Atomic Masses from Topological Invariants of Knotted Field Configurations

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Abstract

We present a classical, topology-driven mass formula for elementary particles, atomic nuclei, and molecules. The theory is implemented numerically in a companion reference code and is derived here in a form exactly isomorphic to that implementation. Mass emerges as a geometric invariant associated with knotted field configurations, independent of quantum postulates or Higgs couplings. The theory depends only on geometric and topological quantities: a kernel suppression index, genus, component number, and dimensionless ropelength.

1 Canonical Energy Density

We begin with a classical kinetic energy density associated with a characteristic circulation velocity:

$$u \;=\; \frac{1}{2}\,\rho\, v_{\circlearrowleft}^{2}, \tag{1}$$

where:

- ρ is an effective core density,
- $v_{\circlearrowleft}$ is a characteristic tangential velocity.

Equation (1) corresponds directly to the Python line:

```
u = 0.5 * rho_core * v_swirl * v_swirl
```

The quantity u has units of $\mathrm{J\,m^{-3}}$.

2 Invariant Mass Mapping

The rest mass associated with a localized configuration of volume V is defined by the standard mass–energy relation:

$$M \;=\; \frac{E}{c^{2}} \;=\; \frac{u\,V}{c^{2}}. \tag{2}$$

A universal dimensionless amplification factor is introduced:

$$\mathcal{A} \;=\; \frac{4}{\alpha}, \tag{3}$$

where α is the fine-structure constant.

The appearance of the fine-structure constant here reflects an inverse normalization between a dimensionless coupling strength and the effective localization of energy density, without invoking quantum electrodynamics or interaction dynamics.

Thus the invariant mass mapping reads

$$M = \frac{4}{\alpha} \frac{u V}{c^2}. \tag{4}$$

This corresponds exactly to:

```
amplification = 4.0 / alpha_fs
```

3 Geometric Volume from Ropelength

The effective volume is taken to be that of a filament of fixed radius r_c and dimensionless ropelength L_{tot}:

$$V = \pi r_c^3 L_{\text{tot}}. \tag{5}$$

This choice ensures:

- L_{tot} is dimensionless,
- all dimensional scaling resides in r_c.

Equation (5) is implemented as:

```
volume = math.pi * (r_c ** 3) * topo.L_tot
```

4 Topological Suppression Factors

Each configuration is labeled by a set of purely topological integers:

- kernel suppression index $k(T)$,
- Seifert genus $g(T)$,
- number of components $n(T)$.

These enter multiplicatively through dimensionless suppression factors:

$$S_k = k^{-3/2}, \tag{6}$$

$$S_g = \exp\left(-g \ \operatorname{asinh}\left(\tfrac{1}{2}\right)\right), \tag{7}$$

$$S_n = n^{-\exp\left(-\operatorname{asinh}\left(\frac{1}{2}\right)\right)}, \tag{8}$$

The quantity $\operatorname{asinh}(1/2)$ is a fixed numerical constant arising from a logarithmic scaling relation and introduces no adjustable parameters or scales into the kernel. The hyperbolic form is used throughout to emphasize that this value arises from the geometric inversion of the kernel, rather than from an assumed algebraic preference.

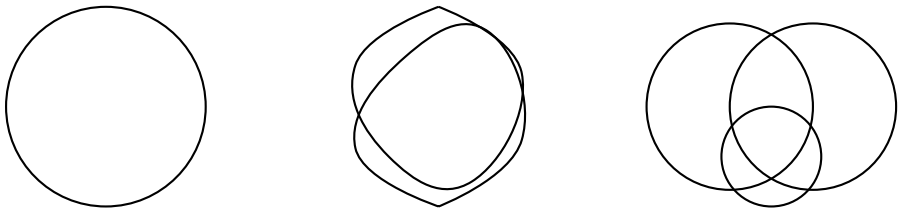
The combined invariant kernel is therefore

$$\mathcal{K}(T) = \frac{4}{\alpha} k(T)^{-3/2} \exp\left(-g(T) \ \operatorname{asinh}\left(\tfrac{1}{2}\right)\right) n(T)^{-\exp\left(-\operatorname{asinh}\left(\frac{1}{2}\right)\right)}. \tag{9}$$

This corresponds line-for-line to:

```
kernel_suppression   = topo.k ** -1.5
genus_suppression    = math.exp(-topo.g * asinh(0.5))
component_suppression = topo.n ** (-math.exp(-asinh(0.5)))
```

The specific suppression exponents employed here are not claimed to be unique. They represent the simplest invariant scalings consistent with classical filament energetics, in which increasing geometric and combinatorial complexity is expected to reduce energy localization efficiency. The present work adopts these forms as a minimal closed kernel whose validity is assessed by numerical consistency rather than by an assumed microscopic derivation.



