CHAPTER 3

Arbitrage and Financial Decision Making

Chapter Synopsis

3.1 Valuing Decisions

When considering an investment opportunity, a financial manager must systematically compare the costs and benefits associated with the project in order to determine whether it is worthwhile. Determining the cash value today of the costs and benefits is one way to make such a comparison.

In a **competitive market,** a good can be bought and sold at the same price, so the market price can be used to determine the cash value today of the good. Because competitive markets exist for many assets, such as commodities and financial securities, they can be used to determine cash values and evaluate decisions in many situations. For example, if gold trades at $250/ounce in a competitive market, then 20 ounces of gold have a cash value today of $5000. A buyer wouldn't need to pay more, and a seller wouldn't need to accept less, so individual preferences are not relevant.

If a manager can observe competitive market prices, he may be able to use them to determine the current cash value of different costs and benefits so they can be compared. For example, if someone offers to trade the manager 20 ounces of gold for 10 ounces of platinum, which trades at $550 per ounce in a competitive market, he should reject the trade. The benefit (the cash value today of the gold, $5000) is smaller than the cost (the cash value today of the platinum, $5500).

By evaluating cost and benefits using competitive market prices, we can determine whether a decision will make the firm and its investors wealthier. This point is one of the central and most powerful ideas in finance, which we call the Valuation Principle:

> The value of an asset to the firm or its investors is determined by its competitive market price. The benefits and costs of a decision should be evaluated using these

> market prices, and when the value of the benefits exceeds the value of the costs, the decision will increase the market value of the firm.

The Valuation Principle provides the basis for decision making throughout this text.

3.2 Interest Rates and the Time Value of Money

Many financial problems require the valuation of cash flows occurring at different times. However, money received in the future is worth less than money received today because the money received today can be invested to grow to have a larger value in the future. Thus, money has **time value,** and it is only possible to compare cash flows occurring at different times by bringing them to the same point in time.

For example, suppose that there is an annual **risk-free rate**, r_f, of 7% at which you can borrow or lend without risk. If you have the opportunity to lend $100,000 dollars to receive $105,000 in one year, you should not accept this opportunity. The benefit (the cash value today of the $105,000 in 1 year = $105,000 / (1.07) = $98,131) is smaller than the cost (the cash value today of the $100,000, which is just $100,000). You would be better off investing the $100,000 at the risk-free rate and receiving $107,000 in one year.

3.3 Present Value and the NPV Decision Rule

When the value of a cost or benefit is computed in terms of cash today, it is referred to as the **present value** (PV). The **net present value** (NPV) of a project or investment is the difference between the present value of its benefits and the present value of its costs:

$$\text{Net Present Value (NPV)} = \text{PV(Benefits)} - \text{PV(Costs)}$$

Because the NPV represents a project in terms of cash today, it simplifies decision making and leads to the **net present value rule**:

> When making an investment decision, take the alternative with the highest NPV. Choosing this alternative is equivalent to receiving its NPV in cash today.

Regardless of individual preferences for cash today versus cash in the future, everyone should always maximize NPV first. Investors can then borrow or lend to shift cash flows through time to achieve their preferred pattern of cash flows.

3.4 Arbitrage and the Law of One Price

In a competitive market, the price of a good cannot trade in two different markets at different prices. Such a price discrepancy represents an **arbitrage opportunity** because you can make a riskless profit without making an investment by buying in the low price market and selling in the high price market. Because an arbitrage opportunity has a positive NPV, whenever an arbitrage opportunity appears in financial markets, investors will quickly take advantage of it. The presence of such arbitrage activity leads to the **Law of One Price**:

> If equivalent goods or securities trade simultaneously in different competitive markets, then they will trade for the same price in both markets.

A competitive market in which there are no arbitrage opportunities can be referred to as a **normal market**.

3.5 No-Arbitrage and Security Prices

The Law of One Price has implications for valuing securities, such as a bond. (A bond is a security issued by governments and corporations to raise money from investors today in exchange for future payments.)

