Math 252 Exam 3 Review (Problems)

1. Using polar coordinates, evaluate $\iint_R (x^2+y^2)^{3/2} dA$ where R is the region bounded by the circle of radius a centered at the origin.

2. a. Show that $\mathbf{F}(x,y) = (3x^2y+2x)\hat{\imath}+(x^3+2y)\hat{\jmath}$ is conservative and find a function f such that $\nabla f = \mathbf{F}$.

   b. Let $\mathbf{F}$ be as in part *a.* and $\mathbf{r}(t) = \langle t^4, t^2+1\rangle$, $0 \le t \le 1$. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{x}$.

3. Find the curl and divergence of $\mathbf{F} = \langle xz^2, 2yz, 3xy^2\rangle$.

4. Evaluate $\int_{-1}^{2}\int_{1}^{x}\int_{0}^{x+y}(3x^2y)dz\ dy\ dx$.

5. Find a conservative vector field $\mathbf{F}$ that has the potential $f(x,y,z) = 4x^2y - 2y^2z^3$¿

6. For the solid bounded in the first octant by the plane $4x+2y+z = 12$ with density $\delta(x,y,z) = 5x^3$,

   a. find it's mass.

   b. set up (but don't solve) the integral to find M_{xz}.

7. Find the equation of the tangent plane to the surface given by $x = u^2+2v^2$, $y = uv$ and $z = 3u-v$ when $u = 2$ and $v = -1$.

8. Evaluate $\int_{(1,1,1)}^{(2,3,4)}(4xy)dx + (2x^2+2yz^3)dy + (3y^2z^2+3)dz$.

9. The force at a point (x,y,z) in three dimensions is given by $\mathbf{F} = \langle y,z,x\rangle$. Find the work done by $\mathbf{F}$ along the twisted cubic $x = t$, $y = t^2$ and $z = t^3$ from $(0,0,0)$ to $(2,4,8)$.

10. For the solid bounded by $z = \sqrt{16x^2+16y^2}$, $x^2+y^2 = 16$, and $z = 0$,

    a. find its volume.

    b. find the center of mass if $\delta = \sqrt{x^2+y^2}$.

11. Find the volume of the solid that lies outside the cone $z^2 = x^2+y^2$ and inside the sphere $x^2+y^2+z^2 = 18$

12. Find the center of mass of the lamina that has the shape of the region bounded by $y = x^2$ and $y = 9$ with density $\delta(x,y) = 12x^2y^2$.

13. Find the volume of the solid bounded by $z = \sqrt{x^2+y^2}$ and $x^2+y^2+z^2 = 16$.

14. Find the surface area of S, the part of the paraboloid $z = x^2+y^2$ under the plane $z = a, a > 0$.

15. Use polar coordinates to evaluate the integral $\iint_R x\sqrt{x^2+y^2}\ dA$ where R si the region bounded by the semicircle $x = \sqrt{36-y^2}$.

16. Evaluate $\iint_R \frac{2y+x}{y-2x}dA$ where R is the region bounded by the trapezoid with vertices $(-1,0)$, $(-2,0)$, $(0,4)$, $(0,2)$.

17. Use an appropriate change of variables to find $\iint_R \frac{x-y}{2x+3y}\ dA$ where R is the region bounded by the lines $x-y = -1$, $x-y = 2$, $2x+3y = 1$, and $2x+3y = 3$.

18. Find the curl and divergence of $\mathbf{F} = \langle -3\sin x + \cos y, 6xz^2, 3y+z\rangle$.

19. Use Green's theorem to evaluate $\oint_C (6y)dx + (\frac{5}{3}x^3)dy$, where C is the boundary of the first quadrant region bounded by $y = 36-x^2$ and the x-axis.

20. Evaluate the line integral $\int_C (xy^2)dx + (4xy^3)dy$ along C: $x = y^2$ from $(0,0)$ to $(4,2)$.

21. Find the surface area of the part of the paraboloid $z = f(x,y) = 20-x^2-y^2$ above $z = 4$.

22. Evaluate $\iint_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = 2x\hat{\imath} - xy\hat{\jmath} + xz\hat{k}$ and S is the surface of the paraboloid $x = y^2+z^2$ with $x \le 1$ and the disk $y^2+z^2 = 1$ at $x = 1$.

