

Evolution of Cooperation based on Reputation on Dynamical Networks

Linlin Tian¹, Mingchu Li¹, Weifeng Sun¹, Xiaowei Zhao¹, Baohui Wang², Jianhua Ma³

¹School of software, Dalian University of Technology, Dalian, China, 116621

²School of software, Beijing University of Aeronautics & Astronautics, Beijing, China, 100191

³Faculty of Computer & Information Sciences, Hosei University, Tokyo, Japan, 184-8584
lelele247@yahoo.com.cn, mingchul@dlut.edu.cn, wfsun@dlut.edu.cn, vivian_dlut@163.com

Abstract—Cooperation within selfish individuals can be promoted by natural selection only in the presence of an additional mechanism. In this paper, we focus on an indirect reciprocity mechanism in dynamical structured populations. In social networks rational individuals update their strategies and adjust their social relationships. We propose a three-strategy prisoner's dilemma game model to investigate the evolution of cooperation on dynamical networks. In the coevolution of state and structure process, reciprocators adapt their behaviors and switch their partners based on reputation. Simulation results show that the dynamics of strategies and links can promote cooperation provided the partners switch proceeds much faster than the strategy updating.

Keywords- coevolutionary game; reputation; cooperation; reciprocal strategy;

I. INTRODUCTION

The emergence of cooperation among selfish individuals is a fascinating topic that has attracted scientists in different fields such as sociology, biology, physics and computer science. It is usually investigated in terms of prisoner's dilemma (PD) and analyzed in the frame of evolutionary game theory (EGT) [1,2]. The PD game presents a paradigmatic example of the social dilemma which each individual adopts cooperative or defective strategy. Cooperators contribute benefits to the opponents and pay costs for the interaction while defectors gain the more without paying. The successful strategy measured in terms of higher payoff spreads in social networks by either biological evolution or cultural evolution. In case the population is infinitely large and well-mixed, the framework can be investigated based on mean-field theory and the replicator equation [3]. It is well known that defection becomes the evolutionarily stable strategy (ESS) [4] as rational individuals maximize their utilities. Hence the evolution leads to the extinction of cooperators unlike the situation in human society or biological population.

In order to escape this dilemma, specific social control mechanisms are demanded to inspire cooperation, such as punishment, reward and reciprocity [5]. Reciprocity means the action of an individual depends on what the opponent do with it and to others [6]. Tit-for-tat (TFT) is the simplest strategy that performs direct reciprocity and indirect reciprocity usually works through reputation [7, 8]. For spatial reciprocity, each individual interacts with their neighbors and the utility depends on the strategy and link behavior. As a result, cooperators form clusters to resist

defectors under some social control mechanisms.

Some studies extend the classic PD to three-strategy game supplemented by reciprocal strategy and find similar results with rock-scissors-paper game. Reciprocators refuse to deal with defectors not only to gain better utility but also motivated by just emotion. However reciprocators have to pay additional costs to punish defectors. Reference [9] has discussed the phase diagrams for evolutionary game on stationary networks and draws a conclusion that reciprocal strategy can prevent the extinction of cooperators on the condition of the reasonable reciprocal cost. Ohsuki also has found such indirect reciprocity provides limit efficiency for costly punishment under second-order social norm in [10].

For cooperation in the context of social dilemmas, the outcome of evolutionary games in structured population can be very different from the well-mixed case. Many studies have explored how population structure affects the evolution of cooperation and revealed that spatial structure promoted cooperation to some extent. Especially the dynamics of structure favors cooperation and cooperators can survive even with high defection temp and in the absence of mutation [11,12].

Recent studies focus on the coevolution of strategies and networks. Reference [13] has explored the evolution of cooperation on weighted network and found adaptive weight promoted the cooperation level under mean-field approximation. The dynamical networks may be influenced by coevolutionary rules, such as the reproduction capability of players, their reputations and mobility. Fu discussed the effect of reputation on partner-switching process in structure population [14]. However he adopts a simple reputation model under image score that is not robust in real system.

