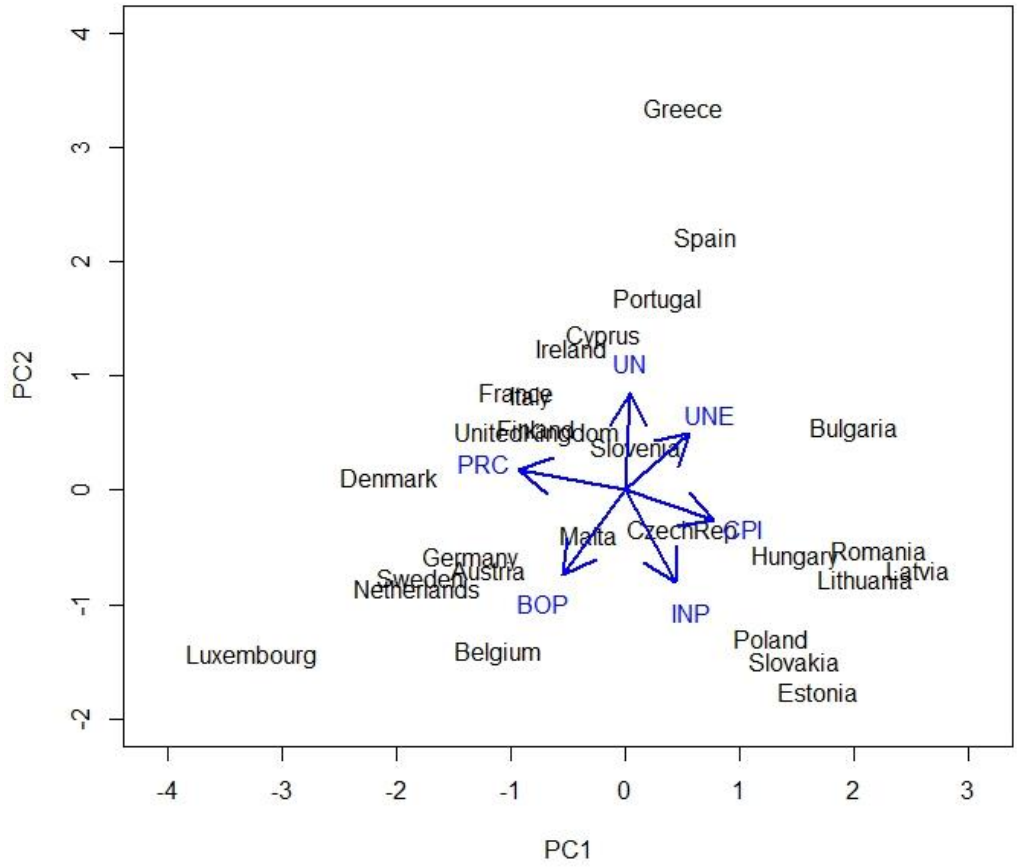
```
### Exercise 2

remove(list = ls())

euro  = read.table(file = "Europe.txt", header = T, dec =".")
attach(euro)
europe = data.frame(euro)
europe = na.omit(europe)
ls(europe)

## [1] "BOP"        "CPI"        "ï..Country" "INP"        "PRC"
## [6] "UN"         "UNE"

Country = ï..Country
numbers = cbind(CPI, UNE, INP, BOP, PRC, UN)

# a)
R = cor(numbers)
E = eigen(R)$vectors

X = scale(numbers, center = T, scale = T)
Yhat = X%*%E
y1hat = Yhat[,1]*(-1)
y2hat = Yhat[,2]

plot(y1hat, y2hat, type = "n", asp = 1, xlab = "PC1", ylab = "PC2", xlim = c(
-4,3), ylim = c(-2, 4))
text(y1hat, y2hat, Country)

arrows(0,0,-E[,1]*1.5,E[,2]*1.5, lwd = 2, col = "blue")
text(-E[,1]*2, E[,2]*2, c("CPI","UNE","INP","BOP","PRC","UN"), col = "blue")
```

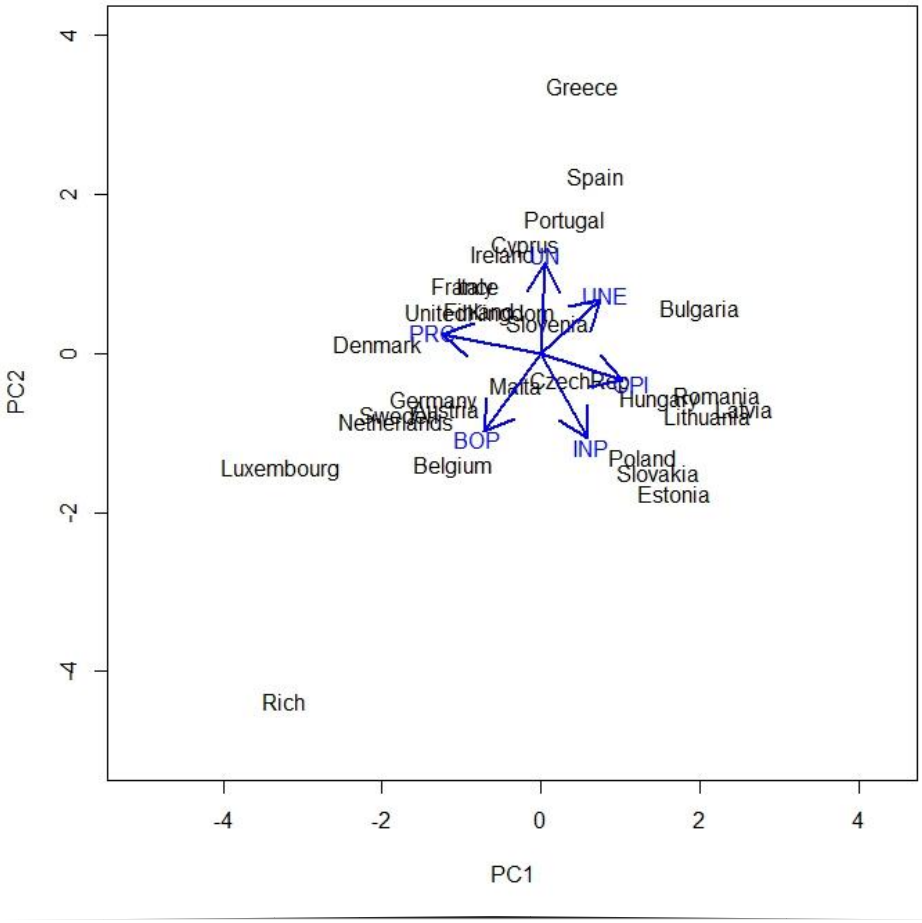



```
# b)
Rich = data.frame(ï..Country = "Rich", CPI = mean(CPI), UNE = mean(UNE), INP
= mean(INP), BOP = max(BOP)*3, PRC = mean(PRC), UN = mean(UN))
rbind.data.frame(europe, Rich)
```

```
##       ï..Country      CPI       UNE      INP     BOP       PRC         UN
## 1        Belgium 116.0300  4.770000 125.5900   908.6  6716.500 -1.6000000
## 2       Bulgaria 141.2000  7.310000 102.3900    27.8  1094.700  3.5000000
## 3       CzechRep 116.2000  4.880000 119.0100  -277.9  2616.400 -0.6000000
## 4        Denmark 114.2000  6.030000  88.2000  1156.4  7992.400  0.5000000
## 5        Germany 111.6000  4.630000 111.3000   499.4  6774.600 -1.3000000
## 6        Estonia 135.0800  9.710000 111.5000   153.4  2194.100 -7.7000000
## 7        Ireland 106.8000 10.200000 111.2000  -166.5  6525.100  2.0000000
## 8         Greece 122.8300 11.300000  