

Bibliography

- [1] A. Abdulle. Fourth order Chebyshev methods with recurrence relation. *SIAM J. Sci. Comput.*, 23:2041–2054, 2002. (cited on 176)
- [2] A. Abdulle and A. A. Medovikov. Second order Chebyshev methods based on orthogonal polynomials. *Numer. Math.*, 90:1–18, 2001. (cited on 176)
- [3] L. M. Adams, R. J. LeVeque, and D. M. Young. Analysis of the SOR iteration for the 9-point Laplacian. *SIAM J. Numer. Anal.*, 25:1156–1180, 1988. (cited on 77)
- [4] U. Ascher, R. Mattheij, and R. Russell. *Numerical Solution of Boundary Value Problems for Ordinary Differential Equations*. Prentice–Hall, Englewood Cliffs, NJ, 1988. (cited on 38, 52, 55)
- [5] U. M. Ascher and L. R. Petzold. *Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations*. SIAM, Philadelphia, 1998. (cited on 113, 129)
- [6] U. M. Ascher, S. J. Ruuth, and R. J. Spiteri. Implicit-explicit Runge-Kutta methods for time-dependent partial differential equations. *Appl. Numer. Math.*, 25:151–167, 1997. (cited on 240)
- [7] U. M. Ascher, S. J. Ruuth, and B. T. R. Wetton. Implicit-explicit methods for time-dependent partial differential equations. *SIAM J. Numer. Anal.*, 32:797–823, 1995. (cited on 240)
- [8] G. Beylkin, J. M. Keiser, and L. Vozovoi. A new class of time discretization schemes for the solution of nonlinear pdes. *J. Comput. Phys.*, 147:362–387, 1998. (cited on 241)
- [9] A. Bourlioux, A. T. Layton, and M. L. Minion. High-order multi-implicit spectral deferred correction methods for problems of reactive flow. *J. Comput. Phys.*, 189:651–675, 2003. (cited on 239)
- [10] J. P. Boyd. *Chebyshev and Fourier Spectral Methods*. Dover, New York, 2001. (cited on 58)
- [11] W. L. Briggs, V. Emden Henson, and S. F. McCormick. *A Multigrid Tutorial, 2nd ed.* SIAM, Philadelphia, 2000. (cited on 103)

- [12] J. C. Butcher, *Numerical Methods for Ordinary Differential Equations*, John Wiley, Chichester, UK, 2003. (cited on 113)
- [13] J. C. Butcher. *The Numerical Analysis of Ordinary Differential Equations: Runge-Kutta and General Linear Methods*. John Wiley, Chichester, UK, 1987. (cited on 128, 171)
- [14] C. Canuto, M. Y. Hussaini, A. Quarteroni, and T. A. Zang. *Spectral Methods in Fluid Dynamics*. Springer, New York, 1988. (cited on 58)
- [15] E. A. Coddington and N. Levinson. *Theory of Ordinary Differential Equations*. McGraw-Hill, New York, 1955. (cited on 116)
- [16] S. D. Conte and C. de Boor. *Elementary Numerical Analysis*. McGraw-Hill, New York, 1980. (cited on 264)
- [17] R. Courant, K. O. Friedrichs, and H. Lewy. Über die partiellen Differenzengleichungen der mathematischen Physik. *Math. Ann.*, 100:32–74, 1928. (cited on 216)
- [18] R. Courant, K. O. Friedrichs, and H. Lewy. On the partial difference equations of mathematical physics. *IBM Journal*, 11:215–234, 1967. (cited on 216)
- [19] S. M. Cox and P. C. Matthews. Exponential time differencing for stiff systems. *J. Comput. Phys.*, 176:430–455, 2002. (cited on 241, 242)
- [20] C. F. Curtiss and J. O. Hirschfelder. Integration of stiff equations. *Proc. Nat. Acad. Sci. USA*, 38:235–243, 1952. (cited on 173)
- [21] G. Dahlquist. A special stability problem for linear multistep methods. *BIT*, 3:27–43, 1963. (cited on 171)
- [22] G. Dahlquist, Convergence and stability in the numerical integration of ordinary differential equations, *Math. Scand.*, 4:33–53, 1956. (cited on 147)
- [23] G. Dahlquist and R. LeVeque. Linear difference equations and matrix theorems. Lecture Notes, Royal Institute of Technology (KTH), Stockholm, <http://www.amath.washington.edu/~rjl/pubs/kth81>, (1981). (cited on 309, 310)
- [24] T. A. Davis. *Direct Methods for Sparse Linear Systems*. SIAM, Philadelphia, 2006. (cited on 68)
- [25] J. R. Dormand and P. J. Prince. A family of embedded Runge-Kutta formulas. *J. Comput. Appl. Math.*, 6:19–26, 1980. (cited on 130)
- [26] J. Douglas and H. H. Rachford. On the numerical solution of heat conduction problems in two and three space variables. *Trans. AMS*, 82:421–439, 1956. (cited on 199)
- [27] D. R. Durran. *Numerical Methods for Wave Equations in Geophysical Fluid Dynamics*. Springer, New York, 1999. (cited on xv)

