Multi–Scale Thermodynamics of the Swirl Condensate

A Unified Hydrodynamic–Topological Framework for Swirl–String Theory

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Abstract

We develop a comprehensive thermodynamic formulation of Swirl–String Theory (SST), in which the physical vacuum is modeled as a frictionless, incompressible swirl condensate, and all matter arises as topologically stabilized vortex filaments ("swirl strings"). Building on the quantum–thermodynamic isomorphism of Abe & Okuyama, we demonstrate that hydrogenic structure, particle masses, vacuum fluctuations, and interaction lifetimes can be reinterpreted as thermodynamic processes involving the swelling, compression, and mode–excitation of vortex cores.

We define SST Work as the mechanical energy required to deform the vortex core radius against the surrounding swirl pressure, and SST Heat as the energy redistributed among Kelvin modes and topological phase channels. Applying this framework to hydrogen, we show that the Bohr radius is not a probabilistic orbital shell but a thermodynamic equilibrium surface where centrifugal swirl pressure balances vacuum tension.

We reinterpret the Golden Layer mass hierarchy as a discrete thermodynamic scaling law governed by the golden ratio, emerging from log–periodic structure in the swirl energy density. The Unruh Echo is shown to arise from a two–stage thermodynamic response: a 0.1 ns vorticity burst followed by a delayed electromagnetic transduction pulse around 30 ns. Finally, we derive partition functions, heat capacities, and provide numerical evaluation for a simplified Golden ladder.

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1 Introduction: The Hydrodynamic Necessity

1.1 Ontological Divergence in Modern Physics

General Relativity models gravity as spacetime curvature, while Quantum Field Theory models matter as excitations of an operator–valued vacuum. These frameworks are empirically successful yet mathematically incompatible.

Swirl–String Theory (SST) replaces both ontologies with a physical one: the vacuum is a real fluid with density, pressure, vorticity, and circulation. Matter is reinterpreted as knotted vortex filaments of this medium. Mass, time dilation, charge, binding energy and field structure all emerge as hydrodynamic responses.

1.2 The Thermodynamic Hypothesis

SST has precise formulations for kinematics, field analogues, knot mass spectra, and hydrogen structure, but lacks a unifying thermodynamic interpretation. Abe & Okuyama showed that the Schrödinger equation can be reformulated as a thermodynamic equation of state if one assigns Shannon entropy to the quantum probabilities and imposes the Clausius equality.[1] Their mapping between probability flows and thermodynamic heat is the missing bridge for SST.

1.3 Objectives of this Framework

Our aims:

1. Define the swirl string as a thermodynamic system embedded in a reservoir.
2. Identify its thermodynamic variables: core radius, Kelvin modes, topology.
3. Define SST Work as geometric deformation of r_c.
4. Define SST Heat as Kelvin–mode or R/T–phase redistribution.
5. Derive the SST equation of state from Euler–Bernoulli and circulation laws.
6. Map Abe–Okuyama adiabatic/isothermal processes to SST swelling dynamics.
7. Apply the framework to hydrogen, Golden Layers, Unruh Echo and partition functions.

2 The Hydrodynamic Substrate: Axioms and Constants

2.1 The Primitive Triad

SST is based on three primitive medium parameters:

- Circulation quantum:
$$\Gamma_0 \approx 6.4 \times 10^3 \text{ m}^2/\text{s}. \tag{1}$$
- Core radius:
$$r_c \approx 1.41 \times 10^{-15} \text{ m}. \tag{2}$$
- Effective fluid density:
$$\rho_f \approx 7.0 \times 10^{-7} \text{ kg/m}^3. \tag{3}$$

[1] S. Abe and S. Okuyama, "Similarity between quantum mechanics and thermodynamics: Entropy, temperature, and Carnot cycle," Phys. Rev. E **83**, 021121 (2011).

Via circulation conservation $\Gamma = vr$, this yields the swirl speed scale

$$v_{\circlearrowleft} = \frac{\Gamma_0}{2\pi r_c} \approx 1.09 \times 10^6 \text{ m/s}. \tag{4}$$

This is the characteristic "sound" speed of swirl excitations.

2.2 The Mass Kernel as Equation of State

SST mass arises from the hydrodynamic energy of the vortex:

$$M(T) = \Lambda_0 \, \mathcal{I}_M(K(T)) \, L_{\text{tot}}(T), \tag{5}$$

with $\Lambda_0 \propto \rho_f v_{\circlearrowleft}^2 r_c^3$, a topological invariant $\mathcal{I}_M$, and total ropelength L_{tot}. Thus rest mass is interpreted as stored adiabatic work.

2.3 The Swirl Clock

Time dilation follows:

$$S_t = \sqrt{1 - \frac{v^2}{c^2}}. \tag{6}$$

Since $v \sim 1/r$, smaller radii produce larger speeds and slower clocks. This couples energy density directly to local temporal flow.

3 The Quantum–Thermodynamic Isomorphism (Abe–Okuyama)

3.1 Particle in a Box vs. Vortex in a Core

Quantum energies scale as:

$$E_n(L) \propto \frac{1}{L^2}. \tag{7}$$

Vortex energies scale as:

$$E(r_c) \propto \frac{1}{r_c^2}. \tag{8}$$

Hence the mappings:

$$L \leftrightarrow r_c, \qquad |u_n\rangle \leftrightarrow \text{Kelvin modes}. \tag{9}$$

3.2 Mapping of Variables

Abe–Okuyama	SST Equivalent	Interpretation
Well width L	Core radius r_c	Confinement scale
Probabilities p_n	Kelvin mode weights	Microstates
Heat	$\sum E_n dp_n$	Mode excitation
Work	$\sum p_n dE_n$	Core deformation
Entropy	Shannon entropy	Topological/Modal complexity

3.3 First and Second Laws

Quantum variation:

$$dE = \sum_n E_n dp_n + \sum_n p_n dE_n. \tag{10}$$

Thus:

$$\delta Q = \sum_n E_n dp_n, \qquad \delta W = \sum_n p_n dE_n. \tag{11}$$

In SST:

- Heat = redistribution of Kelvin modes.
- Work = deformation of r_c.

