Thermodynamic Origin of Quantization in Swirl–String Theory:
From Clausius Work–Heat Structure to Parameter-Free Constants

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Abstract

We present a thermodynamic foundation for quantization within Swirl–String Theory (SST), a hydrodynamic model in which matter is represented by closed circulation-carrying filaments ("swirl strings") embedded in an inviscid, incompressible vacuum-like medium. The central claim is that Planck-scale quantization need not be postulated: discretization arises from a topological circulation invariant together with a Clausius-consistent work/heat decomposition. Using the Abe–Okuyama mapping between Clausius equality and the Shannon/von Neumann structure, we formalize a controlled route from pure-state mechanics to quantum thermodynamics and identify the mechanical meaning of SST work (geometric deformation) and SST heat (redistribution over internal Kelvin-wave modes). We then define a minimal primitive set of medium parameters and show how familiar constants may be expressed as derived quantities under a "zero-free-parameter" program, with explicit dimensional checks and numerical anchors. Finally, we state falsifiable predictions, including a low-temperature heat-capacity scaling tied to filament mode structure and an acceleration-induced echo channel analogous to Unruh response.

Keywords: thermodynamics, quantization, circulation, hydrodynamic quantum analogs, Clausius equality

1

1 Scope, notation, and claim taxonomy

1.1 Terminology

To minimize ambiguity we use:

- **Swirl medium:** an inviscid, incompressible condensate-like vacuum model (effective density ρ_f).
- **Swirl string:** a closed, circulation-carrying filament (topological defect) supporting internal wave modes.
- **Circulation:** $\Gamma \equiv \oint \mathbf{v} \cdot d\mathbf{l}$.
- **Zero-free-parameter principle:** after fixing a minimal primitive set of mechanical inputs, all other constants (e.g. h, e, m_e) are derived without additional tuning.

1.2 Claim taxonomy

We distinguish:

Postulates: medium properties and invariants (incompressible, inviscid, circulation invariants).

Derivations: results that follow from postulates (thermodynamic mapping, derived constants, scaling laws).

Speculations: consistent hypotheses lacking direct experimental validation (e.g. log-periodic "Golden Layer" selection).

1.3 What is proven here (and what remains an assumption)

For clarity we separate theorem-level statements (conditional on explicit assumptions) from SST-specific identifications that are currently postulated or calibrated.

Proved statements (conditional).

Under the assumptions listed in Sec. 4.2 and the mediator EFT of Sec. 4.5, the paper establishes:

1. **(Monopole uniqueness)** If the static mediator equation outside the core is Laplacian, $\nabla^2 \delta p = 0$ for $r > r_c$, and the far field is isotropic and decays at infinity, then the unique solution is $\delta p(r) \propto 1/r$ (Appendix B).
2. **(Screened variant)** If the mediator has a quadratic mass term in the EFT, then the static equation is Helmholtz and the unique isotropic tail is Yukawa, $\delta p(r) \propto e^{-r/\lambda}/r$ with $\lambda = m_\phi^{-1}$ (Sec. 4.5, Appendix B).
3. **(Flux carrier)** Time-dependent sources in the quadratic EFT necessarily carry energy and momentum flux $S_i = T_{0i} = Z(\partial_t \phi)(\partial_i \phi)$, enabling a concrete wave/echo channel (Appendix C).
4. **(Closed constant chain, given calibrated micro-inputs)** Given the calibrated micro-scale inputs $(\rho_{\text{core}}, r_c, \|\mathbf{v}_\circlearrowleft\|)$ and the identification $\alpha = 2\|\mathbf{v}_\circlearrowleft\|/c$, the chain $(\rho_{\text{core}}, r_c, \|\mathbf{v}_\circlearrowleft\|) \to A_C^{(\text{SST})}$ and $(A_C^{(\text{SST})}, \alpha) \to (\hbar, h)$ reproduces the SI values numerically.

The following are treated as SST identifications or calibrations to be justified elsewhere: (i) $\alpha \equiv 2\|\mathbf{v}_\circlearrowleft\|/c$; (ii) the existence and numerical value of ρ_{core} as a microphysical stiffness density; (iii) the pressure-work interaction model $V = -V_{\mathrm{eff}}\delta p$ as the correct coarse-grained two-body energy functional; (iv) the selection principle that maps Γ_0 (and topology) to the observed particle mass spectrum.

2 Primitive inputs and parameter logic (normalized triad)

2.1 The primitive triad

In this paper we normalize the SST primitive set to

$$\boxed{\{\rho_f,\ r_c,\ \|\mathbf{v}_\circlearrowleft\|\}} \tag{1}$$

where ρ_f is the effective inertial density of the medium, r_c is the filament core radius, and $\|\mathbf{v}_\circlearrowleft\|$ is the characteristic tangential swirl speed.

