

# Wave–Particle Duality in Swirl–String Theory: Toroidal Circulation, Knot Collapse, and Photon-Induced Transitions

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## Abstract

We present a Swirl–String Theory (SST) interpretation of the electron’s wave–particle duality based on Canon v0.3.1. The electron is modeled as a swirl-string admitting two phases: an unknotted, delocalized toroidal circulation  $\mathcal{R}$  (wave-like) and a localized, knotted soliton (trefoil)  $\mathcal{T}$  (particle-like). We define an effective energy functional including bulk swirl density, line tension, near-contact interactions, and helicity terms. Photon-induced transitions  $\mathcal{R} \rightleftharpoons \mathcal{T}$  occur at resonant frequencies determined by impedance matching between electromagnetic modes and topological excitations. We show how circulation quantization on  $\mathcal{R}$  recovers de Broglie wave relations, while  $\mathcal{T}$  provides a stable, localized excitation. This framework yields falsifiable predictions in atomic absorption spectra, Rydberg scaling, and polarization-dependent selection rules.

## 1 Introduction

Wave–particle duality remains a central puzzle in quantum mechanics. The hydrodynamic representation of the Schrödinger equation by Madelung [4], the matter-wave hypothesis of de Broglie [7], and Bohm’s hidden-variable interpretation [8, 9] all highlight fluid analogies. In superfluids, Onsager’s circulation quantization [10] and Feynman’s analysis of helium vortices [11] demonstrate how phase coherence enforces quantized flow. Earlier, Helmholtz [12] and Kelvin [13] established the conservation of vorticity and proposed atoms as vortex rings.

Within SST, the Canon postulates a universal swirl condensate with effective density  $\rho_f$  and characteristic swirl velocity  $\|\mathbf{v}_\odot\|$ . Electrons are interpreted not as point particles but as filamentary swirl-strings. We propose here that the dual character of the electron arises from two distinct phases of the same underlying string.

## 2 Two Phases of the Electron String

### 2.1 Unknotted toroidal ring $\mathcal{R}$

The delocalized phase is an unknotted ring circulation  $\mathcal{R}$  with radius  $R$  and length  $L(\mathcal{R}) = 2\pi R$ . Its circulation is quantized:

$$\Gamma_n = \oint_{\mathcal{R}} \mathbf{v} \cdot d\boldsymbol{\ell} = n \frac{h}{m_e}, \quad n \in \mathbb{Z}. \quad (1)$$

Hence the tangential speed is

$$v_\theta(R) = \frac{\Gamma_n}{2\pi R}, \quad (2)$$

and the phase around the ring is  $e^{in\theta}$ , yielding the de Broglie relation

$$\lambda_{ring} = \frac{2\pi R}{n} = \frac{h}{p_\theta}, \quad p_\theta = m_e v_\theta. \quad (3)$$

Thus  $\mathcal{R}$  naturally supports interference and standing-wave behavior.

## 2.2 Knotted trefoil soliton

The localized phase  $\mathcal{T}$  is a knotted filament (e.g. trefoil  $3_1$ ) with enhanced curvature and helicity. Its invariants satisfy  $C(\mathcal{T}) > 0$ ,  $\mathcal{H}(\mathcal{T}) \neq 0$ , and typically  $L(\mathcal{T}) > L(\mathcal{R})$  for equal scale.

## 2.3 Effective energy functional

We postulate an effective energy functional

$$\mathcal{E}_{eff}[K] = \underbrace{\epsilon_0 A L(K)}_{\text{bulk swirl energy}} + \underbrace{\beta L(K)}_{\text{line tension}} + \underbrace{\alpha C(K)}_{\text{near-contact}} + \underbrace{\gamma \mathcal{H}(K)}_{\text{helicity}} \quad (4)$$

where  $K$  is the filament curve,  $A = \pi r_c^2$  is the core cross-sectional area, and  $\epsilon_0$  is the Canon bulk energy density

$$\epsilon_0 = \frac{4}{\alpha_{fs} \varphi} \left( \frac{1}{2} \rho_f \|\mathbf{v}_\odot\|^2 \right). \quad (5)$$

## 2.4 Dimensional analysis

- $[\epsilon_0] = \text{J m}^{-3}$ ,  $[A] = \text{m}^2$ ,  $[L] = \text{m} \Rightarrow [\epsilon_0 A L] = \text{J}$ .
- $[\beta] = \text{J m}^{-1}$ , hence  $[\beta L] = \text{J}$ .
- $\alpha C$ ,  $\gamma \mathcal{H}$  contribute as energy terms.