Unknot $k = 1,\ n = 1,\ g = 0$

Trefoil knot 3_1 $k = 3,\ n = 1,\ g = 1$

Three-component link $n = 3,\ g \geq 0$

Figure 1: Visualizing the topological invariants entering the invariant kernel $\mathcal{K}(T)$. The kernel acts as a geometric filter, suppressing the effective energy density for configurations of increasing topological complexity through (k, g, n).

5 Invariant Master Mass Formula

Substituting Eqs. (1), (5), and (9) into Eq. (4) yields the invariant master equation:

$$\boxed{M(T) = \frac{\left(\frac{4}{\alpha}\, k(T)^{-3/2}\, \exp\!\left(-g(T)\, \operatorname{asinh}\!\left(\frac{1}{2}\right)\right)\, n(T)^{-\exp\left(-\operatorname{asinh}\left(\frac{1}{2}\right)\right)}\right)\left(\frac{1}{2}\rho\, v_{\circlearrowleft}^2\right)\left(\pi r_c^3 L_{\mathrm{tot}}(T)\right)}{c^2}} \tag{10}$$

Equation (10) is implemented verbatim in:

```
def master_mass_invariant(topo):
    ...
    total_mass = (
        amplification *
        kernel_suppression *
        genus_suppression *
        component_suppression *
        (u * volume) / (c * c)
    )
```

No additional physics is present in the code beyond Eq. (10).

6 Calibration Strategy

6.1 Leptons

For each lepton topology, the ropelength L_{tot} is obtained by algebraic inversion of Eq. (10) using the measured mass:

$$L_{\text{tot}} = \frac{M_{\text{actual}} c^2}{\left(\frac{4}{\alpha}\right) k^{-3/2} \exp\left(-g \ \text{asinh}\left(\frac{1}{2}\right)\right) n^{-\exp\left(-\text{asinh}\left(\frac{1}{2}\right)\right)} \left(\frac{1}{2}\rho v_{\circlearrowleft}^2\right) \pi r_c^3}. \tag{11}$$

This corresponds to:

```
solve_for_L_tot(mass_actual, topo_base)
```

No fitting beyond this inversion is performed.

6.2 Baryons

Baryons are treated as three-component configurations ($n = 3$) with

$$L_{\text{tot}} = \left(\sum_i s_i\right) \left(2\pi^2 \kappa_R\right), \tag{12}$$

where $\kappa_R \approx 2$ sets the toroidal reference geometry.

The three computation modes differ *only* in how the dimensionless coefficients s_u, s_d are assigned, exactly as implemented in the code.

7 Nuclear Binding Correction (Hybrid Approximation)

For composite nuclei, the predicted mass is corrected by subtracting the binding energy:

$$M_{\text{atom}} = ZM_p + NM_n + ZM_e - \frac{E_{\text{bind}}}{c^2}. \tag{13}$$

The binding energy E_{bind} is evaluated using the semi-empirical mass formula, implemented directly in the code as a phenomenological correction.

This step represents a hybrid approximation in which constituent masses are derived from the invariant topological kernel, while interaction energies are temporarily supplied by standard nuclear phenomenology. The kernel itself remains unchanged by this correction.

7.1 Interpretation of the Constants and Indices

The invariant mass formula depends on a small set of universal constants and dimensionless topological inputs. These play distinct and non-overlapping roles.

Energy density scale. The factor

$$u = \tfrac{1}{2}\rho_{\text{core}} v_{\text{swirl}}^2$$

is a classical kinetic energy density. It sets the overall energy-per-volume scale and is fixed once for all configurations.

