ELECTROMAGNETISM ASSIGNMENT FOR THE THIRD TIME

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Abstract. Here is the electromagnetism assignment for the third time which is for the corse given by professor Weichao Liu. In order to practise the expertise in scientific film of physics, students need to practise using LaTeXto composing their own work, even if this is only a ordinary homework.

Main Text

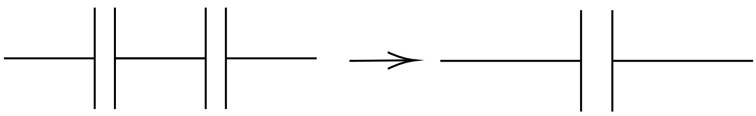
2-4.

(1). C, D plates are not charged, so the sum of the electric field vectors they generate is **0** and does not change the spatial potential distribution.

$$U_{AC} = U_{CD} = U_{DB} = \frac{1}{3}U_0$$

$$E_{AC} = E_{CD} = E_{DB} = \frac{U_0}{d}$$

(2). The opposite charges between the CD plate are neutralized, but the opposite charges between the AC plate and the DB plate remain unchanged, and the result is that the middle two plates become equivalent to one plate.



Equivalent To One Plate

As a result:

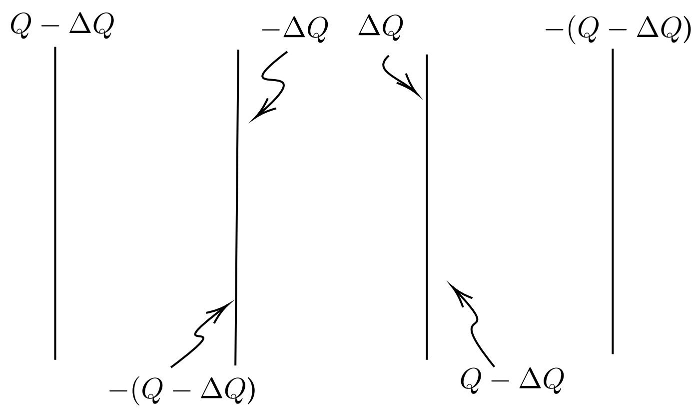
$$U_{AC} = U_{DB} = \frac{1}{3}U_0$$

$$E_{AC} = E_{DB} = \frac{U_0}{d}$$

$$U_{CD} = 0$$

$$E_{CD} = 0$$

(3). Let the final charge distribution be as shown.



Charge Q satisfies the relation:

$$Qd \propto U_0$$

What's more, plates A and B have the same potential.

$$(Q-\Delta Q)\frac{d}{3} - \Delta Q\frac{d}{3} + (Q-\Delta Q)\frac{d}{3} = 0$$

So $\Delta Q = \frac{2}{3}Q$. And as result:

$$U_{AC} = U_{DB} = \frac{1}{9}U_0$$

$$E_{AC} = E_{DB} = \frac{1}{3}\frac{U_0}{d}$$

$$U_{CD} = -\frac{2}{9}U_0$$

$$E_{CD} = -\frac{2}{3}\frac{U_0}{d}$$

(4). The answer of (1) is not change. The answer of (2) will change to

$$U_{AC} = U_{DB} = \frac{1}{2}U_0$$

$$E_{AC} = E_{DB} = \frac{3}{2}\frac{U_0}{d}$$

$$U_{CD} = 0$$

$$E_{CD} = 0$$

So as to remain maintain the voltage between A and B is still U_0. And the answer of (3) will change to

$$U_{AC} = U_{DB} = \frac{1}{6}U_0$$

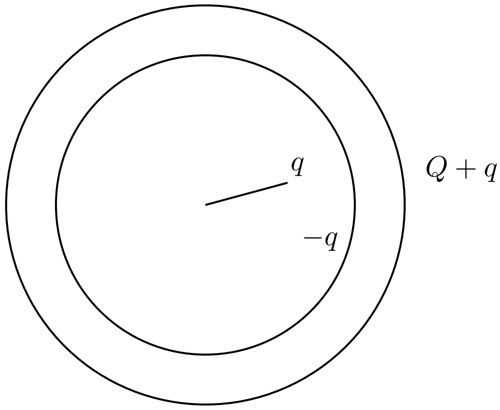
$$E_{AC} = E_{DB} = \frac{1}{2}\frac{U_0}{d}$$

$$U_{CD} = -\frac{1}{3}U_0$$

$$E_{CD} = -\frac{U_0}{d}$$

Because it just like the initial charge changes to 1.5 times.

