



CHAPTER 13

Investor Behavior and Capital Market Efficiency

Chapter Synopsis

13.1 Competition and Capital Markets

When the market portfolio is efficient, all stocks are on the security market line and have an alpha of zero. When a stock's alpha is not zero, investors can improve upon the performance of the market portfolio. As we saw in Chapter 11, the Sharpe ratio of a portfolio will increase if we buy stocks whose expected return exceeds their required return—that is, if we buy stocks with positive alphas. Similarly, we can improve the performance of our portfolio by selling stocks with negative alphas.

Thus, as savvy investors attempt to trade to improve their portfolios, they raise the price and lower the expected return of the positive-alpha stocks, and they depress the price and raise the expected return of the negative-alpha stocks, until the stocks are once again on the security market line and the market portfolio is efficient.

Notice that the actions of investors have two important consequences. First, while the CAPM conclusion that the market is always efficient may not literally be true, competition among savvy investors who try to “beat the market” and earn a positive alpha should keep the market portfolio close to efficient much of the time. In that sense, we can view the CAPM as an approximate description of a competitive market. Second, trading strategies that take advantage of non-zero alpha stocks may exist, and by doing so actually can beat the market.

13.2 Information and Rational Expectations

If news that alters a stock's expected return is publically announced, there are likely to be a large number of investors who receive this news and act on it. Similarly, anybody who hears the news will not want to sell at the old prices. The only way to remove this imbalance is for the price to rise so that the alpha is zero. Note that in this case it is quite possible for the new

prices to come about without trade. That is, the competition between investors is so intense that prices move before any investor can actually trade at the old prices, so no investor can profit from the news.

In order to profit by buying a positive-alpha stock, there must be someone willing to sell it. Under the CAPM assumption of homogenous expectations, which states that all investors have the same information, it would seem that all investors would be aware that the stock had a positive alpha and none would be willing to sell. In reality, investors have different information and spend varying amounts of effort researching stocks. Consequently, we might expect that sophisticated investors would learn that a stock has a positive alpha, and that they would be able to purchase shares from more naïve investors.

However, even differences in the quality of investors' information will not necessarily be enough to generate trade in this situation. An important conclusion of the CAPM is that investors should hold the market portfolio (combined with risk-free investments), and this investment advice does not depend on the quality of an investor's information or trading skill. Even naïve investors with no information can follow this investment advice, and as the following example shows, by doing so they can avoid being taken advantage of by more sophisticated investors.

Because the average portfolio of all investors is the market portfolio, the average alpha of all investors is zero. If no investor earns a negative alpha, then no investor can earn a positive alpha, and the market portfolio must be efficient. As a result, the CAPM does not depend on the assumption of homogeneous expectations. Rather it requires only that investors have **rational expectations**, which means that all investors correctly interpret and use their own information, as well as information that can be inferred from market prices or the trades of others.

For an investor to earn a positive alpha and beat the market, some investors must hold portfolios with negative alphas. Because these investors could have earned a zero alpha by holding the market portfolio, we reach the following important conclusion:

The market portfolio can be inefficient (so it is possible to beat the market) only if a significant number of investors either:

1. Do not have rational expectations so that they misinterpret information and believe they are earning a positive alpha when they are actually earning a negative alpha, or
2. Care about aspects of their portfolios other than expected return and volatility, and so are willing to hold inefficient portfolios of securities.

13.3 The Behavior of Individual Investors

One of the most important implications of our discussion of risk and return is the benefit of diversification. By appropriately diversifying their portfolios, investors can reduce risk without reducing their expected return. In that sense, diversification is a “free lunch” that all investors should take advantage of.

Despite this benefit, evidence suggests that individual investors fail to diversify their portfolios adequately. Evidence from the U.S. Survey of Consumer Finances shows that, for households that held stocks, the median number of stocks held by investors in 2001 was four, and 90% of investors held fewer than ten different stocks.

