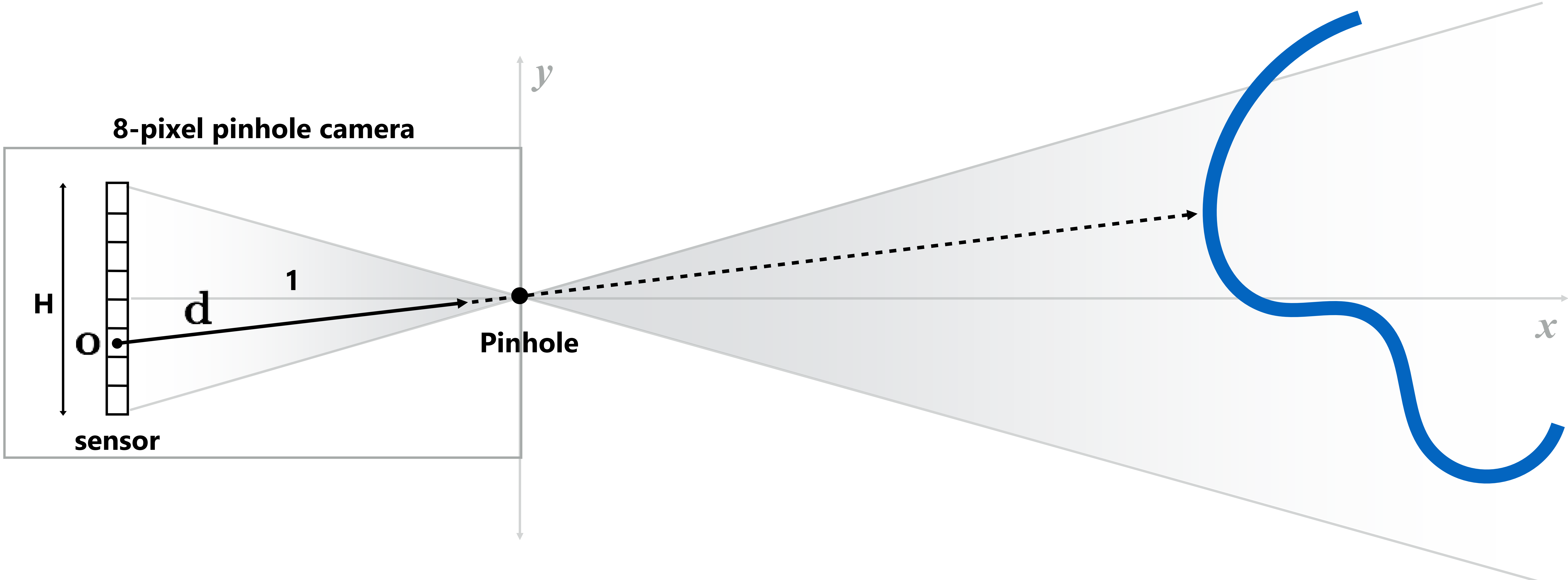
Light, Color and
The Rendering Equation

Simulating a pinhole camera

Imprecise statements you've heard so far…

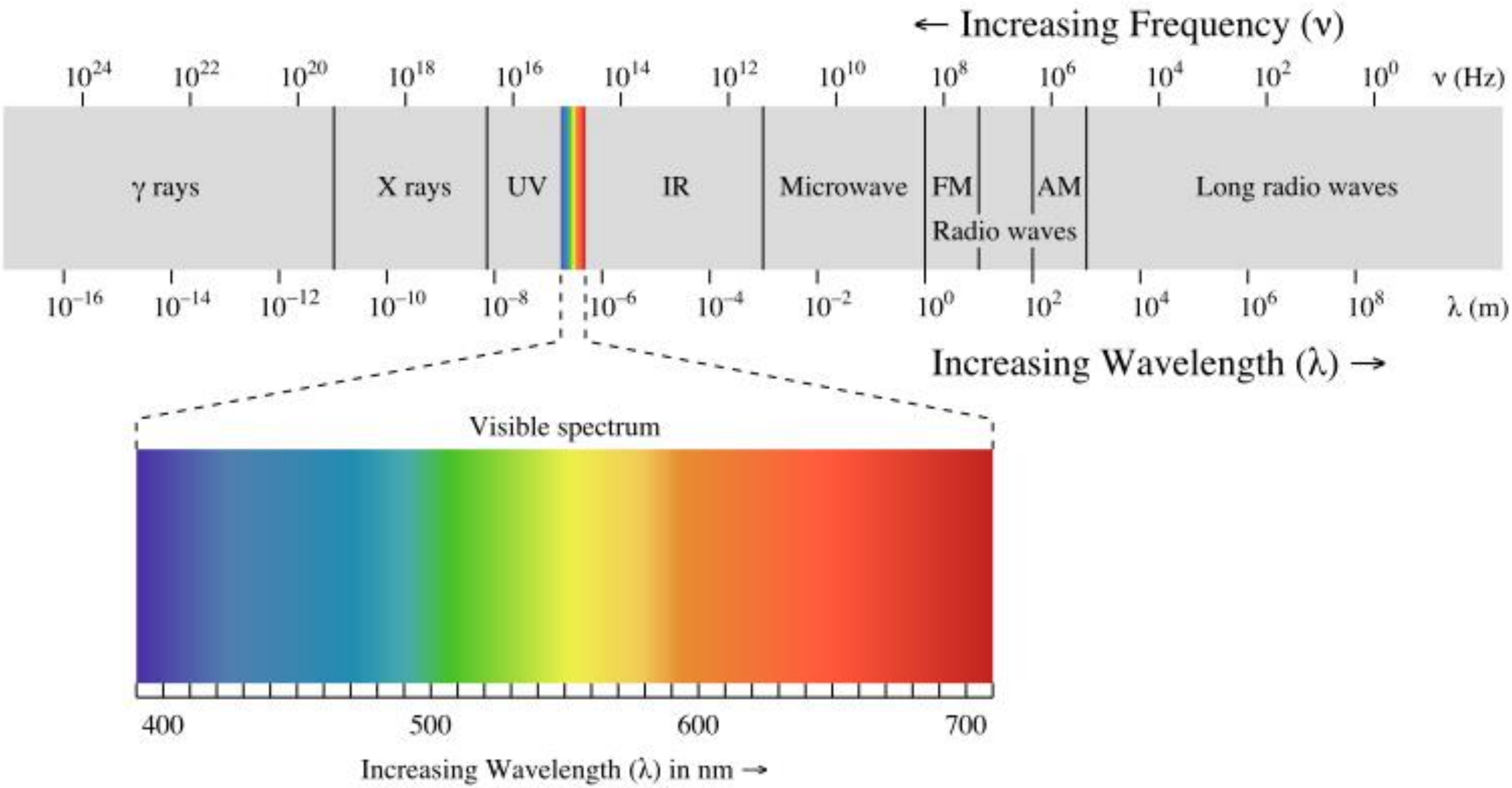
What is the "color" of the surface visible at a sample point?



Need a more precise way to describe and measure the light that arrives at sensor through pinhole.

