Cumulative Review (Problems)

1. Find the exact value of the function

   $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

2. Find the exact value of the function

   $\operatorname{arccsc}(-2)$

3. Evaluate or simplify

   $\arccos\left(\cos\left(-\frac{\pi}{3}\right)\right)$

4. Evaluate or simplify

   $\cos(2\arccos x)$

5. Find the derivative of y with respect to x:

   $y = 3\arcsin(4x^3)$

6. $$\int \frac{1}{x\sqrt{9x^2-6}}dx$$

7. $$\int_0^1 \frac{1}{\sqrt{16-x^2}}dx$$

8. Verify the identity using the definitions of hyperbolic functions

   $$\coth x = \frac{e^{2x}+1}{e^{2x}-1}$$

9. Compute $\frac{dy}{dx}$ for the function

   $y = \sinh^2 7x$

10. Compute $\frac{dy}{dx}$ for the function

    $y = \ln\sinh 7x$

11. $$\int \frac{\sinh x}{1+\cosh x}dx$$

12. Evaluate the expression without a calculator to a value or to show that the value does not exist. Simplify the answer to the extent possible

    $\sinh(2\ln 5)$

13. Find the length of the curve

    $y = 3x^{\frac{3}{2}};$ from $x = 0$ to $x = \frac{5}{9}$

14. Find the area of the surface generated when the given curve is revolved about the x-axis

    $y = \frac{x^3}{3} + \frac{1}{4x};$ from $x = 1$ to $x = 2$

15. Find the general solution of the equation. Express the solution explicitly as a function of the independent variable

    $e^{9t}y'(t) = -2$

16. A conservation organization releases 40 coyotes into a preserve. After 4 years, there are 70 coyotes in the preserve. The preserve has a carrying capacity of 175.

    1. Write a logistic function that models the population, $P(t)$, of coyotoes in the preserve.
    2. Use your answer from (a) to find $\lim_{t\to\infty} P(t)$

17. Find the function $y = f(t)$ passing through the point $(0, 15)$ with the first derivate

    $$\frac{dy}{dt} = \frac{1}{4}t$$

18. $$\lim_{x\to-\infty} 4\sinh x$$

19. Find the equation of the line tangent to the curve

$$2x + \arctan y = y^2 - 1; \text{ at the point } P(\frac{-\pi}{8}, -1)$$

20.

$$\int \frac{x^2+3}{x\sqrt{x^2-4}}dx$$

21. Use l'Hopital's rule to evaluate the limit

$$\lim_{x\to\frac{\pi}{3}} \frac{\cos(x)-\frac{1}{2}}{x-\frac{x}{3}}$$

22. Evaluate the limit

$$\lim_{x\to\infty} x\left(\frac{\pi}{2} - \arctan x\right)$$

23. Evaluate

$$\int x^3 e^{2x} dx$$

24. Evaluate

$$\int \sin^3(2x)dx$$

25. Evaluate

$$\int \frac{\sqrt{x^2-9}}{x}dx$$

$$\frac{\sqrt{x^2-9}}{3} - \arctan\frac{x}{3} + C$$

26. Evaluate

$$\int \frac{3x-1}{x^2-5x+4}dx$$

27. Evaluate

$$\int_1^\infty \frac{1}{\sqrt{x+2}}dx$$

28. Evaluate

$$\int_2^5 \frac{1}{\sqrt{x-2}}dx$$

29. How would you approach the following?

$$\int \frac{1}{x^2+1}dx$$

30. How would you approach the following?

$$\int \frac{x}{x^2+1}dx$$

31. How would you approach the following?

$$\int \frac{1}{\sqrt{x^2+1}}dx$$

32. Prove the reduction formula:

$$\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1}dx$$

33. Evaluate

$$\int \frac{e^x}{(e^{2x}+1)(e^x-1)}dx$$

34. Evaluate

$$\int \frac{e^x}{(e^x+4)^{-4}}dx$$

35. Evaluate

$$\int_1^{e^2} \frac{\ln^2(x^3)}{x}dx$$

36. Evaluate

$$\int \frac{1}{\sqrt{x^2-10x+21}}dx$$

37.

