Class Exercise (Kalman Filter)
Introduction to Data Assimilation

Exercise 8a (Linear Advection Equation)

Consider a one-dimensional linear advection model on a periodic domain of length 360 m ($L_a = -180 \leq x_{axis} < L_b = 180$). The model has a constant advection speed, μ=1 m/s, the grid spacing is Δx = 1 m and the time step Δt = 1 s.

The true initial state $\mathbf{x}_0^t$ is sampled from a normal distribution with mean equal to a square wave (use sqrwv.m in D2L), variance equal to zero and spatial de-correlation length of 45 m. Yes, we assume here that the true state, $\mathbf{x}^\mathbf{t}$ is without system noise.

The initial forecast states $\mathbf{x}_0^f$, however are generated by drawing a sample from a normal distribution with mean $\mathbf{x}_0^t$, variance equal to 0.5 and spatial de-correlation length of 45 m similar to the length scale of $\mathbf{x}_0^t$.

1) Construct $\mathbf{x}_0^t$, $\mathbf{x}_0^f$. Use gcorr.m (and gauss.m) – can be found in D2L -- to generate the correlation matrix, $\boldsymbol{\rho}$. Also use mvnrnd.m to draw the sample.

2) Construct a time-independent state transition matrix **M** using Lax-Wendorff scheme to solve the advection equation:

$$\frac{\partial a}{\partial t} = -\mu \frac{\partial a}{\partial x}$$

$$a_i^{k+1} = a_i^k - \frac{\mu \Delta t}{2\Delta x}\left(a_{i+1}^k - a_{i-1}^k\right) + 2\left(\frac{\mu \Delta t}{2\Delta x}\right)^2 \left(a_{i+1}^k + a_{i-1}^k - 2a_i^k\right)$$

   where *k* and *i* denote time and position, respectively.

3) Calculate the true and forecast states from time=0 (initial) to time=300 seconds.

4) Plot the true and forecast states at time 5, 100 and 240 seconds.