Comparative Thermodynamics of Topological Defects and Quantum Ensembles: Structural Divergence between Swirl–String Theory and the Abe–Okuyama Framework

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Abstract

This article presents a comparative thermodynamic analysis of two conceptually distinct frameworks for microscopic physics: the information-theoretic thermodynamics of Abe and Okuyama (AO), and the hydrodynamic thermodynamics of Swirl–String Theory (SST). AO operates entirely within standard quantum mechanics, defining temperature and entropy from the Shannon information of pure-state expansion coefficients in a fixed Hilbert space. SST, by contrast, posits a physical, incompressible swirl condensate and identifies particles with knotted vortex filaments; thermodynamic variables are reinterpreted as geometric and topological properties of these defects. We formulate and compare the definitions of temperature, heat, work, and entropy, and derive the respective equations of state. SST predicts a quadratic energy law $E \propto T_{\mathbf{swirl}}^2$ and a linear low-temperature heat capacity $C_V \propto T_{\mathbf{swirl}}$, in contrast to the gapped/exponential or equipartition-type behavior of the AO particle-in-a-box model. We further discuss the Golden Layer hypothesis in SST, which endows the vacuum enthalpy with discrete scale invariance, and contrast it with the smooth Shannon entropy structure in AO. The comparison highlights an ontological schism: AO treats thermodynamics as an emergent information calculus over quantum amplitudes, whereas SST treats it as the elastic mechanics of a continuous fluid substrate. We identify falsifiable differences in low-temperature heat capacity, stability mechanisms, and possible temperature dependence of decay rates, and outline experimental and analogue-gravity tests that could discriminate between the two pictures.

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1 Introduction: The Ontological Crisis of the Quantum Vacuum

Thermodynamics and quantum mechanics use superficially similar mathematical structures, yet differ profoundly in ontology. In classical statistical mechanics, entropy and temperature arise from coarse-graining over a continuum of microstates in phase space. In quantum mechanics, the state is a vector in a Hilbert space, and pure states have vanishing von Neumann entropy. The work of Abe and Okuyama demonstrates that, by using the Shannon entropy of expansion coefficients in an energy eigenbasis and imposing the Clausius relation, one can reconstruct the canonical ensemble and even a quantum Carnot cycle within this framework [1]. Thermodynamics is thereby understood as an emergent information-theoretic structure on Hilbert space.

Swirl–String Theory (SST) represents a qualitatively different response to the same conceptual tension. Instead of treating the vacuum as a passive geometric manifold or a probabilistic field, SST posits a physical, frictionless, incompressible fluid condensate filling three-dimensional Euclidean space. Elementary particles are topologically stable knotted filaments in this medium ("swirl strings") with quantized circulation and well-defined geometric cores. The SST Canon derives mass, charge, and time dilation from the hydrodynamics of this substrate, governed by a primitive triplet (Γ_0, ρ_f, r_c) and a characteristic swirl velocity scale $\|\mathbf{v}_{\circlearrowleft}\|$.

This article develops a comparative thermodynamics of these two frameworks. We ask: *What does "temperature" mean in a quantum ensemble vs. in a knotted defect? How do energy, heat, and work scale with this temperature? What are the stability mechanisms that prevent atomic collapse?* The answers diverge sharply.

1.1 Abe–Okuyama baseline: thermodynamics as isomorphism

The AO approach assumes the standard quantum-mechanical ontology: a Hamiltonian $\hat{H}$, energy eigenstates $\{|u_n\rangle\}$ with eigenvalues E_n, and a pure state

$$|\psi\rangle = \sum_n c_n |u_n\rangle, \qquad p_n = |c_n|^2. \tag{1}$$

The internal energy is

$$E = \langle\psi|\hat{H}|\psi\rangle = \sum_n p_n E_n. \tag{2}$$

Because $S_{\mathrm{vN}} = 0$ for pure states, AO identify the entropy with the Shannon functional of $\{p_n\}$:

$$S^{\mathrm{S}} = -k_{\mathrm{B}} \sum_n p_n \ln p_n. \tag{3}$$

They decompose an infinitesimal change in E as

$$dE = \sum_n E_n \, dp_n + \sum_n p_n \, dE_n, \tag{4}$$

and identify

$$\delta' Q = \sum_n E_n \, dp_n, \tag{5}$$

$$\delta' W = \sum_n p_n \, dE_n, \tag{6}$$

as heat and work, respectively, for quasi-static processes. Imposing the Clausius equality for reversible changes,

$$dS^{\mathrm{S}} = \frac{\delta' Q}{T}, \tag{7}$$

forces the probabilities p_n to take the canonical form

$$p_n = \frac{1}{Z} \exp\left(-\frac{E_n}{k_B T}\right), \tag{8}$$

with partition function Z. No physical medium is introduced; temperature T appears as a Lagrange multiplier implementing an information constraint. The "particle-in-a-box" provides a concrete example with $E_n \propto n^2/L^2$ and work performed by varying L.

1.2 Swirl–String baseline: thermodynamics as mechanics

SST rejects an information-theoretic interpretation of thermodynamics. The vacuum is modeled as an incompressible fluid with effective density ρ_f, circulation quantum Γ_0, and an intrinsic core radius r_c. A stable particle is a closed filament with core radius $r \simeq r_c$, embedded in this fluid; its mass originates from the kinetic energy of the surrounding swirl and from vacuum displacement energy. The primitive SST constant set fixes a characteristic swirl speed

$$\|\mathbf{v}_{\circlearrowleft}\| \sim \frac{\Gamma_0}{2\pi r_c}, \tag{9}$$

which, together with ρ_f and r_c, calibrates the electron mass and the hydrogen spectrum in the Canon.

Within this ontology:

- *Heat* is the energy exchanged via excitation and damping of Kelvin modes and topological phase transitions at fixed core geometry.
- *Work* is the mechanical energy required to swell or compress the vortex core against the ambient vacuum pressure.
- *Temperature* is defined as a geometric strain variable measuring the deviation of the vortex radius from its equilibrium value.

These definitions are grounded in Euler–Bernoulli hydrodynamics and a specific enthalpy functional H_{swirl} for the vortex configuration.

2 Temperature: Lagrange multiplier vs. geometric strain

2.1 AO temperature as a spectral parameter

In the AO framework, temperature T emerges from maximizing S^{S} under constraints on E and normalization. It is not introduced as a property of space or matter, but as the multiplier relating the entropy gradient to energy redistribution:

$$dS^{\mathrm{S}} = \frac{\delta' Q}{T}, \qquad \delta' Q = \sum_n E_n \, dp_n. \tag{10}$$

Physically, larger T corresponds to broader occupation of high-energy eigenstates in the chosen basis; smaller T corresponds to concentration near the ground state. A single eigenstate has no intrinsic temperature beyond this statistical interpretation.

2.2 SST swirl temperature as radial strain

SST introduces a two-scale description of the electron in hydrogen: a core radius r (vortex thickness) and an orbital radius R (torus major radius), with ground state

$$r_0 = r_c, \qquad R_0 = a_0. \tag{11}$$

The SST Canon relates these scales via a constitutive relation

$$a_0 = \frac{c^2}{2\|\mathbf{v}_{\circlearrowleft}\|^2}\, r_c, \tag{12}$$

so small perturbations satisfy

$$\frac{\delta a_0}{a_0} \simeq \frac{\delta r_c}{r_c}. \tag{13}$$

SST defines a unified strain

$$\epsilon \equiv \frac{r - r_c}{r_c} \simeq \frac{R - a_0}{a_0}, \tag{14}$$

and sets the swirl temperature as

$$T_{\text{swirl}} = \Theta\, \epsilon, \tag{15}$$

with Θ an effective stiffness scale (in Kelvin) of the swirl condensate. Here T_{swirl} is an intensive observable of a *single* vortex configuration: any deformed state (R, r) has a well-defined temperature, even without statistical mixing.

