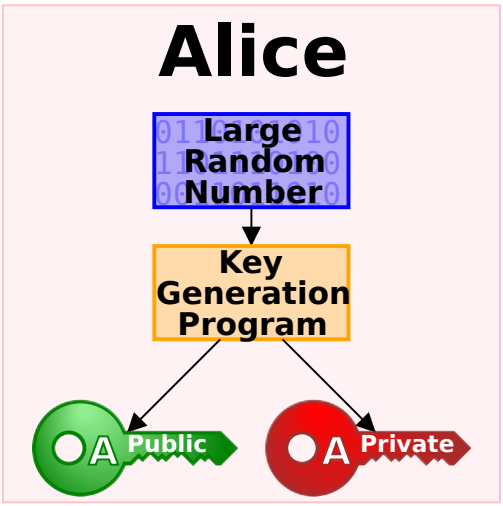
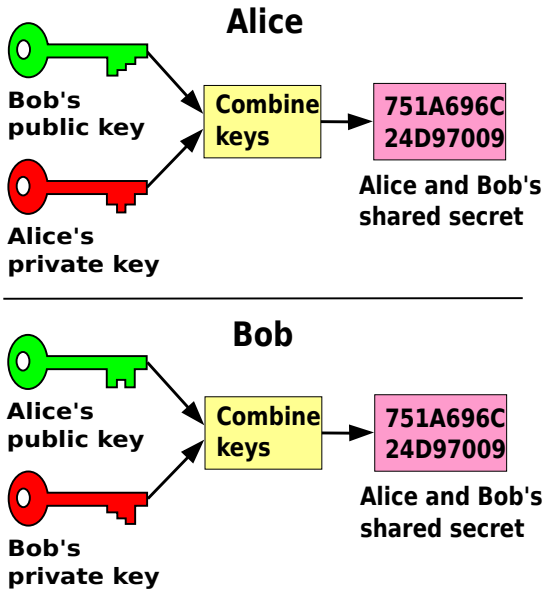
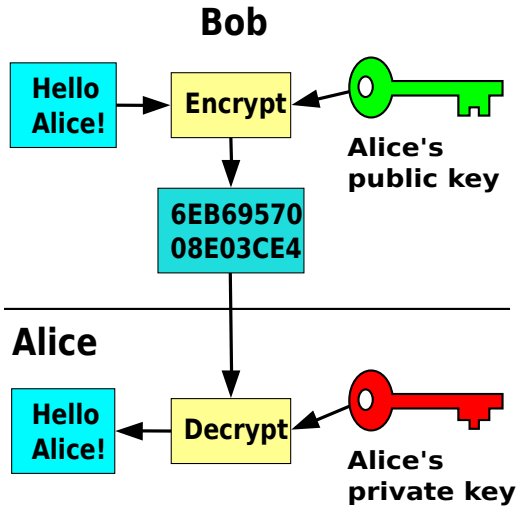
Public-key cryptography



An unpredictable (typically large and random) number is used to begin generation of an acceptable pair of keys suitable for use by an asymmetric key algorithm.



In the Diffie–Hellman key exchange scheme, each party generates a public/private key pair and distributes the public key. After obtaining an authentic copy of each other's public keys, Alice and Bob can compute a shared secret offline. The shared secret can be used, for instance, as the key for a symmetric cipher.



In an asymmetric key encryption scheme, anyone can encrypt messages using the public key, but only the holder of the paired private key can decrypt. Security depends on the secrecy of the private key.

Public-key cryptography, also known as **asymmetric cryptography**, is a class of cryptographic algorithms which requires two separate keys, one of which is *secret* (or *private*) and one of which is *public*. Although different, the two parts of this key pair are mathematically linked. The public key is used to encrypt plaintext or to verify a digital signature; whereas the private key is used to decrypt ciphertext or to create a digital signature. The term "asymmetric" stems from the use of different keys to perform these opposite functions, each the inverse of the other – as contrasted with conventional ("symmetric") cryptography which relies on the same key to perform both.

Public-key algorithms are based on mathematical problems which currently admit no efficient solution that are inherent in certain integer factorization, discrete logarithm, and elliptic curve relationships. It is computationally easy for a user to generate their own public and private key-pair and to use them for encryption and decryption. The strength lies in the fact that it is "impossible" (computationally infeasible) for a properly generated private key to be determined from its corresponding public key. Thus the public key may be published without compromising security, whereas the private key must not be revealed to anyone not authorized to read messages or perform digital signatures. Public key algorithms, unlike symmetric key algorithms, do *not* require a secure initial exchange of one (or more) secret keys between the par-

ties.

