1 Exam 3 Material

1.1 Dot product

$$\vec{v}, \vec{u} \in \mathbf{R}^n$$
$$\vec{v} \cdot \vec{u} \equiv v_1 u_1 + \cdots + v_n u_n$$

In the vector space of polynomials and continuous functions, another dot product can also be defined by using integral over [-1,1]

$$f \cdot g = \int_{-1}^{1} f(x) \cdot g(x) dx$$

(What's the point of this definition?)

Example

$$\begin{aligned} f(x) = x; g(x) = e^x& \\ \int_{-1}^{1} xe^x dx &= [xe^x - e^x] \,|_{-1}^{1} \\ &= 2e^{-1} \doteq 0.736 \end{aligned}$$

Unit vector of $\vec{v} \equiv \frac{1}{\|\vec{v}\|}\vec{v}$

1.1.1 Dot product properties

$$\begin{aligned} \text{Positive definite property } \vec{v} \cdot \vec{v} &= 0 | v = 0 \\ \text{Distributive } (\vec{v} + \vec{u}) \cdot \vec{w} &= \vec{v} \cdot \vec{w} + \vec{u} \cdot \vec{w} \end{aligned}$$

1.2 Angle between two vectors

The angle between two vectors $\vec{v}$ and $\vec{u}$ in $\mathbf{R}^n$, where θ is the smallest angle between them, is

$$\cos\theta = \frac{\vec{v} \cdot \vec{u}}{\|\vec{v}\| \cdot \|\vec{u}\|}$$

1.2.1 Derivation

Given the law of cosines $a^2 = c^2 + b^2 - 2bc\cos A$ and the following definitions:

1.3 Ortho-normal basis

Construct an ortho-normal basis given vectors $\vec{v}$ and $\vec{u}$ of non-standard basis $S_1\{\vec{v}, \vec{u}\}$.

1.3.1 Gram-Schmidt process