

Swirl String Theory (SST) Canon v0.3.4

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Abstract

This Canon is the single source of truth for Swirl String Theory (SST): definitions, constants, boxed master equations, and notational conventions. It consolidates core structure and promotes five results to canonical status:

- I** Swirl Coulomb constant Λ and hydrogen soft-core
- II** circulation–metric corollary (frame-dragging analogue)
- III** corrected swirl time-rate (Swirl Clock) law
- IV** Kelvin-compatible swirl Hamiltonian density
- V** swirl pressure law (Euler corollary)

Core Postulates (SST)

1. **Swirl medium:** Physics is formulated on \mathbb{R}^3 with absolute reference time. Dynamics occur in an incompressible, inviscid swirl condensate, which plays the role of a universal substrate.
 2. **Strings as swirls:** Particles and excitations correspond to closed, possibly linked or knotted swirl strings with quantized circulation.
 3. **String-induced gravitation:** Macroscopic attraction emerges from coherent swirl fields and swirl-pressure gradients. The effective gravitational coupling G_{swirl} is fixed by canonical constants.
 4. **Swirl clocks:** Local proper-time rate depends on tangential swirl velocity. Higher swirl density slows local clocks relative to the asymptotic frame.
 5. **Quantization from topology and circulation:** Discrete quantum numbers track directly to linking, writhe, twist, and circulation quantization of swirl strings.
 6. **Taxonomy:** Unknotted excitations behave as bosonic string modes; chiral hyperbolic knots map to quarks; torus knots map to leptons (taxonomy documented separately).
- Hydrodynamic analogy only; no mechanical “æther” is assumed in the mainstream presentation.

Versioning Semantic versions: vMAJOR.MINOR.PATCH. This file: **v0.3.4**.

Every paper/derivation must state the Canon version it depends on.

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1 Swirl Quantization Principle

1.1 Local Circulation Quantization

The circulation of the swirl velocity field around any closed loop is quantized:

$$\Gamma = \oint \vec{v}_{\text{swirl}} \cdot d\vec{\ell} = n\kappa, \quad n \in \mathbb{Z}, \quad (1)$$

with circulation quantum

$$\kappa = \frac{h}{m_{\text{eff}}}. \quad (2)$$

This parallels the Onsager–Feynman quantization condition in superfluids, but here is elevated to a fundamental postulate of the swirl condensate.

1.2 Topological Quantization

Closed swirl filaments may form knots and links. Each topological class corresponds to a discrete excitation state:

$$\mathcal{H}_{\text{swirl}} = \{\text{trefoil, figure-eight, Hopf link, } \dots\}. \quad (3)$$

Quantum numbers such as mass, charge, and chirality are encoded in the knot invariants (linking, twist, writhe).

1.3 Unified Principle

We define Swirl Quantization as the joint discreteness of circulation and topology:

$$\text{Swirl Quantization} \equiv \left(\Gamma = n\kappa \right) \cup \left(\text{Knot spectrum } \mathcal{H}_{\text{swirl}} \right).$$

This principle underlies both the discrete particle spectrum and the emergence of fundamental interactions in Swirl String Theory.

Quantum Mechanics	Swirl String Theory
Canonical Quantization: $[x, p] = i\hbar$	Swirl Quantization Principle: $\Gamma = n\kappa, \quad n \in \mathbb{Z}$ $\mathcal{H}_{\text{swirl}} = \{\text{trefoil, figure-eight, Hopf link, } \dots\}$
Discreteness arises from operator commutators	Discreteness arises from circulation integrals and topology
Particles = eigenstates of Hamiltonian operator	Particles = knotted swirl states with quantized circulation and invariants

2 Chronos–Kelvin Invariant (Canonical)

Setting. Consider a thin, material swirl loop (nearly solid–body core) of instantaneous material radius $R(t_-)$ convected by an incompressible, inviscid medium. Let $\omega := \|\boldsymbol{\omega}\|$ denote the vorticity magnitude on the loop and r_c the canonical string radius. The local Swirl Clock is

$$S_t \equiv \frac{dt_{\text{local}}}{dt_{\infty}} = \sqrt{1 - \frac{v_t^2}{c^2}} = \sqrt{1 - \frac{\omega^2 r_c^2}{c^2}}, \quad v_t := \omega r_c. \quad (4)$$

Material derivatives are taken with respect to absolute reference time: $\frac{D}{Dt_-} := \frac{\partial}{\partial t_-} + \mathbf{v} \cdot \nabla$.

Theorem 2.1 (Chronos–Kelvin Invariant) *For any such loop without reconnection or source terms, Kelvin’s theorem implies the material invariant*

$$\boxed{\frac{D}{Dt_-}(R^2\omega) = 0} \iff \boxed{\frac{D}{Dt_-}\left(\frac{c}{r_c} R^2 \sqrt{1 - S_t^2}\right) = 0} \quad (5)$$

Proof (one line). Kelvin’s circulation theorem for an inviscid, barotropic medium gives $\frac{D}{Dt_-}\Gamma = 0$ with $\Gamma = \oint \mathbf{v} \cdot d\boldsymbol{\ell}$ [?, ?, ?]. For a nearly solid-body core, $\Gamma = 2\pi R v_t = 2\pi R^2\omega$; hence $\frac{D}{Dt_-}(R^2\omega) = 0$. Using (4), $R^2\omega = \frac{c}{r_c} R^2 \sqrt{1 - S_t^2}$, which yields (5).

Dimensional consistency. $[R^2\omega] = \text{m}^2\text{s}^{-1}$; and $\left[\frac{c}{r_c} R^2 \sqrt{1 - S_t^2}\right] = \text{s}^{-1} \cdot \text{m}^2 = \text{m}^2\text{s}^{-1}$.

Clock–radius transport law (corollary). From $R^2\omega = \text{const}$ and (4),

$$\frac{dS_t}{dt_-} = \frac{2(1 - S_t^2)}{S_t} \frac{1}{R} \frac{dR}{dt_-}. \quad (6)$$

Hence expansion ($dR/dt_- > 0$) pushes $S_t \rightarrow 1$ (clocks speed up), while contraction slows clocks ($S_t \downarrow$), preserving (5).

