

swirl speed magnitude

Suppression of Kelvin-Mode Thermodynamics in Atomic Orbitals: A Gapped Vortex-Filament Resolution in Swirl-String Theory

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Abstract

Swirl-String Theory (SST) models elementary particles as topologically stable vortex filaments embedded in an incompressible, inviscid condensate. In this framework, atomic orbitals arise as hydrodynamic equilibrium configurations rather than probabilistic wavefunctions. A natural concern is whether internal Kelvin-wave excitations of the electron filament would introduce thermodynamic corrections large enough to destabilize the precisely observed hydrogen spectrum. In this work, we show that naive Kelvin thermodynamics would indeed catastrophically distort atomic energy levels. We then demonstrate that a topologically induced excitation gap in the Kelvin spectrum—of order $\mathcal{O}(\text{keV})$ —naturally suppresses all Kelvin contributions at atomic energies. As a result, the Schrödinger equation emerges as an effective equation of state, while Kelvin dynamics remain frozen except under extreme acceleration or high-energy conditions. This resolves a key consistency constraint in SST and sharply delineates its low-energy and high-energy regimes.

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1 Introduction

Hydrodynamic and vortex-based models of matter have a long history, dating back to the vortex-atom theories of Kelvin and Helmholtz [1, 2]. Modern developments in superfluid dynamics and quantum turbulence have revived interest in vortex filaments as fundamental excitations [3, 4, 6].

Swirl–String Theory (SST) adopts this perspective at a fundamental level, positing that electrons correspond to knotted vortex filaments embedded in a real, incompressible condensate. In previous work, it has been shown that the hydrogenic energy spectrum emerges from hydrodynamic force balance, with the Schrödinger equation arising as a variational condition on a free-energy functional.

A potential objection to this picture is the existence of *Kelvin waves*—helical excitations of vortex filaments [5, 6]. If such modes thermally couple to orbital degrees of freedom, they could introduce large corrections to atomic energy levels. The purpose of this paper is to analyze this issue quantitatively and show how SST resolves it.

2 Kelvin Waves on Vortex Filaments

For a thin vortex filament of circulation Γ and core radius ξ , small-amplitude Kelvin waves obey the dispersion relation [5, 6]:

$$\omega(k) \simeq \frac{\Gamma}{4\pi} k^2 \left[\ln\left(\frac{1}{|k|\xi}\right) + C_0 \right], \quad (1)$$

where k is the wavenumber along the filament and C_0 is an $\mathcal{O}(1)$ constant determined by the core model.

For a closed filament of length L , the allowed modes satisfy

$$k_m = \frac{2\pi m}{L}, \quad m = 1, 2, \dots \quad (2)$$

so that

$$\omega_m \propto \frac{\Gamma m^2}{L^2}. \quad (3)$$

In SST, the orbital radius of the electron scales as

$$r_n = a_0 n^2, \quad (4)$$

implying a filament length

$$L_n \sim 2\pi a_0 n^2. \quad (5)$$

Thus Kelvin-mode frequencies soften rapidly with increasing n .

3 Thermodynamic Constraint from Atomic Spectroscopy

If Kelvin modes were thermally excited with an effective temperature T , the internal energy would scale generically as

$$U_{\text{Kelvin}} \sim \sum_m \hbar \omega_m f(\omega_m, T), \quad (6)$$

where f is a thermal occupation factor.

Modeling this contribution phenomenologically as a correction to orbital energies,

$$E_n^{\text{eff}} = E_n^{(0)} - a_n T^2 + \dots, \quad (7)$$

one finds that consistency with hydrogen spectroscopy requires

$$a_n \lesssim 10^{-62} \text{ J K}^{-2} \quad (8)$$

for low-lying states.

By contrast, a naive elastic estimate using SST core parameters yields coefficients exceeding this bound by more than twenty orders of magnitude. Therefore, Kelvin modes must be effectively inert in ordinary atomic states.

4 Gapped Kelvin Spectrum

We propose that the Kelvin spectrum of the electron filament is *topologically gapped*. Specifically, the lowest Kelvin excitation has energy Δ_K , with all higher modes satisfying

$$E_{m,n} \geq \Delta_K. \quad (9)$$

The Kelvin Hamiltonian for a given orbital n may be written as

$$H_K^{(n)} = \sum_m \left[(\Delta_K + \delta E_{m,n}) b_{mn}^\dagger b_{mn} + \frac{1}{2} (\Delta_K + \delta E_{m,n}) \right]. \quad (10)$$

This structure naturally arises in knotted filaments, where reconnection constraints, curvature, and torsion introduce discrete stability thresholds [7].

