

Spiraling Light in Swirl–String Theory: Transverse Orbital Angular Momentum and Off-Axis Tweezer Traps as a Maxwell-Limit Benchmark

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Abstract

We provide a Swirl–String Theory (SST) interpretation of the spiraling light and optical Magnus-effect phenomenology developed by Spreeuw [1]. In standard electromagnetism, a tightly focused, linearly polarized beam excites a circular dipole whose radiation carries transverse orbital angular momentum (OAM), behaves as if emitted from a source displaced by $k^{-1} = \lambda/2\pi$, and produces spin-dependent off-axis equilibrium positions for atoms trapped in optical tweezers. In SST, photons are modeled as quantized swirl strings in an incompressible substrate; Maxwell electrodynamics appears as the long-wavelength ($\lambda \gg r_c$) limit of the swirl-field equations. We show that (i) the circular-dipole spiral field corresponds to an $\ell = \pm 1$ transverse swirl-string mode, (ii) the apparent source displacement is a geometric centroid of the swirl energy flux, and (iii) the off-axis tweezer displacement is a direct example of matter following gradients of the coarse-grained swirl energy density. The numerical size of possible SST corrections, scaling as $(kr_c)^2$, is shown to be $\mathcal{O}(10^{-16})$ for optical wavelengths, so that current experiments probe the pure Maxwell limit of SST while still providing a precise Rosetta case for photon topology and spin–orbit coupling.

1 Motivation

The analysis in Ref. [1] revisits the angular momentum content of tightly focused beams and the resulting forces on atoms. A key result is that a nominally linearly polarized beam, when focused to large numerical aperture, acquires a nontrivial spatial structure in both polarization and phase. This generates transverse spin, transverse OAM, and an effective source displacement of order k^{-1} , together producing an optical Magnus effect and spin-dependent off-axis trapping in optical tweezers.

Swirl–String Theory (SST) postulates that electromagnetic fields arise as collective excitations of quantized swirl strings in a flat, incompressible fluid-like substrate characterized by an effective density ρ_f , a characteristic swirl speed \mathbf{v}_\odot , and a core length r_c . At wavelengths $\lambda \gg r_c$, the coarse-grained swirl dynamics reproduces Maxwell’s equations. The spiraling light configuration is therefore an ideal Maxwell-limit benchmark: it is experimentally sharp, heavily constrained by angular-momentum conservation, and geometrically transparent.

In this note we map the main constructions of Ref. [1] into SST language and identify where potential SST-specific corrections could, in principle, enter.

2 Maxwell-Limit Summary of Spiraling Light

We briefly recall the core standard-electromagnetic results; detailed derivations are given in [1, 2, 3].

2.1 Circular dipole and transverse OAM

Consider an oscillating electric dipole

$$\mathbf{p}(t) = p_0(\hat{\mathbf{x}} \pm i\hat{\mathbf{z}})e^{-i\omega t}, \quad (1)$$

with angular frequency ω and wavenumber $k = \omega/c = 2\pi/\lambda$. The \pm signs correspond to opposite senses of rotation. In the dipole plane ($y = 0$), the radiated field is (locally) linearly polarized but acquires a spiral phase dependence of the form $e^{\pm i\theta}$, where θ is the azimuthal angle in the xz -plane [1]. This phase winding corresponds to a transverse OAM of $\ell = \pm 1$ per photon for observers in the dipole plane, while on-axis observers see predominant spin.

The field behaves as if emitted by an effective source displaced by

$$\mathbf{R}_{\text{eff}} \simeq \pm \frac{1}{k} \hat{\mathbf{y}} = \pm \frac{\lambda}{2\pi} \hat{\mathbf{y}}, \quad (2)$$

a purely geometric offset set by the wavelength and the dipole rotation sense [1]. Equation (2) is dimensionally consistent, with k^{-1} having units of length.