To investigate the evolution of cooperation in social dilemma, we propose a three-strategy PD game model based on reputation. On small world networks agents learn successful strategies from their neighbors and alternate local environment by updating partners. Reciprocators adapt their behavior and linking way against different partners. They distinguish opponents according to reputation under a stern-judge rule. The cooperation dynamics is attributed to the interplay between the evolution of agents' strategies and that of networks structure.

The remainder of this paper is organized as follows. Section II describes the evolutionary game model based on reputation. In section III we study the coevolutionary dynamics by simulation and discuss the experiment results. Finally we summarize the model and our observations.

II. MODEL DESCRIPTION

A. Three-strategy PD Game

Considering a population in network with the constant size N , each agent represents an individual who only interacts with its neighbors. In classical PD, an agent makes a strategy choice either cooperation (C) or defection (D) and the latter is the only Nash equilibrium strategy. As reciprocity is an available mechanism to promote cooperation in social networks, we extended the two-strategy spatial PD game supplemented by reciprocal strategy (R). Strategy set can be presented as $S = \{C, R, D\}$ and the agent i adopt the strategy s_i denoted as either $(1,0,0)^T, (0,1,0)^T$ or $(0,0,1)^T$ respectively. The worker-consumer model is used to explain these strategies. As workers, cooperators provide service to any player. Defectors as consumers only consume resources and never contribute to others. Reciprocators serve friendly neighbors who adopt C or R strategy. To avoid the reduction of their incomes and get better utilities, reciprocators refuse to offer service for defectors. In a reputation system, agents with reciprocal strategy can discern opponents. Therefore they have to pay additional costs for seeking information and making decisions by reputation scores.

Here we study the interaction rule for evolutionary PD game with the three-strategy. If agent j is one of its neighbors of agent i , p_{ij} is defined in (1) to describe the cooperation inclination, i.e. the probability that the agent i with s_i provides service for the agent j . The agent using C or D strategy always adopts a consistent action and never takes into account opponents' characters. Contrarily, reciprocators distinguish neighbors and take the corresponding actions according the strategies of opponents. The formula of decision function $f(s_j)$ depends on the given reciprocal mechanism, such as TFT or reputation. Here we use reputation r_j described in the part C to measure $f(s_j)$ simply.

$$p_{ij} = \begin{cases} 1 & s_i \in C \\ 0 & s_i \in D \\ f(s_j) & s_i \in R \end{cases} \quad (1)$$

After each interaction between two agents, agent i gains a benefit b and provides service for agent j at cost of c and maybe pay reciprocal cost c_r . The expected utility function u_{ij} is defined as

$$u_{ij} = bp_{ji} - cp_{ij} - c_r \delta(i) \quad (2)$$

where $\delta(i)$ is the Kronecker delta i.e. $\delta(i) = 1$ if $s_i \in R$, else $\delta(i) = 0$. Only reciprocators pay c_r to judge the behavior of their opponents. The parameters of the payoff matrix are adopted according to Nowak and May [9] as $b > c + c_r$, where reciprocal cost is usually considered to be very small. On condition that $b - c = 1$, the simplified payoff matrix is described as $U = [u_{ij}]$ in table I.

TABLE I. PAYOFF MATRIX OF THREE-STRATEGY PD GAME

	C	R	D
C	1	1	0
R	$1 - c_r$	$1 - c_r$	$-c_r$
D	b	0	0

B. Evolutionary Dynamics

In dynamical networks, an individual takes the proper strategy to interact with beneficial neighbors to get better utility. The dynamical interaction in a structured population means each individual learns from another who gain higher fitness. On the other hand, individuals adjust their social relationships and select beneficial neighbors to change the local environment. Adaptive networks can be used to describe the dynamical links. Apparently the underlying network affects the dynamic of agents' states and the evolution of the topology depends on the strategies of agents. It is interesting to study the interplay between local states and networks topology by agent-based evolutionary model in small world networks.

