SST-Rosetta-v0.6: Translation Guide for Symbols, Macros, and Constants

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Abstract

This note provides a rigorous nomenclature concordance between the legacy VAM presentation and the Swirl–String Theory (SST) house style. It establishes a one-to-one mapping of symbols and terminology while preserving the underlying kinematics, operators, and calibrated constants. In particular, it fixes the canonical SST equalities

$$\rho_E = \tfrac{1}{2}\,\rho_f\,\|\mathbf{v}_{\circlearrowleft}\|^2, \qquad \rho_m = \rho_E/c^2, \qquad K = \frac{\rho_{\text{core}}\,r_c}{\|\mathbf{v}_{\circlearrowleft}\|\big|_{r=r_c}} = \frac{\rho_{\text{core}}\,r_c}{\|\mathbf{v}_{\circlearrowleft}\|\big|_{r=r_c}}, \quad \rho_f = K\,\Omega,$$

and records that all published numerical values for $\|\mathbf{v}_{\circlearrowleft}\|\big|_{r=r_c}$ (defined here as $\|\mathbf{v}_{\circlearrowleft}\|\big|_{r=r_c}$), r_c, ρ_{core}, the background density, and the sectoral force bounds carry over unchanged. The document includes compact translation tables (fields/kinematics/operators; densities/velocities/coarse–graining; global scales) and a minimal macro layer (`\rhoF`, `\rhoE`, `\rhoM`, `\rhoC`, `\vswirl`, `\vnorm`) to prevent notation drift in large projects. Legacy wording is restricted to historical citations; narrative prose adopts the neutral SST vocabulary (e.g., *foliation*, *swirl string*) without altering the mathematics. Compatibility is ensured both for standalone use (title page + metadata) and for modular inclusion (`\providecommand` guards and no additional package requirements). The result is a drop-in "translation guide" that guarantees dimensional consistency, unambiguous symbol usage, and reproducible cross-referencing across manuscripts that span the VAM→SST transition.

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1 SST–VAM Translation and Constant Overlaps (Extended)

Canonical equalities (SST form)

$$\rho_E = \tfrac{1}{2}\rho_f\,\|\mathbf{v}_{\circlearrowleft}\|^2, \qquad \rho_m = \rho_E/c^2,$$
$$K = \frac{\rho_{\rm core}\,r_c}{\|\mathbf{v}_{\circlearrowleft}\|\,|_{r=r_c}} = \frac{\rho_{\rm core}\,r_c}{\|\mathbf{v}_{\circlearrowleft}\|\,|_{r=r_c}}, \qquad \rho_f = K\,\Omega.$$

Dimensional check

$$\begin{aligned}
[\rho_f] &= \mathrm{kg\,m^{-3}}\\
[\|\mathbf{v}_{\circlearrowleft}\|] &= \mathrm{m\,s^{-1}}\\
[\rho_E] &= \mathrm{J\,m^{-3}}\\
[\rho_m] &= \mathrm{kg\,m^{-3}}\\
[K] &= \mathrm{kg\,m^{-3}\,s}
\end{aligned}$$

Chronos–Kelvin invariant (added for completeness)

$$\frac{D}{Dt}(R^2\omega) = 0 \quad \text{(incompressible, inviscid, barotropic, no reconnection).}$$

(Kelvin/Helmholtz circulation conservation in SST wording; see [8, 9, 6, 7].)

Temporal Ontology in SST

We distinguish absolute parameter time $\mathcal{N}$ (preferred foliation label), external observer time τ, and internal clocks carried by swirl strings: a phase accumulator $S(t)$ and a loop "proper time" T_s. These appear in the field equations and separate global synchronization from local rotational dynamics.