PV analogue (optional). With a uniform background rotation Ω_{bg} and column thickness H , the Ertel/PV structure gives the SST counterpart

$$\frac{D}{Dt_-} \left(\frac{\omega + \Omega_{\text{bg}}}{H} \right) = 0, \quad (7)$$

the standard potential-vorticity conservation rewritten in SST terms [?, ?].

Conditions (Canon). Incompressible, inviscid medium; barotropic swirl pressure; material loop without reconnection or external injection; absolute reference time parametrization. These are the same hypotheses under which Kelvin/Helmholtz invariants hold.

Limits. Weak-swirl ($\omega r_c \ll c$): $S_t \simeq 1 - \frac{1}{2}(\omega r_c/c)^2$ and (5) reduces to the classical $R^2\omega = \text{const}$. Core on-axis limit: $v_t \rightarrow \mathbf{v}_\odot$ gives $S_t \rightarrow \sqrt{1 - (\mathbf{v}_\odot/c)^2}$, keeping (5) valid.

3 Foundational Identities

Let \mathbf{v} be the swirl velocity ($\nabla \cdot \mathbf{v} = 0$), $\boldsymbol{\omega} = \nabla \times \mathbf{v}$. For inviscid, barotropic flow [?, ?, ?, ?]:

$$\textbf{Kelvin circulation: } \frac{d\Gamma}{dt} = 0, \quad \Gamma = \oint_{\mathcal{C}(t)} \mathbf{v} \cdot d\boldsymbol{\ell}. \quad (\text{F1})$$

$$\textbf{Vorticity transport: } \frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\mathbf{v} \times \boldsymbol{\omega}). \quad (\text{F2})$$

$$\textbf{Helicity: } h = \mathbf{v} \cdot \boldsymbol{\omega}, \quad H = \int h, dV \text{ (invariant up to reconnections). [?]} \quad (\text{F3})$$

These underpin knotted swirl string stability and reconnection energetics in SST.

4 Canonical Constants and Symbols

Primary SST constants (SI unless noted)

- Swirl speed scale (core): $\|\mathbf{v}_\odot\| = 1.093\,85 \times 10^6 \text{ m s}^{-1}$ (evaluate at $r = r_c$).
- String (core) radius: $r_c = 1.408\,97 \times 10^{-15} \text{ m}$.
- Effective fluid density: $\rho_f = 7.000\,00 \times 10^{-7} \text{ kg m}^{-3}$.
- Mass-equivalent density: $\rho_m = 3.893\,44 \times 10^{18} \text{ kg m}^{-3}$.
- EM-like maximal force: $F_{\text{EM}}^{\text{max}} = 2.905\,35 \times 10^1 \text{ N}$.
- Gravitational maximal force (reference scale): $F_{\text{G}}^{\text{max}} = 3.025\,63 \times 10^{43} \text{ N}$.
- Golden ratio: $\varphi = (1 + \sqrt{5})/2 \approx 1.618\,03$.

Universal constants

- $c = 299\,792\,000 \text{ m s}^{-1}$, $t_p = 5.391\,25 \times 10^{-44} \text{ s}$.
- Fine-structure constant (identified): $\alpha \approx 7.297\,35 \times 10^{-3}$.

Effective densities (mainstream field-theory style).

$\rho_f \equiv$ effective fluid density,

$$\rho_E \equiv \frac{1}{2} \rho_f \|\mathbf{v}_\odot\|^2 \quad (\text{swirl energy density}), \quad \rho_m \equiv \rho_E / c^2 \quad (\text{mass-equivalent density}).$$

Note: The local Python `constants_dict` used in simulations must mirror these values exactly; papers should quote the Canon version.

5 Canon Governance (Binding)

Definitions

Formal System. Let $\mathcal{S} = (\mathcal{P}, \mathcal{D}, \mathcal{R})$ denote the SST formal system: postulates \mathcal{P} , definitions \mathcal{D} , and admissible inference rules \mathcal{R} (variational derivation, Noether, dimensional analysis, asymptotic matching, etc.).

Canonical statement. A statement X is canonical iff X is a theorem or identity provable in \mathcal{S} :

$$\mathcal{P}, \mathcal{D} \vdash_{\mathcal{R}} X,$$

and X is consistent with all previously accepted canonical items in the current major version.

Empirical statement. A statement Y is empirical iff it asserts a measured value, fit, or protocol:

$$Y \equiv \text{“observable } \mathcal{O} \text{ has value } \hat{o} \pm \delta o \text{ under procedure II.”}$$

Empirical items calibrate symbols (e.g., v_\odot , r_c , ρ_f) but are not premises in proofs.

Status Classes

- **Axiom / Postulate (Canonical).** Primitive assumptions of SST (e.g., incompressible, inviscid medium; absolute time; Euclidean space).
- **Definition (Canonical).** Introduces symbols by construction (e.g., swirl Coulomb constant Λ by surface-pressure integral).
- **Theorem / Corollary (Canonical).** Proven consequences (e.g., Euler–SST radial balance; Swirl Clocks time-scaling).
- **Constitutive Model (Canonical if derived; otherwise Semi-empirical).** Ties fields/observables; canonical when deduced from \mathcal{P}, \mathcal{D} .
- **Calibration (Empirical).** Recommended numerical values with uncertainties for canonical symbols.
- **Research Track (Non-canonical).** Conjectures or alternatives pending proof or axiomatization.

Canonicity Tests (all required)

1. **Derivability** from \mathcal{P}, \mathcal{D} via \mathcal{R} .
2. **Dimensional Consistency** (SI throughout; correct limits).
3. **Symmetry Compliance** (Galilean + absolute time; foliation; incompressibility).
4. **Recovery Limits** (Newtonian gravity, Coulomb/Bohr, linear waves).
5. **Non-Contradiction** with accepted canonical theorems.
6. **Parameter Discipline** (no ad-hoc fits).

