Macroeconometrics
Group Assignment 3

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Abstract

This assignment has the goal of studying the relationship between inflation and inflation uncertainty for Finland, using data that ranges from 1995 to 2021. For such purpose, GARCH and Threshold GARCH will be used to generate measures of inflation uncertainty. After this task has been completed, a Granger causality test will be performed in order to investigate the relationship between average inflation and inflation uncertainty.

Our findings suggest that inflation in Finland is on the one hand (granger) caused by inflation itself as well as by the uncertainty. These test are robust disregarding whether inflation uncertainty is estimated by an GARCH(1,1) or GJR-GARCH(1,1,1).

1 Introduction

As previously stated, this paper aims to study the relationship between Finland's' inflation and it's inflation uncertainty, a similar assessment to the one Grier and Perry performed for the G7 countries for the period between 1948 and 1993.

In this study, however, the selected time period ranges from 1995 to 2021, discarding the 1950–1970 time span because of a apparently spoilt data which disturbs data analysis and the conclusions that could be taken from it 4. This situation may be caused by a myriad of reasons, probably the lack of data quality regarding that period (or other data collection issues or statistical constraints). But one can only speculate about this, since the only certainty involves the disrupting influence this may exert over the analysis. Additionally, the exclusion of the 1970-1995 period is due to the fact that, in 1995, Finland became a member of the European Union, which not only meant the loss of monetary policy sovereignty but also implied the transfer of decision-making power from the national central's bank to the ECB .Therefore, Maastrich Treaty's stability criteria (which involved price stability preoccupation) had to be followed and guaranteed, which evidently sets another policy-setting regarding inflation. Thus, if that structural break was ignored and the 1950-2021 era was considered as a whole, this research would be would lead to biased results. By scratching that period out of this assessment, the relationship between inflation and inflation uncertainty becomes clearer and coherent. Both of these assumptions aforementioned may be verified in a robustness check, displayed in the appendix, demonstrating that the selected timespan (1995-2007) is the most adequate and fitted for the scrutiny at stake. In order to engage in such a scrutiny, both variables have to be obtained This simple introduction should make clear how the values for inflation and inflation uncertainty were obtained.

To obtain a credible and reliable inflation rate, the most well known indicator of inflation, the consumer price index data was used so that Finland's annualized monthly inflation rate was approximated by $inflation = 12 * 100 * \Delta \log CPI$. This computation formula not only granted a suitable approximation to the annualized monthly inflation rate but also resulted in a stationary time series, which would not be the case for the simple CPI time series. This method of calculating inflation based on the consumer price index is widely adopted by economists. However, we observe that our data sample and observations may vary and be significantly different from the inflation obtained with other calculation methods (e.g. Laspeyres Price Index).

When it comes to inflation uncertainty, the methodology used was the same as the one used in the paper by Grier and Perry. This means that a GARCH(q,p) model was used to estimate the inflation's time-varying residual variance, which was then used as a proxy for inflation uncertainty.

For the purpose of this paper, values of $p = q = 1$ were selected, as it is the most commonly used GARCH specification. However, this model assumes that positive and negative shocks have the same impact on the conditional variance, and in practice it is found that that is not very often the case. For that reason, it is later on used a GJR model, which accounts for the asymmetric impacts that shocks have on the conditional variance, i.e negative shocks have a bigger impact than positive shocks.

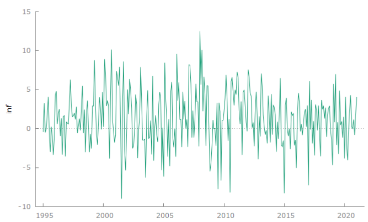
2 Finland's Results

2.1 Preliminary Analysis

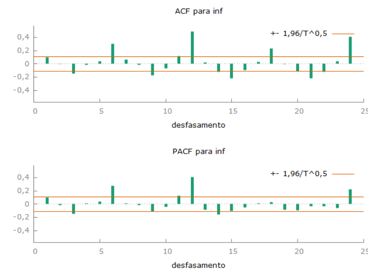
The analysis performed was of the CPI, after plotting the time series (appendix), there was a suspicion of non-stationarity, so an ADF test with a trend was performed were the null is $H_0 = unit - root$ and the resulting p-value was of 0,2279 which, on a 5% or 10% significance level, meant non-stationarity. This problem was taken care of when the approximation of the annualized monthly inflation rate was computed. A plot of the annualized monthly inflation rate 1a, should tip off the reader that the series is stationary. This observation is supported by both the ADF including a constant for non-stationarity with a $p-value = 0,0023$ and the Phillips-Perron test with a $p - value = 0,0000$. Both results suggest to reject $H_0 = unit - root$ and it can be assumed that the series is stationary.

