Overconfidence and welfare in a differentiated duopoly*

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Abstract

We examine whether owners' decisions to delegate responsibilities to overconfident managers improve welfare. We develop a duopoly model with product differentiation, where firms compete in research and development and output (R&D). Before firms compete, each owner makes a strategic decision whether to hire an overconfident manager. The results reveal that under not too much productive R&D technology, it is optimal for owners to hire overconfident managers. Owners will hire overconfident managers who over-invest in R&D, and this decision improves welfare if either R&D productivity is low, or spillovers are large, or products are less substitutes.

Keywords: Delegation, Product differentiation, Overconfidence, R&D, Welfare

JEL: D43, L13, L21, O32

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1. Introduction

Elon Musk, the CEO of *Tesla Inc.*, is one of the most prominent recent examples of how a top manager's personal characteristics could benefit a company. On September 23, 2020, at *Tesla*'s Battery Day event, Elon Musk outlined plans for a $25,000 electric vehicle using cheaper, more powerful batteries, and he targets a goal of eventually producing 20 million electric cars a year (Higgins, 2020). Tough, Elon Musk's project seems ambitious and perhaps unrealistic, he offers an individual trait common among top executives in corporations. If *Tesla Inc.* meets its goals, customers could benefit from a wide range of affordable electric vehicles.

The literature suggests that corporate executives are generally *overconfident* in the sense that they overestimate the expected returns to their corporate decisions on outcomes in which they are highly committed.[2] A manager's excessive confidence in his or her abilities to reduce production costs through innovation, and boost demand and diversity of products may benefit both the corporation and society. We thus explore analytically in this paper how and under which conditions overconfidence could affect research and development (R&D) investments, profits, and welfare (the aggregate sum of consumer surplus and producer profits) in a dynamic setting in which owners or board of directors delegate firms' managerial decisions to overconfident managers who compete in output in the market stage.

Our work builds from and contributes to the literature along four lines: organizational structure, strategic delegation theory, innovation, and welfare in oligopolistic

[2]See Malmendier and Tate (2015) for a literature survey on managers' overconfidence. Empirical studies include Einhorn (1980), Weinstein (1980), Eberhart et al. (2004), Malmendier and Tate (2005, 2008), Galasso and Simcoe (2011), Hirshleifer et al. (2012), Schrand and Zechman (2012), Ben-David et al. (2013), Deshmukh et al. (2013), Phua et al. (2018), and Chen et al. (2019). For theoretical contributions, see Kyle and Wang (1997), Goel and Thakor (2008), Englmaier (2010, 2011), Englmaier and Reisinger (2014), Chen et al. (2014), Yu (2014), Li et al. (2017), Pu et al. (2017), Li et al. (2019), Xu et al. (2019), Tondji (2020), and Schroeder et al. (2021).

industries. Pioneering by Schelling (1980), a strategic delegation in a corporation is a situation in which the owner or the board of directors of the company entrusts a top executive to undertake decisions involving conflicting interests. Schelling (1980) and other studies show that firms could benefit from delegating their managerial decisions to top managers who may have different visions from the owner. The rationale is that in a strategic framework, the manager may induce the board of directors to commit to a credible and observable device that profitably shapes the expectations and behavior of rivals.

Previous studies display a number of course of actions that are profitable in imperfectly competitive markets. The seminal studies by Vickers (1985), Sklivas (1987), and Fershtman and Judd (1987), and a subsequent paper by Lambertini (2000a) in oligopolies postulate that a managerial incentive scheme that constraints a manager to optimize a weighted average of profit and sales volume (revenue) generates greater profits for the owner than by maximizing over simple profit alone. In addition to the use of a contract design, subsequent findings by Zhang and Zhang (1997), Lambertini (2000b), Kräkel (2004), Mitrokostas and Petrakis (2014) show the robustness of profitable strategic delegation by adding an R&D stage before the product-market competition. In this same setting, Overvest and Veldman (2008) show that a contract that provides direct monetary incentives for cost-reducing R&D can proved profitable to the firm. Recent studies use managers' bias, typically overconfidence or aggressiveness as a commitment device to potentially improve owner profit. For instance, Englmaier (2010, 2011) and Tondji (2020) examine the effect of CEO overoptimism (or excessive confidence) on R&D activity in Cournot and Bertrand models; Englmaier and Reisinger (2014) and Yu (2014) study the effect of the type of competition on CEO confidence bias and profit; Pu et al. (2017) address the effect of CEO overconfidence in the presence of yield uncertainty on profit in a Cournot setting; Li et al. (2019) investigate the effect of CEO overconfidence on R&D spillover and profit in a Cournot model; Xu et al. (2019) explores the impact of overconfidence on pricing decision in a duopolistic supply chain context; and Schroeder et al. (2021) explores the effect of CEO bias regarding his or her marketing ability on advertising and profit. We consider managerial overconfidence as a commitment device and explore the welfare implications of hiring an overconfident manager who undertakes cost-reducing R&D in a Cournot framework. To make welfare analyses, we introduce the product differentiation that measures the competitiveness of markets, the R&D spillovers which capture the benefits from over-investments, and the R&D productivity that captures the cost of over-investment.

Our work is also related to organizational structure in markets with product differentiation. More recent studies by Lambertini (2000a), Lambertini (2000b), and Mitrokostas and Petrakis (2014) addressed the issue of full delegation—the manager undertakes market competition (price or quantity) and R&D investment decisions—, and partial delegation—the manager undertakes only market competition (price or quantity) decision—in duopolies. In our setting, an owner can make only two types of binding contracts with shareholders: the delegation contract and the non-delegation contract. If he or she chooses the delegation contract, this means that the firm will have to delegate the R&D investments and production decisions to a manager whatever action the competitor takes. If a firm chooses the non-delegation contract, it commits to making its strategic decisions without hiring a manager independently of the action of the competitor. Consider a four-stage game where firms first simultaneously commit themselves to a type of contract and afterward compete depending upon the chosen models of contracts and the three-stage game under quantity competition. Unlike previous studies that considered a combination of profits and sales volumes as the commitment device, we consider managerial overconfidence, and firm compete in quantities in the market stage.[3] Restricting attention to subgame perfect equilibria of the dynamic game, we show that it is a strictly dominant strategy for an owner to choose the delegation contract and hire an overconfident manager. This result indicates that

[3]In order to simplify the analysis, we assume output competition, but the result holds whether firms compete in output or price.

full delegation and overconfidence may persist and survive in the long run. This finding also implies that adding a stage to model the firm's organizational structure would lead to the conclusion similar to Singh and Vives (1984), Lambertini (2000a), and Lambertini (2000b), namely, that in case of substitutability between products, firms empowered overconfident managers to decide upon the quantity level at the market stage. Moreover, the emergence of full delegation in equilibrium is independent of the size of the pre-innovation cost instead of Mitrokostas and Petrakis (2014).

Overall, we consider a dynamic duopoly model that uses contributions from the organizational structure and strategic delegation theory, innovation, and welfare economics to determine whether owners' decisions to delegate managerial responsibilities to managers who overestimate the intercept of their future market demands can yield better welfare for society. Before firms compete, each owner or board of directors makes a strategic decision: whether or not to hire an overconfident manager. Once owners hire managers, they are responsible for choosing the levels of R&D expenditures and output. We ignore the agency conflict between firms and managers and assume that managers choose R&D activity to maximize firms' profits (according to their prior beliefs on market demands) net of R&D expenditures.[4] The results show that in a strategic setting, an owner's decision to hire an overconfident manager translates into higher welfare if R&D spillovers, the degree of product differentiation, and R&D technology satisfy some mild assumptions. To our knowledge, this is the first formal analysis of the welfare effects of managerial overconfidence in oligopolistic industries. Inspired by previous studies, one can argue that if an overconfident manager is optimal for a firm competing in output or price and the prices decrease, we can infer that welfare will be improving. We will show that this statement only holds if products are less substitutes (or R&D spillovers are large) or R&D technology is less productive.

[4]We follow the existing literature; see, for example, Malmendier and Tate (2005), Englmaier (2010, 2011), Yu (2014), and Li et al. (2019). Also, our incentive scheme does not provide a positive premium for sales volume.

We organize the remainder of the paper as follows. In Section 2, we introduce the model. It augments the Qiu (1997) model by including the delegation stage with potential overconfident managers. Then, we present the predictions and welfare implications of our results. In Section 3, we provide a comparative static analysis. Section 4 describes the four-stage game, and we conclude in Section 5. For clarity, we relegate the proofs of some results in the Appendix.

