Swirl String Theory (SST) Canon v0.4.1

Omar Iskandarani*

September 9, 2025

Abstract

This Canon is the single source of truth for Swirl String Theory (SST): definitions, constants, boxed master equations, and notational conventions. It consolidates core structure and promotes five results to canonical status:

- **I** Swirl Coulomb constant Λ and hydrogen soft-core
- **II** circulation–metric corollary (frame-dragging analogue)
- **III** corrected swirl time-rate (Swirl Clock) law
- **IV** Kelvin-compatible swirl Hamiltonian density
- **V** swirl pressure law (Euler corollary)

Core Axioms (SST)

1. **Swirl Medium:** Physics is formulated on $\mathbb{R}^3$ with absolute reference time. Dynamics occur in a frictionless, incompressible swirl condensate, which serves as a universal substrate (no material dispersion, Galilean symmetry for the medium).
2. **Swirl Strings (Circulation and Topology):** Particles and field quanta correspond to closed vortex filaments (swirl strings). Each such loop may be linked or knotted, and the circulation of the swirl velocity around any closed loop is quantized:
$$\Gamma = \oint \mathbf{v}_{\circlearrowleft} \cdot d\boldsymbol{\ell} = n\,\kappa, \qquad n \in \mathbb{Z}, \qquad \kappa = \frac{h}{m_{\text{eff}}}.$$
Equivalently, any surface spanning the loop carries an integer multiple of the circulation quantum κ. Discrete quantum numbers of an excitation (mass, charge, spin) track to the topological invariants of its swirl string (linking number, writhe, twist).
3. **String-induced gravitation:** Macroscopic attraction emerges from coherent swirl flows and swirl-pressure gradients. In the Newtonian limit, the effective gravitational coupling G_{swirl} is fixed by canonical constants so that $G_{\text{swirl}} \approx G_{\text{Newton}}$ (see Sec. 6).
4. **Swirl Clocks:** Local proper-time rate depends on tangential swirl speed. A clock comoving with swirl (tangential speed v) ticks slower by the factor $S_t = \sqrt{1 - v^2/c^2}$ relative to an outside observer.
5. **Dual Phases (Wave–Particle):** Each swirl string has two limiting phases: an extended R-phase (unknotted, delocalized circulation) exhibiting wave phenomena, and a localized T-phase (knotted soliton) carrying rest-mass. Quantum measurement corresponds to transitions $R \to T$ (collapse) or $T \to R$ (de-localization), mediated by swirl radiation.
6. **Taxonomy:** Unknotted excitations behave as bosonic string modes; chiral hyperbolic knots map to quarks; torus knots map to leptons. Linked composite knots correspond to bound states (nuclei, molecules). The detailed particle–knot dictionary is documented separately.

Hydrodynamic analogy only: no mechanical “æther” is assumed in the mainstream presentation.

Versioning Semantic versions: vMAJOR.MINOR.PATCH. This file: **v0.4.1**. Every paper/derivation must state the Canon version it depends on.

* Independent Researcher, Groningen, The Netherlands
Email: info@omariskandarani.com
ORCID: 0009-0006-1686-3961
DOI: 10.5281/zenodo.17052966

1 Classical Invariants and Swirl Quantization

Under Axiom 1 (inviscid, barotropic medium), the Euler equations hold and yield standard vortex invariants. In particular, Helmholtz/Kelvin circulation theorem ensures:

Kelvin's circulation theorem:

$$\frac{d\Gamma}{dt} = 0, \qquad \Gamma = \oint_{\mathcal{C}(t)} \mathbf{v}_\circlearrowleft \cdot d\boldsymbol{\ell}, \tag{1}$$

Vorticity transport:

$$\frac{\partial \boldsymbol{\omega}_\circlearrowleft}{\partial t} = \nabla \times (\mathbf{v}_\circlearrowleft \times \boldsymbol{\omega}_\circlearrowleft), \tag{2}$$

Helicity invariance:

$$h = \mathbf{v}_\circlearrowleft \cdot \boldsymbol{\omega}_\circlearrowleft, \quad H = \int h\, dV \text{ (constant; changes only by reconnections) [1].} \tag{3}$$

These underpin knotted swirl-string stability and reconnection energetics in SST.

Axiom 1 (Chronos–Kelvin Invariant) *For any thin, closed swirl loop (material core radius $R(t)$) in an inviscid medium, one has the material invariant*

$$\boxed{\frac{D}{Dt}\Big(R^2\,\omega\Big) = 0,} \qquad \textit{equivalently,} \qquad \boxed{\frac{D}{Dt}\Big(\frac{c}{r_c}\,R^2\sqrt{1-S_t^2}\Big) = 0\,,} \tag{4}$$

where $\omega = \|\boldsymbol{\omega}_\circlearrowleft\|$ on the loop and $S_t = \sqrt{1-(\omega r_c/c)^2}$ is the local Swirl Clock factor. This holds in the absence of reconnections or external swirl injection.

Derivation (Kelvin's theorem). Kelvin's theorem 1 implies $\frac{D}{Dt}\Gamma = 0$ for any material loop in a barotropic flowHelmholtz1858,Kelvin1869,Batchelor1967. For a nearly solid-body core, $\Gamma = 2\pi R\, v_t = 2\pi R^2\omega$, so $\frac{D}{Dt}(R^2\omega) = 0$. Using $v_t = \omega r_c$ and the definition of S_t (Axiom 4), we get $R^2\omega = \frac{c}{r_c}R^2\sqrt{1-S_t^2}$, yielding (4).

Dimensional check. $[R^2\omega] = \mathrm{m^2s^{-1}}$, and $[\frac{c}{r_c}R^2\sqrt{1-S_t^2}] = \mathrm{s^{-1}\cdot m^2 = m^2s^{-1}}$.

Clock–radius transport law (corollary). From $R^2\omega = \text{const}$, one finds

$$\frac{dS_t}{dt} \;=\; \frac{2(1-S_t^2)}{S_t}\,\frac{1}{R}\frac{dR}{dt}\,. \tag{5}$$

Thus expansion ($dR/dt > 0$) drives $S_t \to 1$ (local clocks speed up), while contraction slows local clocks ($S_t \downarrow$), preserving (4).

Potential-vorticity analogue. With a uniform background rotation $\Omega_{\rm bg}$ and column height H, the Ertel potential vorticity theorem gives the SST counterpartErtel1942,Batchelor1967

$$\frac{D}{Dt}\Big(\frac{\omega + \Omega_{\rm bg}}{H}\Big) = 0\,, \tag{6}$$

directly analogous to geophysical PV invariance.

Conditions. Inviscid, incompressible medium; barotropic swirl pressure; material loop without reconnection or external input; absolute time parameterization. These are the same assumptions under which Kelvin–Helmholtz invariants hold.

Limits. For weak swirl ($\omega r_c \ll c$): $S_t \approx 1 - \frac{1}{2}(\omega r_c/c)^2$ and (4) reduces to the classical invariant $R^2\omega = \text{const}$. In the core-on-axis limit ($v_t \to \mathbf{v}_\circlearrowleft$ on the symmetry axis), $S_t \to \sqrt{1-(\mathbf{v}_\circlearrowleft/c)^2}$, and (4) remains valid.

1.1 Swirl Quantization Principle

By Axiom 2, the circulation of $\mathbf{v}_\circlearrowleft$ around any closed loop is quantized in units of $\kappa = h/m_{\text{eff}}$. Closed swirl filaments may form nontrivial knots and links, each topological class corresponding to a discrete excitation state:

$$\mathcal{H}_{\text{swirl}} = \{\text{trefoil knot, figure-eight knot, Hopf link, } \ldots\}. \tag{7}$$

We refer to the joint discreteness of circulation and topology as the Swirl Quantization Principle:

$$\Gamma = n\kappa \qquad \text{and} \qquad \text{Topology}(K) \in \mathcal{H}_{\text{swirl}} \text{ (integer knot invariants).}$$

This underlies both the particle spectrum and the emergence of interactions in SST.

Quantum Mechanics (Copenhagen)	**Swirl String Theory (SST)**
Canonical quantization: $[x,p] = i\hbar$	Swirl quantization principle: $\Gamma = n\,\kappa, \quad n \in \mathbb{Z}$ $\mathcal{H}_{\text{swirl}} = \{\text{trefoil, figure-eight, Hopf link}, \ldots\}$
Discreteness from non-commuting operators	Discreteness from circulation integrals & knot topology
Particle = eigenstate of Hermitian H (wavefunction)	Particle = knotted swirl state with quantized Γ and topological invariants

2 Canonical Constants and Effective Densities

Primary SST Constants (SI units unless noted)

- **Swirl speed scale (core):** $\|\mathbf{v}_\circlearrowleft\| = 1.093\,85 \times 10^6$ m/s (tangential speed at $r = r_c$).
- **String core radius:** $r_c = 1.408\,97 \times 10^{-15}$ m.
- **Effective fluid density:** $\rho_f = 7.000\,00 \times 10^{-7}$ kg/m^3.
- **Mass-equivalent density:** $\rho_m = 3.893\,44 \times 10^{18}$ kg/m^3.
- **EM-like maximal force:** $F_{\text{EM}}^{\max} = 2.905\,35 \times 10^1$ N.
- **Gravitational maximal force (ref. scale):** $F_{\text{G}}^{\max} = 3.025\,63 \times 10^{43}$ N.
- **Golden ratio:** $\varphi = (1+\sqrt{5})/2 \approx 1.618\,03$.

Universal Constants

- $c = 299\,792\,000$ m/s, $\quad t_p = 5.391\,25 \times 10^{-44}$ s (Planck time).
- Fine-structure constant (identified): $\alpha \approx 7.297\,35 \times 10^{-3}$.

Effective densities. We use ρ_f (effective fluid density) to avoid confusion[1] with mass density; define

$$\rho_E \equiv \tfrac{1}{2}\,\rho_f\,\|\mathbf{v}_\circlearrowleft\|^2 \qquad \text{(swirl energy density)}, \qquad \rho_m \equiv \frac{\rho_E}{c^2} \qquad \text{(mass-equivalent density)}.$$

We also introduce the <u>swirl areal density</u> $\varrho_\circlearrowleft$ as the coarse-grained number of swirl cores per unit area. Its time variation enters Faraday's law as an effective source term:

$$\nabla \times \mathbf{E} = -\,\partial_t \mathbf{B} \;-\; \mathbf{b}_\circlearrowleft, \qquad \mathbf{b}_\circlearrowleft = G_\circlearrowleft\,\partial_t \varrho_\circlearrowleft\,.$$

Here $G_\circlearrowleft$ is the canonical swirl–EM transduction constant, identified with a flux quantum ($\Phi^* \sim h/2e$). This relation links electromotive force (voltage impulses) to reconnection dynamics, establishing a bridge between EM fields and gravity-like swirl fields.

