Math 252 Exam 2 Review (Problems)

1. Reverse the order of integration of $\int_1^e \int_0^{\ln x} y \, dy \, dx$ and evaluate.

2. Evaluate $\int_0^4 \int_{x^2}^{4x} (6x + 12y) dy \, dx$.

3. Use Lagrange multipliers to find any extrema of $f(x, y, z) = 3x^2 - y^2 + 2z^2$ subject to $3x + z + 50 = 4y$.

4. Using $f(x, y) = 3x^2 + 4y^2$, $P(4, -2)$ and $Q(10, 6)$:

   a. Find the gradient of f at P.

   b. Find the directional derivative of f at P in the direction from P to Q.

   c. Find the maximum value of the directional derivative of f at P.

5. Using $w = f(x, y, z) = 2xy^2 - 4x^3z$,

   a Find an equation of the tangent plane of w at $(1, 3, 2)$.

   b Estimate $f(1.02, 3.01, 1.98)$.

6. For $f(x, y) = \sqrt{x^2 - y^2}$ find the domain of f and describe the level curves.

7. Find f_{xy} for $f(x, y) = \ln(xy + y^2)$.

8. A flat metal plate lies on an xy-plane such that the temperature T at (x, y) is given by $T = 10(x^2 + y^2)^2$, where T is in degrees and x and y are in centimeters. Find the instantaneous rate of change of T with respect to distance at $(1, 2)$ in the direction of the x-axis.

9. The total resistance R of three resistances R_1, R_2 and R_3 connected in parallelis given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. If measurements of R_1, R_2 and R_3 are 100, 200 and 400 ohms respectively, with a maximum error of $\pm 1\%$ in each measurement, estimate the maximum error in the calculated value of R.

10. Find the limit $\lim_{(x,y)\to(4,3)} \frac{\sqrt{x} - \sqrt{y+1}}{x - y - 1}, x \neq y + 1$.

11. Describe the domain of $f(x, y) = \frac{\ln(x-y)}{\sqrt{xy}}$

12. Find an equation of the level surface of $f(x, y, z) = xy \sin z + 3xy^2 e^z$ at $P(1, 2, 0)$

13. Using $f(x, y) = \frac{x-y}{x+y}$ and $P(2, -1)$,

    a. Find the directional derivative of f in the direction of $\mathbf{v} = \langle 4, -8 \rangle$.

    b. Find the direction in which f increases most rapidly.

    c. Find the direction in which f decreases most rapidly.

    d. Find the maximum value of the directional derivative.

14. Use partial derivatives to find $\dfrac{dy}{dx}$ if $4x^2y + 2y^3 = 5x^3y^4$.

15. Without using Lagrange multipliers, find any extrema or saddle points of $f(x, y) = x^3 + 12xy - 3y^2 - 27x + 34$.

16. If $w = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that $f_x^2 = f_y^2 = (\frac{\delta w}{\delta r})^2 + \frac{1}{r^2}(\frac{\delta w}{\delta \theta})^2$.

17. Find the volume of the largest rectangular box that has three of its vertices on the positive x, y and z-axes respectively, and a fourth vertex on the plane $3x + 4y + 2z = 24$.

18. Find the volume of the solid bounded by $y = x^3$, $y = x^4$, $z - x - y = 4$, and $z = 0$.

19. For $f(x, y, z) = 4x^z + z^3 \sin y$ find $\dfrac{\delta^3 f}{\delta x \delta y^2}$.

20. Determine if the following limit exists; if it does also state the value of the limit: $\lim_{(x,y)\to(2,1)} \frac{x^2 - xy - 2y^2}{x^2 - 4y^2}$

21. Find the maximum and minimum values of $f(x, y) = 5 + 4x - 2x^2 + 3y - y^2$ over the triangular region with vertices $(0, 0)$, $(2, 0)$ and $(2, 2)$.

22. Using $x^3 - 2xy + z^3 + 7y + 6 = 0$ and $P(1, 4, -3)$,

    a. Find an equation of the tangent plane at P.

    b. Find equations of the normal line at P.

23. For $f(x, y) = 3x^4y^2 - x\cos y + 4x^3y^3$, find f_x, f_y, f_{xx} and f_{xy}.

Math 252 Exam 2 Review (Answers)

1. (Math-252 Exam 2)
   ANSWER

2. (Math-252 Quiz 15)
   ANSWER

3. (Math-252 Quiz 14)
   ANSWER

4. (Math-252 Quiz 11)
   a. $\nabla f(P) = \langle 24, -16 \rangle$
   b. $\mathbf{u} = \frac{1}{\|\overrightarrow{PQ}\|}\overrightarrow{PQ}$; $D_{\mathbf{u}}f(P) = \nabla f(P) \cdot \mathbf{u} = \frac{16}{10}$
   c. $\|\nabla f(p)\| = 8\sqrt{13}$

5. (Math-252 Quiz 12)
   a ANSWER
   b ANSWER

6. (Math-252 Exam 2)
   ANSWER

7. (Math-252 Exam 2)
   ANSWER

8. (Math-252 Exam 2)
   ANSWER

9. (Math-252 Exam 2)
   ANSWER

10. (Math-252 Exam 2)
    ANSWER

11. (Math-252 Quiz 8)
    $\{(x, y) : x > y, xy > 0\}$

12. (Math-252 Quiz 8)
    $xy \sin z + 3xy^2 e^z$

13. (Math-252 Exam 2)
    a. ANSWER
    b. ANSWER
    c. ANSWER
    d. ANSWER

14. (Math-252 Quiz 10)
    $\frac{dy}{dx} = \frac{15x^2y^4 - 8xy}{20x^3y^3 - 4x^2 + 6y^2}$

15. (Math-252 Quiz 13)
    ANSWER

16. (Math-252 Exam 2)
    ANSWER

17. (Math-252 Exam 2)
    ANSWER

18. (Math-252 Exam 2)
    ANSWER

19. (Math-252 Quiz 10)
    $\frac{\delta^3 f}{\delta x \delta y^2} = 0$

20. (Math-252 Quiz 8)
    $\frac{3}{4}$

21. (Math-252 Exam 2)
    ANSWER

22. (Math-252 Exam 2)
    a. ANSWER
    b. ANSWER

23. (Math-252 Quiz 9)
    $f_x = 12x^3y^2 - \cos y + 12x^2y^3$
    $f_y = 6x^4y + x \sin y + 12x^3y^2$
    $f_{xx} = 36x^2y^2 + 24xy^3$
    $f_{yy} = 6x^4 + x \cos y + 24x^3y$
    $f_{xy} = 24x^3y + \sin y + 36x^2y^2$