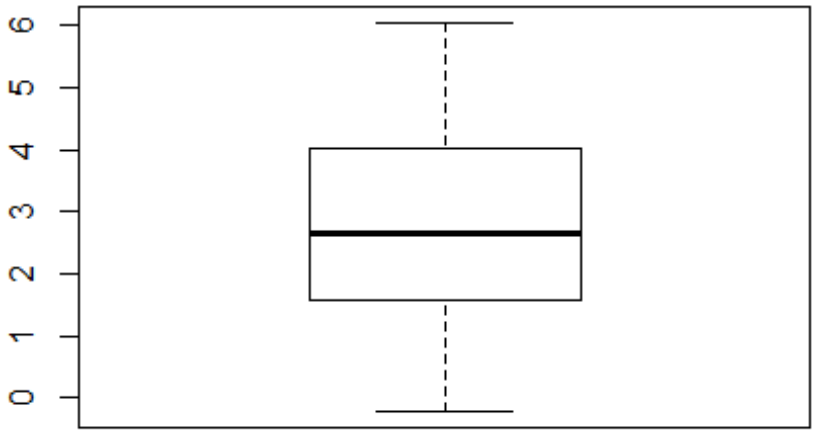
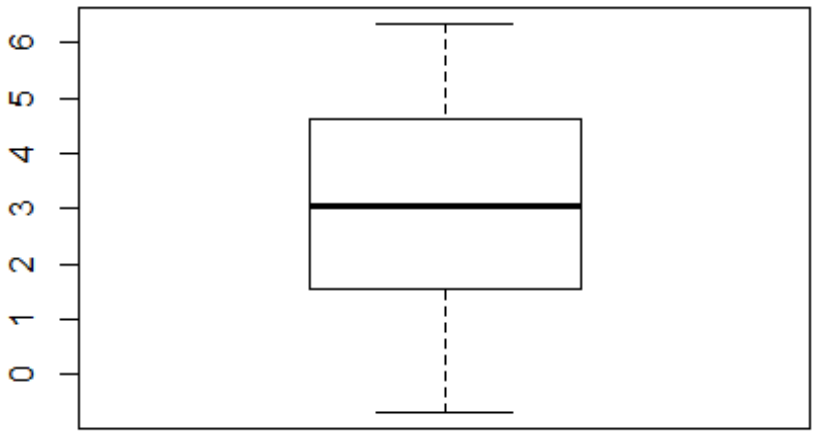
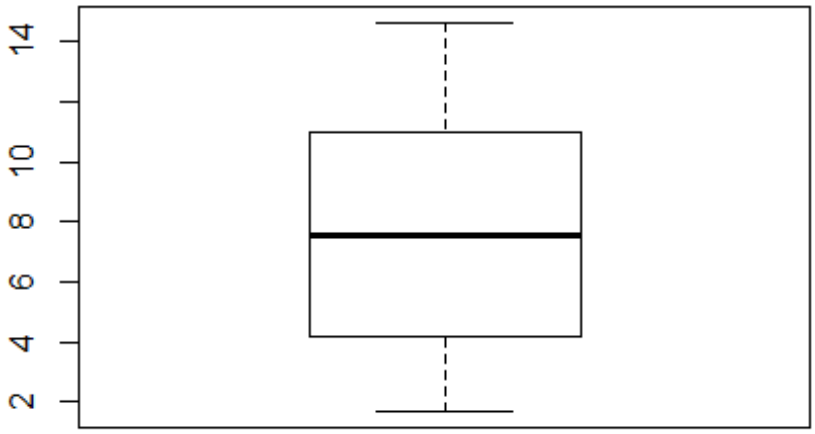
```
### Exercise 5

remove(list = ls())

outlier3dim <-
  structure(list(x1 = c(6.03715164059163, 0.658893396127709,
                        -0.209427598650197, 1.49037743943188, 5.10139004612
713, 2.52660473655032,
                        4.08270263854686, 1.6396297328328, 2.00195903263504
, 2.77458657610558,
                        1.28741521441567, 2.16712480728223, 3.9402013799695
2, 3.66671226097114,
                        5.54307038464309, 2.99996304016723),
                 x2 = c(4.66175971455103,
                        2.49020943909436, 0.686000191260295, 4.747219548586
, 2.92391034468324,
                        2.30272543343906, 3.35099338246828, -0.676869585898
721, 5.43329622565632,
                        3.87047176119487, 3.16424600806285, 0.8286114173131
06, 6.34731657802144,
                        0.683996758387633, 4.61239921442428, 2.597890915626
