# 1 Symmetry Classification of Knot-based Vortex Structures in the Vortex Æther Model (VAM)

*In VAM, these symmetries classify the invariance properties of knotted ætheric filaments, constraining their physical stability, energy quantization, and possible transformation pathways. Remarks.*
Any $D_{2k}$ symmetry ($k \geq 2$) implies $D_2(r)$ symmetry; if $k$ is even, period 2 is also present. $D_{2k}$ symmetry further implies $D_{2j}$ for divisors $j$ of $k$. $Z_{2k}$ symmetry entails positive amphichirality; $D_2(r)$ guarantees reversibility. $I_2$ symmetry implies negative amphichirality. These properties map directly to constraints on vortex knot energy spectra, fusion/interconversion rules, and topological charge conservation in VAM. The "full symmetry group" (FSG) is tabulated for comparison, though it may not capture all VAM-relevant invariances.

*Note.*
Knots such as $8_{10}$, $8_{16}$, $8_{17}$, and $8_{20}$, for which period 2 is absent, also uniquely have FSG $D_2$ among prime knots with 8 or fewer crossings, reflecting special restrictions on allowable vortex periodicities and energy levels in the æther. Exceptional knots $12a_{1202}$ and 15331 are included for their rare $Z_2$ symmetry, potentially corresponding to novel or unanticipated ætheric field states.

# 2 Glossary of Symmetry Table Symbols

$D_2(r)$ **Order-2 Dihedral (Reflectional) Symmetry.** The knot (or vortex structure) admits a dihedral symmetry of order 2, meaning it is invariant under a 180° rotation and a reflection; this often guarantees *reversibility* (the knot is equivalent to itself with reversed orientation).

$D_{2k}$ **Higher-Order Dihedral Symmetry.** The knot is invariant under the full dihedral group of order $2k$, i.e., all rotations by $2\pi/k$ and reflections. In VAM, this corresponds to invariance under both cyclic flows and chirality-reversing operations.

$Z_{2k}$ **Cyclic Symmetry of Order** $2k$**.** The knot admits rotational symmetry by $2\pi/(2k)$ (and its multiples), but not necessarily reflection symmetry. In VAM, such symmetry is associated with periodic vortex phase cycling and often positive amphichirality.

$I$ **Icosahedral Symmetry or Inversion.** $I$ often indicates additional point group symmetries (such as icosahedral, dodecahedral, or inversion symmetries), depending on the context. In tables, it may specify inversion axes or particular symmetry orders, e.g., $I_8$, $I_4$.

**reversible Reversible Knot (Vortex).** The knot is topologically equivalent to itself with the orientation reversed; in VAM, this reflects invariance under reversal of circulation or "vortex time".

**amphichiral Amphichiral (Mirror-Image) Symmetry.** The knot is equivalent to its mirror image:

- *Positive amphichirality* usually corresponds to cyclic ($Z_{2k}$) symmetry.
- *Negative amphichirality* is sometimes indicated by special inversion ($I_2$).

Table 1: **Known Symmetries of Prime Knots as VAM Vortex Structures.** This table catalogs the discrete symmetries of low-crossing-number prime knots, interpreted as possible stable knotted vortex configurations in the Vortex Æther Model (VAM). Columns show the principal symmetry groups ($D_2(r)$, $D_{2k}$, $Z_{2k}$, $I$), reversibility, amphichirality, allowed periods, and the full symmetry group (FSG).

| | $D_2(r)$ | $D_{2k}$ | $Z_{2k}$ | $I$ | reversible | amphichiral | periods | FSG |
|---|---|---|---|---|---|---|---|---|
| $3_1$ | ✓ | $D_4, D_6$ | | | ✓ | | 2, 3 | $Z_2$ |
| $4_1$ | ✓ | $D_4$ | $Z_4$ | $I_8$ | ✓ | ✓ | 2 | $D_8$ |
| $5_1$ | ✓ | $D_4, D_{10}$ | | | ✓ | | 2, 5 | $Z_2$ |
| $5_2, 6_1, 6_2$ | ✓ | $D_4$ | | | ✓ | | 2 | $D_4$ |
| $6_3$ | ✓ | $D_4$ | $Z_4$ | | ✓ | ✓ | 2 | $D_8$ |
| $7_1$ | ✓ | $D_4, D_{14}$ | | | ✓ | | 2, 7 | $Z_2$ |
| $7_2, 7_3$ | ✓ | $D_4$ | | | ✓ | | 2 | $D_4$ |
| $7_4$ | ✓ | $D_4$ | | | ✓ | | 2 | $D_8$ |
| $7_5, 7_6$ | ✓ | $D_4$ | | | ✓ | | 2 | $D_4$ |
| $7_7$ | ✓ | $D_4$ | | | ✓ | | 2 | $D_8$ |
| $8_1, 8_2$ | ✓ | $D_4$ | | | ✓ | | 2 | $D_4$ |
| $8_3$ | ✓ | $D_4$ | $Z_4$ | $I_8$ | ✓ | ✓ | 2 | $D_8$ |
| $8_4, 8_5, 8_6, 8_7, 8_8$ | ✓ | $D_4$ | | | ✓ | | 2 | $D_4$ |
| $8_9$ | ✓ | $D_4$ | | $I_4$ | ✓ | ✓ | 2 | $D_8$ |
| $8_{10}$ | ✓ | | | | ✓ | | none | $D_2$ |
| $8_{11}$ | ✓ | $D_4$ | | | ✓ | | 2 | $D_4$ |
| $8_{12}$ | ✓ | $D_4$ | $Z_4$ | | ✓ | ✓ | 2 | $D_8$ |
| $8_{13}, 8_{14}, 8_{15}$ | ✓ | $D_4$ | | | ✓ | | 2 | $D_4$ |
| $8_{16}$ | ✓ | | | | ✓ | | none | $D_2$ |
| $8_{17}$ | | | | | | ✓ | none | $D_2$ |
| $8_{18}$ | ✓ | $D_4, D_8$ | $Z_8$ | | ✓ | ✓ | 2, 4 | $D_{16}$ |
| $8_{19}$ | ✓ | $D_4, D_6, D_8$ | | | ✓ | | 2, 3, 4 | $Z_2$ |
| $8_{20}$ | ✓ | | | | ✓ | | none | $D_2$ |
| $8_{21}$ | ✓ | $D_4$ | | | ✓ | | 2 | $D_4$ |
| $12a_{1202}$ | ✓ | | $Z_2, Z_6$ | | ✓ | ✓ | | $D_{12}$ |
| 15331 | | | $Z_2$ | | | ✓ | | |

**periods Periods of Symmetry.** Lists the possible orders of cyclic symmetry—i.e., the integer $n$ for which the knot is invariant under a $2\pi/n$ rotation. In VAM, this relates to allowed quantum/vortex mode numbers.

**FSG Full Symmetry Group (FSG).** The maximal discrete symmetry group of the knot, encoding all rotational, reflectional, and inversion symmetries. In VAM, the FSG constrains the topological conservation laws and fusion/annihilation selection rules for knotted vortex states.