2-5. In order for there to be no charge inside the conductor, the field lines generated by $-q$ must all converge to the induced charge inside the shell, so the induced charge inside the shell is q.

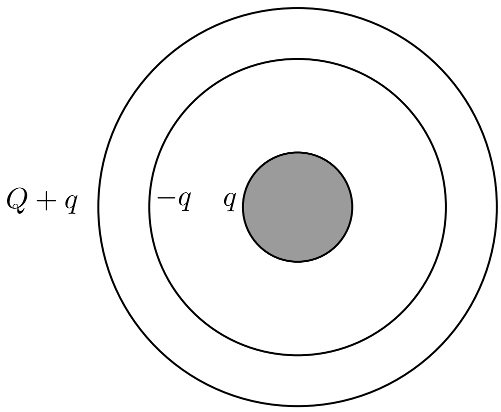


In order for there to be no charge inside the conductor, the field lines generated by -q must all converge to the induced charge inside the shell, so that the distance from each tiny point charge on the shell to the center of the sphere is equal to the local radius, so:

$$U_O = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} - \frac{q}{a} + \frac{Q+q}{b}\right)$$

2-10.

(1). Just like above.



So:

$$\varphi_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{R_3}$$

And there is no electronic field between R_3 and R_2 so:

$$\varphi_1 = \varphi_2 + \frac{1}{4\pi\varepsilon_0}q\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R_1} - \frac{q}{R_2} + \frac{Q+q}{R_3}\right)$$

(2).

$$\Delta\varphi = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R_1} - \frac{q}{R_2}\right)$$

(3). The $-q$ and q will be combined together as 0, and $\varphi_1 = \varphi_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{R_3}$ and $\Delta\varphi = 0$.

(4). The $Q+q$ will go into the ground and disappear. So $\varphi_2 = 0$ and $\varphi_1 = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. As a result $\Delta\varphi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

(5). The inter ball and the ground become equipotential bodies so $\varphi_1 = 0$. Consider the inter ball draw Δq from the ground and because the outer ball is far away from ground so

$$\varphi_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q+\Delta q}{R_3}$$

What's more, accoring to electronic field we can get

$$\Delta\varphi = \frac{q+\Delta q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

And $\varphi_1 = \varphi_2 + \Delta\varphi = 0$. So

$$\Delta q = \frac{\left(-R_1R_2 + R_2R_3 - R_3R_1\right)q - R_1R_2Q}{R_1R_2 - R_2R_3 + R_3R_1}$$

Take back so we get

$$\varphi_2 = -\Delta\varphi = \frac{R_2 - R_1}{R_2R_3 - R_1R_3 + R_1R_2}\frac{Q}{4\pi\varepsilon_0}$$

2-12.

(1). It equals to make the to plate become closer so

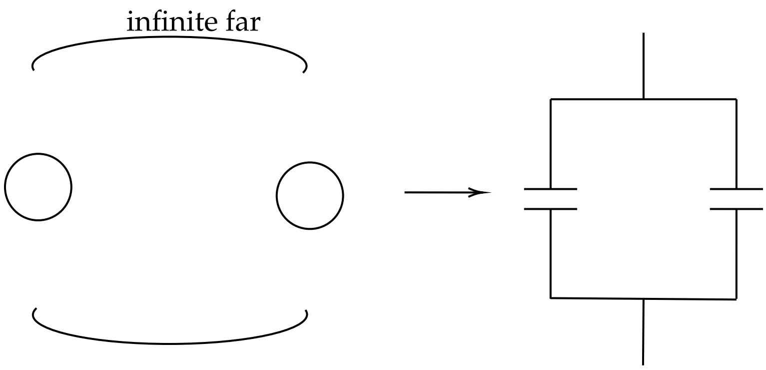
$$C = \varepsilon_0\frac{S}{d-t}$$

(2). Ignoring the boundary effect, it can be seen that there is no effect according to the above equation.