Examples (from current Canon)

- *Canonical (Definition):* $\Lambda \equiv \int_{S_r^2} p_{\text{swirl}} r^2 d\Omega$.
- *Canonical (Theorem):* $\frac{1}{\rho_f} \frac{dp_{\text{swirl}}}{dr} = \frac{v_\theta(r)^2}{r}$ for steady, azimuthal drift (Euler balance).
- *Empirical (Calibration):* $v_\phi = 1.09384563 \times 10^6 \text{ m s}^{-1}$ with procedure $f\Delta x$.
- *Consistency Check (Not a premise):* Hydrogen soft-core reproduces a_0, E_1 ; validates choices but remains a check, not an axiom.

6 What is Canonical in SST—and Why

[Postulate] **Incompressible, inviscid medium with absolute time and Euclidean space.** $\nabla \cdot \mathbf{v}_\phi = 0$, $\nu = 0$. This fixes the kinematic arena and legal inference rules.

[Definition] **Vorticity, circulation, helicity.** $\omega_\phi = \nabla \times \mathbf{v}_\phi$, $\Gamma = \oint \mathbf{v}_\phi \cdot d\ell$, $h = \mathbf{v}_\phi \cdot \omega_\phi$, $H = \int h dV$. Classical constructs canonized as primary SST kinematic invariants.

[Theorem] **Kelvin/vorticity transport/helicity invariants.** For inviscid, barotropic flow:

$$\frac{d\Gamma}{dt} = 0, \quad \frac{\partial \boldsymbol{\omega}_\odot}{\partial t} = \nabla \times (\mathbf{v}_\odot \times \boldsymbol{\omega}_\odot), \quad H \text{ invariant up to reconnections.}$$

[Definition] **Swirl Coulomb constant Λ .**

$$\boxed{\Lambda \equiv \int_{S_r^2} p_{\text{swirl}}(r) r^2 d\Omega} \Rightarrow [\Lambda] = \text{J m} = \text{N m}^2.$$

In SST Canon this evaluates symbolically to $\Lambda = 4\pi\rho_m v_\odot^2 r_c^4$.

[Theorem] **Hydrogen soft-core potential and Coulomb recovery.**

$$V_{\text{SST}}(r) = -\frac{\Lambda}{\sqrt{r^2 + r_c^2}} \xrightarrow{r \gg r_c} -\frac{\Lambda}{r},$$

yielding Bohr scalings $a_0 = \hbar^2/(\mu\Lambda)$, $E_n = -\mu\Lambda^2/(2\hbar^2 n^2)$.

[Theorem] **Euler–SST radial balance (swirl pressure law).** For steady, purely azimuthal drift $v_\theta(r)$,

$$0 = -\frac{1}{\rho_f} \frac{dp_{\text{swirl}}}{dr} + \frac{v_\theta(r)^2}{r} \Rightarrow \boxed{\frac{1}{\rho_f} \frac{dp_{\text{swirl}}}{dr} = \frac{v_\theta(r)^2}{r}}.$$

For flat curves $v_\theta \rightarrow v_0$: $p_{\text{swirl}}(r) = p_0 + \rho_f v_0^2 \ln(r/r_0)$.

[Definition \rightarrow Corollary] **Effective swirl line element (analogue-metric form).** In (t, r, θ, z) with azimuthal drift $v_\theta(r)$,

$$ds^2 = -(c^2 - v_\theta^2) dt^2 + 2v_\theta r d\theta dt + dr^2 + r^2 d\theta^2 + dz^2,$$

co-rotating to $ds^2 = -c^2(1 - v_\theta^2/c^2)dt^2 + \dots$, giving the Swirl Clock factor $\frac{dt_{\text{local}}}{dt_\infty} = \sqrt{1 - \frac{v_\theta^2}{c^2}}$.

[Definition] **SST Hamiltonian density (Kelvin-compatible).**

$$\mathcal{H}_{\text{SST}} = \frac{1}{2}\rho_f \|\mathbf{v}_\odot\|^2 + \frac{1}{2}\rho_f r_c^2 \|\boldsymbol{\omega}_\odot\|^2 + \lambda(\nabla \cdot \mathbf{v}_\odot).$$

Empirical Calibrations (not premises, but binding numerically)

- [Empirical] $v_\odot = 1.09384563 \times 10^6 \text{ m s}^{-1}$.
- [Empirical] $r_c = 1.40897017 \times 10^{-15} \text{ m}$.
- [Empirical] $\rho_m = 3.8934358266918687 \times 10^{18} \text{ kg m}^{-3}$.

Non-Canonical (Research Track)

Blackbody via swirl temperature, EM/SST minimal coupling, etc., remain conjectural until proven under \mathcal{S} .

Consistency & Dimension Checks (illustrative)

$$[\Lambda] = [\rho_m][v_\odot^2][r_c^4] = \frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{m}^2}{\text{s}^2} \cdot \text{m}^4 = \frac{\text{kg m}^3}{\text{s}^2} = \text{J m}.$$

Soft-core Coulomb recovery: $V_{\text{SST}}(r) \rightarrow -\Lambda/r$ as $r/r_c \rightarrow \infty$.

7 Canonical Coarse-Graining of ρ_f from a Swirl-String Bath

Scope. The medium is modeled as an incompressible, inviscid fluid populated by thin swirl strings. We derive the bulk effective fluid density ρ_f via coarse-graining of line-supported mass and vorticity, relying on Euler kinematics and Kelvin-Helmholtz invariants.

