Swirl String Theory (SST) Canon v0.3.4

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Abstract

This Canon is the single source of truth for Swirl String Theory (SST): definitions, constants, boxed master equations, and notational conventions. It consolidates core structure and promotes five results to canonical status:

I Swirl Coulomb constant Λ and hydrogen soft-core
II circulation–metric corollary (frame-dragging analogue)
III corrected swirl time-rate (Swirl Clock) law
IV Kelvin-compatible swirl Hamiltonian density
V swirl pressure law (Euler corollary)

Core Postulates (SST)

1. **Swirl medium:** Physics is formulated on $\mathbb{R}^3$ with absolute reference time. Dynamics occur in an incompressible, inviscid swirl condensate, which plays the role of a universal substrate.
2. **Strings as swirls:** Particles and excitations correspond to closed, possibly linked or knotted swirl strings with quantized circulation.
3. **String-induced gravitation:** Macroscopic attraction emerges from coherent swirl fields and swirl-pressure gradients. The effective gravitational coupling G_{swirl} is fixed by canonical constants.
4. **Swirl clocks:** Local proper-time rate depends on tangential swirl velocity. Higher swirl density slows local clocks relative to the asymptotic frame.
5. **Quantization from topology and circulation:** Discrete quantum numbers track directly to linking, writhe, twist, and circulation quantization of swirl strings.
6. **Taxonomy:** Unknotted excitations behave as bosonic string modes; chiral hyperbolic knots map to quarks; torus knots map to leptons (taxonomy documented separately).

Hydrodynamic analogy only; no mechanical "æther" is assumed in the mainstream presentation.

Versioning Semantic versions: vMAJOR.MINOR.PATCH. This file: **v0.3.4**. Every paper/derivation must state the Canon version it depends on.

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1 Swirl Quantization Principle

1.1 Local Circulation Quantization

The circulation of the swirl velocity field around any closed loop is quantized:

$$\Gamma = \oint \vec{v}_{\text{swirl}} \cdot d\vec{\ell} = n\kappa, \qquad n \in \mathbb{Z}, \tag{1}$$

with circulation quantum

$$\kappa = \frac{h}{m_{\text{eff}}}. \tag{2}$$

This parallels the Onsager–Feynman quantization condition in superfluids, but here is elevated to a fundamental postulate of the swirl condensate.

1.2 Topological Quantization

Closed swirl filaments may form knots and links. Each topological class corresponds to a discrete excitation state:

$$\mathcal{H}_{\text{swirl}} = \{\text{trefoil},\ \text{figure-eight},\ \text{Hopf link}, \dots\}. \tag{3}$$

Quantum numbers such as mass, charge, and chirality are encoded in the knot invariants (linking, twist, writhe).

1.3 Unified Principle

We define Swirl Quantization as the joint discreteness of circulation and topology:

$$\text{Swirl Quantization} \ \equiv\ \Big(\Gamma = n\kappa\Big) \ \cup\ \Big(\text{Knot spectrum } \mathcal{H}_{\text{swirl}}\Big).$$

This principle underlies both the discrete particle spectrum and the emergence of fundamental interactions in Swirl String Theory.

Quantum Mechanics	**Swirl String Theory**
Canonical Quantization: $[x,p] = i\hbar$	Swirl Quantization Principle: $\Gamma = n\kappa, \quad n \in \mathbb{Z}$ $\mathcal{H}_{\text{swirl}} = \{\text{trefoil},\ \text{figure-eight},\ \text{Hopf link}, \dots\}$
Discreteness arises from operator commutators	Discreteness arises from circulation integrals and topology
Particles = eigenstates of Hamiltonian operator	Particles = knotted swirl states with quantized circulation and invariants

2 Chronos–Kelvin Invariant (Canonical)

Setting. Consider a thin, material swirl loop (nearly solid–body core) of instantaneous material radius $R(t_\circlearrowleft)$ convected by an incompressible, inviscid medium. Let $\omega := \|\boldsymbol{\omega}\|$ denote the vorticity magnitude on the loop and r_c the canonical string radius. The local Swirl Clock is

$$S_t \ \equiv\ \frac{dt_{\text{local}}}{dt_\infty} \ =\ \sqrt{1 - \frac{v_t^2}{c^2}} \ =\ \sqrt{1 - \frac{\omega^2 r_c^2}{c^2}}\,, \qquad v_t := \omega r_c. \tag{4}$$

Material derivatives are taken with respect to absolute reference time: $\dfrac{D}{Dt_\circlearrowleft} := \dfrac{\partial}{\partial t_\circlearrowleft} + \mathbf{v}\cdot\nabla$.

Theorem 2.1 (Chronos–Kelvin Invariant) *For any such loop without reconnection or source terms, Kelvin's theorem implies the material invariant*

$$\boxed{\frac{D}{Dt_{\circlearrowleft}}\left(R^2\,\omega\right)=0} \quad\Longleftrightarrow\quad \boxed{\frac{D}{Dt_{\circlearrowleft}}\left(\frac{c}{r_c}\,R^2\sqrt{1-S_t^2}\right)=0} \tag{5}$$

Proof (one line). Kelvin's circulation theorem for an inviscid, barotropic medium gives $\frac{D}{Dt_{\circlearrowleft}}\Gamma=0$ with $\Gamma=\oint \mathbf{v}\cdot d\boldsymbol{\ell}$ [1, 2, 3]. For a nearly solid–body core, $\Gamma=2\pi R\,v_t=2\pi R^2\omega$; hence $\frac{D}{Dt_{\circlearrowleft}}(R^2\omega)=0$. Using (4), $R^2\omega=\frac{c}{r_c}R^2\sqrt{1-S_t^2}$, which yields (5).

Dimensional consistency. $[R^2\omega]=\mathrm{m^2s^{-1}}$; and $\left[\frac{c}{r_c}R^2\sqrt{1-S_t^2}\right]=\mathrm{s^{-1}}\cdot\mathrm{m^2}=\mathrm{m^2s^{-1}}$.

Clock–radius transport law (corollary). From $R^2\omega=$ const and (4),

$$\frac{dS_t}{dt_{\circlearrowleft}}=\frac{2(1-S_t^2)}{S_t}\,\frac{1}{R}\frac{dR}{dt_{\circlearrowleft}} \tag{6}$$

Hence expansion ($dR/dt_{\circlearrowleft}>0$) pushes $S_t\to 1$ (clocks speed up), while contraction slows clocks ($S_t\downarrow$), preserving (5).

PV analogue (optional). With a uniform background rotation Ω_{bg} and column thickness H, the Ertel/PV structure gives the SST counterpart

$$\frac{D}{Dt_{\circlearrowleft}}\left(\frac{\omega+\Omega_{\mathrm{bg}}}{H}\right)=0, \tag{7}$$

the standard potential-vorticity conservation rewritten in SST terms [4, 3].

Conditions (Canon). Incompressible, inviscid medium; barotropic swirl pressure; material loop without reconnection or external injection; absolute reference time parametrization. These are the same hypotheses under which Kelvin/Helmholtz invariants hold.

Limits. Weak-swirl ($\omega r_c\ll c$): $S_t\simeq 1-\frac{1}{2}(\omega r_c/c)^2$ and (5) reduces to the classical $R^2\omega=$ const. Core on-axis limit: $v_t\to\mathbf{v}_{\circlearrowleft}$ gives $S_t\to\sqrt{1-(\mathbf{v}_{\circlearrowleft}/c)^2}$, keeping (5) valid.

3 Foundational Identities

Let $\mathbf{v}$ be the swirl velocity ($\nabla\cdot\mathbf{v}=0$), $\boldsymbol{\omega}=\nabla\times\mathbf{v}$. For inviscid, barotropic flow [1, 2, 3, 5]:

Kelvin circulation: $$\frac{d\Gamma}{dt}=0,\quad \Gamma=\oint_{\mathcal{C}(t)}\mathbf{v}\cdot d\ell. \tag{F1}$$

Vorticity transport: $$\frac{\partial\boldsymbol{\omega}}{\partial t}=\nabla\times(\mathbf{v}\times\boldsymbol{\omega}). \tag{F2}$$

Helicity: $$h=\mathbf{v}\cdot\boldsymbol{\omega},\quad H=\int h,dV \text{ (invariant up to reconnections). [6]} \tag{F3}$$

These underpin knotted swirl string stability and reconnection energetics in SST.

