```
In [56]: import pandas as pd
         import numpy as np
         import seaborn as sns
         import matplotlib.pyplot as plt
         from sklearn.preprocessing import MinMaxScaler, StandardScaler, RobustScaler #use std scaler only when the data is normal
         from sklearn.preprocessing import PowerTransformer
         from warnings import filterwarnings
         filterwarnings('ignore')
         import matplotlib.cm as cm
         from statsmodels.graphics.gofplots import qqplot
         from statsmodels.api import add_constant
         import statsmodels
         import statsmodels.api as sm
         import statsmodels.stats.api as sms
         from statsmodels.graphics.gofplots import qqplot
         from statsmodels.stats.outliers_influence import variance_inflation_factor
         import statsmodels.tsa.api as smt
         from scipy import stats
         np.set_printoptions(suppress = True)
         from sklearn.preprocessing import StandardScaler
         from sklearn.metrics import silhouette_score
         from sklearn.cluster import AgglomerativeClustering
         from sklearn.metrics.pairwise import euclidean_distances
         from sklearn.cluster import DBSCAN
         from scipy.cluster.hierarchy import linkage
         from scipy.cluster.hierarchy import dendrogram
         from scipy.cluster.hierarchy import cophenet
         from sklearn.model_selection import train_test_split
         from numpy.linalg import eig
         from sklearn.decomposition import PCA
         from sklearn.discriminant_analysis import LinearDiscriminantAnalysis as LDA
         from sklearn.tree import DecisionTreeClassifier
         from sklearn.metrics import accuracy_score, roc_auc_score
         from sklearn.cluster import KMeans
         from sklearn.metrics import silhouette_score, silhouette_samples
```

# In [2]: df = pd.read\_csv('student\_evaluation\_reduced (1).csv') df

_		instr	class	nb.repeat	attendance	difficulty	Q1	Q2	Q3	Q4	Q5	 Q19	Q20	Q21	Q22	Q23	Q24	Q25	Q26	Q27	Q28
	0	3	8	1	0	1	5	5	5	5	5	 5	5	5	5	5	5	5	5	5	5
	1	3	8	1	4	3	3	3	3	3	3	 3	3	3	3	3	3	3	3	3	3
	2	3	13	1	3	3	3	3	3	3	3	 3	3	3	3	3	3	3	3	3	3
	3	3	5	1	0	1	1	1	1	1	1	 1	1	1	1	1	1	1	1	1	1
	4	3	3	1	3	4	2	2	4	2	3	 4	4	3	4	3	3	3	3	2	4
	195	3	13	1	0	1	2	2	2	2	2	 3	2	2	2	2	2	2	2	2	3
4	196	2	1	1	3	4	4	4	4	5	4	 5	3	4	4	5	4	4	4	4	4
4	197	1	10	1	1	3	4	4	1	3	5	 3	3	3	3	3	3	3	3	3	3
4	198	3	3	1	3	1	5	5	5	5	5	 5	5	5	5	5	5	5	5	5	5
4	199	1	7	3	1	4	3	3	3	3	3	 3	3	3	3	3	3	3	3	3	3

