ETH zürich
Departement Informatik

Theoretical Computer Science

Prof. Dr. Juraj Hromkovič
Dr. Hans-Joachim Böckenhauer

Second Midterm Exam

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Exercise 1

(a) Show that $L_{\mathrm{U}} \leq_{\mathrm{R}} L_{\mathrm{H}}$ by giving a concrete reduction and proving its correctness.

(b) We consider the language

$$L = \{\mathrm{Kod}(M_1)\#\mathrm{Kod}(M_2)\#\mathrm{Kod}(M_3) \mid M_1, M_2, M_3 \text{ are TM and } (L(M_1) \cup L(M_2)) \cap L(M_3) \neq \emptyset\}\,.$$

Prove that $L \in \mathcal{L}_{\mathrm{RE}}$.

6+4 points

Exercise 2

We consider the language

$$L_{\leq 10} = \{\mathrm{Kod}(M) \mid M \text{ is a TM accepting at most 10 words}\}\,.$$

Which of the following statements is true?

(i) $L_{\leq 10} \in \mathcal{L}_{\mathrm{R}}$.

(ii) $L_{\leq 10} \in \mathcal{L}_{\mathrm{RE}} - \mathcal{L}_{\mathrm{R}}$.

(iii) $L_{\leq 10} \notin \mathcal{L}_{\mathrm{RE}}$.

Prove the statement you found out to be true. **10 points**

(please turn the page)

Exercise 3

(a) Let E3SAT denote the set of all CNF formulas with exactly three literals (of pairwise different variables) per clause that have a satisfying assignment. Prove that E3SAT is NP-hard.

(b) We call a clause of a CNF formula *monotone* if it either does not contain any negated variables or if it contains only negated variables. We consider the set non-3-monotone-3SAT of all satisfiable CNF formulas that consist of clauses of length at most 3 and do not contain monotone clauses of length exactly 3 (monotone clauses of length 2 or 1 are still allowed).

Prove that non-3-monotone-3SAT is NP-complete.

Hint: For your proofs, you may assume that the problems SAT, 3SAT, CLIQUE, VC, SCP, DS, MONO-SAT und SUBSET-SUM, as considered in the lecture or in the exercises, are NP-hard. **4+6 points**

Exercise 4

Let $s\colon \mathbb{N} \to \mathbb{N}$ with $s(n) \geq \log_2(n)$ be time- and space-constructible.
Prove that $\mathrm{NTIME}(s(n)) \subseteq \mathrm{SPACE}(s(n))$.

Hint: You may assume that, for any time-constructible function $t\colon \mathbb{N} \to \mathbb{N}$ and for every nondeterministic MTM M with $\mathrm{Time}_M(n) \leq t(n)$, there exist an equivalent nondeterministic MTM M' and a constant $d \in \mathbb{N}$ such that all computations of M' on arbitrary inputs of length n are of length at most $d \cdot t(n)$. **10 points**