There are a number of potential explanations for this behavior. One is that investors suffer from a familiarity bias, so that they favor investments in companies they are familiar with. Another is that investors have relative wealth concerns and care most about the performance

of their portfolio relative to that of their peers. This desire to “keep up with the Joneses” can lead investors to choose undiversified portfolios that match those of their colleagues or neighbors. In any case, this underdiversification is one important piece of evidence that individual investors may choose suboptimal portfolios.

Another bias comes from the finding that uninformed individuals tend to overestimate the precision of their knowledge. An implication of this overconfidence hypothesis is that, assuming they have no true ability, investors who trade more will not earn higher returns. Instead, their performance will be worse once we take into account the costs of trading (due to both commissions and bid-ask spreads). An implication of this overconfidence hypothesis is that, assuming they have no true ability, investors who trade more will not earn higher returns. Instead, their performance will be worse once we take into account the costs of trading (due to both commissions and bid-ask spreads).

In order for the behavior of uninformed investors to have an impact on the market, there must be patterns to their behavior that lead them to depart from the CAPM in systematic ways, thus imparting systematic uncertainty into prices. For investors' trades to be correlated in this way, they must share a common motivation.

13.4 Systematic Trading Biases

Investors tend to hold on to stocks that have lost value and sell stocks that have risen in value since the time of purchase. We call this tendency to hang on to losers and sell winners the **disposition effect**.

Studies also show that individuals are more likely to buy stocks that have recently been in the news, engaged in advertising, experienced exceptionally high trading volume, or have had extreme (positive or negative) returns.

In addition, investors appear to put too much weight on their own experience rather than considering all the historical evidence. As a result, people who grow up and live during a time of high stock returns are more likely to invest in stocks than people who grow up and live during a time of low stock returns.

An alternative reason why investors make similar trading errors is that they are actively trying to follow each other's behavior. This phenomenon, in which individuals imitate each other's actions, is referred to as **herd behavior**. Traders might herd in their portfolio choices because they might believe others have superior information that they can take advantage of by copying their trades; they may choose to herd in order to avoid the risk of underperforming their peers; and professional fund managers may face reputational risk if they stray too far from the actions of their peers.

Regardless of why individual investors choose not to protect themselves by holding the market portfolio, the fact that they don't has potential implications for the CAPM. If individual investors are engaging in strategies that earn negative alphas, it may be possible for a few more sophisticated investors to take advantage of this behavior and earn positive alphas.

13.5 The Efficiency of the Market Portfolio

In order for sophisticated investors to profit from investor mistakes, two conditions must hold. First, the mistakes must be sufficiently pervasive and persistent to affect stock prices. Second, there must be limited competition to exploit these non-zero alpha opportunities. If competition is too intense, these opportunities will be quickly eliminated before any trader can take advantage of them in a significant way.

If enough other investors are not paying attention, perhaps one can profit from public news announcements. However, investors should not expect this to be the case. Evidence shows that investors trying to profit from such news announcements quickly incorporate the implications of the news into the market prices. Since the average investor earns an alpha of zero, before including trading costs, beating the market should require special skills, such as better analysis of information or lower trading costs.

Numerous studies report that the actual returns to investors of the average mutual fund have a negative alpha. This suggests that mutual fund managers do not have the special skills, such as better analysis of information, necessary to find stocks that consistently beat the market and suggests that the market is largely efficient. Further, mutual funds generate trading costs and charge management fees, which further erode investors' returns.

Some researchers further categorize such tests as *weak form*, *semi-strong form*, and *strong form efficiency*. Weak form efficiency states that it should not be possible to profit by trading on information in past prices. Semi-strong form efficiency states that it should not be possible to consistently profit by trading on any public information, such as news announcements or analysts' recommendations. Finally, strong form efficiency states that it should not be possible to consistently profit even by trading on private information.

13.6 Style-Based Anomalies and the Market Efficiency Debate

- Portfolios of small stocks (those with a low market capitalizations = stock price x shares outstanding) have higher average returns. This empirical result is called the size effect.
- Portfolios of stocks with high book-to-market ratios (the ratio of the book value of equity to the market value of equity) have higher average returns.
- Portfolios of stocks that performed well in the previous year have higher average returns in the following year. Trading to take advantage of this relation is called a momentum strategy.