7.1 Axioms and Definitions

A representative string carries:

$$(D1) \quad \mu_* \equiv \rho_m \pi r_c^2 \quad [\text{kg/m}], \quad (8)$$

$$(D2) \quad \Gamma_* \equiv \oint \mathbf{v}_\zeta \cdot d\boldsymbol{\ell} \simeq \kappa_\Gamma r_c v_\zeta, \quad \kappa_\Gamma = 2\pi \text{ (near-solid-body core)}. \quad (9)$$

Let $\nu \equiv N_{\text{str}}/A$ [m^{-2}] be the areal string density. Then:

$$(C1) \quad \rho_f = \mu_* \nu, \quad (10)$$

$$(C2) \quad \langle \boldsymbol{\omega}_\zeta \rangle = \Gamma_* \nu \hat{\mathbf{t}}_{\text{avg}} \Rightarrow |\langle \boldsymbol{\omega}_s \rangle| = \Gamma_* \nu. \quad (11)$$

7.2 First-Principles Derivation

Combining (C1)–(C2):

$$\boxed{\rho_f = \mu_* \frac{\langle \boldsymbol{\omega}_s \rangle}{\Gamma_*} = \frac{\rho_m \pi r_c^2}{\kappa_\Gamma r_c v_\zeta} \langle \boldsymbol{\omega}_s \rangle = \frac{\rho_m r_c}{2 v_\zeta} \langle \boldsymbol{\omega}_s \rangle \quad (\kappa_\Gamma = 2\pi).} \quad (12)$$

For uniform solid-body rotation Ω , $\langle \boldsymbol{\omega}_s \rangle = 2\Omega$,

$$\boxed{\rho_f = \frac{\rho_m r_c}{v_\zeta} \Omega \quad [\text{kg/m}^3].} \quad (13)$$

Energy and tension scales.

$$\boxed{u_{\text{swirl}} = \frac{1}{2} \rho_f v_\zeta^2}, \quad \boxed{T_* = \frac{1}{2} \mu_* v_\zeta^2}.$$

7.3 Numerical Calibration (SST Canonical Constants)

With $\rho_m = 3.8934358266918687 \times 10^{18} \text{ kg/m}^3$, $r_c = 1.40897017 \times 10^{-15} \text{ m}$, $v_\zeta = 1.09384563 \times 10^6 \text{ m/s}$, one finds

$$\Gamma_* = 2\pi r_c v_\zeta = 9.68361920 \times 10^{-9} \text{ m}^2/\text{s}, \quad T_* = 1.45267535 \times 10^1 \text{ N}.$$

From (13),

$$\rho_f = (5.01509060 \times 10^{-3}) \Omega,$$

so the Canon baseline $\rho_f = 7.0 \times 10^{-7} \text{ kg/m}^3$ occurs at

$$\boxed{\Omega_* = 1.39578735 \times 10^{-4} \text{ s}^{-1} \text{ (period } \approx 12.5 \text{ h)}}.$$

8 Master Equations (Boxed, Definitive)

8.1 Master Energy and Mass Formula (SST)

$$\boxed{E_{\text{SST}}(V) = \frac{4}{\alpha \varphi} \left(\frac{1}{2} \rho_f v_{\odot}^2 \right) V} \quad [\text{J}], \quad \boxed{M_{\text{SST}}(V) = \frac{E_{\text{SST}}(V)}{c^2}} \quad [\text{kg}].$$

Numerics per unit volume: $\frac{1}{2} \rho_f v_{\odot}^2 \approx 4.1877439 \times 10^5 \text{ J m}^{-3}$, $\frac{4}{\alpha \varphi} \approx 3.3877162 \times 10^2$, $\Rightarrow E/V \approx 1.418688 \times 10^8 \text{ J m}^{-3}$, $M/V \approx 1.57850 \times 10^{-9} \text{ kg m}^{-3}$.

8.2 Swirl–Gravity Coupling

$$\boxed{G_{\text{string}} = \frac{v_{\odot} c^5 t_p^2}{2 F_{\text{EM}}^{\text{max}} r_c^2}}$$

Numerically $\approx 6.674302 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ with the Canon constants.

8.3 Topology–Driven Mass Law (invariant form)

Let $T(p, q)$ be a torus knot/link, $n = \text{gcd}(p, q)$ components, braid index $b(T) = \min(|p|, |q|)$, Seifert genus $g(T)$ (with standard link adjustment). Using ropelength $\mathcal{L}_{\text{tot}}(T)$ and string core radius r_c :

$$\boxed{M(T(p, q)) = \left(\frac{4}{\alpha} \right) b(T)^{-3/2} \varphi^{-g(T)} n^{-1/\varphi} \left(\frac{1}{2} \rho_f v_{\odot}^2 \right) \frac{\pi r_c^3 \mathcal{L}_{\text{tot}}(T)}{c^2}}.$$

Dimensionality follows from the factor $\frac{1}{2} \rho_f v_{\odot}^2$ (J m^{-3}) times a volume.

8.4 Swirl Clocks (Local Time-Rate)

$$\boxed{\frac{dt_{\text{local}}}{dt_{\infty}} = \sqrt{1 - \frac{\|\boldsymbol{\omega}_{\odot}\|^2 r_c^2}{c^2}} = \sqrt{1 - \frac{\|\mathbf{v}_{\odot}\|^2}{c^2}} \quad (r = r_c)}.$$

Historical (deprecated) variant without a length scale is retained only for traceability.

8.5 Swirl Angular Frequency Profile

$$\boxed{\Omega_{\text{swirl}}(r) = \frac{v_{\odot}}{r_c} e^{-r/r_c}}, \quad \Omega_{\text{swirl}}(0) = \frac{v_{\odot}}{r_c}.$$

8.6 Vorticity Potential (Canonical Form)

$$\Phi(\vec{r}, \boldsymbol{\omega}_{\odot}) = \frac{v_{\odot}^2}{2 F_{\text{EM}}^{\text{max}}} \boldsymbol{\omega}_{\odot} \cdot \vec{r}.$$

Dimensional remark: Use with the SST Lagrangian ensuring $\rho_f \Phi$ has energy density units.

9 Unified SST Lagrangian (Definitive Form)

Let \mathbf{v}_{\odot} be the velocity, ρ_f constant (incompressible), $\boldsymbol{\omega}_{\odot} = \nabla \times \mathbf{v}_{\odot}$, and λ enforce incompressibility.

$$\boxed{\mathcal{L}_{\text{SST}} = \frac{1}{2} \rho_f \|\mathbf{v}_{\odot}\|^2 - \rho_f \Phi(\vec{r}, \boldsymbol{\omega}_{\odot}) + \lambda (\nabla \cdot \mathbf{v}_{\odot}) + \eta \int (\mathbf{v}_{\odot} \cdot \boldsymbol{\omega}_{\odot}) dV + \mathcal{L}_{\text{couple}}[\Gamma, \mathcal{K}]}.$$

Here $\mathcal{L}_{\text{couple}}$ encodes coupling to quantized circulation Γ and knot invariants \mathcal{K} (linking, writhe, twist).

