Hydrodynamic Origin of the Hydrogen Ground State

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Abstract

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1 Primitive Circulation-Based Canon

1.1 Primitive dimensional constants

We take as primitive the following three dimensional quantities:

$$\Gamma_0: \text{ circulation quantum of a single swirl string}, \qquad [\Gamma_0] = L^2T^{-1}, \tag{1}$$
$$\rho_f: \text{ effective fluid density}, \qquad [\rho_f] = ML^{-3}, \tag{2}$$
$$r_c: \text{ core radius (electron-scale reference length)}, \qquad [r_c] = L. \tag{3}$$

All other dimensional quantities in SST are defined as derived combinations of (Γ_0, ρ_f, r_c) and dimensionless coefficients.

1.2 Kinematic axiom: Kelvin circulation theorem

We assume an incompressible, inviscid, barotropic fluid with no external body forces. The velocity field $\mathbf{v}(\mathbf{x}, t)$ obeys the Euler equations, and the circulation

$$\Gamma(C,t) = \oint_{C(t)} \mathbf{v} \cdot d\boldsymbol{\ell} \tag{4}$$

around any material contour $C(t)$ advected by the flow is conserved:

$$\frac{D\Gamma}{Dt} = 0. \tag{5}$$

Equation (5) is the kinematic backbone of SST, encoding the frozen-in character of swirl strings.

1.3 Quantization axiom: circulation quanta

Swirl strings are modeled as thin-tube regions of concentrated vorticity. For each closed string K we assign an integer circulation quantum n_K, and postulate

$$\Gamma_K = \oint_K \mathbf{v} \cdot d\boldsymbol{\ell} = n_K \Gamma_0, \qquad n_K \in \mathbb{Z}. \tag{6}$$

This is the circulation analogue of the quantized vortex condition in superfluids,

$$\Gamma_n = n\kappa, \qquad \kappa = \frac{h}{m}, \tag{7}$$

as established by Onsager and Feynman.[1] In SST, Γ_0 is taken as a universal topological constant.

1.4 Derived swirl speed scale

We define a canonical swirl speed at the core boundary of a reference string by

$$\|\mathbf{v}$$

where χ_v is a dimensionless geometrical factor that encodes the deviation from a pure Rankine vortex profile. Dimensional consistency is manifest: $[\Gamma_0/r_c] = L^2T^{-1}/L = LT^{-1}$.

For a string K with circulation quantum n_K, the characteristic swirl speed scales as

$$\|\mathbf{v}$$

[1]See Onsager [?], Feynman [?], and Donnelly [?] for quantized circulation in superfluid helium.

1.5 Derived force (tension) and energy scales

The effective force or tension scale associated with swirl strings is defined by combining the fluid density and circulation:

$$F_{\text{swirl}}^{\max} \equiv \chi_F \, \rho_f \, \Gamma_0^2, \tag{10}$$

with χ_F dimensionless. The dimensions are $[\rho_f \, \Gamma_0^2] = (ML^{-3})(L^4T^{-2}) = MLT^{-2}$. Equation (10) replaces the earlier postulation of $F_{\text{swirl}}^{\max}$ as primitive; in this canon, it is a derived quantity.

A characteristic energy scale for the core region is defined as

$$E_{\text{core}} \equiv \chi_E \, \rho_f \, \Gamma_0^2 r_c, \tag{11}$$

which has dimensions of energy $[E] = ML^2T^{-2}$. This can be interpreted as the integrated rotational kinetic energy density of a minimally structured core.

1.6 Electron as reference knot

We identify the electron with a specific knot type K_e and circulation quantum n_e. The electron rest energy sets

$$E_{\text{core}}(K_e) = m_e c^2, \tag{12}$$

with

$$E_{\text{core}}(K_e) = \Xi_e \, E_{\text{core}} = \Xi_e \, \chi_E \rho_f \Gamma_0^2 r_c. \tag{13}$$

Here Ξ_e is a dimensionless factor encoding the detailed geometry of K_e (tube length, curvature, knot type). Equation (12) constrains the combination $\Xi_e \chi_E \rho_f \Gamma_0^2 r_c$ to reproduce $m_e c^2$.

