Math 252 Exam 2 Review (Problems)

1. For $f(x,y) = 3x^4y^2 - x\cos y + 4x^3y^3$, find f_x, f_y, f_{xx} and f_{xy}.

2. Find the maximum and minimum values of $f(x,y) = 5 + 4x - 2x^2 + 3y - y^2$ over the triangular region with vertices $(0,0)$, $(2,0)$ and $(2,2)$.

3. Describe the domain of $f(x,y) = \frac{\ln(x-y)}{\sqrt{xy}}$

4. For the integral $\int_0^4 \int_{x^2}^{4x} (6x + 12y)dy\ dx$,

   a. evaluate.

   b. rewrite by reversing the order of integration.

5. Using $f(x,y) = \frac{x-y}{x+y}$ and $P(2,-1)$,

   a. Find the directional derivative of f in the direction of $\mathbf{v} = \langle 4, -8\rangle$.

   b. Find the direction in which f increases most rapidly.

   c. Find the direction in which f decreases most rapidly.

   d. Find the maximum value of the directional derivative.

6. Determine if the following limit exists; if it does also state the value of the limit: $\lim_{(x,y)\to(2,1)} \frac{x^2-xy-2y^2}{x^2-4y^2}$

7. The total resistance R of three resistances R_1, R_2 and R_3 connected in parallelis given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. If measurements of R_1, R_2 and R_3 are 100, 200 and 400 ohms respectively, with a maximum error of $\pm 1\%$ in each measurement, estimate the maximum error in the calculated value of R.

8. If $w = f(x,y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that $f_x^2 + f_y^2 = \left(\frac{\delta w}{\delta r}\right)^2 + \frac{1}{r^2}\left(\frac{\delta w}{\delta \theta}\right)^2$.

9. Using $f(x,y) = 3x^2 + 4y^2$, $P(4,-2)$ and $Q(10,6)$:

   a. Find the gradient of f at P.

   b. Find the directional derivative of f at P in the direction from P to Q.

   c. Find the maximum value of the directional derivative of f at P.

10. Use Lagrange multipliers to find any extrema of $f(x,y,z) = 3x^2 - y^2 + 2z^2$ subject to $3x + z + 50 = 4y$.

11. Using $x^3 - 2xy + z^3 + 7y + 6 = 0$ and $P(1,4,-3)$,

    a. Find an equation of the tangent plane at P.

    b. Find equations of the normal line at P.

12. Find the volume of the solid bounded by $y = x^3$, $y = x^4$, $z - x - y = 4$, and $z = 0$.

13. Find f_{xy} for $f(x,y) = \ln(xy + y^2)$.

14. Find the limit:

$$\lim_{(x,y)\to(4,3)} \frac{\sqrt{x} - \sqrt{y+1}}{x - y - 1}, x \neq y + 1$$

15. Use partial derivatives to find $\frac{dy}{dx}$ if $4x^2y + 2y^3 = 5x^3y^4$.

16. Find an equation of the level surface of $f(x,y,z) = xy\sin z + 3xy^2e^z$ at $P(1,2,0)$

17. For $f(x,y) = \sqrt{x^2 - y^2}$ find the domain of f and describe the level curves.

18. Reverse the order of integration of $\int_1^e \int_0^{\ln x} y\ dy\ dx$ and evaluate.

19. Using $w = f(x,y,z) = 2xy^2 - 4x^3z$,

    a Find an equation of the tangent plane of w at $(1,3,2)$.

    b Estimate $f(1.02, 3.01, 1.98)$.

20. Find the volume of the largest rectangular box that has three of its vertices on the positive x, y and z-axes respectively, and a fourth vertex on the plane $3x + 4y + 2z = 24$.

21. A flat metal plate lies on an xy-plane such that the temperature T at (x, y) is given by $T = 10(x^2 + y^2)^2$, where T is in degrees and x and y are in centimeters. Find the instantaneous rate of change of T with respect to distance at $(1, 2)$ in the direction of the x-axis.