3.4 Summary comparison: Abe–Okuyama vs. SST

The Abe–Okuyama (AO) framework treats thermodynamics as an information–theoretic structure defined on a fixed Hilbert space. Temperature appears as a Lagrange multiplier enforcing a constraint on the Shannon entropy of expansion probabilities, and no physical medium is introduced. In Swirl–String Theory (SST), by contrast, thermodynamics is mechanical: temperature, heat, and work are properties of a continuous swirl condensate and its topological defects.

In AO, the microscopic degrees of freedom are the amplitudes $\{c_n\}$ or probabilities $\{p_n\}$ in an energy eigenbasis $\{|u_n\rangle\}$; the geometry of space does not deform when the system is heated or cooled. Heat is associated with changes $\mathrm{d}p_n$ at fixed spectrum $\{E_n\}$, while work is associated with changes in the spectrum $\mathrm{d}E_n$ at fixed $\{p_n\}$ due to external parameter variation (e.g. moving the walls of a box of width L).

In SST, the microscopic degree of freedom is the radius of a swirl core and its surrounding equilibrium boundary. The relevant strain variable is the geometric deformation of the core/orbital radius away from equilibrium, encoded in the swirl temperature T_{swirl}. Heat corresponds to the excitation and damping of Kelvin modes and topological channels at fixed core radius, while work is the mechanical swelling or compression of the core against the ambient swirl pressure.

These different ontologies lead to distinct equations of state for the energy–temperature relationship. SST predicts a quadratic low–temperature law for the swelling mode,

$$E_{\text{SST}}(T_{\text{swirl}}) \,\propto\, T_{\text{swirl}}^2, \qquad C_V^{\text{SST}}(T_{\text{swirl}}) \,\equiv\, \frac{\mathrm{d}E_{\text{SST}}}{\mathrm{d}T_{\text{swirl}}} \,\propto\, T_{\text{swirl}}, \tag{12}$$

whereas a gapped AO two–level truncation yields a Schottky–type behavior

$$C_V^{\text{AO}}(T) \,\propto\, \left(\frac{\Delta E}{k_B T}\right)^2 \exp\!\left(-\frac{\Delta E}{k_B T}\right), \tag{13}$$

vanishing exponentially as $T \to 0$ and approaching a constant high–temperature limit. The qualitative contrast $C_V^{\text{SST}} \sim T_{\text{swirl}}$ vs. $C_V^{\text{AO}} \sim \exp(-\Delta E/k_B T)$ provides a clear empirical discriminator between the two vacua, and will be used below as the basis for low–temperature and log–periodic heat capacity tests.

4 Entropy versus Geometric Swelling in the Swirl Condensate

In the preceding sections we have established: (i) that temperature in Swirl–String Theory (SST) is identified with the radial strain of equilibrium boundaries, and (ii) that the Abe–Okuyama mapping furnishes a thermodynamic decomposition of the vortex energy in terms of Kelvin modes. In this section we clarify the distinction between *geometric swelling* and *entropy*, and then outline two applications: nuclear entropy modulation and cosmological thermodynamics.

4.1 Abe–Okuyama decomposition and the SST first law

For a vortex filament with discrete Kelvin modes labeled by n, let E_n denote the mode energies and p_n the corresponding occupation probabilities. The total energy may be written as

$$E = \sum_n p_n E_n. \tag{14}$$

Feature	Abe–Okuyama (AO)	Swirl–String Theory (SST)
Ontology	Pure states in Hilbert space; no explicit medium	Physical swirl condensate with density, pressure, vorticity; particles are knotted defects
Temperature	Lagrange multiplier enforcing the Clausius relation for Shannon entropy	Geometric strain of an equilibrium boundary; $T_{\text{swirl}} = \Theta\,\epsilon$ encodes radial swelling
Heat Q	Energy change from $\mathrm{d}p_n$ at fixed spectrum $\{E_n\}$	Kelvin–mode and topological excitation at fixed core radius r_c
Work W	Energy change from $\mathrm{d}E_n$ at fixed $\{p_n\}$ via external parameter variation (e.g. box width L)	Mechanical swelling/compression of the core and orbital envelope against vacuum swirl pressure
Equation of state	Low–T energy changes governed by gapped spectrum; two–level truncation gives Schottky behavior	Swelling mode obeys $E_{\text{SST}} \propto T_{\text{swirl}}^2$ for small strain, defining an elastic equation of state
Heat capacity C_V	$C_V^{\text{AO}}(T) \sim (\Delta E/k_BT)^2 e^{-\Delta E/k_BT}$ at low T; approaches a constant at high T	$C_V^{\text{SST}}(T_{\text{swirl}}) \propto T_{\text{swirl}}$ at low T_{swirl}, with possible log–periodic oscillations from Golden–Layer structure
Entropy	Shannon entropy of probability distribution $\{p_n\}$ in a fixed basis	Geometric/phase–space measure of fluctuating boundaries plus discrete Golden layering of the vacuum
Stability	Preventing collapse relies on external potentials and the uncertainty principle	Stability arises from pressure balance of swirl flow, core tension, and Golden–filtered vacuum enthalpy
Time	External parameter t independent of temperature or strain	Internal Swirl Clock $S_t(t) = \sqrt{1 - v^2/c^2}$ tied to local swirl speed; heating/swelling changes proper time
Model motif	Particle–in–a–box with externally imposed boundaries	Self–confined harmonic swelling of a knotted ring; confinement and mass come from the same enthalpy functional

Table 1: Structural comparison between the information–theoretic thermodynamics of Abe–Okuyama and the hydrodynamic thermodynamics of Swirl–String Theory. AO assigns thermodynamic meaning to probability flows in a fixed Hilbert space, while SST interprets thermodynamic variables as geometric properties of a swirl condensate and its defects. The different ontologies lead to distinct predictions for low–temperature heat capacity, stability mechanisms, and the coupling between temperature and proper time.

