ELECTROMAGNETISM ASSIGNMENT FOR THE EIGHTTH TIME

HAIXUAN LIN - 23307110267

ABSTRACT. Here is the electromagnetism assignment for the eightth time which is for the course given by professor Weitao Liu. In order to practise the expertise in scientific film of physics, students need to practise using LaTeXto composing their own work, even if this is only a ordinary homework.

MAIN TEXT

8-4. According to Kirchhoff's voltage equation

$$\xi = Ri\left(t\right) + L\frac{\mathrm{d}}{\mathrm{d}t}i\left(t\right) + \frac{1}{C}\int_0^t i\left(t\right)\mathrm{d}t$$

Take $t = 0^+$ into

$$\frac{\mathrm{d}}{\mathrm{d}t}i\left(0^+\right) = 0$$

In order to solve this differential equation we take Laplace Transform on it

$$0 = sRI\left(s\right) - Ri\left(0^+\right) + s^2LI\left(s\right) - sLi\left(0^+\right) - L\frac{\mathrm{d}}{\mathrm{d}t}i\left(0^+\right) + \frac{1}{C}I\left(s\right)$$

As a result

$$sI\left(s\right) = i\left(0^+\right)\frac{s^2 + \frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Take the inverse Laplace transform

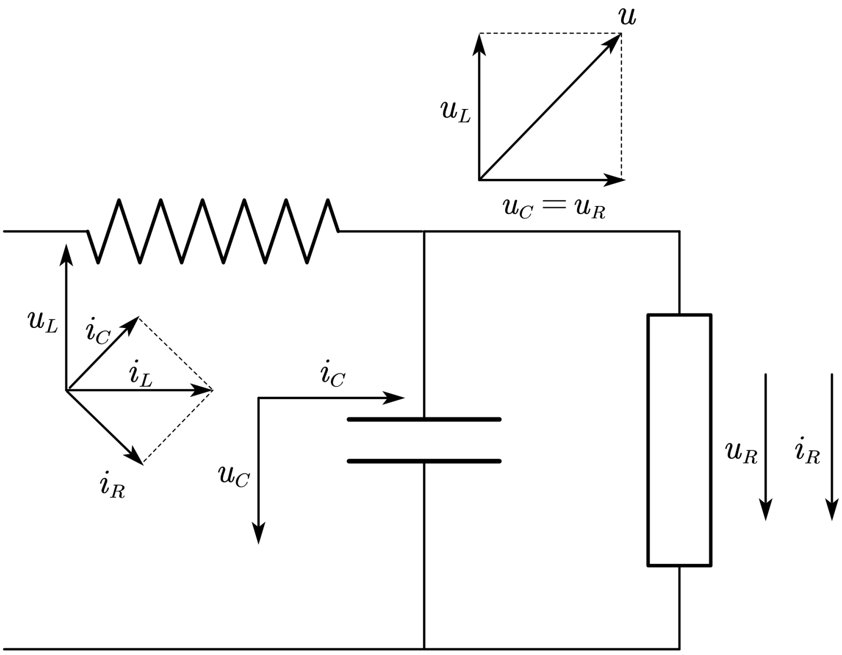
$$\frac{\mathrm{d}}{\mathrm{d}t}i\left(t\right) = -\frac{i\left(0^+\right)}{LC\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}e^{-\frac{R}{2L}t}\sin\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}t \qquad \left(t > 0^+\right)$$

Do a approxmation that we consider when

$$t = \frac{1}{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}\frac{\pi}{2}$$

We have

$$V_{\textbf{out max}} = nL\frac{i\left(0^+\right)}{LC\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}e^{-\frac{R}{2L}\frac{1}{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}\frac{\pi}{2}} = 2.1 \times 10^4\,\mathrm{V}$$



8-4

8-7.

(1).

$$\angle u_C - \angle i_R = 0$$

(2).

$$\angle i_C - \angle i_R = \frac{\pi}{2}$$

(3).

$$\angle u_R - \angle u_L = -\frac{3}{4}\pi$$

(4).

$$\angle u - \angle i = \frac{1}{4}\pi$$

8-9. According to the principle of linear superposition, the contributions of current source and voltage source are considered separately.

When there is no capacitance.

$$U_{ab \text{ direct}} = -E$$

$$U_{ab \text{ alternative}} = iR_r$$

As a result

$$U_{ab} = -E + iR_r = -5.5 \sim -6.0\,\mathrm{V}$$

When there is capacitance.

$$U_{ab\ \text{direct}} = -E$$

$$U_{ab\ \text{alternative}} = \frac{i}{\sqrt{(2\pi f C)^2 + \frac{1}{R_r^2}}}$$

$$U_{ab} = -E + \frac{i}{\sqrt{(2\pi f C)^2 + \frac{1}{R_r^2}}} = -5.85 \sim -6.0\,\mathrm{V}$$

8-12. We resolved the circuit using complex numbers:

$$\tilde{u}_1 = \frac{R}{R + R_L + \mathrm{i}\omega L}\tilde{u}$$

$$\tilde{u}_2 = \frac{R_1}{R_1 + R_2}\tilde{u}$$

$$\tilde{v} = \tilde{u}_1 - \tilde{u}_2$$

$$v = |\tilde{v}| = \sqrt{\left(\frac{R\left(R + R_L\right)}{\left(R + R_L\right)^2 + \omega^2 L^2} - \frac{R_1}{R_1 + R_2}\right)^2 + \left(\frac{R\omega L}{\left(R + R_L\right)^2 + \omega^2 L^2}\right)^2} u$$

