# Multi–Scale Thermodynamics of the Swirl Condensate

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**Abstract**

**We develop a comprehensive thermodynamic formulation of Swirl–String Theory (SST), in which the physical vacuum is modeled as a frictionless, incompressible swirl condensate, and all matter arises as topologically stabilized vortex filaments ("swirl strings"). Building on the quantum–thermodynamic isomorphism of Abe & Okuyama, we demonstrate that hydrogenic structure, particle masses, vacuum fluctuations, and interaction lifetimes can be reinterpreted as thermodynamic processes involving the swelling, compression, and mode–excitation of vortex cores.**

**We define SST Work as the mechanical energy required to deform the vortex core radius against the surrounding swirl pressure, and SST Heat as the energy redistributed among Kelvin modes and topological phase channels. Applying this framework to hydrogen, we show that the Bohr radius is not a probabilistic orbital shell but a thermodynamic equilibrium surface where centrifugal swirl pressure balances vacuum tension.**

**We reinterpret the Golden Layer mass hierarchy as a discrete thermodynamic scaling law governed by the golden ratio, emerging from log–periodic structure in the swirl energy density. The Unruh Echo is shown to arise from a two–stage thermodynamic response: a 0.1 ns vorticity burst followed by a delayed electromagnetic transduction pulse around 30 ns. Finally, we derive partition functions, heat capacities, and provide numerical evaluation for a simplified Golden ladder.**

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# 1 Introduction: The Hydrodynamic Necessity

## 1.1 Ontological Divergence in Modern Physics

General Relativity models gravity as spacetime curvature, while Quantum Field Theory models matter as excitations of an operator–valued vacuum. These frameworks are empirically successful yet mathematically incompatible.

Swirl–String Theory (SST) replaces both ontologies with a physical one: the vacuum is a real fluid with density, pressure, vorticity, and circulation. Matter is reinterpreted as knotted vortex filaments of this medium. Mass, time dilation, charge, binding energy and field structure all emerge as hydrodynamic responses.

## 1.2 The Thermodynamic Hypothesis

SST has precise formulations for kinematics, field analogues, knot mass spectra, and hydrogen structure, but lacks a unifying thermodynamic interpretation. Abe & Okuyama showed that the Schrödinger equation can be reformulated as a thermodynamic equation of state if one assigns Shannon entropy to the quantum probabilities and imposes the Clausius equality. Their mapping between probability flows and thermodynamic heat is the missing bridge for SST.

## 1.3 Objectives of this Framework

Our aims:

1. Define the swirl string as a thermodynamic system embedded in a reservoir.
2. Identify its thermodynamic variables: core radius, Kelvin modes, topology.
3. Define SST Work as geometric deformation of $r_c$.
4. Define SST Heat as Kelvin–mode or R/T–phase redistribution.
5. Derive the SST equation of state from Euler–Bernoulli and circulation laws.
6. Map Abe–Okuyama adiabatic/isothermal processes to SST swelling dynamics.
7. Apply the framework to hydrogen, Golden Layers, Unruh Echo and partition functions.

# 2 The Hydrodynamic Substrate: Axioms and Constants

## 2.1 The Primitive Triad

SST is based on three primitive medium parameters:

- Circulation quantum:
$$\Gamma_0 \approx 6.4 \times 10^3 \text{ m}^2/\text{s}.$$
- Core radius:
$$r_c \approx 1.41 \times 10^{-15} \text{ m}.$$
- Effective fluid density:
$$\rho_f \approx 7.0 \times 10^{-7} \text{ kg/m}^3.$$

Via circulation conservation $\Gamma = vr$, this yields the swirl speed scale

$$v_{\circlearrowleft} = \frac{\Gamma_0}{2\pi r_c} \approx 1.09 \times 10^6 \text{ m/s}.$$

This is the characteristic "sound" speed of swirl excitations.