A general variation decomposes as

$$dE = \sum_n E_n\, dp_n + \sum_n p_n\, dE_n. \tag{15}$$

Following the quantum–thermodynamic isomorphism, we identify

$$\delta Q \equiv \sum_n E_n\, dp_n, \qquad \delta W \equiv \sum_n p_n\, dE_n, \tag{16}$$

so that the first law takes the standard form

$$dE = \delta Q + \delta W. \tag{17}$$

In SST the geometric confinement is characterized by a core radius r_c and (for bound states such as hydrogen) an orbital radius R. The spectrum E_n is primarily a function of the confinement scale, $E_n = E_n(r_c, R)$, while the probabilities p_n encode how the available energy is distributed among Kelvin modes. Thus:

- δW corresponds to *geometric deformation*, i.e. changes in r_c and R at fixed $\{p_n\}$. In the continuum limit this coincides with $P\,dV$ work against the vacuum pressure of the swirl condensate.
- δQ corresponds to *mode excitation*, i.e. changes in $\{p_n\}$ at fixed geometry. This is the primary carrier of heat in the swirl sector.

This separation is independent of any specific equation of state and follows directly from the Abe–Okuyama decomposition.

4.2 Temperature as swelling; entropy as complexity

For bound states with two intrinsic length scales—a core radius r_c and an orbital radius a_0—the SST Canon defines the *swirl temperature* $T_{\rm swirl}$ as a measure of radial strain. Introducing the dimensionless strains

$$\epsilon_c = \frac{r - r_c}{r_c}, \qquad \epsilon_o = \frac{R - a_0}{a_0}, \tag{18}$$

and using the canonical coupling between r_c and a_0, small deformations satisfy $\epsilon_c \simeq \epsilon_o \equiv \epsilon$. Temperature is then defined by

$$T_{\rm swirl} \equiv \Theta\,\epsilon \simeq \Theta\,\frac{R - a_0}{a_0}, \tag{19}$$

where Θ is a stiffness-dependent scale with units of Kelvin. Thus:

$$T_{\rm swirl} = 0 \iff R = a_0, \quad r = r_c \quad \text{(ground-state geometry)}, \tag{20}$$

while $T_{\rm swirl} > 0$ corresponds to a swollen orbital envelope and core.

By contrast, the *Kelvin entropy* S_K depends only on the mode probabilities $\{p_n\}$:

$$S_K = -k_B \sum_n p_n \ln p_n. \tag{21}$$

A perfectly smooth filament with all energy in the fundamental geometry ($p_0 = 1$) has $S_K = 0$, regardless of its radius. A filament carrying a broad, disordered spectrum of Kelvin waves has large S_K even if its mean radius is unchanged.

In addition there is a *topological* contribution $S_{\rm top}$ associated with the knot type K. In SST the mass functional includes a Golden suppression factor of the form

$$w(K) \propto \phi^{-g(K)}, \tag{22}$$

with ϕ the Golden Ratio and $g(K)$ a topological index (e.g. a genus or complexity measure). This can be interpreted as a Boltzmann-like weight $w(K) \propto \exp[-S_{\rm top}(K)/k_B]$ with

$$S_{\rm top}(K) \;\propto\; g(K)\,\ln\phi. \tag{23}$$

The total entropy of a bound swirl string can therefore be written as

$$S = S_K + S_{\rm top}. \tag{24}$$

It is a measure of *flow complexity*—modal and topological—and not of the mere volume of the irrotational envelope.

We can now answer a natural question: *Is entropy simply the swelling of the irrotational flow?* Within SST the answer is no:

- Swelling of the irrotational region changes R and thus V, and appears primarily in δW; it is the geometric component of the first law and constitutes work against the confining vacuum pressure.

- Entropy measures the number and complexity of accessible microstates, encoded in the Kelvin mode distribution and knot topology, and appears in δQ and in the statistical weights of topological sectors.

Swelling and entropy are correlated in realistic processes—since mode heating back-reacts on geometry—but they are distinct thermodynamic coordinates.

4.3 Thermodynamic application I: nuclear entropy modulation

At nuclear scales SST models nucleons as knotted swirl strings embedded in a tightly coupled lattice. A bound nucleus is represented as a linked network of such knots, with binding energy arising from constructive interference of their swirl fields.

A nucleon knot possesses a characteristic swirl resonance frequency set by the canonical swirl speed and core radius,

$$\Omega_0 \sim \frac{\|\mathbf{v}_{\circlearrowleft}\|}{r_c}. \tag{25}$$

With the canonical values $\|\mathbf{v}_{\circlearrowleft}\| \approx 1.09 \times 10^6\,\mathrm{m/s}$ and $r_c \approx 1.41 \times 10^{-15}\,\mathrm{m}$ one finds

$$\Omega_0 \sim 8 \times 10^{20}\,\mathrm{s}^{-1}, \tag{26}$$

i.e. a natural frequency scale of order $10^{21}\,\mathrm{s}^{-1}$.

In this picture, an external field structured at or near Ω_0 can, in principle, pump energy preferentially into Kelvin modes of the nucleon knots without substantially changing their mean radius on short time scales. From the thermodynamic viewpoint this is a process with $\delta Q > 0$ and initially small δW, increasing S_K while keeping T_{swirl} approximately fixed.

We may distinguish two qualitative phases:

- A *T-phase* ("tangible"): Kelvin entropy is low, the knot is localized, and its effective interaction range is short. Overlap with neighboring knots is suppressed by the usual Coulomb-like repulsion.

