Golden Hyperbolic Swirl-Angle and Tangential Swirl Speed in Swirl–String Theory (SST)

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Abstract

This paper introduces a hyperbolic, kinematics-only construction used to define a distinguished "golden layer" for tangential swirl motion in Swirl–String Theory (SST). We adopt a hyperbolic definition of the golden ratio, $\phi := e^{\,\operatorname{asinh}(1/2)}$, based on standard inverse-hyperbolic identities. [1] Defining the *golden hyperbolic swirl-angle* $\xi_g := \frac{3}{2}\operatorname{asinh}(1/2)$, we prove the closed identity $\tanh(\xi_g) = e^{\,-\operatorname{asinh}(1/2)} = \phi^{-1}$ using standard hyperbolic-function relations. [2] We then define a dimensionless tangential swirl fraction $\beta := \|\mathbf{v}\|/v_{\circlearrowleft} \in [0, 1)$, where $v_{\circlearrowleft} = \|\mathbf{v}_{\circlearrowleft}\|$ is the SST canonical reference swirl speed. [6] A *hyperbolic swirl-angle* ξ is introduced purely as a bijective re-parameterization of β via $\beta = \tanh\xi$. [2] At the golden layer $\xi = \xi_g$, the tangential fraction is fixed to $\beta_g = \phi^{-1}$, implying $\|\mathbf{v}\|_g = v_{\circlearrowleft}/\phi$ and $\Omega_g = v_{\circlearrowleft}/(\phi r_c)$. Numerical evaluation is given using the SST constants $v_{\circlearrowleft} = 1.093\,845\,63 \times 10^{6}\,\mathrm{m\,s^{-1}}$ and $r_c = 1.408\,970\,17 \times 10^{-15}\,\mathrm{m}$. [6]

1 Introduction and motivation

Hyperbolic functions provide a natural coordinate system for any quantity constrained to the open interval $[0, 1)$, because $\tanh : \mathbb{R} \to (-1, 1)$ is smooth, monotone, and saturating. [2] In many areas of mathematical physics, a bounded "fraction" β is therefore re-parameterized by a hyperbolic angle ξ such that $\beta = \tanh\xi$; this is a coordinate choice, not a dynamical assumption. [2, 3]

In SST, a canonical tangential swirl-speed scale $v_{\circlearrowleft}$ and a core length scale r_c are treated as fundamental constants in the theory's kinematic and energetic bookkeeping. [6] When discussing tangential swirl fractions $\beta = \|\mathbf{v}\|/v_{\circlearrowleft}$, it is therefore useful to introduce a *hyperbolic swirl-angle* ξ as a compact, analytic parameterization of β.

This paper adds one further ingredient: a hyperbolic definition of the golden ratio and a corresponding *golden swirl-angle* ξ_g at which $\tanh\xi_g = \phi^{-1}$. Standard properties of ϕ are classical and widely documented; here we emphasize a hyperbolic-first presentation. [4, 5]

2 Definitions and notation

2.1 SST kinematic quantities

Let $\mathbf{v}_{\circlearrowleft}$ denote the SST canonical characteristic swirl velocity vector, and define the reference speed

$$v_{\circlearrowleft} \;\equiv\; \|\mathbf{v}_{\circlearrowleft}\|\,. \tag{1}$$

Let $\mathbf{v}$ denote a local tangential swirl velocity (e.g. associated with a swirl-string segment). Define the dimensionless tangential fraction

$$\boxed{\beta \;\equiv\; \frac{\|\mathbf{v}\|}{v_{\circlearrowleft}} \in [0,1).} \tag{2}$$

The interval restriction $\beta < 1$ is definitional: $v_{\circlearrowleft}$ is used as the sector reference scale. [6]

2.2 Hyperbolic swirl-angle

We introduce a *hyperbolic swirl-angle* ξ by the bijection

$$\boxed{\beta = \tanh\xi, \qquad \xi = \operatorname{artanh}(\beta).} \tag{3}$$

The functions tanh and artanh and their standard identities are classical. [2]