Message authentication involves processing a message with a private key to produce a digital signature. Thereafter anyone can verify this signature by processing the signature value with the signer's corresponding public key and comparing that result with the message. Success confirms the message is unmodified since it was signed, and – presuming the signer's private key has remained secret to the signer – that the signer, and no one else, intentionally performed the signature operation. In practice, typically only a hash or digest of the message, and not the message itself, is encrypted as the signature.

Public-key algorithms are fundamental security ingredients in cryptosystems, applications and protocols. They underpin various Internet standards, such as Transport Layer Security (TLS), S/MIME, PGP, and GPG. Some public key algorithms provide key distribution and secrecy (e.g., Diffie–Hellman key exchange), some provide digital signatures (e.g., Digital Signature Algorithm), and some provide both (e.g., RSA).

Public-key cryptography finds application in, amongst others, the IT security discipline information security. Information security (IS) is concerned with all aspects of protecting electronic information assets against security threats.[1] Public-key cryptography is used as a method of assuring the confidentiality, authenticity and non-repudiability of electronic communications and data storage.

1 Understanding

Public-key cryptography is often used to secure electronic communication over an open networked environment such as the internet. Open networked environments are susceptible to a variety of communication security problems such as man-in-the-middle attacks and other security threats. Sending a secure communication means that the communication being sent must not be readable during transit (preserving confidentiality), the communication must not be modified during transit (preserving the integrity of the communication) and to enforce non-repudiation or non-denial of the sending of the communication. Combining public-key cryptography with an Enveloped Public Key Encryption (EPKE)[2] method, allows for the secure sending of a communication over an open networked environment.

The distinguishing technique used in public-key cryptography is the use of asymmetric key algorithms, where the key used to encrypt a message is not the same as the key used to decrypt it. Each user has a pair of cryptographic keys – a **public encryption key** and a **private decryption key**. Similarly, a key pair used for digital signatures consists of a **private signing key** and a **public verification key**. The public key is widely distributed, while the private key is known only to its proprietor. The keys are related mathematically, but the parameters are chosen so that calculating the private key from the public key is either impossible or prohibitively expensive.

In contrast, symmetric-key algorithms – variations of which have been used for thousands of years – use a *single* secret key, which must be shared and kept private by both the sender and the receiver, for both encryption and decryption. To use a symmetric encryption scheme, the sender and receiver must securely share a key in advance.

Because symmetric key algorithms are nearly always much less computationally intensive than asymmetric ones, it is common to exchange a key using a key-exchange algorithm, then transmit data using that key and a symmetric key algorithm. PGP and the SSL/TLS family of schemes use this procedure, and are thus called *hybrid cryptosystems*.

2 Description

There are two main uses of public-key cryptography:

- *Public-key encryption*, in which a message is encrypted with a recipient's public key. The message cannot be decrypted by anyone who does not possess the matching private key, who is thus presumed to be the owner of that key and the person associated with the public key. This is used in an attempt to ensure confidentiality.

- *Digital signatures*, in which a message is signed with the sender's private key and can be verified by anyone who has access to the sender's public key. This verification proves that the sender had access to the private key, and therefore is likely to be the person associated with the public key. This also ensures that the message has not been tampered with, as any manipulation of the message will result in changes to the encoded message digest, which otherwise remains unchanged between the sender and receiver.

An analogy to public-key encryption is that of a locked mail box with a mail slot. The mail slot is exposed and accessible to the public – its location (the street address) is, in essence, the public key. Anyone knowing the street address can go to the door and drop a written message through the slot. However, only the person who possesses the key can open the mailbox and read the message.

An analogy for digital signatures is the sealing of an envelope with a personal wax seal. The message can be opened by anyone, but the presence of the unique seal authenticates the sender.

A central problem with the use of public-key cryptography is confidence/proof that a particular public key is authentic, in that it is correct and belongs to the person

or entity claimed, and has not been tampered with or replaced by a malicious third party. The usual approach to this problem is to use a public-key infrastructure (PKI), in which one or more third parties – known as certificate authorities – certify ownership of key pairs. PGP, in addition to being a certificate authority structure, has used a scheme generally called the "web of trust", which decentralizes such authentication of public keys by a central mechanism, and substitutes individual endorsements of the link between user and public key. To date, no fully satisfactory solution to the "public key authentication problem" has been found.