Over the years since the discovery of the CAPM, it has become increasingly clear to researchers and practitioners alike that by forming portfolios based on market capitalization, book-to-market ratios, and past returns, investors can construct trading strategies that have a positive alpha. Given these results, we are left to draw one of two conclusions.

1. Investors are systematically ignoring positive-NPV investment opportunities. That is, the CAPM correctly computes required risk premiums, but investors are ignoring opportunities to earn extra returns without bearing any extra risk, either because they are unaware of them or because the costs to implement the strategies are larger than the NPV of undertaking them.
2. The positive-alpha trading strategies contain risk that investors are unwilling to bear but the CAPM does not capture. That is, a stock's beta with the market portfolio does not adequately measure a stock's systematic risk, and so the CAPM does not correctly compute the risk premium.

The only way a positive-NPV opportunity can persist in a market is if some barrier to entry restricts competition. The existence of these trading strategies has been widely known for more than fifteen years. Not only is the information required to form the portfolios readily available, but many mutual funds follow momentum-based and market capitalization/book-to-market-based strategies. Hence, the first conclusion does not seem likely.

That leaves the second possibility: The market portfolio is not efficient, and therefore a stock's beta with the market is not an adequate measure of its systematic risk. Stated

another way, the profits (positive alphas) from the trading strategy are really returns for bearing risk that investors are averse to and the CAPM does not capture.

13.7 Multifactor Models of Risk

The expected return of any marketable security can be written as a function of the expected return of the efficient portfolio:

$$E[R_i] = r_i = r_f + \beta_i^{\text{efficient portfolio}} (E[R_{\text{efficient portfolio}}] - r_f)$$

However, identifying an efficient portfolio is difficult to measure because expected returns and the standard deviations of a portfolio cannot be measured with great accuracy. Fortunately, it is not actually necessary to identify the efficient portfolio itself; all that is required is to identify a collection of portfolios from which the efficient portfolio can be constructed. The risk premium of any marketable security can then be written as the sum of the risk premium of each portfolio multiplied by the sensitivity of the stock with that portfolio—the factor betas, β_s^1 and β_s^2 :

$$E[R_s] = r_f + \beta_s^1 (E[R_1] - r_f) + \beta_s^2 (E[R_2] - r_f)$$

where beta is measured relative to portfolios 1 and 2 which capture different **risk factors**. Thus, when more than one portfolio is used to measure market risk, the model is known as a **multifactor model**, and the model can be extended to any number of portfolios. The portfolios themselves can be thought of as either the risk factors or a portfolio of stocks correlated with an unobservable risk factor. This particular form of the multifactor model was originally developed by Stephen Ross and is referred to as the **Arbitrage Pricing Theory (APT)**.

The most common portfolio to use in a multifactor model is the market portfolio, which just needs to capture at least some components of systematic risk. Other popular portfolios are the **small-minus-big (SMB) portfolio**, the **high-minus-low (HML) portfolio**, and the **momentum portfolio**, which are generally constructed as follows.

- **SMB.** Firms below the median market value of NYSE firms each month form an equally weighted portfolio, S, and firms above the median market value form an equally weighted portfolio, B. A trading strategy that each year buys portfolio S and finances this position by short selling portfolio B has historically produced positive risk-adjusted returns.
- **HML.** Each year firms with book-to-market ratios less than the 30th percentile of NYSE firms are used to form an equally weighted portfolio called the low portfolio, L. Firms with book-to-market ratios greater than the 70th percentile of NYSE firms form an equally weighted portfolio called the high portfolio, H. A trading strategy that each year takes a long position in portfolio H, which it finances with a short position in portfolio L, has produced positive risk-adjusted returns.
- **Momentum.** Each year stocks are ranked by their return over the last year, and a portfolio is constructed that goes long on the top 30% of stocks and short on the bottom 30%. This trading strategy requires holding this portfolio for a year, and this process is repeated annually. The resulting portfolio is known as the **prior one-year momentum portfolio (PR1YR)**.

