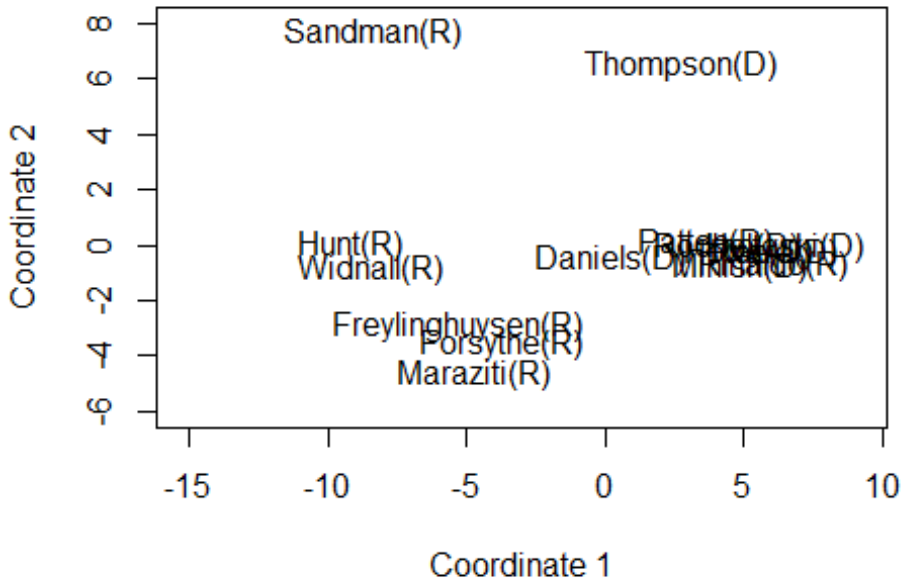
```
### Exercise 2

remove(list = ls())

library(tools)
library(HSAUR)

## Warning: package 'HSAUR' was built under R version 3.4.4

data("voting")

# a)
DelDis = voting

I  = diag(1, nrow = 15, ncol = 15)
J = matrix(c(rep(1, 15*15)), nrow = 15, ncol = 15)
n = 1/15
H = I-n*J
DelDis2 = DelDis^2
B = -0.5*H%*%DelDis2%*%H
round(eigen(B)$values, digits = 4)

##  [1] 497.7608 146.1762 102.9131  76.8776  55.1154  24.7437   8.0050
##  [8]   6.1717   2.3582   0.0000  -2.0261 -15.2141 -18.6943 -20.4015
## [15] -33.9858

### B has negativ eigenvalues, so it is not nonnegativ definite. Therfor it i
s not Euclidean.

# b)
Q = -0.5*H%*%DelDis2%*%H
Qe = eigen(Q)$vectors[,1:2]
Qlam = eigen(Q)$values[1:2]
QLam = diag(sqrt(Qlam), nrow = 2, ncol = 2)
Yhat = Qe%*%QLam
CM = c("Hunt(R)", "Sandman(R)", "Howard(D)", "Thompson(D)",
                "Freylinghuysen(R)", "Forsythe(R)", "Widnall(R)",
                "Roe(D)", "Heltoski(D)", "Rodino(D)", "Minish(D)",
                "Rinaldo(R)", "Maraziti(R)", "Daniels(D)", "Patten(D)")
plot(Yhat, type = "n", asp = 1, xlab = "Coordinate 1", ylab = "Coordinate 2",
xlim = c(-13,7), ylim = c(-6,8))
text(Yhat[,1],Yhat[,2], CM)
```




```
# c)
data.frame("C1" = cmdscale(voting, k = 2)[,1],"Yhat1" = Yhat[,1],"C2" = cmdsc
ale(voting, k = 2)[,2],"Yhat2" = Yhat[,2])
```

```
##                          C1      Yhat1          C2       Yhat2
## Hunt(R)           -9.1640883 -9.1640883  0.02161894  0.02161894
## Sandman(R)        -8.3699537 -8.3699537  7.68023459  7.68023459
## Howard(D)          5.6277025  5.6277025 -0.26582292 -0.26582292
## Thompson(D)        2.7528216  2.7528216  6.55124865  6.55124865
## Freylinghuysen(R) -5.3440596 -5.3440596 -2.89073549 -2.89073549
## Forsythe(R)       -3.7133046 -3.7133046 -3.49671135 -3.49671135
## Widnall(R)        -8.4431079 -8.4431079 -0.83225871 -0.83225871
## Roe(D)             5.6935834  5.6935834 -0.22380571 -0.22380571
## Heltoski(D)        6.5311040  6.5311040 -0.05545261 -0.05545261
## Rodino(D)          4.4214984  4.4214984 -0.02052953 -0.02052953
## Minish(D)          4.8940977  4.8940977 -0.78542948 -0.78542948
## Rinaldo(R)         6.0315595  6.0315595 -0.71851563 -0.71851563
## Maraziti(R)       -4.7595652 -4.7595652 -4.64131141 -4.64131141
## Daniels(D)         0.2098827  0.2098827 -0.42931460 -0.42931460
## Patten(D)          3.6318295  3.6318295  0.10678526  0.10678526

