Kelvin Mode Suppression in Atomic Orbitals: A Vortex-Filament Gap

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Abstract

In hydrodynamic models where electrons are represented by closed vortex filaments in an incompressible medium, atomic orbitals arise as equilibrium configurations. A key consistency issue is whether internal Kelvin-wave excitations could add thermodynamic corrections large enough to destabilize the hydrogenic spectrum. We show that, absent additional structure, such corrections would exceed spectroscopic bounds by many orders of magnitude. A topological excitation gap in the Kelvin spectrum—of order $\boldsymbol{\mathcal{O}(10^2\text{–}10^3\,\mathrm{eV})}$—naturally suppresses these effects. In this regime, the Schrödinger equation functions as a low-energy equation of state, while Kelvin dynamics are inert except under extreme acceleration or high-energy conditions. This separation of scales resolves a central constraint on vortex-filament descriptions of atomic structure.

Keywords: Kelvin modes, vortex filaments, hydrodynamic models, atomic spectroscopy, excitation gap

1 Introduction

Hydrodynamic and vortex-based models of matter trace back to Kelvin and Helmholtz [1, 2]. Modern superfluid dynamics and quantum turbulence have renewed interest in vortex filaments as fundamental excitations [3, 4, 6].

We consider a conservative vortex-filament picture in which closed, knotted filaments embedded in an incompressible medium encode electron degrees of freedom. Prior work indicates that the hydrogenic spectrum can follow from hydrodynamic force balance, with the Schrödinger equation emerging as the Euler–Lagrange condition of an appropriate free-energy functional.

A natural objection is the presence of *Kelvin waves*—helical filament excitations [5, 6]. If these modes couple thermally to orbital degrees of freedom, they could produce level shifts in conflict with precision spectroscopy. Here I quantify this concern and show how a gapped Kelvin spectrum evades it.

This connects to a broader hydrodynamic tradition: the Madelung formulation, pilot-wave ideas, and droplet analogs [11–13], where fluid variables can mirror quantum structure at low energies.

Didactic recap. Plain summary. Internal filament waves could, in principle, ruin hydrogen's spectrum. Naively, they do. A topological gap renders them inert at atomic energies, recovering standard quantum behavior as a low-energy limit.

2 Kelvin Waves on Vortex Filaments

For a thin filament with circulation Γ and core radius ξ, small-amplitude Kelvin waves satisfy [5, 6]

$$\omega(k) \simeq \frac{\Gamma}{4\pi} k^2 \left[\ln\left(\frac{1}{|k|\xi} \right) + C_0 \right], \tag{1}$$

with $C_0 = \mathcal{O}(1)$. For a closed filament of length L,

$$k_m = \frac{2\pi m}{L}, \quad m = 1, 2, \ldots, \qquad \omega_m \propto \frac{\Gamma m^2}{L^2}. \tag{2}$$

For hydrogenic orbitals $r_n = a_0 n^2$, implying $L_n \sim 2\pi a_0 n^2$, Kelvin frequencies soften rapidly with principal quantum number n.

Didactic recap. Intuition. Kelvin modes are bending/twisting ripples on the filament. Longer loops (higher n) mean softer modes: $\omega_m \propto m^2/L^2$.

3 Thermodynamic Constraint from Atomic Spectroscopy

If Kelvin modes are thermally excited at temperature T,

$$U_{\text{Kelvin}} \sim \sum_m \hbar\omega_m f(\omega_m, T), \tag{3}$$

with f a thermal factor. Modeling their net influence as

$$E_n^{\text{eff}} = E_n^{(0)} - a_n T^2 + \ldots, \tag{4}$$

hydrogen spectroscopy imposes

$$a_n \lesssim 10^{-62}\,\mathrm{J\,K^{-2}} \tag{5}$$

for low-lying states. Naive elastic estimates using plausible filament-core parameters overshoot this by > 20 orders of magnitude. Kelvin modes must therefore be inert in ordinary atoms.

Didactic recap. Key point. Without extra structure, thermalized Kelvin waves would shift lines far beyond observed limits.

4 Gapped Kelvin Spectrum

We propose a *topological gap* in the Kelvin spectrum: the first excitation costs Δ_K, and higher modes obey $E_{m,n} \geq \Delta_K$. A convenient Hamiltonian is

$$H_K^{(n)} = \sum_m \left[(\Delta_K + \delta E_{m,n})\, b^\dagger_{mn} b_{mn} + \frac{1}{2}(\Delta_K + \delta E_{m,n}) \right]. \tag{6}$$

Knotted filaments naturally produce thresholds through reconnection constraints, curvature, and torsion [7].