500 rows × 33 columns

Out[2]:

```
In [3]: | df.info()
           <class 'pandas.core.frame.DataFrame'>
           RangeIndex: 500 entries, 0 to 499
           Data columns (total 33 columns):
                                Non-Null Count Dtype
                 Column
            0
                 instr
                                 500 non-null
                                                     int64
                                 500 non-null
            1
                 class
                                                     int64
                                500 non-null
                                                     int64
            2
                 nb.repeat
                 attendance
                                500 non-null
                                                     int64
            3
            4
                 difficulty
                                500 non-null
                                                     int64
                                 500 non-null
                                                     int64
            5
                 Q1
                 Q2
                                 500 non-null
                                                     int64
            6
            7
                 Q3
                                 500 non-null
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                 Q4
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                                                     int64
            9
                                 500 non-null
                                                     int64
                 Q5
                 Q6
                                 500 non-null
                                                     int64
            10
                 Q7
            11
                                 500 non-null
                                                     int64
                                 500 non-null
            12
                 Q8
                                                     int64
            13
                 Q9
                                 500 non-null
                                                     int64
In [4]: |round(df.describe(),3)
Out[4]:
                      instr
                               class nb.repeat attendance
                                                             difficulty
                                                                           Q1
                                                                                    Q2
                                                                                             Q3
                                                                                                      Q4
                                                                                                                Q5 ...
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                                         1.214
                                                      1.530
                                                                2.742
                                                                          2.98
                                                                                  3.134
                                                                                           3.222
                                                                                                    3.124
                                                                                                                          3.344
                                                                                                                                   3.344
                                                                                                                                             3.362
                                                                                                                                                      3.360
                                                                                                                                                               3.2
            mean
              std
                      0.694
                               3.765
                                         0.534
                                                      1.488
                                                                 1.359
                                                                          1.38
                                                                                  1.314
                                                                                           1.301
                                                                                                    1.345
                                                                                                             1.322 ...
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              min
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                                                                                                                                                      5.000
                                                                                                                                                               5.0
             max
           8 rows × 33 columns
In [5]: df.isna().sum()
Out[5]: instr
                             0
           class
                             0
                             0
           nb.repeat
           attendance
                             0
           difficulty
                             0
           Q1
                             0
                             0
           Q2
           Q3
                             0
           Q4
                             0
           Q5
                             0
                             0
           Q6
           Q7
                             0
                             0
           Q8
                             0
           Q9
                             0
           Q10
           Q11
                             0
           Q12
                             0
           Q13
                             0
                             0
           Q14
In [7]: df1 = df.columns
           df1
Out[7]: Index(['instr', 'class', 'nb.repeat', 'attendance', 'difficulty', 'Q1', 'Q2', 'Q3', 'Q4', 'Q5', 'Q6', 'Q7', 'Q8', 'Q9', 'Q10', 'Q11', 'Q12', 'Q13', 'Q14', 'Q15', 'Q16', 'Q17', 'Q18', 'Q19', 'Q20', 'Q21', 'Q22', 'Q23', 'Q24', 'Q25', 'Q26', 'Q27', 'Q28'],
                   dtype='object')
```

```
In [9]: t=1
            plt.figure(figsize = (17,15))
            for i in df1:
                plt.subplot(9,4,t)
                sns.boxplot(df[i])
                t+=1
           plt.show()
                                                            7.5
                                                                 10.0
                   1.5
                          2.0
                                2.5
                                                       5.0
                                                                      12.5
                                                                                      1.5
                                                                                            2.0
                                                                                                         3.0
In [108]: df.skew()
Out[108]: instr
                          -1.161289
            class
                           0.086780
            nb.repeat
                           2.447698
            attendance
                           0.348447
            difficulty
                           0.008813
            Q1
                          -0.042147
            Q2
                          -0.206242
                          -0.307886
            Q3
                          -0.202412
            Q4
                          -0.193517
            Q5
                          -0.270950
            Q6
            Q7
                          -0.172344
            Q8
                          -0.108285
            Q9
                          -0.268426
                          -0.229773
            Q10
                          -0.276114
            Q11
            Q12
                          -0.130354
                          -0.378527
            Q13
                          -0.447448
            Q14
 In [10]:
          df.corr()
 Out[10]:
                            instr
                                          nb.repeat attendance
                                                                difficulty
                                                                                Q1
                                                                                         Q2
                                                                                                   Q3
                                                                                                             Q4
                                                                                                                       Q5 ...
                                                                                                                                   Q19
                                                                                                                                             Q2
                                     class
                  instr
                        1.000000
                                 -0.023370
                                            0.076143
                                                      -0.118317
                                                                -0.066634
                                                                          -0.170904
                                                                                    -0.152960
                                                                                             -0.124389
                                                                                                       -0.152383 -0.157924 ... -0.104894
                                                                -0.023370
                                  1.000000
                                            0.080765
                                                      -0.056917
                 class
                        0.076143
                                  0.080765
                                            1.000000
              nb.repeat
                                                      -0.024509
                                                                 0.208909 -0.092166
                                                                                   -0.083825 -0.062802 -0.081736 -0.075036 ... -0.085625 -0.07638
            attendance -0.118317 -0.056917 -0.024509
                                                       1.000000
                                                                 0.456263
                                                                          0.012985
                                                                                    0.061991
                                                                                              0.123399
                                                                                                        0.095300
                                                                                                                  0.088504 ...
                                                                                                                              0.138929
                                                                                                                                        0.15619
                                           0.208909
                                                                 1.000000 -0.031614
                       -0.066634 -0.031623
                                                       0.456263
                                                                                    -0.037823 -0.007212 -0.036190 -0.027249 ...
                                                                                                                              0.004936
                                                                                                                                        0.03957
                                                                          1.000000
                   Q1 -0.170904 -0.069923 -0.092166
                                                                -0.031614
                                                                                    0.835876
                                                                                                        0.853620
                                                       0.012985
                                                                                              0.738357
                                                                                                                  0.811582 ...
                                                                                                                              0.688258
                                                                                                                                        0.68929
                   Q2 -0.152960 -0.090322 -0.083825
                                                       0.061991
                                                                -0.037823
                                                                          0.835876
                                                                                    1.000000
                                                                                              0.848773
                                                                                                        0.881800
                                                                                                                  0.893332 ...
                                                                                                                              0.803523
                                                                                                                                         0.79932
                                                                                             1.000000
                                                                                                        0.818276  0.888248  ...
                   Q3 -0.124389 -0.066082 -0.062802
                                                       0.123399 -0.007212 0.738357
                                                                                    0.848773
                                                                                                                              0.835292 0.84722
                    Q4 -0.152383 -0.070919 -0.081736
                                                       0.095300 \quad -0.036190 \quad 0.853620 \quad 0.881800 \quad 0.818276 \quad 1.000000 \quad 0.875099 \quad \dots \quad 0.781639 \quad 0.77417
                   Q5 -0.157924 -0.084938 -0.075036  0.088504 -0.027249  0.811582  0.893332  0.888248  0.875099  1.000000 ...  0.831881  0.82743
                   Q6 -0.139502 -0.072400 -0.071724 0.104051 -0.017220 0.783685 0.852389 0.829586 0.845379 0.918232 ... 0.804755 0.80865 ▼
```

```
In [11]: plt.figure(figsize=(20, 20))
           sns.heatmap(df.corr(), annot = True)
Out[11]: <AxesSubplot:>
                instr - 1 0.0230.076-0.12-0.067-0.17-0.15-0.12-0.15-0.16-0.14-0.15-0.17-0.16-0.14-0.15-0.17-0.12-0.16-0.14-0.15-0.17-0.12-0.086-0.11-0.16-0.073-0.15-0.1-0.0780.0810.084-0.14-0.16-0.11-0.14-0.14-0.14-0.11
                           nb.repeat -0.0760.081 1
                             attendance --0.12-0.0570.025 1
                                0.46 0.0130.062 0.12 0.0950.089 0.1 0.0780.052 0.12 0.094 0.13 0.079 0.13 0.15 0.14 0.069 0.2 0.093 0.14 0.16 0.17 0.16 0.1 0.077 0.15 0.13 0.087 0.16
             difficulty -0.0670.0320.21 0.46 1 0.0320.0380.00720.0360.0270.017-0.02-0.045000580.0470.011-0.020.0130.0230.0290.0240.0690.0210.00490.04 0.0220.0150.00470.020.0280.0230.00210.0370
                 Q1 -0.17 -0.07 -0.0920.0130.032 1 0.84 0.74 0.85 0.81 0.78 0.8 0.8 0.76 0.8 0.73 0.8 0.71 0.69 0.71 0.74 0.63 0.71 0.69 0.69 0.69 0.69 0.69 0.74 0.74 0.69 0.67 0.73 0.6
                                                                                                                                             0.8
                 Q3 --0.12-0.0660.0630.120.007<sup>2</sup>0.74 0.85 1 0.82 0.89 0.83 0.82 0.81 0.81 0.84 0.8 0.8 0.85 0.85 0.85 0.85 0.82 0.81 0.84 0.84 0.84 0.85 0.83 0.83 0.84 0.82 0.84 0.83 0.8 0.8
                 Q4 -0.15-0.0710.0820.0950.036 0.85 0.88 0.82 1 0.88 0.85 0.83 0.82 0.81 0.83 0.8 0.81 0.78 0.79 0.81 0.81 0.81 0.81 0.84 0.77 0.78 0.78 0.78 0.78 0.81 0.79 0.78 0.76 0.78 0.76
                 Q5 --0.16-0.0850.0750.0890.027 0.81 0.89 0.89 0.88 1 0.92 0.9 0.88 0.86 0.89 0.88 0.80 0.83 0.86 0.88 0.87 0.87 0.89 0.89 0.85 0.83 0.83 0.81 0.81 0.86 0.85 0.81 0.83 0.84 0.79
                 Q6 --0.14-0.0720.072 0.1 -0.017 0.78 0.85 0.83 0.85 0.92 1 0.91 0.86 0.85 0.87 0.82 0.83 0.84 0.84 0.85 0.85 0.85 0.89 0.81 0.79 0.81 0.8 0.81 0.79 0.79 0.82 0.81 0.79 0.82 0.81 0.79 0.82 0.81 0.79 0.82
In [12]: #sns.pairplot(df)
Out[12]: <seaborn.axisgrid.PairGrid at 0x22757169d90>
                        عنبا
                            سال ال
                                to the
                                        عللي
                                            an IIIa
                                                    عللبا
In [13]: #3
           cov_mat = np.cov(df.T)
           # as 'cov_mat' is a numpy array, select first five observations with [0:5]
           print(cov_mat[0:5])
           [[ 0.48193988 -0.06109018  0.02820842 -0.12220441 -0.06286974 -0.16368737
             -0.13956713 \ -0.11232866 \ -0.14225251 \ -0.1448978 \ -0.12737475 \ -0.13653707
             -0.1572986 -0.14336673 -0.12349499 -0.1368016 -0.15529459 -0.10636473
             -0.07716232 -0.09950301 -0.14908216 -0.06638076 -0.13381964 -0.0951984
             -0.0711503 -0.07272946 -0.07567134 -0.12891383 -0.15112625 -0.10334269
             -0.12776754 -0.1301483 -0.10449699]
            -0.44701002 -0.32367535 -0.35909419 -0.4227014 -0.35855711 -0.3268016
             -0.2817515 -0.14735471 -0.34803206 -0.18266132 -0.41201202 -0.48192786
             -0.31929459 -0.25493788 -0.26581964 -0.26937475 -0.16459319 -0.20506613
             -0.31528657 -0.31000802 -0.24913828 -0.18215631 -0.30066934 -0.28271743
             -0.33816032 -0.2172505 -0.42045691]
            [ 0.02820842  0.16228858  0.28477355 -0.01945892  0.15151503 -0.06785571
             -0.05879359 -0.04359519 -0.05865331 -0.05292184 -0.05034068 -0.07197194
             -0.04736273 -0.0687976 -0.0532024 -0.03171944 -0.02932665 -0.05731864
              -0.06374349 -0.0710501 -0.05347495 -0.04668938 -0.05260922 -0.05973547
             -0.05372345 -0.07161122 -0.06717435 -0.05662525 -0.05305411 -0.05858918
             -0.036998 -0.04947495 -0.05428457]
            \lceil -0.12220441 \ -0.31885772 \ -0.01945892 \ \ 2.21352705 \ \ 0.92258517 \ \ 0.02665331
```

```
In [14]: eig_val, eig_vec = np.linalg.eig(cov_mat)
        print('Eigenvalues:','\n','\n', eig_val,"\n")
        print('Eigenvectors:','\n','\n',eig_vec,'\n')
        Eigenvalues:
         [40.8817616 14.10595168 3.11863913 1.93425971 1.11758087 0.66853536
          0.6372752  0.56522642  0.45319838  0.39400576  0.3644077
                                                               0.32626336
          0.29905634 0.27865971 0.26782365 0.24016007 0.23125354 0.04696903
          0.15230792 0.14858998 0.14263192]
        Eigenvectors:
         [[ 0.01573661  0.0070146  0.0242403  ... -0.00885556  0.05299544
          -0.00859442]
         [ 0.06068539 -0.99714175 -0.02229391 ... -0.00029104 -0.01105689
           0.00362151]
         [ \ 0.00742362 \ -0.01026389 \ -0.02958134 \ \dots \ \ 0.09497082 \ \ 0.07093429
          -0.02535958]
          \hbox{ $[-0.18736572 -0.00918177 -0.04415304 ... -0.12628904 -0.12660605 ] } 
In [15]: # create a list of eigenvalues
        eig_val = list(eig_val)
        # 'sort(reverse = True)' will sort the eigenvalues in the descending order
        eig_val.sort(reverse = True)
        # print the sorted list
        print(eig_val)
```

