

# Golden Hyperbolic Swirl-Angle and Tangential Swirl Speed in Swirl-String Theory (SST)

Omar Iskandarani

December 21, 2025

## Abstract

This paper introduces a hyperbolic, kinematics-only construction used to define a distinguished “golden layer” for tangential swirl motion in Swirl-String Theory (SST). We adopt a hyperbolic definition of the golden ratio,  $\phi := e^{\text{asinh}(1/2)}$ , based on standard inverse-hyperbolic identities. [1] Defining the *golden hyperbolic swirl-angle*  $\xi_g := \frac{3}{2} \text{asinh}(1/2)$ , we prove the closed identity  $\tanh(\xi_g) = e^{-\text{asinh}(1/2)} = \phi^{-1}$  using standard hyperbolic-function relations. [2] We then define a dimensionless tangential swirl fraction  $\beta := \|\mathbf{v}\|/v_\circlearrowright \in [0, 1]$ , where  $v_\circlearrowright = \|\mathbf{v}_\circlearrowright\|$  is the SST canonical reference swirl speed. [6] A *hyperbolic swirl-angle*  $\xi$  is introduced purely as a bijective re-parameterization of  $\beta$  via  $\beta = \tanh \xi$ . [2] At the golden layer  $\xi = \xi_g$ , the tangential fraction is fixed to  $\beta_g = \phi^{-1}$ , implying  $\|\mathbf{v}\|_g = v_\circlearrowright/\phi$  and  $\Omega_g = v_\circlearrowright/(\phi r_c)$ . Numerical evaluation is given using the SST constants  $v_\circlearrowright = 1.093\,845\,63 \times 10^6 \text{ m s}^{-1}$  and  $r_c = 1.408\,970\,17 \times 10^{-15} \text{ m}$ . [6]

## 1 Introduction and motivation

Hyperbolic functions provide a natural coordinate system for any quantity constrained to the open interval  $[0, 1)$ , because  $\tanh : \mathbb{R} \rightarrow (-1, 1)$  is smooth, monotone, and saturating. [2] In many areas of mathematical physics, a bounded “fraction”  $\beta$  is therefore re-parameterized by a hyperbolic angle  $\xi$  such that  $\beta = \tanh \xi$ ; this is a coordinate choice, not a dynamical assumption. [2, 3]

In SST, a canonical tangential swirl-speed scale  $v_\circlearrowright$  and a core length scale  $r_c$  are treated as fundamental constants in the theory’s kinematic and energetic bookkeeping. [6] When discussing tangential swirl fractions  $\beta = \|\mathbf{v}\|/v_\circlearrowright$ , it is therefore useful to introduce a *hyperbolic swirl-angle*  $\xi$  as a compact, analytic parameterization of  $\beta$ .

This paper adds one further ingredient: a hyperbolic definition of the golden ratio and a corresponding *golden swirl-angle*  $\xi_g$  at which  $\tanh \xi_g = \phi^{-1}$ . Standard properties of  $\phi$  are classical and widely documented; here we emphasize a hyperbolic-first presentation. [4, 5]

## 2 Definitions and notation

### 2.1 SST kinematic quantities

Let  $\mathbf{v}_\circlearrowright$  denote the SST canonical characteristic swirl velocity vector, and define the reference speed

$$v_\circlearrowright \equiv \|\mathbf{v}_\circlearrowright\|. \quad (1)$$

Let  $\mathbf{v}$  denote a local tangential swirl velocity (e.g. associated with a swirl-string segment). Define the dimensionless tangential fraction

$$\boxed{\beta \equiv \frac{\|\mathbf{v}\|}{v_\circ} \in [0, 1].} \quad (2)$$

The interval restriction  $\beta < 1$  is definitional:  $v_\circ$  is used as the sector reference scale. [6]

## 2.2 Hyperbolic swirl-angle

We introduce a *hyperbolic swirl-angle*  $\xi$  by the bijection

$$\boxed{\beta = \tanh \xi, \quad \xi = \operatorname{artanh}(\beta).} \quad (3)$$

The functions  $\tanh$  and  $\operatorname{artanh}$  and their standard identities are classical. [2]

**Limit checks.** From standard expansions,  $\tanh \xi \sim \xi$  as  $\xi \rightarrow 0$ , so  $\beta \approx \xi$  for small tangential fractions. [2] As  $\beta \rightarrow 1^-$ , one has  $\xi = \operatorname{artanh}(\beta) \rightarrow +\infty$ , consistent with the open interval  $[0, 1)$ . [2]

## 3 Hyperbolic definition of the golden ratio

We adopt a hyperbolic-first definition:

$$\boxed{\phi \equiv e^{\operatorname{asinh}(0.5)}, \quad \operatorname{asinh}(0.5) \equiv \operatorname{asinh}\left(\frac{1}{2}\right).} \quad (4)$$