Canonical numerical values (SST).

We will use the canonical SST values (SI):

$$\rho_f = 7.0{\times}10^{-7}\ \mathrm{kg\,m^{-3}}, \qquad r_c = 1.40897017{\times}10^{-15}\ \mathrm{m}, \qquad \|\mathbf{v}_\circlearrowleft\| = 1.09384563{\times}10^6\ \mathrm{m\,s^{-1}}. \tag{2}$$

2.2 Derived circulation quantum (removes an input)

A derived circulation quantum consistent with (1) is

$$\boxed{\Gamma_0 \equiv 2\pi r_c\,\|\mathbf{v}_\circlearrowleft\|} \tag{3}$$

with units $[\Gamma_0] = \mathrm{m^2\,s^{-1}}$. Using (2),

$$\Gamma_0 = 9.683619203 \times 10^{-9}\ \mathrm{m^2\,s^{-1}}. \tag{4}$$

Legacy note.

Some earlier drafts used Γ_0 as an independent primitive input. Equation (3) eliminates that redundancy by treating Γ_0 as a derived invariant once $(r_c, \|\mathbf{v}_\circlearrowleft\|)$ are fixed.

2.3 A natural dimensionless constant: α from the swirl speed

A central numerical anchor is the dimensionless ratio $\|\mathbf{v}_\circlearrowleft\|/c$. In SST the fine-structure constant is identified as

$$\boxed{\alpha \equiv \frac{2\|\mathbf{v}_\circlearrowleft\|}{c}} \tag{5}$$

so that $\|\mathbf{v}_\circlearrowleft\| = (\alpha/2)c$. With $c = 299\,792\,458$ m/s,

$$\alpha = 7.297352557 \times 10^{-3}, \qquad \alpha^{-1} = 137.035999312. \tag{6}$$

This will be used below to derive $\hbar$ and h from a single coupling scale.

2.4 Two-density structure and microphysical stiffness scale

While ρ_f governs the far-field inertial response of the medium, SST also employs a microphysical core stiffness density ρ_{core} for the filament core. We treat ρ_{core} as a fixed (canonically calibrated) material property of the string core:

$$\rho_{\text{core}} = 3.8934358266918687 \times 10^{18}\ \text{kg}\,\text{m}^{-3}. \tag{7}$$

Operationally:

- ρ_f controls large-scale inertia/transport and thermodynamic bookkeeping of the medium.
- ρ_{core} controls near-core pressures/energetics relevant for the interaction coupling scale.

This separation is not an added free parameter once ρ_{core} is fixed by independent SST calibration; it is a two-scale constitutive structure.

3 Mechanical theory of heat in SST

3.1 Clausius structure and the Abe–Okuyama mapping

A key structural input is the Clausius equality for reversible processes,

$$\oint \frac{\delta Q}{T} = 0, \tag{8}$$

viewed here as a constraint on admissible state transformations of filament excitations. Following Abe and Okuyama, imposing a Clausius-consistent definition of "heat" on Shannon entropy yields a formal coincidence with von Neumann entropy of a canonical density matrix, providing an isomorphism-like bridge between pure-state mechanics and "quantum thermodynamics" [3, 4].

3.2 Work–heat decomposition from spectral data

Let $\{E_n\}$ denote a discrete internal mode spectrum (e.g. Kelvin-wave eigenmodes on the filament) and $\{p_n\}$ their occupation probabilities. Define

$$\delta W \equiv \sum_n p_n\, dE_n, \qquad \delta Q \equiv \sum_n E_n\, dp_n. \tag{9}$$

This matches standard quantum-thermodynamic decompositions of energy change into "level shift" and "population change" parts [4].

4

3.3 SST interpretation

Within SST we interpret:

- **Work δW:** geometric deformation energy required to change filament geometry (core radius r_c, envelope size R, curvature/torsion class) against medium pressure.
- **Heat δQ:** redistribution of energy over internal filament modes (Kelvin-wave degrees of freedom), at fixed instantaneous spectrum.

3.4 Temperature as geometric strain parameter

We introduce a swirl-temperature T_{swirl} as a parameter controlling the mode population distribution, and we posit it correlates with a geometric strain measure ϵ (radial swelling or envelope deformation):

$$T_{\text{swirl}} \propto \epsilon. \tag{10}$$

The equilibrium vacuum corresponds to $\epsilon = 0$ and $T_{\text{swirl}} = 0$.

