

Midterm exam

Num. CSE, D-INFK/D-MATH

HS 2015

Prof. R. Hiptmair

Name		Grade
Surname		
Department		
Legi Nr.		
Date	23.10.2015	

1	2	3	4	5	6	Total
8	8	8	8	10	12	54

- Only writing material and Legi on the table
- Keep mobile phones, tablets, smartwatches, etc. **turned off** in your bag
- Fill in this cover sheet first
- Turn the cover sheet only when instructed to do so
- Then, put your name and Legi Nr. on each page
- Carefully read the rules for the multiple-choice problems
- Do not write with red/green colour or with pencil
- Write your solution in the predefined tick boxes
- Before handing-in the sheets, make sure you wrote the name on each sheet
- Duration: 30 min
- Additional material: none

Good luck!

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Rules of multiple-choice (MC):

- *(Clearly) put a cross in the circle with the answer you deem correct.*
- *Motivation for the choices is **not** necessary. Remarks and computations have **no** influence on the total number of points.*
- *Wrong answers give negative points.*
- *Mutually exclusive answers are weighted such that the expectation of a random choice is zero.*
- *The total points for a single MC-problem is the maximum between the points achieved and zero.*
- *Mutually exclusive answers are highlighted in the same box.*
- *Any unclear marking will be considered an error.*

1. Cancellation errors [8 P.]

Which expressions are likely to be affected by cancellation errors for some of the x s in the indicated ranges?

(a) $y = (\sin(x) + x)/x$ for x in $[-0.1, 0.1]$

☐ Affected by cancellation. ☒ Not affected by cancellation.

(b) $z = \exp(x)$, $y = z - 1/z$ for x in $[-1, 1]$

☒ Affected by cancellation. ☐ Not affected by cancellation.

(c) $y = \sqrt{x + 1} - \sqrt{x}$ for x in $[0.9, 1.1]$

☐ Affected by cancellation. ☒ Not affected by cancellation.

(d) $y = \log(x) - \log(x - 1)$ for x in $[2, +\infty)$

☒ Affected by cancellation. ☐ Not affected by cancellation.

Scratch space (not evaluated):

2. The Newton iteration [8 P.]

Take $a \in \mathbb{R}$. Let $F: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $F(x) = xe^x - a$ for $x \in \mathbb{R}$. Establish which of the following iterations of the Newton method are correct for solving $F(x) = 0$.

(a) $x^{(k+1)} = \frac{x^{(k)}}{x^{(k)}+1} \left(x^{(k)} + \frac{ae^{-x^{(k)}}}{x^{(k)}} \right).$

☒ Correct iteration. ☐ Incorrect iteration.

+ 2

- 2

(b) $x^{(k+1)} = x^{(k)} - e^{x^{(k)}}(a - x^{(k)}e^{x^{(k)}})(x^{(k)} + 1).$

☐ Correct iteration. ☒ Incorrect iteration.

- 2

+ 2

(c) $x^{(k+1)} = x^{(k)} + \frac{a - x^{(k)}e^{x^{(k)}}}{x^{(k)}e^{x^{(k)}} + e^{x^{(k)}}}.$

☒ Correct iteration. ☐ Incorrect iteration.

+ 2

- 2

(d) $x^{(k+1)} = x^{(k)} - \frac{x^{(k)}e^{x^{(k)}} + e^{x^{(k)}}}{x^{(k)}e^{x^{(k)}} - a}.$

☐ Correct iteration. ☒ Incorrect iteration.

- 2

+ 2

Scratch space (not evaluated):

3. Numerical stability [8 P.]

Let $\tilde{F}: X \rightarrow Y$ be an algorithm for the problem $F: X \rightarrow Y$. Let $w(x)$ denote the computational effort required for evaluating \tilde{F} with input x . Establish which of the following is the correct definition of numerical stability. Here eps represents the machine precision.