The defining inverse-hyperbolic identity used here is standard:  $\operatorname{asinh}(x) = \ln(x + \sqrt{x^2 + 1})$ . [1]

**Lemma 1** (Algebraic corollary). *The hyperbolic definition (4) implies the familiar algebraic relation*

$$\phi^2 = \phi + 1, \quad \text{hence} \quad \phi = \frac{1 + \sqrt{5}}{2}. \quad (5)$$

*Proof.* Let  $t := e^{\operatorname{asinh}(0.5)} > 0$ . Since  $\operatorname{asinh}(0.5) = \operatorname{asinh}(1/2)$ , we have  $\sinh(\operatorname{asinh}(0.5)) = 1/2$ . [1, 2] Using  $\sinh(\operatorname{asinh}(0.5)) = \frac{t-t^{-1}}{2}$ , [2] we obtain

$$\frac{t-t^{-1}}{2} = \frac{1}{2} \implies t - \frac{1}{t} = 1 \implies t^2 - t - 1 = 0 \implies t^2 = t + 1.$$

By definition  $\phi = t$ , so  $\phi^2 = \phi + 1$ . The quadratic solution gives  $\phi = (1 + \sqrt{5})/2$  since  $\phi > 0$ . Classical properties of  $\phi$  are standard. [4, 5]  $\square$

## 4 Golden hyperbolic swirl-angle identity

Define the *golden hyperbolic swirl-angle*

$$\boxed{\xi_g \equiv \frac{3}{2} \operatorname{asinh}(0.5) = \frac{3}{2} \operatorname{asinh}\left(\frac{1}{2}\right).} \quad (6)$$

**Theorem 1** (Golden swirl-angle implies golden tangential fraction). *With  $\phi = e^{(\operatorname{asinh}(1/2))}$  and  $\xi_g$  defined by (6), the following identity holds:*

$$\boxed{\tanh(\xi_g) = e^{-\operatorname{asinh}(0.5)} = \phi^{-1}.} \quad (7)$$

*Proof.* Let  $t := e^{\operatorname{asinh}(0.5)}$ . From Lemma (5) we have  $t^2 = t + 1$ . Using the standard identity  $\tanh y = \frac{e^{2y}-1}{e^{2y}+1}$ , [2] with  $y = \xi_g = \frac{3}{2}\operatorname{asinh}(0.5)$ ,

$$\tanh(\xi_g) = \frac{e^{3\operatorname{asinh}(0.5)} - 1}{e^{3\operatorname{asinh}(0.5)} + 1} = \frac{t^3 - 1}{t^3 + 1}.$$

Since  $t^2 = t + 1$ , we obtain  $t^3 = t(t + 1) = t^2 + t = 2t + 1$ . Therefore

$$\tanh(\xi_g) = \frac{(2t + 1) - 1}{(2t + 1) + 1} = \frac{2t}{2t + 2} = \frac{t}{t + 1} = \frac{t}{t^2} = \frac{1}{t} = e^{-\operatorname{asinh}(0.5)}.$$

Finally,  $\phi = e^{\operatorname{asinh}(0.5)}$  by (4), hence  $\tanh(\xi_g) = \phi^{-1}$ .  $\square$

## 5 SST golden layer for tangential swirl kinematics

Combining the SST kinematic mapping (3) with Theorem (7), the golden layer is defined by  $\xi = \xi_g$  and yields the fixed fraction

$$\boxed{\beta_g \equiv \tanh(\xi_g) = \phi^{-1}.} \quad (8)$$

Thus, for any SST sector employing the hyperbolic swirl-angle coordinate  $\xi$ , the golden layer implies the tangential speed scale

$$\boxed{\|\mathbf{v}\|_g = \beta_g v_{\odot} = \frac{v_{\odot}}{\phi}.} \quad (9)$$

If  $r_c$  denotes the SST core length scale, a corresponding angular-frequency scale can be introduced by dimensional normalization,

$$\boxed{\Omega \equiv \frac{v_{\odot}}{r_c}, \quad \Omega_g \equiv \frac{\|\mathbf{v}\|_g}{r_c} = \frac{1}{\phi} \frac{v_{\odot}}{r_c} = \frac{\Omega}{\phi}.} \quad (10)$$

This step is purely dimensional: it does not assert a specific dynamical equation for  $\Omega$ . The constants  $v_{\odot}$  and  $r_c$  are taken from SST canon. [6]

**Dimensional consistency.** Equation (9) has units of speed, and (10) has units of  $\text{s}^{-1}$ , since  $r_c$  has units of length. [6]

