A First-Principles Origin of the Inverse-Square Law in Swirl–String Theory: Three Derivations from Local Field Mediation and Momentum-Flux Conservation

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Abstract

A recurrent objection to emergent or flat-background gravity programs is that the inverse-square distance law is often imported rather than derived. In the static, weak-field, spherically symmetric (monopole) sector, we close this gap by exhibiting three independent derivations of the $1/r$ potential and $1/r^2$ flux. (I) A minimal Gauss-law scalar effective field theory (EFT) for the far-field mediator yields the Poisson equation, whose Green's function in $\mathbb{R}^3$ is $1/r$. (II) We identify the SST far-field carrier as a foliation ("Swirl-Clock") scalar perturbation, write its quadratic EFT, compute its spatial stress tensor T_{ij}, and show that its conserved radial momentum-flux density satisfies $\mathcal{F}_r \propto 1/r^2$ with total charge $Q \propto \int \rho_m d^3x$. (III) We give an SST-compatible replacement of an auxiliary scalar χ by the foliation scalar, showing that the static weak-field monopole sector reduces to the Gauss-law form of (I), so the $1/r^2$ flux follows automatically. The remaining model-dependent content is the overall coupling normalization, to be matched to the SST expression G_{swirl} in a separate step.

1 Setup and scope

We work on a flat operational background with Minkowski causal structure and consider the weak-field, static, spherically symmetric (monopole) sector. Let $\rho_m(\mathbf{x})$ be the rest-mass density of a compact source with total mass

$$M \;=\; \int_{\mathbb{R}^3} \rho_m(\mathbf{x})\, d^3x. \tag{1}$$

The objective is to derive, from local field mediation and momentum-flux conservation, that the far-field influence generated by M must exhibit

$$\Phi(\mathbf{x}) \sim \frac{1}{r}, \qquad \nabla\Phi \sim \frac{1}{r^2}, \qquad r = \|\mathbf{x}\|. \tag{2}$$

2 Derivation I: Gauss-law scalar EFT $\Rightarrow$ $1/r$ Green's function

2.1 Minimal mediator and quadratic static EFT

In the static weak-field monopole sector, the minimal local mediator is a scalar field $\phi(\mathbf{x})$ coupled linearly to the source density. The most general rotationally invariant quadratic functional is

$$S_{\text{stat}}[\phi] \;=\; \int_{\mathbb{R}^3} d^3x\, \left[\frac{\kappa}{2}\,(\nabla\phi)^2 \;-\; \lambda\,\phi\,\rho_m(\mathbf{x})\right], \tag{3}$$

with constants $\kappa > 0$ and coupling λ.

2.2 Euler–Lagrange equation: Poisson form

Varying (3),

$$\delta S_{\text{stat}} = \int d^3x \; [\kappa\, \nabla\phi \cdot \nabla(\delta\phi) \;-\; \lambda\, \rho_m\, \delta\phi] \tag{4}$$

$$= \int d^3x \; \left[-\kappa\, (\nabla^2\phi)\, \delta\phi \;-\; \lambda\, \rho_m\, \delta\phi\right] \quad \text{(integrate by parts, drop boundary term)}, \tag{5}$$

so stationarity for arbitrary $\delta\phi$ gives

$$\kappa\, \nabla^2\phi(\mathbf{x}) \;=\; -\,\lambda\, \rho_m(\mathbf{x}). \tag{6}$$

2.3 Green's function and the far-field $1/r$ solution

Let $G(\mathbf{x})$ satisfy

$$\nabla^2 G(\mathbf{x}) = -4\pi\, \delta^{(3)}(\mathbf{x}). \tag{7}$$

On $\mathbb{R}^3$, the spherically symmetric fundamental solution is

$$G(\mathbf{x}) = \frac{1}{\|\mathbf{x}\|}, \tag{8}$$

a standard result in potential theory [1, 2]. Convolving (6) with (7) gives

$$\phi(\mathbf{x}) = \frac{\lambda}{4\pi\kappa} \int d^3x' \, \frac{\rho_m(\mathbf{x}')}{\|\mathbf{x} - \mathbf{x}'\|}. \tag{9}$$