2-14.

(1). We can think that this is similar to the parallel operation of capacitors.

$$C = C_1 \mathbin{//} C_2 = 4\pi\varepsilon_0(a+b)$$



(2). The amount of charge on a capacitor is proportional to its size.

$$q_1 = \frac{a}{a+b}Q$$

$$q_2 = \frac{b}{a+b}Q$$

2-19.

(1). Let's consider $\varphi_A = 0$ and $\varphi_B = U$. So $Q_4 = 0$, $Q_1 = C_1 U$, $Q_2 + Q_3 = (C_2 \mathbin{//} C_3)\,U$

$$C_{AB} = \frac{Q_1 + Q_2 + Q_3 + Q_4}{U} = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_2 + C_3}$$

(2). Just same as above.

$$C_{DE} = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_2 + C_3}$$

(3). Obiously it gets short!

$$C_{AE} = +\infty$$

2-23.

(1). For the A sphere, this corresponds to A scalar superposition of potential.

$$\Delta\varphi_1 = \varphi_2$$

For B, its outer surface plus q_1 so

$$\Delta\varphi_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_2} = \frac{q_1}{q_2}\varphi_2$$

(2).

$$W = q_1\Delta\varphi_1 = q_1\varphi_2$$

3-1. Microscopic determination of current:

$$I = neSv$$

So $n = \dfrac{I}{eSv} = 1.3 \times 10^5\,\mathrm{mm}^{-3}$.

3-7.

(1). Consider a very thin ball surface.

$$\mathrm{d}R = \rho\frac{\mathrm{d}r}{4\pi r^2} \Longrightarrow R = \int_{r_a}^{r_b} \mathrm{d}R = \frac{\rho}{4\pi}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

(2).

$$J(r) = \frac{I}{S(r)} = \frac{U}{RS(r)} = \frac{r_a r_b U}{\rho\,(r_b - r_a)\,r_2}$$

3-10. Examine the definition of resistance.

$$R_{AB} = \frac{d_1}{\gamma_1 S} \qquad R_{BC} = \frac{d_2}{\gamma_2 S}$$

Ignoring the marginal effect, the electric field generated by the surface density of charge can be used to find the potential difference.

$$U_{AB} = E_{AB}d_1 = \frac{\sigma_A - \sigma_B - \sigma_C}{2\varepsilon_0}d_1 \qquad U_{BC} = E_{BC}d_1 = \frac{\sigma_A + \sigma_B - \sigma_C}{2\varepsilon_0}d_2$$

Ohm's law:

$$U_{AB} = IR_{AB} \qquad U_{BC} = IR_{BC}$$

And no net charge accumulates on electronic components.

$$\sigma_A + \sigma_B + \sigma_C = 0$$

So:

$$\sigma_A = \frac{\varepsilon_0 l}{\gamma_1 S} \qquad \sigma_B = \frac{\varepsilon_0 l}{S}\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right) \qquad \sigma_C = -\frac{\varepsilon_0 l}{\gamma_2 S}$$

3-16.

(1). By the definition of current.