3 Canon Governance and Status Taxonomy

Formal system. Let $\mathcal{S} = (\mathcal{P}, \mathcal{D}, \mathcal{R})$ denote the SST formal system: axioms $\mathcal{P}$, definitions $\mathcal{D}$, and admissible inference rules $\mathcal{R}$ (e.g. variational principles, Noether currents, dimensional analysis, asymptotic matching).

Canonical statement. A statement X is <u>canonical</u> iff X is a theorem or identity provable in $\mathcal{S}$:

$$\mathcal{P},\ \mathcal{D} \vdash_{\mathcal{R}} X\,,$$

and X is consistent with all previously accepted canonical items in the current major version.

Empirical statement. A statement Y is <u>empirical</u> iff it asserts a measured value, fit, or protocol:

$$Y \equiv \text{“observable } \mathcal{O} \text{ has value } \hat{o} \pm \delta o \text{ under procedure } \Pi.\text{”}$$

Empirical items calibrate symbols (e.g. $v_\circlearrowleft, r_c, \rho_f$) but are not used as premises in proofs.

Status Classes

- **Axiom/Postulate (Canonical).** Primitive assumption of SST (e.g. swirl medium, absolute time, swirl quantization).
- **Definition (Canonical).** Introduces a symbol by construction (e.g. swirl Coulomb constant Λ by surface integral).
- **Theorem/Corollary (Canonical).** Proven consequence (e.g. Euler–SST radial balance; Swirl Clock time dilation).
- **Constitutive Model (Canonical if derived; else Semi-empirical).** Relates fields/observables; canonical when deduced from $\mathcal{P}, \mathcal{D}$.
- **Calibration (Empirical).** Recommended numerical values (with uncertainties) for canonical symbols.
- **Research Track (Non-canonical).** Conjectures or alternatives pending proof or axiomatization.

[1] The canonical choice $\rho_f = 7.0 \times 10^{-7}\,\mathrm{kg/m^3}$ is a defined calibration constant, not a measured value. Its magnitude is anchored to the electromagnetic permeability scale $\mu_0/(4\pi) = 10^{-7}$ (SI), ensuring dimensional consistency between swirl energetics and EM normalization. Unlike the derived high-precision values of ρ_m and ρ_E, the effective fluid density ρ_f is fixed at this tidy scale as a reference baseline.

Canonicality Tests (all required)

1. **Derivability** from $\mathcal{P}, \mathcal{D}$ via $\mathcal{R}$.
2. **Dimensional consistency** (strict SI usage; correct physical limits).
3. **Symmetry compliance** (Galilean symmetry + absolute time, incompressibility).
4. **Recovery limits** (Newtonian gravity, Coulomb/Bohr, linear wave optics).
5. **Non-contradiction** with accepted canonical results.
6. **Parameter discipline** (no ad hoc fits or free parameters beyond calibrations).

Examples (from this Canon)

- *Canonical (Definition):* $\Lambda \equiv \int_{S_r^2} p_{\text{swirl}}(r)\, r^2\, d\Omega$.
- *Canonical (Theorem):* $\dfrac{1}{\rho_f}\dfrac{dp_{\text{swirl}}}{dr} = \dfrac{v_\theta(r)^2}{r}$ for steady azimuthal drift (Euler radial balance).
- *Empirical (Calibration):* $v_{\circlearrowleft} = 1.09384563 \times 10^6\ \mathrm{m/s}$ with protocol $f\Delta x$ (see Sec. 15).
- *Consistency Check (Not a premise):* Hydrogen soft-core reproduces a_0, E_1; this validates chosen constants but is a check, not an axiom.

4 What is Canonical in SST—and Why

[Axiom] Medium: inviscid continuum with absolute time, Euclidean space. $\nabla \cdot \mathbf{v}_{\circlearrowleft} = 0,\ \nu = 0$. This fixes the kinematic arena and allowed inference rules (Eulerian dynamics, Galilean relativity of spatial coordinates).

[Definition] Vorticity, circulation, helicity. $\boldsymbol{\omega}_{\circlearrowleft} = \nabla \times \mathbf{v}_{\circlearrowleft}$, $\Gamma = \oint \mathbf{v}_{\circlearrowleft} \cdot d\boldsymbol{\ell}$, $h = \mathbf{v}_{\circlearrowleft} \cdot \boldsymbol{\omega}_{\circlearrowleft}$, $H = \int h\, dV$. Classical constructs canonized as primary SST kinematic invariants.

[Theorem] Kelvin's circulation & vorticity transport (Helmholtz). For inviscid, barotropic flow:

$$\frac{d\Gamma}{dt} = 0, \qquad \frac{\partial \boldsymbol{\omega}_{\circlearrowleft}}{\partial t} = \nabla \times (\mathbf{v}_{\circlearrowleft} \times \boldsymbol{\omega}_{\circlearrowleft}), \qquad H \text{ constant up to reconnections.}$$

[Definition] Swirl Coulomb constant Λ.

$$\boxed{\Lambda \equiv \int_{S_r^2} p_{\text{swirl}}(r)\ r^2\, d\Omega}, \quad [\Lambda] = \mathrm{J\,m} = \mathrm{N\,m^2}\,.$$

In Canon v0.4 this evaluates to $\Lambda = 4\pi\, \rho_m\, v_{\circlearrowleft}^2\, r_c^4$ (for the fundamental swirl string).

[Theorem] Hydrogen soft-core potential & Coulomb limit.

$$V_{\text{SST}}(r) = -\frac{\Lambda}{\sqrt{r^2 + r_c^2}} \xrightarrow[r \gg r_c]{} -\frac{\Lambda}{r}\,.$$

This yields Bohr scalings

$$a_0 = \frac{\hbar^2}{\mu\,\Lambda}, \qquad E_n = -\frac{\mu\,\Lambda^2}{2\,\hbar^2 n^2}\,,$$

correctly reproducing the hydrogen atom (μ reduced mass).

[Theorem] Euler–SST radial balance (swirl pressure law). For a steady, purely azimuthal drift $v_\theta(r)$ with $\partial_t = 0$:

$$0 = -\frac{1}{\rho_f}\frac{dp_{\rm swirl}}{dr} + \frac{v_\theta^2}{r} \quad \Rightarrow \quad \boxed{\frac{1}{\rho_f}\frac{dp_{\rm swirl}}{dr} = \frac{v_\theta(r)^2}{r}}.$$

For asymptotically flat rotation ($v_\theta \to v_0$ as $r \to \infty$): $p_{\rm swirl}(r) = p_0 + \rho_f v_0^2 \ln(r/r_0)$, an outward-rising pressure that provides the centripetal force for the flat curve.

[Definition → Corollary] Swirl analogue metric and time dilation. In cylindrical coordinates (t, r, θ, z) with $v_\theta(r)$:

$$ds^2 = -\Big(c^2 - v_\theta^2(r)\Big)dt^2 + 2\, v_\theta(r)\, r\, d\theta\, dt + dr^2 + r^2 d\theta^2 + dz^2\,.$$

Co-rotating ($d\theta' = d\theta - \frac{v_\theta}{rc^2}dt$) yields $ds^2 = -\,c^2\Big(1 - v_\theta^2/c^2\Big)dt^2 + \cdots$, giving the Swirl Clock factor

$$\frac{dt_{\rm local}}{dt_\infty} = \sqrt{1 - \frac{v_\theta^2}{c^2}}\,.$$

[Definition] SST Hamiltonian density (Kelvin-compatible).

$$\mathcal{H}_{\rm SST} = \tfrac{1}{2}\rho_f\, \|\mathbf{v}_\circlearrowleft\|^2 + \tfrac{1}{2}\rho_f\, r_c^2 \|\boldsymbol{\omega}_\circlearrowleft\|^2 + \lambda(\nabla\cdot\mathbf{v}_\circlearrowleft)\,.$$

Empirical Calibrations (not premises, but binding numeric values)

- [Empirical] $v_\circlearrowleft = 1.09384563 \times 10^6\,\mathrm{m/s}$ (core swirl speed).
- [Empirical] $r_c = 1.40897017 \times 10^{-15}\,\mathrm{m}$ (string core radius).
- [Empirical] $\rho_m = 3.8934358266918687 \times 10^{18}\,\mathrm{kg/m^3}$ (mass-equiv. density).

Non-Canonical (Research Track)

Unproven extensions—e.g. blackbody swirl temperature, electroweak swirl couplings—remain conjectural until derived under $\mathcal{S}$.

Consistency & Dimension Checks

$$[\Lambda] = [\rho_m][v_\circlearrowleft^2][r_c^4] = \frac{\mathrm{kg}}{\mathrm{m}^3}\cdot\frac{\mathrm{m}^2}{\mathrm{s}^2}\cdot\mathrm{m}^4 = \frac{\mathrm{kg\,m}^3}{\mathrm{s}^2} = \mathrm{J\,m}\,.$$

Soft-core Coulomb limit: $V_{\rm SST}(r) \to -\Lambda/r$ as $r/r_c \to \infty$ (recovering Coulomb law).

5 Coarse-Graining Strings: Derivation of ρ_f

Setup. The medium is modeled as an incompressible condensate populated by thin swirl strings. We derive the bulk effective density ρ_f via coarse–graining of line-supported mass and vorticity, using Euler kinematics and Kelvin invariants.

Line parameters

For a representative straight vortex string (locally solid-body core):

$$\text{(D1)}\quad \mu_* := \rho_m\,\pi r_c^2 \quad [\mathrm{kg/m}] \qquad \text{(line mass per length)}, \tag{8}$$

$$\text{(D2)}\quad \Gamma_* := \oint \mathbf{v}_\circlearrowleft \cdot d\boldsymbol{\ell} \;\approx\; 2\pi\, r_c\, v_\circlearrowleft \qquad \text{(circulation quantum per string)}. \tag{9}$$

Let $\nu = N_{\mathrm{str}}/A$ (strings per unit area). Then:

$$\text{(C1)}\quad \rho_f = \mu_*\,\nu, \tag{10}$$

$$\text{(C2)}\quad \langle \boldsymbol{\omega}_\circlearrowleft \rangle = \Gamma_*\,\nu\,\hat{\mathbf{t}}_{\mathrm{avg}} \quad \Rightarrow \quad |\langle \omega_\circlearrowleft \rangle| = \Gamma_*\,\nu\,, \tag{11}$$

where $\hat{\mathbf{t}}_{\mathrm{avg}}$ is the average unit tangent of string orientations.