3 History

During the early history of cryptography, two parties would rely upon a key that they would exchange between themselves by means of a secure, but non-cryptographic, method. For example, a face-to-face meeting or an exchange, via a trusted courier, could be used. This key, which both parties kept absolutely secret, could then be used to exchange encrypted messages. A number of significant practical difficulties arise with this approach to distributing keys.

In 1874, a book by William Stanley Jevons[3] described the relationship of one-way functions to cryptography, and went on to discuss specifically the factorization problem used to create a trapdoor function. In July 1996, mathematician Solomon W. Golomb said: "Jevons anticipated a key feature of the RSA Algorithm for public key cryptography, although he certainly did not invent the concept of public key cryptography."[4]

In 1970 James H. Ellis a British cryptographer at the UK Government Communications Headquarters(GCHQ) conceived of the possibility of "non-secret encryption", (now called public-key cryptography), but could see no way to implement it. In 1973 his colleague Clifford Cocks invented what has become known as the RSA encryption algorithm, giving a practical method of implementation, and in 1974 another GCHQ mathematician and cryptographer, Malcolm J. Williamson developed what is now known as Diffie–Hellman key exchange. None of these appear to have been put to practical use, and their early invention did not become public knowledge until the research was declassified by the British government in 1997.[5]

In 1976 an asymmetric-key cryptosystem was published by Whitfield Diffie and Martin Hellman who, influenced by Ralph Merkle's work on public-key distribution, disclosed a method of public-key agreement. This method of key exchange, which uses exponentiation in a finite field, came to be known as Diffie–Hellman key exchange. This was the first published practical method for establishing a shared secret-key over an authenticated (but not private) communications channel without using a prior shared secret. Merkle's "public-key-agreement technique" became known as Merkle's Puzzles, and was invented in 1974 and published in 1978.

In 1977 a generalization of Cocks's scheme was independently invented by Ron Rivest, Adi Shamir and Leonard Adleman, all then at MIT. The latter authors published their work in 1978, and the algorithm came to be known as RSA, from their initials. RSA uses exponentiation modulo a product of two very large primes, to encrypt and decrypt, performing both public key encryption and public key digital signature. Its security is connected to the extreme difficulty of factoring large integers, a problem for which there is no known efficient general technique. In 1979, Michael O. Rabin published a related cryptosystem that is probably secure as long as the factorization of the public key remains difficult – it remains an assumption that RSA also enjoys this security.

Since the 1970s, a large number and variety of encryption, digital signature, key agreement, and other techniques have been developed in the field of public-key cryptography. The ElGamal cryptosystem, invented by Taher ElGamal relies on the similar and related high level of difficulty of the discrete logarithm problem, as does the closely related DSA, which was developed at the US National Security Agency (NSA) and published by NIST as a proposed standard.

The introduction of elliptic curve cryptography by Neal Koblitz and Victor Miller, independently and simultaneously in the mid-1980s, has yielded new public-key algorithms based on the discrete logarithm problem. Although mathematically more complex, elliptic curves provide smaller key sizes and faster operations for approximately equivalent estimated security.

4 Security

Some encryption schemes can be proven secure on the basis of the presumed difficulty of a mathematical problem, such as factoring the product of two large primes or computing discrete logarithms. Note that "secure" here has a precise mathematical meaning, and there are multiple different (meaningful) definitions of what it means for an encryption scheme to be "secure". The "right" definition depends on the context in which the scheme will be deployed.

The most obvious application of a public key encryption system is confidentiality – a message that a sender encrypts using the recipient's public key can be decrypted only by the recipient's paired private key. This assumes, of course, that no flaw is discovered in the basic algorithm used.

Another type of application in public-key cryptography is that of digital signature schemes. Digital signature schemes can be used for sender authentication and non-repudiation. In such a scheme, a user who wants to

send a message computes a digital signature for this message, and then sends this digital signature (together with the message) to the intended receiver. Digital signature schemes have the property that signatures can be computed only with the knowledge of the correct private key. To verify that a message has been signed by a user and has not been modified, the receiver needs to know only the corresponding public key. In some cases (e.g., RSA), a single algorithm can be used to both encrypt and create digital signatures. In other cases (e.g., DSA), each algorithm can only be used for one specific purpose.

To achieve both authentication and confidentiality, the sender should include the recipient's name in the message, sign it using his private key, and then encrypt both the message and the signature using the recipient's public key.

These characteristics can be used to construct many other (sometimes surprising) cryptographic protocols and applications, such as digital cash, password-authenticated key agreement, multi-party key agreement, time-stamping services, non-repudiation protocols, etc.