For example, suppose you can either (1) buy a riskless bond paying $1000 in one year, or (2) invest money in a riskless bank account that pays 5%. It would require a $1000 / (1.05) = $952.38 investment in the bank account to generate $952.38(1.05) = $1000 in one year. Thus, the price of the bond must be $952.38 or an arbitrage opportunity would exist:

- When the bond is priced below $952.38, the arbitrage strategy involves buying the bond and borrowing $952.38 from the bank. You will owe the bank $1000 in one year, but you can use the bond's payment to pay that back. Your profit today = $952.38 – P > 0.
- When the bond is priced above $952.38, the arbitrage strategy involves selling the bond and investing $952.38 of the proceeds in the bank account. [Note that if you do not own the bond you can **short sell** the bond by borrowing it from your broker and selling it with the promise to replace it in the future.] You will still receive $1000 (from the bank now instead of from the bond) in one year. Your profit today = P – $952.38 > 0.

Thus, the existence of investors trying to exploit such opportunities leads to the existence of the **no-arbitrage price,** $952.38.

When securities trade at no-arbitrage prices, then investing in securities is a zero NPV investment. Thus, in normal markets, trading securities neither creates nor destroys value. Instead, value is created by the real investment projects made by firms, such as developing new products, opening new stores, or creating more efficient production methods. It follows that the firm's investment decision can be separated from its financing choice. This concept is referred to as the **separation principle.**

The Law of One Price has implications for packages of securities as well. Consider two securities, A and B. Suppose a third security, C, has the same cash flows as A and B combined. Because security C is equivalent to the portfolio of A and B, by the Law of One Price, they must have the same price; otherwise, an obvious arbitrage opportunity would exist. This relationship is known as **Value Additivity**:

$$\text{Price(C)} = \text{Price(A)} + \text{Price(B)}.$$

Value additivity has an important consequence for the value of an entire firm. Since the cash flows of the firm are equal to the total cash flows of all projects and investments within the firm, the value of the firm equals the sum of the values of all of its projects and other assets. Thus, to maximize the value of the firm, managers should make decisions that maximize the NPV of each project, which represents the project's contribution to the firm's total value.

Appendix: The Price of Risk and Arbitrage with Transaction Costs

Thus far we have considered only cash flows that have no risk. However, in many settings, cash flows are risky. Intuitively, investors will pay less to receive a risky cash flow in the future than they would to receive a certain cash flow because they don't like risk. The notion that investors prefer to have a safe income rather than a risky one of the same average amount is called **risk aversion.**

Because investors care about risk, we cannot use the risk-free interest rate to compute the present value of a risky future cash flow. The increase in the discount rate over the risk-free rate that investors use to value risky cash flows is called the **risk premium.** *When a cash flow is risky, to compute its present value you must discount the expected cash flow at a rate equal to the risk-free interest rate plus an appropriate risk premium.*

The risk of a security cannot be evaluated in isolation. Even when a security's returns are quite variable, if the returns vary in a way that offsets other risks investors are holding, the security may reduce rather than increase investors' risk. As a result, risk can only be assessed relative to the other risks that investors face, so *the risk of a security must be*

evaluated in relation to the fluctuations of other investments in the economy. A security's risk premium will be higher the more its returns tend to vary with the overall economy and the market index. If the security's returns vary in the opposite direction of the market index, it offers insurance and will have a lower or even a negative risk premium.

In most markets, you must pay **transaction costs** to trade securities. First, you must pay your broker a commission on the trade. Second, because you will generally pay a slightly higher price when you buy a security (the ask price) than you receive when you sell (the bid price), you will also pay the bid-ask spread. *Thus, when there are transaction costs, arbitrage keeps prices of equivalent goods and securities close to each other. However, prices can deviate, but not by more than the transaction costs of the arbitrage.*

Selected Concepts and Key Terms

Competitive Market

A market in which goods can be bought and sold at the same price. Because competitive markets exist for most commodities and financial assets, we can use them to determine cash values and evaluate decisions in many situations.

Time Value of Money

The idea that it is only possible to compare cash flows occurring at different times by bringing them to the same point in time. When the expected rate of return on invested cash is positive, cash received in the future is worth less than cash received today because less cash can be invested today to equal the future amount. Thus, the **present value** of a future cash flow is less than the amount received in the future, and the **future value** of a cash flow invested in a previous period is worth more than the amount invested in the past.

Risk-Free Interest Rate

The interest rate at which you can borrow or lend without risk. $(1 + r_f)$ is the **interest rate factor** for risk-free cash flows; it defines the exchange rate across time.