[40.88176160469996, 14.105951675873316, 3.118639128970837, 1.93425971494834, 1.1175808718301066, 0.6685353562147048, 0.6372751955247451, 0.5652264237808393, 0.4531983832978018, 0.39400575824459955, 0.364407697734742, 0.32626336243796 816, 0.299056344673724, 0.2786597085119883, 0.26782365222512594, 0.2401600678023966, 0.2312535447555718, 0.203394553 74206933, 0.18607919593248723, 0.1846887839790378, 0.15230791687013553, 0.14858997676709662, 0.14263192311471656, 0. 1261695142399705, 0.12343929031820433, 0.1000582665914212, 0.09709277868218312, 0.0883634450000908, 0.08068443204694 195, 0.07668341821635118, 0.06432787709401604, 0.05904627357123884, 0.0469690326480002]

a) **Kaiser criterion**: This criterion considers the number of pricipal components for which the eigenvalue is greater than 1. This criterion suffers a drawback of selecting more number of components as the eigenvalues very close to 1 may not contribute significantly in explaining the variation in the data. Here the first five eigenvalues are greater than 1. Thus we can consider 5 principal components using kaiser criterion.

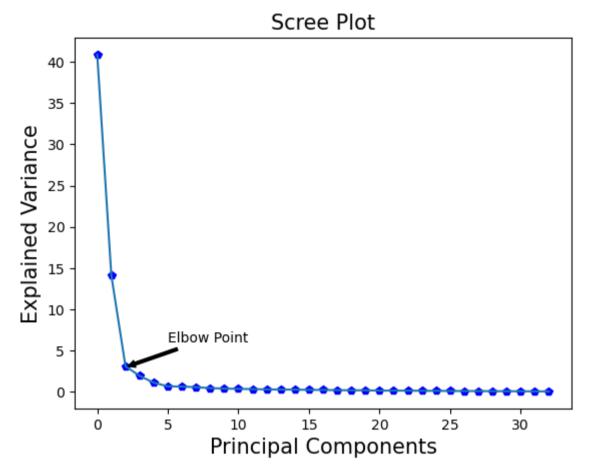
```
In [26]: #b
plt.plot(eig_val,'bp')

plt.plot(eig_val)

plt.title('Scree Plot', fontsize = 15)
plt.xlabel('Principal Components', fontsize = 15)
plt.ylabel('Explained Variance', fontsize = 15)

plt.annotate(text='Elbow Point', xy=(2,3), xytext=(5,6), arrowprops=dict(facecolor='black', arrowstyle = 'simple'))

plt.show()
```



**Interpretation**: It can be observed that, after the elbow point, the principal components do not contribute much to the variance in the data. The Kaiser criterion considers the number of principal components as 5, but the scree plot shows that only first three components explains most of the variation.

```
In [24]: #c Percentage of Explained Variation
         percent_var = []
         for i in eig_val:
             variation = (i/sum(eig_val))*100
             percent_var.append(variation)
         percent_var
Out[24]: [60.32909594581732,
           20.816111602269824,
          4.602166634874946,
          2.8543814000870302,
          1.6492108215830268,
           0.9865556684722538,
           0.9404251408354645,
           0.8341029792479687,
           0.6687835277949257,
           0.5814331442510599,
           0.5377553729853515,
           0.4814658890301411,
           0.4413165725459428,
           0.41121731625969177,
           0.3952265797125358,
           0.354403509146707,
           0.3412601791544458,
           0.30014874765453625,
           0.2745965248141599,
```

