

Exam 2 Review (Problems)

1. Evaluate

$$\int \frac{1}{\sqrt{9x^2 + 4}} dx$$

2. Evaluate

$$\int \frac{e^x}{(e^{2x} + 1)(e^x - 1)} dx$$

3. Evaluate

$$\int \frac{8x^3 + 13x}{(x^2 + 2)^2} dx$$

4. Evaluate

$$\int \arcsin x \, dx$$

5. Evaluate

$$\int \sin^3 x \cos x \ln(\sin x) dx$$

using the reduction formula

$$\int x^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} (-1 + (n+1) \ln u) + C, n \neq -1$$

6. Evaluate

$$\int \frac{x+1}{\sqrt{3x^2+6x}} dx$$

7. Evaluate

$$\int \frac{1}{1+e^x} dx$$

8. Evaluate Solve the differential equation

$$\frac{dy}{dx} = \tan^3 x \sec x; \quad y(\pi/3) = 0$$

9. Evaluate

$$\int \sqrt{9-4x^2} dx$$

10. Evaluate

$$\int e^{2x} x^2 \, dx$$

11. Find the volume of the solid formed by revolving the region bounded by $y = e^{-2x}$ and the x-axis from $[0, \infty)$ about the x-axis.

12. Evaluate Find the area of the region bounded by the curves:

$$y = \sin^2(\pi \cdot x); \quad y = 0; \quad x = 0; \quad x = 1$$

13. Evaluate

$$\int \frac{x^3 e^{x^2}}{(x^2 + 1)^2} dx$$

14. Evaluate

$$\int \frac{1}{\sqrt{1-4x-x^2}} dx$$

15. Evaluate

$$\int x \sin x^2 \, dx$$

16. Evaluate

$$\int x^2 e^{5x} dx$$

17. Evaluate

$$\int_0^{\pi/4} x \sin 2x \, dx$$

18. Evaluate

$$\int \sin^3 x \cos^4 x \, dx$$

19. Evaluate $\int \frac{1}{x^2 + 2x - 3} dx$ via:

1. Trigonometric substitution
2. Partial fraction decomposition

20. Evaluate

$$\int_1^4 \frac{1}{(x-2)^{2/3}} dx$$

21. Evaluate the integral

$$\int \cot^4 4x \, dx$$

using the reduction formula

$$\int \cot^m(u) du = -\frac{\cot^{m-1}(u)}{m-1} - \int \cot^{m-2}(u) du + C$$

22. Evaluate

$$\int \sin^4 \theta \, d\theta$$

23. Evaluate $\int_1^\infty \frac{1}{x^p} dx$ converges if _____, otherwise it diverges.

24. Evaluate

$$\int \frac{3}{2x^2 - 7x - 4} dx$$

25. Evaluate

$$\int_0^{\pi/4} \sin^3 4x \, dx$$

26. Evaluate $\int_0^1 \frac{1}{x^p} dx$ converges if _____, otherwise it diverges.

27. Evaluate Find the volume of the solid formed when the region bounded by the curves

$$y = \cos \frac{x}{2}; y = \sin \frac{x}{2}; x = 0; x = \frac{\pi}{2}$$

is revolved about the x-axis.

28. Evaluate

$$\int \frac{1}{4 + 9x^2} dx$$

29. Evaluate

$$\int \frac{1}{x^2 - 9} dx$$

30. Evaluate

$$\int \frac{1}{\cos \theta - 1} d\theta$$

31. Evaluate

$$\int_0^1 \frac{1}{(x^2 + 1)^{3/2}} dx$$

32. Evaluate

$$\int x^4 \sin 2x \, dx$$

33. Evaluate

$$\int \sqrt{1 - x^2} dx$$

34. Evaluate

$$\int \frac{-\csc \theta}{\csc \theta - \cot \theta} d\theta$$

35. Evaluate

$$\int \frac{x + 4}{x^2 + 5x + 6} dx$$

36. Evaluate

$$\int \sin^3 x \, dx$$

37. Evaluate

$$\int_{13/2}^{13} \sqrt{169 - x^2} dx$$

38. Evaluate

$$\int \frac{3x - 1}{x^2 - 5x + 4} dx$$

39. Evaluate

$$\int_2^4 8x \ln x \, dx$$

40. Evaluate

$$\int \sqrt{25 - 4x^2} dx$$

41. How would you approach the following?

$$\int \frac{x}{x^2 + 1} dx$$

42. Evaluate

$$\int e^{2x} \cos x \, dx$$

43. Use integration by parts to establish a reduction formula for the integral

$$\int x^n e^x dx$$

44.

$$\int_1^\infty \frac{4}{(1+x^2) \arctan x} dx$$

45. Evaluate

$$\int \frac{1}{(1+25x^2)^{3/2}} dx$$

46. Use trig substitution to evaluate

$$\int \frac{1}{\sqrt{4x^2 + 1}} dx$$

47. Evaluate

$$\int x^3 \cos 2x \, dx$$

48. Evaluate

$$\int \frac{\sqrt{x^2 - 9}}{x} dx$$
$$\frac{\sqrt{x^2 - 9}}{3} - \arctan \frac{x}{3} + C$$

