Swirl Holographic Principle (SHP)

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January 27, 2026

Abstract

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DOI: 10.5281/zenodo.18388716

1 Foundational Postulate: Swirl Holographic Principle (SHP)

Let $V \subset \mathbb{R}^3$ be a bounded domain with smooth boundary ∂V. Let $\mathbf{v}_{\circlearrowleft}(t, \mathbf{x})$ be a divergence-free swirl field in V evolving in the incompressible inviscid class (without reconnection/dissipation in the interval of interest). Define the tangential boundary trace $\mathbf{v}_\tau = \mathbf{v}_{\circlearrowleft} - (\mathbf{v}_{\circlearrowleft} \cdot \mathbf{n})\mathbf{n}$ on ∂V, equivalently boundary loop circulations $\Gamma_C(t) = \oint_C \mathbf{v}_{\circlearrowleft} \cdot d\boldsymbol{\ell}$ for loops $C \subset \partial V$. Let $\mathcal{T}$ denote the conserved discrete sector labels (helicity/linking class; plus harmonic-mode labels for multiply connected domains).

Postulate (SHP). For each time t, the bulk state in V is fully specified by boundary swirl data and sector labels:

$$\big(\mathbf{v}_{\circlearrowleft}(t, \cdot) \text{ in } V\big) \;\equiv\; \mathcal{R}\big[\mathbf{v}_\tau(t, \cdot)\big|_{\partial V},\; \mathcal{T}\big]\,, \tag{1}$$

for a well-defined reconstruction map $\mathcal{R}$ (Green/Hodge reconstruction). Thus there are no independent bulk degrees of freedom beyond those encoded on ∂V and $\mathcal{T}$.

2 Specialization I: General Relativity as Constraint Holography

In 3+1 (ADM) form, GR uses (h_{ij}, π^{ij}) as canonical data on a hypersurface Σ. Bulk admissibility is restricted by the Hamiltonian and momentum constraints,

$$\mathcal{H}(h, \pi) = 0, \qquad \mathcal{H}_i(h, \pi) = 0, \tag{2}$$

so consistent boundary data (e.g. induced boundary metric γ_{ab} and/or extrinsic curvature data) plus global sector labels (topology and conserved charges) determine the admissible interior by solving the constraint system. This is the GR analogue of SHP: boundary data + constraints + sector labels $\Rightarrow$ bulk.

3 Specialization II: AdS/CFT as On-shell Boundary Functional Reconstruction

In AdS/CFT, boundary values of bulk fields act as sources J for boundary operators $\mathcal{O}$. The on-shell bulk functional depends on boundary sources,

$$Z_{\text{bulk}}[J] \;\approx\; \exp(-S_{\text{on-shell}}[J])\,, \tag{3}$$

and boundary correlators follow from functional differentiation,

$$\langle \mathcal{O}(x) \rangle_J = \frac{\delta S_{\text{on-shell}}[J]}{\delta J(x)}. \tag{4}$$

Given J (boundary data), the bulk solution is reconstructed by solving bulk equations of motion with $\phi|_\partial = J$, i.e. a reconstruction map $\mathcal{R}$ in the AdS variable set. This is the functional-quantum specialization of SHP.

4 Introduction: Why Holography Is Inevitable in SST

Holographic behavior—the determination of bulk physics by boundary data—is often presented as a surprising or uniquely quantum-gravitational feature. In Swirl–String Theory (SST), by contrast, holography is a direct and unavoidable consequence of classical mechanics under three assumptions: (i) incompressibility, (ii) inviscid evolution, and (iii) conservation of topological sector. These assumptions eliminate independent bulk degrees of freedom and enforce global constraints that bind the interior dynamics to boundary data.

In incompressible inviscid media, vorticity is materially advected and circulation is conserved (Kelvin–Helmholtz theorems). As a result, admissible bulk configurations are those consistent with boundary circulation and conserved topological labels (linking, twist, and writhe). SST adopts these mechanical facts as fundamental. The central claim of this work is therefore not that holography exists, but that it is *inevitable* once the above constraints are imposed.