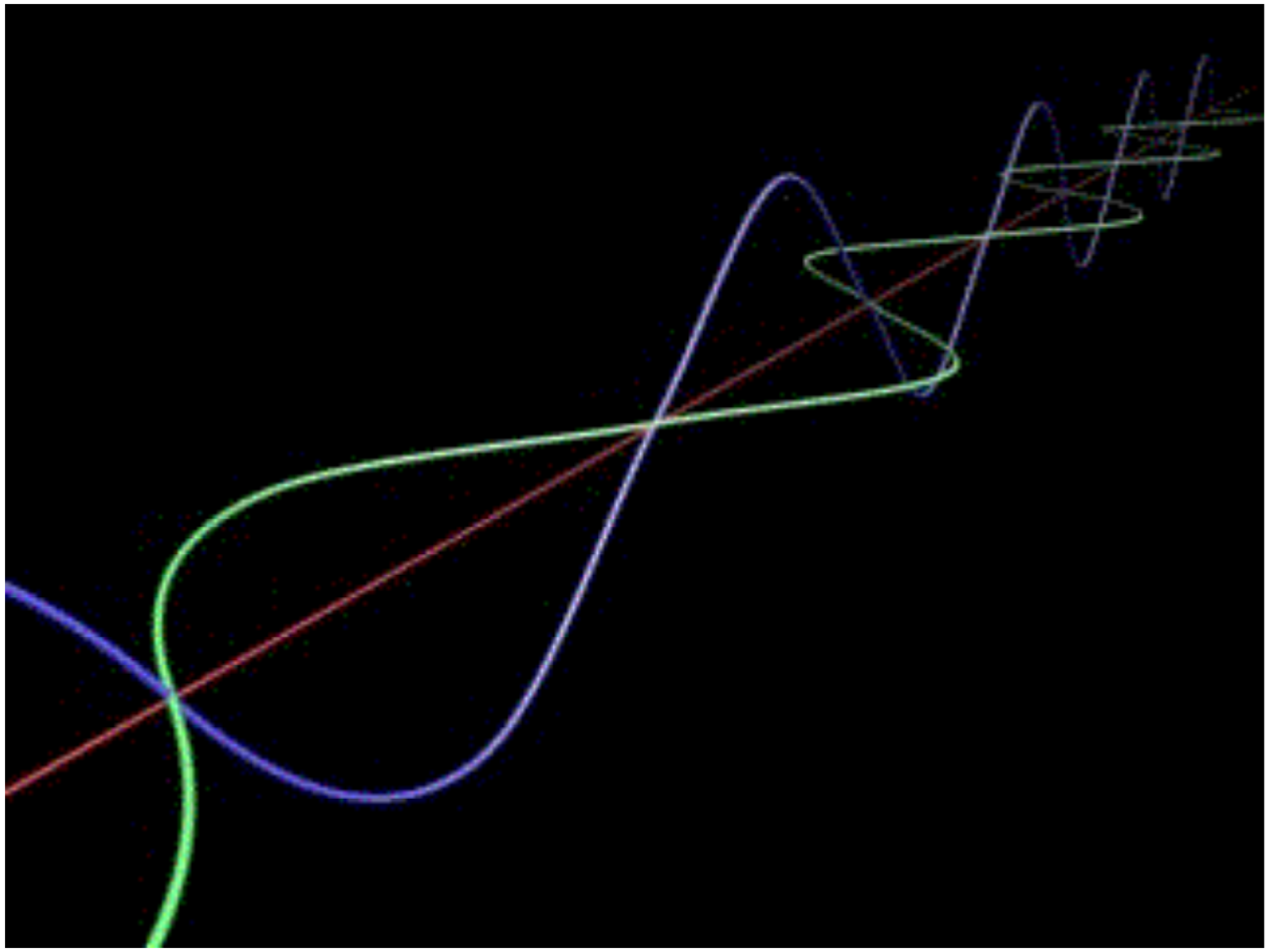
What is color?

