Appendix: Derivation of Entropic Force in the Vortex Æther Model (VAM)

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June 27, 2025

Abstract

This appendix explores the convergence of thermodynamic gravity, variable light speed theories, and gauge torsion with the Vortex Æther Model (VAM). By mapping Verlinde's entropic force law to pressure gradients in swirling æther, we reinterpret inertia and gravitation as emergent phenomena from vorticity-induced entropy flows. The speed of light, rather than a fixed invariant, emerges as a density-dependent wave speed modulated by local ætheric swirl, offering a VAM explanation of cosmological horizon problems. We further connect torsion in gauge gravity theories to the antisymmetric vorticity tensor in the æther, grounding spacetime torsion in vortex topology. Experimental effects—including the Sagnac shift, Rømer delay, and Schwarzschild collapse—are rederived in fluid-dynamic terms. Finally, we extend the VAM framework to account for the cosmological evolution of physical constants and noncommutative black hole geometries as manifestations of vortex quantization and minimal circulation domains. This unified approach offers new fluid-based insights into the fundamental architecture of gravitational and quantum fields.

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In this appendix, we derive a VAM-consistent analog of Verlinde's entropic force equation:

$$F = T\frac{\Delta S}{\Delta x}, \tag{1}$$

and reinterpret all terms within the framework of vortex-based æther dynamics. The goal is to ground the entropic force in the structured vorticity fields of the Vortex Æther Model (VAM), using fundamental fluid variables such as local angular velocity Ω, æther density $\rho_{æ}$, and swirl-clock phase memory $S(t)$.

1. Swirl Clock as Entropy

In VAM, the swirl clock $S(t)$ tracks the phase evolution of a vortex:

$$S(t) = \int \Omega(r(t'))\, dt'. \tag{2}$$

This acts as an angular-memory or identity phase. The entropy gradient in Verlinde's formulation becomes:

$$\frac{\Delta S}{\Delta x} \sim \frac{d\Omega}{dx} \cdot \Delta t. \tag{3}$$

This defines a local change in phase memory across a spatial distance Δx.

2. Effective Temperature from Swirl Energy

We define an effective temperature as a thermal analogue of the local rotational energy per degree of freedom:

$$T_{\text{eff}} = \frac{1}{2k_B}\rho_{æ}\Omega^2 r^2. \tag{4}$$

This connects the kinetic swirl energy to a thermodynamic-like quantity usable in the entropic force expression.

3. Entropic Force in VAM

Substituting into the original equation:

$$\begin{aligned} F &= T_{\text{eff}} \cdot \frac{\Delta S}{\Delta x} \qquad (5)\\ &= \left(\frac{1}{2k_B}\rho_{æ}\Omega^2 r^2\right) \cdot \left(\frac{d\Omega}{dx} \cdot \Delta t\right) \qquad (6)\\ &\sim \frac{\rho_{æ}}{2k_B}\Omega^2 r^2 \cdot \frac{d\Omega}{dx} \cdot \Delta t. \qquad (7) \end{aligned}$$

4. Comparison with Pressure Gradient Force

VAM models forces as arising from Bernoulli-type swirl pressure:

$$F_{\text{vortex}} = -\nabla P = -\frac{1}{2}\rho_{æ}\nabla|\vec{\omega}|^2. \tag{8}$$

Letting $\vec{\omega} = \Omega(r)\hat{\theta}$, we compute:

$$\begin{aligned} F &\sim -\frac{1}{2}\rho_{æ}\nabla(\Omega^2 r^2) \qquad (9)\\ &= -\rho_{æ}\left(\Omega\frac{d\Omega}{dr}r^2 + \Omega^2 r\right). \qquad (10) \end{aligned}$$

This matches the qualitative structure of the entropic force when $\frac{\Delta S}{\Delta x} \sim \frac{d\Omega}{dx}$.

5. Final VAM Entropic Force Expression

We summarize the derived VAM-compatible expression as:

$$F = \left(\frac{1}{2}\rho_{æ}\Omega^2 r^2\right) \cdot \left(\frac{d\Omega}{dx} \cdot \Delta t\right) \tag{11}$$

or, in full gradient form:

$$F = -\rho_{æ}\left(\Omega \frac{d\Omega}{dr} r^2 + \Omega^2 r\right). \tag{12}$$

This shows that entropy-driven forces emerge naturally from structured angular motion in the æther.

