Physics Homework

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Abstract

In order to improve my computer and English skills, please allow me to complete this physics homework in English context with LaTeX, so as to improve my professional level. Sorry for the inconvenience!

1 Question 6-3

Define M as the mass of the complete big disc. So

$$\frac{m}{1} = \frac{M}{1 - 3 \times \frac{1}{9}} \tag{1}$$

The moment of inertia of the complete disk is

$$J_0 = \frac{1}{2} M R^2 \tag{2}$$

According to the parallel-axis theorem, the moment of inertia of each cut small disk is

$$J_i = \frac{1}{2}\frac{M}{9}\left(\frac{R}{3}\right)^2 + \frac{M}{9}\left(\frac{R}{2}\right)^2 \tag{3}$$

As a result

$$J = J_0 - 3J_i = \frac{43}{72} m R^2$$

2 Question 6-8

(1)

Take any thin ring element in the upper disk

$$\mathrm{d}m = \frac{3m}{\pi R^2} \cdot 2\pi r \mathrm{d}r \tag{4}$$

Do force analysis of this element

$$\mathrm{d}f = \mu \mathrm{d}m g \tag{5}$$

$$\mathrm{d}M_f = r\mathrm{d}f \tag{6}$$

So

$$M = \int_0^R 6\mu m g \frac{r^2}{R^2} \mathrm{d}r = 2\mu m g R \tag{7}$$

The discs both above and below has own moment of inertia

$$J_1 = \frac{1}{2} 3 m R^2 \tag{8}$$

$$J_2 = \frac{1}{2} m R^2 \tag{9}$$

According to rotation law

$$\beta_1 = \frac{M}{J_1} \tag{10}$$

$$\beta_2 = \frac{M}{J_2} \tag{11}$$

As a result

$$\Delta t = \frac{\omega_0}{\beta_1 + \beta_2} = \frac{3}{16}\frac{R\omega_0}{\mu g}$$

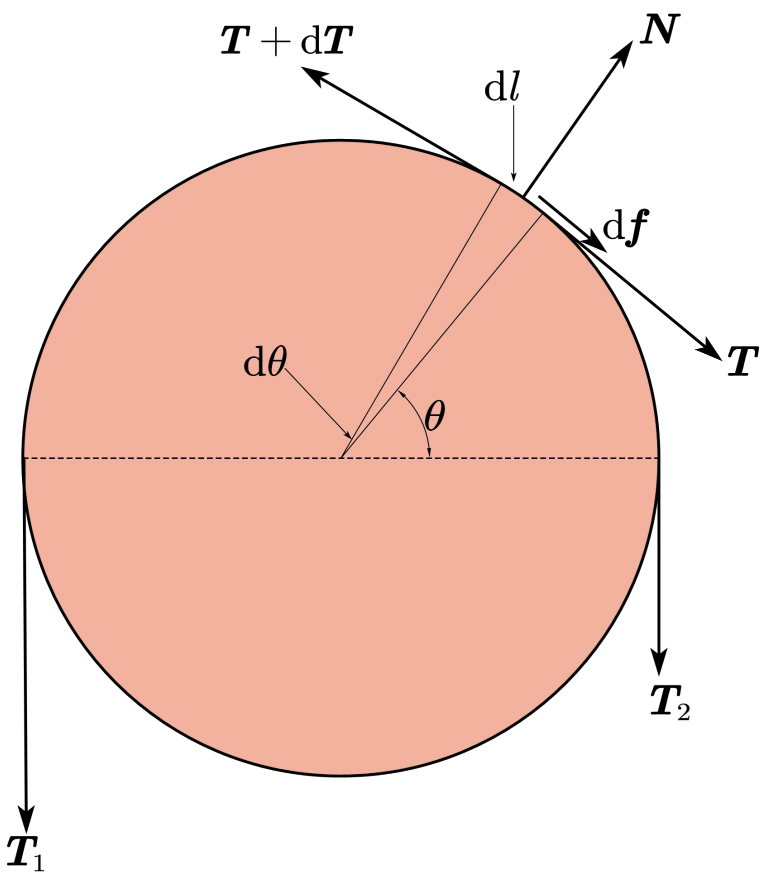
(2)