We say that \tilde{F} is *numerically stable* if there exists $C \approx 1$ such that

(a) $\forall x \in X. \quad \|F(x) - \tilde{F}(x)\|_Y \leq Cw(x)\text{eps}\|\tilde{F}(x)\|_Y.$

(b) $\forall x \in X. \quad \exists \tilde{x} \in X. \quad \|\tilde{x} - x\|_X \leq Cw(x)\text{eps}\|x\|_X \wedge \tilde{F}(\tilde{x}) = F(x).$

(c) $\forall x \in X. \quad \exists \tilde{x} \in X. \quad \|F(x) - \tilde{F}(\tilde{x})\|_Y \leq Cw(x)\text{eps}\|F(x)\|_Y.$

(d) $\forall x \in X. \quad \exists \tilde{x} \in X. \quad \|x - \tilde{x}\|_X \leq Cw(x)\text{eps}\|x\|_X \wedge \tilde{F}(x) = F(\tilde{x}).$

Which one is the correct definition?

<input type="radio"/> (a)	<input type="radio"/> (b)	<input type="radio"/> (c)	<input checked="" type="radio"/> (d)
- 8 / 3	- 8 / 3	- 8 / 3	+ 8

Scratch space (not evaluated):

4. Fixed point iteration [8 P.]

A fixed point iteration for solving $F(x) = 0$, $F = [F_1, F_2]^T: D \subset \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is known to be **affine invariant**. Which of the following conclusions are true?

- (a) The iterates for F and $\tilde{F} = [F_2, F_1]^T$ agree if the initial guesses are the same.

☒ True. ☐ False.

- (b) When applied to $\tilde{F} = [F_1 + F_2, 0]^T$, the method produces the same iterates.

☐ True. ☒ False.

- (c) The method produces the same iterates for F and for \tilde{F} defined by $\tilde{F}(x_1, x_2) = F(x_2, x_1)$.

☐ True. ☒ False.

- (d) Let $x^{(k)}$ and $\tilde{x}^{(k)}$ be two different sequences generated by the fixed point method applied to F . If $\tilde{x}^{(0)} = Ax^{(0)}$ for some regular matrix $A \in \mathbb{R}^{2,2}$, then $\tilde{x}^{(k)} = Ax^{(k)}$ for every k .

☐ True. ☒ False.

Scratch space (not evaluated):

5. C++ code [10 P.]

Consider the C++ and Eigen code:

```

1  #include <Eigen/Dense>
   using Eigen;
3  void fn(const Matrix<double, Dynamic, Dynamic, RowMajor> &A,
         const VectorXd &v, VectorXd &w) {
5     const double *a = A.data(); // ptr. to linear data of A
     const std::size_t n = A.rows();
7     assert( ( n == v.size() ) && ( n == w.size() ) );
     for(std::size_t i = 0; i < n; ++i) {
9         a += i;
         w(i) = 0.;
11        for(std::size_t j = i; j < n; ++j) {
            w(i) += (*a)*v(j);
13            a++;
        }
15    }
}

```

For each of the following lines of Matlab code, decide whether it is algebraically equivalent to the function `fn` or not.

HINT: The functions `triu(A)` and `tril(A)` return the upper and lower triangular parts of A , respectively.

HINT: Have a look at the figure in the next page for an picture hint.

Algebraically equivalent?