2.2 Off-axis tweezer trapping

In an optical tweezer formed by a tightly focused beam, atoms experience a conservative potential $U(\mathbf{r}) \propto -\alpha I(\mathbf{r})$, where α is the scalar polarizability and I the local intensity. When the beam is configured to excite the circular dipole described above, the interference of the incident and re-radiated fields produces a transverse asymmetry of the intensity and Poynting vector. Ref. [1] shows that this leads to a stable equilibrium position displaced by $\mathcal{O}(\lambda)$ from the nominal optical axis, with opposite Zeeman sublevels trapped at opposite offsets.

Thus, the spiraling light construction provides:

- a tightly constrained mapping between dipole helicity, transverse OAM, and apparent source displacement;
- a specific, experimentally accessible force pattern on trapped atoms.

3 Swirl–String Photon Model in the Maxwell Limit

In SST, a single photon of frequency ω is modeled as a quantized swirl string: a closed filament of concentrated vorticity with core radius r_c and swirl speed \mathbf{v}_\odot , embedded in an incompressible background of density ρ_f . The swirl energy density ρ_E is related to ρ_f and the local swirl speed, and the mass-equivalent density is $\rho_m = \rho_E/c^2$.

At long wavelengths ($\lambda \gg r_c$), the coarse-grained dynamics of these swirl strings reduces to linear wave equations for effective fields that can be identified with the electromagnetic field tensor $F_{\mu\nu}$. In this limit, SST predicts that:

1. the dispersion relation is $\omega = ck$ up to corrections of order $(kr_c)^2$;
2. the field carries total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$, with $L_z = \ell\hbar$ and $S_z = \sigma\hbar$ emerging from the topology and internal twist of the swirl string;
3. the leading corrections to Maxwell electrodynamics are suppressed by a dimensionless factor

$$\epsilon(k) \sim (kr_c)^2 = \left(\frac{2\pi r_c}{\lambda}\right)^2. \quad (3)$$

Numerically, taking $r_c = 1.40897017 \times 10^{-15}$ m and a representative optical wavelength $\lambda = 1.0 \times 10^{-6}$ m, we obtain

$$\epsilon(k) = \left(\frac{2\pi r_c}{\lambda} \right)^2 \approx 7.8 \times 10^{-17}, \quad (4)$$

a dimensionless correction that is far below the sensitivity of current optical experiments. Thus the spiraling light configurations studied in [1] lie deep inside the Maxwell regime of SST.

4 SST Translation of Spiraling Light

4.1 Circular dipole as a localized swirl-string source

In the Maxwell description, the circular dipole is specified by $\mathbf{p}(t)$ and produces a complex field $\mathbf{E}(\mathbf{r}, t)$ with spiral phase. In SST, we instead specify a localized swirl-string source whose internal degree of freedom (the SwirlClock S_t^ϕ) precesses with frequency ω and helicity $\sigma = \pm 1$. The far-field, after coarse-graining, is described by an effective complex scalar phase

$$\Phi_{\text{SST}}(\mathbf{r}, t) = kz - \omega t + \ell\theta, \quad (5)$$

with $\ell = \pm 1$ encoding the transverse phase winding inherited from the swirl-string topology. The emergent electromagnetic field is then reconstructed from gradients and time-derivatives of Φ_{SST} , reproducing the same $e^{\pm i\theta}$ phase structure found in [1].

In this mapping:

- the internal swirl-string twist maps to photon spin σ ;
- the spatial winding of the swirl string in the transverse plane maps to photon OAM ℓ ;
- the dipole plane corresponds to an observation slice where OAM dominates over spin in the measured angular-momentum density, consistent with the transverse OAM interpretation.

4.2 Effective source displacement as a swirl energy centroid

Equation (2) can be rephrased as a statement about the intensity-weighted centroid of the field in the dipole plane. Let $I(\mathbf{r}_\perp)$ be the time-averaged Poynting flux magnitude through a transverse plane, with $\mathbf{r}_\perp = (x, z)$ and y the transverse offset. Define the effective emission point as

$$\mathbf{R}_{\text{eff}} = \frac{\int \mathbf{r}_\perp I(\mathbf{r}_\perp) d^2 r_\perp}{\int I(\mathbf{r}_\perp) d^2 r_\perp}. \quad (6)$$

In the Maxwell analysis of [1], the special structure of $I(\mathbf{r}_\perp)$ for the circular dipole leads directly to $\mathbf{R}_{\text{eff}} \simeq \pm k^{-1} \hat{\mathbf{y}}$.