According to kinematic formula of angle

$$\omega = \omega_0 - \beta_1 \Delta t = \frac{3}{4}\omega_0$$

3 Question 6-9

(1)



Take a tiny element of the rope above the Atwood machine to analyse.

$$\mathrm{d}N = T\mathrm{d}\theta \tag{12}$$

$$\mathrm{d}f = \mu\mathrm{d}N = \mu T\mathrm{d}\theta \tag{13}$$

Combine these two equations and we get a differential equation

$$\frac{\mathrm{d}T}{T} = \mu\mathrm{d}\theta \tag{14}$$

The solution is that

$$T = T_2\mathrm{e}^{\mu\theta} \tag{15}$$

Further have

$$f = \int_0^\pi \mu T_2\mathrm{e}^{\mu\theta}\mathrm{d}\theta = T_2(\mathrm{e}^{\mu\pi} - 1) \tag{16}$$

The system consists of m_1 and m_2 and the rope has the same acceleration with the part of only m_2. So according to the Newton's second law

$$\frac{(m_1 - m_2)\,g - f}{m_1 + m_2} = \frac{T_2}{m_2} - g = a \tag{17}$$

So the solution is that

$$a = \frac{m_1 - m_2\mathrm{e}^{\mu\pi}}{m_1 + m_2\mathrm{e}^{\mu\pi}}g$$

(2)

The moment of inertia of the Atwood machine is

$$J = \frac{1}{2}mR^2 \tag{18}$$

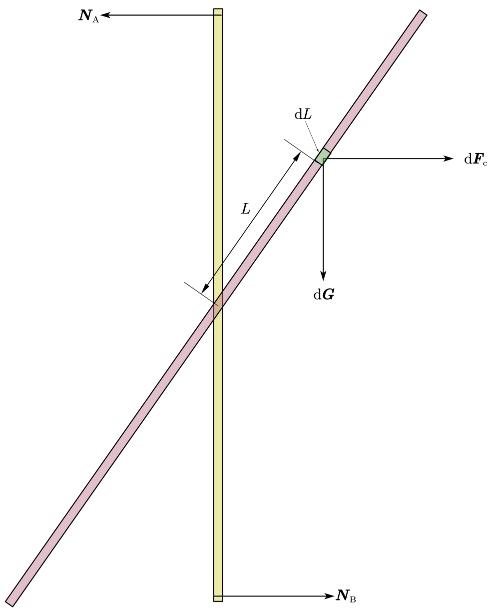
So according to the law of rotation

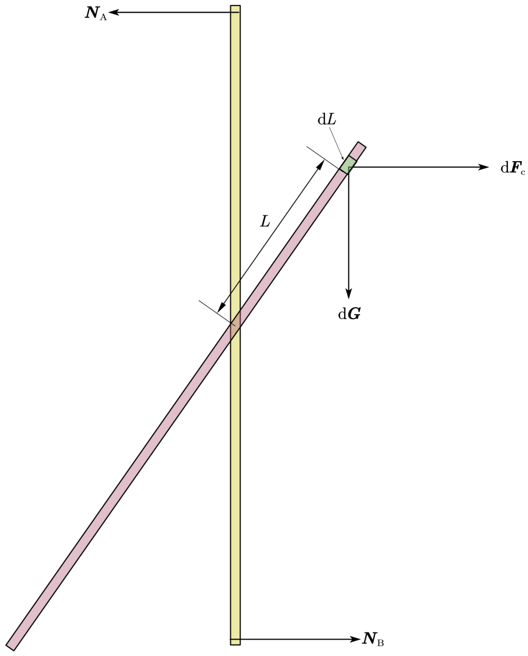
$$\alpha = \frac{Rf}{J} = \frac{4m_1m_2g(\mathrm{e}^{\mu\pi} - 1)}{mR(m_1 + m_2\mathrm{e}^{\mu\pi})}$$

4 Question 6-12

(1)

As the figure below, the moment of force $\boldsymbol{N}_{\mathrm{A}}$ and $\boldsymbol{F}_{\mathrm{c}}$ of the above half are equivalent.