2.3 Interpretational divergence

The two definitions differ qualitatively:

- AO temperature is a property of the ensemble of amplitudes $\{c_n\}$; it encodes informational spread over energy eigenstates and has no direct geometric meaning.
- SST temperature is a property of the geometric state (R, r) of a single defect; it measures physical swelling of the core and envelope against vacuum pressure.

In SST, a "hotter" atom is literally larger; in AO, a "hotter" state need not have any spatial deformation, only a different superposition structure.

3 Enthalpy and the equation of state

3.1 Swirl enthalpy functional and harmonic swelling

In SST the relevant thermodynamic potential for a hydrogenic vortex ring in a background vacuum pressure P_∞ is the enthalpy

$$H_{\text{swirl}}(R, r) = E_{\text{kin}}(R, r) + E_{\text{vac}}(R, r) + E_{\text{surf}}(R, r), \tag{16}$$

where

$$E_{\text{kin}}(R, r) \simeq \frac{1}{2}\rho_f\, \Gamma^2 R \left[\ln\left(\frac{8R}{r}\right) - \alpha\right], \tag{17}$$

$$E_{\text{vac}}(R, r) = P_\infty\, V(R, r), \qquad V(R, r) = 2\pi^2 R r^2, \tag{18}$$

$$E_{\text{surf}}(R, r) \sim \sigma\, 4\pi^2 R r, \tag{19}$$

with $\alpha = \mathcal{O}(1)$ and an effective surface tension σ [2]. Expanding $H_{\rm swirl}$ around the ground state $(R_0, r_0) = (a_0, r_c)$ and projecting along the unified strain direction $\delta R = a_0\epsilon$, $\delta r = r_c\epsilon$ yields

$$H_{\rm swirl}(T_{\rm swirl}) \simeq H_0 + \frac{1}{2}K_{\rm eff}\left(\frac{T_{\rm swirl}}{\Theta}\right)^2, \tag{20}$$

with $K_{\rm eff}$ a positive stiffness coefficient combining the second derivatives of $H_{\rm swirl}$ with respect to R and r. The thermodynamic excitation energy above the ground state is therefore

$$\Delta E_{\rm SST}(T_{\rm swirl}) = E(T_{\rm swirl}) - E(0) \propto T_{\rm swirl}^2. \tag{21}$$

3.2 AO internal energy scaling

In the AO particle-in-a-box, eigenvalues scale as

$$E_n(L) = \frac{h^2 n^2}{8mL^2}. \tag{22}$$

For a canonical distribution at temperature T one has

$$E_{\rm AO}(T) = \sum_n p_n(T) E_n(L), \qquad p_n(T) = \frac{1}{Z}\exp\left(-\frac{E_n(L)}{k_{\rm B}T}\right). \tag{23}$$

At low T, the internal energy is dominated by the ground state plus exponentially suppressed contributions from the first excited level:

$$\Delta E_{\rm AO}(T) \sim \Delta E \, \exp\left(-\frac{\Delta E}{k_{\rm B}T}\right), \tag{24}$$

with $\Delta E = E_2 - E_1$. At high T, the classical limit recovers $E \propto T$.

3.3 Structural conflict

SST thus predicts a quadratic power law $\Delta E \propto T_{\rm swirl}^2$ near the ground state, characteristic of a harmonic elastic mode. AO predicts either exponential activation (gapped discrete spectrum) at low T or linear scaling (classical equipartition) at high T. This difference originates in the choice of microscopic degrees of freedom: a continuous breathing mode of a core radius vs. a discrete ladder of standing waves in a rigid box.

4 Heat capacity as a vacuum diagnostic

4.1 Linear heat capacity in SST

From $\Delta E_{\rm SST} \propto T_{\rm swirl}^2$ one obtains

$$C_V^{\rm SST}(T_{\rm swirl}) \equiv \frac{dE}{dT_{\rm swirl}} \propto T_{\rm swirl}, \tag{25}$$

i.e. a heat capacity that vanishes linearly with temperature. This behavior is typical of gapless excitations with a linear density of states (e.g. certain fermionic or phononic systems), and suggests that the swirl condensate provides a continuum of low-energy deformation modes.