2.2 Modelling

The first step to be taken in the modelling process is to plot the annualized monthly inflation rate correlogram 1b and see what ARMA model could caption the dynamics of the series. Both the ACF and the PACF



(a) annualized monthly inflation rate time series

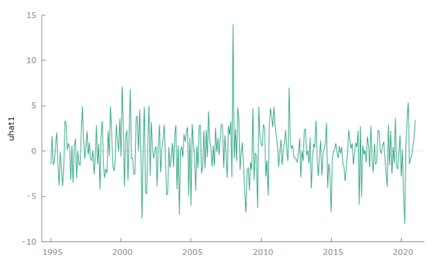


(b) annualized monthly inflation rate correlogram

show significant lags upon the 24th with high significance every 6 lags. This suggests to model either an ARMA(6,6), ARMA(12,12), AR(6) or an AR(12). This specifications also takes into account that the data are monthly and that there could be a semi-annual or annual sesonality in the data. In order to select from the possible models the one that best models the dynamics of the underlying time series and thereby avoids over-fitting, we estimate both ARMA models and compare them by using the Akaike Information Criteria. The AIC obtained for both AR-models is bigger than the one for the ARMA(6,6). Here the $AIC = 1600.708$, while for the ARMA(12,12) $AIC = 1577.761$. Since the AIC is smaller for the ARMA(12,12) we consider it for the further analysis.

The following GARCH models are fit on the resulting errors ϵ_t of the mean process. It's assumed that they can be split into a stochastic component z_t and the time-dependent standard derivation σ_t. $\epsilon_t = \sigma_t z_t$.

Given that GARCH models are used to model the conditional variance of the residuals, our attention must now turn to the residuals coming from the ARMA(12,12) framework used to model the mean of the annualized monthly inflation rate. The residuals are shown in 2a. From a visual observation they appear



(a) residuals time series

(b) residuals correlogram

Figure 2: Residuals plots

to have a constant mean of zero. The ACF and PACF 2b indicates that they are as well uncorrelated. We further apply a Breusch-Pagan test for heteroskedasticity. $H_0 : \sigma_t^2 = \sigma^2$. We reject the null hypothesis of homoskedasticity with $p - value = 0.0000$ and decide for the alternative hypothesis of homoskedasticity. This time time varying variance in the time series is gonna be captured with the different GARCH- and GJR-GARCH-models.

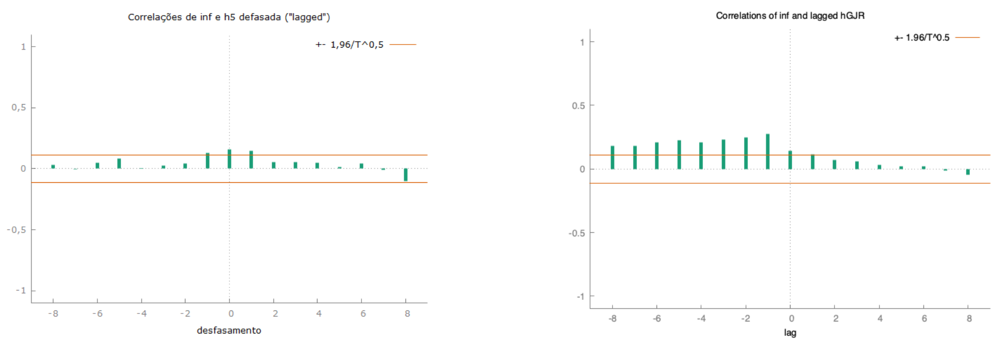
For modeling the GARCH(1,1) we assume two different distribution of the error terms which are common in financial econometrics. The first using a normal distribution in the errors and the second using a t-skewed distribution. Given that the first had a lower AIC we opted for that. And not all the coefficients seem to be relevant, since the constant and α have p-values a bit over 0,10.

For comparison reasons the conditional variance of a GARCH(1,1) was also implemented using the inflation variable and 12 of its' lags (AR(12)), it would seem that the omission of the MA dynamics makes a big difference in the results (5). Let's just note that given the GARCH equation, $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$ this would mean that the previous errors do not impact on the conditional variance.

The residuals of the GARCH model appear to have mean zero and that they are uncorrelated except for 2 lags that seem to be correlated 6, but we believe that may be due to seasonality. We performed an ARCH test on the squared residuals, regressing them on a contant using OLS, the null hypothesis is that there are no ARCH effects present. The result was $p = P(Qui - quadrado(12) > 0,3891) = 1$, meaning we don't reject the null and that our GARCH(1,1) captures all the ARCH effects.

