Math 252 Exam 1 Review (Problems)

1. Indentify via cross-sections the surface defined by $2y^2 = 3z^2 = 12$.

2. A baseball is thrown from the stands 128 feet above the field at an angle of 30 degrees up from the horizontal with an initial speed of 64 feet per second.
   a. Give the position vector for any time t.
   b. When will the ball strike the ground?
   c. How far away will the ball strike the ground?
   d. What is the speed of the ball when it strikes the ground?

3. Find the center and radius of the sphere given by $x^2 + y^2 + z^2 - 8x + 6x = 0$

4. Using $P(-2, 0, 3), Q(1, 2, 4), R(-3, 1, 0)$,
   a. Find a vector orthogonal to the plane determined by P, Q and R.
   b. Find an equation of the plane passing through P, Q and R.
   c. Find the set of parametric equations for the line through Q and parallel to $\mathbf{a} = \langle 4, -3, -2\rangle$.
   d. Find the distance from the point $(-4, -1, 5)$ to the plane passing through P, Q and R.

5. Using $\mathbf{r}(t) = \langle 4\cos(2t), 4\sin(2t), 6t\rangle$,
   a. Find $\mathbf{T}(t)$
   b. Find $\mathbf{N}(t)$
   c. Find the curvature

6. Identify the surface $2x^2 - 3y^2 + 6z^2 = 6$.

7. Identify the surface $x^2 - 6y + 5z^2 = 0$.

8. Using $\mathbf{r}(t) = \langle t\cos t, t\sin t, t^2\rangle$ at $t = 0$,
   a. Find $\mathbf{v}$ and $\mathbf{a}$.
   b. Find $\mathbf{T}$ and $\mathbf{N}$.
   c. Find K.
   d. By first finding $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$, express $a = a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$.

9. Using $\mathbf{u} = \langle -4, 6, 5\rangle$ and $\mathbf{v} = \langle 2, -3, 1\rangle$,
   a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
   b. Find $\mathbf{u} \cdot \mathbf{v}$.
   c. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.
   d. Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.
   e. Find $\mathbf{u} \times \mathbf{v}$.

10. Indentify via cross-sections the surface defined by $x = 3y^2 + 5z^2$.

11. Indentify via cross-sections the surface defined by $3^2 - y^2 + 3z^2 + 9 = 0$.

12. Identify the surface $x = y^2$.

13. Identify the surface $4x^2 + 4y^2 + z^2 = 4$.

14. A projectile is fired at a speed of 448 feet per second at and angle of 30 degrees from a tower 512 feet above the ground.
   a. Give the position vector for any time t.
   b. How far away will the object strike?

15. Indentify via cross-sections the surface defined by $y = x^2$.

16. Using $\mathbf{u} = \langle 8, 3, -5\rangle, \mathbf{v} = \langle 4, -4, -2\rangle$,
   a. Find $3u - 4v$.
   b. Find $\|\mathbf{u}\|, \|\mathbf{v}\|$.

17. Using $\mathbf{u} = \langle 8, -4, 1\rangle$ and $\mathbf{v} = \langle -4, 4, 2\rangle$,
   a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
   b. Find $\mathbf{u} \cdot \mathbf{v}$.
   c. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.
   d. Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.
   e. Find $\mathbf{u} \times \mathbf{v}$.

18. Using $P(-4, 1, 2)$, $Q(1, -3, 4)$, $R(-1, 0, 2)$,
   a. Find an equation of the plane passing through the points.
   b. Find parametric equations for the line through P and parallel to $a = \langle 2, -1, 4\rangle$.
   c. Find the distance from the point $(5, -3, 2)$ to the plane.
   d. Find the area of the parallelogram determined by P, Q, and R.

19. Using $r(t) = \langle \cos t, \sin t, t^2\rangle$, $t = \frac{\pi}{2}$:
   a. Find the velocity vector.
   b. Find the acceleration vector.

20. Find the tangential and normal components of acceleration for the curve $\mathbf{r}(t) = \langle 3t^2, 4t^2, 10t\rangle$ at $t = 2$ and express a in terms of T and N.

Math 252 Exam 1 Review (Answers)

1. Elliptical cylinder

2. ANSWER

3. $C(4,-3,0), \rho = 5$

4. a. $\mathbf{n} = \mathbf{PQ} \times \mathbf{PR} = \langle -7, 8, 5 \rangle$
   b. $-7x + 8y + 5z = 29$
   c. $x = 1 + 4t, y = 2 - 3t, z = 4 - 2t; t \in \mathbb{R}$
   d. $D = \frac{16}{\sqrt{138}}$

5. a. $\mathbf{T}(t) = \langle -\frac{4}{5}\sin(2t), \frac{4}{5}\cos(2t), \frac{3}{5} \rangle$
   b. $\mathbf{N}(t) = \langle -\cos(2t), \sin(2t), 0 \rangle$
   c. $k = \frac{4}{25}$

6. ANSWER

7. ANSWER

8. a. $\mathbf{v} = \langle -t\sin t + \cos t, t\cos t + \sin t, 2t \rangle$
      $\mathbf{a} = \langle -t\cos t - 2\sin t, -t\sin t + 2\cos t, 2 \rangle$
   b. $\|\mathbf{v}\| =$
      $\mathbf{T} =$
      $\mathbf{N} =$

9. a. $\|\mathbf{u}\| = \sqrt{77}$
      $\|\mathbf{v}\| = \sqrt{14}$
   b. $\mathbf{u} \cdot \mathbf{v} = -21$
   c. $\theta = \arccos\left(\frac{-21}{7\sqrt{22}}\right)$

10. Elliptical paraboloid

11. Circular hyperboloid of two sheets

12. Parabolic cylinder

13. ANSWER

14. a. $\mathbf{r}(t) = \langle 224\sqrt{3}t, -16t^2 + 224t + 512 \rangle$
    b. $T = 16,\ x(16) = 224\sqrt{3}(16) \doteq 6207.7$ feet

15. Parabolic cylinder

16. a. $\langle 8, 25, -7 \rangle$.
    b. $\|\mathbf{u}\| = 7\sqrt{2}, \|\mathbf{v}\| = 6$.

17. a. $\|\mathbf{u}\| = 9, \|\mathbf{v}\| = 6$
    b. $\mathbf{u} \cdot \mathbf{v} = -46$
    c. $\theta = \arccos\left(-\frac{23}{27}\right) = 148.4°$
    d. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(-\frac{23}{18}\right)\langle -4, 4, 2 \rangle = \langle -\frac{46}{9}, -\frac{46}{9}, -\frac{23}{9} \rangle$
    e. $\mathbf{u} \times \mathbf{v} = \langle -12, -20, 16 \rangle$

18. a. $2x + 6y + 7z - 12 = 0$
    b. $x = 2t - 4,\ y = -t + 1,\ z = 4t + 2$
    c. $D = \frac{6}{\sqrt{89}}$
    d. $A = \sqrt{89}$

19. a. $\mathbf{v}(t) = \langle -\sin t, \cos t, 2t \rangle, \quad \mathbf{v}(\frac{\pi}{2}) = \langle -1, 0, \pi \rangle$
    b. $\mathbf{a}(t) = \langle -\cos t, -\sin t, 2 \rangle, \quad \mathbf{a}(\frac{\pi}{2}) = \langle 0, -1, 2 \rangle$

20. $\mathbf{a} = 4\sqrt{5}\mathbf{T} + 2\sqrt{5}\mathbf{N}$
    (correction?) $\mathbf{a} = \frac{20}{\sqrt{5}}\mathbf{T} + \frac{10}{\sqrt{5}}\mathbf{N}$