Limit checks. From standard expansions, $\tanh\xi \sim \xi$ as $\xi \to 0$, so $\beta \approx \xi$ for small tangential fractions. [2] As $\beta \to 1^-$, one has $\xi = \operatorname{artanh}(\beta) \to +\infty$, consistent with the open interval $[0,1)$. [2]

3 Hyperbolic definition of the golden ratio

We adopt a hyperbolic-first definition:

$$\boxed{\phi \;\equiv\; e^{\,\operatorname{asinh}(0.5)}, \qquad \operatorname{asinh}(0.5) \equiv \operatorname{asinh}\!\left(\tfrac{1}{2}\right).} \tag{4}$$

The defining inverse-hyperbolic identity used here is standard: $\operatorname{asinh}(x) = \ln\!\big(x + \sqrt{x^2+1}\big)$. [1]

Lemma 1 (Algebraic corollary)**.** *The hyperbolic definition* (4) *implies the familiar algebraic relation*

$$\phi^2 = \phi + 1, \qquad \textit{hence} \quad \phi = \frac{1+\sqrt{5}}{2}. \tag{5}$$

Proof. Let $t := e^{\,\operatorname{asinh}(0.5)} > 0$. Since $\operatorname{asinh}(0.5) = \operatorname{asinh}(1/2)$, we have $\sinh(\operatorname{asinh}(0.5)) = 1/2$. [1, 2] Using $\sinh(\operatorname{asinh}(0.5)) = \frac{t-t^{-1}}{2}$, [2] we obtain

$$\frac{t-t^{-1}}{2} = \frac{1}{2} \implies t - \frac{1}{t} = 1 \implies t^2 - t - 1 = 0 \implies t^2 = t + 1.$$

By definition $\phi = t$, so $\phi^2 = \phi + 1$. The quadratic solution gives $\phi = (1+\sqrt{5})/2$ since $\phi > 0$. Classical properties of ϕ are standard. [4, 5] □

4 Golden hyperbolic swirl-angle identity

Define the *golden hyperbolic swirl-angle*

$$\boxed{\xi_g \;\equiv\; \frac{3}{2}\operatorname{asinh}(0.5) \;=\; \frac{3}{2}\operatorname{asinh}\!\left(\tfrac{1}{2}\right).} \tag{6}$$

Theorem 1 (Golden swirl-angle implies golden tangential fraction)**.** *With* $\phi = e^{(}\ \operatorname{asinh}(1/2))$ *and* ξ_g *defined by* (6)*, the following identity holds:*

$$\boxed{\tanh(\xi_g) \;=\; e^{-\operatorname{asinh}(0.5)} \;=\; \phi^{-1}.} \tag{7}$$

Proof. Let $t := e^{\ \operatorname{asinh}(0.5)}$. From Lemma (5) we have $t^2 = t + 1$. Using the standard identity $\tanh y = \frac{e^{2y}-1}{e^{2y}+1}$, [2] with $y = \xi_g = \frac{3}{2}\operatorname{asinh}(0.5)$,

$$\tanh(\xi_g) = \frac{e^{\ 3\operatorname{asinh}(0.5)} - 1}{e^{\ 3\operatorname{asinh}(0.5)} + 1} = \frac{t^3 - 1}{t^3 + 1}.$$

Since $t^2 = t + 1$, we obtain $t^3 = t(t+1) = t^2 + t = 2t + 1$. Therefore

$$\tanh(\xi_g) = \frac{(2t+1) - 1}{(2t+1) + 1} = \frac{2t}{2t+2} = \frac{t}{t+1} = \frac{t}{t^2} = \frac{1}{t} = e^{\ -\operatorname{asinh}(0.5)}.$$

Finally, $\phi = e^{\ \operatorname{asinh}(0.5)}$ by (4), hence $\tanh(\xi_g) = \phi^{-1}$. □