We investigate the coevolution of strategy and networks based evolution graph theory. The vertices of dynamical graph represent agents in the networks and each edge denotes the social relationship between the agents. The number of vertices and edges are assumed to remain invariable during the evolution. Neighbors are defined as familiar agents who have chances to trade. The immediate neighbors of an agent have direct links to it and an indirect neighbor means neighbors' neighbor.

The evolutionary game mode usually includes the following phases: interaction phase, evolution phase and mutation phase. In the interaction stage agents play pairwise trade with their immediate neighbors according to a specific rule. Agent i plays with partners and obtains an income as

$$g_i = \sum_{j \in N_i} s_i^T U s_j \quad (3)$$

where N_i represents the neighborhood set of agent i . In the evolution stage, agents update their strategy and make partner-switch. We adopt an analogy of death-birth (DB) update to model the pairwise comparable process. After agents interact for some rounds, they select one of its neighbors randomly and compare their incomes. Agent i attempts to change its strategy and enforce another strategy of agent j with likelihood given by the Femi function as

$$\Phi(s_i \leftarrow s_j) = \frac{1}{1 + e^{\beta(g_i - g_j)}} \quad (4)$$

where β ($\beta \geq 0$) is explained as the amplitude of noise or the intensity of selection. If $\beta \rightarrow 0$ social learning leads to random drift while $\beta \rightarrow \infty$ means the deterministic imitation. The update process depends on the efficiency of the strategy in current local environment. We pay more attention to the utility gained per round but not on the accumulated payoff, because agents may not have the same

strategy and neighborhood. In mutation stage, agents change their strategies incorporating innovation. The mutation probability is usually regarded so small that an agent rarely transfers to another strategy different from their own and the selected neighbor.

In the dynamic networks, the utility of each agent depends not only on the strategy but also the local environment. To maximize the income individuals adjust their links by searching new contacts with friendly neighbors and abandoning adverse social ties. We assume agents have the basic cognitive ability to switch the partners according to given link rules. Considering the cost of seeking information and judging neighbors' reputation, reciprocal agents can witness the past interaction of their neighbors and are assumed that they can acquire local information about the immediate neighbors and indirect neighbors. To improve the interaction environment, reciprocators keep the relationships with agents with C or R strategy and break the links with defectors. The specific link rule determines the manner to update neighbors. For example, a break in adverse relation occurs with a certain probability depending on the reputation score of the other player. To maintain the constant size of edges during the evolution of network topology, reciprocators search friendly partners and try to create cooperative links after breaking links with defectors. New neighbors are either selected randomly or recommended according to their reputation.

The reciprocal strategy is considered as a behavior pair combining both interaction game and linking manner. The behavior of distinguishing opponents leads to coevolution of network structure and individual state. Considering the frequency of evolution, two parameters in our model are the time scale for strategy updating τ_s and that for link adjustment τ_n . Depending on the time ratio $W = \tau_s / \tau_n$, the two aspects of coevolution are updated asynchronously. In each interaction round, strategy updating occurs with the probability $1/(1+W)$ and reciprocators select new neighbors with the probability $W/(1+W)$. In an extreme case $W \rightarrow 0$, the strategy is much faster than the evolution of topology. Therefore the evolution of strategy occurs on static graphs. On the other hand, the structure changes faster than the strategy updating with increasing the ratio. If W is large enough, agents are regarded to adopt fixed strategies while their links are adjusted promptly.

C. Reputation Mechanism

Reciprocal agents adopt different actions and link behaviors based on the reputation of opponents. The incentive mechanism in dynamic social networks can accelerate the evolution of cooperation. Reputation scores are used to estimate the behaviors of the neighbors and make correct choices to adopt proper behavior. Agents with higher reputation have more opportunities for advantage interaction therefore the cooperative population can obtain better fitness. On the other hand, defectors with bad

reputations are refused by reciprocal agents and get less utility due to losing some links.