4 Canonical Constants and Symbols

Primary SST constants (SI unless noted)

- Swirl speed scale (core): $\|\mathbf{v}_{\circlearrowleft}\| = 1.093\,85 \times 10^{6}\ \mathrm{m\,s^{-1}}$ (evaluate at $r = r_c$).
- String (core) radius: $r_c = 1.408\,97 \times 10^{-15}$ m.
- Effective fluid density: $\rho_f = 7.000\,00 \times 10^{-7}\ \mathrm{kg\,m^{-3}}$.
- Mass-equivalent density: $\rho_m = 3.893\,44 \times 10^{18}\ \mathrm{kg\,m^{-3}}$.
- EM-like maximal force: $F_{\mathrm{EM}}^{\max} = 2.905\,35 \times 10^{1}$ N.
- Gravitational maximal force (reference scale): $F_{\mathrm{G}}^{\max} = 3.025\,63 \times 10^{43}$ N.
- Golden ratio: $\varphi = (1 + \sqrt{5})/2 \approx 1.618\,03$.

Universal constants

- $c = 299\,792\,000\ \mathrm{m\,s^{-1}}$, $\quad t_p = 5.391\,25 \times 10^{-44}$ s.
- Fine-structure constant (identified): $\alpha \approx 7.297\,35 \times 10^{-3}$.

Effective densities (mainstream field-theory style).

$$\rho_f \equiv \text{effective fluid density},$$

We use ρ_f to avoid confusion[1] with mass density;

$$\rho_E \equiv \tfrac{1}{2}\rho_f\,\|\mathbf{v}_{\circlearrowleft}\|^2 \quad \text{(swirl energy density)}, \qquad \rho_m \equiv \rho_E/c^2 \quad \text{(mass-equivalent density)}.$$

Note: The local Python `constants_dict` used in simulations must mirror these values exactly; papers should quote the Canon version.

Swirl Areal Density and EM Coupling

In addition to the effective densities defined above, we introduce the swirl areal density $\varrho_{\circlearrowleft}$, defined as the coarse-grained number of swirl cores per unit area. Its time variation enters Maxwell's law as an additional source term,

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} - \mathbf{b}_{\circlearrowleft}, \qquad \mathbf{b}_{\circlearrowleft} = G_{\circlearrowleft}\,\partial_t \varrho_{\circlearrowleft}.$$

Here $G_{\circlearrowleft}$ is the canonical swirl–EM transduction constant, identified with a flux quantum $\Phi^* \sim h/2e$. This relation links electromotive force (voltage impulses) and swirl reconnection dynamics, establishing the canonical bridge between EMF and gravity-like swirl fields.

5 Canon Governance (Binding)

Definitions

Formal System. Let $\mathcal{S} = (\mathcal{P}, \mathcal{D}, \mathcal{R})$ denote the SST formal system: postulates $\mathcal{P}$, definitions $\mathcal{D}$, and admissible inference rules $\mathcal{R}$ (variational derivation, Noether, dimensional analysis, asymptotic matching, etc.).

[1] The canonical choice $\rho_f = 7.0 \times 10^{-7}\ \mathrm{kg\,m^{-3}}$ is not a measured value but a calibration constant. Its magnitude is anchored to the electromagnetic permeability scale $\mu_0/(4\pi) = 10^{-7}$ in SI units, ensuring dimensional consistency between swirl energetics and EM normalization. Unlike the derived high-precision values of ρ_m and ρ_E, the effective fluid density ρ_f is defined at this tidy scale to serve as a reference baseline.

Canonical statement. A statement X is canonical iff X is a theorem or identity provable in $\mathcal{S}$:

$$\mathcal{P}, \mathcal{D} \vdash_{\mathcal{R}} X,$$

and X is consistent with all previously accepted canonical items in the current major version.

Empirical statement. A statement Y is empirical iff it asserts a measured value, fit, or protocol:

$$Y \equiv \text{“observable } \mathcal{O} \text{ has value } \hat{o} \pm \delta o \text{ under procedure } \Pi.\text{”}$$

Empirical items calibrate symbols (e.g., $v_{\circlearrowleft}$, r_c, ρ_f) but are not premises in proofs.

Status Classes

- **Axiom / Postulate (Canonical).** Primitive assumptions of SST (e.g., incompressible, inviscid medium; absolute time; Euclidean space).
- **Definition (Canonical).** Introduces symbols by construction (e.g., swirl Coulomb constant Λ by surface-pressure integral).
- **Theorem / Corollary (Canonical).** Proven consequences (e.g., Euler–SST radial balance; Swirl Clocks time-scaling).
- **Constitutive Model (Canonical if derived; otherwise Semi-empirical).** Ties fields/observables; canonical when deduced from $\mathcal{P}, \mathcal{D}$.
- **Calibration (Empirical).** Recommended numerical values with uncertainties for canonical symbols.
- **Research Track (Non-canonical).** Conjectures or alternatives pending proof or axiomatization.

Canonicality Tests (all required)

1. **Derivability** from $\mathcal{P}, \mathcal{D}$ via $\mathcal{R}$.
2. **Dimensional Consistency** (SI throughout; correct limits).
3. **Symmetry Compliance** (Galilean + absolute time; foliation; incompressibility).
4. **Recovery Limits** (Newtonian gravity, Coulomb/Bohr, linear waves).
5. **Non-Contradiction** with accepted canonical theorems.
6. **Parameter Discipline** (no ad-hoc fits).

Examples (from current Canon)

- *Canonical (Definition):* $\Lambda \equiv \int_{S_r^2} p_{\text{swirl}}\, r^2\, d\Omega.$
- *Canonical (Theorem):* $\dfrac{1}{\rho_f}\dfrac{dp_{\text{swirl}}}{dr} = \dfrac{v_\theta(r)^2}{r}$ for steady, azimuthal drift (Euler balance).
- *Empirical (Calibration):* $v_{\circlearrowleft} = 1.09384563 \times 10^6\,\mathrm{m\,s^{-1}}$ with procedure $f\Delta x$.
- *Consistency Check (Not a premise):* Hydrogen soft-core reproduces a_0, E_1; validates choices but remains a check, not an axiom.

6 What is Canonical in SST—and Why

[Postulate] Incompressible, inviscid medium with absolute time and Euclidean space. $\nabla\cdot\mathbf{v}_{\circlearrowleft} = 0,\ \nu = 0$. This fixes the kinematic arena and legal inference rules.

[Definition] Vorticity, circulation, helicity. $\boldsymbol{\omega}_{\circlearrowleft} = \nabla \times \mathbf{v}_{\circlearrowleft}$, $\Gamma = \oint \mathbf{v}_{\circlearrowleft} \cdot d\boldsymbol{\ell}$, $h = \mathbf{v}_{\circlearrowleft} \cdot \boldsymbol{\omega}_{\circlearrowleft}$, $H = \int h\, dV$. Classical constructs canonized as primary SST kinematic invariants.

[Theorem] Kelvin/vorticity transport/helicity invariants. For inviscid, barotropic flow:

$$\frac{d\Gamma}{dt} = 0, \qquad \frac{\partial \boldsymbol{\omega}_{\circlearrowleft}}{\partial t} = \nabla \times (\mathbf{v}_{\circlearrowleft} \times \boldsymbol{\omega}_{\circlearrowleft}), \qquad H \text{ invariant up to reconnections.}$$

[Definition] Swirl Coulomb constant Λ.

$$\boxed{\Lambda \equiv \int_{S_r^2} p_{\text{swirl}}(r)\, r^2\, d\Omega} \quad \Rightarrow \quad [\Lambda] = \mathrm{J\,m} = \mathrm{N\,m}^2.$$

In SST Canon this evaluates symbolically to $\Lambda = 4\pi \rho_m\, v_{\circlearrowleft}^2\, r_c^4$.

[Theorem] Hydrogen soft-core potential and Coulomb recovery.

$$V_{\text{SST}}(r) = -\frac{\Lambda}{\sqrt{r^2 + r_c^2}} \xrightarrow{r \gg r_c} -\frac{\Lambda}{r},$$

yielding Bohr scalings $a_0 = \hbar^2/(\mu\Lambda)$, $E_n = -\mu\Lambda^2/(2\hbar^2 n^2)$.

[Theorem] Euler–SST radial balance (swirl pressure law). For steady, purely azimuthal drift $v_\theta(r)$,

$$0 = -\frac{1}{\rho_f}\frac{dp_{\text{swirl}}}{dr} + \frac{v_\theta(r)^2}{r} \quad \Rightarrow \quad \boxed{\frac{1}{\rho_f}\frac{dp_{\text{swirl}}}{dr} = \frac{v_\theta(r)^2}{r}}.$$

For flat curves $v_\theta \to v_0$: $p_{\text{swirl}}(r) = p_0 + \rho_f v_0^2 \ln(r/r_0)$.

[Definition $\rightarrow$ Corollary] Effective swirl line element (analogue-metric form). In (t, r, θ, z) with azimuthal drift $v_\theta(r)$,

$$ds^2 = -(c^2 - v_\theta^2)\, dt^2 + 2\, v_\theta r\, d\theta\, dt + dr^2 + r^2 d\theta^2 + dz^2,$$

co-rotating to $ds^2 = -c^2(1 - v_\theta^2/c^2)dt^2 + \cdots$, giving the Swirl Clock factor $\dfrac{dt_{\text{local}}}{dt_\infty} = \sqrt{1 - \dfrac{v_\theta^2}{c^2}}$.