$$N = \frac{I\Delta t}{e} = 3.12 \times 10^{11}$$

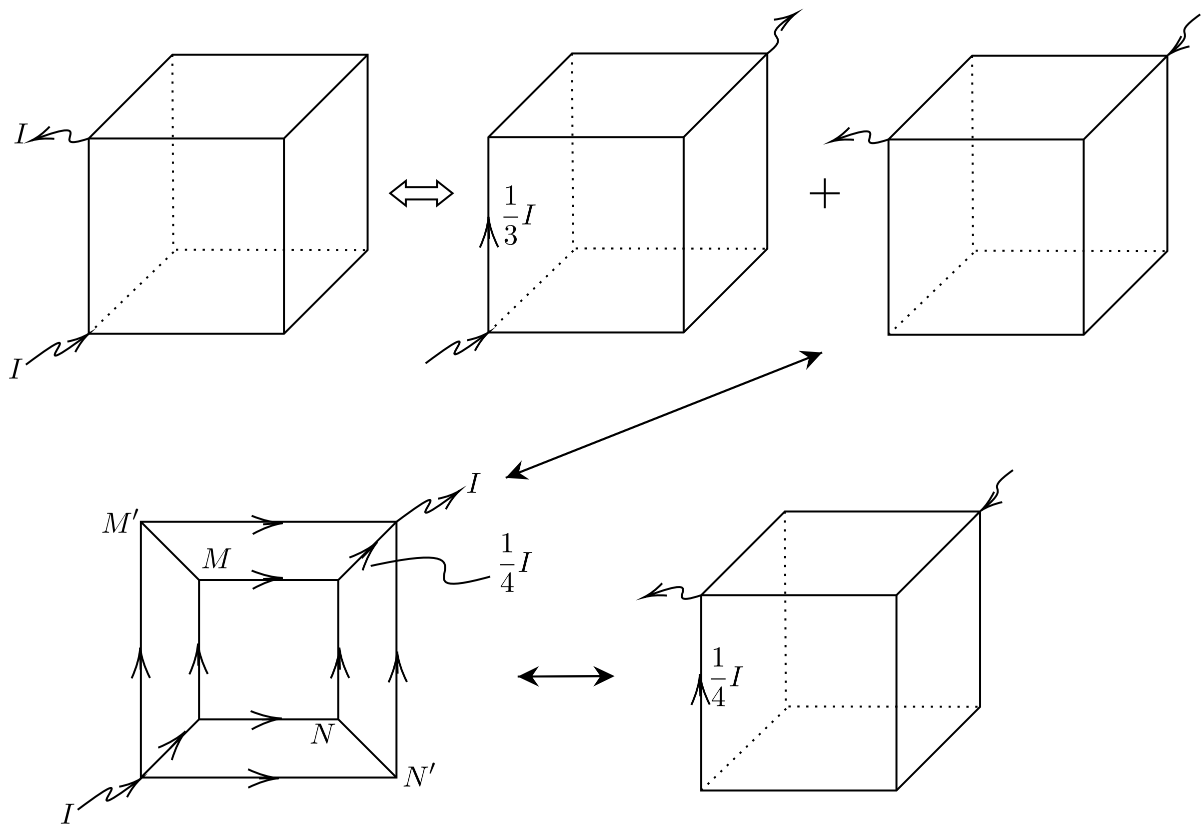
(2).

$$\bar{I} = I \times \frac{500 \times 0.1 \times 10^{-6}}{1} = 25\,\mu\text{A}$$

(3).

$$\bar{P} = \frac{IW_i}{e} = 1250\,\text{W}$$

3-20.



As shown in the figure, a current is given to point A and discharged from point B, and then decomposed into the sum of two parts according to the superposition principle.

For the first case, the system is symmetrical diagonally, so each adjacent edge bisecting the current from the point A, $I_1 = \dfrac{1}{3}I$.

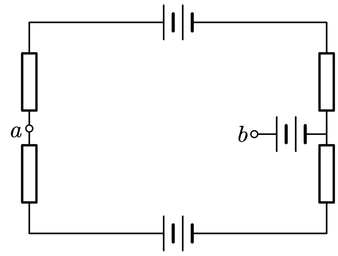
For the second case, there is no current between MM and NN, otherwise the inverse sign of the input current can be derived contradictory, $I_2 = \dfrac{2 \mathbin{//} 2}{(1+1+2 \mathbin{//} 2) + 2 \mathbin{//} 2} = \dfrac{1}{4}I$.

So it's easy to get that the current on line AB is $\dfrac{7}{12}I$.

$$R_{AB} = \frac{U_{AB}}{I_{AB}} = \frac{7}{12}R$$

3-21.

