spBeamer **Demo**

Sweet Pastry

Fudan University, Shanghai, China

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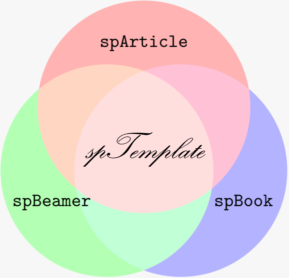
Summary

Introduction

Logo



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Math

`uncover` command

Definition (Linear Functional)

A linear functional on a vector space X over $\mathbb{R}$ or $\mathbb{C}$ is a map $f : X \to \mathbb{R}$ (or $\mathbb{C}$) such that:

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y) \quad \forall x, y \in X,\ \alpha, \beta \in \mathbb{R} \text{ (or } \mathbb{C})$$

Definition (Sublinear Functional)

A map $p : X \to \mathbb{R}$ is called sublinear if:

$$p(x+y) \leq p(x) + p(y) \quad \text{and} \quad p(\lambda x) = \lambda p(x) \quad \forall x, y \in X,\ \lambda \geq 0$$

`uncover` command

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Theorem (Hahn-Banach, Normed Space Version)

Let X be a normed space, M a subspace of X, and $f : M \to \mathbb{R}$ a bounded linear functional. Suppose $p : X \to \mathbb{R}$ is a sublinear functional such that:

$$f(x) \leq p(x) \quad \forall x \in M$$

Then, there exists an extension $F : X \to \mathbb{R}$ of f such that:

$$F(x) \leq p(x) \quad \forall x \in X$$

and F is linear and bounded.

The proof uses Zorn's Lemma and proceeds by extending f step by step to larger subspaces. The key idea is to define:

$$F(x + \alpha y) = F(x) + \alpha f(y)$$

for some $y \notin M$ and ensure the extension satisfies the sublinear constraint:

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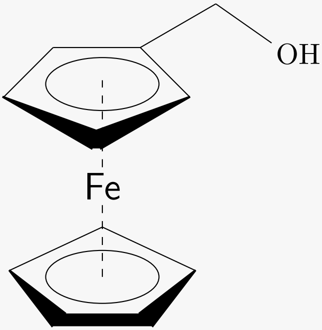
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References

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