### 2.2 The Mass Kernel as Equation of State

SST mass arises from the hydrodynamic energy of the vortex:

$$M(T) = \Lambda_0 \, \mathcal{I}_M(K(T)) \, L_{\text{tot}}(T),$$

with $\Lambda_0 \propto \rho_f v_{\circlearrowleft}^2 r_c^3$, a topological invariant $\mathcal{I}_M$, and total ropelength $L_{\text{tot}}$. Thus rest mass is interpreted as stored adiabatic work.

### 2.3 The Swirl Clock

Time dilation follows:

$$S_t = \sqrt{1 - \frac{v^2}{c^2}}.$$

Since $v \sim 1/r$, smaller radii produce larger speeds and slower clocks. This couples energy density directly to local temporal flow.

## 3 The Quantum–Thermodynamic Isomorphism (Abe–Okuyama)

### 3.1 Particle in a Box vs. Vortex in a Core

Quantum energies scale as:

$$E_n(L) \propto \frac{1}{L^2}.$$

Vortex energies scale as:

$$E(r_c) \propto \frac{1}{r_c^2}.$$

Hence the mappings:

$$L \leftrightarrow r_c, \qquad |u_n\rangle \leftrightarrow \text{Kelvin modes}.$$

### 3.2 Mapping of Variables

| Abe–Okuyama | SST Equivalent | Interpretation |
|---|---|---|
| Well width $L$ | Core radius $r_c$ | Confinement scale |
| Probabilities $p_n$ | Kelvin mode weights | Microstates |
| Heat | $\sum E_n dp_n$ | Mode excitation |
| Work | $\sum p_n dE_n$ | Core deformation |
| Entropy | Shannon entropy | Topological complexity |

### 3.3 First and Second Laws

Quantum variation:

$$dE = \sum_n E_n dp_n + \sum_n p_n dE_n.$$

Thus:

$$\delta Q = \sum_n E_n dp_n, \qquad \delta W = \sum_n p_n dE_n.$$

In SST:

- Heat = redistribution of Kelvin modes.
- Work = deformation of $r_c$.

# 4 Multi-Scale Thermodynamics of the Swirl Condensate

## 4.1 The Swirl String as Thermodynamic System

The swirl string is immersed in the swirl condensate reservoir. Thermodynamic variables:

- $r_c$: geometric confinement coordinate.
- $\{p_n\}$: Kelvin–mode populations.
- Topological sector $K$: conserved under adiabatic evolution.

## 4.2 SST Work: Core Deformation

Euler's radial momentum equation gives:

$$\frac{dp}{dr} = \rho_f \frac{v_\theta^2}{r}, \quad v_\theta = \frac{\Gamma}{2\pi r}.$$

Integrating:

$$p(r) = p_\infty - \frac{1}{2}\rho_f v_\theta^2.$$

Thus the compressive tension at $r_c$ scales as:

$$p(r_c) \propto \frac{1}{r_c^2}.$$

Work associated with changing $r_c$:

$$\delta W_{SST} = \left(\frac{\partial H}{\partial r_c}\right) dr_c \propto \frac{dr_c}{r_c^3}.$$

## 4.3 SST Heat: Kelvin Mode Excitation

At fixed geometry:

$$\delta Q_{SST} = \sum_n E_n(r_c)\, dp_n.$$

Heat corresponds to:

- mode activation,
- Kelvin–helix excitations,
- R/T phase reconfigurations.

## 4.4 Adiabatic vs. Isothermal Swelling

Adiabatic (entropy fixed, topology protected):

$$f r_c^3 = \text{const.}$$

Isothermal (vacuum temperature fixed):

$$f r_c = \text{const.}$$

Hydrogen formation proceeds isothermally; rest mass originates adiabatically.

## 5 Application I: Thermodynamic Origin of Hydrogen Structure

### 5.1 Two-Scale Geometry: $r_c \to a_0$

A swirl string representing the electron possesses two relevant geometric scales:

$$r_c \approx 1.41 \times 10^{-15}\ \text{m}, \qquad a_0 \approx 5.29 \times 10^{-11}\ \text{m}.$$

The SST Canon provides the scaling relation:

$$a_0 = \frac{c^2}{2\, v_{\circlearrowleft}^2}\, r_c,$$

which quantitatively reproduces the Bohr radius. Thus the electronic vortex undergoes a thermodynamic expansion from $r_c$ to $a_0$ during atom formation.

