Math 252 Exam 1 Review (Problems)

- 1. Find the center and radius of the sphere given by $x^2 + y^2 + z^2 8x + 6x = 0$
- 2. Using $\mathbf{u} = \langle 8, 3, -5 \rangle, \mathbf{v} = \langle 4, -4, -2 \rangle,$
 - a. Find 3u 4v.
 - b. Find $\|\mathbf{u}\|$, $\|\mathbf{v}\|$.
- 3. Using $\mathbf{u} = \langle 8, -4, 1 \rangle$ and $\mathbf{v} = \langle -4, 4, 2 \rangle$,
 - a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
 - b. Find $\mathbf{u} \cdot \mathbf{v}$.
 - c. Find the angle θ between **u** and **v**.
 - d. Find $\operatorname{proj}_{\mathbf{v}}\mathbf{u}$.
 - e. Find $\mathbf{u} \times \mathbf{v}$.
- 4. Using P(-2,0,3), Q(1,2,4), R(-3,1,0),
 - a. Find a vector orthogonal to the plane determined by P, Q and R.
 - b. Find an equation of the plane passing through P, Q and R.
 - c. Find the set of parametric equations for the line through Q and parallel to $\mathbf{a} = \langle 4, -3, -2 \rangle$.
 - d. Find the distance from the point (-4, -1, 5) to the plane passing through P, Q and R.
- 5. Indentify via cross-sections the surface defined by $3^2 y^2 + 3z^2 + 9 = 0$.
- 6. Indentify via cross-sections the surface defined by $x = 3y^2 + 5z^2$.
- 7. Indentify via cross-sections the surface defined by $y = x^2$.
- 8. Indentify via cross-sections the surface defined by $2y^2 = 3z^2 = 12$.
- 9. Using $r(t) = \langle \cos t, \sin t, t^2 \rangle$, $t = \frac{\pi}{2}$:
 - a. Find the velocity vector.
 - b. Find the acceleration vector.
- 10. A projectile is fired at a speed of 448 feet per second at and angle of 30 degrees from a tower 512 feet above the ground.
 - a. Give the position vector for any time t.
 - b. How far away will the object strike?

- 11. Using $\mathbf{r}(t) = \langle 4\cos(2t), 4\sin(2t), 6t \rangle$,
 - a. Find $\mathbf{T}(t)$
 - b. Find $\mathbf{N}(t)$
 - c. Find the curvature
- 12. Find the tangential and normal components of acceleration for the curve $\mathbf{r}(t) = \langle 3t^2, 4t^2, 10t \rangle$ at t=2 and express a in terms of T and N.
- 13. Using $\mathbf{u} = \langle -4, 6, 5 \rangle$ and $\mathbf{v} = \langle 2, -3, 1 \rangle$,
 - a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
 - b. Find $\mathbf{u} \cdot \mathbf{v}$.
 - c. Find the angle θ between **u** and **v**.
 - d. Find proj_vu.
 - e. Find $\mathbf{u} \times \mathbf{v}$.
- 14. Using P(-4,1,2), Q(1,-3,4), R(-1,0,2),
 - a. Find an equation of the plane passing through the points.
 - b. Find parametric equations for the line through P and parallel to $a = \langle 2, -1, 4 \rangle$.
 - c. Find the distance from the point (5, -3, 2) to the plane.
 - d. Find the area of the parallelogram determined by P, Q, and R.
- 15. Identify the surface $x = y^2$.
- 16. Identify the surface $4x^2 + 4y^2 + z^2 = 4$.
- 17. Identify the surface $2x^2 3y^2 + 6z^2 = 6$.
- 18. Identify the surface $x^2 6y + 5z^2 = 0$.
- 19. A baseball is thrown from the stands 128 feet above the field at an angle of 30 degrees up from the horizontal with an initial speed of 64 feet per second.
 - a. Give the position vector for any time t.
 - b. When will the ball strike the ground?
 - c. How far away will the ball strike the ground?
 - d. What is the speed of the ball when it strikes the ground?
- 20. Using $\mathbf{r}(t) = \langle t \cos t, t \sin t, t^2 \rangle$ at t = 0,
 - a. Find **v** and **a**.
 - b. Find **T** and **N**.
 - c. Find K.
 - d. By first finding $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$, express $a = a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$.