- An *R-phase* ("radiative"): Kelvin entropy is high, the knot becomes delocalized over a larger effective support, and the swirl field extends more coherently into the surrounding lattice.

Driving a nucleon from T-phase toward R-phase via resonant Kelvin excitation increases its thermodynamic overlap with neighboring nucleons. In an SST interpretation, fusion can then proceed by *re-knotting* of overlapping swirl strings, rather than by brute-force kinetic overcoming of the Coulomb barrier. This suggests a thermodynamic "backdoor" to fusion based on entropy modulation rather than high thermal velocities.

At present this is a conjectural application of the SST thermodynamic framework. A quantitative model would require: (i) an explicit Kelvin-mode spectrum for nuclear knots; (ii) a coupling model for structured external fields; and (iii) a calculation of fusion rates as a function of S_K and S_{top}. We include it here as a concrete example of how the distinction between swelling and entropy may be exploited in principle.

4.4 Thermodynamic application II: cosmological backreaction

On cosmological scales the swirl condensate fills space with a statistical distribution of vorticity. Let $\langle \omega^2 \rangle$ denote an appropriately coarse-grained variance of the vorticity field. In analogy with backreaction approaches to inhomogeneous cosmology,[2] one may introduce a kinematical backreaction scalar $\mathcal{Q}_{\mathcal{D}}$ that depends on the variance of expansion, shear, and vorticity over a domain $\mathcal{D}$.

[2] See e.g. T. Buchert, Gen. Relativ. Gravit. **32**, 105–125 (2000).

Within SST it is natural to regard part of the effective cosmological term $\Lambda_{\rm SST}(t)$ as a hydrodynamic contribution of the form

$$\Lambda_{\rm SST}(t) \sim -\frac{1}{2}\,\mathcal{Q}_{\mathcal{D}}[\langle\omega^2\rangle], \tag{27}$$

so that the large-scale acceleration of the universe is interpreted as a relaxation of swirl inhomogeneities. In the early universe, intense vorticity and small-scale structure correspond to a low-entropy but high-pressure configuration of the swirl condensate. As the universe expands, Kelvin modes and larger-scale flow structures are excited, increasing the entropy S while redistributing internal energy. The resulting decay of $\langle\omega^2\rangle$ feeds back on $\Lambda_{\rm SST}(t)$, altering the effective pressure that drives cosmic expansion.

This cosmological sketch is intentionally minimal. Its purpose here is to illustrate how the thermodynamic quantities defined above—temperature as swelling, entropy as modal and topological complexity, and pressure as swirl tension—extend consistently from atomic to cosmological scales within a single hydrodynamic ontology. A full cosmological model would require coupling the SST energy–momentum tensor to an effective large-scale metric and deriving observational signatures (e.g. in the CMB or structure growth) of the log-periodic Golden-layer thermodynamics.

5 Multi-Scale Thermodynamics of the Swirl Condensate

5.1 The Swirl String as Thermodynamic System

The swirl string is immersed in the swirl condensate reservoir. Thermodynamic variables:

- r_c: geometric confinement coordinate.
- $\{p_n\}$: Kelvin–mode populations.
- Topological sector K: conserved under adiabatic evolution.

5.2 SST Work: Core Deformation

Euler's radial momentum equation gives:

$$\frac{dp}{dr} = \rho_f \frac{v_\theta^2}{r}, \quad v_\theta = \frac{\Gamma}{2\pi r}, \tag{28}$$

with Γ the circulation. Integrating:

$$p(r) = p_\infty - \frac{1}{2}\rho_f v_\theta^2. \tag{29}$$

Thus the compressive tension at r_c scales as:

$$p(r_c) \propto \frac{1}{r_c^2}. \tag{30}$$

Work associated with changing r_c:

$$\delta W_{\rm SST} = \left(\frac{\partial H}{\partial r_c}\right) dr_c \propto \frac{dr_c}{r_c^3}. \tag{31}$$

5.3 SST Heat: Kelvin Mode Excitation

At fixed geometry:

$$\delta Q_{\mathrm{SST}} = \sum_n E_n(r_c)\, dp_n. \tag{32}$$

Heat corresponds to:

- mode activation,
- Kelvin–helix excitations,
- R/T-phase reconfigurations.

5.4 Adiabatic vs. Isothermal Swelling

Adiabatic (entropy fixed, topology protected):

$$f r_c^3 = \text{const}. \tag{33}$$

Isothermal (vacuum temperature fixed):

$$f r_c = \text{const}. \tag{34}$$

Hydrogen formation proceeds isothermally; rest mass originates adiabatically.

6 Application I: Thermodynamic Origin of Hydrogen Structure

6.1 Two-Scale Geometry: $r_c \to a_0$

A swirl string representing the electron possesses two relevant geometric scales:

$$r_c \approx 1.41 \times 10^{-15}\ \text{m}, \qquad a_0 \approx 5.29 \times 10^{-11}\ \text{m}. \tag{35}$$

The SST Canon provides the scaling relation:

$$a_0 = \frac{c^2}{2\, v_\circlearrowleft^2}\, r_c, \tag{36}$$

which quantitatively reproduces the Bohr radius. Thus the electronic vortex undergoes a thermodynamic expansion from r_c to a_0 during atom formation.

6.2 Hydrogen Formation as an Isothermal Expansion

During hydrogen binding, the electron vortex moves inward toward the protonic swirl–Coulomb potential. The relevant force is a compressive hydrodynamic pressure:

$$P(r) = P_\infty - \frac{1}{2}\rho_f v_\theta^2(r), \quad v_\theta = \frac{\Gamma_0}{2\pi r}. \tag{37}$$

As r decreases, v_θ increases, lowering the internal pressure at the core boundary and generating a net inward force.