5 Practical considerations

5.1 Enveloped Public Key Encryption

Enveloped Public Key Encryption (EPKE) is the method of applying public-key cryptography and ensuring that an electronic communication is transmitted confidentially, has the contents of the communication protected against being modified (communication integrity) and cannot be denied from having been sent (non-repudiation). This is often the method used when securing communication on an open networked environment such by making use of the Transport Layer Security (TLS) or Secure Sockets Layer (SSL) protocols.

EPKE consists of a two-stage process that includes both Forward Public Key Encryption (FPKE) also known as symmetric encryption and Inverse Public Key Encryption (IPKE). Both Forward Public Key Encryption and Inverse Public Key encryption make up the foundation of Enveloped Public Key Encryption (these two processes are described in full in their own sections).

For EPKE (asymmetric encryption) to work effectively, it is required that:

- Every participant in the communication has their own unique pair of keys. The first key that is required is a public key and the second key that is required is a private key.
- Each person's own private and public keys must be mathematically related where the private key is used to decrypt a communication sent using a public key and vice versa. The RSA algorithm is a well known asymmetric encryption algorithm.
- The private key must be kept absolutely private by the owner though the public key can be published in a public directory such as with a certification authority.

To send a message using EPKE, the sender of the message first encrypts the message using their own private key, this ensures non-repudiation of the message (Inverse Public Key Encryption). The sender then encrypts their digitally signed message using the receivers public key (Forward Public Key Encryption) thus applying a digital envelope to the message. This step ensures confidentiality during the transmission of the message. The receiver of the message then uses their private key to decrypt the message thus removing the digital envelope and then uses the sender's public key to decrypt the sender's digital signature. At this point, if the message has been unaltered during transmission, the message will be clear to the receiver.

Due to the computationally complex nature of the RSA asymmetric encryption algorithm, the time taken to encrypt a large documents or files to be transmitted can take an increased amount of time to complete. To speed up the process of transmission, instead of applying the sender's digital signature to the large documents or files, the sender can rather hash the documents or files and then encrypt the generated hash key with their digital signature therefore enforcing non-repudiation. Hashing is a much faster computation to complete as opposed to using the RSA encryption algorithm alone. The sender would then encrypt the newly generated hash key and encrypt the original documents or files with the receiver's public key. The transmission would then take place securely and with confidentiality and non-repudiation still intact. The receiver would then decrypt both the encrypted hash key and the encrypted documents or files with their private key. Finally, to ensure that the message being sent was in fact sent by the appropriate sender, the receiver would then use the sender's public key to decrypt the hash key and then the receiver would hash the documents or files using the same hashing algorithm as the sender and see if the hash key that is generated matches the hash key generated by the sender. If both hash keys match, the documents or files are the same as the sender's and non-repudiation has been preserved.

Note: The sender and receiver do not usually carry out the process mentioned above manually though rather rely on sophisticated software to automatically complete the EPKE process.

5.1.1 Forward Public Key Encryption

The goal of Forward Public Key Encryption (FPKE) encryption is to ensure that the communication being sent is kept confidential during transit.

To send a message using FPKE, the sender of the message uses the public key of the receiver to encrypt the contents of the message. The encrypted message is then transmitted electronically to the receiver and the receiver can then use their own matching private key to decrypt the message.

The encryption process of using the receivers public key is useful for preserving the confidentiality of the message as only the receiver has the matching private key to decrypt the message. Therefore, the sender of the message cannot decrypt the message once it has been encrypted using the receivers public key. However, FPKE does not address the problem of non-repudiation, as the message could have been sent by anyone that has access to the receivers public key.

5.1.2 Inverse Public Key Encryption

The goal of Inverse Public Key Encryption (IPKE) is to ensure that the sender of the communication that is being sent is known to the receiver and that the sender of the message cannot refute that the message that they have sent was not sent by them. Therefore, the purpose of IPKE is to ensure the non-repudiation of the message being sent. This is useful in a practical setting where a sender wishes to make an electronic purchase of shares and the receiver wants to confirm that it was indeed the actual sender requesting the purchase and not someone else. IPKE is also known as a digital signature.

To send a message using IPKE, the message is signed using the sender's private key which serves as the sender's digital signature. The digitally "signed" message is then sent to the receiver who can then use the sender's public key to verify the signature.

IPKE is useful for applying ones digital signature to a message thus enforcing non-repudiation however, it does not enforce the confidentiality of the message being sent.