Numerically, with Canon constants,

$$\epsilon_0 \approx 1.4187 \times 10^8 \text{ J m}^{-3}, \quad A \approx 6.24 \times 10^{-30} \text{ m}^2, \quad (6)$$

so bulk energy per length is

$$\epsilon_0 A \approx 8.85 \times 10^{-22} \text{ J m}^{-1}. \quad (7)$$

# 3 Photon-Driven Transitions

## 3.1 Resonance condition

The transition  $\mathcal{R} \rightarrow \mathcal{T}$  changes the energy by

$$\Delta E = (\epsilon_0 A + \beta) \Delta L + \alpha C(\mathcal{T}) + \gamma \mathcal{H}(\mathcal{T}). \quad (8)$$

Resonance occurs when

$$\boxed{\hbar \omega \approx \Delta E}. \quad (9)$$

## 3.2 Selection rules

- Angular momentum matching: photon helicity must match the winding number (e.g.  $\Delta n = \pm 3$  for trefoil).
- Radius dependence: larger  $R$  reduces  $\Delta L$ , lowering resonance energy (Rydberg scaling).
- Polarization dependence: circular polarization aligned with knot chirality enhances coupling.

## 4 Effective Field Theory Formulation

At the EFT level, the string world-sheet  $\Sigma$  enters a Lagrangian

$$\mathcal{L} = \frac{1}{2}\rho_f \|\mathbf{v}\|^2 - \rho_E - \beta\ell[\Sigma] - \alpha\mathcal{C}[\Sigma] - \gamma\mathcal{H}[\Sigma] + \mathcal{L}_{\text{EM}}^{\text{int}}[A_\mu; \Sigma]. \quad (10)$$

In the static limit,  $\mathcal{L} \rightarrow -\mathcal{E}_{\text{eff}}$ . Time-dependent solutions yield Rabi-like oscillations between  $\mathcal{R}$  and  $\mathcal{T}$  under monochromatic drive  $\omega \approx \Delta E/\hbar$ .

## 5 Photoelectric and Compton Effects from Canonical Photon Modes

### 5.1 Canonical Photon Mode Recap

From Sec. X (Canonical photon derivation), the photon is described as a *pulsed unknot swirl-string*, with action

$$S[\xi] = \frac{1}{2}\rho_f A_{\text{eff}} \int dt \int_0^L ds [(\partial_t \xi)^2 - c^2(\partial_s \xi)^2],$$

normal modes  $\xi_m(s, t)$  of frequency  $\omega_m = ck_m$ , and single-quantum amplitude

$$a_m = \sqrt{\frac{\hbar}{\rho_f A_{\text{eff}} L \omega_m}}. \quad (11)$$

This constitutes the *canonical* photon description in SST: delocalized circulation with no rest-mass density ( $\rho_m = 0$ ), but finite energy density  $\rho_E$ .

### 5.2 Swirl-EM Mapping (Empirical Consistency)

To compare with classical electrodynamics, we adopt the empirical swirl-EM map

$$\mathbf{E} = \sqrt{\frac{\rho_f}{\varepsilon_0}} \mathbf{v}, \quad \mathbf{B} = \sqrt{\frac{\rho_f}{\varepsilon_0}} \mathbf{b}, \quad \omega = ck, \quad (12)$$

so that the quadratic energy-momentum balance reproduces Maxwell:

$$u = \frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2, \quad \mathbf{S} = \mu_0^{-1} \mathbf{E} \times \mathbf{B},$$

with  $c^{-2} = \varepsilon_0 \mu_0$  [18]. Equation (11) is then consistent with the cavity-QED single-photon field amplitudes [19, 20].

### 5.3 Photoelectric Effect as $\mathcal{R} \rightarrow \mathcal{T}$ Transition

Within the  $\mathcal{R}/\mathcal{T}$  two-phase electron model (SST-1), absorption of a canonical photon mode by a delocalized electron  $\mathcal{R}$  can trigger localization to a knotted  $\mathcal{T}$  state if

$$\hbar\omega \geq \Phi + \Delta E_{RT}, \quad (13)$$

where  $\Phi$  is the material work function and  $\Delta E_{RT} = E[\mathcal{T}] - E[\mathcal{R}]$  the localization gap. In metals,  $\Delta E_{RT}$  is negligible or renormalized into an effective  $\Phi_{\text{eff}}$ , giving Einstein's law

$$K_{\text{max}} = \hbar\omega - \Phi_{\text{eff}}. \quad (14)$$

This is thus recovered as an *empirical consequence* of the canonical photon.