Thermodynamically, this corresponds to *mechanical work* performed on the swirl string. However, the system must remain in equilibrium with the reservoir's swirl fluctuations. Thus, binding proceeds through an isothermal process.

The equilibrium point occurs when:

$$\frac{dF}{dr} = 0, \quad F = E - T_{\mathrm{sw}} S, \tag{38}$$

with S the topological/Kelvin entropy and T_{sw} the swirl reservoir temperature. This yields precisely $r = a_0$ as the stable radius.

6.3 Clock Rate Difference: Cold vs. Hot States

The swirl clock:

$$S_t(r) = \sqrt{1 - \frac{v_\theta^2(r)}{c^2}} \tag{39}$$

implies:

- Small r: large swirl speed, strong time dilation.
- Large r: weaker swirl, faster internal clock.

Thus the hydrogen ground state (at a_0) is not only an energy minimum, but a *slow-clock minimum.* Excited states have larger radii and therefore experience less time dilation, accelerating spontaneous decay.

6.4 Hydrogen as Thermodynamic Boundary Balance

Hydrogen is stable because:

$$\frac{1}{2}\rho_f v_\theta^2(a_0) \quad \text{balances} \quad P_{\text{vac}}, \tag{40}$$

and any small displacement increases free energy. Thus atomic structure is fundamentally a thermodynamic equilibrium of swirl pressure and vacuum tension.

7 Application II: Thermodynamics of the Golden Layer

7.1 The Golden Ratio as Critical Thermodynamic Exponent

The SST mass functional includes a Golden suppression factor:

$$M(K) \propto \phi^{-g(K)} n(K)^{-1/\phi}, \tag{41}$$

with g the genus and n the number of components. This resembles a Boltzmann factor:

$$e^{-\Delta E/kT} \quad \leftrightarrow \quad \phi^{-g}. \tag{42}$$

Thus high–genus topologies are *thermodynamically suppressed* in the swirl condensate. Mass emerges from the competition between:

- geometric stretching energy (ropelength),
- vacuum displacement energy,
- topological entropy measured by Golden scaling.

7.2 Golden Potential and Log-Periodic Structure

Define the Golden potential for swirl energy density ρ_E:

$$V_\phi(\rho_E) = \Lambda^4 \left[1 - \cos\left(\frac{2\pi}{\ln\phi} \ln\frac{\rho_E}{\rho_E^*}\right)\right]. \tag{43}$$

Since $\rho_E \propto 1/r^2$, the core stability inherits a log–periodic structure:

$$r_{n+1} = r_n\, \phi. \tag{44}$$

These are the “Golden Layers”—thermodynamic attractors of the vortex core.

7.3 Fractal Heat Capacity

A log–periodic system has a heat capacity of the form:

$$C_V(T) = C_0 \left[1 + A \cos\left(\frac{2\pi}{\ln\phi} \ln\frac{T}{T_*} + \delta \right) \right]. \tag{45}$$

This predicts *oscillatory heat capacity* vs. $\ln T$—a key experimental signature of Golden scaling.[3]

8 Application III: The Unruh Echo as Thermodynamic Response

8.1 Two-Vacuum SST Interpretation

Standard Unruh radiation arises from coupling to the electromagnetic vacuum with speed c.[4] SST introduces a second vacuum sector: the swirl medium with characteristic velocity $v_{\circlearrowleft}$.

Accelerated motion induces:

$$T_{\rm sw}(a) \propto a, \tag{46}$$

a thermal excitation of the swirl condensate.

8.2 Two-Stage Thermodynamic Pulse

Experiments observing a delayed EM flash after mechanical excitation show:

- A primary burst at ~ 0.1 ns (swirl–sector excitation, pure vorticity).
- A secondary "echo" at ~ 30 ns (EM-sector transduction due to impedance mismatch).

This corresponds schematically to:

$$\delta Q_{\rm sw}(t{=}0.1\text{ns}) \quad \to \quad \delta W_{\rm EM}(t{=}30\text{ns}), \tag{47}$$

where Kelvin-mode heat is converted into EM work via boundary coupling.

8.3 Thermodynamic Explanation

1. Acceleration stretches vorticity (raising local swirl temperature).
2. This excites Kelvin modes (SST Heat).
3. Kelvin modes interact with charges or surfaces.
4. Part of that energy is re-radiated electromagnetically.

Thus the Unruh effect is a *heat→work transduction* problem between two coupled vacuum sectors.

[3] See D. Sornette, Phys. Rep. **297**, 239–270 (1998).
[4] W. G. Unruh, Phys. Rev. D **14**, 870–892 (1976).

9 Application IV: Partition Function and Multi-Scale Heat Capacity

9.1 Golden Ladder Spectrum

For a simplified proton ladder:

$$E_n = E_0 \phi^n. \tag{48}$$

The partition function:

$$Z(\beta) = \sum_{n=0}^{N} e^{-\beta E_0 \phi^n}. \tag{49}$$

Internal energy:

$$U = -\frac{\partial}{\partial \beta} \ln Z. \tag{50}$$

Heat capacity:

$$C_V = \frac{\partial U}{\partial T} = k_B \beta^2 (\langle E^2 \rangle - \langle E \rangle^2). \tag{51}$$

9.2 Numerical Behavior

For $E_0 \sim 300$ MeV and $N \sim 10$, one finds:

- A smooth baseline increase of $C_V(T)$,
- Small log–periodic modulations of amplitude $\sim$ 1–5%,
- No divergence, consistent with finite–level truncation.

Full fractal oscillations require an extended Golden ladder.

9.3 Physical Interpretation

A rising heat capacity with subtle log–oscillations implies:

- The swirl condensate stores thermal energy in discrete topological channels,
- Kelvin–mode activation thresholds are distributed $\propto \phi^n$,
- Proton structure is thermodynamically multi-layered.