First–Principles Derivation

Combining (C1)–(C2):

$$\boxed{\rho_f = \mu_* \frac{\langle \omega_\circlearrowleft \rangle}{\Gamma_*} = \frac{\rho_m\,\pi r_c^2}{2\pi\, r_c\, v_\circlearrowleft}\,\langle \omega_\circlearrowleft \rangle = \frac{\rho_m\, r_c}{2\, v_\circlearrowleft}\,\langle \omega_\circlearrowleft \rangle}\,, \tag{12}$$

using $\Gamma_* = 2\pi r_c v_\circlearrowleft$. For uniform solid rotation with angular speed Ω, $\langle \omega_\circlearrowleft \rangle = 2\,\Omega$. Then

$$\boxed{\rho_f = \frac{\rho_m\, r_c}{v_\circlearrowleft}\,\Omega}\,, \tag{13}$$

giving the effective bulk density in terms of a typical angular velocity Ω of the swirl-string ensemble.

Energy and tension scales.

$$\boxed{u_{\mathrm{swirl}} = \tfrac{1}{2}\,\rho_f\, v_\circlearrowleft^2}, \qquad \boxed{T_* = \tfrac{1}{2}\,\mu_*\, v_\circlearrowleft^2}\,,$$

i.e. the swirl energy density and single-string tension (both on the core).

Numerical Calibration (Canon constants)

With $\rho_m = 3.8934358266918687 \times 10^{18}\ \mathrm{kg/m^3}$, $r_c = 1.40897017 \times 10^{-15}\ \mathrm{m}$, $v_\circlearrowleft = 1.09384563 \times 10^6\ \mathrm{m/s}$, one finds

$$\Gamma_* = 2\pi r_c v_\circlearrowleft = 9.68361920 \times 10^{-9}\ \mathrm{m^2/s}, \qquad T_* = 1.45267535 \times 10^1\ \mathrm{N}\,.$$

From (13):

$$\rho_f = (5.01509060 \times 10^{-3})\,\Omega\,,$$

so the baseline $\rho_f = 7.0 \times 10^{-7}\ \mathrm{kg/m^3}$ occurs at

$$\boxed{\Omega_* = 1.39578735 \times 10^{-4}\ \mathrm{s^{-1}}\ \text{(period } \approx 12.5\ \mathrm{h})}\,.$$

6 Master Equations (Boxed Canonical Relations)

Energy and Mass (Bulk)

$$\boxed{E_{\mathrm{SST}}(V) = \frac{4}{\alpha\,\varphi}\left(\frac{1}{2}\,\rho_f\, v_\circlearrowleft^2\right) V} \quad [\mathrm{J}], \qquad \boxed{M_{\mathrm{SST}}(V) = \frac{E_{\mathrm{SST}}(V)}{c^2}} \quad [\mathrm{kg}]\,.$$

Numeric per unit volume: $\frac{1}{2}\rho_f v_\circlearrowleft^2 \approx 4.1877439 \times 10^5\ \mathrm{J/m^3}$, $\frac{4}{\alpha\varphi} \approx 3.3877162 \times 10^2$, so $E/V \approx 1.418688 \times 10^8\ \mathrm{J/m^3}$, $M/V \approx 1.57850 \times 10^{-9}\ \mathrm{kg/m^3}$.

Swirl–Gravity Coupling

$$\boxed{G_{\text{swirl}} = \frac{v_{\circlearrowleft}\, c^5\, t_p^2}{2\, F_{\text{EM}}^{\max}\, r_c^2}}\,.$$

Numerically $G_{\text{swirl}} \approx 6.67430 \times 10^{-11}$ $\text{m}^3/\text{kg}/\text{s}^2$, matching Newton's G to within calibration precision.

Topology–Driven Mass Law (Invariant Form)

For a torus knot $T(p, q)$ (with $n = \gcd(p, q)$ components, braid index $b = \min(|p|, |q|)$, Seifert genus g), using ropelength $\mathcal{L}_{\text{tot}}(T)$ and core radius r_c:

$$\boxed{M\Big(T(p,q)\Big) = \left(\frac{4}{\alpha}\right) b^{-3/2}\, \varphi^{-g}\, n^{-1/\varphi} \left(\frac{1}{2}\, \rho_f v_{\circlearrowleft}^2\right) \frac{\pi\, r_c^3\, \mathcal{L}_{\text{tot}}(T)}{c^2}}\,.$$

(Dimension comes from the factor $\frac{1}{2}\, \rho_f v_{\circlearrowleft}^2$ [J/m^3] times a volume.)

Swirl Clocks (Local Time Rate)

$$\boxed{\frac{dt_{\text{local}}}{dt_{\infty}} = \sqrt{1 - \frac{\|\boldsymbol{\omega}_{\circlearrowleft}\|^2\, r_c^2}{c^2}} = \sqrt{1 - \frac{\|\mathbf{v}_{\circlearrowleft}\|^2}{c^2}} \quad (r = r_c)}\,.$$

Note: An earlier variant without a length scale (r_c) is deprecated, retained only for historical traceability.

Swirl Angular Frequency Profile

$$\boxed{\Omega_{\text{swirl}}(r) = \frac{v_{\circlearrowleft}}{r_c}\, e^{-r/r_c}\,, \qquad \Omega_{\text{swirl}}(0) = \frac{v_{\circlearrowleft}}{r_c}}\,.$$

Vorticity Potential (Canonical Form)

$$\Phi_{\text{swirl}}(\mathbf{r}, \boldsymbol{\omega}_{\circlearrowleft}) = \frac{v_{\circlearrowleft}^2}{2\, F_{\text{EM}}^{\max}}\, (\boldsymbol{\omega}_{\circlearrowleft} \cdot \mathbf{r})\,.$$

(Use with the SST Lagrangian so that $\rho_f\, \Phi_{\text{swirl}}$ has units of energy density.)

6.1 Empirical Anchoring of Gauge Sector (Canonical Calibration)

The $\text{SU}(3) \oplus \text{SU}(2) \oplus \text{U}(1)$ sector is anchored to experiment by the following empirical values:

$$m_W = 80.377 \text{ GeV}, \tag{14}$$

$$m_Z = 91.1876 \text{ GeV}, \tag{15}$$

$$\sin^2\theta_W = 0.23121 \pm 0.00004, \tag{16}$$

$$\alpha_s(M_Z) = 0.1181 \pm 0.0011. \tag{17}$$

These imply a canonical electroweak symmetry-breaking scale

$$v_{\Phi}^{\exp} = \frac{2m_W}{g} \approx 246.22 \text{ GeV}. \tag{18}$$

This scale is treated as an empirical calibration. Any Swirl–String reinterpretation in terms of fluid constants $(\rho_f, r_c, \|\mathbf{v}_{\circlearrowleft}\|)$ belongs to Canon 4R (Research) until it reproduces this value.

7 Standard Gauge Sector (Canonical Core)

Bundle and connections (Canonical). Let $P \to \mathbb{R}^3 \times \mathbb{R}$ be a principal bundle with $G = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. Local gauge potentials are

$$\mathcal{A}_\mu = g_s\, A^a_\mu T^a \;\oplus\; g\, W^i_\mu \tau^i \;\oplus\; g'\, B_\mu\, Y,$$

with $T^a \in \mathfrak{su}(3)$, $\tau^i \in \mathfrak{su}(2)$, $Y \in \mathfrak{u}(1)$.

Field strengths (Canonical).

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu, \tag{19}$$
$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk} W^j_\mu W^k_\nu, \tag{20}$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{21}$$

Yang–Mills Lagrangian (Canonical).

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} G^a_{\mu\nu}\, G^{a\,\mu\nu} - \frac{1}{4} W^i_{\mu\nu}\, W^{i\,\mu\nu} - \frac{1}{4} B_{\mu\nu}\, B^{\mu\nu}. \tag{22}$$

Dimensional check (natural units): $[A_\mu] = \text{mass}$, $[F_{\mu\nu}] = \text{mass}^2$, so $[\mathcal{L}_{\mathrm{YM}}] = \text{mass}^4$ (SI $\to$ J/m^3 by $\hbar c$).

Matter, covariant derivative (Canonical). For any gauge-charged field Φ in representation R,

$$D_\mu \Phi = \Big(\partial_\mu + \mathrm{i} g_s A^a_\mu T^a + \mathrm{i} g W^i_\mu \tau^i + \mathrm{i} g' B_\mu Y\Big)\,\Phi, \qquad \mathcal{L}^{\mathrm{kin}}_\Phi = (D_\mu\Phi)^\dagger (D^\mu \Phi). \tag{23}$$

Electroweak mixing and masses (Canonical).

$$A_\mu = \sin\theta_W\, W^3_\mu + \cos\theta_W\, B_\mu, \tag{24}$$
$$Z_\mu = \cos\theta_W\, W^3_\mu - \sin\theta_W\, B_\mu, \qquad \tan\theta_W = \frac{g'}{g}. \tag{25}$$

If a scalar doublet (or SST-equivalent) develops a vacuum value v_Φ via a gauge-invariant potential $V(\Phi)$,

$$m_W = \tfrac{1}{2} g v_\Phi, \qquad m_Z = \tfrac{1}{2}\sqrt{g^2 + g'^2}\, v_\Phi, \qquad m_\gamma = 0. \tag{26}$$

Empirical anchoring: v_Φ is fixed by data, cf. Sec. 6 (Empirical Anchoring).

Currents and anomaly constraint (Canonical). Noether currents:

$$J^{a\,\mu}_{(3)} = \sum \bar\Psi \gamma^\mu T^a \Psi, \quad J^{i\,\mu}_{(2)} = \sum \bar\Psi \gamma^\mu \tau^i \Psi, \quad J^\mu_{(1)} = \sum \bar\Psi \gamma^\mu Y \Psi,$$

with minimal-coupling interaction $\mathcal{L}_{\mathrm{int}} = -A^a_\mu J^{a\,\mu}_{(3)} - W^i_\mu J^{i\,\mu}_{(2)} - B_\mu J^\mu_{(1)}$. **Anomaly cancellation** for one generation must hold; this constrains any knot$\to$rep mapping used elsewhere.

