Math 252 Exam 1 Review (Problems)

1. Using $\mathbf{u} = \langle 8, 3, -5\rangle, \mathbf{v} = \langle 4, -4, -2\rangle$,
   a. Find $3u - 4v$.
   b. Find $\|\mathbf{u}\|, \|\mathbf{v}\|$.

2. A baseball is thrown from the stands 128 feet above the field at an angle of 30 degrees up from the horizontal with an initial speed of 64 feet per second.
   a. Give the position vector for any time t.
   b. When will the ball strike the ground?
   c. How far away will the ball strike the ground?
   d. What is the speed of the ball when it strikes the ground?

3. Identify the surface $x^2 - 6y + 5z^2 = 0$.

4. Using $P(-2, 0, 3), Q(1, 2, 4), R(-3, 1, 0)$,
   a. Find a vector orthogonal to the plane determined by P, Q and R.
   b. Find an equation of the plane passing through P, Q and R.
   c. Find the set of parametric equations for the line through Q and parallel to $\mathbf{a} = \langle 4, -3, -2\rangle$.
   d. Find the distance from the point $(-4, -1, 5)$ to the plane passing through P, Q and R.

5. Indentify via cross-sections the surface defined by $x = 3y^2 + 5z^2$.

6. Using $\mathbf{r}(t) = \langle t\cos t, t\sin t, t^2\rangle$ at $t = 0$,
   a. Find $\mathbf{v}$ and $\mathbf{a}$.
   b. Find $\mathbf{T}$ and $\mathbf{N}$.
   c. Find K.
   d. By first finding $a_{\mathbf{T}}$ and $._{\mathbf{N}}$, express $a = a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$.

7. Using $P(-4, 1, 2)$, $Q(1, -3, 4)$, $R(-1, 0, 2)$,
   a. Find an equation of the plane passing through the points.
   b. Find parametric equations for the line through P and parallel to $a = \langle 2, -1, 4\rangle$.
   c. Find the distance from the point $(5, -3, 2)$ to the plane.
   d. Find the area of the parallelogram determined by P, Q, and R.

8. Indentify via cross-sections the surface defined by $y = x^2$.

9. A projectile is fired at a speed of 448 feet per second at and angle of 30 degrees from a tower 512 feet above the ground.
   a. Give the position vector for any time t.
   b. How far away will the object strike?

10. Using $\mathbf{u} = \langle -4, 6, 5\rangle$ and $\mathbf{v} = \langle 2, -3, 1\rangle$,
    a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
    b. Find $\mathbf{u} \cdot \mathbf{v}$.
    c. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.
    d. Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.
    e. Find $\mathbf{u} \times \mathbf{v}$.

11. Using $\mathbf{r}(t) = \langle 4\cos(2t), 4\sin(2t), 6t\rangle$,
    a. Find $\mathbf{T}(t)$
    b. Find $\mathbf{N}(t)$
    c. Find the curvature

12. Indentify via cross-sections the surface defined by $3^2 - y^2 + 3z^2 + 9 = 0$.

13. Identify the surface $2x^2 - 3y^2 + 6z^2 = 6$.

14. Identify the surface $x = y^2$.

15. Find the center and radius of the sphere given by $x^2 + y^2 + z^2 - 8x + 6x = 0$

16. Find the tangential and normal components of acceleration for the curve $\mathbf{r}(t) = \langle 3t^2, 4t^2, 10t\rangle$ at $t = 2$ and express a in terms of T and N.

17. Identify the surface $4x^2 + 4y^2 + z^2 = 4$.

18. Indentify via cross-sections the surface defined by $2y^2 = 3z^2 = 12$.

19. Using $\mathbf{u} = \langle 8, -4, 1\rangle$ and $\mathbf{v} = \langle -4, 4, 2\rangle$,

    a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

    b. Find $\mathbf{u} \cdot \mathbf{v}$.

    c. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

    d. Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.

    e. Find $\mathbf{u} \times \mathbf{v}$.

20. Using $r(t) = \langle \cos t, \sin t, t^2\rangle$, $t = \frac{\pi}{2}$:

    a. Find the velocity vector.

    b. Find the acceleration vector.

Math 252 Exam 1 Review (Answers)

1. (Math-252 Quiz 1)
   a. $\langle 8, 25, -7\rangle$.
   b. $\|\mathbf{u}\| = 7\sqrt{2}$, $\|\mathbf{v}\| = 6$.

