Math 252 Cumulative Review (Problems)

1. For the solid bounded by $z = \sqrt{16x^2 + 16y^2}$, $x^2 + y^2 = 16$, and $z = 0$,
   a. find its volume.
   b. find the center of mass if $\delta = \sqrt{x^2 + y^2}$.

2. Using $r(t) = \langle \cos t, \sin t, t^2 \rangle$, $t = \frac{\pi}{2}$:
   a. Find the velocity vector.
   b. Find the acceleration vector.

3. Using $\mathbf{r}(t) = \langle 4\cos(2t), 4\sin(2t), 6t \rangle$,
   a. Find $\mathbf{T}(t)$
   b. Find $\mathbf{N}(t)$
   c. Find the curvature

4. For $f(x, y) = 4x^2 + 2y^2$, find the equation of the level curve that contains the point $P(-2, 4)$ and describe its shape.

5. Find the surface area for the surface given by the parametric equations $x = u + v$, $y = uv$ and $z = u - v$ with $u^2 + v^2 \leq 4$.

6. Without using Lagrange multipliers, find any extrema or saddle points of $f(x, y) = x^3 + 12xy - 3y^2 - 27x + 34$.

7. Identify the surface $x^2 - 6y + 5z^2 = 0$.

8. For the solid bounded in the first octant by the plane $4x + 2y + z = 12$ with density $\delta(x, y, z) = 5x^3$,
   a. find it's mass.
   b. set up (but don't solve) the integral to find M_{xz}.

9. Find a conservative vector field $\mathbf{F}$ that has the potential $f(x, y, z) = 4x^2y - 2y^2z^3$¿

10. Find the maximum and minimum values of $f(x, y) = 5 + 4x - 2x^2 + 3y - y^2$ over the triangular region with vertices $(0, 0)$, $(2, 0)$ and $(2, 2)$.

11. Indentify via cross-sections the surface defined by $2y^2 = 3z^2 = 12$.

12. Find f_{xy} for $f(x, y) = \ln(xy + y^2)$.

13. Evaluate $\iint_R \frac{2y+x}{y-2x} dA$ where R is the region bounded by the trapezoid with vertices $(-1, 0)$, $(-2, 0)$, $(0, 4)$, $(0, 2)$.

14. Use partial derivatives to find $\frac{dy}{dx}$ if $4x^2y + 2y^3 = 5x^3y^4$.

15. Using $P(-2, 0, 3), Q(1, 2, 4), R(-3, 1, 0)$,
    a. Find a vector orthogonal to the plane determined by P, Q and R.
    b. Find an equation of the plane passing through P, Q and R.
    c. Find the set of parametric equations for the line through Q and parallel to $\mathbf{a} = \langle 4, -3, -2 \rangle$.
    d. Find the distance from the point $(-4, -1, 5)$ to the plane passing through P, Q and R.

16. Find the volume of the solid bounded by $y = x^3$, $y = x^4$, $z - x - y = 4$, and $z = 0$.

17. Find the curl and divergence of $\mathbf{F} = \langle -3\sin x + \cos y, 6xz^2, 3y + z \rangle$.

18. Use Lagrange multipliers to find any extrema of $f(x, y, z) = 3x^2 - y^2 + 2z^2$ subject to $3x + z + 50 = 4y$.

19. Use Green's theorem to evaluate $\oint_C (6y)dx + (\frac{5}{3}x^3)dy$, where C is the boundary of the first quadrant region bounded by $y = 36 - x^2$ and the x-axis.

20. Find the surface area of S, the part of the paraboloid $z = x^2 + y^2$ under the plane $z = a, a > 0$.

21. Using $\mathbf{r}(t) = \langle 4\cos t, 3t, 4\sin 5 \rangle$ at $t = 0$,
    a. Find $\mathbf{v}(t)$ and $\mathbf{a}(t)$.
    b. Find $\mathbf{T}(t)$ and $\mathbf{N}(t)$.
    c. Find K, $a_{\mathbf{T}}$, and $a_{\mathbf{N}}$.