## 5.4 Compton Scattering with a Delocalized Photon Packet

Treating the incident photon as a canonical delocalized mode of momentum  $(\hbar\omega/c, \hbar\mathbf{k})$  and the target electron as a localized  $\mathcal{T}$  of mass  $m_e$ , energy–momentum conservation gives

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos\theta), \quad (15)$$

the standard Compton shift [16]. The equality follows because the swirl–EM map (12) preserves stress–energy flux.

## 5.5 Consistency, Limits, and Corrections

Finite core structure of the electron string introduces corrections  $\mathcal{O}((kr_c)^2)$ , negligible for  $kr_c \ll 1$  (X-rays) but potentially testable at  $\gamma$ -ray energies.

**Interpretation.**

1. **Photoelectric:** canonical photon  $\rightarrow$  delocalized  $\mathcal{R}$  electron  $\rightarrow$  localized  $\mathcal{T}$  emission.
2. **Compton:** canonical photon scattering on  $\mathcal{T}$  electron, kinematics as in QED.
3. **Polarization:** photon helicity corresponds to swirl-clock orientation, affecting cross sections and selection rules.

## 5.6 Notes on Provenance

- Einstein’s photoelectric law [14, 15].
- Compton scattering and Klein–Nishina [16, 17].
- Swirl–EM map consistency with Maxwell [18].
- Cavity QED amplitudes [19, 20].

## 6 Predictions and Tests

1. **Spectral lines:** new resonances in absorption spectra, not coinciding with standard atomic transitions.
2. **Rydberg atoms:** red-shifted knotting lines with increasing principal quantum number.
3. **Pump–probe:** suppression of interference fringes coincident with localization after resonant pump.
4. **Polarization dependence:** transition rates sensitive to photon chirality.

## 7 Conclusion

In SST, the same electron string supports both a delocalized circulation  $\mathcal{R}$  (wave aspect) and a localized knot  $\mathcal{T}$  (particle aspect). Photon-induced transitions between these phases provide a dynamical explanation of wave–particle duality consistent with the Canon and with hydrodynamic analogies established since Helmholtz and Kelvin. The framework yields concrete, falsifiable predictions.

