Physics Homework

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Abstract

In order to improve my computer and English skills, please allow me to complete this physics homework in English context with LaTeX, so as to improve my professional level. Sorry for the inconvenience!

1 Question 4-2

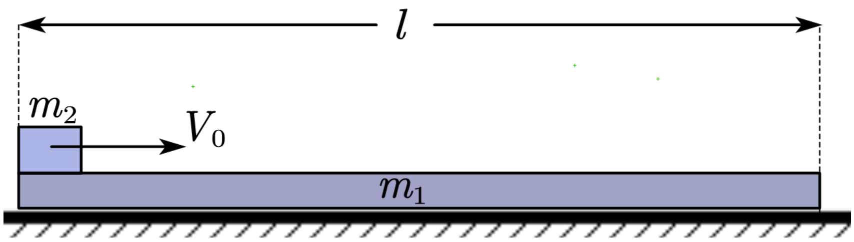

Figure for 4-2

1.1

For plank and small piece, by Newton's second law:

$$m_1 a_1 = \mu m_2 g \tag{1}$$

$$m_2 a_2 = \mu m_2 g. \tag{2}$$

Using the plank as the frame of reference, according to kinematic formula, we have:

$$v_0{}^2 = 2\left(a_1 + a_2\right) l \tag{3}$$

Hence：

$$\mu = \frac{m_1}{m_1 + m_2} \frac{v_0{}^2}{2gl}$$

1.2

We can calculate the length of time they are in relative motion:

$$t = \frac{v_0}{a_1 + a_2} \tag{4}$$

The displacement of plank is

$$x_1 = \frac{1}{2} a_1 t^2 \tag{5}$$

That is

$$x_1 = \frac{m_2}{m_1 + m_2} l$$

2 Question 4-7

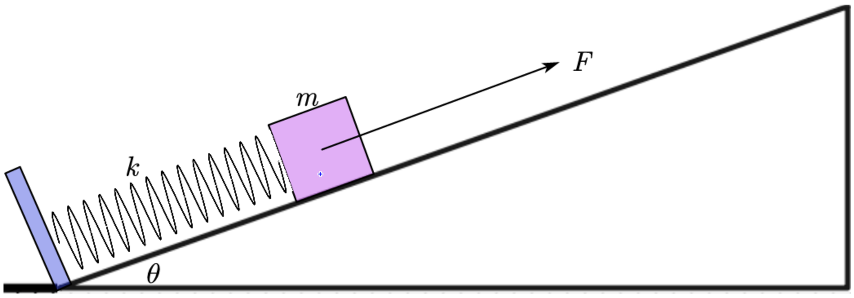

Figure for 4-7

2.1

If the process of m from static motion to the highest point is regarded as a harmonic motion of half a period, the position of the origin of motion relative to the spring origin obtained by considering the friction force, gravity slope component and tension is:

$$k\Delta x = F - mg\sin\theta - \mu mg\cos\theta \tag{6}$$

So the W can calculate by the definition of work:

$$W = Fl = F \cdot 2\Delta x \tag{7}$$

Hence:

$$W = \frac{2F}{k}\left(F - mg\cos\theta - \mu mg\sin\theta\right)$$

2.2

Obviously m get v_m when it is at the origin of motion. According to the Kinetic energy theorem:

$$\frac{1}{2}m{v_m}^2 = \frac{1}{2}W - mg\sin\theta\Delta x - \mu mg\cos\theta\Delta x - \frac{1}{2}k\Delta x^2 \tag{8}$$

Hence:

$$v_m = \frac{F - mg\sin\theta - \mu mg\cos\theta}{\sqrt{mk}}$$

3 Question 4-10

Given a probe particle m on the surface of the moon, according to the law of gravitation:

$$F_G = \frac{Gm_m m}{R^2} \tag{9}$$

Hence:

$$g_m = \frac{F_G}{m} = \frac{Gm_m}{R^2} = 1.7m/s^2$$

Escape velocity allows particles to travel from the surface of the moon to infinity.

