Physics Homework

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Abstract

In order to improve my computer and English skills, please allow me to complete this physics homework in English context with LaTeX, so as to improve my professional level. Sorry for the inconvenience!

1 Question 5-2

For satellite, according to the law of conservation of angular momentum, we have

$$3R \cdot m4v_0 = r \cdot mv_0 \tag{1}$$

So

$$r = 12R$$

2 Question 5-5

(1)

Before and after a small rocket launch, the system's orbital energy is

$$\frac{1}{2}mv_0^2 - G\frac{Mm}{R_0} = -G\frac{Mm}{2R_0} \tag{2}$$

$$\frac{1}{2}m\left(v_0 + \Delta v_1\right)^2 - G\frac{Mm}{R_0} = -G\frac{Mm}{\frac{8}{3}R_0} \tag{3}$$

So

$$\frac{\Delta v_1}{v_0} = \frac{\sqrt{5}}{2} - 1$$

(2)

In order to accomplish the same effect, Δv_2 subjuets to

$$\frac{1}{2}m\left(v_0 + \Delta v_1\right)^2 = \frac{1}{2}m\left(v_0^2 + \Delta v_2^2\right) \tag{4}$$

So

$$\frac{\Delta v_2}{v_0} = \frac{1}{2}$$

3 Question 5-8

Let J be the moment of inertia of the rod, according to the law of conservation of angular momentum, we have

$$J\omega = mv_0 l \tag{5}$$

The mechanical energy of the system is conserved, so

$$\frac{1}{2}J\omega^2 = \left(mgl + m'2l\right)\left(1 - \cos\theta\right) \tag{6}$$

Finally

$$J = m'l^2 + \left(m + m'\right)l^2 + m'4l^2 \tag{7}$$

So

$$\theta = \arccos\left(1 - \frac{v_0^2}{2g}\frac{m^2}{\left(m + 6m'\right)\left(m + 2m'\right)}\right)$$

4 Question 5-9

(1)

According to the defination of barycenter, we choose the middle point of the rope and

$$x_C = \frac{m_1\frac{-l}{2} + m_2\frac{l}{2}}{m_1 + m_2} \tag{8}$$

The above equation takes a derivative with respect to time

$$v_C = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \tag{9}$$

In the system of barycenter

$$v_1' = v_1 - v_C \tag{10}$$

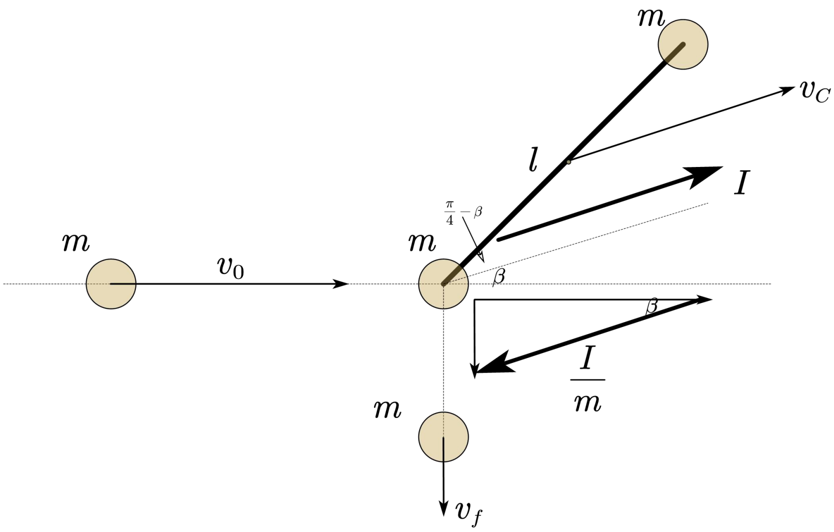

Figure for 5-10

$$\omega = \frac{v_1'}{x_C + \frac{l}{2}} \tag{11}$$

So

$$\begin{aligned} L &= [m_1(x_C + \frac{l}{2})^2 + m_2(\frac{l}{2} - x_C)^2]\omega \\ &= \frac{m_1 m_2}{m_1 + m_2}(v_1 + v_2)l \end{aligned}$$

(2)

In the system of barycenter, tension provides centripetal force

$$T = m_1\omega^2 r_1 = \frac{m_1 m_2}{m_1 + m_2}\frac{(v_1 - v_2)^2}{l}$$

5 Question 5-10

(1)

