Rotating–Frame Unification in the SST Canon: From Swirl Density to Swirl–EMF, and a Canonical Derivation of the Coupling $\mathcal{G}_{\circlearrowleft}$

Omar Iskandarani

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Abstract

We derive, directly from the Swirl–String Theory (SST) Canon, a rotating–frame unification in which centrifugal and gravitational (swirl) contributions merge into a single effective source term that modifies Faraday's law in matter. The core objects are the swirl (vortex-line) areal density $\boldsymbol{\varrho}_{\circlearrowleft}$ and a swirl-induced electromotive source $\mathbf{b}_{\circlearrowleft}$ that appears in the curl equation of $\mathbf{E}$. We prove the canonical relation

$$\boxed{\nabla \times \mathbf{E} \;=\; -\,\partial_t \mathbf{B} \;-\; \mathbf{b}_{\circlearrowleft}, \qquad \mathbf{b}_{\circlearrowleft} \;=\; \mathcal{G}_{\circlearrowleft}\, \partial_t \boldsymbol{\varrho}_{\circlearrowleft}}$$

with $\mathcal{G}_{\circlearrowleft}$ a material/topological transduction constant. Using SST "electron logic" (the toroidal ring phase $\mathcal{R}$), circulation quantization, and a flux-pumping pillbox argument, we show that $\mathcal{G}_{\circlearrowleft}$ is naturally quantized in Weber units and, under minimal assumptions, is fixed by a single-flux normalization $\mathcal{G}_{\circlearrowleft} \simeq \Phi_\star$ with $\Phi_\star$ a flux quantum (*a priori* h/e; in superconducting analogues $h/2e$) [1–4]. We give a rotating-frame derivation, dimensional checks, and experimental predictions (EMF spikes coincident with vortex nucleation during plate-area compression; integrated EMF $\simeq \Phi_\star \Delta N$).

1 Canonical objects and rotating foliation

SST adopts absolute time t and Euclidean space on leaves Σ_t, with a preferred congruence u^μ orthogonal to Σ_t. The Canon's chronos–Kelvin invariant enforces conservation of circulation at fixed topology,

$$\frac{D}{Dt}\Big(R^2\omega\Big) = 0 \quad \Longrightarrow \quad \Gamma \equiv \oint_{\mathcal{C}} \mathbf{v}\cdot d\boldsymbol{\ell} \;=\; N\,\kappa, \qquad N \in \mathbb{Z}, \tag{1}$$

where κ is the circulation quantum. Coarse graining over an area $A \subset \Sigma_t$ defines the *swirl (vortex-line) areal density vector*

$$\boldsymbol{\varrho}_{\circlearrowleft}(\mathbf{x},t) \;\equiv\; n_v(\mathbf{x},t)\,\hat{\mathbf{n}} \;=\; \frac{1}{A}\sum_{\ell \in A} \hat{\mathbf{t}}_\ell, \qquad [\boldsymbol{\varrho}] = \mathrm{m}^{-2}, \tag{2}$$

whose flux counts vortex lines through A:

$$\Phi_{\circlearrowleft}(t;A) = \int_A \boldsymbol{\varrho}_{\circlearrowleft}\cdot d\mathbf{A} = N(A,t). \tag{3}$$

Rotating frame merger. In a frame rotating with angular velocity $\boldsymbol{\Omega}$, the standard decomposition of absolute vorticity $\boldsymbol{\zeta}_a = \boldsymbol{\zeta}_r + 2\boldsymbol{\Omega}$ and the effective gravity $\mathbf{g}_{\text{eff}} = \mathbf{g} - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$ imply that centrifugal and gravitational contributions enter through *one* potential. In SST, this translates to a long-range *swirl gravity* channel: time-varying $\boldsymbol{\varrho}_{\circlearrowleft}$ couples to electromotive response via a single effective source $\mathbf{b}_{\circlearrowleft}$, i.e. the "centrifugal + gravity" merger manifests as

$$\nabla \times \mathbf{E} = -\,\partial_t \mathbf{B} - \mathbf{b}_{\circlearrowleft}, \qquad \mathbf{b}_{\circlearrowleft} = (\text{long-range response to } \partial_t \boldsymbol{\varrho}_{\circlearrowleft}). \tag{4}$$

2 Constitutive closure in matter (local tier)

At laboratory scales we assume two local, linear constitutive maps:

$$\mathbf{D} = \varepsilon\,\mathbf{E}, \qquad \mathbf{B} = \mu\,\mathbf{H}, \tag{5}$$

$$\boldsymbol{\varrho}_{\circlearrowleft} = \chi_H\,\mathbf{H}, \qquad [\chi_H] = \mathrm{m}^{-1}\mathrm{A}^{-1}, \tag{6}$$

where χ_H is a *swirl susceptibility*: stronger $\mathbf{H}$ aligns/admits more vortex lines per area in the medium. This is the right-hand (magnetic/swirl) mirror of Ohm's law on the left (electric/conduction) side,

$$\mathbf{j} = \sigma\,\mathbf{E}, \qquad [\sigma] = \mathrm{S\,m}^{-1}. \tag{7}$$