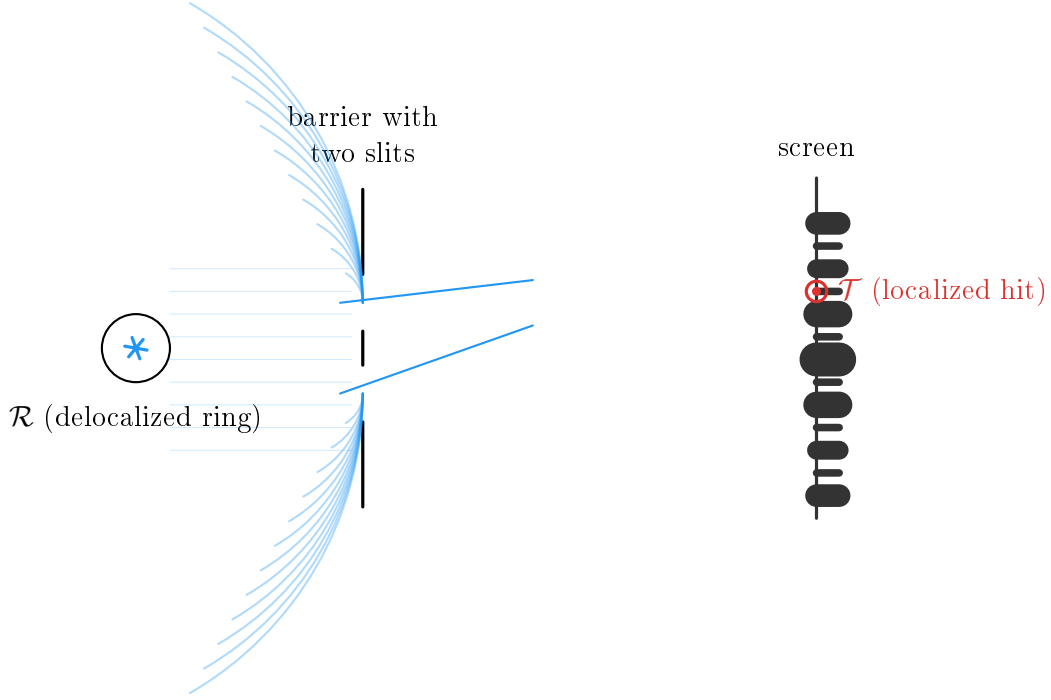


Figure 1: SST double slit schematic. The electron approaches as a delocalized toroidal circulation  $\mathcal{R}$  (ring), whose phase bifurcates at two slits to produce coherent downstream fields that interfere. At the screen, interaction triggers collapse into a localized knotted state  $\mathcal{T}$ , yielding discrete impacts while the ensemble reproduces the fringe intensity.

## Appendix: Popular Summary

A swirl-ring can ripple smoothly like a hula-hoop (wave). If twisted into a knot, the swirl bunches up in one spot (particle). A photon of just the right frequency can flip the string between these two states.

### A Quantitative Fringe Geometry and Visibility

Consider a double slit with center-to-center separation  $s$  and distance  $L$  from slits to screen. Let the electron approach in the delocalized ring phase  $\mathcal{R}$  with mean axial momentum  $p_z$  (de Broglie wavelength  $\lambda = h/p_z$ ). In the Fraunhofer regime ( $L \gg s^2/\lambda$ ), the transverse intensity on the screen is well approximated by

$$I(x) \propto I_1(x) + I_2(x) + 2\sqrt{I_1(x)I_2(x)} \cos\left(\frac{2\pi s}{\lambda} \frac{x}{L} + \phi_0\right), \quad (16)$$

with fringe spacing

$$d = \frac{\lambda L}{s}. \quad (17)$$

Within SST, the phase  $\phi_0$  is the circulation phase offset inherited from  $\mathcal{R}$ ; any path-dependent coupling to the environment adds a random phase  $\delta\phi$  that reduces the fringe visibility

$$\mathcal{V} \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = |\langle e^{i\delta\phi} \rangle|. \quad (18)$$

### A.1 Ring-phase mapping

For the ring state  $\mathcal{R}$  with quantized azimuthal phase  $e^{in\theta}$ , the two slits act as partial projectors of this phase onto two spatially separated downstream wavefronts. The resulting interference encodes the *same* azimuthal winding through the optical path difference  $\Delta\ell(x) \approx s x/L$ , hence the cosine argument above.

## B Which-Way Coupling and Decoherence in SST

In SST language, “which-way” monitoring is an *EM impedance* to the ring phase, parameterized by a coupling rate  $\Gamma$  (net photon or field-interaction rate that carries path information). Let  $\tau$  be the transit time through the interferometer. For weak, memoryless monitoring the phase undergoes a random walk, giving the standard exponential visibility law [21]

$$\boxed{\mathcal{V}(\Gamma, \tau) = e^{-\Gamma\tau}}. \quad (19)$$

Equivalently, define a *monitoring strength*  $\eta \in [0, 1]$  as the single-pass which-way information; then  $\mathcal{V} = \sqrt{1 - \eta}$  (two-outcome, information-balance form). Both parameterizations are compatible at small  $\eta$  via  $\eta \simeq 2(1 - e^{-\Gamma\tau})$ .

**SST mechanism.** Microscopically, the EM coupling stochastically seeds premature  $\mathcal{R} \rightarrow \mathcal{T}$  collapses *upstream*, terminating coherent superposition. Equation (19) thus measures the survival probability of coherence before the knotting transition.

### B.1 Dimensional check

$\Gamma$  has units  $s^{-1}$  and  $\tau$  has units  $s$ , so  $\Gamma\tau$  is dimensionless, consistent with (19).

## C Delayed-Choice and Quantum Eraser (SST View)

In a Wheeler-type delayed choice, the interferometer is reconfigured *after* the electron passes the slits [22]. In SST, this reconfiguration alters whether the downstream optics *allow* continued  $\mathcal{R}$ -phase recombination or instead force local  $\mathcal{T}$ -phase collapse:

- **Interference mode on:** downstream optics preserve the  $\mathcal{R}$  phase coherence until the screen  $\Rightarrow$  fringes.
- **Which-way mode on:** downstream optics couple EM impedance strongly, enforcing premature  $\mathcal{R} \rightarrow \mathcal{T}$  collapse  $\Rightarrow$  no fringes.