4 Derived constants: strategy and closed consistency chain

4.1 The Coulomb coupling scale as a near-core pressure invariant

Empirical SI comparison scale.

For comparison with standard SI electromagnetism, define the empirical Coulomb coupling scale

$$A_C^{(\text{SI})} \equiv \frac{e^2}{4\pi\epsilon_0}, \tag{11}$$

with units $[\text{J m}]$. This quantity is *not* taken as an SST primitive; it is used only as a numerical benchmark.

SST prediction (magnitude).

Section 4.2 yields the SST coupling magnitude

$$A_C^{(\text{SST})} = 4\pi\,\rho_{\text{core}}\,\|\mathbf{v}_\circlearrowleft\|^2\,r_c^4,$$

which is dimensionally $[\text{J m}]$ and subsequently fixes $\hbar$ and h via Eq. (25) once the identification step below is made.

Identification step (comparison to nature).

The "zero-free-parameter" program asserts that after calibrating $(\rho_{\text{core}}, r_c, \|\mathbf{v}_\circlearrowleft\|)$, the predicted magnitude should match the empirical SI benchmark within uncertainty,

$$A_C^{(\text{SST})} \stackrel{!}{\approx} A_C^{(\text{SI})}. \tag{12}$$

Numerical value (SST prediction).

With (2) and (7),

$$A_C^{(\mathrm{SST})} \;=\; 2.307077328 \times 10^{-28} \ \mathrm{J\,m}. \tag{13}$$

This is a parameter-free anchor once $(\rho_{\mathrm{core}}, r_c, \|\mathbf{v}_{\circlearrowleft}\|)$ are fixed.

4.2 Far-field monopole from EFT (Laplace/Helmholtz) and core matching (fixing the 4π)

We now derive Eq. (22) from a minimal far-field construction.

Assumptions (explicit).

(i) The medium is inviscid and incompressible; (ii) we consider a quasi-static interaction regime in which the relevant mediator is a scalar pressure/foliation perturbation $\delta p(\mathbf{x})$ generated by a compact swirl-string core of radius r_c; (iii) at distances $r \gg r_c$ the closed loop coarse-grains to an isotropic monopole in δp (all multipoles beyond $\ell = 0$ are neglected).

Mediator equation outside the core (from the quadratic EFT).

The quasi-static far-field is controlled by the scalar mediator ϕ introduced in Sec. 4.5, with $\delta p \equiv g_p\,\phi$ [Eq. (30)]. Outside the compact core ($r > r_c$) the source vanishes, $J = 0$, and the static limit of Eq. (32) gives

$$(\nabla^2 - m_\phi^2)\phi = 0 \qquad (r > r_c). \tag{14}$$

Hence the physical pressure/foliation perturbation obeys

$$(\nabla^2 - \lambda^{-2})\,\delta p = 0 \qquad (r > r_c), \qquad \lambda \equiv m_\phi^{-1}, \tag{15}$$

with the unscreened Laplacian limit recovered as $\lambda \to \infty$:

$$\nabla^2 \delta p = 0 \qquad (r > r_c,\ \lambda \to \infty). \tag{16}$$

The uniqueness of the isotropic $1/r$ tail (and its screened Yukawa generalization) is shown explicitly in Appendix B.

Isotropic monopole tail (unscreened).

In the unscreened case, spherical symmetry implies the unique decaying solution

$$\delta p(r) = -\chi_1\, p_\star\, \frac{r_c}{r}, \qquad r \geq r_c, \qquad \chi_1 \in \{+1, -1\}, \tag{17}$$

where the discrete sign χ_1 labels the two polarity classes of the source (the "charge" analogue), and boundary matching fixes $|\delta p(r_c)| = p_\star$.

Core matching: fixing $p_\star$ by a momentum-flux stress scale.

The near-core interaction scale is set by a normal stress (momentum flux density) rather than a purely kinetic-energy density. We therefore match to

$$p_\star \equiv \sigma_\star \equiv \rho_{\mathrm{core}} \, \|\mathbf{v}_\circlearrowleft\|^2, \tag{18}$$

which has units of pressure and is the canonical stress scale associated with a tangential swirl speed $\|\mathbf{v}_\circlearrowleft\|$ in a stiff core medium of density ρ_{core}. (Using the aerodynamic "dynamic pressure" $\frac{1}{2}\rho_{\mathrm{core}}\|\mathbf{v}_\circlearrowleft\|^2$ would shift the final prefactor by $1/2$; SST fixes the choice (18) by matching to momentum-flux normalization.)

Effective excluded volume from isotropic stress normalization.

