Unification of Relativistic Kinematics and Gravitational Dynamics from a Single Covariant Action

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Abstract

Special relativity, general relativity, and the universality of free fall are usually introduced as conceptually distinct principles: Lorentz invariance as a kinematical symmetry, Einstein gravity as a dynamical theory of spacetime, and the equivalence principle as an independent postulate. In this work we show that these elements are not logically independent.

We consider a minimal covariant action consisting of the Einstein–Hilbert term, a unit timelike vector field subject to a normalization constraint, and minimally coupled matter. Without assuming Lorentz invariance, geodesic motion, or the equivalence principle *a priori*, we demonstrate that all three arise as necessary consequences of diffeomorphism invariance and the variational structure of the theory.

We prove that local kinematics reduce to special relativity in the tangent space at every spacetime point, yielding the standard Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$. We further show that the field equations take the Einstein form with a covariantly conserved total stress–energy tensor, and that the universality of free fall follows directly from minimal coupling and the Bianchi identity. General relativity is recovered as a consistent sector of the theory, while controlled deviations are naturally parameterized by the additional timelike sector.

These results clarify the minimal structural conditions under which relativistic physics emerges and demonstrate that special-relativistic kinematics, Einstein gravity, and the weak equivalence principle can be unified within a single covariant framework, rather than postulated as independent foundations.

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1 Introduction

Special relativity, general relativity, and the universality of free fall are traditionally presented as logically distinct pillars of modern physics. Special relativity is introduced as a kinematical symmetry of spacetime, general relativity as a dynamical theory of the metric, and the equivalence principle as an independent postulate relating gravity to inertia. In this work we show that these elements are not independent.

We consider the minimal covariant extension of Einstein gravity obtained by augmenting the metric $g_{\mu\nu}$ with a unit timelike vector field u^μ subject to the normalization constraint $u^\mu u_\mu = -1$, coupled to matter only through the metric. Without assuming Lorentz invariance, geodesic motion, or the equivalence principle as axioms, we demonstrate that all three emerge as necessary consequences of a single variational principle. While related structures have appeared in modified gravity and preferred-frame theories, the present work emphasizes the logical unification of relativistic kinematics, gravitational dynamics, and free-fall universality within a single minimal variational framework. Related effective-field-theoretic approaches to Lorentz symmetry and gravity, including work by Kostelecký and collaborators, provide complementary perspectives on symmetry breaking and emergence at low energies.

Our main result can be stated as follows.

Theorem (Unification of Relativistic Principles). *Let $(\mathcal{M}, g_{\mu\nu})$ be a four-dimensional Lorentzian manifold with a diffeomorphism-invariant action consisting of the Einstein–Hilbert term, a covariant sector for a unit timelike field u^μ, and minimally coupled matter. Then:*

1. *Local kinematics reduce to special relativity in the tangent space of every spacetime point, yielding the standard Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$.*

2. *The gravitational field equations take the Einstein form $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, up to additional covariantly conserved stress contributions from u^μ.*

3. *The universality of free fall follows from diffeomorphism invariance and minimal coupling, implying that structureless matter follows metric geodesics.*

The derivation relies only on diffeomorphism invariance, the existence of a Lorentzian metric, the unit-timelike constraint, and minimal coupling of matter. No additional symmetry assumptions are imposed. The present work does not introduce a new phenomenological modification of gravity, but instead clarifies the minimal variational structure required to recover known relativistic physics.

Thus, special-relativistic kinematics, Einsteinian dynamics, and the weak equivalence principle arise from the same underlying structure and need not be postulated independently. The normalization of the timelike sector enforces local Lorentzian behavior, while the Bianchi identity ensures covariant conservation and geodesic motion of matter.

This result reframes relativity as a unified effective structure rather than a collection of separate assumptions. It clarifies the minimal conditions under which relativistic physics emerges and provides a natural framework for systematically classifying controlled deviations from general relativity that remain compatible with current observational constraints.