$\mathcal{N}$	Absolute time (foliation)	Global causal parameter
ν_0	Now-point	Localized synchronization label
τ	External/chronos time	Measured time of external observer
$S(t)$	Swirl clock	Internal phase memory along a string
T_s	String proper time	Loop-duration functional
$\mathbb{K}$	Kairos event	Topological/phase transition moment

Fields, kinematics, operators (mapping)

VAM (legacy)	SST (house)	Meaning	Units	Overlap
"æther time"	absolute time parametrization	foliation time label	—	Yes
$T(x)$	$T(x)$	scalar clock field	—	Yes
u_μ (unit "æther" vector)	u_μ (unit time-like field)	$u_\mu = \partial_\mu T/\sqrt{-g^{\alpha\beta}\partial_\alpha T\partial_\beta T}$	—	Yes
"vortex line(s)"	swirl string(s)	object name only	—	Yes
$B_{\mu\nu},\ H_{\mu\nu\rho}$	same	Kalb–Ramond 2-form; $H = \partial_{[\mu}B_{\nu\rho]}$	—	Yes
W_μ	W_μ	coarse-grained frame connection	—	Yes
$C(K),\ L(K),\ \mathcal{H}(K)$	same	crossing #, ropelength, hyperbolic proxy	—	Yes

Densities, velocities, coarse–graining (mapping)

VAM (legacy)	SST (macro)	Meaning	Units	Overlap
$\rho_0,\ \rho_{\text{æ}}^{\rm (fluid)},\ \rho_{\text{æ}}^{\rm (vacuum)}$	$\rho_f,\ \rho_f^{\rm bg}$ or $\rho_f^{(0)}$	effective fluid density	$\mathrm{kg\,m^{-3}}$	Yes
$\rho_{\text{æ}}^{\rm (core)},\ \rho_{\text{æ}}^{\rm (mass)}$	$\rho_{\rm core}$	core/material density	$\mathrm{kg\,m^{-3}}$	Yes
$\rho_{\text{æ}}^{\rm (energy)}$	ρ_E (or $\rho_{\rm core}c^2$)	energy density	$\mathrm{J\,m^{-3}}$	Yes
C_e (tangential)	$\|\mathbf{v}_{\circlearrowleft}\|\,|_{r=r_c}$	characteristic swirl speed (= $\|\mathbf{v}_{\circlearrowleft}\|$ at $r=r_e$)	$\mathrm{m\,s^{-1}}$	Yes
C_e (field form)	$\mathbf{v}_{\circlearrowleft}$	swirl-velocity *vector field*	$\mathrm{m\,s^{-1}}$	Add
C_e (scalar use)	$\|\mathbf{v}_{\circlearrowleft}\|\,|_{r=r_c}$	core magnitude of $\mathbf{v}_{\circlearrowleft}$	$\mathrm{m\,s^{-1}}$	Add
$K = \dfrac{\rho^{\rm (mass)}r_c}{C_e}$	$K = \dfrac{\rho_{\rm core}\,r_c}{\|\mathbf{v}_{\circlearrowleft}\|\,|_{r=r_c}}$	coarse–graining coefficient	$\mathrm{kg\,m^{-3}\,s}$	Add
Ω	Ω	leaf angular rate	$\mathrm{s^{-1}}$	Yes

Global scales and bounds

VAM (legacy)	SST (house)	Meaning	Units	Overlap
$F^{\max}$ (Coulomb)	$\mathrm{F}^{\max}_{\mathrm{EM}}$	Coulomb-sector bound	N	Yes
$F^{\max}_{\mathrm{gr}}$ (Universal)	$\mathrm{F}^{\max}_{\mathrm{G}}$	gravitational/universal bound	N	Yes
Γ	Γ	loop circulation	$\mathrm{m}^2\,\mathrm{s}^{-1}$	Yes
Ω_R, Ω_c	same	outer rigid vs. core spin	s^{-1}	Yes

Numeric overlaps (published values)