$$N_{\mathrm{A}}R = \int_0^{\frac{1}{2}l} \frac{\mathrm{d}L}{l} m\omega^2 L\cos\theta L\sin\theta \tag{19}$$

And according to the symmetry, $\boldsymbol{N}_{\mathrm{A}} = -\boldsymbol{N}_{\mathrm{B}}$, so

$$N_{\mathrm{A}} = N_{\mathrm{B}} = \frac{1}{24R} m\omega^2 l^2 \sin\theta\cos\theta$$

(2)

The basic idea doesn't change, but because of the broken symmetry, we have to consider the torque of gravity.

$$\mathrm{d}G = \frac{\mathrm{d}L}{l} mg \tag{20}$$

$$\mathrm{d}M_{\mathrm{G}} = L\sin\theta \cdot \mathrm{d}G = L\sin\theta \cdot \frac{\mathrm{d}L}{l} mg \tag{21}$$

Get by integrating.

$$M_{\mathrm{G}} = \int \mathrm{d}M_{\mathrm{G}} = \frac{mg\sin\theta}{l} \int_{\frac{1}{4}l}^{\frac{1}{2}l} L\mathrm{d}L = \frac{3}{32} mg\sin\theta \tag{22}$$

Now consider centrifugal force.

$$\mathrm{d}F_{\mathrm{c}} = \frac{\mathrm{d}L}{l} m\omega^2 L\sin\theta \mathrm{d}M_{\mathrm{c}} = L\cos\theta \mathrm{d}F_{\mathrm{c}} = \frac{\mathrm{d}L}{l} m\omega^2 L^2 \sin\theta\cos\theta \tag{23}$$

Get by integrating.

$$F_{\mathrm{c}}=\int_{\frac{1}{4}l}^{\frac{1}{2}l}\frac{\mathrm{d}L}{l}m\omega^2L\sin\theta=\frac{3}{32}m\omega^2L\sin\theta \tag{24}$$

$$M_{\mathrm{c}}=\int_{\frac{1}{4}l}^{\frac{1}{2}l}\frac{\mathrm{d}L}{l}m\omega^2L^2\sin\theta\cos\theta=\frac{3}{64}m\omega^2l^2\sin\theta\cos\theta \tag{25}$$

According to the equations of mechanical equilibrium we have

$$\left(N_{\mathrm{A}}+N_{\mathrm{B}}\right)R=M_{\mathrm{c}}-M_{\mathrm{G}} \tag{26}$$

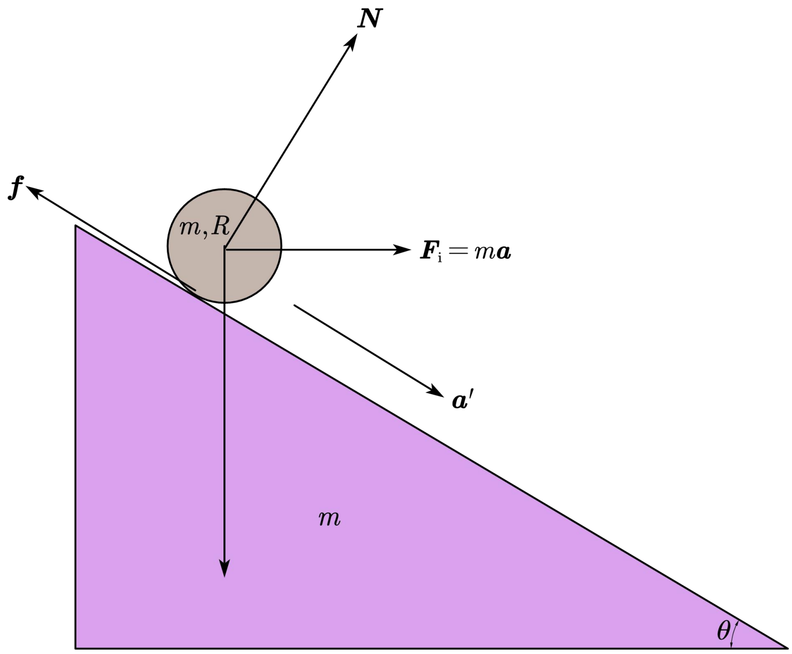
$$N_{\mathrm{B}}-N_{\mathrm{A}}=F_{\mathrm{c}} \tag{27}$$