For a spherical core the geometric volume is $V_{\mathrm{core}} = \frac{4\pi}{3}r_c^3$. In an isotropic 3D medium, the scalar pressure corresponds to the trace of the momentum-flux tensor, and the appropriate normalization implies an effective factor of 3 in the pressure-work channel (derivation in Appendix A):

$$V_{\mathrm{eff}} \equiv 3\,V_{\mathrm{core}} = 3 \cdot \frac{4\pi}{3}r_c^3 = 4\pi r_c^3. \tag{19}$$

Two-body sign and magnitude.

To separate the *magnitude* of the coupling from the *sign* (attraction/repulsion), we assign each swirl-string core a discrete polarity $\chi \in \{+1, -1\}$. Core 1 generates $\delta p_1(r)$ as in Eq. (17). The interaction energy of core 2 placed in that background is modeled by pressure-work on an excluded effective volume,

$$V_{12}(r) \; = \; -\chi_2\, V_{\mathrm{eff}}\, \delta p_1(r), \tag{20}$$

so that like polarities ($\chi_1\chi_2 = +1$) give repulsion and opposite polarities ($\chi_1\chi_2 = -1$) give attraction.

Using Eqs. (17), (18), and (19) we obtain

$$V_{12}(r) = -\chi_2\Big(4\pi r_c^3\Big)\Big(-\chi_1\,\rho_{\mathrm{core}}\|\mathbf{v}_\circlearrowleft\|^2\,\frac{r_c}{r}\Big) = \chi_1\chi_2\,\frac{A_C^{(\mathrm{SST})}}{r}, \tag{21}$$

where the positive coupling magnitude is

$$\boxed{A_C^{(\mathrm{SST})} = 4\pi\,\rho_{\mathrm{core}}\,\|\mathbf{v}_\circlearrowleft\|^2\,r_c^4.} \tag{22}$$

Screened (Yukawa) variant.

If $m_\phi > 0$ (finite λ), the isotropic tail is Yukawa (Appendix B), and the two-body potential becomes

$$V_{12}(r) = \chi_1\chi_2\,\frac{A_C^{(\mathrm{SST})}}{r}\,e^{-r/\lambda}. \tag{23}$$

Origin of the 4π ***(summary).***

The overall coefficient in Eq. (22) is fixed without continuous tuning by two standard normalizations: (i) δp has an isotropic monopole tail determined by the Laplace/Helmholtz Green's function and boundary matching (Appendix B); (ii) the pressure-work channel couples to the trace part of the isotropic stress, which yields $V_{\text{eff}} = 3V_{\text{core}} = 4\pi r_c^3$ (Appendix A). Together with the core stress scale $p_\star = \rho_{\text{core}}\|\mathbf{v}_\circlearrowleft\|^2$, this fixes A_C uniquely.

Sign and "charge" assignment.

Equation (22) fixes the positive coupling magnitude $A_C^{(\text{SST})}$. The interaction sign is encoded by the discrete polarity labels $\chi_1, \chi_2 \in \{+1, -1\}$ via Eq. (21). Mapping χ to a microscopic SST chirality/polarization invariant is deferred to the topological selection sector.

4.3 Planck's constant as a derived quantity (no postulate)

In SI, the fine-structure constant satisfies

$$\alpha = \frac{A_C^{(\text{SI})}}{\hbar c}. \tag{24}$$

Using (5) and identifying $A_C^{(\text{SST})} \approx A_C^{(\text{SI})}$ as in Eq. (12), we obtain

$$\boxed{\hbar = \frac{A_C^{(\text{SST})}}{\alpha c}}, \qquad \boxed{h = 2\pi\hbar = \frac{2\pi A_C^{(\text{SST})}}{\alpha c}}. \tag{25}$$

This is the precise sense in which SST "bypasses Planck": h becomes a derived constant fixed by (i) a topological/kinematic dimensionless ratio $\alpha = 2\|\mathbf{v}_\circlearrowleft\|/c$ and (ii) a mechanically predicted coupling magnitude $A_C^{(\text{SST})}$.

Numerical value.

Using (13) and (6),

$$\hbar = 1.0545717167 \times 10^{-34} \text{ J s}, \qquad h = 6.6260695157 \times 10^{-34} \text{ J s}. \tag{26}$$

4.4 Action from circulation: the structural identity

A robust identity in superfluid circulation quantization is

$$\kappa = \frac{h}{m}, \tag{27}$$

where κ is a circulation quantum [9, 10]. SST retains the structural form

$$h \sim m_{\text{eff}} \, \Gamma_0, \tag{28}$$

but Γ_0 is now fixed by (3) and h by (25). Hence (28) becomes an *equation that fixes* m_{eff} for the relevant excitation class:

$$m_{\text{eff}} \ \sim \ \frac{h}{\Gamma_0}. \tag{29}$$

The "selection problem" is then: which filament configuration realizes that m_{eff} as a stable extremum?