2. (Math-252 Practice Exam 1)
   a. $\mathbf{r}(t) = \langle 32\sqrt{3}\,t, -16t^2 + 32t + 128\rangle$
   b. in 4 seconds
   c. $128\sqrt{3}$ feet away
   d. $64\sqrt{3}$ feet per second

3. (Math-252 Practice Exam 1)
   Elliptical cone

4. (Math-252 Quiz 3)
   a. $\mathbf{n} = \mathbf{PQ} \times \mathbf{PR} = \langle -7, 8, 5\rangle$
   b. $-7x + 8y + 5z = 29$
   c. $x = 1 + 4t, y = 2 - 3t, z = 4 - 2t; t \in \mathbb{R}$
   d. $D = \frac{16}{\sqrt{138}}$

5. (Math-252 Quiz 4)
   Elliptical paraboloid

6. (Math-252 Practice Exam 1)
   a. $\mathbf{v} = \langle -t\sin t + \cos t, t\cos t + \sin t, 2t\rangle$
      $\mathbf{a} = \langle -t\cos t - 2\sin t, -t\sin t + 2\cos t, 2\rangle$
   b. $\mathbf{T}(t) = \left\langle \frac{-t\sin 5 + \cos t}{\sqrt{5t^2+1}}, \frac{t\cos t + \sin 5}{\sqrt{5t^2+1}}, \frac{2t}{\sqrt{5t^2+1}} \right\rangle$
      $\mathbf{T}(0) = \langle 1, 0, 0\rangle$
      $\mathbf{N}(0) = \left\langle 0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

7. (Math-252 Practice Exam 1)
   a. $2x + 6y + 7z - 12 = 0$
   b. $x = 2t - 4,\ y = -t + 1,\ z = 4t + 2$
   c. $D = \dfrac{6}{\sqrt{89}}$
   d. $A = \sqrt{89}$

8. (Math-252 Quiz 4)
   Parabolic cylinder

9. (Math-252 Quiz 5)
   a. $\mathbf{r}(t) = \langle 224\sqrt{3}\,t$
      $-16t^2 + 224t + 512\rangle$
   b. $T = 16$
      $x(16) = 224\sqrt{3}(16) \doteq 6207.7$ feet

10. (Math-252 Practice Exam 1)
    a. $\|\mathbf{u}\| = \sqrt{77}$
       $\|\mathbf{v}\| = \sqrt{14}$
    b. $\mathbf{u} \cdot \mathbf{v} = -21$
    c. $\theta = \arccos\left(\frac{-21}{7\sqrt{22}}\right)$

11. (Math-252 Quiz 6)
    a. $\mathbf{T}(t) = \langle -\frac{4}{5}\sin(2t), \frac{4}{5}\cos(2t), \frac{3}{5}\rangle$
    b. $\mathbf{N}(t) = \langle -\cos(2t), \sin(2t), 0\rangle$
    c. $k = \frac{4}{25}$

12. (Math-252 Quiz 4)
    Circular hyperboloid of two sheets

13. (Math-252 Practice Exam 1)
    Hyperbaloid (one sheet)

14. (Math-252 Practice Exam 1)
    Parabolic cylinder

15. (Math-252 Quiz 1)
    $C(4, -3, 0), \rho = 5$

16. (Math-252 Quiz 7)
    $\mathbf{a} = 4\sqrt{5}\,\mathbf{T} + 2\sqrt{5}\,\mathbf{N}$

17. (Math-252 Practice Exam 1)
    Circular ellipsoid

18. (Math-252 Quiz 4)
    Elliptical cylinder

19. (Math-252 Quiz 2)
    a. $\|\mathbf{u}\| = 9$, $\|\mathbf{v}\| = 6$
    b. $\mathbf{u} \cdot \mathbf{v} = -46$
    c. $\theta = \arccos\left(-\frac{23}{27}\right) = 148.4°$

d. $\text{proj}_{\mathbf{v}}\mathbf{u} = \langle -\frac{46}{9}, -\frac{46}{9}, -\frac{23}{9} \rangle$

e. $\mathbf{u} \times \mathbf{v} = \langle -12, -20, 16 \rangle$

20. (Math-252 Quiz 5)

a. $\mathbf{v}(t) = \langle -\sin t, \cos t, 2t \rangle$
$\mathbf{v}(\frac{\pi}{2}) = \langle -1, 0, \pi \rangle$

b. $\mathbf{a}(t) = \langle -\cos t, -\sin t, 2 \rangle$
$\mathbf{a}(\frac{\pi}{2}) = \langle 0, -1, 2 \rangle$