Quantity	Symbol (SST)	Value	Units
Characteristic swirl speed	$\|\mathbf{v}_{\circlearrowleft}\|\|_{r=r_c}$ $(\equiv \|\mathbf{v}_{\circlearrowleft}\|\|_{r=r_c})$	1,093,845.63	$\mathrm{m\,s}^{-1}$
Core radius	r_c	$1.40897017 \times 10^{-15}$	m
Core density	ρ_{core}	$3.8934358266918687 \times 10^{18}$	$\mathrm{kg\,m}^{-3}$
Background density	ρ_f^{bg}	7.0×10^{-7}	$\mathrm{kg\,m}^{-3}$
Max Coulomb force	$\mathrm{F}^{\max}_{\mathrm{EM}}$	29.053507	N
Max universal force	$\mathrm{F}^{\max}_{\mathrm{G}}$	3.02563×10^{43}	N

Macro glossary (house style)

Use the macros to avoid drift:

$$\rho_f \text{ (effective density)}, \quad \rho_E \text{ (energy density)}, \quad \rho_m \text{ (mass-equivalent)}$$

ρ_{core} (core density), $\mathbf{v}_{\circlearrowleft}$ (swirl velocity vector), $\|\mathbf{v}_{\circlearrowleft}\| = \|\mathbf{v}_{\circlearrowleft}\|$ (speed magnitude at a point).

Energy vs mass-equivalent (clarification). ρ_E is an *energy density*; $\rho_m = \rho_E/c^2$ is the corresponding local mass-equivalent. *Note.* ρ_{core} is a calibration *constant*. The mass-equivalent density is a *field* $\rho_m(x) = \rho_E(x)/c^2$. In the core-saturation evaluation $\rho_E^{\mathrm{core}} = \rho_{\mathrm{core}}\, c^2$, one has $\rho_m^{\mathrm{core}} = \rho_{\mathrm{core}}$.

Prose guardrails (rebrand policy)

Use *foliation* and *swirl string(s)* in narrative text. Reserve legacy words ("æther", "vortex") strictly for quoting historical titles or citations. Retain *vorticity* as standard.

Sentence rewrites (examples)

Legacy: "The æther sector fixes the vortex core density."
SST: "The *foliation* sector fixes the *core density* ρ_{core} of the swirl string."
Legacy: "Kelvin's vortex theorem implies conserved $R^2\omega$."
SST: "Kelvin's *circulation* theorem implies $\frac{D}{Dt}(R^2\omega) = 0$ under incompressible, inviscid, barotropic flow."

2 Rosetta Concordance of SST and Mainstream Terminology

Chronometric and Metric Layer

SST Term	Mainstream Equivalent	Context / Reference
Swirl clock $S_t^{\circlearrowleft}$	Khronon / preferred-foliation scalar $T(x)$	Einstein–Æther, Hořava–Lifshitz gravity [1, 2]
Swirl time dilation $d\tau = dt\sqrt{1 - \|\mathbf{v}_{\circlearrowleft}\|^2/c^2}$	Æther lapse factor N	ADM decomposition
Chronos–Kelvin invariant	Kelvin's circulation / helicity conservation	Classical fluid invariants [7]
Swirl-foliation metric $g_{\mu\nu}^{(\mathrm{swirl})}$	Acoustic / analogue-gravity metric	Unruh–Visser emergent metric [12]

Fluid–Dynamic and Field Layer

SST Term	Mainstream Name / Concept	Relation
Swirl velocity field $\mathbf{v}_\circlearrowleft$	Superfluid phase-gradient velocity	$\mathbf{v} = (\hbar/m)\nabla\phi$ analogue
Effective density ρ_f	Superfluid or effective mass density	Incompressible limit
Swirl energy density ρ_E	Kinetic-energy density $\frac{1}{2}\rho v^2$	Barotropic flow
Swirl pressure gradient	Hydrodynamic pressure field $p(\rho)$	Euler acceleration source
Swirl tensor $\omega_{ij} = \partial_i v_j - \partial_j v_i$	Vorticity tensor	Identical operator