4.5 Quadratic EFT for the pressure/foliation mediator (deriving Helmholtz/Yukawa)

To make the screened alternative of Appendix B structurally unavoidable (rather than optional), we introduce a minimal quadratic effective field theory (EFT) for a scalar mediator ϕ that encodes the relevant pressure/foliation perturbation channel.

Field content and calibration.

We treat ϕ as a coarse-grained scalar degree of freedom sourced by a compact swirl-string core. The physical pressure perturbation is taken to be proportional to the field,

$$\delta p \equiv g_p \, \phi, \tag{30}$$

where the proportionality constant g_p is fixed by one calibration condition (e.g. matching the boundary value $\delta p(r_c) = -p_\star$ used in Sec. 4.2).

Minimal quadratic action.

In a flat background and at leading order in derivatives, the most general stable quadratic Lagrangian density is

$$\mathcal{L}_\phi = \frac{Z}{2} \left(\frac{1}{c_\phi^2} (\partial_t \phi)^2 - (\nabla \phi)^2 - m_\phi^2 \phi^2 \right) + \phi \, J, \tag{31}$$

with wave speed $c_\phi > 0$, normalization $Z > 0$, mass parameter $m_\phi \geq 0$, and a source J representing the compact core.

Equation of motion and the static limit.

Varying (31) gives

$$\frac{1}{c_\phi^2} \partial_t^2 \phi - \nabla^2 \phi + m_\phi^2 \phi = \frac{J}{Z}. \tag{32}$$

In the quasi-static regime relevant for the Coulomb-like interaction, $\partial_t \phi \approx 0$, so (32) reduces to the screened Poisson (Helmholtz) equation

$$(\nabla^2 - m_\phi^2)\phi = -\frac{J}{Z}. \tag{33}$$

9

For a pointlike monopole source $J = q_\phi\, \delta^{(3)}(\mathbf{x})$, the Green's function solution is

$$\phi(r) = \frac{q_\phi}{4\pi Z}\, \frac{e^{-m_\phi r}}{r}. \tag{34}$$

Using $\delta p = g_p \phi$ in (30) gives precisely the Yukawa form $\delta p(r) \propto e^{-r/\lambda}/r$ of Appendix B, with identification

$$\boxed{\lambda = m_\phi^{-1}} \tag{35}$$

and effective monopole strength $Q_p = (g_p q_\phi / Z)$.

Unscreened limit.

When $m_\phi \to 0$ (equivalently $\lambda \to \infty$), (34) reduces to $\phi \propto 1/r$, recovering the Laplacian monopole channel and the $1/r$ interaction potential.

Why this matters for the "echo" channel.

Equation (32) is hyperbolic for time-dependent sources. Hence acceleration pulses can launch propagating ϕ-waves that carry energy and momentum to boundaries. This supplies a concrete dynamical carrier for the "Unruh-echo" phenomenology, independent of whether $m_\phi = 0$ or $m_\phi > 0$.

10-year-old analogy (brief).

Imagine a stretchy drum skin (ϕ). If you press it and hold it (static), the dent spreads like $1/r$. If the skin is heavier or stiffer (a "mass term"), the dent fades away faster with distance (the exponential). If you tap it (time-dependent), waves run outward and carry the tap's energy away.

5 Consistency anchors (numerical checks and SI bookkeeping)

5.1 Kinematic and pressure scales

Two useful derived scales from the triad are:

$$\omega_\star \equiv \frac{\|\mathbf{v}_\circlearrowleft\|}{r_c}, \qquad q_f \equiv \frac{1}{2}\rho_f \|\mathbf{v}_\circlearrowleft\|^2, \qquad q_{\text{core}} \equiv \frac{1}{2}\rho_{\text{core}} \|\mathbf{v}_\circlearrowleft\|^2. \tag{36}$$

Numerically,

$$\omega_\star = 7.763440655\times10^{20}\ \text{s}^{-1}, \qquad q_f = 4.187743918\times10^5\ \text{Pa}, \qquad q_{\text{core}} = 2.329244600\times10^{30}\ \text{Pa}. \tag{37}$$

5.2 Closed chain summary

The central "parameter-free" chain, in its leanest form, is:

$$(r_c, \|\mathbf{v}_\circlearrowleft\|) \Rightarrow \Gamma_0,\ \alpha; \qquad (\rho_{\text{core}}, r_c, \|\mathbf{v}_\circlearrowleft\|) \Rightarrow A_C^{(\text{SST})}; \qquad (A_C^{(\text{SST})}, \alpha) \Rightarrow \hbar,\ h. \tag{38}$$

5.3 Anchor table

Table 1 collects the above anchors. Where available we also list reference values for comparison.