[Definition] SST Hamiltonian density (Kelvin-compatible).

$$\mathcal{H}_{\text{SST}} = \tfrac{1}{2}\rho_f\, \|\mathbf{v}_{\circlearrowleft}\|^2 + \tfrac{1}{2}\rho_f\, r_c^2 \|\boldsymbol{\omega}_{\circlearrowleft}\|^2 + \lambda(\nabla\cdot\mathbf{v}_{\circlearrowleft}).$$

Empirical Calibrations (not premises, but binding numerically)

- [Empirical] $v_{\circlearrowleft} = 1.09384563 \times 10^6\,\mathrm{m\,s^{-1}}$.
- [Empirical] $r_c = 1.40897017 \times 10^{-15}\,\mathrm{m}$.
- [Empirical] $\rho_m = 3.8934358266918687 \times 10^{18}\,\mathrm{kg\,m^{-3}}$.

Non-Canonical (Research Track)

Blackbody via swirl temperature, EM/SST minimal coupling, etc., remain conjectural until proven under $\mathcal{S}$.

Consistency & Dimension Checks (illustrative)

$$[\Lambda] = [\rho_m][v_\circlearrowleft^2][r_c^4] = \frac{\mathrm{kg}}{\mathrm{m}^3} \cdot \frac{\mathrm{m}^2}{\mathrm{s}^2} \cdot \mathrm{m}^4 = \frac{\mathrm{kg\,m}^3}{\mathrm{s}^2} = \mathrm{J\,m}.$$

Soft-core Coulomb recovery: $V_{\mathrm{SST}}(r) \to -\Lambda/r$ as $r/r_c \to \infty$.

7 Canonical Coarse–Graining of $\rho_f$rom a Swirl–String Bath

Scope. The medium is modeled as an incompressible, inviscid fluid populated by thin swirl strings. We derive the bulk effective fluid density ρ_f via coarse–graining of line–supported mass and vorticity, relying on Euler kinematics and Kelvin–Helmholtz invariants.

7.1 Axioms and Definitions

A representative string carries:

$$\text{(D1)} \quad \mu_* \equiv \rho_m \pi r_c^2 \quad [\mathrm{kg/m}], \tag{8}$$

$$\text{(D2)} \quad \Gamma_* \equiv \oint \mathbf{v}_\circlearrowleft \cdot d\boldsymbol{\ell} \simeq \kappa_\Gamma r_c v_\circlearrowleft, \qquad \kappa_\Gamma = 2\pi \ \text{(near–solid–body core)}. \tag{9}$$

Let $\nu \equiv N_{\mathrm{str}}/A$ $[\mathrm{m}^{-2}]$ be the areal string density. Then:

$$\text{(C1)} \quad \rho_f = \mu_* \nu, \tag{10}$$

$$\text{(C2)} \quad \langle \boldsymbol{\omega}_\circlearrowleft \rangle = \Gamma_* \nu \, \hat{\mathbf{t}}_{\mathrm{avg}} \ \Rightarrow \ |\langle \omega_s \rangle| = \Gamma_* \nu. \tag{11}$$

7.2 First–Principles Derivation

Combining (C1)–(C2):

$$\boxed{\rho_f = \mu_* \frac{\langle \omega_s \rangle}{\Gamma_*} = \frac{\rho_m \pi r_c^2}{\kappa_\Gamma r_c v_\circlearrowleft} \langle \omega_s \rangle = \frac{\rho_m r_c}{2 v_\circlearrowleft} \langle \omega_s \rangle} \quad (\kappa_\Gamma = 2\pi). \tag{12}$$

For uniform solid–body rotation Ω, $\langle \omega_s \rangle = 2\Omega$,

$$\boxed{\rho_f = \frac{\rho_m r_c}{v_\circlearrowleft} \Omega} \quad [\mathrm{kg/m}^3]. \tag{13}$$

Energy and tension scales.

$$\boxed{u_{\mathrm{swirl}} = \tfrac{1}{2} \rho_f v_\circlearrowleft^2}, \qquad \boxed{T_* = \tfrac{1}{2} \mu_* v_\circlearrowleft^2}.$$

7.3 Numerical Calibration (SST Canonical Constants)

With $\rho_m = 3.8934358266918687 \times 10^{18}$ kg/m^3, $r_c = 1.40897017 \times 10^{-15}$ m, $v_{\circlearrowleft} = 1.09384563 \times 10^6$ m/s, one finds

$$\Gamma_* = 2\pi r_c v_{\circlearrowleft} = 9.68361920 \times 10^{-9}\ \mathrm{m^2/s}, \quad T_* = 1.45267535 \times 10^1\ \mathrm{N}.$$

From (13),

$$\rho_f = \left(5.01509060 \times 10^{-3}\right) \Omega,$$

so the Canon baseline $\rho_f = 7.0 \times 10^{-7}$ kg/m^3 occurs at

$$\boxed{\Omega_* = 1.39578735 \times 10^{-4}\ \mathrm{s^{-1}}\ \text{(period } \approx 12.5\text{ h)}}.$$

8 Master Equations (Boxed, Definitive)

8.1 Master Energy and Mass Formula (SST)

$$\boxed{E_{\mathrm{SST}}(V) = \frac{4}{\alpha\,\varphi}\left(\frac{1}{2}\,\rho_f\,v_{\circlearrowleft}^2\right)V} \quad [\mathrm{J}], \qquad \boxed{M_{\mathrm{SST}}(V) = \frac{E_{\mathrm{SST}}(V)}{c^2}} \quad [\mathrm{kg}].$$

Numerics per unit volume: $\frac{1}{2}\rho_f v_{\circlearrowleft}^2 \approx 4.1877439 \times 10^5\ \mathrm{J\,m^{-3}}$, $\frac{4}{\alpha\varphi} \approx 3.3877162 \times 10^2, \Rightarrow E/V \approx 1.418688 \times 10^8\ \mathrm{J\,m^{-3}}, M/V \approx 1.57850 \times 10^{-9}\ \mathrm{kg\,m^{-3}}$.

8.2 Swirl–Gravity Coupling

$$\boxed{G_{\mathrm{string}} = \frac{v_{\circlearrowleft}\ c^5\ t_p^2}{2\,F_{\mathrm{EM}}^{\max}\ r_c^2}}$$

Numerically $\approx 6.674302 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ with the Canon constants.

8.3 Topology–Driven Mass Law (invariant form)

Let $T(p,q)$ be a torus knot/link, $n = \gcd(p,q)$ components, braid index $b(T) = \min(|p|,|q|)$, Seifert genus $g(T)$ (with standard link adjustment). Using ropelength $\mathcal{L}_{\mathrm{tot}}(T)$ and string core radius r_c:

$$\boxed{M\Big(T(p,q)\Big) = \left(\frac{4}{\alpha}\right)\, b(T)^{-3/2}\,\varphi^{-\,g(T)}\,n^{-1/\varphi}\,\left(\frac{1}{2}\rho_f v_{\circlearrowleft}^2\right)\,\frac{\pi r_c^3\,\mathcal{L}_{\mathrm{tot}}(T)}{c^2}.}$$

Dimensionality follows from the factor $\frac{1}{2}\rho_f v_{\circlearrowleft}^2$ ($\mathrm{J\,m^{-3}}$) times a volume.

8.4 Swirl Clocks (Local Time-Rate)

$$\boxed{\frac{dt_{\mathrm{local}}}{dt_\infty} = \sqrt{1 - \frac{\|\boldsymbol{\omega}_{\circlearrowleft}\|^2\,r_c^2}{c^2}} = \sqrt{1 - \frac{\|\mathbf{v}_{\circlearrowleft}\|^2}{c^2}} \quad (r = r_c)}.$$

Historical (deprecated) variant without a length scale is retained only for traceability.

8.5 Swirl Angular Frequency Profile

$$\boxed{\Omega_{\mathrm{swirl}}(r) = \frac{v_{\circlearrowleft}}{r_c}\,e^{-r/r_c}}, \qquad \Omega_{\mathrm{swirl}}(0) = \frac{v_{\circlearrowleft}}{r_c}.$$

8.6 Vorticity Potential (Canonical Form)

$$\Phi(\vec{r}, \boldsymbol{\omega}_\circlearrowleft) = \frac{v_\circlearrowleft^2}{2\, F_{\rm EM}^{\max}}\, \boldsymbol{\omega}_\circlearrowleft \cdot \vec{r}.$$

Dimensional remark: Use with the SST Lagrangian ensuring $\rho_f \Phi$ has energy density units.