2 Emergence of Special Relativity (Equations Only)

2.1 Kinematic setup

Let $(\mathcal{M}, g_{\mu\nu})$ be a 4D Lorentzian manifold with signature $(-, +, +, +)$. Introduce a unit timelike field u^μ obeying

$$u^\mu u_\mu = -1. \tag{1}$$

Define the spatial projector orthogonal to u^μ:

$$h_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu, \qquad h_{\mu\nu}u^\nu = 0, \qquad h^\mu{}_\alpha h^\alpha{}_\nu = h^\mu{}_\nu. \tag{2}$$

2.2 Local inertial reduction

At any point $p \in \mathcal{M}$ choose Riemann normal coordinates such that

$$g_{\mu\nu}(p) = \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1), \qquad \partial_\alpha g_{\mu\nu}(p) = 0. \tag{3}$$

Choose a local frame so that

$$u^\mu(p) = (1, 0, 0, 0), \qquad u_\mu(p) = (-1, 0, 0, 0). \tag{4}$$

Then

$$h_{\mu\nu}(p) = \mathrm{diag}(0, 1, 1, 1). \tag{5}$$

2.3 Timelike worldlines and the Lorentz factor

Let $x^\mu(\lambda)$ be a timelike curve with tangent $\dot{x}^\mu \equiv dx^\mu/d\lambda$. Define proper time τ by

$$d\tau^2 = -\frac{1}{c^2}\, g_{\mu\nu}\, dx^\mu dx^\nu. \tag{6}$$

In the local inertial frame at p (with $x^0 = ct$) this becomes

$$d\tau^2 = dt^2 - \frac{1}{c^2}\, d\mathbf{x}^2 = dt^2 \left(1 - \frac{v^2}{c^2}\right), \qquad v^2 \equiv \left\| \frac{d\mathbf{x}}{dt} \right\|^2. \tag{7}$$

Hence

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}}, \qquad \gamma \equiv \frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{8}$$

2.4 Energy–momentum relations

Define the four-velocity and four-momentum:

$$U^\mu \equiv \frac{dx^\mu}{d\tau}, \qquad p^\mu \equiv m U^\mu. \tag{9}$$

Normalization gives

$$g_{\mu\nu} U^\mu U^\nu = -c^2, \qquad g_{\mu\nu} p^\mu p^\nu = -m^2 c^2. \tag{10}$$

In the local inertial frame,

$$U^\mu = \gamma(c, \mathbf{v}), \qquad p^\mu = (\gamma m c, \gamma m \mathbf{v}), \tag{11}$$

and the invariant yields

$$E^2 = (pc)^2 + (mc^2)^2, \qquad E \equiv p^0 c, \qquad p \equiv \|\mathbf{p}\|. \tag{12}$$

3 Single-Action Construction (SR Kinematics + GR Dynamics + Universality)

3.1 Action

$$S \;=\; S_g + S_u + S_m \;=\; \int d^4x\,\sqrt{-g}\left[\frac{1}{16\pi G}\,R \;+\; \mathcal{L}_u(g_{\mu\nu}, u^\mu, \nabla u)\right] \;+\; S_m[\psi,\, g_{\mu\nu}], \tag{13}$$

$$u^\mu u_\mu = -1, \tag{14}$$

$$\mathcal{L}_u = -c_1\,(\nabla_\mu u_\nu)(\nabla^\mu u^\nu) - c_2\,(\nabla_\mu u^\mu)^2 - c_3\,(\nabla_\mu u_\nu)(\nabla^\nu u^\mu) - c_4\,u^\mu u^\nu(\nabla_\mu u_\alpha)(\nabla_\nu u^\alpha) + \lambda\,(u^\mu u_\mu + 1). \tag{15}$$

The dimensionless coefficients c_i are assumed to lie within observationally allowed bounds consistent with gravitational-wave and post-Newtonian constraints [4].

Although the Lagrangian structure overlaps with Einstein–Æther and related Lorentz-violating gravity theories, the emphasis of the present work differs. Previous studies have primarily focused on phenomenological consequences or constraints associated with preferred-frame effects. Here, the central result is instead the logical unification of special-relativistic kinematics, Einsteinian dynamics, and the weak equivalence principle as necessary consequences of a single covariant variational framework.

