

Math 252 Final Review (Problems)

- Using $\mathbf{u} = \langle -2, 4, 4 \rangle$ and $\mathbf{v} = \langle 0, 3, -4 \rangle$,
 - Find $\mathbf{u} \cdot \mathbf{v}$.
 - Find $\text{Proj}_{\mathbf{u}} \mathbf{v}$.
 - Find the angle θ between \mathbf{u} and \mathbf{v} .
 - Find $\mathbf{u} \times \mathbf{v}$.
- Find the length of the helix $\mathbf{r}(t) = \langle 6 \sin(2t), -5t, -6 \cos(2t) \rangle$ for $0 \leq t \leq 4\pi$.
- Using $\mathbf{r}(t) = \langle 4 \cos t, 3t, 4 \sin 5 \rangle$ at $t = 0$,
 - Find $\mathbf{v}(t)$ and $\mathbf{a}(t)$.
 - Find $\mathbf{T}(t)$ and $\mathbf{N}(t)$.
 - Find K , $a_{\mathbf{T}}$, and $a_{\mathbf{N}}$.
- Given the points $P(2, 1, 2)$, $Q(6, -2, 1)$ and $R(-1, 4, 5)$,
 - Find an equation of the plane passing through the points.
 - Find an equation of the line perpendicular to the plane, passing through the point $(8, 2, -1)$.
 - Find the distance from the point $(-5, -2, 7)$ to the plane.
 - Find the area of the parallelogram determined by the points.
- A Projectile is launched at an angle of 30° , with speed 224 feet per second, and from a platform 128 feet above the ground,
 - Find the position vector of the object at time t .
 - How far away will it hit the ground?
 - What is the speed upon impact?
- Identify each surface by identifying the cross sections in each plane of \mathbb{R}^3 space:
 - $2x^2 - 3y^2 + 6z^2 = 1$
 - $4x - 2y^2 - 2z^2 = 9$
 - $4x - 2y^2 = 9$
- Determine if the planes $ax + by + cz + d_1 = 0$ and $(ak)x + (bk)y + (ck)z + d_2 = 0$ intersect. If they do not, find the distance between them.
- Prove that all lines and circles (in the xy-plane) have constant curvature.
- Describe the domain of $f(x, y) = \frac{\ln(xy)}{\sqrt{x+y}}$
- Evaluate $\int_0^1 \int_{y^2}^{\sqrt{y}} 4xy \, dx \, dy$
- For $f(x, y) = 4x^2 + 2y^2$, find the equation of the level curve that contains the point $P(-2, 4)$ and describe its shape.
- Use Lagrange multipliers to find the extrema of $f(x, y, z) = x - 2y - 4z$ subject to the constraint $z = 4x^2 + y^2$
- Evaluate $\int_C 6xt \, dx + x^2y \, dy$ where C is the graph $y = x^2 + 3$ from $(0, 3)$ to $(3, 12)$.
- Find the mass of the solid with the density $\delta(x, y, z) = 8xy$ whose base in the xy-plane is bounded by $y = x$, $y = 0$ and $x = 3$ and bounded above by $z = 9 - x^2$.
- Find the surface area of the paraboloid $z = x^2 + y^2$ between the planes $z = 6$ and $z = 30$.
- Evaluate $\oint_C (-2xy^2 \, dx + 4x^2y \, dy)$ where C is the boundary of the region in the first quadrant bounded by the x-axis, the y-axis and the semicircle $y = \sqrt{16 - x^2}$.
- Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 36$, the cone $z = \sqrt{x^2 + y^2}$, and the plane $z = 12$.
- For $\mathbf{F} = \langle 3x^2y^3, 2y^3z, 4xz^2 \rangle$ find each of the following:
 - $\text{Div } \mathbf{F}$
 - $\text{Curl } \mathbf{F}$
 - $\text{Div}(\text{Curl } \mathbf{F})$
- Find the work done by $\mathbf{F} = \langle xy, y, -yz \rangle$, $\mathbf{r}(t) = \langle t, t^2, t \rangle$, $0 \leq t \leq 4$.

20. Evaluate $\iint_R (x-y)^2(x+y)dA$ where the boundary of R is the rectangle with vertices $(4,0)$, $(8,4)$, $(4,8)$ and $(0,4)$.
21. Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} \cos(x^2+y^2)dy dx$.
22. Determine if $\mathbf{F}(x,y,z) = \langle 18x^2 \sin y, 6x^3 \cos y - 3e^y \cos z, 3e^y \sin z + 2 \cos z \rangle$, is conservative.

Math 252 Final Review (Answers)

1. (Math-252 Exam 1)

a. $\mathbf{u} \cdot \mathbf{v} = -4$

b. $\text{Proj}_{\mathbf{u}} \mathbf{v} = \langle \frac{2}{9}, \frac{-4}{9}, \frac{-4}{9} \rangle$

c. $\theta = \cos^{-1} \left(\frac{-2}{15} \right) \doteq 1.705 \text{ rad}$

d. $\mathbf{u} \times \mathbf{v} = \langle -28, -8, -6 \rangle$

2. (Math-252 Exam 1)

$s = 52\pi$

3. (Math-252 Exam 1)

a. $\mathbf{v}(t) = \langle -4 \sin t, 3, 4 \cos t \rangle$

$\mathbf{v}(0) = \langle 0, 3, 4 \rangle$

$\mathbf{a}(t) = \langle -4 \cos t, 0, -4 \sin t \rangle$

$\mathbf{a}(0) = \langle -4, 0, 0 \rangle$

b. $\mathbf{T}(t) = \langle -\frac{4 \sin t}{5}, \frac{3}{5}, \frac{4 \cos t}{5} \rangle$

