

Symbol reference

Symbol	Description	L ^A T _E X
\sim	Negation	<code>\neg</code>
\wedge	And	<code>\wedge</code>
\vee	Or	<code>\vee</code>
\rightarrow	If...then	<code>\rightarrow</code>
\leftrightarrow	If and only if...then	<code>\leftrightarrow</code>
\in	In	<code>\in</code>
\subset	Subset	<code>\subset</code>
\supset	Superset	<code>\supset</code>
\emptyset	Empty set	<code>\emptyset</code>

(February 7th)

1 Logic

1.1 Operations

In order of precedence (what is evaluated first)

Negation (\sim) Unary operator whose argument follows on the right. Changes true to false and false to true. Also has basically highest precedence in evaluation order.

And (\wedge) Says if left and right, then true, else false.

Or (\vee) Says if left or right, then true, else false.

If...then (\rightarrow) A logical statement about correlation and causation. Formally expressed as “if p , then q ” ($p \rightarrow q$) where p is the *hypothesis* and q is the *conclusion*. If the hypothesis p is true, then the conclusion q must then be true. Note that causation in the opposite direction is not implied: it is not stated that if not p then not q , so q can be true even if p is not true. This is known as *inverse error* (Ch 2.3).

If and only if...then (\leftrightarrow) Bi-conditional if says if p and q have the same values. Only if both are true or both are false is bi-conditional if true.

1.2 Truth Tables

Used to evaluate all the potential outcomes of a logical statement given all possible combinations of inputs. Given two inputs p and q then would be four potential input combinations:

$$(p, q) \in \{T, F\} \times \{T, F\} = \{(T, T), (T, F), (F, T), (F, F)\}$$

Here is a table of the potential outputs for the previously introduced logical operators (not, and, or, if, iff).

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	F	F	T	T	F
F	F	F	F	F	F	T

1.3 De Morgan's Laws

Method for negating a statement involving an **and** or an **or** operation.

$$\sim(p \wedge q) \equiv \sim p \vee \sim q \quad (1)$$

$$\sim(p \vee q) \equiv \sim p \wedge \sim q \quad (2)$$

1.4 Argument

Invalid An argument is invalid if its conclusion can be true while at the same time have false premises. If a logical argument were mapped in a logic table, any row where all premises are true is called a *critical row*.

Valid An argument is valid if its conclusion is true only when all of its premises are true.

Sound A

Unsound A

1.4.1 Syllogism

An argument that consists of two premises is called a syllogism. The premises are called the major and minor premises respectively. Some forms of syllogisms are:

- Modus ponens: “if p then q ; p ; therefore q ”
- Modus tollens: “if not p then not q ; not p ; therefore not q ”

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1.5 Conditional arguments

Conditional	$p \rightarrow q$
Bi-conditional	$p \leftrightarrow q$
Converse	$q \rightarrow p$ Logically equivalent to “if and only if p then q ” (contrapositive), but not to “if p then q ”
Inverse	$\sim p \rightarrow \sim q$
Contrapositive	$\sim q \rightarrow \sim p$ Don't have conclusion, therefore don't have hypothesis
Negation	$p \wedge \sim q \equiv \sim(p \rightarrow q)$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Examples

- “Sam will be allowed on Jane’s boat only if he is an expert sailor”
 - Let “Sam is an expert sailor” be q
 - Let “Sam will be allowed on Jane’s boat” be p
 - $\sim q \rightarrow \sim p \equiv p \rightarrow q$
 - Converse: Is an expert sailor, will be let on boat.
 - Inverse: $\sim p \rightarrow \sim q$
 - Contrapositive: $\sim q \rightarrow \sim p$
- “If someone is eligible to vote, then they are at least 18 years old”
 - Let “Someone is eligible to vote” be p
 - Let “They are at least 18 years old” be q
 - Converse: If 18 years old, then eligible to vote.
 - Inverse: If not eligible, then younger than 18.
 - Contrapositive: If not 18, then not eligible to vote.