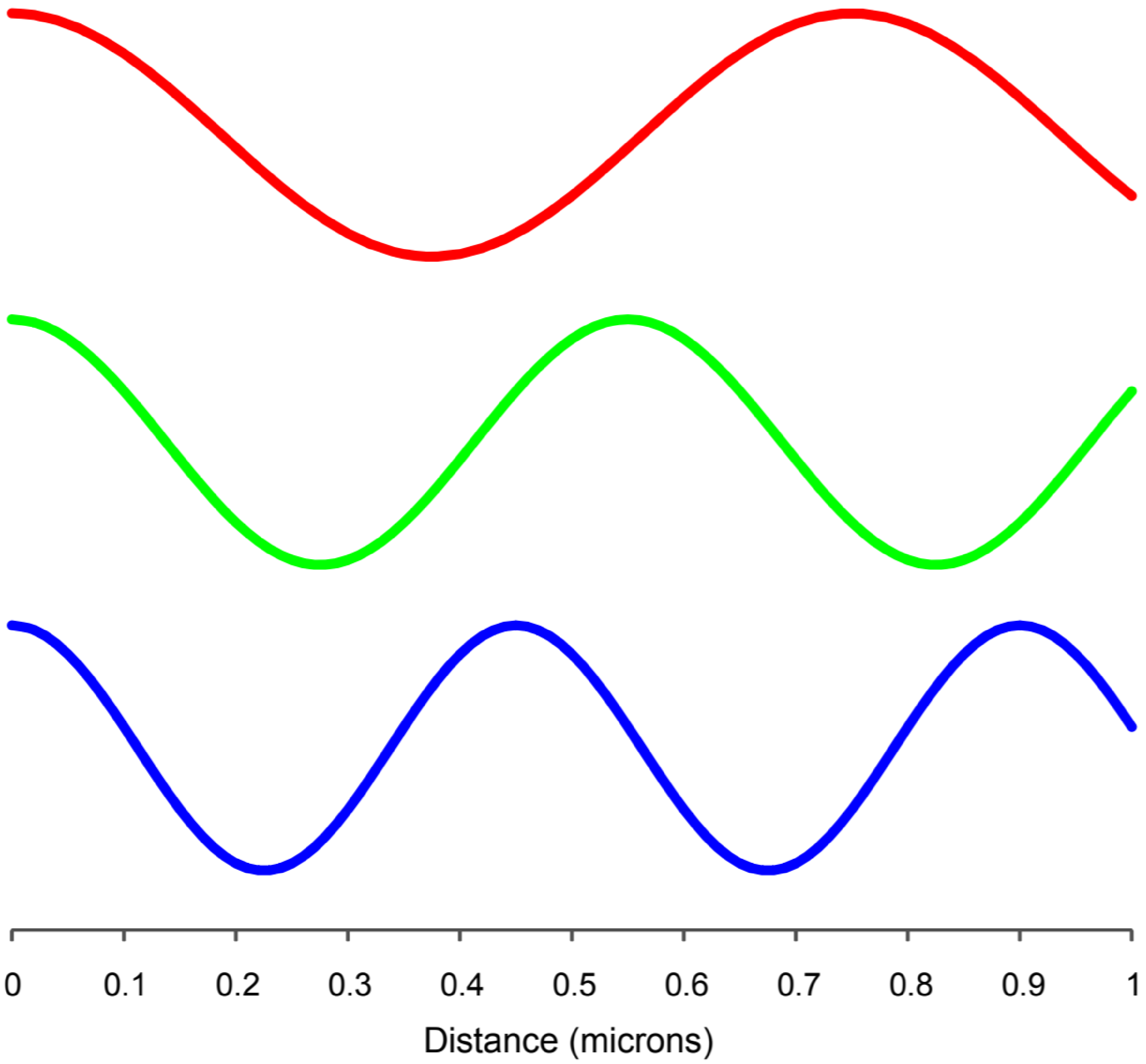
Light is electromagnetic radiation



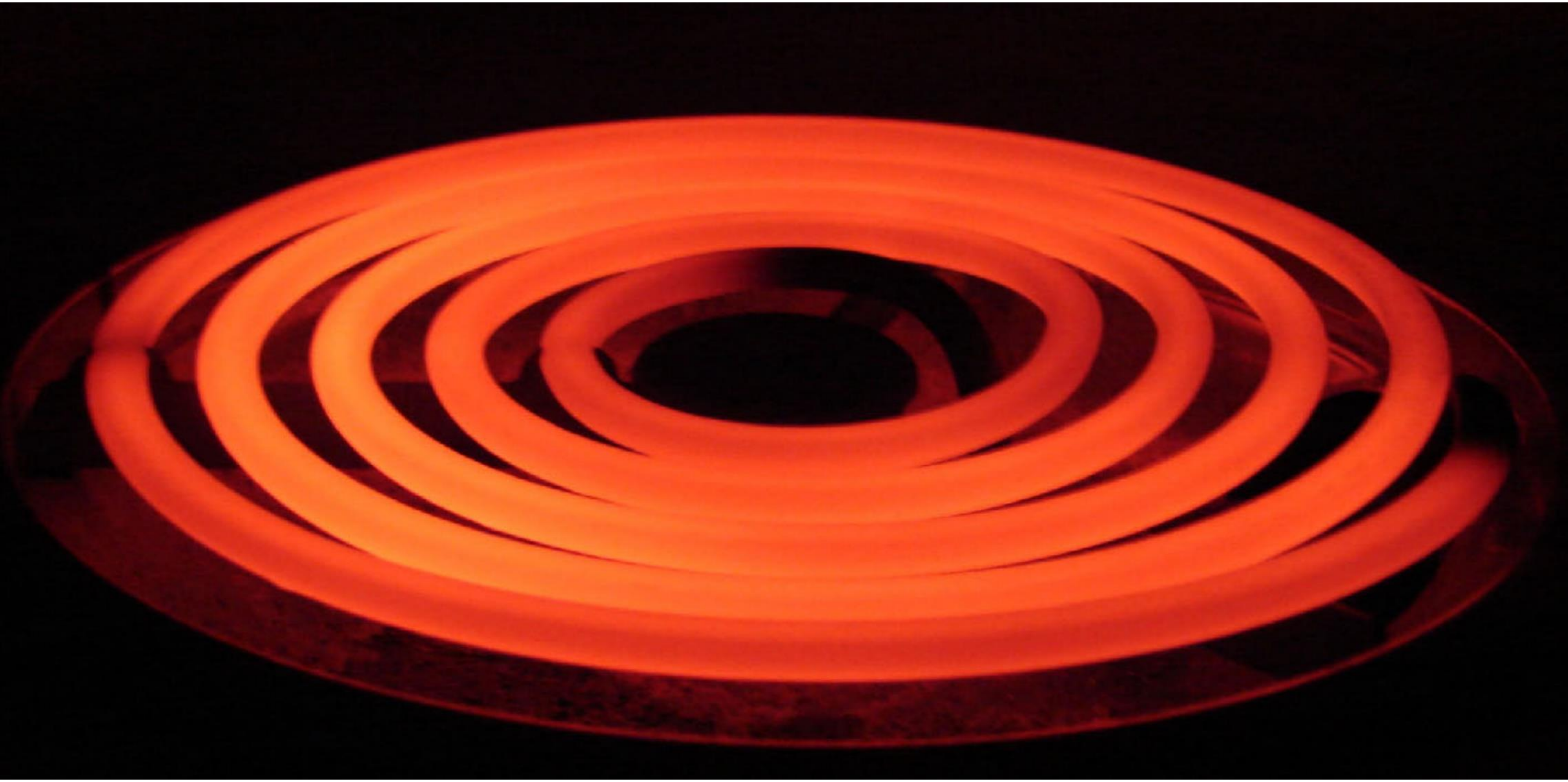
Light is EM radiation; Color is Frequency

- Light is oscillating electric and magnetic field
- Frequency determines color of light
- Q: What is the difference between frequency and wavelength?