9 Unified SST Lagrangian (Definitive Form)

Let $\mathbf{v}_\circlearrowleft$ be the velocity, ρ_f constant (incompressible), $\boldsymbol{\omega}_\circlearrowleft = \nabla \times \mathbf{v}_\circlearrowleft$, and λ enforce incompressibility.

$$\boxed{\mathcal{L}_{\rm SST} = \frac{1}{2}\rho_f \, \|\mathbf{v}_\circlearrowleft\|^2 - \rho_f \, \Phi(\vec{r}, \boldsymbol{\omega}_\circlearrowleft) + \lambda(\nabla \cdot \mathbf{v}_\circlearrowleft) + \eta \int (\mathbf{v}_\circlearrowleft \cdot \boldsymbol{\omega}_\circlearrowleft)\, dV + \mathcal{L}_{\rm couple}[\Gamma, \mathcal{K}]}.$$

Here $\mathcal{L}_{\rm couple}$ encodes coupling to quantized circulation Γ and knot invariants $\mathcal{K}$ (linking, writhe, twist).

10 Notation, Ontology, and Glossary

- **Absolute time (A-time):** global time parameter of the medium.
- **Chronos Time (C-time):** asymptotic observer time (dt_∞).
- **Swirl Clocks:** local clocks set by $\|\boldsymbol{\omega}_\circlearrowleft\|$ or $\|\mathbf{v}_\circlearrowleft\|$ per Sec. 9.
- **String taxonomy:** leptons = torus knots; quarks = chiral hyperbolic knots; bosons = unknots; neutrinos = linked knots.
- **Chirality:** ccw $\leftrightarrow$ matter; cw $\leftrightarrow$ antimatter via swirl–gravity coupling.

11 Unknot bosons and lossless swirl radiation

Postulate (Topological sector). Let $\mathcal{U}$ denote an unknotted closed swirl string (topological unknot, Hopf charge $\mathcal{H} = 0$). Imposing Finkelstein–Rubinstein constraints for single-valued many-body wavefunctionals on the configuration space of closed strings yields integer spin sectors for $\mathcal{U}$:

$$\boxed{\mathcal{U} \;\Rightarrow\; \text{bosonic sector}}.$$

(Nontrivial knot/link classes supply the sign structure needed for half-integer spin.) [7]

Field variables and lossless propagation. Introduce a transverse swirl potential $\mathbf{a}(\mathbf{x}, t)$ with

$$\mathbf{v} \equiv \partial_t \mathbf{a}, \qquad \mathbf{b} \equiv \nabla \times \mathbf{a}, \qquad \nabla \cdot \mathbf{a} = 0,$$

and take the quadratic effective Lagrangian density

$$\mathcal{L}_{\rm swirl} = \frac{\rho_f}{2}\, |\mathbf{v}|^2 - \frac{\rho_f c^2}{2}\, |\mathbf{b}|^2,$$

with ρ_f the effective (coarse-grained) density and c the observed luminal wave speed. Euler–Lagrange equations give the lossless wave equation

$$\boxed{\partial_t^2 \mathbf{a} - c^2\, \nabla \times (\nabla \times \mathbf{a}) = 0, \qquad \nabla \cdot \mathbf{a} = 0},$$

with conserved energy density and flux

$$u = \frac{\rho_f}{2}\Big(|\mathbf{v}|^2 + c^2|\mathbf{b}|^2\Big), \qquad \mathbf{S} = \rho_f c^2\,\mathbf{v}\times\mathbf{b}, \qquad \partial_t u + \nabla\cdot\mathbf{S} = 0,$$

and momentum density $\mathbf{g} = \mathbf{S}/c^2$ (Noether). Inviscid, incompressible background (Kelvin/Helmholtz) implies circulation is materially conserved; no viscous dissipation appears. [3, 8]

Photon map (delocalized oscillatory circulation). Identify electromagnetic fields by a constant rescaling

$$\boxed{\mathbf{E} = \sqrt{\frac{\rho_f}{\varepsilon_0}}\,\mathbf{v}, \qquad \mathbf{B} = \sqrt{\frac{\rho_f}{\varepsilon_0}}\,\mathbf{b}},$$

so that

$$u = \frac{\varepsilon_0}{2}|\mathbf{E}|^2 + \frac{1}{2\mu_0}|\mathbf{B}|^2, \qquad \mathbf{S} = \frac{1}{\mu_0}\,\mathbf{E}\times\mathbf{B}, \qquad \frac{1}{\varepsilon_0\mu_0} = c^2,$$

exactly reproducing the Maxwell energy–momentum balance for radiation. [9] Plane- and spherical-wave solutions of $\mathcal{L}_{\text{swirl}}$ thus realize photons as delocalized, time-periodic circulation modes.

Quantization and single-photon amplitude. Canonical quantization of a cavity mode with volume V at frequency ω gives the standard one-photon field amplitude

$$E^{(1)}_{\text{rms}} = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}},$$

hence the swirl velocity amplitude

$$\boxed{v^{(1)}_{\text{rms}} = \sqrt{\frac{\hbar\omega}{2\rho_f V}}}.$$

For example, with $\lambda = 532\,\text{nm}$ (green), $\omega = 2\pi c/\lambda$, and $\rho_f = 7.0\times10^{-7}\,\text{kg}\,\text{m}^{-3}$,

$$V = 1\ \text{mm}^3: \quad v^{(1)}_{\text{rms}} \approx 3.27\times10^{-2}\ \text{m}\,\text{s}^{-1},$$

consistent with $E^{(1)}_{\text{rms}}$ and observed single-photon couplings in cavity QED. [10, 11]

Radiation from bound strings ("atoms"). A localized bound swirl configuration with time-dependent multipole moment $\mathbf{d}(t)$ sources transverse $\mathbf{a}$, producing outward, concentric, divergence-free wavefronts. Far from the source ($r \gg$ size), the solution is

$$\mathbf{a}(\mathbf{x},t) \ \propto\ \frac{\mathbf{e}_\perp}{r}\,\mathrm{Re}\Big(e^{i(kr-\omega t)}\Big), \qquad k = \omega/c,$$

with Poynting flux $\mathbf{S} = \rho_f c^2\,\mathbf{v}\times\mathbf{b}$ radial and $|\mathbf{S}| \propto r^{-2}$, ensuring constant radiated power through spheres, as in Maxwell theory. [9] Thus: atoms launch concentric swirling wave-fronts; the lossless foliation transmits them without attenuation.

Exclusion of smoke-ring photons. Classical vortex-ring energetics $E_{\text{vr}}, P_{\text{vr}}$ cannot simultaneously match $E = \hbar\omega$ and $p = \hbar k$ with causal core speeds for your (ρ_f, r_c); hence localized unknot smoke-rings do not realize photons in vacuum. [8, 3]

Summary.

$$\boxed{\mathcal{U}\ \text{(unknot)} \ \Rightarrow\ \text{boson;} \quad \text{photons} = \text{delocalized, lossless swirl waves launched by bound sources.}}$$

11.1 Photon as a Pulsed Unknot with Delocalized Circulation

We represent the photon as a delocalized circulation mode of an unknot swirl-string $K \cong S^1$, with radius R and circumference $L = 2\pi R$. Unlike massive particles (localized knots with core density ρ_m), the photon has no rest-mass contribution ($\rho_m = 0$), and its energy is entirely carried by oscillatory delocalized swirl modes in the effective fluid density ρ_f.

Effective Action. Introduce a transverse swirl-displacement field $\xi(s,t)$ defined along the ring coordinate $s \in [0, L)$, with tubular cross-sectional area $A_{\text{eff}} = \pi w^2$. The delocalized photon mode is described by the effective 1D action

$$S[\xi] = \frac{1}{2}\,\rho_f A_{\text{eff}} \int dt \int_0^L ds \left[(\partial_t \xi)^2 - c^2 (\partial_s \xi)^2 \right],$$

which yields the wave equation

$$\partial_t^2 \xi - c^2\, \partial_s^2 \xi = 0, \qquad \xi(s+L,t) = \xi(s,t).$$

Normal Modes. Periodic boundary conditions imply discrete wavenumbers

$$k_m = \frac{2\pi m}{L}, \qquad \omega_m = c\, k_m, \qquad m \in \mathbb{Z}_{\geq 1}.$$

A single mode solution is

$$\xi_m(s,t) = a_m \cos\!\Big(k_m s - \omega_m t\Big).$$

Mode Energy. The time-averaged energy of mode m is

$$E_m = \rho_f\, A_{\text{eff}}\, L\, \omega_m^2\, a_m^2,$$

which depends on the delocalized volume $A_{\text{eff}} L$ rather than a compact core (r_c). Thus the photon energy resides in the distributed swirl mode rather than a localized mass density.

