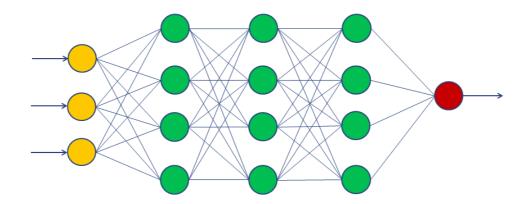


Workshop: Machine Learning and Prediction Modelling

Neural Networks



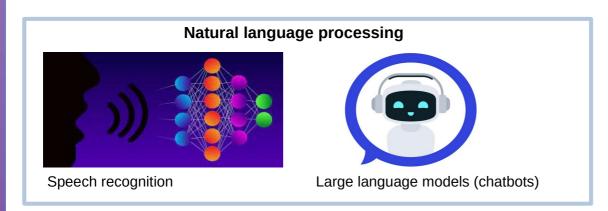
Yannick Rothacher

SPF, HS2025

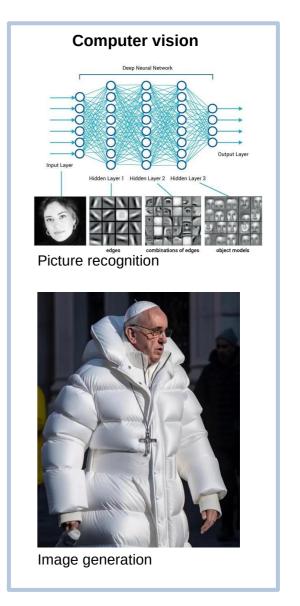


Artificial Neural Networks (today)

- (Deep) Neural Networks are very popular!
 - Also referred to as "Deep Learning"
- Successfully applied in many fields:









Artificial Neural Networks (yesterday)

- ► **Heureka** exhibition in Zürich, Brunau (**1991**)
- Presented the "Forschungsstandort Schweiz"



 $https://www.e-pics.ethz.ch/index/ethbib.bildarchiv/ETHBIB.Bildarchiv_Com_FC24-8002-0196_24364.html$



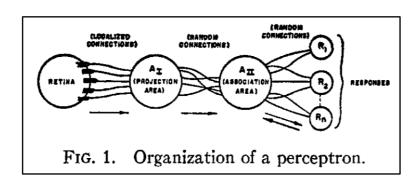
Einzelworterkennung mit
neuronalen Netzen (6.3.1)
Neuronale Netze werden seit kurzem in einer Vielzahl verschiedener
Gebiete verwendet: Bildverarbeitung, Signalbereinigung, Trendanalyse, Regelungstechnik usw.
Daneben werden sie weltweit auf

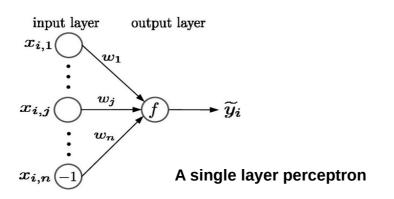
ihre Tauglichkeit zur Erkennung gesprochener Sprache untersucht. Das Problem dabei ist, dass die Erkennung ganzer Wörter sprecherunabhängig sein soll. Als Beispiel suchen wir in einem Computer ein Dokument anstatt mit einer Maus mittels gesprochener Schlüsselwörter.

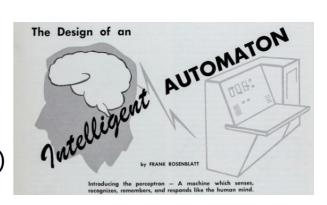


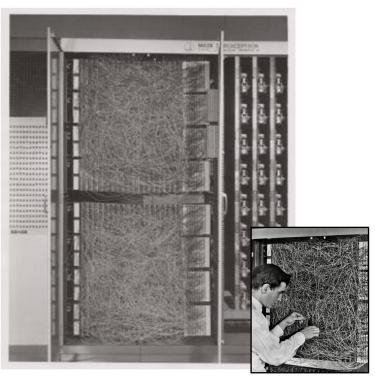
Artificial Neural Networks (yesterday)

- The idea of artificial neural networks is very old
 - First reference dates back to 1944 (Warren S. McCulloch and Walter Pitts)
- The "perceptron" (the first "modern" neural network)
 - Invented by psychologist F. Rosenblatt (1958)







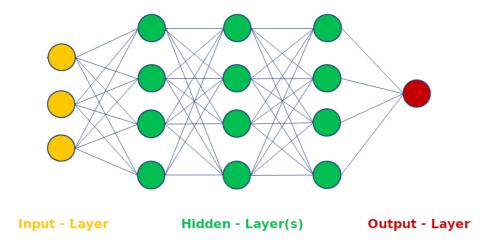


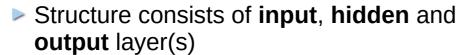
Mark I Perceptron machine

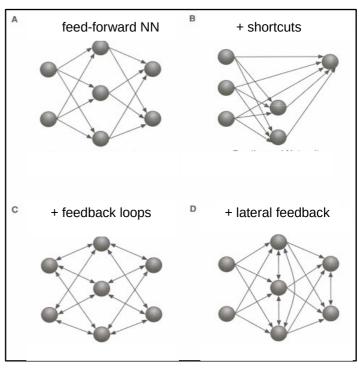


Structure of Neural Networks (NN)