5.2 Certification authority

In order for Enveloped Public Key Encryption to be as secure as possible, there needs to be a "gatekeeper" of public and private keys, or else anyone could publish their public key and masquerade as the intended sender of a communication. This digital key "gatekeeper" is known as a certification authority. A certification authority is a trusted third party that can issue public and private keys thus certifying public keys.It also works as a depository to store key chain and emphorse the trust factor.

5.3 A postal analogy

An analogy that can be used to understand the advantages of an asymmetric system is to imagine two people, Alice and Bob, who are sending a secret message through the public mail. In this example, Alice wants to send a secret message to Bob, and expects a secret reply from Bob.

With a symmetric key system, Alice first puts the secret message in a box, and locks the box using a padlock to which she has a key. She then sends the box to Bob through regular mail. When Bob receives the box, he uses an identical copy of Alice's key (which he has somehow obtained previously, maybe by a face-to-face meeting) to open the box, and reads the message. Bob can then use the same padlock to send his secret reply.

In an asymmetric key system, Bob and Alice have separate padlocks. First, Alice asks Bob to send his open padlock to her through regular mail, keeping his key to himself. When Alice receives it she uses it to lock a box containing her message, and sends the locked box to Bob. Bob can then unlock the box with his key and read the message from Alice. To reply, Bob must similarly get Alice's open padlock to lock the box before sending it back to her.

The critical advantage in an asymmetric key system is that Bob and Alice never need to send a copy of their keys to each other. This prevents a third party – perhaps, in this example, a corrupt postal worker that will open unlocked boxes – from copying a key while it is in transit, allowing the third party to spy on all future messages sent between Alice and Bob. So, in the public key scenario, Alice and Bob need not trust the postal service as much. In addition, if Bob were careless and allowed someone else to copy *his* key, Alice's messages to Bob would be compromised, but Alice's messages to other people would remain secret, since the other people would be providing different padlocks for Alice to use.

Another kind of asymmetric key system, called a three-pass protocol, requires neither party to even touch the other party's padlock (or key); Bob and Alice have separate padlocks. First, Alice puts the secret message in a box, and locks the box using a padlock to which only she has a key. She then sends the box to Bob through regular mail. When Bob receives the box, he adds his own padlock to the box, and sends it back to Alice. When Alice receives the box with the two padlocks, she removes her padlock and sends it back to Bob. When Bob receives the box with only his padlock on it, Bob can then unlock the box with his key and read the message from Alice. Note that, in this scheme, the order of decryption is NOT the same as the order of encryption – this is only possible if commutative ciphers are used. A commutative cipher is one in which the order of encryption and decryption is interchangeable, just as the order of multiplication is interchangeable (i.e., A*B*C = A*C*B = C*B*A). This method is secure for certain choices of commutative ciphers, but insecure for others (e.g., a simple XOR). For example, let $E_1()$ and $E_2()$ be two encryption functions, and let "M" be the message so that if Alice encrypts it using $E_1()$ and sends $E_1(M)$ to Bob. Bob then again encrypts the message as $E_2(E_1(M))$ and sends it to Alice.

Now, Alice decrypts $E_2(E_1(M))$ using $E_1()$. Alice will now get $E_2(M)$, meaning when she sends this again to Bob, he will be able to decrypt the message using $E_2()$ and get "M". Although none of the keys were ever exchanged, the message "M" may well be a key (e.g., Alice's Public key). This three-pass protocol is typically used during key exchange.

5.4 Actual algorithms: two linked keys

Not all asymmetric key algorithms operate in precisely this fashion. The most common ones have the property that Alice and Bob each own *two* keys, one for encryption and one for decryption. In a secure asymmetric key encryption scheme, the private key should not be deducible from the public key. This is known as public-key encryption, since an encryption key can be published without compromising the security of messages encrypted with that key.

In the analogy above, Bob might publish instructions on how to make a lock ("public key"). However, the workings of the lock are such that it is impossible (so far as is known) to deduce from the instructions given just exactly how to make a key that will open that lock (e.g., a "private key"). Those wishing to send messages to Bob must use the public key to encrypt the message, then Bob can use his private key to decrypt it.

Another example has Alice and Bob each choosing a key at random, and then contacting each other to compare the depth of each notch on their keys. Having determined the difference, a locked box is built with a special lock that has each pin inside divided into 2 pins, matching the numbers of their keys. Now the box will be able to be opened with either key, and Alice and Bob can exchange messages inside the box in a secure fashion.