22. Prove that all lines and circles (in the xy-plane) have constant curvature.

23. Find an equation of the level surface of $f(x,y,z) = xy\sin z + 3xy^2e^z$ at $P(1,2,0)$

24. Use an appropriate change of variables to find $\iint_R \frac{x-y}{2x+3y}\,dA$ where R is the region bounded by the lines $x-y=-1$, $x-y=2$, $2x+3y=1$, and $2x+3y=3$.

25. A baseball is thrown from the stands 128 feet above the field at an angle of 30 degrees up from the horizontal with an initial speed of 64 feet per second.

    a. Give the position vector for any time t.

    b. When will the ball strike the ground?

    c. How far away will the ball strike the ground?

    d. What is the speed of the ball when it strikes the ground?

26. Use Stokes' theorem to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = xy\hat{\imath} + yz\hat{\jmath} + xz\hat{k}$ and C is the positively oriented triangle with vertices $(3,0,0)$, $(0,2,0)$, $(0,0,6)$.

27. Reverse the order of integration of $\int_1^e \int_0^{\ln x} y\;dy\;dx$ and evaluate.

28. Given the points $P(2,1,2)$, $Q(6,-2,1)$ and $R(-1,4,5)$,

    a. Find an equation of the plane passing through the points.

    b. Find an equation of the line perpendicular to the plane, passing through the point $(8,2,-1)$.

    c. Find the distance from the point $(-5,-2,7)$ to the plane.

    d. Find the area of the parallelogram determined by the points.

29. Use Lagrange multipliers to find the extrema of $f(x,y,z) = x-2y-4z$ subject to the constraint $z = 4x^2+y^2$

30. Evaluate the line integral $\int_C (xy^2)dx + (4xy^3)dy$ along C: $x = y^2$ from $(0,0)$ to $(4,2)$.

31. Using $x^3 - 2xy + z^3 + 7y + 6 = 0$ and $P(1,4,-3)$,

    a. Find an equation of the tangent plane at P.

    b. Find equations of the normal line at P.

32. Describe the domain of $f(x,y) = \frac{\ln(xy)}{\sqrt{x+y}}$

33. Using polar coordinates, evaluate $\iint_R (x^2+y^2)^{3/2}dA$ where R is the region bounded by the circle of radius a centered at the origin.

34. A projectile is fired at a speed of 448 feet per second at and angle of 30 degrees from a tower 512 feet above the ground.

    a. Give the position vector for any time t.

    b. How far away will the object strike?

35. Find the volume of the solid that lies outside the cone $z^2 = x^2+y^2$ and inside the sphere $x^2+y^2+z^2 = 18$

36. A projectile is launched at an angle of 30°, with speed 224 feet per second, and from a platform 128 feet above the ground,

    a. Find the position vector of the object at time t.

    b. How far away will it hit the ground?

    c. What is the speed upon impact?

37. Find the limit:

$$\lim_{(x,y)\to(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1}, x \neq y+1$$

38. Find the surface area of the part of the paraboloid $z = f(x,y) = 20-x^2-y^2$ above $z=4$.

39. Determine if the planes $ax+by+cz+d_1 = 0$ and $(ak)x+(bk)y+(ck)z+d_2 = 0$ intersect. If they do not, find the distance between them.

40. Evaluate $\int_{-1}^{2}\int_{1}^{x}\int_{0}^{x+y}(3x^2y)dz\;dy\;dx$.

41. Find the tangential and normal components of acceleration for the curve $\mathbf{r}(t) = \langle 3t^2, 4t^2, 10t\rangle$ at $t = 2$ and express a in terms of T and N.

42. Identify the surface $x = y^2$.

43. Find the center and radius of the sphere given by $x^2 + y^2 + z^2 - 8x + 6x = 0$

44. Find the volume of the solid bounded by $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 = 16$.

45. Indentify via cross-sections the surface defined by $x = 3y^2 + 5z^2$.

46. Find the equation of the tangent plane to the surface given by $x = u^2 + 2v^2$, $y = uv$ and $z = 3u - v$ when $u = 2$ and $v = -1$.

