

# A Topology-Driven Invariant Mass Kernel in Swirl–String Theory (SST): Reference Implementation and Benchmark Against Atomic and Molecular Masses

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## Abstract

This manuscript documents a canonical reference implementation of the Swirl–String Theory (SST) invariant mass kernel, as encoded in `SST_Atom_Mass_Invariant.py`. The kernel maps a topological object  $T$  (specified by braid index  $b_T$ , Seifert genus  $g_T$ , component count  $n_T$ , and a dimensionless total ropelength  $L_{\text{tot}}(T)$ ) into an inertial mass  $M(T)$  via a fixed, mode-independent functional. Three computation paths are supported: *canonical* (strict evaluation with fixed quark-geometry factors), *sector\_norm* (single baryon-sector normalization fixed by the proton), and *exact\_closure* (analytic closure in which the proton and neutron are matched exactly without modifying the kernel). A semi-empirical nuclear binding correction is used to map nucleon-level predictions to atomic masses. In the *exact\_closure* mode (electron, proton, neutron exact by construction), the benchmark over elements H–U yields mean absolute percentage error  $\approx 1.960\,00 \times 10^{-1}\%$  with maximum absolute error  $\approx 1.840\,00\%$  for the fixed  $(Z, N)$  table used. A small molecule set (19 formulas) yields mean absolute percentage error  $\approx 1.560\,00 \times 10^{-1}\%$ .

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# 1 Scope and conventions

The goal is to present (i) the invariant kernel used in code, (ii) the calibration/closure steps that feed its geometric inputs, and (iii) a transparent benchmark against a fixed table of elemental  $(Z, N)$  choices and standard atomic weights (molar masses).

This document is a *methods + benchmark* paper: it does not attempt to derive the SST kernel from first principles beyond the canonical master equation assumption.

## 2 Canonical constants and reference values

**Golden (hyperbolic):**  $\ln \phi = \operatorname{asinh}(0.5)$ , hence  $\phi = e^{\operatorname{asinh}(0.5)}$ .

(Algebraic form  $\phi = (1 + \sqrt{5})/2$  is equivalent.)

The implementation uses the following constants (as literal numeric values in code):

$$\phi = \frac{1 + \sqrt{5}}{2}, \quad (1)$$

$$\alpha \equiv \alpha_{\text{fs}} = 7.297\,35 \times 10^{-3}, \quad (2)$$

$$c = 2.997\,92 \times 10^8 \text{ m s}^{-1}, \quad (3)$$

$$\mathbf{v}_{\text{O}} = 1.093\,85 \times 10^6 \text{ m s}^{-1}, \quad (4)$$

$$r_c = 1.408\,97 \times 10^{-15} \text{ m}, \quad (5)$$

$$\rho_{\text{core}} = 3.893\,44 \times 10^{18} \text{ kg m}^{-3}, \quad (6)$$

$$N_A = 6.022\,14 \times 10^{23} \text{ mol}^{-1}. \quad (7)$$

Particle rest masses used for calibration and reporting are taken as fixed numerical targets in the code (electron, muon, tau, proton, neutron).

## 3 Invariant mass kernel

### 3.1 Energy density and effective volume

Define the characteristic kinetic energy density

$$u = \frac{1}{2} \rho_{\text{core}} \mathbf{v}_{\text{O}}^2, \quad (8)$$

with units  $\text{J m}^{-3}$ .

The code uses an effective volume proxy

$$V(T) = \pi r_c^3 L_{\text{tot}}(T), \quad (9)$$

where  $L_{\text{tot}}(T)$  is dimensionless.

### 3.2 Kernel definition

For each topological object  $T$  with invariants  $(b_T, g_T, n_T, L_{\text{tot}}(T))$ , the invariant mass is

$$M(T) = \left(\frac{4}{\alpha}\right) b_T^{-3/2} \phi^{-g_T} n_T^{-1/\phi} \frac{u V(T)}{c^2}. \quad (10)$$

All factors preceding  $uV/c^2$  are dimensionless; therefore the mapping is dimensionally consistent:  $uV$  is energy and division by  $c^2$  yields mass.

