Physics Homework

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Abstract

In order to improve my computer and English skills, please allow me to complete this physics homework in English context with LaTeX, so as to improve my professional level. Sorry for the inconvenience!

Contents

Question 10-3

Calculate time in the Earth reference frame.

$$t = \frac{S}{v} \tag{1}$$

According to the clock slowing effect, the time in the spacecraft's reference frame is.

$$t' = \frac{S}{v}\sqrt{1-\frac{v^2}{c^2}} \tag{2}$$

So.

$$\Delta t = t - t' \tag{3}$$

The solution is.

$$v = \frac{\frac{2\Delta t}{S}}{\frac{\Delta t^2}{S^2}+\frac{1}{c^2}} \approx 0.198c \approx 5.932\times 10^7\ \mathrm{m/s} \tag{4}$$

Note that here we consider $c = 299792458\ \mathrm{m/s}$.

Question 10-7

(1)

As far as the people in the carriage are concerned, the length of the car is constant.

$$\Delta t_1' = \frac{L}{c}$$

$$\Delta t' = \frac{2L}{c}$$

(2)

According to the scaling effect, the observer in the ground will measure the car's length as below.

$$L' = L\sqrt{1-\frac{v^2}{c^2}} \tag{5}$$

The photon is chasing the right end of the car.

$$L' + v\Delta t_1 = c\Delta t_1 \tag{6}$$

The solution is.

$$\Delta t_1 = \frac{L}{c}\sqrt{\frac{c+v}{c-v}}$$

According to the clock slowing effect.

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2L}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Question 10-9

(1)

According to symmetry analysis, the two observers would have the opposite situation. B 棒上的观察者会看到两棒右端先重合，再左端重合.

(2)

According to the scaling effect.

$$l' = l_0\sqrt{1-\frac{v^2}{c^2}} \tag{7}$$

A simple kinematic formula is obtained.

$$l_0 - l' = v\Delta t \tag{8}$$

We solve that.

$$v = \frac{\frac{2\Delta t}{l_0}}{\frac{\Delta t^2}{l_0^2}+\frac{1}{c^2}}$$

(3)

According to symmetry analysis, the two observers would have the opposite situation. 两个端点同时重合.

Question 10-12

According to the clock slowing effect.

$$v = \frac{1}{\sqrt{\frac{t_0^2}{S^2}+\frac{1}{c^2}}} \approx 0.998c \approx 2.997\times 10^8\ \mathrm{m/s} \tag{9}$$

Question 10-17

(1)

The length of the projection of the rod in the y direction is constant, define it as H. According to the Lorentz transformation.

$$v_{S'}^{(O)} = \frac{v_{S'}^{(S)} - v_{S'}^{(S)}}{1 - \frac{v_{S'}^{(S)} v_{S'}^{(S)}}{c^2}} = \frac{0.6c - v}{1 - \frac{0.6v}{c}} \tag{10}$$

Based on simple geometry and scaling effect.

$$L_x^{(S)} = H \cot \theta^{(S)} = H \cot 45^\circ = L_x^{(O)} \sqrt{1 - \left[\frac{v_S^{(O)}}{c}\right]^2} \tag{11}$$

$$L_x^{(S')} = H \cot \theta^{(S')} = H \cot 35^\circ = L_x^{(O)} \sqrt{1 - \left[\frac{v_{S'}^{(O)}}{c}\right]^2} \tag{12}$$

And we know $v_{S'}^{(O)} = v$, so we get.

$$\left(\frac{\cot 45^\circ}{\cot 35^\circ}\right)^2 = \frac{1 - \left(\frac{v}{c}\right)^2}{1 - \left(\frac{0.6 - \frac{v}{c}}{1 - \frac{0.6v}{c}}\right)^2} \tag{13}$$

This is a quartic equation with one variable, the analytical solution is very complex, but the numerical solution can be calculated.

$$v_O^{(S)} = v \approx 0.73c$$

(2)

Solve the euqations above we can get.

$$v_O^{(S')} \approx 0.24c$$

(3)

Do a tansformation of equation above.

$$L_x^{(S)} = H \cos \theta^{(S)} = L_x^{(O)} \sqrt{1 - \frac{v^2}{c^2}} = H \cos \theta^{(O)} \sqrt{1 - \frac{v^2}{c^2}} \tag{14}$$

As a result.

$$\theta^{(O)} = \arctan\left(\tan\theta^{(O)}\sqrt{1-\frac{v^2}{c^2}}\right) \approx 34.2^\circ$$

Question 10-18

(1)

By symmetry analysis.

$$v_B^{(A)} = v_A^{(B)} = 0.7c$$

It is easily obtained by the Lorentz transformation.

$$v_C^{(A)} = \frac{v_C^{(B)} - v_A^{(B)}}{1 - \dfrac{v_C^{(B)} v_A^{(B)}}{c^2}} = \frac{140}{149}c \approx 0.94c$$

(2)

It is easily obtained by the Lorentz transformation.