23. Evaluate $\iint_S (x^2 z + y^2 z)\, dS$, where S is the part of the plane $z = 4 + x + y$ that lies inside the cylinder $x^2 + y^2 = 4$.

24. Use Stokes' theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = xy\hat{\imath} + yz\hat{\jmath} + xz\hat{k}$ and C is the positively oriented triangle with vertices $(3, 0, 0)$, $(0, 2, 0)$, $(0, 0, 6)$.

25. Evaluate $\iint_R (2x + y)e^{(2y-x)} dA$, where R is the rectangle with vertices $(2, 1)$, $(6, 3)$, $(4, 7)$ and $(0, 5)$.

26. Find the surface area for the surface given by the parametric equations $x = u + v$, $y = uv$ and $z = u - v$ with $u^2 + v^2 \leq 4$.

27. Verify Green's Theorem is true for $\int_C xy^2\, dx - x^2 y\, dy$, where C consists of the parabola $y = x^2$ from $(-1, 1)$ to $(1, 1)$ and the line segment from $(1, 1)$ to $(-1, 1)$.
(i.e. evaluate directly and using Green's Theorem)

Math 252 Exam 3 Review (Answers)

1. (Math-252 Exam 3 Practice)

   (ANSWER)

2. (Math-252 Some Exam 3 Practice)

   6

3. (Math-252 Quiz 21)

   - Curl: $\langle 6xy - 2y, 2xz - 3y^2, 0 \rangle$
   - Divergence: $z^2 + 2z$

4. (Math-252 Exam 3 Practice)

   (ANSWER)

5. (Math-252 Quiz 21)

   $\mathbf{F} = \nabla f = \langle 8xy, 4x^2 - 4yz^3, -6y^2z^2 \rangle$

6. (Math-252 Quiz 18)

   a. $m = 243$

   b. $M_{xz} = \int_0^3 \int_0^{6-2x} \int_0^{12-4x-2y} (5x^3) y \, dz \, dy \, dx$

7. (Math-252 Some Exam 3 Practice)

   -14

8. (Math-252 Exam 3 Practice)

   (ANSWER)

9. (Math-252 Exam 3 Practice)

   (ANSWER)

10. (Math-252 Exam 3 Practice)

    a. (ANSWER)

    b. (ANSWER)

11. (Math-252 Quiz 19)

    $V = 72\pi$

12. (Math-252 Exam 3 Practice)

    (ANSWER)

13. (Math-252 Exam 3 Practice)

    (ANSWER)

14. (Math-252 Exam 3 Practice)

    (ANSWER)

15. (Math-252 Quiz 16)

    $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^6 r \cos\theta \sqrt{r^2}\, r \; dr \; d\theta = 648$

16. (Math-252 Exam 3 Practice)

    (ANSWER)

17. (Math-252 Some Exam 3 Practice)

    $\frac{3}{10} \ln 3$

18. (Math-252 Exam 3 Practice)

    (ANSWER)

19. (Math-252 Exam 3 Practice)

    (ANSWER)

20. (Math-252 Quiz 21)

    $\int_C (xy^2) dx + (4xy^3) dy = \int_0^2 (6t^5) dt = 64$

21. (Math-252 Quiz 17)

    $S = \frac{\pi}{6}(65^{3/2} - 1)$

22. (Math-252 Some Exam 3 Practice)

    3π

23. (Math-252 Some Exam 3 Practice)

    $32\pi\sqrt{3}$

24. (Math-252 Some Exam 3 Practice)

    -25

25. (Math-252 Quiz 20)

    $\int_5^{15} \int_0^{10} u e^v \left(\frac{5}{25}\right) dv \, du = 20(e^{10} - 1)$

26. (Math-252 Some Exam 3 Practice)

    $\frac{\pi}{3}(12^{3/2} - 8)$

27. (Math-252 Some Exam 3 Practice)

    0 (but what does this mean?)