The reputation $r_i(n)$ reflects the cooperative degree of agent i in the social networks during a period. After agent i interacts with all its neighbors in n -th round, reputation increment $\Delta r_i(n)$ is accumulated to the old reputation $r_i(n-1)$ given in last round. The score in current round involved history records is updated as

$$r_i(n) = \eta r_i(n-1) + (1-\eta) \Delta r_i(n) \quad (5)$$

$$\Delta r_i(n) = \sin\left(\frac{\pi}{2} * \frac{N_{good}}{N_{sum}} * \frac{(N_{sum} - N_{bad})}{N_{sum}}\right) \quad (6)$$

where η ($0 \leq \eta \leq 1$) represents history factor. if $\eta = 0$ the reciprocal strategy conforms to TFT while η is close to 1, more information on past interaction is incorporated into the current reputation score. N_{good} and N_{bad} represent the number of good actions and bad actions respectively and their sum may not equal to the total amount N_{sum} that agent takes interactions in a round.

Under second-order social norm, good action is determined by whether provide service or not. It also depends on the reputation of the consumer. The reputation model in this paper adopts stern-judging rule [15], i.e. helping cooperative agents or refusing to serve defectors leads to good reputation and contrary actions reduce reputation. Reputation of any agent increases when it does good to agents with R and C strategy or reject defectors. Here we use $\sin()$ function instead of linear function to suppress the rise rate. The decline of reputation results from bad actions taken by uncooperative agents. Compared with a generous rule, the indirect reciprocity mechanism based on reputation under a stern judge rule punishes selfish agents more severely and encourages them to become cooperators.

We assume that interactions between providers and consumers can be observed by other agents in the local network. The reputation of an agent is evaluated by its neighbors according to the past interaction. The characters of agents can be judged by their reputation as such rule: agent i is considered as a cooperative individual if $r_i(n) \geq r_g$, it is likely to be a defector as $r_i(n) < r_b$ and otherwise it is treated as uncertain one for which reciprocators provide service with the probability $r_i(n)$. The threshold parameters ($r_g \geq r_b$) are taken as constant values or variables according to the local networks. For the sake of simplicity we use the parameters $r_g = 0.7$ and $r_b = 0.4$ with initialization reputation is 0.5.

Reputation scores can be used to measure the decision function $f(s_j)$ in formula (1) and determine link manner. An agent evaluates the reputations of neighbors and breaks the links between the partners with the lower reputation. It tries to contact with new partners according to their reputation. Considering recommendation given by the neighborhood, the agent switches another partner k from the indirect

neighbors with probability $r_k(n)$. Otherwise the agent randomly selects neighbors from the network except its immediate neighbors. Therefore the agent process with cognitive ability can break up the uncooperative ties and choose cooperative partners with good reputations.

III. SIMULATION RESULTS

To study the evolution of cooperation in social networks, simulations are performed in structured populations of size $N = 10^4$. We construct the dynamic networks by small world networks according to the classical Watts-Strogatz model. Starting with a regular lattice with the degree of each node as $k = 50$, edges are rewired with probability $p = 0.05$. Each agent tries to abandon origin neighbors and links to another one randomly except its neighborhood. Initially, agents adopt one of three strategies with equal probability. In each step, agents play games with neighbors and receive payoff given by formula (2). Evaluating the utilities and reputations of their neighbors, agents update new strategies under the weak selection $\beta = 0.05$ and update partners asynchronously. A mutation probability ($p' = 10^{-5}$) is introduced that a few agents change strategy randomly after every round. The experiment results of the Monte Carlo (MC) simulations are obtained through $10^3 \sim 10^5$ rounds (MC steps, MCS) and got the average values through 100 times.