The solution is that

$$N_{\mathrm{A}}=\frac{3}{128}ml\sin\theta\left[\frac{1}{R}\left(l\omega^2\cos\theta-2g\right)-2\omega^2\right]$$

$$N_{B}=\frac{3}{128}ml\sin\theta\left[\frac{1}{R}\left(l\omega^2\cos\theta-2g\right)+2\omega^2\right]$$

5 Question 6-19



Taking the slope as the reference frame, the force of the cylinder is considered, and the inertia force $\boldsymbol{F}_{\mathrm{i}}$ is corrected. According to the Newton's second law

$$ma' = mg\sin\theta + ma\cos\theta - f \tag{28}$$

$$mg\cos\theta = N + ma\sin\theta \tag{29}$$

Because it is rolling without slipping, the axis of rotation is tangent to the cylinder and the bevel , according to the law of rotation and parallel-axis theorem

$$mgR\sin\theta + maR\cos\theta = \left(\frac{1}{2}mR^2 + mR^2\right)\beta = \left(\frac{1}{2}mR^2 + mR^2\right)\frac{a'}{R} \tag{30}$$

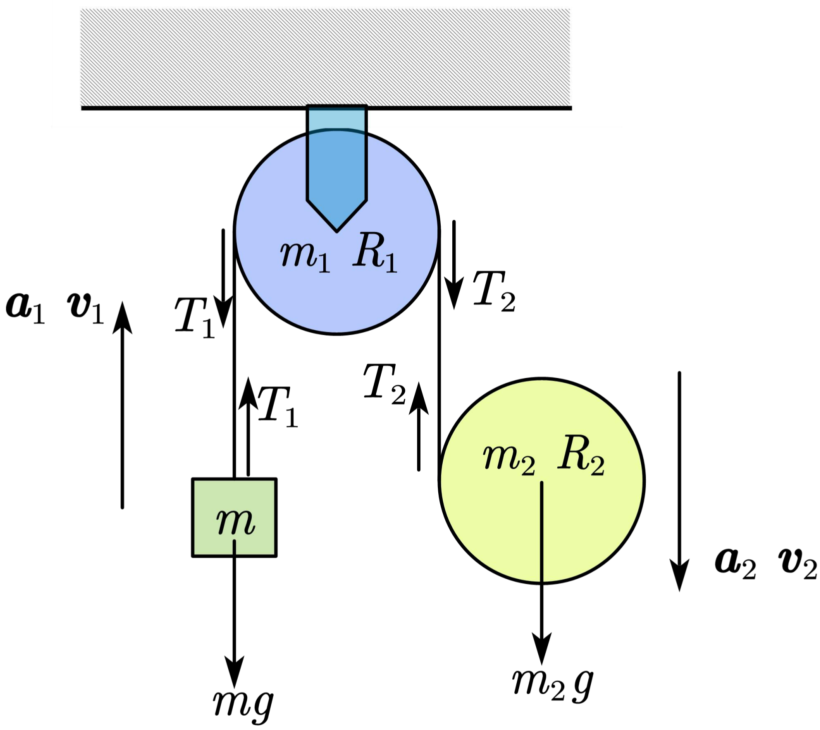
do force analysis of slope, according to the Newton's second law

$$ma = N\sin\theta - f\cos\theta \tag{31}$$

The solution is that

$$a = \frac{g}{3\tan\theta + 2\cot\theta}$$

6 Question 6-22



According to the angular momentum theorem

$$\frac{\mathrm{d}L}{\mathrm{d}t} == m_2 g \left(R_1 + R_2\right) - mgR_1 \tag{32}$$

$$L = mv_1 R_1 + m_2 v_2 \left(R_1 + R_2\right) + \frac{1}{2} m_1 R_1^2 \omega_1 + \frac{1}{2} m_2 R_2^2 \omega_2 \tag{33}$$

