Swirl-Strings: Bridging Blocks for Modern Physics and String Methods in Fluid Dynamics

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Abstract

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Contents

1 Introduction

Parameter Dictionary (cross-referencing A–I)

Cutoff / EFT (A): $\mathrm{E}_c \equiv \hbar, \Omega_c = \hbar, |\mathbf{v}_{!\circlearrowleft}|/r_c$.
Misalignment (A3): $\xi = v^2/f^2$.
Projector flavour (A4):
ASG (B): $\mathrm{f}_g,\ f_y;\quad yt,*^2 - ya, \hat{\mathbf{v}}$
ASG (B): $\mathrm{f}_g,\ f_y;\quad yt,*^2 - yb,*^2 = (Q_t^2 - Q_b^2), gY,^2/;$(SM: $= \frac{1}{3}g_{Y,}{}^{2}$).
EW kernels (C): $\Delta\rho,\ \Delta r,\ a_\mu;\quad F_0,\ F^{\mathrm{C,N}}$
EJA ladder (D): $\sqrt{m}$-eigenvalues $s-\delta,\ s,\ s+\delta,\ \delta^2 = 3/8$.
R2M2 (E): $\mathrm{M}_f(\mu) = y_f(\mu), \alpha(\mu)\alpha^\dagger(\mu),\ \alpha : S^2$ curve Γ; $\omega \equiv |\mathrm{d}\alpha/\mathrm{d}\ln\mu|$.
Safe PS (F): GPS, $\mathrm{v}_R,\ N_{Fi},\ A_i = 3,\ (\bar{\lambda}_i) \leftarrow (\lambda_j),\ m_b \simeq (M_F m_t)/(\sqrt{2}m_B)$.
Sterile/modulino (G): $(\Delta m^2, \theta)41, m\tilde{z} \sim [(M_P R)^2, 4\pi^2 R]^{-1} f(\omega),\ m_{3/2} = \omega/R$.
Entanglement (H): $\mathrm{E}^\perp$
Unified flavor (I): GUFG(T), local groups, CP inner/outer; T (swirl modulus).

Synthesis Matrix

Theme	**Overlap (who constrains whom)**
hline	(D) EJA fixes adjacent $\sqrt{m}$ ratios via $s \pm \delta$; (E) R2M2 generates hierarchies/mixings from $\alpha(\mu)$ rotation; (H) EMH selects Cabibbo/mixing minima via $E^{\perp}_{\min}$; (I) local flavor groups set allowed textures/CP via T. Together: choose Γ, T, and an interference weight y so that (D) $\Rightarrow$ ratios, (E) $\Rightarrow$ leakage consistent with (D), and (H) $\Rightarrow$ observed small quark vs large lepton mixings.
Flavor backbone	
Top–bottom split	(B) demands $y_t > y_b$ by $\Delta y^2 = (Q_t^2 - Q_b^2) g^2_{Y,*}$; (F) generates $m_b \ll m_t$ via $(10, 1, 1)$ mixing. Compatible if $m_b/m_t \simeq M_F/(\sqrt{2}m_B)$ saturates the required IR split after running from the ASG-determined UV difference.
EW integrity	(C) $\lvert\Delta\rho\rvert \lesssim 10^{-3}$ on any new doublet; (A3) pNGB misalignment with custodial symmetry preserves $\Delta\rho \approx 0$. Use (C) as a veto on spectrum choices introduced by (E,F,H,I).
Naturalness/EFT	(A1,A2) cap power growth at E_c and require fine-tuning bookkeeping for any bridging scalar; dovetails with pNGB option in (A3) and with UV safety (F) or gravitational fixed points (B).
Sterile window	(G) defines a decoupled sterile sector (aetherino), consistent with (C) and (A) since couplings are Planck-suppressed; does not disturb the flavor backbone but offers experimental hooks (FPF).
CP origin	(E) converts strong θ to CKM phase through rotation; (I) makes CP inner (exact) at enhancement loci and outer (spontaneously broken) away; both mechanisms can co-exist: R2M2 provides the phase while T controls where CP is protected/broken.

Compatibility Lemmas

Lemma 1 (EJA $\leftrightarrow$ R2M2). For a fermion species f, suppose R2M2 yields (small-angle) leakage

$$m_2^{(f)}/m_3^{(f)} \simeq c_f\, \Delta\theta_{23}^2, \quad m_1^{(f)}/m_2^{(f)} \simeq c'_f\, \Delta\theta_{12}^2.$$

The EJA ladder (D6) gives

$$\left(\frac{m_2^{(f)}}{m_3^{(f)}}\right)^{1/2} = \frac{s_f}{s_f + \delta}, \qquad \left(\frac{m_1^{(f)}}{m_2^{(f)}}\right)^{1/2} = \frac{s_f - \delta}{s_f}.$$

Then the two descriptions are mutually consistent if

$$\Delta\theta_{23} = \frac{1}{\sqrt{c_f}}\frac{s_f}{s_f + \delta}, \qquad \Delta\theta_{12} = \frac{1}{\sqrt{c'_f}}\frac{s_f - \delta}{s_f} \tag{1}$$

for some real $c_f, c'_f > 0$. Thus EJA sets *targets* for the arc-lengths along Γ; R2M2 supplies them geometrically.

Lemma 2 (EMH $\leftrightarrow$ Cabibbo). With the two-generation entangling-power kernel (H2), there exists an interference weight $y \in \mathbb{R}$ such that

$$\left.\frac{\partial E_{ud}^{\perp}}{\partial \theta_C}\right|_{\theta_C=\theta^{\star}} = 0$$

holds at a desired $\theta^{\star} \in (0, \pi/4)$. In particular, deforming y away from its SM value by small anisotropies in neutral vs charged-current swirl permeability moves the minimum continuously from $\sim 6^{\circ}$ toward $\sim 13^{\circ}$, thereby matching the projector-driven Cabibbo in (A4).

Lemma 3 (ASG $\leftrightarrow$ PS split). Take $\tan\beta = 1$ at the PS scale so $y_t \simeq y_b$. ASG then enforces a UV offset $\Delta y^2 = \frac{1}{3}g_{Y,*}^2$. After RG flow and PS vector-like mixing, the IR ratio obeys

$$\frac{m_b}{m_t} \simeq \frac{M_F}{\sqrt{2}m_B}\left[1 - \frac{\Delta y^2}{2y_t^2} + \mathcal{O}(\text{running})\right] \tag{2}$$

so choosing M_F/m_B fixes m_b/m_t while respecting the ASG-induced inequality $y_t > y_b$.

Lemma 4 (EW integrity under misalignment). If custodial symmetry is preserved (pNGB map A3 with custodial coset), then new contributions to $\Delta\rho$ vanish at leading order; any extra doublets (C1) must satisfy $|\Delta\rho| \lesssim 10^{-3}$. Coupling deviations scale as $\delta g_{hVV}/g_{hVV} = -\frac{1}{2}\xi$, hence $\xi \ll 1$ suffices to keep both Higgs-coupling and W-mass constraints under control.

Minimal Canon Workflow (bridging layer)

1. **Choose local flavor groups** per sector via T (I); decide CP inner/outer at that scale.
2. **Pick a rotation curve** $\Gamma(\mu; a)$ (E) with one curvature parameter a; this sets leakages and mixing via small arc lengths.
3. **Fix EJA targets** by selecting (s_f, δ) (D) (canonical $\delta^2 = 3/8$) and equate with the R2M2 arc relations of Lemma 1 to determine a and c_f, c_f'.
4. **Tune Cabibbo** by the EMH selector (H): adjust the interference weight y (swirl permeability) to place the $E_{ud}^{\perp}$ minimum at the desired θ_C.
5. **Impose ASG/PS** via Lemma 3 for third family, then extend to lighter families with the projector basis (A4) and R2M2 geometry.
6. **Check EW kernels** (C): $\Delta\rho$, a_μ, and Δr for any extra excitations introduced; ensure ξ is small (A3).
7. **Optionally add a sterile window** (G): record $(\Delta m^2, \theta)$ consistent with cosmology toggles; keep Planck-suppressed couplings to avoid conflicts with (C).

Tensions & Resolutions

- **Cabibbo** 6° **vs** 13°**.** Resolved by Lemma 2: small y-deformations (effective neutral/charged-current balance) shift the EMH minimum to the projector target.
- **EJA fixed** δ **vs R2M2 flexibility.** Lemma 1 equates EJA ratios to arc lengths, absorbing differences into species coefficients c_f, c'_f (expected from differing y_f normalizations).
- **ASG inequality vs PS** $\tan\beta = 1$**.** Lemma 3 shows vector-like mixing accomplishes the IR split while keeping $y_t > y_b$ as required in the UV.
- **EW bounds vs new excitations.** Apply the Custodial Integrity Rule (C1) and small ξ; veto spectra that overshoot $|\Delta\rho|$.

Dimensional & Limit Checks

All synthesis relations are dimensionally consistent: arc lengths are angles, mass ratios are dimensionless, loop kernels dimensionless, and ξ dimensionless. Limits: $\omega \to 0$ (no rotation) $\Rightarrow$ no mixing; $\delta \to 0$ (EJA collapse) $\Rightarrow$ degenerate families; $y \to 0$ (no neutral-current interference) $\Rightarrow$ EMH prefers small Cabibbo.

(D6) EJA ratios: $\frac{\sqrt{m}_{\text{mid}}}{\sqrt{m}_{\text{heavy}}}$

(E) Rotating rank-one: $M_f(\mu) =$

$m_2/m_3 \simeq c_f\,\Delta\theta_{23}^2, \quad m_1/m_2 \simeq c'_f\,\Delta\theta_{12}^2.$

(H2) EMH kernel: $E^{\perp}_{ud}(\theta_C; y$

$(\mathrm{B3_{SM}})ASGdifference:$ $(\mathrm{B3_{SM}})ASGdifference:$ $(\mathrm{B3_{SM}})ASGdifference:$ $(\mathrm{B3_{SM}})ASGdifference: y_{t,*}^2 - y_{b,*}^2 = \frac{1}{3}g_{Y,*}^2.$

$\frac{M_F\,m_t}{\sqrt{2}\,m_B}$ (at leading order).

(C & A3) EW/Higgs bounds: $|\Delta\rho| \lesssim 10^{-3}, \quad \delta g_{hVV}/g_{hVV} = -\frac{1}{2}\xi, \quad \xi \ll 1.$

Fundamental Lagrangian (schematic form)

1

$$\mathcal{L}_{\text{SST}} = \underbrace{\left(\frac{1}{2}^2 - \Phi_{\text{swirl}}\right)}_{\text{fluid swirl field}} + \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Electromagnetism}} + \underbrace{\alpha C(K) + \beta L(K) + \gamma\mathcal{H}(K)}_{\text{Mass from topology}} + \underbrace{\ln\sqrt{1 - \frac{|\omega|^2}{c^2}}}_{\text{Time dilation (swirl-clock)}} + \underbrace{\Delta p(}_{\text{Gravity via}}$$

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Addendum E: Naturalness bounds and pNGB misalignment (bridging)

E1. Load-bearing statements (from Wulzer)

(i) Naturalness as a quantitative fine-tuning bound. Split the Higgs mass just below the SM cutoff:

$$m_H^2 \;=\; \underbrace{\delta^{\mathrm{SM}} m_H^2}_{E<\Lambda_{\mathrm{SM}}} \;+\; \underbrace{\delta^{\mathrm{BSM}} m_H^2}_{E\geq\Lambda_{\mathrm{SM}}}. \tag{3}$$

With SM quadratic sensitivity,

$$\delta^{\mathrm{SM}} m_H^2 = \frac{3y_t^2}{4\pi^2}\Lambda_{\mathrm{SM}}^2 - \frac{3g_W^2}{8\pi^2}\left(\frac{1}{4} + \frac{1}{8\cos^2\theta_W}\right)\Lambda_{\mathrm{SM}}^2 - \frac{3\lambda}{8\pi^2}\Lambda_{\mathrm{SM}}^2, \tag{4}$$

the minimum fine-tuning obeys

$$\Delta \;\gtrsim\; \frac{\delta^{\mathrm{SM}} m_H^2}{m_H^2} \;\approx\; \left(\frac{\Lambda_{\mathrm{SM}}}{450\,\mathrm{GeV}}\right)^2. \tag{5}$$

(ii) No-Lose Theorems post-Higgs. After EWSB, no remaining EW/QCD amplitude exhibits forced power growth below $\sim 4\pi v$; only gravity ensures new physics at M_{Pl}.

(iii) Composite Higgs misalignment. For $SO(5)/SO(4)$ pNGB Higgs,

$$v = f\,\sin\!\left(\frac{V}{f}\right), \qquad \xi \equiv \frac{v^2}{f^2}, \qquad \frac{g_{hVV}}{g_{hVV}^{\mathrm{SM}}} = \sqrt{1-\xi}, \quad \frac{g_{hhVV}}{g_{hhVV}^{\mathrm{SM}}} = 1-2\xi, \tag{6}$$

with SM recovered as $\xi \to 0$.

(iv) If Nature is un-natural. Fallback origins for m_H^2: environmental (anthropic) selection; dynamical relaxation (relaxion).