Berk (1995) provides the following explanation for the size effect. When the market portfolio is not efficient, some stocks will plot above the SML, and some will plot below this line. All else equal, a positive alpha implies that the stock also has a relatively higher expected return. A higher expected return implies a lower price—the only way to offer a higher expected

return is for investors to buy the stock's dividend stream at a lower price. A lower price means a lower market capitalization. Thus, when the market portfolio is not efficient, you should expect to observe the size effect.

All portfolios except the market portfolio are referred to as **self-financing** portfolios because the long position is financed by taking a short position, and because they require no net investment.

The collection of these four portfolios is currently the most popular choice for the multifactor model and is sometimes referred to as the **Fama-French-Carhart (FFC) factor model**, where:

$$E[R_s] = r_f + \beta_s^{\text{Mkt}}(E[R_{\text{Mkt}}] - r_f) + \beta_s^{\text{SMB}}E[R_{\text{SMB}}] + \beta_s^{\text{HML}}E[R_{\text{HML}}] + \beta_s^{\text{PR1YR}}E[R_{\text{PR1YR}}]$$

The FFC factor specification was identified a little more than ten years ago. Although it is widely used in academic literature to measure risk, much debate persists about whether it really is a significant improvement over the CAPM.

13.8 Methods Used in Practice

All of the techniques are imprecise, and there is no model of expected returns that gives an exact estimate of the cost of capital. John Graham and Campbell Harvey surveyed 392 CFOs and found that 73.5% of the firms use the CAPM to calculate the cost of capital. They also found that larger firms were more likely to use the CAPM. About one third reported using a multifactor model to calculate the cost of capital.

Appendix: Building a Multifactor Model

If an efficient portfolio can be constructed out of a collection of well-diversified portfolios, the collection of portfolios will correctly price assets. To keep things simple, assume that we have identified two portfolios that we can combine to form an efficient portfolio called **factor portfolios** and denote their returns by R_{F1} and R_{F2} . The efficient portfolio consists of some (unknown) combination of these two factor portfolios, represented by portfolio weights x_1 and x_2 :

$$R_{\text{eff}} = x_1 R_{F1} + x_2 R_{F2}$$

To see that we can use these factor portfolios to measure risk, consider regressing the excess returns of some stock s on the excess returns of *both* factors:

$$R_s - r_f = \alpha_s + \beta_s^{F1}(R_{F1} - r_f) + \beta_s^{F2}(R_{F2} - r_f) + \varepsilon_s$$

We write the excess return of stock s as the sum of a constant, α_s , plus the variation in the stock that is related to each factor, and an error term ε_s that has an expectation of zero and is uncorrelated with either factor. The error term represents the risk of the stock that is unrelated to either factor.

If we can use the two factor portfolios to construct the efficient portfolio, then the constant term α_s is zero. To see why, consider a portfolio in which we buy stock s , then sell a fraction β_s^{F1} of the first factor portfolio and β_s^{F2} of the second factor portfolio, and invest the proceeds from these sales in the risk-free investment. This portfolio, which we call P , has return

$$\begin{aligned} R_p &= R_s - \beta_s^{F1}R_{F1} - \beta_s^{F2}R_{F2} + (\beta_s^{F1} + \beta_s^{F2})r_f \\ &= R_s - \beta_s^{F1}(R_{F1} - r_f) - \beta_s^{F2}(R_{F2} - r_f) \end{aligned}$$

The return of this portfolio is

$$R_p = r_f + \alpha_s + \varepsilon_s.$$

That is, portfolio P has a risk premium of α_s and risk given by ε_s . Now, because ε_s is uncorrelated with each factor, it must be uncorrelated with the efficient portfolio. But recall from Chapter 11 that *risk that is uncorrelated with the efficient portfolio is firm-specific risk that does not command a risk premium*. Therefore, the expected return of portfolio P is r_f , which means α_s must be zero.

Setting α_s equal to zero and taking expectations of both sides of the equation we get the following two-factor model of expected returns:

$$E[R_s] = r_f + \beta_s^{F1} (E[R_{F1}] - r_f) + \beta_s^{F2} (E[R_{F2}] - r_f).$$

Selected Concepts and Key Terms

Alpha

The difference between a security's expected return and its CAPM required return from the security market line. According to the CAPM, all stocks and securities should be on the security market line and have an alpha of zero. If some securities have a non-zero alpha, the market portfolio is not efficient, and its performance can be improved by buying securities with positive alphas and selling those with negative alphas.