$$\frac{1}{2}m{v_2}^2 = \frac{Gm_m m}{R_m} \tag{10}$$

Hence:

$$v_2 = \sqrt{\frac{2Gm_m}{R_m}} = 2.4 \times 10^3 m/s$$

4 Question 4-16

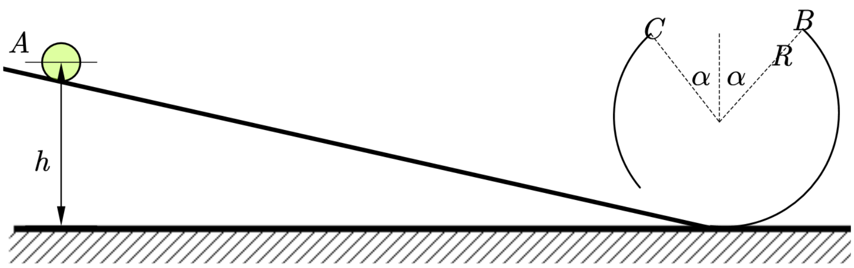

Figure for 4-16

The motion from point B to point C is given by the kinematic formula:

$$2R\sin\alpha = vt\cos\alpha \tag{11}$$

$$gt = 2v\sin\alpha \tag{12}$$

From point A to point B, according to the conservation of mechanical energy:

$$\frac{1}{2}mv^2 = mgh - mgR\left(1 + \cos\alpha\right) \tag{13}$$

Hence:

$$h = R(1 + cos\alpha + \frac{1}{2cos\alpha})$$

5 Question 4-17

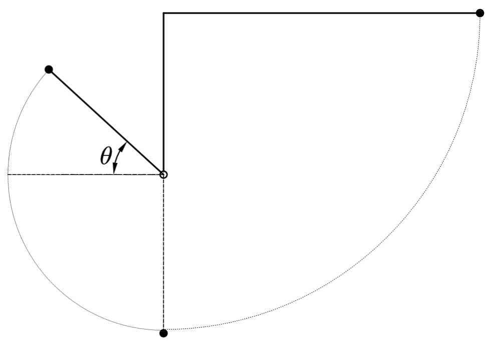

Figure 1 for 4-17

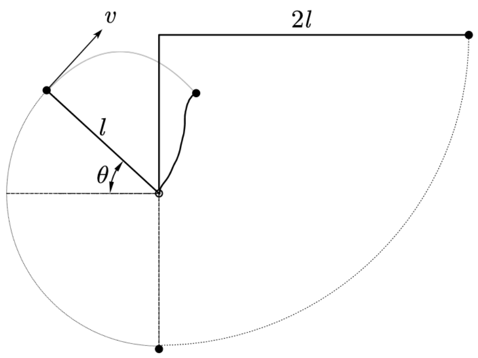

Figure 2 for 4-17

As shown in Figure 2, the ball will fall out of the orbit at the position shown in Figure 2, at that moment, the centripetal force is provided entirely by the component of gravity.

$$\frac{1}{2}mv^2 = mgl\left(1 - \sin\alpha\right) \tag{14}$$

$$m\frac{v^2}{l} = mg\sin\alpha \tag{15}$$

Hence:

$$H_{max} = l(1 + sin\alpha) + \frac{(vcos\alpha)^2}{2g} = \frac{50}{27}l$$

6 Question 4-19

6.1

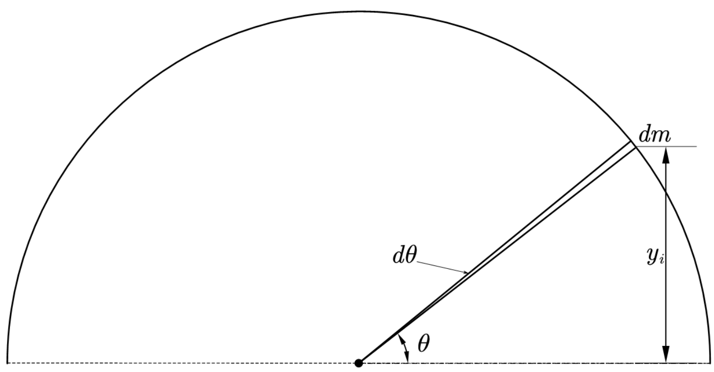

Figure for 4-19

According to the definition of the center of mass:

$$y_C = \frac{1}{\rho_l \pi R}\int_0^{\pi} R\sin\theta \cdot \rho_l R d\theta \tag{16}$$

