ELECTROMAGNETISM ASSIGNMENT FOR THE SECOND TIME

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ABSTRACT. Here is the electromagnetism assignment for the first time which is for the corse given by professor Weichao Liu. In order to practise the expertise in scientific film of physics, students need to practise using LaTeXto composing their own work, even if this is only a ordinary homework.

MAIN TEXT

1-16. Let the point charge q on the central axis gets a potential φ from the thin ring when the distance from the center of a thin ring with charge Q is z:

$$\varphi = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{a^2+z^2}}$$

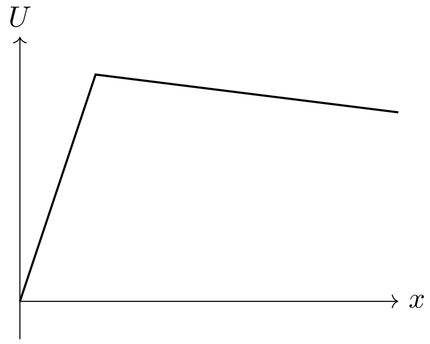
That because every tiny electronic part of thin ring has the same form of potential so the result is like above. The potential is a scalar quantity, according to the superposition principle:

$$\Phi_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{a}+\frac{q_2}{\sqrt{a^2+b^2}}\right) = \frac{A_1}{q}$$
$$\Phi_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\sqrt{a^2+b^2}}+\frac{q_2}{a}\right) = \frac{A_2}{q}$$

So:

$$q_1 = \frac{4\pi\varepsilon_0}{b^2 q}a\sqrt{a^2+b^2}\left(\sqrt{a^2+b^2}A_1 - aA_2\right)$$
$$q_1 = \frac{4\pi\varepsilon_0}{b^2 q}a\sqrt{a^2+b^2}\left(\sqrt{a^2+b^2}A_2 - aA_1\right)$$

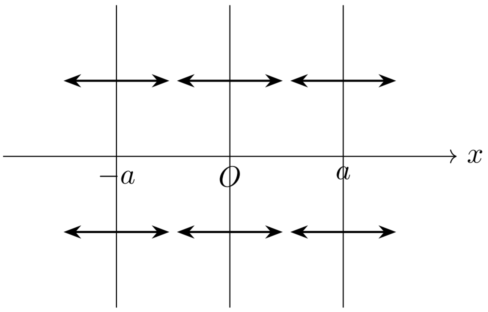
1-19.
(1). According to the assumption, the potential gradient between the grid is uniform, that is, the potential varies linearly with distance, and the cathode is the zero point of the potential.



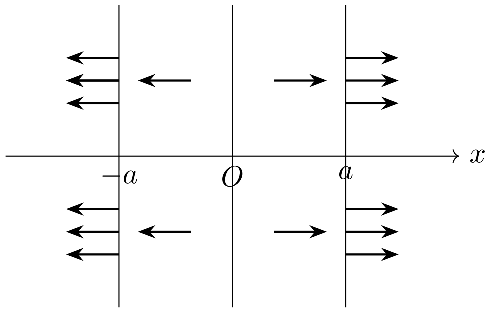
(2). According to classical mechanics, the kinetic energy theorem is listed.

$$eU_+ = \frac{1}{2}m_{\mathrm{e}}v^2$$

So $v = \sqrt{\dfrac{2eU_+}{m_{\mathrm{e}}}} \approx 2.30\times 10^6\, m/s$.



Befor each electric field
is superimposed



After each electric field
is superimposed

1-20. It's easy to draw the electric field given by each plate.

Where each face gives its half of the electric field strength is

$$E_0 = \frac{\sigma}{2\varepsilon_0}$$

So

$$E(x) = \begin{cases} -\dfrac{3\sigma}{2\varepsilon_0} & x < -a \\ -\dfrac{\sigma}{2\varepsilon_0} & -a < x < 0 \\ \dfrac{\sigma}{2\varepsilon_0} & 0 < x < a \\ \dfrac{3\sigma}{2\varepsilon_0} & a < x \end{cases}$$

Integrate this function immediately:

$$\varphi(x) = \begin{cases} -\dfrac{\sigma}{2\varepsilon_0}(a+3x) & x < -a \\ -\dfrac{\sigma}{2\varepsilon_0}x & -a < x < 0 \\ \dfrac{\sigma}{2\varepsilon_0}x & 0 < x < a \\ \dfrac{\sigma}{2\varepsilon_0}(a+3x) & a < x \end{cases}$$

1-21. First prove that the electric field on the plane has no horizontal component:

Assuming that there is a horizontal component, then the negative half axis of z is supplemented with half a spherical shell to form a complete spherical shell, then according to the symmetry, the lower half of the spherical shell also contributes the same horizontal electric field, then the interior of the entire spherical shell has a horizontal electric field in the Oxy plane, which is contradictory to the conclusion that there is no electric field inside the uniform charged spherical shell.

