OPENBURST Summary of Monostatic, Bistatic and Multistatic Radar Theory

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1 Monostatic Basics

1.1 Bandwidth

From: [6] Bandwidth is the difference between the upper and lower cut-off frequencies of a radar receiver, and is typically measured in hertz.

1.2 Antenna Gain

From: [6] Independent of the use of a given antenna for transmitting or receiving, an important characteristic of this antenna is the gain. Some antennas are highly directional; that is, more energy is propagated in certain directions than in others. The ratio between the amount of energy propagated in these directions compared to the energy that would be propagated if the antenna were not directional (Isotropic Radiation) is known as its gain. When a transmitting antenna with a certain gain is used as a receiving antenna, it will also have the same gain for receiving.

Most radiators emit (radiate) stronger radiation in one direction than in another. A radiator such as this is referred to as anisotropic. However, a standard method allows the positions around a source to be marked so that one radiation pattern can easily be compared with another.

The energy radiated from an antenna forms a field having a definite radiation pattern. A radiation pattern is a way of plotting the radiated energy from an antenna. This energy is measured at various angles at a constant distance from the antenna. The shape of this pattern depends on the type of antenna used.

To plot this pattern, two different types of graphs, rectangular-and polar-coordinate graphs are used. The polar-coordinated graph has proved to be of great use in studying radiation patterns. In the polar-coordinate graph, points are located by projection along a rotating axis (radius) to an intersection with one of several concentric, equally-spaced circles.

The main beam (or main lobe) is the region around the direction of maximum radiation (usually the region that is within 3 dB of the peak of the main beam).

The sidelobes are smaller beams that are away from the main beam. These sidelobes are usually radiation in undesired directions which can never be completely eliminated. The sidelobe level (or sidelobe ratio) is an important parameter used to characterize radiation patterns. It is the maximum value of the sidelobes away from the main beam and is expressed in Decibels. One sidelobe is called backlobe. This is the portion of radiation pattern that is directed opposing the main beam direction.

1.3 Beamwidth

From: [6] The angular range of the antenna pattern in which at least half of the maximum power is still emitted is described as a „Beam With". Bordering points of this main lobe are therefore the points at which the field strength has fallen in the room around 3 dB regarding the maximum field strength. This angle is then described as beam width or aperture angle or half power (- 3 dB) angle - with notation Θ. The angle Θ can be determined in the horizontal plane as well as in the vertical plane.

1.4 Aperture

From: [6]

The effective aperture of an antenna A_e is the area presented to the radiated or received signal. It is a key parameter, which governs the performance of the antenna. The gain is related to the effective area by the following relationship:

$$G = \frac{4\pi A_e}{\lambda^2} \tag{1}$$

where $A_e = K_a A$, λ is wavelength, A_e effective antenna aperture, A physical area of antenna, K_a antenna aperture efficiency. The aperture efficiency depends on the distribution of the illumination across the aperture. If this is linear then $K_a = 1$. This high efficiency is offset by the relatively high level of sidelobes obtained with linear illumination. Therefore, antennas with more practical levels of sidelobes have an antenna aperture efficiency less than one $(Ae < A)$.

1.5 Monostatic range resolution

From: [6]

The target resolution of a radar is its ability to distinguish between targets that are very close in either range or bearing. Resolution is usually divided into two categories; range resolution and bearing (angular) resolution.

Range resolution is the ability of a radar system to distinguish between two or more targets on the same bearing but at different ranges. The degree of range resolution depends on the width of the transmitted pulse, the types and sizes of targets, and the efficiency of the receiver and indicator. Pulse width is the primary factor in range resolution. A well-designed radar system, with all other factors at maximum efficiency, should be able to distinguish targets separated by one-half the pulse width time τ. Therefore, the theoretical range resolution cell of a radar system can be calculated from the following equation:

$$S_r \geq \frac{c_0 \tau}{2} \tag{2}$$

where τ is pulsewidth [s], c_0 is lightspeed.

In a pulse compression system, the range-resolution of the radar is given by the length of the pulse at the output-jack of the pulse compressing stage. The ability to compress the pulse depends on the bandwidth of the transmitted pulse (BW_{tx}) not by its pulse width. As a matter of course the receiver needs at least the same bandwidth to process the full spectrum of the echo signals.

$$S_r \geq \frac{c_0}{2BW_{tx}} \tag{3}$$

This allows very high resolution (and a small radar range resolution cell) to be obtained with long pulses, thus with a higher average power. An 1.5 m resolution will be achieved with a -3 dB bandwidth of 100 MHz theoretically.