78.2200  -764.1  5620.100  6.4000000
## 9          Spain 116.9700 15.790000  83.4400  -280.8  4955.800  0.7000000
## 10        France 111.5500  6.770000  92.6000  -337.1  6828.500 -0.9000000
## 11         Italy 115.0000  5.050000  87.8000  -366.2  5996.600 -0.5000000
## 12        Cyprus 116.4400  5.140000  86.9100 -1090.6  5310.300 -0.4000000
## 13        Latvia 144.4700 12.110000 110.3900    42.3  1968.300 -3.6000000
## 14     Lithuania 135.0800 11.470000 114.5000   -77.4  2130.600 -4.3000000
## 15    Luxembourg 118.1900  3.140000  85.5100  2016.5 10051.600 -3.0000000
## 16       Hungary 134.6600  6.770000 115.1000   156.2  1954.800 -0.1000000
```

```
## 17         Malta 117.6500  4.150000 101.6500   359.4  3378.300 -0.6000000
## 18   Netherlands 111.1700  3.230000 103.8000  1156.6  6046.000 -0.4000000
## 19       Austria 114.1000  2.990000 116.8000    87.8  7045.500 -1.5000000
## 20        Poland 119.9000  6.280000 146.7000   -74.8  2124.200 -1.0000000
## 21      Portugal 113.0600  9.680000  89.3000  -613.4  4073.600  0.8000000
## 22       Romania 142.3400  4.760000 131.8000  -128.7  1302.200  3.2000000
## 23      Slovenia 118.3300  5.560000 105.4000    39.4  3528.300  1.8000000
## 24      Slovakia 117.1700  9.190000 156.3000    16.0  2515.300 -2.1000000
## 25       Finland 114.6000  5.920000 101.0000  -503.7  7198.800 -1.3000000
## 26        Sweden 112.7100  6.100000 100.5000  1079.1  7476.700 -2.3000000
## 27 UnitedKingdom 120.9000  6.110000  90.3600   -24.3  6843.900 -0.8000000
## 28          Rich 120.6752  7.001481 106.1952  6049.5  4824.563 -0.5592593
```

```
XRich = c(0,0,0,(3*max(BOP)-mean(BOP))/sd(BOP),0,0)
YhatRich = XRich%*%E
