Laplace Transform

Sweet Pastry

(FDU Physics)

Abstract

In order to improve my computer and English skills, please allow me to complete this physics homework in English context with LaTeX, so as to improve my professional level. Sorry for the inconvenience!

Contents

1 Defination

Define $\mathcal{L}$ as the Laplacian change operator.

$$F(s)=\mathcal{L}\left[f(t)\right]=\int_{0^-}^{+\infty} f\left(t\right)\mathrm{e}^{-st}\mathrm{d}t$$

2 Properties

1. (Linear Property) if $F\left(s\right)=\mathcal{L}\left[f\left(t\right)\right]$, we have

$$\mathcal{L}[A_1 f_1(t)+A_2 f_2(t)]=A_1\mathcal{L}\left[f_1\left(t\right)\right]+A_2\mathcal{L}\left[f_2(t)\right]$$

2. (Differential Property) if $F\left(s\right)=\mathcal{L}\left[f\left(t\right)\right]$, we have

$$\mathcal{L}\left[f'\left(t\right)\right]=sF\left(s\right)-f\left(0^-\right)$$

3. (Integral Property) if $F\left(s\right)=\mathcal{L}\left[f\left(t\right)\right]$, we have

$$\mathcal{L}\left[\int_{0^-}^{+\infty} f\left(t\right)\mathrm{d}t\right]=\frac{F\left(s\right)}{s}$$

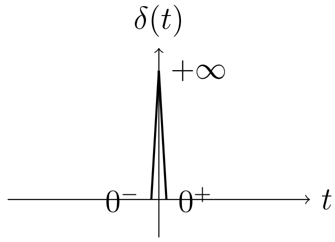
4. (Delayed Property) if $F\left(s\right)=\mathcal{L}\left[f\left(t\right)\right]$, we have

$$\mathcal{L}[f(t-t_0)\varepsilon(t-t_0)]=\mathrm{e}^{-st_0}F(s)$$

3 Common Transformation

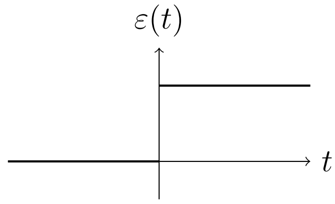
$$\begin{array}{lll}
\mathcal{L}\left[\delta\left(t\right)\right]=1 & \mathcal{L}\left[\varepsilon\left(t\right)\right]=\dfrac{1}{s} & \mathcal{L}\left[t\right]=\dfrac{1}{s^2} \\
\mathcal{L}\left[\sin\omega t\right]=\dfrac{\omega}{s^2+\omega^2} & \mathcal{L}\left[\cos\omega t\right]=\dfrac{s}{s^2+\omega^2} & \mathcal{L}\left[\mathrm{e}^{-\alpha t}\cos\omega t\right]=\dfrac{s+\alpha}{\left(s+\alpha\right)^2+\omega^2} \\
\mathcal{L}\left[\mathrm{e}^{-\alpha t}\right]=\dfrac{1}{s+a} & \mathcal{L}\left[t\mathrm{e}^{-\alpha t}\right]=\dfrac{1}{\left(s+\alpha\right)^2} & \mathcal{L}\left[1-\mathrm{e}^{-\alpha t}\right]=\dfrac{\alpha}{\alpha\left(s+\alpha\right)}
\end{array}$$

(Impulse Function)



$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

(Step Function)



$$\varepsilon(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$