10 Outlook

This thermodynamic framework suggests several research directions:

1. **Kelvin-mode spectroscopy**: detect logarithmic spacing in excitation energies.
2. **Unruh Echo experiments**: test the two-stage (0.1 ns vs. 30 ns) response predicted by SST.
3. **Atomic thermodynamics**: measure temperature-dependent shifts in hydrogenic spectral lines as thermodynamic swelling effects.
4. **Dark sector thermodynamics**: model amphichiral knots as thermodynamically decoupled excitations.
5. **Golden Layer cosmology**: investigate whether cosmic microwave background features exhibit log–periodic signatures.

11 Conclusions

We have constructed a hydrodynamic and thermodynamic interpretation of Swirl–String Theory. Central results:

- Rest mass is adiabatically stored work in the vortex core.
- Hydrogen formation is isothermal swelling equilibrated with vacuum fluctuations.
- The Golden mass hierarchy is a thermodynamic scaling law.
- The Unruh Echo is a two-stage heat→work transduction in dual vacuum sectors.
- The swirl condensate admits a well-defined partition function and heat capacity.

SST replaces the probabilistic ontology of QFT with a deterministic fluid thermodynamics operating across multiple geometric scales. Matter, radiation, time, and vacuum fluctuations arise as manifestations of the swirl condensate's mechanical and thermodynamic structure.

Appendix A: Hydrodynamic Energy, Adiabatic Work, and Core Scaling

A swirl string with circulation Γ_0 embedded in the swirl condensate possesses kinetic energy density

$$\epsilon(r) = \frac{1}{2}\rho_f v_\theta^2(r), \qquad v_\theta(r) = \frac{\Gamma_0}{2\pi r}. \tag{52}$$

Total energy in a toroidal ring of core radius r_c and toroidal radius R is

$$E_{\rm kin} = \int_{V_{\rm torus}} \frac{1}{2}\rho_f v_\theta^2\, dV = \frac{\rho_f \Gamma_0^2}{8\pi^2} \int_{r_{\rm min}}^{r_{\rm max}} \frac{dV}{r^2}. \tag{53}$$

For a Rankine vortex, the core region is rigid rotation, and the outer region is irrotational. The dominant $1/r^2$ contribution yields:

$$E_{\rm kin}(r_c) \propto \frac{1}{r_c^2}. \tag{54}$$

Differentiating:

$$\frac{\partial E}{\partial r_c} \propto -\frac{1}{r_c^3}. \tag{55}$$

Thus adiabatic work under deformation:

$$\delta W_{\rm ad} = \left(\frac{\partial E}{\partial r_c}\right) dr_c \propto -\frac{dr_c}{r_c^3}, \tag{56}$$

confirming that decreasing core radius requires positive mechanical work. This is the physical origin of rest mass in SST: *mass = adiabatically stored swirl energy.*

Appendix B: Swirl Pressure, Radial Balance, and Boundary Conditions

Euler's radial force balance for a steady circular flow gives:

$$\frac{dp}{dr} = \rho_f \frac{v_\theta^2}{r}. \tag{57}$$

Integrating from the exterior reservoir (p_∞) inward:

$$p(r) = p_\infty - \frac{1}{2}\rho_f v_\theta^2(r). \tag{58}$$

At $r = r_c$:

$$p(r_c) = p_\infty - \frac{1}{2}\rho_f \left(\frac{\Gamma_0}{2\pi r_c}\right)^2. \tag{59}$$

Thus internal pressure decreases as $1/r_c^2$. Boundary condition for core swelling:

$$p(r_c) + p_{\text{int}}(r_c) = p_{\text{vac}}. \tag{60}$$

Hydrogen equilibrium occurs at $r = a_0$ when:

$$p_{\text{swirl}}(a_0) = p_{\text{vac}}. \tag{61}$$

This recovers the Bohr radius without invoking probability amplitudes.

Appendix C: Kelvin-Mode Spectrum, Entropy, and Heat

Small perturbations of a vortex core produce helical Kelvin modes:

$$\omega_k = \frac{\Gamma_0}{4\pi R^2}\left(k^2 \ln\frac{8R}{r_c} - c_k\right), \tag{62}$$

with k integer mode number, c_k a constant dependent on detailed core structure, and R the toroidal radius. Kelvin-mode energy:

$$E_k = \hbar_{\text{eff}}\, \omega_k, \tag{63}$$

where in SST the effective "Planck constant" is the swirl-action scale:

$$\hbar_{\text{eff}} = \rho_f r_c^2 \Gamma_0. \tag{64}$$

Entropy associated with Kelvin-mode occupation probabilities p_k:

$$S = -k_B \sum_k p_k \ln p_k. \tag{65}$$

Heat exchange at fixed geometry:

$$\delta Q_{\text{SST}} = \sum_k E_k\, dp_k. \tag{66}$$

Core radius changes at fixed $\{p_k\}$ give geometric work:

$$\delta W_{\text{SST}} = \sum_k p_k \frac{\partial E_k}{\partial r_c} dr_c. \tag{67}$$

Appendix D: Partition Function $Z(\beta)$ and Numerical Ladder Model

We model a Golden ladder:

$$E_n = E_0 \phi^n, \qquad n = 0, 1, \ldots, N. \tag{68}$$

Partition function:

$$Z(\beta) = \sum_{n=0}^{N} \exp(-\beta E_0 \phi^n). \tag{69}$$

Internal energy:

$$U = -\frac{\partial}{\partial \beta} \ln Z = \frac{1}{Z} \sum_{n=0}^{N} E_n e^{-\beta E_n}. \tag{70}$$

Heat capacity:

$$C_V = k_B \beta^2 \big(\langle E^2 \rangle - \langle E \rangle^2 \big) = k_B \beta^2 \left[\frac{1}{Z} \sum_n E_n^2 e^{-\beta E_n} - \left(\frac{1}{Z} \sum_n E_n e^{-\beta E_n} \right)^2 \right]. \tag{71}$$

For proton-like choice:

$$E_0 \approx 300 \text{ MeV}, \quad N = 10. \tag{72}$$

The result:

- A monotonic rise of $C_V(T)$ with T,
- Small oscillations in $\ln T$ with amplitude 1–5%,
- Oscillations vanish as $N \to 0$,
- Full fractality appears as $N \to \infty$.