76),
                 x3 = c(13.0501573302298, 3.00057699240579, 1.7037328970179
3, 8.9782687954901, 9.26777654464302,
                        5.08300843101136, 11.0413483929881, 2.0813759646612
6, 10.9661939544758,
                        4.31909969951773, 5.88679514311043, 4.0750110711623
1, 14.6272252027742,
                        7.1921458497291, 14.5033586402903, 7.85822188801989
)),
            class = "data.frame", row.names = c(NA,-16L))
A = outlier3dim
x1 = outlier3dim$x1
x2 = outlier3dim$x2
x3 = outlier3dim$x3
boxplot(x1)
```



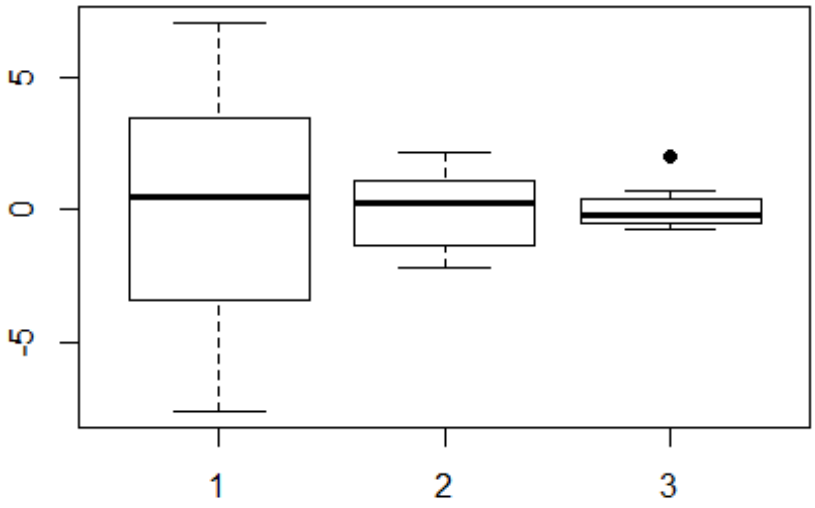

```
boxplot(x2)
```



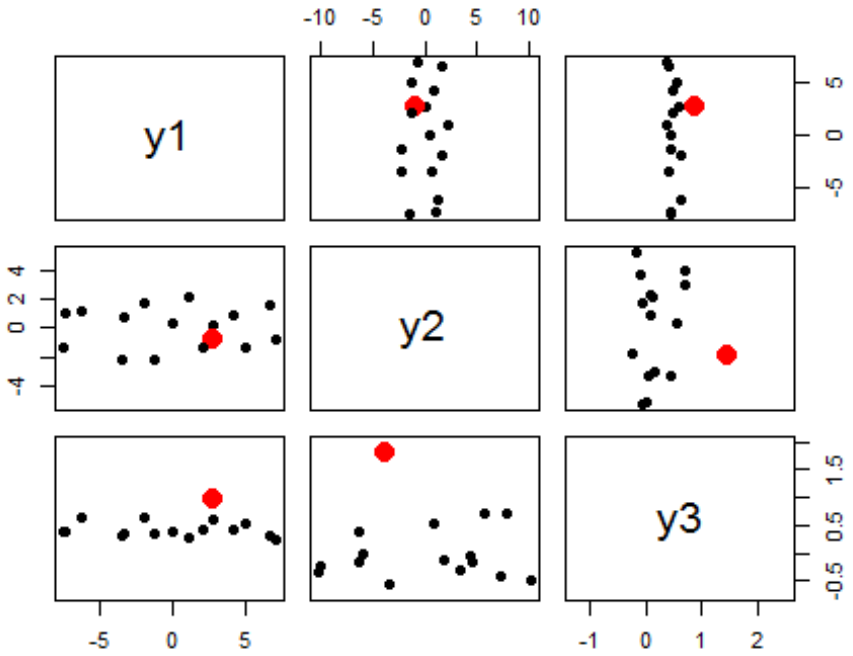

```
boxplot(x3)
```




```
S = cov(outlier3dim)
Sinv = solve(S)
R = cov2cor(S)

lam1 = eigen(S)$values[1]
lam2 = eigen(S)$values[2]
lam3 = eigen(S)$values[3]

X = scale(A, TRUE, FALSE)

y1 = X%*%eigen(S)$vectors[,1]
y2 = X%*%eigen(S)$vectors[,2]
y3 = X%*%eigen(S)$vectors[,3]