- Usually represented as connected nodes (neurons) organized in layers
 - Different structures are possible
- We will focus on feed-forward networks







Source: Holling & Schmitz (2010)

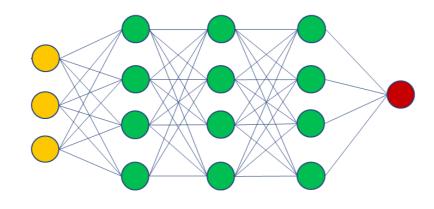
- Connections are associated with specific weights (strength of connection)
 - Nodes are associated with specific activation functions (later)
- The structure is loosely inspired by real-life neurons

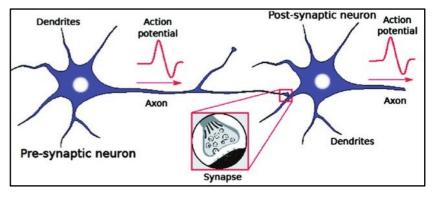


Input - Layer

Structure of Neural Networks (NN)

- Usually represented as connected nodes (neurons) organized in layers
 - Different structures are possible
- We will focus on feed-forward networks





Source: Huang et al. (2018)

Structure consists of input, hidden and output layer(s)

Hidden - Layer(s)

Connections are associated with specific weights (strength of connection)

Output - Layer

- Nodes are associated with specific activation functions (later)
- The structure is loosely inspired by real-life neurons



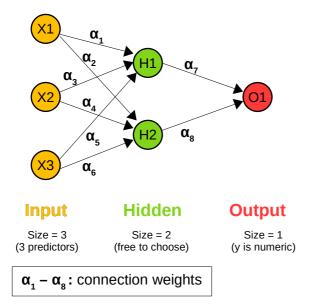
Single hidden layer NN

Supervised learning with NNs (predict target variable)

Exemplary regression data: Target variable (y)

aining time (X1)	Sleep time (X2)	Body height (X3)	Performance (0-100)
2	8	1.8	60
8	9	1.6	100
5	4	1.7	85
8	5	1.8	79
7	7	1.8	62
1	2 8 5 8 7	2 8 8 9 5 4 8 5 7 7	2 8 1.8 8 9 1.6 5 4 1.7 8 5 1.8 7 7 1.8

Applied (single hidden layer) NN:





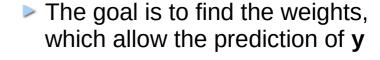
Single hidden layer NN

Supervised learning with NNs (predict target variable)

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7	7	1.8	62

Applied (single hidden layer) NN:



How the prediction works (slightly simplified, see later):

X1	X2	Х3	Performance (0-100)
3	5	1.7	?

Value at H1: H1 =
$$\alpha_1^*$$
3 + α_3^* 5 + α_5^* 1.7

Value at
$$H2$$
: $H2 = \alpha_2 * 3 + \alpha_4 * 5 + \alpha_6 * 1.7$

Value at
$$O1$$
 (= Prediction): $O1 = \alpha_7^* H1 + \alpha_8^* H2$

α_1
α_3 H1 α_7
$\alpha_{\rm s}$ $\alpha_{\rm s}$
α_5 α_8 α_6

Input

Size = 3 (3 predictors)

Hidden

Output

Size = 1 (y is numeric)

 $\alpha_1 - \alpha_8$: connection weights



Single hidden layer NN

Supervised learning with NNs (predict target variable)

Exemplary regression data:

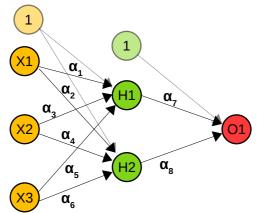
___Target variable (y)

Training time (X1)	Sleep time (X2)	Body height (X3)	Performance (0-100)
2	8	1.8	60
8	9	1.6	100
5	4	1.7	85
8	5	1.8	79
7	7	1.8	62

The goal is to find the weights, which allow the prediction of y

How the prediction works (slightly simplified, see later):

Applied (single hidden layer) NN:



X1	X2	Х3	Performance (0-100)
3	5	1.7	?

