

Vacuum Stress–Energy Engineering via Dynamic Toroidal Multipoles

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January 27, 2026

Abstract

We investigate whether structured electromagnetic near fields with dominant toroidal multipole moments can induce measurable modifications of the quantum vacuum stress–energy tensor. In quantum electrodynamics (QED), vacuum polarization introduces nonlinear corrections to Maxwell electrodynamics described by the Euler–Heisenberg effective Lagrangian. We show that counter-rotating, phase-locked toroidal current configurations (anapole states) maximize these nonlinear field invariants while suppressing radiative losses, thereby concentrating stress in the electromagnetic near field. Interpreted within the polarizable-vacuum representation of gravity, such localized vacuum polarization corresponds to an effective refractive-index gradient and hence to a weak emergent metric perturbation. The predicted effects are extremely small and do not constitute macroscopic gravity control. Instead, the proposal defines a falsifiable laboratory framework for probing the coupling between topological electrodynamics, vacuum stress–energy, and analog gravitational metrics.

1 Introduction

In classical Maxwell electrodynamics, the vacuum is a linear, structureless medium. Quantum electrodynamics (QED), however, predicts that virtual electron–positron fluctuations endow the vacuum with nonlinear dielectric and magnetic properties. These effects are encapsulated in the Euler–Heisenberg effective Lagrangian, which provides the leading-order correction to the electromagnetic action at field strengths well below the Schwinger limit. [1, 2]

While direct observation of QED vacuum nonlinearity typically requires extreme field amplitudes, recent advances in topological and near-field electrodynamics suggest that specific source geometries can concentrate the relevant Lorentz invariants locally. [6, 7]

This work explores whether electromagnetic configurations dominated by toroidal multipole moments can act as controlled sources of localized vacuum stress–energy perturbations.

2 Nonlinear QED Vacuum Response

2.1 Euler–Heisenberg Effective Lagrangian

In the weak-field limit, the QED-corrected electromagnetic Lagrangian density is given by

$$\mathcal{L}_{\text{EH}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{2\alpha^2}{45m_e^4} \left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \right], \quad (1)$$

where α is the fine-structure constant, m_e the electron mass, and $\tilde{F}^{\mu\nu}$ the dual field tensor.

The two Lorentz invariants are

$$F_{\mu\nu}F^{\mu\nu} = 2(B^2 - E^2), \quad (2)$$

$$F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\mathbf{E} \cdot \mathbf{B}. \quad (3)$$

The second invariant, proportional to $\mathbf{E} \cdot \mathbf{B}$, plays a central role in topologically nontrivial field configurations. [?]

2.2 Vacuum Stress–Energy Contribution

The nonlinear correction induces a shift in the vacuum expectation value of the stress–energy tensor,

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu}^{\text{EM}} + \langle T_{\mu\nu}^{\text{vac}} \rangle_{\text{EH}} \right), \quad (4)$$

where $\langle T_{\mu\nu}^{\text{vac}} \rangle_{\text{EH}}$ is derived from \mathcal{L}_{EH} . This work concerns only localized perturbations of this vacuum contribution.

3 Toroidal Multipoles and Anapole Fields

3.1 Definition of the Toroidal Moment

Beyond electric and magnetic multipoles, localized current distributions admit a toroidal dipole moment

$$\mathbf{T} = \frac{1}{10c} \int [(\mathbf{r} \cdot \mathbf{J})\mathbf{r} - 2r^2\mathbf{J}] d^3r. \quad (5)$$

Toroidal moments correspond to poloidal current flow on a torus and are associated with finite magnetic helicity and longitudinal vector potentials. [5, 6]

3.2 Anapole-Dominant Configurations

By driving counter-rotating current paths in anti-phase, it is possible to engineer configurations for which the net magnetic dipole moment vanishes, $\mathbf{m} \approx 0$, while the toroidal moment \mathbf{T} is maximized. Such anapole states suppress far-field radiation through multipole cancellation, concentrating electromagnetic stress in the near field. [4, 7]

4 Polarizable Vacuum Interpretation

In the polarizable-vacuum representation of gravity, spacetime curvature is mathematically equivalent to a spatially varying vacuum refractive index $n(\mathbf{r})$, defined by

$$n(\mathbf{r}) = \sqrt{\varepsilon_r(\mathbf{r}) \mu_r(\mathbf{r})}. \quad (6)$$

A weak effective gravitational acceleration may be written as

$$\mathbf{g}_{\text{eff}} = -c^2 \nabla \ln n. \quad (7)$$

Within this representation only, a localized QED-induced modification of the vacuum constitutive parameters corresponds to an effective metric perturbation. Outside this formalism, refractive-index gradients are not generally equivalent to gravitational fields. [?, ?]

5 Numerical Simulation Framework

5.1 Maxwell–Euler–Heisenberg System

To quantitatively assess the feasibility of vacuum stress–energy modulation, we consider classical Maxwell fields supplemented by perturbative Euler–Heisenberg (EH) corrections. The field equations follow from variation of the effective action,

$$S = \int \mathcal{L}_{\text{EH}} d^4x, \quad (8)$$

leading to modified constitutive relations:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \frac{\partial \Delta \mathcal{L}_{\text{EH}}}{\partial \mathbf{E}}, \quad (9)$$

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \frac{\partial \Delta \mathcal{L}_{\text{EH}}}{\partial \mathbf{B}}. \quad (10)$$

In the weak-field regime, these relations may be linearized around the classical solution $(\mathbf{E}_0, \mathbf{B}_0)$, yielding effective susceptibilities $\chi_{\text{vac}}^{(E)}$ and $\chi_{\text{vac}}^{(B)}$ that depend on the local Lorentz invariants $E_0^2 - B_0^2$ and $\mathbf{E}_0 \cdot \mathbf{B}_0$.