Arbitrage Pricing Theory (APT), Multifactor Model

When more than one portfolio is used to measure systematic risk, the model is known as a multifactor model. The portfolios themselves can be thought of as either the risk factors or as portfolios of stocks correlated with unobservable risk factors. The Arbitrage Pricing Theory is a form of multifactor model originally developed by Stephen Ross.

Disposition Effect

The empirically documented tendency for investors to hold on to stocks that have lost value and sell stocks that have risen in value since the time of purchase.

Fama-French-Carhart (FFC) Factor Specification

Because using the portfolios, SMB, HML, and momentum, along with the market, were identified by Eugene Fama, Kenneth French, and Mark Carhart, this specification of the multifactor model is sometimes referred to as the Fama-French-Carhart (FFC) factor model.

Herd Behavior

The tendency of individual investors to imitate each other's actions. Traders might herd in their portfolio choices because they might believe others have superior information that they can take advantage of by copying their trades; they may choose to herd in order to avoid the risk of underperforming their peers; and professional fund managers may face reputational risk if they stray too far from the actions of their peers.

High-Minus-Low (HML) Portfolio

A portfolio often used in factor models that equals to the return on high book-to-market firms minus the return on low book-to-market firms. Each year firms with book-to-market ratios less than the 30th percentile of NYSE firms are used to form an equally weighted portfolio called the low portfolio, L. Firms with book-to-market ratios greater than the 70th percentile of NYSE firms form an equally weighted portfolio called the high portfolio, H. A trading strategy that each year takes a long position in portfolio H, which it finances with a short position in portfolio L, has produced positive risk-adjusted returns.

Momentum Strategy

Buying stocks that performed well in the prior period, often six months or a year, and holding them for the next period.

Prior One-Year Momentum (PR1YR) Portfolio

A portfolio often used in factor models. To form this portfolio, each year stocks are ranked by their return over the last year and a portfolio is constructed that goes long on the top 30% of stocks and short on the bottom 30%. This trading strategy requires holding the portfolio for a year, and this process is repeated annually.

Rational Expectations

An economic theory that can be used to model how investors impound information in stock prices. In the context of the CAPM, it implies that although investors may have different information regarding expected returns, correlations, and volatilities, they correctly interpret the information contained in market prices and adjust their estimates of expected returns in a rational way.

Semi-Strong Form Efficiency

The theory that consistent profits should not be possible from trading on any public information, such as news announcements or analysts' recommendations.

Size Effect

The observation that small stocks have positive CAPM alphas, which was first discovered in 1981 by Rolf Banz in "The Relationship between Return and Market Values of Common Stock," *Journal of Financial Economics* 9 (March 1981): 3–18.

Small-Minus-Big (SMB) Portfolio

A portfolio often used in factor models equal to the return on small firms minus the return on big firms. Firms below the median market value of NYSE firms each month form an equally weighted portfolio, S, and firms above the median market value form an equally weighted portfolio, B. A trading strategy that buys portfolio S and finances this position by short selling portfolio B is the SMB portfolio has historically produced positive risk-adjusted returns.

Strong Form Efficiency

The theory that it should not be possible to consistently profit even by trading on private information.

Weak Form Efficiency

The theory that it should not be possible to profit by trading on information in past prices by, for example, selling winners and hanging on to losers.

Concept Check Questions and Answers

13.1.1. If investors attempt to buy a stock with a positive alpha, what is likely to happen to its price and expected return? How will this affect its alpha?

As savvy investors attempt to trade to improve their portfolios, they raise the price and lower the expected return of the positive-alpha stocks, and they depress the price and raise the expected return of the negative-alpha stocks, until the stocks are once again on the security market line, the market portfolio is efficient, and all stocks have zero alpha.

13.1.2. What is the consequence of investors exploiting non-zero alpha stocks for the efficiency of the market portfolio?

Competition among savvy investors who try to “beat the market” and earn a positive alpha should keep the market portfolio close to efficient much of the time.

