$$z = c_{00} + c_{01} y + c_{02} y^2 + c_{03} y^3 + c_{10} x + c_{11} x y + c_{12} x y^2 + c_{13} x y^3 + c_{20} x^2 + c_{21} x^2 y + c_{30} x^3 + c_{31} x^3 y$$

we know 4 points : a(0,0) ; b(1,0) ; c(0,1) ; d(1,1)

Step 1 solving a 3rd degree polynomial function.

$$P(x) = a x^3 + b x^2 + c x + d$$
$$P'(x) = 3 a x^2 + 2 b x + c$$

with know x_0, x_0', x_1 and x_1'

So :

$$P(0) = a 0^3 + b 0^2 + c 0 + d = d = x_0$$
$$P'(0) = 3 a 0^2 + 2 b 0 + c = c = x_0'$$
$$P(1) = a 1^3 + b 1^2 + c 1 + d = a + b + c + d = x_1$$
$$P'(1) = 3 a 1^2 + 2 b 1 + c = 3 a + 2 b + c = x_1'$$

So :

$$c = x_0'$$
$$a + b + x_0' + x_0 = x_1$$
$$3 a + 2 b + x_0' = x_1'$$

$$b = x_1 - x_0' - x_0 - a$$

$$3 a + 2 (x_1 - x_0' - x_0 - a) + x_0' = x_1'$$
$$3 a + 2 x_1 - 2 x_0' - 2 x_0 - 2 a + x_0' = x_1'$$
$$a + 2 x_1 - x_0' - 2 x_0 = x_1'$$
$$a = x_1' - 2 x_1 + x_0' + 2 x_0$$

$$b = x_1 - x_0' - x_0 - (x_1' - 2 x_1 + x_0' + 2 x_0)$$
$$b = x_1 - x_0' - x_0 - x_1' + 2 x_1 - x_0' - 2 x_0$$
$$b = 3 x_1 - 2 x_0' - 3 x_0 - x_1'$$

Recap :

$$a = 2 x_0 + x_0' - 2 x_1 + x_1'$$
$$b = -3 x_0 - 2 x_0' + 3 x_1 - x_1'$$
$$c = x_0'$$
$$d = x_0$$

Verified in geogebra : OK

Step 2 : y=0 and x=0

For y=0 :

$$z=c_{00}+c_{01}0+c_{02}0^2+c_{03}0^3+c_{10}x+c_{11}x0+c_{12}x0^2+c_{13}x0^3+c_{20}x^2+c_{21}x^2 0+c_{30}x^3+c_{31}x^3 0$$

$$z=c_{30}x^3+c_{20}x^2+c_{10}x+c_{00}$$

So (using step 1 results):

$$c_{30}=2a+a'_x-2b+b'_x$$
$$c_{20}=-3a-2a'_x+3b-b'_x$$
$$c_{10}=a'_x$$
$$c_{00}=a$$

For x=0 :

$$z=c_{03}y^3+c_{02}y^2+c_{01}y+c_{00}$$

So :

$$c_{03}=2a+a'_y-2c+c'_y$$
$$c_{02}=-3a-2a'_y+3c-c'_y$$
$$c_{01}=a'_y$$
$$c_{00}=a$$

Recap :

$$z=c_{00}+c_{01}y+c_{02}y^2+c_{03}y^3+c_{10}x+c_{11}xy+c_{12}xy^2+c_{13}xy^3+c_{20}x^2+c_{21}x^2y+c_{30}x^3+c_{31}x^3y$$

Step 3 : y=1 and x=1

For y=1 :

$$z = c_{00} + c_{01} + c_{02} + c_{03} + c_{10}x + c_{11}x + c_{12}x + c_{13}x + c_{20}x^2 + c_{21}x^2 + c_{30}x^3 + c_{31}x^3$$

$$z'_x = c_{10} + c_{11} + c_{12} + c_{13} + 2c_{20}x + 2c_{21}x + 3c_{30}x^2 + 3c_{31}x^2$$

For x=0 :

$z = c_{00} + c_{01} + c_{02} + c_{03} = c$ ok with previous results.