5 Low-Temperature Thermodynamics

The partition function for a single gapped bosonic mode is [8]:

$$Z = \frac{1}{1 - e^{-\beta \Delta_K}}, \quad (11)$$

with $\beta = (k_B T)^{-1}$. The internal energy is

$$U = \frac{\Delta_K}{e^{\beta \Delta_K} - 1}. \quad (12)$$

In the low-temperature limit $k_B T \ll \Delta_K$,

$$U \approx \Delta_K e^{-\Delta_K/(k_B T)}, \quad (13)$$

and both entropy and heat capacity are exponentially suppressed.

For a finite number of Kelvin modes, the total Kelvin contribution satisfies

$$U_K^{(n)}(T) \lesssim N_K \Delta_K \exp\left(-\frac{\Delta_K}{k_B T}\right). \quad (14)$$

This exponential suppression replaces the dangerous polynomial behavior found in the ungapped case.

6 Required Gap Scale

Requiring the effective Kelvin-induced coefficient to satisfy

$$a_n^{\text{eff}} \lesssim 10^{-62} \text{ J K}^{-2} \quad (15)$$

leads to the condition

$$\frac{\Delta_K}{k_B T_{\text{eff}}} \gtrsim 60. \quad (16)$$

Taking a conservative upper bound for the effective microphysical temperature seen by Kelvin modes,

$$T_{\text{eff}} \lesssim 10^5 \text{ K}, \quad (17)$$

yields

$$\Delta_K \gtrsim 5 \times 10^2 \text{ eV}. \quad (18)$$

Such a gap is small compared to the electron rest energy (511 keV) but enormous relative to atomic binding energies (~ 10 eV). Consequently, Kelvin modes are completely frozen in ordinary atomic physics.

7 Relation to the Schrödinger Equation

With Kelvin modes suppressed, the relevant free-energy functional reduces to

$$\mathcal{F}[\psi] = \int d^3r \left[\frac{\hbar^2}{2m_e} |\nabla\psi|^2 + V_{\text{SST}}(r) |\psi|^2 \right], \quad (19)$$

where $V_{\text{SST}}(r) \propto -1/r$ arises from hydrodynamic pressure gradients.

Variation under normalization yields

$$-\frac{\hbar^2}{2m_e} \nabla^2 \psi + V_{\text{SST}}(r) \psi = E \psi, \quad (20)$$

i.e. the stationary Schrödinger equation. Thus quantum mechanics appears as a low-energy, Kelvin-frozen limit of vortex-filament thermodynamics, consistent with the quantum–thermodynamic correspondence of Abe and Okuyama [9].

8 Discussion and Outlook

The existence of a Kelvin excitation gap resolves a central consistency requirement of Swirl–String Theory. Atomic orbitals remain sharply quantized because Kelvin modes are exponentially suppressed, while the internal structure of the electron filament becomes dynamically relevant only in extreme environments.

Potential regimes where Kelvin dynamics may activate include:

- ultra-high accelerations (Swirl–Unruh regimes) [10],
- keV–MeV scattering processes,
- astrophysical and cosmological vortex interactions.

Future work will explore whether the Kelvin gap can be derived directly from knot topology and whether its opening correlates with lepton generation structure.

Abstract

Swirl–String Theory (SST) models electrons as knotted vortex filaments embedded in an incompressible, inviscid condensate. Hydrogenic orbitals arise as hydrodynamic equilibrium configurations rather than probabilistic wavefunctions. A central concern in such a framework is whether internal Kelvin-wave excitations of the electron filament would introduce thermodynamic corrections large enough to destabilize the observed atomic spectrum. In this work, we show that naive Kelvin thermodynamics would catastrophically distort hydrogenic energy levels by many orders of magnitude. We then demonstrate that a topologically induced gap in the Kelvin spectrum naturally suppresses these contributions. A Kelvin excitation gap of order $\mathcal{O}(10^2\text{--}10^3 \text{ eV})$ is sufficient to render Kelvin modes thermodynamically inert at atomic energies. This establishes a clear separation of scales in SST: the Schrödinger equation emerges as a low-energy equation of state, while Kelvin dynamics activate only in extreme acceleration or high-energy regimes.

A Hydrodynamic Origin of the Schrödinger Equation

In SST, the electron is modeled as a closed, knotted vortex filament. At low energies, internal excitations are assumed frozen, and the relevant free-energy functional reduces to

$$\mathcal{F}[\psi] = \int d^3r \left[\frac{\hbar^2}{2m_e} |\nabla\psi|^2 + V_{\text{SST}}(r) |\psi|^2 \right], \quad (21)$$

with normalization $\int |\psi|^2 d^3r = 1$.

Here $V_{\text{SST}}(r)$ arises from the radial pressure deficit generated by swirl circulation and reduces asymptotically to a $-1/r$ potential. Variation of \mathcal{F} yields

$$-\frac{\hbar^2}{2m_e} \nabla^2 \psi + V_{\text{SST}}(r) \psi = E\psi, \quad (22)$$

i.e. the stationary Schrödinger equation. This realizes the quantum–thermodynamic correspondence identified by Abe and Okuyama [9] in a concrete hydrodynamic setting.