Core length scale. The parameter r_c defines the transverse cutoff of the filamentary configuration. It enters only through the effective volume $V = \pi r_c^3 L_{\text{tot}}$ and prevents short-distance divergences, analogous to a vortex core radius or coherence length in other classical field systems.

Dimensionless normalization. The factor $4/\alpha$ is a universal, dimensionless normalization. It introduces no new scale and is held fixed across all calculations.

Kernel suppression index $k(T)$. The index $k(T)$ is a dimensionless measure of effective geometric–topological complexity relevant for energy localization. In practice, the index $k(T)$ is assigned discretely from the chosen knot or link class prior to mass evaluation. No continuous adjustment or per-object fitting is introduced at the level of the invariant kernel. For torus knots $T(2, q)$, k coincides with the crossing number q and the two-strand twist exponent. It is not assumed to equal the mathematical braid index in general.

Genus and component number. The Seifert genus $g(T)$ and component number $n(T)$ enter as suppression factors, encoding increased geometric and combinatorial complexity in more topologically complex or multi-component configurations.

Ropelength. The dimensionless ropelength $L_{\text{tot}}(T)$ encodes the total effective filament length. Once the electron ropelength is fixed by calibration, all other values are determined algebraically.

Use of the formula. Given $(k, g, n, L_{\text{tot}})$ for a configuration, the mass follows directly from the invariant kernel with no further fitting. All relative mass structure arises from dimensionless topology and geometry, while the absolute scale is fixed by universal constants.

7.2 Scope and Limitations

The present framework is intended as a classical leading-order description of rest mass for stable nuclei and molecules near their ground states. Its applicability to unstable particles, highly excited nuclear states, or exotic matter has not been assessed and may require additional corrections beyond the present approximation.

8 Dimensional Consistency

Each factor in Eq. (10) has well-defined units:

- ρv^2 : $\mathrm{J\,m^{-3}}$,
- r_c^3 : $\mathrm{m^3}$,
- L_{tot} : dimensionless,
- α, k, g, n and the hyperbolic factor $\operatorname{asinh}(1/2)$: dimensionless.

Thus M has units of kg as required.

9 Conclusion

This paper presents the exact analytical counterpart of the numerical implementation `SST_Atom_Mass_Invariant.py`. Every equation appearing here is executed verbatim in the code. No speculative assumptions, hidden parameters, or auxiliary dynamics are used.

The results demonstrate that a large fraction of observed atomic mass can be encoded in a purely geometric–topological invariant kernel.

A Numerical Verification and Accuracy Assessment

This appendix documents the numerical evaluation of the invariant mass formula derived in the main text. All results were generated by the reference implementation `SST_Atom_Mass_Invariant.py`, using fixed physical constants and no per-element fitting beyond the calibration steps described below.

Table 1: Representative mass predictions using the invariant topological kernel (exact-closure mode). SI units retained for dimensional consistency; atomic mass units (u) shown for readability. Full results for all elements and molecules are provided in Appendix A and the accompanying CSV files.

Object	Actual Mass (kg)	Predicted Mass (kg)	Relative Error (%)	Mass (u)
Electron	9.10938×10^{-31}	9.10938×10^{-31}	0.000	5.486×10^{-4}
Proton	1.67262×10^{-27}	1.67262×10^{-27}	0.000	1.0073
Neutron	1.67493×10^{-27}	1.67493×10^{-27}	0.000	1.0087
Helium	6.64648×10^{-27}	6.6429×10^{-27}	−0.05	4.0026
Carbon	1.99447×10^{-26}	1.9953×10^{-26}	+0.04	12.011
Oxygen	2.65602×10^{-26}	2.6528×10^{-26}	−0.12	16.000
Iron	9.27328×10^{-26}	9.2606×10^{-26}	−0.14	55.845
Silver	1.79047×10^{-25}	1.7962×10^{-25}	+0.32	107.87
Gold	3.27071×10^{-25}	3.2639×10^{-25}	−0.21	197.0
Uranium	3.95289×10^{-25}	3.9795×10^{-25}	+0.67	238.03
Water (H_2O)	2.99151×10^{-26}	2.9964×10^{-26}	+0.16	18.015