Net Present Value (NPV)

The difference between the present value of an investment's benefits and the present value of its costs. The NPV of a project represents its value in terms of cash today.

NPV Decision Rule

Select all projects that have a positive NPV. When choosing among mutually exclusive alternatives, take the alternative with the highest NPV. Choosing this alternative is equivalent to receiving its NPV in cash today.

Arbitrage

The practice of buying and selling equivalent goods in different markets to take advantage of a price difference. A situation in which it is possible to make a profit without taking any risk or making any investment is an **arbitrage opportunity.**

Normal Market

A competitive market in which there are no arbitrage opportunities. The term *efficient market* is also sometimes used to describe a market that, along with other properties, is without arbitrage opportunities.

Law of One Price

If equivalent goods or securities trade simultaneously in different competitive markets, then they will trade for the same price in both markets.

Separation Principle

The idea that a firm's investment decision can be separated from its financing choice. This follows from the idea that, in normal markets, trading securities neither creates nor destroys value. Instead, value is created by the real investment projects made by firms, such as developing new products, opening new stores, or creating more efficient production methods.

Concept Check Questions and Answers

3.1.1. In order to compare the costs and benefits of a decision, what must we determine?

In order to compare the costs and benefits, we need to evaluate them in the same terms—cash today.

3.1.2. If crude oil trades in a competitive market, would an oil refiner that has a use for the oil value it differently than another investor?

No, if crude oil trades in a competitive market, the value of crude oil depends only on the current market price. The personal opinion of an oil refiner or any investor does not alter the value of the decision today.

3.2.1. How do you compare costs at different points in time?

We can compare costs at different points in time by converting the costs in the future to dollars today using the interest rate.

3.2.2. If interest rates rise, what happens to the value *today* of a promise of money in one year?

When interest rates rise, the value of money today to be received in one year is lower. In other words, the higher the discount rate, the lower the value of money today.

3.3.1. What is the NPV decision rule?

The NPV decision rule states that when choosing among alternatives, we should take the alternative with the highest NPV. Choosing this alternative is equivalent to receiving its NPV in cash today.

3.3.2. Why doesn't the NPV decision rule depend on the investor's preferences?

Regardless of our preferences for cash today versus cash in the future, we should always maximize NPV first. We can then borrow or lend to shift cash flows through time and find our most preferred pattern of cash flows.

3.4.1. If the Law of One Price were violated, how could investors profit?

If the Law of One Price were violated, investors can profit by arbitrage. They buy goods or securities at a lower price in one market and simultaneously resell the goods or securities at a higher price in a different market to take advantage of a price difference.

3.4.2. When investors exploit an arbitrage opportunity, how do their actions affect prices?

Investors exploit an arbitrage opportunity when taking advantage of price differences in two separate markets. In doing so, investors will buy in the market where it is cheap and simultaneously sell in the market where it is expensive. As more and more investors compete, the price will rise with increased buy orders in one market and fall with increased sell orders in the other. Arbitrage activities will continue until the prices in the two markets are equal.

3.5.1. If a firm makes an investment that has a positive NPV, how does the value of the firm change?

If a firm makes an investment that has a positive NPV, the value of the firm will increase by the NPV amount today.

3.5.2. What is the separation principle?

The separation principle states that security transactions in a normal market neither create nor destroy value on their own. Therefore, we can evaluate the NPV of an investment decision separately from any security transactions the firm is considering. That is, we can separate the firm's investment decision from its financing choice.

3.5.3. In addition to trading opportunities, what else do liquid markets provide?

Competitive markets depend upon liquidity because liquid markets allow market prices to be determined. When markets become illiquid, it may not be possible to trade at the posted price. As a consequence, we can no longer rely on market prices as a measure of value.

3.A.1 Why does the expected return of a risky security generally differ from the risk-free interest rate? What determines the size of its risk premium?

The expected return of a risky security generally differs from the risk-free interest rate because the expected return includes a risk premium. The higher the variability of returns, the higher the risk premium demanded by investors.

3.A.2. Explain why the risk of a security should not be evaluated in isolation.

The risk of a security must be evaluated in relation to the fluctuations of other investments in the economy. A security's risk premium will be higher the more its returns tend to vary with the overall economy and the market index.