**Interpretation**: It can be seen that the first principal component explains 60.32% variation in the data.

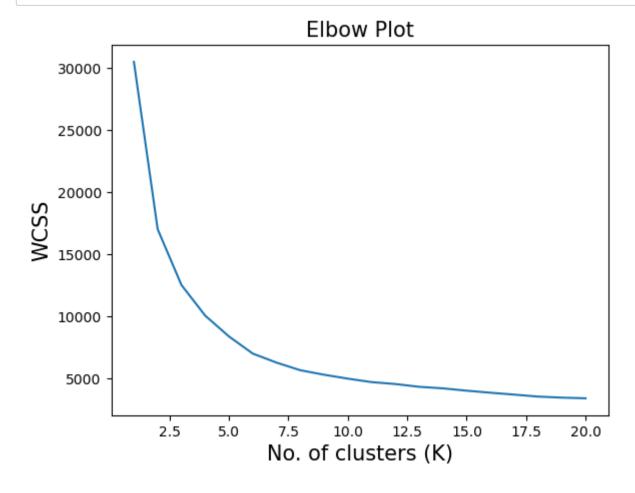
```
In [38]: |np.cumsum(percent_var)
Out[38]: array([ 60.32909595, 81.14520755, 85.74737418,
                                                            88.60175558,
                 90.2509664 , 91.23752207, 92.17794721,
                                                            93.01205019,
                                                            95.28148813,
                 93.68083372, 94.26226687, 94.80002224,
                 95.7228047 , 96.13402202, 96.5292486 ,
                                                            96.8836521 ,
                 97.22491228, 97.52506103, 97.79965756, 98.07220225,
                 98.2969626 , 98.5162364 , 98.72671791, 98.91290589,
                 99.09506489, 99.24272058, 99.38600011, 99.51639779,
                 99.63546357, 99.74862506, 99.84355352, 99.93068794,
                100.
                            ])
In [39]: pca = PCA(n_components = 5, random_state = 10)
         components = pca.fit_transform(df)
In [40]: df_pca = pd.DataFrame(data = components, columns = ['PC1', 'PC2', 'PC3', 'PC4', 'PC5'])
         df_pca.head()
Out[40]:
                 PC1
                          PC2
                                  PC3
                                           PC4
                                                   PC5
            -9.262208 -1.246360 -2.567580 0.266860 0.200472
             1.168775 -0.483509
                              1.756509 -1.088134 1.346846
             1.502789 -5.490850 1.147391 -0.939926 0.756805
            10.736984 2.965870 -2.411270 -0.465814 0.747689
            -0.252974 4.416256 2.389900 1.076906 0.145288
In [37]: df_pca.shape
Out[37]: (500, 5)
```

**Interpretation**: In the above step, we obtained the data with reduced dimensions. The new dataset has 500 observations and 5 columns, i.e. we have decreased the number of features from 33 to 5.

# K-Means Clustering

```
In [78]: wcss = []
    for i in range(1,21):
        kmeans = KMeans(n_clusters = i, random_state = 10)
        kmeans.fit(df_pca)
        wcss.append(kmeans.inertia_)

In [79]: plt.plot(range(1,21), wcss)
    plt.title('Elbow Plot', fontsize = 15)
    plt.xlabel('No. of clusters (K)', fontsize = 15)
    plt.ylabel('WCSS', fontsize = 15)
    plt.show()
```

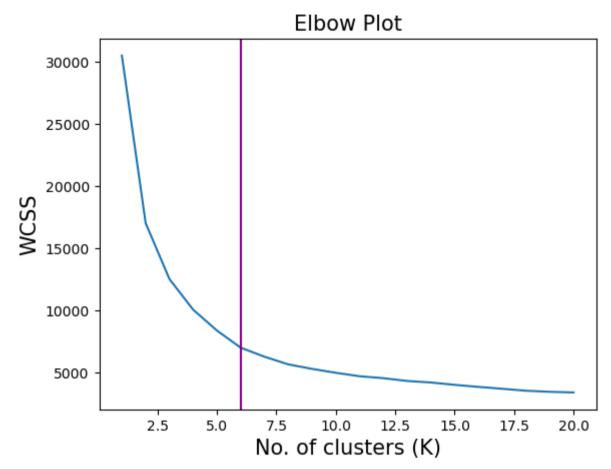


```
In [80]: plt.plot(range(1,21), wcss)

# set the axes and plot labels
# set the font size using 'fontsize'
plt.title('Elbow Plot', fontsize = 15)
plt.xlabel('No. of clusters (K)', fontsize = 15)
plt.ylabel('WCSS', fontsize = 15)

# plot a vertical line at the elbow
plt.axvline(x = 6, color = 'purple')

# display the plot
plt.show()
```



**Interpretation:** We can see that the for K = 6, there is an elbow in the plot. Before this elbow point, the WCSS is decreasing rapidly and after K = 6, the WCSS is decreasing slowly.

Now, let us use the silhouette score method to identify the optimal value of K.