2. The Model and Cournot Equilibrium

Consider a sector of an economy with two firms (or owners) $i = 1, 2$ producing differentiated goods q_1 and q_2 respectively. Following Singh and Vives (1984), we assume that the representative consumer's utility function is

$$U(q_1, q_2) = a(q_1 + q_2) - \frac{q_1^2 + 2bq_1q_2 + q_2^2}{2},$$

where $a > 0$ is the true market size, and $b \in (0, 1)$ is the degree of product differentiation, with differentiation increasing as b is close to zero. The inverse market demands are linear:

$$p_i = a - q_i - bq_j, \ i, j = 1, 2, \ i \neq j. \tag{1}$$

Firms may invest in cost-reducing R&D. The pre-innovation cost for each firm is c, with $a > c$. If firm i engages in R&D, then by spending $V(x_i)$ on R&D, it lowers its marginal cost by $x_i + \beta x_j$: $c_i(x_i, x_j) = c - x_i - \beta x_j$, where the parameter $\beta \in [0, 1]$ captures the extent of output spillovers. A number of papers adopt the linear form of cost-reduction with spillovers (see, for example, Qiu (1997), Amir (2000), Englmaier (2011), Gama et al. (2019), Li et al. (2019), among others). We assume that the R&D expenditure function is quadratic: $V(x_i) = v\frac{x_i^2}{2}$, where v relates to the productivity of the R&D technology (higher v means lower productivity).[5]

[5] See, for example, d'Aspremont and Jacquemin (1988), Qiu (1997), Amir and Wooders (1999), and the references therein.

Timing is the following.

Stage 1. Each firm hires a manager to take charge of R&D investments and production decisions. Managers have different personalities (types or beliefs) and may over-estimate or under-estimate the size of the market. We assume that there is at least two managers for each type so that the matching between firms and managers is always perfect. Managers' types are observable both to firms and competing managers. If firm i hires manager of type α_i, he or she believes that the inverse market demand of firm i is:

$$p_i = \alpha_i - q_i - bq_j,\ i, j = 1, 2,\ i \neq j. \tag{2}$$

An ***overconfident*** manager over-estimate the true (or real) intercept of the demand function. This means that an overconfident manager of type α_i believes that α_i is greater than a: $\alpha_i > a$. We can also view α_i as manager i's prior belief about a.

Stage 2. Managers simultaneously and independently undertake cost-reducing R&D based on their prior beliefs about the market. As mentioned in the Introduction, we assume that the objectives of the managers are aligned with shareholders' goals after being hired.

Stage 3. The real market size is realized, and both firms produce and sell their products to consumers.

We consider Cournot competition, where firms compete by setting quantities in the market stage, and we derive subgame perfect Nash equilibria using backward induction. We present the problem and most of our solutions only from firm i's perspective for simplicity. The expression for firm j is entirely analogous. For clarity, we relegate some of the proofs of our results in the Appendix.

Assume:

A1. $v > 2a/c$.

Assumption **A1** ensures that every subgame after the initial stage has a unique Nash equilibrium with both firms (and managers) in the quantity competition.

Let π_C^i denote firm i's market profit (profit excluding R&D costs) after the real market is revealed in stage 3. Given any R&D outcome (x_i, x_j),

$$\pi_C^i(q_i, q_j; x_i, x_j) = p_i q_i - (c - x_i - \beta x_j) q_i. \tag{3}$$

Firms choose outputs to maximize their respective market profits, and the Cournot-Nash equilibrium is

$$q_i^*(x_i, x_j) = \frac{1}{(4-b^2)}[(a-c)(2-b) + (2-b\beta)x_i + (2\beta - b)x_j], \tag{4}$$

and firm i's overall market profit $\Pi_C^i(x_i, x_j) = \pi_C^i(q_i^*, q_j^*; x_i, x_j) - V(x_i)$ is:

$$\frac{[(2-b)(a-c) + (2-b\beta)x_i + (2\beta - b)x_j]^2}{(4-b^2)^2} - v\frac{x_i^2}{2}. \tag{5}$$

Turning to the second stage, based on their beliefs about the market in (2), each manager undertakes an R&D level to maximize the firm's expected overall profit

$$\Pi_C^i(x_i, x_j) = \frac{[(2-b)(\alpha_i - c) + (2-b\beta)x_i + (2\beta - b)x_j]^2}{(4-b^2)^2} - v\frac{x_i^2}{2}. \tag{6}$$

Using the first-order conditions, manager i's best R&D strategy in response to firm j's choice of R&D level x_j is:

$$x_i(\alpha_i, x_j) = \frac{2(2-b\beta)}{v(4-b^2)^2 - 2(2-b\beta)^2}[(2-b)(\alpha_i - c) + (2\beta - b)x_j]. \tag{7}$$

Equation (7) and the fact that the best response functions $x_i(\alpha_i, x_j)$ and $x_j(\alpha_j, x_i)$ are symmetric yield $x_i^*(\alpha_i, \alpha_j)$ as firm i's optimal R&D investment in the second stage:

$$x_i^*(\alpha_i, \alpha_j) = \frac{2(2-b)(2-b\beta)}{\Lambda}\, \overline{x}_i(\alpha_i, \alpha_j), \tag{8}$$

where $\overline{x}_i(\alpha_i, \alpha_j) = [(4-b^2)^2 v - 2(2-b\beta)^2]\alpha_i + 2(2-b\beta)(2\beta-b)\alpha_j + [2(2-b\beta)(2+b)(1-\beta) - (4-b^2)^2 v]c$, for $i, j = 1, 2$ and $i \neq j$, and $\Lambda = [(4-b^2)^2 v - 2(2-b\beta)^2]^2 - 4(2\beta-b)^2(2-b\beta)^2$.

2.1. The Cournot-Benchmark Model

In the benchmark model, there is no delegation in the first stage. Then, $\alpha_i = \alpha_j = a$, and the symmetric equilibrium outcomes are:

$$x_{nc} = \frac{2(2-b\beta)(a-c)}{\Delta},\ q_{nc} = \frac{1}{\Delta} v(a-c)(4-b^2),\ \text{and } p_{nc} = a - (1+b)q_{nc},$$

where $\Delta = (2+b)(4-b^2)v - 2(1+\beta)(2-b\beta) > 0$ under **A1**. Therefore, the total producer surplus $[\Pi = \Pi_1 + \Pi_2 = 2(q_{nc})^2 - v(x_{nc})^2]$, the consumer surplus $[CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2 = (1+b)(q_{nc})^2]$, and welfare $[W = CS + \Pi]$ are respectively:

$$\Pi_{nc} = \frac{2(a-c)^2 v}{\Delta^2}[(4-b^2)^2 v - 2(2-b\beta)^2],$$

$$CS_{nc} = \frac{(1+b)(4-b^2)^2(a-c)^2 v^2}{\Delta^2},\ \text{and}$$

$$W_{nc} = \frac{(a-c)^2 v}{\Delta^2}[(3+b)(4-b^2)^2 v - 4(2-b\beta)^2].$$

2.2. The Cournot Model With Delegation and Overconfidence

In the initial stage, each firm chooses a manager of type α_i to maximize its overall expected profits, $\Pi_C^i(\alpha_i, \alpha_j) = \pi_C^i(q_i^*, q_j^*, x_i^*, x_j^*; \alpha_i, \alpha_j) - V(x_i^*(\alpha_i, \alpha_j))$. By the first-order conditions and symmetry, firms hire a manager with a confidence level

$$\alpha_C = a + \frac{2(a-c)v}{\Delta_C}(2+b)(2\beta-b)^2, \tag{9}$$

where $\Delta_C = 4(1-\beta)(2-b\beta)(1+\beta)^2 - 2(2+b)[4-b+2(1-b)\beta](2-b\beta)v + (2-b)^2(2+b)^3 v^2$, the latter being positive under **A1**. Equation (9) yields the symmetric equilibrium R&D investment level:

$$x_C = \left\{1 + \frac{2(2+b)(2\beta-b)^2 v}{\Delta_C}\right\} x_{nc}. \tag{10}$$

A1 guarantees positive post-innovation costs of production in Cournot competition, i.e., $c - (1+\beta)x_C > 0$. Note that if R&D investments are very productive, the firms will invest more to gain a competitive advantage in the market game which will lead to zero, and even negative post-innovation costs.[6]

Substituting x_C into (4) and (1) yield the (symmetric) equilibrium output and price:

$$q_C = \frac{(a-c)v}{\Delta_C}\left\{(4-b^2)^2 v - 2(2-b\beta)^2\right\}, \text{ and } p_C = a - (1+b)q_C. \tag{11}$$

Finally, equilibrium consumer surplus CS_C, total producer surplus Π_C, and welfare W_C in Cournot competition with delegation and overconfidence are given as:

$$CS_C = \frac{(1+b)(a-c)^2 v^2}{\Delta_C^2}\left\{(4-b^2)^2 v - 2(2-b\beta)^2\right\}^2,$$

$$\Pi_C = \frac{2(a-c)^2 v}{\Delta_C^2}\left\{\begin{array}{l} v[(4-b^2)^2 v - 2(2-b\beta)^2]^2 \\ -2(2-b\beta)^2[2(1-\beta^2) - (4-b^2)v]^2 \end{array}\right\}, \text{ and}$$

$$W_C = \frac{(a-c)^2 v}{\Delta_C^2}\left\{\begin{array}{l} (1+b)v[(4-b^2)^2 v - 2(2-b\beta)^2]^2 + 2[v((4-b^2)^2 v \\ -2(2-b\beta)^2)^2 - 2(2-b\beta)^2(2-2\beta^2-(4-b^2)v)^2] \end{array}\right\}.$$

An immediate analysis of (9) and (10) yields the following proposition.