6. Interpretation and Outlook

This derivation grounds Verlinde's concept of gravity as an entropic force in concrete fluid dynamics. Rather than invoking information bits on holographic screens, the VAM replaces them with physical vortex swirl memory $S(t)$ and energy gradients. This paves the way for testable predictions linking vorticity and inertia.

Next steps may include:

- Deriving the vortex potential energy corresponding to this force.
- Exploring entropy production during vortex reconnection events $K(x, \tau)$.
- Comparing with Unruh temperature analogs in accelerated vortex frames.

7. Entropy as Function of Vortex Area and Swirl Phase

To connect entropy with structured vorticity, we define entropy in VAM as a function of vortex energy per unit phase memory over the core area:

$$\mathcal{S}(t) = \frac{1}{T} \cdot \frac{E_{\text{vortex}}(A_v)}{S(t)} \tag{13}$$

where:

- $E_{\text{vortex}} = \int_{A_v} \frac{1}{2}\rho_{æ}\Omega^2 \, dA$
- $S(t) = \int_0^t \Omega(t') \, dt'$
- $A_v = \pi r_c^2$: vortex core area

Assuming cylindrical symmetry,

$$E_{\text{vortex}} = \frac{1}{2}\rho_{æ} \int_0^{r_c} \Omega^2(r) \cdot 2\pi r \, dr \tag{14}$$

So the entropy becomes:

$$\boxed{\mathcal{S}(t) = \frac{\rho_{æ}\pi}{T} \cdot \frac{\int_0^{r_c} \Omega^2(r) r \, dr}{\int_0^t \Omega(t') dt'}} \tag{15}$$

This gives a field-based entropy grounded in æther swirl dynamics, rather than microstate statistics.

Interpretation

- Numerator: Total stored energy in the vortex cross-section.
- Denominator: Accumulated swirl phase — the memory of angular identity.
- $\mathcal{S}(t)$: Rotational entropy density, indicating information per phase per unit area.

This provides a vortex-theoretic alternative to Boltzmann entropy and supports future modeling of irreversible dynamics such as bifurcation (Kairos) events.

8. Mapping Verlinde's Screen Bits to Vortex Topology

In Verlinde's framework, a holographic screen encodes information as discrete "bits," with the number of bits scaling as:

$$N = \frac{Ac^3}{G\hbar} \quad \Rightarrow \quad S = k_B N. \tag{16}$$

In VAM, the screen corresponds to the vortex core surface, and bits are replaced by physical vortex characteristics:

- **Bit** $\leftrightarrow$ **Winding number** $n \in \mathbb{Z}$
- **Total bits** $\leftrightarrow$ $N_{\text{vortex}} = 2\pi n$
- **Entropy** $\leftrightarrow$ $S = k_B \cdot H$: helicity quantization

The vortex helicity is given by:

$$H = \sum_i \Gamma_i^2 (\mathcal{T}^{(i)} + \mathcal{W}^{(i)}) \tag{17}$$

where $\Gamma_i = 2\pi n_i \kappa$ is circulation, and $\mathcal{T}, \mathcal{W}$ represent twist and writhe respectively.

Thus, entropy becomes:

$$\boxed{S_{\text{vortex}} = k_B \sum n_i^2 (\mathcal{T}^{(i)} + \mathcal{W}^{(i)})} \tag{18}$$

Each quantized swirl structure carries information via its topological configuration. This replaces Verlinde's flat screen with a knotted, rotating geometry storing information in winding and linking.

Interpretation

- Bits are *phase loops*, not area pixels.
- Helicity H measures stored topological information.
- $N_{\text{bits}}^{(\text{VAM})} = H/\hbar$ gives a quantized information count.

This mapping allows Verlinde's emergent gravity picture to be implemented with tangible fluid structures in the æther.

9. Side-by-Side Comparison: Verlinde vs VAM

Concept	Verlinde	VAM (Vortex Æther Model)
Information unit	Bit on holographic screen	Vortex winding number $n \in \mathbb{Z}$
Screen area	$A = 4\pi r^2$	Vortex core cross-section $A_v = \pi r_c^2$
Total bits	$N = \frac{Ac^3}{G\hbar}$	$N = \sum_i 2\pi n_i$ from circulation Γ_i
Entropy	$S = k_B N$	$S = k_B \sum n_i^2 (\mathcal{T}^{(i)} + \mathcal{W}^{(i)})$
Force law	$F = T\frac{\Delta S}{\Delta x}$	$F = T_{\text{eff}} \cdot \frac{d\Omega}{dx} \cdot \Delta t$
Temperature	$T = \frac{\hbar a}{2\pi c k_B}$ (Unruh)	$T_{\text{eff}} = \frac{1}{2k_B}\rho_{æ}\Omega^2 r^2$
Storage medium	Holographic surface	Toroidal vortex topology and swirl phase memory $S(t)$
Underlying mechanism	Information displacement near screen	Vorticity-induced energy gradients in fluid æther
Quantization basis	Area-encoded bits	Circulation and helicity: $H = \sum \Gamma^2 (\mathcal{T} + \mathcal{W})$

Table 1: Comparison of core concepts and equations in Verlinde's entropic gravity and the Vortex Æther Model.

