Sections covered:

- Partial derivatives
- Chain rule (partial derivatives)
- Gradient and directional derivative
- Tangent planes, linear approximation
- Multivariable extrema
- Lagrange multipliers
- Double integration
- Double integration (polar coordinates)
- Surface area

Things I did wrong last exam

- Confused an equation of a plane for a line
- Didn't simplify ($\frac{84}{\sqrt{126}} \Rightarrow 2\sqrt{14}$, $\sqrt{126} \Rightarrow 3\sqrt{14}$)

Shapes and surfaces

$$\begin{aligned}
\text{Tangent plane:} &\quad (f_x)(x - x_0) - (f_y)(y - y_0) - (f_z)(z - z_0) = 0 \\
\text{Circle:} &\quad ax^2 + ay^2 = c, c \geq 0 \\
\text{Ellipse:} &\quad ax^2 + by^2 = c, c \geq 0 \\
\text{Hyperbola:} &\quad ax^2 - by^2 = \begin{cases} k^2 + c : \text{of one sheet} \\ k^2 - c : \text{of two sheets} \end{cases}
\end{aligned}$$

Function manipulation

2 to 3 variables: $\quad z = f(x, y) \Rightarrow g(x, y, z) = f(x, y) - z$

Partial Derivatives

- Chain rule of $z(x, y)$ with one dependent variable (parameter):

$$\frac{dz}{dt} = \frac{\delta z}{\delta x}\frac{dx}{dt} + \frac{\delta z}{\delta y}\frac{dy}{dt}$$

- (?) Chain rule of $z = z(x, y) = z(x, y(x))$ with respect to x:

$$\frac{\delta z}{\delta x} = \frac{\delta z}{\delta y}\left(\frac{\delta y}{\delta x}\right)$$

- Implicit differentiation of y with respect to x:

$$g(x, y) = h(x, y) \Rightarrow F(x, y) = g(x, y) - h(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

Gradient and directional derivative

- Gradient of f:

$$\nabla f = \langle f_x, f_y, f_z \rangle$$

- Directional derivative of f in the direction of unit vector $\mathbf{u}$:

$$D_{\mathbf{u}} f = \nabla f \cdot \langle u_1, u_2 \rangle$$