Proposition 1 *For any $b \in (0,1)$ and $\beta \in [0,1]$, $\alpha_C > a$ and $x_C > x_{nc}$ under **A1** and $b \neq 2\beta$. If $b = 2\beta$, then $\alpha_C = a$ and $x_C = x_{nc}$.* □

[6] Note that Qiu (1997) uses a weaker assumption for Cournot competition, $v > \frac{a}{c}$, and a more robust assumption $v > \frac{2(2-b^2-b\beta)^2}{(1-b^2)(4-b^2)^2}$ for Bertrand competition. Also, Semenov and Tondji (2019) use the assumption $v > \frac{3\alpha}{c}$ for Cournot-Bertrand game.

In words, Proposition 1 states that firms hire overconfident managers who undertake over-investment in R&D. Using the first-order conditions and the Envelope Theorem, we can decompose each firm's overconfidence effect into four parts as follows.

$$\frac{\partial \Pi^i}{\partial \alpha_i} = \text{Strategic effect} + \text{Spillover effect} + \text{Size effect} + \text{Cost effect}, \text{ where}$$

$$\text{Strategic effect} = \frac{\partial \pi^i}{\partial q_j}\left\{\frac{\partial q_j}{\partial x_i}\frac{\partial x_i}{\partial \alpha_i} + \frac{\partial q_j}{\partial x_j}\frac{\partial x_j}{\partial \alpha_i}\right\},$$

$$\text{Spillover effect} = \frac{\partial \pi^i}{\partial c_i}\frac{\partial c_i}{\partial x_j}\frac{\partial x_j}{\partial x_i}\frac{\partial x_i}{\partial \alpha_i} = \beta q_i \frac{\partial x_j}{\partial \alpha_i},$$

$$\text{Size effect} = \frac{\partial \pi^i}{\partial c_i}\frac{\partial c_i}{\partial x_i}\frac{\partial x_i}{\partial \alpha_i} = q_i \frac{\partial x_i}{\partial \alpha_i}, \text{ and}$$

$$\text{Cost effect} = -\frac{\partial V_i}{\partial x_i}\frac{\partial x_i}{\partial \alpha_i}.$$

We discuss the four components of the decomposition in reverse order. We assume that **A1** holds. Note that whatever the degree of product differentiation and the extent of spillover effects, overconfidence level is positively correlated to R&D investment: $\frac{\partial x_i}{\partial \alpha_i} > 0$. First, investing in R&D is costly, and hiring overconfident managers increases the cost of R&D investment, implying that the cost effect $(-\frac{\partial V_i}{\partial x_i}\frac{\partial x_i}{\partial \alpha_i})$ is negative for both firms. The cost effect gives the firm disincentive to hire overconfident managers. Second, a firm's R&D lowers its marginal cost of production. Therefore, *ceteris paribus*, the larger is the production, the more the firm benefits. Moreover, the higher is the manager's confidence level higher is the size effect. Thus, the size effect $(q_i\frac{\partial x_i}{\partial \alpha_i})$ is positive for both firms. The size effect gives the firm incentive to hire overconfident managers who undertake R&D investments. Third, an overconfidence level α_i increases a firm's R&D x_i, which lowers both the firm and its rival's cost. In response to a tougher competitor that hires an overconfident manager, the rival prefers to hire a less overconfident manager who undertakes less R&D investment when spillovers are small or products are sufficiently differentiated ($b > 2\beta$). Therefore, the spillover effect is positive when spillovers are large: $\beta q_i \frac{\partial x_j}{\partial \alpha_i} > 0$ when $\beta > \frac{b}{2}$, and it is negative when

Table 1: Decomposition of Overconfidence Incentives

	Strategic effect	Spillover effect	Size effect	Cost effect
Small spillovers ($\beta < \frac{b}{2}$)	+	-	+	-
Large spillovers ($\beta > \frac{b}{2}$)	-	+	+	-

spillovers are small. Finally, hiring an overconfident manager indirectly reduces its production costs, and consequently affects the rival's strategic choice. This is the strategic effect. It is straightforward to see that firm i is hurt when its rival increases its quantity: $\frac{\partial \pi^i}{\partial q_j} < 0$. A firm's rival increases its quantity due to the firm hiring an overconfident manager if spillovers are large: $\frac{\partial q_j}{\partial \alpha_i} > 0$ when $\beta > \frac{b}{2}$. It follows that the strategic effect is positive when spillovers are small, and it is negative when spillovers are large. Table 1 summarizes the above discussion.

We now turn to the quantity, price, and welfare comparison. Directly comparing market equilibrium outcomes under quantity competition with possible overconfident managers with those obtained in a market without delegation (Section 2.1) yields the following result.

Proposition 2 *Assume **A1** holds. For any $b \in (0,1)$ and $\beta \in (0,1)$, with $\beta \neq \frac{b}{2}$.*

(a) $q_C > q_{nc}$, and $p_C < p_{nc}$. Consequently, $CS_C > CS_{nc}$.

(b) $\Pi_C > \Pi_{nc}$ if and only if $\beta > \frac{b}{2}$.

(c) There exists a unique $v_1 = v_1(b, \beta)$ such that $W_C - W_{nc} = \begin{cases} > 0 & if \quad v > v_1 \\ < 0 & if \quad v < v_1 \end{cases}$. □

Proof See Appendix. ■

Proposition 2 implies that under *not too much* productive R&D technology (**A1** is satisfied), overconfident managers make decisions that induce higher productions, lower prices, and higher consumer surplus. If the extent of spillover effects is sufficiently strong ($\beta > \frac{b}{2}$), then delegation increases profits, and therefore welfare. In the case of weak spillover effects ($\beta < \frac{b}{2}$), the cumulative strategic and size effects are not enough to make delegation profitable for owners who fire overconfident managers. The resulting overconfidence level and R&D investments at the equilibrium yield lower profits. However, according to case *(c)* in Proposition 2, overconfidence can still improve welfare if the R&D technology is less productive ($v > v_1$). The gains from consumer surplus outweigh the reduction in profits. In fact, for any $b \in (0, 1)$ and $\beta \in (0, 1)$ so that $\beta > \frac{b}{2}$, the inequality $v > v_1$ always holds under **A1**, and therefore $W_C > W_{nc}$. Furthermore, under **A1**, $v < v_1$ only holds when the R&D spillover effects are relatively low (i.e., β is small), and the product market is almost homogeneous (i.e., b is sufficiently high and close to 1). We illustrate this situation in Figure 1 for parameters: $a = 1$, $c = 0.955$, $\beta = 0.001$, $b = 0.93$, and $v_1 = 2.49068 > \frac{2a}{c} = 2.09424$. From Proposition 2, it results that under sufficiently product differentiation, and assuming that the R&D technology is *not too much* productive (i.e., **A1** is satisfied), even if spillovers are low, we expect society to be better off (i.e., the difference in welfare to be positive). We illustrate this situation in Figure 2 for parameters: $a = 1$, $c = 0.955$, $\beta = 0.001$, and $b = 0.9$.

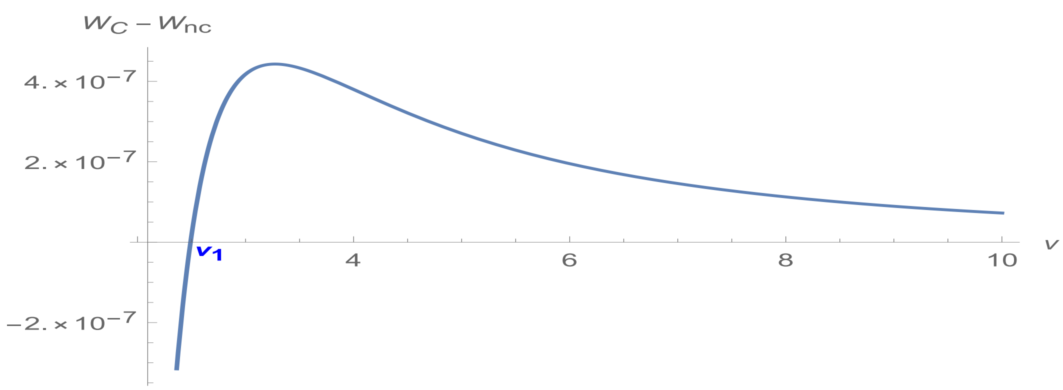


Figure 1: Difference in welfare in a market with less productive R&D technology, weak spillover effects, and homogeneous products.