B Kelvin Waves on Vortex Filaments

Kelvin waves are helical perturbations propagating along vortex filaments. For a thin filament of circulation Γ and core radius ξ , their dispersion relation is [5, 6]

$$\omega(k) \simeq \frac{\Gamma}{4\pi} k^2 \left[\ln\left(\frac{1}{|k|\xi}\right) + C_0 \right]. \quad (23)$$

For a closed filament of length L_n , the allowed modes satisfy

$$k_m = \frac{2\pi m}{L_n}, \quad m = 1, 2, \dots \quad (24)$$

so that $\omega_m \propto \Gamma m^2 / L_n^2$.

In SST, the orbital radius scales as $r_n = a_0 n^2$, implying

$$L_n \sim 2\pi a_0 n^2. \quad (25)$$

Thus Kelvin-mode frequencies soften rapidly with increasing principal quantum number.

C Thermodynamic Catastrophe from Ungapped Kelvin Modes

If Kelvin modes couple thermodynamically to orbital degrees of freedom, their contribution to the internal energy may be parameterized as

$$U_{\text{Kelvin},n}(T_{\text{swirl}}) = a_n T_{\text{swirl}}^2 + \mathcal{O}(T_{\text{swirl}}^4), \quad (26)$$

leading to an effective level shift

$$E_n^{\text{eff}} = E_n^{(0)} - a_n T_{\text{swirl}}^2. \quad (27)$$

Using canonical SST constants, a naive elastic estimate yields

$$a_n^{\text{naive}} \sim 10^{-39} \text{ J/K}^2. \quad (28)$$

However, spectroscopy of hydrogen requires

$$a_n \lesssim 10^{-62} \text{ J/K}^2 \quad (29)$$

for low-lying states, implying a mismatch of more than twenty orders of magnitude. Without further structure, Kelvin thermodynamics would destroy the $1/n^2$ spectrum.

D Topologically Gapped Kelvin Spectrum

We therefore posit that the Kelvin spectrum of the electron filament is topologically gapped. Specifically, the lowest allowed Kelvin excitation lies at energy Δ_K , with all higher modes satisfying

$$E_{m,n} \geq \Delta_K. \quad (30)$$

The Kelvin Hamiltonian may be written as

$$H_K^{(n)} = \sum_m \left[(\Delta_K + \delta E_{m,n}) b_{mn}^\dagger b_{mn} + \frac{1}{2} (\Delta_K + \delta E_{m,n}) \right]. \quad (31)$$

Such gaps are natural in knotted filaments, where curvature, torsion, and topological constraints restrict admissible excitations [7].

E Low-Temperature Suppression of Kelvin Thermodynamics

For a bosonic mode of gap Δ_K , the partition function is [8]

$$Z = \frac{1}{1 - e^{-\beta \Delta_K}}, \quad \beta = (k_B T)^{-1}. \quad (32)$$

The internal energy is

$$U = \frac{\Delta_K}{e^{\beta \Delta_K} - 1} \approx \Delta_K e^{-\Delta_K/(k_B T)} \quad (k_B T \ll \Delta_K). \quad (33)$$

Thus all Kelvin thermodynamic quantities are exponentially suppressed. For a finite number of modes,

$$U_{\text{Kelvin}}^{(n)}(T) \lesssim N_K \Delta_K \exp\left(-\frac{\Delta_K}{k_B T}\right). \quad (34)$$

The effective quadratic coefficient a_n^{eff} inherits this exponential suppression.

F Required Gap Scale

Demanding $a_n^{\text{eff}} \lesssim 10^{-62} \text{ J/K}^2$ yields the condition

$$\frac{\Delta_K}{k_B T_{\text{eff}}} \gtrsim 60. \quad (35)$$

Taking a conservative upper bound $T_{\text{eff}} \sim 10^5 \text{ K}$ gives

$$\Delta_K \gtrsim 5 \times 10^2 \text{ eV}. \quad (36)$$

This scale is negligible compared to $m_e c^2 = 511 \text{ keV}$ but enormous relative to atomic binding energies, rendering Kelvin modes inert in ordinary atomic physics.

G Discussion

The existence of a Kelvin excitation gap resolves a central consistency constraint in SST. Atomic orbitals remain sharply quantized because Kelvin modes are thermodynamically frozen, while internal filament dynamics are reserved for extreme regimes such as strong acceleration or high-energy scattering.

This establishes a clear separation of scales:

- **Low-energy regime:** Schrödinger equation as an equation of state.
- **High-energy / high-acceleration regime:** Kelvin modes activate, enabling novel SST phenomenology.

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