5.5 Weaknesses

Of course, there is a possibility that someone could "pick" Bob's or Alice's lock. Among symmetric key encryption algorithms, only the one-time pad can be proven to be secure against any adversary – no matter how much computing power is available. However, there is no public-key scheme with this property, since all public-key schemes are susceptible to a "brute-force key search attack". Such attacks are impractical if the amount of computation needed to succeed – termed the "work factor" by Claude Shannon – is out of reach of all potential attackers. In many cases, the work factor can be increased by simply choosing a longer key. But other algorithms may have much lower work factors, making resistance to a brute-force attack irrelevant. Some special and specific algorithms have been developed to aid in attacking some public key encryption algorithms – both RSA and ElGamal encryption have known attacks that are much faster than the brute-force approach. These factors have changed dramatically in recent decades, both with the decreasing cost of computing power and with new mathematical discoveries.

Aside from the resistance to attack of a particular key pair, the security of the certification hierarchy must be considered when deploying public key systems. Some certificate authority – usually a purpose-built program running on a server computer – vouches for the identities assigned to specific private keys by producing a digital certificate. Public key digital certificates are typically valid for several years at a time, so the associated private keys must be held securely over that time. When a private key used for certificate creation higher in the PKI server hierarchy is compromised, or accidentally disclosed, then a "man-in-the-middle attack" is possible, making any subordinate certificate wholly insecure.

Major weaknesses have been found for several formerly promising asymmetric key algorithms. The 'knapsack packing' algorithm was found to be insecure after the development of a new attack. Recently, some attacks based on careful measurements of the exact amount of time it takes known hardware to encrypt plain text have been used to simplify the search for likely decryption keys (see "side channel attack"). Thus, mere use of asymmetric key algorithms does not ensure security. A great deal of active research is currently underway to both discover, and to protect against, new attack algorithms.

Another potential security vulnerability in using asymmetric keys is the possibility of a "man-in-the-middle" attack, in which the communication of public keys is intercepted by a third party (the "man in the middle") and then modified to provide different public keys instead. Encrypted messages and responses must also be intercepted, decrypted, and re-encrypted by the attacker using the correct public keys for different communication segments, in all instances, so as to avoid suspicion. This attack may seem to be difficult to implement in practice, but it is not impossible when using insecure media (e.g., public networks, such as the Internet or wireless forms of communications) – for example, a malicious staff member at Alice or Bob's Internet Service Provider (ISP) might find it quite easy to carry out. In the earlier postal analogy, Alice would have to have a way to make sure that the lock on the returned packet really belongs to Bob before she removes her lock and sends the packet back. Otherwise, the lock could have been put on the packet by a corrupt postal worker pretending to be Bob, so as to fool Alice.

One approach to prevent such attacks involves the use of a certificate authority, a trusted third party responsible for verifying the identity of a user of the system. This authority issues a tamper-resistant, non-spoofable digital certificate for the participants. Such certificates are signed data blocks stating that this public key belongs to that person, company, or other entity. This approach also has its weaknesses – for example, the certificate authority issuing the certificate must be trusted to have properly

checked the identity of the key-holder, must ensure the correctness of the public key when it issues a certificate, and must have made arrangements with all participants to check all their certificates before protected communications can begin. Web browsers, for instance, are supplied with a long list of "self-signed identity certificates" from PKI providers – these are used to check the *bona fides* of the certificate authority and then, in a second step, the certificates of potential communicators. An attacker who could subvert any single one of those certificate authorities into issuing a certificate for a bogus public key could then mount a "man-in-the-middle" attack as easily as if the certificate scheme were not used at all. Despite its theoretical and potential problems, this approach is widely used. Examples include SSL and its successor, TLS, which are commonly used to provide security for web browsers, for example, so that they might be used to securely send credit card details to an online store.

5.6 Computational cost

The public key algorithms known thus far are relatively computationally costly compared with most symmetric key algorithms of apparently equivalent security. The difference factor is the use of typically quite large keys. This has important implications for their practical use. Most are used in hybrid cryptosystems for reasons of efficiency – in such a cryptosystem, a shared secret key ("session key") is generated by one party, and this much briefer session key is then encrypted by each recipient's public key. Each recipient then uses the corresponding private key to decrypt the session key. Once all parties have obtained the session key, they can use a much faster symmetric algorithm to encrypt and decrypt messages. In many of these schemes, the session key is unique to each message exchange, being pseudo-randomly chosen for each message.