47. The total resistance R of three resistances R_1, R_2 and R_3 connected in parallelis given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. If measurements of R_1, R_2 and R_3 are 100, 200 and 400 ohms respectively, with a maximum error of $\pm 1\%$ in each measurement, estimate the maximum error in the calculated value of R.

48. The force at a point (x, y, z) in three dimensions is given by $\mathbf{F} = \langle y, z, x\rangle$. Find the work done by $\mathbf{F}$ along the twisted cubic $x = t$, $y = t^2$ and $z = t^3$ from $(0, 0, 0)$ to $(2, 4, 8)$.

49. For the integral $\int_0^4\int_{x^2}^{4x}(6x + 12y)dy\;dx$,

   a. evaluate.

   b. rewrite by reversing the order of integration.

50. Using $w = f(x, y, z) = 2xy^2 - 4x^3z$,

   a Find an equation of the tangent plane of w at $(1, 3, 2)$.

   b Estimate $f(1.02, 3.01, 1.98)$.

51. Using $f(x, y) = \frac{x-y}{x+y}$ and $P(2, -1)$,

   a. Find the directional derivative of f in the direction of $\mathbf{v} = \langle 4, -8\rangle$.

   b. Find the direction in which f increases most rapidly.

   c. Find the direction in which f decreases most rapidly.

   d. Find the maximum value of the directional derivative.

52. Identify each surface by identifing the cross sections in each plane of $\mathbb{R}^3$ space:

   a. $2x^2 - 3y^2 + 6z^2 = 1$

   b. $4x - 2y^2 - 2z^2 = 9$

   c. $4x - 2y^2 = 9$

53. Indentify via cross-sections the surface defined by $y = x^2$.

54. Verify Green's Theorem is true for $\int_C xy^2\,dx - x^2y\,dy$, where C consists of the parabola $y = x^2$ from $(-1, 1)$ to $(1, 1)$ and the line segment from $(1, 1)$ to $(-1, 1)$.
(i.e. evaluate directly and using Green's Theorem)

55. a. Show that $\mathbf{F}(x, y) = (3x^2y + 2x)\hat{\imath} + (x^3 + 2y)\hat{\jmath}$ is conservative and find a function f such that $\nabla f = \mathbf{F}$.

   b. Let $\mathbf{F}$ be as in part *a.* and $\mathbf{r}(t) = \langle t^4, t^2+1\rangle$, $0 \le t \le 1$. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{x}$.

56. Using $\mathbf{u} = \langle -4, 6, 5\rangle$ and $\mathbf{v} = \langle 2, -3, 1\rangle$,

   a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

   b. Find $\mathbf{u}\cdot\mathbf{v}$.

   c. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

   d. Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.

   e. Find $\mathbf{u}\times\mathbf{v}$.

57. Identify the surface $4x^2 + 4y^2 + z^2 = 4$.

58. Find the volume of the largest rectangular box that has three of its vertices on the positive x, y and z-axes respectively, and a fourth vertex on the plane $3x + 4y + 2z = 24$.

59. Using $\mathbf{u} = \langle 8, 3, -5\rangle, \mathbf{v} = \langle 4, -4, -2\rangle$,

    a. Find $3u - 4v$.

    b. Find $\|\mathbf{u}\|, \|\mathbf{v}\|$.

60. Evaluate $\int_0^1 \int_{y^2}^{\sqrt{y}} 4xy\, dx\, dy$

61. Using $\mathbf{u} = \langle -2, 4, 4\rangle$ and $\mathbf{v} = \langle 0, 3, -4\rangle$,

    a. Find $\mathbf{u} \cdot \mathbf{v}$.

    b. Find $\text{proj}_{\mathbf{u}}\mathbf{v}$.

    c. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

    d. Find $\mathbf{u} \times \mathbf{v}$.