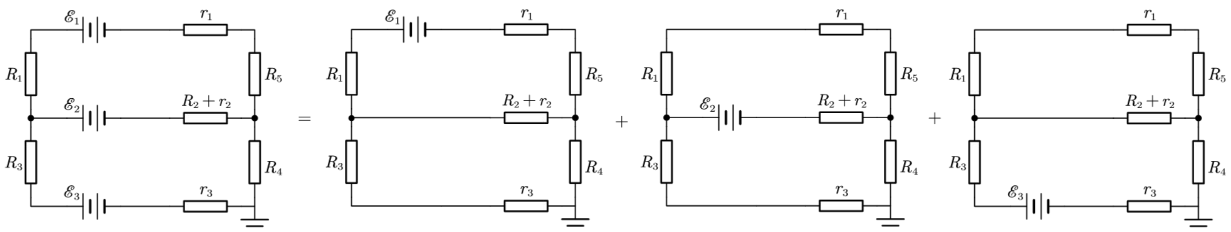
(1). No current flows through R_2 so we have a equivalent circuit.



And as a result:

$$U_{ab} = I(R_3 + r_3 + R_4) + \mathscr{E}_3 - \mathscr{E}_2 = \frac{\mathscr{E}_1 - \mathscr{E}_2}{R_1 + R_3 + R_4 + R_5 + r_1 + r_3}(R_3 + r_3 + R_4) + \mathscr{E}_3 - \mathscr{E}_2 = 1\,\mathrm{V}$$

(2). Based on the principle of independent superposition of currents:



$$
\begin{aligned}
I_1 &= \frac{\mathscr{E}_1}{(R_2+r_2)\,//\,(R_3+r_3+R_4)+(R_1+r_1+R_5)}\frac{R_3+r_3+R_4}{(R_3+r_3+R_4)+(R_2+r_2)}\\
I_2 &= -\frac{\mathscr{E}_2}{(R_2+r_2)+(R_1+r_1+R_5)\,//\,(R_3+r_3+R_4)}\\
I_3 &= \frac{\mathscr{E}_3}{(R_3+r_3+R_4)+(R_1+r_1+R_5)\,//\,(R_2+r_2)}\frac{R_1+r_1+R_5}{(R_1+r_1+R_5)+(R_2+r_2)}
\end{aligned}
$$

$$
\begin{aligned}
I &= I_1 + I_2 + I_3\\
&= \frac{\mathscr{E}_1}{(R_2+r_2)\,//\,(R_3+r_3+R_4)+(R_1+r_1+R_5)}\frac{R_3+r_3+R_4}{(R_3+r_3+R_4)+(R_2+r_2)}\\
&\quad - \frac{\mathscr{E}_2}{(R_2+r_2)+(R_1+r_1+R_5)\,//\,(R_3+r_3+R_4)}\\
&\quad + \frac{\mathscr{E}_3}{(R_3+r_3+R_4)+(R_1+r_1+R_5)\,//\,(R_2+r_2)}\frac{R_1+r_1+R_5}{(R_1+r_1+R_5)+(R_2+r_2)}\\
&= \frac{2}{13}\,\mathrm{A}
\end{aligned}
$$

3-26. Do as above we can have

$$\mathscr{E}_1 = 18\,\mathrm{V} \qquad \mathscr{E}_2 = 7\,\mathrm{V} \qquad Uab = 13\,\mathrm{V}$$

3-28.

(1). Consider using the full circuit Ohm's law after the capacitor is disconnected.

$$I = \frac{10\,\mathrm{V} + 20\,\mathrm{V} - 24\,\mathrm{V}}{2.0\,\Omega + 10\,\Omega + 3.0\,\Omega + 17\,\Omega + 18\,\Omega} = 0.12\,\mathrm{A}$$

So $U_A = -(18 + 2.0)\,\Omega \times 0.12\,\mathrm{A} + 10\,\mathrm{V} = 7.6\,\mathrm{V}$.

(2). Consider the node is U_x.

$$(U_x - U_A) \times 20\,\mathrm{\mu F} + (U_x - U_B) \times 20\,\mathrm{\mu F} + U_x \times 10\,\mathrm{\mu F} = 0$$

As a result.

$$Q = U_x \times 10\,\mathrm{\mu F} = 136\,\mathrm{\mu C}$$

That means the $U_x > 0$, so the plane is $-136\,\mathrm{\mu C}$.

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