4.2 Schottky-type behavior in AO

For a low-temperature two-level truncation in the AO picture, the heat capacity follows a Schottky-type form

$$C_V^{\rm AO}(T) \propto \left(\frac{\Delta E}{k_{\rm B}T}\right)^2 \exp\left(-\frac{\Delta E}{k_{\rm B}T}\right), \tag{26}$$

vanishing exponentially as $T \to 0$, and exhibiting a finite peak at intermediate temperatures before approaching a constant high-T value.

4.3 Falsifiable difference

The qualitative difference,

$$C_V^{\mathrm{SST}} \sim T \quad \text{vs.} \quad C_V^{\mathrm{AO}} \sim e^{-\Delta E/k_{\mathrm{B}}T}, \tag{27}$$

constitutes a clear empirical discriminator, in principle accessible in precision low-temperature spectroscopy or analogue systems designed to emulate the SST swelling mode.

5 Entropy, Golden Layer, and fractal thermodynamics

5.1 Geometric entropy and Boltzmann–swirl distribution

SST proposes a probability density for the core radius

$$P(r) \propto \exp\left[-\frac{H_{\mathrm{swirl}}(r)}{k_{\mathrm{SST}}\Theta_{\mathrm{vac}}}\right], \tag{28}$$

with an effective swirl Boltzmann constant k_{SST} and vacuum noise temperature Θ_{vac}. Near equilibrium, with $H_{\mathrm{swirl}}(r) \simeq H_0 + \frac{1}{2}K_r(r-r_c)^2$, this yields Gaussian fluctuations

$$\langle (r-r_c)^2 \rangle = \frac{k_{\mathrm{SST}}\Theta_{\mathrm{vac}}}{K_r}, \tag{29}$$

and entropy as a measure of geometric phase-space volume of the fluctuating boundary. The spatial "fuzziness" of the electron is interpreted as thermal fluctuation of a vortex interface rather than intrinsic indeterminacy.

5.2 Golden potential and discrete scale invariance

The Golden Principle in SST introduces a log-periodic vacuum potential for the swirl energy density ρ_E,

$$V_\phi(\rho_E) = \Lambda^4 \left[1 - \cos\left(\frac{2\pi}{\ln\phi}\,\ln\frac{\rho_E}{\rho_E^*}\right)\right], \tag{30}$$

where ϕ is the Golden Ratio and ρ_E^* a reference density. Because $\rho_E \propto v^2 \propto 1/r^2$, this induces a sequence of preferred radii corresponding to minima of V_ϕ, leading to discrete scale invariance in core swelling. Summing over such Golden-layer states produces log-periodic corrections to thermodynamic quantities, including a fractal heat capacity with oscillatory dependence on $\ln T$ [3].

5.3 Contrast with smooth Shannon entropy

In AO the entropy is smooth in $\{p_n\}$ and, through the canonical distribution, produces thermodynamic functions without log-periodic structure. There is no number-theoretic structure analogous to ϕ in the underlying Hilbert-space formalism. SST thereby embeds number theory directly into thermodynamics through the Golden Layer, whereas AO remains purely analytic and smooth.

6 Stability: Uncertainty principle vs. hydrodynamic balance

6.1 Kinematic stability in AO/QM

In standard quantum mechanics, and implicitly in AO, atomic stability is ensured by Heisenberg's uncertainty relation. Confining a particle to a narrower spatial region increases momentum variance and kinetic energy, preventing collapse of the wavefunction into the nucleus. The potential well is imposed externally; stability is kinematic and probabilistic.

6.2 Mechanical stability and Golden filter in SST

SST explains stability through the equilibrium of pressures:

- Centrifugal swirl pressure $P_{\mathrm{cent}}(r)$ from circulation around the core, tending to expand the loop.
- Vacuum confining pressure P_{vac} from the background condensate, tending to contract it.

