Endterm exam

Num. CSE, D-INFK/D-MATH

HS 2016

A

Prof. R. Hiptmair

Family name		Grade
First name		
Department		
Legi Nr.		
Mark if you did *not* pass the midterm:		
Date	23.12.2016	

1	2	3	4	5	Total
4	6	6	4	4	24

- **Keep only writing material and Legi on the table.**
- Keep mobile phones, tablets, smartwatches, etc. **turned off** in your bag.
- Fill in this cover sheet first.
- **Turn the cover sheet only when instructed to do so.**
- **Read the rules on the next page carefully.**
- Do not write with red/green color or with pencil.
- **Make sure to hand in every sheet.**
- **Duration: 30 min.**
- Additional material: none.

Wish you much success!

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Rules:

- Motivation for the answers is **not** necessary. Remarks and computations have **no** influence on the total number of points.
- Wrong answers (for multiple choice problems) give negative points. The minimum number of points for each problem is 0.
- All notes outside the predefined boxes will not be considered.
- Each multiple choice box has one and only one correct answer.
- If required, write your solution in the predefined box:

your text here

- Any unclear marking will be considered an error.

1. *LT-FIR* [4 P.]

   A linear time-invariant channel (LT-FIR) with finite impulse response

   $$(\dots, 0, h_0, \dots, h_m, 0, \dots)\ , \quad h_i \in \mathbb{R},\ m \in \mathbb{N}\ ,$$

   is fed with an n-periodic signal $(x_j)_{j\in\mathbb{Z}}$ $(n \in \mathbb{N})$. The output signal $(y_j)_{j\in\mathbb{Z}}$ will be n-periodic again and is given by the formula:

   $$y_j = \sum_{l=0}^{n-1} p_l x_{j-l}\ , \quad j = 0, \dots, n-1\ ,$$

   where the coefficients p_l, $l = 0, \dots, n-1$, may depend on h_i, m and n.

   Which of the following formulas for p_l, $l \in \{0, \dots, n-1\}$, is/are correct: [+1P correct, -1P wrong, min: 0P]

   - $p_l = \sum_{k\in\mathbb{Z}} h_{l+km}$ ○ correct ● wrong
   - $p_l = \sum_{k\in\mathbb{Z}} h_{l+kn}$ ● correct ○ wrong
   - $p_l = \sum_{k\in\mathbb{Z}} h_{l-kn}$ ○ correct ● wrong
   - $p_l = \sum_{k=0}^{\infty} h_{l+kn}$ ● correct ○ wrong

Scratch space (not evaluated):

2. *Discrete convolution* [6 P.]

A discrete convolution of two vectors $\mathbf{p}, \mathbf{x} \in \mathbb{R}^n$ is defined as:

$$(\mathbf{p} * \mathbf{x})_j := \sum_{l=0}^{j} (\mathbf{x})_l (\mathbf{p})_{j-l} \,, \quad j = 0, \dots, n-1 \,.$$

Note that vector indices start from 0!

For which of the following matrices $\mathbf{A} \in \mathbb{R}^{n,n}$ does there exist a vector $\mathbf{p} \in \mathbb{R}^n$ such that

$$\mathbf{p} * \mathbf{x} = \mathbf{A}\mathbf{x}, \quad \forall \mathbf{x} \in \mathbb{R}^n \qquad ?$$

If such $\mathbf{p}$ exists, specify its components. [+1P correct "p exists"?, -1P wrong "p exists"?, +3P correct p entries if p exists, min: 0P]

(a)

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & & & \\ 1 & 2 & & & \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & 0 \\ & & & 1 & 2 \end{bmatrix}$$

$\mathbf{p}$ exists? ● true ○ false

If $\mathbf{p}$ exists, $\mathbf{p} =$ $(2, 1, \dots, 0, \dots)^\top$

(b)

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & & & \\ 0 & 2 & -1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & -1 \\ -1 & & & 0 & 2 \end{bmatrix}$$

$\mathbf{p}$ exists? ○ true ● false

If $\mathbf{p}$ exists, $\mathbf{p} =$

(c)

$$\mathbf{A} = \begin{bmatrix} 1 & & & & 0 \\ 2 & 2 & & & \\ 3 & 3 & 3 & & \\ \vdots & & & \ddots & \\ n & n & n & n & n \end{bmatrix}$$

p exists? ○ true ● false

If **p** exists, **p** = $\boxed{}$

3. *Interpolation error* [6 P.]