5 SST golden layer for tangential swirl kinematics

Combining the SST kinematic mapping (3) with Theorem (7), the golden layer is defined by $\xi = \xi_g$ and yields the fixed fraction

$$\boxed{\beta_g \;\equiv\; \tanh(\xi_g) = \phi^{-1}.} \tag{8}$$

Thus, for any SST sector employing the hyperbolic swirl-angle coordinate ξ, the golden layer implies the tangential speed scale

$$\boxed{\|\mathbf{v}\|_g = \beta_g\, v_{\circlearrowleft} = \frac{v_{\circlearrowleft}}{\phi}.} \tag{9}$$

If r_c denotes the SST core length scale, a corresponding angular-frequency scale can be introduced by dimensional normalization,

$$\boxed{\Omega \equiv \frac{v_{\circlearrowleft}}{r_c}, \qquad \Omega_g \equiv \frac{\|\mathbf{v}\|_g}{r_c} = \frac{1}{\phi}\,\frac{v_{\circlearrowleft}}{r_c} = \frac{\Omega}{\phi}.} \tag{10}$$

This step is purely dimensional: it does not assert a specific dynamical equation for Ω. The constants $v_{\circlearrowleft}$ and r_c are taken from SST canon. [6]

Dimensional consistency. Equation (9) has units of speed, and (10) has units of s^{-1}, since r_c has units of length. [6]

6 Numerical evaluation using SST constants

We evaluate ϕ, ξ_g, and the corresponding kinematic scales using

$$v_{\circlearrowleft} = 1.093\,845\,63 \times 10^{6}\,\mathrm{m\,s^{-1}}, \qquad r_c = 1.408\,970\,17 \times 10^{-15}\,\mathrm{m},$$

as specified in SST canon. [6]

Using the hyperbolic definitions and standard functions, [1, 2] we obtain

$$\phi = e^{\,(\operatorname{asinh}(1/2))} \approx 1.618033988749895, \tag{11}$$
$$\operatorname{asinh}(0.5) = \operatorname{asinh}(1/2) \approx 0.4812118250596035, \tag{12}$$
$$\xi_g = \tfrac{3}{2}\operatorname{asinh}(0.5) \approx 0.7218177375894053, \tag{13}$$
$$\beta_g = \tanh(\xi_g) \approx 0.6180339887498948 = \phi^{-1}. \tag{14}$$

Therefore,

$$\|\mathbf{v}\|_g = \frac{v_{\circlearrowleft}}{\phi} \approx 6.760\,337\,777\,855\,416 \times 10^5\,\mathrm{m\,s^{-1}}, \tag{15}$$
$$\Omega = \frac{v_{\circlearrowleft}}{r_c} \approx 7.763\,440\,655\,383\,073 \times 10^{20}\,\mathrm{s^{-1}}, \tag{16}$$
$$\Omega_g = \frac{\Omega}{\phi} \approx 4.798\,070\,194\,669\,498 \times 10^{20}\,\mathrm{s^{-1}}. \tag{17}$$

7 Interpretation and mainstream-facing remarks

7.1 What is "new" here

The hyperbolic function identities and the classical properties of ϕ used above are not new. [1, 2, 4] The SST-specific contribution is the *interpretive packaging*:

- introduce ξ as a *hyperbolic swirl-angle* coordinating the SST tangential fraction β,
- define the golden layer ξ_g and note that $\beta_g = \phi^{-1}$ becomes an SST-internal, dimensionless marker,
- express the associated speed and frequency scales in terms of SST canonical constants $v_{\circlearrowleft}$ and r_c. [6]

7.2 Why use the hyperbolic coordinate at all

The mapping $\beta = \tanh\xi$ has two practical advantages: (i) it automatically enforces $\beta \in [0, 1)$ for all real ξ; [2] (ii) it linearizes certain algebraic manipulations because exponentials appear directly in tanh identities. [2] In SST, this makes it convenient to talk about "layers" in ξ rather than repeatedly carrying bounded fractions β.

7.3 Analogy (age 10)

Imagine a speed slider that can never go above 100%. The hyperbolic swirl-angle is like a special knob behind the slider: turning the knob by equal steps changes the slider smoothly, and there is one special knob position that always lands exactly at the same famous fraction (about 62%), the golden one.

Acknowledgements

The numerical values used for $v_{\circlearrowleft}$ and r_c are taken from the SST canon. [6]

References

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