In the far field $r \gg$ source size,

$$\phi(r) \;\simeq\; \frac{\lambda}{4\pi\kappa}\, \frac{1}{r} \int \rho_m(\mathbf{x}')\, d^3x' \;=\; \frac{\lambda}{4\pi\kappa}\, \frac{M}{r}. \tag{10}$$

Therefore

$$\nabla\phi(r) \;\simeq\; -\frac{\lambda M}{4\pi\kappa}\, \frac{\hat{\mathbf{r}}}{r^2}. \tag{11}$$

2.4 Gauss law and inverse-square flux

Define the flux density

$$\mathbf{J} := -\,\kappa\, \nabla\phi. \tag{12}$$

Then (6) becomes $\nabla \cdot \mathbf{J} = \lambda\rho_m$. Integrating over a ball B_R and using the divergence theorem:

$$\oint_{S_R} \mathbf{J} \cdot d\mathbf{A} = \lambda \int_{B_R} \rho_m\, d^3x \;\xrightarrow{R\to\infty}\; \lambda M. \tag{13}$$

Spherical symmetry implies $\mathbf{J} = J_r(r)\hat{\mathbf{r}}$, so

$$4\pi r^2\, J_r(r) = \lambda M \quad \Rightarrow \quad J_r(r) = \frac{\lambda M}{4\pi}\, \frac{1}{r^2}. \tag{14}$$

Thus the inverse-square law follows from locality and the $\mathbb{R}^3$ Green's function structure.

Analogy (for a child). If something spreads out equally in all directions, it must cover a bigger sphere as you go farther away, so each square meter gets less and less—like $1/r^2$.

3 Derivation II: Foliation scalar as far-field carrier; T_{ij} and $\mathcal{F}_r \propto 1/r^2$

3.1 Field identification and quadratic EFT

In SST language, the long-range static degree of freedom is taken to be a foliation ("Swirl-Clock") scalar $T(x)$ that labels preferred-time hypersurfaces. Consider perturbations about an inertial foliation:

$$T(x) = t + \tau(x), \tag{15}$$

where t is the operational background time coordinate and τ is a weak perturbation sourced by matter. At quadratic order, the minimal Lorentzian EFT is

$$S[\tau] = \int d^4x \left[\frac{\kappa}{2}\,\partial_\mu\tau\,\partial^\mu\tau - \lambda\,\tau\,\rho_m(\mathbf{x})\right], \tag{16}$$

with $\partial_t\rho_m = 0$ (static sources). The static sector reduces to (3) with $\phi \equiv \tau$.

3.2 Stress-energy tensor

For the free part of (16), the symmetric stress-energy tensor is

$$T^{(\tau)}_{\mu\nu} = \kappa\left(\partial_\mu\tau\,\partial_\nu\tau - \frac{1}{2}\eta_{\mu\nu}\,\partial_\alpha\tau\,\partial^\alpha\tau\right). \tag{17}$$

In the static regime $\partial_0\tau = 0$, hence $\partial_\alpha\tau\,\partial^\alpha\tau = -(\nabla\tau)^2$ and

$$T^{(\tau)}_{ij} = \kappa\left(\partial_i\tau\,\partial_j\tau - \frac{1}{2}\delta_{ij}(\nabla\tau)^2\right), \qquad T^{(\tau)}_{00} = \frac{\kappa}{2}(\nabla\tau)^2. \tag{18}$$

3.3 Monopole solution, Gauss charge, and radial momentum flux

The field equation from (16) is $\kappa\nabla^2\tau = -\lambda\rho_m$, so in the exterior region the monopole solution is

$$\tau(r) = \frac{\lambda}{4\pi\kappa}\,\frac{Q}{r}, \qquad Q := \int \rho_m\,d^3x \;=\; M, \tag{19}$$

and

$$\partial_r\tau(r) = -\frac{\lambda Q}{4\pi\kappa}\,\frac{1}{r^2}. \tag{20}$$

Define the conserved far-field momentum-flux density (Gauss flux) carried by the foliation scalar:

$$\mathcal{F}_r(r) := -\kappa\,\partial_r\tau(r). \tag{21}$$

Using (20),

$$\boxed{\mathcal{F}_r(r) = \frac{\lambda Q}{4\pi}\,\frac{1}{r^2}, \qquad Q = \int \rho_m\,d^3x.} \tag{22}$$

This is the requested monopole scaling.