Similarly, the electron-scale relations involving the classical radius, Compton frequency, and hydrogen ground-state energy select a consistent set of dimensionless parameters $\{\chi_v, \chi_F, \chi_E, \Xi_e, \dots\}$ given (Γ_0, ρ_f, r_c).

1.7 Swirl energy density and effective mass density

Locally, the swirl energy density is taken as

$$\rho_E = \frac{1}{2} \rho_f \| \mathbf{v}$$

and the associated mass-equivalent density is

$$\rho_m = \frac{\rho_E}{c^2}. \tag{15}$$

Substituting Eq. (**??**) into Eq. (**??**) yields

$$\rho_E = \frac{1}{2} \rho_f \left(\chi_v \frac{\Gamma_0}{2\pi r_c} \right)^2 = \frac{\chi_v^2}{8\pi^2} \, \rho_f \, \frac{\Gamma_0^2}{r_c^2}. \tag{16}$$

1.8 Gravitational coupling from (Γ_0, ρ_f, r_c)

The swirl-based gravitational coupling G_{swirl} is taken to be a dimensionally consistent combination of the primitive constants and the speed of light:

$$G_{\text{swirl}} = \lambda_G \, \frac{\Gamma_0^a \, \rho_f^b \, r_c^d}{c^e}, \tag{17}$$

with integers (or rationals) (a, b, d, e) chosen such that $[G_{\text{swirl}}] = M^{-1}L^3T^{-2}$. The dimensionless coefficient λ_G is then determined by matching G_{swirl} to Newton's constant G. In this formulation, gravity emerges as a coarse-grained response to the circulation-controlled swirl energy density ρ_E.

Figure 1: **SST energy spectrum as hydrodynamic flow regimes.** The classical Bohr energy levels of hydrogen are reinterpreted in Swirl–String Theory (SST) as distinct hydrodynamic flow regimes of an incompressible background medium rather than as abstract "shells" in configuration space. **Left:** The usual bound states $n = 1, 2, 3, \dots$ are organised as a ladder in which each level is characterised by an orbital fluid speed $v_n = \alpha c/n$, with α the fine-structure constant and c the speed of light. As n increases, the orbital swirl weakens and the effective flow speed decreases, $v_n \to 0$ as $n \to \infty$, corresponding to ionisation. **Right:** The ground state $n = 1$ is shown as a geometric stability boundary of the vacuum "engine". The green orbit marks the innermost stable laminar flow, at which the electron circulates with speed $v_1 = \alpha c \approx 2.19 \times 10^6$ m/s. The red inner region indicates radii for which a hypothetical orbit would require $v > \alpha c$. In SST, such a regime lies beyond the Mach limit of the vacuum texture: the flow would become hydrodynamically unstable (turbulent or singular), and no stationary laminar swirl-string configuration exists. The Bohr ground state thereby acquires a hydrodynamic interpretation: the electron cannot "fall into the nucleus" because the medium admits no stable laminar orbit at radii smaller than $n = 1$.

1.9 Knot–particle dictionary in circulation language

For a general knot K, we assign:

- circulation quantum n_K via Eq. (6),
- geometric invariants: tube length $L(K)$, crossing number $C(K)$, helicity $\mathcal{H}(K)$,
- physical attributes: mass m_K, spin s_K, charge q_K.

The effective mass is modeled as

$$m_K = \frac{1}{c^2} \left[\alpha\, C(K) + \beta\, L(K) + \gamma\, \mathcal{H}(K) \right] \rho_f\, \Gamma_0^2 r_c, \tag{18}$$

where (α, β, γ) are dimensionless coefficients and we have factored out the canonical energy scale $\rho_f \Gamma_0^2 r_c$. Additional discrete factors (e.g. golden-layer weights) can be included as multiplicative corrections to the bracketed term. This realizes a circulation-based mass functional in which Γ_0 and ρ_f provide the dimensional backbone and K's topology selects the dimensionless multipliers.

1.10 Hydrodynamic Origin of the Hydrogen Ground State

In conventional quantum mechanics the discrete spectrum of the Hydrogen atom,

$$E_n = -\frac{13.6 \text{ eV}}{n^2}, \qquad n = 1, 2, 3, \dots, \tag{19}$$

is regarded as a purely spectral property of the Coulomb Hamiltonian. Within Swirl–String Theory (SST), the same spectrum is reinterpreted as a hierarchy of *stationary incompressible flow regimes* sustained by the orbital swirl structure of the electron string around the protonic core.