Value at (H1): H1 = α_1^* 3 + α_3^* 5 + α_5^* 1.7

Value at (H2): $H2 = \alpha_2 * 3 + \alpha_4 * 5 + \alpha_6 * 1.7$

Value at 01 (= Prediction): O1 = α_7 *H1 + α_8 *H2

"Biases" are often added at each layer (think of them as "intercepts")

Input

Size = 3 (3 predictors)

Hidden Size = 2

(free to choose)

Output
Size = 1
(y is numeric)

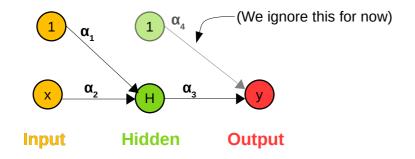
 $\alpha_1 - \alpha_8$: connection weights

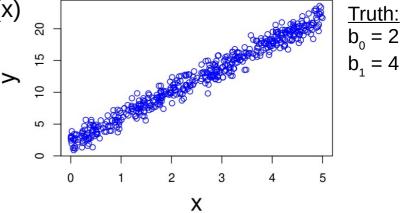


Univariate regression: Only one predictor (x)

$$y = b_0 + b_1 x + \varepsilon$$
 $(\varepsilon \sim N(0, \sigma^2))$

Applied Neural Network:



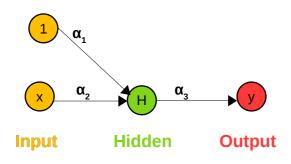


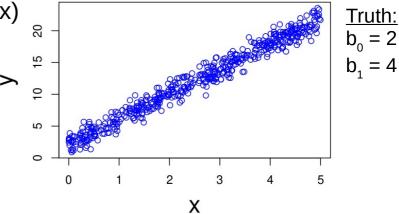


Univariate regression: Only one predictor (x)

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Applied Neural Network:



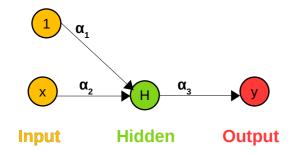




Univariate regression: Only one predictor (x)

$$y = b_0 + b_1 x + \varepsilon (\varepsilon \sim N(0, \sigma^2))$$

Applied Neural Network:



Prediction of y:

$$H = \alpha_1 + \alpha_2 * x$$

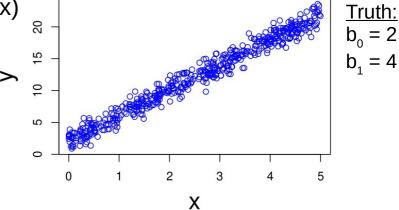
$$y = \alpha_3 * H$$

$$\rightarrow \text{ Ideal solution:}$$

$$\alpha_1 = b_0 = 2$$

$$\alpha_2 = b_1 = 4$$

$$\alpha_3 = 1$$





Fitted with nnet()

!?

function:

 $\alpha_{1} = -0.19$

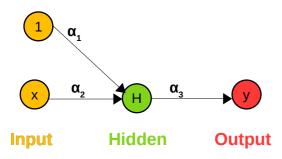
 $\alpha_{2} = 0.14$

 $\alpha_{3} = 115.93$

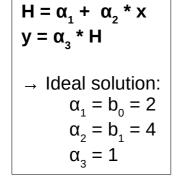
Univariate regression: Only one predictor (x)

$$y = b_0 + b_1 x + \varepsilon$$
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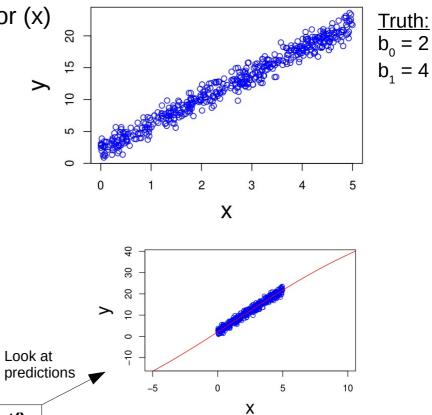
Applied Neural Network:



Prediction of y:



Let's compare with solution of NN ...

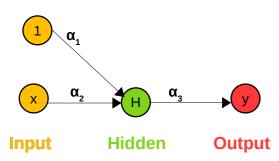




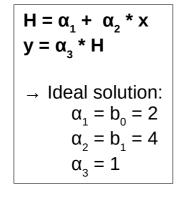
Univariate regression: Only one predictor (x)

$$y = b_0 + b_1 x + \varepsilon$$
 $(\varepsilon \sim N(0, \sigma^2))$

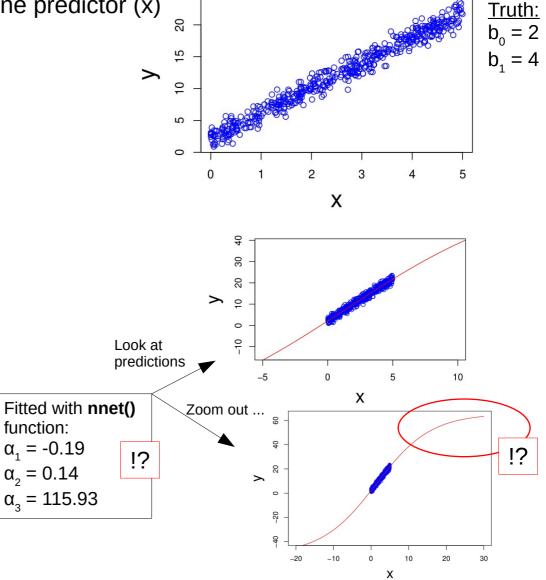
Applied Neural Network:



Prediction of y:



Let's compare with solution of NN ...