62. Describe the domain of $f(x, y) = \frac{\ln(x-y)}{\sqrt{xy}}$

63. Identify the surface $2x^2 - 3y^2 + 6z^2 = 6$.

64. Use polar coordinates to evaluate the integral $\iint_R x\sqrt{x^2 + y^2}\, dA$ where R si the region bounded by the semicircle $x = \sqrt{36 - y^2}$.

65. Evaluate $\int_{(1,1,1)}^{(2,3,4)} (4xy)dx + (2x^2 + 2yz^3)dy + (3y^2z^2 + 3)dz$.

66. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = 2x\hat{\imath} - xy\hat{\jmath} + xz\hat{k}$ and S is the surface of the paraboloid $x = y^2 + z^2$ with $x \leq 1$ and the disk $y^2 + z^2 = 1$ at $x = 1$.

67. Using $\mathbf{u} = \langle 8, -4, 1\rangle$ and $\mathbf{v} = \langle -4, 4, 2\rangle$,

    a. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

    b. Find $\mathbf{u} \cdot \mathbf{v}$.

    c. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

    d. Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.

    e. Find $\mathbf{u} \times \mathbf{v}$.

68. Find the center of mass of the lamina that has the shape of the region bounded by $y = x^2$ and $y = 9$ with density $\delta(x, y) = 12x^2y^2$.

69. Find the length of the helix $\mathbf{r}(t) = \langle 6\sin(2t), -5t, -6\cos(2t)\rangle$ for $0 \leq t \leq 4\pi$.

70. Evaluate $\iint_R (2x + y)e^{(2y-x)}dA$, where R is the rectangle with vertices $(2, 1)$, $(6, 3)$, $(4, 7)$ and $(0, 5)$.

71. A flat metal plate lies on an xy-plane such that the temperature T at (x, y) is given by $T = 10(x^2 + y^2)^2$, where T is in degrees and x and y are in centimeters. Find the instantaneous rate of change of T with respect to distance at $(1, 2)$ in the direction of the x-axis.

72. For $f(x, y, z) = 4x^z + z^3 \sin y$ find $\frac{\delta^3 f}{\delta x \delta y^2}$.

73. Using $\mathbf{r}(t) = \langle t\cos t, t\sin t, t^2\rangle$ at $t = 0$,

    a. Find $\mathbf{v}$ and $\mathbf{a}$.

    b. Find $\mathbf{T}$ and $\mathbf{N}$.

    c. Find K.

    d. By first finding $a_{\mathbf{T}}$ and $._{\mathbf{N}}$, express $a = a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$.

74. Using $f(x, y) = 3x^2 + 4y^2$, $P(4, -2)$ and $Q(10, 6)$:

    a. Find the gradient of f at P.

    b. Find the directional derivative of f at P in the direction from P to Q.

    c. Find the maximum value of the directional derivative of f at P.

75. If $w = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that $f_x^2 + f_y^2 = (\frac{\delta w}{\delta r})^2 + \frac{1}{r^2}(\frac{\delta w}{\delta \theta})^2$.

76. Indentify via cross-sections the surface defined by $3^2 - y^2 + 3z^2 + 9 = 0$.

77. For $f(x, y) = \sqrt{x^2 - y^2}$ find the domain of f and describe the level curves.

78. Find the curl and divergence of $\mathbf{F} = \langle xz^2, 2yz, 3xy^2\rangle$.

79. Determine if the following limit exists; if it does also state the value of the limit: $\lim_{(x,y)\to(2,1)} \frac{x^2-xy-2y^2}{x^2-4y^2}$

80. Using $P(-4, 1, 2)$, $Q(1, -3, 4)$, $R(-1, 0, 2)$,

   a. Find an equation of the plane passing through the points.

   b. Find parametric equations for the line through P and parallel to $a = \langle 2, -1, 4 \rangle$.

   c. Find the distance from the point $(5, -3, 2)$ to the plane.

   d. Find the area of the parallelogram determined by P, Q, and R.

81. Evaluate $\iint_S (x^2 z + y^2 z)\, dS$, where S is the part of the plane $z = 4 + x + y$ that lies inside the cylinder $x^2 + y^2 = 4$.