A quantum eraser removes the stored which-way information, effectively *post-selecting* runs where  $\mathcal{R}$ -coherence survived to the recombination stage; sorted sub-ensembles then display fringes.

## D Spectroscopic Rabi Drive Between $\mathcal{R}$ and $\mathcal{T}$

We model the  $\mathcal{R} \leftrightarrow \mathcal{T}$  manifold as a driven two-level system with detuning  $\Delta = \omega - \omega_0$  (where  $\hbar\omega_0 = \Delta E_{\mathcal{R} \rightarrow \mathcal{T}}$ ) and coupling  $\Omega_R$  set by the EM impedance overlap:

$$\dot{c}_{\mathcal{R}} = -\frac{i}{2}\Omega_R c_{\mathcal{T}} - \frac{\gamma_{\mathcal{R}}}{2} c_{\mathcal{R}}, \quad (20)$$

$$\dot{c}_{\mathcal{T}} = -\frac{i}{2}\Omega_R c_{\mathcal{R}} - \left(\frac{\gamma_{\mathcal{T}}}{2} + i\Delta\right) c_{\mathcal{T}}. \quad (21)$$

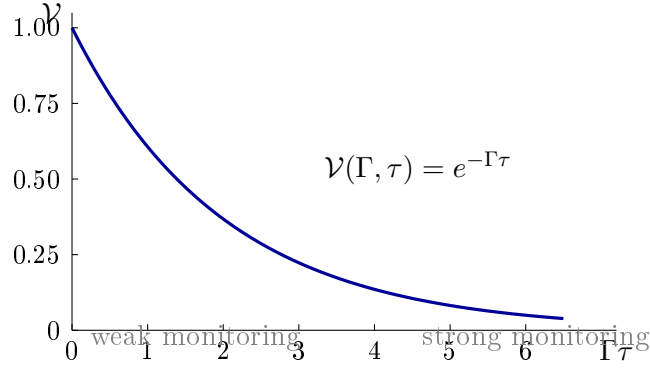


Figure 2: Fringe visibility vs. which-way coupling in SST. The parameter  $\Gamma$  encodes EM impedance to the  $\mathcal{R}$  phase (path information rate),  $\tau$  is the transit time. Exponential decay of visibility reflects coherence loss prior to recombination.

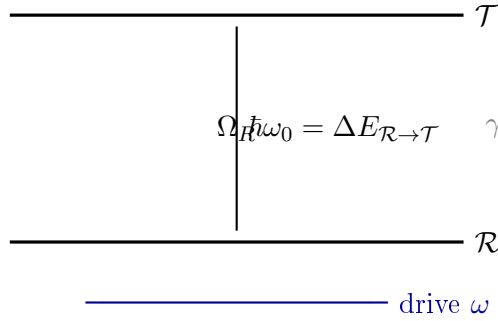


Figure 3: Two-level SST manifold for  $\mathcal{R} \leftrightarrow \mathcal{T}$  with Rabi drive. At resonance  $\omega = \omega_0$ , the knotting probability oscillates at  $\Omega_R$  and decays at rate  $\gamma$ , enabling pump-probe control of collapse within an interferometer.

At resonance ( $\Delta = 0$ ) and for  $\gamma_{\mathcal{R}}, \gamma_{\mathcal{T}} \ll \Omega_R$ ,

$$P_{\mathcal{T}}(t) = |c_{\mathcal{T}}(t)|^2 = \sin^2\left(\frac{\Omega_R t}{2}\right) e^{-\gamma t}, \quad \gamma \equiv \frac{\gamma_{\mathcal{R}} + \gamma_{\mathcal{T}}}{2}. \quad (22)$$

A *pump-probe* synchronized to the flight time across the interferometer can therefore *gate* the knotting probability and modulate fringe visibility in a time-resolved fashion.

**Boxed Result (SST double slit).** An electron is a single swirl-string with two phases:  $\mathcal{R}$  (delocalized ring) for propagation and  $\mathcal{T}$  (knotted soliton) for detection. Interference arises from coherent splitting and recombination of  $\mathcal{R}$ ; localized impacts result from  $\mathcal{R} \rightarrow \mathcal{T}$  collapse at the screen. Which-way monitoring increases the EM impedance, raising the premature knotting rate  $\Gamma$ ; the fringe visibility obeys  $\mathcal{V} = e^{-\Gamma\tau}$  and vanishes in the strong-monitoring limit.

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