3.A.3. In the presence of transactions costs, why might different investors disagree about the value of an investment opportunity?

In the presence of transaction costs, different investors might disagree about the value of an investment opportunity because investors with high transaction costs will value the investment opportunity less.

3.A.4. By how much could this value differ?

When there are transaction costs, arbitrage keeps prices of equivalent securities close to each other. Prices can deviate, but not by more than the transaction costs of the arbitrag

Examples with Step-by-Step Solutions

Solving Problems

Problems using the ideas in this chapter generally involve:

- Finding the NPV of an investment (or comparing different investment alternatives) by calculating the benefits minus the costs using either competitive market prices, the risk-free rate, or currency exchange rates; or
- Determining no arbitrage prices based on current market prices.

Below are examples of finding the net present value of an investment by calculating the benefits minus the costs using the risk-free rate, finding the net present value of an investment by calculating the benefits minus the costs using competitive market prices, and determining a no-arbitrage price based on current market prices.

Examples

1. You are going to retire in one year, and your defined benefit retirement plan will pay you $3 million on the date you retire. If you work another year, you will get one more year's salary of $100,000 (paid one year from today). Your firm has offered to pay you an early retirement package of $2.9 million if you quit today. The risk-free interest rate is 5%, and there are no income tax effects. Ignoring the fact that one option involves having to show up at work one more year:

[A] Which option should you take if you compare them based on dollars today?

[B] Which option should you take if you compare them based on dollars in one year?

[C] What is the lowest retirement value today that would make you indifferent between the two options?

Step 1. Determine the value of each option in today's dollars. Since there are no cash outflows, the NPV is just the present value of the cash inflows.

Present value of retiring today = $2,900,000

$$\text{Present value of retiring in one year} = \frac{\$3{,}000{,}000 + \$100{,}000}{1.05} = \$2{,}952{,}381$$

Thus, you should keep working.

Step 2. Determine the value of each option in next year's dollars.

Future value of retiring today = $2,900,000(1.05) = $3,045,000

Future value of retiring in one year = $3,000,000 + $200,000 = $3,200,000

Thus, you should keep working.

The two approaches, whether comparing the options based on present values or future values, will always provide the same answer.

Step 3. Determine the lowest retirement package today that would make you indifferent between the two options.

You would accept the present value of retiring in one year, $2,952,381. This is equivalent to having $3,200,000 in one year since $2,952,381(1.05) = $3,200,000. In other words, you can take the $2,952,381 payment and invest it at the risk-free rate and end up with $3,200,000.

2. Suppose your employer offers you a choice between a $20,000 bonus and 30 ounces of gold. Whichever one you choose will be awarded today. Gold is trading today at $500 per ounce. Ignoring income tax implications:
[A] Which form of the bonuses should you choose?
[B] What do you tell your broker, who advises you to take the gold because he predicts that the price of gold is going to double in value this year?

Step 1. Determine the value of each option.

Value of bonus = $20,000

$$\text{Value of the gold} = 30\left(\frac{\$500}{1 \text{ ounce of gold}}\right) = \$15{,}000 < \$20{,}000 \text{ cash bonus.}$$

So, you should take the cash bonus.

Step 2. Address the concern that gold is a good investment.

The reason you can compare the two options above is because gold trades in a competitive market and you can buy it and sell it for the same price. Thus, you would be better of taking the bonus and buying 30 ounces of gold for $15,000; you would still have $5,000 left over.

3. A hedge fund has a portfolio consisting of 1 million shares of Microsoft, which trades at $30 per share, and 1 million shares of Intel, which trades at $20. A stockholder in the hedge fund, which has 500,000 shares outstanding, has offered to sell you 1,000 shares for $75 per share. Does this represent an arbitrage opportunity? If so, how can you exploit it?

Step 1. Determine the no-arbitrage price.

Because the hedge fund is equivalent to a portfolio of Microsoft and Intel, by the Law of One Price, they must have the same price, so:

Value(hedge fund) = Value(Microsoft stock) + Value(Intel Stock)

= $30(1 million) + $20(1 million) = $50 million.

So, the Value(hedge fund) per share $= \frac{\$50{,}000{,}000}{500{,}000} = \$100 > \$75 \Rightarrow$ an arbitrage opportunity.

Step 2. Determine how you would exploit it.