### These are the same results.
```

```
### Exercise 3

remove(list = ls())
I  = diag(1, nrow = 23, ncol = 23)
J = matrix(c(rep(1, 23*23)), nrow = 23, ncol = 23)
n = 1/230
H = I-n*J

# a)
#  i)
pref = read.table(file = "Preferences.txt", header = T)
Pref =  cbind(Name = pref[,1],pref[,2:21])
Preferences = as.matrix(Pref[,2:21])
X = scale(Preferences, center = T, scale = F)
Q = X%*%t(X)
Qlam = eigen(Q)$values[1:2]
QLam = diag(sqrt(Qlam), nrow = 2, ncol = 2)
Qe = eigen(Q)$vectors[,1:2]
Yhat = Qe%*%QLam
head(Yhat,4)

##            [,1]        [,2]
## [1,] -0.6713242 -0.06986646
## [2,] -0.4397291 -1.67075200
## [3,]  1.5584607 -1.47549195
## [4,]  0.4263072  2.37895190

#  ii)
S = cov(Preferences)
E = eigen(S)$vectors
Yhat = X%*%E
head(Yhat[,1:2],4)

##            [,1]        [,2]
## [1,]  0.6713242  0.06986646
## [2,]  0.4397291  1.67075200
## [3,] -1.5584607  1.47549195
## [4,] -0.4263072 -2.37895190

# b)
X = scale(Preferences, center = T, scale = F)
S = cov(X)
Lam = eigen(S)$values[1:2]
E = eigen(S)$vectors[,1:2]
Yhat = X%*%E
plot(Yhat, type = "n", asp = 1, xlab = "PC1", ylab = "PC2")
text(Yhat[,1],Yhat[,2], pref[,1])

# c)
```

```
E_1 = E[,1]
E_2 = E[,2]
lqa = sqrt(E_1^2+E_2^2)
ra = order(lqa, decreasing = T)
arrows(0,0,2.5*E_1[ra[1:4]],2.5*E_2[ra[1:4]], col="red")
text(3*E_1[ra[1:4]],3*E_2[ra[1:4]], labels = c("Q4","Q19","Q1","Q18"), col =
"red")

# d)
colMeans(Preferences)
```

```
##          Q1          Q2          Q3          Q4          Q5          Q6
##  0.30434783  0.04347826 -0.91304348 -0.04347826 -0.21739130  0.39130435
##          Q7          Q8          Q9         Q10         Q11         Q12
## -0.47826087  0.13043478 -0.39130435 -0.13043478  0.39130435 -0.56521739
##         Q13         Q14         Q15         Q16         Q17         Q18
##  0.13043478  0.13043478  0.30434783 -0.21739130 -0.65217391  0.04347826
##         Q19         Q20
##  0.13043478 -0.47826087
```

```
Conformist =  c(1,1,-1,-1,-1,1,-1,1,-1,-1,1,-1,1,1,1,-1,-1,1,1,-1)
Individualist = (-1)*Conformist
means = colMeans(Preferences, na.rm = F, dims = 1)
XC = Conformist-means
XI = Individualist-means
YhatC = XC%*%E
YhatI = XI%*%E

text(YhatC[1,1],YhatC[1,2], "conformist", col = "green")
text(YhatI[1,1],YhatI[1,2], "individualist", col = "green")
```