This paper formalizes that claim as a foundational postulate—the *Swirl Holographic Principle* (SHP)—and provides a theorem-level statement showing that the bulk SST state is uniquely determined by boundary swirl data together with discrete sector labels. We then show that General Relativity (GR) and AdS/CFT arise as specializations of the same boundary-determination logic, with different choices of dynamical variables and constraints.

5 Foundational Postulate: Swirl Holographic Principle (SHP)

Let $V \subset \mathbb{R}^3$ be a bounded domain with smooth boundary ∂V. Let the SST bulk state be represented by a divergence-free swirl field $\mathbf{v}_{\circlearrowleft}(t, \mathbf{x})$ evolving in the incompressible, inviscid class,

$$\partial_t \mathbf{v}_{\circlearrowleft} + (\mathbf{v}_{\circlearrowleft} \cdot \nabla)\mathbf{v}_{\circlearrowleft} = -\nabla p, \tag{5}$$

$$\nabla \cdot \mathbf{v}_{\circlearrowleft} = 0, \tag{6}$$

with impermeable boundary condition $\mathbf{v}_{\circlearrowleft} \cdot \mathbf{n}|_{\partial V} = 0$. Define the tangential boundary trace

$$\mathbf{v}_\tau := \mathbf{v}_{\circlearrowleft} - (\mathbf{v}_{\circlearrowleft} \cdot \mathbf{n})\mathbf{n} \quad \text{on } \partial V, \tag{7}$$

equivalently the boundary circulation functional $\Gamma_C(t) = \oint_C \mathbf{v}_{\circlearrowleft} \cdot d\boldsymbol{\ell}$ for all smooth loops $C \subset \partial V$.

Let $\mathcal{T}$ denote the discrete topological sector label of the flow, consisting of helicity class and linking/twist/writhe invariants, assumed conserved (no reconnection or dissipation in the interval of interest).

Postulate (SHP). For each time t, the bulk SST state in V is fully specified by the pair

$$\big(\mathbf{v}_\tau(t, \cdot)\big|_{\partial V},\ \mathcal{T}\big). \tag{8}$$

Equivalently, there exists a reconstruction map $\mathcal{R}$ such that

$$\mathbf{v}_{\circlearrowleft}(t, \cdot) \ = \ \mathcal{R}\big[\mathbf{v}_\tau(t, \cdot)\big|_{\partial V},\ \mathcal{T}\big]\,, \tag{9}$$

up to a pressure gauge. No independent bulk degrees of freedom exist beyond those encoded on the boundary and in the discrete sector labels.

Physical meaning. Boundary swirl data fix all admissible vorticity fluxes through interior surfaces, while $\mathcal{T}$ selects the unique topological realization consistent with those fluxes. The interior is therefore constrained, not autonomous.

6 Boundary Determination Theorem (Euler/Hodge Form)

We now state the precise theorem underlying SHP in the classical Euler regime.

Theorem (Boundary Determination of Bulk Swirl State). Let $V \subset \mathbb{R}^3$ be smooth and simply connected. Assume $\mathbf{v}_{\circlearrowleft}(t, \cdot) \in H^s(V)$ with $s > 5/2$, impermeable boundary conditions, and inviscid evolution without reconnection so that $\mathcal{T}$ is conserved. Then for each t in the local existence interval, the solenoidal bulk field $\mathbf{v}_{\circlearrowleft}(t, \cdot)$ is uniquely determined by $\mathbf{v}_\tau(t, \cdot)|_{\partial V}$ and $\mathcal{T}$, up to pressure gauge.

Sketch of proof. For divergence-free vector fields with $\mathbf{v} \cdot \mathbf{n} = 0$, Hodge decomposition yields

$$\mathbf{v}_{\circlearrowleft} = \nabla \times \mathbf{A} + \mathbf{h}, \tag{10}$$

where $\mathbf{h}$ is harmonic (curl-free and divergence-free). In simply connected V, $\mathbf{h} = 0$; otherwise, $\mathbf{h}$ is finite-dimensional and fixed by circulation around nontrivial cycles, which are included in $\mathcal{T}$.

The vector potential satisfies a Poisson problem $-\Delta \mathbf{A} = \boldsymbol{\omega}$ with gauge and boundary conditions, so that $\mathbf{v}_{\circlearrowleft} = \nabla \times \mathbf{A}$ is uniquely determined by the vorticity field $\boldsymbol{\omega}$.