Table 1 SST numerical anchors derived from the normalized primitive set.

Quantity	Formula	Value (SST)	Units
ρ_f	input	7.0×10^{-7}	$\mathrm{kg\,m^{-3}}$
r_c	input	$1.40897017 \times 10^{-15}$	m
$\|\mathbf{v}_\circlearrowleft\|$	input	1.09384563×10^{6}	$\mathrm{m\,s^{-1}}$
Γ_0	$2\pi r_c\|\mathbf{v}_\circlearrowleft\|$	$9.683619203 \times 10^{-9}$	$\mathrm{m^2\,s^{-1}}$
α	$2\|\mathbf{v}_\circlearrowleft\|/c$	$7.297352557 \times 10^{-3}$	(dimensionless)
α^{-1}	$1/\alpha$	137.035999312	(dimensionless)
ρ_{core}	input (micro-scale)	$3.8934358267 \times 10^{18}$	$\mathrm{kg\,m^{-3}}$
$A_C^{(\mathrm{SST})}$	$4\pi\rho_{\mathrm{core}}\|\mathbf{v}_\circlearrowleft\|^2 r_c^4$	$2.307077328 \times 10^{-28}$	J m
$A_C^{(\mathrm{SI})}$	$e^2/(4\pi\epsilon_0)$	(benchmark)	J m
$\hbar$	$A_C^{(\mathrm{SST})}/(\alpha c)$	$1.0545717167 \times 10^{-34}$	J s
h	$2\pi\hbar$	$6.6260695157 \times 10^{-34}$	J s
$\omega_\star$	$\|\mathbf{v}_\circlearrowleft\|/r_c$	$7.763440655 \times 10^{20}$	$\mathrm{s^{-1}}$
q_f	$\frac{1}{2}\rho_f\|\mathbf{v}_\circlearrowleft\|^2$	$4.187743918 \times 10^{5}$	Pa
q_{core}	$\frac{1}{2}\rho_{\mathrm{core}}\|\mathbf{v}_\circlearrowleft\|^2$	$2.329244600 \times 10^{30}$	Pa

5.4 Hydrogen ground state energy (external comparison; not an SST input)

For later cross-checking, standard hydrogen binding energy can be written as

$$E_B = \frac{1}{2}m_e\alpha^2 c^2. \tag{39}$$

Using $\alpha = 2\|\mathbf{v}_\circlearrowleft\|/c$ from (5) gives the equivalent form

$$E_B = 2m_e\|\mathbf{v}_\circlearrowleft\|^2. \tag{40}$$

With the standard electron mass m_e (used here *only* for comparison, not as an SST primitive), (40) evaluates to $E_B \approx 13.6057\,\mathrm{eV}$, consistent with the known value. In SST, the objective is to derive m_e (or its effective analog) from topological/thermodynamic selection, at which point (40) becomes a genuine parameter-free prediction.

10-year-old analogy (brief).
Think of a rubber ring in water: you can stretch the ring (work) or make it wiggle (heat). The "quantum numbers" are like special wiggle patterns the ring is allowed to have, because the ring is closed and cannot wiggle any random way.

6 Predictions and experimental tests

6.1 Low-temperature heat capacity scaling

If internal excitations are effectively one-dimensional gapless modes on a filament, then the low-T specific heat generically scales linearly,

$$C_V^{\mathrm{SST}} \propto T_{\mathrm{swirl}}, \tag{41}$$

in contrast with exponentially suppressed heat capacity in gapped spectra. A decisive falsifier is the observation of exponential suppression in regimes where SST predicts a mode continuum.

6.2 A direct falsifier of the monopole pressure-mediator hypothesis

The derivation of Eq. (22) assumes that the relevant far-field mediator is an isotropic monopole in δp, implying $\delta p(r) \propto r^{-1}$ and $V(r) \propto r^{-1}$. A direct falsifier is any robust observation (in an SST-analog system or a proposed proxy experiment) that the dominant far-field scaling is instead r^{-2} (or faster), which would indicate that the leading monopole channel is absent or screened and that the coupling must arise from higher multipoles or a different mediator sector.