10. VAM Action for Global Swirl Field $S(x,t)$

Inspired by the JT gravity scalar field formulation $X(u,v)$, we define an analogous action in the Vortex Æther Model (VAM) based on the swirl clock phase field $S(x,t)$. This action governs how temporal structure and gravitational analogues emerge from vorticity phase memory.

Field Definition

We define $S(x,t)$ as the swirl phase memory field, tracking accumulated rotation in the fluid æther. In 1D, this field determines:

- Temporal swirl energy via $\partial_t S$
- Spatial swirl pressure via $\partial_x S$

VAM Action Functional

We propose the following action:

$$\boxed{S_{\text{VAM}}[S] = \int dx\, dt \left[\frac{1}{2}\rho_{æ} (\partial_t S)^2 - \frac{1}{2}\rho_{æ} C_e^2 (\partial_x S)^2 + \Lambda \left\{ S(t), t \right\} \right]} \tag{19}$$

Interpretation

- $(\partial_t S)^2$: Kinetic energy of phase change (swirl clock ticking)
- $(\partial_x S)^2$: Spatial swirl gradient or vortex tension
- $\{S(t), t\}$: Schwarzian term—measures chaotic swirl deformation
- C_e: Core swirl speed constant
- Λ: Coupling constant for time chaos sensitivity

Equations of Motion

Variation of the action yields:

$$\rho_{æ}(\partial_t^2 S - C_e^2 \partial_x^2 S) = -\Lambda \frac{d}{dt}\left[\frac{S'''(t)}{S'(t)} - \frac{3}{2}\left(\frac{S''(t)}{S'(t)}\right)^2\right] \tag{20}$$

This shows that swirl dynamics in VAM can exhibit Schwarzian-instability when the clock phase becomes nonuniform.

Toward 2D or 3D Generalization

In higher dimensions, the action can include:

- Laplacian terms: $|\nabla^2 S|^2$ for vortex tension
- Helicity-based terms: $\mathcal{H} = \vec{v} \cdot \vec{\omega}$
- Topological charges: Hopf index or linking number

This provides a vortex-dynamic variational basis for emergent gravity in the Vortex Æther framework, rooted in physical swirl phase evolution.

11. Cosmological Evolution of Constants in VAM

Stanyukovich [1] proposed that fundamental constants—such as the gravitational constant G, the Planck constant $\hbar$, and the speed of light c—may evolve cosmologically through their dependence on scalar curvature R. In his approach, the Compton wavelength of a nucleon is expressed as:

$$\lambda^3 = \frac{2G\hbar}{c^2 H}, \tag{21}$$

where H is the Hubble constant. This relation links microphysics (via λ) to large-scale structure.

In the Vortex Æther Model (VAM), this suggests that physical constants emerge from the dynamical properties of the æther itself. Since G in VAM is derived from circulation parameters and vortex interaction, it is plausible that the large-scale swirl configuration of the cosmos modifies the apparent values of these constants.

The proposed scaling laws:

$$\hbar \sim R,$$
$$G \sim R^{-1/2},$$

can be recast in VAM as emergent constants from varying background swirl curvature:

$$G_{\text{swirl}} = \frac{C_e c^5 t_p^2}{2F_{\max} r_c^2} \cdot f(R), \tag{22}$$

where $f(R)$ encodes global ætheric swirl modulation. This supports the view that constants are not fundamental, but phase-state parameters of the superfluid æther.

Interpretation

- Cosmological evolution of constants maps to large-scale swirl evolution in VAM.
- Constants such as G and $\hbar$ emerge from collective topological vorticity structure.
- VAM provides a physical mechanism for Stanyukovich's curvature-driven evolution.

Unified Time Dilation as Ætheric Relative Motion

Within the Vortex Æther Model (VAM), time dilation is reinterpreted as a unified consequence of relative motion through a dynamical æther flow. The local clock rate is expressed as:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{|\vec{u} - \vec{v}_g|^2}{c^2}}$$

where $\vec{u}$ is the local æther flow velocity and $\vec{v}_g$ is the object's velocity. This formulation subsumes both special and general relativistic effects into a single framework based on effective swirl-relative velocity.