Orbital swirl velocity as the principal quantum number

The Bohr orbital velocity,

$$v_n = \frac{\alpha c}{n}, \tag{20}$$

is taken to represent the coarse–grained swirl speed of the electron string along a circular streamline at radius

$$r_n = \frac{n^2 a_0}{1}, \tag{21}$$

with a_0 the Bohr radius. As n increases, the swirl becomes progressively weaker and more diffuse. In the limit $n \to \infty$, $v_n \to 0$ and the flow approaches the unbound (ionised) regime.

The principal quantum number labels discrete *laminar* flow patterns supported by the medium.

The electron-scale constraint

Independently, the SST electron-scale derivation relates the core radius r_c, the swirl speed $\|\mathbf{v}_{\circlearrowleft}\|$, and the swirl energy density ρ_E via

$$\rho_E = \frac{1}{2}\rho_f \|\mathbf{v}_{\circlearrowleft}\|^2. \tag{22}$$

Using the canonical swirl speed value established in the SST Canon,

$$\|\mathbf{v}_{\circlearrowleft}\| \approx 1.0938 \times 10^6 \text{ m/s} \approx \frac{1}{2}\alpha c, \tag{23}$$

we identify the internal vorticity scale as exactly half the vacuum Mach limit. This factor of 1/2 is characteristic of the dipole topology of the vortex loop.

Ground-state stability from a hydrodynamic speed limit

Combining the orbital relation $v_n = \alpha c/n$ with the electron-scale constraint reveals a hydrodynamic interpretation of the Bohr ground state.

At $n = 1$, the orbital velocity reaches the vacuum limit:

$$v_1 = \alpha c = 2\|\mathbf{v}_{\circlearrowleft}\|. \tag{24}$$

This velocity $v_1 = \alpha c$ represents the *maximum laminar translation speed* permitted by the vacuum flow texture (the transverse Mach limit). Thus the $n = 1$ state sits at the boundary between admissible laminar flow and a regime in which the required flow speed would exceed the vacuum stability limit.

For any hypothetical "sub-Bohr" value $n < 1$, the orbital velocity would satisfy

$$v_n = \frac{\alpha c}{n} > \alpha c, \tag{25}$$

forcing the flow into a non-laminar (turbulent or singular) regime analogous to a sonic boom. In SST such a configuration cannot sustain a stationary swirl string, and therefore *no bound state exists for* $r < a_0$.

The ground state is not imposed by abstract quantisation but arises dynamically as the innermost stable laminar flow configuration permitted by the fluid properties of the vacuum.

1.11 Hydrodynamic Derivation of the Rydberg Constant

In the SST framework, the ionization of the Hydrogen atom corresponds to the acceleration of the electron vortex string from its stable ground-state orbit ($n = 1$) to the unbound vacuum flow regime ($n \to \infty$).

The energy required for this transition—the Rydberg energy E_{Ry}—is identifiable not as an electrostatic potential difference, but as the *kinetic energy* of the electron vortex traveling at the vacuum stability limit.

Rydberg Energy as Vacuum Kinetic Limit

From the preceding section, the ground-state orbital velocity v_1 is defined by the transverse Mach limit of the vacuum:

$$v_1 = \alpha c. \tag{26}$$

The classical kinetic energy T_1 of the electron mass m_e moving at this limit is:

$$T_1 = \frac{1}{2} m_e v_1^2 = \frac{1}{2} m_e (\alpha c)^2. \tag{27}$$

In standard theory, the Rydberg energy is defined as hcR_∞. Equating the hydrodynamic kinetic energy to the spectral energy yields:

$$hcR_\infty = \frac{1}{2} m_e \alpha^2 c^2. \tag{28}$$

SST Substitution: The Swirl Velocity Relation

We now substitute the SST canonical relation between the fine structure constant and the intrinsic swirl velocity, $\alpha c = 2\mathbf{v}_{\circlearrowleft}$:

$$hcR_\infty = \frac{1}{2} m_e (2\mathbf{v}_{\circlearrowleft})^2 = 2 m_e \mathbf{v}_{\circlearrowleft}^2. \tag{29}$$

Solving for the Rydberg constant R_∞:

$$R_\infty = \frac{2 m_e \mathbf{v}_{\circlearrowleft}^2}{hc}. \tag{30}$$

Physical Interpretation

Equation (30) provides a purely kinematic definition of the Rydberg constant. It states that the fundamental wavenumber of atomic spectroscopy is determined by the ratio of the *vortex swirl energy* ($m_e \mathbf{v}_{\circlearrowleft}^2$) to the *action-speed product* (hc).