13.2.1. How can an uninformed or unskilled investor guarantee themselves a non-negative alpha?

Naïve investors with no information can hold the market portfolio (combined with risk-free investments) to ensure a non-negative alpha.

13.2.2. Under what conditions will it be possible to earn a positive alpha and beat the market?

For an investor to earn a positive alpha and beat the market, some investors must hold portfolios with negative alphas.

13.3.1. Do investors hold well-diversified portfolios?

There is much evidence that individual investors fail to diversify their portfolios adequately. Evidence from the U.S. Survey of Consumer Finances shows that, for households that held stocks, the median number of stocks held by investors in 2001 was four, and 90% of investors held fewer than ten different stocks.

13.3.2. Why is the high trading volume observed in markets inconsistent with the CAPM equilibrium?

Because the market portfolio is a value-weighted portfolio, it is also a passive portfolio in the sense that an investor does not need to trade in response to daily price changes in order to maintain it. Thus, if all investors held the market, we would see relatively little trading volume in financial markets.

13.3.3. What must be true about the behavior of small, uninformed investors for them to have an impact on market prices?

In order for the behavior of uninformed investors to have an impact on the market, there must be patterns to their behavior that lead them to depart from the CAPM in systematic ways, thus imparting systematic uncertainty into prices. For investors' trades to be correlated in this way, they must share a common motivation.

13.4.1. What are several systematic behavioral biases that individual investors fall prey to?

Investors tend to hold on to stocks that have lost value and sell stocks that have risen in value since the time of purchase. We call this tendency to hang on to losers and sell winners the disposition effect. Studies also show that individuals are more likely to buy

stocks that have recently been in the news, engaged in advertising, experienced exceptionally high trading volume, or have had extreme (positive or negative) returns. In addition, investors appear to put too much weight on their own experience rather than considering all the historical evidence. As a result, people who grow up and live during a time of high stock returns are more likely to invest in stocks than people who grow up and live during a time of low stock returns.

13.4.2. What implication might these behavioral biases have for the CAPM?

Regardless of why individual investors choose not to protect themselves by holding the market portfolio, the fact that they don't has potential implications for the CAPM. If individual investors are engaging in strategies that earn negative alphas, it may be possible for a few more sophisticated investors to take advantage of this behavior and earn positive alphas.

13.5.1. Should uninformed investors expect to make money by trading based on news announcements?

If enough other investors are not paying attention, perhaps one can profit from public news announcements. However, investors should not expect this to be the case. Evidence shows that investors trying to profit from such news announcements quickly incorporate the implications of the news into the market prices.

13.5.2. If fund managers are talented, why do the returns of their funds to investors not have positive alphas?

Numerous studies report that the actual returns to investors of the average mutual fund have a negative alpha. This suggests that mutual fund managers do not have the special skills, such as better analysis of information, necessary to find stocks that consistently beat the market and suggests that the market is largely efficient. Further, mutual funds generate trading costs and charge management fees, which further erode investors' returns.

13.6.1. What does the existence of a positive-alpha trading strategy imply?

There are two different implications. 1) The positive-alpha trading strategies contain risk that investors are unwilling to bear but the CAPM does not capture. That is, a stock's beta with the market portfolio does not adequately measure a stock's systematic risk, and so the CAPM does not correctly compute the risk premium. 2) Investors are systematically ignoring positive-NPV investment opportunities. That is, the CAPM correctly computes required risk premiums, but investors are ignoring opportunities to earn extra returns without bearing any extra risk, either because they are unaware of them or because the costs to implement the strategies are larger than the NPV of undertaking them.

13.6.2. If investors have a significant amount of non-tradeable (but risky) wealth, why might the market portfolio not be efficient?

If investors have a significant amount of non-tradeable wealth, this wealth will be an important part of their portfolios, but will not be part of the market portfolio of tradeable securities. In such a world, the market portfolio of tradeable securities will likely not be efficient.

13.7.1. What is the advantage of a multifactor model over a single factor model?

Multifactor models have a distinct advantage over single-factor models in that it is much easier to identify a collection of portfolios that captures systematic risk than just a single portfolio.