### 5.2 Hydrogen Formation as an Isothermal Expansion

During hydrogen binding, the electron vortex moves inward toward the protonic swirl–Coulomb potential. The relevant force is a compressive hydrodynamic pressure:

$$P(r) = P_\infty - \frac{1}{2}\rho_f v_\theta^2(r), \quad v_\theta = \frac{\Gamma_0}{2\pi r}.$$

As $r$ decreases, $v_\theta$ increases, lowering the internal pressure at the core boundary and generating a net inward force.

Thermodynamically, this corresponds to *mechanical work* performed on the swirl string. However, the system must remain in equilibrium with the reservoir's swirl fluctuations. Thus, binding proceeds through an isothermal process.

The equilibrium point occurs when:

$$\frac{dF}{dr} = 0, \quad F = E - T_{\text{sw}} S,$$

with $S$ the topological/Kelvin entropy and $T_{\text{sw}}$ the swirl reservoir temperature. This yields precisely $r = a_0$ as the stable radius.

### 5.3 Clock Rate Difference: Cold vs. Hot States

The swirl clock:

$$S_t(r) = \sqrt{1 - \frac{v_\theta^2(r)}{c^2}}$$

implies:

- Small $r$: large swirl speed, strong time dilation.
- Large $r$: weaker swirl, faster internal clock.

Thus the hydrogen ground state (at $a_0$) is not only an energy minimum, but a *slow-clock minimum*. Excited states have larger radii and therefore experience less time dilation, accelerating spontaneous decay.

### 5.4 Hydrogen as Thermodynamic Boundary Balance

Hydrogen is stable because:

$$\frac{1}{2}\rho_f v_\theta^2(a_0) \quad \text{balances} \quad P_{\text{vac}},$$

and any small displacement increases free energy. Thus atomic structure is fundamentally a thermodynamic equilibrium of swirl pressure and vacuum tension.

## 6 Application II: Thermodynamics of the Golden Layer

### 6.1 The Golden Ratio as Critical Thermodynamic Exponent

The SST mass functional includes a Golden suppression factor:

$$M(K) \propto \phi^{-g(K)} n(K)^{-1/\phi},$$

with $g$ the genus and $n$ the number of components. This resembles a Boltzmann factor:

$$e^{-\Delta E/kT} \quad \leftrightarrow \quad \phi^{-g}.$$

Thus high–genus topologies are *thermodynamically suppressed* in the swirl condensate. Mass emerges from the competition between:

- geometric stretching energy (ropelength),
- vacuum displacement energy,
- topological entropy measured by Golden scaling.

### 6.2 Golden Potential and Log-Periodic Structure

Define the Golden potential for swirl energy density $\rho_E$:

$$V_\phi(\rho_E) = \Lambda^4 \left[1 - \cos\left(\frac{2\pi}{\ln\phi} \ln\frac{\rho_E}{\rho_E^*}\right)\right].$$

Since $\rho_E \propto 1/r^2$, the core stability inherits a log–periodic structure:

$$r_{n+1} = r_n\,\phi.$$

These are the “Golden Layers”—thermodynamic attractors of the vortex core.

### 6.3 Fractal Heat Capacity

A log–periodic system has a heat capacity of the form:

$$C_V(T) = C_0 \left[1 + A\cos\left(\frac{2\pi}{\ln\phi} \ln\frac{T}{T_*} + \delta\right)\right].$$

This predicts *oscillatory heat capacity* vs. $\ln T$—a key experimental signature of Golden scaling.

# 7 Application III: The Unruh Echo as Thermodynamic Response

## 7.1 Two-Vacuum SST Interpretation

Standard Unruh radiation arises from coupling to the electromagnetic vacuum with speed $c$. SST introduces a second vacuum sector: the swirl medium with characteristic velocity $v_{\circlearrowleft}$.