Didactic recap. What the gap does. It imposes a threshold energy for *any* Kelvin excitation, preventing low-energy thermal activation.

A dimensional estimate supports the required magnitude of Δ_K. For a filament with circulation Γ and core radius ξ, the tension $T_f \sim \rho\,\Gamma^2/(4\pi\xi^2)$ sets a characteristic bending energy for the first Kelvin mode. Taking the mode wavelength to be of order the loop length L, one obtains $\Delta_K \sim \Gamma^2/(4\pi\xi L)$ up to logarithmic corrections. With $\Gamma = h/m_e$, $\xi \sim 10^{-15}\,\mathrm{m}$, and $L \sim 10^{-10}\,\mathrm{m}$, this gives $\Delta_K = \mathcal{O}(10^2\text{–}10^3\,\mathrm{eV})$, consistent with the phenomenological scale used below.

5 Low-Temperature Thermodynamics

The dimensionless temperature ratio

$$\varepsilon_K = \frac{k_B T}{\Delta_K} \ll 1$$

characterizes the low-temperature (Kelvin-frozen) regime. For a single gapped bosonic mode [8],

$$Z = \frac{1}{1 - e^{-\beta\Delta_K}}, \qquad U = \frac{\Delta_K}{e^{\beta\Delta_K} - 1}, \qquad \beta = (k_B T)^{-1}. \tag{7}$$

In the limit $\varepsilon_K \ll 1$,

$$U \simeq \Delta_K e^{-1/\varepsilon_K}, \tag{8}$$

so entropy and heat capacity are exponentially suppressed.

For finitely many Kelvin modes,

$$U_K^{(n)}(T) \lesssim N_K \Delta_K \exp\left(-\frac{\Delta_K}{k_B T} \right), \tag{9}$$

replacing the dangerous polynomial scaling of the ungapped case.

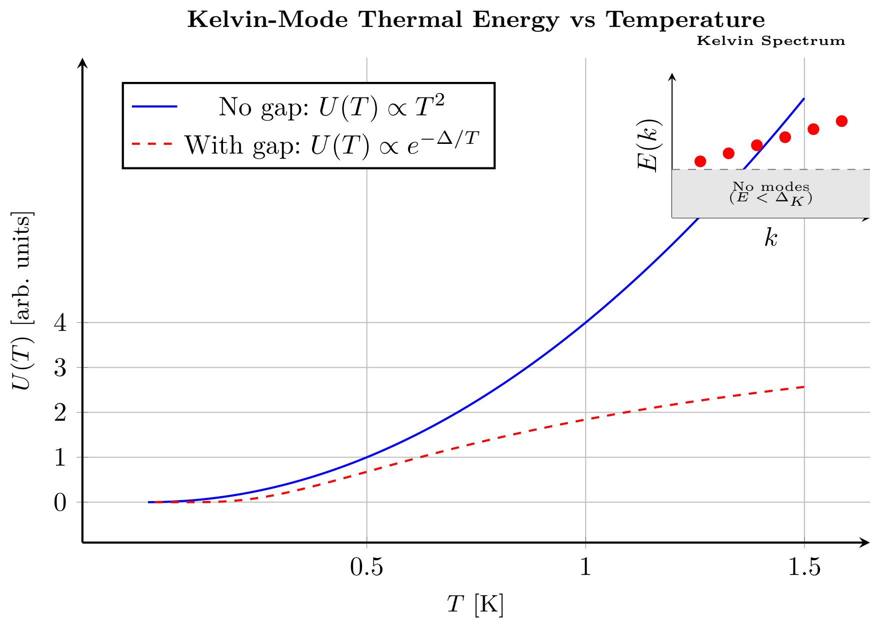

Fig. 1: Comparison of Kelvin-mode thermal energy with and without a topological excitation gap Δ_K. Inset: discrete spectrum with a forbidden band $E < \Delta_K$.

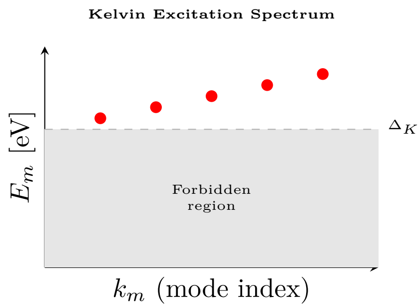

Fig. 2: Illustrative Kelvin excitation spectrum with a gap $\Delta_K = 500\,\mathrm{eV}$. Shaded band: $E < \Delta_K$.

Didactic recap. Takeaway. With a gap, thermal activation is exponentially small: $U \sim e^{-\Delta_K/k_B T}$, not $\sim T^2$.