1.6 Monostatic angular resolution

From: [6]

Angular resolution is the minimum angular separation at which two equal targets can be separated when at the same range. The angular resolution characteristics of a radar are determined by the antenna beam width represented by the -3 dB angle Θ which is defined by the half-power (-3 dB) points. The half-power points of the antenna radiation pattern (i.e. the -3 dB beam width) are normally specified as the limits of the antenna beam width for the purpose of defining angular resolution; two identical targets at the same distance are, therefore, resolved in angle if they are separated by more than the antenna -3 dB beam width.

An important remark has to be made immediately: the smaller the beam width Θ, the higher the directivity of the radar antenna. The angular resolution as a distance between two targets calculate the following formula:

$$S_A \geq 2 \cdot R \cdot sin(\frac{\Theta}{2}) \tag{4}$$

where S_A is angular resolution as a distance between two targets, Θ is antenna beamwidth and R is slant range aim - antenna [m].

The quantity ΔCR is called the cross-range resolution and ΔDR the down-range resolution (the same as S_r from chapter 1.5). It can be easily verified that:

$$\Delta CR = \theta \cdot R \tag{5}$$

this can be derived from the simple proportion calculation (perimeter of circle/360deg):

$$\frac{2\pi R}{2\pi} = \frac{\Delta CR}{\theta} \tag{6}$$

2 Bistatic Theoretical background

2.1 Derivation of the bistatic radar equation ([2] p.2)

Energy density E_p of the outgoing pulse measured at the target location at range R_t from an isotropic transmitting antenna:

$$E_p = \frac{E_t}{4\pi R_t^2} \qquad [J/m^2] \tag{7}$$

where E_t [joules] is the energy of the transmitted pulse and $4\pi R_t^2$ the area of the sphere of radius R_t centered at the transmitter.

For an assumed rectangular pulse of width $\tau[s]$ and peak power $P_t[W]$, $E_t = P_t\tau[J]$. With transmitting antenna gain G_t we write:

$$\begin{aligned} E_p &= \frac{E_t G_t}{4\pi R_t^2} \\ &= \frac{P_t \tau G_t}{4\pi R_t^2} \qquad [J/m^2] \end{aligned} \tag{8}$$

After reflection from a target with bistatic radar cross section σ $[m^2]$, the energy density of the echo incident on the receiver antenna at range R_r from target is:

$$\begin{aligned} E_a &= \frac{E_p \sigma}{4\pi R_r^2} \\ &= \frac{P_t \tau G_t \sigma}{4\pi R_t^2 4\pi R_r^2} \qquad [J/m^2] \end{aligned} \tag{9}$$

The energy actually received by the receiver depends on the effective aperture A_r $[m^2]$:

$$\begin{aligned} E &= E_a A_r \\ &= \frac{P_t \tau G_t \sigma A_r}{4\pi R_t^2 4\pi R_r^2} \qquad [J] \end{aligned} \tag{10}$$

With $A_r = \frac{G_r \lambda^2}{4\pi}$ (G_r is receiver antenna gain and λ is the signal wavelength, we obtain the equation for the signal energy at the output port of the receiver for a pulsed signal:

$$E = \frac{P_t \tau G_t G_r \lambda^2 \sigma}{(4\pi)^3 R_r^2 R_t^2} \qquad [J] \tag{11}$$

To allow transmission losses L_1 (> 1) we write:

$$E = \frac{P_t \tau G_t G_r \lambda^2 \sigma}{(4\pi)^3 R_r^2 R_t^2 L_1} \qquad [J] \tag{12}$$

2.2 Maximum available Signal-to-Noise ratio (SNR)

The noise spectral density N_0 $(W/Hz\ or J)$ at the receiver is defined as:

$$N_0 = kT_s \qquad [W/Hz\ or J] \tag{13}$$

where $k = 1.38 * 10^{-23}$ (J/K) is Boltzmann's constant, and T_s is system noise temperature in kelvins (K). Combining equations 12 and 13 we obtain the maximum available SNR:

$$\begin{aligned} \left(\frac{S}{N}\right)_{max} &= \frac{E}{N_0} \\ &= \frac{P_t \tau G_t G_r \lambda^2 \sigma}{(4\pi)^3 R_r^2 R_t^2 k T_s L_1} \end{aligned} \tag{14}$$

This equation is valid also for the monostatic case, where we use $R_t = R_r$ and the monostatic RCS replaces the bistatic one.