plot(y1hat, y2hat, type = "n", asp = 1, xlab = "PC1", ylab = "PC2",ylim = c(-
5,4))
text(y1hat,y2hat, Country)
text(-YhatRich[1,1],YhatRich[1,2], "Rich")
arrows(0,0,-E[,1]*2,E[,2]*2, col = "blue", lwd = 2)
text(-E[,1]*2.2,E[,2]*2.2, c("CPI","UNE","INP","BOP","PRC","UN"), col = "blue
")
```



```
### Exercise 3

remove(list = ls())

euro  = read.table(file = "Europe.txt", header = T, dec =".")
attach(euro)
europe = data.frame(CPI, UNE, INP, BOP, PRC, UN)
europe = na.omit(europe)
ls(europe)
```

```
## [1] "BOP" "CPI" "INP" "PRC" "UN"  "UNE"
```

```
X = scale(europe, center = T, scale = T)
R = cor(europe)
E = eigen(R)$vectors

#  i)
A1hat = X%*%E[,1]%*%t(E[,1])
head(A1hat,5)
```

```
##         [,1]       [,2]       [,3]       [,4]       [,5]        [,6]
## 1 -0.5660436 -0.4131536 -0.3217987  0.4032920  0.6883677 -0.02347820
## 2  1.0154376  0.7411651  0.5772815 -0.7234740 -1.2348774  0.04211804
## 3  0.2549705  0.1861023  0.1449520 -0.1816601 -0.3100706  0.01057559
## 4 -1.0526220 -0.7683059 -0.5984211  0.7499670  1.2800975 -0.04366036
## 5 -0.6885770 -0.5025904 -0.3914596  0.4905940  0.8373810 -0.02856060
```

```
AE1 = (dim(X)[1]-1)*sum(eigen(R)$values[2:6])
AE1
```

```
## [1] 97.11576
```

```
#  ii)
A2hat = X%*%E[,1:2]%*%t(E[,1:2])
head(A2hat, 5)
```

```
##         [,1]        [,2]        [,3]        [,4]       [,5]        [,6]
## 1 -0.3242375 -0.89125343  0.43842950  1.10504305  0.5172357 -0.82318795
## 2  0.9231616  0.92361347  0.28716965 -0.99127032 -1.1695714  0.34729651
## 3  0.3139868  0.06941505  0.33049678 -0.01038758 -0.3518378 -0.18460519
## 4 -1.0729105 -0.72819144 -0.66220720  0.69108732  1.2944562  0.02343844
## 5 -0.5872209 -0.70299193 -0.07280039  0.78474171  0.7656489 -0.36376896
```