Remark 1 Instead of quantity competition in the market stage, we obtain similar results in equilibrium if we consider Bertrand competition, a framework in which firms compete in prices. Under price competition in the market stage with differentiated products and less productive R&D technology, firms hire overconfident managers, and they make decisions that induce lower prices, higher productions and higher consumer

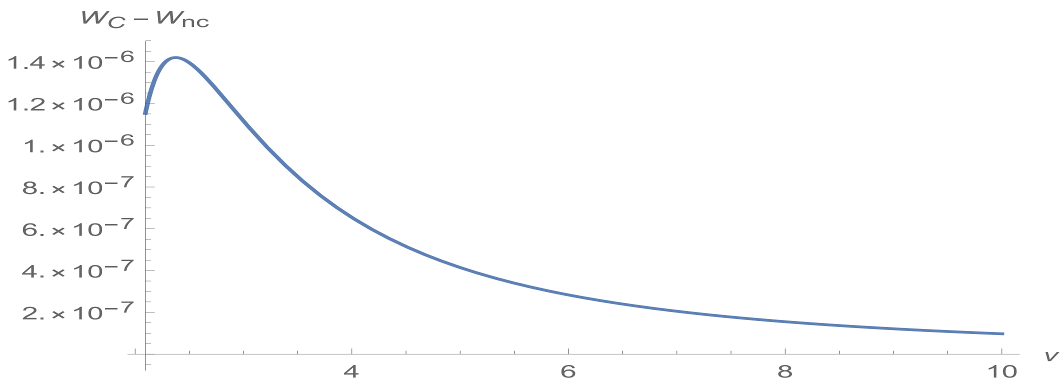


Figure 2: Difference in welfare in a market with *not too much* productive R&D technology, weak spillover effects, and sufficiently product differentiation.

surplus. Moreover, if the spillovers are large ($\beta > \frac{b}{2-b^2}$), then delegation and overconfidence increase profits and welfare. In the case of small spillovers ($\beta < \frac{b}{2}$), then, as in case *(c)* in Proposition 2, overconfidence improves welfare if the R&D technology is less productive (higher v).[7] $\square$

3. Comparative Statics

In this section, we examine how the degree of product differentiation (b), the productivity of R&D technology (v), the pre-innovation cost (c), and the extent of spillovers (β) affect the level of overconfidence, R&D investments, profits, and welfare in equilibrium.

3.1. Product Differentiation

Using (9) and (10), we derive:

$$\frac{\partial \alpha_C}{\partial b} = \frac{4(2\beta - b)(a-c)}{\Delta_C^2} A(b,\beta,v), \text{ and } \frac{\partial x_C}{\partial b} = \frac{2(a-c)}{\Delta_C^2} B(b,\beta,v), \tag{12}$$

where expressions $A(b,\beta,v)$ and $B(b,\beta,v)$ are provided in the Appendix. The analysis of (12) yields the following proposition.

[7]See Tondji (2020).

Proposition 3 *Assume **A1** holds and* $\beta \in (0,1)$*. There exists* $b_1 = b_1(\beta)$*,* $v_2 = v_2(b,\beta)$*, and* $v_3 = v_3(b,\beta)$ *such that:*

1. *If* $0 < b < b_1 < 1$*, then*

   (i) $\frac{\partial \alpha_C}{\partial b} < 0$ *if and only if* $b < 2\beta$*, and*

   (ii) $\frac{\partial x_C}{\partial b} = \begin{cases} > 0 & if \quad v_2 < v < v_3 \\ < 0 & if \qquad v > v_3 \end{cases}.$

2. *If* $0 < b_1 < b < 1$*, then*

   (iii) $\frac{\partial \alpha_C}{\partial b} > 0$*, and*

   (iv) $\frac{\partial x_C}{\partial b} > 0$ *whenever* $v > v_2$*.* □

Proof See Appendix. ■

Proposition 3 unfolds two important messages. First, firms' incentive to hire overconfident managers diminishes when there is less substitutability between products or when innovation is easily accessible by competitors. Second, product differentiation's effects on R&D investments depend on the efficiency of R&D technology and spillovers. These two points convey a key result that, with small spillovers, managers' overconfidence levels and over-investment decisions have a U-shaped relationship with the degree of product differentiation. Initially, they tend to decrease when products are less substitutes and increase with product differentiation. When there is sufficiently strong substitutability between goods, and small spillovers, the strategic and size effects from hiring overconfident managers who undertake R&D investments outweigh the cumulative negative effects from higher production costs and spillover effects. Thus, in equilibrium, the size of R&D investments increases due to managers' overconfidence as long as the R&D technology is less efficient. The latter result also holds with less substitutability between products if the R&D technology is not sufficiently efficient. However, with large spillovers, less substitutability between products induces a decrease in R&D investments when the R&D technology is less productive. Note, however, that over-investment in R&D from overconfident managers can be detrimental to the firm's profitability. For instance, for $b > b_1 > 2\beta$, it holds that $\Pi_C < \Pi_{nc}$ [see Figure 3c for $\beta = 0.1$]. In fact, for $b > b_1$, we can show that the marginal rate of overconfidence ($\frac{\partial \alpha_C}{\partial b}$), which is positive, decreases with spillovers: $\frac{\partial^2 \alpha_C}{\partial \beta \partial b} < 0$. Therefore, the extent of spillovers plays a critical role in the choice of delegating innovative and production activities to overconfident managers. It results that in the equilibrium, firms are more inclined to hire managers with overconfidence levels that induce greater profits [case *(b)* in Proposition 2], and this strategic managerial decision generally translates into higher welfare [case *(b)* in Proposition 2, and for illustration, see Figures 1, 2, and 3d].[8]

Using Proposition 3, we illustrate in Figure 3, the changes in overconfidence levels, R&D investments, profits, and welfare with the degree of product differentiation, for small ($\beta = 0.1$), intermediate ($\beta = 0.5$), and large ($\beta = 0.85$) spillovers. In each case, we consider $a = 1$, $c = 0.95$, and $v = 2.11 > \frac{2a}{c}$. The changes in equilibrium outcomes are monotonic and decreasing as products are highly substitutes for intermediate and large spillovers. When spillovers are small, the U-shaped relationship between overconfidence levels and over-investment decisions with product differentiation imply that firms may be hurt from delegation. In fact, given $\beta = 0.1$, $\Pi_C > \Pi_{nc}$ only if $b < 0.2$. However, consumers benefit the most from delegation, and society overall is better off.

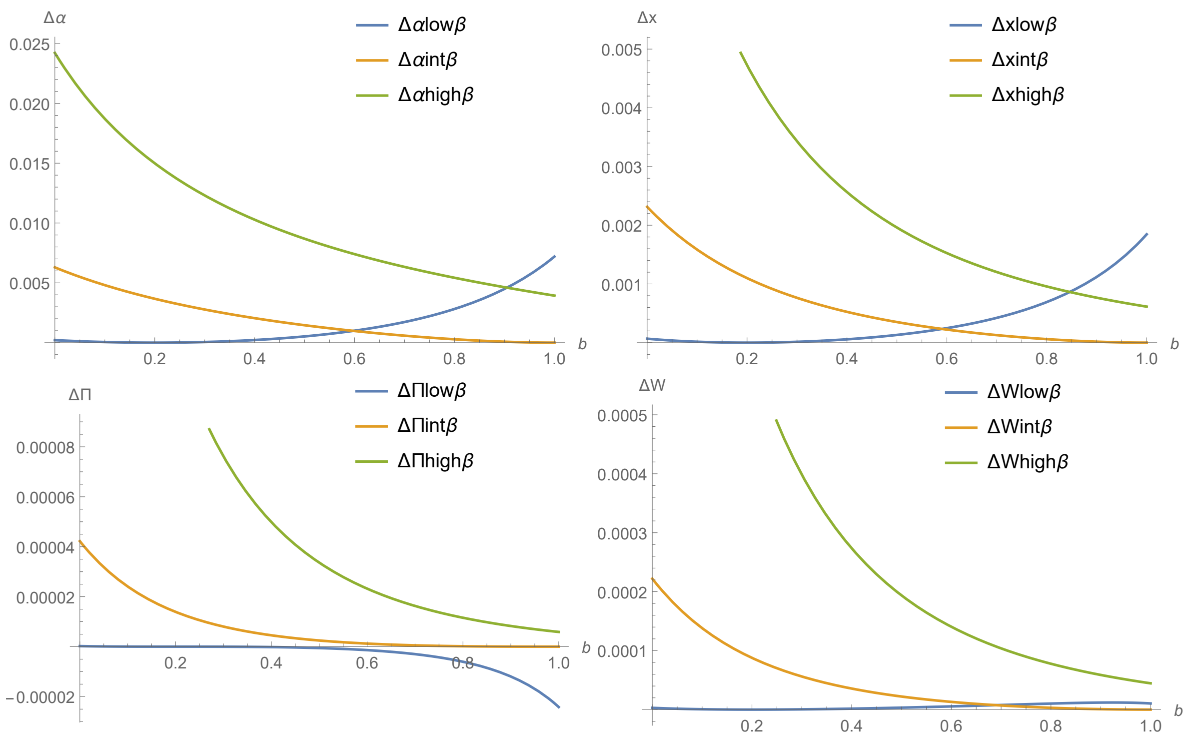
3.2. Productivity of the firm's technology

Differentiating (9) and (10) with respect to the parameter v yield:

$$\frac{\partial \alpha_C}{\partial v} = -\frac{2(2+b)(b-2\beta)^2(a-c)}{\Delta_C^2}[4(1-\beta)(1+\beta)^2(2-b\beta) - (2-b)^2(2+b)^3v^2],$$

[8] Consider that firms compete in prices instead of quantities in the market stage, Tondji (2020) shows that with large spillovers ($\beta > \frac{b}{2-b^2}$) and *not too much* productive R&D technology (Assumption A1), the equilibrium level of overconfidence and R&D investments decrease if products are closer substitutes. Assuming product differentiation and no R&D technology, Englmaier and Reisinger (2014) also find that the level of overconfidence increases with closer substitute products when firms compete in quantities, and it decreases under price competition.