boxplot(c(y1),c(y2),c(y3))
points(3,max(y3),pch = 16 )
```

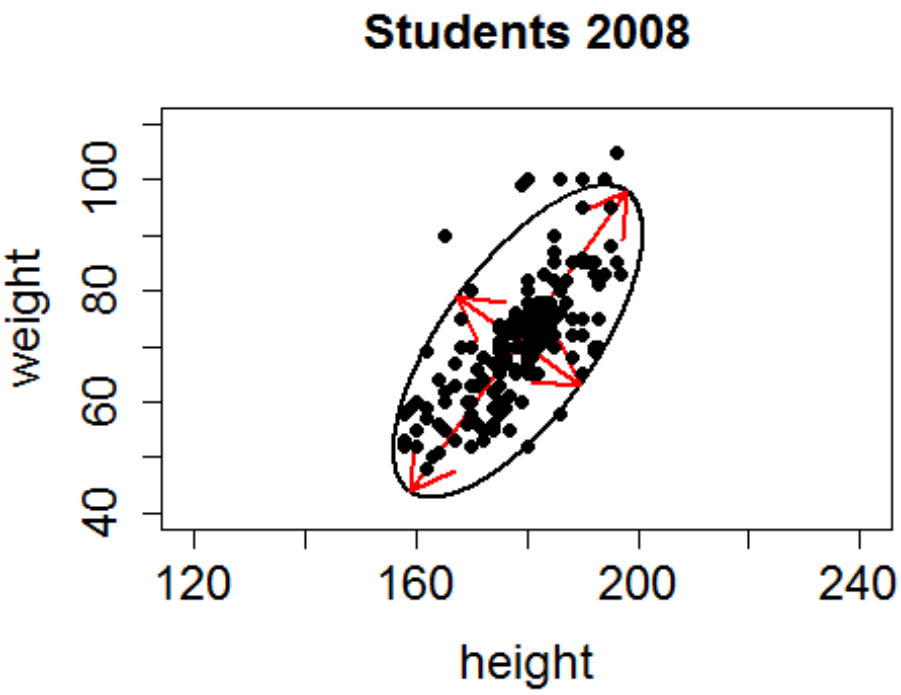



```
fab = c(rep("black",9),"red",rep("black"))
siz = c(rep(1, 9), 2, rep(1))
pairs(~y1+y2+y3, pch = 16, asp = 1, col = fab, cex = siz)
```



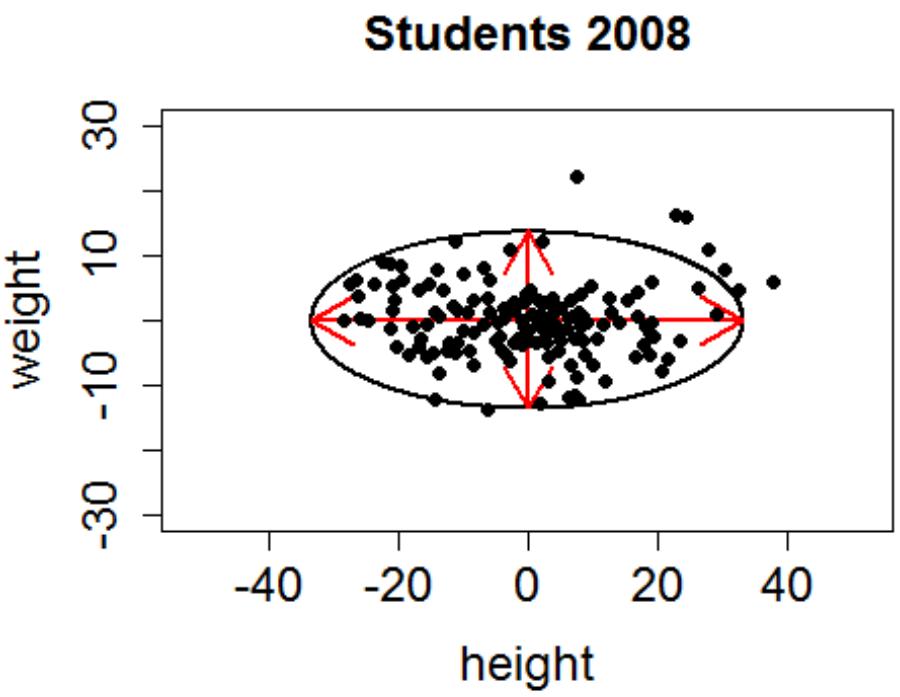
```
### Exercise 6

remove(list = ls())

students  = read.table(file = "students2008.txt", header = T, dec =",")
attach(students)

## The following object is masked from package:datasets:
##
##     sleep

heightweight=data.frame(height,weight)
heightweight=na.omit(heightweight)
attach(heightweight)

## The following objects are masked from students:
##
##     height, weight

m1=mean(height)
m2=mean(weight)
c2=qchisq(0.95,2)
S=cov(heightweight)
Sinv=solve(S)
a11=Sinv[1,1]
a12=Sinv[1,2]
a22=Sinv[2,2]
x1=seq(150,210,le=1000)
x2=seq(40,110,le=1000)
f=function(x1,x2)
{
  a11*(x1-m1)^2+a22*(x2-m2)^2+2*a12*(x1-m1)*(x2-m2)
}
z=outer(x1,x2,f)

# a)

contour(x1,x2,z,asp=1,levels=c2,drawlabels=FALSE,lwd=2,