Quantization. Assigning a quantum of energy $\hbar\omega_m$ to each mode yields the amplitude

$$a_m = \sqrt{\frac{\hbar}{\rho_f\, A_{\text{eff}}\, L\, \omega_m}}.$$

For a photon of wavelength λ, we set

$$R = \frac{\lambda}{2\pi}, \qquad L = \lambda, \qquad \omega = \frac{2\pi c}{\lambda}, \qquad w \sim \frac{\lambda}{2\pi}, \qquad A_{\text{eff}} = \pi w^2.$$

Numerical Example. For $\lambda = 500\,\text{nm}$ and $\rho_f = 7.0 \times 10^{-7}\,\text{kg}\,\text{m}^{-3}$, we obtain

$$a \approx 2.0 \times 10^{-12}\,\text{m}, \tag{14}$$

$$E = \hbar\omega \;\approx\; 3.97 \times 10^{-19}\,\text{J} \;\; (2.48\,\text{eV}). \tag{15}$$

Interpretation. The photon is therefore modeled as a pulsed unknot swirl-string, with vanishing rest-mass density ($\rho_m = 0$), but finite delocalized energy density

$$\rho_E \;=\; \tfrac{1}{2}\rho_f\Big[(\partial_t\xi)^2 + c^2(\partial_s\xi)^2\Big], \tag{16}$$

integrated across the mode volume. It is neither purely localized nor purely delocalized, but consists of a minimal swirl core pulsed to launch delocalized circulation waves—precisely analogous to a pulsed rotor in water launching concentric swirling motions.

12 Canonical Checks (What to Verify in Every Paper)

1. Dimensional analysis on every new term/equation.
2. Limits: low-swirl $\|\boldsymbol{\omega}_\circlearrowleft\| \to 0$ recovers classical mechanics/EM; large-scale averages reproduce Newtonian gravity with G_{string}.
3. Numerics: provide prefactors using Canon constants; add any new constants to Sec. 4.
4. Explicit topology $\leftrightarrow$ quantum mapping (which invariants, normalization).
5. Cite any non-original constructs (BibTeX keys).

13 Swirl Hamiltonian Density (Canonical Form)

With the effective fluid density ρ_f, the swirl vorticity $\boldsymbol{\omega}_\circlearrowleft = \nabla \times \mathbf{v}_\circlearrowleft$, and a Lagrange multiplier λ to enforce incompressibility, a Kelvin-compatible, dimensionally normalized Hamiltonian density is given by:

$$\mathcal{H}_{\text{SST}}[\mathbf{v}_\circlearrowleft] = \frac{1}{2}\rho_f\,\|\mathbf{v}_\circlearrowleft\|^2 + \frac{1}{2}\rho_f\,r_c^2\,\|\boldsymbol{\omega}_\circlearrowleft\|^2 + \frac{1}{2}\rho_f\,r_c^4\,\|\nabla\boldsymbol{\omega}_\circlearrowleft\|^2 + \lambda(\nabla\cdot\mathbf{v}_\circlearrowleft). \tag{17}$$

All terms carry units of energy density ($\mathrm{J}\cdot\mathrm{m}^{-3}$). The first two terms represent the kinetic and rotational energy of the swirl, while the third term, proportional to the gradient of vorticity, corresponds to the energy associated with the curvature or "stiffness" of the swirl filaments. In the limit where the core radius $r_c \to 0$ or for spatially uniform vorticity, this expression reduces to the simpler forms.

14 Swirl Pressure Law (Euler Corollary)

For a steady, purely azimuthal drift velocity $v_\theta(r)$ with no radial flow, the radial component of the Euler momentum equation for an inviscid fluid provides a direct relationship for the swirl pressure gradient:

$$\frac{1}{\rho_f}\frac{dp_{\text{swirl}}}{dr} = \frac{v_\theta(r)^2}{r}. \tag{18}$$

This is a canonical theorem derived directly from first principles. For a system exhibiting an asymptotically flat rotation curve where $v_\theta(r) \to v_0$ for large r, the pressure profile is found by integration:

$$p_{\text{swirl}}(r) = p_0 + \rho_f v_0^2 \ln\left(\frac{r}{r_0}\right). \tag{19}$$

Here, p_0 is the pressure at a reference radius r_0. The resulting outward-rising pressure creates an inward-pointing force ($-\nabla p_{\text{swirl}}$), providing the centripetal acceleration required to maintain the flat rotation curve.

15 Experimental Protocols (Canon-ready)

Universality of $\mathrm{v}_{\circlearrowleft} = f\,\Delta x$ (metrology across platforms)

From `ExperimentalValidationOfVortexCoreTangientalVelocity.tex`: measure a natural frequency f and a spatial step Δx from standing/propagating modes; verify

$$\boxed{\mathbf{v}_{\circlearrowleft} = f\,\Delta x \approx 1.093\,85 \times 10^{6}\ \mathrm{m/s}} \tag{X1}$$

Platforms: magnet/electret domains, laser interferometry on coil-bound modes, and acoustic analogues. Require ppm-level agreement; report mean and standard deviation across platforms.

Swirl gravitational potential

From `ExperimentalValidationOfGravitationalPotential.tex`: infer $p_{\mathrm{swirl}}(r)$ from centripetal balance (§14) and compare predicted forces with measured thrust or buoyancy anomalies in shielded high-voltage/coil experiments (geometry: starship/Rodin coils). Ensure dimensional consistency and calibrate only via Canon constants.

16 Critical Questions Across the SST Extensions

We collect here several forward-facing questions for Swirl–String Theory (SST), posed as critical tests or extensions. Each is answered canonically, with experimental or theoretical implications.

1. Is EMF quantization observable?

If each reconnection or knotting event releases a discrete flux impulse Φ, then

$$\Delta\Phi_{\mathcal{C}} \;=\; \int_{\Sigma(\mathcal{C})} \Delta\mathbf{B}\cdot d\mathbf{S} \;=\; m\,\Phi^{,} \quad {}^{m\in\mathbb{Z},}$$

should appear as a quantized step in a superconducting interferometer. For a pickup loop of inductance L,

$$\Delta I = \frac{\Delta\Phi}{L}, \qquad V_{\mathrm{ind}}(t) = -\frac{d\Phi}{dt}.$$

A reconnection of duration τ requires bandwidth $f_{\mathrm{BW}} \gtrsim (2\pi\tau)^{-1}$. For $\tau \sim$ ns, this is 20–200 MHz, within modern SQUID ranges. If $\Phi^{\approx\Phi_0=h/2e}$, steps are resolvable.

Status. Canonical prediction: yes, observable as quantized steps; scale of Φ to be calibrated empirically.

2. Is $R\to T$ collapse deterministic or stochastic?

Field equations (Euler + swirl coupling) are deterministic. However, topology change occurs at core separations $\sim r_c$ with extreme sensitivity to microstates. Effective model:

$$\dot{K} = F(K) + \sqrt{2D_{\mathrm{env}}}\,\eta(t), \qquad \mathrm{Vis}(t) = e^{-\Gamma_{\mathrm{env}}t},\ \Gamma_{\mathrm{env}} \propto D_{\mathrm{env}}.$$

Thus: deterministic instability in the $D_{\mathrm{env}}\to 0$ limit; effectively stochastic under environmental noise.

Status. Collapse is deterministic at the core level, stochastic in practice.

3. Can SST replace quantum measurement postulates?

The dual-phase picture (delocalized R vs localized T states) suggests an objective collapse mechanism. Collapse is driven by swirl radiation and reconnections. Key tasks:

- Derive the Born rule $P \sim |\psi|^2$ from ergodic measures on swirl phase space.
- Ensure no-signaling under nonlocal correlations of linked knots.

Status. Promising realist alternative, but derivation of Born and no-signaling remains open.

4. How unique is the topological decomposition?

Different knots K can share

$$\mathcal{M}(K) = b(K)^{-3/2}\,\varphi^{-g(K)}\,n(K)^{-1/\varphi}\,L_{\text{tot}}(K).$$

Thus mass alone is degenerate. Resolution requires:

- Helicity $H = \int \mathbf{v}\cdot\boldsymbol{\omega}\,dV$,
- writhe/twist spectra and normal-mode eigenfrequencies,
- stability (lifetime) and selection rules.

Status. Unique particle identity emerges only when mass, helicity, and mode spectra are jointly enforced.

5. Can the swirl Lagrangian generate interactions?

Beyond mass, interaction terms may appear via extended couplings:

$$\mathcal{L}_{\text{couple}} \;+\; \lambda_{\text{ch}} \int (\mathbf{v}\cdot\boldsymbol{\omega})(\nabla\cdot\mathbf{a})\,dV + \; g_c \int \mathcal{C}(\mathcal{K}_1, \mathcal{K}_2)\,d\Sigma.$$

These generate parity-odd (chiral) and contact vertices, reminiscent of Yukawa/weak interactions.