**10-year-old analogy (one sentence).** Think of  $u$  as how “packed” the motion-energy is in a material, and  $V(T)$  as how much of that packed region the string-shape occupies; more packed energy times more occupied volume means more mass.

## 4 Topology-to-geometry assembly and computation paths

### 4.1 Lepton calibration

A base topology for each lepton is specified as:

$$\text{electron base: } (b, g, n) = (2, 1, 1), \quad (11)$$

$$\text{muon base: } (b, g, n) = (5, 2, 1), \quad (12)$$

$$\text{tau base: } (b, g, n) = (7, 3, 1). \quad (13)$$

Given a target mass  $M_{\text{target}}$  and fixed  $(b, g, n)$ , the code solves for the required ropelength by rearranging (10):

$$L_{\text{tot}}(T) = \frac{M_{\text{target}} c^2}{\left(\frac{4}{\alpha}\right) b^{-3/2} \phi^{-g} n^{-1/\phi} u \pi r_c^3}. \quad (14)$$

This step makes each listed lepton mass exact for its chosen base topology (it is a calibration, not a prediction).

### 4.2 Baryon-sector ropelength mapping

The baryon sector uses fixed  $(b, g, n) = (3, 2, 3)$  and constructs baryon ropelengths from two geometric factors  $(s_u, s_d)$  via

$$L_{\text{tot}}(p) = \lambda_b (2\pi^2 \kappa_R) (2s_u + s_d), \quad (15)$$

$$L_{\text{tot}}(n) = \lambda_b (2\pi^2 \kappa_R) (s_u + 2s_d), \quad (16)$$

with  $\kappa_R \approx 2$  (default  $\kappa_R = 2$  in code).

**Modes.**

- **canonical:**  $(s_u, s_d)$  fixed by constants (hyperbolic-volume assignments external to this script);  $\lambda_b = 1$ .
- **sector\_norm:**  $(s_u, s_d)$  fixed as in canonical;  $\lambda_b$  chosen so the proton mass is exact.
- **exact\_closure (default):**  $(s_u, s_d)$  solved analytically so that *both* proton and neutron are exact with  $\lambda_b = 1$ .

### 4.3 Analytic closure used in exact\_closure

Let  $A(b, g, n)$  denote the kernel prefactor multiplying  $L_{\text{tot}}$ :

$$A(b, g, n) = \left(\frac{4}{\alpha}\right) b^{-3/2} \phi^{-g} n^{-1/\phi} \frac{u \pi r_c^3}{c^2}. \quad (17)$$

Define  $S \equiv 2\pi^2 \kappa_R$  and  $K \equiv A(3, 2, 3) S$ . Then

$$M_p = K(2s_u + s_d), \quad (18)$$

$$M_n = K(s_u + 2s_d). \quad (19)$$

Solving yields

$$s_u = \frac{2M_p - M_n}{3K}, \quad (20)$$

$$s_d = \frac{M_p}{K} - 2s_u. \quad (21)$$

This is the exact algebra implemented by `fit_quark_geom_factors_for_baryons`.

## 5 From nucleons to atoms and molecules

### 5.1 Atomic mass assembly and binding correction

For a chosen nuclide  $(Z, N)$  with  $A = Z + N$ , the script constructs

$$M_{\text{sum}}(Z, N) = Z M_p^{\text{pred}} + N M_n^{\text{pred}} + Z M_e^{\text{pred}}. \quad (22)$$

It then subtracts a semi-empirical mass defect (binding energy divided by  $c^2$ ),

$$M_{\text{atom}}^{\text{pred}}(Z, N) = M_{\text{sum}}(Z, N) - \Delta m(Z, N), \quad (23)$$

where  $\Delta m(Z, N)$  is computed via the semi-empirical mass formula (SEMF)

$$B(Z, N) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \delta(A, Z) a_p A^{-1/2}, \quad (24)$$

$$\Delta m(Z, N) = \frac{B(Z, N)}{c^2}. \quad (25)$$

The code uses a MeV→kg conversion factor to implement  $\Delta m$  directly in kilograms. Coefficients are fixed in the script:

$$(a_v, a_s, a_c, a_a, a_p) = (15.75, 17.8, 0.711, 23.7, 11.18) \text{ MeV}.$$

### 5.2 Molecules

Chemical binding energies are neglected (eV scale). A molecular mass prediction is the sum of predicted atomic masses of constituents:

$$M_{\text{mol}}^{\text{pred}}(\text{formula}) = \sum_i \nu_i M_{\text{atom}}^{\text{pred}}(Z_i, N_i), \quad (26)$$

where  $\nu_i$  are stoichiometric coefficients.