        cex.axis=1.5,cex.lab=1.5,cex.main=1.5,
        xlab="height",ylab="weight",main="Students 2008")
reseigen=eigen(S)
lambda=reseigen$values
E=reseigen$vectors
lambda1=lambda[1]
lambda2=lambda[2]
e1=E[,1]
e2=E[,2]
arrows(m1,m2,sqrt(c2*lambda1)*e1[1]+m1,sqrt(c2*lambda1)*e1[2]+m2,lwd=2, col
= "red")
arrows(m1,m2,sqrt(c2*lambda2)*e2[1]+m1,sqrt(c2*lambda2)*e2[2]+m2,lwd=2, col
= "red")
arrows(m1,m2,-sqrt(c2*lambda1)*e1[1]+m1,-sqrt(c2*lambda1)*e1[2]+m2,lwd=2, c
ol = "red")
arrows(m1,m2,-sqrt(c2*lambda2)*e2[1]+m1,-sqrt(c2*lambda2)*e2[2]+m2,lwd=2, c
ol = "red")
```

```
points(height,weight,pch=16)
```

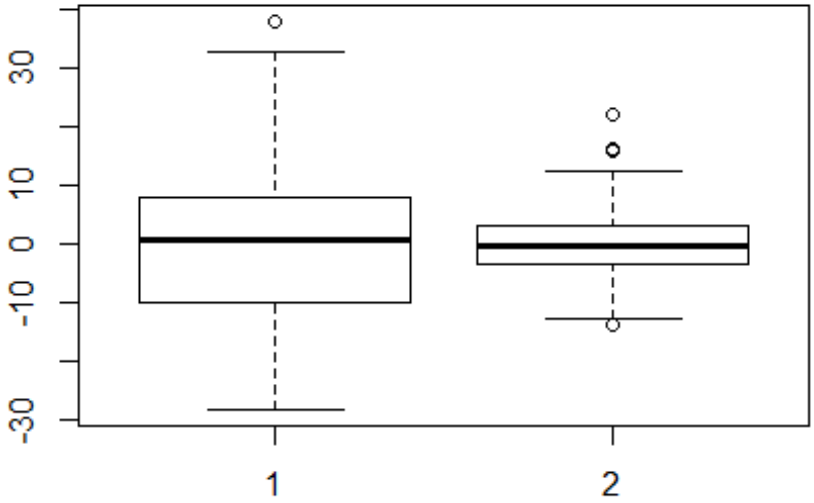



```
# ii)
X = scale(heightweight, TRUE, FALSE)
S = cov(heightweight)
Ee = eigen(S)
S = matrix(c(Ee$values[1],0,0,Ee$values[2]),2,2)
Sinv = solve(S)
lam1 = eigen(S)$values[1]
lam2 = eigen(S)$values[2]

c2=qchisq(0.95,2)

x1 = seq(-40,40, le = 1000)
x2 = seq(-30,30, le = 1000)

f=function(x1,x2)