8 Unified SST Lagrangian (Definitive Form)

Let $\mathbf{v}_{\circlearrowleft}(\mathbf{x}, t)$ be the velocity, ρ_f constant (incompressible), $\boldsymbol{\omega}_{\circlearrowleft} = \nabla \times \mathbf{v}_{\circlearrowleft}$, and λ enforce $\nabla \cdot \mathbf{v}_{\circlearrowleft} = 0$. Then

$$\boxed{\mathcal{L}_{\text{SST+Gauge}} = \underbrace{\frac{1}{2}\rho_f \, \|\mathbf{v}_{\circlearrowleft}\|^2 - \rho_f \, \Phi_{\text{swirl}}(\mathbf{r}, \boldsymbol{\omega}_{\circlearrowleft}) + \lambda(\nabla \cdot \mathbf{v}_{\circlearrowleft}) + \chi_h \, \rho_f \, (\mathbf{v}_{\circlearrowleft} \cdot \boldsymbol{\omega}_{\circlearrowleft})}_{\text{SST (Kelvin-compatible, local)}} + \underbrace{\mathcal{L}_{\text{YM}}}_{\text{Yang–Mills, Eq. (22)}} + \underbrace{(D_\mu \Phi)^\dagger (D^\mu \Phi)}_{\text{Gauge-charged sca}}}$$

Units. All terms carry energy-density units $[\mathcal{L}] = \mathrm{J\,m^{-3}}$; the helicity coupling is dimensionless ($[\chi_h] = 1$). (In natural units $\hbar = c = 1$, $[\mathcal{L}] = \text{mass}^4$.)
Governance: $\mathcal{L}_{\text{YM}}$, minimal coupling, and EW mixing/masses (§7) are Canonical; any SST-specific mapping $v_\Phi(\rho_f, r_c, \|\mathbf{v}_{\circlearrowleft}\|)$ is tracked in Canon 4R (Research).

Here $\Phi_{\text{swirl}}(\mathbf{r}, \boldsymbol{\omega}_{\circlearrowleft})$ is a prescribed swirl potential (Sec. 6); the term $\chi_h \, \rho_f \, (\mathbf{v}_{\circlearrowleft} \cdot \boldsymbol{\omega}_{\circlearrowleft})$ is the local helicity density (dimensionless χ_h); and $\mathcal{L}_{\text{couple}}$ encodes coupling to quantized circulation Γ and knot invariants $\mathcal{K}$ (linking, writhe, twist). For the scalar sector one may take $V(\Phi) = \lambda_\Phi(|\Phi|^2 - v_\Phi^2)^2$ with v_Φ fixed empirically (Sec. 6).

Constraints (data). Running couplings g_s, g, g' obey standard β functions; numerically we anchor $\alpha_s(M_Z)$ in Sec. 6. Electroweak precision observables (LEP/SLD) constrain $\sin^2\theta_W$ and thus the A–Z mixing in (24).

9 Wave–Particle Duality in SST

The dual phases introduced in Axiom 5 formalize the wave–particle duality in SST. An unknotted swirl string in R-phase behaves as a coherent circulation wave (delocalized, diffraction-capable), whereas a knotted T-phase is localized and particle-like. We outline how standard quantum phenomena emerge from these phases:

de Broglie relation from circulation. Consider a ring-like swirl string of radius R carrying circulation $\Gamma = nh/m_e$ (assuming $m_{\text{eff}} = m_e$ for an electron). The tangential momentum is $p_\theta \approx m_e v_\theta$. Quantization gives $v_\theta = n\hbar/(m_e 2\pi R)$, hence

$$p_\theta = \frac{n\,h}{2\pi R}\,.$$

The de Broglie wavelength $\lambda = h/p_\theta$ follows as

$$\lambda = \frac{2\pi R}{n}\,,$$

i.e. the circumference $2\pi R$ is an integer multiple of the wavelength, consistent with wave coherence around the loop.

Interference and R→T collapse. In a double-slit experiment, an electron's swirl string travels in R-phase through both slits as a distributed vortex loop. The intensity pattern arises from the phase difference of the two path segments around the loop, yielding interference fringes. No which-way information is embedded in the R-phase itself. If a detection attempt (e.g. a photon scattering) forces a T-phase localization, the swirl string knots or collapses to one side, appearing particle-like at a single slit.

Photon-induced collapse (measurement). A photon of frequency ω impinging on an R-phase swirl loop can deposit energy $\hbar\omega$. If this matches the gap ΔE_{eff} between the delocalized state and the nearest knotted state, it triggers

$$\hbar\omega \approx \Delta E_{\text{eff}} ,$$

causing the loop to knot (transition to T-phase). Thus measurements involving photons inherently induce collapse by exciting the swirl into a localized mode.

Fringe visibility decay. Environmental interactions cause gradual swirl-string collapse. If Γ_{collapse} is the net knotting rate (transitions per second from R to T), the interference fringe visibility decays as

$$V(t) = \exp(-\Gamma_{\text{collapse}}\, t) ,$$

analogous to decoherence with a coherence time $\tau_c = \Gamma_{\text{collapse}}^{-1}$. Low-noise experiments correspond to $\Gamma_{\text{collapse}} \to 0$ (long-lived R-phase), preserving interference, whereas any information leak ($\Gamma_{\text{collapse}} > 0$) will diminish V.

10 Notation, Ontology, and Glossary

- **Absolute time (A-time):** The universal time parameter t of the swirl condensate (preferred foliation).
- **Chronos time (C-time):** Time measured at infinity or far outside any swirl field (dt_∞).
- **Swirl Clocks:** Local proper-time scale factors set by $\|\boldsymbol{\omega}_\circlearrowleft\|$ or $\|\mathbf{v}_\circlearrowleft\|$ (see Sec. 8); high swirl intensity (large ω) slows down these clocks relative to A-time.
- **R-phase vs. T-phase:** "Ring" phase (unknotted, extended) versus "Torus-knot" phase (knotted, localized). R-phase excitations superpose and interfere (bosonic behavior), while T-phase excitations manifest particle individuality (fermionic behavior via topological sign rules [2]).
- **String taxonomy:** Leptons are associated with torus knots; quarks with chiral hyperbolic knots; gauge bosons with unknots; neutrinos with linked loops (Hopf links). Family structure and conservation laws correspond to topological properties (e.g. genus, chirality, linking number).
- **Chirality:** Counter-clockwise (ccw) swirl orientation corresponds to matter; clockwise (cw) corresponds to antimatter, through the sign of swirl–gravity interaction.

11 Unknot Bosons and Lossless Swirl Radiation

Postulate (Topological sector). Let $\mathcal{U}$ denote an unknotted closed swirl string ($\mathcal{H} = 0$ Hopf invariant). Finkelstein–Rubinstein single-valuedness on multi-string configuration space enforces integer spin for $\mathcal{U}$FinkelsteinRubinstein1968:

$$\boxed{\mathcal{U} \implies \text{bosonic sector} .}$$

(Nontrivial knot classes supply the topological phase needed for half-integer spin.)

Field variables and lossless propagation. Introduce a transverse swirl potential $\mathbf{a}(\mathbf{x},t)$ such that

$$\mathbf{v}=\partial_t\mathbf{a}, \qquad \mathbf{b}=\nabla\times\mathbf{a}, \qquad \nabla\cdot\mathbf{a}=0\,.$$

Consider the quadratic Lagrangian

$$\mathcal{L}_{\text{wave}}=\frac{\rho_f}{2}\,|\mathbf{v}|^2-\frac{\rho_f c^2}{2}\,|\mathbf{b}|^2\,,$$

where c is the observed luminal wave speed (from Axiom 1). The Euler–Lagrange equations give a lossless wave equation:

$$\boxed{\partial_t^2\mathbf{a}-c^2\,\nabla\times(\nabla\times\mathbf{a})=0, \qquad \nabla\cdot\mathbf{a}=0}\,,$$

with conserved energy density u and Poynting flux $\mathbf{S}$:

$$u=\frac{\rho_f}{2}\Big(|\mathbf{v}|^2+c^2|\mathbf{b}|^2\Big), \qquad \mathbf{S}=\rho_f c^2\,(\mathbf{v}\times\mathbf{b}), \qquad \partial_t u+\nabla\cdot\mathbf{S}=0\,,$$

and momentum density $\mathbf{g}=\mathbf{S}/c^2$. Inviscid, dissipation-free background (Kelvin's theorem) means no swirl circulation is lost; the waves propagate without attenuationBatchelor1967,Saffman1992.

Photon identification. We identify electromagnetic field variables by the linear mapping

$$\boxed{\mathbf{E}=\sqrt{\frac{\rho_f}{\varepsilon_0}}\,\mathbf{v}, \qquad \mathbf{B}=\sqrt{\frac{\rho_f}{\varepsilon_0}}\,\mathbf{b}}\,,$$

yielding

$$u=\frac{\varepsilon_0}{2}|\mathbf{E}|^2+\frac{1}{2\mu_0}|\mathbf{B}|^2, \qquad \mathbf{S}=\frac{1}{\mu_0}\,(\mathbf{E}\times\mathbf{B}), \qquad \frac{1}{\varepsilon_0\mu_0}=c^2\,,$$

exactly the Maxwell energy–momentum in vacuumJackson1999. Plane- and spherical-wave solutions of $\mathcal{L}_{\text{wave}}$ thus describe <u>photons</u> as <u>delocalized</u>, divergence-free swirl oscillations.

Quantization and single-photon amplitude. Quantizing a cavity mode (volume V, frequency ω) gives the standard one-photon field amplitude

$$E^{(1)}_{\text{rms}}=\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\,,$$

hence swirl velocity amplitude

$$\boxed{v^{(1)}_{\text{rms}}=\sqrt{\frac{\hbar\omega}{2\,\rho_f\,V}}}\,.$$

For $\lambda=532$ nm (green, $\omega=2\pi c/\lambda$) and $\rho_f=7.0\times10^{-7}\,\text{kg/m}^3$:

$$V=1\ \text{mm}^3: \quad v^{(1)}_{\text{rms}}\approx 3.27\times10^{-2}\ \text{m/s}\,,$$

consistent with $E^{(1)}_{\text{rms}}$ and observed cavity QED couplingsHarocheRaimond2006,ScullyZubairy1997.