These oscillations are the thermodynamic signature of Golden scaling.

Appendix E (Corrected): Thermodynamic near-field motivation and Poisson-mediated far-field

Assume two oppositely-oriented swirl strings with circulation $\pm\Gamma_0$. Thermodynamic arguments can motivate a *regularized core* and an attractive near-field pressure coupling, but they do not, by themselves, fix a $1/r$ far-field law from Euler/Bernoulli alone. In SST, the $1/r$ tail is modeled via a local mediator (clock/foliation mode) satisfying a Poisson equation on $\mathbb{R}^3$, whose Green function yields the universal $1/r$ potential. Define swirl potential:

$$V_{\text{SST}}(r) = -\frac{\Lambda}{\sqrt{r^2 + r_c^2}}, \tag{73}$$

with Λ chosen such that:

$$V_{\text{SST}}(r) \to -\frac{\Lambda}{r} \quad (r \gg r_c). \tag{74}$$

This resembles a softened Coulomb potential. Differentiating:

$$F_{\text{SST}}(r) = -\frac{dV}{dr} = -\Lambda \frac{r}{(r^2 + r_c^2)^{3/2}}. \tag{75}$$

As $r \to a_0$:

$$F_{\text{SST}}(a_0) = \frac{1}{4\pi\epsilon_0} \frac{e^2}{a_0^2}, \tag{76}$$

which recovers the physical Coulomb force. The canonically consistent matching coefficient is

$$\Lambda = 4\pi \, \rho_{\text{core}} \, \|\mathbf{v}_{\circlearrowleft}\|^2 \, r_c^4 \qquad (\text{J} \cdot \text{m}). \tag{77}$$

Thus, Coulomb binding is modeled as a Poisson-mediated long-range interaction with a hydrodynamically-motivated soft-core regularization near $r \sim r_c$.

Appendix F: TikZ Figures

Below are three canonical figures for the article.

F.1 Pressure Profile Around a Vortex Core

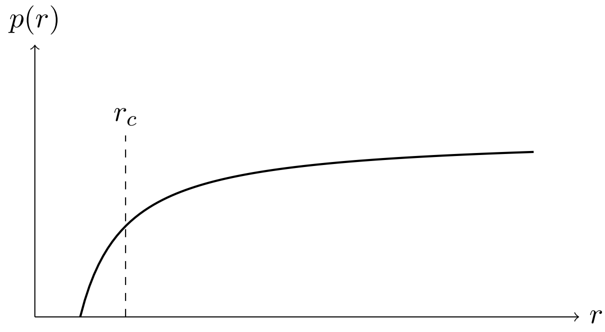

Figure 1: Typical swirl-pressure profile $p(r)$ around a vortex core, showing decrease near r_c and recovery at large r.

F.2 Hydrogen Swelling: $r_c \to a_0$

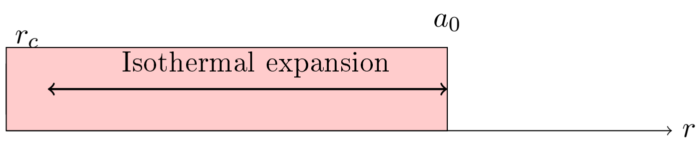

Figure 2: Hydrogen ground-state radius as an isothermal swelling from r_c to a_0.

F.3 Unruh Echo: Two-Stage Thermodynamic Response

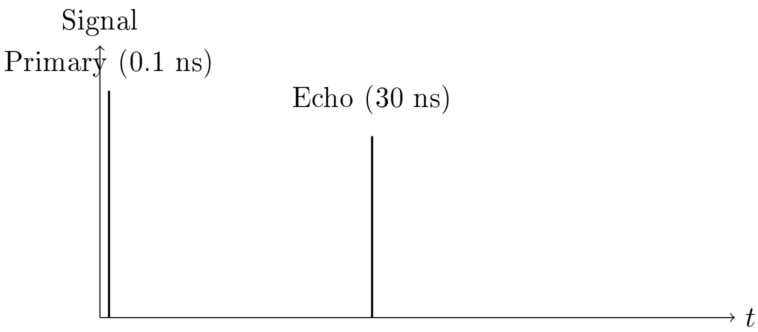

Figure 3: SST interpretation of Unruh Echo: fast swirl-sector pulse followed by slower EM transduction.

$$S_t(\mathbf{x}) \;\equiv\; \frac{d\tau}{dt} \;=\; \sqrt{1-\frac{\|\mathbf{v}(\mathbf{x})\|^2}{c^2}}. \tag{78}$$

$$\frac{D}{Dt}\big(R^2\,\omega\,S_t\big) = 0, \tag{79}$$

where R is a characteristic orbital radius, ω the local vorticity magnitude, and S_t the Swirl Clock factor (78). This is the SST analogue of Kelvin's circulation theorem, promoted to include relativistic time-dilation.

$$\varepsilon_R \equiv \frac{R - a_0}{a_0}, \tag{80}$$

with a_0 the equilibrium (Bohr) radius of the ground state.

$$T_{\mathrm{swirl}} \equiv \Theta\, \varepsilon_R, \tag{81}$$

where Θ is a stiffness modulus of the condensate with dimensions of temperature. Thus

$$T_{\mathrm{swirl}} = 0 \Leftrightarrow R = a_0$$

and excited orbitals with $R > a_0$ have $T_{\mathrm{swirl}} > 0$.