## 6 Numerical evaluation using SST constants

We evaluate  $\phi$ ,  $\xi_g$ , and the corresponding kinematic scales using

$$v_{\odot} = 1.093\,845\,63 \times 10^6 \text{ m s}^{-1}, \quad r_c = 1.408\,970\,17 \times 10^{-15} \text{ m},$$

as specified in SST canon. [6]

Using the hyperbolic definitions and standard functions, [1, 2] we obtain

$$\phi = e^{(\operatorname{asinh}(1/2))} \approx 1.618033988749895, \quad (11)$$

$$\operatorname{asinh}(0.5) = \operatorname{asinh}(1/2) \approx 0.4812118250596035, \quad (12)$$

$$\xi_g = \frac{3}{2} \operatorname{asinh}(0.5) \approx 0.7218177375894053, \quad (13)$$

$$\beta_g = \tanh(\xi_g) \approx 0.6180339887498948 = \phi^{-1}. \quad (14)$$

Therefore,

$$\|\mathbf{v}\|_g = \frac{v_\odot}{\phi} \approx 6.760337777855416 \times 10^5 \text{ m s}^{-1}, \quad (15)$$

$$\Omega = \frac{v_\odot}{r_c} \approx 7.763440655383073 \times 10^{20} \text{ s}^{-1}, \quad (16)$$

$$\Omega_g = \frac{\Omega}{\phi} \approx 4.798070194669498 \times 10^{20} \text{ s}^{-1}. \quad (17)$$

## 7 Interpretation and mainstream-facing remarks

### 7.1 What is “new” here

The hyperbolic function identities and the classical properties of  $\phi$  used above are not new. [1, 2, 4] The SST-specific contribution is the *interpretive packaging*:

- introduce  $\xi$  as a *hyperbolic swirl-angle* coordinating the SST tangential fraction  $\beta$ ,
- define the golden layer  $\xi_g$  and note that  $\beta_g = \phi^{-1}$  becomes an SST-internal, dimensionless marker,
- express the associated speed and frequency scales in terms of SST canonical constants  $v_\odot$  and  $r_c$ . [6]

### 7.2 Why use the hyperbolic coordinate at all

The mapping  $\beta = \tanh \xi$  has two practical advantages: (i) it automatically enforces  $\beta \in [0, 1)$  for all real  $\xi$ ; [2] (ii) it linearizes certain algebraic manipulations because exponentials appear directly in  $\tanh$  identities. [2] In SST, this makes it convenient to talk about “layers” in  $\xi$  rather than repeatedly carrying bounded fractions  $\beta$ .

### 7.3 Analogy (age 10)

Imagine a speed slider that can never go above 100%. The hyperbolic swirl-angle is like a special knob behind the slider: turning the knob by equal steps changes the slider smoothly, and there is one special knob position that always lands exactly at the same famous fraction (about 62%), the golden one.

## Acknowledgements

The numerical values used for  $v_\odot$  and  $r_c$  are taken from the SST canon. [6]

## References

- [1] F. W. J. Olver, A. B. Olde Daalhuis, D. W. Lozier, B. I. Schneider, R. F. Boisvert, C. W. Clark, B. R. Miller, and B. V. Saunders (eds.) (2010–present), *NIST Digital Library of Mathematical Functions*, National Institute of Standards and Technology. Section 4.37 “Inverse Hyperbolic Functions” (definition and properties of  $\text{asinh}$ ). Permalink: <https://dlmf.nist.gov/4.37>.
- [2] F. W. J. Olver, A. B. Olde Daalhuis, D. W. Lozier, B. I. Schneider, R. F. Boisvert, C. W. Clark, B. R. Miller, and B. V. Saunders (eds.) (2010–present), *NIST Digital Library of Mathematical Functions*, National Institute of Standards and Technology. Section 4.35 “Hyperbolic Functions” (identities for  $\tanh$ ,  $\sinh$ , and  $\text{artanh}$ ). Permalink: <https://dlmf.nist.gov/4.35>.
- [3] W. Rindler (2006), *Relativity: Special, General, and Cosmological*, 2nd ed., Oxford University Press, Oxford. (Reference for the mainstream use of hyperbolic-angle parameterizations of bounded velocity fractions in mathematical physics.)
- [4] T. Koshy (2001), *Fibonacci and Lucas Numbers with Applications*, Wiley-Interscience, New York. (Reference for classical identities of the golden ratio, including  $\phi^2 = \phi + 1$ .)
- [5] M. Livio (2002), *The Golden Ratio: The Story of Phi, the World’s Most Astonishing Number*, Broadway Books, New York. (General reference for historical and mathematical context of  $\phi$ .)
- [6] O. Iskandarani (2025), *Swirl String Theory (SST) Canon v0.6.0*, Zenodo. DOI: [10.5281/zenodo.17899592](https://doi.org/10.5281/zenodo.17899592).