Physics Homework

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Abstract

In order to improve my computer and English skills, please allow me to complete this physics homework in English context with LaTeX, so as to improve my professional level. Sorry for the inconvenience!

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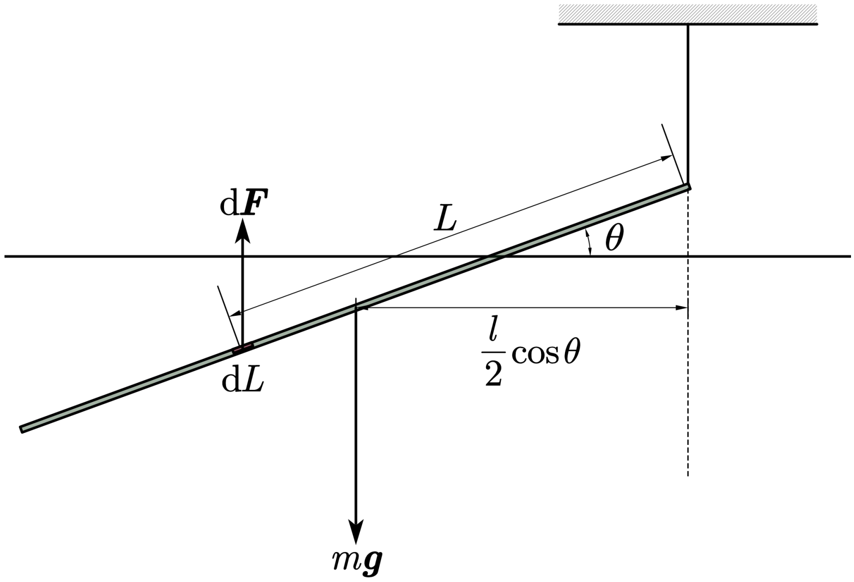
Question 7-5



(1)

Let's take a tiny element to analyse. The force and moment can get with point where rod and rope tied together as reference point.

$$\mathrm{d}F = \rho_0 g \mathrm{d}V = \rho_0 g S \mathrm{d}L \tag{1}$$

$$\mathrm{d}M_F = \mathrm{d}F L \cos\theta = \rho_0 g S \cos\theta L \mathrm{d}L \tag{2}$$

And we can get the moment of $\boldsymbol{F}$ by integrating.

$$M_F = \int \mathrm{d}M_F = \frac{1}{2}\rho_0 g S \cos\theta L^2 \Big|_{d\csc\theta}^{l} = \frac{1}{2}\rho_0 g S \cos\theta (l^2 - d^2 \csc^2\theta) \tag{3}$$

And the moment of gravity is easy to get.

$$M_G = mg\frac{l}{2}\cos\theta = \rho g l S \frac{l}{2}\cos\theta \tag{4}$$

And the torque equilibrium equation will get.

$$M_G = M_F \tag{5}$$

The solution is below.

$$\theta = \arcsin \frac{d}{l\sqrt{1 - \dfrac{\rho}{\rho_0}}}$$

(2)

We can get the $\boldsymbol{F}$ by integrating.

$$F = \int \mathrm{d}F = \rho_0 gSL\bigg|_{d\csc\theta}^{l} = \rho_0 gS\left(l - d\csc\theta\right) \tag{6}$$

According to the force balance, we can get $\boldsymbol{F}_{\mathrm{T}}$.

$$F_{\mathrm{T}} = mg - F = \left(\rho - \rho_0 + \rho_0\sqrt{1 - \frac{\rho}{\rho_0}}\right) lSg$$

Question 7-9

(1)

Equation of continuity is below.

$$Sv_A = Sv_B = Sv_C \tag{7}$$

According to Torricelli's theorem.

$$v_B = \sqrt{2g\left(h_A - h_B\right)} \tag{8}$$

Compare point A with liquid level of the same height, according to the Bernoulli's equation,

$$p_0 = p_A + \frac{1}{2}\rho v_A^2 \tag{9}$$

The same method.

