ETH zürich
Departement Informatik

Theoretical Computer Science

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First Midterm Exam

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Exercise 1

(a) Construct a deterministic finite automaton in graphical representation that accepts the language

$$L = \Big\{ w \in \{0,1\}^* \mid w = a_1 a_2 \dots a_n \text{ for some } n \in \mathbb{N} \text{ and } a_1, \dots, a_n \in \{0,1\} \text{ and } \Big(\sum_{\substack{1 \le i \le n \\ i \text{ odd}}} \text{Number}(a_i) \Big) \bmod 3 = 0 \Big\}.$$

(b) For each state q of your automaton as constructed in exercise part (a), give the class $\text{Kl}[q]$.

5+5 points

Exercise 2

(a) Prove, using the method of Kolmogorov complexity, that the language

$$L = \{ w \in \{0,1\}^* \mid w = uvu \text{ and } |u| = |v| \}$$

is not regular.

(b) Prove that the language

$$L = \{ w \in \{a,b\}^* \mid w = aub \text{ for some } u \in \{a,b\}^* \text{ and } |w|_a \text{ divides } |w|_b \}$$

is not regular.

For your proof, you may choose among the following two proof methods.

(i) Using Lemma 3.12 from the English book (i. e., Lemma 3.3 from the German book) or a direct argumentation about the automaton, or

(ii) using the pumping lemma.

4+6 points

(please turn the page)

Exercise 3

We consider the language

$$L = \{x00y \mid x \in \{0,1\}^* \text{ and } y \in \{0,1\}\}\,.$$

(a) Construct a nondeterministic finite automaton with at most 4 states that accepts L.

(b) Prove that every deterministic finite automaton accepting L needs at least 5 states.

4+6 points

Exercise 4

(a) Prove that the language $(L_{\text{empty}})^{\complement}$ is recursively enumerable.

(b) Prove that $(L_{\text{empty}})^{\complement}$ is not recursive by giving a concrete reduction from L_{U} and showing the correctness of this reduction.

4+6 points