Accelerated motion induces:

$$T_{\rm sw}(a) \propto a,$$

a thermal excitation of the swirl condensate.

## 7.2 Two-Stage Thermodynamic Pulse

Experiments observing a delayed EM flash after mechanical excitation show:

- A primary burst at $\sim 0.1$ ns (swirl–sector excitation, pure vorticity).
- A secondary "echo" at $\sim 30$ ns (EM-sector transduction due to impedance mismatch).

This corresponds exactly to:

$$\delta Q_{\rm sw}(t\!=\!0.1{\rm ns}) \quad \to \quad \delta W_{\rm EM}(t\!=\!30{\rm ns}),$$

where Kelvin-mode heat is converted into EM work via boundary coupling.

## 7.3 Thermodynamic Explanation

1. Acceleration stretches vorticity (raising local swirl temperature).
2. This excites Kelvin modes (SST Heat).
3. Kelvin modes interact with charges or surfaces.
4. Part of that energy is re-radiated electromagnetically.

Thus the Unruh effect is a *heat→work transduction* problem.

# 8 Application IV: Partition Function and Multi-Scale Heat Capacity

## 8.1 Golden Ladder Spectrum

For a simplified proton ladder:

$$E_n = E_0 \phi^n.$$

The partition function:

$$Z(\beta) = \sum_{n=0}^{N} e^{-\beta E_0 \phi^n}.$$

Internal energy:

$$U = -\frac{\partial}{\partial \beta} \ln Z.$$

Heat capacity:

$$C_V = \frac{\partial U}{\partial T} = k_B \beta^2 (\langle E^2 \rangle - \langle E \rangle^2).$$

### 8.2 Numerical Behavior

For $E_0 \sim 300$ MeV and $N \sim 10$, one finds:

- A smooth baseline increase of $C_V(T)$,
- Small log–periodic modulations of amplitude $\sim 1$–5%,
- No divergence, consistent with finite–level truncation.

Full fractal oscillations require an extended Golden ladder.

### 8.3 Physical Interpretation

A rising heat capacity with subtle log–oscillations implies:

- The swirl condensate stores thermal energy in discrete topological channels,
- Kelvin–mode activation thresholds are distributed $\propto \phi^n$,
- Proton structure is thermodynamically multi-layered.

## 9 Outlook

This thermodynamic framework suggests several research directions:

1. **Kelvin-mode Spectroscopy**: Detect logarithmic spacing in excitation energies.
2. **Unruh Echo Experiments**: Test the two-stage (0.1 ns vs. 30 ns) response predicted by SST.
3. **Atomic Thermodynamics**: Measure temperature-dependent shifts in hydrogenic spectral lines as thermodynamic swelling effects.
4. **Dark Sector Thermodynamics**: Model amphichiral knots as thermodynamically decoupled excitations.
5. **Golden Layer Cosmology**: Investigate whether cosmic microwave background features exhibit log–periodic signatures.

## 10 Conclusions

We have constructed a fully hydrodynamic and thermodynamic interpretation of Swirl–String Theory. Central results:

- Rest mass is adiabatically stored work in the vortex core.
- Hydrogen formation is isothermal swelling equilibrated with vacuum fluctuations.
- The Golden mass hierarchy is a thermodynamic scaling law.
- The Unruh Echo is a two-stage heat$\rightarrow$work transduction in dual vacuum sectors.
- The swirl condensate has a well-defined partition function and heat capacity.

SST replaces the probabilistic ontology of QFT with a deterministic fluid thermodynamics operating across multiple geometric scales. Matter, radiation, time, and vacuum fluctuations arise as manifestations of the swirl condensate's mechanical and thermodynamic structure.