The core equilibrium radius r_c satisfies $P_{\mathrm{cent}}(r_c) = P_{\mathrm{vac}}$. For composite knots such as the proton, deeper layers of topology are protected by Golden suppression factors in the mass functional, rendering inner structures thermally inaccessible and dynamically stable over cosmological times.

7 Chronometric dimension and inverse-time cooling

SST relates local proper time τ to swirl speed via the Swirl Clock,

$$S^{\circlearrowleft}_{(t)} = \frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}}, \qquad v = v_\theta(r), \tag{31}$$

with $v_\theta(r) \sim \Gamma/(2\pi r)$. Cooling a bound state drives $r \to r_c$, increasing v and decreasing $S^{\circlearrowleft}_{(t)}$: cold, compact states have more time dilation and slower internal clocks. Heating swells the core, reduces v, and moves $S^{\circlearrowleft}_{(t)}$ toward unity; excited states age faster.

AO, in contrast, uses an external time parameter t independent of temperature or internal deformation. No chronometric thermodynamics is present.

8 From particle-in-a-box to harmonic swelling

In AO the confinement length L is an externally set parameter of the potential; work is defined by moving the box walls via an external agent. In SST, the confinement scale a_0 is tied to the core radius r_c and arises from hydrodynamic balance. One cannot change the "box" without deforming the "particle": the system is self-confining.

The SST harmonic swelling model thus internalizes the boundary conditions that in AO are imposed by hand. Confinement and mass emerge from the same enthalpy functional, rather than being split into separate kinematic and potential terms.

9 Outlook and experimental directions

The structural differences between AO and SST thermodynamics offer concrete, though challenging, experimental tests:

- **Low-temperature specific heat**: SST predicts $C_V \propto T$ for the swelling mode, while AO predicts exponentially small C_V for isolated, gapped atomic degrees of freedom.
- **Log-periodic modulations**: Golden layering implies subtle, log-periodic oscillations of $C_V(T)$ on a logarithmic temperature scale; AO predicts smooth, monotonic dependence.
- **Temperature-dependent decay**: SST's inverse-time cooling implies that decay rates of unstable states should depend on their geometric swelling, and hence on temperature; AO ties decay primarily to coupling strengths and spectral gaps with no explicit geometric strain.
- **Analogue systems**: Superfluid and BEC vortices, as well as high-Q cavities probing Unruh-type excitations, may provide experimental platforms where SST-inspired swelling and transduction dynamics can be tested.

A decisive discrimination between information-theoretic and hydrodynamic thermodynamics at the microscopic scale would have far-reaching consequences for our understanding of the quantum vacuum.

Appendix: Summary comparison table

Feature	Abe–Okuyama (AO)	Swirl–String Theory (SST)
Ontology	Hilbert space, pure states	Fluid condensate, vortex defects
Temperature	Lagrange multiplier on S^{S}	Geometric strain $T_{\mathrm{swirl}} = \Theta\epsilon$
Heat Q	Energy from dp_n at fixed E_n	Mode/topology excitation at fixed r_c
Work W	Energy from dE_n at fixed p_n	Mechanical swelling/compression of core
Equation of state	$\Delta E \sim e^{-\Delta E/k_{\mathrm{B}}T}$ (low T)	$\Delta E \propto T_{\mathrm{swirl}}^2$
Heat capacity C_V	Schottky/exponential, then constant	Linear: $C_V \propto T_{\mathrm{swirl}}$
Entropy	Shannon entropy of $\{p_n\}$	Geometric phase-space, Golden layering
Stability	Uncertainty principle, external well	Pressure balance, Golden filter
Time	External parameter t	Swirl Clock $S_{(t)}^{\circlearrowleft} = \sqrt{1 - v^2/c^2}$
Model motif	Particle-in-a-box	Harmonic swelling of self-confined ring

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