(a) $w = (A+A')*v;$

<input type="radio"/> yes.	<input checked="" type="radio"/> no.
-5/3	+5/3

(b) $w = \text{triu}(A)*v;$

<input checked="" type="radio"/> yes.	<input type="radio"/> no.
+5/3	-5/3

(c) $w = \text{triu}(A')*v;$

<input type="radio"/> yes.	<input checked="" type="radio"/> no.
-5/3	+5/3

(d) $w = \text{tril}(A)*v;$

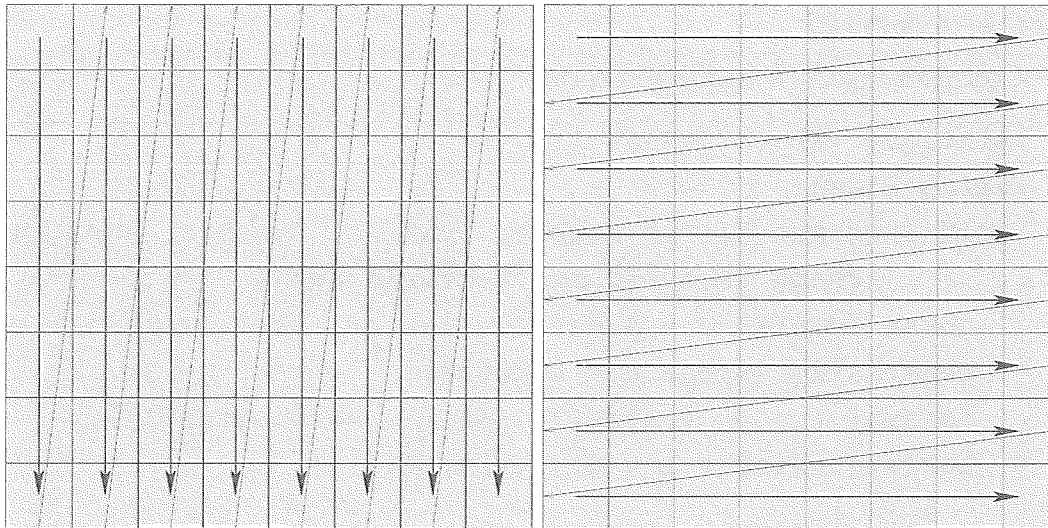
<input type="radio"/> yes.	<input checked="" type="radio"/> no.
-5/3	+5/3

(e) $w = \text{tril}(A')*v;$

<input type="radio"/> yes.	<input checked="" type="radio"/> no.
-5/3	+5/3

(f) $w = A*v;$

<input type="radio"/> yes.	<input checked="" type="radio"/> no.
-5/3	+5/3



Column major

Row major

Scratch space (not evaluated):

6. *QR-decomposition* [12 P.]

Consider the MATLAB code:

```

function [x,r] = solveandmore(A,b)
2   [n,m] = size(A);
    if(n ~= m), error('A must be square'); end
4   [Q,R] = qr(A)
    r = min(diag(R));
6   x = A\b;
end

```

For each of the following lines of code, decide whether it is algebraically equivalent to line 6 above or not. Provide the asymptotic complexity of the substitute line, if A is a generic dense square matrix.

HINT: Matlab's \ can detect triangular system matrices.

(a) $x = R \setminus (Q \setminus b)$;

<input checked="" type="radio"/> algebraically equivalent.	<input type="radio"/> not algebraically equivalent.
<input type="radio"/> $O(n)$.	<input type="radio"/> $O(n^2)$.
<input checked="" type="radio"/> $O(n^3)$.	

(b) $x = R \setminus (Q' * b)$;

<input checked="" type="radio"/> algebraically equivalent.	<input type="radio"/> not algebraically equivalent.
<input type="radio"/> $O(n)$.	<input checked="" type="radio"/> $O(n^2)$.
<input type="radio"/> $O(n^3)$.	

(c) $x = (Q * R) \setminus b$;

<input checked="" type="radio"/> algebraically equivalent.	<input type="radio"/> not algebraically equivalent.
<input type="radio"/> $O(n)$.	<input type="radio"/> $O(n^2)$.
<input checked="" type="radio"/> $O(n^3)$.	

(d) $x = Q \setminus (R \setminus b)$;

<input type="radio"/> algebraically equivalent.	<input checked="" type="radio"/> not algebraically equivalent.
<input type="radio"/> $O(n)$.	<input type="radio"/> $O(n^2)$.
<input checked="" type="radio"/> $O(n^3)$.	

(e) $x = (Q \setminus R) \setminus b$;

<input type="radio"/> algebraically equivalent.	<input checked="" type="radio"/> not algebraically equivalent.
<input type="radio"/> $O(n)$.	<input type="radio"/> $O(n^2)$.
<input checked="" type="radio"/> $O(n^3)$.	

(f) $x = Q' * (R \setminus b)$;

<input type="radio"/> algebraically equivalent.	<input checked="" type="radio"/> not algebraically equivalent.
<input type="radio"/> $O(n)$.	<input checked="" type="radio"/> $O(n^2)$.
<input type="radio"/> $O(n^3)$.	

Scratch space (not evaluated):