E2. SST/VAM translations (canon add-ons)

E2.1 No-Lose (SST version). If a swirl–EFT operator of dimension $\mathcal{D} > 4$ induces $2\to2$ growth $\mathcal{M} \sim (E/\Lambda)^{\mathcal{D}-4}$, then new swirl microstructure must appear below

$$E \;\lesssim\; 4\pi\,\Lambda, \qquad \Lambda^{(E)} \equiv E_c = \hbar\,\Omega_c = \hbar\,\frac{\|\mathbf{v}_{\circlearrowleft}\|}{r_c}. \tag{7}$$

Canon: restate (7) next to Addendum A1 and use E_c as the default UV saturation scale for swirl loops.

E2.2 Naturalness bookkeeping (bridging sectors). For any SST mass-like parameter μ^2,

$$\mu^2 = \delta^{\rm IR}\mu^2 + \delta^{\rm UV}\mu^2, \qquad \Delta \equiv \frac{\delta^{\rm IR}\mu^2}{\mu^2} \gtrsim \kappa\left(\frac{E_c}{E_*}\right)^2, \tag{8}$$

with $E_c = \hbar\|\mathbf{v}_{\circlearrowleft}\|/r_c$ and E_* the physical scale set by μ^2; κ is the largest loop coefficient. *Canon:* track Δ in any scalar *bridging* module (see Addendum A2).

E2.3 Optional pNGB–Higgs map (SST↔EWSB). If the observed Higgs is emergent from $G_{\rm sw} \to H_{\rm sw}$, adopt

$$v = f\,\sin\!\left(\frac{\Theta}{f}\right), \quad \xi = \frac{v^2}{f^2}, \quad \frac{g_{hVV}}{g_{hVV}^{\rm SM}} = \sqrt{1-\xi}, \quad \frac{g_{hhVV}}{g_{hhVV}^{\rm SM}} = 1 - 2\xi, \tag{9}$$

with $\xi \ll 1$ from LHC coupling fits. *Canon:* include (9) in the "SST↔EWSB map" (bridging).

E2.4 Fallback origins for μ^2. *Environmental:* allow μ^2 to scan across æther vacua (Weinberg-style selection). *Dynamical:* a slow "swirl-axion" $a(x)$ that couples to helicity density $\mathbf{v}\cdot\boldsymbol{\omega}$ and halts via backreaction (relaxion analogue).

E3. Minimal math blocks (dimension-checked)

E3.1 Swirl UV saturation. For naive quadratic integrals,

$$\int^{\Lambda} \frac{d^3k}{(2\pi)^3}\,\frac{1}{k^2+m^2} \xrightarrow[\Lambda\to k_c=1/r_c]{\text{SST}} \frac{k_c}{4\pi^2} = \frac{1}{4\pi^2 r_c}, \tag{10}$$

and the associated correction scales as

$$\delta\mu^2 \;\sim\; \kappa\,(\hbar\Omega_c)^2 = \kappa\left(\frac{\hbar\|\mathbf{v}_{\circlearrowleft}\|}{r_c}\right)^2, \qquad [\delta\mu^2] = \text{energy}^2. \tag{11}$$

E3.2 pNGB misalignment identities. Collect the coupling relations:

$$\boxed{v = f\sin(\Theta/f), \quad \xi = \frac{v^2}{f^2}, \quad \frac{g_{hVV}}{g_{hVV}^{\rm SM}} = \sqrt{1-\xi}, \quad \frac{g_{hhVV}}{g_{hhVV}^{\rm SM}} = 1-2\xi} \tag{12}$$

with the SM limit $\xi \to 0$.

E4. Canon to-do (practical)

- Add (7) next to A1 as the *No-Lose (swirl)* lemma and set E_c as the default loop cutoff in Bridging modules.
- Enforce the bookkeeping rule (8) wherever μ^2 appears; record Δ alongside fit parameters.
- If adopting an emergent/pNGB Higgs, register ξ in the Canon constants and expose coupling ratios for phenomenology tables.
- Log the environmental/relaxion options as allowable origins for any remaining scalar mass terms (Bridging-only note).

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Addendum D: EJA ladder — $\sqrt{\text{mass}}$ ratios and CKM root–sum rules (bridging)

D1. Universal inputs from $J_3(\mathbb{O}_\mathbb{C})$

Charged-sector Jordan spectrum. In each charged sector (ℓ, u, d), the right-handed flavor matrix $X \in J_3(\mathbb{O}_\mathbb{C})$ admits three Jordan eigenvalues

$$\lambda \in \{\, s-\delta,\ s,\ s+\delta \,\}, \qquad \delta^2 = \frac{3}{8}, \tag{D1}$$

with *family centre* $s \equiv \sqrt{m}$ fixed by a trace choice per sector. In the canonical normalization (octonionic entries of norm $1/8$), the EJA invariants are

$$T = \operatorname{tr} X = 3s, \qquad S = 3s^2 - \frac{3}{8}, \qquad D = s^3 - \frac{3s}{8}, \tag{D2}$$

and the characteristic polynomial factorizes as

$$\chi_X(\lambda) = (\lambda - s)^3 - \delta^2(\lambda - s), \qquad \delta^2 = S - 3s^2 = D/s - s^2 = \frac{3}{8}. \tag{D3}$$

Dimensional check: s, δ carry $[\sqrt{\text{mass}}]$; with the stated normalization, δ is *sector-universal*.

D2. $\mathrm{Sym}^3(\mathbf{3})$ minimal ladder and edge–universality

Seed rung (fixed Clebsches). Work in the corner basis $\{|E\rangle, |B\rangle, |C\rangle\}$ of $\mathrm{Sym}^3(\mathbf{3})$. The unique minimal top rung satisfying outward–edge universality is

$$|\psi_0\rangle = \alpha|E\rangle + \beta|B\rangle + \gamma|C\rangle, \qquad (\alpha : \beta : \gamma) = (2:1:1). \tag{D4}$$

Edge projection identity (rung cancellation). Let P_{XY} project to the XY edge. Then

$$P_{XY}\big(|\psi_0\rangle^{\otimes_s 3}\big) = \big(\alpha_X|X\rangle + \alpha_Y|Y\rangle\big)^{\otimes_s 3}, \tag{D5}$$

so outward–edge profiles depend *only* on their two legs; the third leg cancels. For $(2:1:1)$ the two outward edges from E are identical at leading order (edge–universality), while the base BC edge is the symmetric binomial.

Adjacent $\sqrt{m}$ ratios (edge–only). Feeding the EJA eigenvalues *(D1)* into the ladder and using edge–universality yields rung–independent adjacent ratios

$$\boxed{\frac{\sqrt{m}_{\text{light}}}{\sqrt{m}_{\text{mid}}} = \frac{s-\delta}{s}, \qquad \frac{\sqrt{m}_{\text{mid}}}{\sqrt{m}_{\text{heavy}}} = \frac{s}{s+\delta}, \qquad \frac{\sqrt{m}_{\text{light}}}{\sqrt{m}_{\text{heavy}}} = \frac{s-\delta}{s+\delta}}. \tag{D6}$$

Note. The ladder does not determine δ; it consumes $s \pm \delta$ and, thanks to *(D5)*, outputs edge–universal adjacent ratios.

D3. Sector mapping and the trace split

Adopt the sector trace split

$$\mathrm{tr}\, X_\ell : \mathrm{tr}\, X_u : \mathrm{tr}\, X_d = 1 : 2 : 3, \tag{D7}$$

so that $s_\ell : s_u : s_d = 1 : 2 : 3$. Then the lightest–generation $\sqrt{m}$ relation follows immediately:

$$\boxed{\sqrt{m_e} : \sqrt{m_u} : \sqrt{m_d} = 1 : 2 : 3} \tag{D8}$$

(up to a common normalization), while the three families arise from a single ladder: the down edge, its Dynkin $\mathbb{Z}_2$ swap (leptons), and the opposite outward edge (up sector).

D4. CKM root–sum rules and geometric phases

In the adjacent–edge approximation the CKM moduli obey

$$|V_{us}| \simeq \left| \sqrt{\tfrac{m_d}{m_s}} - e^{i\varphi_{12}} \sqrt{\tfrac{m_u}{m_c}} \right|, \tag{D9}$$

$$|V_{cb}| \simeq \kappa_{23} \left| \sqrt{\tfrac{m_s}{m_b}} - e^{i\varphi_{23}} \sqrt{\tfrac{m_c}{m_t}} \right|, \tag{13}$$

with a *geometric* Cabibbo phase $\varphi_{12} = \pi/2$ from Fano–oriented rotor overlaps, a mild cross–family normalization $\kappa_{23} \approx 0.55$, and a single up–leg tilt ε (a phase on the e_1 channel) that shifts $\varphi_{12} \to \varphi_{12} + \varepsilon$ without altering rung magnitudes. Choosing $\varepsilon \approx -26.123^\circ$ aligns $|V_{us}|$ with experiment at a common renormalization scale. Leading consequences include

$$|V_{ub}| \sim \sqrt{m_u/m_t}, \qquad \frac{|V_{td}|}{|V_{ts}|} \text{ predicted with no extra freedom.} \tag{D10}$$

Dimensional check: all CKM relations are dimensionless and scheme–consistent when evaluated at a common scale.

D5. SST/VAM adapter (bridging)

- **Dictionary.** Interpret the EJA rank–1 idempotents as *swirl–knot classes* (points of $\mathbb{OP}^2$). The family centre $s \equiv \sqrt{m}$ maps to a swirl–invariant scalar in the RH mass frame (e.g. a knot–energy or time–scaling density); only *ratios* enter predictions, so absolute normalization is model–dependent.

- **Selection rule.** The Dynkin $\mathbb{Z}_2$ swap enforcing down↔lepton edge matching is canonized as a *topological reflection* in the swirl–charge dictionary.

- **Usage.** To test an SST flavour realization: (i) choose $s_\ell : s_u : s_d = 1 : 2 : 3$; (ii) set $\delta = \sqrt{3/8}$; (iii) evaluate *(D6)* for each sector; (iv) feed PDG masses at a common scale into *(D9)* to check CKM.

D6. Implementation & sanity checks

- Tag D1–D5 as *Bridging*. They supply a representation–theoretic kernel and phase geometry; no change to core SST postulates.
- Dimensions: *(D1)–(D3)* carry $[\sqrt{m}]$; *(D6)–(D10)* are dimensionless ratios.
- Limits: $\delta \to 0$ collapses ladders to degenerate families; outward–edge universality remains well defined.

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Addendum A: EFT Growth, Naturalness, and Flavour Projectors

A1. SST "No–Lose" Lemma (EFT growth $\Rightarrow$ new swirl microstructure)

Statement. Consider any SST effective operator of swirl–dimension $\mathcal{D} > 4$ that induces a $2 \to 2$ amplitude $\mathcal{M}(E) \sim (E/\Lambda)^{\mathcal{D}-4}$. Perturbative unitarity then demands the onset of new microstructure at

$$E_{\text{new}} \lesssim 4\pi\,\Lambda\,.$$

Here we choose the geometric swirl cutoff in energy units

$$\Lambda^{(E)} \equiv E_c = \hbar\,\Omega_c = \hbar\,\frac{\|\mathbf{v}_{\circlearrowleft}\|}{r_c}\,.$$

Dimensional check: $[\Omega_c] = \mathrm{s}^{-1}$, $[E_c] = \mathrm{J}$. Using the Canon constants $\|\mathbf{v}_{\circlearrowleft}\| = 1.09384563 \times 10^{6}\,\mathrm{m\,s^{-1}}$, $r_c = 1.40897017 \times 10^{-15}\,\mathrm{m}$, $\hbar = 1.054571817 \times 10^{-34}\,\mathrm{J\,s}$, we get

$$\Omega_c \approx 7.761 \times 10^{20}\,\mathrm{s}^{-1}, \qquad E_c \approx 8.19 \times 10^{-14}\,\mathrm{J} \approx 0.511\,\mathrm{MeV}.$$

Interpretation: Quadratic/power growth in any swirl–EFT amplitude must be cut off at (or below) the electron mass scale.

A2. Naturalness bookkeeping rule (quadratic sensitivity cap)

For any *bridging* scalar–sector parameter with mass dimension two, μ^2, enforce the split

$$\mu^2 = \delta^{\mathrm{IR}}\mu^2 + \delta^{\mathrm{UV}}\mu^2,$$

with a minimal fine–tuning measure

$$\Delta \equiv \frac{\delta^{\mathrm{IR}}\mu^2}{\mu^2} \gtrsim \kappa\left(\frac{E_c}{E_*}\right)^2,$$

where $E_c = \hbar\,\|\mathbf{v}_{\circlearrowleft}\|/r_c$ (from A1), E_* is the physical scale governed by μ^2, and κ is the largest loop coefficient (SST analogue of the top loop). *Dimensional check:* $[\Delta] = 1$.

Remark. This rule either (i) motivates symmetry protection (pseudo–Nambu–Goldstone origin), or (ii) defers to an environmental/dynamical setting (relaxion–style) if protection is absent.

A3. Optional pNGB–misalignment adapter for EWSB (bridging)

If the observed Higgs is realized as an emergent pNGB of a swirl–internal breaking $G_{\mathrm{sw}} \to H_{\mathrm{sw}}$, adopt the one–parameter misalignment map

$$\boxed{v = f\,\sin\!\left(\frac{\Theta}{f}\right), \quad \xi \equiv \frac{v^2}{f^2}, \quad \frac{g_{hVV}}{g_{hVV}^{\mathrm{SM}}} = \sqrt{1-\xi}, \quad \frac{g_{hhVV}}{g_{hhVV}^{\mathrm{SM}}} = 1 - 2\xi}$$

with $\xi \ll 1$ required by Higgs–coupling fits. *Dimensional check:* $[v] = [f] =$ energy.

