Math 252 Exam 1 Review (Problems)

1. Find the center and radius of the sphere given by $x^2 + y^2 + z^2 - 8x + 6x = 0$
2. Using $\mathbf{u} = \langle 8, 3, -5\rangle, \mathbf{v} = \langle 4, -4, -2\rangle$,
   a. Find $3u - 4v$.
   b. Find $\|\mathbf{u}\|, \|\mathbf{v}\|$.
3. Using $\mathbf{u} = \langle 8, -4, 1\rangle$ and $\mathbf{v} = \langle -4, 4, 2\rangle$,
   a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
   b. Find $\mathbf{u} \cdot \mathbf{v}$.
   c. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.
   d. Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.
   e. Find $\mathbf{u} \times \mathbf{v}$.
4. Using $P(-2, 0, 3), Q(1, 2, 4), R(-3, 1, 0)$,
   a. Find a vector orthogonal to the plane determined by P, Q and R.
   b. Find an equation of the plane passing through P, Q and R.
   c. Find the set of parametric equations for the line through Q and parallel to $\mathbf{a} = \langle 4, -3, -2\rangle$.
   d. Find the distance from the point $(-4, -1, 5)$ to the plane passing through P, Q and R.
5. Indentify via cross-sections the surface defined by $3^2 - y^2 + 3z^2 + 9 = 0$.
6. Indentify via cross-sections the surface defined by $x = 3y^2 + 5z^2$.
7. Indentify via cross-sections the surface defined by $y = x^2$.
8. Indentify via cross-sections the surface defined by $2y^2 = 3z^2 = 12$.
9. Using $r(t) = \langle \cos t, \sin t, t^2\rangle$, $t = \frac{\pi}{2}$:
   a. Find the velocity vector.
   b. Find the acceleration vector.
10. A projectile is fired at a speed of 448 feet per second at and angle of 30 degrees from a tower 512 feet above the ground.
    a. Give the position vector for any time t.
    b. How far away will the object strike?
11. Using $\mathbf{r}(t) = \langle 4\cos(2t), 4\sin(2t), 6t\rangle$,
    a. Find $\mathbf{T}(t)$
    b. Find $\mathbf{N}(t)$
    c. Find the curvature
12. Find the tangential and normal components of acceleration for the curve $\mathbf{r}(t) = \langle 3t^2, 4t^2, 10t\rangle$ at $t = 2$ and express a in terms of T and N.

13. Using $\mathbf{u} = \langle -4, 6, 5 \rangle$ and $\mathbf{v} = \langle 2, -3, 1 \rangle$,

    a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

    b. Find $\mathbf{u} \cdot \mathbf{v}$.

    c. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

    d. Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.

    e. Find $\mathbf{u} \times \mathbf{v}$.

14. Using $P(-4, 1, 2)$, $Q(1, -3, 4)$, $R(-1, 0, 2)$,

    a. Find an equation of the plane passing through the points.

    b. Find parametric equations for the line through P and parallel to $a = \langle 2, -1, 4 \rangle$.

    c. Find the distance from the point $(5, -3, 2)$ to the plane.

    d. Find the area of the parallelogram determined by P, Q, and R.

15. Identify the surface $x = y^2$.

16. Identify the surface $4x^2 + 4y^2 + z^2 = 4$.

17. Identify the surface $2x^2 - 3y^2 + 6z^2 = 6$.

18. Identify the surface $x^2 - 6y + 5z^2 = 0$.

19. A baseball is thrown from the stands 128 feet above the field at an angle of 30 degrees up from the horizontal with an initial speed of 64 feet per second.

    a. Give the position vector for any time t.

    b. When will the ball strike the ground?

    c. How far away will the ball strike the ground?

    d. What is the speed of the ball when it strikes the ground?

20. Using $\mathbf{r}(t) = \langle t\cos t, t\sin t, t^2 \rangle$ at $t = 0$,

    a. Find $\mathbf{v}$ and $\mathbf{a}$.

    b. Find $\mathbf{T}$ and $\mathbf{N}$.

    c. Find K.

    d. By first finding $a_{\mathbf{T}}$ and $._{\mathbf{N}}$,
    express $a = a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$.