A. State Evolution

For rational individuals, defection is always the best choice in the tow-strategy social dilemma. Strategy D is the unique Nash equilibrium and also an evolutionarily stable strategy (ESS). However, reference [16] proofs mixed population with three strategies finally converges to full cooperation with small reciprocation cost and mutation probability limited to zero. While R and D form a bistable community and hence a population of R cannot be invaded by defectors. We discuss evolutionarily stable state in structured populations under different values of benefit b and reciprocal cost c_r . To simplify the problem, we assume the time ratio $W \rightarrow 0$. The results in this part are reasoned on the condition that agents update strategy on the stationary small world network.

Considering reciprocal strategy with c_r , the ESS is neither unique nor bistable state and presents different states. Fig.1 features the b - c_r phase diagram obtained via simulations of the evolutionary PD game. There are four regions in the diagram, represented as mixed states (C+D and C+D+R), absorbed state (D) and also oscillatory state (O) where the three strategies coexist in an oscillatory manner. The population cycles from C to D, D to R and back to C under corresponding parameters of benefit b and reciprocal cost c_r . The evolutionary process of stable stationary depends on the state. For example, with small b and high c_r values strategy C and D coexist stably only after

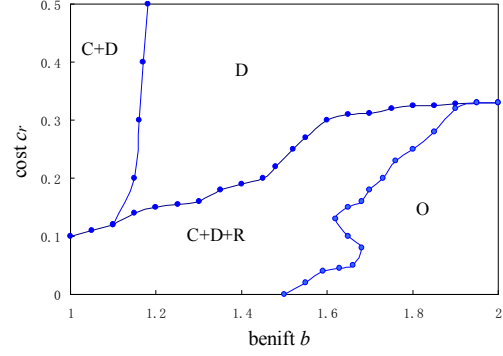


Figure 1. Phase diagram of the three-strategy evolutionary game on the small world network

2000 MCS. Moreover agents with strategy C and R become extinct fast within 1000 steps when b and c_r are both high. However 10^4 MCS are sufficient to reach the stationary state C+D+R and more MCS need to be discarded at the border of O state. The results reveal that the third strategy R induces different strategies coexist and promote cooperation to some extent. Provided reciprocal cost c_r is low enough, the survival barrier between D and the other states (C+D+R and O) is formed even by higher temptation to defect. It is obvious that higher b directly supports defection and lower c_r facilitates cooperation and reciprocal agents.

Fig.2 shows the average time of various strategies for small value of reciprocal cost c_r (0~0.3). As reciprocal agents have the ability to restrain the adverse behaviors by refusing to interact with defectors, the average time centralizes on the strategy of R and C. Accounting for addition cost c_r , reciprocal strategy is not as advantageous as the other strategies. With the increasing of c_r , the average time of R reduces sharply. Consequently the percentage of cooperator decreases and the population are invaded by defectors. As soon as reciprocators become extinct, strategy D remains the evolutionarily stable strategy. The experiment draws the same conclusion with [16] only if the reciprocation cost and mutation probability are both small enough, the average time of the oscillations can be mostly dominated by reciprocal strategy.

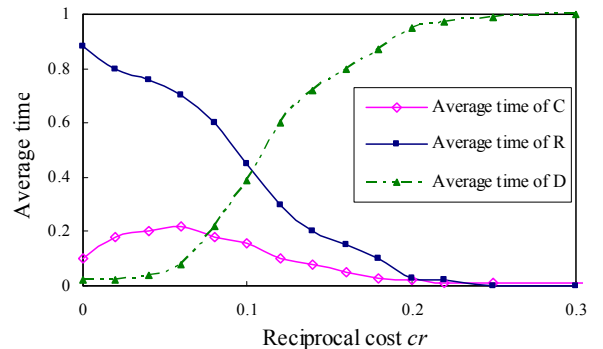


Figure 2. Average time of various strategies for lower value of reciprocal cost c_r (0~0.3) with $b=2$