Specifically, R_∞ represents the spatial frequency of a wave associated with a vortex loop accelerating to twice its intrinsic spin velocity. The factor of 2 arises from the geometry of the loop: the coherent translation of a dipole structure requires twice the energy of a monopole flow of equivalent velocity.

Thus, in SST, spectral lines are not transitions between abstract probability clouds, but are acoustic resonance shifts caused by the deceleration of the electron knot from its maximum laminar speed (αc) to lower harmonic velocities ($v_n = \alpha c/n$).

1.12 Hydrodynamic Derivation of the Compton Wavelength

The Compton wavelength λ_c defines the fundamental length scale of quantum interaction for a particle of mass m_e. In SST, this emerges from the helical geometry of the vortex string trajectory.

We interpret λ_c as the *longitudinal spatial period* (or pitch) of the vortex filament as it translates at the speed of light c, governed by the internal gearing ratio α.

The Geometric Pitch Relation

Standard electrodynamics establishes the relationship between the Classical Electron Radius (r_e), the Fine Structure Constant (α), and the Compton Wavelength (λ_c):

$$r_e = \alpha \frac{\lambda_c}{2\pi}. \tag{31}$$

In the SST Canon, the geometric Core Radius r_c is exactly half the classical radius ($r_e = 2r_c$), reflecting the dipole (loop) topology of the knot. Substituting $r_e = 2r_c$:

$$2r_c = \alpha \frac{\lambda_c}{2\pi} \quad \Longrightarrow \quad \lambda_c = \frac{4\pi r_c}{\alpha}. \tag{32}$$

This equation states that the Compton wavelength is the circumference of the vortex core ($2\pi r_c$) amplified by the inverse Mach number ($1/\alpha$) and a topological factor of 2.

SST Substitution: The Helical Pitch Formula

We now substitute the SST canonical definition of α derived from the swirl velocity ($\alpha = 2\mathbf{v}_{\circlearrowleft}/c$) into Eq. (32):

$$\lambda_c = \frac{4\pi r_c}{(2\mathbf{v}_{\circlearrowleft}/c)} = \frac{2\pi r_c c}{\mathbf{v}_{\circlearrowleft}}. \tag{33}$$

Numerical Verification

Using the canonical values:

- $r_c \approx 1.409 \times 10^{-15}$ m
- $c \approx 3.00 \times 10^{8}$ m/s
- $\mathbf{v}_{\circlearrowleft} \approx 1.094 \times 10^{6}$ m/s

$$\lambda_c \approx \frac{2\pi(1.409 \times 10^{-15})(2.998 \times 10^{8})}{1.094 \times 10^{6}} \approx 2.426 \times 10^{-12}\,\text{m}. \tag{34}$$

This matches the CODATA value for the Compton wavelength of the electron (2.42631×10^{-12} m).

Physical Interpretation: The Vacuum Screw

Equation (33) reveals the mechanical nature of mass transport in the vacuum. The term $\frac{2\pi r_c}{\mathbf{v}_{\circlearrowleft}}$ represents the *period of one internal rotation* of the vortex core. Multiplying by c gives the distance traveled during one rotation.

Thus, the electron behaves as a **Self-Propelling Screw**:

1. It spins internally at speed $\mathbf{v}_{\circlearrowleft}$.
2. It moves forward at speed c.
3. The "thread pitch" of this motion is exactly λ_c.

Mass, in this view, is the resistance to changing this pitch. A shorter wavelength (higher mass) implies a "tighter" screw thread that requires more energy to accelerate.

References

[1] A. Einstein, *Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?*, newblock Ann. Phys. **18**, 639–641 (1905).