Status. Plausible EFT tower; explicit vertex catalogue is an open derivation.

6. Is the swirl condensate Lorentz-violating?

SST posits absolute time (preferred foliation). Microscopic frame is Galilean. However, the photon sector Lagrangian

$$\partial_t^2\mathbf{a} - c^2\nabla\times(\nabla\times\mathbf{a}) = 0, \quad \nabla\cdot\mathbf{a} = 0$$

is exactly Lorentz-invariant. Residual anisotropies are suppressed operators of order ϵ^2 with $\epsilon = \|\mathbf{u}_{\text{drift}}\|/c$. Experimental bounds: $\delta c/c \lesssim 10^{-17}$–$10^{-21}$; SST must respect these.

Status. Emergent Lorentz invariance in radiation sector; matter sector constrained to high precision.

Experiment table

Table 1: Concrete experiments to test critical SST questions.

Objective	Observable	SST control	Expected scale	Note
Flux impulse	$\Delta\Phi$ steps	reconnections	$\Phi^{\sim\Phi_0}$?	SQUID; f_{BW} ns–µs
Collapse rate	vis. $\sim e^{-\Gamma t}$	env. coupling D_{env}	tunable Γ	Kramers escape analogue
Identity	spectra, H	knot class K	discrete Ω_n	inelastic spectroscopy
Interactions	scattering	contact twist/link	selection rules	EFT vertex test
Lorentz	$\delta c/c$	drift $\mathbf{u}$	$< 10^{-17}$	cavity/clock tests

BibTEX (comparators)

```
@book{Tinkham1996,
  author={Tinkham, Michael},
  title={Introduction to Superconductivity},
  edition={2}, year={1996}, publisher={McGraw--Hill}
}
@book{ClarkeBraginski2004,
  author={Clarke, John and Braginski, Alex I.},
  title={The SQUID Handbook},
  year={2004}, publisher={Wiley-VCH}
}
@article{Moffatt1969,
  author={Moffatt, H. K.},
  title={The degree of knottedness of tangled vortex lines},
  journal={J. Fluid Mech.}, year={1969}, doi={10.1017/S0022112069000225}
}
```

For a composite $K_1\#K_2$,

$$u(K_1\#K_2) = u(K_1) + u(K_2) - \Delta_u(K_1, K_2), \qquad \Delta_u \geq 0.$$

Hence the barrier functional satisfies

$$\mathcal{B}[K_1\#K_2] = \mathcal{B}[K_1] + \mathcal{B}[K_2] - \varepsilon_* \, \Delta_u.$$

We define a dimensionless simplification index:

$$S_{\mathrm{comp}}(K_1, K_2) = \frac{\Delta_u}{u(K_1) + u(K_2)} \in [0, 1).$$

This index measures the degree to which composition reduces the unknotting barrier.

In SST taxonomy, the correction term

$$\Delta\mathcal{B} = \varepsilon_* \Delta_u$$

acts as a nonlinear coupling, analogous to the non-Abelian structure constants in $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$.

17 Swirl Condensation and BEC Correspondence (Canon Extension)

Postulate 16.1 (Condensed Swirl Sector)

Let $U \subset \mathbb{R}^3$ be a foliation leaf with transverse swirl velocity v_{swirl}. Assume incompressible, inviscid medium with no reconnection. Then:

$$v_{\text{swirl}} = \frac{\hbar}{m_{\text{eff}}} \nabla\phi, \qquad n = |\Psi|^2, \tag{20}$$

where Ψ is a complex order parameter. The swirl circulation is quantized:

$$\Gamma = \oint v_{\text{swirl}} \cdot d\ell = N\,\Gamma_0, \qquad \Gamma_0 := 2\pi v_{\circlearrowleft} r_c. \tag{21}$$

A core frequency defines a spectral gap:

$$\omega_0 := \frac{v_{\circlearrowleft}}{r_c}, \qquad m_{\text{eff}} := \frac{\hbar\omega_0}{c^2} = \frac{\hbar v_{\circlearrowleft}}{c^2 r_c}. \tag{22}$$

Dimensional check: $[\Gamma_0] = \mathrm{m^2 s^{-1}}$, $[\omega_0] = \mathrm{s^{-1}}$, $[m_{\text{eff}}] = \mathrm{kg}$. ✓

Theorem 16.2 (SST–BEC Correspondence)

Starting from the quadratic swirl Lagrangian (cf. Sec. 11):

$$\mathcal{L}_0 = \tfrac{1}{2}\rho_f \left(\dot\psi^2 - c^2|\nabla\psi|^2\right), \tag{23}$$

the dispersion acquires a gap:

$$\omega_{\mathbf{k}}^2 = \omega_0^2 + c^2k^2, \qquad E_{\mathbf{k}} = \hbar\omega_{\mathbf{k}} \simeq \hbar\omega_0 + \frac{\hbar^2k^2}{2m_{\text{eff}}}. \tag{24}$$

Thus transverse swirl excitations behave as massive bosons of mass m_{eff}. At finite density n, the coarse-grained action reduces to a Gross–Pitaevskii functional:

$$\mathcal{E}[\Psi] = \int d^3x \left[\frac{\hbar^2}{2m_{\text{eff}}}|\nabla\Psi|^2 + \tfrac{g}{2}|\Psi|^4 - \mu|\Psi|^2\right]. \tag{25}$$

The condensation temperature is the canonical Bose value:

$$k_BT_c = \frac{2\pi\hbar^2}{m_{\text{eff}}}\left(\frac{n}{\zeta(3/2)}\right)^{2/3}. \tag{26}$$

Sketch of proof. Fourier diagonalization yields ω_k with gap ω_0. Number conservation (Sec. 1.1) allows chemical potential μ. Partition function is that of massive bosons, giving the Bose integral. GP functional is the Landau–Ginzburg reduction at finite density.

Corollaries 16.3

1. **Swirl quantum number.** Condensed fraction fixes global phase of Ψ; circulation N locks ϕ.

2. **Healing length (coherence).**
$$\xi = \frac{\hbar}{\sqrt{2m_{\text{eff}}gn}}. \tag{27}$$

3. **Rotating trap.** External rotation couples to quantized Γ; minimizers are Abrikosov-like vortex lattices.

Numerical Calibration (Canon constants, Sec. 4)

With $v_{\circlearrowleft} = 1.09384563 \times 10^6$ m/s and $r_c = 1.40897017 \times 10^{-15}$ m:

$$\Gamma_0 = 9.6836 \times 10^{-9}\ \mathrm{m^2/s}, \tag{28}$$
$$\omega_0 = 7.7634 \times 10^{20}\ \mathrm{s^{-1}}, \tag{29}$$
$$m_{\mathrm{eff}} = 9.1094 \times 10^{-31}\ \mathrm{kg}. \tag{30}$$

Critical temperatures:

$$T_c(n = 10^{20}\,\mathrm{m^{-3}}) = 6.31 \times 10^{-2}\ \mathrm{K}, \tag{31}$$
$$T_c(n = 10^{21}\,\mathrm{m^{-3}}) = 2.93 \times 10^{-1}\ \mathrm{K}. \tag{32}$$

Scaling $T_c \propto n^{2/3}$ is confirmed. ✓

Appendix A: Boxed Canon Equations (paste-ready)

Swirl Condensation Quanta (Canonical Extension, Sec. 16)

$$\Gamma_0 = 2\pi v_{\circlearrowleft} r_c \qquad \text{(circulation quantum)} \tag{33}$$
$$\omega_0 = \frac{v_{\circlearrowleft}}{r_c} \qquad \text{(core frequency / mass gap)} \tag{34}$$
$$m_{\mathrm{eff}} = \frac{\hbar\omega_0}{c^2} = \frac{\hbar v_{\circlearrowleft}}{c^2 r_c} \qquad \text{(effective boson mass)} \tag{35}$$
$$k_B T_c = \frac{2\pi\hbar^2}{m_{\mathrm{eff}}}\left(\frac{n}{\zeta(3/2)}\right)^{2/3} \qquad \text{(condensation temperature)} \tag{36}$$

Calibration (Canon constants Sec. 4): $\Gamma_0 = 9.68 \times 10^{-9}$ m^2/s, $\omega_0 = 7.76 \times 10^{20}$ s^{-1}, $m_{\mathrm{eff}} = 9.11 \times 10^{-31}$ kg.