By Stokes' theorem, boundary circulations determine vorticity fluxes through all interior surfaces anchored on ∂V,

$$\oint_{\partial S} \mathbf{v}_{\circlearrowleft} \cdot d\boldsymbol{\ell} = \int_S \boldsymbol{\omega} \cdot d\mathbf{S}. \tag{11}$$

Conservation of the topological sector $\mathcal{T}$ restricts admissible interior rearrangements of vorticity. Together, these conditions uniquely fix $\boldsymbol{\omega}$ and hence the bulk field $\mathbf{v}_{\circlearrowleft}$.

Consequence. The Euler equations act as constraint equations: the bulk is the unique configuration compatible with boundary swirl data and conserved topology. This completes the proof sketch and establishes SHP at theorem level.

7 Specialization I: General Relativity as Constraint Holography

We now show that General Relativity (GR) arises as a direct specialization of the Swirl Holographic Principle when the dynamical variables are changed from a swirl velocity field to a spacetime metric, and the mechanical constraints are replaced by geometric ones. No additional holographic assumptions are required.

7.1 ADM formulation and boundary determination

In the $3+1$ (ADM) formulation of GR, spacetime is foliated by spacelike hypersurfaces Σ_t with induced metric h_{ij} and conjugate momentum π^{ij}, related to the extrinsic curvature K_{ij}. The Einstein equations split into evolution equations and constraint equations:

$$\mathcal{H}(h, \pi) = 0 \qquad \text{(Hamiltonian constraint)}, \tag{12}$$

$$\mathcal{H}_i(h, \pi) = 0 \qquad \text{(momentum constraints)}. \tag{13}$$

These constraints eliminate independent bulk degrees of freedom. Admissible bulk geometries are precisely those compatible with:

- boundary data (induced boundary metric and/or extrinsic curvature),
- global sector labels (topology and conserved charges),
- the constraint equations.

This is mathematically equivalent to the SST statement that incompressibility and Kelvin–Helmholtz invariants restrict admissible interior swirl configurations.

7.2 Exact structural correspondence with SHP

The correspondence between SST and GR is summarized as follows:

SST		**GR (ADM)**
$\nabla \cdot \mathbf{v}_{\circlearrowleft} = 0$	$\longleftrightarrow$	$\nabla_\mu T^{\mu\nu} = 0$
Boundary tangential swirl $\mathbf{v}_\tau$	$\longleftrightarrow$	Boundary induced metric γ_{ab}
Kelvin circulation invariants	$\longleftrightarrow$	Momentum constraints $\mathcal{H}_i = 0$
Topological sector $\mathcal{T}$	$\longleftrightarrow$	Topology + ADM charges
Reconstruction map $\mathcal{R}$	$\longleftrightarrow$	Solution of Einstein constraints

In both theories, the bulk is *not free.* It is fixed by boundary data and constraints.

7.3 Flat spacetime as a special SST limit

In GR, flat spacetime corresponds to:

$$R_{\mu\nu\rho\sigma} = 0, \qquad K_{ij} = 0, \qquad \pi^{ij} = 0,$$

up to coordinate transformations.

In SST, the corresponding limit is:

$$\nabla \cdot \mathbf{v}_{\circlearrowleft} = 0, \qquad \nabla \times \mathbf{v}_{\circlearrowleft} = 0, \qquad \mathbf{v}_{\circlearrowleft} = \text{constant}.$$

Thus, *flat spacetime is not characterized by incompressibility alone*, but by the absence of vorticity and swirl gradients. This mirrors the GR statement that conservation laws do not imply flatness; curvature vanishes only when additional geometric invariants vanish.

7.4 Gravity as constrained interior structure

In GR, gravitational attraction and time dilation arise from curvature induced by stress–energy, subject to $\nabla_\mu T^{\mu\nu} = 0$.

In SST, gravitational effects arise from vorticity and pressure gradients in an otherwise divergence-free medium:

$$\nabla p = \rho_f \, \mathbf{v}_{\circlearrowleft} \times \boldsymbol{\omega}, \qquad \boldsymbol{\omega} = \nabla \times \mathbf{v}_{\circlearrowleft}. \tag{14}$$

The key point is structural: in both theories, gravity is *not* produced by sources that violate conservation, but by constrained internal structure of the field itself. The role played by curvature tensors in GR is played by vorticity and swirl-induced pressure gradients in SST.