Radiation from bound strings ("atoms"). A localized bound swirl configuration with time-varying multipole moment $\mathbf{d}(t)$ launches outward transverse $\mathbf{a}$ waves. Far from the source ($r\gg$ source size), the solution is

$$\mathbf{a}(\mathbf{x},t)\ \propto\ \frac{\mathbf{e}_\perp}{r}\ \text{Re}\Big(e^{i(kr-\omega t)}\Big), \qquad k=\omega/c\,,$$

with flux $\mathbf{S}=\rho_f c^2\,(\mathbf{v}\times\mathbf{b})$ directed radially and $|\mathbf{S}|\propto r^{-2}$, ensuring constant power through spheresJackson1999. Thus, <u>atoms (knotted strings) emit concentric swirl waves; the lossless medium transmits them without attenuation.</u>

Exclusion of smoke-ring photons. A localized vortex-ring (smoke ring) of core radius r_c and energy $E_{\rm vr}$ carrying momentum $p_{\rm vr}$ cannot simultaneously satisfy $E_{\rm vr} = \hbar\omega$ and $p_{\rm vr} = \hbar k$ with subluminal core speedSaffman1992,Batchelor1967. Hence, photons in vacuum are not toroidal vortex rings, but rather extended swirl modes as above.

Summary.

$$\boxed{\mathcal{U}\ \text{(unknot)} \implies \text{boson}; \qquad \text{photons} = \text{delocalized, lossless swirl waves launched by bound sources.}}$$

11.1 Photon as a Pulsed Unknot with Delocalized Circulation

We can model the photon as a pulsed, unknotted swirl-string $K \cong S^1$ of radius R (circumference $L = 2\pi R$). Unlike massive particles (localized knots with core density ρ_m), the photon has no rest-mass contribution ($\rho_m = 0$); its energy resides entirely in oscillatory swirl motion within the effective fluid ρ_f.

Effective 1D action. Define a transverse displacement $\xi(s,t)$ along the ring (parametrized by $s \in [0, L)$) with cross-sectional area $A_{\rm eff} = \pi w^2$. The photon's delocalized mode is described by

$$S[\xi] = \frac{1}{2}\rho_f A_{\rm eff} \int dt \int_0^L ds \left[(\partial_t \xi)^2 - c^2 (\partial_s \xi)^2\right],$$

yielding the wave equation

$$\partial_t^2 \xi - c^2\, \partial_s^2 \xi = 0, \qquad \xi(s+L,t) = \xi(s,t)\,.$$

Normal modes. Periodic boundary conditions give discrete wavenumbers

$$k_m = \frac{2\pi m}{L}, \qquad \omega_m = c\, k_m, \qquad m \in \mathbb{Z}_{>0}\,.$$

A single-mode solution:

$$\xi_m(s,t) = a_m \cos(k_m s - \omega_m t)\,.$$

Mode energy. The time-averaged energy of mode m is

$$E_m = \frac{1}{2}\, \rho_f A_{\rm eff} L\, \omega_m^2 a_m^2\,,$$

which depends on the delocalized volume $A_{\rm eff} L$ rather than a massive core. Thus, photon energy is carried by the distributed swirl field, not a localized mass density.

Quantization. Assigning energy $\hbar\omega_m$ to each mode yields amplitude

$$a_m = \sqrt{\frac{\hbar}{\rho_f A_{\rm eff} L\, \omega_m}}\,.$$

For a photon of wavelength λ, set $R = \lambda/(2\pi)$ so $L = \lambda$, and choose $w \sim \lambda/(2\pi)$ (so $A_{\rm eff} = \pi w^2$):

$$a = \sqrt{\frac{\hbar}{\rho_f\, A_{\rm eff}\, L\, \omega}} = \sqrt{\frac{\hbar}{\rho_f\, \pi w^2\, \lambda\, \omega}}\,,$$

with $\omega = 2\pi c/\lambda$. For $\lambda = 500\,\text{nm}$ and $\rho_f = 7.0 \times 10^{-7}\,\text{kg/m}^3$,

$$a \approx 2.0 \times 10^{-12}\,\text{m}\,, \qquad E = \hbar\omega \approx 3.97 \times 10^{-19}\ \text{J}\ \ (2.48\,\text{eV})\,.$$

Interpretation. The photon is thus a pulsed unknot swirl-string, with vanishing rest-mass density ($\rho_m = 0$) but finite distributed energy density

$$\rho_E = \tfrac{1}{2}\rho_f\Big((\partial_t\xi)^2 + c^2(\partial_s\xi)^2\Big)$$

integrated over its volume. It is neither pointlike nor bound to a core, but consists of a minimal swirl loop excited to launch delocalized waves—akin to a momentarily perturbed vortex ring radiating ripples in a fluid.

12 Canonical Checks (Verification in Practice)

1. **Dimensional analysis:** Verify SI consistency for every new term and equation introduced.
2. **Limiting cases:** Show that low-swirl limits ($\|\boldsymbol{\omega}_{\circlearrowleft}\| \to 0$) recover classical mechanics and Maxwell electrodynamics; large-scale averages reproduce Newtonian gravity with G_{swirl}.
3. **Numerical evaluation:** Provide numeric factors using Canon constants (Sec. 2) for any new formula. If new constants are needed, add them to Sec. 2 for consistency.
4. **Topology–quantum mapping:** Explicitly state which knot invariants correspond to which quantum numbers and how they are normalized (linking number $\leftrightarrow$ baryon number, etc.).
5. **Citations:** Cite any non-original constructs or standard results (e.g. Kelvin's theorem, Planck's law) using the provided BibTeX keys.

13 Swirl Hamiltonian Density (Canonical Form)

Given effective density ρ_f and swirl vorticity $\boldsymbol{\omega}_{\circlearrowleft} = \nabla \times \mathbf{v}_{\circlearrowleft}$, a Kelvin-compatible, dimensionally consistent Hamiltonian density is:

$$\mathcal{H}_{\text{SST}}[\mathbf{v}_{\circlearrowleft}] = \frac{1}{2}\rho_f\,\|\mathbf{v}_{\circlearrowleft}\|^2 + \frac{1}{2}\rho_f\,r_c^2\,\|\boldsymbol{\omega}_{\circlearrowleft}\|^2 + \frac{1}{2}\rho_f\,r_c^4\,\|\nabla\boldsymbol{\omega}_{\circlearrowleft}\|^2 + \lambda\,(\nabla\cdot\mathbf{v}_{\circlearrowleft})\,, \tag{27}$$

which has units of energy density (J/m^3). The first term is kinetic energy of swirl motion; the second term $\sim r_c^2\|\boldsymbol{\omega}_{\circlearrowleft}\|^2$ represents core rotational energy (rest-mass analogue); the third term $\sim r_c^4\|\nabla\boldsymbol{\omega}_{\circlearrowleft}\|^2$ penalizes curvature of the vortex filaments (string tension). In the limit $r_c \to 0$ or for spatially uniform vorticity, the higher-order terms vanish, reducing $\mathcal{H}_{\text{SST}}$ to the usual fluid kinetic energy density $\frac{1}{2}\rho_f v^2$ with incompressibility constraint.

14 Swirl Pressure Law (Euler Corollary)

For a steady, purely azimuthal flow ($v_r = v_z = 0$, $\partial_t = 0$), the radial component of the Euler momentum equation ($\rho_f\,v_\theta^2/r = dp_{\text{swirl}}/dr$) provides a direct relationship for the swirl pressure gradient:

$$\frac{1}{\rho_f}\frac{dp_{\text{swirl}}}{dr} = \frac{v_\theta(r)^2}{r}\,. \tag{28}$$

This is a canonical theorem derived directly from first principles. For a system exhibiting an asymptotically flat rotation curve where $v_\theta(r) \to v_0$ for large r, the pressure profile is found by integration:

$$p_{\text{swirl}}(r) = p_0 + \rho_f v_0^2 \ln\left(\frac{r}{r_0}\right)\,. \tag{29}$$

Here, p_0 is the pressure at a reference radius r_0. The resulting outward-rising pressure creates an inward-pointing force ($-\nabla p_{\text{swirl}}$), providing the centripetal acceleration required to maintain the flat rotation curve.

15 Experimental Protocols (Canon-Ready Tests)

Universality of $v_{\circlearrowleft} = f\,\Delta x$ (multi-platform metrology)

(From `ExperimentalValidationOfVortexCoreTangentialVelocity.tex`*)*—In diverse systems (magnetized plasmas, superconducting vortices, optical ring modes, acoustic vortices), measure a natural frequency f and a spatial period Δx of a standing or traveling swirl mode. Verify:

$$\boxed{v_{\circlearrowleft} = f\,\Delta x \approx 1.09384563 \times 10^6\ \mathrm{m/s}}\,. \tag{X1}$$

Achieve sub-ppm agreement across platforms; report mean and standard deviation. This confirms a universal quantum of circulation speed.

Swirl-induced gravitational potential

(From `ExperimentalValidationOfGravitationalPotential.tex`*)*—Infer $p_{\mathrm{swirl}}(r)$ from centripetal balance (§14) and compare predicted forces with measured thrust or buoyancy anomalies in shielded high-voltage/coil experiments (geometry: starship/Rodin coils). Ensure dimensional consistency and calibrate only via Canon constants.

16 Critical Questions Across SST Extensions

We collect here several forward-facing questions for Swirl–String Theory (SST), posed as critical tests or extensions. Each is answered canonically, with experimental or theoretical implications.

1. Is EMF quantization observable?

If each reconnection or knotting event releases a discrete flux impulse $\mathbf{\Phi}^*$, then

$$\Delta\Phi_{\mathcal{C}} = \int_{\Sigma(\mathcal{C})} \Delta\mathbf{B}\cdot d\mathbf{S} = m\,\mathbf{\Phi}^*, \quad m \in \mathbb{Z},$$

should appear as a <u>quantized step</u> in a superconducting interferometer. For a pickup loop of inductance L,

$$\Delta I = \frac{\Delta\Phi}{L}, \qquad V_{ind}(t) = -\frac{d\Phi}{dt}.$$

A reconnection of duration τ requires bandwidth $f_{\mathrm{BW}} \gtrsim (2\pi\tau)^{-1}$. For $\tau \sim$ ns, this is 20–200 MHz, within modern SQUID ranges. If $\mathbf{\Phi}^* \approx \mathbf{\Phi}_0 = h/2e$, steps are resolvable.

Status. Canonical prediction: <u>yes</u>, observable as quantized steps; scale of $\mathbf{\Phi}^*$ to be calibrated empirically.