$$p_0 = p_C + \frac{1}{2}\rho v_C^2 + \rho g(h_C - h_A) \tag{10}$$

So we solve it.

$$p_A = p_0 - \rho g(h_A - h_B) \approx 0.915 \times 10^5\ \mathrm{Pa}$$

$$p_B = p_0 \approx 1.013 \times 10^5\ \mathrm{Pa}$$

$$p_C = p_0 - \rho g(h_C - h_B) \approx 0.866 \times 10^5\ \mathrm{Pa}$$

(2)

According to the definition of fluid flow.

$$Q = Sv \tag{11}$$

So we have.

$$Q = Sv \approx 3.10 \times 10^{-3}\ \mathrm{m^3/s}$$

(3)

The critical condition requires $p_C = 0$, so.

$$p_0 = \rho g h_{C_{\max}} + \frac{1}{2}\rho v_C^2 \tag{12}$$

Here we consider $p_0 = 1.00 \times 10^5$ Pa. The solution is that.

$$h_{C_{\max}} = \frac{p_0 - \rho g\left(h_A - h_B\right)}{\rho g} \approx 9.20 \text{ m}$$

Question 7-10

Examine the fluid in the pipe. According to the equation of continuity.

$$S_A v_A = S_B v_B = Q \tag{13}$$

According to Bernoulli's equation.

$$p_A + \frac{1}{2}\rho v_A^2 = p_B + \frac{1}{2}\rho v_B^2 \tag{14}$$

And the pipe B is conect with air.

$$p_B = p_0 \tag{15}$$

At the same time, the pipe A and the tiny pipe below form a communication device.

$$p_A + \rho g h = p_0 \tag{16}$$

So.

$$h = \frac{Q^2}{2g}\left(\frac{1}{S_A^2} - \frac{1}{S_B^2}\right)$$

Questuon 7-11

Static pressure changes to dynamic pressure when the fluid passes through point B.

$$\frac{1}{2}\rho\left(v_B^2 - v_A^2\right) = \rho' g h \tag{17}$$

And we have the equation of continuity.

$$S_A v_A = S_B v_B \tag{18}$$

So.

$$v_A = \sqrt{\frac{2\rho' g h}{\rho\left(\frac{S_A^2}{S_B^2} - 1\right)}} \approx 0.56 \text{ m/s}$$

Question 7-12

According to Bernoulli's equation.

$$p_0 + \Delta p = p_0 + \rho g h + \frac{1}{2}\rho v^2 \tag{19}$$

So.

$$\Delta p = \rho g h + \frac{1}{2}\rho v^2 \approx 6.025 \times 10^3 \text{ Pa}$$

Question 7-13

(1)

It is easy to know that the outflow velocity of water is below.

$$v_0 = \sqrt{2gH} \tag{20}$$

So.

$$Q = \pi \left(\frac{d}{2}\right)^2 v_0 = \frac{1}{4}\pi d^2 \sqrt{2gH}$$

(2)

According to the Bernoulli's equation.

$$p_{\text{up}} + \frac{1}{2}\rho v_0^2 + \rho g h = p_{\text{down}} + \frac{1}{2}\rho v_{\text{down}}^2 \tag{21}$$

According to the equation of continuity.

$$\pi \left(\frac{D}{2}\right)^2 v_{\text{down}} = Q \tag{22}$$

And.

$$p_{\text{up}} = p_0 \tag{23}$$

So.

$$p_{\text{down}} = p_0 + \rho g h + \rho g H \left(1 - \frac{d^4}{D^4}\right)$$

Question 7-14

(1)

According to the communicator principle.

$$h_1 = h_A$$

(2)

The static pressure at B after discharge becomes p_0. According to the communicator principle.

$$h_2 = h_C$$

Question 7-17

According to the Bernoulli's equation and continuity equation.