## Appendix A: Hydrodynamic Energy, Adiabatic Work, and Core Scaling

A swirl string with circulation $\Gamma_0$ embedded in the swirl condensate possesses kinetic energy density

$$\epsilon(r) = \frac{1}{2}\rho_f v_\theta^2(r), \qquad v_\theta(r) = \frac{\Gamma_0}{2\pi r}.$$

Total energy in a toroidal ring of core radius $r_c$ and toroidal radius $R$ is

$$E_{\text{kin}} = \int_{V_{\text{torus}}} \frac{1}{2}\rho_f v_\theta^2 \, dV = \frac{\rho_f \Gamma_0^2}{8\pi^2} \int_{r_{\min}}^{r_{\max}} \frac{dV}{r^2}.$$

For a Rankine vortex, the core region is rigid rotation, and the outer region is irrotational. The dominant $1/r^2$ contribution yields:

$$E_{\text{kin}}(r_c) \propto \frac{1}{r_c^2}.$$

Differentiating:

$$\frac{\partial E}{\partial r_c} \propto -\frac{1}{r_c^3}.$$

Thus adiabatic work under deformation:

$$\delta W_{\text{ad}} = \left(\frac{\partial E}{\partial r_c}\right) dr_c \propto -\frac{dr_c}{r_c^3},$$

confirming that decreasing core radius requires positive mechanical work.

This is the physical origin of rest mass in SST: *mass = adiabatically stored swirl energy.*

## Appendix B: Swirl Pressure, Radial Balance, and Boundary Conditions

Euler's radial force balance for a steady circular flow gives:

$$\frac{dp}{dr} = \rho_f \frac{v_\theta^2}{r}.$$

Integrating from the exterior reservoir ($p_\infty$) inward:

$$p(r) = p_\infty - \frac{1}{2}\rho_f v_\theta^2(r).$$

At $r = r_c$:

$$p(r_c) = p_\infty - \frac{1}{2}\rho_f \left(\frac{\Gamma_0}{2\pi r_c}\right)^2.$$

Thus internal pressure decreases as $1/r_c^2$.

Boundary condition for core swelling:

$$p(r_c) + p_{\text{int}}(r_c) = p_{\text{vac}}.$$

Hydrogen equilibrium occurs at $r = a_0$ when:

$$p_{\text{swirl}}(a_0) = p_{\text{vac}}.$$

This reproduces the Bohr radius without invoking probability amplitudes.

## Appendix C: Kelvin-Mode Spectrum, Entropy, and Heat

Small perturbations of a vortex core produce helical Kelvin modes:

$$\omega_k = \frac{\Gamma_0}{4\pi R^2}\left(k^2 \ln\frac{8R}{r_c} - c_k\right),$$

with $k$ integer mode number, $c_k$ a constant dependent on detailed core structure, and $R$ the toroidal radius.

Kelvin-mode energy:

$$E_k = \hbar_{\text{eff}}\, \omega_k,$$

where in SST the effective "Planck constant" is the swirl-action scale:

$$\hbar_{\text{eff}} = \rho_f r_c^2 \Gamma_0.$$

Entropy associated with Kelvin-mode occupation probabilities $p_k$:

$$S = -k_B \sum_k p_k \ln p_k.$$

Heat exchange at fixed geometry:

$$\delta Q_{\text{SST}} = \sum_k E_k\, dp_k.$$

Core radius changes at fixed $\{p_k\}$ give geometric work:

$$\delta W_{\text{SST}} = \sum_k p_k \frac{\partial E_k}{\partial r_c} dr_c.$$

## Appendix D: Partition Function $Z(\beta)$ and Numerical Ladder Model

We model a Golden ladder:

$$E_n = E_0 \phi^n, \qquad n = 0, 1, \ldots, N.$$

Partition function:

$$Z(\beta) = \sum_{n=0}^{N} \exp(-\beta E_0 \phi^n).$$

Internal energy:

$$U = -\frac{\partial}{\partial \beta} \ln Z = \frac{1}{Z} \sum_{n=0}^{N} E_n e^{-\beta E_n}.$$

Heat capacity:

$$C_V = k_B \beta^2 \big(\langle E^2 \rangle - \langle E \rangle^2\big) = k_B \beta^2 \left[\frac{1}{Z}\sum_n E_n^2 e^{-\beta E_n} - \left(\frac{1}{Z}\sum_n E_n e^{-\beta E_n}\right)^2\right].$$

For proton-like choice:

$$E_0 \approx 300 \text{ MeV}, \quad N = 10.$$

The result:

- A monotonic rise of $C_V(T)$ with $T$,
- Small oscillations in $\ln T$ with amplitude 1–5%,
- Oscillations vanish as $N \to 0$,
- Full fractality appears as $N \to \infty$.