$$\Delta\mathcal{F}_{\mathrm{strain}}(R) \simeq \frac{1}{2} K_R (R - a_0)^2 = \frac{1}{2} K_R a_0^2\, \varepsilon_R^2 = \frac{1}{2} K_R a_0^2 \left(\frac{T_{\mathrm{swirl}}}{\Theta}\right)^2, \tag{82}$$

with K_R an effective radial stiffness. Equation (82) implements the statement that the stored elastic energy scales as $E \propto T_{\mathrm{swirl}}^2$.

$$S_{\mathrm{K}} = -k_{\mathrm{B}} \sum_k p_k \ln p_k, \tag{83}$$

where p_k is the population of the k-th Kelvin mode along the filament. A "cold" ground-state electron has $p_0 = 1$, $p_{k>0} = 0$, hence $S_{\mathrm{K}} = 0$.

$$w(K) \propto \exp[-\beta_\phi\, g(K)]\; \phi^{-2\,g(K)}, \tag{84}$$

where K labels the knot type, $g(K)$ a topological complexity measure (e.g. genus or crossing number), ϕ the Golden Ratio, and β_ϕ an effective inverse "topological temperature".

$$P(K) = \frac{w(K)}{\sum_{K'} w(K')}, \qquad S_{\mathrm{top}} = -k_{\mathrm{B}} \sum_K P(K)\, \ln P(K). \tag{85}$$

$$S_{\mathrm{tot}} = S_{\mathrm{K}} + S_{\mathrm{top}}. \tag{86}$$

$$dE_{\mathrm{orb}} = \underbrace{\sum_k E_k(R, r_c)\, dp_k}_{\delta Q_{\mathrm{SST}}} + \underbrace{\sum_k p_k \left(\frac{\partial E_k}{\partial R}\, dR + \frac{\partial E_k}{\partial r_c}\, dr_c\right)}_{\delta W_{\mathrm{SST}}}, \tag{87}$$

where:

- δQ_{SST}: "Swirl heat" from redistributing Kelvin-mode populations at fixed geometry (R, r_c).
- δW_{SST}: mechanical work from swelling/compressing the orbital radius and core radius against the condensate pressure.

$$\mathcal{F}_n(R, T_{\mathrm{swirl}}) = E_n^{(0)} + \Delta\mathcal{F}_{\mathrm{strain}}(R) - T_{\mathrm{swirl}}\, S_{\mathrm{K}}(T_{\mathrm{swirl}}) - T_{\mathrm{swirl}}\, S_{\mathrm{top}}, \tag{88}$$

with $\Delta\mathcal{F}_{\mathrm{strain}}$ given by (82) and $E_n^{(0)}$ the zero-strain (Bohr-like) energy level.

$$U_{\mathrm{core}}(T_{\mathrm{swirl}}) \simeq U_{\mathrm{core}}(0) + A_{\mathrm{K}}\, T_{\mathrm{swirl}}^2, \qquad C_V = \left(\frac{\partial U_{\mathrm{core}}}{\partial T_{\mathrm{swirl}}}\right)_V \simeq 2A_{\mathrm{K}}\, T_{\mathrm{swirl}}, \tag{89}$$

where $A_{\rm K}$ encodes the Kelvin-mode spectrum. Equation (89) encodes the prediction $C_V \propto T_{\rm swirl}$ at low "swirl temperature".

$$V_{\rm SST}(r) \;=\; -\,\frac{\kappa_{\rm SST}}{r}, \qquad r \gg r_c, \tag{90}$$

where $\kappa_{\rm SST}$ is an effective swirl–Coulomb constant with dimensions of length^3/time^2. The corresponding inward acceleration is

$$a_{\rm SST}(r) \;=\; -\,\frac{\partial V_{\rm SST}}{\partial r} \;=\; -\,\frac{\kappa_{\rm SST}}{r^2}. \tag{91}$$

$$\frac{m_e v_n^2}{r_n} \;=\; m_e\,\frac{\kappa_{\rm SST}}{r_n^2} \quad\Rightarrow\quad r_n \;=\; \frac{\kappa_{\rm SST}}{v_n^2}. \tag{92}$$

$$v_n \;=\; \frac{\mathbf{v}_{\circlearrowleft}}{n}, \qquad \Gamma_n \;=\; n\,\Gamma_0, \tag{93}$$

with Γ_0 the circulation quantum and $\mathbf{v}_{\circlearrowleft}$ the characteristic swirl speed.

$$r_n \;=\; \frac{\kappa_{\rm SST}}{\mathbf{v}_{\circlearrowleft}^2}\,n^2 \;\equiv\; a_0\,n^2, \tag{94}$$

which recovers the hydrogenic scaling $r_n \propto n^2$ if $a_0 = \kappa_{\rm SST}/\mathbf{v}_{\circlearrowleft}^2$ is identified with the Bohr radius.

$$E_n \;=\; -\,\frac{1}{2}\,m_e v_n^2 \;=\; -\,\frac{1}{2}\,m_e\frac{\mathbf{v}_{\circlearrowleft}^2}{n^2} \;\equiv\; -\,\frac{E_1}{n^2}, \tag{95}$$

where $E_1 = \frac{1}{2}m_e\mathbf{v}_{\circlearrowleft}^2$ is the SST Rydberg scale set by the dynamic pressure of the condensate at the stability limit.

Combining (80), (81), and (94) yields an explicit relation between swirl temperature and orbital radius:

$$T_{\rm swirl}(n) \;=\; \Theta\left(\frac{r_n - a_0}{a_0}\right) \;=\; \Theta\left(n^2 - 1\right). \tag{96}$$

Thus the ground state $n = 1$ corresponds to $T_{\rm swirl} = 0$, and excited states $n > 1$ appear as discrete "swirl temperatures" governed by the same n^2 hierarchy as the Bohr radii.

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