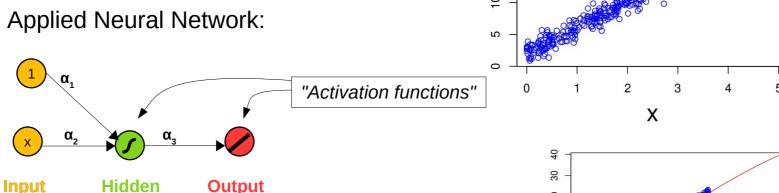




Univariate regression: Only one predictor (x)

$$y = b_0 + b_1 x + \varepsilon$$
 $(\varepsilon \sim N(0, \sigma^2))$

Applied Neural Network:



Prediction of y:

$$H = \alpha_1 + \alpha_2 * x$$

$$y = \alpha_3 * H$$

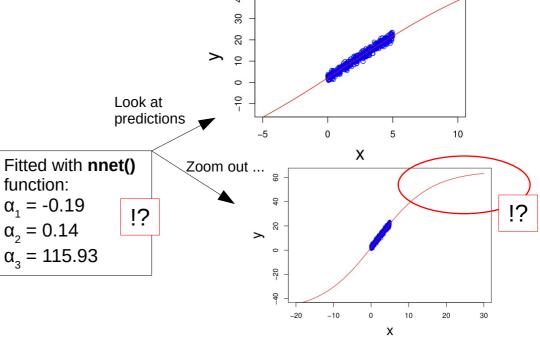
$$\rightarrow \text{ Ideal solution:}$$

$$\alpha_1 = b_0 = 2$$

$$\alpha_2 = b_1 = 4$$

$$\alpha_3 = 1$$

Let's compare with solution of NN ...



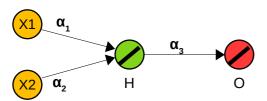
Truth: $b_0 = 2$

 $b_1 = 4$



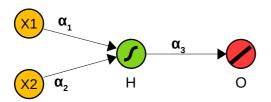
Activation functions

- Activation functions transform the input of a neuron
- Generally, activation functions in the hidden layer are non-linear (e.g. logistic or step function)
 - Identity function:



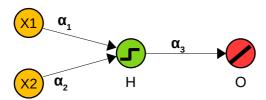
$$H_{\mathit{output}} = g(H_{\mathit{input}}) = H_{\mathit{input}}$$

Logistic function:



$$H_{output} = g(H_{input}) = \frac{1}{1 + e^{-H_{input}}}$$

> **Step** function:

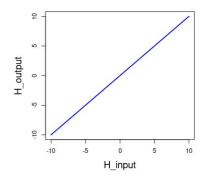


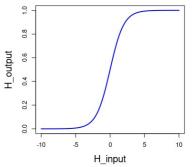
$$H_{output} = g(H_{input}) = \begin{cases} 1 & \text{if } H_{input} > 0 \\ 0 & \text{if } H_{input} \le 0 \end{cases}$$

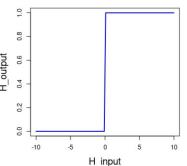
Activation function g()

$$H_{Input} = \alpha_1^* X 1 + \alpha_2^* X 2$$

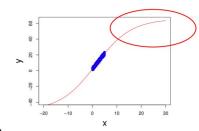
$$H_{Output} = \mathbf{g(H_{Input})}$$







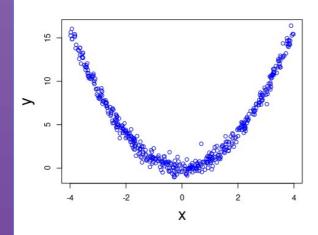
Only with non-linear activation functions can we model non-linear patterns!



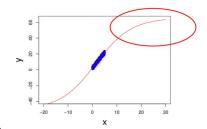
- If only the identity function is used the output will always be a linear function (no matter how complex/deep the NN is)
- Example: Non-linear univariate regression