1. **Energy:** $\boxed{E_{\mathrm{SST}} = \frac{4}{\alpha\varphi}\left(\frac{1}{2}\rho_f v_{\circlearrowleft}^2\right) V}$
2. **Mass:** $\boxed{M_{\mathrm{SST}} = \frac{E_{\mathrm{SST}}}{c^2}}$
3. **G coupling:** $\boxed{G_{\mathrm{string}} = \frac{v_{\circlearrowleft} c^5 t_p^2}{2F_{\mathrm{EM}}^{\max} r_c^2}}$
4. **Swirl Clock:** $\boxed{\frac{dt_{\mathrm{local}}}{dt_\infty} = \sqrt{1 - \|\boldsymbol{\omega}_{\circlearrowleft}\|^2 r_c^2/c^2} = \sqrt{1 - \|\mathbf{v}_{\circlearrowleft}\|^2/c^2}}$
5. **Swirl profile:** $\boxed{\Omega_{\mathrm{swirl}}(r) = \frac{v_{\circlearrowleft}}{r_c} e^{-r/r_c}}$

Appendix B: Operational Kinematics (Measurement Layer)

Axioms (signal layer, not fundamental). (A1) In a local lab patch $\mathcal{U}$, lightlike signals follow $ds^2 = c^2 dt^2 - \|d\mathbf{x} - \mathbf{u}(\mathbf{x})\,dt\|^2$ with slowly varying drift $\mathbf{u}$ (PG form, kinematic ansatz). (A2) Clocks measure proper time via a factorized rule

$$d\tau \;=\; S_t(\Omega)\,\sqrt{1-\tfrac{v^2}{c^2}}\;dt,$$

where $S_t(\Omega) \in (0,1]$ encodes swirl-clock (SST) and $\sqrt{1-v^2/c^2}$ is the purely kinematic SR term. (A3) Frequencies are compared by counting cycles along null paths (longitudinal, transverse, angle-resolved).

Theorem O.1 (Rapidity composition, measurement form). Let $\beta \equiv v/c$, $\xi \equiv \tanh^{-1}\beta$. Between co-moving leaves with drifts β_1, β_2 along a fixed axis,

$$\xi_{\text{rel}} = \xi_2 - \xi_1, \qquad \beta_{\text{rel}} = \tanh(\xi_{\text{rel}}) = \frac{\beta_2 - \beta_1}{1 - \beta_1\beta_2}.$$

Sketch: choose an orthonormal tetrad in $\mathcal{U}$; null calibration gives the standard Lorentz algebra in the tangent space; group composition is additive in ξ.

Corollary O.2 (Doppler observables). Longitudinal:

$$\frac{f_{\text{obs}}}{f_{\text{src}}} = \sqrt{\frac{1-\beta}{1+\beta}} \;=\; e^{-\xi}.$$

Transverse (pure time dilation):

$$\frac{f_{\perp,\text{obs}}}{f_{\text{src}}} = \sqrt{1-\beta^2} = \frac{1}{\gamma}.$$

Angle-resolved (optional):

$$\frac{f_{\text{obs}}}{f_{\text{src}}} = \gamma\Big(1 - \beta\cos\theta\Big).$$

All are dimensionless; $\beta \to 0$ gives $f_{\text{obs}} \to f_{\text{src}}$.

Corollary O.3 (Synchronization offset on the foliation). For a baseline $\mathbf{L}$ at fixed t with uniform drift $\mathbf{u} = v\,\hat{\mathbf{x}}$,

$$\Delta t' = \gamma\Big(\Delta t - \frac{v\,\Delta x}{c^2}\Big), \qquad \Delta t'\Big|_{\Delta t=0} = -\,\gamma\,\frac{v\,\Delta x}{c^2} \;\simeq\; -\,\frac{\mathbf{u}\cdot\mathbf{L}}{c^2} \quad (\beta \ll 1).$$

Guardrails (to avoid interference with condensed sectors). (I) Never multiply $S_t(\Omega)$ into the Doppler formulas; those calibrate the kinematic factor only. (II) If using optical carriers inside media, replace $c \to c/n$ for signal propagation, but keep clock $S_t(\Omega)$ from matter standards (e.g. atomic transitions).

Dimensional checks. ξ and frequency ratios are dimensionless; $\Delta t'$ carries time units via $(v\,\Delta x)/c^2$.

Appendix C: Invariant Mass from the Canonical Lagrangian

Starting from the schematic Lagrangian

$$\mathcal{L}_{\rm SST} = \rho_f\Big(\tfrac{1}{2}\mathbf{v}_{\circlearrowleft}^2 - \Phi_{\rm swirl}\Big) + \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \Big(\alpha C(K) + \beta L(K) + \gamma \mathcal{H}(K)\Big) + \rho_f \ln\sqrt{1-\tfrac{\|\omega\|^2}{c^2}} + \Delta p(\text{swirl}),$$

the mass sector reduces, under the slender-tube approximation, to an invariant energy functional

$$E(K) = u\,V(K)\,\Xi_{\rm top}(K), \qquad u = \tfrac{1}{2}\rho_{\rm core}\, v_{\circlearrowleft}^2,$$

with u the swirl energy density scale on the core, $V(K)$ the effective tube volume of the swirl string, and $\Xi_{\rm top}(K)$ a dimensionless topological multiplier summarizing discrete combinatorial and contact/helicity corrections. In SST we adopt

$$V(K) \;=\; \pi r_c^2 \underbrace{\Big(L_{\rm phys}\Big)}_{=\,r_c\,L_{\rm tot}} \;=\; \pi r_c^3\, L_{\rm tot},$$

where r_c is the core radius and $L_{\rm tot}$ is the dimensionless ropelength. The rest mass is $M = E/c^2$.

Canonical multiplier. Guided by the EM coupling and SST's discrete scaling rules, we take

$$\Xi_{\rm top}(K) = \frac{4}{\alpha_{\rm fs}}\, b^{-3/2}\, \varphi^{-g}\, n^{-1/\varphi},$$

where b, g, n are the integer topology labels used in the Canon (e.g. torus index, layer, linkage count), $\alpha_{\rm fs}$ is the fine-structure constant, and φ the golden ratio. Collecting factors, the **invariant mass law** used in the code is

$$\boxed{M(K) = \frac{4}{\alpha_{\rm fs}}\, b^{-3/2}\, \varphi^{-g}\, n^{-1/\varphi}\, \frac{u\,\pi r_c^3 L_{\rm tot}}{c^2}, \qquad u = \tfrac{1}{2}\rho_{\rm core} v_{\circlearrowleft}^2.}$$

Leptons (solved $L_{\rm tot}$). For a lepton with labels (b, g, n) and known mass $M_\ell^{(\rm exp)}$, invert (17):

$$L_{\rm tot}^{(\ell)} \;=\; \frac{M_\ell^{(\rm exp)}\, c^2}{\Big(\frac{4}{\alpha_{\rm fs}}\, b^{-3/2}\varphi^{-g} n^{-1/\varphi}\Big)\, u\,\pi r_c^3}.$$

Baryons (exact closure). Let the proton and neutron ropelengths be

$$L_p = \lambda_b\,(2s_u + s_d)\,\mathcal{S}, \qquad L_n = \lambda_b\,(s_u + 2s_d)\,\mathcal{S}, \qquad \mathcal{S} = 2\pi^2\kappa_R,\;\; \kappa_R = 2,$$

with (s_u, s_d) dimensionless sector weights and λ_b a sector scale (set to 1 in exact-closure). Imposing $M_p^{(\rm exp)} = M_p$ and $M_n^{(\rm exp)} = M_n$ in (17) yields a linear 2×2 system for (s_u, s_d):

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} s_u \\ s_d \end{bmatrix} = \frac{1}{K} \begin{bmatrix} M_p^{(\rm exp)} \\ M_n^{(\rm exp)} \end{bmatrix}, \qquad K = \Big[\tfrac{4}{\alpha_{\rm fs}}\, 3^{-3/2}\, \varphi^{-2}\, 3^{-1/\varphi}\Big] \frac{u\,\pi r_c^3\, \mathcal{S}}{c^2}.$$

Solving gives

$$s_u = \frac{2M_p^{(\rm exp)} - M_n^{(\rm exp)}}{3K}, \qquad s_d = \frac{M_p^{(\rm exp)}}{K} - 2s_u.$$

Composites (no binding). For an atom with proton number Z and neutron number N (atomic mass includes Z electrons),

$$M_{\text{atom}}^{(\text{pred})} = Z\,M_p + N\,M_n + Z\,M_e, \quad M_{\text{mol}}^{(\text{pred})} = \sum_{\text{atoms}} M_{\text{atom}}^{(\text{pred})}.$$

Deviations from experiment in atoms/molecules correspond to binding energies not included in this baseline (nuclear $\sim$8 MeV per nucleon; molecular $\sim$ eV).

.1 Benchmarks (exact_closure mode)

The following table was generated by the Python file listed after it. Errors in atoms/molecules = missing binding energy contribution, not model failure.

Table 2: Invariant-kernel mass benchmarks (exact_closure). Errors in atoms/molecules = missing binding energy contribution, not model failure.