82. For $f(x, y) = 3x^4 y^2 - x \cos y + 4x^3 y^3$, find f_x, f_y, f_{xx} and f_{xy}.

Math 252 Cumulative Review (Answers)

1. (Math-252 Exam 3 Practice)

    a. (ANSWER)

    b. (ANSWER)

2. (Math-252 Quiz 5)

    a. $\mathbf{v}(t) = \langle -\sin t, \cos t, 2t\rangle$
    $\mathbf{v}(\frac{\pi}{2}) = \langle -1, 0, \pi\rangle$

    b. $\mathbf{a}(t) = \langle -\cos t, -\sin t, 2\rangle$
    $\mathbf{a}(\frac{\pi}{2}) = \langle 0, -1, 2\rangle$

3. (Math-252 Quiz 6)

    a. $\mathbf{T}(t) = \langle -\frac{4}{5}\sin(2t), \frac{4}{5}\cos(2t), \frac{3}{5}\rangle$

    b. $\mathbf{N}(t) = \langle -\cos(2t), \sin(2t), 0\rangle$

    c. $k = \frac{4}{25}$

4. (Math-252 Exam 2)

    $4x^2 + 2y^2 = 48$ (an ellipse)

5. (Math-252 Some Exam 3 Practice)

    $\frac{\pi}{3}(12^{3/2} - 8)$

6. (Math-252 Quiz 13)

    Saddle point $f(1, 2) = 20$, local max $f(-9, -18) = 520$

7. (Math-252 Practice Exam 1)

    Elliptical cone

8. (Math-252 Quiz 18)

    a. $m = 243$

    b. $M_{xz} = \int_0^3 \int_0^{6-2x} \int_0^{12-4x-2y} (5x^3) y \, dz \, dy \, dx$

9. (Math-252 Quiz 21)

    $\mathbf{F} = \nabla f = \langle 8xy, 4x^2 - 4yz^3, -6y^2z^2\rangle$

10. (Math-252 Exam 2 Practice)

    absolute max $\frac{37}{4}$ at $(1, \frac{3}{2})$

11. (Math-252 Quiz 4)

    Elliptical cylinder

12. (Math-252 Exam 2 Practice)

    $f_{xy} = -\frac{1}{(x+y)^2}$

13. (Math-252 Exam 3 Practice)

    (ANSWER)

14. (Math-252 Quiz 10)

    $\frac{dy}{dx} = \frac{15x^2y^4 - 8xy}{20x^3y^3 - 4x^2 + 6y^2}$

15. (Math-252 Quiz 3)

    a. $\mathbf{n} = \mathbf{PQ} \times \mathbf{PR} = \langle -7, 8, 5\rangle$

    b. $-7x + 8y + 5z = 29$

    c. $x = 1 + 4t, y = 2 - 3t, z = 4 - 2t; t \in \mathbb{R}$

    d. $D = \frac{16}{\sqrt{138}}$

16. (Math-252 Exam 2 Practice)

    $V = \frac{157}{630}$

17. (Math-252 Exam 3 Practice)

    (ANSWER)

18. (Math-252 Quiz 14)

    Absolute minimum $f(4, 16, 2) = -200$

19. (Math-252 Exam 3 Practice)

    (ANSWER)

20. (Math-252 Exam 3 Practice)

    (ANSWER)

21. (Math-252 Exam 1)

    a. $\mathbf{v}(t) = \langle -4\sin t, 3, 4\cos t\rangle$
    $\mathbf{v}(0) = \langle 0, 3, 4\rangle$
    $\mathbf{a}(t) = \langle -4\cos t, 0, -4\sin t\rangle$
    $\mathbf{a}(0) = \langle -4, 0, 0\rangle$

    b. $\mathbf{T}(t) = \left\langle -\frac{4\sin t}{5}, \frac{3}{5}, \frac{4\cos t}{5}\right\rangle$
    $\mathbf{T}(0) = \left\langle 0, \frac{3}{5}, \frac{4}{5}\right\rangle$
    $\mathbf{N}(t) = \langle -\cos t, 0, -\sin t\rangle$
    $\mathbf{N}(0) = \langle -1, 0, 0\rangle$

    c. $K = \frac{4}{25}$ $a_{\mathbf{T}} = 0$ $a_{\mathbf{N}} = 4$

22. (Math-252 Exam 1)