- [28] A. Dutt, L. Greengard, and V. Rokhlin. Spectral deferred correction methods for ordinary differential equations. *BIT*, 40:241–266, 2000. (cited on 58, 239)
- [29] B. Fornberg. *A Practical Guide to Pseudospectral Methods*. Cambridge University Press, London, 1996. (cited on 58)
- [30] B. Fornberg. Calculation of weights in finite difference formulas. *SIAM Rev.*, 40:685–691, 1998. (cited on 11)
- [31] F. G. Friedlander and M. Joshi. *Introduction to the Theory of Distributions*. Cambridge University Press, London, 1998. (cited on 24)
- [32] E. Gallopoulos and Y. Saad. Efficient solution of parabolic equations by Krylov approximation methods. *SIAM J. Sci. Statist. Comput.*, 13:1236–1264, 1992. (cited on 242)
- [33] C. W. Gear. *Numerical Initial Value Problems in Ordinary Differential Equations*. Prentice–Hall, Englewood Cliffs, NJ, 1971. (cited on 113, 173)
- [34] A. George and J. W. H. Liu. *Computer Solution of Large Sparse Positive-Definite Systems*. Prentice–Hall, Englewood Cliffs, NJ, 1981. (cited on 68)
- [35] G. H. Golub and C. F. Van Loan. *Matrix Computations*, 3rd ed. Johns Hopkins University Press, Baltimore, 1996. (cited on 67, 250, 271)
- [36] G. H. Golub and D. P. O’Leary. Some history of the conjugate gradient and Lanczos algorithms: 1948–1976. *SIAM Rev.*, 31:50–102, 1989. (cited on 86)
- [37] G. H. Golub and J. M. Ortega. *Scientific Computing and Differential Equations: An Introduction to Numerical Methods*. Academic Press, New York, 1992. (cited on 76)
- [38] D. Gottlieb and S. A. Orszag. *Numerical Analysis of Spectral Methods*. CBMS-NSF Regional Conference Series in Applied Mathematics, 26, SIAM, Philadelphia, 1977. (cited on 58)
- [39] A. Greenbaum. *Iterative Methods for Solving Linear Systems*. SIAM, Philadelphia, 1997. (cited on 78, 87, 88, 99, 100)
- [40] B. Gustafsson, H.-O. Kreiss, and J. Oliger. *Time Dependent Problems and Difference Methods*. John Wiley, New York, 1995. (cited on xv, 191, 214, 228)
- [41] W. Hackbusch. *Multigrid Methods and Applications*. Springer-Verlag, Berlin, 1985. (cited on 103)
- [42] L. A. Hageman and D. M. Young. *Applied Iterative Methods*. Academic Press, New York, 1981. (cited on 76)
- [43] E. Hairer, S. P. Nørsett, and G. Wanner. *Solving Ordinary Differential Equations I. Nonstiff Problems*. Springer-Verlag, Berlin, Heidelberg, 1987. (cited on 113, 128, 129, 148)

