CS131

MATHEMATICS FOR COMPUTER SCIENCE II

Sample Questions

Antonio Brito

for Warwick.Guide

NOTE TO STUDENTS:

This document tries to cover all possible bases of knowledge, but cannot possibly cover all types of question that you COULD be given. This is not a bible. Use other sources of revision to supplement your learning and problem solving skills.

NOTE TO DCS STAFF:

This document was made in its entirety by me. For concerns of cheating, malpractice, plagiarism or copyright infringement, please contact me at

antonio.brito@warwick.ac.uk

NOTE TO EVERYONE:

I do a lot of stuff, so I have office hours, where I will respond to queries quicker. Please take note of them, alongside my other projects here:

https://github.com/SwiftfoxStudios

1 Integers, Reals & Complex Numbers

1. Convert 12_{10} to base 8.
2. In what base is 26^2 equal to 6400_{10}?
3. Find $gcd(7544, 115)$.
4. Find $gcd(72, 126, 162)$.
5. Find x and y such that $gcd(2322, 654) = 2322x + 654y$.
6. Solve $24 \equiv x \mod 5$ such that $0 \leq x < 5$.
7. Solve $7^{1383921} \equiv m \mod 4$ such that $0 \leq m < 4$.
8. Solve $2022^{2023} \equiv f \mod 20$ such that $0 \leq f < 20$.
9. Prove that every bounded set must have a least upper bound.
10. Determine the real and imaginary parts of the following product:
$$(2 + 3i)(2 - i)$$
11. Determine the modulus and argument of $6 - 8i$.
12. Determine the complex conjugate of the expression:
$$(3 + i)(i - 10)$$
13. Express the complex number $5 + 2i$ in polar form.
14. Put the complex number
$$\frac{12-5i}{3+2i}$$
in the form $x + yi$ with $x, y \in \mathbb{R}$.
15. Solve the following expression:
$$(\sin \tfrac{\pi}{6} + i \cos \tfrac{\pi}{6})^{18}$$
16. Find all $z \in \mathbb{C}$ such that $z^3 = 1$.
17. Prove, by induction or otherwise, that $\forall n \in \mathbb{Z}^+$,
$$(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$$
holds.
18. Express $z^6 - 4z^3 + 8$ as the product of three factors with real coefficients, where $z \in \mathbb{C}$.

2 Linear & Matrix Algebra

1. Determine the length of the following vector

$$(3, 4, 5)$$

2. Find the unit vector in $\mathbb{R}^2$ with the same direction as $(2, -1)$.

3. Find the angle between the vectors

$$A = (4, 2)$$
$$B = (9, 3)$$

   in $\mathbb{R}^2$.

4. Find all vectors in $\mathbb{R}^3$ orthogonal to $(3, 6, 1)$ and $(6, 8, 1)$.

5. Express $\underline{v} = (9, 11, 27)$ as a linear combination of the vectors

$$\underline{p} = (3, -1, 2) \text{ and } \underline{q} = (3, 1, 5)$$

6. Is the set

$$Q = \{(1, 2, 3), (1, -1, -1), (5, 4, 3)\}$$

   linearly independent? If not, express this set as a linear combination of its vectors.

7. Is the set

$$Q = \{(1, 0, 1), (1, 1, 0), (0, 1, -1)\}$$

   linearly independent? If not, express this set as a linear combination of its vectors.

8. Express the plane $x - 3y + 4z = 0$ as a span of vectors.

9. Determine if

$$S = \{(x, y, z) \in \mathbb{R}^3 | x = 4s - t, y = s + 3t, z = 3s\} \text{ with } s, t \in \mathbb{R}$$

   is a subspace in $\mathbb{R}^3$.

10. Determine a basis for the subspace

$$S = \{(x, y, z) \in \mathbb{R}^3 | z = 2x - 3y\}$$

11. Prove that a set of vectors, S, in an vector space, V with dimension $|S|$ spans V, then it forms a basis.

12. Prove that the set containing the basis vectors of $\mathbb{R}^n$ is linearly independent $\forall n \in \mathbb{Z}^+$.

13. What is the determinant of the following matrix?

$$\begin{bmatrix} 4 & -3 & 0 \\ 2 & -1 & 2 \\ 1 & 5 & 7 \end{bmatrix}$$

14. Check whether the following matrix is invertible and find its matrix if it exists.

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 3 & 2 & 4 \\ 2 & 10 & 2 & 0 \end{bmatrix}$$

15. Check whether the following matrix is invertible and find its matrix if it exists.

$$\begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

16. Determine the result of the calculation below.

$$\begin{bmatrix} 2 & 0 \\ 1 & 9 \end{bmatrix} \times \begin{bmatrix} 3 & 9 \\ 4 & 7 \end{bmatrix}$$

17. Find the co-ordinates of the vector

$$[1, 2, 4]$$

(in the standard basis) with respect to the basis

$$\{[1, 4, 6], [0, 1, 4], [0, 0, 1]\}.$$

18. Let $\underline{u} = (2, 3)$ with respect to the basis $V = \{(1, 1), (1, -1)\}$. Following the transformation

$$T(x, y) = (y, x + y, x),$$

what are the coordinates of $\underline{u}$ with respect to the new basis $W = \{(1, 2, 0), (2, 1, 0), (0, 0, 1)\}$?

19. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix}$$

20. Find a diagonal matrix D and an invertible matrix P such that $P^{-1}BP = D$, where

$$B = \begin{bmatrix} 4 & 2 & 2 \\ -1 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

3 Sequences, Recurrences & Series

1. Determine the limit, if it exists, of the sequence:

$$a_n = \frac{4n^3 - 2n^2 + n - 2}{6n^2 - 3n + 2n^3}$$

2. Evaluate

$$\lim_{n \to \infty} \frac{n!}{n^n}$$

3. Use the fact that:

$$\Sigma \frac{1}{4n^2 - 1}$$

converges, as $n \to \infty$, to prove that

$$\Sigma \frac{1}{n^k}$$

converges $\forall k \in \mathbb{R}, k \geq 2$, as $n \to \infty$.

4. Determine if

$$\sum_{n=1}^{\infty} \left(\frac{n+2}{n^3 - n^2 + 1}\right)$$

converges.

5. Determine if

$$\sum_{n=1}^{\infty} \left(\frac{10^n}{n!}\right)$$

converges.

6. Suppose a_n and b_n are sequences such that $0 \leq a_n \leq b_n$. Is it true that if b_n converges, then a_n converges? If false, give a counter-example.

7. Suppose a sequence a_n converges. Is it true that the series $\sum a_n$ converges?

8. Determine if

$$\sum_{n=1}^{\infty} \left(\frac{\arctan(n)}{2^n + n^2}\right)$$

converges.

9. What is the general solution to the recurrence:

$$4x_n - 12x_{n-1} + 9x_{n-2} = 0$$

10. Find a particular solution to the recurrence:

$$a_n - a_{n-1} - 6a_{n-2} = 0$$

when $a_0 = 1$ and $a_1 = 8$.

11. Determine the interval of convergence of the power series:

$$\sum_{n=1}^{\infty} n^n x^n$$

12. Determine the interval and radius of convergence of the power series:

$$\sum_{n=0}^{\infty} (\sqrt{n}(x-1)^n)$$

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4 Calculus

1. Show that $3x^5 - 4x^2 = 3$ is solvable on the interval $[0, 2]$ without directly solving the equation.

2. Without solving, show that $3x^3 - 8x^2 + x = -3$ has a root between 0 and 1.

3. State the quotient rule, then using it, or otherwise, find

$$\frac{d}{dx}(\frac{x^2}{5x})$$

4. Differentiate the following expression with respect to x:

$$\sin(\cos x)$$

5. Find $\frac{\delta f}{\delta x}$ of the following expression:

$$f(x, y) = \sin\sqrt{\frac{y}{x^3}}$$

6. Find $\frac{dy}{dx}$ when:

$$x = \frac{at}{1-t^3}$$

and

$$y = \frac{at^2}{1-t^3}$$

7. Sketch the curve represented by the function:

$$y(x) = \frac{x^3}{x-1}$$

labelling all stationary points, asymptotes, limits and intersections with the axes.

8. Sketch the curve represented by the function:

$$f(x) = \frac{x^2 - 2x + 2}{x-1}$$

labelling all stationary points, asymptotes, limits and intersections with the axes.

9. Find:

$$\lim_{x \to 0} \left(\frac{\sin 7x}{\sin 4x}\right)$$

10. Use L'Hôpital's rule to determine:

$$\lim_{x \to 0} \left(\frac{\sin 2x + 7x^2 - 2x}{x^2(x+1)^2}\right)$$

11. Find $\frac{dy}{dx}$ where:

$$y \sin x + 5y = 0$$

12. State the differentiation with respect to x of the following functions:

   (a)
   $$\arcsin x$$

   (b)
   $$\arccos x$$

   (c)
   $$\arctan x$$

13. Find:

$$\int 3x^2 \, dx$$

14. Using integration by parts, or otherwise, find:

$$\int x^4 \ln x \, dx$$

15. Using integration by substitution, or otherwise, find:

$$\int (x+4)^5 \, dx$$

16. Find:

$$\int \left(\frac{2x+5}{x^2+5x-8} + \frac{1}{\sin x}\right) dx$$

17. Find:

$$\int \frac{x^2 + 2x + 2}{(x+1)^3}\, dx$$

18. Find:

$$\int_0^{3\pi} |\sin x|\, dx$$

19. Find the first three non-zero terms in the Taylor series expansion of $\sin^2 x$ around the point $x = \pi$.

20. Find the first three terms in the Taylor series expansion of $\ln(3 + 4x)$ around the point $x = 0$.

21. Solve the following separable differential equation:

$$x(y^2 - 1) - y(x^2 - 1)\frac{dy}{dx} = 0$$

22. Find y' in the expression:

$$7y^2 + \sin 3x = 12 - y^4$$

23. Solve the following second order differential equation:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 10 \sin x$$

with $y = 6$ and $y' = 5$ at $x = 0$.