In SST, $I(\mathbf{r}_\perp)$ is proportional to the swirl energy flux, derived from $\rho_E(\mathbf{r})$ and the local swirl velocity. In the $\lambda \gg r_c$ regime, the proportionality is linear and isotropic, so that the centroid definition (6) is unchanged. Consequently, SST predicts the same universal offset (2) in the Maxwell limit, with any deviation only entering through the tiny $(kr_c)^2$ corrections of Eq. (3).

4.3 Off-axis tweezer displacement as motion in a swirl potential

The optical tweezer potential $U(\mathbf{r}) \propto -\alpha I(\mathbf{r})$ can be interpreted in SST as a potential energy landscape generated by the swirl energy density $\rho_E(\mathbf{r})$ and its coupling to the internal degrees of freedom of the trapped atom. The force is

$$\mathbf{F}(\mathbf{r}) = -\nabla U(\mathbf{r}) \propto \nabla I(\mathbf{r}), \quad (7)$$

identical in form to the Maxwell description. The spin-dependent displacement arises because the circular dipole component—and therefore the local swirl-string mode that is most strongly excited—changes with the internal state of the atom. Different Zeeman sublevels couple to different helicities σ , thus sampling slightly different swirl energy landscapes.

In SST terms:

1. the internal two-level system of the atom is treated as a localized swirl-clock degree of freedom $S_t^\mathcal{O}$,
2. the focused beam prepares a structured swirl-string field with nontrivial ℓ and σ ,
3. the equilibrium positions are minima of the effective potential defined by the swirl energy density around the atom.

Because the underlying field is in the Maxwell limit, the resulting equilibrium displacements are of order λ and match those computed in [1]. SST neither modifies nor conflicts with these predictions at currently accessible optical parameters.

5 Outlook and SST-Specific Opportunities

The spiraling light construction offers a clean Rosetta example for SST:

- It provides a concrete mapping between swirl-string topology (ℓ, σ) and measurable spin-orbit effects in focused beams.
- It realizes a simple example of matter moving in a structured swirl energy landscape, with spin-dependent equilibrium positions.
- It probes the regime $\lambda \gg r_c$, where SST must reproduce Maxwell’s equations to extremely high precision.

A natural next step is to consider regimes where the beam waist approaches microscopic scales or where multiple swirl-string modes interfere in strongly nonparaxial geometries. Within SST, higher-derivative corrections suppressed by $(kr_c)^2$ could then, in principle, generate small deviations from the pure Maxwell predictions in angular momentum flow or equilibrium positions. For $\lambda \sim 1\,\mu\text{m}$ and $r_c \sim 10^{-15}\text{m}$, the estimated correction $\epsilon(k) \sim 8 \times 10^{-17}$ implies that such effects are currently unobservable, but they offer a well-defined target for future high-precision tests.

In summary, the spiraling light and optical Magnus-effect phenomenology of Ref. [1] is fully compatible with the SST Canon. It provides a useful benchmark problem for the SST photon model, a laboratory example of matter–swirl coupling in the Maxwell limit, and a potential springboard for constraining SST corrections beyond Maxwell electrodynamics.

References

- [1] R. J. C. Spreeuw, Spiraling light: from donut modes to a Magnus-effect analogy, *Preprint* (2021), arXiv:2109.03937.
- [2] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Orbital angular momentum of light and the transformation of Laguerre–Gaussian laser modes, *Phys. Rev. A* **45**, 8185–8189 (1992). doi:10.1103/PhysRevA.45.8185.
- [3] K. Y. Bliokh, M. A. Alonso, E. A. Ostrovskaya, and A. Aiello, Angular momenta and spin–orbit interaction of nonparaxial light in free space, *Phys. Rev. A* **82**, 063825 (2010). doi:10.1103/PhysRevA.82.063825.