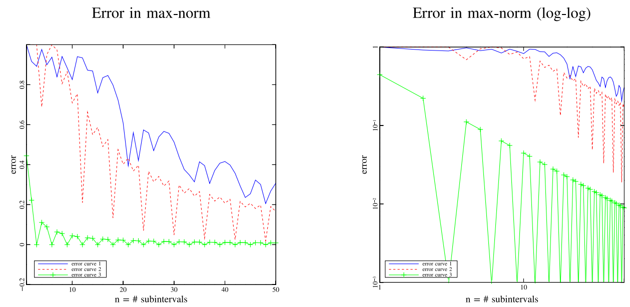
   The following plots show the maximum norms of the interpolation errors of piecewise linear interpolation of the three functions on $[0, 1]$

   (a) $f(t) = \left|\sin(6\pi t)\right|$

   (b) $f(t) = \left|x - \frac{1}{3}\right|$

   (c) $f(t) = \left|\sin\left(6\pi t^2\right)\right|$

   on the equidistant node set $\mathcal{J}_n = \left\{\frac{j}{n} : j = 0, \ldots, n\right\}$, with $n \in \mathbb{N}$.



Assign the error curves to the individual functions.

[+2P correct assignment, -1P wrong assignment, min: 0P]

Function $f(t)$	error curve 1	error curve 2	error curve 3
$\left\|\sin(6\pi t)\right\|$	○	●	○
$\left\|x - \frac{1}{3}\right\|$	○	○	●
$\left\|\sin\left(6\pi t^2\right)\right\|$	●	○	○

4. *Newton method* [4 P.]

Given a regular matrix $\mathbf{A} \in \mathbb{R}^{n,n}$, which of the following formulas describe the Newton iteration for solving $F(\mathbf{X}) = \mathbf{0}_{n,n}$ with

$$F : \mathbb{R}^{n,n} \to \mathbb{R}^{n,n}, \quad F(\mathbf{X}) := \mathbf{A}\mathbf{X} - \mathbf{I}_{n,n} \qquad ?$$

[+1P correct, -1P wrong, min: 0P]

- $\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + \mathbf{A}^{-1}\left(\mathbf{I} - \mathbf{A}\mathbf{X}^{(k)}\right)$ — ● true ○ false
- $\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)}\left(\mathbf{I} - \mathbf{A}\mathbf{X}^{(k)}\right)$ — ○ true ● false
- $\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + \mathbf{A}\mathbf{X}^{(k)} - \mathbf{I}$ — ○ true ● false
- $\mathbf{X}^{(k+1)} = \mathbf{A}^{-1}$ — ● true ○ false

Scratch space (not evaluated):

5. *Convergence* [4 P.]

Which of the following statements hold true for a linearly convergent sequence $\left(\mathbf{x}^{(k)}\right)_{k\in\mathbb{N}_0}$, $\mathbf{x}^{(k)} \in \mathbb{R}^n$, with limit $\mathbf{x}^\star = \lim\limits_{k\to\infty} \mathbf{x}^{(k)}$? [+1P correct, -1P wrong, min: 0P]

- $\exists L \in [0,1[: \ \left\|\mathbf{x}^{(k)} - \mathbf{x}^\star\right\| \le L^k \left\|\mathbf{x}^{(0)} - \mathbf{x}^\star\right\|$ — ● true ○ false
- $\exists L \in [0,1[: \ \left\|\mathbf{x}^{(k+1)}\right\| \le L \left\|\mathbf{x}^{(k)}\right\|$ — ○ true ● false
- $\exists L \in [0,1[: \ \left\|\mathbf{x}^{(k+1)} - \mathbf{x}^\star\right\| \le L \left\|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\right\|$ — ○ true ● false
- $\exists C \ge 0: \ \left\|\mathbf{x}^{(k+1)} - \mathbf{x}^\star\right\| \le C \left\|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\right\|$ — ● true ○ false

$\|\cdot\|$ is a vector norm on $\mathbb{R}^n$.

Scratch space (not evaluated):