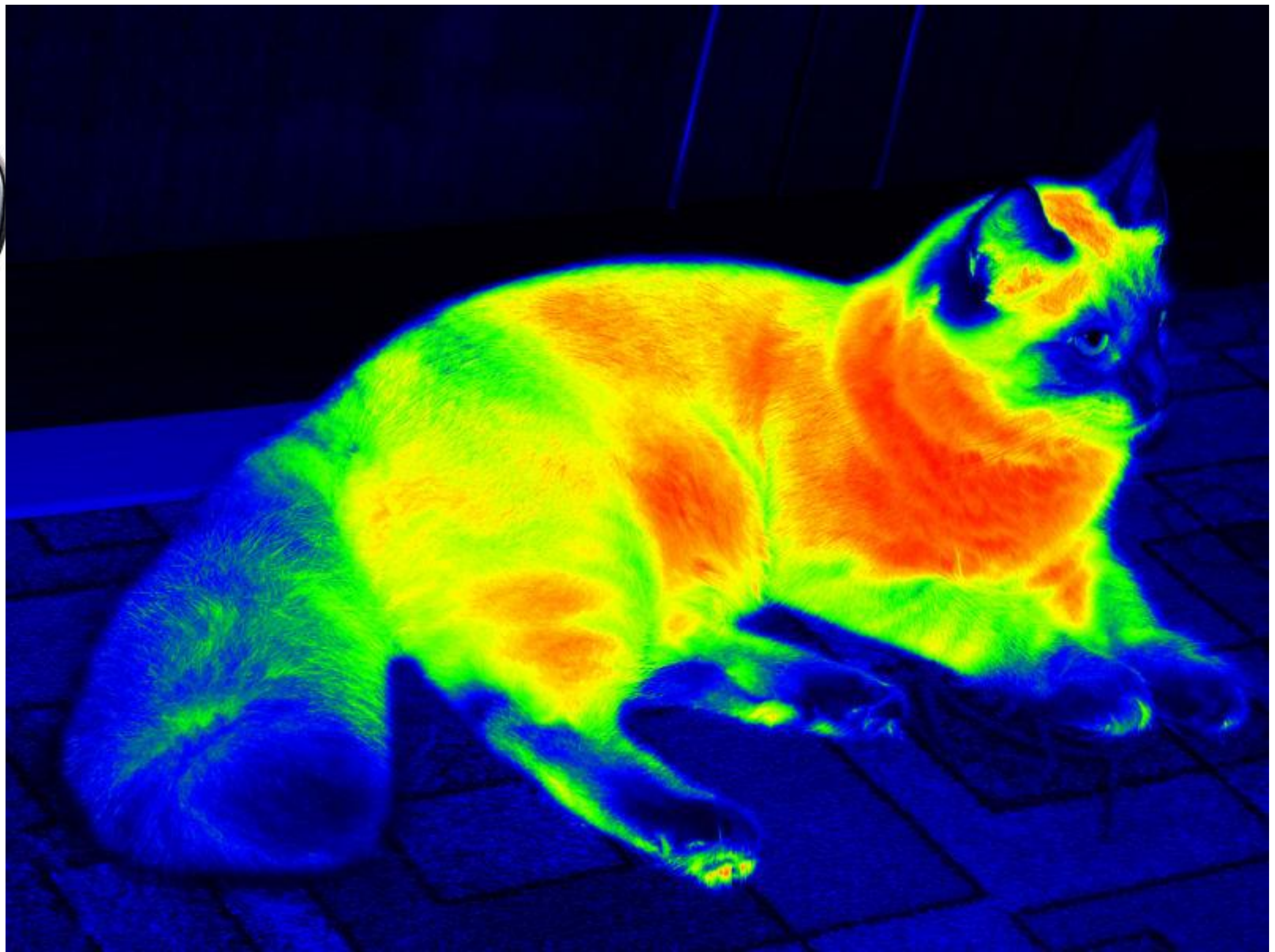


Why does your stove turn red when it heats up?