6.3 Acceleration-induced echo channel (Unruh-analog)

A uniformly accelerating detector in relativistic QFT experiences thermal response at temperature

$$T_U = \frac{\hbar a}{2\pi c k_B}, \tag{42}$$

the Unruh effect [12]. SST posits that an accelerating filament couples to medium degrees of freedom in a way that can produce a delayed boundary transduction "echo" under suitable impedance mismatch between characteristic speeds. This is presented as a phenomenological prediction; a full calculation requires the quadratic EFT of the mediating SST field and its stress tensor (planned follow-up).

7 Speculative extension: Golden-layer selection

The "Golden Layer" hypothesis introduces a log-periodic weighting

$$w(K) \propto \varphi^{-g(K)},$$

with φ the golden ratio and $g(K)$ a topological index, acting as a thermodynamic filter selecting stable mass islands. This is explicitly classified as speculation until a concrete mapping $g(K)$ and an experimental discriminator are supplied.

8 Discussion and paper map

This step-2 version makes the "bypass Planck" claim concrete: h is not assumed but obtained from the SST-normalized primitive set via the chain $(\rho_{\text{core}}, r_c, \|\mathbf{v}_\circlearrowleft\|) \to A_C^{(\text{SST})}$ and $(\|\mathbf{v}_\circlearrowleft\|/c) \to \alpha$, then $(A_C^{(\text{SST})}, \alpha) \to \hbar, h$. Future work must (i) derive ρ_{core} and (ii) supply the selection principle for m_{eff} that ties circulation Γ_0 to particle masses.

A Isotropic stress normalization and the factor 3 in V_{eff}

This appendix justifies the factor 3 used in Eq. (19) using only standard continuum-mechanics identities (no group-theory assumptions).

A.1 Pressure as the trace of the momentum-flux tensor

In an inviscid medium the Cauchy stress tensor is

$$\sigma_{ij} = -p\,\delta_{ij}, \tag{43}$$

so the scalar pressure is related to the trace by

$$p = -\frac{1}{3}\,\text{tr}(\sigma) = -\frac{1}{3}\,\sigma_{ii}. \tag{44}$$

Equivalently, a pressure perturbation δp corresponds to an isotropic perturbation in the normal stresses $\delta\sigma_{ii} = -3\,\delta p$.

A.2 Work of inserting an excluded core: why the trace contributes

Consider inserting a small spherical excluded region (the core) into a background stress field. The incremental mechanical work can be written as a contraction of stress with an isotropic strain/volume change. For a purely isotropic insertion channel, the relevant scalar work is proportional to the trace part of the stress, hence to $\delta\sigma_{ii}$, not to a single component $\delta\sigma_{rr}$ alone.

A convenient way to package this into a scalar pressure-work form is to write the interaction energy shift as

$$\Delta E \equiv -\delta p\, V_{\text{eff}}, \tag{45}$$

and then absorb the conversion between δp and $\delta\sigma_{ii}$ into V_{eff}. Using Eq. (44), $\delta\sigma_{ii} = -3\,\delta p$, so the isotropic channel effectively counts three equal normal-stress contributions (one per spatial axis) in the trace.

A.3 Result: $V_{\text{eff}} = 3V_{\text{core}}$

If V_{core} is the geometric excluded volume, then the trace-normalized scalar work channel corresponds to

$$V_{\text{eff}} = 3\,V_{\text{core}}, \tag{46}$$

which yields Eq. (19): $V_{\text{eff}} = 3 \cdot \frac{4\pi}{3} r_c^3 = 4\pi r_c^3$.

A.4 Scope and limitation

This argument fixes only the *normalization* of the isotropic pressure-work channel under the assumptions of (i) inviscid isotropic stress, (ii) monopole far-field $\delta p \propto 1/r$, and (iii) a compact spherical core. If the core is strongly anisotropic, or if higher multipoles dominate, the factorization into δp times a single V_{eff} must be revisited.

B Monopole solution for δp: Green's function derivation and screened variant

B.1 Laplace equation and uniqueness of the $1/r$ far field

Outside a compact source region (the core), the quasi-static pressure/foliation perturbation satisfies

$$\nabla^2 \delta p = 0 \qquad (r > r_c). \tag{47}$$

The general spherically symmetric harmonic function in 3D is

$$\delta p(r) = a + \frac{b}{r}. \tag{48}$$

Imposing $\delta p(\infty) = 0$ gives $a = 0$, hence $\delta p(r) = b/r$. Matching the boundary magnitude $|\delta p(r_c)| = p_\star$ fixes the amplitude up to a discrete polarity $\chi_1 \in \{+1, -1\}$, yielding

$$\delta p(r) = -\chi_1 \, p_\star \, \frac{r_c}{r}, \qquad r \geq r_c, \qquad \chi_1 \in \{+1, -1\}, \tag{49}$$

which is Eq. (17). This establishes the uniqueness of the $1/r$ tail given (i) Laplace outside, (ii) spherical symmetry, and (iii) decay at infinity.