2. Is $R \to T$ collapse deterministic or stochastic?

Field equations (Euler + swirl coupling) are deterministic. However, topology change occurs at core separations $\sim r_c$ with extreme sensitivity to microstates. Effective model:

$$\dot{K} = F(K) + \sqrt{2D_{env}}\,\eta(t), \qquad \mathrm{Vis}(t) = e^{-\Gamma_{env}t},\ \Gamma_{env} \propto D_{env}.$$

Thus: deterministic instability in the $D_{env} \to 0$ limit; effectively stochastic under environmental noise.

Status. Collapse is <u>deterministic at the core level</u>, <u>stochastic in practice</u>.

3. Can SST replace quantum measurement postulates?

The dual-phase picture (delocalized R vs localized T states) suggests an objective collapse mechanism. Collapse is driven by swirl radiation and reconnections. Key tasks:

- Derive the Born rule $P \sim |\psi|^2$ from ergodic measures on swirl phase space.
- Ensure no-signaling under nonlocal correlations of linked knots.

Status. Promising realist alternative, but derivation of Born and no-signaling remains open.

4. How unique is the topological decomposition?

Different knots K can share

$$\mathcal{M}(K) = b(K)^{-3/2}\,\varphi^{-g(K)}\,n(K)^{-1/\varphi}\,L_{tot}(K).$$

Thus mass alone is degenerate. Resolution requires:

- Helicity $H = \int \mathbf{v}\cdot\boldsymbol{\omega}\,dV$,
- writhe/twist spectra and normal-mode eigenfrequencies,
- stability (lifetime) and selection rules.

Status. Unique particle identity emerges only when mass, helicity, and mode spectra are jointly enforced.

5. Can the swirl Lagrangian generate interactions?

Beyond mass, interaction terms may appear via extended couplings:

$$\mathcal{L}_{couple} \;+\; \lambda_{ch}\int(\mathbf{v}\cdot\boldsymbol{\omega})(\nabla\cdot\mathbf{a})\,dV +\; g_c\int \mathcal{C}(\mathcal{K}_1,\mathcal{K}_2)\,d\Sigma.$$

These generate parity-odd (chiral) and contact vertices, reminiscent of Yukawa/weak interactions.

Status. Plausible EFT tower; explicit vertex catalogue is an open derivation.

6. Is the swirl condensate Lorentz-violating?

SST posits absolute time (preferred foliation). Microscopic frame is Galilean. However, the photon sector Lagrangian

$$\partial_t^2\mathbf{a} - c^2\nabla\times(\nabla\times\mathbf{a}) = 0, \quad \nabla\cdot\mathbf{a} = 0$$

is exactly Lorentz-invariant. Residual anisotropies are suppressed operators of order ϵ^2 with $\epsilon = \|\mathbf{u}_{drift}\|/c$. Experimental bounds: $\delta c/c \lesssim 10^{-17}$–$10^{-21}$; SST must respect these.

Status. Emergent Lorentz invariance in radiation sector; matter sector constrained to high precision.

Experiment table

Table 1: Concrete experiments to test critical SST questions.

Objective	Observable	SST control	Expected scale	Note
Flux impulse	$\Delta\Phi$ steps	reconnections	$\Phi^* \sim \Phi_0$?	SQUID; f_{BW} ns–µs
Collapse rate	vis. $\sim e^{-\Gamma t}$	env. coupling D_{env}	tunable Γ	Kramers escape analogue
Identity	spectra, H	knot class K	discrete Ω_n	inelastic spectroscopy
Interactions	scattering	contact twist/link	selection rules	EFT vertex test
Lorentz	$\delta c/c$	drift $\mathbf{u}$	$< 10^{-17}$	cavity/clock tests

Governance Note. Definitions of ε_*, $\mathcal{B}[K]$, and S_{comp} are Canonical. Their interpretation as renormalizing $g_{2,3}$ and the representation map $t(K) \to (\mathrm{SU}(3), \mathrm{SU}(2), Y)$ are Research until anomaly cancellation and empirical coupling fits are demonstrated.

Canon 4R: Research Extensions (Non-Canonical)

The following conjectural relations are recorded for future calibration. They are dimensionally consistent but not yet anchored to empirical values.

- $v_\Phi^{\mathrm{SST}} \sim \sqrt{\rho_f}\, r_c \|\mathbf{v}_{\circlearrowleft}\| / \hbar$
- Swirl-helicity × Chern–Simons couplings
- Knot → gauge representation map beyond anomaly checks

BibTeX (comparators)

```
@book{Tinkham1996,
  author={Tinkham, Michael},
  title={Introduction to Superconductivity},
  edition={2}, year={1996}, publisher={McGraw--Hill}
}
@book{ClarkeBraginski2004,
  author={Clarke, John and Braginski, Alex I.},
  title={The SQUID Handbook},
  year={2004}, publisher={Wiley-VCH}
}
@article{Moffatt1969,
  author={Moffatt, H. K.},
  title={The degree of knottedness of tangled vortex lines},
  journal={J. Fluid Mech.}, year={1969}, doi={10.1017/S0022112069000225}
}
```

For a composite $K_1 \# K_2$,

$$u(K_1 \# K_2) = u(K_1) + u(K_2) - \Delta_u(K_1, K_2), \qquad \Delta_u \geq 0.$$

Hence the barrier functional satisfies

$$\mathcal{B}[K_1 \# K_2] = \mathcal{B}[K_1] + \mathcal{B}[K_2] - \varepsilon_*\, \Delta_u.$$

We define a dimensionless simplification index:

$$S_{comp}(K_1, K_2) = \frac{\Delta_u}{u(K_1) + u(K_2)} \in [0, 1).$$

This index measures the degree to which composition reduces the unknotting barrier.

In SST taxonomy, the correction term

$$\Delta\mathcal{B} = \varepsilon_* \Delta_u$$

acts as a nonlinear coupling, analogous to the non-Abelian structure constants in $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$.

Appendix C: Invariant Mass from the Canonical Lagrangian

Starting from the schematic Lagrangian

$$\mathcal{L}_{\text{SST}} = \rho_f\Big(\tfrac{1}{2}\mathbf{v}_{\circlearrowleft}^2 - \Phi_{\text{swirl}}\Big) + \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \Big(\alpha C(K) + \beta L(K) + \gamma \mathcal{H}(K)\Big) + \rho_f \ln\sqrt{1 - \tfrac{\|\omega\|^2}{c^2}} + \Delta p(\text{swirl}),$$

the mass sector reduces, under the slender-tube approximation, to an invariant energy functional

$$E(K) = u\, V(K)\, \Xi_{\text{top}}(K), \qquad u = \tfrac{1}{2}\rho_{\text{core}}\, v_{\circlearrowleft}^2,$$

with u the swirl energy density scale on the core, $V(K)$ the effective tube volume of the swirl string, and $\Xi_{\text{top}}(K)$ a dimensionless topological multiplier summarizing discrete combinatorial and contact/helicity corrections. In SST we adopt

$$V(K) \;=\; \pi r_c^2 \underbrace{\Big(L_{phys}\Big)}_{= r_c\, L_{tot}} \;=\; \pi r_c^3\, L_{\text{tot}},$$

where r_c is the core radius and L_{tot} is the dimensionless ropelength. The rest mass is $M = E/c^2$.

Canonical multiplier. Guided by the EM coupling and SST's discrete scaling rules, we take

$$\Xi_{\text{top}}(K) = \frac{4}{\alpha_{\text{fs}}}\, b^{-3/2}\, \varphi^{-g}\, n^{-1/\varphi},$$

where b, g, n are the integer topology labels used in the Canon (e.g. torus index, layer, linkage count), α_{fs} is the fine-structure constant, and φ the golden ratio. Collecting factors, the **invariant mass law** used in the code is

$$\boxed{M(K) = \frac{4}{\alpha_{\text{fs}}}\, b^{-3/2}\, \varphi^{-g}\, n^{-1/\varphi}\, \frac{u\,\pi r_c^3 L_{\text{tot}}}{c^2}, \qquad u = \tfrac{1}{2}\rho_{\text{core}} v_{\circlearrowleft}^2.}$$

Leptons (solved L_{tot}). For a lepton with labels (b, g, n) and known mass $M_\ell^{(\exp)}$, invert (16):

$$L_{\text{tot}}^{(\ell)} \;=\; \frac{M_\ell^{(\exp)}\, c^2}{\Big(\frac{4}{\alpha_{\text{fs}}}\, b^{-3/2}\varphi^{-g} n^{-1/\varphi}\Big)\, u\,\pi r_c^3}.$$

Baryons (exact closure). Let the proton and neutron ropelengths be

$$L_p = \lambda_b\,(2s_u + s_d)\,\mathcal{S}, \qquad L_n = \lambda_b\,(s_u + 2s_d)\,\mathcal{S}, \qquad \mathcal{S} = 2\pi^2\kappa_R,\;\; \kappa_R = 2,$$

with (s_u, s_d) dimensionless sector weights and λ_b a sector scale (set to 1 in exact-closure). Imposing $M_p^{(\text{exp})} = M_p$ and $M_n^{(\text{exp})} = M_n$ in (16) yields a linear 2×2 system for (s_u, s_d):

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} s_u \\ s_d \end{bmatrix}$$

$$= 1\frac{}{K\begin{bmatrix} M_p^{(\text{exp})} \\ M_n^{(\text{exp})} \end{bmatrix}, \qquad K=\left[\frac{4}{\alpha_{\text{fs}}}\,3^{-3/2}\,\varphi^{-2}\,3^{-1/\varphi}\right]\frac{u\,\pi r_c^3\,\mathcal{S}}{c^2}.}$$ Solving gives

$$s_u = \frac{2M_p^{(\text{exp})} - M_n^{(\text{exp})}}{3K}, \qquad s_d = \frac{M_p^{(\text{exp})}}{K} - 2s_u.$$

Composites (no binding). For an atom with proton number Z and neutron number N (atomic mass includes Z electrons),

$$M_{atom}^{(pred)} = Z\,M_p + N\,M_n + Z\,M_e, \quad M_{\text{mol}}^{(pred)} = \sum_{\text{atoms}} M_{\text{atom}}^{(pred)}.$$

Deviations from experiment in atoms/molecules correspond to binding energies not included in this baseline (nuclear $\sim 8\,\text{Me}V$ per nucleon; molecular $\sim$ eV).

16.1 Benchmarks (exact_closure mode)

The following table was generated by the Python file listed after it. Errors in atoms/molecules = missing binding energy contribution, not model failure.

Table 2: Invariant-kernel mass benchmarks (exact_closure). Errors in atoms/molecules = missing binding energy contribution, not model failure.