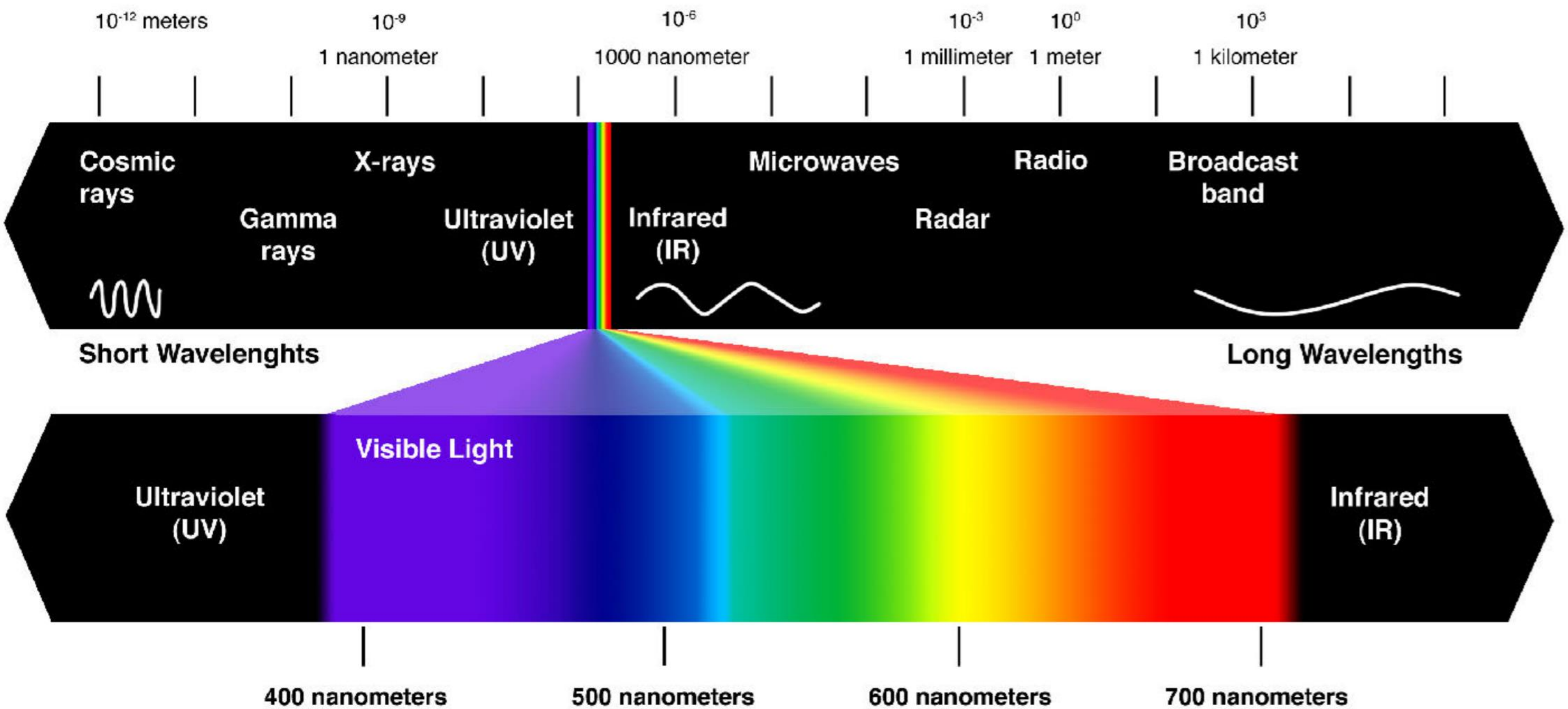
Heat generates light

- One of many ways light (EM radiation) is created
  - Maxwell: motion of charged particle creates EM field
  - Thermodynamics: particles jiggle around
- Hence, anything that's moving generates light!
