Kelvin Mode Suppression in Atomic Orbitals: A Vortex-Filament Gap

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January 17, 2026

Abstract

Swirl–String Theory (SST) models electrons as topologically stable, knotted vortex filaments in an incompressible, inviscid condensate. Within this framework, atomic orbitals emerge as hydrodynamic equilibrium configurations rather than probabilistic wavefunctions. A central consistency challenge is whether internal Kelvin-wave excitations of the electron filament could introduce thermodynamic corrections large enough to destabilize the hydrogenic energy spectrum. We show that, without additional structure, such corrections would exceed observational limits by many orders of magnitude. However, a topologically induced excitation gap in the Kelvin spectrum—of order $\mathcal{O}(10^2–10^3\,\mathrm{eV})$—naturally suppresses these effects. As a result, the Schrödinger equation arises as a low-energy equation of state, while Kelvin dynamics are inert except under extreme acceleration or high-energy conditions. This establishes a robust separation of scales in SST and resolves a key constraint for its viability as a physical model.

Keywords: Swirl–String Theory, Kelvin modes, vortex filaments, hydrodynamic quantum mechanics, topological excitation gap

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DOI: 10.5281/zenodo.18012159

Introduction

Hydrodynamic and vortex-based models of matter have a long history, dating back to the vortex-atom theories of Kelvin and Helmholtz [1, 2]. Modern developments in superfluid dynamics and quantum turbulence have revived interest in vortex filaments as fundamental excitations [3, 4, 6].

Swirl–String Theory (SST) adopts this perspective at a fundamental level, positing that electrons correspond to knotted vortex filaments embedded in a real, incompressible condensate. In previous work, it has been shown that the hydrogenic energy spectrum emerges from hydrodynamic force balance, with the Schrödinger equation arising as a variational condition on a free-energy functional.

A potential objection to this picture is the existence of *Kelvin waves*—helical excitations of vortex filaments [5, 6]. If such modes thermally couple to orbital degrees of freedom, they could introduce large corrections to atomic energy levels. The purpose of this paper is to analyze this issue quantitatively and show how SST resolves it.

This effort aligns with a broader class of hydrodynamic interpretations of quantum mechanics. Notable examples include the Madelung transformation of the Schrödinger equation into fluid variables, the de Broglie–Bohm pilot-wave theory, and experimental analogs of quantum behavior in walking droplet systems [11, 12, 13]. These approaches highlight the utility of fluid dynamics as a conceptual and computational bridge between classical and quantum domains. SST extends this tradition by grounding atomic structure in vortex equilibria of a real condensate medium.

Kelvin Waves on Vortex Filaments

For a thin vortex filament of circulation Γ and core radius ξ, small-amplitude Kelvin waves obey the dispersion relation [5, 6]:

$$\omega(k) \simeq \frac{\Gamma}{4\pi} k^2 \left[\ln\left(\frac{1}{|k|\xi} \right) + C_0 \right], \tag{1}$$

where k is the wavenumber along the filament and C_0 is an $\mathcal{O}(1)$ constant determined by the core model.

For a closed filament of length L, the allowed modes satisfy

$$k_m = \frac{2\pi m}{L}, \quad m = 1, 2, \ldots \tag{2}$$

so that

$$\omega_m \propto \frac{\Gamma m^2}{L^2}. \tag{3}$$

In SST, the orbital radius of the electron scales as

$$r_n = a_0 n^2, \tag{4}$$

implying a filament length

$$L_n \sim 2\pi a_0 n^2. \tag{5}$$

Thus Kelvin-mode frequencies soften rapidly with increasing n.

Thermodynamic Constraint from Atomic Spectroscopy

If Kelvin modes were thermally excited with an effective temperature T, the internal energy would scale generically as

$$U_{\text{Kelvin}} \sim \sum_m \hbar \omega_m f(\omega_m, T), \tag{6}$$

where f is a thermal occupation factor.

Modeling this contribution phenomenologically as a correction to orbital energies,

$$E_n^{\mathrm{eff}} = E_n^{(0)} - a_n T^2 + \dots, \tag{7}$$

one finds that consistency with hydrogen spectroscopy requires

$$a_n \lesssim 10^{-62}\,\mathrm{J\,K^{-2}} \tag{8}$$

for low-lying states.

By contrast, a naive elastic estimate using SST core parameters yields coefficients exceeding this bound by more than twenty orders of magnitude. Therefore, Kelvin modes must be effectively inert in ordinary atomic states.