(a) Overconfidence ($\Delta\alpha = \alpha_C - a \geq 0$) (b) Difference in R&D Investments ($\Delta x = x_C - x_{nc} \geq 0$)



(c) Difference in Profits ($\Delta\Pi = \Pi_C - \Pi_{nc} \lesseqqgtr 0$) (d) Differences in Welfare ($\Delta W = W_C - W_{nc} \geq 0$)

Figure 3: Changes in the differences of equilibrium outcomes with the degree of production differentiation when spillovers are small (lowβ), intermediate (intβ), and large (highβ).

and

$$\frac{\partial x_C}{\partial v} = \frac{2(2-b\beta)(a-c)}{\Delta_C^2}[-4(2+b)(2-b\beta)^2(1-\beta^2)+4(2-b)^2(2+b)^3(1-\beta^2)v-(2-b)^3(2+b)^4v^2].$$

Under **A1**, we can show that for any $b \in (0,1)$ and $\beta \in (0,1)$, $4(1-\beta)(1+\beta)^2(2-b\beta)-(2-b)^2(2+b)^3v^2 < 0$, and $-4(2+b)(2-b\beta)^2(1-\beta^2)+4(2-b)^2(2+b)^3(1-\beta^2)v-(2-b)^3(2+b)^4v^2 < 0$. Therefore, the result below unfolds.

Proposition 4 *Assume* ***A1*** *holds. Then, for any $b \in (0,1)$ and $\beta \in (0,1)$: $\frac{\partial \alpha_C}{\partial v} \geq 0$ and $\frac{\partial x_C}{\partial v} < 0$.* □

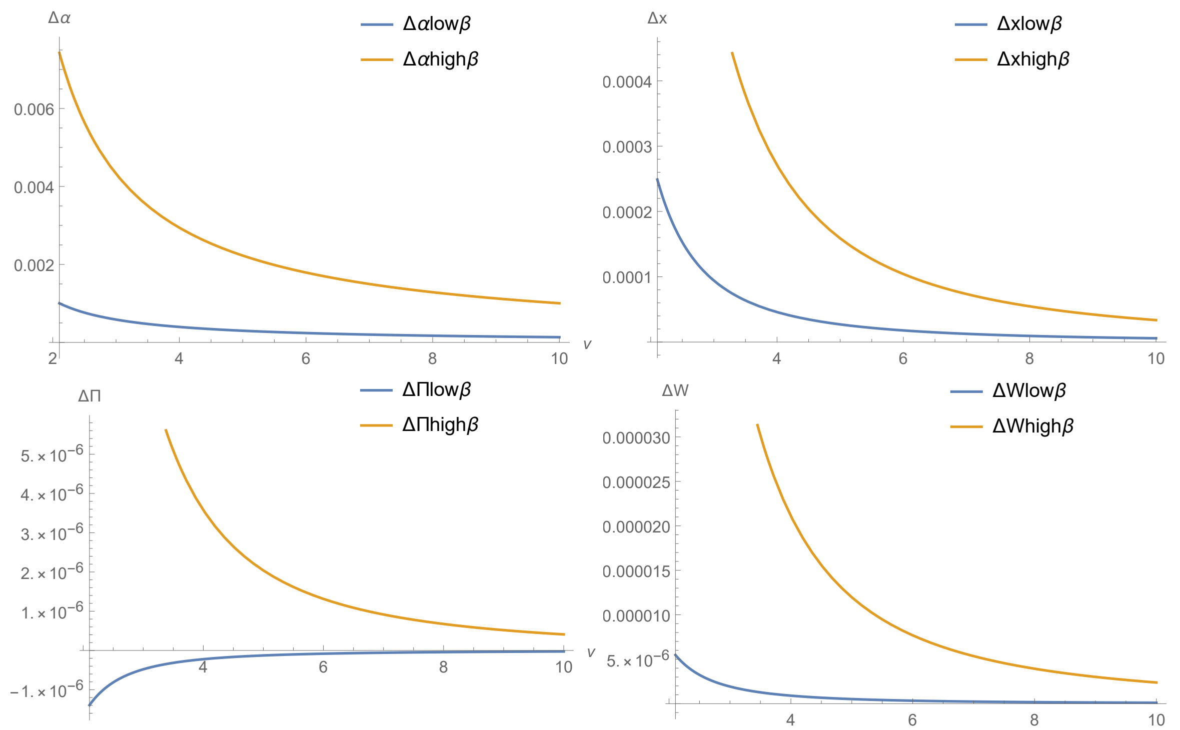
Proposition 4 illustrates the dilemma that firms face in the market competition. Firms are more likely to delegate managerial decisions to overconfident managers who are capable to provide innovative solutions when the R&D technology is less productive. But, investing in R&D is more costly when the technology is not sufficiently productive. Therefore, the cost effects induce overconfident managers to reduce investment in R&D as the technology becomes less effective. Proposition 1 shows that overconfident managers over-invest in innovation. However, the degree of over-investment decreases when the R&D technology is less productive: $\frac{\partial x_i}{\partial \alpha_i} > 0$ and $\frac{\partial^2 x_i}{\partial v \partial \alpha_i} < 0$. Under less productive R&D technology, product differentiation and spillovers affect firm's managerial decisions. For instance, with small spillovers, the incentive for firms to hire overconfident managers as a result of less effective R&D technology declines when the degree of product differentiation between products increases: $\frac{\partial^2 \alpha_C}{\partial b \partial v} < 0$ if and only if $\beta < \frac{b}{2}$. Similarly, when products are highly substitutes, the incentive for firms to hire overconfident managers as a result of less effective R&D technology increases when the extent of spillovers increases: $\frac{\partial^2 \alpha_C}{\partial \beta \partial v} > 0$ if and only if $b > 2\beta$. In Figure 4, we illustrate the changes in the differences in equilibrium outcomes with the productivity of the R&D technology when spillovers are small ($\beta = 0.1$), and large ($\beta = 0.85$). We consider $a = 1$, $c = 0.95$, and $b = 0.6$. Thus, products are sufficiently substitutes. We observe that the changes in overconfidence levels, R&D investments, and welfare with R&D productivity are monotonic and decreasing whatever the size of spillovers. However, as discussed previously, the changes in differences in profits with R&D productivity are non-monotonic. The changes are positive and decreasing when spillovers are large, and they are non-positive and increasing if spillovers are small. Also, the equilibrium outcomes with delegation are closer to the benchmark model when spillovers are small.

3.3. Pre-innovation Costs

Using (9) and (10), we note that:

$$\frac{\partial \alpha_C}{\partial c} = -\frac{2(2+b)(2\beta - b)^2 v}{\Delta_C}, \text{ and } \frac{\partial x_C}{\partial c} = -\frac{2(2-b\beta)\left\{(4-b^2)v - 2(1-\beta^2)\right\}}{\Delta_C}.$$

(a) Overconfidence ($\Delta\alpha = \alpha_C - a \geq 0$) (b) Difference in R&D Investments ($\Delta x = x_C - x_{nc} \geq 0$)



(c) Difference in Profits ($\Delta\Pi = \Pi_C - \Pi_{nc} \lesseqgtr 0$) (d) Differences in Welfare ($\Delta W = W_C - W_{nc} \geq 0$)

Figure 4: Changes in the differences of equilibrium outcomes with R&D productivity when spillovers are small (lowβ), and large (highβ). On the horizontal axis $v > \frac{2a}{c}$.

Under **A1**, whatever the degree of product of differentiation $b \in (0, 1)$, and the extent of spillovers $\beta \in [0, 1]$, the value of Δ_C is positive. Therefore, we have the following result.

Proposition 5 *The equilibrium level of overconfidence and R&D investments decrease in pre-innovation costs.* □

Proposition 5 states that the equilibrium level of managers' overconfidence and R&D investments decrease with pre-innovation cost c: $\frac{\partial \alpha_C}{\partial c} \leq 0$ and $\frac{\partial x_C}{\partial c} < 0$. This result shows that cost-reducing is not necessarily the main incentive that drives firms to hire overconfident managers. In fact, the markup that firms enjoy under market competition decreases with initial production costs, and it is relatively small compared to the one with no delegation. Thus, under *not too much* productive R&D technology (i.e., **A1** is satisfied) and higher production costs, the size effects are not enough to outweigh the cost effects, resulting in firms hiring less confident managers who undertake fewer R&D investments.[9] In Figure 5, we illustrate the changes in the differences in equilibrium outcomes with the pre-innovation cost when spillovers are small ($\beta = 0.1$), and large ($\beta = 0.85$). We consider $a = 1$, $b = 0.6$, and under **A1** we must have $a > c > \frac{2a}{v}$. We observe that the changes in overconfidence levels, R&D investments, outputs, and welfare follow the same pattern described in Figure 4.