A4. Flavour: rank–1 projector basis for neutrinos and up sector (bridging)

Normalized family directions

$$\hat{\mathbf{v}}_a = \frac{1}{\sqrt{2}}(0,1,1), \qquad \hat{\mathbf{v}}_b = \frac{1}{\sqrt{21}}(1,4,2), \qquad \hat{\mathbf{v}}_c = (0,0,1).$$

(These unit vectors are *not* mutually orthogonal; orthogonality is not required for a rank–1 projector sum.)

Swirl–mass ansatz (neutrinos). Normal hierarchy with a single quantized relative phase $\Delta\phi = 4\pi/5$:

$$\boxed{m_\nu = \alpha\,\hat{\mathbf{v}}_a\hat{\mathbf{v}}_a^\top + \beta\,e^{i\,4\pi/5}\,\hat{\mathbf{v}}_b\hat{\mathbf{v}}_b^\top + \gamma\,\hat{\mathbf{v}}_c\hat{\mathbf{v}}_c^\top}$$

with $\alpha, \beta, \gamma > 0$. For suitable β/α (fixed by m_2/m_3), this reproduces $\theta_{12} \approx 34^\circ$, $\theta_{23} \approx 40^\circ$, $\theta_{13} \approx 9^\circ$ and a leptonic CP phase $\delta_\ell \approx 260^\circ$.

Up–sector reuse and Cabibbo. Reusing $\hat{\mathbf{v}}_b$ as the dominant column in Y_u yields at leading order $\theta_C \approx 13$–14°, with quark mixing generated predominantly from the up sector.

Matching conditions (down vs. charged leptons). Impose the Georgi–Jarlskog relations at the Pati–Salam matching scale as boundary data for the *down*/charged–lepton sectors:

$$\boxed{m_e = \tfrac{1}{3} m_d, \qquad m_\mu = 3 m_s, \qquad m_\tau = m_b}$$

(This functions as a *selection rule* in the SST coupling dictionary.)

Selection–rule note. The factor $1/3$ effectively suppresses the third–family contribution in the neutrino channel; in SST this is encoded as a topological coupling ratio (e.g., twist parity or circulation quantum), making the hierarchy robust.

A5. Implementation notes

- Tag A1–A4 as *Bridging* in the Canon. They do not alter the core postulates; they constrain the EFT and provide a compact flavour template.
- Place A3 in the "SST $\leftrightarrow$ EWSB Map" subsection. Place A4 in the "Flavour" subsection, with a cross reference to your knot taxonomy rules.
- Record constants and numerics in the Canon constants table (no new constants introduced).

A6. Sanity checks (dimensions and limits)

- A1: $[E_c] = \mathrm{J}$; decoupling limit $r_c \to 0$ with fixed $\|\mathbf{v}_{\circlearrowleft}\|$ pushes $E_c \to \infty$, restoring pure contact EFT.
- A2: Δ is dimensionless; protection via symmetry (pNGB) corresponds to $\kappa \to 0$ at leading order.
- A3: $\xi \to 0$ recovers SM–like couplings; $\xi \to 1$ approaches the maximally composite limit.
- A4: The $\hat{\mathbf{v}}_i$ are unit vectors; orthogonality is not assumed—rank–1 projectors suffice for the texture.

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5 Swirl-Strings: Bridging Blocks for Modern Physics and String Methods in Fluid Dynamics

Addendum B: ASG-Induced Yukawa Splitting and Charge Selection

B1. Setup and one-loop structure with gravity-induced anomalous dimensions

We introduce dimensionless, approximately scale-constant (in the trans–cutoff regime) anomalous-dimension parameters $f_g \geq 0$ and $f_y \geq 0$ to encode universal gravity effects on gauge and Yukawa sectors, respectively. The gauge beta functions are modeled as

$$\beta_{g_i} \;=\; \frac{b_{0,i}}{16\pi^2}\, g_i^3 \;-\; f_g\, g_i, \qquad i \in \{3, 2, Y\}, \tag{14}$$

with $b_{0,3} = -7$, $b_{0,2} = -19/6$, and

$$b_{0,Y} \;=\; \frac{1}{6}\Big(19 + 36\,(Y_b^2 + 2Y_Q^2 + Y_t^2)\Big), \tag{15}$$

where Y_t, Y_b, Y_Q denote the (model-normalized) hypercharges of the RH top, RH bottom, and LH doublet. A nontrivial Abelian fixed point appears at

$$g_{Y,*}^2 \;=\; \frac{16\pi^2}{b_{0,Y}}\, f_g\,. \tag{B1}$$

The Yukawa beta functions (suppressing family indices) in the trans–cutoff regime (with f_g, f_y effectively constant) read

$$\beta_{y_{t(b)}} \;=\; \frac{y_{t(b)}}{16\pi^2}\Big(\tfrac{3}{2}y_{b(t)}^2 + \tfrac{9}{2}y_{t(b)}^2 - \tfrac{9}{4}g_2^2 - 8g_3^2\Big) \;-\; f_y\, y_{t(b)} \;-\; \frac{3y_{t(b)}}{16\pi^2}\,(Y_Q^2 + Y_{t(b)}^2)\, g_Y^2. \tag{B2}$$

At the interacting UV fixed point with $g_2 \to 0$, $g_3 \to 0$ and $g_Y \to g_{Y,*}$, Eqs. (B1)–(B2) yield nonzero Yukawa fixed points $y_{t,*}, y_{b,*}$.

B2. Fixed-point relation and its generalization

Eliminating f_y between the t and b equations at the fixed point gives the *universal difference law*

$$y_{t,*}^2 \; - \; y_{b,*}^2 \; = \; (Q_t^2 - Q_b^2)\, g_{Y,*}^2, \tag{B3}$$

using $Y_t = Q_t$, $Y_b = Q_b$, $Y_Q = Q_t - \frac{1}{2}$ and the normalization $Q = T_3 + Y$. For the Standard-Model charges $Q_t = \frac{2}{3}$, $Q_b = -\frac{1}{3}$, Eq. (B3) reduces to

$$\boxed{y_{t,*}^2 - y_{b,*}^2 \; = \; \tfrac{1}{3}\, g_{Y,*}^2}\,. \tag{B3-SM}$$

Thus $y_{t,*} > y_{b,*}$ is enforced by hypercharge alone. IR running (below the gravity-dominated regime) preserves the inequality, thereby generating $M_t > M_b$ without ad hoc Yukawa hierarchies.

B3. Consequences and selection rules (SST interpretation)

- **Charge selection:** Varying Q_b/Q_t away from $-\frac{1}{2}$ spoils the simultaneous retrodiction of (g_Y, y_t, y_b). In SST, this acts as a *topological selection rule*: the emergent hypercharge map from swirl–string data must fix $Q_b/Q_t = -1/2$.
- **Universality of gravity–induced anomalous dimensions:** The observed (g_Y, y_t, y_b) favor gauge-group *independence* of f_g. In SST bridging language: the swirl microstructure that induces antiscreening couples *universally* to all gauge sectors.
- **IR robustness:** Once set by (B3) at the UV fixed point, Abelian screening (the $-g_Y^2$ term) drives y_t/y_b further from unity, maintaining $y_t > y_b$ down to the electroweak scale.

B4. Canon Lemma (bridging): ASG quark–mass splitter

Lemma. In any SST bridging EFT where (i) a universal antiscreening term induces an Abelian fixed point (B1) and (ii) Yukawa flows include gravity-induced damping as in (B2), the UV fixed point implies the exact relation (B3). For SM charges, (B3-SM) holds and enforces $M_t > M_b$ after IR evolution.

Dimensional/limit checks. All quantities in (B1)–(B3) are dimensionless. The limit $g_{Y,*} \to 0$ (no fixed point) collapses (B3) to the trivial $y_{t,*} = y_{b,*}$. The result is independent of the absolute UV scale so long as the fixed-point regime exists.

B5. SST mapping and practical use

- **Hypercharge from swirl topology:** Implement the emergent U(1) map so that Q_t, Q_b (hence Y) arise from discrete swirl invariants; enforce $Q_b/Q_t = -1/2$ as a canon constraint (bridging) consistent with (B3).

- **Numerical pipeline:** Calibrate f_g from the IR value of g_Y using (B1), then use (B3) plus IR running to fit y_b with a single f_y. This mirrors the two-parameter retrodiction in the ASG analysis and provides a compact SST flavour module.
- **Universality test:** Penalize any SST scenario that requires f_g to differ between $SU(3)$ and $SU(2)$ to hit (g_Y, y_t, y_b), in line with the universality hint.

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Addendum C: SUSY–era constraints (bridging kernels for SST)

C1. Custodial breaking and the W mass (generic doublets)

Master relation. From muon decay, the (tree + loop) relation is

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}\,G_\mu}\,(1+\Delta r), \qquad \Delta r_{\rm new} \simeq -\frac{c_W^2}{s_W^2}\,\Delta\rho, \tag{16}$$

with $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$. For an $SU(2)_L$ doublet with mass splitting, the oblique shift is

$$\Delta\rho = \frac{3G_F}{8\sqrt{2}\,\pi^2}\,F_0(m_1^2,m_2^2), \qquad F_0(x,y) = x+y-\frac{2xy}{x-y}\ln\frac{x}{y}, \tag{17}$$

and with mixing between weak and mass eigenstates (θ) a convenient template is

$$\Delta\rho = \frac{3G_F}{8\sqrt{2}\,\pi^2}\Big[-\sin^2\theta\,\cos^2\theta\,F_0(m_1^2,m_2^2)+\cos^2\theta\,F_0(m_0^2,m_2^2)+\sin^2\theta\,F_0(m_0^2,m_1^2)\Big], \tag{18}$$

where m_0 denotes the neutral partner mass in the doublet. Therefore the induced W-mass shift is

$$\boxed{\Delta M_W \simeq \frac{M_W}{2}\,\frac{c_W^2}{c_W^2-s_W^2}\,\Delta\rho}. \tag{19}$$

Dimensional check: $[\Delta\rho]=1$, so $[\Delta M_W]=[M_W]$. *SST constraint:* any SST doublet excitation that breaks custodial symmetry must satisfy $|\Delta\rho| \lesssim \mathcal{O}(10^{-3})$ to respect EW precision fits; canonize as a *Custodial Integrity Rule*.

C2. Muon $g-2$ (model–independent one–loop kernels)

Adopt two universal Yukawa structures for a charged lepton ℓ (here μ):

$$\mathcal{L}_1 = -\,\bar{\mu}\,(A_S+A_P\gamma_5)\,F\,\phi+\text{h.c.} \quad (\phi:\ \text{neutral scalar},\ F:\ \text{charged fermion}), \tag{20}$$

$$\mathcal{L}_2 = -\,\bar{\mu}\,(B_S+B_P\gamma_5)\,N\,\phi^-+\text{h.c.} \quad (\phi^-:\ \text{charged scalar},\ N:\ \text{neutral fermion}). \tag{21}$$

The one–loop corrections are

$$a_\mu^{(1)} = \frac{m_\mu^2}{8\pi^2}\int_0^1 dx\,\frac{|A_S|^2\,P_S(x)+|A_P|^2\,P_P(x)}{(1-x)\,m_F^2+x\,m_\phi^2}, \tag{22}$$

$$a_\mu^{(2)} = \frac{m_\mu^2}{8\pi^2}\int_0^1 dx\,\frac{|B_S|^2\,Q_S(x)+|B_P|^2\,Q_P(x)}{(1-x)\,m_N^2+x\,m_\phi^2}, \tag{23}$$

with dimensionless polynomials $P_{S,P}, Q_{S,P}$ (canon loop library). In the chargino/neutralino limit these reduce to the standard kernels $F^{\rm C}_{1,2}(x)$, $F^{\rm N}_{1,2}(x)$, with schematic approximations

$$a_\mu^{(\rm chg)} \simeq \frac{g_2^2\, m_\mu^2}{16\pi^2}\, \frac{\mu_H M_2\, \tan\beta}{m_{\tilde\nu}^2}\, F_2^{\rm C}\!\left(\frac{M_2^2}{m_{\tilde\nu}^2}, \frac{\mu_H^2}{m_{\tilde\nu}^2}\right), \tag{24}$$

$$a_\mu^{(\rm neu)} \simeq \frac{g'^2\, m_\mu^2}{16\pi^2}\, \frac{m_{\chi^0}}{m_{\tilde\mu_1}^2 m_{\tilde\mu_2}^2}\, \sin 2\theta_{\tilde\mu}\, \left[m_{\tilde\mu_2}^2\, F_2^{\rm N}\!\left(\frac{m_{\chi^0}^2}{m_{\tilde\mu_2}^2}\right) - m_{\tilde\mu_1}^2\, F_2^{\rm N}\!\left(\frac{m_{\chi^0}^2}{m_{\tilde\mu_1}^2}\right)\right], \tag{25}$$

which are positive in the large-$\tan\beta$ regime for appropriate parameter signs. *Dimensional check:* $[a_\mu] = 1$.

SST adapter. Any SST lepton–adjoint or lepton–knot excitation fitting $\mathcal{L}_{1,2}$ contributes via the same kernels. Use this block as a plug-in when mapping swirl excitations to Δa_μ; require $\Delta a_\mu \sim \mathcal{O}(10^{-9})$ to address the reported anomaly.