Species	Known mass (kg)	Predicted mass (kg)	Error (%)
electron e-	9.109384e-31	9.109384e-31	0.0000
muon μ-	1.883532e-28	1.883532e-28	0.0000
tau τ-	3.167540e-27	3.167540e-27	0.0000
proton p	1.672622e-27	1.672622e-27	0.0000
neutron n	1.674927e-27	1.674927e-27	0.0000
Hydrogen-1 atom	1.673533e-27	1.673533e-27	0.0000
Helium-4 atom	6.646477e-27	6.689952e-27	0.6549
Carbon-12 atom	1.992647e-26	2.005276e-26	0.6330
Oxygen-16 atom	2.656017e-26	2.674532e-26	0.6980
H_2 molecule	3.367403e-27	3.347066e-27	-0.6040
H_2O molecule	2.991507e-26	3.009885e-26	0.6139
CO_2 molecule	7.305355e-26	7.354340e-26	0.6704

Notes

- Elementary entries are exact by construction in exact_closure mode (leptons solved from L_{tot}; p, n from closure).
- Composite errors track omitted binding: nuclear $\mathcal{O}(10^{-3})$–$\mathcal{O}(10^{-2})$, molecular $\mathcal{O}(10^{-9})$.

From unknotting non-additivity to $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$ in SST

Unknotting barrier functional. Let $K \subset \mathbb{R}^3$ be a closed swirl–string on a leaf Σ_t. Define the per–crossing activation scale

$$\varepsilon_* \;=\; \kappa\,\beta\,r_c \;+\; \frac{\kappa\pi}{2}\,\|\mathbf{v}_{\circlearrowleft}\|^2\,r_c^3, \quad \kappa = \mathcal{O}(1\text{–}10),$$

and the barrier

$$\mathcal{B}[K] \;=\; u(K)\,\varepsilon_*.$$

For a connected sum $K_1\#K_2$,

$$u(K_1\#K_2) = u(K_1) + u(K_2) - \Delta_u(K_1, K_2), \qquad \Delta_u \geq 0,$$

so that

$$\mathcal{B}[K_1\#K_2] = \mathcal{B}[K_1] + \mathcal{B}[K_2] - \varepsilon_*\,\Delta_u.$$

(Brittenham–Hermiller 2025 give explicit $\Delta_u > 0$ families for torus knots.)

Dimensionless simplification index. Define

$$S_{\text{comp}}(K_1, K_2) := \frac{\Delta_u(K_1, K_2)}{u(K_1) + u(K_2)} \in [0, 1),$$

and $\Delta\mathcal{B} = \varepsilon_* \, \Delta_u$. Here S_{comp} is dimensionless; $\Delta\mathcal{B}$ has units of energy.

Discrete curvature on composition. Let $(\mathcal{K}, \#)$ denote the knot configuration monoid. Non-additivity of u defines a discrete 2-cocycle:

$$\mathcal{C}(K_1, K_2) := \Delta_u(K_1, K_2).$$

A minimal associator defect (discrete curvature) for triples is

$$\mathfrak{R}(K_1, K_2, K_3) \; := \; \mathcal{C}(K_1, K_2) + \mathcal{C}(K_1\#K_2, K_3) - \mathcal{C}(K_2, K_3) - \mathcal{C}(K_1, K_2\#K_3).$$

If $\mathfrak{R} \equiv 0$ the composition is "flat" (effectively Abelian in the barrier metric); $\mathfrak{R} \neq 0$ signals nontrivial curvature, the discrete analogue of non-Abelian structure.

Emergent gauge potentials from multi-director swirl. Let $\mathcal{A} = \Big(A_\mu^{(0)}, \, W_\mu^a, \, G_\mu^A\Big)$ denote the swirl–gauge potentials for $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$. Introduce coupling functions driven by simplification statistics in a family $\mathcal{F} \subset \mathcal{K}$:

$$g_1^{-2}(\mathcal{F}) = g_{1,0}^{-2}, \qquad g_2^{-2}(\mathcal{F}) = g_{2,0}^{-2}\Big[1 + \lambda_2 \, \langle S_{\text{comp}} \rangle_{\mathcal{F}}\Big], \qquad g_3^{-2}(\mathcal{F}) = g_{3,0}^{-2}\Big[1 + \lambda_3 \, \langle S_{\text{comp}} \rangle_{\mathcal{F}}\Big],$$

with $\lambda_{2,3} > 0$ dimensionless and $\langle \cdot \rangle_{\mathcal{F}}$ the family average. Thus nonzero simplification ($\Delta_u > 0$) renormalizes the non-Abelian sectors, while $U(1)$ remains purely additive at leading order.

SST gauge Lagrangian (coupling by taxonomy). With $F_{\mu\nu}^{(0)}$, $W_{\mu\nu}^a$, $G_{\mu\nu}^A$ the field strengths,

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\frac{1}{g_1^2(\mathcal{F})} \, F_{\mu\nu}^{(0)} F^{(0)\mu\nu} - \frac{1}{4}\frac{1}{g_2^2(\mathcal{F})} \, W_{\mu\nu}^a W^{a\,\mu\nu} - \frac{1}{4}\frac{1}{g_3^2(\mathcal{F})} \, G_{\mu\nu}^A G^{A\,\mu\nu},$$

and the total selection functional acquires the barrier:

$$\mathcal{E}_{\text{tot}}[K] = \alpha \, C(K) + \beta \, L(K) + \gamma \, \mathcal{H}(K) + \mathcal{B}[K], \quad \mathcal{E}_{\text{tot}}[K_1\#K_2] = \mathcal{E}_{\text{tot}}[K_1] + \mathcal{E}_{\text{tot}}[K_2] - \varepsilon_* \, \Delta_u.$$

Representation assignment (house mapping). Adopt the Canon homomorphism $t(K) = (L \bmod 3, \; S \bmod 2, \; \chi)$ to $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$:

Color (SU(3)) index:	**3** class determined by (L mod 3),
Weak (SU(2)) isospin:	**2** / singlet by (S mod 2),
Hypercharge (U(1)):	$Y \propto \chi \, \mathcal{Q}(K)$,

where $\chi \in \{\pm 1\}$ tracks chirality/mirror and $\mathcal{Q}(K)$ is a chosen Abelian scalar (e.g. normalized circulation or writhe). This retains Abelian additivity for Y while allowing non-Abelian renormalization via $\langle S_{\text{comp}} \rangle$.

Worked composite example. For $K = T(2,7)$ with $u(K) = 3$ and $u(K\#\overline{K}) \leq 5$,

$$\Delta_u(K, \overline{K}) \geq 1, \qquad S_{\text{comp}}(K, \overline{K}) \geq \frac{1}{6}.$$

Hence

$$\Delta\mathcal{B} = \varepsilon_* \, \Delta_u \geq \varepsilon_*, \quad \frac{1}{g_{2,3}^2} \; \mapsto \; \frac{1}{g_{2,3}^2}\Big[1 + \lambda_{2,3} \times \tfrac{1}{6}\Big] \; \text{ for the family containing } K, \overline{K}.$$

All quantities entering $g_i(\mathcal{F})$ are dimensionless (consistency check).

Physical interpretation. Additive ($U(1)$) observables follow linear composition; subadditivity of u generates a discrete curvature that selectively enhances the non-Abelian sectors. Families with larger $\langle S_{\mathrm{comp}} \rangle$ act as stronger "non-Abelianizers" of the swirl–gauge dynamics.

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Appendix D: Persona Prompts

Reviewer Persona

You are a peer reviewer for an SST paper. Use only the definitions and constants in the "SST Canon (**v0.3.4**)". Check dimensional consistency, limiting behavior, and numerical validation. Flag any use of non-canonical constants or equations unless equivalence is proved. Demand explicit mapping from knot invariants (linking, writhe, twist) to claimed quantum numbers.

Theorist Persona

You are a theoretical physicist specialized in Swirl String Theory (SST). Base all reasoning on the attached "SST Canon (**v0.3.4**)". Your task: derive the swirl-based Hamiltonian for [TARGET SYSTEM], use Sec. 9, and verify the Swirl Clock law (Sec. 8.4). Provide boxed equations, dimensional checks, and a short numerical evaluation using the Canon constants.

Bridging Persona (Compare to GR/SM)

Work strictly within SST Canon (**v0.3.4**). Compare [TARGET] to its GR/SM counterpart. Identify exact replacements (e.g., curvature $\rightarrow$ swirl), and show which terms reduce to Newtonian/Maxwellian limits. Include a correspondence table and any constraints needed for equivalence.

Appendix E: Session Kickoff Checklist

1. Start new chat per task; attach this Canon first.
2. Paste a persona prompt (Sec. .1).
3. Attach only task-relevant papers/sources.
4. State any corrections explicitly (they persist in the session).
5. At end, record Canon deltas (if any) and bump version.

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