Note $\mathrm{i} = \sqrt{-1}$. In order to minimize v, let

$$\frac{R\left(R + R_L\right)}{\left(R + R_L\right)^2 + \omega^2 L^2} - \frac{R_1}{R_1 + R_2} = 0$$

$$V = v_{\min} = \frac{R\omega L}{\left(R + R_L\right)^2 + \omega^2 L^2} u_{\text{effect}}$$

As a result

$$R_L = 14\,\Omega$$

$$L = 89.1\,\mathrm{mH}$$

Or we can use graph and geometry:

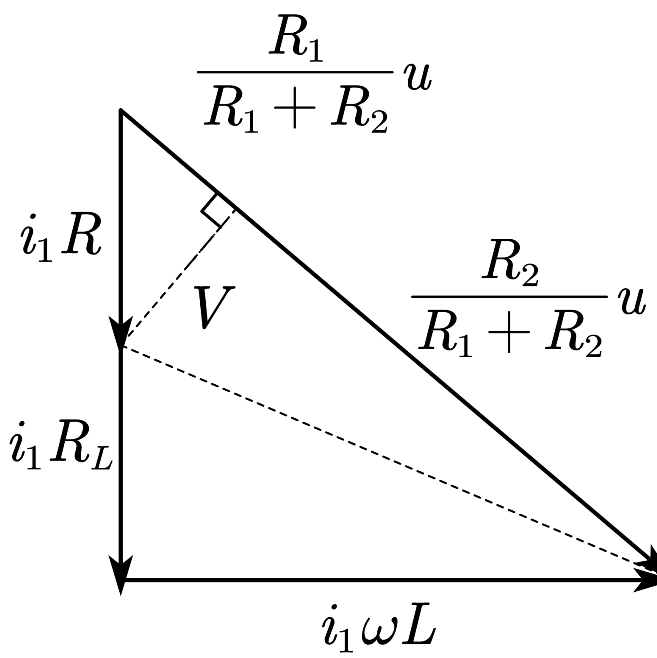
$$\left(\frac{R_1}{R_1 + R_2}u\right)^2 + V^2 = \left(i_1 R\right)^2$$

$$\left(i_1\left(R + R_L\right)\right)^2 + \left(i_1 \omega L\right)^2 = u^2$$

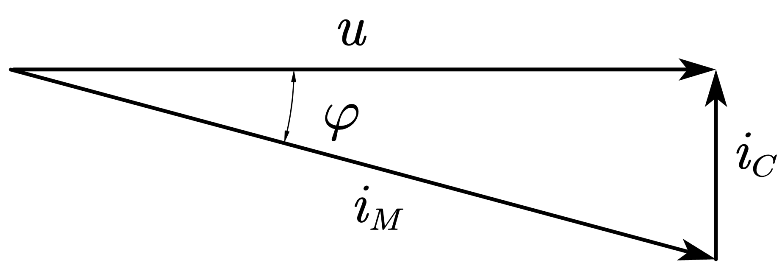
$$\left(i_1 R_L\right)^2 + \left(i_1 \omega L\right)^2 = V^2 + \left(\frac{R_2}{R_1 + R_2}u\right)^2$$

We can solve this too!

8-15.



8-12



8-15

(1). Use defination

$$P_W = ui\cos\varphi = 24.2\,\mathrm{W}$$

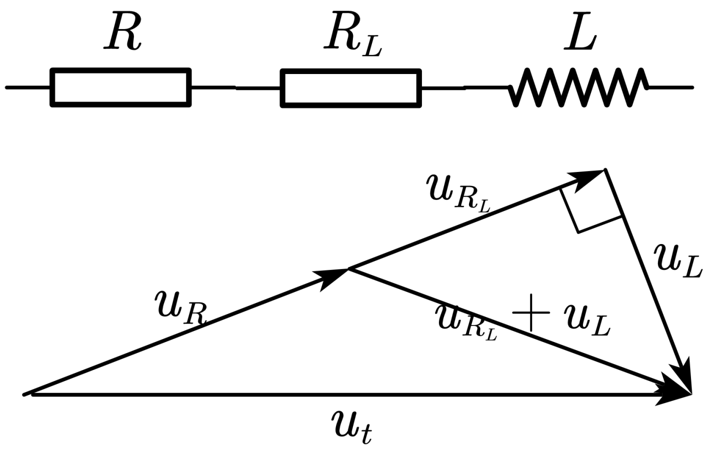
(2).

$$i_C = u\omega C = i_M \sin\varphi$$

$$C = \frac{i_M \sin\varphi}{u\omega} = 2.76\,\mu\mathrm{F}$$

8-16. Use defination

$$\cos\varphi = \frac{P_{\mathrm{in}}}{\sqrt{3}ui} = 0.69$$



8-18

so

$$P_{\text{var}} = P_{\text{in}} \tan\varphi = 5.76\,\text{kvar}$$

8-18.

$$(iR_L)^2 + (\omega L i)^2 = (iR)^2$$

$$(iR_L + iR)^2 + (\omega L i)^2 = u_t^2$$

So

$$R_L = 20\,\Omega$$

$$L = 34.6\,\text{mH}$$

Email address: 23307110267@m.fudan.edu.cn

Fudan University, Physics Department, China