1. Diverge if $|r| \ge 1$.
    2. Converge to $S = \dfrac{a}{1-r}$ if $0 < |r| < 1$.