And there is no relative slip

$$R_1 \omega_1 = v_1 \tag{34}$$

$$R_2 \omega_2 = v_2 - v_1 \tag{35}$$

By combining these equations

$$mR_1 a_1 + m_2 \left(R_1 + R_2\right) a_2 + \frac{1}{2} m_1 R_1 a_1 + \frac{1}{2} m_2 R_2 \left(a_2 - a_1\right) = m_2 g \left(R_1 + R_2\right) - mgR_1 \tag{36}$$

Study the rotation of the fixed pulley.

$$fR_1 = \frac{1}{2} m_1 R_1^2 \beta_1 = \frac{1}{2} m_1 R_1^2 \frac{a_1}{R_1} = \frac{1}{2} m_1 R_1 a_1 \tag{37}$$

Force analysis of disk and weight perspectively by isolation method, according to Newton's second law

$$m_2 a_2 = m_2 g - T_2 \tag{38}$$

$$T_1 = T_2 - f \tag{39}$$

$$ma_1 = T_1 - mg \tag{40}$$

By combining these equations

$$m_2 g - mg = ma_1 + m_2 a_2 + \frac{1}{2} m_1 a_1 \tag{41}$$

By combining the (36) and (41) can we solve this problem.

$$a_1 = \frac{2(m_2 - 3m)}{6m + 3m_1 + m_2} g$$

$$a_2 = \frac{2(m + m_1 + m_2)}{6m + 3m_1 + m_2} g$$

Note that the a_1 can be both positive and negtive, if it is negetive, that means the direction of a_1 is opposite with the original suppose.

7 Question 6-25

Before reaching pure rolling, under the action of sliding friction, the center of mass of the ball accelerates and the speed slows down. According to the Newton's second law and the rotation law with the center of mass as the reference axis

$$ma = \mu mg\cos\theta - mg\sin\theta \tag{42}$$

$$\frac{2}{5}mr^2\beta = r\mu mg\cos\theta \tag{43}$$

Define t as the time when ball reach pure rolling.

$$r(\omega_0 - \beta t) = at \tag{44}$$

According to kinematic formula

$$h_1 = x_1\sin\theta = \frac{1}{2}at^2 \tag{45}$$

After reaching pure roll, the ball will roll and rise. According to conservation of mechanical energy

$$\frac{1}{2}m(at)^2 + \frac{1}{2}\frac{2}{5}mr^2(\frac{at}{r})^2 = mgh_2 \tag{46}$$

We can have

$$h = h_1 + h_2 = \frac{2r^2\omega_0^2}{5g}\frac{\mu\cos\theta - \sin\theta}{7\mu\cos\theta - 2\sin\theta}$$

Note that there must be $\mu > \tan\theta$ or the ball can't move above because the fiction isn't enough.

8 Question 6-27

(1)

(i)

Since the direction of acceleration and angular acceleration is not determined, but will change with the adjustment of the parameters, it is best to take the right and clockwise as the positive direction, so that the positive and negative values can represent the direction. According to the Newton's second law and the rotation law with the middle axis as the reference axis.

$$ma = f - F\cos\theta \tag{47}$$

$$J\beta = Rf + rF \tag{48}$$

And the pure rolling need

$$a = R\beta \tag{49}$$

Combine (49) with others and kill the f by liner calculate between equations.

$$(mR^2 + J)a = R^2 F\cos\theta - rRF \tag{50}$$

The direction of a is required to be negative.

$$a < 0 \tag{51}$$

The solution is that

$$\theta > \arccos\frac{r}{R} \tag{52}$$

(ii)

By solving equations we get

$$f = -\frac{mrR + J\cos\theta}{mR^2 + J}F < 0 \tag{53}$$

The f is always negtive, it means the direction of it is alwanys behind. Here is a restirction

$$|f| < \mu N = \mu(mg - F\sin\theta) \tag{54}$$

The solution is that

$$\mu \geqslant \frac{mrR + J\cos\theta}{mR^2 + J}\frac{F}{\mu\,(mg - F\sin\theta)}$$

(iii)

We can kill f by liner calculate between equations. And to work out a.

$$a = \frac{(R\cos\theta - r)\,RF}{mR^2 + J} < 0$$

(2)

We know that the numerical value indicate the vector's direction so the anwsers above still correct[1].

$$\theta < \arccos\frac{r}{R}$$

[1]The θ just take the opposite componet of (1).

$$\mu \geqslant \frac{mrR + J\cos\theta}{mR^2 + J}\frac{F}{\mu\left(mg - F\sin\theta\right)}$$

$$a = \frac{\left(R\cos\theta - r\right)RF}{mR^2 + J} > 0$$

9 Question 6-32

(1)

According to the momentum conservation law.

$$mv_0 = mu_1 + mu_2 \tag{55}$$

Because the collision is elastic and ignores friction, mechanical energy is conserved.