```
AE2 = (dim(X)[1]-1)*sum(eigen(R)$values[3:6])
AE2
```

```
## [1] 57.18878
```

```
#  iii)
A3hat = X%*%E[,1:3]%*%t(E[,1:3])
head(A3hat, 5)
```

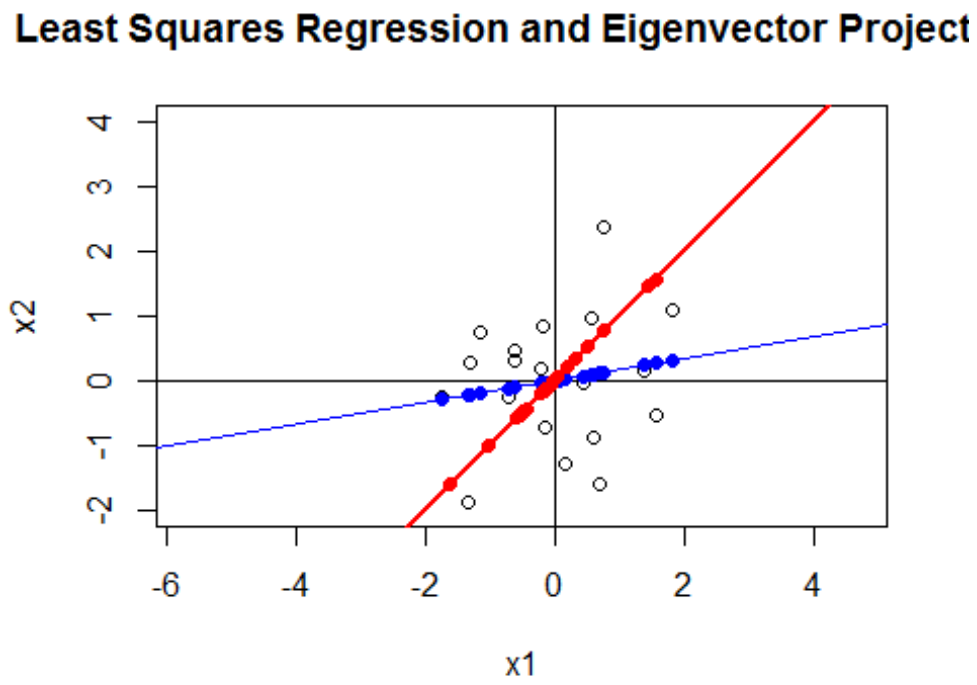
```
##         [,1]       [,2]       [,3]       [,4]       [,5]       [,6]
## 1 -0.3856722 -1.1109001  0.5970611  1.0433610  0.4527950 -0.5999865
## 2  0.7990121  0.4797432  0.6077384 -1.1159197 -1.2997955  0.7983505
## 3  0.1447466 -0.5356673  0.7674949 -0.1803092 -0.5293588  0.4302699
## 4 -1.0107256 -0.5058625 -0.8227760  0.7535226  1.3596837 -0.2024887
## 5 -0.6380826 -0.8848371  0.0585305  0.7336752  0.7122985 -0.1789808
```

```
AE3 = (dim(X)[1]-1)*sum(eigen(R)$values[4:6])
AE3
```

```
## [1] 33.29049
```

```
### Exercise 4

remove(list = ls())

appro = read.table(file = "Approx2dim.txt", header = T, dec =".")
attach(appro)
x1 = ï..x1
Approx2 = data.frame(x1,x2)
Approx2 = na.omit(Approx2)

plot(x1,x2, asp = 1, xlim = c(-4,3), ylim = c(-2, 4), main = "Least Squares R
egression and Eigenvector Projection")
abline(h = 0, v = 0)

reslm = lm(x2 ~ x1 - 1)
x2hatlm = reslm$fitted.values

abline(reslm, col= "blue")
points(x1,x2hatlm, pch = 16, col = "blue")

S = cov(Approx2)
E = eigen(S)$vectors
X = scale(Approx2, center = F, scale = T)

Ahat = X%*%E[,1]%*%t(E[,1])
reslm2 = lm(Ahat[,2] ~ Ahat[,1] -1)
x2hatlm2 = reslm2$fitted.values
```

```
abline(reslm2, col = "red", lwd = 2)
points(Ahat[,1], x2hatlm2, pch = 16, col ="red")
```




```
#Approximation Error for OLS = sum of residuals
sum((reslm$residuals)^2)

## [1] 18.46061

# Approximation Error
AE = (dim(X)[1]-1)*sum(eigen(S)$values[2])
AE

## [1] 15.79869
```