2.3 Maximum available Signal-to-Noise ratio (SNR) for monostatic Radar

For monostatic radar we can simplify equation 14 as :

$$\begin{aligned}\left(\frac{S}{N}\right)_{max} &= \frac{E}{N_0} \\ &= \frac{P_t \tau G^2 \lambda^2 \sigma F_t F_r}{(4\pi)^3 R^4 k T_s L_1}\end{aligned} \tag{15}$$

which corresponds to the equation in [9] equation 2.6 p2.6 where F_t and F_r are equal to 1 for free space loss and omni transmitting antenna.

According to [12] p.17 variation 5, we assume: $T_S = N_f T_0$, where T_0 is reference ambient temperature by IEEE Standard of 290K, and N_f is noise figure. Using this and $F_t = F_r = 1$ we rewrite:

$$\left(\frac{S}{N}\right)_{max} = \frac{P_t \tau G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k N_f T_0 L_1} \tag{16}$$

we multiply both denominator and nominator with bandwidth B and get:

$$\left(\frac{S}{N}\right)_{max} = \frac{P_t G^2 \lambda^2 \sigma \tau B}{(4\pi)^3 R^4 k T_0 B N_f L_1} \tag{17}$$

For coherent pulse integration and for pulse compression we use the gains G_d and G_p and the equation is therefore:

$$\left(\frac{S}{N}\right)_{max} = \frac{P_t G^2 \lambda^2 \sigma G_d G_p}{(4\pi)^3 R^4 k T_0 B N_f L_1} \tag{18}$$

where G_d the integration gain is simply the number of integrated pulses and $G_p = \tau B$ is the pulse compression gain where τ is the width of the uncompressed pulse.

The computation of probability of detection pd using the above SNR for a given range and a given probability of false alarm p_{fa} is described for example in [10]. We implemented the Matlab code given in [10] in Python in BURST to compute the maximum range or the pd.

We compute the propagation losses using the splat tool [13] and introduce the propagation loss due to terrain shielding in the losses L_1.

2.4 Maximum range for bistatic radar

For a given SNR we can compute the maximum range from equation 14:

$$(R_t R_r)_{max} = \sqrt{\frac{P_t \tau G_t G_r \lambda^2 \sigma}{(4\pi)^3 k T_s L_1 (S/N)}} \tag{19}$$

2.5 Derivation of maximum range for one-way transmission

We now consider the simpler case of a transmitter and a receiver at range R_t from the transmitter, and compute the available energy density at the receiver. From equation 7 we have the emitted power at range R_t:

$$E_p = \frac{E_t G_t}{4\pi R_t^2} \qquad [J/m^2] \tag{20}$$

With the receiver effective aperture $A_r = \frac{G_r \lambda^2}{4\pi}[m^2]$ (G_r is receiver antenna gain and λ is the signal wavelength) we compute the energy actually received by the receiver after the one-way transmission:

$$\begin{aligned} E_{ow} &= E_p A_r \\ &= \frac{E_t G_t G_r \lambda^2}{(4\pi)^2 R_t^2} \qquad [J] \end{aligned} \tag{21}$$

This value converted into decibels can be used as a measure of antenna $sensitivity$ of a one-way receiver antenna (e.g. in the coverage calculation HTZ Warfare).

In BURST, for computing the PET max detection range (or equivalently the Richtstrahl max detection range) we remove the antenna gain of the receiver by setting it to 1 and use $P_t \tau = E_t$ to get:

$$E_{ow} = \frac{P_t \tau G_t \lambda^2}{(4\pi)^2 R_t^2} \qquad [J] \tag{22}$$

where $\tau[s]$ is the pulsewidth, $P_t[W]$ is peak power and G_t is transmitting antenna gain.

Now we divide the equation by τ to get the expression for signal strength S in $[J/s]$ = $[W]$:

$$S = \frac{P_t G_t \lambda^2}{(4\pi)^2 R_t^2} \qquad [J/s] \qquad [W] \tag{23}$$

$P_t G_t$ is known as the Effective Radiated Power [ERP], the units whereof is $[W]$ or $[J/s]$. Using this and $\lambda = c/f$ we get (c is speed of light in $[m/s]$ and f is frequency in Hz):

$$S = \frac{ERP}{(4\pi R_t^2)} \frac{c^2}{f^2} \frac{1}{4\pi} \qquad [J/s] \qquad [W] \tag{24}$$

Now e.g. for a given ELINT sensor with detection sensitivity S in $[W]$ we can compute the maximum detection range by solving for $R_t[m]$ in the above equation:

$$R_t = \sqrt{\frac{ERP}{S}} \frac{c}{f 4\pi} \qquad [m] \tag{25}$$

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