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2 \tag{56}$$

Define I_f as the impulse of friction from ground. According to the theorem of impulse and the angular momentum theorem, for ball-first we have

$$mv_1 = mu_1 + I_f \tag{57}$$

$$J\omega_1 = J\omega_0 - RI_f \tag{58}$$

For ball-second we have

$$mv_2 = mu_2 - I_f \tag{59}$$

$$J\omega_2 = RI_f \tag{60}$$

Pure rolling requres

$$\omega_0 = \frac{v_0}{R} \tag{61}$$

$$\omega_1 = \frac{v_1}{R} \tag{62}$$

$$\omega_2 = \frac{v_2}{R} \tag{63}$$

So

$$v_1 = \frac{2}{7}v_0$$

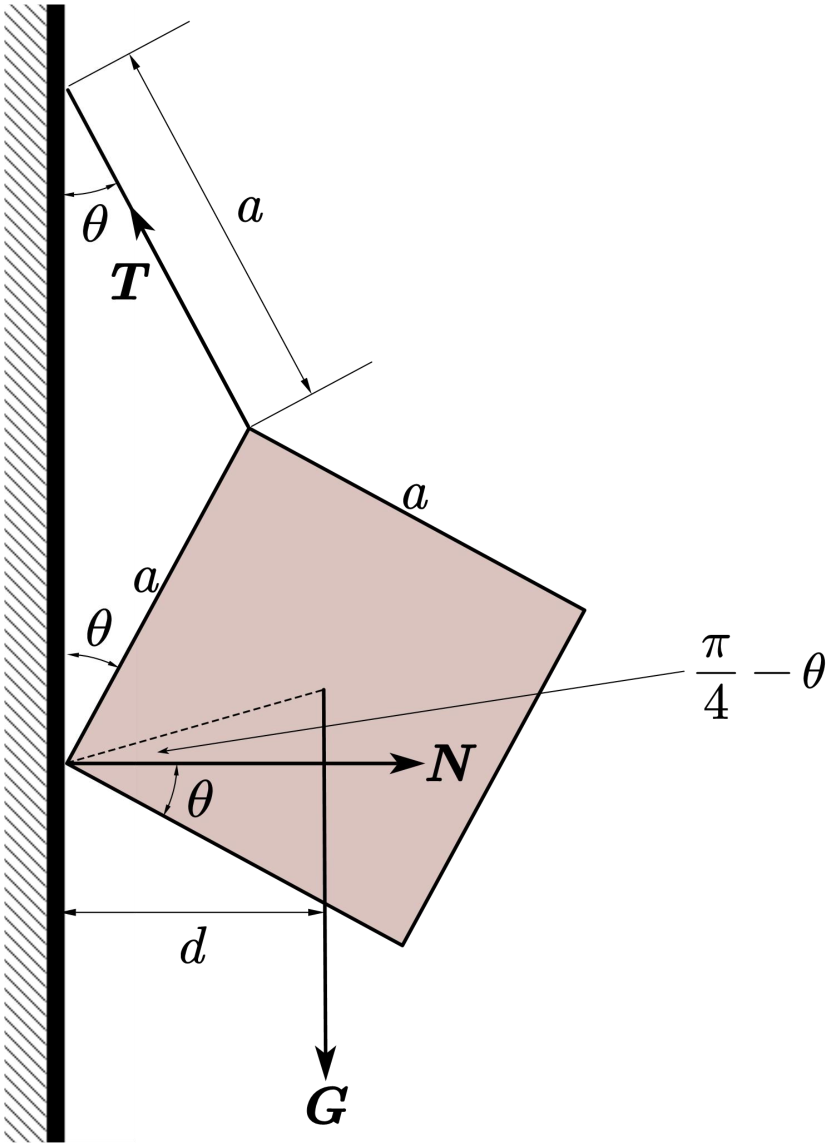
$$v_2 = \frac{5}{7}v_0$$

(2)