$$\int \arccos x\ dx$$

38. Evaluate

$$\int_1^3 \ln 2x\ dx$$

39. Evaluate

$$\int 2\cos^4 5x \; dx$$

40. Evaluate

$$\int \tan^4 9t \; dt$$

41. Evaluate

$$\int \frac{1}{(25x^2+1)^2} dx$$

42. Evaluate

$$\int \frac{x+5}{x^2+3x} dx$$

43. Evaluate

$$\int \frac{cost}{\sin^2 t - 9\sin t + 18} dt$$

44. Evaluate

$$\int_0^\infty \frac{4(1+\arctan x)}{1+x^2} dx$$

45. Evaluate

$$\int_{-\infty}^{e} 23e^{-x} dx$$

46. Evaluate

$$\arccos\left(-\frac{\sqrt{3}}{2}\right)$$

47. Evaluate

$$\tan\left(\arccos\left(\frac{1}{2}\right)\right)$$

48. Differentiate

$$y = \frac{1}{2}\left[x\sqrt{4-x^4} + 4\arcsin\frac{x}{2}\right]$$

49. Evaluate the integral

$$\int \frac{2x}{x^2+6x+13} dx$$

50. Evaluate

$$\int \frac{9}{\sqrt{64-81x^2}} dx$$

51. Compute $\frac{dy}{dx}$ for the function

$$y = \sinh 7x$$

52. Compute $\frac{dy}{dx}$ for the function:

$$y = \sinh^2 4x$$

53. Evaluate

$$\int -\operatorname{csch}^2 x \coth x \; dx$$

54. Evaluate

$$\int_0^{\ln 2} \cosh x \; dx$$

55. Evaluate the following without use of a calculator

$$\coth(\ln 6)$$

56. Determine if the given function y is a solution of the differential equation y''. Assume that C is an arbitrary constant.

$$y = C_1 \sin 5t + C_2 \cos 5t; \quad y''(t) + 25y = 0$$

57. Find the general solution of the equation

$$y'(t) - \frac{y}{16} = -11$$

58. Evaluate

$$\int \frac{-\csc\theta}{\csc\theta - \cot\theta} d\theta$$

59. Evaluate

$$\int e^{2x}x^2\ dx$$

60. Evaluate

$$\int_2^4 8x\ln x\ dx$$

61. Use integration by parts to establish a reduction formula for the integral

$$\int x^n e^x dx$$

62. Evaluate

$$\int_0^{\pi/4} \sin^3 4x\ dx$$

63. Evaluate

$$\int \cos^2\theta \sin 2\theta\ d\theta$$

64. Use trig substitution to evaluate

$$\int \frac{1}{\sqrt{4x^2+1}}dx$$

65. Evaluate

$$\int \frac{8x^3+13x}{(x^2+2)^2}dx$$

66.

$$\int_1^\infty \frac{4}{(1+x^2)\arctan x}dx$$

67. Evaluate the integral

$$\int \cot^4 4x\ dx$$

using the reduction formula

$$\int \cot^m(u)du = -\frac{\cot^{m-1}(u)}{m-1} - \int \cot^{m-2}(u)du + C$$

68. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n$$

69. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=0}^{\infty}(1.2)^n$$

70. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty}\frac{n}{n+3}$$

71. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty}\frac{1}{n(n+3)}$$

72. Determine the convergence or divergence of the series using the *integral test*

$$\sum_{n=1}^{\infty}\frac{n}{n^2+1}$$

73. Determine the convergence or divergence of the series using the *limit comparison test*

$$\sum_{n=1}^{\infty}\frac{n}{n^2+1}$$

74. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{-2, 5, 12, 19, \dots\}$$

75. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{3, 6, 12, 24, 48, \dots\}$$

