

Assignment_3

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Objective Function Minimize $TC = 622x_{11} + 614x_{12} + 630x_{13} + 641x_{21} + 645x_{22} + 649x_{23}$

These are subject to following constraints supply constraints: $x_{11} + x_{12} + x_{13} \geq 100$

$x_{21} + x_{22} + x_{23} \geq 120$ demand constraints: $x_{11} + x_{21} \geq 80$ $x_{12} + x_{22} \geq 60$ $x_{13} + x_{23} \geq 70$

Now all are subjected to non-negativity where $x_{ij} \geq 0$ where $i=1,2$ and $j=1,2,3$

#loading packages

```
library(Matrix)
library("lpSolve")

## Warning: package 'lpSolve' was built under R version 4.1.3

display <- matrix(c(22,14,30,600,100,
                    16,20,24,625,120,
                    80,60,70,"-", "210/220"), ncol=5, nrow=3, byrow=TRUE)
colnames(display) <- c("Warehouse1", "Warehouse2", "Warehouse3", "Production
Cost", "Production Capacity")
rownames(display) <- c("PlantA", "PlantB", "Monthly Demand")
display <- as.table(display)
display

##              Warehouse1 Warehouse2 Warehouse3 Production Cost
## PlantA              22          14          30          600
## PlantB              16          20          24          625
## Monthly Demand      80          60          70          -
##              Production Capacity
## PlantA              100
## PlantB              120
## Monthly Demand    210/220

display1 <- matrix(c(622,614,630,0,100,
                    641,645,649,0,120,
                    80,60,70,10,220), ncol=5, nrow=3, byrow=TRUE)
colnames(display1) <-
c("Warehouse1", "Warehouse2", "Warehouse3", "Dummy", "Production Capacity")
rownames(display1) <- c("PlantA", "PlantB", "Monthly Demand")
display1 <- as.table(display1)
display1

##              Warehouse1 Warehouse2 Warehouse3 Dummy Production Capacity
## PlantA              622          614          630          0          100
```

## PlantB	641	645	649	0	120
## Monthly Demand	80	60	70	10	220

#The balanced problem will be satisfied by this table. Below is a cost totals matrix that we created.

```
totalcosts <- matrix(c(622,614,630,0,
                      641,645,649,0),nrow=2, byrow = TRUE)
```

#finding the production capacity in the matrix's row

```
row.rhs <- c(100,120)
row.signs <- rep("<=", 2)
```

#using the double variable 10 at the end to determine the monthly demand.

```
col.rhs <- c(80,60,70,10)
col.signs <- rep(">=", 4)
```

#now we are ready to run LP Transport command

```
lp.transport(totalcosts,"min",row.signs,row.rhs,col.signs,col.rhs)
```

Success: the objective function is 132790

solution matrix

```
lp.transport(totalcosts, "min", row.signs, row.rhs, col.signs,
col.rhs)$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

We can conclude from this that $Z = \$132790$. The results are as follows for each of the variables: 60x12 which is the Warehouse 2 from plant A. 40x13 which is the Warehouse 3 from plant A. 80x21 which is the Warehouse 1 from plant B. 30x23 which is the Warehouse 3 from plant B. and because "10" shows up in the 4th Variable 10x24 is a "throw away variable".

Question 2) We are aware that the number of constants in dual and the number of variables in primal are the same. The primary of the LP is asked in the first question. We shall maximize in the dual because we choose to minimize in the primal. Let's utilize "m" and "n" as our variables for the dual problem.

```
display2 <- matrix(c(622,614,630,100,"m_1",
                    641,645,649,120,"m_2",
                    80,60,70,220,"-",
                    "n_1","n_2","n_3","-","-"),ncol=5,nrow=4,byrow=TRUE)
colnames(display2) <- c("W1","W2","W3","Prod Cap","Supply (Dual)")
rownames(display2) <- c("PlantA","PlantB","Monthly Demand","Demand (Dual)")
```

```
display2 <- as.table(display2)
display2

##           W1  W2  W3  Prod Cap Supply (Dual)
## PlantA      622 614 630 100      m_1
## PlantB      641 645 649 120      m_2
## Monthly Demand 80  60  70  220      -
## Demand (Dual)  n_1 n_2 n_3  -      -
```

Now we are going to create our objective function based on the constraints from the primal. Later we will use the objective function from the primal to find the constants of the dual.

Maximize $Z = 100m_1 + 120m_2 + 80n_1 + 60n_2 + 70n_3$

This objective function is subject to following constraints:

$m_1 + n_1 \leq 622$

$m_1 + n_2 \leq 614$

$m_1 + n_3 \leq 630$

$m_2 + n_1 \leq 641$

$m_2 + n_2 \leq 645$

$m_2 + n_3 \leq 649$

These variables were extracted from the linear programming function's transposed primal matrix. Transposing the f.con into the matrix and comparing it to the primal's above constants is an easy way to verify. These are confined where $m=1,2$ & $n=1,2,3$ and m_k, n_l

#Constants of the primal are now the objective function variables.