3.2 Field equations

$$G_{\mu\nu} \;=\; 8\pi G\left(T^{(m)}_{\mu\nu} + T^{(u)}_{\mu\nu}\right), \qquad T^{(m)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}, \qquad T^{(u)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_u}{\delta g^{\mu\nu}}. \tag{16}$$

$$\frac{\delta S}{\delta u^\mu} = 0, \qquad \frac{\delta S}{\delta \lambda} = 0 \;\Rightarrow\; u^\mu u_\mu = -1. \tag{17}$$

Unlike previous treatments that emphasize phenomenological modifications or preferred-frame effects, the present analysis focuses on the logical interdependence of relativistic kinematics, gravitational dynamics, and free-fall universality.

3.3 Noether identity and covariant conservation

$$\nabla_\mu G^{\mu\nu} \equiv 0 \;\Rightarrow\; \nabla_\mu\left(T^{(m)\mu\nu} + T^{(u)\mu\nu}\right) = 0. \tag{18}$$

If the matter action is diffeomorphism invariant and minimally coupled,

$$\nabla_\mu T^{(m)\mu\nu} = 0 \quad\Rightarrow\quad \nabla_\mu T^{(u)\mu\nu} = 0. \tag{19}$$

3.4 Local SR limit

As shown in Sec. 2, the local inertial limit yields the standard special-relativistic kinematics. Consistency with the observed equality of gravitational-wave and electromagnetic signal speeds further constrains combinations of the coefficients c_i, notably requiring $c_1 + c_3 \approx 0$. Additional bounds arise from stability, causality, and post-Newtonian analyses (see, e.g., [4] and references therein), which collectively restrict the viable parameter space without affecting the structural results derived here.

3.5 Universality of free fall (geodesic limit from minimal coupling)

For a structureless point particle with minimal coupling,

$$S_{\mathrm{pp}} = -mc \int d\tau = -mc \int \sqrt{-g_{\mu\nu}\, dx^\mu dx^\nu}, \tag{20}$$

variation yields

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0. \tag{21}$$

Equivalently, for dust $T^{(m)}_{\mu\nu} = \rho\, U_\mu U_\nu$ with $U^\mu U_\mu = -c^2$,

$$\nabla_\mu T^{(m)\mu\nu} = 0 \;\Rightarrow\; U^\mu \nabla_\mu U^\nu = 0. \tag{22}$$

3.6 GR recovery as a sector

If u^μ is in a configuration such that

$$T^{(u)}_{\mu\nu} = 0, \tag{23}$$

then

$$G_{\mu\nu} = 8\pi G\, T^{(m)}_{\mu\nu}, \tag{24}$$

and in vacuum

$$G_{\mu\nu} = 0. \tag{25}$$

3.7 Weak-field (Newtonian) limit

Let

$$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right), \qquad g_{0i} = 0, \qquad g_{ij} = \delta_{ij}\left(1 - \frac{2\Phi}{c^2}\right), \qquad \left|\frac{\Phi}{c^2}\right| \ll 1, \tag{26}$$

and take nonrelativistic matter $T^{(m)}_{00} \approx \rho c^2$. Then to leading order

$$\nabla^2 \Phi = 4\pi G\, \rho \qquad (\text{when } T^{(u)}_{\mu\nu} \text{ contributes negligibly in this limit}). \tag{27}$$

4 Outlook and Discussion: Possible Departures from General Relativity

The construction presented in this work is intentionally conservative: by design it reproduces special-relativistic kinematics, Einstein gravity, and the universality of free fall in all experimentally tested regimes. Nevertheless, the framework admits controlled departures from general relativity, which become relevant only when additional dynamical structure in the timelike sector is activated. We briefly outline these possibilities without committing to a specific microscopic interpretation.

4.1 High-curvature and strong-field regimes

While general relativity is recovered whenever the stress–energy contribution of the timelike sector vanishes or remains subdominant, deviations may arise in regions of large curvature where gradients of the unit vector field become significant. In such regimes, the effective field equations take the form

$$G_{\mu\nu} = 8\pi G \left(T^{(m)}_{\mu\nu} + T^{(u)}_{\mu\nu}\right), \tag{28}$$

with $T^{(u)}_{\mu\nu}$ no longer negligible. Observable consequences may include small corrections to:

- the innermost stable circular orbit (ISCO) around compact objects,
- strong-field lensing and time-delay observables,
- quasi-normal mode spectra of black holes.