Topological and Knot Layer

SST Term	Mainstream Equivalent	Connection
Swirl string	Quantized vortex filament / Nielsen–Olesen string	Fluid–string duality
Knot-energy functional $\mathcal{E}_{\text{eff}} = \alpha C + \beta L + \gamma \mathcal{H}$	Moffatt–Faddeev–Skyrme functional	Topological soliton energy
Swirl helicity $\mathcal{H} = \int \mathbf{v}\cdot\boldsymbol{\omega}\, dV$	Fluid / magnetic helicity	Linkage and twist invariant [3, 4]
Hopf charge H_{vortex}	Hopf invariant	$\pi_3(S^2)$ topological index [5]
Golden-layer factor φ^{-2k}	Discrete-scale-invariance factor	Renormalization-group analog

Gauge and Quantum Layer

SST Concept	Mainstream Analogue	Mapping
Multi-director swirl symmetry	Gauge algebra $\mathfrak{su}(3)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1)$	Emergent gauge structure
Swirl-string excitations	Unknotted field quanta / gauge bosons	Photon, gluon, . . . as unknots
Chiral swirl orientation	CPT-conjugate sectors	Matter antimatter dual
Swirl potential $\Phi = \frac{1}{2}\lVert\mathbf{v}_\circlearrowleft\rVert^2$	Kinetic / gravitational potential analog	Replaces GR curvature scalar
Mass functional $M = \frac{1}{\varphi}\frac{4}{\alpha}(\frac{1}{2}\rho_f\lVert\mathbf{v}_\circlearrowleft\rVert^2 V)$	Energy–mass equivalence	Mechanical derivation of rest mass

Gravitational and Large-Scale Layer

SST Term	Mainstream Name	Comment
Swirl-gravity constant G_{swirl}	Newtonian G (effective)	Derived from swirl mechanics
Pressure-well curvature	Gravitational potential well	Fluid-mechanical analog of curvature
Swirl potential waves	Gravitational / acoustic waves	Analogue-gravity mode
Torsional shocks	Nonlinear vorticity / spin-density waves	Possible new radiation class

Thermodynamic and Entropic Layer

SST Feature	Mainstream Analog	Interpretation
Swirl entropy growth	Enstrophy / helicity cascade	Entropy production in turbulence
Swirl-dissipation arrow	Thermodynamic arrow of time	Irreversibility via reconnection
Coherence factor $\xi(n) = 1 + \beta \log n$	Quantum-coherence correction	Many-body renormalization analog

Summary Table

SST Structural Layer	Physics Discipline	Closest Mainstream Equivalent
Chronometric	Lorentz-violating gravity	Khronon field / preferred foliation
Fluid–Dynamic	Superfluid hydrodynamics	Velocity, pressure, density fields
Topological	Soliton and knot theory	Hopfions, Skyrmions, helicity
Gauge–Quantum	Quantum field theory	Gauge algebra and field quanta
Gravitational	Analogue gravity / GR limit	Metric potentials, G analog
Thermodynamic	Non-equilibrium physics	Entropic and causal arrows

Scale-dependent Effective Densities in SST

Effective densities (house style).

$$\rho_f \equiv \text{effective fluid density}, \qquad \rho_E \equiv \tfrac{1}{2}\,\rho_f\,\|\mathbf{v}_{\circlearrowleft}\|^2 \quad \text{(swirl energy density)},$$

$$\rho_m \equiv \rho_E/c^2 \quad \text{(mass-equivalent density)}.$$

Background value: $\rho_f^{\text{bg}} \approx 7.0\times10^{-7}\ \text{kg}\,\text{m}^{-3}$. Core (material) density: $\rho_{\text{core}} \approx 3.8934358267\times10^{18}\ \text{kg}\,\text{m}^{-3}$. Hence core energy density

$$\rho_E^{\text{core}} = \rho_{\text{core}}\,c^2 \approx 3.499\times10^{35}\ \text{J}\,\text{m}^{-3}.$$

Radial profile (phenomenology). It is convenient to model the near-core energy density with an exponential relaxation to the background:

$$\rho_E(r) = \rho_E^{\text{bg}} + \left(\rho_E^{\text{core}} - \rho_E^{\text{bg}}\right) e^{-r/r_*},$$

with a microscopic decay scale r_* (fit parameter). This empirical profile does not replace the exact tube energetics below.

