Vectors

Magnitude: $\|\mathbf{u}\| = \sqrt{(u_x)^2 + (u_y)^2 + (u_z)^2}$

Dot product: $\mathbf{u} \cdot \mathbf{v} = (u_x v_x) + (u_y v_y) + (u_z v_z) = \|\mathbf{u}\| \|\mathbf{v}\| \cos\theta$

$\mathbf{u} \cdot \mathbf{v} = 0 \mid \mathbf{u} \perp \mathbf{v}$

Projection of u onto v: $\text{proj}_{\mathbf{v}} \mathbf{u} = \|\mathbf{u}\| \cos\theta \left(\frac{\mathbf{u}}{\|\mathbf{u}\|} \right) = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{u}\|} \mathbf{v}$

Cross product: $\mathbf{u} \times \mathbf{v} = \langle (u_y v_z - u_z v_y), -(u_x v_z - u_z v_x), (u_x v_y - u_y v_x) \rangle$

$\mathbf{u} \times \mathbf{v} = 0 \mid \mathbf{u} \parallel \mathbf{v}$

Equation of PQR plane: $\mathbf{n} = \mathbf{u} \times \mathbf{v} = \langle a, b, c \rangle$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$ax + by + cz + d = 0$

Area of PQR parallelogram: $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \|\mathbf{v}\| \sin\theta$

Line through P, $\parallel$ to $\mathbf{a}$: $x = (a_x t + P_x),\ y = (a_y t + P_y),\ z = (a_z t + P_z)$

Distance D of point P from plane $\mathbf{a}$: $\mathbf{a} = ax + by + cz + d = 0,\ P = \langle x_1, y_1, z_1 \rangle$

$D = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$

Surfaces

Parabola: $ax + by^2 = c$

Circle: $ax^2 + ay^2 = c$

Ellipse: $ax^2 + by^2 = c$

Hyperbola (one-sheet): $ax^2 - by^2 = k^2 + c$

Hyperbola (two-sheets): $ax^2 - by^2 = k^2 - c$

Vector-value functions

Arc length: $L = \int_a^b \|\mathbf{v}(t)\| dt$

Trajectory

Initial velocity: $\langle u_0, v_0 \rangle = \langle \|\mathbf{v}_0\| \cos\alpha, \|\mathbf{v}_0\| \sin\alpha \rangle$

Position: $\mathbf{r}(t) = \langle x(t), y(t) \rangle = \langle u_0 t + x_0, -\frac{1}{2} g t^2 + v_0 t + y_0 \rangle$

Time of flight: $T = (-\frac{1}{2} g t^2 + v_0 t + y_0 = 0) \mid t > 0$

Motion

Position: $\mathbf{d}(t) = \mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$

Velocity: $\mathbf{v}(t) = \mathbf{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$

Acceleration: $\mathbf{a}(t) = \mathbf{r}''(t) = \langle x''(t), y''(t), z''(t) \rangle$

$\mathbf{a}(t) = a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$

Unit tangent: $\mathbf{T}(t) = \frac{1}{\|\mathbf{v}(t)\|}\mathbf{v}(t)$

Unit normal: $\mathbf{N}(t) = \frac{1}{\|\mathbf{T}'(t)\|}\mathbf{T}'(t),\ \mathbf{N}(s) = \frac{1}{k}\mathbf{T}'(s)$

$\mathbf{T} \cdot \mathbf{N} = 0$

Curvature: $k = \|\mathbf{T}'(s)\| = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{v}(t) \times \mathbf{a}(t)\|}{\|\mathbf{v}(t)\|^3}$

Linear component: $a_{\mathbf{T}} = \frac{d^2s}{dt^2} = \frac{\mathbf{v} \cdot \mathbf{a}}{\|\mathbf{v}\|}$

Angular component: $a_{\mathbf{N}} = \left(\frac{ds}{dt}\right)^2 k = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|}$