The kinetic energy theorem tells us:

$$\frac{1}{2}mv^2 = mg(y_C + \frac{1}{2}\pi R) \tag{17}$$

Hence:

$$v = \sqrt{2gR(\frac{\pi}{2} + \frac{2}{\pi})}$$

6.2

Follow the example above:

$$\frac{1}{2}mv^2 = \frac{1}{3}mg\left(\frac{1}{\frac{1}{3}\rho_l \pi R}\int_{\frac{2}{3}\pi}^{\pi} R\sin\theta \cdot \rho_l R d\theta + \frac{1}{6}\pi R\right) \tag{18}$$

Hence:

$$v = \sqrt{\frac{gR}{3}(\frac{\pi}{3} + \frac{3}{\pi})}$$

Consider the most general case, the length of the chain that slides out $x = \theta R$.

$$\frac{1}{2}mv^2 = \frac{\theta}{\pi}mg\left(\frac{1}{\rho_l \theta R}\int_{\pi-\theta}^{\pi} R\sin\theta \cdot \rho_l R d\theta + \frac{\theta}{2}\pi R\right) \tag{19}$$

Hence:

$$v = \sqrt{2gR\left(\frac{1-\cos\theta}{\pi} + \frac{\theta^2}{2\pi}\right)} \tag{20}$$

Notice that:

$$R \cdot \frac{d}{dt}\theta = v \tag{21}$$

Hence:

$$a = \frac{d}{dt}v = g\left(\frac{\sin\theta}{\pi} + \frac{\theta}{\pi}\right) \tag{22}$$

Let $\theta = \frac{\pi}{3}$, we get:

$$a = g\left(\frac{\sqrt{3}}{2\pi} + \frac{1}{3}\right)$$

7 Question 4-21

7.1

According to conservation of momentum and conservation of mechanical energy.

$$m_1v_1 = m_2v_2 \tag{23}$$

$$m_1gl = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 \tag{24}$$

Hence:

$$v_2 = \sqrt{\frac{{m_1}^2}{(m_1 + m_2)\, m_2}2gl}$$

7.2

Follow the example above:

$$m_1gl\cos 60^o = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 \tag{25}$$

$$m_1v_1\cos 60^o = m_2v_2 \tag{26}$$

Hence:

$$v_2 = \sqrt{\frac{gl}{3{m_1}^2 + 7m_1m_2 + 4{m_2}^2}}$$

7.3

Obviously the answer is YES.

$$W = \frac{1}{2}mv_1^2 - m_1gl = -\frac{m_1^2}{m_1 + m_2}gl < 0$$

8 Question 4-28

When m_2 happens to leave the ground:

$$k\Delta x = m_2g \tag{27}$$

After m and m_1 are adhered to, the distance between the center of motion of their simple harmonic motion and the original length of the spring is:

$$k\Delta x_0 = (m + m_1)g \tag{28}$$

The energy of the new spring oscillator meets:

$$\frac{1}{2}k\left(\Delta x+\Delta x_0\right)^2=\frac{1}{2}\left(m+m_1\right)v^2+\frac{1}{2}k{\Delta x_1}^2 \tag{29}$$

And:

$$k\Delta x_1=m_1 g \tag{30}$$

For a completely inelastic collision:

$$\frac{1}{2}\left(m+m_1\right)v^2=\frac{1}{2}m{v_0}^2-\frac{1}{2}\mu{v_0}^2 \tag{31}$$

And μ is called reduced mass:

$$\mu=\frac{mm_1}{m+m_1} \tag{32}$$

The kinetic energy theorem of the falling process of m states:

$$\frac{1}{2}m{v_0}^2=mgh \tag{33}$$

Hence:

$$h=\frac{g}{2m^2k}\left(m+m_1\right)\left(m_1+m_2\right)\left(2m+m_1+m_2\right)$$