49. How would you approach the following?

$$\int \frac{1}{\sqrt{x^2 + 1}} dx$$

50. Evaluate

$$\int x^3 e^{2x} dx$$

51. Prove the reduction formula:

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$$

52. Solve the differential equation

$$xy \frac{dy}{dx} = 1 - \ln x; \text{quady}(1) = 2$$

53. Evaluate

$$\int \sin(10x) \cos(3x) dx$$

by using one of the following identities:

$$\sin(mx) \sin(nx) = \frac{1}{2} (\cos[(m-n)x] - \cos[(m+n)x])$$

$$\sin(mx) \cos(nx) = \frac{1}{2} (\sin[(m-n)x] + \sin[(m+n)x])$$

$$\cos(mx) \cos(nx) = \frac{1}{2} (\cos[(m-n)x] + \cos[(m+n)x])$$

54. Evaluate

$$\int \frac{4x+7}{(x+1)^2} dx$$

55. Evaluate

$$\int \frac{x^3 + x - 3}{x^2 - 4} dx$$

56. Evaluate

$$\int_2^5 \frac{1}{\sqrt{x-2}} dx$$

57. Evaluate

$$\int_0^1 \frac{x}{(x^2 + 1)^{3/2}} dx$$

58. Evaluate

$$\int \tan^2 2x \, dx$$

59. Evaluate

$$\int \cos^2 \theta \sin 2\theta \, d\theta$$

60. Evaluate

$$\int_1^2 \frac{1}{(x-1)^2} dx$$

61. Evaluate

$$\int \frac{4x+1}{x^2+9} dx$$

62. Evaluate

$$\int \frac{1}{x^2 \sqrt{4-x^2}} dx$$

63. Evaluate

$$\int \cos^2 \left(\frac{x}{c} \right) dx$$

64. Evaluate

$$\int \sin 2x \cos 3x \, dx$$

by using one of the following identities:

$$\sin(mx) \sin(nx) = \frac{1}{2} (\cos[(m-n)x] - \cos[(m+n)x])$$

$$\sin(mx) \cos(nx) = \frac{1}{2} (\sin[(m-n)x] + \sin[(m+n)x])$$

$$\cos(mx) \cos(nx) = \frac{1}{2} (\cos[(m-n)x] + \cos[(m+n)x])$$

65. Evaluate

$$\int \frac{\ln x}{x^2} dx$$

66. Evaluate

$$\int \tan^3 4x \, dx$$

67. Derive the reduction formula

$$\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$$

68. Evaluate

$$\int x^2 \ln 3x \, dx$$

69. Evaluate

$$\int x \sin x \, dx$$

70. Evaluate

$$\int \frac{2x+3}{x^3-2x^2+3x-6} dx$$

71. How would you approach the following?

$$\int \frac{1}{x^2+1} dx$$

72. Evaluate

$$\int \frac{\sin x}{\cos x + \cos^2 x} dx$$

73. Evaluate

$$\int_0^{\pi/6} \ln(2 \sec x) dx$$

74. Evaluate

$$\int_1^{\infty} \frac{1}{e^x} dx$$

75. Evaluate

$$\int x e^{3x} dx$$

76. Evaluate

$$\int_0^{\infty} \frac{1}{1+x^2} dx$$

77. Evaluate

$$\int \sin^3(2x) dx$$

78. Evaluate

$$\int_1^{\infty} \frac{1}{\sqrt{x+2}} dx$$

79. Evaluate

$$\int \frac{4x^2}{x^2 + 9} dx$$

80. Evaluate

$$\int \sin(6x) \sin(4x) dx$$

by using one of the following identities:

$$\sin(mx) \sin(nx) = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$

$$\sin(mx) \cos(nx) = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$

$$\cos(mx) \cos(nx) = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

81. Evaluate

$$\int x \sqrt{5 - 4x^4} dx$$

using the reduction formula

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left(x \sqrt{a^2 - x^2} + a^2 \arcsin \frac{x}{a} \right) + C$$

82. Evaluate

$$\int x^2 \sin 2x \, dx$$

83. Evaluate

$$\int \frac{2x - 1}{4x^2 - 9} dx$$

Exam 2 Review (Answers)

2. (Exam 2 Studyguide)

$$\frac{1}{2} \ln |e^x - 1| - \frac{1}{4} \ln |e^{2x} + 1| - \frac{1}{2} \arctan(e^x) + C$$

3. (Quiz 4)

$$4 \ln(x^2 + 2) + \frac{3}{2(x^2 + 2)} + C$$

10. (Quiz 3)

$$\frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + C$$

21. (Quiz 4)

$$-\frac{1}{12} \cot^3(4x) + \frac{1}{4} \cot(4x) + x + C$$

25. (Quiz 3)

$$\frac{1}{3}$$

34. (Quiz 3)

$$\cot \theta + \csc \theta + C$$

39. (Quiz 3)

$$(64 \ln 4 - 32) - (16 \ln 2 - 8) \approx 53.6$$

41. (Exam 2 Studyguide)

Use u-substitution

43. (Quiz 3)

$$x^n e^x - n \int x^{n-1} e^x dx$$

44. (Quiz 4)

$$4 \ln 2 \rightarrow \text{Converges}$$

46. (Quiz 4)

$$\frac{1}{2} \ln \left| \sqrt{4x^2 + 1} + 2x \right| + C$$

49. (Exam 2 Studyguide)

Use trig-substitution

50. (Exam 2 Studyguide)

$$\frac{1}{2} x^3 e^{2x} - \frac{3}{4} x^2 e^{2x} + \frac{3}{4} x e^{2x} - \frac{3}{8} e^{2x} + C$$

56. (Exam 2 Studyguide)

$$2\sqrt{3} \rightarrow \text{converges}$$

59. (Quiz 3)

$$-\frac{1}{2} \cos^4 \theta + C$$

71. (Exam 2 Studyguide)

Use the inverse tangent integration formula

77. (Exam 2 Studyguide)

$$-\frac{1}{2} \left(\cos(2x) - \frac{1}{3} \cos^3(2x) \right) + C$$

78. (Exam 2 Studyguide)

$$\infty \rightarrow \text{diverges}$$