13.7.2. How can you use the Fama-French-Carhart factor specification to estimate the cost of capital?

The Fama-French-Carhart factor specification identifies the collection of four portfolios that are most commonly used in a multifactor model. These portfolios include the market portfolio, small-minus-big portfolio, high-minus-low portfolio, and prior one-year momentum portfolio.

13.8.1. Which is the most popular method used by corporations to calculate the cost of capital?

John Graham and Campbell Harvey surveyed 392 CFOs and found that 73.5% of the firms use the CAPM to calculate the cost of capital and that larger firms were more likely to use the CAPM.

13.8.2. What other techniques do corporations use to calculate the cost of capital?

John Graham and Campbell Harvey surveyed 392 CFOs and found that about one third reported using a multifactor model to calculate the cost of capital. Two other methods that some firms in the survey reported using are historical average returns (40%) and the dividend discount model (16%).

Examples with Step-by-Step Solutions

Solving Problems

Quantitative problems using the concepts in this chapter may require determining if stocks are over- or undervalued according to the CAPM and interpreting stock alphas. They may also involve finding the cost of capital using a factor model, such as the Fama-French-Carhart model. Below are examples of each.

Examples

- Suppose that a firm with a stock price of \$80 just announced that it expects to pay a \$100 per share liquidating dividend in 1 year, although the exact amount of the dividend depends on the performance of the company this year. Assume that the CAPM is a good description of stock price returns and that the stock's beta is 1.5, the market's expected return is 12%, and the risk-free rate is 5%.

[A] Is the stock priced correctly now?

[B] What is the alpha of the stock?

[C] What would you expect to happen to the stock price in the market after the announcement?

Step 1. Determine the stock's expected return.

$$E[R] = r_f + \beta_1^{\text{Mkt}}(R_{\text{Mkt}} - r_f) = .05 + 1.5(.12 - .05) = 0.148 = 15.5\%$$

Step 2. Determine the value of the stock.

$$P_0 = \frac{\$100}{1.155} = \$86.58$$

Thus, the stock is undervalued by $\$86.58 - \$80 = \$6.58$.

Step 3. To determine the stock's alpha, find the stock's expected return.

$$P_0 = \$86.58 = \frac{\$100}{1+r} \Rightarrow 1+r = \frac{\$100}{\$80} = 1.25 \Rightarrow r = 25\%$$

Thus, the alpha is $25\% - 15.5\% = 9.5\%$.

Step 4. Predict what would happen in an efficient market.

Since the stock has a positive alpha, investors should now increase the market price to \$86.58. Thus, when the stock begins trading after the announcement, you should not expect to be able to buy the stock for a price below \$86.58.

- 2. Assume that the CAPM is a good description of stock price returns. You observe the following information for three stocks:**

	Beta	Expected Return
Ford	1.4	18%
General Electric	1.0	12%
Yahoo!	2.8	22%

Based on the stocks' alphas, which should you buy if the market's expected return is 12% and the risk-free rate is 5%?

Step 1. Calculate the CAPM expected returns on each stock.

$$E[R_{\text{Ford}}] = r_f + \beta_i^{\text{Mkt}}(R_{\text{Mkt}} - r_f) = .05 + 1.4(.12 - .05) = 0.148 = 14.8\%$$

$$E[R_{\text{GE}}] = r_f + \beta_i^{\text{Mkt}}(R_{\text{Mkt}} - r_f) = .05 + 1.0(.12 - .05) = 0.120 = 12.0\%$$

$$E[R_{\text{Yahoo!}}] = r_f + \beta_i^{\text{Mkt}}(R_{\text{Mkt}} - r_f) = .05 + 2.8(.12 - .05) = 0.246 = 24.6\%$$

Step 2. Calculate the alpha of each stock.

$$\alpha_i = E[R_i] - r_i = E[R_i] - (r_f + \beta_i^{\text{Mkt}}(E[R_{\text{Mkt}}] - r_f))$$

$$\alpha_{\text{Ford}} = 18\% - 14.8\% = 3.2\%$$

$$\alpha_{\text{GE}} = 12\% - 12\% = 0\%$$

$$\alpha_{\text{Yahoo!}} = 22\% - 24.6\% = -2.6\%$$

So you should buy Ford because it has a positive alpha.