$$y = x^2 + \varepsilon$$
 $(\varepsilon \sim N(0, \sigma^2))$

Data Neural Network Prediction

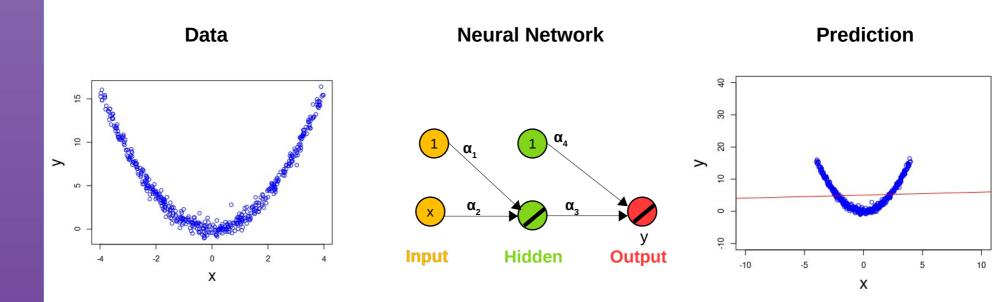




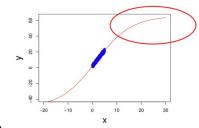


- If only the identity function is used the output will always be a linear function (no matter how complex/deep the NN is)
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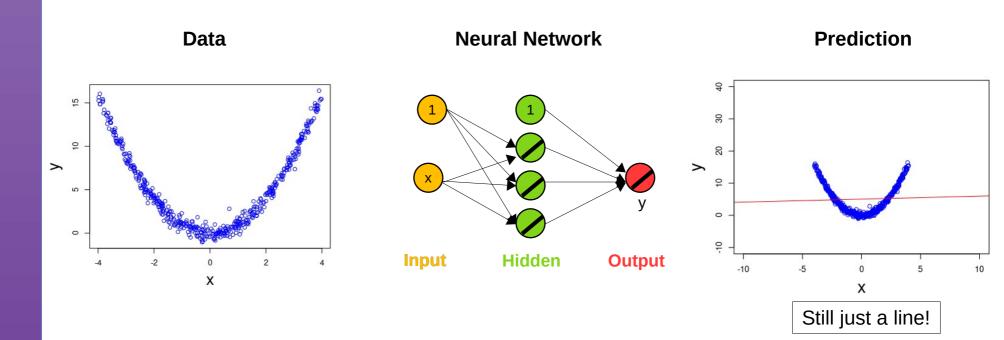




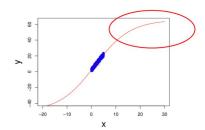


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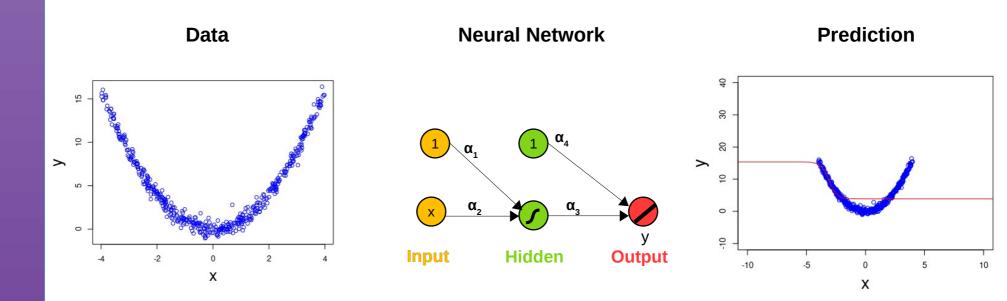




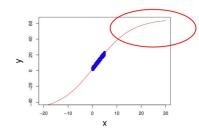


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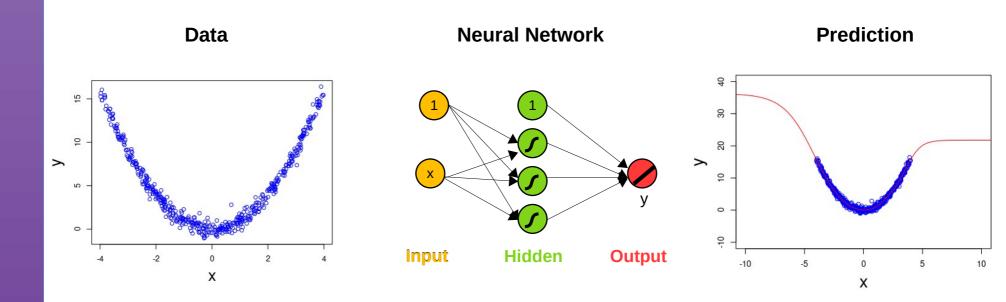






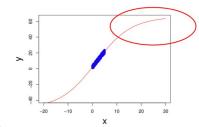
- If only the identity function is used the output will always be a linear function (no matter how complex/deep the NN is)
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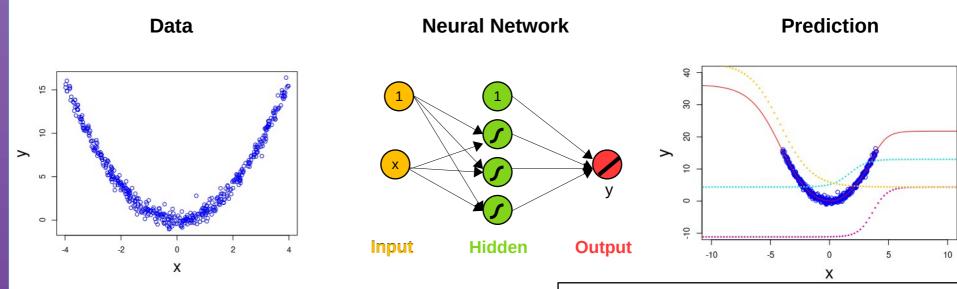


Only with non-linear activation functions can we model non-linear patterns!