{
  1/(lam1)*(x1)^2+1/lam2*(x2)^2
}
z=outer(x1,x2,f)

contour(x1,x2,z,asp=1,levels=c2,drawlabels=FALSE,lwd=2,
        cex.axis=1.5,cex.lab=1.5,cex.main=1.5,
        xlab="height",ylab="weight",main="Students 2008")

reseigen=eigen(S)

lambda=reseigen$values
E=reseigen$vectors
lambda1=lambda[1]
lambda2=lambda[2]
e1=E[,1]
```

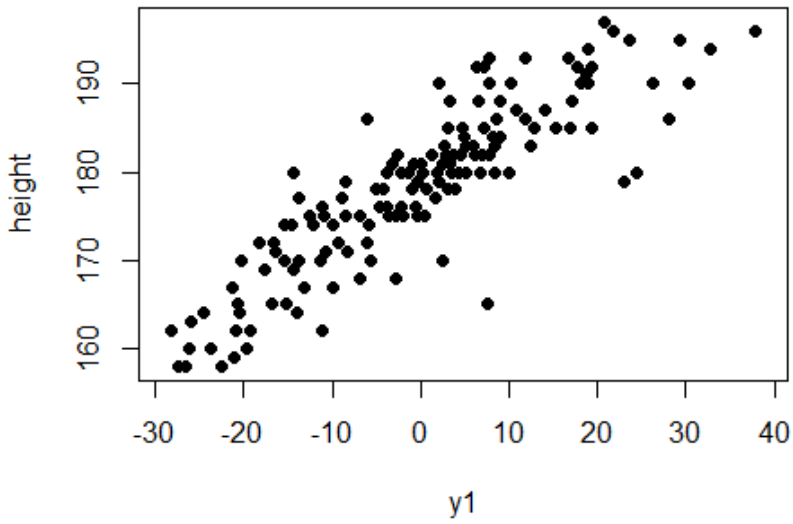
```
e2=E[,2]
m1 = 0
m2 = 0
arrows(m1,m2,sqrt(c2*lambda1)*e1[1]+m1,sqrt(c2*lambda1)*e1[2]+m2,lwd=2, col
= "red")
arrows(m1,m2,sqrt(c2*lambda2)*e2[1]+m1,sqrt(c2*lambda2)*e2[2]+m2,lwd=2, col
= "red")
arrows(m1,m2,-sqrt(c2*lambda1)*e1[1]+m1,sqrt(c2*lambda1)*e1[2]+m2,lwd=2, co
l = "red")
arrows(m1,m2,sqrt(c2*lambda2)*e2[1]+m1,-sqrt(c2*lambda2)*e2[2]+m2,lwd=2, co
l = "red")

y1 = X%*%Ee$vectors[,1]
y2 = X%*%Ee$vectors[,2]

Y = cbind(y1,y2)
points(Y, pch = 16)
```



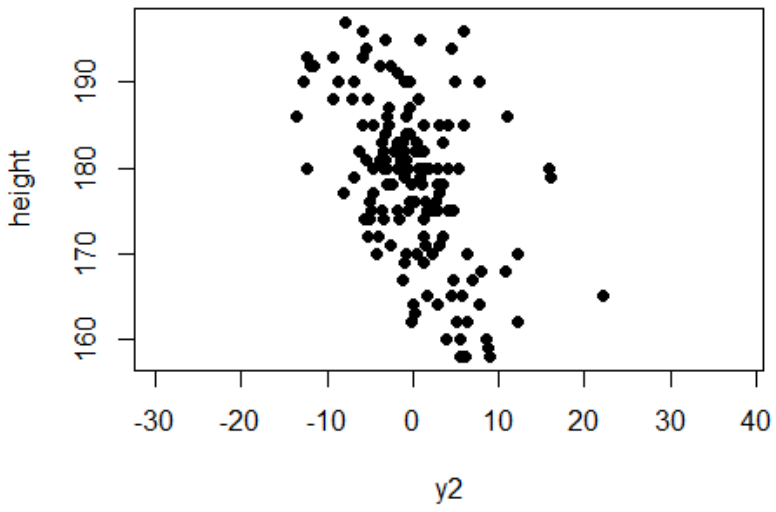

```
boxplot(c(y1),c(y2))
```



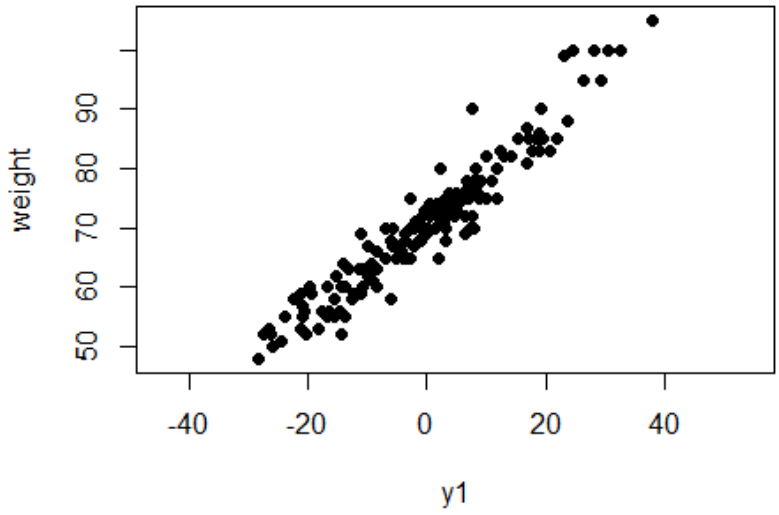

```