22. Without using Lagrange multipliers, find any extrema or saddle points of $f(x, y) = x^3 + 12xy - 3y^2 - 27x + 34$.

23. For $f(x, y, z) = 4x^z + z^3 \sin y$ find $\frac{\delta^3 f}{\delta x \delta y^2}$.

Math 252 Exam 2 Review (Answers)

1. (Math-252 Quiz 9)

   $f_x = 12x^3y^2 - \cos y + 12x^2y^3$

   $f_y = 6x^4y + x\sin y + 12x^3y^2$

   $f_{xx} = 36x^2y^2 + 24xy^3$

   $f_{yy} = 6x^4 + x\cos y + 24x^3y$

   $f_{xy} = 24x^3y + \sin y + 36x^2y^2$

2. (Math-252 Exam 2 Practice)

   absolute max $\frac{37}{4}$ at $(1, \frac{3}{2})$

3. (Math-252 Quiz 8)

   $\{(x, y) : x > y, xy > 0\}$

4. (Math-252 Quiz 15)

   a. $\frac{4736}{5}$

   b. $\int_0^{16} \int_{\frac{1}{4}y}^{\sqrt{y}} (6x + 12y)dx\ dy$

5. (Math-252 Exam 2 Practice)

   a. $\nabla f(x, y) = \langle \frac{2y}{(x_y)^2}, \frac{-2x}{(x_y)^2} \rangle$
   $\nabla f(2, -1) = \langle -2, -4 \rangle$
   $\mathbf{u} = \mathbf{v}\frac{1}{|\mathbf{v}|} = \langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \rangle$
   $D_{\mathbf{u}}f(2, -1) = \frac{6\sqrt{5}}{5}$

   b. $\nabla f \frac{1}{|\nabla f|} = \langle \frac{-\sqrt{5}}{5}, \frac{-2\sqrt{5}}{5} \rangle$

   c. $\langle \frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5} \rangle$

   d. $|\nabla f| = 2\sqrt{5}$

6. (Math-252 Quiz 8)

   $L = \frac{3}{4}$

7. (Math-252 Exam 2 Practice)

   $\left|\frac{dR}{R}\right| = \frac{400}{7}(\frac{1}{100^2} + \frac{2}{200^2} + \frac{4}{400^2}) = 0.01$

8. (Math-252 Exam 2 Practice)

   $\frac{\delta w}{\delta r} = f_x(\cos\theta) + f_y(\sin\theta)$
   $\frac{\delta w}{\delta \theta} = f_x(-r\sin\theta) + f_y(r\cos\theta)$
   $(\frac{\delta w}{\delta r})^2 + \frac{1}{r^2}(\frac{\delta w}{\delta \theta})^2 = f_x^2 + f_y^2$

9. (Math-252 Quiz 11)

   a. $\nabla f(P) = \langle 24, -16 \rangle$

   b. $\mathbf{u} = \frac{1}{\|\overrightarrow{PQ}\|}\overrightarrow{PQ};\ D_{\mathbf{u}}f(P) = \nabla f(P) \cdot \mathbf{u} = \frac{16}{10}$

   c. $\|\nabla f(p)\| = 8\sqrt{13}$

10. (Math-252 Quiz 14)

    Absolute minimum $f(4, 16, 2) = -200$

11. (Math-252 Exam 2 Practice)

    a. $-5x + 5y + 27z + 66 = 0$

    b. $\langle x, y, z \rangle = \langle 1, 4, -3 \rangle + t\langle -5, 5, 27 \rangle$
    $x = -5t + 1;\ y = 5t + 4;\ z = 27t - 3$

12. (Math-252 Exam 2 Practice)

    $V = \frac{157}{630}$

13. (Math-252 Exam 2 Practice)

    $f_{xy} = -\frac{1}{(x+y)^2}$

14. (Math-252 Exam 2 Practice)

    $L = \frac{1}{4}$

15. (Math-252 Quiz 10)

    $\frac{dy}{dx} = \frac{15x^2y^4 - 8xy}{20x^3y^3 - 4x^2 + 6y^2}$

16. (Math-252 Quiz 8)

    $xy\sin z + 3xy^2e^z$

17. (Math-252 Exam 2 Practice)

    $D = \{(x, y) : |x| \geq |y|\}$
    Hyperbola in xy-plane

18. (Math-252 Exam 2 Practice)

    $\int_0^1 \int_{e^y}^{e} y\ dx\ dy = \frac{e}{2} - 1$

19. (Math-252 Quiz 12)

    a. $-6x + 12y - 4z - 22 = 0$

    b. $f(1.02, 3.01, 1.98) \approx 10.08$

20. (Math-252 Exam 2 Practice)

    $V = \frac{64}{3}$

21. (Math-252 Exam 2 Practice)

    $T_x = 200$ degrees per centimeters

22. (Math-252 Quiz 13)

    Saddle point $f(1,2) = 20$, local max $f(-9,-18) = 520$

23. (Math-252 Quiz 10)

    $\frac{\delta^3 f}{\delta x \delta y^2} = 0$