ELECTROMAGNETISM ASSIGNMENT FOR THE SEVENTH TIME

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ABSTRACT. Here is the electromagnetism assignment for the seventh time which is for the course given by professor Weitao Liu. In order to practise the expertise in scientific film of physics, students need to practise using LaTeXto composing their own work, even if this is only a ordinary homework.

MAIN TEXT

6-21. We need to calculate the change in the electric field energy across space, and according to the conservation of energy, this is equal to the work done by the external force.

$$\Delta W_e = \int_{R_1}^{R_2} \left(\frac{1}{2}\boldsymbol{D}\cdot\boldsymbol{E} - \frac{1}{2}\varepsilon_0\boldsymbol{E}^2\right)\cdot 4\pi r^2 \mathrm{d}r$$

$$= \int_{R_1}^{R_2} \frac{1}{2}\varepsilon_0\boldsymbol{E}^2\left(\varepsilon_r - 1\right)\cdot 4\pi r^2 \mathrm{d}r$$

$$= \int_{R_1}^{R_2} \frac{1}{2}\varepsilon_0\frac{Q^2}{4\pi\varepsilon_0\varepsilon_r r^4}\left(\varepsilon_r - 1\right)\cdot 4\pi r^2 \mathrm{d}r$$

$$= \frac{Q^2\left(\varepsilon_r - 1\right)}{8\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

7-1. For a uniformly charged rigid body, its contribution to the magnetic moment is studied by taking its upper microelement.

$$\begin{aligned}
\mathrm{d}m_{es} &= \frac{\mathrm{d}Q}{T}\pi r^2 = \frac{\omega}{2\pi}\pi r^2\mathrm{d}Q = \frac{\omega}{2\pi}\frac{\rho_e}{\rho}\pi r^2\mathrm{d}m = \frac{\omega}{2\pi}\frac{Q}{m}\pi r^2\mathrm{d}m \\
&= \frac{Q}{2m}\cdot r^2\mathrm{d}m\cdot\omega = \frac{Q}{2m}\cdot\mathrm{d}J\cdot\omega \\
m_{es} &= \frac{Q}{2m}J\omega = \frac{Q}{2m}L_s
\end{aligned}$$

This is different from what quantum mechanics tells us, which is $m_{es} = \dfrac{Q}{m}L_s$.

7-2. The Lorente force give the contribute:

$$qv'B + m\frac{v'^2}{r} = m\frac{v^2}{r}$$

Base on the opinion that r is a constant. So

$$\Delta m_{el} = \frac{1}{2} q \Delta v r = 3.94 \times 10^{-29}\,\mathrm{A} \cdot \mathrm{m}^2$$

$$\frac{\Delta m_{el}}{m_{el}} = 4.2 \times 10^{-6}$$

7-7.

(1). According to Ohm's law in differential form

$$J = \frac{I}{\pi R^2} = \gamma E$$

So $E = \dfrac{I}{\pi R^2 \gamma}$.
According to Ampere's loop theorem modified by Maxwell

$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = \oint \frac{\boldsymbol{B}}{\mu_0 \mu_r} \cdot \mathrm{d}\boldsymbol{l} = \frac{B}{\mu_0 \mu_r} \cdot 2\pi r = \sum I_c = I \frac{r^2}{R^2}$$

We have $B = \dfrac{\mu_0 \mu_r I}{2\pi R^2} r$.

(2). According to the definition of the magnetization vector

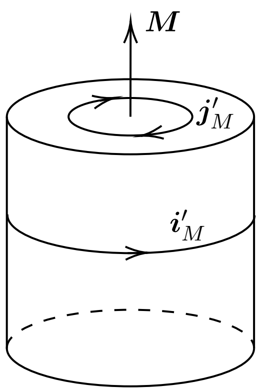
$$\boldsymbol{M} = \frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{H} = \frac{1}{\mu_0}\left(1 - \frac{1}{\mu_r}\right)\boldsymbol{B} = \frac{1}{\mu_0}\left(1 - \frac{1}{\mu_r}\right)\frac{\mu_0 \mu_r I}{2\pi R^2} r \hat{\boldsymbol{z}}$$

According to vector $\boldsymbol{M}$ we can calculate magnetizing current

$$\boldsymbol{j}'_M = \nabla \times \boldsymbol{M} = \frac{1}{\mu_0}\left(1 - \frac{1}{\mu_r}\right)\frac{\mu_0 \mu_r I}{2\pi R^2} \cdot \frac{1}{r} \begin{vmatrix} \hat{\boldsymbol{r}} & r\hat{\boldsymbol{\theta}} & \hat{\boldsymbol{z}} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial z} \\ 0 & 0 & r \end{vmatrix} = -\frac{(\mu_r - 1) I}{2\pi R^2} \hat{\boldsymbol{\theta}}$$

