Notes for Discrete Math (MATH-245)
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Symbol reference

Symbol	Description	LaTeX
$\neg$	Negation	`\neg`
$\wedge$	And	`\wedge`
$\vee$	Or	`\vee`
$\rightarrow$	If...then	`\rightarrow`
$\leftrightarrow$	If and only if...then	`\leftrightarrow`
$\in$	In	`\in`
$\subset$	Subset	`\subset`
$\supset$	Superset	`\supset`
$\emptyset$	Empty set	`\emptyset`

(February 7th)

1 Logic

1.1 Operations

In order of precedence (what is evaluated first)

Negation ($\neg$) Unary operator whose argument follows on the right. Changes true to false and false to true. Also has basically highest precedence in evaluation order.

And ($\wedge$) Says if left and right, then true, else false.

Or ($\vee$) Says if left or right, then true, else false.

If...then ($\rightarrow$) A logical statement about correlation and causation. Formally expressed as "if p, then q" ($p \rightarrow q$) where p is the *hypothesis* and q is the *conclusion*. If the hypothesis p is true, then the conclusion q must then be true. Note that causation in the opposite direction is not implied: it is not stated that if not p then not q, so q can be true even if p is not true. This is known as *inverse error* (Ch 2.3).

If and only if...then ($\leftrightarrow$) Bi-conditional if says if p and q have the same values. Only if both are true or both are false is bi-conditional if true.

1.2 Truth Tables

Used to evaluate all the potential outcomes of a logical statement given all possible combinations of inputs. Given two inputs p and q then would be four potential input combinations:

$$(p, q) \in \{T, F\} \times \{T, F\} = \{(T, T), (T, F), (F, T), (F, F)\}$$

Here is a table of the potential outputs for the previously introduced logical operators (not, and, or, if, iff).

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	F	F	T	T	F
F	F	F	F	F	F	T

1.3 De Morgan's Laws

Method for negating a statement involving an `and` or an `or` operation.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q \tag{1}$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q \tag{2}$$

1.4 Argument

Invalid An argument is invalid if its conclusion can be true while at the same time have false premises. If a logical argument were mapped in a logic table, any row where all premises are true is called a *critical row*.

Valid An argument is valid if its conclusion is true only when all of its premises are true.

Sound A

Unsound A

1.4.1 Syllogism

An argument that consists of two premises is called a syllogism. The premises are called the major and minor premises respectively. Some forms of syllogisms are:

- Modus ponens: "if p then q; p; therefore q"
- Modus tollens: "if not p then not q; not p; therefore not q"

(February 12th)

1.5 Conditional arguments

Conditional	$p \to q$
Bi-conditional	$p \leftrightarrow q$
Converse	$q \to p$
	Logically equivalent to "if and only if p then q" (contrapositive), but not to "if p then q"
Inverse	$\neg p \to \neg q$
Contrapositive	$\neg q \to \neg p$
	Don't have conclusion, therefore don't have hypothesis
Negation	$p \land \neg q \equiv \neg(p \to q)$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Examples

- "Sam will be allowed on Jane's boat only if he is an expert sailor"
  - Let "He is an expert sailor" be q
  - Let "Sam will be allowed on Jane's boat" by p
  - $\neg q \to \neg p \equiv p \to q$
  - Inverse: $\neg p \to \neg q$
  - Contrapositive: $\neg q \to \neg p$
- "If someone is eligible to vote, then they are at least 18 years old"
  - Let "Someone is eligible to vote" be p
  - Let "They are at least 18 years old" be q
  - Converse: If 18 years old, then eligible to vote.
  - Inverse: If not eligible, then younger than 18.
  - Contrapositive: If not 18, then not eligible to vote.