3.4. The effect of investment spillovers

To analyze the direct effect of investment spillovers on overconfidence, we differentiate (9) with respect to the parameter β:

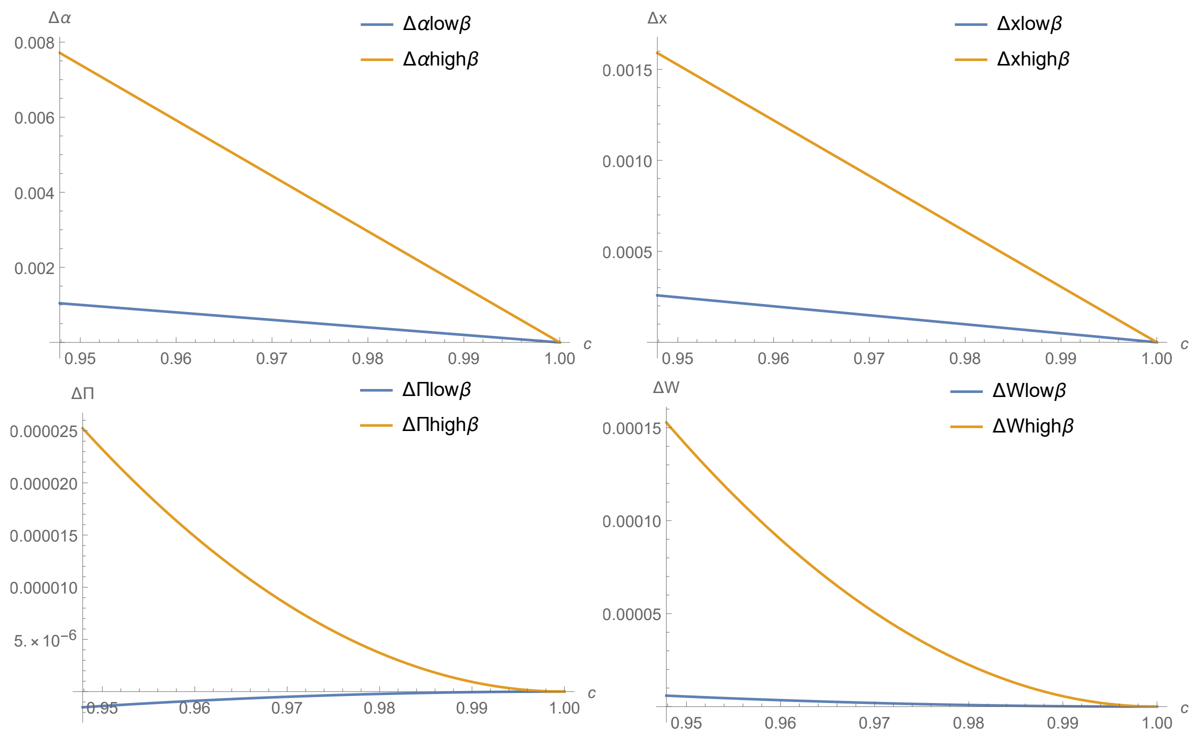
$$\frac{\partial \alpha_C}{\partial \beta} = \frac{4(2+b)(b-2\beta)(a-c)}{\Delta_C^2} C(b,\beta,v), \text{ where} \tag{13}$$

$$C(b,\beta,v) = v \left\{ \begin{array}{l} 2(1+\beta)[-8-2b+b^2+(4+b(8+b))\beta] \\ +\,2(1+\beta)[-2(2+b+2b^2)\beta^2+4b\beta^3] \\ +\,(-2+b)(2+b)^2(b+4b\beta-2(4+\beta))v \\ -\,2(-2+b)^2(2+b)^3v^2 \end{array} \right\}.$$

Using firm i's optimal R&D investment $x_i^*(\alpha_i, \alpha_j)$ in the second stage (8), and equation (13), we can write the derivative of x_i^* with respect to β as $\frac{\partial x_i^*}{\partial \beta} = \frac{\partial x_i^*}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial \beta}$. Under **A1**, we show that for any values of b and β, $C(b,\beta,v)$ is negative, and the sign of $\frac{\partial \alpha_C}{\partial \beta}$ depends on $b - 2\beta$ and $C(b,\beta,v)$. Thus, $\frac{\partial \alpha_C}{\partial \beta} < 0$ if and only if $\beta < \frac{b}{2}$. Given that

[9] Tondji (2020) finds a similar result under price competition in the market stage if spillovers are large and R&D technology is less efficient. Li et al. (2019) find a similar result when Cournot competition in the market stage involves homogeneous products, and the productivity of the R&D technology is fixed at $v = 1$. Assuming product differentiation and no R&D technology, Englmaier and Reisinger (2014) find that the level of overconfidence decreases in production costs under Cournot competition, and it increases with price competition.

(a) Overconfidence ($\Delta\alpha = \alpha_C - a \geq 0$) (b) Difference in R&D Investments ($\Delta x = x_C - x_{nc} \geq 0$)



(c) Difference in Profits ($\Delta\Pi = \Pi_C - \Pi_{nc} \lesseqgtr 0$) (d) Differences in Welfare ($\Delta W = W_C - W_{nc} \geq 0$)

Figure 5: Changes in the differences of equilibrium outcomes with pre-innovation cost when spillovers are small (lowβ), and large (highβ). On the horizontal axis $a > c > \frac{2a}{v}$.

overconfidence is positively correlated to over-investment ($\frac{\partial x_i^*}{\partial \alpha_i} > 0$), it follows that the direction of the effect of investment spillovers on equilibrium overconfidence (α_C) and R&D levels (x_C) is the same, and the proposition below holds.[10]

Proposition 6 *Assume that the firm's technology is not too productive. In the equilibrium, managers' overconfidence levels and over-investment decisions have a U-shaped relationship with the degree of spillovers.* □

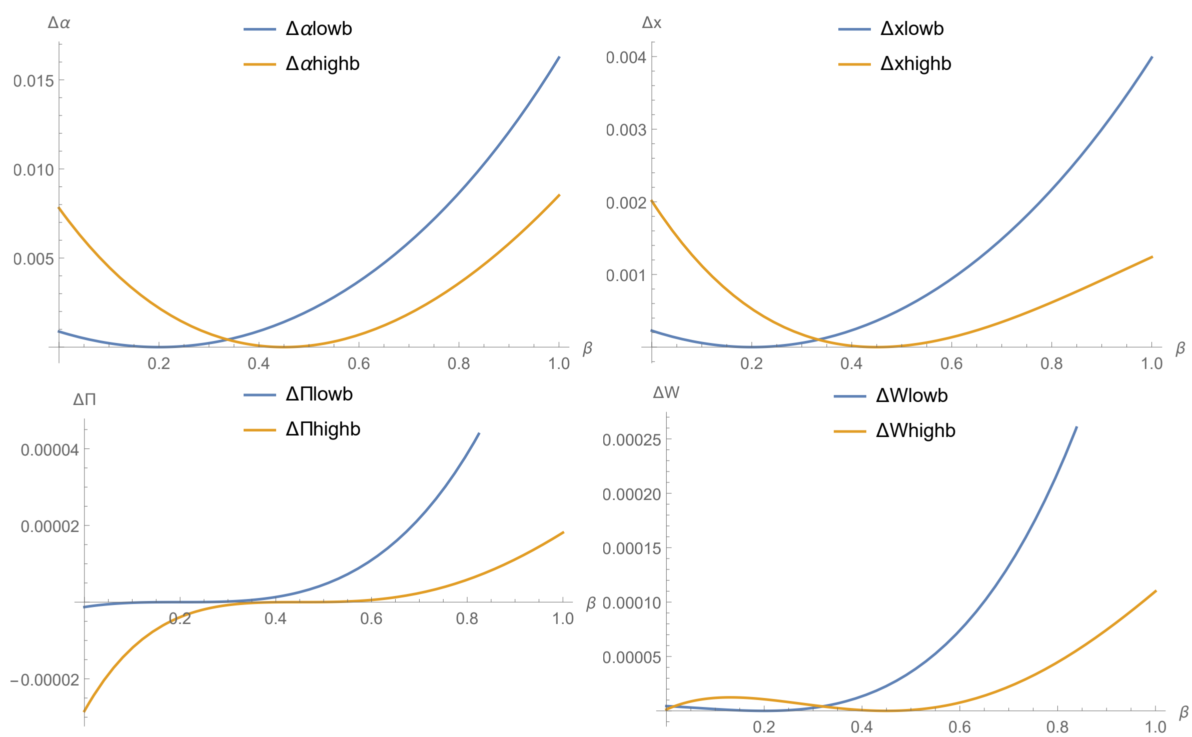
Figure 6 illustrates the changes in equilibrium outcomes with spillovers when products are either highly substitutes ($b = 0.9$) or less substitutes ($b = 0.4$). We consider $a = 1$,

[10]With homogeneous products and relatively productive R&D technology ($v = 1$), managers' overconfidence levels and over-investment decisions also have a U-shaped relationship with the degree of spillovers (see Li et al. (2019)).