5.2 Simulation Strategy

The numerical workflow is as follows:

1. Solve the classical Maxwell problem for the driven toroidal coil geometry using finite-element or finite-difference time-domain (FDTD) methods.
2. Compute the local invariants $F_{\mu\nu} F^{\mu\nu}$ and $F_{\mu\nu} \tilde{F}^{\mu\nu}$ in the near field.
3. Evaluate the EH correction $\Delta \mathcal{L}_{\text{EH}}$ perturbatively.
4. Derive the induced vacuum energy density $\Delta \rho_{\text{vac}} = -\Delta \mathcal{L}_{\text{EH}}$.
5. Reconstruct the effective refractive-index perturbation $\delta n(\mathbf{r})$.

The perturbative nature of the EH term ensures numerical stability and avoids the need for full quantum-field simulations. [2, 3]

6 Quantitative Order-of-Magnitude Estimate

Consider optimistic laboratory field amplitudes achievable in pulsed or resonant near-field systems:

$$E \sim 10^7 \text{ V m}^{-1}, \quad B \sim 10 \text{ T}. \quad (11)$$

The Euler–Heisenberg vacuum energy density correction scales as

$$\Delta \rho_{\text{vac}} \sim \frac{2\alpha^2}{45m_e^4} [(E^2 - B^2)^2 + 7(\mathbf{E} \cdot \mathbf{B})^2]. \quad (12)$$

Substituting numerical values yields

$$\Delta \rho_{\text{vac}} \sim 10^{-24} - 10^{-22} \text{ J m}^{-3}, \quad (13)$$

corresponding to a fractional modification of the background vacuum energy density at the level

$$\frac{\Delta \rho_{\text{vac}}}{\rho_{\text{vac}}^{(0)}} \lesssim 10^{-36}. \quad (14)$$

For a characteristic spatial scale $L \sim 10^{-2}$ m, the associated effective acceleration in the polarizable-vacuum picture is

$$|\mathbf{g}_{\text{eff}}| \sim c^2 \frac{\delta n}{L} \lesssim 10^{-18} \text{ m s}^{-2}. \quad (15)$$

This magnitude is far below terrestrial gravity but comparable to sensitivity limits of state-of-the-art null-force experiments.

7 Experimental Proposal (PRD / CQG Style)

7.1 Objective

The experiment aims to detect or constrain QED-induced vacuum stress–energy perturbations generated by an anapole-dominant electromagnetic source.

7.2 Apparatus

- **Emitter:** A topology-optimized toroidal or torus-knot coil with dual counter-wound conductors.
- **Drive:** Multi-phase current excitation producing a slowly rotating toroidal moment $\mathbf{T}(t)$.
- **Shielding:** Magnetic and electrostatic shielding to suppress conventional Lorentz forces.

7.3 Measurement Channels

1. **Precision force balance:** Search for phase-locked force signals at the drive frequency.
2. **Vacuum birefringence probe:** Interferometric measurement of polarization rotation through the torus center.
3. **Parity-sensitive photodetection:** Detection of weak circular or elliptically polarized emission correlated with drive phase.

All observables are evaluated under current phase reversal (CW \leftrightarrow CCW), implementing a strict null-test protocol.

7.4 Expected Outcome

The experiment is expected to yield either:

- a null result establishing improved upper bounds on vacuum stress–energy modulation by electromagnetic topology, or
- a statistically significant phase-locked anomaly consistent with QED vacuum polarization.

8 Experimental Considerations

We propose a laboratory-scale null-test experiment employing a topology- optimized toroidal emitter:

- high-writhe toroidal or torus-knot winding geometry,
- dual counter-wound conductors driven in anti-phase,
- multi-phase excitation producing a slow rotation of the toroidal moment.

Observable signatures include phase-locked force anomalies in precision force balances, differential vacuum birefringence under phase reversal, and weak parity-odd polarization components in emitted photons. All measurements are designed as differential null tests. [8]

9 Discussion and Limitations

Order-of-magnitude estimates based on the Euler–Heisenberg Lagrangian indicate that any effective metric perturbation produced by laboratory electromagnetic fields is many orders of magnitude weaker than terrestrial gravity. The present proposal therefore does not enable macroscopic gravitational control. Its value lies instead in providing a controlled platform for probing the coupling between electromagnetic topology, vacuum polarization, and stress–energy in the semi-classical regime.

10 Conclusion

Dynamic toroidal multipole fields offer a theoretically well-defined method for concentrating QED vacuum nonlinearities in localized regions of space. When interpreted through the polarizable-vacuum representation, such configurations correspond to extremely weak but in-principle measurable effective metric perturbations. The framework developed here establishes a conservative, falsifiable pathway toward experimental studies of vacuum stress–energy engineering using topological electrodynamics.

A Relation to Analog Gravity

The present work is situated within the broader context of analog gravity, in which effective spacetime metrics emerge from field-dependent constitutive relations. In optical, acoustic, and condensed-matter systems, variations in refractive index or sound speed are formally equivalent to curved spacetime metrics for perturbations propagating in the medium. [9]

In the polarizable-vacuum representation of gravity, a weak gravitational field may be expressed as a spatial variation of the vacuum refractive index $n(\mathbf{r})$. The QED-induced modification of vacuum constitutive parameters considered here therefore admits an interpretation as an effective metric perturbation.

Limitation. This equivalence holds only within the isotropic, dispersion-free, weak-field limit of the polarizable-vacuum formalism. Outside this restricted context, refractive-index gradients are not generally equivalent to spacetime curvature, and no claim of direct gravitational engineering is implied.

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