5.7 Associating public keys with identities

The binding between a public key and its "owner" must be correct, or else the algorithm may function perfectly and yet be entirely insecure in practice. As with most cryptography applications, the protocols used to establish and verify this binding are critically important. Associating a public key with its owner is typically done by protocols implementing a public key infrastructure – these allow the validity of the association to be formally verified by reference to a trusted third party in the form of either a hierarchical certificate authority (e.g., X.509), a local trust model (e.g., SPKI), or a web of trust scheme, like that originally built into PGP and GPG, and still to some extent usable with them. Whatever the cryptographic assurance of the protocols themselves, the association between a public key and its owner is ultimately a matter of subjective judgment on the part of the trusted third party, since the key is a mathematical entity, while the owner – and the connection between owner and key – are not. For this reason, the formalism of a public key infrastructure must provide for explicit statements of the policy followed when making this judgment. For example, the complex and never fully implemented X.509 standard allows a certificate authority to identify its policy by means of an object identifier, which functions as an index into a catalog of registered policies. Policies may exist for many different purposes, ranging from anonymity to military classifications.

5.8 Relation to real world events

A public key will be known to a large and, in practice, unknown set of users. All events requiring revocation or replacement of a public key can take a long time to take full effect with all who must be informed (i.e., all those users who possess that key). For this reason, systems that must react to events in real time (e.g., safety-critical systems or national security systems) should not use public-key encryption without taking great care. There are four issues of interest:

5.8.1 Privilege of key revocation

A malicious (or erroneous) revocation of some (or all) of the keys in the system is likely, or in the second case, certain, to cause a complete failure of the system. If public keys can be revoked individually, this is a possibility. However, there are design approaches that can reduce the practical chance of this occurring. For example, by means of certificates, we can create what is called a "compound principal" – one such principal could be "Alice and Bob have Revoke Authority". Now, only Alice and Bob (in concert) can revoke a key, and neither Alice nor Bob can revoke keys alone. However, revoking a key now requires both Alice *and* Bob to be available, and this creates a problem of reliability. In concrete terms, from a security point of view, there is now a "single point of failure" in the public key revocation system. A successful Denial of Service attack against either Alice or Bob (or both) will block a required revocation. In fact, any partition of authority between Alice and Bob will have this effect, regardless of how it comes about.

Because the principle allowing revocation authority for keys is very powerful, the mechanisms used to control it should involve **both** as many participants as possible (to guard against malicious attacks of this type), while at the same time as few as possible (to ensure that a key can be revoked without dangerous delay). Public key certificates that include an expiration date are unsatisfactory in that the expiration date may not correspond with a real-world revocation need – but at least such certificates need not all be tracked down system-wide, nor must all users be in constant contact with the system at all times.

5.8.2 Distribution of a new key

After a key has been revoked, or when a new user is added to a system, a new key must be distributed in some predetermined manner. Assume that Carol's key has been **revoked** (e.g., by exceeding its expiration date, or because of a compromise of Carol's matching private key). Until a new key has been distributed, Carol is effectively "out of contact". No one will be able to send her messages without violating system protocols (i.e., without a valid public key, no one can encrypt messages to her), and messages from her cannot be signed, for the same reason. Or, in other words, the "part of the system" controlled by Carol is, in essence, unavailable. Security requirements have been ranked higher than system availability in such designs.

One could leave the power to create (and certify) keys (as well as to revoke them) in the hands of each user – the original PGP design did so – but this raises problems of user understanding and operation. For security reasons, this approach has considerable difficulties – if nothing else, some users will be forgetful, or inattentive, or confused. On the one hand, a message revoking a public key certificate should be spread as fast as possible, while on the other hand, parts of the system might be rendered inoperable *before* a new key can be installed. The time window can be reduced to zero by always issuing the new key together with the certificate that revokes the old one, but this requires co-location of authority to both revoke keys and generate new keys.

It is most likely a system-wide failure if the (possibly combined) principal that issues new keys fails by issuing keys improperly. This is an instance of a "common mutual exclusion" – a design can make the reliability of a system high, but only at the cost of system availability (and *vice versa*).

5.8.3 Spreading the revocation

Notification of a key certificate revocation must be spread to all those who might potentially hold it, and as rapidly as possible.

There are but two means of spreading information (i.e., a key revocation) in a distributed system: either the information is "pushed" to users from a central point (or points), or else it is "pulled" from a central point(or points) by the end users.

Pushing the information is the simplest solution, in that a message is sent to all participants. However, there is no way of knowing whether all participants will actually *receive* the message. If the number of participants is large, and some of their physical or network distances are great, then the probability of complete success (which is, in ideal circumstances, required for system security) will be rather low. In a partly updated state, the system is particularly vulnerable to "denial of service" attacks as security has been breached, and a vulnerability window will continue to exist as long as some users have not "gotten the word". Put another way, pushing certificate revocation messages is neither easy to secure, nor very reliable.

