Math 252 Exam 1 Practice Test (Problems)

1. Given $\mathbf{u} = \langle 8, -4, 1 \rangle$ and $\mathbf{v} = \langle -4, 4, 2 \rangle$,
   find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

2. Indentify via cross-sections the surface defined by the following:
$$3^2 - y^2 + 3z^2 + 9 = 0$$

3. Using $P(-4, 1, 2)$, $Q(1, -3, 4)$, $R(-1, 0, 2)$,

   a. Find an equation of the plane passing through the points.
   b. Find parametric equations for the line through P and parallel to $a = \langle 2, -1, 4 \rangle$.
   c. Find the distance from the point $(5, -3, 2)$ to the plane.
   d. Find the area of the parallelogram determined by P, Q, and R.

4. Find an equation of the plane passing through the points $P(-2, 0, 3)$, $Q(1, 2, 4)$, and $R(-3, 1, 0)$.

5. Indentify via cross-sections the surface defined by the following:
$$2y^2 = 3z^2 = 12$$

6. Indentify via cross-sections the surface defined by the following:
$$y = x^2$$

7. Identify the surface $2x^2 - 3y^2 + 6z^2 = 6$.

8. Find the center and radius of the sphere given by $x^2 + y^2 + z^2 - 8x + 6x = 0$

9. Using $\mathbf{u} = \langle 8, 3, -5 \rangle, \mathbf{v} = \langle 4, -4, -2 \rangle$,
   find $\|\mathbf{u}\|, \|\mathbf{v}\|$.

10. Identify the surface $x^2 - 6y + 5z^2 = 0$.

11. Given $\mathbf{u} = \langle 8, -4, 1 \rangle$ and $\mathbf{v} = \langle -4, 4, 2 \rangle$,
    font $\mathbf{u} \times \mathbf{v}$.

12. Using $\mathbf{r}(t) = \langle t \cos t, t \sin t, t^2 \rangle$ at $t = 0$,

    a. Find $\mathbf{v}$ and $\mathbf{a}$.
    b. Find $\mathbf{T}$ and $\mathbf{N}$.
    c. Find K.
    d. By first finding $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$,
       express $a = a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$.

13. Indentify via cross-sections the surface defined by the following:
$$x = 3y^2 + 5z^2$$

14. Identify the surface $4x^2 + 4y^2 + z^2 = 4$.

15. Given $\mathbf{u} = \langle 8, -4, 1 \rangle$ and $\mathbf{v} = \langle -4, 4, 2 \rangle$,
    find $\text{proj}_{\mathbf{v}} \mathbf{u}$.

16. Given $\mathbf{u} = \langle 8, -4, 1 \rangle$ and $\mathbf{v} = \langle -4, 4, 2 \rangle$,
    find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

17. Find the set of parametric equations for the line through $Q(1, 2, 4)$ and parallel to $a = \langle 4, -3, -2 \rangle$.

18. A baseball is thrown from the stands 128 feet above the field at an angle of 30 degrees up from the horizontal with an initial speed of 64 feet per second.

    a. Give the position vector for any time t.
    b. When will the ball strike the ground?
    c. How far away will the ball strike the ground?
    d. What is the speed of the ball when it strikes the ground?

19. Using $\mathbf{u} = \langle 8, 3, -5 \rangle, \mathbf{v} = \langle 4, -4, -2 \rangle$,
    find $3u - 4v$.

20. Identify the surface $x = y^2$.

21. Find the distance from the point $(-4, -1, 5)$ to the plane determined by the points $P(-2, 0, 3)$, $Q(1, 2, 4)$, and $R(-3, 1, 0)$.

22. Using $\mathbf{u} = \langle -4, 6, 5 \rangle$ and $\mathbf{v} = \langle 2, -3, 1 \rangle$,

    a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
    b. Find $\mathbf{u} \cdot \mathbf{v}$.
    c. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.
    d. Find $\text{proj}_{\mathbf{v}} \mathbf{u}$.
    e. Find $\mathbf{u} \times \mathbf{v}$.

23. Find a vector orthogonal to the plane determined by the points $P(-2, 0, 3)$, $Q(1, 2, 4)$, and $R(-3, 1, 0)$.

24. Given $\mathbf{u} = \langle 8, -4, 1 \rangle$ and $\mathbf{v} = \langle -4, 4, 2 \rangle$,
    find $\mathbf{u} \cdot \mathbf{v}$.

Math 252 Exam 1 Practice Test (Answers)

1. $\theta = \arccos\left(-\frac{23}{27}\right) = 148.4°$

2. Circular hyperboloid of two sheets

3. a. $2x + 6y + 7z - 12 = 0$
    b. $x = 2t - 4, \ y = -t + 1, \ z = 4t + 2$
    c. $D = \frac{6}{\sqrt{89}}$
    d. $A = \sqrt{89}$

4. $-7x + 8y + 5z = 29$

5. Elliptical cylinder

6. Parabolic cylinder

7. ANSWER

8. $C(4, -3, 0), \rho = 5$

9. $\|\mathbf{u}\| = 7\sqrt{2}, \|\mathbf{v}\| = 6$

10. ANSWER

11. $\mathbf{u} \times \mathbf{v} = \langle -12, -20, 16 \rangle$

12. a. $\mathbf{v} = \langle -t\sin t + \cos t, t\cos t + \sin t, 2t \rangle$
        $\mathbf{a} = \langle -t\cos t - 2\sin t, -t\sin t + 2\cos t, 2 \rangle$
    b. $\|\mathbf{v}\| =$

        $\mathbf{T} =$
        $\mathbf{N} =$

13. Elliptical paraboloid

14. ANSWER

15. $\text{proj}_{\mathbf{v}}\mathbf{u} = -\frac{23}{18}\langle -4, 4, 2 \rangle = \langle -\frac{46}{9}, -\frac{46}{9}, -\frac{23}{9} \rangle$

16. $\|\mathbf{u}\| = 9, \|\mathbf{v}\| = 6$

17. $x = 1 + 4t, y = 2 - 3t, z = 4 - 2t; t \in \mathbb{R}$

18. ANSWER

19. $\langle 8, 25, -7 \rangle$

20. Parabolic cylinder

21. $h = \frac{16}{\sqrt{138}}$

22. a. $\|\mathbf{u}\| = \sqrt{77}$
        $\|\mathbf{v}\| = \sqrt{14}$
    b. $\mathbf{u} \cdot \mathbf{v} = -21$
    c. $\theta = \arccos\left(\frac{-21}{7\sqrt{22}}\right)$

23. $\mathbf{n} = \mathbf{PQ} \times \mathbf{PR} = \langle -7, 8, 5 \rangle$

24. $\mathbf{u} \cdot \mathbf{v} = -46$