1. **简答题**:

(1) **行波法**：$u(x,t)=f(x+t/b)+g(x-t/b)$，**但** $g(x-t/b)$ **不满足泛定方程**，

故：$u(x,t)=f(x+t/b)$ (4**分**)

比较初条得：$u(x,t)=\psi(x+t/b)$ (4**分**)

(2) $(n+1)z_n+xz_n'=xz_{n-1}$, $\implies$ $(n+2)z_{n+1}+xz_{n+1}'=xz_n$ (2**分**)

$-nz_n+xz_n'=-xz_{n+1}$, $\implies$ $xz_n''+z_n'=nz_n'-z_{n+1}-xz_{n+1}'$ (2**分**)

$\implies$ $x^2z_n''+2xz_n'+[x^2-n(n+1)]z_n=0$ (4**分**)

(3) **原式** $=\displaystyle\int_{-1}^{1}P_l(x)P_0(x)\mathrm{d}x$ (4**分**)

故：**当** $l\neq 0$ **时**，**原式** $=0$，$l=0$ **时**，**原式** $=2$。 (4**分**)

(4) $-ku_x(l,t)=Ku(l,t)$ (4**分**)

$-ku_x(0,t)=q$ (4**分**)

(5) $u_{tt}=a^2u_{xx}$ (4**分**)

$u(0,t)=0,\quad YSu_x(l,t)+mu_{tt}(l,t)+mg=0$ $(1+1$**分**$)$

$u(x,0)=0,\quad u_t(x,t)=0$ $(1+1$**分**$)$

(6) $u=A\ln r+B$ (4**分**)

$A\ln a+B=u_0,\quad A\ln 2a+B=2u_0$ $(1+1$**分**$)$

$A=u_0/\ln 2,\quad B=u_0-u_0\ln a/\ln 2$ (2**分**)

2. $u_{tt}=a^2u_{xx}$ (3**分**)

$YSu_x(0,t)=F,\quad YSu_x(1,t)=-ku(l,t),$ (3**分**)

$u_t(x,0)=,\quad u(x,0)=h(1-x)$ $(2+2$**分**$)$

$v(x,t)=F(x-l)/(YS)-F/k$ (5**分**)

3. $u=\displaystyle\sum_{n=0}^{\infty}c_n\cos\frac{(n+\frac{1}{2})\pi x}{l}\exp[-(n+\tfrac{1}{2})^2\pi^2t/l^2]$ $(5+5$**分**$)$

$\displaystyle\sum_{n=0}^{\infty}c_n\cos\frac{(n+\frac{1}{2})\pi x}{l}=h(1-\frac{x}{l})$ (5**分**)

$c_n=\dfrac{2h}{(n+\frac{1}{2})^2\pi^2}$ (5**分**)

4. $\nabla^2 u = 0$ (2分)

$u\Big|_{\theta=0,\,\pi}$ 有限 (1分)

$u\Big|_{\phi+2\pi} = u\Big|_{\phi}$ (1分)

$u\Big|_{r=0}$ 有限 (1分)

$$u\Big|_{r=1} = \begin{cases} u_0\cos\theta, & \cos\theta > h \\ -u_0, & \cos\theta < h \end{cases} \qquad (2分)$$

$$u = \sum_{l=1}^{+\infty} A_l r^l P_l(\cos\theta) \qquad (5分)$$

$$\sum_{l=1}^{+\infty} A_l P_l(\cos\theta) = \begin{cases} u_0\cos\theta, & \cos\theta > h \\ -u_0, & \cos\theta < h \end{cases} \qquad (3分)$$

$$u\Big|_{r=0} = A_0 = \int_0^{\cos^{-1}h} u_0\cos\theta\sin\theta\mathrm{d}\theta - \int_{\cos^{-1}h}^{\pi} u_0\sin\theta\mathrm{d}\theta = -\frac{u_0}{4}\left(h+1\right)^2 \qquad (5分)$$

5. $u_t = a^2\nabla^2 u, \qquad u\Big|_{t=0} = u_0, \qquad u\Big|_{r=b} = u_1$ (1 + 1 + 1 分)

$u = v + w, \quad v = u_1, \quad w$ 满足齐次方程、齐次边条

$w_t = a^2\nabla^2 w, \quad w\Big|_{t=0} = u_0 - u_1, \quad w\Big|_{r=b} = 0$ (3分)

$w = R(r)T(t)$

$$r^2R''(r) + rR(r) + \mu^2 r^2 R(r) = 0 \implies R_n(r) = J_0\left(\frac{x_n^{(0)}}{b}r\right) \qquad (4分)$$

$$T''(z) + a^2\mu^2 T(z) = 0 \implies T(t) = \exp\left[-\left(\frac{x_n^{(0)}}{b}a\right)^2 t\right] \qquad (4分)$$

$$w(r,t) = \sum_{n=1}^{\infty} c_n J_0\left(\frac{x_n^{(0)}}{b}r\right)\exp\left[-\left(\frac{x_n^{(0)}}{b}a\right)^2 t\right] \qquad (2分)$$

应用初条：$\displaystyle\sum_{n=1}^{\infty} c_n J_0\left(\frac{x_n^{(0)}}{b}r\right) = (u_0 - u_1)$ (2分)

$$c_n = \frac{(u_0-u_1)\displaystyle\int_0^b J_0\left(\frac{x_n^{(0)}}{b}r\right) r\mathrm{d}r}{\displaystyle\int_0^b J_0^2\left(\frac{x_n^{(0)}}{b}r\right) r\mathrm{d}r} \qquad (2分)$$