Math 252 Exam 1 Review (Answers)

1. $C(4, -3, 0), \rho = 5$

2. a. $\langle 8, 25, -7 \rangle$.
   b. $\|\mathbf{u}\| = 7\sqrt{2}, \|\mathbf{v}\| = 6$.

3. a. $\|\mathbf{u}\| = 9, \|\mathbf{v}\| = 6$
   b. $\mathbf{u} \cdot \mathbf{v} = -46$
   c. $\theta = \arccos\left(-\frac{23}{27}\right) = 148.4°$
   d. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(-\frac{23}{18}\right)\langle -4, 4, 2\rangle = \langle -\frac{46}{9}, -\frac{46}{9}, -\frac{23}{9}\rangle$
   e. $\mathbf{u} \times \mathbf{v} = \langle -12, -20, 16\rangle$

4. a. $\mathbf{n} = \mathbf{PQ} \times \mathbf{PR} = \langle -7, 8, 5\rangle$
   b. $-7x + 8y + 5z = 29$
   c. $x = 1 + 4t, y = 2 - 3t, z = 4 - 2t; t \in \mathbb{R}$
   d. $D = \frac{16}{\sqrt{138}}$

5. Circular hyperboloid of two sheets

6. Elliptical paraboloid

7. Parabolic cylinder

8. Elliptical cylinder

9. a. $\mathbf{v}(t) = \langle -\sin t, \cos t, 2t\rangle,\ \mathbf{v}(\frac{\pi}{2}) = \langle -1, 0, \pi\rangle$
   b. $\mathbf{a}(t) = \langle -\cos t, -\sin t, 2\rangle,\ \mathbf{a}(\frac{\pi}{2}) = \langle 0, -1, 2\rangle$

10. a. $\mathbf{r}(t) = \langle 224\sqrt{3}t, -16t^2 + 224t + 512\rangle$
    b. $T = 16,\ x(16) = 224\sqrt{3}(16) \doteq 6207.7$ feet

11. a. $\mathbf{T}(t) = \langle -\frac{4}{5}\sin(2t), \frac{4}{5}\cos(2t), \frac{3}{5}\rangle$
    b. $\mathbf{N}(t) = \langle -\cos(2t), \sin(2t), 0\rangle$
    c. $k = \frac{4}{25}$

12. $\mathbf{a} = 4\sqrt{5}\mathbf{T} + 2\sqrt{5}\mathbf{N}$
    (correction?) $\mathbf{a} = \frac{20}{\sqrt{5}}\mathbf{T} + \frac{10}{\sqrt{5}}\mathbf{N}$

13. a. $\|\mathbf{u}\| = \sqrt{77}$
       $\|\mathbf{v}\| = \sqrt{14}$
    b. $\mathbf{u} \cdot \mathbf{v} = -21$
    c. $\theta = \arccos\left(\dfrac{-21}{7\sqrt{22}}\right)$

14. a. $2x + 6y + 7z - 12 = 0$
    b. $x = 2t - 4,\ y = -t + 1,\ z = 4t + 2$
    c. $D = \dfrac{6}{\sqrt{89}}$
    d. $A = \sqrt{89}$

15. Parabolic cylinder

16. Circular ellipsoid

17. Hyperbaloid (one sheet)

18. Elliptical cone

19. a. $\mathbf{r}(t) = \langle 32\sqrt{3}t, -16t^2 + 32t + 128 \rangle$
    b. in 4 seconds
    c. $128\sqrt{3}$ feet away
    d. $64\sqrt{3}$ feet per second

20. a. $\mathbf{v} = \langle -t\sin t + \cos t, t\cos t + \sin t, 2t \rangle$
    $\mathbf{a} = \langle -t\cos t - 2\sin t, -t\sin t + 2\cos t, 2 \rangle$
    b. $\mathbf{T} = \left\langle \dfrac{-t\sin 5 + \cos t}{\sqrt{5t^2+1}}, \dfrac{t\cos t + \sin 5}{\sqrt{5t^2+1}}, \dfrac{2t}{\sqrt{5t^2+1}} \right\rangle$
    $\mathbf{N} = \langle \rangle$