- [44] E. Hairer, S. P. Nørsett, and G. Wanner. *Solving Ordinary Differential Equations II. Stiff and Differential-Algebraic Problems*. Springer-Verlag, New York, 1993. (cited on 113, 128, 166, 171, 173)
- [45] P. Henrici. *Discrete Variable Methods in Ordinary Differential Equations*. John Wiley, New York, 1962. (cited on 113)
- [46] M. R. Hestenes and E. Stiefel. Method of conjugate gradients for solving linear equations. *J. Res. Nat. Bureau Standards*, 49:409–436, 1952. (cited on 86)
- [47] D. J. Higham and L. N. Trefethen. Siffness of ODEs. *BIT*, 33:285–303, 1993. (cited on 156, 228)
- [48] M. Hochbruck, C. Lubich, and H. Selhofer. Exponential integrators for large systems of differential equations. *SIAM J. Sci. Comput.*, 19:1552–1574, 1998. (cited on 241, 242)
- [49] A. Iserles. *Numerical Analysis of Differential Equations*. Cambridge University Press, Cambridge, UK, 1996. (cited on xv)
- [50] A. Iserles. Think globally, act locally: Solving highly-oscillatory ordinary differential equations. *Appl. Numer. Math.*, 43:145–160, 2002. (cited on 169)
- [51] A. Iserles and S. P. Norsett. *Order Stars*. Chapman and Hall, London, 1991. (cited on 166, 171)
- [52] D. C. Jespersen. Multigrid methods for partial differential equations. In *Studies in Numerical Analysis*, G. H. Golub, ed. MAA Studies in Mathematics, Vol. 24, 1984, pages 270–317. (cited on 103)
- [53] A.-K. Kassam and L. N. Trefethen. Fourth-order time-stepping for stiff PDEs. *SIAM J. Sci. Comput.*, 26:1214–1233, 2005. (cited on 241, 242, 286)
- [54] H. B. Keller. *Numerical Solution of Two Point Boundary Value Problems*. SIAM, Philadelphia, 1976. (cited on 38, 43, 55)
- [55] J. Kevorkian. *Partial Differential Equations*. Wadsworth & Brooks/Cole, Pacific Corove, CA, 1990. (cited on 14, 43, 216, 311, 314, 328)
- [56] J. Kevorkian and J. D. Cole. *Perturbation Methods in Applied Mathematics*. Springer, New York, 1981. (cited on 43)
- [57] D. A. Knoll and D. E. Keyes. Jacobian-free Newton-Krylov methods: A survey of approaches and applications. *J. Comput. Phys.*, 193:357–397, 2004. (cited on 102)
- [58] D. Kröner. *Numerical Schemes for Conservation Laws*. Wiley-Teubner, New York, 1997. (cited on 201)
- [59] J. D. Lambert. *Computational Methods in Ordinary Differential Equations*. John Wiley, New York, 1973. (cited on 113, 173)