String energetics (Rankine core + irrotational envelope). For a core of radius r_c and length ℓ with solid-body rotation $v_\phi(r) = \Omega r$ for $r \le r_c$,

$$E_{\text{core}} = \int_0^{r_c} \frac{1}{2}\,\rho_f\,(\Omega r)^2\,(2\pi r\,\ell)\,dr = \frac{\pi}{4}\,\rho_f\,\Omega^2\,r_c^4\,\ell.$$

Outside the core, $v_\phi(r) = \Gamma/(2\pi r)$ with $\Gamma = 2\pi\Omega r_c^2$, giving the slender-tube envelope term

$$E_{\text{env}} \simeq \frac{\rho_f\,\Gamma^2}{4\pi}\,\ell\,\ln\frac{R}{r_c},$$

where R is an outer cutoff set by the nearest boundary or neighboring strings. Both contributions are standard in vortex-tube energetics (core + Biot–Savart envelope).

Coarse-graining. At macroscales, we use the canonical identity

$$K = \frac{\rho_{\text{core}}\,r_c}{\|\mathbf{v}_{\circlearrowleft}\|\,|_{r=r_c}} = \frac{\rho_{\text{core}}\,r_c}{\|\mathbf{v}_{\circlearrowleft}\|\,|_{r=r_c}}, \qquad \rho_f = K\,\Omega_{\text{leaf}}.$$

where Ω_{leaf} is a coarse-grained (leaf-averaged) angular rate. Numerically, $\Omega_{\text{leaf}} \sim 10^{-4}\,\text{s}^{-1}$ in the Canon fit; it must not be confused with the microscopic core rate below.

3 Layered Time Scaling from Swirl Dynamics

Adopt the SR-like local rule

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v_\phi^2(r)}{c^2}}.$$

With a Rankine profile,

$$v_\phi(r) = \begin{cases} \Omega_{\text{core}}\,r, & r \le r_c, \\ \dfrac{\Gamma}{2\pi r}, & r \ge r_c, \end{cases} \qquad \Gamma = 2\pi\Omega_{\text{core}}\,r_c^2.$$

Continuity at $r = r_c$ gives $v_\phi(r_c) = \Omega_{\rm core}\, r_c \equiv \|\mathbf{v}_{\circlearrowleft}\||_{r=r_c} = \|\mathbf{v}_{\circlearrowleft}\||_{r=r_c}$, hence

$$\Omega_{\rm core} = \frac{\|\mathbf{v}_{\circlearrowleft}\||_{r=r_c}}{r_c} = \frac{\|\mathbf{v}_{\circlearrowleft}\||_{r=r_c}}{r_c} \approx \frac{1.09384563 \times 10^6}{1.40897017 \times 10^{-15}} \approx 7.763 \times 10^{20}\ \mathrm{s}^{-1}.$$

Thus

$$\frac{d\tau}{dt} = \begin{cases} \sqrt{1 - \dfrac{\Omega_{\rm core}^2\, r^2}{c^2}}, & r \le r_c, \\ \sqrt{1 - \dfrac{\Gamma^2}{4\pi^2 c^2 r^2}}, & r \ge r_c. \end{cases}$$

The earlier ansatz $d\tau/d\bar{t} = e^{-r/r_c}$ can be used only as a phenomenological fit; it does not follow from the SR-like form unless one imposes a special $v_\phi(r)$ inconsistent with Rankine.