76. Write the first four terms of the sequence

$$a_{n+1} = 2a_n + 1,\ a_1 = 1$$

77. Evaluate

$$\int \frac{4x+1}{x^2+9} dx$$

78. Evaluate

$$\int \frac{4x^2}{x^2+9} dx$$

79. Evaluate

$$\int \frac{1}{1+e^x} dx$$

80. Evaluate

$$\int \tan^2 2x \ dx$$

81. Evaluate

$$\int \frac{1}{\sqrt{1-4x-x^2}} dx$$

82. Solve the differential equation

$$xy\frac{dy}{dx} = 1 - \ln x; quady(1) = 2$$

83. :: Section 8.2 Evaluate

$$\int xe^{3x} dx$$

84. Evaluate

$$\int \frac{\ln x}{x^2} dx$$

85. Evaluate

$$\int \arcsin x \ dx$$

86. Evaluate

$$\int x \sin x \ dx$$

87. Evaluate

$$\int x \sin x^2 \ dx$$

88. Evaluate

$$\int x^2 \sin 2x \ dx$$

89. Evaluate

$$\int x^3 \cos 2x \ dx$$

90. Evaluate

$$\int e^{2x} \cos x \ dx$$

91. Evaluate

$$\int_0^{\pi/4} x \sin 2x \ dx$$

92. Evaluate

$$\int \frac{x+1}{\sqrt{3x^2+6x}} dx$$

93. Evaluate

$$\int \frac{1}{\cos\theta - 1} d\theta$$

94. Evaluate

$$\int \frac{x^3 e^{x^2}}{(x^2+1)^2} dx$$

95. Evaluate

$$\int x^2 \ln 3x \ dx$$

96. Evaluate

$$\int x^4 \sin 2x \ dx$$

97. Evaluate

$$\int \sin^3 x \cos^4 x \; dx$$

98. Evaluate

$$\int \cos^2\left(\frac{x}{c}\right) dx$$

99. Evaluate

$$\int \sin^4 \theta \; d\theta$$

100. Evaluate

$$\int \tan^3 4x \; dx$$

101. Evaluate Solve the differential equation

$$\frac{dy}{dx} = \tan^3 x \sec x; \quad y(\pi/3) = 0$$

102. Evaluate Find the area of the region bounded by the curves:

$$y = \sin^2(\pi \cdot x); \; y = 0; \; x = 0; \; x = 1$$

103. Evaluate Find the volume of the solid formed when the region bounded by the curves

$$y = \cos\frac{x}{2}; y = \sin\frac{x}{2}; x = 0; x = \frac{\pi}{2}$$

is revolved about the x-axis.

104. Evaluate

$$\int \sin 2x \cos 3x \; dx$$

by using one of the following identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$
$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$
$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

105. Evaluate

$$\int_0^{\pi/6} \ln(2\sec x) dx$$

106. Evaluate

$$\int \sin(10x)\cos(3x) dx$$

by using one of the following identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$
$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$
$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

107. Evaluate

$$\int \sin(6x)\sin(4x) dx$$

by using one of the following identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$
$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$
$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