### 5.3 Reference conversion from molar mass

For each table entry with molar mass  $M_{\text{mol}}$  (g/mol), the script uses

$$M_{\text{ref}} = \frac{M_{\text{mol}} \times 10^{-3}}{N_A} \text{ [kg]}, \quad (27)$$

to obtain a per-entity reference mass.

## 6 Results (from the provided run)

This section records the numerical outcomes printed by the script for the run corresponding to *exact\_closure* (the observed command-line mode string in the log is non-canonical but triggers the exact-closure branch in the current implementation).

## 6.1 Fitted/derived geometric quantities

With  $\kappa_R = 2$ ,

$$S = 2\pi^2 \kappa_R = 3.947\,84 \times 10^1, \quad (28)$$

$$A(3, 2, 3) = 4.653\,60 \times 10^{-30}, \quad (29)$$

$$K = A(3, 2, 3) S = 1.837\,17 \times 10^{-28}. \quad (30)$$

Exact-closure geometric factors (dimensionless):

$$s_u = 3.030\,60, \quad (31)$$

$$s_d = 3.043\,15, \quad (32)$$

$$\lambda_b = 1. \quad (33)$$

Calibrated ropelengths (dimensionless):

$$L_{\text{tot}}(e) = 3.339\,63 \times 10^{-2}, \quad (34)$$

$$L_{\text{tot}}(\mu) = 4.416\,51 \times 10^1, \quad (35)$$

$$L_{\text{tot}}(\tau) = 1.990\,71 \times 10^3, \quad (36)$$

$$L_{\text{tot}}(p) = 3.594\,25 \times 10^2, \quad (37)$$

$$L_{\text{tot}}(n) = 3.599\,21 \times 10^2. \quad (38)$$

Reported ratios:

$$\frac{L_\mu}{L_e} \approx 1.322\,46 \times 10^3, \quad (39)$$

$$\frac{L_\tau}{L_\mu} \approx 4.507\,43 \times 10^1. \quad (40)$$

## 6.2 Benchmark summary statistics

Define percentage error as

$$\varepsilon = 100\% \times \frac{M^{\text{pred}} - M^{\text{ref}}}{M^{\text{ref}}}.$$

For the elements table (92 entries, H–U) in *exact\_closure*:

$$\text{MAE}(|\varepsilon|) \approx 1.960\,00 \times 10^{-1} \%, \quad (41)$$

$$\text{median}(|\varepsilon|) \approx 6.900\,00 \times 10^{-2} \%, \quad (42)$$

$$\text{RMSE}(|\varepsilon|) \approx 3.740\,00 \times 10^{-1} \%, \quad (43)$$

$$\max |\varepsilon| \approx 1.840\,00 \%, \quad (44)$$

$$\text{p95}(|\varepsilon|) \approx 8.080\,00 \times 10^{-1} \%. \quad (45)$$

For the molecules table (19 entries):

$$\text{MAE}(|\varepsilon|) \approx 1.560\,00 \times 10^{-1} \%, \quad (46)$$

$$\text{median}(|\varepsilon|) \approx 3.860\,00 \times 10^{-2} \%, \quad (47)$$

$$\max |\varepsilon| \approx 1.330\,00 \%. \quad (48)$$

## 6.3 Selected examples

Table 1 lists representative entries (*exact\_closure*).

Table 1: Selected benchmark entries in *exact\_closure*.