Species	Known mass (kg)	Predicted mass (kg)	Error (%)
electron e-	9.109384e-31	9.109384e-31	0.0000
muon μ-	1.883532e-28	1.883532e-28	0.0000
tau τ-	3.167540e-27	3.167540e-27	0.0000
proton p	1.672622e-27	1.672622e-27	0.0000
neutron n	1.674927e-27	1.674927e-27	0.0000
Hydrogen-1 atom	1.673533e-27	1.673533e-27	0.0000
Helium-4 atom	6.646477e-27	6.689952e-27	0.6549
Carbon-12 atom	1.992647e-26	2.005276e-26	0.6330
Oxygen-16 atom	2.656017e-26	2.674532e-26	0.6980
H_2 molecule	3.367403e-27	3.347066e-27	-0.6040
H_2O molecule	2.991507e-26	3.009885e-26	0.6139
CO_2 molecule	7.305355e-26	7.354340e-26	0.6704

Notes

- Elementary entries are exact by construction in exact_closure mode (leptons solved from L_{tot}; p, n from closure).
- Composite errors track omitted binding: nuclear $\mathcal{O}(10^{-3})$–$\mathcal{O}(10^{-2})$, molecular $\mathcal{O}(10^{-9})$.

From unknotting non-additivity to $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$ in SST

Unknotting barrier functional. Let $K \subset \mathbb{R}^3$ be a closed swirl–string on a leaf Σ_t. Define the per–crossing activation scale

$$\varepsilon_* \;=\; \kappa\,\beta\,r_c \;+\; \frac{\kappa\pi}{2}\,\rho_f\,\|\mathbf{v}_{\circlearrowleft}\|^2\,r_c^3, \quad \kappa = \mathcal{O}(1\text{–}10),$$

and the barrier

$$\mathcal{B}[K] \;=\; u(K)\,\varepsilon_*.$$

For a connected sum $K_1\#K_2$,

$$u(K_1\#K_2) = u(K_1) + u(K_2) - \Delta_u(K_1, K_2), \qquad \Delta_u \geq 0,$$

so that

$$\mathcal{B}[K_1\#K_2] = \mathcal{B}[K_1] + \mathcal{B}[K_2] - \varepsilon_*\,\Delta_u.$$

(Brittenham–Hermiller 2025 give explicit $\Delta_u > 0$ families for torus knots.)

Dimensionless simplification index. Define

$$S_{comp}(K_1, K_2) := \frac{\Delta_u(K_1, K_2)}{u(K_1) + u(K_2)} \in [0, 1),$$

and $\Delta\mathcal{B} = \varepsilon_*\,\Delta_u$. Here S_{comp} is dimensionless; $\Delta\mathcal{B}$ has units of energy.

Discrete curvature on composition. Let $(\mathcal{K}, \#)$ denote the knot configuration monoid. Non-additivity of u defines a discrete 2-cocycle:

$$\mathcal{C}(K_1, K_2) := \Delta_u(K_1, K_2).$$

A minimal associator defect (discrete curvature) for triples is

$$\mathfrak{R}(K_1, K_2, K_3) \;:=\; \mathcal{C}(K_1, K_2) + \mathcal{C}(K_1\#K_2, K_3) - \mathcal{C}(K_2, K_3) - \mathcal{C}(K_1, K_2\#K_3).$$

If $\mathfrak{R} \equiv 0$ the composition is "flat" (effectively Abelian in the barrier metric); $\mathfrak{R} \neq 0$ signals nontrivial curvature, the discrete analogue of non-Abelian structure.

Emergent gauge potentials from multi-director swirl. Let $\mathcal{A} = \left(A_\mu^{(0)},\, W_\mu^a,\, G_\mu^A\right)$ denote the swirl–gauge potentials for $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$. Introduce coupling functions driven by simplification statistics in a family $\mathcal{F} \subset \mathcal{K}$:

$$g_1^{-2}(\mathcal{F}) = g_{1,0}^{-2}, \qquad g_2^{-2}(\mathcal{F}) = g_{2,0}^{-2}\Big[1 + \lambda_2\,\langle S_{comp}\rangle_{\mathcal{F}}\Big], \qquad g_3^{-2}(\mathcal{F}) = g_{3,0}^{-2}\Big[1 + \lambda_3\,\langle S_{comp}\rangle_{\mathcal{F}}\Big],$$

with $\lambda_{2,3} > 0$ dimensionless and $\langle\cdot\rangle_{\mathcal{F}}$ the family average. Thus nonzero simplification ($\Delta_u > 0$) renormalizes the non-Abelian sectors, while $U(1)$ remains purely additive at leading order.

SST gauge Lagrangian (coupling by taxonomy). With $F_{\mu\nu}^{(0)}$, $W_{\mu\nu}^a$, $G_{\mu\nu}^A$ the field strengths,

$$\mathcal{L}_{gauge} = -\frac{1}{4}\frac{1}{g_1^2(\mathcal{F})}\,F_{\mu\nu}^{(0)}F^{(0)\mu\nu} - \frac{1}{4}\frac{1}{g_2^2(\mathcal{F})}\,W_{\mu\nu}^a W^{a\,\mu\nu} - \frac{1}{4}\frac{1}{g_3^2(\mathcal{F})}\,G_{\mu\nu}^A G^{A\,\mu\nu},$$

and the total selection functional acquires the barrier:

$$\mathcal{E}_{tot}[K] = \alpha\,C(K) + \beta\,L(K) + \gamma\,\mathcal{H}(K) + \mathcal{B}[K], \quad \mathcal{E}_{tot}[K_1\#K_2] = \mathcal{E}_{tot}[K_1] + \mathcal{E}_{tot}[K_2] - \varepsilon_*\,\Delta_u.$$

Representation assignment (house mapping). Adopt the Canon homomorphism $t(K) = (L \bmod 3,\ S \bmod 2,\ \chi)$ to $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$:

$$\begin{aligned}
&\text{Color (SU(3)) index:} && \mathbf{3} \text{ class determined by } (L \bmod 3),\\
&\text{Weak (SU(2)) isospin:} && \mathbf{2} \text{ / singlet by } (S \bmod 2),\\
&\text{Hypercharge (U(1)):} && Y \propto \chi\, \mathcal{Q}(K),
\end{aligned}$$

where $\chi \in \{\pm 1\}$ tracks chirality/mirror and $\mathcal{Q}(K)$ is a chosen Abelian scalar (e.g. normalized circulation or writhe). This retains Abelian additivity for Y while allowing non-Abelian renormalization via $\langle S_{comp} \rangle$.

Worked composite example. For $K = T(2,7)$ with $u(K) = 3$ and $u(K\#\overline{K}) \leq 5$,

$$\Delta_u(K, \overline{K}) \geq 1, \qquad S_{comp}(K, \overline{K}) \geq \frac{1}{6}.$$

Hence

$$\Delta\mathcal{B} = \varepsilon_* \,\Delta_u \geq \varepsilon_*, \quad \frac{1}{g_{2,3}^2} \;\mapsto\; \frac{1}{g_{2,3}^2}\left[1 + \lambda_{2,3} \times \tfrac{1}{6}\right] \text{ for the family containing } K, \overline{K}.$$

All quantities entering $g_i(\mathcal{F})$ are dimensionless (consistency check).

Physical interpretation. Additive ($U(1)$) observables follow linear composition; subadditivity of u generates a discrete curvature that selectively enhances the non-Abelian sectors. Families with larger $\langle S_{comp} \rangle$ act as stronger "non-Abelianizers" of the swirl–gauge dynamics.

References

[1] M. Brittenham and S. Hermiller, Unknotting number is not additive under connected sum, arXiv:2506.24088 (2025).

Appendix D: Persona Prompts

Reviewer Persona

You are a peer reviewer for an SST paper. Use only the definitions and constants in the "SST Canon (**v0.4.1**)". Check dimensional consistency, limiting behavior, and numerical validation. Flag any use of non-canonical constants or equations unless equivalence is proved. Demand explicit mapping from knot invariants (linking, writhe, twist) to claimed quantum numbers.

Theorist Persona

You are a theoretical physicist specialized in Swirl String Theory (SST). Base all reasoning on the attached "SST Canon (**v0.4.1**)". Your task: derive the swirl-based Hamiltonian for [TARGET SYSTEM], use Sec. 8, and verify the Swirl Clock law (Sec. 1). Provide boxed equations, dimensional checks, and a short numerical evaluation using the Canon constants.

Bridging Persona (Compare to GR/SM)

Work strictly within SST Canon (**v0.4.1**). Compare [TARGET] to its GR/SM counterpart. Identify exact replacements (e.g., curvature → swirl), and show which terms reduce to Newtonian/Maxwellian limits. Include a correspondence table and any constraints needed for equivalence.

Appendix E: Session Kickoff Checklist

1. Start new chat per task; attach this Canon first.
2. Paste a persona prompt (Sec. 16.1).
3. Attach only task-relevant papers/sources.
4. State any corrections explicitly (they persist in the session).
5. At end, record Canon deltas (if any) and bump version.

Appendix F: Dimensional Cross-Check for $\mathcal{L}_{\mathrm{SST+Gauge}}$

Unit conventions. We present SI checks. Where convenient, we also note the natural-unit assignment ($\hbar = c = 1$), with $[A_\mu] = \text{mass}$, $[F_{\mu\nu}] = \text{mass}^2$, ensuring $[\mathcal{L}] = \text{mass}^4$; conversion to SI energy density uses $\hbar c$.

Term	**Expression**	**Primary units**
Kinetic (swirl)	$\frac{1}{2}\,\rho_f\,\lVert \mathbf{v}_\circlearrowleft \rVert^2$	$[\rho_f] = \mathrm{kg\,m^{-3}}$, $[\mathbf{v}_\circlearrowleft] = \mathrm{m\,s^{-1}}$
Swirl potential	$-\,\rho_f\,\Phi_{\mathrm{swirl}}(\mathbf{r}, \boldsymbol{\omega}_\circlearrowleft)$	$[\Phi_{\mathrm{swirl}}] = \mathrm{m^2\,s^{-2}}$
Incompressibility constraint	$\lambda\,(\nabla\cdot\mathbf{v}_\circlearrowleft)$	$[\nabla\cdot\mathbf{v}_\circlearrowleft] = \mathrm{s^{-1}}$
Helicity density (local)	$\chi_h\,\rho_f\,(\mathbf{v}_\circlearrowleft\cdot\boldsymbol{\omega}_\circlearrowleft)$	$[\boldsymbol{\omega}_\circlearrowleft] = \mathrm{s^{-1}}$
Yang–Mills (gauge)	$-\frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$	$\hbar = c = 1:\ [F] = \text{mass}^2$
Scalar kinetic	$(D_\mu\Phi)^\dagger(D^\mu\Phi)$	$\hbar = c = 1:\ [D_\mu] = \text{mass},\ [\Phi] = \text{mass}$
Scalar potential	$V(\Phi) = \lambda_\Phi\Big(\lvert\Phi\rvert^2 - v_\Phi^2\Big)^2$	$\hbar = c = 1:\ [V] = \text{mass}^4$
Minimal coupling	$\mathcal{L}_{\mathrm{int}} = -A^a_\mu J^{a\,\mu}_{(3)} - W^i_\mu J^{i\,\mu}_{(2)} - B_\mu J^\mu_{(1)}$	$\hbar = c = 1:\ [A_\mu] = \text{mass},\ [J^\mu] = \text{mass}$

Notes for reviewers.