108. Evaluate

$$\int \sqrt{1-x^2} dx$$

109. Evaluate

$$\int \frac{1}{x^2\sqrt{4-x^2}} dx$$

110. Evaluate

$$\int \frac{1}{\sqrt{9x^2+4}} dx$$

111. Evaluate

$$\int_0^1 \frac{1}{(x^2+1)^{3/2}} dx$$

112. Evaluate

$$\int_0^1 \frac{x}{(x^2+1)^{3/2}} dx$$

113. Evaluate

$$\int \frac{1}{4+9x^2}dx$$

114. Evaluate

$$\int \sqrt{25-4x^2}dx$$

115. Evaluate

$$\int_{13/2}^{13} \sqrt{169-x^2}dx$$

116. Evaluate

$$\int \frac{1}{(1+25x^2)^{3/2}}dx$$

117. Evaluate

$$\int \frac{3}{2x^2-7x-4}dx$$

118. Evaluate

$$\int \frac{x+4}{x^2+5x+6}dx$$

119. Evaluate

$$\int \frac{2x-1}{4x^2-9}dx$$

120. Evaluate

$$\int \frac{4x+7}{(x+1)^2}dx$$

121. Evaluate

$$\int \frac{2x+3}{x^3-2x^2+3x-6}dx$$

122. Evaluate

$$\int \frac{x^3+x-3}{x^2-4}dx$$

123. Evaluate $\int \frac{1}{x^2+2x-3}dx$ via:

    1. Trigonometric substitution
    2. Partial fraction decomposition

124. Evaluate

$$\int x^2e^{5x}dx$$

125. Evaluate

$$\int \sin^3 x \ dx$$

126. Evaluate

$$\int \sqrt{9-4x^2}dx$$

127. Evaluate

$$\int \frac{1}{x^2-9}dx$$

128. Evaluate

$$\int \frac{\sin x}{\cos x+\cos^2 x}dx$$

129. Evaluate

$$\int \sin^3 x\cos x\ln(\sin x)dx$$

using the reduction formula

$$\int x^n \ln u \ du = \frac{u^{n+1}}{(n+1)^2}(-1+(n+1)\ln u)+C, n\neq -1$$

130. Evaluate

$$\int x\sqrt{5-4x^4}dx$$

using the reduction formula

$$\int \sqrt{a^2-x^2}dx = \frac{1}{2}\left(x\sqrt{a^2-x^1}+a^2\arcsin\frac{x}{a}\right)+C$$

131. Derive the reduction formula

$$\int u^n\cos u \ du = u^n\sin u - n\int u^{n-1}\sin u \ du$$

132. Evaluate

$$\int_1^\infty \frac{1}{e^x} dx$$

133. Evaluate

$$\int_1^2 \frac{1}{(x-1)^2} dx$$

134. Evaluate $\int_1^\infty \frac{1}{x^p} dx$ converges if _____, otherwise it diverges.

135. Evaluate $\int_0^1 \frac{1}{x^p} dx$ converges if _____, otherwise it diverges.

136. Evaluate

$$\int_0^\infty \frac{1}{1+x^2} dx$$

137. Evaluate

$$\int_1^4 \frac{1}{(x-2)^{2/3}} dx$$

138. Find the volume of the solid formed by revolving the region bounded by $y = e^{-2x}$ and the x-axis from $[0, \infty)$ about the x-axis.

139. Write the nth-term formula for the following sequences

    1. $\{3, 7, 11, 15, \dots\}$
    2. $\{2, -1, \frac{1}{2}, -\frac{1}{4}, \dots\}$
    3. $\{1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \dots\}$

140. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \frac{n}{n+1}$$

141. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = 2 + (-1)^n$$

142. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \left(1 + \frac{1}{n}\right)^n$$

143. Find the limit of the sequence and state whether the sequence converges or diverges

$\{1, 4/3, 9/7, 16/15, 25/31\}$

144. A sequence is *monotonic* if all of its terms are entirely either:

    1. _____ $(a_1 \_ \cdots \_ a_n)$, or
    2. _____ $(a_1 \_ \cdots \_ a_n)$.

Cumulative Review (Answers)

23. *(Exam 2 Studyguide)*

$$\frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x} + C$$

24. *(Exam 2 Studyguide)*

$$-\frac{1}{2}\left(\cos(2x) - \frac{1}{3}\cos^3(2x)\right) + C$$

27. *(Exam 2 Studyguide)*

$\infty \to$ diverges

28. *(Exam 2 Studyguide)*

$2\sqrt{3} \to$ converges

29. *(Exam 2 Studyguide)*
Use the inverse tangent integration formula

30. *(Exam 2 Studyguide)*
Use u-substitution

31. *(Exam 2 Studyguide)*
Use trig-substitution

33. *(Exam 2 Studyguide)*

$$\frac{1}{2}\ln|e^x - 1| - \frac{1}{4}\ln|e^{2x} + 1| - \frac{1}{2}\arctan(e^x) + C$$