$z'_x = c_{10} + c_{11} + c_{12} + c_{13} = c'_x$

For x=1 :

$$z = c_{00} + c_{01} + c_{02} + c_{03} + c_{10} + c_{11} + c_{12} + c_{13} + c_{20} + c_{21} + c_{30} + c_{31} = d$$

$$z = c + c'_x + c_{20} + c_{21} + c_{30} + c_{31} = d$$

$$z'_x = c_{10} + c_{11} + c_{12} + c_{13} + 2c_{20} + 2c_{21} + 3c_{30} + 3c_{31} = d'_x$$

$$z'_x = c'_x + 2c_{20} + 2c_{21} + 3c_{30} + 3c_{31} = d'_x$$

For x=1 :

$$z = c_{00} + c_{01}y + c_{02}y^2 + c_{03}y^3 + c_{10} + c_{11}y + c_{12}y^2 + c_{13}y^3 + c_{20} + c_{21}y + c_{30} + c_{31}y$$

$$z = c_{00} + c_{10} + c_{20} + c_{30} + c_{01}y + c_{11}y + c_{21}y + c_{31}y + c_{02}y^2 + c_{12}y^2 + c_{03}y^3 + c_{13}y^3$$

$$z'_y = c_{01} + c_{11} + c_{21} + c_{31} + 2c_{02}y + 2c_{12}y + 3c_{03}y^2 + 3c_{13}y^2$$

For y=0 :

$z = c_{00} + c_{10} + c_{20} + c_{30} = b$ ok with previous results.

$z'_y = c_{01} + c_{11} + c_{21} + c_{31} = b'_y$

For y=1 :

$$z = b + b'_y + c_{02} + c_{12} + c_{03} + c_{13} = d$$

$$z'_y = c_{01} + c_{11} + c_{21} + c_{31} + 2c_{02} + 2c_{12} + 3c_{03} + 3c_{13} = d'_y$$

$$z'_y = b'_y + 2c_{02} + 2c_{12} + 3c_{03} + 3c_{13} = d'_y$$

Step 4 : resolution

Finding c_{11} :

$z = b + b'_y + c_{02} + c_{12} + c_{03} + c_{13} = d$

$c_{12} + c_{13} = d - b - b'_y - c_{02} - c_{03}$

We plug this into $z'_x = c_{10} + c_{11} + c_{12} + c_{13} = c'_x$:

$c_{10} + c_{11} + d - b - b'_y - c_{02} - c_{03} = c'_x$

$c_{11} = c'_x - c_{10} - d + b + b'_y + c_{02} + c_{03}$

Finding c_{12} and c_{13} :

$c_{12} + c_{13} = d - b - b'_y - c_{02} - c_{03}$

$c_{13} = d - b - b'_y - c_{02} - c_{03} - c_{12}$

We plug this into $z'_y = b'_y + 2c_{02} + 2c_{12} + 3c_{03} + 3c_{13} = d'_y$:

$b'_y + 2c_{02} + 2c_{12} + 3c_{03} + 3(d - b - b'_y - c_{02} - c_{03} - c_{12}) = d'_y$

$b'_y + 2c_{02} + 2c_{12} + 3c_{03} + 3d - 3b - 3b'_y - 3c_{02} - 3c_{03} - 3c_{12} = d'_y$

$-c_{02} - c_{12} + 3d - 3b - 2b'_y = d'_y$

$c_{12} = -c_{02} - d'_y + 3d - 3b - 2b'_y$

Finding c_{21} and c_{31} :

$z = c + c'_x + c_{20} + c_{21} + c_{30} + c_{31} = d$

$c_{31} = d - c - c'_x - c_{20} - c_{30} - c_{21}$

We plug this into $z'_x = c'_x + 2c_{20} + 2c_{21} + 3c_{30} + 3c_{31} = d'_x$:

$c'_x + 2c_{20} + 2c_{21} + 3c_{30} + 3d - 3c - 3c'_x - 3c_{20} - 3c_{30} - 3c_{21} = d'_x$

$-c_{20} - c_{21} + 3d - 3c - 2c'_x = d'_x$

$c_{21} = -c_{20} - d'_x + 3d - 3c - 2c'_x$

Yeah !

Step 5 : Recap

$$c_{00}=a$$
$$c_{01}=a'_y$$
$$c_{02}=-3a-2a'_y+3c-c'_y$$
$$c_{03}=2a+a'_y-2c+c'_y$$
$$c_{10}=a'_x$$
$$c_{11}=b+b'_y+c+c'_x-a-a'_x-a'_y-d$$
$$c_{12}=-c_{02}-d'_y+3d-3b-2b'_y$$
$$c_{13}=b'_y-c_{03}+d'_y-2d+2b$$
$$c_{20}=-3a-2a'_x+3b-b'_x$$
$$c_{21}=-c_{20}-d'_x+3d-3c-2c'_x$$
$$c_{30}=2a+a'_x-2b+b'_x$$
$$c_{31}=c'_x-c_{30}+d'_x-2d+2c$$