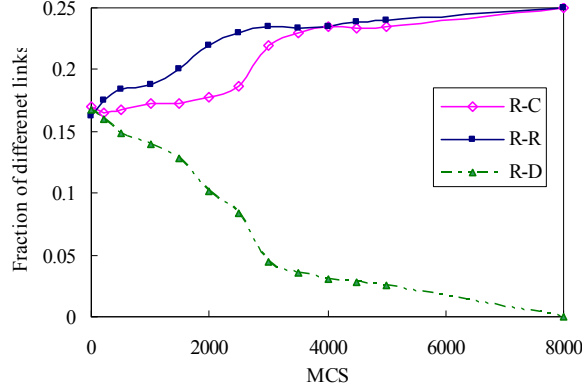


Figure 3. The fraction of different links evolves with $b=1.5$ and $c_r = 0.05$

B. Structure Evolution

In the well-mixed population, the probability that an agent meets an i -strategy player is equal to its global frequency x_i . However the fitness of an agent on structured population is locally determined by the adjacent agents. Therefore agents would create advantageous links by switching partners. On the condition $W \gg 1$, the frequency of partner switching is much faster than that of agents' state updating.

Irrespective of changing of strategy, we analyze the evolution of network structure when $b = 1.5$, $c_r = 0.05$ and $W \rightarrow \infty$. Initially nearly half of the links between reciprocal agents and others include R-R, R-C and R-D in the networks. During the structure evolution, most R-D links are broken and instead more cooperative relationship comes forth as reciprocal agents select new neighbors according to their reputations. As a result, the fractions of R-C and R-R links increase and that of R-D decreases dramatically. The results in fig.3 display that R can suppress adverse ties effectively and the link dynamics facilitates cooperative relationships. The evolution of structure favors agents with R or C strategies to form clusters and keep the cooperative population invaded by defectors.

Reciprocal agents adopt various actions and linking behavior against different partners. When the state of agents and structure of networks update at comparable time scales,

the strategy and link relationship evolve asynchronously. When $b = 1.5$ and $c_r = 0.05$, we observe the results in fig.4 that the stationary fraction of different strategies varies with the time ratio W . When W is relative small, agents update their strategies faster and reciprocators can not suppress defectors effectively by partner-switch mechanism. With the increase of W , defective agents receive less and the fraction of D reduces while the fraction of other strategies raises. The reciprocates get better fitness than cooperators and remain superiority as they can distinguish the opponents. When $W > 12$ defectors vanish gradually as reciprocators update neighbors in time. Taking reciprocal cost into account, cooperation becomes dominance and the fraction of C exceeds that of R when defectors extinct in the population.

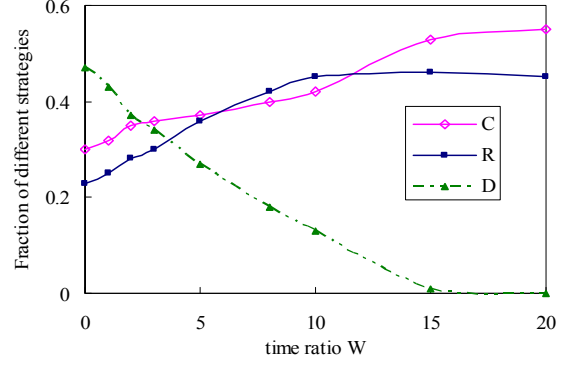


Figure 4. The fraction of different strategies with various time ratio W

C. Influence of Reputation

The indirect reciprocity mechanism based on reputation play a crucial role in the evolution of cooperation. The heterogeneity of networks structure results in a high level of cooperation as reciprocators select cooperative neighbors with high reputation. The number of an individuals' partners is restricted because of the limited capabilities of individuals in social networks. On the assumption that the size of neighbors doesn't exceed \sqrt{N} , we study the relation between agents' degree and their reputations through degree-dependent reputation function $r(k)$ which represents the average reputation of agents with degree k .