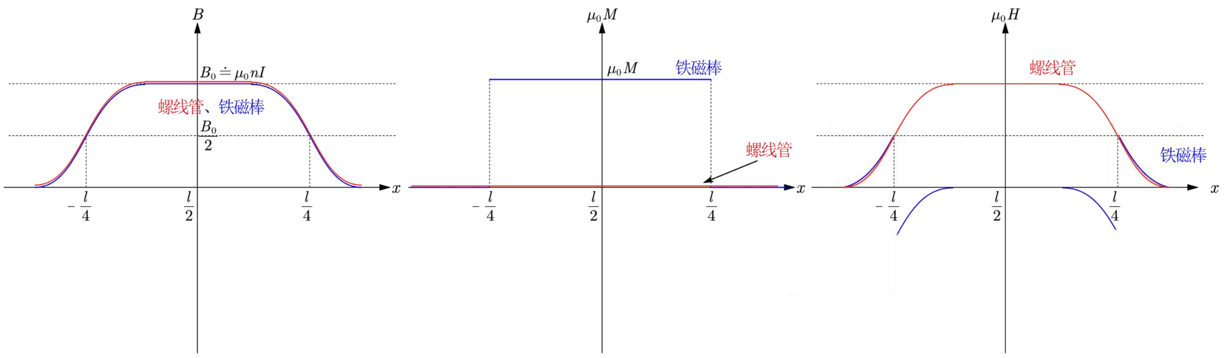
$$\boldsymbol{i}'_M = \boldsymbol{M} \times \hat{\boldsymbol{n}} = M\hat{\boldsymbol{\theta}} = \frac{(\mu_r - 1) I}{2\pi R} \hat{\boldsymbol{\theta}}$$

The direction is about the picture below:



7-9. The graph is as follow:

7-14.



Graph for 7-9

(1). Let the Angle between the magnetic induction vector and the normal line in the medium be β according to the magnetic induction refraction formula:

$$\frac{\mu_r}{1}=\frac{\tan\beta}{\tan\theta}$$

And the normal component of the magnetic induction vector is always continuous:

$$B'\cos\beta=B\cos\theta$$

As a result:

$$\beta=\arctan\mu_r\tan\theta$$

$$B'=\frac{\cos\theta}{\cos\arctan\mu_r\tan\theta}B=\sqrt{\mu_r^2\sin^2\theta+\cos^2\theta B}$$

(2). According to the definition of magnetization:

$$M'_\tau=\frac{B'\sin\beta}{\mu_0}\left(1-\frac{1}{\mu_r}\right)$$

With it property:

$$i'_M=M'_\tau=\frac{1}{\mu_0}\left(\mu_r-1\right)B\sin\theta$$

8-2. So

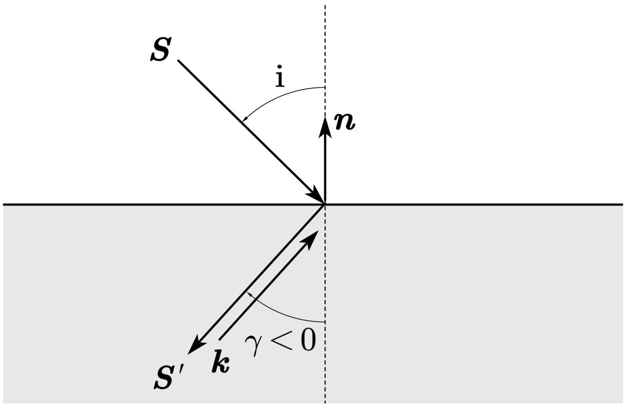
$$X_C=X_L$$

$$\frac{1}{\omega C}=\omega L$$

As a result $\omega=\dfrac{1}{\sqrt{LC}}$.

$$f=2\pi\omega=1678\,\mathrm{Hz}$$

$$X_C = X_L = 31.6\,\Omega$$



Left-Handed Metamateria

Left-Handed Metamateria.

Proof the $\gamma < 0$. By the refraction formula of electric field intensity, we can assert that the $\gamma < 0$.

$$\frac{\tan \mathrm{i}}{\tan \gamma} = \frac{1}{\varepsilon_r}$$

Proof $\boldsymbol{S'} // -\boldsymbol{k}$. In the material we have

$$\boldsymbol{S'} = \boldsymbol{E} \times \boldsymbol{H} = \boldsymbol{E} \times \left(\frac{1}{\mu_0 \mu_r} \boldsymbol{B} \right) = \boldsymbol{E} \times \left(\frac{1}{\mu_0 \mu_r \omega} \boldsymbol{k} \times \boldsymbol{E} \right) = \frac{1}{\mu_0 \mu_r \omega} E^2 \boldsymbol{k}$$

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