Since you can buy it at a lower price than the components are worth, you should buy the hedge fund shares and sell the components. You can short sell the stocks by borrowing them from your broker and selling them with the promise to replace them in the future.

To take advantage of the situation, you should buy the 1,000 shares for $75(1,000) = $75,000.

Next, short sell $\left(\frac{\$30(1{,}000{,}000)}{\$30(1{,}000{,}000)+\$20(1{,}000{,}000)}\right)(\$75{,}000) = \$45{,}000$ in Microsoft, and short sell $75,000 – $45,000 = $30,000 of Intel. This amounts to short selling $\frac{\$45{,}000}{\$30}$ = 1,500 shares of Microsoft and $\frac{\$30{,}000}{\$20}$ = 1,500 shares of Intel. The NPV of the transactions is $25,000.

Questions and Problems

1. Suppose a Treasury bill with a risk-free cash flow of $10,000 in one year trades for $9,615 today. If there are no arbitrage opportunities, what is the current risk-free interest rate?
2. You have an investment opportunity in Italy. It requires an investment of $1 million today and will produce a cash flow of 1 million euros in one year with no risk. Suppose the risk-free interest rate in the United States is 4%, the risk-free interest rate in Italy is 5%, and the current competitive exchange rate is € 1.2 per $1. What is the NPV of this investment? Is it a good opportunity?
3. Suppose the risk-free interest rate is 5.5%.
   [A] Having $100,000 today is equivalent to having what amount in one year?
   [B] Having $100,000 in one year is equivalent to having what amount today?
4. Your firm has identified three potential investment projects. All of the projects would use the same a tract of land, so you can only select one of them. All of the projects would generate risk-free cash flows and the risk-free rate is 5%.
   - Project 1 costs $1 and pays $1 million in 1 year
   - Project 2 costs $10 million and pays $12 million in 1 year
   - Project 3 has a cash inflow of $10 million today but a cash outflow of $11 million in 1 year

   What should the firm do?
5. Your employer has notified you that in 1 year you will be lose your job that pays $200,000 per year when you reach mandatory retirement age. They have offered you the choice of leaving today and keeping your company car, a 2005 Mercedes SLK with a Kelley blue book market value of $40,000. If you stay, you don't keep the car. You believe that the most you could make elsewhere is $100,000 next year if you quit and forego your current salary. The risk-free rate is 6%. Ignoring taxes, and assuming that you are paid your salary at the end of the year, what should you do?

Solutions to Questions and Problems

1. The PV of the security's cash flow is ($10,000) / (1 + *r*), where *r* is the one-year risk-free interest rate. If there are no arbitrage opportunities, this PV equals the security's price of $9,615 today. Therefore,

$$\$9{,}615 = \frac{\$10{,}000}{(1+r)} \Rightarrow (1+r) = \frac{\$10{,}000}{\$9{,}615} = 1.04 \Rightarrow r = .04, \text{ or } 4\%.$$

2. Cost = $1 million today

$$\text{Benefit} = €1 \text{ million in one year}$$

$$\text{PV} = \left(\frac{€1 \text{ million in one year}}{1.05}\right) = €0.9524 \text{ million today}$$

$$= €0.95 \text{ million today} \times \left(\frac{€1.2}{\$ \text{ today}}\right) = \$1.142857 \text{ million today}$$

$$\Rightarrow \text{NPV} = \$1.142857 - \$1 \text{ million} = \$142{,}857$$

The NPV is positive, so it is a good investment opportunity.

3. [A] Having $100,000 today is equivalent to having 100,000 × 1.055 = $105,500 in 1 year.
   [B] Having $100,000 in one year is equivalent to having 100,000/ 1.055 = $94,787 today.

4. Projects 2 and 3 are equally as valuable—select either one.

$$NPV_1 = -\$1 + \frac{\$1,000,000}{1.05} = \$952,381$$

$$NPV_2 = -\$10,000,000 + \frac{\$12,000,000}{1.05} = \$1,428,571.$$

$$NPV_3 = \$10,000,000 + \frac{-\$9,000,000}{1.05} = \$1,428,571$$

5. NPV of staying = $\frac{\$200,000}{1.06} = \$186,679$

   NPV of leaving = $\frac{\$100,000}{1.06} + 40,000 = \$134,340$

   So, you should stay one more year.