$c = 0.95$, and $v = 2.11 > \frac{2a}{c}$. Figures 6a and 6b confirm the fact that α_C and x_C have U-shaped relationship with β. Figure 6c is consistent with previous analyses and Proposition 2. When products are less substitutes, $\Pi_C > \Pi_{nc}$ if $\beta > \frac{0.4}{2} = 0.2$, and with closer substitute products, $\Pi_C > \Pi_{nc}$ if $\beta > \frac{0.9}{2} = 0.45$. From Figure 6d, whatever the degree of product differentiation, the changes in the differences in welfare are non-negative.

(a) Overconfidence ($\Delta\alpha = \alpha_C - a \geq 0$)

(b) Difference in R&D Investments ($\Delta x = x_C - x_{nc} \geq 0$)



(c) Difference in Profits ($\Delta\Pi = \Pi_C - \Pi_{nc} \lesseqgtr 0$)

(d) Differences in Welfare ($\Delta W = W_C - W_{nc} \geq 0$)

Figure 6: Changes in the differences of equilibrium outcomes with spillovers when the degree of product differentiation is low (low$b$), and high (high$b$).

4. The Four-stage game

In this section, we suppose that each firm (or owner) can make only two types of binding contracts with shareholders: the delegation contract (D) and the non-delegation contract (ND). If firm i chooses the delegation contract, this means that it will have to delegate the R&D investments and production decisions to a manager whatever action the rival, firm j, $j \neq i$, takes. If firm i chooses the non-delegation contract, it commits to making its strategic decisions without hiring a manager independently of the action of the competitor. Consider a four-stage game where firms first simultaneously commit themselves to a type of contract and afterward compete depending upon the chosen types of arrangements and the three-stage game under quantity competition described in Section 2. Restricting attention to subgame perfect equilibria of the four-stage game, we show that it is a dominant strategy for firm i to choose the delegation contract and hire an overconfident manager.[11]

Let consider firm i, $i = 1, 2$. In the first stage, if both firms choose the delegation contract, D, then both firms enjoy the Cournot profits Π_C. If they decide to choose the non-delegation contract, then each firm receives its profit Π_{nc}. Denote Π_{C-ND}, a firm i's profit when i chooses non-delegation contract, and the rival firm j chooses the delegation contract, and Π_{C-D} firm i's profit when i chooses the delegation contract and the competitor firm j chooses the non-delegation contract. In the first stage, both firms face the payoff matrix described in Table 2.

		Firm j	
		D	ND
Firm i	D	$(\Pi_C; \Pi_C)$	(Π_{C-D}, Π_{C-ND})
	ND	(Π_{C-ND}, Π_{C-D})	$(\Pi_{nc}; \Pi_{nc})$

Table 2: Payoff Matrix (Four-stage game)

When firms choose different strategies in the first stage, the firm that commits to

[11] The same result holds if the post-contract framework's market stage is price competition with product differentiation or the Cournot-Bertrand model (see, for example, Sato (1996), Tremblay and Tremblay (2011), and Semenov and Tondji (2019)). Proofs can be obtained upon request.

delegate its investment and production decisions chooses the equilibrium level of confidence:

$$\alpha_D = a + \frac{2(2+b)(b-2\beta)^2(a-c)v}{\Delta_D}\left\{(2-b)^2(2+b)v - 2(1-\beta)(2-b\beta)\right\}$$

with $\Delta_D = -8(2-b\beta)^2(1-\beta^2)^2 + 4(2-b\beta)^2(12-4b\beta-8\beta^2-b^2(2-3\beta^2))v - 6(4-b^2)^2(2-b\beta)^2v^2 + (4-b^2)^4v^3$. Note that under **A1**, $\Delta_D > 0$, and $\alpha_D > a$. After computation, we report that:

$$\Pi_{C-D} = \frac{(a-c)^2v}{\Delta_D}\left\{(2-b)^2(2+b)v - 2(1-\beta)(2-b\beta)\right\}^2, \text{ and}$$

$$\Pi_{C-ND} = \frac{v(a-c)^2(v(4-b^2)^2 - 2(2-b\beta)^2)}{\Delta_D^2}\left\{\begin{array}{l} 4(1-\beta)^2(1+\beta)(2-b\beta) \\ -2(2-b)(2-b\beta)(4+b-2(1+b)\beta)v \\ +(2-b)^3(2+b)^2v^2 \end{array}\right\}^2.$$

In Appendix, we show that, under **A1**, $\Pi_C > \Pi_{C-ND}$, and $\Pi_{C-D} > \Pi_{nc}$.[12] It follows that it is a dominant strategy for firms to choose delegation, and therefore, hire overconfident managers. Proposition 7 states the result.

Proposition 7 *In the four-stage game, it is a dominant strategy for owners to choose the delegation contract.* □

Proposition 7 indicates that full delegation and overconfidence may persist and survive in the long run. This finding also implies that adding a stage to model the firm's organizational structure would lead to the conclusion similar to Singh and Vives (1984), Lambertini (2000a), and Lambertini (2000b), namely, that in case of substitutability between products, firms empowered overconfident managers to decide upon the quantity level at the market stage. Unlike Mitrokostas and Petrakis (2014), our result holds

[12] A comparison of profits Π_{C-D} and Π_{C-ND} shows that $\Pi_C > \Pi_{C-ND} > \Pi_{C-D} > \Pi_{nc}$ if and only if $b < 2\beta$.

whatever the size of pre-innovation cost.[13] The survival of overconfidence in the long run is due to the fact that it acts as a commitment device to aggressive managers. Kyle and Wang (1997) consider a Cournot duopoly model of informed speculation with two traders, and show that overconfidence can persist and survive in the long run since it may strictly dominate rationality. Overconfidence can also survived in imperfectly markets for other strategic reasons. For instance, Gervais et al. (2002) find that overconfident managers can make better decisions than rational managers in coordinating the interests of the shareholders; and Gervais et al. (2011) show that overconfident managers are more attractive to firms than their rational counterparts because overconfidence commits them to exert effort to learn about projects. In an imperfectly competitive market, information about demand is a crucial advantage for profit-maximizing firms. A firm might be motivated to hire an overconfident and risk-taking manager because owner expects the hired manager to use his or her personal attributes to overcome market uncertainty. In that setting, delegating strategic decisions to overconfident managers could be an optimal managerial decision for owners (see, for example, Bhardwaj (2001), Acemoglu et al. (2007), Mackey (2008), Bloom et al. (2012), Liozu and Hinterhuber (2013), Kala (2019), and Schroeder et al. (2021)). On the other side, delegation can be detrimental to the firm if, for instance, the commitment device does not constraint the manager to use his or her superior informational advantage to make decisions against the objectives of the principal (see, for example, Acemoglu et al. (2007), Alonso et al. (2008), and Ruzzier (2018)).

[13] Mitrokostas and Petrakis (2014) add the possibility for owners to delegate R&D activity to managers in the classic studies by Vickers (1985), Sklivas (1987), and Fershtman and Judd (1987), and find that with homogeneous product-market competition, full delegation emerges in the subgame perfect equilibrium if the pre-innovation cost is high. In our model, we use a different course of action, namely, managerial overconfidence, and products are differentiated.

5. Conclusion

This paper provides a framework to examine the relationship between managers' characteristics in corporations and welfare in product markets. For strategic reasons, an owner or board of directors of a company decides to delegate innovation and production decisions to managers who over-estimate demands in a differentiated duopoly with R&D competition. We show that the owner hires an overconfident manager under *not too much* productive R&D technology who over-invests in R&D and produces more output in markets. If R&D productivity is low, and spillovers are large, or products are less substitutes, strategic delegation in corporations induces higher welfare relative to imperfectly competitive modes with no delegation.

Appendix A. Proof of Results

Proof (Proposition 2) (a) Comparing quantities give

$$q_C = \left\{1 + \frac{4(2\beta - b)^2(1+\beta)(2-b\beta)}{(4-b^2)\Delta_C}\right\} q_{nc}.$$

Under **A1**,$\Delta_C > 0$, and given that $\beta \neq \frac{b}{2}$, it is immediate that $q_C > q_{nc}$. Given that $p_C - p_{nc} = -(1+b)(q_C - q_{nc})$ and $CS_C - CS_{nc} = (1+b)(q_C^2 - q_{nc}^2)$, it follows that $p_C < p_{nc}$ and $CS_C > CS_{nc}$.