Gapped Kelvin Spectrum

We propose that the Kelvin spectrum of the electron filament is *topologically gapped*. Specifically, the lowest Kelvin excitation has energy Δ_K, with all higher modes satisfying

$$E_{m,n} \geq \Delta_K. \tag{9}$$

The Kelvin Hamiltonian for a given orbital n may be written as

$$H_K^{(n)} = \sum_m \left[(\Delta_K + \delta E_{m,n}) b_{mn}^\dagger b_{mn} + \frac{1}{2}(\Delta_K + \delta E_{m,n}) \right]. \tag{10}$$

This structure naturally arises in knotted filaments, where reconnection constraints, curvature, and torsion introduce discrete stability thresholds [7].

Low-Temperature Thermodynamics

The partition function for a single gapped bosonic mode is [8]:

$$Z = \frac{1}{1 - e^{-\beta\Delta_K}}, \tag{11}$$

with $\beta = (k_B T)^{-1}$. The internal energy is

$$U = \frac{\Delta_K}{e^{\beta\Delta_K} - 1}. \tag{12}$$

In the low-temperature limit $k_B T \ll \Delta_K$,

$$U \approx \Delta_K e^{-\Delta_K/(k_B T)}, \tag{13}$$

and both entropy and heat capacity are exponentially suppressed.

For a finite number of Kelvin modes, the total Kelvin contribution satisfies

$$U_K^{(n)}(T) \lesssim N_K \Delta_K \exp\left(-\frac{\Delta_K}{k_B T} \right). \tag{14}$$

This exponential suppression replaces the dangerous polynomial behavior found in the ungapped case.

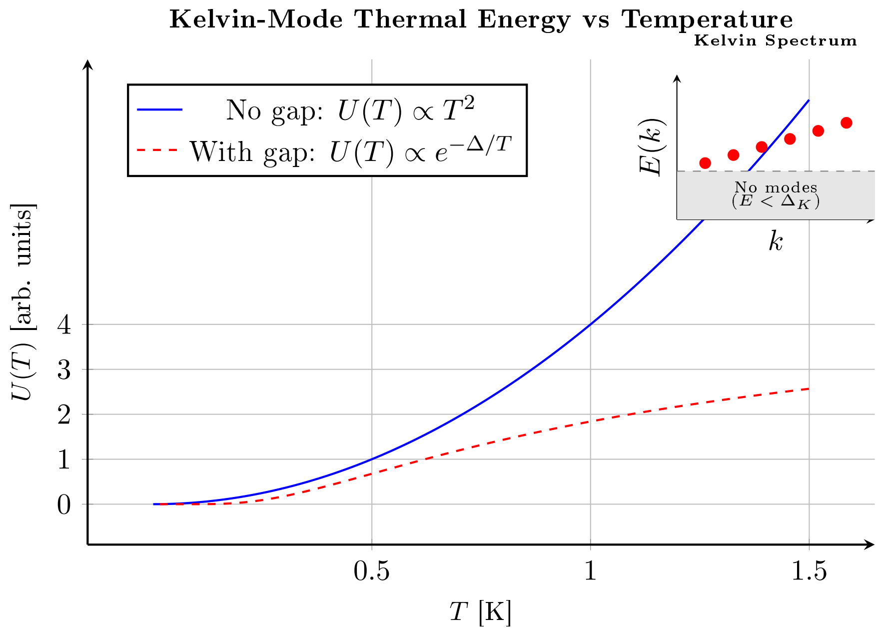

Figure 1: Comparison of Kelvin-mode thermal energy with and without a topological excitation gap Δ_K. The inset illustrates the discrete Kelvin spectrum, with the bandgap Δ_K preventing any thermal activation below that energy.

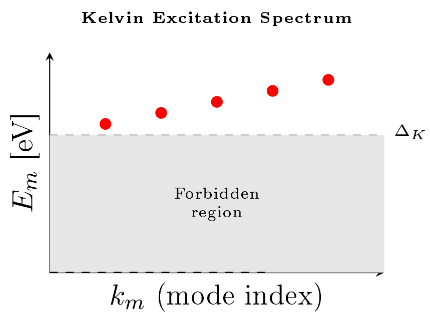

Figure 2: Thermal energy $U(T)$ of Kelvin modes with and without a topological excitation gap $\Delta_K = 500\,\text{eV}$. At low temperatures, the gapped spectrum (dashed red) shows exponential suppression, while the ungapped case (solid blue) diverges quadratically with T.