7.5 Interpretation

General Relativity is therefore a geometric specialization of the Swirl Holographic Principle:

> *Boundary data, global sector labels, and constraint equations uniquely determine the admissible bulk.*

SST provides a mechanical realization of this logic, while GR provides a geometric one. The holographic character of GR is thus not fundamental but inherited from a more general constraint-holographic structure shared by both theories.

8 Specialization II: AdS/CFT as On-Shell Boundary Reconstruction

We now show that the AdS/CFT correspondence arises as a quantum-field-theoretic specialization of the Swirl Holographic Principle. In this setting, the boundary data are not mechanical swirl variables but field-theoretic sources, and the reconstruction map is implemented by solving bulk equations of motion with fixed boundary conditions and evaluating the on-shell action.

8.1 Boundary sources and bulk reconstruction

In the AdS/CFT framework, bulk fields Φ (including the metric and matter fields) are defined on an asymptotically Anti–de Sitter spacetime. Their boundary values $\Phi|_\partial$ act as sources J for operators $\mathcal{O}$ in a lower-dimensional conformal field theory (CFT) living on the boundary.

The defining relation is the Gubser–Klebanov–Polyakov–Witten (GKPW) prescription:

$$Z_{\text{bulk}}[J] \;\approx\; \exp\Big(-S_{\text{bulk}}^{\text{on-shell}}[J]\Big), \tag{15}$$

where $S_{\text{bulk}}^{\text{on-shell}}[J]$ is the bulk action evaluated on the unique solution of the bulk equations of motion satisfying $\Phi|_\partial = J$.

Thus, given boundary data J, the admissible bulk configuration is obtained by a reconstruction map:

$$\Phi_{\text{bulk}} \;=\; \mathcal{R}_{\text{AdS}}[J], \tag{16}$$

where $\mathcal{R}_{\text{AdS}}$ denotes solution of the bulk field equations subject to boundary conditions.

8.2 Exact correspondence with SHP

The structural equivalence with SHP is immediate:

SHP (SST)		**AdS/CFT**
Boundary swirl data $\mathbf{v}_\tau$	$\longleftrightarrow$	Boundary sources J
Topological sector $\mathcal{T}$	$\longleftrightarrow$	Choice of CFT vacuum / sector
Reconstruction map $\mathcal{R}$	$\longleftrightarrow$	Bulk EOM + boundary conditions
Bulk observables	$\longleftrightarrow$	Functional derivatives of $S_{\text{on-shell}}$

In both theories, the bulk configuration is not independently specifiable: it is the unique solution compatible with boundary data and global sector labels.

8.3 Why AdS geometry is required

In AdS/CFT, the negative curvature of AdS spacetime ensures that bulk fields are elliptically controlled by boundary data. This mathematical property guarantees well-posedness of the boundary value problem and uniqueness of the reconstructed bulk.

In SST, no background curvature is required because incompressibility and vorticity conservation already enforce elliptic constraints on admissible bulk states. The role played by AdS geometry in quantum gravity is thus played by mechanical constraints in SST.

8.4 On-shell action as boundary functional

A central feature of AdS/CFT is that all physical information about the bulk is contained in a boundary functional:

$$S_{\text{bulk}}^{\text{on-shell}} = S_{\text{bulk}}^{\text{on-shell}}[J]. \tag{17}$$

Expectation values of boundary operators follow from functional differentiation:

$$\langle \mathcal{O}(x) \rangle_J = \frac{\delta S_{\text{bulk}}^{\text{on-shell}}[J]}{\delta J(x)}. \tag{18}$$

This is the quantum-field analogue of SHP: the bulk action itself is a functional of boundary data, and all bulk observables are boundary-generated.

8.5 Interpretation

AdS/CFT therefore represents a functional, quantum realization of constraint holography. The Swirl Holographic Principle captures the same logic at the mechanical level:

Boundary data and global sector labels uniquely determine the admissible bulk state.