## **Optimal Value of K Using Silhouette Score**

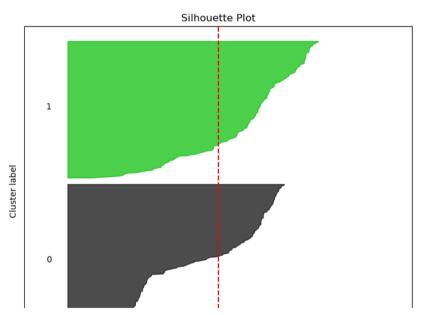
```
In [83]: n_clusters = [2, 3, 4, 5, 6,7]

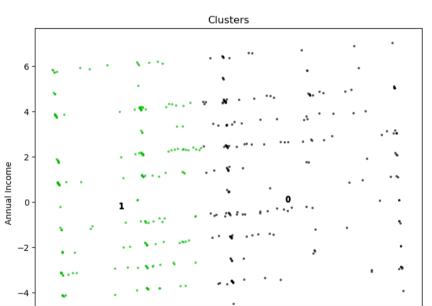
for K in n_clusters:
    cluster = KMeans (n_clusters= K, random_state= 10)
    predict = cluster.fit_predict(df_pca)
    score = silhouette_score(df_pca, predict, random_state= 10)
    print ("For {} clusters the silhouette score is {})".format(K, score))

For 2 clusters the silhouette score is 0.3503302858536722)
    For 3 clusters the silhouette score is 0.3031171189178978)
    For 4 clusters the silhouette score is 0.3156622058518483)
    For 5 clusters the silhouette score is 0.2994635218729511)
    For 6 clusters the silhouette score is 0.3076228107561279)
    For 7 clusters the silhouette score is 0.30251036232888245)
```

```
In [84]: |n_clusters = [2,3,4,5,6,7]
         X = np.array(df_pca)
         for K in n_clusters:
             fig, (ax1, ax2) = plt.subplots(1, 2)
             fig.set_size_inches(18, 7)
             model = KMeans(n_clusters = K, random_state = 10)
             cluster_labels = model.fit_predict(X)
             silhouette_avg = silhouette_score(X, cluster_labels)
             sample_silhouette_values = silhouette_samples(X, cluster_labels)
             y_{lower} = 10
             for i in range(K):
                 ith_cluster_silhouette_values = sample_silhouette_values[cluster_labels == i]
                 ith_cluster_silhouette_values.sort()
                 size_cluster_i = ith_cluster_silhouette_values.shape[0]
                 y_upper = y_lower + size_cluster_i
                 color = cm.nipy_spectral(float(i) / K)
                 ax1.fill_betweenx(np.arange(y_lower, y_upper),
                                    0, ith_cluster_silhouette_values,
                                    facecolor=color, edgecolor=color, alpha=0.7)
                 ax1.text(-0.05, y_lower + 0.5 * size_cluster_i, str(i))
                 y_{\text{lower}} = y_{\text{upper}} + 10
             ax1.set_title("Silhouette Plot")
             ax1.set_xlabel("Silhouette coefficient")
             ax1.set_ylabel("Cluster label")
             ax1.axvline(x=silhouette_avg, color="red", linestyle="--")
             ax1.set_yticks([])
             ax1.set_xticks([-0.1, 0, 0.2, 0.4, 0.6, 0.8])
             colors = cm.nipy_spectral(cluster_labels.astype(float) / K)
             ax2.scatter(X[:, 0], X[:, 1], marker='.', s=30, lw=0, alpha=0.7, c=colors, edgecolor='k')
             centers = model.cluster_centers_
             for i, c in enumerate(centers):
                 ax2.scatter(c[0], c[1], marker='$%d$' % i, alpha=1, s=50, edgecolor='k')
             ax2.set_title("Clusters")
             ax2.set_xlabel("Spending Score")
             ax2.set_ylabel("Annual Income")
             plt.suptitle(("Silhouette Analysis for K-Means Clustering with n_clusters = %d" % K), fontsize=14,fontweight='bold
         plt.show()
```

#### Silhouette Analysis for K-Means Clustering with n\_clusters = 2





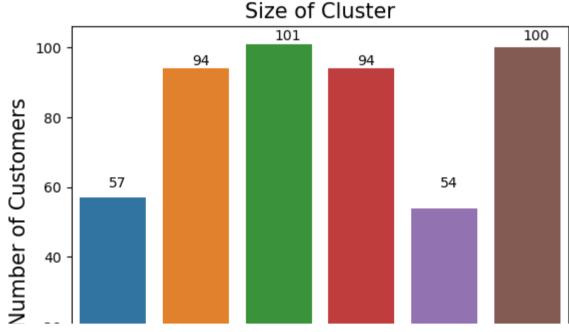
```
In [87]: # build a K-Means model with 5 clusters
   new_clust = KMeans(n_clusters = 6, random_state = 10)
   new_clust.fit(df_pca)
   df_pca['Cluster'] = new_clust.labels_
```

### In [88]: df\_pca.head()

### Out[88]:

	PC1	PC2	PC3	PC4	PC5	Cluster
0	-9.262208	-1.246360	-2.567580	0.266860	0.200472	1
1	1.168775	-0.483509	1.756509	-1.088134	1.346846	5
2	1.502789	-5.490850	1.147391	-0.939926	0.756805	5
3	10.736984	2.965870	-2.411270	-0.465814	0.747689	0
4	-0.252974	4.416256	2.389900	1.076906	0.145288	2

```
Out[89]: 2
              101
              100
         1
               94
               94
         3
               57
         0
               54
         Name: Cluster, dtype: int64
In [95]: sns.countplot(data= df_pca, x = 'Cluster')
         plt.title('Size of Cluster', fontsize = 15)
         plt.xlabel('Clusters', fontsize = 15)
         plt.ylabel('Number of Customers', fontsize = 15)
         plt.text(x = -0.05, y = 60, s = np.unique(new_clust.labels_, return_counts=True)[1][0])
         plt.text(x = 0.95, y =95, s = np.unique(new_clust.labels_, return_counts=True)[1][1])
         plt.text(x = 1.95, y =102, s = np.unique(new_clust.labels_, return_counts=True)[1][2])
         plt.text(x = 2.95, y = 95, s = np.unique(new_clust.labels_, return_counts=True)[1][3])
         plt.text(x = 3.95, y = 60, s = np.unique(new_clust.labels_, return_counts=True)[1][4])
         plt.text(x = 4.95, y = 102, s = np.unique(new_clust.labels_, return_counts=True)[1][5])
         plt.show()
```



#### Cluster 2

In [89]: df\_pca.Cluster.value\_counts()

```
In [96]: len(df_pca[df_pca['Cluster'] == 0])
```

Out[96]: 57

In [97]: df\_pca[df\_pca.Cluster==0].describe()

Out[97]:

	PC1	PC2	PC3	PC4	PC5	Cluster
count	57.000000	57.000000	57.000000	57.000000	57.000000	57.0
mean	8.754803	3.693102	-0.497815	-0.217525	0.010422	0.0
std	2.466693	1.639907	1.770061	1.446855	1.109417	0.0
min	4.674983	-0.265743	-3.767152	-4.901782	-2.936649	0.0
25%	6.265543	2.667615	-2.173110	-0.731551	-0.453207	0.0
50%	8.838590	3.781217	-0.813832	-0.064762	0.464963	0.0
75%	11.542602	4.962079	0.925016	0.438758	0.590147	0.0
max	11.752290	7.035331	3.276542	4.535248	2.917115	0.0