C3. Proton decay selection rule (dimension–5/6 template)

In SU(5)–like embeddings, integrating out colored Higgsinos yields the superpotential

$$W_5 \;=\; \frac{\kappa}{2M_{H_C}}\,(f^u e^{i\phi})(f^d V^*)\, QQQL \;+\; \frac{\kappa}{M_{H_C}}\,(f^u e^{i\phi})(f^d V^*)\, U^c E^c U^c D^c\,, \tag{26}$$

leading, after dressing, to effective dimension–6 operators for $p \to K^+\bar\nu_j$ with loop kernel $F(x,y,z)$ and renormalization factor A_R. A practical lifetime estimate is

$$\boxed{\tau(p\to K^+\bar\nu) \;\simeq\; 4\times 10^{35}\,\mathrm{yr}\times \sin^4 2\beta\, \left(\frac{0.1}{A_R}\right)^2 \left[2\,F(\mu_H^2, m_{\tilde t_R}^2, m_{\tilde\tau_R}^2)\right]^{-1} \left(\frac{M_{H_C}/\kappa}{10^{16}\,\mathrm{GeV}}\right)^2} \tag{27}$$

(up to order–one hadronic and phase–space factors). *SST selection rule:* impose a *topological suppression* $\kappa \ll 1$ (e.g., orbifold/origin selection or twist parity) *or* raise the colored–mediator scale to protect $\tau_p > 10^{34}$ yr, dovetailing with Canon flavour projectors and GJ matching.

C4. Implementation & cross–checks

- Tag C1–C3 as *Bridging*. They provide kernel libraries and constraints; no change to core SST postulates.
- Cross–link C1 to the Canon EW precision section; C2 to the lepton sector and time–scaling notes; C3 to the unification/selection–rule subsection.
- Dimensional sanity: F_0 and $F^{\rm C,N}_{1,2}$ are dimensionless; all prefactors carry the required mass dimensions.

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7 Swirl-Strings: Bridging Blocks for Modern Physics and String Methods in Fluid Dynamics

Addendum D: CSD4 projectors and Cabibbo–PMNS linkage (bridging)

D1. Technical synopsis (King, 2014)

The "tetra-model" employs an A_4 family symmetry with Pati–Salam $SU(4)_{\rm PS} \times SU(2)_L \times U(1)_R$ and a single vacuum-alignment vector $(1,4,2)$ (CSD4) that controls both the lepton and up-quark sectors. Down-quark and charged-lepton Yukawas are diagonal with Georgi–Jarlskog (GJ) factors; quark mixing originates from Y_u (equal to Y_ν up to Clebsch factors). The Cabibbo angle then follows at leading order from the same $(1,4,2)$ column: $\theta_C \simeq 1/4 \approx 14^\circ$.

At the PS matching scale (LR convention),

$$Y_d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad Y_e = \begin{pmatrix} y_d/3 & 0 & 0 \\ 0 & 3y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \tag{28}$$

$$Y_u = \begin{pmatrix} 0 & b\varepsilon & 0 \\ a\varepsilon^2 & 4b\varepsilon & 0 \\ a\varepsilon^2 & 2b\varepsilon & c \end{pmatrix}, \quad Y_\nu = \begin{pmatrix} 0 & b\varepsilon & 0 \\ a\varepsilon^2 & 4b\varepsilon & 0 \\ a\varepsilon^2 & 2b\varepsilon & c/3 \end{pmatrix}, \quad M_R = \mathrm{diag}\big(\varepsilon^4 \tilde{M}_1,\ \varepsilon^2 \tilde{M}_2,\ \tilde{M}_3\big). \tag{29}$$

These yield the GJ relations $m_e = m_d/3$, $m_\mu = 3m_s$, $m_\tau = m_b$.

After see-saw, the light-neutrino matrix is a sum of three rank-1 pieces aligned with $(0,1,1)$, $(1,4,2)$ and the third-family direction, with a fixed phase 2η:

$$m_\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b\, e^{2i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{30}$$

with $m_a \propto |a|^2 v_u^2/\tilde{M}_1$, $m_b \propto |b|^2 v_u^2/\tilde{M}_2$, $m_c \propto |c|^2 v_u^2/(9\tilde{M}_3)$. Choosing $\eta = 2\pi/5$ reproduces (normal hierarchy)

$$\theta_{12} \approx 34^\circ, \quad \theta_{23} \approx 40^\circ, \quad \theta_{13} \approx 9^\circ, \quad \delta_\ell \approx 260^\circ, \qquad |m_{ee}| \approx 1.5\ \mathrm{meV}.$$

RH-neutrino masses are hierarchical with $M_1 : M_2 : M_3 \sim m_u^2 : m_c^2 : m_t^2$ (e.g. $M_1 \sim$ 10 TeV, $M_2 \sim 10^{10}$ GeV, $M_3 \sim 10^{16}$ GeV).

D2. Bridging primitives for SST/VAM

We import King's structure as projector primitives in the swirl–string (SST) / vorticity–anomaly (VAM) canon.

(i) Family projector basis. Define unit (not mutually orthogonal) directions

$$\hat{\mathbf{v}}_a = \frac{1}{\sqrt{2}}(0,1,1), \qquad \hat{\mathbf{v}}_b = \frac{1}{\sqrt{21}}(1,4,2), \qquad \hat{\mathbf{v}}_c = (0,0,1), \tag{31}$$

to be used as rank-1 projectors $\hat{\mathbf{v}}_i \hat{\mathbf{v}}_i^\top$.

(ii) Swirl–mass ansatz (neutrinos). Adopt the CSD4 phase as a quantized circulation offset,

$$m_\nu = \alpha\, \hat{\mathbf{v}}_a \hat{\mathbf{v}}_a^\top + \beta\, e^{i\,4\pi/5}\, \hat{\mathbf{v}}_b \hat{\mathbf{v}}_b^\top + \gamma\, \hat{\mathbf{v}}_c \hat{\mathbf{v}}_c^\top, \tag{32}$$

with $\alpha, \beta, \gamma > 0$. Tuning β/α to m_2/m_3 and setting γ fixes m_1; the angles and δ_ℓ then follow as in King.

(iii) Up-sector reuse and Cabibbo. Reuse $\hat{\mathbf{v}}_b$ as the dominant column in Y_u,

$$Y_u \;\propto\; \left[\, \hat{\mathbf{v}}_a \quad \sqrt{21}\,\hat{\mathbf{v}}_b \quad \hat{\mathbf{v}}_c \,\right] \begin{pmatrix} a\varepsilon^2 \\ b\varepsilon \\ c \end{pmatrix}^{\top} \qquad \text{(up to row relabellings)}, \tag{33}$$

to obtain $\theta_C \simeq 1/4$ at leading order, with CKM generated by small deformations.

(iv) Clebsch as a selection rule. Encode the $c/3$ entry of Y_ν as a *topological coupling ratio* in the neutrino channel (e.g. twist parity or circulation quantum) that suppresses the third-family right-swirl mode and enforces a normal hierarchy.

D3. Canon notes and implementation

- Tag D2 as *Bridging*. It constrains flavour textures but does not modify core SST postulates.
- Cross-link to Addendum A4 (projector template) and your knot taxonomy rules for how $\hat{\mathbf{v}}_i$ arise from overlap integrals on distinct knot/link classes.
- Keep GJ relations as boundary conditions at the PS scale for (Y_d, Y_e); propagate to pole masses in your mass tables.

D4. Quick checks (VAM pipeline)

- Fit β/α to m_2/m_3, choose γ for $m_1 \simeq 0.3$ meV, and verify $(\theta_{12}, \theta_{23}, \theta_{13}, \delta_\ell) \approx (34^\circ, 40^\circ, 9^\circ, 260^\circ)$.
- Verify $\theta_C \simeq 14^\circ$ from the $\hat{\mathbf{v}}_b$ column at LO.
- Record the implied RH scales (M_1, M_2, M_3) in the Canon constants table for see-saw benchmarks.

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Addendum E: Framons and the Rotating Rank–One Mass (R2M2)

E1. Framon fields as internal frames (dictionary)

FSM statement. Introduce frame vectors (*framons*) carrying both local and dual global indices under $G = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and $\tilde{G} = \widetilde{\mathrm{SU}}(3) \times \widetilde{\mathrm{SU}}(2) \times \widetilde{\mathrm{U}}(1)$. The weak framon is an SU(2) doublet (the Higgs), the strong framon is a colour triplet, and invariance under $G \times \tilde{G}$ endows a global $\widetilde{\mathrm{SU}}(3)$ acting as generations.

SST/VAM adapter. Treat a framon as a *swirl frame field* $\mathcal{F}(x)$ mapping a global flavour triad to the local vortex–knot basis. Invariants built from $\mathcal{F}$ generate the scalar sector, with the observed Higgs identified with the weak framon (no change to Core). Strong framons correspond to colour–charged swirl excitations confined inside hadronic bound states.

E2. Universal rank–one mass matrix (tree level)

FSM form. For each fermion species $f \in \{u, d, \ell, \nu\}$,

$$\boxed{M_f(\mu) \;=\; y_f(\mu)\,\alpha(\mu)\,\alpha^\dagger(\mu)} \tag{34}$$

with a *universal* unit vector $\alpha \in \mathbb{R}^3$ in generation space and a species–dependent scalar coefficient y_f. At tree level, (34) yields one massive generation and two zero eigenvalues. *Dimensional check:* $[M_f] = \text{mass}$, $[y_f] = \text{mass}$, $[\alpha] = 1$.

SST/VAM adapter. Identify α with a normalized swirl–projector orientation. In a fixed orthonormal projector basis $\{\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, \hat{\mathbf{v}}_3\}$ (cf. Addendum A4), take $\alpha(\mu) = \sum_i a_i(\mu)\,\hat{\mathbf{v}}_i$, with $\sum_i |a_i|^2 = 1$.

E3. RG–induced rotation and its kinematics

FSM mechanism. Renormalization by *strong framon* loops rotates the vacuum orientation in generation space; hence $\alpha(\mu)$ becomes scale–dependent. Denote by $\Gamma : \ln\mu \mapsto \alpha(\mu) \in \mathbb{S}^2$ a unit–speed curve on the unit sphere, with speed $\omega(\mu) = \|\mathrm{d}\alpha/\mathrm{d}\ln\mu\|$ and geodesic curvature $\kappa_g(\mu)$ on $\mathbb{S}^2$.

Consequences (model–independent):

- *Mixing from misalignment.* For example, $V_{tb} = \alpha(m_t)\cdot\alpha(m_b) = \cos\Delta\theta_{tb}$, where $\Delta\theta_{tb}$ is the arc length on Γ between m_t and m_b. Small–angle limit: $1 - |V_{tb}| \simeq \frac{1}{2}\,\Delta\theta_{tb}^2$.
- *Hierarchy by leakage.* With α rotating, lower–generation masses are induced at their own scales: $m_2/m_3 \sim \mathcal{O}(\Delta\theta_{23}^2)$, $m_1/m_2 \sim \mathcal{O}(\Delta\theta_{12}^2)$, up to species coefficients.
- *Texture features from geometry.* Corner suppression ($|V_{ub}|, |V_{td}| \ll |V_{us}|, |V_{cb}|$) and the pattern $|V_{us}| > |V_{cb}|$ emerge when the geodesic torsion on $\mathbb{S}^2$ vanishes (as it does) and κ_g dominates sideways bending near one segment of Γ.

SST/VAM adapter (operational rule). To compute spectra and mixings at leading order: (i) choose an admissible curve family $\Gamma(\mu; a)$ with one curvature parameter a; (ii) set heavy–generation state vectors by $\mathbf{t} = \alpha(m_t)$, $\mathbf{b} = \alpha(m_b)$, etc.; (iii) define the lighter states as orthonormal directions following the local Frenet frame of Γ; (iv) assemble V_{CKM} from dot products $V_{ij} = \mathbf{u}_i\cdot\mathbf{d}_j$. This realizes R2M2 inside SST without altering its core.

E4. $\theta_{\mathrm{QCD}} \to$ CKM phase (bridging)

Because rank $M_f = 1$ at each μ, two chiral rotations are available to remove the QCD θ term at any fixed scale while keeping M_f real. The *scale variation* of $\alpha(\mu)$ transmits this rotation as a physical Kobayashi–Maskawa phase at nearby scales, providing a bridge from strong CP to CKM CP violation within the R2M2 scheme (no change to SST postulates).

E5. Phenomenology hooks (to be tested in SST)

1. **Qualitative CKM/PMNS features from geometry:** corner suppression, $|V_{us}| \gg |V_{cb}|$ for quarks, and the possibility $m_u < m_d$ despite $m_t \gg m_b$. These map to curvature properties of Γ and a fixed–point structure at high μ.
2. **Higgs couplings and LFV decays (bridging signal):** if the weak framon mixes with strong–framon composites, second–generation Higgs decays ($H \to \mu^+\mu^-$, $c\bar{c}$) can be suppressed and a LFV mode $H \to \tau\mu$ at $\sim 10^{-4}$ may occur. SST

can treat such effects via small misalignment terms in the swirl–Higgs sector (cf. Addendum A3).

3. **Strong–framon composites (H_K) as dark matter candidates:** neutral, colour–singlet framon–antiframon bound states (notably H_4, H_5) may be stable. In SST this corresponds to confined colour–swirl excitations with vanishing weak charge; interaction rates are naturally suppressed.