3.4 From T_{ij} to integrated inverse-square force

The radial traction (normal stress) from (18) is

$$t_r(r) := \hat{r}_i \, T^{(\tau)}_{ij} \, \hat{r}_j = \frac{\kappa}{2} (\partial_r \tau)^2 \propto \frac{1}{r^4}. \tag{23}$$

The integrated force across a sphere of radius r is

$$F(r) = \int_{S_r} t_r \, dA \sim 4\pi r^2 \times \frac{1}{r^4} \sim \frac{1}{r^2}, \tag{24}$$

consistent with inverse-square behavior.

Analogy (for a child). The field's slope gets weaker like $1/r^2$; stress depends on slope squared, so it weakens even faster, but when you multiply by the sphere's area you get back $1/r^2$.

4 Derivation III: SST replacement of χ by foliation scalar; static weak-field $\Rightarrow$ Gauss-law form

4.1 Static EFT inevitability from symmetry and locality

Any SST-compatible long-range mediator in the weak-field static monopole sector must be described by a local scalar functional whose leading term is quadratic in first derivatives. Denoting the relevant scalar by τ, the leading static effective Lagrangian must take the Gauss-law form

$$\mathcal{L}_{\text{eff,stat}} = \frac{\kappa}{2} (\nabla \tau)^2 - \lambda \, \tau \, \rho_m \; + \; \mathcal{O}(\nabla^4, \tau^3). \tag{25}$$

Higher-derivative terms renormalize the near-zone but do not change the far-field $1/r$ Green's function behavior in the monopole sector.

4.2 Replacement map $\chi \mapsto \tau$ and recovery of Poisson

Let χ be an auxiliary scalar used to package the weak-field influence. Replace it by a rescaled foliation perturbation:

$$\chi := \alpha \, \tau, \tag{26}$$

where α is a constant fixed by matching to the Newtonian limit. Then (25) becomes

$$\mathcal{L}_{\text{eff,stat}} = \frac{\kappa}{2\alpha^2} (\nabla \chi)^2 - \frac{\lambda}{\alpha} \, \chi \, \rho_m. \tag{27}$$

Varying yields

$$\nabla^2 \chi = -\left(\frac{\alpha \lambda}{\kappa} \right) \rho_m. \tag{28}$$

Imposing the standard Poisson normalization

$$\frac{\alpha \lambda}{\kappa} = 4\pi G, \tag{29}$$

we recover

$$\chi(r) = -\frac{GM}{r}, \qquad \nabla \chi(r) = +GM \, \frac{\hat{\mathbf{r}}}{r^2}. \tag{30}$$

Hence, once the foliation scalar reduces to the Gauss-law static form, the inverse-square flux follows automatically.

Analogy (for a child). If two different "field names" obey the same rule, they make the same shape—so both give $1/r$.

5 Discussion and outlook (normalization and G_{swirl} kept separate)

The distance-law problem admits a purely structural solution: locality, rotational invariance, and propagation in three spatial dimensions force the monopole solution of the Laplacian to behave as $1/r$, yielding $1/r^2$ flux. What remains model-dependent is the overall normalization: in the EFT language, the coupling combination $\alpha\lambda/\kappa$ in (29). In SST, this normalization is to be matched to a derived gravitational coupling G_{swirl} constructed from microphysical constants. That matching is orthogonal to the inverse-square derivation itself: it fixes the coefficient of the already-determined Green's function.

Appendix A: Connection to other SST manuscripts (informational, not cited in main text)

This appendix outlines how the present structural result interfaces with other SST lines of development, without relying on those manuscripts as premises. First, Kelvin-mode suppression supports treating ordinary atomic sources as effectively rigid in the far zone, justifying a monopole approximation for $\rho_m(\mathbf{x})$ in (6). Second, SST thermodynamic formulations provide a route to compute the EFT stiffness κ and coupling λ by coarse-graining a microscopic equation of state and identifying the foliation mode's kinetic term. Third, variational selection principles for stable configurations can constrain admissible couplings and potentially fix α in (26), thereby enabling an explicit match to G_{swirl}.

References

[1] J. D. Jackson, *Classical Electrodynamics*, 3rd ed., Wiley (1999).

[2] G. B. Arfken, H. J. Weber, and F. E. Harris, *Mathematical Methods for Physicists*, 7th ed., Academic Press (2013).