Exam 3 Review (Problems)

1. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \frac{1}{\sqrt[3]{25}} + \cdots$$

2. A sequence is bounded ______ if there is a number M such that $a_n \geq M$ for all n.

3. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} 3(\pi)^{-n}$$

4. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} e^{-n}$$

5. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{3, 6, 12, 24, 48, \dots\}$$

6. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$$

7. Find the explicit nth term formula for the following sequence

$$\{a_n\} = \{-2, 5, 12, 19, \dots\}$$

8. A sequence is bounded if it is ________________________________.

9. Consider the repeating decimal $0.1313\cdots$; Convert this decimal to a fraction.

10. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

11. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(5x)^n}{n^2}$$

12. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$$

13. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} \frac{(\ln n)(x-3)^n}{n}$$

14. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

15. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{n!}{3^n}$$

16. Determine the convergence or divergence of the series using the *limit comparison test*

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

17. Given the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^3+1}$$

   1. Use the first 4 terms to approximate the sum; then explain what the maximum error is in this approximation.
   2. Determine how many terms are required to estimate the sum with an error of no more than 0.0001.

18. Determine convergence or divergence, including absolute/conditional convergence if applicable; state clearly what test is being used; and determine the interval of convergence for x if applicable.

$$\sum_{n=1}^{\infty} n!(n-1)^n$$

19. A sequence is bounded ______ if there is a number M such that $a_n \leq M$ for all n.

20. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \frac{1}{2^n}$$

21. A geometric series is a series of the form

$$\sum_{n=0}^{\infty} \underline{\quad ? \quad}$$

22. Write the first four terms of the sequence

$$a_{n+1} = 2a_n + 1, \; a_1 = 1$$

23. Does the following series converge or diverge? If it converges, find the sum.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^2$$

24. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3}{2n+1}$$

25. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

26. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^n}$$

27. Find the limit of the sequence and state whether the sequence converges or diverges

$$\{1, {}^{4}\!/_{3}, {}^{9}\!/_{7}, {}^{16}\!/_{15}, {}^{25}\!/_{31}\}$$

28. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$$

29. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=0}^{\infty} (1.2)^n$$

30. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = 2 + (-1)^n$$

31. Determine convergence or divergence, including absolute/conditional convergence if applicable. State clearly what test is being used.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$$

32. Does the series converge or diverge, and if it converges then find the sum (use the geometric series test, the telescoping series test, or the nth term test for divergence)

$$\sum_{n=1}^{\infty} \frac{n}{n+3}$$

33.

34. Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$$

35. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \frac{n}{n+1}$$

36. Write the nth-term formula for the following sequences

    1. $\{3, 7, 11, 15, \dots\}$
    2. $\{2, -1, \frac{1}{2}, -\frac{1}{4}, \dots\}$
    3. $\{1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \dots\}$

37. Determine the convergence or divergence of the series using the *integral test*

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

38. Find the limit of the sequence and state whether the sequence converges or diverges

$$a_n = \left(1 + \frac{1}{n}\right)^n$$

39. A geometric series with ratio r will:

    1. Diverge if ______.
    2. Converge to $S =$ ______ if ______.

40. A sequence is *monotonic* if all of its terms are entirely either:

    1. ____________ $(a_1 \_ \cdots \_ a_n)$, or
    2. ____________ $(a_1 \_ \cdots \_ a_n)$.

Exam 3 Review (Answers)

1. *(Quiz 6)*

   (P-series) Because $p = {}^2/_3 < 1$ the series diverges.

2. *(Section 9.1–9.3)*

   below

3. *(Exam 3 Studyguide)*

   (Geometric series) The series has the ratio $r = {}^1/_\pi$, which because $|r| < 0$ converges. The series converges to:

   $a_1 = {}^3/_\pi;\ r = {}^1/_\pi$

   $$S_n = \frac{{}^3/_\pi}{1 - {}^1/_\pi} \approx 1.4008$$

4. *(Section 9.1–9.3)*

   Converges, and the sum equals $\frac{e}{e-1}$.

5. *(Quiz 5)*

   $$a_n = 3(2)^{n-1}$$

6. *(Quiz 5)*

   Because $r = |{}^3/_4| < 1$ the series converges (geometric series), and converges to the value 3.

7. *(Quiz 5)*

   $$a_n = 7n - 9$$

8. *(Section 9.1–9.3)*

   bounded both above and below

9. *(Section 9.1–9.3)*

   $$0.1313\cdots = \frac{13}{99}$$

10. *(Quiz 5)*

    By means of partial fraction decomposition and evaluation of a telescopic series, the series converges to the value ${}^{11}/_{18}$.

11. *(Exam 3 Studyguide)*

    The series converges for $-{}^1/_5 \le x \le {}^1/_5$.