$$v_A^{(O)} = \frac{v_A^{(B)} + v_B^{(O)}}{1 + \dfrac{v_A^{(B)} v_B^{(O)}}{c^2}} = \frac{20}{37}c \approx 0.54c$$

$$v_C^{(O)} = \frac{v_C^{(B)} + v_B^{(O)}}{1 + \dfrac{v_C^{(B)} v_B^{(O)}}{c^2}} = \frac{160}{163}c \approx 0.98c$$

Question 10-27

According to the mass-energy equation.

$$E_{\mathrm{k}} = mc^2 - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} - 1\right) = \frac{1}{2}m_0c^2 \tag{15}$$

So.

$$v = \frac{\sqrt{5}}{3}c \approx 2.2 \times 10^8 \text{ m/s}$$

Question 10-30

According to the law of conservation of momentum and conservation of energy.

$$p_0 = Mv_M \tag{16}$$

$$m_0c^2 + 4m_0c^2 = Mv_M^2 \tag{17}$$

And we know that.

$$4m_0c^2 = \frac{m_0c^2}{\sqrt{1 - \frac{v_m^2}{c^2}}} \tag{18}$$

$$p_0 = mv_m = \frac{m_0v}{\sqrt{1 - \frac{v_m^2}{c^2}}} \tag{19}$$

According to quality and efficiency.

$$M = \frac{M_0}{\sqrt{1 - \frac{v_M^2}{c^2}}} \tag{20}$$

The solution is.

$$M_0 = \sqrt{10}m_0 \approx 3.16m_0$$

Question 10-31

(1)

According to the Lorentz transformation.

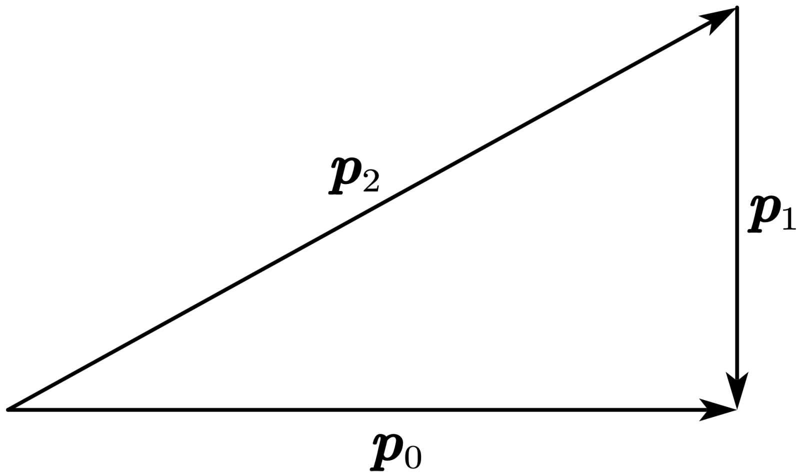
$$p_0 = m'v' = \frac{m_0'}{\sqrt{1 - \frac{0.8^2c^2}{c^2}}}0.8c = \frac{4}{3}m_0'c \tag{21}$$

$$p_1 = m_1v_1 = \frac{m_0}{\sqrt{1 - \frac{0.6^2c^2}{c^2}}}0.6c = \frac{3}{4}m_0c \tag{22}$$

$$p_2 = m_2v_2 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}v \tag{23}$$

According to the Pythagorean theorem.

$$p_2^2 = p_0^2 + p_1^2 \tag{24}$$



At the same time, the energy is also conservative.

$$E_0 = E_1 + E_2 \tag{25}$$

The Lorentz transformation tells us.

$$E_0 = \frac{m_0' c^2}{\sqrt{1 - \frac{0.8^2 c^2}{c^2}}} = \frac{5}{3} m_0' c^2 \tag{26}$$

$$E_1 = \frac{m_0 c^2}{\sqrt{1 - \frac{0.6^2 c^2}{c^2}}} = \frac{5}{4} m_0 c^2 \tag{27}$$

$$E_2 = \sqrt{m_0^2 c^4 + p_2^2 c^2} \tag{28}$$

We solve this.

$$v = \sqrt{\frac{40729}{42025}} c \approx 0.984c$$

$$\theta = \arctan \frac{p_1}{p_0} = \arctan \frac{27}{200} = \arctan 0.135$$

(2)

We solve it above.

$$\frac{m_0}{m_0'} = \frac{6}{25} \approx 0.24$$

Question 10-32

The moment is conservative.

$$p_\gamma = p_{\text{ship}} = \frac{m_0'}{\sqrt{1 - \frac{0.6^2 c^2}{c^2}}} 0.6c \tag{29}$$

The energy of photon is.

$$E_\gamma = p_\gamma c \tag{30}$$

The energy of moving ship is.

$$E_{\text{ship}} = \frac{m_0'}{\sqrt{1 - \frac{0.6^2 c^2}{c^2}}} c^2 \tag{31}$$

The energy is conservative.

$$E_\gamma + E_{\text{ship}} = m_0 c^2 \tag{32}$$

As a result.

$$\frac{m_0'}{m_0} = \frac{1}{2}$$