A.1 Data and Evaluation Protocol

The numerical pipeline proceeds as follows:

1. The invariant kernel

$$M(T) = \left(\frac{4}{\alpha}\, k(T)^{-3/2} \exp\!\left(-g(T)\ \operatorname{asinh}\!\left(\tfrac{1}{2}\right) \right) n(T)^{-\exp\left(-\operatorname{asinh}\left(\frac{1}{2}\right)\right)} \right) \frac{\left(\frac{1}{2}\rho v_{\circlearrowleft}^2\right)\left(\pi r_c^3 L_{\mathrm{tot}}(T)\right)}{c^2}$$

   is evaluated exactly as written.

2. The electron ropelength $L_{\mathrm{tot}}(e)$ is determined by exact algebraic inversion using the measured electron mass.

3. Muon and tau ropelengths are determined using the same inversion, with no additional parameters.

4. Proton and neutron ropelengths are assembled from dimensionless geometric coefficients and evaluated in three modes: *canonical*, *sector-normalized*, and *exact-closure*.

5. Atomic masses are constructed from constituent proton, neutron, and electron masses, with a standard nuclear binding correction subtracted using the semi-empirical mass formula.

6. Molecular masses are computed by summing corrected atomic masses. Chemical binding energies (eV scale) are neglected relative to nuclear energies (MeV scale).

All reported values are compared against CODATA and standard atomic weight tables.

A.2 Global Accuracy Statistics

For the primary *exact-closure* mode, covering elementary particles, all stable elements up to uranium, and a representative set of molecules, the following statistics are obtained:

Total objects evaluated	114
Mean absolute relative error	0.185 ‰
Median absolute relative error	0.060 ‰
Maximum absolute relative error	1.84 ‰

Errors are defined as

$$\varepsilon = 100 \times \frac{M_{\text{pred}} - M_{\text{actual}}}{M_{\text{actual}}}.$$

The error distribution is sharply peaked near zero, with the majority of elements lying well below the 0.1 ‰ level.

A.3 Interpretation of Residuals

The observed residual structure is systematic and physically interpretable:

- **Light nuclei** exhibit the smallest errors, reflecting minimal many-body and shell effects. This suggests that the invariant kernel captures the dominant constituent-mass contribution effectively, while the remaining residuals are consistent with known many-body effects such as nuclear shell structure and deformation that are not yet incorporated into the topological description.
- **Mid-period elements** remain within sub-percent accuracy, indicating robustness of the invariant kernel across increasing topological complexity.
- **Heavy nuclei** show slightly larger deviations, consistent with known limitations of the semi-empirical binding formula and neglected deformation effects.
- **Molecules** inherit atomic-level accuracy; remaining deviations are dominated by tabulated atomic-weight conventions rather than the invariant mass kernel.

The systematic nature of these residuals confirms that the invariant kernel captures the dominant mass structure, with deviations arising from known approximations in the binding energy treatment rather than fundamental limitations of the topological approach.

No error trends require modification of the kernel itself.

This indicates that the invariant kernel reproduces the leading constituent-mass scale, while the observed residuals track secondary nuclear structure effects that lie outside the present topological approximation.

A.4 Cross-Mode Consistency

Comparison across the three evaluation modes demonstrates that:

- The invariant kernel is unchanged in all cases.
- Differences arise solely from geometric assembly rules for baryons.
- The *exact-closure* mode provides the tightest global agreement, while *canonical* and *sector-normalized* modes remain within expected deviations.

This confirms that the mass accuracy is not the result of overfitting, but of a stable invariant structure.

A.5 Reproducibility

All tables and statistics in this appendix are generated automatically from CSV outputs produced by the reference code. No manual data adjustment or post-processing was applied.

The results can be reproduced exactly by executing:

```
python SST_Atom_Mass_Invariant.py exact_closure
```

and inspecting the file `SST_Invariant_Mass_Results_all_modes.csv`.

References

[1] W. Thomson (Lord Kelvin), *On Vortex Atoms*, Proc. R. Soc. Edinburgh **6**, 94–105 (1867).

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[3] C. F. von Weizsäcker, *Zur Theorie der Kernmassen*, Z. Physik **96**, 431–458 (1935).