E6. Compact SST ruleset (RR1–SST)

- **Rule E.1 (rank–one ansatz):** Use (34) with a *common* $\alpha(\mu)$ for all species at the *same* μ.
- **Rule E.2 (scale–matching):** Define the heavy state of a species at its pole/running scale $\mu = m_3$; lighter states are evaluated at their own μ with the same $\alpha(\mu)$.
- **Rule E.3 (mixing from rotation):** For leading estimates, use small–angle expansions in the arc distances between the μ intervals of neighbouring masses.
- **Rule E.4 (geometry prior):** Adopt a one–parameter $\Gamma(\mu; a)$ with a sharp bend near one region and a UV fixed point where $\omega \to 0$. Fit a and species y_f to a minimal input set; predict the rest.

E7. Dimensions and limits

- $[\alpha] = 1$, $[y_f] =$ mass. If rotation is switched off ($\omega \to 0$), mixing vanishes and only one family is massive per species.
- A UV fixed point with $\omega \to 0$ implies increasing mass ratios m_2/m_3 across ℓ, d, u in the order observed if ω grows towards the IR.

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Addendum F: Asymptotically Safe Pati–Salam as an SST Bridge

F1. Group, matching, and scale landmarks

Gauge group. $G_{\text{PS}} = SU(4) \otimes SU(2)_L \otimes SU(2)_R$. Matching at the PS breaking scale v_R:

$$g_3 = g_4, \qquad g_{B-L} = \sqrt{\tfrac{3}{8}}\, g_3. \tag{35}$$

Phenomenological bound. Kaon–decay constraints are encoded as a hard lower limit

$$v_R \gtrsim 2 \times 10^3 \text{ TeV}. \tag{36}$$

SST map. Identify the $U(1)_{B-L}$ generator with a swirl–circulation charge; the PS–breaking vev v_R sets a swirl–EFT matching scale in the Canon tables.

F2. Safety via large N_F and bubble resummation

Introduce sets of vector–like multiplets

$$N_{F4}\,(\mathbf{4},\mathbf{1},\mathbf{1}) \;\oplus\; N_{F2L}\,(\mathbf{1},\mathbf{3},\mathbf{1}) \;\oplus\; N_{F2R}\,(\mathbf{1},\mathbf{1},\mathbf{2}),$$

with $N_{Fi} \gg 1$. Define

$$\alpha_i \equiv \frac{g_i^2}{(4\pi)^2}, \qquad A_i = 4\,\alpha_i\,T_R\,N_{Fi}, \qquad i \in \{2L, 2R, 4\}. \tag{37}$$

The higher–order (bubble) contribution to the gauge β functions reads schematically

$$\beta_i^{\text{ho}} \;=\; \frac{2\,A_i\,\alpha_i^2}{3\,N_{Fi}}\;H_{1i}(A_i), \tag{38}$$

with a nonperturbative pole yielding an interacting UV fixed point at

$$A_i \;=\; 3. \tag{39}$$

Unification selector. Choose reps such that $A_{2L} = A_{2R} = A_4$ to drive g_L, g_R, g_4 to a common interacting fixed point ("safe unification"). A representative UV crossover scale for sample $(N_{F2L}, N_{F2R}, N_{F4})$ choices sits near

$$\mu \sim 5 \times 10^8 \text{ GeV}.$$

SST interpretation. Vector–like families act as dense swirl microstructure (vortex–knot multiplets) whose net effect is to anti–screen all three non–abelian sectors equally when their swirl charges are matched, implementing an SST analogue of fixed–point unification without enlarging the gauge group.

F3. Scalar (bi–doublet) $\to$ 2HDM matching

At PS breaking, the scalar bi–doublet $\Phi \sim (1, 2, 2)$ matches onto a two–Higgs–doublet potential with couplings $\bar{\lambda}_i$ related to PS quartics λ_j via

$$\bar{\lambda}_1 = \lambda_1, \qquad \bar{\lambda}_2 = \lambda_1, \qquad \bar{\lambda}_3 = 2\lambda_1, \qquad \bar{\lambda}_4 = 4(-2\lambda_2 + \lambda_4), \tag{40}$$

$$\bar{\lambda}_5 = 4\lambda_2, \qquad \bar{\lambda}_6 = -\lambda_3, \qquad \bar{\lambda}_7 = \lambda_3. \tag{41}$$

Placement. Record this map in the Canon "SST$\leftrightarrow$EWSB" dictionary; it composes cleanly with the misalignment adapter (Addendum A3) for pNGB scenarios.

F4. Yukawas, seesaw, and m_b–m_t splitting

With Yukawas y, y_c and

$$\langle \Phi \rangle = \mathrm{diag}(u_1, u_2), \qquad \tan\beta \equiv \frac{u_1}{u_2}, \qquad v = \sqrt{u_1^2 + u_2^2} = 174\,\mathrm{GeV},$$

the third–family masses at tree level are

$$m_t = m_{\nu_\tau} = \big(y \sin\beta + y_c \cos\beta\big)\, v, \qquad m_b = m_\tau = \big(y \cos\beta + y_c \sin\beta\big)\, v. \tag{42}$$

A type–I seesaw with $N_L \sim (1, 1, 1)$ and $M_R = y_\nu v_R$ yields one light ν_τ with

$$m_{\nu_\tau} \;=\; M_N \frac{m_t^2}{m_D^2}, \qquad m_D = \sqrt{m_t^2 + M_R^2}. \tag{43}$$

Bottom–top splitting via vector–like $F \sim (10, 1, 1)$. Mixing with the PS–triplet B and singlet E components (masses $m_B = y_F v_R/\sqrt{2}$, M_F Dirac) generates, to leading order,

$$m_b \;\simeq\; \frac{M_F\, m_t}{\sqrt{2}\, m_B}, \qquad M_B = \frac{M_E}{\sqrt{2}} \simeq m_B, \tag{44}$$

providing a technically natural path to $m_b \ll m_t$ while retaining $y \approx y_c$ near the PS scale. *Dimensional check:* the right–hand side has mass dimension one.

SST encoding. Treat the $(10, 1, 1)$ as a *twist–parity*–odd swirl multiplet that couples only to the right–handed PS sector; the induced m_b emerges from a suppressed cross–circulation between right– and left–handed swirl layers.

F5. Practical "safe window" for Canon tables

- Use $v_R = 2 \times 10^3$ TeV as a conservative default; annotate flows where $v_R \sim 10^4$ TeV aids 125 GeV Higgs fits.
- Example safe–unification choice: $N_{F2L} = 35$, $N_{F2R} = 140$, $N_{F4} = 140$ (equal A_i). Record that the UV fixed point is encountered around $\mu \sim 5 \times 10^8$ GeV.
- For Yukawa fits, a CP–symmetric choice $y = y_c$, $\tan\beta = 1$ gives $m_t = \sqrt{2}\, y\, v$, implying $y \approx 0.66$ at the EW scale after running.

F6. Sanity checks

- All $\bar{\lambda}_i$ maps are dimensionless and linear in PS quartics; custodial–symmetric limits are recovered for $\lambda_2, \lambda_3 \to 0$.
- The m_b relation scales as $1/v_R$; increasing v_R suppresses m_b at fixed M_F, y_F, consistent with decoupling.
- The fixed–point condition $A_i = 3$ is representation–independent once T_R and N_{Fi} are specified; equalization $A_{2L} = A_{2R} = A_4$ enforces safe unification.

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Addendum G: FPF modulino oscillations and Scherk–Schwarz SUSY (bridging)

G1. (3+1) oscillation formalism for an eV–keV modulino

Consider a sterile state $\tilde{z}$ ("modulino") mixing with the three active neutrinos. In the short–baseline limit $\Delta m_{21}^2 L/E \ll 1$, $\Delta m_{31}^2 L/E \ll 1$, the disappearance probability is

$$P_{\alpha\alpha} \;=\; 1 - 4\,|U_{\alpha 4}|^2\big(1-|U_{\alpha 4}|^2\big)\,\sin^2\!\Big(\frac{\Delta m_{41}^2\,L}{4E}\Big), \qquad \alpha\in\{e,\mu,\tau\}. \tag{45}$$

Oscillation maxima occur at $\Delta m_{41}^2 L/(4E) = \pi/2$, i.e.

$$\Delta m_{41}^2 \;\simeq\; \frac{\pi\,E}{2\,L} \quad\Rightarrow\quad \Delta m_{41}^2(\mathrm{eV}^2) \;\simeq\; \frac{\pi}{2\times 1.27}\,\frac{E(\mathrm{GeV})}{L(\mathrm{km})} \approx 1.24\,\frac{E}{L}. \tag{46}$$

For the FPF benchmark $L \simeq 600$ m, $E \simeq 100$ GeV this gives $\Delta m_{41}^2 \sim \mathcal{O}(10^2)$ eV^2. *Dimensional check:* $[\Delta m_{41}^2] = \mathrm{mass}^2$; the sine argument is dimensionless.

G2. Planck–suppressed mixing from SUSY breaking

Assume supergravity–suppressed couplings between $\tilde{z}$ and SM fields after SUSY breaking:

$$\mathcal{L}_{\mathrm{eff}} \supset \lambda_\alpha\,\overline{L}_\alpha H\,\tilde{z} + \mathrm{h.c.}, \qquad \lambda_\alpha \;=\; \alpha_\alpha\,\frac{m_{3/2}}{M_P}, \tag{47}$$

so that

$$m_{\nu_\alpha\tilde{z}} \;\equiv\; \eta\,\lambda_\alpha\,v \;=\; \alpha_\alpha\,\eta\,\frac{m_{3/2}\,v}{M_P}, \tag{48}$$

with $m_{3/2}$ the gravitino mass, M_P the reduced Planck mass, $v \simeq 174$ GeV, and η a renormalization factor. The active–sterile mixing angle obeys (for $m_4 \gg m_{\nu_i}$)

$$\tan 2\theta_{\alpha\tilde{z}} \;=\; \frac{m_{\nu_\alpha\tilde{z}}}{m_4 - m_{\nu_i}} \;\simeq\; \frac{\alpha_\alpha\,\eta}{m_4}\,\frac{m_{3/2}\,v}{M_P}. \tag{49}$$

Dimensional check: $[\lambda_\alpha] = 1$, $[m_{\nu_\alpha\tilde{z}}] = \mathrm{mass}$, $[\theta] = 1$.

G3. Modulino mass from Scherk–Schwarz SUSY breaking (sequestered gravity)

In 5D Scherk–Schwarz breaking along a compact extra dimension of radius R, the gravitino KK spectrum is $M_n(\omega) = m_{3/2} + n/R$ with $m_{3/2} = \omega/R$ ($0 < \omega < 1/2$). One–loop gravitational mediation from the bulk to the brane generates a modulino Majorana mass

$$\boxed{m_{\tilde{z}} \;=\; \frac{1}{R}\,\frac{1}{(M_P R)^2}\,\frac{1}{(4\pi)^2}\,f(\omega), \qquad f(\omega) \;=\; \frac{3}{4\pi^3}\,\mathrm{Im}\Big[\mathrm{Li}_4\big(e^{2\pi i\omega}\big)\Big]} \tag{50}$$

with $f(\omega) \to 0$ at $\omega = \frac{1}{2}$ by an R-symmetry. For $1/R \sim 10^{11\text{–}11.5}$ GeV and $f(\omega) \sim 10^{-2}$, one finds $m_{\tilde{z}} \sim$ 10–500 eV, i.e. $\Delta m^2_{41} \sim 10^2$–$10^5$ eV^2. *Limit:* large-R decoupling: as $R \to \infty$, $m_{\tilde{z}} \propto R^{-3} \to 0$.

G4. Cosmology constraints (BBN/CMB) and longevity

For $m_4 < 2m_e$, the leading decay $\tilde{z} \to 3\nu$ has width

$$\Gamma(\tilde{z} \to 3\nu) \;\simeq\; \frac{G_F^2\, m_4^5}{768\,\pi^3}\,\sin^2(2\theta_{\alpha\tilde{z}}) \;\approx\; 8.7\times 10^{-28}\,\mathrm{s}^{-1}\left(\frac{\sin^2 2\theta}{10^{-2}}\right)\left(\frac{m_4}{100\,\mathrm{eV}}\right)^5, \tag{51}$$

so $\tilde{z}$ is cosmologically long–lived in the eV–keV range. Thermalization before neutrino decoupling raises ΔN_{eff}; a useful empirical fit is

$$\Delta m^2_{41}\,\sin^4 2\theta_{\alpha\tilde{z}} \;\simeq\; 10^{-5}\,\mathrm{eV}^2\,\ln^2\!\big(1 - \Delta N_{\mathrm{eff}}\big), \tag{52}$$

which can be satisfied either by small mixing or by extending the IR spectrum to allow invisible decays that reduce ΔN_{eff}.

G5. SST/VAM adapter

- **Dictionary.** Introduce a neutral *aetherino* ψ (a fermionic swirl singlet). Bridge to the modulino by the identification $\psi \equiv \tilde{z}$ at the EFT level with Planck–suppressed mixing $\propto m_{3/2}/M_P$. No change to Core: ψ is a bridging excitation external to the core hydrodynamic fields.
- **FPF estimator.** For any proposed $(\Delta m^2, |U_{\mu 4}|^2)$, the FPF disappearance amplitude is $\mathcal{A}_{\mu\mu} = 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)$. Oscillation maxima occur near $\Delta m^2 \approx (\pi/2)(E/L)/(1.27)$, providing a direct map from Canon $(\Delta m^2, \theta)$ tables to expected FPF reach.
- **Cosmology toggle.** Flag two options in Canon runs: (i) *CMB-tension-tolerant* (loose ΔN_{eff}) vs. (ii) *CMB-safe* (suppressed $\sin^2 2\theta$ or invisible IR decays of ψ).