Following the same mythology we estimated an GJR-GARCH(1,1,1) $\sigma_t^2 = \omega +^2_{t-1} + I\gamma_1 u^2_{t-1} + \beta\sigma^2_{t-1}$ with the residuals of the ARMA(12,12). I is the indicator variable which becomes 1 if a shock is negative. The resulting model looks as follows: $\sigma_t^2 = 0.3785 + 0.0082u^2_{t-1} + I - 1.1026u^2_{t-1} + 0.9283\sigma^2_{t-1}$ All estimated coefficients apart from α are significant to 5% level. The ARCH-test based on the linear regression with squared residuals and constant proofs again that this model captured all the all the ARCH-effects. $p = P(Qui - quadrado(12) > 0,0455) = 1$. The estimated conditional variance from the GARCH(1,1) and GJR-GARCH(1,1,1) are shown in the appendix 7

The conditional variance is handled as a proxy for inflation uncertainty. The conditional variance is



(a) cross correlogram between inflation rate and uncertainty GARCH

(b) cross correlogram between inflation rate and uncertainty GJR-GARCH

Figure 3: Cross-Correlogram for both models

stationary in both models. Proven by an ADF test p-value is $p = 0.00$.

The findings for cross-correlation differ pending on the model. 3. The cross-correlogram from the GARCH(1,1) indicates that there is no apparent degree of a correlation between lagged inflation uncertainty and inflation except maybe for the first lag of inflation uncertainty, but even in that case it seems to be very small.

This conveys that inflation uncertainty would probably not be good at helping to predict future values of inflation. However, a Granger test must be performed in order to be sure. This test consists in performing an OLS regression in which the inflation rate is regressed on inflation uncertainty lags and on its own lags.

To start, the command –lagselect was used to decide how many lags to use for the test, and according to the Akaike criteria 1 lag should be used. Executing the test, we now turn our attention to the F-test results. For the null hypothesis, we have that all the lagged coefficients of the uncertainty in the regression are equal to zero and for the alternative hypothesis we have that that inflation uncertainty causes inflation. The obtained values for the F-statistics were $F(1, 309) = 5,366$ with a $p - value = 0,021$ meaning we confidently reject the null hypothesis of the F-test and say that inflation uncertainty Granger Causes inflation. It is also note worthy that since we only have one lag of inflation uncertainty, the F-test result is the same as the p-value of the lag of inflation uncertainty which is statistically significant with a $p - value = 0,021$. The results of the regression being $inf_t = 0,078inf_{t-1} + 0,234uncertainty_{t-1}$.

The same regression was performed but for the alternative, that inflation Granger causes inflation uncertainty. The obtained values for the F-statistics were $F(1, 309) = 0,50262$ with a $p - value = 0,4789$ which which means we cannot reject the null of the F-test and that we can confidently say that inflation does not

Granger Cause inflation uncertainty. The regression result was $uncertainty = 3,03341 + 0,0186inf_{t-1} + 0,584uncertainty_{t-1}$ were only the uncertainty lag and the constant are relevant, given that the coefficient of the lagged inflation as a $p - value = 0,479$.

Again for estimating whether uncertainty granger causes inflation in the GJR-GARCH set-up we following the previously described mythology. The lag selection function yields to an integration of 3 lags in the VAR-system. Only the first lag of uncertainty and the third of inflation is significant in the first VAR to explain inflation. But judging by the $p - value(F) = 0.00$ the whole model with 3 lags is significant. Uncertainty on the other hand is granger caused caused by first lag of inflation and the first lag of uncertainty itself. And again even if not all parameters are significant the F-statistic suggests that the model describes the data well $p - value(F) = 0.00$.