3 Pillbox theorem and the mixed topological coupling

Integrate (4) over a surface $S \subset \Sigma_t$ with boundary ∂S and time interval $[t_i, t_f]$:

$$\int_{t_i}^{t_f}\oint_{\partial S}\mathbf{E}\cdot d\boldsymbol{\ell}\,dt = -\,\Delta\Phi_B(S) \;-\; \int_{t_i}^{t_f}\int_S \mathbf{b}_{\circlearrowleft}\cdot d\mathbf{A}\,dt. \tag{8}$$

If the magnetic flux is held fixed ($\Delta\Phi_B = 0$), the *time-integrated EMF* equals minus the spacetime integral of $\mathbf{b}_{\circlearrowleft}$.

Now, by definition (3) the rate of change of swirl flux counts vortex nucleations/escapes through S:

$$\frac{d}{dt}\int_S \boldsymbol{\varrho}_{\circlearrowleft}\cdot d\mathbf{A} \;=\; \dot N(S,t). \tag{9}$$

Postulate the *mixed topological coupling* (EFT level)

$$\boxed{\mathbf{b}_{\circlearrowleft} \;=\; \mathcal{G}_{\circlearrowleft}\,\partial_t \boldsymbol{\varrho}_{\circlearrowleft}}, \qquad [\mathcal{G}_{\circlearrowleft}] = \mathrm{V\,s} = \mathrm{Wb}, \tag{10}$$

which is the unique linear, local-in-time map that (i) respects units ($\mathrm{V\,m}^{-2}$ on both sides of (4)), (ii) vanishes in steady states, and (iii) couples only to *topological* changes (nucleations/reconnections) via (9).

Inserting (10) into (8) and using (9) gives the *flux-pumping quantization*:

$$\boxed{\int_{t_i}^{t_f}\oint_{\partial S}\mathbf{E}\cdot d\boldsymbol{\ell}\,dt \;=\; -\,\mathcal{G}_{\circlearrowleft}\,\Delta N(S)}, \qquad \Delta N(S) = \int_{t_i}^{t_f}\dot N(S,t)\,dt \in \mathbb{Z}. \tag{11}$$

Thus each net vortex line added/removed through S produces a *quantized EMF-time impulse* set by $\mathcal{G}_{\circlearrowleft}$.

4 Electron logic: canonical normalization of $\mathcal{G}$

SST models the electron in its propagation phase as a toroidal ring $\mathcal{R}$ with tangential speed fixed by the Canon,

$$\|\mathbf{v}_{\circlearrowleft}\| \equiv C_e \approx 1.09384563\times 10^{6}\ \mathrm{m\,s}^{-1}, \qquad r_c \approx 1.40897\times 10^{-15}\ \mathrm{m}, \tag{12}$$

and core cross-section $A_c = \pi r_c^2$. When $\mathcal{R}$ knots ($\mathcal{T}$) or unknots, the swirl topology changes by $\Delta N = \pm 1$. The ring guides electromagnetic phase around its core; a minimal and natural normalization is to require that *one topological event* corresponds to *one flux impulse* of size $\Phi_\star$:

$$\int_{t_i}^{t_f}\oint_{\partial S_c}\mathbf{E}\cdot d\boldsymbol{\ell}\,dt \stackrel{!}{=} \;\Phi_\star\,\Delta N, \qquad S_c \sim \text{core disk}. \tag{13}$$

Comparing with (11) fixes

$$\boxed{\mathcal{G}_\circlearrowleft = \Phi_\star}, \tag{14}$$

i.e. the swirl–EMF transduction constant equals a *flux quantum.* For single-charged rings the Aharonov–Bohm quantum suggests $\Phi_\star = h/e$ [1]; for Cooper-paired media, $\Phi_\star = h/2e$ [2]. Which constant is realized is a *material/topology* question; either choice preserves (11) and yields a falsifiable prediction.

Dimensional and energetic consistency. Equation (14) gives $[\mathcal{G}_\circlearrowleft] = \mathrm{V\,s}$ as required by (10). Energetically, the EM work per event is $W = \int dt \oint \mathbf{E}\cdot d\boldsymbol{\ell}\, I_{\mathrm{loop}}(t)$. For weak backaction ($I_{\mathrm{loop}}$ set by readout), (13) predicts an *impulse* independent of drive details—an SST counterpart of flux quantization.