It is found that agents with good reputations are usually inclined to become popular partners in social networks. It is proved in fig.5 that agents with large degrees also achieve high reputation scores. Defectors with bad reputations are deprived of profitable opportunities when R dominates in the network. Under the partners selecting pressure, agents are forced to provide more service for good reputation. Obviously the positive correlation between reputation scores and degrees represented as $r(k)$ constitutes an effective feedback mechanism that can promote the emergence of cooperative behavior.

Errors in reputation estimation rise when agents adjust their strategies frequently. In the case of fast strategy dynamics, defectors beat the cooperative players and reciprocal agents have no advantage over cooperators. If any agent adopts different strategy in each round, reputation given according to interaction experiences does not exactly describe the agents' characters. It is hard to judge the opponents' actions and forecast their behaviors through their reputation, especially when the history factor η is large. When the dynamics of strategy becomes slow, the error of reputation is fairly small and accounted as system noise.

IV. CONCLUSION

In the frame of EGT, we study the coevolution of state and structure on small world networks and formally model the reciprocity mechanism based on reputation. In three-

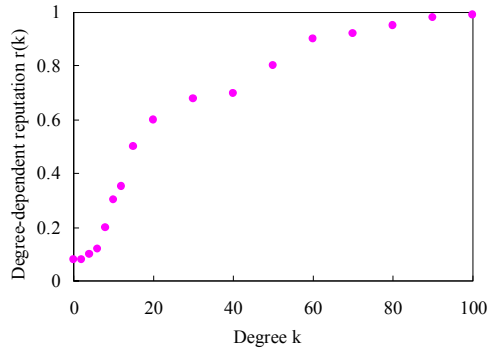


Figure 5. Degree-dependent reputation $r(k)$

strategy PD game, agents adopt such strategies as unconditional cooperation, always defection and reciprocity. Any agent with a base cognitive capability compares one of neighbors randomly and imitates the strategy with better utility. However reciprocal agents are supposed to possess modest cognitive abilities. As a result they can discriminate opponents according to their reputations and enforce different actions and linking behaviors. They dismiss the links for partners with lowest reputations and rewire to agents with good reputation. Most relative studies analyze the reciprocity rules such as tit-for-tat strategy without considering reciprocal cost and rarely discuss the implement of reciprocity. This paper investigates the effect of reciprocity based on a simple reputation model under stern-judge rule.

The evolution process induces self-organization on social networks owing to social learning and partners switching i.e. agents update strategies and alternate local environment asynchronously. Simulation results display that the introduction of the third strategy R favors the emergence of cooperation in structured populations. Irrespective of structure dynamics, the coexistence of the three strategies prevails provided small reciprocal cost c_r . Cooperators remain survival in mixed states (C+D/ C+R+D) even under high defection temptation b . With a tiny mutation probability, the strategy evolution drives oscillating state as rock-paper-scissor game. The experiments show that the average time of each strategy depends on c_r and small reciprocal cost conduces the oscillation centralize on reciprocators. On the other hand, when the time scale W is relatively large, the diversity in neighborhood structure arising from dynamics of networks topology promotes cooperation significantly. The advertise links between reciprocators and defectors are broken down and cooperative links go up instead. Defectors with low reputation are punished by losing interaction opportunity. The correlation of reputation and degree is observed in the simulation and cooperative agents with good reputation usually have large degrees. The indirect reciprocity mechanism based on reputation leads to the structure heterogeneity of networks and promotes the evolution of cooperation. However the errors of reputation may induce estimation mistakes for agents' attribute, especially when agents update their strategies frequently.

Our future work of cooperation evolution will focus on the robustness of the reputation system in social networks.