Object	Actual mass (kg)	Predicted mass (kg)	$\varepsilon$ (%)
H	$1.673\,56 \times 10^{-27}$	$1.673\,53 \times 10^{-27}$	-1.480 00
He	$6.646\,48 \times 10^{-27}$	$6.656\,20 \times 10^{-27}$	1.460 00
C	$1.994\,47 \times 10^{-26}$	$1.993\,10 \times 10^{-26}$	-6.880 00
O	$2.656\,70 \times 10^{-26}$	$2.656\,31 \times 10^{-26}$	-1.450 00
Fe	$9.273\,28 \times 10^{-26}$	$9.287\,66 \times 10^{-26}$	1.550 00
Au	$3.270\,76 \times 10^{-25}$	$3.270\,62 \times 10^{-25}$	-4.350 00
U	$3.952\,58 \times 10^{-25}$	$3.952\,69 \times 10^{-25}$	2.850 00
H <sub>2</sub> O	$2.991\,46 \times 10^{-26}$	$2.991\,02 \times 10^{-26}$	-1.480 00
CO <sub>2</sub>	$7.308\,03 \times 10^{-26}$	$7.305\,73 \times 10^{-26}$	-3.160 00
C <sub>8</sub> H <sub>10</sub> N <sub>4</sub> O <sub>2</sub>	$3.224\,60 \times 10^{-25}$	$3.223\,41 \times 10^{-25}$	-3.690 00

Table 2: Largest absolute percentage errors in *exact\_closure* for the fixed  $(Z, N)$  table used by the script.

Object	Actual mass (kg)	Predicted mass (kg)	$\varepsilon$ (%)
B	$1.795\,04 \times 10^{-26}$	$1.828\,06 \times 10^{-26}$	1.839 00
Cl	$5.886\,61 \times 10^{-26}$	$5.806\,22 \times 10^{-26}$	-1.366 00
Mg	$4.035\,94 \times 10^{-26}$	$3.982\,56 \times 10^{-26}$	-1.323 00
Li	$1.152\,41 \times 10^{-26}$	$1.164\,99 \times 10^{-26}$	1.091 00
Ne	$3.350\,97 \times 10^{-26}$	$3.319\,44 \times 10^{-26}$	-9.410 00
HCl	$6.054\,33 \times 10^{-26}$	$5.973\,57 \times 10^{-26}$	-1.334 00
NaCl	$9.704\,19 \times 10^{-26}$	$9.623\,19 \times 10^{-26}$	-8.350 00

#### 6.4 Largest deviations in the fixed table

Table 2 lists the largest absolute errors over the element and molecule sets.

### 7 Interpretation and limitations

**(1) Table mismatch: isotopes vs standard atomic weights.** The element table fixes a single  $(Z, N)$  per element, while the “actual” mass is computed from a standard molar mass (often an isotopic average). A strict test should compare against isotope-specific atomic masses (e.g., AME tables) for the chosen  $(Z, N)$ .

**(2) Nuclear binding model is semi-empirical.** The SEMF step (24) is a phenomenological correction. The reported accuracy for atoms therefore reflects the combined effect of (i) exact nucleon calibration in *exact\_closure*, and (ii) an empirical nuclear binding proxy.

**(3) Leptons beyond the electron are not predicted in this implementation.** Muon and tau ropelengths are solved from their measured masses via (14); they should be treated as calibrations unless an independent SST rule for lepton  $L_{\text{tot}}$  is introduced.

**(4) Mode-handling robustness.** In the current code, any unrecognized `mode` string falls through to the *exact\_closure* branch. For publication-quality reproducibility, the CLI should validate mode strings and fail loudly on invalid values.

## 8 Falsifiable checks enabled by this kernel

Even within this script’s structure, two immediate falsifiers can be stated:

- **Canonical baryons:** Fix  $(s_u, s_d)$  from hyperbolic-volume assignments and set  $\lambda_b = 1$ . The resulting proton/neutron residuals become a direct test of the Canon mapping (independent of the exact-closure fit).
- **Isotope-resolved benchmark:** Replace standard atomic weights with isotope masses for the script’s chosen  $(Z, N)$  (or update  $(Z, N)$  to match dominant isotopes) and re-evaluate error statistics.

## 9 Reproducibility

The script writes `SST_Invariant_Mass_Results_{mode}.csv` and optionally a cross-mode comparison CSV. For non-interactive environments, the optional prompt should be disabled or guarded by `sys.stdin.isatty()`.

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