```
f.obj <- c(100,120,80,60,70)
#transposed from the constraints matrix in the primal
f.con <- matrix(c(1,0,1,0,0,
                  1,0,0,1,0,
                  1,0,0,0,1,
                  0,1,1,0,0,
                  0,1,0,1,0,
                  0,1,0,0,1),nrow=6, byrow = TRUE)
#these change As we are MAX the dual not min
f.dir <- c("<=",
          "<=",
          "<=",
          "<=",
          "<=", "<=")
f.rhs <- c(622,614,630,641,645,649)
lp ("max", f.obj, f.con, f.dir, f.rhs)
```

```
## Success: the objective function is 139120

lp ("max", f.obj, f.con, f.dir, f.rhs)$solution

## [1] 614 633 8 0 16
```

So $Z=139,120$ dollars and variables are: $m1 = 614$ which represents plant A $m2 = 633$ which represents Plant B $n1 = 8$ which represents Warehouse 1 $n3 = 16$ which represents Warehouse 3

OBSERVATION

The minimal $Z=132790$ (Primal) and the maximum $Z=139120$ (Dual). What are we trying to max/min in this problem. We found that we should not be shipping from Plant(A/B) to all three Warehouses. We should be shipping from:

60x12 which is 60 Units from Plant A to Warehouse 2. 40x13 which is 40 Units from Plant A to Warehouse 3. 80x21 which is 60 Units from Plant B to Warehouse 1. 30x23 which is 60 Units from Plant B to Warehouse 3. Now we want to Max the profits from each distribution in respect to capacity.

Question 3)

$m1 - n1 \leq 622$

then we subtract $n1$ to the other side to get $m1 \leq 622 - n1$

To compute that value it would be $614 \leq (-8+622)$ which is true. We would continue to evaluate these equations:

$m1 \leq 622 - n1 \implies 614 \leq 622 - 8 = 614 = \text{TRUE}$ $m1 \leq 614 - n2 \implies 614 \leq 614 - 0 = 614 = \text{TRUE}$
 $m1 \leq 630 - n3 \implies 614 \leq 630 - 16 = 614 = \text{TRUE}$ $m2 \leq 641 - n1 \implies 633 \leq 641 - 8 = 633 = \text{TRUE}$
 $m2 \leq 645 - n2 \implies 633 \leq 645 - 0 = 645 = \text{NOT TRUE}$ $m2 \leq 649 - n3 \implies 633 \leq 649 - 16 = 633 = \text{TRUE}$

#By updating each column, we can test for the shadow price while also learning from the Duality-and-Sensitivity. We change the 100 to 101 and 120 to 121 in our LP Transport.

```
row.rhs1 <- c(101,120)
row.signs1 <- rep("<=", 2)
col.rhs1 <- c(80,60,70,10)
col.signs1 <- rep(">=", 4)
row.rhs2 <- c(100,121)
row.signs2 <- rep("<=", 2)
col.rhs2 <- c(80,60,70,10)
col.signs2 <- rep(">=", 4)
lp.transport(totalcosts, "min", row.signs, row.rhs, col.signs, col.rhs)

## Success: the objective function is 132790

lp.transport(totalcosts, "min", row.signs1, row.rhs1, col.signs1, col.rhs1)
```

```
## Success: the objective function is 132771
```

```
lp.transport(totalcosts, "min", row.signs2, row.rhs2, col.signs2, col.rhs2)
```

```
## Success: the objective function is 132790
```

Since we are taking the minimum of this particular function, the fact that the number decreases by 19 indicates that the shadow price, which was determined by adding 1 to each of the Plants and the primordial, is 19. Plant B, on the other hand, doesn't have a shadow price. Additionally, we discovered that the dual variable n_2 with the relationship Marginal Revenue (MR) = Marginal Cost (MC). Considering the equation that was It was determined that $m_2 = 645 - n_2 = 633 = 645 - 0 = 645 \neq 633$ = NOT TRUE by utilizing $m_1 - n_1 = 622$.

```
lp ("max", f.obj, f.con, f.dir, f.rhs)$solution
```

```
## [1] 614 633 8 0 16
```

n_2 was = 0.

CONCLUSION: from the primal: 60x12 which is 60 Units from Plant A to Warehouse 2. 40x13 which is 40 Units from Plant A to Warehouse 3. 80x21 which is 60 Units from Plant B to Warehouse 1. 30x23 which is 60 Units from Plant B to Warehouse 3. from the dual We want the MR=MC. Five of the six MR≤MC. The only equation that does not satisfy this requirement is Plant B to Warehouse 2. We can see that from the primal that we will not be shipping any AED device there.