Quiz 24 (Problems)

1. Find $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{F} = \langle 2x, 2y, 4z \rangle$, S is the portion of the paraboloid $z = 16 - x^2 - y^2$ above $z \geq 0$.

Quiz 24 (Answers)

1. (Math 252 Quiz 24)

   Note that the surface is not closed, so cannot use divergence theorem.
   $g(x,y,z) = x^2 + y^2 + z - 16$,
   $\nabla g = \langle 2x, 2y, 1\rangle$,
   $\|\nabla g\| = \sqrt{4x^2 + 4y^2 + 1}$,
   $\mathbf{n} = \frac{1}{\|\nabla g\|}\nabla g$,
   $f(x,y) = 16 - x^2 - y^2 - z$, $f_x = -2x$,
   $f_y = -2y$,
   $dS = \sqrt{f_x^2 + f_y^2 + 1}\, dA = \sqrt{4x^2 + 4y^2 + 1}\, dA$,
   $\iint_S \mathbf{F}\cdot\mathbf{v}\, dS = \iint_D \mathbf{F}\cdot\nabla g\, dA$,
   $= \int_0^{2\pi}\int_0^4 64r\, dr\, d\theta = 1024\pi$