- 3. Based on the Fama-French-Carhart factor model, which of the following stocks has the most systematic risk? If the risk-free rate is 5%, what are the firms' equity costs of capital?**

Factor	Stock A	Stock B
MKT	1.26	0.45
SMB	-0.43	0.33
HML	-0.38	0.23
PR1YR	-0.33	-0.35

Mean Monthly Return 1926-2005

MKT - r_f	0.64%
SMB	0.17%
HML	0.53%
PR1YR	0.76%

Step 1. The stock with the most systematic risk has the highest risk premium, so the risk premiums for stocks A and B must be calculated.

First, the monthly risk premiums:

$$RP_A = 1.26(0.64) + -0.43(0.17) + -0.38(0.53) + -0.33(0.76) = 0.281\%$$

$$RP_B = 0.45(0.64) + 0.33(0.17) + 0.23(0.53) + -0.35(0.76) = 0.200\%$$

Then the annual risk premiums,

$$RP_A = (1 + 0.00281)^{12} - 1 = 3.4\%$$

$$RP_B = (1 + 0.00200)^{12} - 1 = 2.4\%$$

So, stock A has the highest risk premium and thus the most systematic risk.

Step 2. Calculate the equity cost of capital for each firm.

The expected return can be calculated based on the FFC factor model:

$$E[R_S] = r_f + \beta_S^{\text{Mkt}} (E[R_{\text{Mkt}}] - r_f) + \beta_S^{\text{SMB}} E[R_{\text{SMB}}] + \beta_S^{\text{HML}} E[R_{\text{HML}}] + \beta_S^{\text{PR1YR}} E[R_{\text{PR1YR}}]$$

$$E[R_A] = 0.05 + 0.034 = 8.4\%$$

$$E[R_B] = 0.05 + 0.024 = 7.4\%$$

Questions and Problems

1. What are the three types of portfolios discussed in this chapter that have had positive CAPM alphas.
2. Explain the Berk (1995) explanation for the size effect.
3. Describe the factors in the Fama-French-Carhart factor model.
4. How is the small-minus-big (SMB) factor in the Fama-French-Carhart factor model estimated?
5. How is the high-minus-low (HML) factor in the Fama-French-Carhart factor model estimated?

Solutions to Questions and Problems

1. The three types of portfolios that have positive CAPM alphas are small firms (firms with a low stock market capitalization), firms with high book value of equity-to-market value of equity ratios, and firms with positive momentum (i.e. high returns in the prior period).
2. When the market portfolio is not efficient, you should expect to observe the size effect. When the market portfolio is not efficient, some stocks will plot above the SML, and some will plot below this line. All else equal, a positive alpha implies that the stock also has a relatively higher expected return. A higher expected return implies a lower price—the only way to offer a higher expected return is for investors to buy the stock's dividend stream at a lower price. A lower price means a lower market capitalization. Thus, when a financial economist forms a

portfolio of stocks with low market capitalizations, that collection contains stocks that will likely have higher expected returns and, if the market portfolio is not efficient, positive alphas.

3. The factors in the Fama-French-Carhart factor model are as follows.

- **The market.** This is similar to the CAPM market portfolio.
- **SMB (the small-minus-big portfolio).** This factor equals the return on small capitalization firms minus the return on large capitalization firms.
- **HML (the high-minus-low portfolio).** This factor equals to the return on high book-to-market firms minus the return on low book-to-market firms.
- **Momentum.** This factor measures the return on a stock over a prior period, such as one year.

4. Firms below the median market value of NYSE firms each month form an equally weighted portfolio, S, and firms above the median market value form an equally weighted portfolio, B.

5. Each year firms with book-to-market ratios less than the 30th percentile of NYSE firms are used to form an equally weighted portfolio called the low portfolio, L. Firms with book-to-market ratios greater than the 70th percentile of NYSE firms form an equally weighted portfolio called the high portfolio, H.