Math 252 Exam 1 Review (Answers)

1.
$$C(4, -3, 0), \rho = 5$$

2. a.
$$(8, 25, -7)$$
.

b.
$$\|\mathbf{u}\| = 7\sqrt{2}, \|\mathbf{v}\| = 6.$$

3. a.
$$\|\mathbf{u}\| = 9, \|\mathbf{v}\| = 6$$

b.
$$\mathbf{u} \cdot \mathbf{v} = -46$$

c.
$$\theta = \arccos(-\frac{23}{27}) = 148.4^{\circ}$$

d.
$$\text{proj}_{\mathbf{v}}\mathbf{u} = \begin{pmatrix} -\frac{23}{18} \end{pmatrix} \langle -4, 4, 2 \rangle$$

 $\langle -\frac{46}{9}, -\frac{46}{9}, -\frac{23}{9} \rangle$

e.
$$\mathbf{u} \times \mathbf{v} = \langle -12, -20, 16 \rangle$$

4. a.
$$\mathbf{n} = \mathbf{PQ} \times \mathbf{PR} = \langle -7, 8, 5 \rangle$$

b.
$$-7x + 8y + 5z = 29$$

c.
$$x = 1 + 4t, y = 2 - 3t, z = 4 - 2t; t \in \mathbb{R}$$

d.
$$D = \frac{16}{\sqrt{138}}$$

- 5. Circular hyperboloid of two sheets
- 6. Elliptical paraboloid
- 7. Parabolic cylinder
- 8. Elliptical cylinder

9. a.
$$\mathbf{v}(t) = \langle -\sin t, \cos t, 2t \rangle, \quad \mathbf{v}(\frac{\pi}{2}) = \langle -1, 0, \pi \rangle$$

b.
$$\mathbf{a}(t) = \langle -\cos t, -\sin t, 2 \rangle, \quad \mathbf{a}(\frac{\pi}{2}) = \langle 0, -1, 2 \rangle$$

10. a.
$$\mathbf{r}(t) = \langle 224\sqrt{3}t, -16t^2 + 224t + 512 \rangle$$

b.
$$T = 16$$
, $x(16) = 224\sqrt{3}(16) \doteq 6207.7$ feet.

11. a.
$$\mathbf{T}(t) = \langle -\frac{4}{5}\sin(2t), \frac{4}{5}\cos(2t), \frac{3}{5} \rangle$$

b.
$$\mathbf{N}(t) = \langle -\cos(2t), \sin(2t), 0 \rangle$$

c.
$$k = \frac{4}{25}$$

12.
$$\mathbf{a} = 4\sqrt{5}\mathbf{T} + 2\sqrt{5}\mathbf{N}$$

13. a.
$$\|\mathbf{u}\| = \sqrt{77}$$

$$\|\mathbf{v}\| = \sqrt{14}$$

b.
$$\mathbf{u} \cdot \mathbf{v} = -21$$

c.
$$\theta = \arccos\left(\frac{-21}{7\sqrt{22}}\right)$$

14. a.
$$2x + 6y + 7z - 12 = 0$$

b.
$$x = 2t - 4$$
, $y = -t + 1$, $z = 4t + 2$

c.
$$D = \frac{6}{\sqrt{89}}$$

d.
$$A = \sqrt{89}$$

15. Parabolic cylinder

- 17. ANSWER
- 18. ANSWER
- 19. ANSWER

20. a.
$$\mathbf{v} = \langle -t \sin t + \cos t, t \cos t + \sin t, 2t \rangle$$

$$\mathbf{a} = \langle -t\cos t - 2\sin t, -t\sin t + 2\cos t, 2 \rangle$$

b.
$$\|{\bf v}\| =$$

$$T =$$

$$N =$$