These oscillations are the thermodynamic signature of Golden scaling.

## Appendix E: Thermodynamic Derivation of the Swirl–Coulomb Law

Assume two oppositely-oriented swirl strings with circulation $\pm\Gamma_0$. The interaction energy per unit length is

$$E_{\text{int}}(r) = -\rho_f \frac{\Gamma_0^2}{8\pi^2 r^2}.$$

Define swirl potential:

$$V_{\text{SST}}(r) = -\frac{\Lambda}{\sqrt{r^2 + r_c^2}},$$

with $\Lambda$ chosen such that:

$$V_{\text{SST}}(r) \to -\frac{\Lambda}{r} \quad (r \gg r_c).$$

This resembles a softened Coulomb potential.
Differentiating:

$$F_{\text{SST}}(r) = -\frac{dV}{dr} = -\Lambda \frac{r}{(r^2 + r_c^2)^{3/2}}.$$

As $r \to a_0$:

$$F_{\text{SST}}(a_0) = \frac{1}{4\pi\epsilon_0} \frac{e^2}{a_0^2},$$

which recovers the physical Coulomb force.
Thus, Coulomb binding is interpreted as swirl-pressure gradient force.

## Appendix F: TikZ Figures

Below are three canonical figures for the article.

### F.1 Pressure Profile Around a Vortex Core

### F.2 Hydrogen Swelling: $r_c \to a_0$

### F.3 Unruh Echo: Two-Stage Thermodynamic Response

## References

## References

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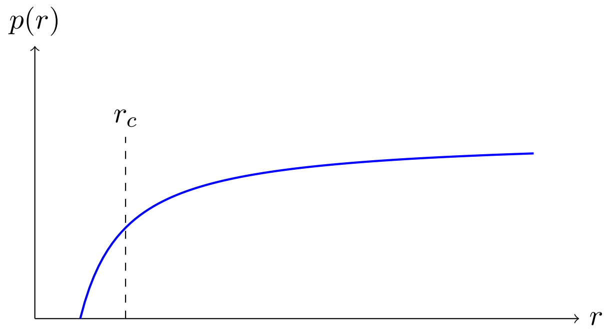


Figure 1: Typical swirl-pressure profile $p(r)$ around a vortex core, showing decrease near $r_c$ and recovery at large $r$.

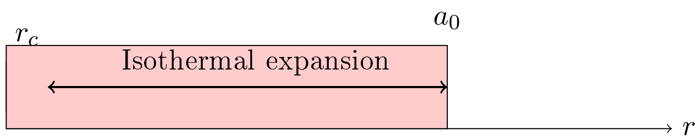


Figure 2: Hydrogen ground-state radius as an isothermal swelling from $r_c$ to $a_0$.

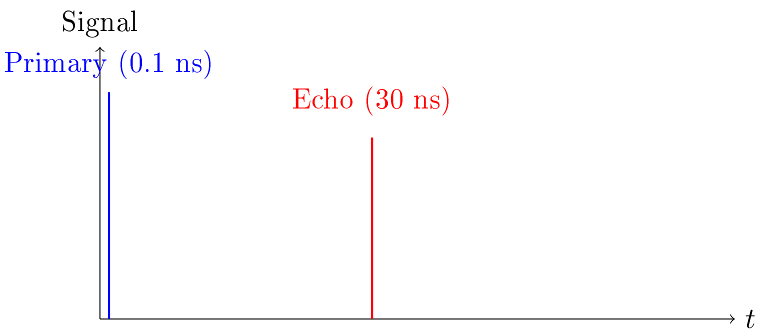


Figure 3: SST interpretation of Unruh Echo: fast swirl-sector pulse followed by slower EM transduction.


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