Appendix

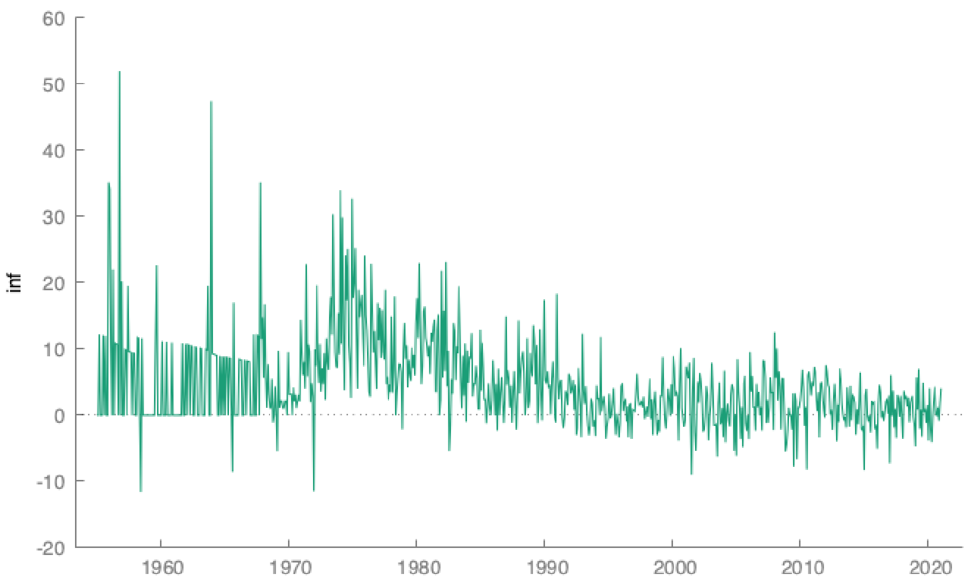


Figure 4: Inflation 1955-2021: visualisation only

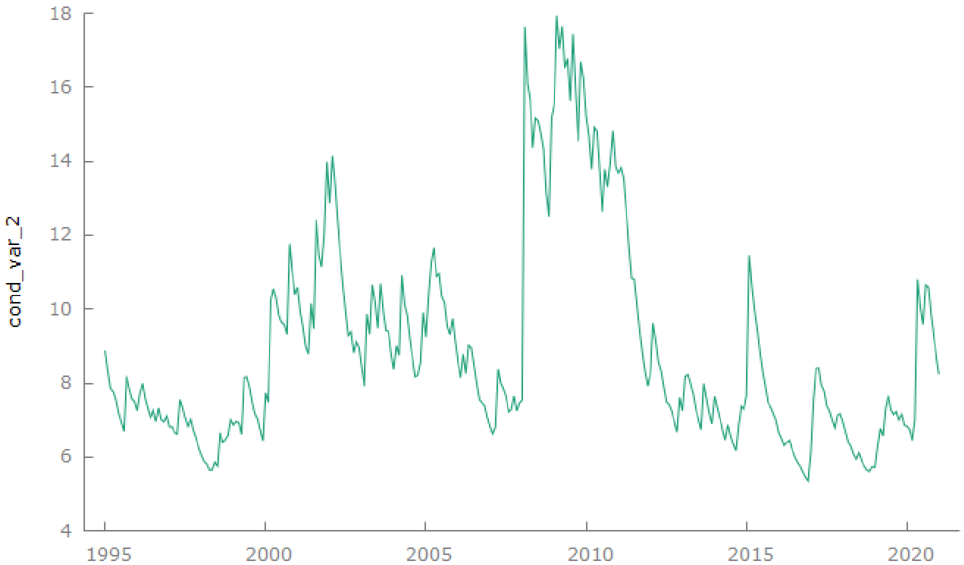
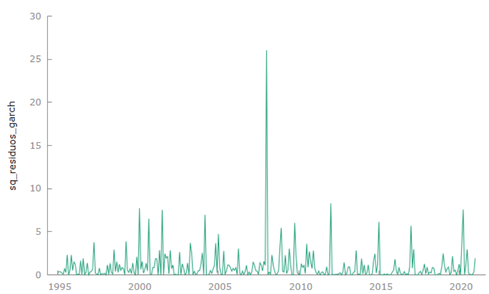
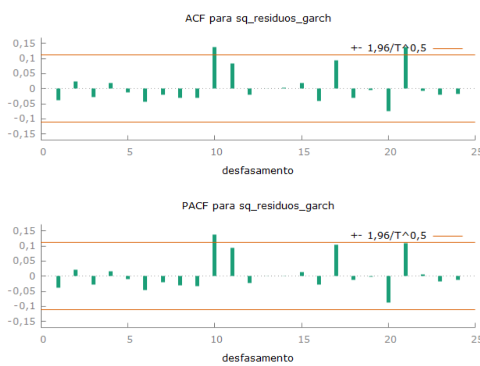


Figure 5: conditional variance AR(12)-GARCH(1,1), i.e uncertainty

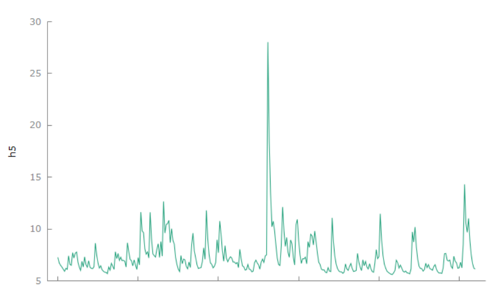


(a) residuals GARCH(1,1)

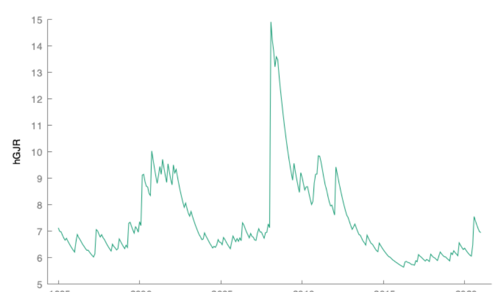


(b) Correlogram residuals GARCH(1,1)

Figure 6: Residuals and residuals correlogram GARCH(1,1)



(a) Conditional SD GARCH(1,1)



(b) Conditional SD GJR-ARCH

Figure 7: Estimated conditional standard derivation