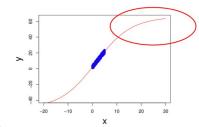
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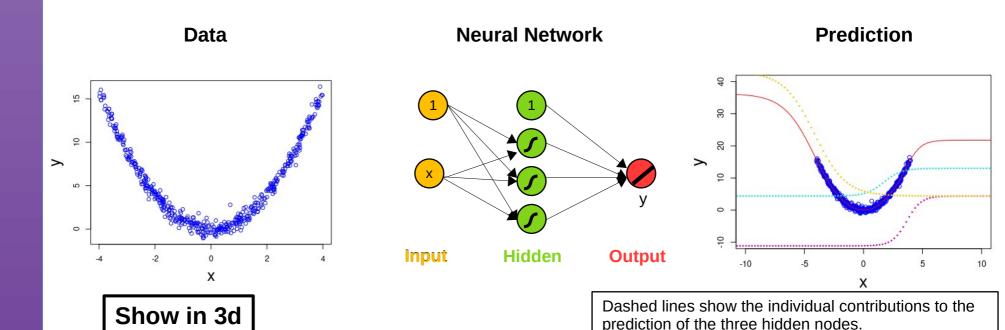
Dashed lines show the individual contributions to the prediction of the three hidden nodes.





- If only the identity function is used the output will always be a linear function (no matter how complex/deep the NN is)
- Example: Non-linear univariate regression

$$y = x^2 + \varepsilon$$
 $(\varepsilon \sim N(0, \sigma^2))$





Neural Networks for classification

► The activation function and/or size in the output layer can be adapted to use NN for classification

Regression

Can use the identity function in the output layer

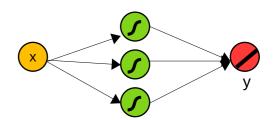
Binary classification

- Can e.g. use the logistic function in the output layer
- The value of the output node represents the probability of y being equal to class 1

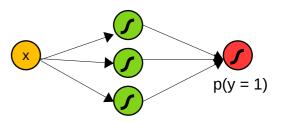
Multiclass classification

- Could use k output nodes for k possible classes and e.g. a logistic function
- The k output nodes represent the probabilities that y is equal to a certain class
- Classically, the values of the k output nodes are forced to sum up to 1 (softmax function)

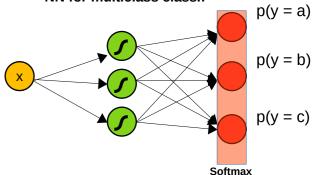
NN for regression:



NN for binary class.:









Neural Networks for classification

► The activation function and/or size in the output layer can be adapted to use NN for classification

Regression

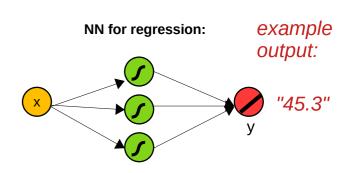
Can use the identity function in the output layer

Binary classification

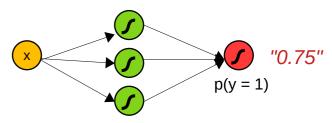
- Can e.g. use the logistic function in the output layer
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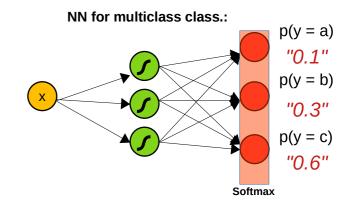
Multiclass classification

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NN for binary class.:

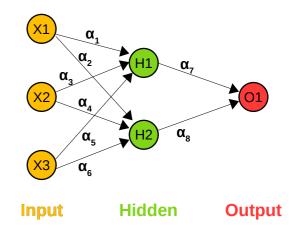




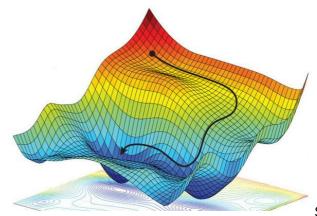


How are the weights found?