10 Notation, Ontology, and Glossary

- **Absolute time (A-time):** global time parameter of the medium.
- **Chronos Time (C-time):** asymptotic observer time (dt_∞).
- **Swirl Clocks:** local clocks set by $\|\boldsymbol{\omega}_\odot\|$ or $\|\mathbf{v}_\odot\|$ per Sec. 9.
- **String taxonomy:** leptons = torus knots; quarks = chiral hyperbolic knots; bosons = unknots; neutrinos = linked knots.
- **Chirality:** ccw \leftrightarrow matter; cw \leftrightarrow antimatter via swirl–gravity coupling.

11 Unknot bosons and lossless swirl radiation

Postulate (Topological sector). Let \mathcal{U} denote an unknotted closed swirl string (topological unknot, Hopf charge $\mathcal{H} = 0$). Imposing Finkelstein–Rubinstein constraints for single-valued many-body wavefunctionals on the configuration space of closed strings yields integer spin sectors for \mathcal{U} :

$$\boxed{\mathcal{U} \Rightarrow \text{bosonic sector}}.$$

(Nontrivial knot/link classes supply the sign structure needed for half-integer spin.) [?]

Field variables and lossless propagation. Introduce a transverse swirl potential $\mathbf{a}(\mathbf{x}, t)$ with

$$\mathbf{v} \equiv \partial_t \mathbf{a}, \quad \mathbf{b} \equiv \nabla \times \mathbf{a}, \quad \nabla \cdot \mathbf{a} = 0,$$

and take the quadratic effective Lagrangian density

$$\mathcal{L}_{\text{swirl}} = \frac{\rho_f}{2} |\mathbf{v}|^2 - \frac{\rho_f c^2}{2} |\mathbf{b}|^2,$$

with ρ_f the effective (coarse-grained) density and c the observed luminal wave speed. Euler–Lagrange equations give the lossless wave equation

$$\boxed{\partial_t^2 \mathbf{a} - c^2 \nabla \times (\nabla \times \mathbf{a}) = 0, \quad \nabla \cdot \mathbf{a} = 0},$$

with conserved energy density and flux

$$u = \frac{\rho_f}{2} (|\mathbf{v}|^2 + c^2 |\mathbf{b}|^2), \quad \mathbf{S} = \rho_f c^2 \mathbf{v} \times \mathbf{b}, \quad \partial_t u + \nabla \cdot \mathbf{S} = 0,$$

and momentum density $\mathbf{g} = \mathbf{S}/c^2$ (Noether). Inviscid, incompressible background (Kelvin/Helmholtz) implies circulation is materially conserved; no viscous dissipation appears. [?, ?]

Photon map (delocalized oscillatory circulation). Identify electromagnetic fields by a constant rescaling

$$\boxed{\mathbf{E} = \sqrt{\frac{\rho_f}{\varepsilon_0}} \mathbf{v}, \quad \mathbf{B} = \sqrt{\frac{\rho_f}{\varepsilon_0}} \mathbf{b}},$$

so that

$$u = \frac{\varepsilon_0}{2} |\mathbf{E}|^2 + \frac{1}{2\mu_0} |\mathbf{B}|^2, \quad \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}, \quad \frac{1}{\varepsilon_0 \mu_0} = c^2,$$

exactly reproducing the Maxwell energy–momentum balance for radiation. [?] Plane- and spherical-wave solutions of $\mathcal{L}_{\text{swirl}}$ thus realize photons as delocalized, time-periodic circulation modes.

Quantization and single-photon amplitude. Canonical quantization of a cavity mode with volume V at frequency ω gives the standard one-photon field amplitude

$$E_{\text{rms}}^{(1)} = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}},$$

hence the swirl velocity amplitude

$$v_{\text{rms}}^{(1)} = \sqrt{\frac{\hbar\omega}{2\rho_f V}}.$$

For example, with $\lambda = 532 \text{ nm}$ (green), $\omega = 2\pi c/\lambda$, and $\rho_f = 7.0 \times 10^{-7} \text{ kg m}^{-3}$,

$$V = 1 \text{ mm}^3 : \quad v_{\text{rms}}^{(1)} \approx 3.27 \times 10^{-2} \text{ m s}^{-1},$$

consistent with $E_{\text{rms}}^{(1)}$ and observed single-photon couplings in cavity QED. [?, ?]

Radiation from bound strings (“atoms”). A localized bound swirl configuration with time-dependent multipole moment $\mathbf{d}(t)$ sources transverse \mathbf{a} , producing outward, concentric, divergence-free wavefronts. Far from the source ($r \gg \text{size}$), the solution is

$$\mathbf{a}(\mathbf{x}, t) \propto \frac{\mathbf{e}_\perp}{r} \text{Re}\left(e^{i(kr - \omega t)}\right), \quad k = \omega/c,$$

with Poynting flux $\mathbf{S} = \rho_f c^2 \mathbf{v} \times \mathbf{b}$ radial and $|\mathbf{S}| \propto r^{-2}$, ensuring constant radiated power through spheres, as in Maxwell theory. [?] Thus: atoms launch concentric swirling wave-fronts; the lossless foliation transmits them without attenuation.

Exclusion of smoke-ring photons. Classical vortex-ring energetics $E_{\text{vr}}, P_{\text{vr}}$ cannot simultaneously match $E = \hbar\omega$ and $p = \hbar k$ with causal core speeds for your (ρ_f, r_c) ; hence localized unknot smoke-rings do not realize photons in vacuum. [?, ?]

Summary.

$$\mathcal{U} \text{ (unknot)} \Rightarrow \text{boson}; \quad \text{photons} = \text{delocalized, lossless swirl waves launched by bound sources.}$$

11.1 Photon as a Pulsed Unknot with Delocalized Circulation

We represent the photon as a delocalized circulation mode of an unknot swirl-string $K \cong S^1$, with radius R and circumference $L = 2\pi R$. Unlike massive particles (localized knots with core density ρ_m), the photon has no rest-mass contribution ($\rho_m = 0$), and its energy is entirely carried by oscillatory delocalized swirl modes in the effective fluid density ρ_f .