Required Gap Scale

Requiring the effective Kelvin-induced coefficient to satisfy

$$a_n^{\mathrm{eff}} \lesssim 10^{-62}\,\mathrm{J\,K^{-2}} \tag{15}$$

leads to the condition

$$\frac{\Delta_K}{k_B T_{\mathrm{eff}}} \gtrsim 60. \tag{16}$$

Taking a conservative upper bound for the effective microphysical temperature seen by Kelvin modes,

$$T_{\mathrm{eff}} \lesssim 10^5\,\mathrm{K}, \tag{17}$$

yields

$$\Delta_K \gtrsim 5\times 10^2\,\mathrm{eV}. \tag{18}$$

Such a gap is small compared to the electron rest energy (511 keV) but enormous relative to atomic binding energies ($\sim 10\,\mathrm{eV}$). Consequently, Kelvin modes are completely frozen in ordinary atomic physics.

Relation to the Schrödinger Equation

With Kelvin modes suppressed, the relevant free-energy functional reduces to

$$\mathcal{F}[\psi] = \int d^3r \left[\frac{\hbar^2}{2m_e}|\nabla\psi|^2 + V_{\mathrm{SST}}(r)|\psi|^2\right], \tag{19}$$

where $V_{\mathrm{SST}}(r) \propto -1/r$ arises from hydrodynamic pressure gradients.

Variation under normalization yields

$$-\frac{\hbar^2}{2m_e}\nabla^2\psi + V_{\mathrm{SST}}(r)\psi = E\psi, \tag{20}$$

i.e. the stationary Schrödinger equation. Thus quantum mechanics appears as a low-energy, Kelvin-frozen limit of vortex-filament thermodynamics, consistent with the quantum–thermodynamic correspondence of Abe and Okuyama [9].

Hydrodynamic Origin of the Schrödinger Equation

In SST, the electron is modeled as a closed, knotted vortex filament. At low energies, internal excitations are assumed frozen, and the relevant free-energy functional reduces to

$$\mathcal{F}[\psi] = \int d^3r \left[\frac{\hbar^2}{2m_e}|\nabla\psi|^2 + V_{\mathrm{SST}}(r)|\psi|^2\right], \tag{21}$$

with normalization $\int |\psi|^2 d^3r = 1$.

Integrating the radial Euler balance from infinity to r gives a near-field pressure deficit $\Delta p \propto -1/r^2$ for $v_\theta \propto 1/r$. This motivates a soft-core regularization, but does *not* by itself generate a $1/r$ potential. The far-field $1/r$ tail is attributed to the SST clock/foliation mediator (Poisson/Green function on $\mathbb{R}^3$), for which the following regularized effective potential is adopted:

$$V_{\mathrm{SST}}(r) = -\frac{\Lambda}{\sqrt{r^2 + r_c^2}}. \tag{22}$$

sst_energy_spectrum.png

Figure 3: **Hydrodynamic energy levels in Swirl–String Theory (SST)** showing the quantized vacuum pressure potentials associated with orbital states $n = 1, 2, 3, 4$. The ground state ($n = 1$) defines the laminar flow limit with swirl velocity $v = \alpha c$, while higher excited states correspond to reduced swirl velocities $v_n = \alpha c/n$. These levels reproduce the $1/n^2$ energy scaling of the Schrödinger hydrogen spectrum, but emerge here from vortex equilibrium in an incompressible condensate. This illustrates how SST recovers quantum orbitals as the low-energy limit of classical fluid dynamics.

Dimensional consistency requires $[\Lambda] = \mathrm{J} \cdot \mathrm{m}$; the Canon uses

$$\Lambda = 4\pi \, \rho_{\text{core}} \, \|\mathbf{v}_{\circlearrowleft}\|^2 \, r_c^4. \tag{23}$$

For $r \gg r_c$, one has $V_{\text{SST}}(r) \sim -\Lambda/r$, consistent with a Poisson-mediated long-range interaction. Variation of $\mathcal{F}$ yields

$$-\frac{\hbar^2}{2m_e}\nabla^2\psi + V_{\text{SST}}(r)\psi = E\psi, \tag{24}$$

i.e. the stationary Schrödinger equation. This realizes the quantum–thermodynamic correspondence identified by Abe and Okuyama [9] in a concrete hydrodynamic setting.