6 Required Gap Scale

Imposing

$$a_n^{\mathrm{eff}} \lesssim 10^{-62}\,\mathrm{J\,K^{-2}} \tag{10}$$

yields

$$\frac{\Delta_K}{k_B T_{\rm eff}} \gtrsim 60. \tag{11}$$

Using a conservative bound

$$T_{\rm eff} \lesssim 10^5\,\mathrm{K}, \tag{12}$$

we obtain

$$\Delta_K \gtrsim 5 \times 10^2\,\mathrm{eV}. \tag{13}$$

This sits far below $m_e c^2 = 511\,\mathrm{keV}$ yet far above atomic binding scales ($\sim 10\,\mathrm{eV}$), so Kelvin modes are frozen in ordinary atomic physics.

Didactic recap. Numbers in context. A few hundred eV is negligible compared to the electron's rest energy but huge on atomic scales—exactly what we need for inert Kelvin modes.

Such a gap implies that Kelvin excitations remain frozen up to effective temperatures corresponding to electron accelerations of order $10^{22}\,\mathrm{m/s^2}$—far beyond terrestrial or typical laboratory regimes. Hence no measurable corrections to atomic spectra are expected under ordinary conditions, while high-acceleration or keV-MeV scattering experiments could in principle probe activation of these modes.

7 Relation to the Schrödinger Equation

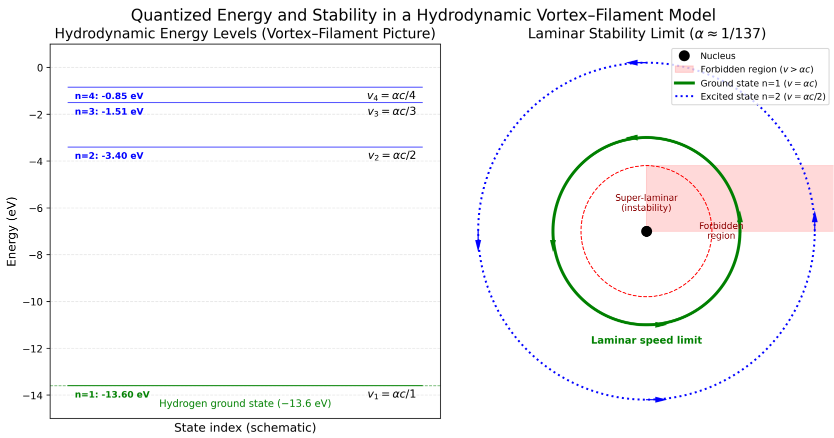

Fig. 3: **Hydrodynamic energy levels** for $n = 1, 2, 3, 4$. The ground state ($n = 1$) sets a laminar baseline; excited states scale approximately as $1/n^2$, matching the hydrogenic spectrum. These levels emerge from vortex equilibrium in an incompressible medium.

With Kelvin modes suppressed, a minimal free-energy functional is

$$\mathcal{F}[\psi] = \int d^3r \left[\frac{\hbar^2}{2m_e} |\nabla\psi|^2 + V(r)\,|\psi|^2 \right]. \tag{14}$$

Here $V(r)$ is taken as an *effective* binding potential with far-field behavior $V(r) \sim -\Lambda/r$. In the corrected SST-consistent picture, a steady swirl with $v_\theta \propto 1/r$ yields a near-field pressure deficit $\Delta p \propto -1/r^2$ (Euler radial balance), while the far-field $1/r$ tail is modeled as Poisson-mediated (clock/foliation mediator on $\mathbb{R}^3$). Variation under normalization gives

$$-\frac{\hbar^2}{2m_e}\nabla^2\psi + V(r)\,\psi = E\,\psi, \tag{15}$$

the stationary Schrödinger equation [9].

Didactic recap. Bridge to QM. In the Kelvin-frozen regime, hydrodynamics reproduces Schrödinger's stationary equation as the Euler–Lagrange condition of $\mathcal{F}$.

8 Hydrodynamic Origin of the Schrödinger Equation

Assuming a closed, knotted filament with internal excitations frozen, consider

$$\mathcal{F}[\psi] = \int d^3r\left[\frac{\hbar^2}{2m_e}|\nabla\psi|^2 + V(r)\,|\psi|^2\right], \qquad \int |\psi|^2 d^3r = 1, \tag{16}$$

with $V(r)$ an effective potential whose far-field asymptotic is $V(r) \sim -\Lambda/r$. The wording "pressure potential" is not meant as a claim that Euler pressure alone produces $1/r$; rather, the $1/r$ tail is attributed to a local mediator (Poisson/Green-function response), while Euler pressure motivates only the regularized core/near-field structure. Varying $\mathcal{F}$ yields

$$-\frac{\hbar^2}{2m_e}\nabla^2\psi + V(r)\,\psi = E\,\psi, \tag{17}$$

i.e., the stationary Schrödinger equation.