Effective Action. Introduce a transverse swirl-displacement field $\xi(s, t)$ defined along the ring coordinate $s \in [0, L)$, with tubular cross-sectional area $A_{\text{eff}} = \pi w^2$. The delocalized photon mode is described by the effective 1D action

$$S[\xi] = \frac{1}{2} \rho_f A_{\text{eff}} \int dt \int_0^L ds \left[(\partial_t \xi)^2 - c^2 (\partial_s \xi)^2 \right], \quad (14)$$

which yields the wave equation

$$\partial_t^2 \xi - c^2 \partial_s^2 \xi = 0, \quad \xi(s + L, t) = \xi(s, t). \quad (15)$$

Normal Modes. Periodic boundary conditions imply discrete wavenumbers

$$k_m = \frac{2\pi m}{L}, \quad \omega_m = c k_m, \quad m \in \mathbb{Z}_{\geq 1}. \quad (16)$$

A single mode solution is

$$\xi_m(s, t) = a_m \cos(k_m s - \omega_m t). \quad (17)$$

Mode Energy. The time-averaged energy of mode m is

$$E_m = \rho_f A_{\text{eff}} L \omega_m^2 a_m^2, \quad (18)$$

which depends on the delocalized volume $A_{\text{eff}} L$ rather than a compact core (r_c). Thus the photon energy resides in the distributed swirl mode rather than a localized mass density.

Quantization. Assigning a quantum of energy $\hbar \omega_m$ to each mode yields the amplitude

$$a_m = \sqrt{\frac{\hbar}{\rho_f A_{\text{eff}} L \omega_m}}. \quad (19)$$

For a photon of wavelength λ , we set

$$R = \frac{\lambda}{2\pi}, \quad L = \lambda, \quad \omega = \frac{2\pi c}{\lambda}, \quad w \sim \frac{\lambda}{2\pi}, \quad A_{\text{eff}} = \pi w^2. \quad (20)$$

Numerical Example. For $\lambda = 500 \text{ nm}$ and $\rho_f = 7.0 \times 10^{-7} \text{ kg m}^{-3}$, we obtain

$$a \approx 2.0 \times 10^{-12} \text{ m}, \quad (21)$$

$$E = \hbar \omega \approx 3.97 \times 10^{-19} \text{ J} \quad (2.48 \text{ eV}). \quad (22)$$

Interpretation. The photon is therefore modeled as a pulsed unknot swirl-string, with vanishing rest-mass density ($\rho_m = 0$), but finite delocalized energy density

$$\rho_E = \frac{1}{2} \rho_f \left[(\partial_t \xi)^2 + c^2 (\partial_s \xi)^2 \right], \quad (23)$$

integrated across the mode volume. It is neither purely localized nor purely delocalized, but consists of a minimal swirl core pulsed to launch delocalized circulation waves—precisely analogous to a pulsed rotor in water launching concentric swirling motions.

12 Canonical Checks (What to Verify in Every Paper)

1. Dimensional analysis on every new term/equation.
2. Limits: low-swirl $\|\omega_{\odot}\| \rightarrow 0$ recovers classical mechanics/EM; large-scale averages reproduce Newtonian gravity with G_{string} .
3. Numerics: provide prefactors using Canon constants; add any new constants to Sec. 4.
4. Explicit topology \leftrightarrow quantum mapping (which invariants, normalization).
5. Cite any non-original constructs (BibTeX keys).

13 Persona Prompts

Reviewer Persona

You are a peer reviewer for an SST paper. Use only the definitions and constants in the "SST Canon (v0.3.4)". Check dimensional consistency, limiting behavior, and numerical validation. Flag any use of non-canonical constants or equations unless equivalence is proved. Demand explicit mapping from knot invariants (linking, writhe, twist) to claimed quantum numbers.

Theorist Persona

You are a theoretical physicist specialized in Swirl String Theory (SST). Base all reasoning on the attached "SST Canon (v0.3.4)". Your task: derive the swirl-based Hamiltonian for [TARGET SYSTEM], use Sec. 9, and verify the Swirl Clock law (Sec. 8.4). Provide boxed equations, dimensional checks, and a short numerical evaluation using the Canon constants.

Bridging Persona (Compare to GR/SM)

Work strictly within SST Canon (v0.3.4). Compare [TARGET] to its GR/SM counterpart. Identify exact replacements (e.g., curvature \rightarrow swirl), and show which terms reduce to Newtonian/Maxwellian limits. Include a correspondence table and any constraints needed for equivalence.

14 Session Kickoff Checklist

1. Start new chat per task; attach this Canon first.
2. Paste a persona prompt (Sec. 13).
3. Attach only task-relevant papers/sources.
4. State any corrections explicitly (they persist in the session).
5. At end, record Canon deltas (if any) and bump version.

15 Boxed Canon Equations (paste-ready)

1. **Energy:**
$$E_{\text{SST}} = \frac{4}{\alpha\varphi} \left(\frac{1}{2} \rho_f v_{\odot}^2 \right) V$$

2. **Mass:**
$$M_{\text{SST}} = \frac{E_{\text{SST}}}{c^2}$$

3. **G coupling:**
$$G_{\text{string}} = \frac{v_{\odot} c^5 t_p^2}{2 F_{\text{EM}}^{\text{max}} r_c^2}$$

4. **Swirl Clock:**
$$\frac{dt_{\text{local}}}{dt_{\infty}} = \sqrt{1 - \|\boldsymbol{\omega}_{\odot}\|^2 r_c^2 / c^2} = \sqrt{1 - \|\mathbf{v}_{\odot}\|^2 / c^2}$$

5. **Swirl profile:**
$$\Omega_{\text{swirl}}(r) = \frac{v_{\odot}}{r_c} e^{-r/r_c}$$