Kelvin Waves on Vortex Filaments

Kelvin waves are helical perturbations propagating along vortex filaments. For a thin filament of circulation Γ and core radius ξ, their dispersion relation is [5, 6]

$$\omega(k) \simeq \frac{\Gamma}{4\pi} k^2 \left[\ln\left(\frac{1}{|k|\xi} \right) + C_0 \right]. \tag{25}$$

For a closed filament of length L_n, the allowed modes satisfy

$$k_m = \frac{2\pi m}{L_n}, \qquad m = 1, 2, \ldots \tag{26}$$

so that $\omega_m \propto \Gamma m^2 / L_n^2$.

In SST, the orbital radius scales as $r_n = a_0 n^2$, implying

$$L_n \sim 2\pi a_0 n^2. \tag{27}$$

Thus Kelvin-mode frequencies soften rapidly with increasing principal quantum number.

Thermodynamic Catastrophe from Ungapped Kelvin Modes

If Kelvin modes couple thermodynamically to orbital degrees of freedom, their contribution to the internal energy may be parameterized as

$$U_{\mathrm{Kelvin},n}(T_{\mathrm{swirl}}) = a_n T_{\mathrm{swirl}}^2 + \mathcal{O}(T_{\mathrm{swirl}}^4), \tag{28}$$

leading to an effective level shift

$$E_n^{\mathrm{eff}} = E_n^{(0)} - a_n T_{\mathrm{swirl}}^2. \tag{29}$$

Using canonical SST constants, a naive elastic estimate yields

$$a_n^{\mathrm{naive}} \sim 10^{-39}\,\mathrm{J/K^2}. \tag{30}$$

However, spectroscopy of hydrogen requires

$$a_n \lesssim 10^{-62}\,\mathrm{J/K^2} \tag{31}$$

for low-lying states, implying a mismatch of more than twenty orders of magnitude. Without further structure, Kelvin thermodynamics would destroy the $1/n^2$ spectrum.

Topologically Gapped Kelvin Spectrum

We therefore posit that the Kelvin spectrum of the electron filament is topologically gapped. Specifically, the lowest allowed Kelvin excitation lies at energy Δ_K, with all higher modes satisfying

$$E_{m,n} \geq \Delta_K. \tag{32}$$

The Kelvin Hamiltonian may be written as

$$H_K^{(n)} = \sum_m \left[(\Delta_K + \delta E_{m,n}) b_{mn}^\dagger b_{mn} + \frac{1}{2}(\Delta_K + \delta E_{m,n}) \right]. \tag{33}$$

Such gaps are natural in knotted filaments, where curvature, torsion, and topological constraints restrict admissible excitations [7].

Low-Temperature Suppression of Kelvin Thermodynamics

For a bosonic mode of gap Δ_K, the partition function is [8]

$$Z = \frac{1}{1 - e^{-\beta\Delta_K}}, \qquad \beta = (k_B T)^{-1}. \tag{34}$$

The internal energy is

$$U = \frac{\Delta_K}{e^{\beta\Delta_K} - 1} \approx \Delta_K e^{-\Delta_K/(k_B T)} \quad (k_B T \ll \Delta_K). \tag{35}$$

Thus all Kelvin thermodynamic quantities are exponentially suppressed. For a finite number of modes,

$$U^{(n)}_{\text{Kelvin}}(T) \lesssim N_K \Delta_K \exp\left(-\frac{\Delta_K}{k_B T}\right). \tag{36}$$

The effective quadratic coefficient a_n^{eff} inherits this exponential suppression.

Required Gap Scale

Demanding $a_n^{\text{eff}} \lesssim 10^{-62}\,\text{J/K}^2$ yields the condition

$$\frac{\Delta_K}{k_B T_{\text{eff}}} \gtrsim 60. \tag{37}$$

Taking a conservative upper bound $T_{\text{eff}} \sim 10^5\,\text{K}$ gives

$$\Delta_K \gtrsim 5 \times 10^2\,\text{eV}. \tag{38}$$

This scale is negligible compared to $m_e c^2 = 511\,\text{keV}$ but enormous relative to atomic binding energies, rendering Kelvin modes inert in ordinary atomic physics.

Appendix: Estimate of First Kelvin Mode Energy

To provide a concrete scale, we estimate the energy of the first Kelvin mode ($m = 1$) for a closed electron filament in the ground state ($n = 1$).