### Cluster 3

```
In [98]: len(df_pca[df_pca['Cluster'] == 1])
```

Out[98]: 94

```
Out[99]:
                                   PC2
                                                                  PC5 Cluster
                        PC1
                                             PC3
                                                        PC4
             count 94.000000 94.000000 94.000000 94.000000
                                                                          94.0
                    -6.173217 -3.535887
                                        -0.255052 -0.095540
                                                              0.135689
                                                                           1.0
                              1.782500
                                                              0.989583
                    2.451086
                                         1.810100
                                                    1.303063
               std
                                                                           0.0
                    -9.340700
                              -6.232069
                                        -3.219197
                                                   -2.901430
                                                             -3.055702
                                                                           1.0
                              -5.834795
                                        -2.329625
              25%
                    -9.051964
                                                   -0.946471
                                                              -0.110508
                                                                           1.0
                    -5.238163
                              -3.160489
                                        -0.131824
                                                              0.298601
              50%
                                                   -0.263121
                                                                           1.0
                    -3.911378 -2.058475
                                         1.062423
              75%
                                                    0.263193
                                                              0.638029
                                                                           1.0
                   -2.884924 -0.726907
                                         3.299267
                                                              2.758766
                                                    4.575728
                                                                           1.0
            Cluster 4
In [100]: len(df_pca[df_pca['Cluster'] == 2])
Out[100]: 101
In [101]: df_pca[df_pca.Cluster==2].describe()
Out[101]:
                                                                       PC5 Cluster
                         PC1
                                     PC2
                                                 PC3
                                                            PC4
             count 101.000000
                               101.000000
                                          101.000000 101.000000
                                                                 101.000000
                                                                              101.0
                                 3.364348
                     0.495415
                                            0.336561
                                                        0.196106
                                                                   -0.026819
                                                                                2.0
             mean
                                 1.584346
                                            1.811519
                                                        1.401206
               std
                     1.691418
                                                                   1.019940
                                                                                0.0
                     -3.254616
                                 0.428823
                                            -2.774788
                                                       -2.635203
                                                                   -2.625741
                                                                                2.0
              min
                                 2.321610
              25%
                     -0.556506
                                            -0.561469
                                                       -0.696026
                                                                   -0.623723
                                                                                2.0
              50%
                     0.957107
                                 3.389242
                                            0.374655
                                                       -0.094560
                                                                   0.106971
                                                                                2.0
              75%
                     1.154803
                                 4.444959
                                            1.697557
                                                        0.842354
                                                                   0.402824
                                                                                2.0
                     4.419544
                                 6.589606
                                            3.579641
                                                        4.800258
                                                                   2.821771
                                                                                2.0
              max
            Cluster 5
In [102]: len(df_pca[df_pca['Cluster'] == 3])
Out[102]: 94
In [103]: |df_pca[df_pca.Cluster==3].describe()
Out[103]:
                        PC1
                                             PC3
                                   PC2
                                                        PC4
                                                                  PC5 Cluster
             count 94.000000 94.000000 94.000000 94.000000
                                                                          94.0
                    -6.730059
                               2.896565
                                        -0.218283
                                                   -0.226355
                                                             -0.107398
                                                                           3.0
             mean
                    2.570382
                                                              0.950654
                               1.805925
                                         1.616934
                                                    1.034805
               std
                                                                           0.0
                                                   -1.597796
                    -9.825877
                              -0.225199
                                         -2.699397
                                                             -3.043514
                                                                           3.0
              min
                    -9.491667
                               1.130301
                                        -1.426663
                                                   -0.884685
                                                             -0.656578
              25%
                                                                           3.0
                    -5.470635
                               2.625061
                                         0.011388
              50%
                                                   -0.494429
                                                              0.177475
                                                                           3.0
                    -4.288576
                               4.135418
                                         1.162170
                                                    0.317419
                                                              0.517612
              75%
                                                                           3.0
              max -3.176419
                               6.180595
                                         2.625427
                                                    4.469229
                                                              2.050280
                                                                           3.0
```

In [99]: |df\_pca[df\_pca.Cluster==1].describe()

Cluster 6

Out[104]: 54

In [104]: len(df\_pca[df\_pca['Cluster'] == 4])

```
Out[105]:
                       PC1
                                 PC2
                                           PC3
                                                     PC4
                                                               PC5 Cluster
            count 54.000000 54.000000 54.000000 54.000000
                                                                       54.0
                                      -0.152446 -0.361505
            mean 10.177791 -3.808935
                                                           0.108067
                                                                        4.0
                                       1.672347
                   2.417262
                            1.702287
                                                           1.174297
              std
                                                 1.054360
                                                                        0.0
                   5.817402 -5.319577
                                      -2.391329 -3.533300
                                                          -2.519543
                                                                        4.0
                   7.230519 -4.991398
                                      -1.950171 -1.063927 -0.343579
             25%
                                                                        4.0
                  11.822600 -4.876219
                                      -0.211746
                                                -0.333412
                                                           0.480868
             50%
                                                                        4.0
             75% 12.031493 -2.877350
                                       1.116334
                                                 0.270227
                                                           0.616434
                                                                        4.0
             max 12.155694
                             0.078102 3.368880
                                                 2.553171
                                                           3.058939
                                                                        4.0
           Cluster 7
In [106]: len(df_pca[df_pca['Cluster'] == 5])
Out[106]: 100
In [107]: | df_pca[df_pca.Cluster==5].describe()
Out[107]:
                                                                    PC5 Cluster
                        PC1
                                   PC2
                                              PC3
                                                         PC4
                              100.000000 100.000000 100.000000 100.000000
            count 100.000000
                                                                           100.0
                                                                            5.0
                    1.142465
                               -2.845273
                                          0.471084
                                                     0.423716
                                                               -0.063803
            mean
                                                                1.157244
              std
                    1.832966
                               2.028479
                                          1.731743
                                                     1.734804
                                                                            0.0
                    -2.961083
                               -5.896807
                                          -2.592574
                                                     -2.465212
                                                               -2.864044
                                                                            5.0
              min
                                          -0.907223
                                                     -0.785322
             25%
                     0.323304
                               -5.498824
                                                               -0.791874
                                                                            5.0
                    1.365484
                               -2.571362
                                          0.534035
                                                                0.059009
             50%
                                                     -0.100642
                                                                            5.0
                                          1.604805
                                                     1.576395
             75%
                     1.736747
                               -0.847540
                                                                0.624910
                                                                            5.0
                     6.034595
                               0.601928
                                          4.912580
                                                     6.264353
                                                                3.436725
                                                                            5.0
             max
           Hierarchical Clustering
In [109]: link_mat = linkage(df_pca, method = 'ward')
           # print first 10 observations of the linkage matrix 'link_mat'
           print(link_mat[0:10])
           [[129. 290.
                                2.]
```

In [105]: |df\_pca[df\_pca.Cluster==4].describe()

[431. 500.

[ 97. 320.

[370. 502.

[375. 503.

[ 83. 283.

[280. 460.

[ 68. 211.

[397. 507.

[371. 451.

0.

0.

0.

0.

0.

0.

0.

0.

3.]

2.]

3.]

4.]

2.]

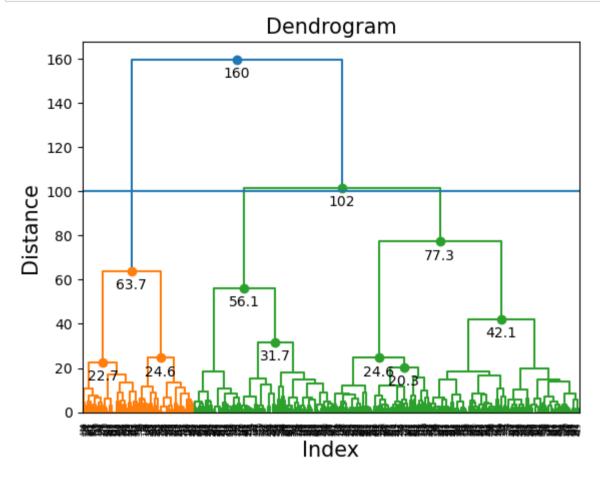
2.]

2.]