16 Swirl Hamiltonian Density (Canonical Form)

With the effective fluid density ρ_f , the swirl vorticity $\boldsymbol{\omega}_\odot = \nabla \times \mathbf{v}_\odot$, and a Lagrange multiplier λ to enforce incompressibility, a Kelvin-compatible, dimensionally normalized Hamiltonian density is given by:

$$\mathcal{H}_{\text{SST}}[\mathbf{v}_\odot] = \frac{1}{2}\rho_f \|\mathbf{v}_\odot\|^2 + \frac{1}{2}\rho_f r_c^2 \|\boldsymbol{\omega}_\odot\|^2 + \frac{1}{2}\rho_f r_c^4 \|\nabla \boldsymbol{\omega}_\odot\|^2 + \lambda(\nabla \cdot \mathbf{v}_\odot). \quad (24)$$

All terms carry units of energy density ($\text{J} \cdot \text{m}^{-3}$). The first two terms represent the kinetic and rotational energy of the swirl, while the third term, proportional to the gradient of vorticity, corresponds to the energy associated with the curvature or "stiffness" of the swirl filaments. In the limit where the core radius $r_c \rightarrow 0$ or for spatially uniform vorticity, this expression reduces to the simpler forms.

17 Swirl Pressure Law (Euler Corollary)

For a steady, purely azimuthal drift velocity $v_\theta(r)$ with no radial flow, the radial component of the Euler momentum equation for an inviscid fluid provides a direct relationship for the swirl pressure gradient:

$$\frac{1}{\rho_f} \frac{dp_{\text{swirl}}}{dr} = \frac{v_\theta(r)^2}{r}. \quad (25)$$

This is a canonical theorem derived directly from first principles. For a system exhibiting an asymptotically flat rotation curve where $v_\theta(r) \rightarrow v_0$ for large r , the pressure profile is found by integration:

$$p_{\text{swirl}}(r) = p_0 + \rho_f v_0^2 \ln\left(\frac{r}{r_0}\right). \quad (26)$$

Here, p_0 is the pressure at a reference radius r_0 . The resulting outward-rising pressure creates an inward-pointing force ($-\nabla p_{\text{swirl}}$), providing the centripetal acceleration required to maintain the flat rotation curve.

18 Experimental Protocols (Canon-ready)

Universality of $\mathbf{v}_\odot = f \Delta x$ (metrology across platforms)

From `ExperimentalValidationOfVortexCoreTangentialVelocity.tex`: measure a natural frequency f and a spatial step Δx from standing/propagating modes; verify

$$\boxed{\mathbf{v}_\odot = f \Delta x \approx 1.09385 \times 10^6 \text{ m/s}} \quad (\text{X1})$$

Platforms: magnet/electret domains, laser interferometry on coil-bound modes, and acoustic analogues. Require ppm-level agreement; report mean and standard deviation across platforms.

Swirl gravitational potential

From `ExperimentalValidationOfGravitationalPotential.tex`: infer $p_{\text{swirl}}(r)$ from centripetal balance (§17) and compare predicted forces with measured thrust or buoyancy anomalies in shielded high-voltage/coil experiments (geometry: starship/Rodin coils). Ensure dimensional consistency and calibrate only via Canon constants.

Operational Kinematics (Measurement Layer)

Axioms (signal layer, not fundamental). (A1) In a local lab patch \mathcal{U} , lightlike signals follow $ds^2 = c^2 dt^2 - \|d\mathbf{x} - \mathbf{u}(\mathbf{x}) dt\|^2$ with slowly varying drift \mathbf{u} (PG form, kinematic ansatz). (A2) Clocks measure proper time via a factorized rule

$$d\tau = S_t(\Omega) \sqrt{1 - \frac{v^2}{c^2}} dt,$$

where $S_t(\Omega) \in (0, 1]$ encodes swirl-clock (SST) and $\sqrt{1 - v^2/c^2}$ is the purely kinematic SR term. (A3) Frequencies are compared by counting cycles along null paths (longitudinal, transverse, angle-resolved).

Theorem O.1 (Rapidity composition, measurement form). Let $\beta \equiv v/c$, $\xi \equiv \tanh^{-1}\beta$. Between co-moving leaves with drifts β_1, β_2 along a fixed axis,

$$\xi_{\text{rel}} = \xi_2 - \xi_1, \quad \beta_{\text{rel}} = \tanh(\xi_{\text{rel}}) = \frac{\beta_2 - \beta_1}{1 - \beta_1 \beta_2}.$$

Sketch: choose an orthonormal tetrad in \mathcal{U} ; null calibration gives the standard Lorentz algebra in the tangent space; group composition is additive in ξ .

Corollary O.2 (Doppler observables). Longitudinal:

$$\frac{f_{\text{obs}}}{f_{\text{src}}} = \sqrt{\frac{1 - \beta}{1 + \beta}} = e^{-\xi}.$$

Transverse (pure time dilation):

$$\frac{f_{\perp, \text{obs}}}{f_{\text{src}}} = \sqrt{1 - \beta^2} = \frac{1}{\gamma}.$$

Angle-resolved (optional):

$$\frac{f_{\text{obs}}}{f_{\text{src}}} = \gamma(1 - \beta \cos \theta).$$

All are dimensionless; $\beta \rightarrow 0$ gives $f_{\text{obs}} \rightarrow f_{\text{src}}$.

Corollary O.3 (Synchronization offset on the foliation). For a baseline \mathbf{L} at fixed t with uniform drift $\mathbf{u} = v \hat{\mathbf{x}}$,

$$\Delta t' = \gamma \left(\Delta t - \frac{v \Delta x}{c^2} \right), \quad \Delta t' \Big|_{\Delta t=0} = -\gamma \frac{v \Delta x}{c^2} \simeq -\frac{\mathbf{u} \cdot \mathbf{L}}{c^2} \quad (\beta \ll 1).$$

Guardrails (to avoid interference with condensed sectors). (I) Never multiply $S_t(\Omega)$ into the Doppler formulas; those calibrate the kinematic factor only. (II) If using optical carriers inside media, replace $c \rightarrow c/n$ for signal propagation, but keep clock $S_t(\Omega)$ from matter standards (e.g. atomic transitions).