Didactic recap. Same result, different route. This re-derivation underlines that the reduction is robust to modeling details once Kelvin modes are inert.

9 Thermodynamic Catastrophe from Ungapped Kelvin Modes

If Kelvin modes couple thermodynamically to orbitals, one can parameterize

$$U_{\mathrm{Kelvin},n}(T_*) = a_n T_*^2 + \mathcal{O}(T_*^4), \qquad E_n^{\mathrm{eff}} = E_n^{(0)} - a_n T_*^2. \tag{18}$$

A naive elastic estimate gives

$$a_n^{\mathrm{naive}} \sim 10^{-39}\,\mathrm{J/K^2}, \tag{19}$$

while spectroscopy requires

$$a_n \lesssim 10^{-62}\,\mathrm{J/K^2}. \tag{20}$$

The gap is therefore essential: without it, the $1/n^2$ spectrum collapses.

Didactic recap. Bottom line. Ungapped Kelvin thermodynamics is a disaster for atoms; the gap prevents it.

10 Discussion and Outlook

A Kelvin excitation gap resolves a central consistency constraint in vortex-filament accounts of atomic structure. Atomic orbitals remain sharply quantized because Kelvin waves are exponentially suppressed at low energies, and the Schrödinger equation emerges as an effective equation of state in this Kelvin-frozen regime. This establishes a clean separation of scales: (1) low-energy, where hydrodynamic equilibrium reproduces quantum results; and (2) high-energy or high-acceleration domains, where Kelvin dynamics activate and new phenomenology may arise.

This delivers a clear separation of scales:

- **Low energy**: Kelvin modes inert; orbital structure from hydrodynamic equilibrium, matching standard quantum results.
- **High energy / high acceleration**: Kelvin dynamics activate, enabling new phenomenology.

One possible Swirl-String framework (SST) realizes this broader picture by modeling electrons as topologically stable, knotted vortex filaments in a real, incompressible condensate. The present mainstream formulation dovetails with SST's claims: the same gapped Kelvin mechanism explains why low-energy atomic physics is effectively quantum-mechanical, while predicting activation of internal modes under extreme conditions.

From an empirical perspective, possible signatures include slight broadening or phase lags in atomic transitions under extreme acceleration, and energy-loss features in high-field scattering consistent with Kelvin-mode activation thresholds near a few hundred eV. Quantum-clock and relativistic interferometry techniques [16, 17] offer a future route to testing such coherence-dependent effects.

Potentially relevant domains include:

- ultra-high accelerations (Unruh-like activation) [10],
- keV-MeV scattering,
- vortex reconnections in astrophysical or cosmological fluids.

More broadly, quantum-clock experiments access coherence-dependent time-dilation signatures [16, 17]. From a foundations standpoint, relational-time formalisms suggest treating clock observables as conditional dynamics within correlations. [14, 15].

Didactic recap. What to test. Look for activation of Kelvin modes under extreme conditions; at ordinary atomic energies they should remain silent. SST provides one concrete framework where these effects can be modeled in detail.

Appendix A: Estimate of First Kelvin Mode Energy

For the first mode ($m = 1$) in the ground state ($n = 1$), $L \sim 2\pi r_1 = 2\pi a_0 \approx 3.3 \times 10^{-10}\,\mathrm{m}$ with $a_0 \approx 5.29 \times 10^{-11}\,\mathrm{m}$. The circulation quantum $\Gamma = h/m_e$ gives $\Gamma \approx 7.27 \times 10^{-4}\,\mathrm{m}^2/\mathrm{s}$.

Then $k_1 = 2\pi/L \approx 1.9 \times 10^{10}\,\mathrm{m}^{-1}$ and

$$\omega_1 \simeq \frac{\Gamma}{4\pi} k_1^2 \left[\ln\left(\frac{1}{k_1 \xi}\right) + C_0 \right], \tag{21}$$

with $C_0 \sim 1$ and $\xi \sim 1 \times 10^{-15}\,\mathrm{m}$, giving $\omega_1 \approx 2.8 \times 10^{17}\,\mathrm{rad/s}$ and

$$E_1 = \hbar\omega_1 \approx 29.5\,\mathrm{eV}. \tag{22}$$

Tighter knots/smaller cores push this higher, consistent with a gap $\Delta_K \sim 500\,\mathrm{eV}$.

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Consent for publication

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Code availability

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Author contributions

O.I. conceived the study, developed the analysis, performed the derivations, and wrote the manuscript.

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