plot(y1,height, pch = 16, asp = 1)
```



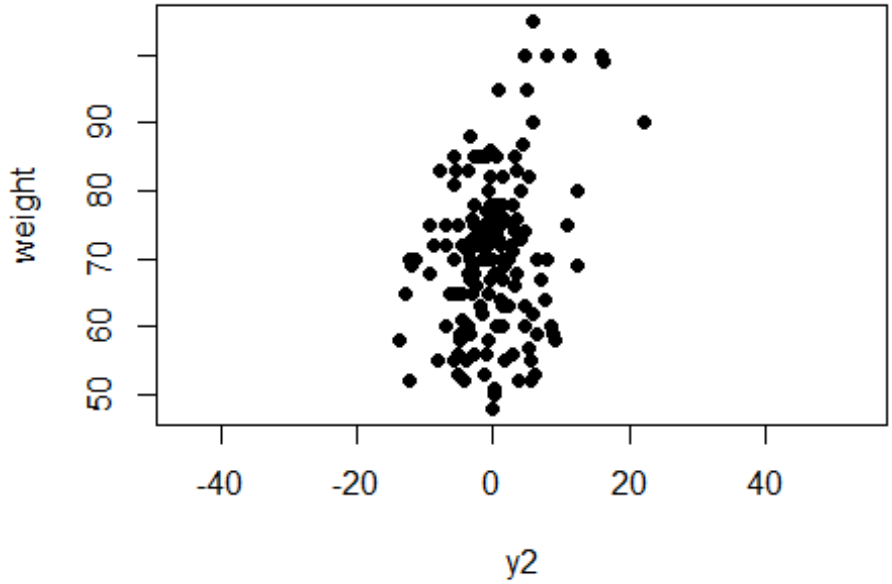

```
plot(y2,height, pch = 16, asp = 1)
```




```
plot(y1, weight, pch = 16, asp = 1)
```




```
plot(y2, weight, pch = 16, asp = 1)
```




```
# b)
round(Ee$vectors, digits = 4)

##         [,1]    [,2]
## [1,] 0.5875 -0.8092
## [2,] 0.8092  0.5875

round(Ee$values, digits = 4)

## [1] 184.7198  30.8903

round(cor(y1,height), digits = 4)

##         [,1]
## [1,] 0.8713

round(cor(y1,weight), digits = 4)
```

```
##         [,1]
## [1,] 0.9586

round(cor(y2,height), digits = 4)

##          [,1]
## [1,] -0.4908

round(cor(y2,weight), digits = 4)

##         [,1]
## [1,] 0.2846

round(Ee$values[1]/sum(Ee$values), digits = 4)

## [1] 0.8567

round(sum(Ee$values)/sum(Ee$values), digits = 4)

## [1] 1
```

Variable	PC1	PC2
X1 = heigth	0,5875 (0,8713)	-0,8092 (-0,4908)
X2 = Weight	0,8092 (0,9586)	0,5875 (0,2846)
lamda $_j$	184,7198	30,8903
Cumulative Prop. of Total Variance	0.8567	1