    (this was extra-credit)
    (ANSWER)

23. (Math-252 Quiz 8)

$xy \sin z + 3xy^2 e^z$

24. (Math-252 Some Exam 3 Practice)

$\frac{3}{10} \ln 3$

25. (Math-252 Practice Exam 1)

a. $\mathbf{r}(t) = \langle 32\sqrt{3}t, -16t^2 + 32t + 128 \rangle$

b. in 4 seconds

c. $128\sqrt{3}$ feet away

d. $64\sqrt{3}$ feet per second

26. (Math-252 Some Exam 3 Practice)

-25

27. (Math-252 Exam 2 Practice)

$\int_0^1 \int_{e^y}^e y \; dx \; dy = \frac{e}{2} - 1$

28. (Math-252 Exam 1)

a. $2x + 3y - z - 5 = 0$

b. $x = -6t + 8, \;\; y = -9t + 2, \;\; z = 3t - 1$

c. $D = 2\sqrt{14}$

d. $A = 3\sqrt{14}$

29. (Math-252 Exam 2)

Absolute max $f(\frac{1}{32}, -\frac{1}{4}, \frac{17}{256}) = \frac{17}{64}$

30. (Math-252 Quiz 21)

$\int_C (xy^2)dx + (4xy^3)dy = \int_0^2 (6t^5)dt = 64$

31. (Math-252 Exam 2 Practice)

a. $-5x + 5y + 27z + 66 = 0$

b. $\langle x, y, z \rangle = \langle 1, 4, -3 \rangle + t\langle -5, 5, 27 \rangle$
$x = -5t + 1; \;\; y = 5t + 4; \;\; z = 27t - 3$

32. (Math-252 Exam 2)

$\{(x, y) : x > 0, y > 0\}$

33. (Math-252 Exam 3 Practice)

(ANSWER)

34. (Math-252 Quiz 5)

a. $\mathbf{r}(t) = \langle 224\sqrt{3}t$
$-16t^2 + 224t + 512 \rangle$

b. $T = 16$
$x(16) = 224\sqrt{3}(16) \doteq 6207.7$ feet

35. (Math-252 Quiz 19)

$V = 72\pi$

36. (Math-252 Exam 1)

a. $\mathbf{r}(t) = \langle 112\sqrt{3}t, -16t^2 + 112t + 128 \rangle$

b. max distance: $893\sqrt{3}$

c. impact speed: $32\sqrt{57}$

37. (Math-252 Exam 2 Practice)

$L = \frac{1}{4}$

38. (Math-252 Quiz 17)

$S = \frac{\pi}{6}(65^{3/2} - 1)$

39. (Math-252 Exam 1)

(this was extra-credit)

$\mathbf{n}_1 = \langle a, b, c \rangle, \;\; \mathbf{n}_2 = \langle ka, kb, kc \rangle = k\langle a, b, c \rangle$

$\mathbf{n}_1 \times \mathbf{n}_2 = 0, \;\; \therefore n_1 \parallel n_2$

point on first plane: $P(0, 0, -\frac{d_1}{c})$

distance from point to second plane:

$$D = |\operatorname{proj}_{\mathbf{n}_1} P||$$
$$= \frac{|ka(0) + kb(0) + kc(-\frac{d_1}{c}) + d_2|}{\sqrt{(ka)^2 + (kb)^2 + (kc)^2}}$$
$$= \frac{|-kd_1 + d_2|}{|k|\sqrt{a^2 + b^2 + c^2}}$$

40. (Math-252 Exam 3 Practice)

(ANSWER)

41. (Math-252 Quiz 7)

$\mathbf{a} = 4\sqrt{5}\mathbf{T} + 2\sqrt{5}\mathbf{N}$

42. (Math-252 Practice Exam 1)