ACKNOWLEDGMENT

This work is supported in part by Natural Science Foundation of China under grant No. 61103233 and No. 61100194, NSFC-JST under grant No.51021140004, NKTRDP 2011BAH11B01 and 2011BAH11B04, the Fundamental Research Funds for the Central Universities DUT12ZD104 and DUT12JR08.

REFERENCES

- [1] J.M.Smith, *Evolution and the Theory of Games*. Cambridge, UK: Cambridge University Press, 1982.
- [2] M.A.Nowak, *Evolutionary Dynamics*. Cambridge, MA: Harvard University Press, 2006.
- [3] H.Ohtsuki, M. A. Nowak, "The replicator equation on graphs," *Theor. Biol.*, vol. 243, Nov. 2006, pp. 86–97, doi:10.1016/j.jtbi.2006.06.004
- [4] J.Hofbauer, K.Sigmund, *Evolutionary Games and Population Dynamics*. London, Cambridge: Cambridge University Press, 1998.
- [5] M.A.Nowak, K.Sigmund, "Evolution of indirect reciprocity," *Nature*, vol. 437, Oct. 2005, pp. 1291–1298, doi:10.1038/nature04131.
- [6] H. Ohtsuki, M.A.Nowak, "Direct reciprocity on graphs," *Theor. Biol.*, vol. 247, Aug. 2007, pp.462–470, doi: 10.1016/j.jtbi.2007.03.018.
- [7] M.A.Nowak, "Five rules for the evolution of cooperation," *Science*, vol. 314, Dec. 2006, pp. 1560–1563, doi: 10.1126/science.1133755.
- [8] M.Santos,D.J.Rankin and C.Wedekind, "The evolution of punishment through reputation," *Proc. R. Soc. B*, vol. 278, Feb. 2011, pp. 371–377, doi: 10.1098/rspb.2010.127.
- [9] A.Szolnoki, M.Perc and G.Szabó, "Phase diagrams for three-strategy evolutionary prisoner's dilemma games on regular graphs," *Phys. Rev.E*. vol. 80, Nov. 2009, doi:10.1103/PhysRevE.80.056104.
- [10] H.Ohtsuki1, Y.Iwasa and M.A. Nowak, "Indirect reciprocity provides only a narrow margin of efficiency for costly punishment," *Nature*, vol. 457, Jan. 2009, pp. 79–82, doi:10.1038/nature07601.
- [11] F. C. Santos, J. M. Pacheco and T. Lenaerts, "Evolutionary dynamics of social dilemmas in structured heterogeneous populations," *PNAS* vol. 103, Feb. 2006, pp 3490–3494, doi: 10.1073/pnas.0508201103.
- [12] C.Zhang, J.Zhang, FJ Weissing, M. Perc, et al. "Different Reactions to Adverse Neighborhoods in Games of Cooperation," *PLoS ONE*, 2012, 7(4): e35183. doi:10.1371/journal.pone.0035183
- [13] L. Cao, H. Ohtsukib, B. Wang and K. Aihara, "Evolution of cooperation on adaptively weighted networks," *Theor. Biol.*, vol. 272, Mar. 2011, pp 8–15, doi:10.1016/j.jtbi.2010.12.008.
- [14] F. Fu, C. Hauert and M.A. Nowak, "Reputation-based partner choice promotes cooperation in social networks," *Physical Review E*, vol. 78 2008, doi: 10.1103/PhysRevE.78.026117.
- [15] J. M. Pacheco1, C. S. Francisco, "Stern-Judging: A simple, successful norm which promotes cooperation under indirect," *PLoS Comput Biol* vol. 2, Dec.2006, e178. doi:10.1371/journal.pcbi.0020178.
- [16] Y.Wang, A.Nakao, A.V.Vasilakos and J.Ma, "P2P soft security: On evolutionary dynamics of P2P incentive mechanism," *Computer Communications*, vol. 34, Mar. 2011, pp. 241–249.