5 Rotating frame: centrifugal + gravity ⇒ b

Let the container rotate at $\boldsymbol{\Omega}$ while the plate area shrinks from A_0 to A. With swirl flux frozen (disconnected electrodes), flux conservation (3) implies

$$\boldsymbol{\varrho}_\circlearrowleft(A) = \frac{N\,\hat{\mathbf{n}}}{A}, \quad a(A) \sim n_v^{-1/2} = \sqrt{\frac{A}{N}}, \tag{15}$$

and nucleation when $a \lesssim \alpha r_c$. The rate $\partial_t \boldsymbol{\varrho}_\circlearrowleft$ is nonzero during nucleation bursts, and by (10) produces a nonzero $\mathbf{b}_\circlearrowleft$. In the rotating foliation, the absolute vorticity merger ensures that centrifugal forcing does not appear as a separate source: its effect is absorbed into the *long-range* channel represented by $\mathbf{b}_\circlearrowleft$. Combining these, we obtain the *two-tier symmetry*:

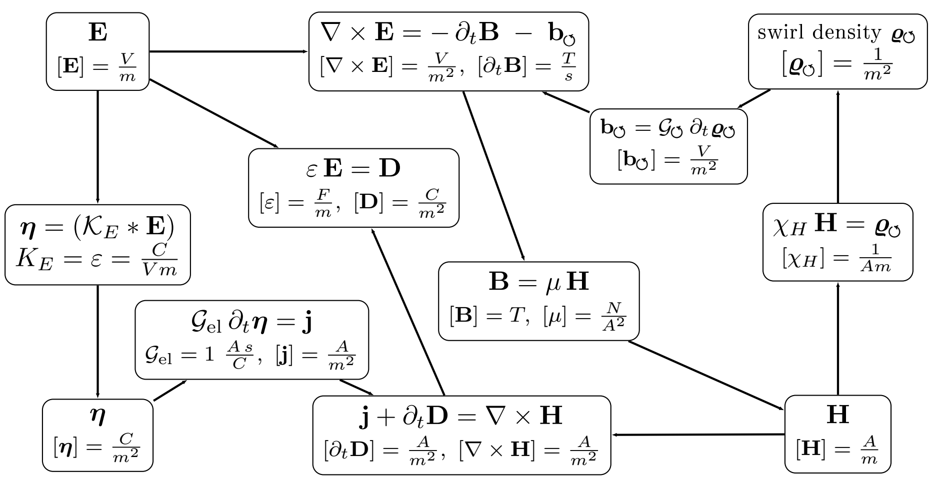
Local tier (mirror):

$$\boxed{\mathbf{j} = \sigma\mathbf{E} \quad \leftrightarrow \quad \boldsymbol{\varrho}_\circlearrowleft = \chi_H \mathbf{H}}.$$

Long-range tier (unification):

$$\boxed{\partial_t \boldsymbol{\varrho}_\circlearrowleft[\ \mathcal{G}_\circlearrowleft\ ] = \ \mathbf{b}_\circlearrowleft, \quad \text{centrifugal + gravity merged}}.$$

6 Complete diagram (with units and the long-range link)



- $\mathbf{b}_\circlearrowleft = \mathcal{G}_\circlearrowleft\,\partial_t\boldsymbol{\varrho}_\circlearrowleft$: swirl–EMF source, with units $[\mathbf{b}_\circlearrowleft]$.
- $\boldsymbol{\varrho}_\circlearrowleft$: swirl density, with units $[\boldsymbol{\varrho}_\circlearrowleft]$.
- Swirl–gravity mediation: $\mathcal{G}_\circlearrowleft = \Phi_\star$.

- $\boldsymbol{\eta}$: conduction accumulation, with units $[\boldsymbol{\eta}]$.

- $\mathcal{K}_E$: a constitutive kernel (electric side), mapping the field $\mathbf{E}$ into an areal charge accumulation $\boldsymbol{\eta}$. In the simplest (local, isotropic) form:

$$\boldsymbol{\eta} = \varepsilon\,\mathbf{E}$$

  but written as $(\mathcal{K}_E * \mathbf{E})$, it allows for spatial/temporal nonlocal response (like a susceptibility kernel).

- χ_H: a swirl susceptibility (magnetic side), mapping the field $\mathbf{H}$ into the swirl density $\boldsymbol{\varrho}_{\circlearrowleft}$:

$$\boldsymbol{\varrho}_{\circlearrowleft} = \chi_H\,\mathbf{H}$$

  Units: $[\chi_H] = \mathrm{m}^{-1}\mathrm{A}^{-1}$. It plays the same role as an electric or magnetic susceptibility, but in the SST Canon it measures how strongly $\mathbf{H}$ seeds swirl line density.