(b) With the help of Wolfram Research Inc. (2019), we can write

$$\Pi_C - \Pi_{nc} = \frac{16(a-c)^2(2-b\beta)(2\beta-b)^3 v^2}{\Delta_C^2\ \Delta^2} f(b,\beta,v),$$

where $f(b,\beta,v) = f_0(b,\beta) + f_1(b,\beta)v + f_2(b,\beta)v^2$, with $f_0(b,\beta) = 2(1+\beta)^2(2-b\beta)(4+b-2(1+b)\beta)$, $f_1(b,\beta) = -(2+b)^2(2-b\beta)(8+6\beta-b(3+4\beta))$, and $f_2(b,\beta) = (2-b)^2(2+b)^4 > 0$.

The function $f(b,\beta,v)$ is convex in v, and there are two real solutions v_1^f and v_2^f (with $v_1^f < v_2^f$) to equation $f(b,\beta,v) = 0$ for any b and β. By definition, $f(b,\beta,v) > 0$ for

$v > v_2^f$. Given that any v that satisfies **A1** is such that $v > v_2^f$, it follows that for any b and β, $f(b, \beta, v) > 0$, and sign $(\Pi_C - \Pi_{nc}) =$ sign $(2\beta - b)$. It is immediate that $\Pi_C > \Pi_{nc}$ if and only if $\beta > \frac{b}{2}$.

(c) In the same manner, as in (b), we can write

$$W_C - W_{nc} = \frac{8(2-b\beta)(2\beta-b)^2(a-c)^2v^2}{\Delta_C^2\ \Delta^2} g(b, \beta, v),$$

where $g(b,\beta,v) = g_0(b,\beta) + g_1(b,\beta)v + g_2(b,\beta)v^2$, with $g_0(b,\beta) = 2(1+\beta)^2(2-b\beta)(8 - b^2(3+b) + 16\beta + 4b\beta - 2(1+b)(6-b^2)\beta^2) < 0$, $g_1(b,\beta) = 2(2+b)^2(-2+b\beta)(8 + b(-6+(-2+b)b) + 28\beta + (-3+b)b(4+3b)\beta + 2(8+b(-5+(-2+b)b))\beta^2) > 0$, and $g_2(b,\beta) = (2-b)^2(2+b)^5(1-b+(3-b)\beta) < 0$.

Note that sign $(W_C - W_{nc}) =$ sign $(g(b,\beta,v))$. The function $g(b,\beta,v)$ is convex in v, and there are two real solutions v_1^g and v_2^g (with $v_1^g < v_2^g$) to equation $g(b,\beta,v) = 0$ for any b and β. Any v that satisfies **A1** is such that $v > v_1^g$. Since $g(b,\beta,v)$ is convex, $g(b,\beta,v) < 0$ if $v_1^g < v < v_2^g$ and $g(b,\beta,v) > 0$ if $v > v_2^g$. Therefore, $W_C < W_{nc}$ if $v < v_2^g$ and $W_C > W_{nc}$ if $v > v_2^g$. Consider $v_1 = v_2^g(b,\beta)$. ∎

Proof (Proposition 3) Rewriting (12):

$$\frac{\partial \alpha_C}{\partial b} = \frac{4(2\beta-b)(a-c)}{\Delta_C^2} A(b,\beta,v) \text{ and } \frac{\partial x_C}{\partial b} = \frac{2(a-c)}{\Delta_C^2} B(b,\beta,v).$$

$\frac{A(b,\beta,v)}{v} = A_0(b,\beta) + A_1(b,\beta)v + A_2(b,\beta)v^2$, where $A_0(b,\beta) = -4(-1+\beta)(1+\beta)^2(-4 - 3b + (2+b+b^2)\beta + 2\beta^2) < 0$, $A_1(b,\beta) = 2(2+b)^2(-1+\beta^2)(b+4b\beta-2(4+\beta)) > 0$, and $A_2(b,\beta) = (-2+b)(2+b)^3(4+b^2-4b\beta) < 0$. The sign of $\frac{\partial \alpha_C}{\partial b}$ depends on $2\beta - b$ and $A(b,\beta,v)$ since Δ_C is positive under **A1**. For a given b and β, $\frac{\partial^2 A(b,\beta,v)}{\partial v^2} = 2A_2(b,\beta) < 0$, then the function $A(b,\beta,v)$ is concave in v. Moreover, there are two real solutions s_1^A and s_2^A (with $s_1^A < s_2^A$) to equation $A(b,\beta,v) = 0$ for any b and β. By definition, $A(b,\beta,v) < 0$ for any $v > s_2^A$. Given that any v that satisfies **A1** is such that $v > s_2^A$, it follows that for any b and β, $A(b,\beta,v) < 0$. Therefore, $\frac{\partial \alpha_C}{\partial b} < 0$ if and only if $2\beta > b$.

Similarly, using (12), we write $\frac{B(b,\beta,v)}{v} = f_0(b,\beta) + f_1(b,\beta)v + f_2(b,\beta)v^2$, with $f_0(b,\beta) = -8(-2+b\beta)^2(-1+\beta+b\beta)(-1+\beta^2) < 0$, $f_1(b,\beta) = 4(2+b)^2(-1+\beta)(2b^3\beta(1+\beta) - 4(2+\beta(2+\beta)) + 4b(3+\beta(5+\beta)) - b^2(5+2\beta(5+3\beta))) > 0$, and $f_2(b,\beta) = -2(-2+b)^2(2+b)^3(2-3b+(2+(-1+b)b)\beta)$. The sign of $\frac{\partial x_C}{\partial b}$ depends on $B(b,\beta,v)$ since Δ_C is positive under **A1**. For a given b and β, $\frac{\partial^2 B(b,\beta,v)}{\partial v^2} = 2f_2(b,\beta)$. However, there exists $b_1 = b_1(\beta) = \frac{3+\beta}{2\beta} - \frac{1}{2}\sqrt{\frac{(1-\beta)(9+7\beta)}{\beta^2}}$ such that for any $\beta \in (0,1)$, $f_2(b,\beta) > 0$ if $0 < b_1 < b < 1$, and $f_2(b,\beta) < 0$ if $0 < b < b_1 < 1$. Furthermore, there are two real solutions t_1^f and t_2^f to equation $B(b,\beta,v) = 0$ for any b and β. We differentiate two cases.

1. Assume $0 < b < b_1 < 1$.

   (i) As shown above, for a given b and β, $A(b,\beta,v) < 0$ under $0 < b < b_1 < 1$. Therefore, $\frac{\partial \alpha_C}{\partial b} > 0$ if and only if $2\beta - b < 0$.

   (ii) We have $f_2(b,\beta) < 0$, the function $B(b,\beta,v)$ is concave in v, and $0 < t_1^f < t_2^f$. Given any b and β, by definition $B(b,\beta,v) > 0$ if $t_1^f < v < t_2^f$, and $B(b,\beta,v) < 0$ if $v < t_1^f$ or $v > t_2^f$. Under **A1**, $v > t_1^f$. Therefore, $\frac{\partial x_C}{\partial b} > 0$ if $t_1^f < v < t_2^f$, and $\frac{\partial x_C}{\partial b} < 0$ if $v > t_2^f$. Consider $v_2 = t_1^f$ and $v_3 = t_2^f$.

2. Assume $0 < b_1 < b < 1$.

   (iii) We proved above that for a given b and β, $A(b,\beta,v) < 0$ under $0 < b_1 < b < 1$. Given that $2\beta - b < 0$ under $0 < b_1 < b < 1$, it follows that $\frac{\partial \alpha_C}{\partial b} > 0$.

   (iv) We have $f_2(b,\beta) > 0$, the function $B(b,\beta,v)$ is convex in v, $t_1^f > 0$, $t_2^f < 0$, and $t_1^f > t_2^f$. By definition $B(b,\beta,v) > 0$ for $v > t_1^f$. Given that under **A1**, $v > t_1^f$, then $\frac{\partial x_C}{\partial b} > 0$. Consider $v_2 = t_1^f$. ∎

PROOF (PROPOSITION 7)

$$\Pi_C - \Pi_{C-ND} = \frac{v(a-c)^2}{\Delta_C^2 \Delta_D^2} O(b,\beta,v),$$

where $O(b,\beta,v) = [-2(2-b\beta)^2(2-2\beta^2-(4-b^2)v)^2+v(-2(2-b\beta)^2+(4-b^2)^2v)^2]\Delta_D^2 - [(-2(2-b\beta)^2+(4-b^2)^2v)(4(1-\beta^2)(1-\beta)(-2+b\beta)+2(2-b)(2-b\beta)(4+b-2(1+b)\beta)v-(2-b)^3(2+b)^2v^2)^2]\Delta_C^2$. Under **A1**, $O(b,\beta,v) > 0$, for all b and β. Therefore, $\Pi_C > \Pi_{C-ND}$.

Similarly,

$$\Pi_{C-D} - \Pi_{nc} = \frac{v(a-c)^2}{\Delta^2\Delta_D^2} 8(b-2\beta)^4(2-b\beta)^2 v.$$

Assuming $\beta \neq \frac{b}{2}$, it is evident that $\Pi_{C-D} > \Pi_{nc}$. ■

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