G6. Implementation & sanity checks

- Tag G1–G5 as *Bridging*. They constrain a sterile sector coupled to SST, but do not alter core postulates (absolute time, incompressible inviscid aether, knot particles, etc.).

- All displayed probabilities are dimensionless; widths have dimension of inverse time; mass formulas carry correct mass dimensions.
- Limits: $\omega \to \frac{1}{2} \Rightarrow f(\omega) \to 0$ (R-symmetry); $\theta \to 0 \Rightarrow P_{\alpha\alpha} \to 1$.

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Addendum H: Electroweak entanglement–minimization (bridging)

H1. Setup: entangling power and perpendicular scattering

Flavor Hilbert space. Let $\mathcal{H}_u \otimes \mathcal{H}_d$ denote the up/down flavor Hilbert space (qutrits for three generations). A bipartite pure state $|\alpha\rangle = \sum_{i,j} \alpha_{ij} \, |i\rangle_u \otimes |j\rangle_d$ has linear–entropy entanglement

$$E(\rho) \equiv \frac{G}{G-1} \left|1 - \operatorname{tr} \rho_R^2\right|, \qquad E(|\alpha\rangle)\big|_{G=2} = 4\lambda_1\lambda_2, \quad E(|\alpha\rangle)\big|_{G=3} = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1), \tag{53}$$

with $\{\lambda_i\}$ the eigenvalues of $\alpha^\dagger\alpha$ (trace one).

Entangling power. For the flavor–space map S_f representing "scattering + projection" (elastic, fixed helicities), define

$$E(S_f) \equiv \overline{E\big(S_f|i\rangle_u \otimes |j\rangle_d\big)} \quad \text{(average over generation product states).} \tag{54}$$

We use *perpendicular entangling power* at $\Theta = \pi/2$:

$$E^{\perp}_{\min} \equiv \min\{E_{ud},\, E_{ud^{+}}\}\Big|_{\Theta=\pi/2,\; t=u=-s/2}. \tag{55}$$

Kinematic window. The relevant ranges are $m_Z \lesssim \sqrt{s} \lesssim m_t$ (quarks) and $m_\tau \lesssim \sqrt{s} \lesssim m_Z$ (leptons). Within each interval, choose the $\sqrt{s}$ that minimizes $E^{\perp}_{\min}$.

SST reading. All quantities above are *dimensionless* functionals of scattering amplitudes; this addendum provides a selection principle usable in the Canon without changing the fluid postulates.

H2. Two–generation analytic kernel (Cabibbo sector)

At leading order for $ud \to ud$ with Z/γ (t–channel) and W (u–channel) exchange, introduce

$$y \;\equiv\; \frac{Y^u Y^d}{\cos^2\theta_W} + \sin^2\theta_W\, Q_u Q_d, \qquad \theta_C \equiv \theta_{12}. \tag{56}$$

Then, for perpendicular scattering the flavor–averaged entangling power is

$$E^{\perp}_{ud}(\theta_C) \;=\; 8y^2 \left[\frac{\cos^4\theta_C}{\left(1+2y+4y^2-2y\cos 2\theta_C\right)^2} + \frac{\sin^4\theta_C}{\left(1+2y+4y^2+2y\cos 2\theta_C\right)^2} \right]. \tag{57}$$

This exhibits a shallow minimum at $\theta_C \approx 6^\circ$ for SM couplings, with controlled dependence on θ_W: as $\sin\theta_W \to 0$ the preferred θ_C decreases; as $\sin\theta_W \to 1$, the minimum drifts to $\pi/4$. *Remark:* suppressing the photon term can shift the minimum toward $\theta_C \approx 13^\circ$.

H3. Three generations: CKM and PMNS patterns

Quarks (CKM). Minimizing $E^{\perp}_{\min}$ for $m_Z \lesssim \sqrt{s} \lesssim m_t$ yields a nearly diagonal CKM with a single small angle: $\theta_{12} \sim \mathcal{O}(\text{few}^\circ)$, while $\theta_{13} \approx \theta_{23} \approx 0$. Energy dependence is mild: far below the EW scale, photon dominance drives maximal 12–mixing; just above m_t the preferred angles compress to $\sim 4^\circ$; for $\sqrt{s} \gg m_t$, the crossed channel prefers vanishing mixing.

Leptons (PMNS). In $m_\tau \lesssim \sqrt{s} \lesssim m_Z$, the minimum favors two large angles and one smaller: a representative minimum occurs near $\sqrt{s} \approx 30\,\text{GeV}$ with $\theta_{12} \approx \theta_{23} \approx 29^\circ$, $\theta_{13} \approx 16^\circ$. The CP phase enters nontrivially; the minima generically prefer *minimal* CP violation ($\delta_{\text{PMNS}} \approx 0, \pi$), and align more closely with normal ordering than inverted ordering.

H4. SST/VAM adapter and usage

- **Principle (optional):** *Entanglement–Minimization Hypothesis (EMH).* At a chosen $\sqrt{s}$ within the canonical windows, select flavor parameters so that $E^{\perp}_{\min}$ is minimized.

- **Interference dictionary:** The single parameter y in Eq. (57) encodes the γ/Z vs. W competition. In SST, treat y as an *effective swirl–interference weight*, i.e. a ratio of neutral–current to charged–current swirl couplings at the probe scale. Small deformations of y (e.g. anisotropic swirl permeability) move the Cabibbo minimum from $\sim 6^\circ$ toward $\sim 13^\circ$, where our rank–1 projector basis (Addendum A4) already sits.
- **How to use:** (i) Fix $\sqrt{s}$ per sector; (ii) compute $E^{\perp}_{\min}$ along your flavor template manifold (e.g. rotating rank–one, projector basis); (iii) pick the point of minimal E. Record the result as a *bridging* selection, not a postulate.

H5. Sanity checks

- All entanglement measures and $E^{\perp}_{\min}$ are dimensionless. The dependence on θ_W enters only through y.
- Limits: $\theta_C \to 0, \pi/2 \Rightarrow E^{\perp}_{ud}$ reduces to a single–denominator term; $\sin\theta_W \to 0$ enhances charged–current dominance; $\sin\theta_W \to 1$ drives $\theta_C \to \pi/4$ at the minimum.
- CKM/PMNS qualitative hierarchies persist across the recommended $\sqrt{s}$ windows.

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Addendum I: Unified Flavor–CP–Modular Symmetry (Narain $\to$ SST) (bridging)

I1. Unified symmetry via outer automorphisms (Narain picture)

Statement. In heterotic orbifolds, the *unified flavor group* $\mathcal{G}_{\mathrm{UFG}}$ arises from the *outer automorphisms* of the Narain space group $\widehat{S}_{\mathrm{Narain}}$. It *contains and unifies* (i) traditional non-Abelian flavor, (ii) modular/T-duality symmetries, and (iii) CP-like transformations. The group is *moduli-dependent* and can be enhanced on lines/points in moduli space.

Lattice-basis automorphism. For $g = (\widehat{\Theta}^k, \widehat{N}) \in \widehat{S}_{\mathrm{Narain}}$, an outer automorphism $\widehat{h} = (\widehat{\Sigma}, \widehat{T}) \notin \widehat{S}_{\mathrm{Narain}}$ acts by $g \mapsto \widehat{h}\, g\, \widehat{h}^{-1} \in \widehat{S}_{\mathrm{Narain}}$ with consistency

$$\widehat{\Sigma}\,\widehat{\Theta}^k\,\widehat{\Sigma}^{-1} = \widehat{\Theta}^{k'}, \qquad \big(\mathbb{1} - \widehat{\Sigma}\,\widehat{\Theta}^k\,\widehat{\Sigma}^{-1}\big)\widehat{T} \in \mathbb{Z}^{2D}.$$

When $\widehat{\Sigma}$ leaves the compactification moduli invariant, it contributes to the *traditional* flavor group at that moduli value.

Local flavor groups. Fields localized at distinct fixed points/sectors can feel *different* subgroups (*local flavor groups*). This supplies a controlled sector-dependent flavor/CP structure.

I2. $T^2/\mathbb{Z}_3$ exemplar and enhancement loci

Let the Kähler modulus be $T = b + i\frac{\sqrt{3}}{2}\, r$. Generically one obtains a non-Abelian flavor group $\Delta(54)$ generated by two lattice translations and a symmetric rotation.

I2.1 Generic region

At generic $\langle T \rangle$, the generators $A = (\mathbb{1}, \widehat{T}_1)$, $B = (\mathbb{1}, \widehat{T}_2)$, and $C = (\widehat{S}_{\mathrm{rot}}(\pi), 0)$ close to $\Delta(54)$. (Space-group selection rules embed into this closure.)

I2.2 Vertical lines $b = \frac{n_B}{2}$ (CP unification)

For integer n_B, a left–right symmetric reflection $D(n_B) = (\widehat{S}_{\mathrm{refl}}(\frac{2\pi}{6}), 0)$ preserves the Narain lattice and acts on the modulus as

$$T \;\longmapsto\; n_B - T.$$

On $\operatorname{Re} T = \frac{n_B}{2}$ this yields an extra $\mathbb{Z}_2$ intertwining barred/unbarred twisted sectors, promoting CP from an outer automorphism of $\Delta(54)$ to an *inner* operation inside the enhanced group. The unified flavor group enhances to $\mathrm{SG}(108, 17)$.

I2.3 Black semicircles $|T| = 1$

On $|T| = 1$ ($\operatorname{Im} T > 0$), an asymmetric reflection $E = (\widehat{A}_{\text{refl}}, 0)$ is unbroken, giving another enhancement to a group isomorphic to $\mathrm{SG}(108, 17)$ (a different extension of $\Delta(54)$).

I2.4 Intersections

At intersections of enhancement loci: two-line crossings (e.g. $b = 0$, $r = 2/\sqrt{3}$) give $\mathrm{SG}(216, 87)$; triple crossings (e.g. $b = \frac{1}{2}$, $r = 1$) give $\mathrm{SG}(324, 39)$. These realize points where CP is exactly conserved inside $\mathcal{G}_{\text{UFG}}$, breaking spontaneously when displaced in moduli.

I3. SST/VAM adapter (bridging)

Dictionary. Replace the Narain lattice by a *swirl lattice* (a discrete lattice of circulation/winding and æ-phase translations) with an effective "swirl modulus" T_{ae} (dimensionless). Outer automorphisms of the swirl lattice unify: (i) discrete swirl flavor (knot classes), (ii) æ-modular transformations (duality of discrete swirl periods), and (iii) a CP-like reflection. *Local flavor groups* correspond to distinct localization of knot excitations in the swirl lattice.

CP-in-UFG selection rule. If a sector's T_{ae} lies on a CP-enhancement locus (analogue of $\operatorname{Re} T = \frac{n}{2}$ or $|T| = 1$), then CP is *inner* and can be exact at that scale; away from the locus, CP is spontaneously broken by T_{ae} displacement.

Sector differentiation. Assign T_{ae} values for u, d, ℓ, ν on *different* loci ("local flavoring"): e.g. quarks near generic $\Delta(54)$ (small mixing) while leptons near an enhancement arc (large mixing). This dovetails with Addenda C (EW kernels), D (EJA ladders), and E (rotating rank-one).

I4. Practical canon rules

- **Rule I.1 (Swirl–Narain closure).** Build the sector's discrete symmetry as the closure of its swirl translations (A, B) and one reflection/rotation (C, D, E) determined by T_{ae}.
- **Rule I.2 (CP bookkeeping).** Mark CP as *inner* if T_{ae} lies on enhancement loci; else tag as *outer* with spontaneous breaking controlled by δT_{ae}.
- **Rule I.3 (Local flavor).** Permit $\mathcal{G}_u \neq \mathcal{G}_d \neq \mathcal{G}_\ell \neq \mathcal{G}_\nu$. Record their residuals after Wilson-line–like swirl translations ("swirl Wilson lines").

I5. Consistency and limits

- Dimensions: T, T_{ae} are dimensionless; all group operations are algebraic.
- Limits: moving off enhancement loci reduces $SG(108, 17) \to \Delta(54)$; triple points flow to $SG(216, 87)$ or $\Delta(54)$ under small deformations.
- Compatibility: The EMH selector (Addendum H) and R2M2 geometry (Addendum E) can be imposed *within* a given local flavor group.

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Addendum K: Intersecting D-brane PS & ϑ-texture bridge (bridging)

K1. Scope and payoff

A Pati–Salam (PS) construction on a Type IIA $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with intersecting D6-branes yields (i) rank-3 Yukawa matrices built from products of Jacobi ϑ-functions, (ii) a selection rule $i+j+k \equiv 0 \pmod 3$ for trilinears, and (iii) textures equivalent to $\Delta(27)$ that naturally give near-tribimaximal lepton mixing. This block translates those ingredients into the SST/VAM language (swirl projectors, EMH selector) and records usable kernel forms.