Entropic Gravity as Swirl-Induced Pressure Gradient

Verlinde's entropic gravity framework [2] is interpreted in VAM as an emergent phenomenon driven by swirl-induced pressure gradients. His proposed entropic force:

$$F = T\frac{\Delta S}{\Delta x}$$

is mapped to pressure gradients in the æther vortex field. The entropy shift associated with displacement:

$$\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x$$

parallels the rotational energy stored in vortex tangential motion.

The equipartition principle,

$$Mc^2 = \frac{1}{2}Nk_B T,$$

is reinterpreted in VAM as quantized energy stored within discrete æther volumes, linking thermodynamic and fluid dynamic perspectives.

Table 2: Conceptual Correspondence Between Entropic Gravity and VAM

Verlinde Concept	VAM Interpretation
Entropy gradient	Swirl-induced pressure drop
Holographic screen	Vortex boundary with helicity content
Equipartition energy	Core quantized swirl energy
Unruh effect	Kinetic swirl temperature
Inertial mass from ΔS	Swirl resistance to displacement

Emergent Speed of Light from Swirl Density

The hypothesis of a variable speed of light [3] is naturally realized in VAM via local æther density effects. Wave propagation speed varies with swirl-induced pressure and density:

$$c^2 \propto \frac{\partial P}{\partial \rho \sim \frac{F_{\max}}{\rho}}$$

In high-density vortex cores ($\rho_{\sim 10^{18}}$ kg/m^3), the local effective speed of light $c_{\text{local}} \ll c_\infty$.

Time dilation in such regions follows:

$$dt_{\text{local}} = dt_\infty \sqrt{1 - \frac{|\vec{\omega}|^2}{c^2}}$$

Gauge Torsion as Ætheric Vorticity

In line with Minkevich's gauge gravity formalism [4], torsion is understood in VAM as a manifestation of ætheric vorticity:

$$T^{\lambda}_{\mu\nu} \sim \epsilon^{\lambda}_{\ \mu\nu\sigma}\omega^{\sigma}$$

This identification directly connects Cartan's geometric torsion to fluidic vorticity, implying gauge-like conservation laws for helicity in the æther.

Rømer Delay and Ætheric Propagation

Ole Rømer's 1676 measurement of light's finite speed [5] is interpreted in VAM as evidence for swirl-mediated propagation:

$$\Delta t = \frac{L}{v_{\text{swirl}}} \approx \frac{L}{c}$$

Here, light is treated as a wave on the æther medium, with c as its asymptotic swirl wave speed.

Sagnac Effect and Circulation in Æther

In rotating frames, the Sagnac effect provides evidence for ætheric circulation:

$$\Delta t_{\text{VAM}} = \frac{4\Gamma_A}{C_e^2}, \quad \Gamma_{=\oint \vec{v}\cdot d\vec{l}}$$

This offers empirical detection of rotational motion in the underlying æther structure.

Schwarzschild Collapse in VAM

Gravitational collapse in VAM results from central pressure depletion due to swirl intensification:

$$P(r) = P_{\infty} - \frac{1}{2}\rho_{\omega^2(r)=0}$$

This leads to a characteristic collapse radius:

$$R_{\text{vam}} = \left(\frac{2F_{\max}}{\rho_{\omega_0^2}}\right)^{1/2}$$

Incompressible Swirl Equilibrium

Stable spherical vortex structures satisfy a pressure equilibrium condition:

$$\nabla P = \rho_{\Omega^2(r)r=\frac{GM(r)}{r^2}\rho}$$

This replaces curvature-based gravitation with a force balance within incompressible æther flow.

Cosmological Evolution of Constants

Drawing from Stanyukovich's evolving constants framework [1], VAM proposes that:

$$\lambda^3 = \frac{2G\hbar}{c^2 H}, \quad \hbar \sim R, \quad G \sim R^{-1/2}$$

Cosmic evolution of swirl parameters induces effective variation in fundamental constants across space-time.

Noncommutative Black Holes and Swirl Quantization

Inspired by Tejeiro and Larrañaga's work [6] on noncommutative black hole models:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}$$

VAM analogously suggests a quantized æther structure, with vortex cores smeared over minimal circulation zones:

$$\rho(r) = \frac{M}{4\pi\theta} e^{-r^2/4\theta}$$

This reflects a natural resolution limit imposed by swirl quantization.

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