- [60] J. D. Lambert. *Numerical Methods for Ordinary Differential Systems: The initial value Problem*. John Wiley, Chichester, 1991. (cited on 113)
- [61] P. D. Lax. *Hyperbolic Systems of Conservation Laws and the Mathematical Theory of Shock Waves*. CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM, Philadelphia, 11, 1973. (cited on 314)
- [62] R. Lehoucq, K. Maschhoff, D. Sorensen, and C. Yang. ARPACK software. <http://www.caam.rice.edu/software/ARPACK/> (1997). (cited on 100)
- [63] S. K. Lele. Compact difference schemes with spectral-like resolution. *J. Comput. Phys.*, 103:16–42, 1992. (cited on 230)
- [64] R. J. LeVeque. CLAWPACK software, 2006. <http://www.amath.washington.edu/~claw>. (cited on 201)
- [65] R. J. LeVeque. Intermediate boundary conditions for time-split methods applied to hyperbolic partial differential equations. *Math. Comput.*, 47:37–54, 1986. (cited on 239)
- [66] R. J. LeVeque. *Finite Volume Methods for Hyperbolic Problems*. Cambridge University Press, London, 2002. (cited on 201, 214, 216, 226, 230, 231, 314)
- [67] R. J. LeVeque and L. N. Trefethen. Fourier analysis of the SOR iteration. *IMA J. Numer. Anal.*, 8:273–279, 1988. (cited on 76)
- [68] D. Levy and E. Tadmor. From semidiscrete to fully discrete: Stability of Runge–Kutta schemes by the energy method. *SIAM Rev.*, 40:40–73, 1998. (cited on 191)
- [69] A. A. Medovikov. High order explicit methods for parabolic equations. *BIT*, 38:372–390, 1998. (cited on 176)
- [70] C. Moler and C. Van Loan. Nineteen dubious ways to compute the exponential of a matrix. *SIAM Rev.*, 20:801–836, 1978. (cited on 241)
- [71] C. Moler and C. Van Loan. Nineteen dubious ways to compute the exponential of a matrix, twenty-five years later. *SIAM Rev.*, 45:3–49, 2003. (cited on 241)
- [72] K. W. Morton and D. F. Mayers. *Numerical Solution of Partial Differential Equations*. Cambridge University Press, Cambridge, UK, 1994. (cited on xv)
- [73] J. D. Murray. *Mathematical Biology*. Springer-Verlag, Berlin, Heidelberg, 1989. (cited on 235)
- [74] L. R. Petzold, L. O. Jay, and J. Yen. Numerical solution of highly oscillatory ordinary differential equations. *Acta Numer.*, 6:437–484, 1997. (cited on 169)
- [75] R. D. Richtmyer and K. W. Morton. *Difference Methods for Initial-Value Problems*. Wiley-Interscience, New York, 1967. (cited on 190, 191, 192, 214, 307)
- [76] J. W. Ruge and K. Stüben. *Algebraic multigrid*, in multigrid methods, S.F. McCormick, ed., SIAM, Philadelphia, 1987, pages 73–103. (cited on 110)

- [77] Y. Saad. Analysis of some Krylov subspace approximations to the matrix exponential operator. *SIAM J. Numer. Anal.*, 29:209–228, 1992. (cited on 242)
- [78] L. F. Shampine and M. W. Reichelt. The MATLAB ODE suite. *SIAM J. Sci. Comput.*, 18:1–22, 1997. (cited on 129, 130, 135)
- [79] J. R. Shewchuk. *An Introduction to the Conjugate Gradient Method Without the Agonizing Pain*. Technical report, available from <http://www.cs.cmu.edu/~jrs/jrspapers.html> (1994). (cited on 78)
- [80] B. P. Sommeijer, L. F. Shampine, and J. G. Verwer. RKC: An explicit solver for parabolic PDEs. *J. Comput. Appl. Math.*, 88:315–326, 1997. (cited on 176, 178)
- [81] M. N. Spijker. On a conjecture by LeVeque and Trefethen related to the Kreiss matrix theorem. *BIT*, 31:551–555, 1991. (cited on 293)
- [82] G. W. Stewart. *Introduction to Matrix Computations*. Academic Press, New York, 1973. (cited on 67)
- [83] G. Strang. On the construction and comparison of difference schemes. *SIAM J. Numer. Anal.*, 5:506–517, 1968. (cited on 238)
- [84] J. C. Strikwerda. *Finite Difference Schemes and Partial Differential Equations*, 2nd ed. SIAM, Philadelphia, 2004. (cited on xv, 191, 192, 228)
- [85] J. C. Strikwerda and B. A. Wade. A survey of the Kreiss matrix theorem for power bounded families of matrices and its extensions. In *Linear Operators*, Banach Center Publ., 38, Polish Acad. Sci., Warsaw, 1997, pages 329–360. (cited on 307)
- [86] K. Stüben. A review of algebraic multigrid. *J. Comput. Appl. Math.*, 128:281–309, 2001. (cited on 110)
- [87] Paul N. Swarztrauber. Fast Poisson Solvers. In *Studies in Numerical Analysis*, volume 24, G. H. Golub, ed., Mathematical Association of America, Washington, D.C., 1984, pp. 319–370. (cited on 68)
- [88] E. F. Toro. *Riemann Solvers and Numerical Methods for Fluid Dynamics*. Springer-Verlag, Berlin, Heidelberg, 1997. (cited on 201)
- [89] L. N. Trefethen. Group velocity in finite difference schemes. *SIAM Rev.*, 24:113–136, 1982. (cited on 228)
- [90] L. N. Trefethen. *Spectral Methods in MATLAB*. SIAM, Philadelphia, 2000. (cited on 58, 264)
- [91] L. N. Trefethen and D. Bau, III, *Numerical Linear Algebra*. SIAM, Philadelphia, 1997. (cited on xiv, 67, 78, 88, 94, 100, 250)
- [92] L. N. Trefethen and M. Embree. *Spectra and Pseudospectra*. Princeton University Press, Princeton, NJ, 2005. (cited on 74, 228, 231, 293, 299, 304, 307)