K2. ϑ-function Yukawa kernel (canonical form)

For three two-tori, the trilinear Yukawa couplings factorize as

$$Y_{ijk} = \mathfrak{h}\, q_u\, \sigma_{abc} \prod_{r=1}^{3} \vartheta\begin{bmatrix} \delta^{(r)} \\ \phi^{(r)} \end{bmatrix}(\kappa^{(r)}), \tag{58}$$

$$\vartheta\begin{bmatrix} \delta \\ \phi \end{bmatrix}(\kappa) \equiv \sum_{\ell \in \mathbb{Z}} \exp\Big\{\pi i\, (\delta+\ell)^2 \kappa \;+\; 2\pi i\, (\delta+\ell)\phi\Big\}, \tag{59}$$

with input parameters (for each torus) $\delta^{(r)}, \phi^{(r)}, \kappa^{(r)}$ determined by intersection numbers, relative brane shifts $\varepsilon^{(r)}_{a,b,c}$, and Kähler data. Define the *total shift*

$$\varepsilon \;\equiv\; \frac{I_{ab}\,\varepsilon_c + I_{ca}\,\varepsilon_b + I_{bc}\,\varepsilon_a}{I_{ab} I_{bc} I_{ca}}.$$

Focusing on the first torus (the others provide an overall factor), choose counting map $s^{(1)} = -i$. Then a generic family mass matrix *decomposes* into odd/even Higgs-VEV sectors:

$$M \;\sim\; \underbrace{\begin{pmatrix} A\,v_1 & B\,v_3 & C\,v_5 \\ C\,v_3 & A\,v_5 & B\,v_1 \\ B\,v_5 & C\,v_1 & A\,v_3 \end{pmatrix}}_{\text{odd-VEV block } (v_{1,3,5})} + \underbrace{\begin{pmatrix} E\,v_4 & F\,v_6 & D\,v_2 \\ D\,v_6 & E\,v_2 & F\,v_4 \\ F\,v_2 & D\,v_4 & E\,v_6 \end{pmatrix}}_{\text{even-VEV block } (v_{2,4,6})}, \tag{60}$$

where $v_k \equiv \langle H_{k+1} \rangle$ and the six coefficients are ϑ-combinations

$$A \equiv \vartheta\left[\begin{smallmatrix}\varepsilon \\ 0\end{smallmatrix}\right](6J/\alpha'), \quad B \equiv \vartheta\left[\begin{smallmatrix}\varepsilon+\frac{1}{3} \\ 0\end{smallmatrix}\right](6J/\alpha'), \quad C \equiv \vartheta\left[\begin{smallmatrix}\varepsilon-\frac{1}{3} \\ 0\end{smallmatrix}\right](6J/\alpha'),$$
$$D \equiv \vartheta\left[\begin{smallmatrix}\varepsilon+\frac{1}{6} \\ 0\end{smallmatrix}\right](6J/\alpha'), \quad E \equiv \vartheta\left[\begin{smallmatrix}\varepsilon+\frac{1}{2} \\ 0\end{smallmatrix}\right](6J/\alpha'), \quad F \equiv \vartheta\left[\begin{smallmatrix}\varepsilon-\frac{1}{6} \\ 0\end{smallmatrix}\right](6J/\alpha').$$

Symmetry loci (shift ε). At $\varepsilon = 0$ or $\frac{1}{2}$: $B = C$ and $D = F$; additionally $A = 1,\ E = 0$ at $\varepsilon = 0$ and $A = 0,\ E = 1$ at $\varepsilon = \frac{1}{2}$. Thus the odd-VEV block ($v_{1,3,5}$) or even-VEV block ($v_{2,4,6}$) can dominate by choice of ε.

K3. $\Delta(27)$ texture emergence

At the symmetry loci above, M takes the $\Delta(27)$-type form

$$M \sim \begin{pmatrix} f_1 v_1 & f_2 v_3 & f_2 v_2 \\ f_2 v_3 & f_1 v_2 & f_2 v_1 \\ f_2 v_2 & f_2 v_1 & f_1 v_3 \end{pmatrix},$$

leading to (near-)tribimaximal lepton mixing when the even-VEV block dominates. This supplies a top-down realization of the projector textures adopted in Addendum A4.

K4. PS breaking and hypercharge (for cross-matching Addendum F)

Brane-splitting on torus 1: $a \to (a_1, a_2) = (6, 2)$, $c \to (c_1, c_2) = (2, 2)$ breaks $SU(4)_C \times SU(2)_L \times SU(2)_R$ to $SU(3)_C \times SU(2)_L \times U(1)_{I_3^R} \times U(1)_{B-L}$, then to hypercharge $U(1)_Y$ via vevs of vectorlike states with

$$Q_Y = \tfrac{1}{6}\big(Q_{a_1} - 3Q_{a_2} - 3Q_{c_1} + 3Q_{c_2}\big).$$

Gauge couplings unify at the string scale with a calculable holomorphic kinetic function; this dovetails with Safe PS (Addendum F).

K5. SST/VAM dictionary

- **Shift parameter** ε $\longleftrightarrow$ *swirl phase-offset* $\varepsilon_{\text{ae}} \equiv \Phi_{\text{swirl}}/(2\pi)$ between the three family projectors: tuning $\varepsilon_{\text{ae}} = 0$ (quarks) favors odd-VEV dominance and small quark mixing; $\varepsilon_{\text{ae}} = \frac{1}{2}$ (leptons) favors even-VEV dominance and large lepton mixing.
- **Odd/even VEV blocks** $\longleftrightarrow$ *helicity-parity sectors* in the knotted-vortex map: odd (co-rotating) vs. even (counter-rotating) excitations.
- $\Delta(27)$ **locus** $\longleftrightarrow$ projector basis of Addendum A4 with a fixed phase $\Delta\phi$; the ϑ-kernel supplies the discrete phase structure without inserting ad hoc symmetries.

K6. Minimal working recipe (quarks vs. leptons)

1. Take $\varepsilon_{\rm ae}^{(q)} = 0$ and arrange $v_{1,3,5}^{U,D} \gg v_{2,4,6}^{U,D}$ (odd-dominance). Then CKM arises from small off-diagonals $B, C, D, F \ll A$ at the $\varepsilon = 0$ locus.

2. Take $\varepsilon_{\rm ae}^{(\ell)} = \frac{1}{2}$ and arrange $v_{2,4,6}^{U,\ell} \gg v_{1,3,5}^{U,\ell}$ (even-dominance). Then the lepton sector inherits the $\Delta(27)$ pattern and near-tribimaximal mixing.

3. Enforce the consistency constraint on shifts $\varepsilon_u + \varepsilon_\ell = \varepsilon_d + \varepsilon_\nu$ and map to Addendum J lemmas (EMH selector for Cabibbo; EJA targets for mass ratios).

K7. Dimensional and limit checks

- Y_{ijk} and ϑ-coefficients are dimensionless; Higgs v_k carry mass dimension 1; entries of M have mass dimension 1.

- Limits: $\varepsilon \to 0, \frac{1}{2}$ recover symmetry-enhanced textures; turning off even (odd) VEVs isolates the quark (lepton) template.

K8. Predictive hooks

- **Mixing angles (illustrative):** near-tribimaximal PMNS with nonzero θ_{13} emerges naturally once the even block dominates.

- **PS gauge unification:** string-scale unification compatible with Addendum F running; hypercharge embedding fixed as above.

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15 Swirl-Strings: Bridging Blocks for Modern Physics and String Methods in Fluid Dynamics

Addendum L: $SU(2) \otimes SU(4)$ gauge self-energy mass map (bridging)

L1. Scope and takeaway

Marsch–Narita propose that light-fermion masses (first family; template for all) arise from gauge-field energy in each fermion's rest frame within a unified $SU(2) \otimes SU(4)$ model. This addendum records their kernels and provides an SST/VAM dictionary so the idea can be tested (or vetoed) inside the Canon without altering core postulates.

L2. Kernels captured from the paper

Unified covariant structure. The 8×8 gauge-field matrix enters the covariant derivative as

$$S_\mu \;=\; Q\,A_\mu \;+\; R\,Z_\mu \;+\; Q_2\,G_\mu^{(2)} \;+\; \frac{1}{\sqrt{3}}\,Q_3\,G_\mu^{(3)} \;+\; I_\mu, \tag{61}$$

where Q is the electromagnetic charge operator, R the weak Z-charge operator, $Q_{2,3}$ are hadronic charge operators, and I_μ contains off-diagonal weak/strong exchange blocks ("leptoquark" V, W, and gluon sub-structure).

Charge operators. With weak/strong couplings g_2, g_4, the electric unit and Z-charges are (paper conventions)

$$e = \frac{g_2\,g_4}{\sqrt{g_4^2 + \frac{2}{3}g_2^2}}, \qquad Q \;=\; e\,\mathrm{diag}\Big(\tfrac{2}{3},\,\tfrac{2}{3},\,\tfrac{2}{3},\,0,\,-\tfrac{1}{3},\,-\tfrac{1}{3},\,-\tfrac{1}{3},\,-1\Big), \tag{62}$$

$$R = \frac{e}{2}\,\mathrm{diag}(q_-, q_-, q_-, \ell_-, q_+, q_+, q_+, \ell_+), \quad \ell_\pm = \pm\sqrt{\tfrac{2}{3}}\frac{g_2}{g_4} \mp \sqrt{\tfrac{3}{2}}\frac{g_4}{g_2}, \quad q_\pm = \pm\sqrt{\tfrac{2}{3}}\frac{g_2}{g_4} + \tfrac{1}{3}\sqrt{\tfrac{3}{2}}\frac{g_4}{g_2}. \tag{63}$$

Classical Coulomb mass heuristic. In natural units, the canonical-momentum replacement $P_\mu = p_\mu - eA_\mu$ and a static Coulomb potential lead heuristically to

$$m_e c^2 \;=\; \frac{e^2}{r_e}, \qquad r_e \;=\; \alpha_e\,\lambda_e, \qquad \lambda_e \;=\; \frac{\hbar}{m_e c}. \tag{64}$$

Short-range Z-Yukawa mass heuristic. Replacing A_0 by a Yukawa potential $Z_0 = Y(r) = e^{-r/\lambda_Z}/r$ gives the species-dependent rest energy

$$m_j \;=\; M_Z \left(\frac{e_j}{e}\right)^2 \alpha_e\,\frac{e^{-x}}{x}, \qquad x \equiv \frac{r}{\lambda_Z}, \qquad \lambda_Z = \frac{\hbar}{M_Z c}. \tag{65}$$

Dimensional checks. α_e and e_j/e are dimensionless; e^{-x}/x is dimensionless; M_Z sets the mass scale; hence $[m_j]$ = mass. The Coulomb form (64) is recovered from (65) in the formal limit $M_Z \to 0$ (i.e. $\lambda_Z \to \infty$).

L3. SST/VAM dictionary and testable map

We reinterpret (64)–(65) as *swirl self-energies* of a knotted-vortex excitation interacting with emergent gauge sectors:

$$\boxed{m_j^{(\mathrm{ae})} = \underbrace{\kappa_E\, q_{E,j}^2\, \frac{1}{r_\star}}_{\text{long-range (EM-like) swirl}} + \underbrace{\kappa_Z\, q_{Z,j}^2\, \alpha_e\, \frac{e^{-r_\star/\lambda_Z}}{r_\star/\lambda_Z}}_{\text{short-range (Z-like) swirl}}} \tag{66}$$

with a single geometric regulator $r_\star \sim r_c$ (vortex-core scale) and sector "swirl-charges" $q_{E,j}, q_{Z,j}$ inheriting the diagonal entries of Q and R. Matching the neutrino fixes κ_Z given λ_Z, while the electron fixes κ_E (up to $r_\star$).

Lemma L.1 (Neutrino calibration). Given a target m_ν (from oscillation/global fits), choose $r_\star$ and κ_Z such that (66) satisfies $m_\nu^{(\mathrm{ae})} = m_\nu$ with $q_{E,\nu} = 0$. This sets the *short-range* normalization that then predicts *minimal* contributions for neutral-current dominated species.

Lemma L.2 (Electron calibration). With $q_{E,e} = -1$ and $q_{Z,e} = \ell_+$, pick κ_E (or $r_\star$) to reproduce m_e. This sets the *long-range* normalization and bounds $r_\star$ relative to r_c (Canon constants).

Cross-links. (i) Insert (66) as an *IR seed* into the R2M2 rotation (Addendum E) and EJA ratios (Addendum D): the absolute masses fix the leakage normalizations c_f, c_f' without changing ratios. (ii) The Z-term scales with M_Z and is thus consistent with the misalignment adapter in A3 (via v). (iii) The diagonal structure is compatible with PS matching (Addendum F) before vectorlike mixing.

L4. Consistency gates

- **Custodial/EW veto:** any extra doublets or altered Z couplings must obey $|\Delta\rho| \lesssim 10^{-3}$ and small ξ_{ae} (Addendum C/A3).
- **UV caution:** (64) is a classical heuristic; within SST it only serves as an *effective IR parametrization* of swirl self-energy, not a fundamental QED self-energy calculation.
- **Limits:** $\alpha_e \to 0 \Rightarrow m_j^{(\mathrm{ae})}$ reduces to the long-range term; $M_Z \to \infty \Rightarrow$ short-range decouples.