In particular, deviations in the quasi-normal mode spectrum or ISCO location provide natural observational targets, as these quantities are increasingly accessible through gravitational-wave observations and high-resolution astrophysical measurements. These effects are suppressed in weak fields and therefore compatible with solar-system and binary-pulsar tests.

4.2 Preferred-time effects beyond the local limit

Although local Lorentz invariance is recovered in the tangent space at each spacetime point, global effects associated with the timelike sector may appear on large scales or in rapidly evolving backgrounds. Such effects do not violate local special relativity, but may manifest as:

$$g_{\mu\nu}^{\text{eff}} = g_{\mu\nu} + \alpha\, u_\mu u_\nu, \tag{29}$$

with α dynamically suppressed in ordinary conditions. Potential observational windows include cosmological evolution, rotating spacetimes, or systems with large vorticity or acceleration.

4.3 Cosmological implications

In homogeneous and isotropic spacetimes, the timelike sector naturally aligns with the cosmological frame. Its stress contribution may therefore act as an effective fluid component in the Friedmann equations,

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_u\right), \tag{30}$$

where ρ_u depends on the dynamics of u^μ. This opens the possibility of explaining certain large-scale phenomena—such as early-universe dynamics or late-time acceleration—without modifying local relativistic physics.

4.4 Relation to microscopic completions

The analysis presented here is strictly effective-field-theoretic. It does not assume any particular microscopic origin for the additional timelike sector. The unit timelike field u^μ should therefore be understood as an effective low-energy degree of freedom encoding a preferred temporal structure, rather than as a fundamental violation of local Lorentz invariance.

Similar structures arise in several well-studied contexts, including Einstein–Æther theories, khronometric gravity, and analogue or emergent spacetime models, where a normalized timelike direction appears as a collective or coarse-grained variable. In the present work no specific ontological interpretation is assumed; the role of u^μ is to identify the minimal covariant structure required for the emergence of relativistic kinematics, gravitational dynamics, and universal free fall. This contrasts with the traditional approach where special relativity, the weak equivalence principle, and Einstein's field equations are postulated independently; here, all three emerge from a single covariant action through diffeomorphism invariance and minimal coupling.

However, the minimal structure identified in this work provides a clear target for possible ultraviolet completions: any such completion must reproduce the unit-timelike constraint, diffeomorphism invariance, and minimal coupling in the infrared in order to recover observed relativistic physics.

4.5 Summary

General relativity and special relativity emerge in this framework as robust low-energy structures, enforced by covariance and variational consistency. Departures from Einstein gravity are confined to regimes where additional dynamical stresses become relevant and are therefore both controlled and potentially testable. This separation between a universal relativistic core and well-defined deviation channels provides a systematic path for exploring extensions of general relativity without sacrificing its experimentally confirmed foundations. In this sense, the usual postulates of relativistic physics may be understood as emergent consequences of a single covariant variational structure, rather than as independent foundational assumptions.

References

[1] A. Einstein (1915), *Die Feldgleichungen der Gravitation*, Sitzungsberichte der Preussischen Akademie der Wissenschaften (Berlin), 844–847. (permalink: https://einsteinpapers.press.princeton.edu/)

[2] C. W. Misner, K. S. Thorne, J. A. Wheeler (1973), *Gravitation*, W. H. Freeman. (ISBN: 978-0716703440)

[3] T. Jacobson, D. Mattingly (2001), *Gravity with a dynamical preferred frame*, Phys. Rev. D **64**, 024028. DOI: 10.1103/PhysRevD.64.024028

[4] T. Jacobson (2008), *Einstein-Æther gravity: a status report*, PoS QG-PH, 020. DOI: 10.22323/1.043.0020

[5] B. P. Abbott *et al.* (LIGO/Virgo) (2017), *GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral*, Phys. Rev. Lett. **119**, 161101. DOI: 10.1103/PhysRevLett.119.161101

[6] B. P. Abbott *et al.* (LIGO/Virgo) (2017), *Multi-messenger Observations of a Binary Neutron Star Merger*, Astrophys. J. Lett. **848**, L12. DOI: 10.3847/2041-8213/aa91c9