Parabolic cylinder

43. (Math-252 Quiz 1)

$C(4, -3, 0), \rho = 5$

44. (Math-252 Exam 3 Practice)

(ANSWER)

45. (Math-252 Quiz 4)

    Elliptical paraboloid

46. (Math-252 Some Exam 3 Practice)

    -14

47. (Math-252 Exam 2 Practice)

    $\left|\frac{dR}{R}\right| = \frac{400}{7}(\frac{1}{100^2} + \frac{2}{200^2} + \frac{4}{400^2}) = 0.01$

48. (Math-252 Exam 3 Practice)

    (ANSWER)

49. (Math-252 Quiz 15)

    a. $\frac{4736}{5}$

    b. $\int_0^{16} \int_{\frac{1}{4}y}^{\sqrt{y}} (6x + 12y) dx\ dy$

50. (Math-252 Quiz 12)

    a. $-6x + 12y - 4z - 22 = 0$

    b. $f(1.02, 3.01, 1.98) \approx 10.08$

51. (Math-252 Exam 2 Practice)

    a. $\nabla f(x, y) = \langle \frac{2y}{(x_y)^2}, \frac{-2x}{(x_y)^2} \rangle$
       $\nabla f(2, -1) = \langle -2, -4 \rangle$
       $\mathbf{u} = \mathbf{v}\frac{1}{|\mathbf{v}|} = \langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \rangle$
       $D_{\mathbf{u}} f(2, -1) = \frac{6\sqrt{5}}{5}$

    b. $\nabla f \frac{1}{|\nabla f|} = \langle \frac{-\sqrt{5}}{5}, \frac{-2\sqrt{5}}{5} \rangle$

    c. $\langle \frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5} \rangle$

    d. $|\nabla f| = 2\sqrt{5}$

52. (Math-252 Exam 1)

    a. Elliptical hyperboloid (one sheet)

    b. Circular hyperboloid

    c. Parabolic cylinder

53. (Math-252 Quiz 4)

    Parabolic cylinder

54. (Math-252 Some Exam 3 Practice)

    0 (but what does this mean?)

55. (Math-252 Some Exam 3 Practice)

    6

56. (Math-252 Practice Exam 1)

    a. $\|\mathbf{u}\| = \sqrt{77}$
       $\|\mathbf{v}\| = \sqrt{14}$

    b. $\mathbf{u} \cdot \mathbf{v} = -21$

    c. $\theta = \arccos\left(\frac{-21}{7\sqrt{22}}\right)$

57. (Math-252 Practice Exam 1)

    Circular ellipsoid

58. (Math-252 Exam 2 Practice)

    $V = \frac{64}{3}$

59. (Math-252 Quiz 1)

    a. $\langle 8, 25, -7 \rangle$.

    b. $\|\mathbf{u}\| = 7\sqrt{2}$, $\|\mathbf{v}\| = 6$.

60. (Math-252 Exam 2)

    $\int_0^1 \int_{y^2}^{\sqrt{y}} 4xy\, dx\, dy = \frac{1}{3}$

61. (Math-252 Exam 1)

    a. $\mathbf{u} \cdot \mathbf{v} = -4$

    b. $\mathrm{proj}_{\mathbf{u}}\mathbf{v} = \left\langle \frac{2}{9}, \frac{-4}{9}, \frac{-4}{9} \right\rangle$

    c. $\theta = \cos^{-1}\left(\frac{-2}{15}\right) \doteq 1.705$ rad

    d. $\mathbf{u} \times \mathbf{v} = \langle -28, -8, -6 \rangle$

62. (Math-252 Quiz 8)

    $\{(x, y) : x > y, xy > 0\}$

63. (Math-252 Practice Exam 1)

    Hyperbaloid (one sheet)

64. (Math-252 Quiz 16)

    $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^6 r\cos\theta\sqrt{r^2}\, r\ dr\ d\theta = 648$

65. (Math-252 Exam 3 Practice)

    (ANSWER)