34. *(Exam 2)*

$$-\frac{1}{3(e^x+4)^3}dx$$

35. *(Exam 2)*

24

36. *(Exam 2)*

$$\ln\left|\frac{x - 5 + \sqrt{(x-5)^2 - 4}}{2}\right| + C$$

37. *(Exam 2)*

$$x\arccos x - \sqrt{1-x^2} + C$$

38. *(Exam 2)*

≈ 2.68

39. *(Exam 2)*

$$\frac{3}{4}x + \frac{1}{10}\sin 10x\frac{1}{80}\sin 10x + C$$

40. *(Exam 2)*

$$\frac{\tan^3 9t}{27} - \frac{\tan 9t}{9} + t + C$$

41. *(Exam 2)*

$$\frac{1}{10}\arctan 5x + \frac{x}{50x^2+2} + C$$

42. *(Exam 2)*

$$\frac{5}{3}\ln|x| - \frac{2}{3}\ln|x+3| + C$$

or

$$\frac{1}{3}\ln\left|\frac{x^5}{(x+3)^2}\right| + C$$

43. *(Exam 2)*

$$\frac{1}{3}\ln|\sin t - 6| - \frac{1}{3}|\sin t - 3| + C$$

44. *(Exam 2)*

$$= 2\pi + \frac{\pi^2}{2}; \quad \therefore \text{converges}$$

45. *(Exam 2)*

$$= \infty; \quad \therefore \text{diverges}$$

46. *(Quiz 1)*

$$\frac{5\pi}{6}$$

47. *(Quiz 1)*

$$\sqrt{3}$$

48. *(Quiz 1)*

$$\sqrt{4 - x^2}$$

49. *(Quiz 1)*

$$\ln\left|x^2 + 6x + 13\right| - 3\arctan\frac{x+3}{2} + C$$

50. *(Quiz 2)*

$$\arcsin\frac{9x}{8} + C$$

51. *(Quiz 2)*

$$7\cosh 7x$$

52. *(Quiz 2)*

$$8\sinh(4x)\cosh(4x)$$

53. *(Quiz 2)*

$$\frac{\coth^2 x}{2} + C$$

54. *(Quiz 2)*

$$\frac{3}{4}$$

55. *(Quiz 2)*

$$\frac{37}{35}$$

56. *(Quiz 2)*

Yes (verified)

57. *(Quiz 2)*

$$y = Ce^{x/16} + 176$$

58. *(Quiz 3)*

$$\cot\theta + \csc\theta + C$$

59. *(Quiz 3)*

$$\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

60. *(Quiz 3)*

$(64\ln 4 - 32) - (16\ln 2 - 8) \approx 53.6$

61. *(Quiz 3)*

$x^n e^x - n\int x^{n-1}e^x dx$

62. *(Quiz 3)*

$\frac{1}{3}$

63. *(Quiz 3)*

$-\frac{1}{2}\cos^4\theta + C$

64. *(Quiz 4)*

$\frac{1}{2}\ln\left|\sqrt{4x^2+1}+2x\right| + C$

65. *(Quiz 4)*

$4\ln(x^2+2) + \frac{3}{2(x^2+2)} + C$

66. *(Quiz 4)*

$4\ln 2 \to$ Converges

67. *(Quiz 4)*

$-\frac{1}{12}\cot^3(4x) + \frac{1}{4}\cot(4x) + x + C$

68. *(Quiz 5)*

Because $r = |3/4| < 1$ the series converges (geometric series), and converges to the value 3.

69. *(Quiz 5)*

Because $r = |1.2| > 1$ the series diverges (geometric series).

70. *(Quiz 5)*

By the nth term test, $\lim_{n\to\infty}\frac{n}{n+3} = 1 \neq 0$, therefore the series diverges.

71. *(Quiz 5)*

By means of partial fraction decomposition and evaluation of a telescopic series, the series converges to the value $11/18$.

72. *(Quiz 5)*

By the integral test, $\lim_{b\to x}\int_1^b \frac{x}{x^2+1}dx = \infty$ which is non-finite, therefore the series diverges.

73. *(Quiz 5)*

By the limit comparison test and selection of the harmonic series/divergent p-series as the comparison series,

$$\lim_{n\to\infty}\frac{n}{n^2+1}\cdot\frac{n}{1} = 1$$

Because the limit exists and is both positive and finite, the original series behaves similarly to the comparison series, and thus diverges.

74. *(Quiz 5)*

$a_n = 7n - 9$

75. *(Quiz 5)*

$a_n = 3(2)^{n-1}$

76. *(Quiz 5)*

$\{1, 3, 7, 15\}$

139. *(Section 9.1–9.3)*

1. $a_n = 4n - 1$
2. $a_n = (-1)^{n+1}2^{2-n}$
3. $a_n = \frac{x^{n-1}}{(n-1)!}$

140. *(Section 9.1–9.3)*

lim = 1, therefore the sequence converges.

141. *(Section 9.1–9.3)*

The function oscillates between the values 1 and 3, thus limit does not exist, and therefore the sequence diverges.

142. *(Section 9.1–9.3)*

lim = e, therefore the sequence converges.

143. *(Section 9.1–9.3)*

lim = 0, therefore the sequence converges.