Let β subjuects to

$$v_f = v_0 tan\beta \tag{12}$$

And impulse I subjects to

$$I cos\beta = m v_0 \tag{13}$$

For the system on the bar, according to the impulse theorem

$$2mv_C = I \tag{14}$$

According to theorem of moment of impulse, for the system on the bar

$$J\omega = I\frac{l}{2}sin(\frac{\pi}{4} - \beta) \tag{15}$$

Because it's an elastic collision, mechanical energy is conserved

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}2mv_C^2 + \frac{1}{2}J\omega^2 \tag{16}$$

So

$$\begin{aligned} v_f &= v_0 tan\left(\frac{1}{2}arctan\frac{51\sqrt{2}+34}{68}\right) \\ &\approx 0.5469181607 v_0 \end{aligned}$$

(2)

We can also have that

$$\omega = \frac{v_0}{\sqrt{2}l}\left[1 - tan\left(\frac{1}{2}arctan\frac{51\sqrt{2}+34}{68}\right)\right]$$
$$\approx 0.320377241\frac{v_0}{l}$$

6 Question 5-11

According to Newton's second law and kinematics

$$T = m_1 g + m_1 a_1 = m_2 g + m_2 a_2 \tag{17}$$

$$t = \sqrt{\frac{2h_1}{g}} = \sqrt{\frac{2h_2}{g}} \tag{18}$$

The solution is that

$$t = \sqrt{\frac{-2}{g}\frac{m_1 h_1 - m_2 h_2}{m_1 - m_2}}$$

7 Question 5-15

By observation, $mg < m\frac{v_0^2}{r_0} = \frac{9}{2}mg$, as a result, the object will go futher and futher from the centre

$$r_{min} = r_0$$

In this process, according to conservation of angular momentum and conservation of mechanical energy

$$mvr = mv_0 r_0 \tag{19}$$

$$\frac{1}{2}m\left(v_0^2 - v^2\right) = mg\left(r - r_0\right) \tag{20}$$

So

$$r_{max} = 3r_0$$

8 Question 5-19

According to conservation of angular momentum and conservation of mechanical energy

$$2ma^2\omega_0 = 2mx^2\omega \tag{21}$$

$$\frac{1}{2}2ma^2\omega_0^2 = \frac{1}{2}2mx^2\omega^2 + 2\frac{1}{2}m\left(\frac{d}{dt}x\right)^2 \tag{22}$$

The x is the distance from the centre to the small ball. The solution can be] write as below

$$\omega = \frac{\omega_0}{\omega_0^2 t^2 + 1}$$
$$\alpha = \frac{d}{dt}\omega = -\frac{2\omega_0^2 t}{(\omega_0^2 t^2 + 1)^2}$$

9 Question 5-20

(1)

According to conservation of angular momentum and conservation of mechanical energy

$$mva = mv_1 L_1 \tag{23}$$

$$3mva = mv_2(4a - L_1) \tag{24}$$

$$2\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \tag{25}$$

So

$$L_1 = \frac{6}{\sqrt{46}\cos\left(\frac{1}{3}\arccos\left(-\frac{121}{23\sqrt{46}}\right)\right) - 1}a$$
$$\approx 1.653165286a$$

(2)

According to conservation of angular momentum and conservation of mechanical energy

$$mrv_{1n} = mav \tag{26}$$

$$m(4a-r)v_{2n}=m3av \tag{27}$$

The length of the rope is constant

$$\frac{d^2r_1}{dt^2}+\frac{d^2r_2}{dt^2}=0 \tag{28}$$

The kinetic equations are

$$m\frac{d^2r_1}{dt^2}=m\frac{v_{1n}^2}{r}-T \tag{29}$$

$$m\frac{d^2r_2}{dt^2}=m\frac{v_{2n}^2}{r}-T \tag{30}$$

We can get the expression of T, and according to $Holder$ inequation

$$\begin{aligned}
T&=\frac{ma^2v^2}{2}\left(\frac{1}{r^3}+\frac{3}{(4a-r)^3}\right)\\
&\geqslant\frac{ma^2v^2}{2}\frac{\left(1+\sqrt{3}\right)^4}{(r+4a-r)^3}\\
&=\frac{\left(1+\sqrt{3}\right)^4}{128}\frac{mv^2}{a}\\
&\approx 0.4352563509\frac{mv^2}{a}
\end{aligned}$$