66. (Math-252 Some Exam 3 Practice)

    3π

67. (Math-252 Quiz 2)

    a. $\|\mathbf{u}\| = 9$, $\|\mathbf{v}\| = 6$

    b. $\mathbf{u} \cdot \mathbf{v} = -46$

    c. $\theta = \arccos\left(-\frac{23}{27}\right) = 148.4°$

d. $\text{proj}_{\mathbf{v}}\mathbf{u} = \langle -\frac{46}{9}, -\frac{46}{9}, -\frac{23}{9} \rangle$

e. $\mathbf{u} \times \mathbf{v} = \langle -12, -20, 16 \rangle$

68. (Math-252 Exam 3 Practice)

(ANSWER)

69. (Math-252 Exam 1)

$s = 52\pi$

70. (Math-252 Quiz 20)

$\int_5^{15} \int_0^{10} ue^v \left(\frac{5}{25}\right) dv\, du = 20(e^{10} - 1)$

71. (Math-252 Exam 2 Practice)

$T_x = 200$ degrees per centimeters

72. (Math-252 Quiz 10)

$\frac{\delta^3 f}{\delta x \delta y^2} = 0$

73. (Math-252 Practice Exam 1)

a. $\mathbf{v} = \langle -t\sin t + \cos t, t\cos t + \sin t, 2t \rangle$

$\mathbf{a} = \langle -t\cos t - 2\sin t, -t\sin t + 2\cos t, 2 \rangle$

b. $\mathbf{T}(t) = \left\langle \frac{-t\sin 5 + \cos t}{\sqrt{5t^2+1}}, \frac{t\cos t + \sin 5}{\sqrt{5t^2+1}}, \frac{2t}{\sqrt{5t^2+1}} \right\rangle$

$\mathbf{T}(0) = \langle 1, 0, 0 \rangle$

$\mathbf{N}(0) = \left\langle 0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

74. (Math-252 Quiz 11)

a. $\nabla f(P) = \langle 24, -16 \rangle$

b. $\mathbf{u} = \frac{1}{\|\overrightarrow{PQ}\|}\overrightarrow{PQ}$; $D_{\mathbf{u}}f(P) = \nabla f(P) \cdot \mathbf{u} = \frac{16}{10}$

c. $\|\nabla f(p)\| = 8\sqrt{13}$

75. (Math-252 Exam 2 Practice)

$\frac{\delta w}{\delta r} = f_x(\cos\theta) + f_y(\sin\theta)$
$\frac{\delta w}{\delta \theta} = f_x(-r\sin\theta) + f_y(r\cos\theta)$
$\left(\frac{\delta w}{\delta r}\right)^2 + \frac{1}{r^2}\left(\frac{\delta w}{\delta \theta}\right)^2 = f_x^2 + f_y^2$

76. (Math-252 Quiz 4)

Circular hyperboloid of two sheets

77. (Math-252 Exam 2 Practice)

$D = \{(x, y) : |x| \geq |y|\}$
Hyperbola in xy-plane

78. (Math-252 Quiz 21)

- Curl: $\langle 6xy - 2y, 2xz - 3y^2, 0 \rangle$
- Divergence: $z^2 + 2z$

79. (Math-252 Quiz 8)

$L = \frac{3}{4}$

80. (Math-252 Practice Exam 1)

a. $2x + 6y + 7z - 12 = 0$

b. $x = 2t - 4,\ y = -t + 1,\ z = 4t + 2$

c. $D = \dfrac{6}{\sqrt{89}}$

d. $A = \sqrt{89}$

81. (Math-252 Some Exam 3 Practice)

$32\pi\sqrt{3}$

82. (Math-252 Quiz 9)

$f_x = 12x^3y^2 - \cos y + 12x^2y^3$
$f_y = 6x^4y + x\sin y + 12x^3y^2$
$f_{xx} = 36x^2y^2 + 24xy^3$
$f_{yy} = 6x^4 + x\cos y + 24x^3y$
$f_{xy} = 24x^3y + \sin y + 36x^2y^2$