The alternative to pushing is pulling. In the extreme, all certificates contain all the keys needed to verify that the public key of interest (i.e., the one belonging to the user to whom one wishes to send a message, or whose signature is to be checked) is still valid. In this case, at least some use of the system will be blocked if a user cannot reach the verification service (i.e., one of the systems that can establish the current validity of another user's key). Again, such a system design can be made as reliable as one wishes, at the cost of lowering security – the more servers to check for the possibility of a key revocation, the longer the window of vulnerability.

Another trade-off is to use a somewhat less reliable, but more secure, verification service, but to include an expiration date for each of the verification sources. How long this "timeout" should be is a decision that requires a trade-off between availability and security that will have to be decided in advance, at the time of system design.

5.8.4 Recovery from a leaked key

Assume that the principal authorized to revoke a key has decided that a certain key must be revoked. In most cases, this happens after the fact – for instance, it becomes known that at some time in the past an event occurred that endangered a private key. Let us denote the time at which it is decided that the compromise occurred as T.

Such a compromise has two implications. First, messages encrypted with the matching public key (now or in the past) can no longer be assumed to be secret. One solution to avoid this problem is to use a protocol that has perfect forward secrecy. Second, signatures made with the *no longer trusted to be actually private key* after time T can no longer be assumed to be authentic without additional information (i.e., who, where, when, etc.) about the events leading up to the digital signature. These will not always be available, and so all such digital signatures will be less than credible. A solution to reduce the impact of leaking a private key of a signature scheme is to use timestamps.

Loss of secrecy and/or authenticity, even for a single user, has system-wide security implications, and a strategy for recovery must thus be established. Such a strategy will determine who has authority to, and under what conditions one must, revoke a public key certificate. One must also decide how to spread the revocation, and ideally, how to deal with all messages signed with the key since time T (which will rarely be known precisely). Messages sent to that user (which require the proper – now compromised – private key to decrypt) must be considered compromised as well, no matter when they were sent.

6 Examples

Examples of well-regarded asymmetric key techniques for varied purposes include:

- Diffie–Hellman key exchange protocol
- DSS (Digital Signature Standard), which incorporates the Digital Signature Algorithm
- ElGamal
- Various elliptic curve techniques
- Various password-authenticated key agreement techniques
- Paillier cryptosystem
- RSA encryption algorithm (PKCS#1)
- Cramer–Shoup cryptosystem
- YAK authenticated key agreement protocol

Examples of asymmetric key algorithms not widely adopted include:

- NTRUEncrypt cryptosystem
- McEliece cryptosystem

Examples of notable – yet insecure – asymmetric key algorithms include:

- Merkle–Hellman knapsack cryptosystem

Examples of protocols using asymmetric key algorithms include:

- S/MIME
- GPG, an implementation of OpenPGP
- Internet Key Exchange
- PGP
- ZRTP, a secure VoIP protocol
- Secure Socket Layer, now codified as the IETF standard Transport Layer Security (TLS)
- SILC
- SSH
- Bitcoin
- Off-the-Record Messaging

7 See also

- Symmetric-key cryptography
- Books on cryptography
- GNU Privacy Guard
- Identity based encryption (IBE)
- Key-agreement protocol
- Key escrow
- PGP word list
- Pretty Good Privacy
- Pseudonymity
- Public key fingerprint
- Public key infrastructure (PKI)
- Quantum cryptography
- Secure Shell (SSH)
- Secure Sockets Layer (SSL)
- Threshold cryptosystem

8 Notes

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9 References

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- IEEE 1363: Standard Specifications for Public-Key Cryptography
- Christof Paar, Jan Pelzl, "Introduction to Public-Key Cryptography", Chapter 6 of "Understanding Cryptography, A Textbook for Students and Practitioners". (companion web site contains online cryptography course that covers public-key cryptography), Springer, 2009.

10 External links

- Oral history interview with Martin Hellman, Charles Babbage Institute, University of Minnesota. Leading cryptography scholar Martin Hellman discusses the circumstances and fundamental insights of his invention of public key cryptography with collaborators Whitfield Diffie and Ralph Merkle at Stanford University in the mid-1970s.
- An account of how GCHQ kept their invention of PKE secret until 1997 Archived June 25, 2008 at the Wayback Machine

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11.1 Text

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