Rosetta Card: Einstein–Æther/Khronon → SST (Swirl–Clock)

Domain: preferred-frame EFTs; GW constraints; PPN.

Dictionary:

EA field	SST counterpart	Notes
Unit timelike u^μ	normalized Swirl–Clock four-velocity	picks foliation
c_i couplings	effective foliation elasticities	mapped by calibration
c_T (spin-2 GW speed)	luminal by construction	enforce $c_{13} = c_1 + c_3 \simeq 0$

Action map (symbolic):

$$S_{æ} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[-R - K_{ab}{}^{mn}\nabla^a u_m \nabla^b u_n + \lambda(u^\mu u_\mu + 1)\Big],$$
$$K_{ab}{}^{mn} = c_1 g_{ab} g^{mn} + c_2 \delta_a{}^m \delta_b{}^n + c_3 \delta_a{}^n \delta_b{}^m + c_4 u_a u_b g^{mn}.$$

GW calibration:

impose $c_{13} = 0 \Rightarrow c_T^2 = 1$ (luminal spin-2). Spin-1 and spin-0 mode speeds are functions of c_i (see table in source); choose parameter ranges that avoid instabilities/Čerenkov bounds.

Numerical anchor (GW170817 class):

$$|c_T - 1| \lesssim 10^{-15} \;\Rightarrow\; |c_{13}| \lesssim 10^{-15} \quad \text{(imposed)}.$$

Predictions & Falsifiers:

- With $c_{13} = 0$, SST's Swirl–Clock foliation is consistent with coincident GW–EM arrival.
- Dipole/monopole radiation channels are suppressed by calibration choices; detection at current pulsar-timing sensitivity would falsify this mapping.

Status: Calibration (GW speed) / Research (spin-0/1 sector) **Version:** SST-Rosetta v0.5.10

Rosetta Card: GR/PPN/GW $\to$ Swirl–String Theory (SST)

Domain: weak–field, stationary backgrounds; lensing, Shapiro delay, PPN.

Symbol Dictionary:

GR object	SST counterpart	Notes
$g_{\mu\nu}$ (weak field)	analogue metric via Swirl–Clock	matter propagation sector
Φ (Newtonian potential)	$\Phi_{\rm SST}$ from swirl energy fraction	defined below
$T^{\mu\nu}$	swirl stress $(\rho_E, p_{\rm swirl}, \ldots)$	barotropic, inviscid
c	c	calibrated to luminal signals

EOM / Metric Map (linearized):

$$ds^2_{\rm GR} \approx -\left(1+\frac{2\Phi}{c^2}\right)c^2dt^2 + \left(1-\frac{2\gamma\,\Phi}{c^2}\right)d\mathbf{x}^2.$$

Define the local swirl energy density and maximum energy density

$$U_{\rm swirl} = \tfrac{1}{2}\rho_f\|\mathbf{v}_{\circlearrowleft}\|^2, \qquad U_{\max} = \rho_{\rm core}c^2, \qquad \chi_{\rm swirl} = \frac{U_{\rm swirl}}{U_{\max}} \text{ (dimensionless)},$$

and map

$$g_{tt} = -(1-\chi_{\rm swirl})\,, \qquad g_{ij} = (1+\gamma\,\chi_{\rm swirl})\,\delta_{ij}.$$

Matching coefficients gives

$$\boxed{\Phi_{\rm SST} \equiv -\frac{U_{\rm swirl}}{2\,\rho_{\rm core}}} \quad\Rightarrow\quad \frac{2\Phi_{\rm SST}}{c^2} = -\chi_{\rm swirl}, \;\; \gamma = 1 \text{ (Calibration)}.$$

Dimensional check: $[U_{\rm swirl}] = \mathrm{J/m^3}$, $[\rho_{\rm core}] = \mathrm{kg/m^3}$, so $U_{\rm swirl}/\rho_{\rm core}$ has units $\mathrm{J/kg} = \mathrm{m^2/s^2}$, as required for Φ.