12. *(Exam 3 Studyguide)*

    (Check for absolute convergence using Ratio Test) Because $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$, the series converges absolutely for all values of x.

13. *(Exam 3 Studyguide)*

    (Check for absolute convergence using Root test; check end points using A.S.T. and L.C.T.) The series converges for $2 \le x < 4$.

14. *(Quiz 6)*

    Converges conditionally.

15. *(Exam 3 Studyguide)*

    (Ratio test) Because $\lim_{n\to\infty} \left|\frac{n+1}{3}\right| = \infty > 1$, the series diverges.

16. *(Quiz 5)*

    By the limit comparison test and selection of the harmonic series/divergent p-series as the comparison series,

    $$\lim_{n\to\infty} \frac{n}{n^2+1} \cdot \frac{n}{1} = 1$$

    Because the limit exists and is both positive and finite, the original series behaves similarly to the comparison series, and thus diverges.

17. *(Exam 3 Studyguide)*

    1. $S_4 = -\frac{1}{3} + \frac{1}{17} - \frac{1}{55} + \frac{1}{129} \approx -0.2849$, and by alternating series remainder theorem, error $= a_5 = \left|\frac{1}{251}\right|$.
    2. $$a_{n+1} = \left|\frac{(-1)^n}{2n^3+1}\right| \le 0.0001$$

       $16.099 \le n$

       $n = 17$

18. *(Exam 3 Studyguide)*

    (Ratio test) Because $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, the series diverges for all x.

19. *(Section 9.1–9.3)*

    above

20. *(Section 9.1–9.3)*

    Converges, and the sum equals 2.

21. *(Section 9.1–9.3)*

$$\sum_{n=0}^{\infty} ar^n$$

22. *(Quiz 5)*

$$\{1, 3, 7, 15\}$$

23. *(Section 9.1–9.3)*

 Diverges

24. *(Exam 3 Studyguide)*

 1. (Check for absolute convergence using L.C.T.) Comparing against $\sum_{n=1}^{\infty} \frac{1}{n}$ (divergent p-series):

 $$\lim_{n\to\infty} \left| \frac{3}{2n+1} \cdot \frac{n}{1} \right| = \frac{3}{2}$$

 Which is both finite and positive, and so diverges similarly. We need to check for conditional convergence.

 2. (A.S.T.)

 (a) $\lim_{n\to\infty} \frac{(-1)^n 3}{2n+1} = 0$ ✓

 (b) $a_{n+1} < a_n$ ✓

25. *(Quiz 6)*

 (Integral test) Because lim $= \infty$ the series diverges.

26. *(Exam 3 Studyguide)*

 (Ratio Test) Because $\lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^n}} = 0 < 1$, the series converges.

27. *(Section 9.1–9.3)*

 lim $= 0$, therefore the sequence converges.

28. *(Exam 3 Studyguide)*

 (Divergence test) lim $= 1/3$ which $\neq 0$ and therefore diverges.

29. *(Quiz 5)*

 Because $r = |1.2| > 1$ the series diverges (geometric series).

30. *(Section 9.1–9.3)*

 The function oscillates between the values 1 and 3, thus limit does not exist, and therefore the sequence diverges.

31. *(Exam 3 Studyguide)*

 (L.C.T.) Comparing against $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ (convergent p-series),

 $$\lim_{n\to\infty} \frac{1}{n\sqrt{n+1}} \cdot \frac{n\sqrt{n}}{1} = 1$$

 Which is both finite and positive, and so converges similarly.

32. *(Quiz 5)*

 By the nth term test, $\lim_{n\to\infty} \frac{n}{n+3} = 1 \neq 0$, therefore the series diverges.

34. *(Quiz 6)*

 (-3,1)

35. *(Section 9.1–9.3)*

 lim $= 1$, therefore the sequence converges.

36. *(Section 9.1–9.3)*

 1. $a_n = 4n - 1$
 2. $a_n = (-1)^{n+1} 2^{2-n}$
 3. $a_n = \frac{x^{n-1}}{(n-1)!}$

37. *(Quiz 5)*

 By the integral test, $\lim_{b\to x} \int_1^b \frac{x}{x^2+1} dx = \infty$ which is non-finite, therefore the series diverges.

38. *(Section 9.1–9.3)*

 lim $= e$, therefore the sequence converges.

39. *(Section 9.1–9.3)*

 1. Diverge if $|r| \geq 1$.
 2. Converge to $S = \dfrac{a}{1-r}$ if $0 < |r| < 1$.

40. *(Section 9.1–9.3)*

 1. <u>non-decreasing</u> $(a_1 \leq \cdots \leq a_n)$, or
 2. <u>non-increasing</u> $(a_1 \geq \cdots \geq a_n)$.