- **Helicity term:** since $[\mathbf{v}_\circlearrowleft\cdot\boldsymbol{\omega}_\circlearrowleft] = \mathrm{m\,s^{-2}}$, multiplying by ρ_f gives $\mathrm{J\,m^{-3}}$, so $[\chi_h] = 1$.
- **Constraint multiplier:** with $(\nabla\cdot\mathbf{v}_\circlearrowleft)$ in $\mathrm{s^{-1}}$, the Lagrange multiplier has units $[\lambda] = \mathrm{Pa\cdot s} = \mathrm{J\,s\,m^{-3}}$. In Euler–Lagrange equations, its spatial gradient divided by time (or $\partial_t\lambda$) plays the role of a pressure field.
- **Swirl potential:** the canonical choice $\Phi_{\mathrm{swirl}} = \frac{v_\circlearrowleft^2}{2\,F_{\mathrm{EM}}^{\max}}(\boldsymbol{\omega}_\circlearrowleft\cdot\mathbf{r})$ has $[\Phi_{\mathrm{swirl}}] = \mathrm{m^2\,s^{-2}}$ provided $F_{\mathrm{EM}}^{\max}$ carries force units, consistent with Table entries.
- **Gauge block:** all gauge-sector rows are standard; in natural units they produce mass^4, hence energy density after restoring $\hbar, c$.

Summary. Each addend in $\mathcal{L}_{\mathrm{SST+Gauge}}$ carries $\mathrm{J\,m^{-3}}$ in SI; coupling constants (g_s, g, g', χ_h) are dimensionless; the incompressibility multiplier has $[\lambda] = \mathrm{Pa\cdot s}$ under the present choice of constraint.

References

[1] H. K. Moffatt. The degree of knottedness of tangled vortex lines. Journal of Fluid Mechanics, 1969.

[2] David Finkelstein and Jerrold Rubinstein. Connection between spin, statistics, and kinks. Journal of Mathematical Physics, 9(11):1762–1779, 1968.

[3] H. von Helmholtz. über integrale der hydrodynamischen gleichungen, welche den wirbelbewegungen entsprechen. Journal für die reine und angewandte Mathematik, 55:25–55, 1858.

[4] W. Thomson (Lord Kelvin). On vortex motion. Transactions of the Royal Society of Edinburgh, 25:217–260, 1869.

[5] Philip G. Saffman. Vortex Dynamics. Cambridge University Press, 1992.

[6] R. P. Feynman. Application of quantum mechanics to liquid helium. In C. J. Gorter, editor, Progress in Low Temperature Physics, volume 1, pages 17–53. North-Holland, 1955.

[7] Russell J. Donnelly. Quantized Vortices in Helium II. Cambridge University Press, 1991.

[8] L. D. Landau and E. M. Lifshitz. Fluid Mechanics. Course of Theoretical Physics, Vol. 6. Pergamon, 2nd edition, 1987.

[9] John David Jackson. Classical Electrodynamics. Wiley, 3rd edition, 1999.

[10] W. Thomson (Lord Kelvin). On vortex atoms. Proc. Royal Society of Edinburgh, 1867.

[11] E. Schrödinger. An undulatory theory of the mechanics of atoms and molecules. Physical Review, 1926.

[12] G. Călugăreanu. L'intégral de gauss et l'analyse des noeuds tridimensionnels. Rev. Math. Pures Appl., 4, 1959.

[13] J. H. White. Self-linking and the gauss integral in higher dimensions. American Journal of Mathematics, 91, 1969.

[14] F. B. Fuller. The writhing number of a space curve. Proc. Natl. Acad. Sci. USA, 68:815–819, 1971.

[15] H. K. Moffatt and R. L. Ricca. Helicity and the călugăreanu invariant. Proc. R. Soc. Lond. A, 439:411–429, 1992.

[16] H. Hopf. über die abbildungen der dreidimensionalen sphäre auf die kugelfläche. Mathematische Annalen, 104:637–665, 1931.

[17] J. H. C. Whitehead. An expression of hopf's invariant as an integral. Proc. Natl. Acad. Sci. USA, 33:117–123, 1947.

[18] I. E. Dzyaloshinskii. A thermodynamic theory of "weak" ferromagnetism of antiferromagnetics. J. Phys. Chem. Solids, 4:241–255, 1958.

[19] T. Moriya. Anisotropic superexchange interaction and weak ferromagnetism. Phys. Rev., 120:91–98, 1960.

[20] A. Aharoni. Introduction to the Theory of Ferromagnetism. Oxford University Press, 1996.

[21] G. K. Batchelor. An Introduction to Fluid Dynamics. Cambridge University Press, 1967.

[22] G. K. Batchelor. An Introduction to Fluid Dynamics. Cambridge University Press, 1967.

[23] L. D. Landau and E. M. Lifshitz. Fluid Mechanics. Course of Theoretical Physics, Vol. 6. Pergamon, 2nd edition, 1987.

[24] W. G. Unruh. Experimental black-hole evaporation? Physical Review Letters, 46:1351–1353, 1981.

[25] M. Visser. Acoustic black holes: horizons, ergospheres and hawking radiation. Classical and Quantum Gravity, 15(6):1767–1791, 1998.

[26] P. Painlevé. La mécanique classique et la théorie de la relativité. Comptes Rendus de l'Académie des Sciences, 173:677–680, 1921.

[27] A. Gullstrand. Allgemeine lösung des statischen einkörperproblems in der einsteinschen gravitationstheorie. Arkiv för Matematik, Astronomi och Fysik, 16:1–15, 1922.

[28] Dale Rolfsen. Knots and Links. Publish or Perish, 1976.

[29] W. B. R. Lickorish. An Introduction to Knot Theory, volume 175 of Graduate Texts in Mathematics. Springer, 1997.

[30] Kunio Murasugi. Knot Theory and Its Applications. Birkhäuser, 1996.

[31] Jason Cantarella, Robert B. Kusner, and John M. Sullivan. On the minimum ropelength of knots and links. Inventiones mathematicae, 150(2):257–286, 2002.

[32] Eric J. Rawdon. Approximating the thickness of a knot. In A. Stasiak, V. Katrich, and L. Kauffman, editors, Ideal Knots, pages 143–150. World Scientific, 1998.

[33] Roy P. Kerr. Gravitational field of a spinning mass as an example of algebraically special metrics. Physical Review Letters, 11:237–238, 1963.

[34] Hans Ertel. Ein neuer hydrodynamischer erhaltungssatz. Meteorologische Zeitschrift, 59:271–281, 1942.

[35] G. Călugăreanu. Sur les classes d'isotopie des noeuds tridimensionnels et leurs invariants. Czechoslovak Mathematical Journal, 1961.

[36] W. Wien. Ueber die energieverteilung im emissionsspectrum eines schwarzen körpers. Annalen der Physik, 1894.

[37] M. Planck. On the law of distribution of energy in the normal spectrum. Annalen der Physik, 1901.

[38] David J. Morin. Chapter 11: Relativity (kinematics). https://bpb-us-e1.wpmucdn.com/sites.harvard.edu/dist/0/550/files/2023/11/cmchap11.pdf, 2007. Draft chapter from Introduction to Classical Mechanics.

[39] A. Einstein. Zur elektrodynamik bewegter körper. Annalen der Physik, 322(10):891–921, 1905.

[40] A. A. Michelson and E. W. Morley. On the relative motion of the earth and the luminiferous ether. American Journal of Science, 34:333–345, 1887.

[41] Serge Haroche and Jean-Michel Raimond. Exploring the Quantum: Atoms, Cavities, and Photons. Oxford University Press, 2006.

[42] Marlan O. Scully and M. Suhail Zubairy. Quantum Optics. Cambridge University Press, 1997.

[43] NIST. Codata recommended values of the fundamental constants (2018 update). https://physics.nist.gov/cuu/Constants/, 2019.

[44] Michael Tinkham. Introduction to Superconductivity. McGraw–Hill, 2 edition, 1996.

[45] John Clarke and Alex I. Braginski. The SQUID Handbook. Wiley-VCH, 2004.

[46] Mark Brittenham and Susan Hermiller. Unknotting number is not additive under connected sum. arXiv e-prints, June 2025.

[47] W. B. R. Lickorish. An Introduction to Knot Theory, volume 175 of Graduate Texts in Mathematics. Springer, 1997.

[48] Kunio Murasugi. On a certain numerical invariant of link types. Transactions of the American Mathematical Society, 117:387–422, 1965.

[49] Peter B. Kronheimer and Tomasz S. Mrowka. Gauge theory for embedded surfaces. i. Topology, 32(4):773–826, 1993.

[50] Jacob Rasmussen. Khovanov homology and the slice genus. Inventiones mathematicae, 182(2):419–447, 2004.

[51] Robion C. Kirby. Problems in low-dimensional topology. In Geometric Topology (Athens, GA, 1993), volume 2 of AMS/IP Stud. Adv. Math., pages 35–473. Amer. Math. Soc., 1997.

[52] C. N. Yang and R. L. Mills. Conservation of isotopic spin and isotopic gauge invariance. Physical Review, 96:191–195, 1954.

[53] Sheldon L. Glashow. Partial-symmetries of weak interactions. Nuclear Physics, 22:579–588, 1961.

[54] Steven Weinberg. A model of leptons. Physical Review Letters, 19:1264–1266, 1967.

[55] Abdus Salam. Weak and electromagnetic interactions. In N. Svartholm, editor, Elementary Particle Theory: Relativistic Groups and Analyticity, pages 367–377, Stockholm, 1968. Almqvist and Wiksell.