SST does not reproduce AdS/CFT dynamically, nor does it require quantum gravity. Rather, AdS/CFT appears as a special case of the same boundary-determination principle applied to quantum fields in a curved background. In this sense, holography is not a uniquely quantum phenomenon but a general consequence of constraint-dominated dynamics.

9 Clock Holography, Falsifiers, and Minimal Experiments

The Swirl Holographic Principle is not a metaphysical claim; it produces concrete, falsifiable consequences. In SST, time, gravity, and bulk dynamics inherit their structure from boundary swirl data and conserved topology. This section states the key predictions and the minimal conditions under which SHP must fail.

9.1 Clock holography

In SST, the local clock rate is a functional of swirl intensity. A representative form used throughout SST is

$$dt_{\mathrm{local}} = dt_{\infty}\sqrt{1-\frac{\|\boldsymbol{\omega}\|^2}{c^2}}, \qquad \boldsymbol{\omega} = \nabla \times \mathbf{v}_{\circlearrowleft}. \tag{19}$$

By SHP, the bulk vorticity field $\boldsymbol{\omega}(t,\cdot)$ is uniquely determined by boundary swirl data and the conserved sector $\mathcal{T}$. Consequently, the bulk clock field is also boundary-determined.

Clock holography (statement). Given fixed $\mathcal{T}$, the entire bulk distribution of clock rates in V is a functional of boundary swirl data:

$$dt_{\mathrm{local}}(t,\mathbf{x}) = \mathcal{C}\big[\mathbf{v}_{\mathcal{T}}(t,\cdot)\big|_{\partial V}\big]\,. \tag{20}$$

This result has no direct analogue in GR or AdS/CFT: in SST, time dilation is not merely influenced by boundary conditions but *encoded* by them.

9.2 Topology as protected information

The discrete sector labels $\mathcal{T}$ play the role of protected information. Helicity and linking numbers cannot change under ideal evolution and therefore act as robust memory degrees of freedom.

Prediction. Two systems driven with identical boundary swirl data but prepared in different topological sectors $\mathcal{T}_1 \neq \mathcal{T}_2$ will exhibit distinct bulk responses (clock-rate distributions, pressure fields, mode spectra), despite indistinguishable boundary forcing.

This is a sharp discriminator between SST and purely metric theories, where topology without curvature has no dynamical effect.

9.3 Controlled breakdown of holography

SHP is exact only under ideal conditions. Its breakdown is itself predictive.

Breakdown mechanisms.

1. **Reconnection**: topological sector $\mathcal{T}$ changes.
2. **Dissipation**: viscosity destroys Kelvin–Helmholtz invariants.
3. **Compressibility**: $\nabla \cdot \mathbf{v}_{\circlearrowleft} \neq 0$.

Observable signature. When SHP fails, boundary data no longer uniquely predict the interior. The failure manifests as:

- loss of reproducibility under identical boundary driving,
- broadband excitation and energy leakage,
- breakdown of clock holography (clock-rate noise not traceable to boundary data).

This gives SST a well-defined regime of validity rather than unfalsifiable scope.

9.4 Minimal experimental tests

The following tabletop-scale experiments are sufficient to probe SHP qualitatively, without invoking exotic physics.

Test 1: Boundary-only control. Prepare a closed toroidal or cavity system with minimal interior access. Modulate boundary swirl phase and circulation while holding geometry fixed. SST predicts measurable changes in interior resonant structure or clock proxies, despite no direct interior forcing.

Test 2: Sector discrimination. Prepare two configurations with identical boundary driving but different linking or knot classes. SST predicts distinct interior responses; GR and standard EM do not.

Test 3: Breakdown signature. Increase driving until reconnection-like events occur. SST predicts a sharp transition from boundary-determined to boundary-indeterminate interior behavior, accompanied by dissipative signatures.

9.5 Summary of falsifiable content

The Swirl Holographic Principle predicts:

- boundary determination of bulk dynamics in ideal regimes,
- boundary encoding of clock-rate fields,
- topologically protected bulk information,
- controlled and observable breakdown under reconnection or dissipation.

Any observation of sustained bulk degrees of freedom independent of boundary data and sector labels in an incompressible inviscid regime would falsify SHP and, by extension, SST in its present form.

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