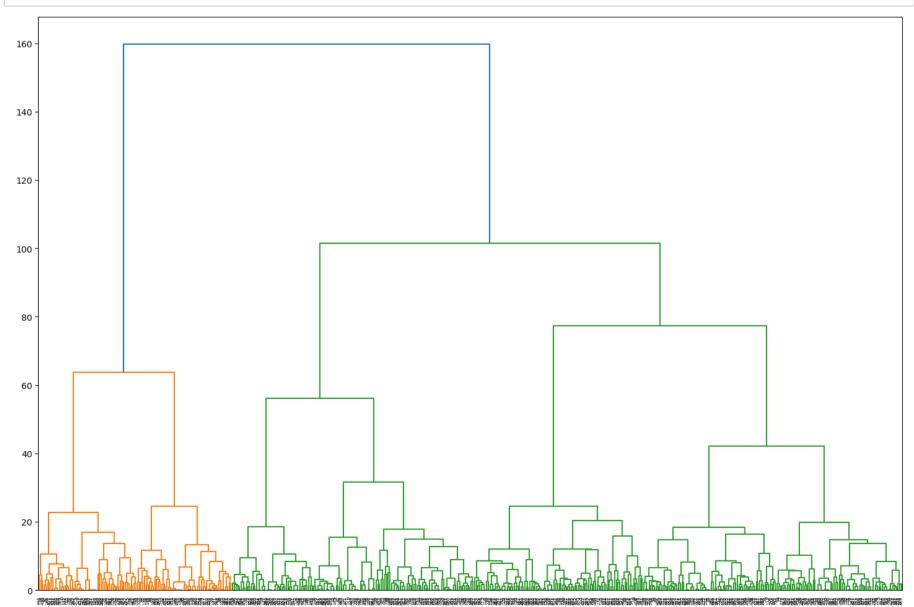
3.]

2.]]

```
In [110]:
    dendro = dendrogram(link_mat)
    for i, d, c in zip(dendro['icoord'], dendro['color_list']):
        x = sum(i[1:3])/2
        y = d[1]
        if y > 20:
            plt.plot(x, y, 'o', c=c)
            plt.annotate("%.3g" % y, (x, y), xytext=(0, -5), textcoords='offset points', va='top', ha='center')
    plt.axhline(y = 100)
    plt.title('Dendrogram', fontsize = 15)
    plt.xlabel('Index', fontsize = 15)
    plt.ylabel('Distance', fontsize = 15)
    plt.show()
```



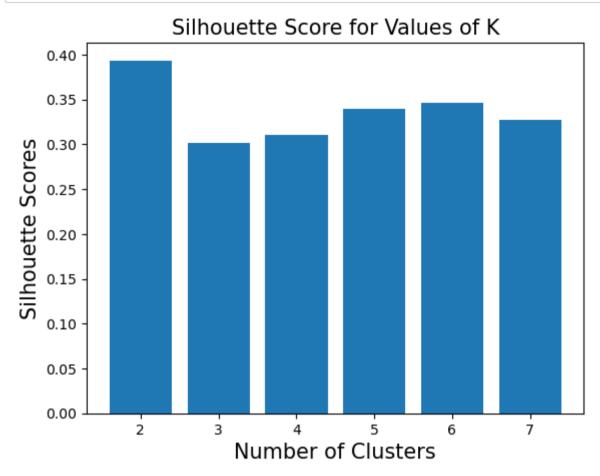
```
In [111]: plt.figure(figsize = (18,12))
    dendrogram(link_mat)
    plt.show()
```



```
In [114]: K = [2,3,4,5,6,7]

silhouette_scores = []
for i in K:
    model = AgglomerativeClustering(n_clusters = i)
    silhouette_scores.append(silhouette_score(df_pca, model.fit_predict(df_pca)))
plt.bar(K, silhouette_scores)

plt.title('Silhouette Score for Values of K', fontsize = 15)
plt.xlabel('Number of Clusters', fontsize = 15)
plt.ylabel('Silhouette Scores', fontsize = 15)
plt.show()
```



```
In [115]: clusters = AgglomerativeClustering(n_clusters=2, linkage='ward')
    clusters.fit(df_pca)
```

Out[115]: AgglomerativeClustering()

Out[116]:

	PC1	PC2	PC3	PC4	PC5	Cluster	Agg Cluster
(	-9.262208	-1.246360	-2.567580	0.266860	0.200472	1	0
1	1.168775	-0.483509	1.756509	-1.088134	1.346846	5	0
2	1.502789	-5.490850	1.147391	-0.939926	0.756805	5	0
3	10.736984	2.965870	-2.411270	-0.465814	0.747689	0	1
4	-0.252974	4.416256	2.389900	1.076906	0.145288	2	0

```
In [117]: df_pca['Agg Cluster'].value_counts()
```

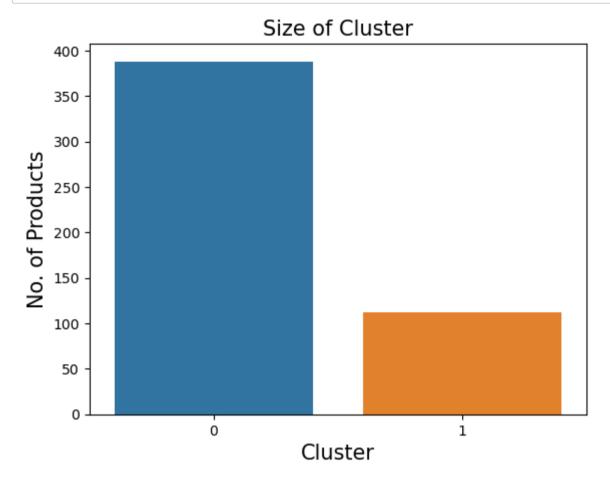
Out[117]: 0 388 1 112

Name: Agg Cluster, dtype: int64

```
In [119]: sns.countplot(data = df_pca, x = 'Agg Cluster')

plt.title('Size of Cluster', fontsize = 15)
plt.xlabel('Cluster', fontsize = 15)
plt.ylabel('No. of Products', fontsize = 15)

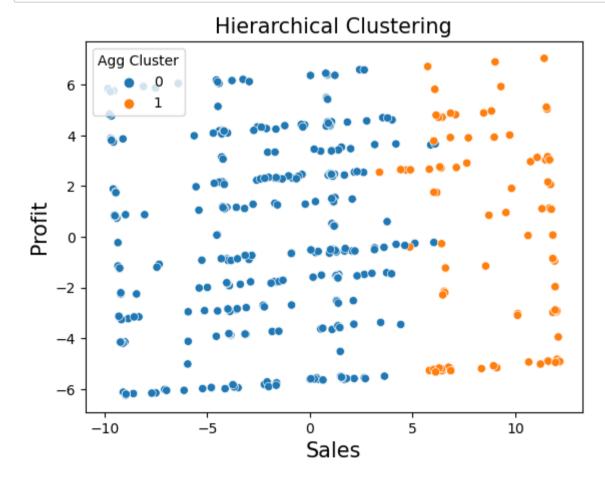
plt.show()
```



```
In [120]: # plot the scatterplot to visualize the clusters
sns.scatterplot(x = 'PC1', y = 'PC2', data = df_pca, hue = 'Agg Cluster')

plt.title('Hierarchical Clustering', fontsize = 15)
plt.xlabel('Sales', fontsize = 15)
plt.ylabel('Profit', fontsize = 15)

# display the plot
plt.show()
```