- [93] A. Tveito and R. Winther. *Introduction to Partial Differential Equations: A Computational Approach*. Springer, New York, 1998. (cited on xv)
- [94] P. J. van der Houwen and B. P. Sommeijer. On the internal stability of explicit m -stage Runge–Kutta methods for large values of m . *Z. Angew. Math. Mech.*, 60:479–485, 1980 (in German). (cited on 178)
- [95] H. A. Van der Vorst. Bi-CGSTAB: A fast and smoothly converging variant of Bi-CG for the solution of nonsymmetric linear systems. *SIAM J. Sci. Statist. Comput.*, 13:631–644, 1992. (cited on 100)
- [96] R. S. Varga. *Matrix Iterative Analysis*. Prentice–Hall, Englewood Cliffs, NJ, 1962. (cited on 76)
- [97] J. G. Verwer. Explicit Runge-Kutta methods for parabolic differential equations. *Appl. Numer. Math.*, 22:359, 1996. (cited on 176, 177, 178)
- [98] G. Wanner. Order stars and stability. In *The State of the Art in Numerical Analysis*, A. Iserles and M. J. D. Powell, eds., Clarendon Press Oxford, UK, 1987, pp. 451–471. (cited on 166)
- [99] G. Wanner, E. Hairer, and S. P. Nørsett. Order stars and stability theorems. *BIT*, 18:475–489, 1978. (cited on 165)
- [100] R. Warming and B. Hyett. The modified equation approach to the stability and accuracy analysis of finite-difference methods. *J. Comput. Phys.*, 14:159–179, 1974. (cited on 219)
- [101] P. Wesseling. *An Introduction to Multigrid Methods*. John Wiley, New York, 1992. (cited on 103)
- [102] G. Whitham. *Linear and Nonlinear Waves*. Wiley-Interscience, New York, 1974. (cited on 314, 323, 328)
- [103] J. H. Wilkinson. *The Algebraic Eigenvalue Problem*. Oxford University Press, London, 1965. (cited on 278)
- [104] T. Wright. Eigtool, 2002. <http://web.comlab.ox.ac.uk/projects/pseudospectra/eigtool/>. (cited on 304)
- [105] D. M. Young. *Iterative Methods for Solving Partial Differential Equations of Elliptic Type*. Ph.D. thesis, Harvard University, Cambridge, 1950. (cited on 76)
- [106] D. M. Young. *Iterative Solution of Large Linear Systems*. Academic Press, New York, 1971. (cited on 76)

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