Numerical validation (SST constants):

$$\kappa = 2\pi r_c\|\mathbf{v}_{\circlearrowleft}\| = 9.68\times10^{-9}\,\mathrm{m^2/s}, \quad U_{\rm swirl} = \tfrac{1}{2}\rho_f\|\mathbf{v}_{\circlearrowleft}\|^2 = 4.19\times10^{5}\,\mathrm{J/m^3},$$

$$U_{\max} = \rho_{\rm core}c^2 = 3.50\times10^{35}\,\mathrm{J/m^3}, \quad \chi_{\rm swirl} = U_{\rm swirl}/U_{\max} = 1.20\times10^{-30},$$

$$\Phi_{\rm SST} = -\frac{U_{\rm swirl}}{2\rho_{\rm core}} = -5.38\times10^{-14}\,\mathrm{m^2/s^2}, \quad \left|\frac{2\Phi_{\rm SST}}{c^2}\right| = 1.20\times10^{-30}.$$

Known-limit check: $|\chi_{\rm swirl}| \ll 1 \Rightarrow$ PPN weak-field holds with $\gamma = 1$.

Predictions & Falsifiers

- Lensing/Shapiro from $\Phi_{\rm SST}$ matches GR to $\mathcal{O}(\chi_{\rm swirl})$; deviations scale with spatial gradients of $U_{\rm swirl}$.
- High-frequency GW propagation luminal (Calib.) $\Rightarrow$ multi-messenger bounds satisfied.
- Falsifier: any measured $\gamma \neq 1$ at 10^{-5}–10^{-6} in quasi-static fields contradicts this mapping.

Status: Calibration **Version:** v0.5.10

Rosetta Card: Maxwell/QED → SST (multi-director swirl)

Domain: radiation sector; vacuum/linear media analogues.

Dictionary:

EM object	SST counterpart	Notes
A_μ	director phase gradient $\partial_\mu\theta$	Abelian sector
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$	swirl curvature of director field	circulation quantized
Charge q	knot/link index (topological)	integer invariants
Flux quantum	$\kappa = 2\pi r_c \lVert \mathbf{v}_\circlearrowleft \rVert$	9.68×10^{-9} m^2/s
Poynting $\mathbf{S}$	energy flux of Kelvin–swirl waves	$\sim U_{\text{swirl}}\, \mathbf{v}_{\text{ph}}$

Lagrangian/EOM map (linearized, uniform background):

$$\mathcal{L}_{\text{EM}} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \;\longleftrightarrow\; \mathcal{L}^{(\theta)}_{\text{SST}} = \tfrac{1}{2}\left[\frac{1}{c^2}(\partial_t\theta)^2 - |\nabla\theta|^2\right] U_{\max},$$

yielding the wave equation

$$\partial_t^2\theta - c^2\nabla^2\theta = 0 \quad \text{(calibrated luminal phase speed).}$$

In inhomogeneous multi-director fields, polarization-dependent phase shifts (vacuum-like birefringence) enter via curvature of the director manifold.

Numerical anchor:

circulation quantum $\kappa = 9.68 \times 10^{-9}$ m^2/s fixes the smallest swirl-flux unit consistent with r_c and $\mathbf{v}_\circlearrowleft$.

Dimensional checks:

$\mathcal{L}^{(\theta)}_{\text{SST}}$ has units of energy density by the factor $U_{\max}$.

Predictions & Falsifiers:

- Plane-wave dispersion $\omega = ck$ in uniform regions; gradients in θ produce tiny polarization-dependent delays $\propto \nabla^2\theta / U_{\max}$.
- Falsifier: vacuum birefringence above current bounds in high-energy astrophysical spectra would contradict the calibration.

Status: Canonical (kinematics) / Research (multi-director birefringence) **Version:** v0.5.10

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