Calculate it directly.

$$\Delta E_k = (\frac{1}{2}mv_0^2 + \frac{1}{2}\frac{2}{5}m\omega_0^2) - (\frac{1}{2}mv_1^2 + \frac{1}{2}\frac{2}{5}m\omega_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}\frac{2}{5}m\omega_2^2) = \frac{2}{7}mv_0^2$$

10 Question 6-35



According to the Mechanical equilibrium equation and some Geometric relation

$$mg = T\cos\theta \tag{64}$$

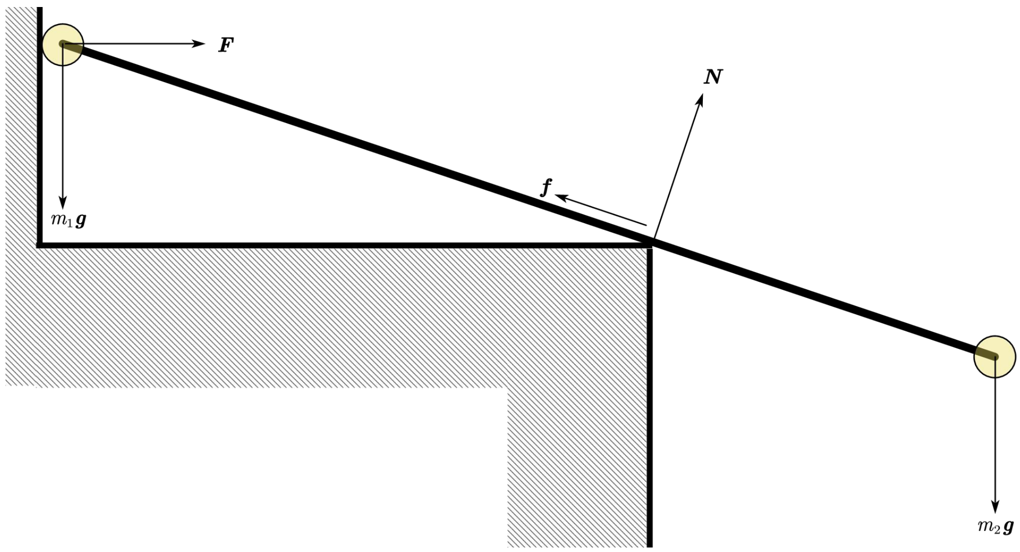
$$N = T\sin\theta \tag{65}$$

$$mg\frac{a}{\sqrt{2}}\cos(\frac{\pi}{4}-\theta) = N2a\cos\theta \tag{66}$$

Easy to get

$$T = \frac{\sqrt{10}}{3}mg$$

11 Question 6-40



According to the Mechanical equilibrium equation and the moment balance with the position of A as the reference point.

$$N\cos\theta + f\sin\theta = (m_1+m_2)g \tag{67}$$

$$f\cos\theta = F + N\sin\theta \tag{68}$$

$$N\frac{a}{\cos\theta} = m_2 gl\cos\theta \tag{69}$$

Take (69) into other equation

$$m_2 g\frac{l}{a}\cos^3\theta + f\sin\theta = (m_1+m_2)g \tag{70}$$

$$f\cos\theta = F + m_2 g\frac{l}{a}\sin\theta\cos^2\theta \tag{71}$$

Consider $f \leqslant \mu N$, the (70) can be write like

$$(m_1+m_2)g \leqslant m_2 g\frac{l}{a}\cos^3\theta + \mu m_2 g\frac{l}{a}\cos^2\theta\sin\theta \tag{72}$$

That is

$$1+\frac{m_1}{m_2}\leqslant\frac{l}{a}\cos^2\theta(\mu\sin\theta+\cos\theta) \tag{73}$$

Considering the critical condition, the supporting force $\boldsymbol{F}$ of the left wall is 0.

$$1+\frac{m_1}{m_2}=\frac{l}{a}\cos\theta \tag{74}$$

Obviously, m_1 is healvier than that case.