7 From the Canon to a value for $\mathcal{G}$

Equation (14) sets the *scale* of $\mathcal{G}_{\circlearrowleft}$; SST "electron logic" refines it:

(i) Topological normalization. The ring $\mathcal{R}$ carries an integer winding N; knotting/unknotting changes $N \to N \pm 1$. A single event thus generates an EMF-time impulse $\Phi_\star$ by (11)–(13).

(ii) Energetic matching. The ring's effective energy change for $\Delta N = \pm 1$ is

$$\Delta E \simeq (\epsilon_0 A_c + \beta)\,\Delta L + \alpha C(\mathcal{T}) + \gamma \mathcal{H}(\mathcal{T}), \tag{16}$$

with Canon bulk term ϵ_0 and line/helicity/contact coefficients (as in the SST Lagrangian). A resonant photon of $\hbar\omega_0 \approx \Delta E$ mediates the transition. The EMF impulse $\Phi_\star$ does *no net* work without a readout current; thus energetic matching does not fix $\Phi_\star$—it fixes *rates* (Rabi), while (14) fixes the *topological size*. This separation is natural in a mixed topological term.

(iii) Choice of $\Phi_\star$. For single-charge matter waves, the Aharonov–Bohm flux quantum h/e is the canonical choice [1]; in superconducting media, $h/2e$ applies [2]. Measuring EMF-time impulses during controlled vortex nucleation discriminates these cases.

8 Predictions & experimental program

- **Plate compression (levitated PG/electret stack).** With electrodes disconnected (frozen charge), shrink the effective plate area so that the swirl flux cannot escape. Monitor a pickup loop around the active region. Prediction:

$$\int dt\,\mathrm{EMF}(t) \;=\; \Phi_\star\,\Delta N, \quad \Delta N \in \mathbb{Z},$$

  with bursts coincident with vortex nucleation (when $a \lesssim \alpha r_c$).

- **Rotating frame.** Repeat while ramping Ω. The threshold area $A_\star(\Omega)$ for first nucleation obeys $N/A_\star \simeq 2\Omega/\kappa$ (Feynman relation), and EMF-time impulse remains quantized by $\Phi_\star$.

- **Pump–probe control.** A resonant optical pump at ω_0 modulates the nucleation rate $\propto |\partial_t \boldsymbol{\varrho}|$; the *integrated* EMF per event remains $\Phi_\star$ (topologically protected), while the *temporal* profile tracks the pump.

9 Boxed summary (SST Canon ⇒ diagram)

$$\boxed{\begin{array}{c} \text{(Kelvin/Canon)} \quad \Gamma = \oint \mathbf{v}\cdot d\boldsymbol{\ell} = N\kappa, \quad \Phi_{\circlearrowleft} = \int_A \boldsymbol{\varrho}\cdot d\mathbf{A} = N \\ \text{(local mirror)} \quad \mathbf{j} = \sigma\mathbf{E} \ \leftrightarrow \ \boldsymbol{\varrho} = \chi_H \mathbf{H} \\ \text{(long-range unification)} \quad \nabla\times\mathbf{E} = -\partial_t\mathbf{B} - \mathbf{b}_{\circlearrowleft}, \quad \mathbf{b}_{\circlearrowleft} = \mathcal{G}_{\circlearrowleft}\,\partial_t\boldsymbol{\varrho} \\ \text{(electron normalization)} \quad \mathcal{G}_{\circlearrowleft} = \Phi_\star \in \{h/e,\ h/2e\}, \quad \int dt \oint \mathbf{E}\cdot d\boldsymbol{\ell} = -\,\Phi_\star\,\Delta N \end{array}}$$

Dimensional checks. $[\boldsymbol{\varrho}] = \mathrm{m}^{-2}$, $[\partial_t\boldsymbol{\varrho}] = \mathrm{m}^{-2}\mathrm{s}^{-1}$; $[\mathcal{G}_{\circlearrowleft}] = \mathrm{V\,s}$ so $[\mathbf{b}_{\circlearrowleft}] = \mathrm{V\,m^{-2}}$ matches $[\partial_t\mathbf{B}] = \mathrm{T\,s^{-1}} = \mathrm{V\,m^{-2}}$; all local maps in (6) use standard SI.

Acknowledgement of canonical constants

Where numerical evaluation is desired, adopt the Canon values C_e, r_c, ρ^{core}, ρ provided in the SST Canon; these enter rate and threshold estimates (via $a \sim \sqrt{A/N}$ and r_c), but not the quantized *magnitude* $\Phi_\star$ of the EMF-time impulse.

References

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