ETH
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Department of Computer Science

Theory of Computing

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Second Midterm Exam

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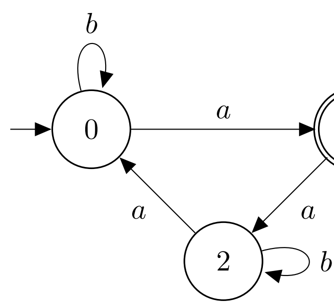
Exercise 1

(a) Let $R = ((a+bb)^*b)^*$ be a regular expression. Construct a λ-NFA A with $L(A) = L(R)$.

(b) Construct an equivalent regular expression for the following finite automaton A. Either use one of the construction methods from the self study or give an informal justification for the correctness of your construction.



4+6 points

Exercise 2

(a) Formulate the pumping lemma for context-free languages.

(b) Use the pumping lemma for context-free languages to prove that the language

$$L = \{a^i b^j c^k \mid i, j, k \in \mathbb{N} \text{ and } k = \min\{i, j\}\}$$

is not context-free.

3+7 points

(please turn the page)

Exercise 3

Consider the two languages

$$L_{011} = \{\text{Code}(M) \mid \text{the TM } M \text{ accepts the word } 011\}$$

and

$$L_{110} = \{\text{Code}(M) \mid \text{the TM } M \text{ accepts the word } 110\}\,.$$

Prove by a concrete reduction that $L_{011} \leq_{\mathrm{R}} L_{110}$. **10 points**

Exercise 4

Let $C = C_1 \wedge C_2 \wedge \ldots \wedge C_m$ be a Boolean formula with clauses $C_1, C_2, \ldots, C_m$, where each clause $C_i = (l_{i,1} \vee l_{i,2} \vee l_{i,3})$ contains exactly 3 literals. For each such clause C_i, one can construct a formula $\Phi(C_i)$ in 2-CNF as follows:

$$\begin{aligned}\Phi(C_i) = {} & (l_{i,1}) \wedge (l_{i,2}) \wedge (l_{i,3}) \\ & \wedge (\overline{l_{i,1}} \vee \overline{l_{i,2}}) \wedge (\overline{l_{i,1}} \vee \overline{l_{i,3}}) \wedge (\overline{l_{i,2}} \vee \overline{l_{i,3}}) \\ & \wedge (y_i) \wedge (l_{i,1} \vee \overline{y_i}) \wedge (l_{i,2} \vee \overline{y_i}) \wedge (l_{i,3} \vee \overline{y_i})\,,\end{aligned}$$

where y_i is a new variable that appears only in $\Phi(C_i)$.

(a) Prove that, for each assignment not satisfying the clause C_i, every possible extension of the assignment by a truth value for y_i leads to at most 6 satisfied clauses in $\Phi(C_i)$.

(b) Prove that every assignment that satisfies the clause C_i can be extended to an assignment for $\Phi(C_i)$ (by defining a truth value for y_i) such that 7 clauses of $\Phi(C_i)$ are satisfied.

It is possible to additionally prove the following:

$$\text{There does not exist any assignment satisfying more than 7 clauses of } \Phi(C_i). \qquad (1)$$

(c) We consider the decision problem Threshold-2Sat that consists of all pairs (Φ, k) such that Φ is a formula in 2-CNF for which there exists an assignment satisfying at least k clauses of Φ. Prove, using the claims from (a), (b), and (1), that Threshold-2Sat is NP-hard by showing a polynomial-time reduction from 3Sat.

(d) **Bonus exercise for 3 bonus points:** Prove (1).

2+4+4 points