spBeamer Demo

Sweet Pastry

Fudan University, Shanghai, China

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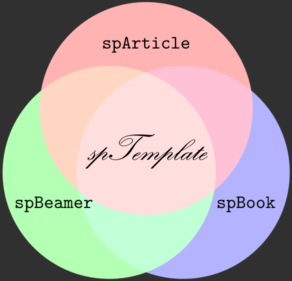
Summary

Introduction

Logo



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Math

uncover command

Definition (Linear Functional)

A linear functional on a vector space X over $\mathbb{R}$ or $\mathbb{C}$ is a map $f : X \to \mathbb{R}$ (or $\mathbb{C}$) such that:

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y) \quad \forall x, y \in X, \ \alpha, \beta \in \mathbb{R} \text{ (or } \mathbb{C}\text{)}$$

Definition (Sublinear Functional)

A map $p : X \to \mathbb{R}$ is called sublinear if:

$$p(x+y) \leq p(x) + p(y) \quad \text{and} \quad p(\lambda x) = \lambda p(x) \quad \forall x, y \in X, \ \lambda \geq 0$$

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Theorem (Hahn-Banach, Normed Space Version)

Let X be a normed space, M a subspace of X, and $f : M \to \mathbb{R}$ a bounded linear functional. Suppose $p : X \to \mathbb{R}$ is a sublinear functional such that:

$$f(x) \leq p(x) \quad \forall x \in M$$

Then, there exists an extension $F : X \to \mathbb{R}$ of f such that:

$$F(x) \leq p(x) \quad \forall x \in X$$

and F is linear and bounded.

The proof uses Zorn's Lemma and proceeds by extending f step by step to larger subspaces. The key idea is to define:

$$F(x + \alpha y) = F(x) + \alpha f(y)$$

for some $y \notin M$ and ensure the extension satisfies the sublinear constraint.

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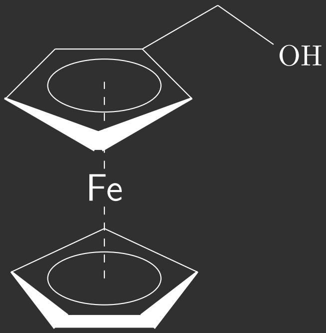
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References

[1] L. Lamport, *LaTeX: A Document Preparation System*, 2nd. Boston, MA: Addison-Wesley, 1994, An excellent introduction to LaTeX., ISBN: 978-0201529838. https://latex-project.org/.

[2] A. Einstein, "Zur elektrodynamik bewegter körper," *Annalen der Physik*, vol. 322, no. 10, pp. 891–921, 1905, This paper introduces the theory of special relativity. DOI: 10.1002/andp.19053221004.

[3] D. E. Knuth, *Knuth: Computers and typesetting*, http://www-cs-faculty.stanford.edu/~knuth/, Accessed: 2025-01-01.

[4] A. M. Turing, "On computable numbers, with an application to the entscheidungsproblem," in *Proceedings of the London Mathematical Society*, A landmark paper in computer science., vol. s2-42, 1936, pp. 230–265. DOI: 10.1112/plms/s2-42.1.230.

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