- "Training" of the Network
 - Start with random weights (initialization)
 - The predictions will be random as well (bad)
- Compare predictions with true y-values (classically using batches of training data)



- Calculate how "wrong" the predictions were (Loss-function)
- Find out how the weights have to be shifted to improve the predictions (backpropagation algorithm to calculate gradients)
- Continue to update the weights until the algorithm converges
 - Stochastic gradient descent



Source: Amini et al. 2018



Single hidden layer NN in R

Single hidden layer NNs can be fitted with nnet R-package

```
> library("nnet")
> nn_smk <- nnet(intention_to_smoke ~ friends_smoke + scale(alcohol_per_month) + scale(age),
              data = dat smoking,
              size=2,
              decay=0,
              linout=FALSE,
              maxit=10000)
> summary(nn_smk)
a 3-2-1 network with 11 weights
options were - entropy fitting
 b->h1 i1->h1 i2->h1 i3->h1
         9.32 2.74 -0.76
 -3.88
 b->h2 i1->h2 i2->h2 i3->h2
 9.16 -34.48 -7.01 2.28
  b->o h1->o h2->o
                                                                  H1
-14.02 15.27 11.78
# Make predictions (here for training data):
                                                                Hidden
                                                      Input
                                                                           Output
> predict(nn_smk, newdata=dat_smoking,
         type='class')
```



Single hidden layer NN in R

Single hidden layer NNs can be fitted with nnet R-package

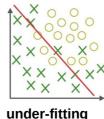
For neural networks it is usually advisable to standardize the predictors!

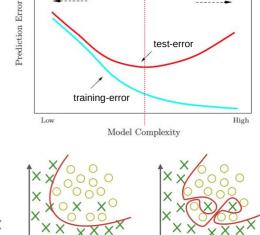
```
> library("nnet")
> nn_smk <- nnet(intention_to_smoke ~ friends_smoke + scale(alcohol_per_month) + scale(age),</pre>
                data = dat smoking,
                                     Number of nodes in hidden layer
                size=2,◀
                decay=0, ◀
                                    "Weight decay" regularization (later)
                linout=FALSE, ←
                                        Use identity activation function in output layer? (default is logistic)
                maxit=10000) _
                                         Number of maximum iterations (how many iterations to adjust weights)
> summary(nn_smk)
a 3-2-1 network with 11 weights
options were - entropy fitting
 b->h1 i1->h1 i2->h1 i3->h1
 -3.88 9.32 2.74 -0.76
 b->h2 i1->h2 i2->h2 i3->h2
  9.16 -34.48 -7.01 2.28
                                                                         H1
  b->o h1->o h2->o
                             To predict labels use:
-14.02 15.27 11.78
                             predict(..., type='class')
# Make predictions (here for training data):
                                                                       Hidden
                                                            Input
                                                                                   Output
> predict(nn_smk, new@ata=dat_smoking,
          type='class')
```



Parameter tuning in NNs

- With the nnet() function two main tuning parameter exists
 - "size" (number of neurons in hidden layer)
 - "decay" (Regularization factor using weight decay)
- The more neurons in the hidden layer the more complex patterns can be modeled
 - Danger of under/over-fitting!





"Just right"

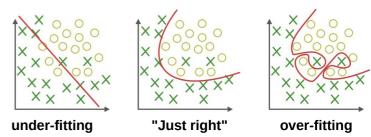
test-error

- Overfitting on the example of the smoking data set:
 - Training error (miscl.rate) for different NN sizes



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training-error

test-error

Model Complexity

Prediction Error

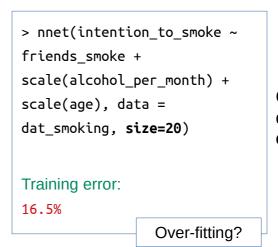
- Overfitting on the example of the smoking data set:
 - Training error (miscl.rate) for different NN sizes

```
> nnet(intention_to_smoke ~
friends_smoke +
scale(alcohol_per_month) +
scale(age), data =
dat_smoking, size=1)

Training error:
22%
Under-fitting?
```

```
> nnet(intention_to_smoke ~
friends_smoke +
scale(alcohol_per_month) +
scale(age), data =
dat_smoking, size=6)

Training error:
19%
Good?
```



Can try to find optimum with crossvalidation



- Caret is an R-package to automatically tune various machine learning models
- Allows the tuning of neural networks from nnet and neuralnet R-packages
- Have to define a search grid of parameter values
 - For each setting caret fits a NN and estimates the **test-error** using a resampling-based estimation (similar to crossvalidation)
 - The model with the lowest test-error is selected as the winner

```
> library("caret")
### Create tuning grid:
> t grid <- expand.grid(size=c(1,5,10),</pre>
                        decay=c(0, 0.5))
> t grid
  size decay
     1
         0.0
         0.0
         0.0
    1 0.5
         0.5
        0.5
    10
### Tune the model ("train" function):
> set.seed(288)
> tune caret <- train(intention to smoke ~ .,
data=dat smoking, method='nnet', tuneGrid=t_grid,
maxit=10000, linout=FALSE, preProcess=c("center","scale"))
```



