

## Math 252 Exam 1 Review (Problems)

1. Find the center and radius of the sphere given by  $x^2 + y^2 + z^2 - 8x + 6y = 0$
2. Identify via cross-sections the surface defined by the following:

$$3^2 - y^2 + 3z^2 + 9 = 0$$

3. Identify the surface  $2x^2 - 3y^2 + 6z^2 = 6$ .
4. Identify the surface  $x^2 - 6y + 5z^2 = 0$ .
5. Identify via cross-sections the surface defined by the following:

$$2y^2 = 3z^2 = 12$$

6. Using  $P(-4, 1, 2)$ ,  $Q(1, -3, 4)$ ,  $R(-1, 0, 2)$ ,
  - a. Find an equation of the plane passing through the points.
  - b. Find parametric equations for the line through P and parallel to  $a = \langle 2, -1, 4 \rangle$ .
  - c. Find the distance from the point  $(5, -3, 2)$  to the plane.
  - d. Find the area of the parallelogram determined by  $P$ ,  $Q$ , and  $R$ .
7. Given  $\vec{u} = \langle 8, -4, 1 \rangle$  and  $\vec{v} = \langle -4, 4, 2 \rangle$ ,  
find  $\|\vec{u}\|$  and  $\|\vec{v}\|$ .
8. Identify the surface  $4x^2 + 4y^2 + z^2 = 4$ .
9. Using  $\mathbf{u} = \langle 8, 3, -5 \rangle$ ,  $\mathbf{v} = \langle 4, -4, -2 \rangle$ ,  
find  $\|\mathbf{u}\|$ ,  $\|\mathbf{v}\|$ .
10. Find the distance from the point  $(-4, -1, 5)$  to the plane determined by the points  $P(-2, 0, 3)$ ,  $Q(1, 2, 4)$ , and  $R(-3, 1, 0)$ .
11. Identify via cross-sections the surface defined by the following:

$$y = x^2$$

12. Using  $\mathbf{u} = \langle -4, 6, 5 \rangle$  and  $\mathbf{w} = \langle 2, -3, 1 \rangle$ ,
  - a. Find  $\|\mathbf{u}\|$  and  $\|\mathbf{w}\|$ .
  - b. Find  $\mathbf{u} \cdot \mathbf{w}$ .
  - c. Find the angle  $\theta$  between  $\mathbf{u}$  and  $\mathbf{w}$ .
  - d. Find  $\text{proj}_{\mathbf{w}} \mathbf{u}$ .
  - e. Find  $\mathbf{u} \times \mathbf{w}$ .
13. Given  $\vec{u} = \langle 8, -4, 1 \rangle$  and  $\vec{v} = \langle -4, 4, 2 \rangle$ ,  
find  $\vec{u} \times \vec{v}$ .
14. A baseball is thrown from the stands 128 feet above the field at an angle of 30 degrees up from the horizontal with an initial speed of 64 feet per second.
  - a. Give the position vector for any time  $t$ .

- b. When will the ball strike the ground?
  - c. How far away will the ball strike the ground?
  - d. What is the speed of the ball when it strikes the ground?
15. Given  $\vec{u} = \langle 8, -4, 1 \rangle$  and  $\vec{v} = \langle -4, 4, 2 \rangle$ ,  
find the angle  $\theta$  between  $\vec{u}$  and  $\vec{v}$ .
16. Identify via cross-sections the surface defined by the following:
- $$x = 3y^2 + 5z^2$$
17. Identify the surface  $x = y^2$ .
18. Given  $\vec{u} = \langle 8, -4, 1 \rangle$  and  $\vec{v} = \langle -4, 4, 2 \rangle$ ,  
find  $\text{proj}_{\vec{v}} \vec{u}$ .
19. Find the set of parametric equations for the line through  $Q(1, 2, 4)$  and parallel to  $a = \langle 4, -3, -2 \rangle$ .
20. Find a vector orthogonal to the plane determined by the points  $P(-2, 0, 3)$ ,  $Q(1, 2, 4)$ , and  $R(-3, 1, 0)$ .
21. Find an equation of the plane passing through the points  $P(-2, 0, 3)$ ,  $Q(1, 2, 4)$ , and  $R(-3, 1, 0)$ .
22. Given  $\vec{u} = \langle 8, -4, 1 \rangle$  and  $\vec{v} = \langle -4, 4, 2 \rangle$ ,  
find  $\vec{u} \cdot \vec{v}$ .
23. Using  $\mathbf{r}(t) = \langle t \cos t, t \sin t, t^2 \rangle$  at  $t = 0$ ,
- a. Find  $\mathbf{v}$  and  $\mathbf{a}$ .
  - b. Find  $\mathbf{T}$  and  $\mathbf{N}$ .
  - c. Find  $K$ .
  - d. By first finding  $a_{\mathbf{T}}$  and  $a_{\mathbf{N}}$ ,  
express  $a = a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$ .
24. Using  $\mathbf{u} = \langle 8, 3, -5 \rangle$ ,  $\mathbf{v} = \langle 4, -4, -2 \rangle$ ,  
find  $3u - 4v$ .

## Math 252 Exam 1 Review (Answers)

1.  $C(4, -3, 0), \rho = 5$
2. Circular hyperboloid of two sheets
3. ANSWER
4. ANSWER
5. Elliptical cylinder
6. ANSWER
7.  $\|\vec{u}\| = 9, \|\vec{v}\| = 6$
8. ANSWER
9.  $\|\mathbf{u}\| = 7\sqrt{2}, \|\mathbf{v}\| = 6$
10.  $h = \frac{16}{\sqrt{138}}$
11. Parabolic cylinder
12. a.  $\|\mathbf{u}\| = \sqrt{77}$   
 $\|\mathbf{w}\| = \sqrt{14}$   
b.  $\mathbf{u} \cdot \mathbf{w} = -21$   
c.  $\theta = \arccos\left(\frac{-21}{7\sqrt{22}}\right)$
13.  $\vec{u} \times \vec{v} = \langle -12, -20, 16 \rangle$
14. ANSWER
15.  $\theta = \arccos\left(-\frac{23}{27}\right) = 148.4^\circ$
16. Elliptical paraboloid
17. Parabolic cylinder
18.  $\text{proj}_{\vec{v}}\vec{u} = -\frac{23}{18}\langle -4, 4, 2 \rangle = \langle -\frac{46}{9}, -\frac{46}{9}, -\frac{23}{9} \rangle$
19.  $x = 1 + 4t, y = 2 - 3t, z = 4 - 2t; t \in \mathbb{R}$
20.  $\vec{n} = \vec{PQ} \times \vec{PR} = \langle -7, 8, 5 \rangle$
21.  $-7x + 8y + 5z = 29$
22.  $\vec{u} \cdot \vec{v} = -46$
23. a.  $\mathbf{v} = \langle -t \sin t + \cos t, t \cos t + \sin t, 2t \rangle$   
 $\mathbf{a} = \langle -t \cos t - 2 \sin t, -t \sin t + 2 \cos t, 2 \rangle$   
b.  $\|\mathbf{v}\| =$   
 $\mathbf{T} =$   
 $\mathbf{N} =$
24.  $\langle 8, 25, -7 \rangle$