

Machine Learning algorithms

Machine learning:

"The field of study that gives computers **the ability to learn** without being **explicitly programmed**." by Arthur Samuel

"A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P , if its performance at tasks in T , as measured by P , improves with experience E "

Linear regression

- **Linear regression:** predict a real-value output based on input value. For example, house price prediction
- **Multivariate linear regression:** predict a real-value output based on multiple input values.
- **Cost function:** mean square error (MSE)

Gradient Descent

Previously ($n=1$):

Repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

$\frac{\partial}{\partial \theta_0} J(\theta)$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

(simultaneously update θ_0, θ_1)

}

New algorithm ($n \geq 1$):

Repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update θ_j for $j = 0, \dots, n$)

}

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

...

- Practical tricks for Gradient descent:

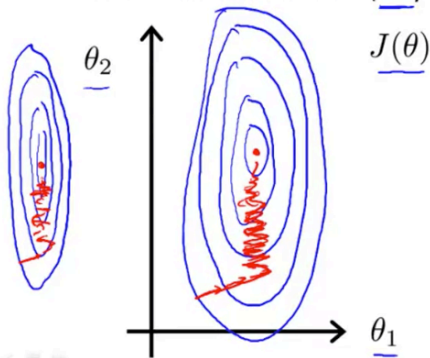
- **Feature scaling:** make sure features are on a similar scale.
 - Gradient descent has a hard time to find its way and take a much **more convoluted** path to the minimum if features are not in the same scale.
 - On the contrary, if the features are in the same scale, Gradient descent takes a **much simpler and more direct** path to the minimum

Feature Scaling

Idea: Make sure features are on a similar scale.

E.g. $x_1 = \text{size (0-2000 feet}^2\text{)}$ ←

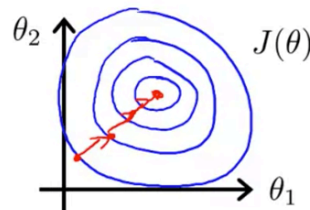
$x_2 = \text{number of bedrooms (1-5)}$ ←



$$\rightarrow x_1 = \frac{\text{size (feet}^2\text{)}}{2000} \quad \checkmark$$

$$\rightarrow x_2 = \frac{\text{number of bedrooms}}{5} \quad \checkmark$$

$$0 \leq x_1 \leq 1 \quad 0 \leq x_2 \leq 1$$



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- Get every feature into approximately a $[-1 \ 1]$ range, $[-3 \ 3]$, $[-1/3 \ 1/3]$ range is still fine, no worries if the features are not in the exact same range
- Mean normalization: to make features with zero mean

Mean normalization

Replace x_i with $x_i - \mu_i$ to make features have approximately zero mean
(Do not apply to $x_0 = 1$).

E.g. $\rightarrow x_1 = \frac{\text{size} - 1000}{2000}$

$$x_2 = \frac{\# \text{bedrooms} - 2}{5}$$

Average size = 1000

1-5 bedrooms

$$-0.5 \leq x_1 \leq 0.5, \quad -0.5 \leq x_2 \leq 0.5$$

$$x_1 \leftarrow \frac{x_1 - \mu_1}{s_1} \quad \left| \quad x_2 \leftarrow \frac{x_2 - \mu_2}{s_2}$$

μ_1 : avg value of x_1 in training set
 s_1 : range (max-min) (or standard deviation)

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- How to make sure **gradient descent is working correctly?**
 - Cost function $J(\theta)$ should decrease after every iteration (plot $J(\theta)$ vs no. of iterations)
 - If $J(\theta)$ is not decreasing, probably learning rate α is too large, should decrease the learning rate α
 - If α is small enough, $J(\theta)$ should decrease on every iteration
 - But if α is too small, then it will take a long time to converge
 - Try a number of α , 0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1..., plot $J(\theta)$ vs no. of iterations
 - Automatic convergence test: declare convergence if $J(\theta)$ decreases by less than a certain threshold after every iteration (the threshold is hard to choose most of the time)
- **Polynomial regression:** We can **change the behavior or curve** of our hypothesis function by making it a quadratic, cubic or square root function (or any other form).
 - **Feature engineering:**
 - create new features based on the problem
 - Feature scaling is important with respect to new features (new features might be in a much larger or smaller magnitude)
- **Normal equation method** is another alternative for gradient descent

$$\theta = (X^T X)^{-1} X^T y$$

Gradient Descent	Normal Equation
Need to choose alpha	No need to choose alpha
Needs many iterations	No need to iterate
$O(kn^2)$	$O(n^3)$, need to calculate inverse of $X^T X$
Works well when n is large	Slow if n is very large

- Noninvertible (singular)
 - Causes
 - redundant features (linearly dependent)
 - Too many features (No. Training samples less than No. of features)
 - Pseudo inverse will solve this problem
 - Inverse
- **Coding steps**

Simple linear regression

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

parameters/weights: θ_0, θ_1

Cost function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^i) - y^i)^2$

Find the parameters that'll minimize cost function $J(\theta_0, \theta_1)$:

★ Gradient descent algorithm

△ starting with random θ_0, θ_1

△ Updates the weights until reach minimum of $J(\theta_0, \theta_1)$

$$\theta_j = \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad \begin{array}{l} j=0, \text{ for } \theta_0 \\ j=1, \text{ for } \theta_1 \end{array}$$

Note: weights should be updated simultaneously.

$$\text{e.g. } \theta_{0, \text{next}} = \theta_{0, \text{current}} - \alpha \frac{\partial}{\partial \theta_0} J(\theta_{0, \text{current}}, \theta_{1, \text{current}})$$

$$\theta_{1, \text{next}} = \theta_{1, \text{current}} - \alpha \frac{\partial}{\partial \theta_1} J(\theta_{0, \text{current}}, \theta_{1, \text{current}})$$

★ Normal equation method

$$\theta = (X^T X)^{-1} X^T y$$