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```
> tune_caret <- train(intention_to_smoke ~ .,</pre>
data=dat smoking, method='nnet', tuneGrid=t_grid,
maxit=10000, linout=FALSE, preProcess=c("center","scale"))
> tune caret
Neural Network
200 samples
 4 predictor
 2 classes: 'no', 'yes'
Pre-processing: centered (4), scaled (4)
Resampling: Bootstrapped (25 reps)
Summary of sample sizes: 200, 200, 200, 200, 200, 200, ...
Resampling results across tuning parameters:
  size decay Accuracy
                          Kappa
        0.0
               0.7257813 0.4456960
   1
        0.5
               0.7231326 0.4456194
        0.0
               0.6825685 0.3546761
        0.5
               0.7219893 0.4428660
        0.0
  10
               0.6467391 0.2792476
        0.5
               0.7207855 0.4403016
  10
```

Accuracy was used to select the optimal model using the largest value.

The final values used for the model were size = 1 and decay = 0.



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 - The (hyper) parameters of the winner model are used to fit a final model to the complete data (this will be the model used for prediction)
 - Can use the predict function with the caret object: predict(tune_caret)

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> tune_caret
Neural Network
200 samples
    4 predictor
    2 classes: 'no', 'yes'

Pre-processing: centered (4), scaled (4)
Resampling: Bootstrapped (25 reps)
Summary of sample sizes: 200, 200, 200, 200, 200, 200, ...
Resampling results across tuning parameters:
```

size	decay	Ассигасу	Карра	Because we evaluated multiple models, the
1	0.0	0.7257813	0.4456960	performance estimate of
1	0.5	0.7231326	0.4456194	the winner configuration can be too optimistic.
5	0.0	0.6825685	0.3546761	Therefore, people often
5	0.5	0.7219893	0.4428660	use an additional
10	0.0	0.6467391	0.2792476	separate data set to get a final performance
10	0.5	0.7207855	0.4403016	estimate after model selection.

Accuracy was used to select the optimal model using the largest value.

The final values used for the model were size = 1 and decay = 0.

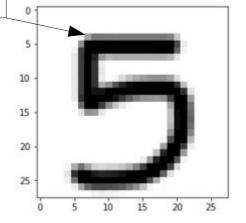


Picture recognition with NNs

- So far in supervised learning: Data is always a table with predictors and target variable
- What if we want to predict the content shown on a picture?
 - A picture is no different! We can translate it into a row of values where each value represents the shade of one pixel.
- Example: Predict/recognize handwritten digits
 - Examplary data table of pictures with 30 x 30 pixels:

The shading of each pixel corresponds to a "grayscale" value ranging from 0 (white) to 255 (black).

	Pix1	Pix2	Pix3	 Pix900	Digit
Picture 1	20	24	60	 44	0
Picture 2	10	94	160	 244	7
Picture 3	220	89	143	 134	3
Picture 4	12	123	70	 230	5



Target variable

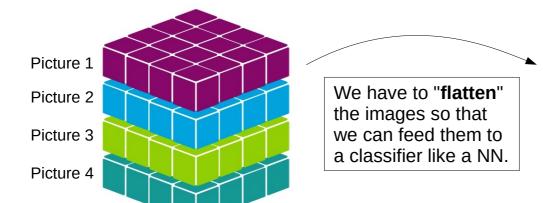


Picture recognition with NNs

- Grayscale images are normally stored as tables with rows and columns indicating the pixel positions
 - ▶ Table storing a grayscale picture with 4 x 4 pixels:

100	0	0	255				
0	0	255	0	Lo the image			
0	255	0	0	Is the image			
255	0	0	100			_	

- A collection of multiple images is normally stored as a 3-dimensional array, which is like a cube with pictures "stacked" as layers
 - Exemplary array storing four 4x4 pixel images:



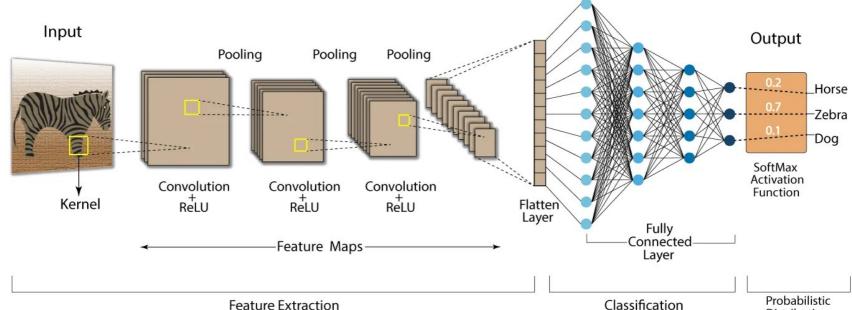
	Pix1	Pix2	Pix3	 Pix16
Picture 1	20	24	60	 44
Picture 2	10	94	160	 244
Picture 3	220	89	143	 134
Picture 4	12	123	70	 230

(table-format we need)



Picture recognition with NNs

- Feeding a "flattened" image into a classification NN works for smaller picture sizes
- For large pictures, however, the NN becomes too complex and complicated (millions of parameters)
- Convolutional NNs try to more efficiently capture the spatial dependencies in a picture by reducing the image to a set of (hopefully) relevant features (feature extraction)
 - The extracted features are subsequently fed into a classification NN



Source: www.developersbreach.com

Distribution



Deep learning with R (book)

Good introduction to deep neural networks with R (by François Chollet and J.J. Allaire)