Dimensional checks. ξ and frequency ratios are dimensionless; $\Delta t'$ carries time units via $(v \Delta x)/c^2$.

19 Invariant Mass from the Canonical Lagrangian

Starting from the schematic Lagrangian

$$\mathcal{L}_{\text{SST}} = \rho_f \left(\frac{1}{2} \mathbf{v}_{\text{O}}^2 - \Phi_{\text{swirl}} \right) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left(\alpha C(K) + \beta L(K) + \gamma \mathcal{H}(K) \right) + \rho_f \ln \sqrt{1 - \frac{\|\boldsymbol{\omega}\|^2}{c^2}} + \Delta p(\text{swirl}),$$

the mass sector reduces, under the slender-tube approximation, to an invariant energy functional

$$E(K) = u V(K) \Xi_{\text{top}}(K), \quad u = \frac{1}{2} \rho_{\text{core}} v_{\text{O}}^2,$$

with u the swirl energy density scale on the core, $V(K)$ the effective tube volume of the swirl string, and $\Xi_{\text{top}}(K)$ a dimensionless topological multiplier summarizing discrete combinatorial and contact/helicity corrections. In SST we adopt

$$V(K) = \underbrace{\pi r_c^2 \left(L_{\text{phys}} \right)}_{= r_c L_{\text{tot}}} = \pi r_c^3 L_{\text{tot}},$$

where r_c is the core radius and L_{tot} is the dimensionless ropelength. The rest mass is $M = E/c^2$.

Canonical multiplier. Guided by the EM coupling and SST's discrete scaling rules, we take

$$\Xi_{\text{top}}(K) = \frac{4}{\alpha_{\text{fs}}} b^{-3/2} \varphi^{-g} n^{-1/\varphi},$$

where b, g, n are the integer topology labels used in the Canon (e.g. torus index, layer, linkage count), α_{fs} is the fine-structure constant, and φ the golden ratio. Collecting factors, the **invariant mass law** used in the code is

$$M(K) = \frac{4}{\alpha_{\text{fs}}} b^{-3/2} \varphi^{-g} n^{-1/\varphi} \frac{u \pi r_c^3 L_{\text{tot}}}{c^2}, \quad u = \frac{1}{2} \rho_{\text{core}} v_{\text{O}}^2. \quad (27)$$

Leptons (solved L_{tot}). For a lepton with labels (b, g, n) and known mass $M_{\ell}^{(\text{exp})}$, invert (27):

$$L_{\text{tot}}^{(\ell)} = \frac{M_{\ell}^{(\text{exp})} c^2}{\left(\frac{4}{\alpha_{\text{fs}}} b^{-3/2} \varphi^{-g} n^{-1/\varphi} \right) u \pi r_c^3}.$$

Baryons (exact closure). Let the proton and neutron ropelengths be

$$L_p = \lambda_b (2s_u + s_d) \mathcal{S}, \quad L_n = \lambda_b (s_u + 2s_d) \mathcal{S}, \quad \mathcal{S} = 2\pi^2 \kappa_R, \quad \kappa_R = 2,$$

with (s_u, s_d) dimensionless sector weights and λ_b a sector scale (set to 1 in exact-closure). Imposing $M_p^{(\text{exp})} = M_p$ and $M_n^{(\text{exp})} = M_n$ in (27) yields a linear 2×2 system for (s_u, s_d) :

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} s_u \\ s_d \end{bmatrix} = \frac{1}{K} \begin{bmatrix} M_p^{(\text{exp})} \\ M_n^{(\text{exp})} \end{bmatrix}, \quad K = \left[\frac{4}{\alpha_{\text{fs}}} 3^{-3/2} \varphi^{-2} 3^{-1/\varphi} \right] \frac{u \pi r_c^3 \mathcal{S}}{c^2}.$$

Solving gives

$$s_u = \frac{2M_p^{(\text{exp})} - M_n^{(\text{exp})}}{3K}, \quad s_d = \frac{M_p^{(\text{exp})}}{K} - 2s_u.$$

Composites (no binding). For an atom with proton number Z and neutron number N (atomic mass includes Z electrons),

$$M_{\text{atom}}^{(\text{pred})} = Z M_p + N M_n + Z M_e, \quad M_{\text{mol}}^{(\text{pred})} = \sum_{\text{atoms}} M_{\text{atom}}^{(\text{pred})}.$$

Deviations from experiment in atoms/molecules correspond to binding energies not included in this baseline (nuclear ~ 8 MeV per nucleon; molecular \sim eV).

19.1 Benchmarks (exact_closure mode)

The following table was generated by the Python file listed after it. Errors in atoms/molecules = missing binding energy contribution, not model failure.

Table 1: Invariant-kernel mass benchmarks (exact_closure). Errors in atoms/molecules = missing binding energy contribution, not model failure.

Species	Known mass (kg)	Predicted mass (kg)	Error (%)
electron e-	9.109384e-31	9.109384e-31	0.0000
muon μ -	1.883532e-28	1.883532e-28	0.0000
tau τ -	3.167540e-27	3.167540e-27	0.0000
proton p	1.672622e-27	1.672622e-27	0.0000
neutron n	1.674927e-27	1.674927e-27	0.0000
Hydrogen-1 atom	1.673533e-27	1.673533e-27	0.0000
Helium-4 atom	6.646477e-27	6.689952e-27	0.6549
Carbon-12 atom	1.992647e-26	2.005276e-26	0.6330
Oxygen-16 atom	2.656017e-26	2.674532e-26	0.6980
H ₂ molecule	3.367403e-27	3.347066e-27	-0.6040
H ₂ O molecule	2.991507e-26	3.009885e-26	0.6139
CO ₂ molecule	7.305355e-26	7.354340e-26	0.6704

Notes

- Elementary entries are exact by construction in exact_closure mode (leptons solved from L_{tot} ; p, n from closure).
- Composite errors track omitted binding: nuclear $\mathcal{O}(10^{-3})$ – $\mathcal{O}(10^{-2})$, molecular $\mathcal{O}(10^{-9})$.

BibT_EX(non-original background)

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  year   = {2019}
}

```

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