B.2 Equivalent source formulation with a monopole strength

The same result can be written using the Poisson equation with a pointlike monopole source:

$$\nabla^2 \delta p(\mathbf{x}) = -Q_p \, \delta^{(3)}(\mathbf{x}), \tag{50}$$

whose solution is

$$\delta p(r) = \frac{Q_p}{4\pi r}. \tag{51}$$

Comparing (51) with $\delta p(r) = -\chi_1 p_\star r_c / r$ gives the effective monopole strength

$$Q_p = -4\pi \, \chi_1 \, p_\star r_c. \tag{52}$$

In this form, the 4π appears directly from the Green's function normalization $\nabla^2(1/r) = -4\pi \delta^{(3)}(\mathbf{x})$ [14].

B.3 Screened mediator: Yukawa tail and a falsifiable length scale

If the mediator sector has a finite correlation length λ (e.g. from a mass term in an effective field theory), then the quasi-static equation becomes the screened Poisson (Helmholtz) equation

$$(\nabla^2 - \lambda^{-2})\,\delta p(\mathbf{x}) = -Q_p\,\delta^{(3)}(\mathbf{x}). \tag{53}$$

The corresponding Green's function yields the Yukawa form

$$\delta p(r) = \frac{Q_p}{4\pi r}\,e^{-r/\lambda}. \tag{54}$$

Substituting into the two-body pressure-work model of Sec. 4.2 gives the Yukawa-screened interaction

$$V_{12}(r) = \chi_1\chi_2\,\frac{A_C^{\text{(SST)}}}{r}\,e^{-r/\lambda}, \tag{55}$$

with the same short-distance coupling magnitude $A_C^{\text{(SST)}} > 0$ but exponential suppression beyond λ.

Falsifier (operational).

The unscreened monopole channel predicts $\lim_{r\to\infty} r\,V_{12}(r) = \chi_1\chi_2\,A_C^{\text{(SST)}}$, whereas the screened model predicts $\lim_{r\to\infty} r\,e^{+r/\lambda}V_{12}(r) = \chi_1\chi_2\,A_C^{\text{(SST)}}$. Equivalently, the magnitude obeys $\lim_{r\to\infty} |rV_{12}(r)| = A_C^{\text{(SST)}}$ (unscreened) and $\lim_{r\to\infty} |re^{+r/\lambda}V_{12}(r)| = A_C^{\text{(SST)}}$ (screened). Any robust evidence for exponential suppression of the effective coupling at large separation would falsify the strictly Laplacian mediator assumption and instead indicate a finite λ.

C Stress tensor of the mediator field and far-field momentum flux

This appendix records the canonical stress-energy tensor for the scalar mediator ϕ and identifies the energy/momentum flux channel relevant for pulse/echo phenomenology.

C.1 Stress-energy tensor

From the quadratic Lagrangian (31), the symmetric stress-energy tensor (in flat space) can be written as

$$T_{\mu\nu} = Z\,\partial_\mu\phi\,\partial_\nu\phi - \eta_{\mu\nu}\,\mathcal{L}_\phi^{(0)}, \tag{56}$$

where $\mathcal{L}_\phi^{(0)}$ denotes $\mathcal{L}_\phi$ without the source term ϕJ, and $\eta_{\mu\nu}$ is the Minkowski metric.

C.2 Energy density and flux

Writing explicitly in $3 + 1$ form, the energy density $u \equiv T_{00}$ and the energy flux (Poynting-like) vector $S_i \equiv T_{0i}$ are

$$u = \frac{Z}{2} \left(\frac{1}{c_\phi^2} (\partial_t \phi)^2 + (\nabla \phi)^2 + m_\phi^2 \phi^2 \right), \tag{57}$$

$$S_i = T_{0i} = Z \, (\partial_t \phi)(\partial_i \phi). \tag{58}$$

Thus, *static* configurations $\partial_t \phi = 0$ carry no flux:

$$\partial_t \phi = 0 \quad \Rightarrow \quad S_i = 0. \tag{59}$$

Time-dependent excitations (pulses) have $S_i \neq 0$ and transport energy and momentum outward.

C.3 Implication for echo phenomenology

If a localized acceleration episode produces a transient source $J(t, \mathbf{x})$, Eq. (32) generates outgoing ϕ-waves whose far-field flux is given by (58). Reflection/transduction at boundaries can then yield delayed responses ("echoes") without modifying the static $1/r$ coupling in the $m_\phi \to 0$ limit.

Data availability

No new experimental data are reported.

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