L5. Minimal procedure

1. Fix λ_Z (input), choose $r_\star \sim r_c$ from Canon constants; solve Lemma L.1 for κ_Z at m_ν.
2. Solve Lemma L.2 for κ_E at m_e; compute *seed* masses m_u, m_d from (66) using $q_{E/Z}$ entries.
3. Feed seeds into Addendum J pipeline (EJA targets $\to$ R2M2 arc-lengths; EMH selector for Cabibbo) and apply EW vetoes.

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Addendum M: SU(6) Grand Gauge–Higgs Unification (GHU) kernel (bridging)

M1. Scope

Record the mass–generation kernel and EWSB structure from 5D SU(6) GHU on $S^1/\mathbb{Z}_2$, and provide an SST/VAM dictionary so it can be used as an *auxiliary seed* for fermion hierarchies and an alternative EWSB parametrization without altering the Core.

M2. Model primitives (captured)

- **Orbifold breaking and Higgs as gauge:** SU(6) on $S^1/\mathbb{Z}_2$ with boundary parities breaking to $SU(5)\times U(1)_X$ (at $y{=}0$) and $SU(2)\times SU(4)$ (at $y{=}\pi R$). The 4D Higgs doublet is a component of A_y. At the unification (near compactification) scale, $g_3{=}g_2{=}\sqrt{5/3}\,g_Y \Rightarrow \sin^2\theta_W = 3/8$.

- **Boundary SM matter & bulk messengers:** SM fermions live on the $y{=}0$ boundary as SU(5) multiplets and couple via boundary mass terms to massive bulk fermions (four SU(6) representations plus mirrors). Integrating out the bulk tower yields nonlocal masses that are exponentially sensitive to the bulk mass parameters.

- **Mass kernel (large-λ limit):** For species $a \in \{u, d, e, \nu\}$, the physical masses scale as
$$m_a^{\text{phys}} \simeq m_W\, e^{-\lambda_a}, \qquad \lambda_a \equiv \pi R\, M_a,$$
up to wavefunction renormalizations encoded by simple coth combinations (explicit ratio formulae below).

- **Explicit ratios vs. bulk masses:**

$$\frac{m_u}{m_W} = \sqrt{1-\coth^2\lambda_{20}}\,\sqrt{1+\frac{\coth\lambda_{20}}{\lambda_{20}}+\frac{\coth\lambda_{56}}{\lambda_{56}}\Big(1+\frac{\coth\lambda_{20}}{\lambda_{20}}\Big)}, \tag{67}$$

$$\frac{m_d}{m_W} = \sqrt{2}\,\sqrt{1-\coth^2\lambda_{56}}\,\sqrt{1+\varepsilon_{21}\frac{\coth\lambda_{56}}{\lambda_{56}}\Big(1+\varepsilon_{22}\frac{\coth\lambda_{56}}{\lambda_{56}}+\varepsilon_{21}\frac{\coth\lambda_{20}}{\lambda_{20}}\Big)}, \tag{68}$$

$$\frac{m_e}{m_W} = \sqrt{1-\coth^2\lambda_{15}}\,\sqrt{1+\frac{\coth\lambda_{15}}{\lambda_{15}}\Big(1+\frac{\coth\lambda_{15}}{\lambda_{15}}+\frac{\coth\lambda_{21}}{\lambda_{21}}\Big)}, \tag{69}$$

$$\frac{m_\nu}{m_W} = \sqrt{2}\,\sqrt{1-\coth^2\lambda_{21}}\,\sqrt{1+\frac{\coth\lambda_{21}}{\lambda_{21}}\Big(1+\frac{\coth\lambda_{21}}{\lambda_{21}}+\frac{\coth\lambda_{15}}{\lambda_{15}}\Big)}. \tag{70}$$

- **Illustrative fit (bulk masses):** A representative set reproducing SM masses (except top) is

$$\lambda_{20} = (5.9,\ 2.55,\ 0.1), \quad \lambda_{56} = (5.65,\ 4.1,\ 1.1), \quad \lambda_{15} = (6.58,\ 3.87,\ 2.4), \quad \lambda_{21} = (13,\ 10,\ 10)$$

  for generations 1–3.

- **EWSB one-loop potential:** With only the four messenger sets, the Coleman–Weinberg potential has its minimum at the origin (no EWSB). Adding a pair of bulk fermions in the **126** of SU(6) triggers EWSB with a small dimensionless VEV $\alpha \sim 10^{-2}$, and a Higgs mass $m_H \approx 147\,g_4\,\mathrm{GeV}$ at $1/R \sim 0.8$ TeV.

M3. SST/VAM dictionary

M3.1 Exponential hierarchy as swirl attenuation. Identify each bulk parameter λ_a with a dimensionless *swirl attenuation length* $\Lambda_a \equiv \sigma_a\, L/L_*$ along the local swirl–moduli geodesic. Then

$$m_a^{(\mathrm{ae})} = m_W\, e^{-\Lambda_a} \quad \text{(seed masses)},$$

providing UV-insensitive IR seeds for Addendum J (EJA targets $\{s,\delta\}$ and R2M2 arc–lengths). Species coefficients σ_a absorb representation-dependent normalization.

M3.2 Hypercharge and PS matching. The SU(6) boundary pattern $SU(5)\times U(1)_X$ delivers $g_3{=}g_2{=}\sqrt{5/3}\,g_Y$ at the compactification scale; this is consistent with our Safe–Pati–Salam (Addendum F) matching and Georgi–Jarlskog boundary conditions in A4.

M3.3 EWSB: misalignment map. The small α solution naturally fits the pNGB misalignment adapter (Addendum A3): use $v = f_{\mathrm{ae}}\sin(\Theta/f_{\mathrm{ae}})$ with $\xi_{\mathrm{ae}}{=}v^2/f_{\mathrm{ae}}^2 \ll 1$ and take the GHU potential as one possible IR completion generating Θ.

M4. Minimal calibration protocol

1. Pick a common geometric scale $L_* \sim r_c$ and initial σ_a. Calibrate σ_e to m_e and σ_ν to a target m_ν.
2. Use the λ-ratio formulas as constraints on $\sigma_{u,d,e,\nu}$ (or on the effective arc lengths) so that the seeds match the observed hierarchies before R2M2 leakage.
3. For EWSB, choose $f_{\rm ae}$ such that $\zeta_{\rm ae} \ll 1$. If a GHU-like periodic potential is desired, encode its periodicity via a higher-representation analogue of the **126** add-on.

M5. Consistency gates

- **Top mass:** the baseline SU(6) setup underproduces m_t unless higher-rank embeddings or localized gauge–kinetic terms are included (outside this addendum). Use our Safe–PS machinery for the third family (Addendum F).
- **Custodial/EW veto:** keep $\zeta_{\rm ae} \ll 1$ (Addendum A3) and enforce $|\Delta\rho| \lesssim 10^{-3}$ (Addendum C).
- **UV: unification and proton decay:** treat SU(6)-GUT coupling running and baryon number as external constraints; this block does not import those UV commitments into the Canon.

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17 Swirl-Strings: Bridging Blocks for Modern Physics and String Methods in Fluid Dynamics

amsmath,amssymb,bm hyperref physics siunitx cleveref

Policy status. *Bridging-only addendum.* This note introduces a fixed-point mechanism for fermion family triplication and an approximate determination of the fine-structure constant, adapted from Lemmon [**?**]. It **does not modify** SST Core postulates or boxed Canon results [**?**], and is intended for phenomenological interfacing with the covariant EFT in [**?**].

18 Executive Summary (Benchmark Targets)

Lemmon [**?**] proposes: (i) treating the UV cutoff as physical, (ii) resumming the divergent fermion self-mass as a geometric formal sum, yielding a self-consistency equation with potentially three fixed points (interpreted as generations), (iii) a QED-only estimate of α, and (iv) an electroweak extension that preserves the α estimate while driving the cutoff to M_P. We port this *formal structure* into SST by replacing the hard cutoff with an SST spectral regulator derived from swirl scales (no change to SST ontology).

19 Canon-Aligned Formal Statements

19.1 Fixed-point self-mass (charged leptons)

The formal resummation of the mass series gives

$$\boxed{m \;=\; m_0 \;+\; \frac{m\,\Delta(m)}{1-\Delta(m)}}\,, \tag{71}$$

with one-loop QED kernel

$$\Delta^{(1)}(m) \;=\; \frac{3\alpha_0}{4\pi}\left(\ln\frac{\Lambda^2}{m^2}+\frac{1}{2}\right) \;+\; \cdots\,, \tag{72}$$

as in [**?**] (see also classic renormalization structures [**?**, **?**]). For suitable (m_0, α_0, Λ), eq:N1,eq:N2 admit three fixed points m, interpretable as (m_e, m_μ, m_τ) [**?**].

19.2 Electroweak extension (leptons)

Including W and Z loops and a physical weak angle θ_W, Lemmon writes (displaying only the integral skeleton)

$$\begin{aligned}\Delta^{(1)}(m) \;=\; &\frac{3\alpha_0}{4\pi}\left(\ln\frac{\Lambda^2}{m^2}+\frac{1}{2}\right) - \frac{\alpha_0}{\pi}\left[\frac{1}{16}\csc^2\theta_W\, I_1\right.\\ &\left.+\left(\frac{1}{32}\cot^2\theta_W+\frac{5}{32}\tan^2\theta_W-\frac{1}{16}\right) I_2+\left(\frac{1}{2}-\frac{1}{2}\tan^2\theta_W\right) I_3\right],\end{aligned} \tag{73}$$

with

$$I_1(m,m_\nu) = 2\int_0^1 z\,\ln\frac{\Lambda^2 z}{M_W^2 z - m^2 z(1-z) + m_\nu^2(1-z)}\,dz,$$
$$I_2(m) = 2\int_0^1 z\,\ln\frac{\Lambda^2 z}{M_Z^2 z + m^2(1-z)^2}\,dz,$$
$$I_3(m) = \int_0^1 \ln\frac{\Lambda^2 z}{M_Z^2 z + m^2(1-z)^2}\,dz, \tag{74}$$

and a chiral averaging rule for the pole mass (see also [?]) as quoted in [?]. The W loop contributes positively to charge renormalization, shifting the preferred cutoff toward M_P while preserving the α estimate [?].

19.3 Charge renormalization and α

At one loop,

$$\alpha \;=\; \alpha_0\left[1 - \frac{\alpha_0}{3\pi}\sum_{\ell=e,\mu,\tau}\ln\frac{\Lambda^2}{m_\ell^2} + \frac{11\alpha_0}{12\pi}\ln\frac{\Lambda^2}{M_W^2}\right] \;-\; \alpha(M_Z^2)\,\Delta\alpha_H(M_Z^2)\,, \tag{75}$$

yielding $\alpha^{-1} \approx 158.7$ (QED-only) and $\alpha^{-1} \approx 164.5$ with EW contributions in the scheme of [?].

19.4 Quark sector and mixing trigger

For up-type quarks (down-type analogous), a one-loop kernel of the form

$$\Delta_U^{(1)}(m_U) \;=\; \frac{\alpha_0}{3\pi}\left(\ln\frac{\Lambda^2}{m_U^2} + \frac{1}{2}\right) + \frac{3\alpha_0^{(s)}}{4\pi}\left(\frac{4}{9}\right)\left(\ln\frac{\Lambda^2}{m_U^2} + \frac{1}{2}\right) - \frac{\alpha_0}{\pi}\left[\cdots\right] \tag{76}$$

is augmented by a mixing necessity once $I_1(m_t, m_b)$ becomes complex, which Lemmon implements by CKM-weighted replacements $I_1 \to \sum_{ij}|V_{ij}|^2 I_1(m_i, m_j)$ [?].

20 SST Bridging Map (No-Core-Change)

To interface eq:N1–eq:N6 with SST:

- Replace the hard cutoff Λ by an *SST spectral regulator* tied to swirl scales,

  $$\Lambda \;\longrightarrow\; \Lambda_{\text{SST}} \sim \Omega_{\max} \equiv \frac{v_{\circlearrowleft}}{r_c}\,,$$

  where $v_{\circlearrowleft}$ and r_c are Canon constants [?]. This keeps Δ dimensionless while grounding the logs in SST kinematics.

- Interpret $\Delta(m)$ as nonlinear backreaction from swirl polarization (vacuum screening) around a knotted filament. The geometric sum in eq:N1 corresponds to iterated medium feedback—consistent with the EFT structure in [?].

- Mixing/CP: complex thresholds in the SST mediator sector (swirl-gauge channels) trigger superpositions, paralleling the CKM-weighted I_1 replacement [**?**].

Note. This addendum does not alter SST ontology (no point-particle fundamentals, no Planckian necessity); it supplies a calculational kernel compatible with the Canon/Lagrangian.

21 Immediate SST Tests

1. **Triplication with $\Lambda_{\mathbf{SST}}$.** Calibrate (α_0, m_0) on (m_e, m_μ) using Λ_{SST} and test whether a third fixed point reproduces m_τ.
2. α **with SST screening.** Evaluate eq:N5 with $\Lambda \to \Lambda_{\mathrm{SST}}$ and quantify the analogue of $\Delta\alpha_H$ via collective swirl modes.
3. **Mixing trigger.** Identify the SST channel whose opening renders the analogue of I_1 complex and verify that CKM-like phase structure follows.

Classification (Canon Governance)

Type: Addendum (Bridging). **Scope:** Family replication, hierarchies, α estimate via fixed points. **Core:** Unchanged (no modification to boxed Canon equations [**?**]).

References

References