So the potential is equal everywhere in the Oxy plane of the hemispherical shell, we just need to find the potential at O.

The potential contribution of any element on the shell is the same, so

$$\varphi = \frac{1}{4\pi\varepsilon_0}\frac{2\pi R^2\sigma}{R} = \frac{R\sigma}{2\varepsilon_0}$$

1-23.

(1). It is obviously the same as the circular electric flux formed by the intersection line between the hemispherical surface and the Oxy plane.

$$\Phi_{\mathrm{E}} = \pi R^2 E$$

(2). In other words, the normal of the circle will be at an Angle of $60°$ to the strength of the electric field.

$$\Phi_{\mathrm{E}}' = \Phi_{\mathrm{E}} \cos 60° = \frac{1}{2}\pi R^2 E$$

References

1-26.

(1). Consider the energy.

$$\frac{1}{2\pi\varepsilon_0}\frac{Q_\alpha Q_{\mathrm{Au}}}{r_{\min}} = \frac{1}{2}m_\alpha v_0^2$$

So $r_{\min} = \dfrac{1}{2\pi\varepsilon_0}\dfrac{Q_\alpha Q_{\mathrm{Au}}}{m_\alpha v_0^2} \approx 4.25 \times 10^{-14}\,\mathrm{m}$.

(2). Consider energy in two phases.

$$(E_{\mathrm{k}})_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{Q_\alpha Q_{\mathrm{Au}}}{r_{\mathrm{Au}}} + \int_0^{r_{\mathrm{Au}}} E\left(r\right)\mathrm{d}r$$

According to Gauss theorem.

$$4\pi r^2 E\left(r\right) = \frac{r^3}{r_{\mathrm{Au}}^3}Q_{\mathrm{Au}}$$

So $(E_{\mathrm{k}})_{\min} = \dfrac{1.5}{4\pi\varepsilon_0}\dfrac{Q_\alpha Q_{\mathrm{Au}}}{r_{\mathrm{Au}}} \approx 7.29 \times 10^{-12}\,\mathrm{J}$.

1-27.

(1). According to Gauss's theorem of free space:

$$\nabla \cdot \boldsymbol{E} = \frac{\rho\left(r\right)}{\varepsilon_0}$$

Where ∇ is the Hamiltonian operator, in spherical coordinates, if the system is spherically symmetric, then

$$\nabla = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\hat{\boldsymbol{r}}$$

Consider Electrons are negatively charged

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2 E\left(r\right) = -\frac{e}{\pi a_0^3}\mathrm{e}^{-\frac{2r}{a_0}}$$

Because the system is spherically symmetric

$$\frac{\partial}{\partial r} = \frac{\mathrm{d}}{\mathrm{d}r}$$

Solving this equation above gives us

$$r^2 E\left(r\right) = \frac{e}{\pi\varepsilon_0}\left(\frac{1}{2a_0^2}r^2 + \frac{1}{2a_0}r + \frac{1}{4}\right)\mathrm{e}^{-\frac{2r}{a_0}}$$

That is

$$E\left(r\right) = \frac{e}{\pi\varepsilon_0}\left(\frac{1}{4}\frac{1}{r^2} + \frac{1}{2a_0}\frac{1}{r} + \frac{1}{2a_0^2}\right)\mathrm{e}^{-\frac{2r}{a_0}}$$

(2). Get by substitution

$$E\left(a_0\right)=\frac{5\mathrm{e}^{-2}e}{4\pi\varepsilon_0 a_0^2}\approx 3.47\times 10^{11}\,\mathrm{N/C}$$

But

$$E_{class}(a_0)=\frac{e^2}{4\pi\varepsilon_0 a_0}\approx 5.13\times 10^{11}\,\mathrm{N/C}$$

So here is

$$E_{class}(a_0)<E(a_0)$$

1-31. Write down the potential energy of the electron

$$U\left(r\right)=\frac{1}{4\pi\varepsilon_0}\frac{\frac{r^3}{R^3}Qe}{r}=\frac{1}{2}\frac{Qe}{2\pi\varepsilon_0 R^3}r^2$$

Compared with the energy of the spring oscillator, the equivalent recovery coefficient $\mathcal{K}$ is obtained.

$$\mathcal{K}=\frac{Qe}{2\pi\varepsilon_0 R^3}$$

Apply the formula of simple harmonic motion

$$f=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{\mathcal{K}}{m}}=\frac{1}{2\pi}\sqrt{\frac{Qe}{2\pi\varepsilon_0 R^3 m}}$$

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