$$v = \sqrt{2gH} \tag{24}$$

$$Sv = \pi \left(\frac{R}{h}H\right)^2 \frac{-\mathrm{d}H}{\mathrm{d}t} \tag{25}$$

Here we ignore the velocity of the free liquid surface. And.

$$-H^{\frac{3}{2}}\mathrm{d}H = \frac{Sh^2}{\pi^2 R^2}\sqrt{2g}\mathrm{d}t \tag{26}$$

$$t = \frac{-\pi^2 R^2}{Sh^2\sqrt{2g}} \int_h^{\frac{1}{2}h} H^{\frac{3}{2}}\mathrm{d}H = \frac{\pi^2 R^2}{20S}\sqrt{\frac{h}{g}}(4\sqrt{2}-1)$$

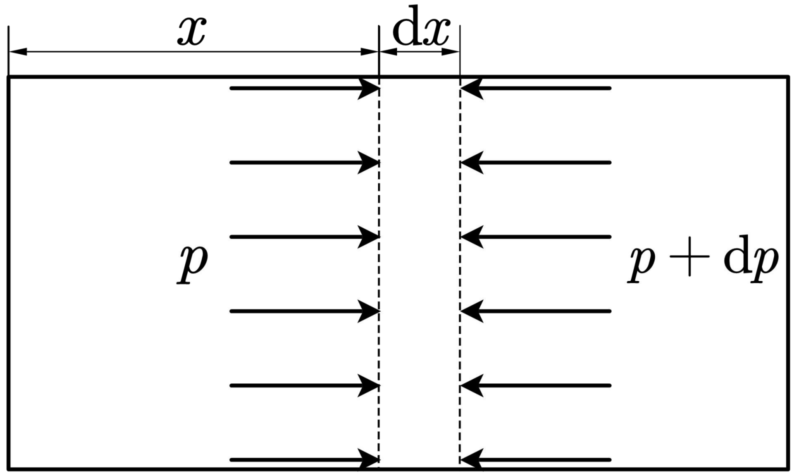
Question 7-23

Because of centrifugal force, air molecules have a tendency to move outward in a radial direction. It is obtained by air thin layer study which length is $\mathrm{d}x$ and force analysis.

$$\mathrm{d}pS = S\mathrm{d}x\rho(x)\omega^2 x \tag{27}$$

The ideal gas equation of state can be derived from the relationship between density and pressure

$$pV = nRT = \frac{m}{M_{\mathrm{mol}}}RT \Longrightarrow pM_{\mathrm{mol}} = \frac{m}{V}RT = \rho RT \tag{28}$$



As a result.

$$\frac{\mathrm{d}p}{p}=\frac{\rho_0\omega^2}{p_0}x\mathrm{d}x \tag{29}$$

The solution is as below.

$$p\left(x\right)=p\left(x=0\right)\mathrm{e}^{\dfrac{\rho_0\omega^2}{2p_0}x^2} \tag{30}$$

And we know that the ω is very very small. So we can do Tailor expand.

$$p(x)\sim p(x=0)\left(\frac{\rho_0\omega^2}{2p_0}x^2+1\right) \tag{31}$$

By the law of conservation of mass.

$$\int_0^L p(x)\mathrm{d}x=\int_0^L p_0\mathrm{d}x \tag{32}$$

And.

$$\int_0^L p(x)\mathrm{d}x\sim\int_0^L p(x=0)\left(\frac{\rho_0\omega^2}{2p_0}x^2+1\right)\mathrm{d}x=p_0L \tag{33}$$

We get.

$$p(x=0)=\frac{p_0}{\dfrac{\rho_0\omega^2}{6p_0}L^2+1}\sim p_0\left(1-\frac{\rho_0\omega^2}{6p_0}L^2\right) \tag{34}$$

According to the communicator principle.

$$p_0-p(x=0)=\rho gh \tag{35}$$

We solve this problem.

$$h = \frac{L^2\omega^2\rho_0}{6\rho g}$$