$$1+\frac{m_1}{m_2}\geqslant\frac{l}{a}\cos\theta \tag{75}$$

As a result.

$$\frac{l}{a}\cos\theta\leqslant 1+\frac{m_1}{m_2}\leqslant\frac{l}{a}\cos^2\theta(\mu\sin\theta+\cos\theta) \tag{76}$$

However, the rightmost end of the inequality is not necessarily greater than or equal to the leftmost end, which requires the following.

$$\frac{l}{a}\cos\theta\leqslant\frac{l}{a}\cos^2\theta(\mu\sin\theta+\cos\theta) \tag{77}$$

That is

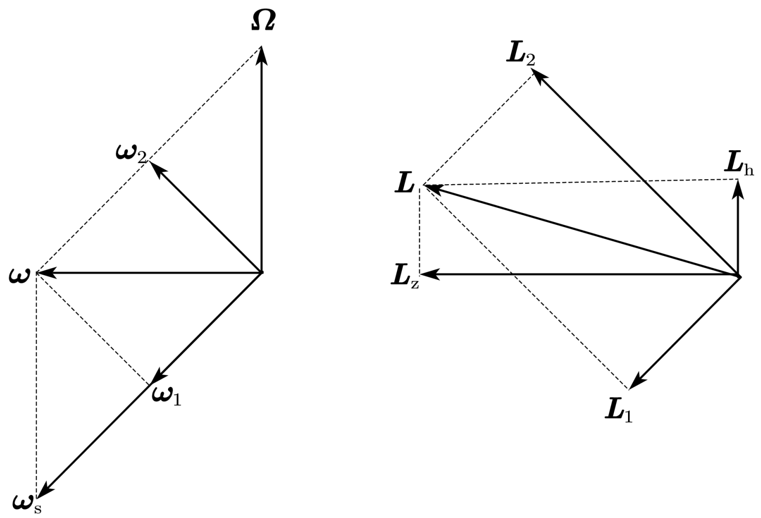
$$\mu\geqslant\tan\theta \tag{78}$$

So

$$\frac{l}{a}\cos\theta\leqslant 1+\frac{m_1}{m_2}\leqslant\frac{l}{a}\cos^2\theta(\mu\sin\theta+\cos\theta)\qquad\mu\geqslant\tan\theta$$

12 Question 6-45

(1)



As the figure, the instantaneous axis of rotation is the line between two points with instantaneous velocity of zero. That is, the line between two points of contact between a rigid body and the ground, it means the instantaneous axis of rotation is horizontal.

Do geometric operations in vector drawings. The directions of $\boldsymbol{\omega_1}$ and $\boldsymbol{\omega_2}$ are the principal axis of inertia.

$$\omega_1 = \omega_2 = \frac{\Omega}{\sqrt{2}} \tag{79}$$

The principal axis of inertia makes the angular momentum and angular velocity go in the same direction.

$$L_1 = J_1\omega_1 \tag{80}$$

$$L_2 = J_2\omega_2 \tag{81}$$

The moment of inertia can be obtained by the parallel axis theorem and the vertical axis theorem.

$$J_1 = \frac{1}{2}mR^2 \tag{82}$$

$$J_2 = \frac{1}{2}J_1 + mR^2 \tag{83}$$

Then the angular momentum is decomposed into the longitudinal and vertical instantaneous rotation directions.

$$L_z = \frac{L_1}{\sqrt{2}} + \frac{L_2}{\sqrt{2}} \tag{84}$$

According to the nutation formula of rigid body.

$$\boldsymbol{M} = \boldsymbol{\Omega} \times \boldsymbol{L}_z \tag{85}$$

Write in scalar form

$$\sqrt{2}RF_N - \frac{\sqrt{2}}{2}Rmg = \frac{1}{\sqrt{2}}\Omega L_z \tag{86}$$

The solution is that

$$F_N = \frac{7\sqrt{2}}{16}mR\Omega^2 + \frac{1}{2}mg$$

(2)

Kinetic energy is examined on the principal axis of inertia.

$$E_k = \frac{1}{2}J_1\omega_1^2 + \frac{1}{2}J_2\omega_2^2 = \frac{7}{16}mR^2\Omega^2$$