# **Analysis of Cluster\_1**

```
In [123]: df_pca['Agg Cluster'].value_counts()[0]
```

Out[123]: 388

```
In [128]: df_pca[df_pca['Agg Cluster'] == 0].head(10)
Out[128]:
                              PC2
                                                           PC5 Cluster Agg Cluster
                    PC1
                                        PC3
                                                 PC4
             0 -9.262208 -1.246360 -2.567580
                                             0.266860
                                                       0.200472
                                                                                 0
               1.168775 -0.483509
                                   1.756509 -1.088134
                                                       1.346846
                                                                     5
                                                                                 0
                 1.502789 -5.490850
                                   1.147391
                                             -0.939926
                                                       0.756805
                                                                     5
                                                                                 0
                -0.252974
                          4.416256
                                   2.389900
                                             1.076906
                                                       0.145288
                                                                     2
                                                                                 0
                -4.183824
                          1.122334
                                   -0.425883
                                             -0.483475
                                                       0.784353
                                                                     3
                                                                                 0
               -9.067464
                         -6.122137
                                    2.268556 -1.495790
                                                       0.145169
                                                                                 0
                4.421797 -3.448411
                                    1.502602
                                             4.871930
                                                      -2.334254
                                                                     5
                                                                                 0
                0.270670
                          1.392619
                                    1.537734
                                             2.137315
                                                      1.536218
                                                                     2
                                                                                 0
                          4.448672
                0.933944
                                   0.233446 -0.561480
                                                      -0.044376
                                                                                 0
                1.109854 2.392100 -2.381492 0.361236 0.378332
                                                                                 0
In [129]: df_pca[df_pca['Agg Cluster'] == 0].describe()
Out[129]:
                        PC1
                                    PC2
                                               PC3
                                                         PC4
                                                                     PC5
                                                                             Cluster Agg Cluster
                              388.000000 388.000000 388.000000 388.000000
            count 388.000000
                                                                          388.000000
                                                                                          388.0
                    -2.704420
                               -0.006016
                                           0.111227
                                                      0.102775
                                                                -0.029595
                                                                            2.755155
                                                                                            0.0
            mean
                     4.242743
                                3.660922
                                           1.762270
                                                      1.438359
                                                                1.028689
              std
                                                                            1.495678
                                                                                            0.0
                               -6.232069
                    -9.825877
                                          -3.219197
                                                     -2.901430
                                                                -3.055702
                                                                            0.000000
                                                                                            0.0
              min
                    -5.021935
                               -3.127874
                                          -1.354316
                                                     -0.823078
                                                                -0.608055
             25%
                                                                            2.000000
                                                                                            0.0
                    -2.657257
              50%
                                0.080340
                                           0.290084
                                                     -0.261956
                                                                 0.186674
                                                                            2.000000
                                                                                            0.0
             75%
                     1.079361
                                3.382530
                                           1.464524
                                                      0.382302
                                                                 0.574676
                                                                            5.000000
                                                                                            0.0
                     6.098243
                                           4.912580
                                                      6.264353
                                                                 3.436725
              max
                                6.589606
                                                                            5.000000
                                                                                            0.0
In [130]: | df_pca[df_pca['Agg Cluster'] == 0].index.value_counts()
Out[130]: 0
           326
                   1
           354
                   1
           353
                   1
           352
           172
           170
           169
           167
                   1
           499
                   1
           Length: 388, dtype: int64
           Analysis of Cluster_2
In [131]: |df_pca['Agg Cluster'].value_counts()[1]
Out[131]: 112
In [132]: df_pca[df_pca['Agg Cluster'] == 1].head(10)
Out[132]:
                                                           PC5 Cluster Agg Cluster
                     PC1
                              PC2
                                        PC3
                                                  PC4
             3 10.736984
                          2.965870 -2.411270 -0.465814
                                                       0.747689
                 6.234561
                         4.746177 -0.984623 -0.050082
                                                       0.329282
                                                                      0
                 6.469278 -5.266479 -0.797090 -0.281165 0.260791
                6.386913 2.715617 -2.255007 0.383144 0.474162
                                                                     0
            15
            17 12.149456 -4.937999 -1.950171 0.270227 0.606786
            21 11.647354 2.092886 -0.046428 -0.417751 1.071840
                                                                     0
            23 11.906714 -0.949432 -2.039346 0.337640 0.588389
            32 12.117289 -4.857791 1.147955 -0.900082 -1.775050
               34
```

0

1

36

9.725568 4.012831 -0.275711 -0.064820 -1.843556

```
In [133]: df_pca[df_pca['Agg Cluster'] == 1].describe()
Out[133]:
                        PC1
                                   PC2
                                              PC3
                                                         PC4
                                                                   PC5
                                                                            Cluster Agg Cluster
            count 112.000000 112.000000 112.000000 112.000000 112.000000
                                                                                         112.0
                    9.368884
                               0.020842
                                         -0.385322
                                                    -0.356040
                                                                0.102524
                                                                          2.008929
                                                                                           1.0
             mean
                                                     1.147831
              std
                    2.625180
                               4.084796
                                          1.731806
                                                                1.149352
                                                                          2.002231
                                                                                           0.0
                                                               -2.936649
                    3.387603
                              -5.319577
                                         -3.767152
                                                    -4.901782
                                                                          0.000000
                                                                                           1.0
              min
             25%
                    6.468852
                               -4.861597
                                         -1.971005
                                                    -1.030685
                                                               -0.398739
                                                                          0.000000
                                                                                           1.0
              50%
                    10.365755
                               0.465917
                                          -0.576058
                                                    -0.203263
                                                                0.469562
                                                                          2.000000
                                                                                           1.0
             75%
                   11.821513
                               3.811919
                                          1.006243
                                                     0.375558
                                                                0.614342
                                                                          4.000000
                                                                                           1.0
                   12.155694
                               7.035331
                                          3.368880
                                                                3.058939
                                                                          5.000000
                                                     2.553171
                                                                                           1.0
             max
In [134]: | df_pca[df_pca['Agg Cluster'] == 1].index.value_counts()
Out[134]: 3
                   1
                   1
           323
                   1
           321
                   1
           317
                   1
           130
                   1
           129
                   1
           126
                   1
           121
           495
                   1
           Length: 112, dtype: int64
  In [ ]:
  In [ ]:
```