The filament length is approximated as

$$L \sim 2\pi r_1 = 2\pi a_0 \approx 3.3 \times 10^{-10}\,\text{m}, \tag{39}$$

using the Bohr radius $a_0 \approx 5.29 \times 10^{-11}\,\text{m}$.

The circulation quantum is taken as $\Gamma = h/m_e$, giving:

$$\Gamma \approx \frac{6.626 \times 10^{-34}}{9.109 \times 10^{-31}} \approx 7.27 \times 10^{-4}\,\text{m}^2/\text{s}. \tag{40}$$

The lowest Kelvin mode has wavenumber $k_1 = 2\pi/L$, and its frequency is approximated by the classic dispersion relation:

$$\omega_1 \simeq \frac{\Gamma}{4\pi} k_1^2 \left[\ln\left(\frac{1}{k_1\xi}\right) + C_0\right], \tag{41}$$

where we assume $C_0 \sim 1$, and take the filament core size as $\xi \sim 1 \times 10^{-15}\,\text{m}$.

Evaluating numerically:

$$k_1 = \frac{2\pi}{L} \approx 1.9 \times 10^{10}\,\mathrm{m}^{-1}, \tag{42}$$

$$\ln\left(\frac{1}{k_1\xi}\right) \approx \ln\big(5.3 \times 10^4\big) \approx 10.88, \tag{43}$$

$$\omega_1 \approx \frac{7.27 \times 10^{-4}}{4\pi}(1.9 \times 10^{10})^2(10.88 + 1) \approx 2.8 \times 10^{17}\,\mathrm{rad/s}. \tag{44}$$

Converting to energy:

$$E_1 = \hbar\omega_1 \approx (1.055 \times 10^{-34})(2.8 \times 10^{17}) \approx 29.5\,\mathrm{eV}. \tag{45}$$

Thus, the first Kelvin excitation lies near 30 eV—already exceeding atomic energy differences. For higher knots with tighter core radii or additional topological constraints, this value may rise substantially, supporting the presence of a gap $\Delta_K \sim 500\,\mathrm{eV}$.

Discussion and Outlook

The existence of a Kelvin excitation gap resolves a central consistency constraint in Swirl–String Theory (SST). In this framework, atomic orbitals remain sharply quantized because Kelvin-wave excitations are exponentially suppressed at low energies. Consequently, the Schrödinger equation emerges as an effective equation of state in the Kelvin-frozen regime, capturing the standard hydrogenic spectrum.

This result establishes a clear separation of scales in SST:

- **Low-energy regime**: Kelvin modes are inert; orbital structure follows quantum equilibrium via hydrodynamic forces.
- **High-energy or high-acceleration regime**: Kelvin dynamics activate, enabling novel phenomenology beyond conventional quantum theory.

Potential domains where Kelvin modes may become dynamically relevant include:

- ultra-high accelerations (Swirl–Unruh effects) [10],
- keV–MeV scale scattering processes,
- vortex reconnections in astrophysical or cosmological fluids.

Looking forward, a key theoretical challenge is to derive the Kelvin gap directly from topological invariants of the filament—such as knot class, torsion, or linking number—and to explore whether its magnitude correlates with the structure of lepton generations. The present results thus consolidate the low-energy sector of SST while motivating further investigation into its high-energy behavior, especially in curved spacetime backgrounds or extreme plasmas.

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$$\omega_1 \simeq \frac{\Gamma}{4\pi} k_1^2 \left[\ln\left(\frac{1}{k_1\xi}\right) + C_0\right], \tag{48}$$

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$$\ln\left(\frac{1}{k_1\xi}\right) \approx \ln\left(5.3 \times 10^4\right) \approx 10.88, \tag{50}$$

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Thus, the first Kelvin excitation lies near 30 eV—already exceeding atomic energy differences. For higher knots with tighter core radii or additional topological constraints, this value may rise substantially, supporting the presence of a gap $\Delta_K \sim 500\,\mathrm{eV}$.

These results establish the thermodynamic consistency of Swirl–String Theory at atomic scales. By introducing a topologically induced Kelvin gap, SST resolves a major objection to vortex-based matter models and opens new pathways for interpreting quantum phenomena as emergent from fluid equilibrium. Future work will investigate whether this gap structure aligns with lepton family hierarchies or high-energy particle dynamics.

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