$\mathbf{T}(0) = \langle 0, \frac{3}{5}, \frac{4}{5} \rangle$

$\mathbf{N}(t) = \langle -\cos t, 0, -\sin t \rangle$

$\mathbf{N}(0) = \langle -1, 0, 0 \rangle$

c. $K = \frac{4}{25} \quad a_{\mathbf{T}} = 0 \quad a_{\mathbf{N}} = 4$

4. (Math-252 Exam 1)

a. $2x + 3y - z - 5 = 0$

b. $x = -6t + 8, \quad y = -9t + 2, \quad z = 3t - 1$

c. $D = 2\sqrt{14}$

d. $A = 3\sqrt{14}$

5. (Math-252 Exam 1)

a. $\mathbf{r}(t) = \langle 112\sqrt{3}t, -16t^2 + 112t + 128 \rangle$

b. max distance: $893\sqrt{3}$

c. impact speed: $32\sqrt{57}$

6. (Math-252 Exam 1)

a. Elliptical hyperboloid (one sheet)

b. Circular hyperboloid

c. Parabolic cylinder

7. (Math-252 Exam 1)

(this was extra-credit)

$\mathbf{n}_1 = \langle a, b, c \rangle, \quad \mathbf{n}_2 = \langle ka, kb, kc \rangle = k\langle a, b, c \rangle$

$\mathbf{n}_1 \times \mathbf{n}_2 = 0, \quad \therefore n_1 \parallel n_2$

point on first plane: $P(0, 0, -\frac{d_1}{c})$

distance from point to second plane:

$D = |\text{Proj}_{\mathbf{n}_1} P|$

$= \frac{|ka(0) + kb(0) + kc(-\frac{d_1}{c}) + d_2|}{\sqrt{(ka)^2 + (kb)^2 + (kc)^2}}$

$= \frac{|-kd_1 + d_2|}{|k|\sqrt{a^2 + b^2 + c^2}}$

8. (Math-252 Exam 1)

(this was extra-credit)

(ANSWER)

9. (Math-252 Exam 2)

$\{(x, y) : x > 0, y > 0\}$

10. (Math-252 Exam 2)

$\int_0^1 \int_{y^2}^{\sqrt{y}} 4xy \, dx \, dy = \frac{1}{3}$

11. (Math-252 Exam 2)

$4x^2 + 2y^2 = 48$ (an ellipse)

12. (Math-252 Exam 2)

Absolute max $f(\frac{1}{32}, -\frac{1}{4}, \frac{17}{256}) = \frac{17}{64}$

13. (Math-252 Exam 3)

$\int_0^3 (2t^5 + 12t^3 + 18t) dt$

14. (Math-252 Exam 3)

$m = \int_0^3 \int_0^x \int_0^{9-x^2} 8xy \, dz \, dy \, dx = 243$

15. (Math-252 Exam 3)

$\iint_R \sqrt{4x^2 + 4y^2 + 1} \, dA = 201\pi$

16. (Math-252 Exam 3)

Using Green's theorem, $\int_0^4 \int_0^{\sqrt{16-x^2}} 12xy \, dy \, dx = 384$.

17. (Math-252 Exam 3)

$V = \int_0^{2\pi} \int_0^6 \int_r^{12} r \, dz \, dr \, d\theta = 288\pi$

18. (Math-252 Exam 3)

- a. $\text{Div } \mathbf{F} = 6xy^3 + y^2z + 8xz$
- b. $\text{Curl } \mathbf{F} = \langle -2y^3, -4z^2, -9x^2y^2 \rangle$
- c. $\text{Div}(\text{Curl } \mathbf{F}) = 0$

19. (Math-252 Exam 3)

$$W = \int \mathbf{F}(t) \cdot \mathbf{r}'(t) dt = 128$$

20. (Math-252 Exam 3)

Let $u = x - y$, $v = x + y$, $x = \frac{1}{2}u + \frac{1}{2}v$, and $y = -\frac{1}{2}u + \frac{1}{2}v$, where $-4 \leq u \leq 4$, $4 \leq v \leq 8$.

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{1}{2}$$
$$\iint_R (x-y)^2(x+y) dA = \frac{1}{2} \int_{-4}^4 \int_4^{12} u^2 v dv du = \frac{4096}{3}$$

21. (Math-252 Exam 3)

$$\int_0^{\pi/2} \int_0^3 \cos(r^2) r dr d\theta = \frac{\pi}{4} \sin 9$$

22. (Math-252 Exam 3)

(Extra credit)

$M_y = 18x^2 \cos y = N_x$, $M_z = 0 = P_x$ and $N_z = 3e^y \sin z = P_y$, therefore \mathbf{F} is conservative.

$$f(x, y, z) = \int 18x^2 \sin y dx = 6x^3 \sin y + g(y, z)$$

$$f_y = 6x^3 \cos y + g_y = 6x^3 \cos y - 3e^y \cos z$$

$$\therefore g(y, z) = -3e^y \cos z + h(z)$$

$$f(x, y, z) = \int 6x^3 y^2 - 3e^y \cos z + h(z)$$

$$f_z = 3e^y \sin z + h_z = 3e^y \sin z + 2 \cos z$$

$$\therefore h_z = 2 \cos z + C$$

$$\therefore f(x, y, z) = 6x^3 \sin y - 3e^y \cos z + 2 \sin z + C$$