  - Every object around you is producing color
  - Frequency determined by temperature



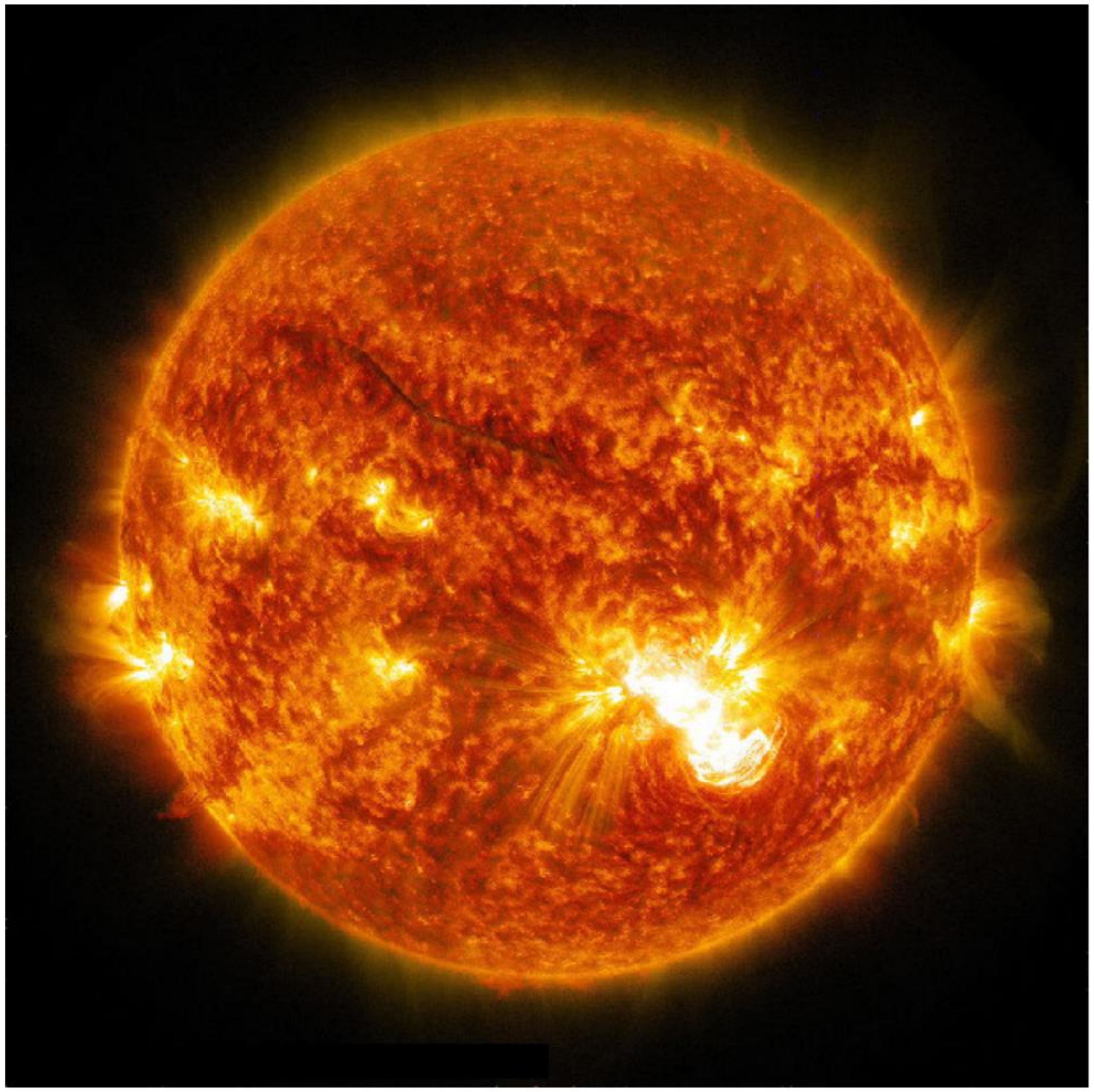
Most light is not visible to human eye

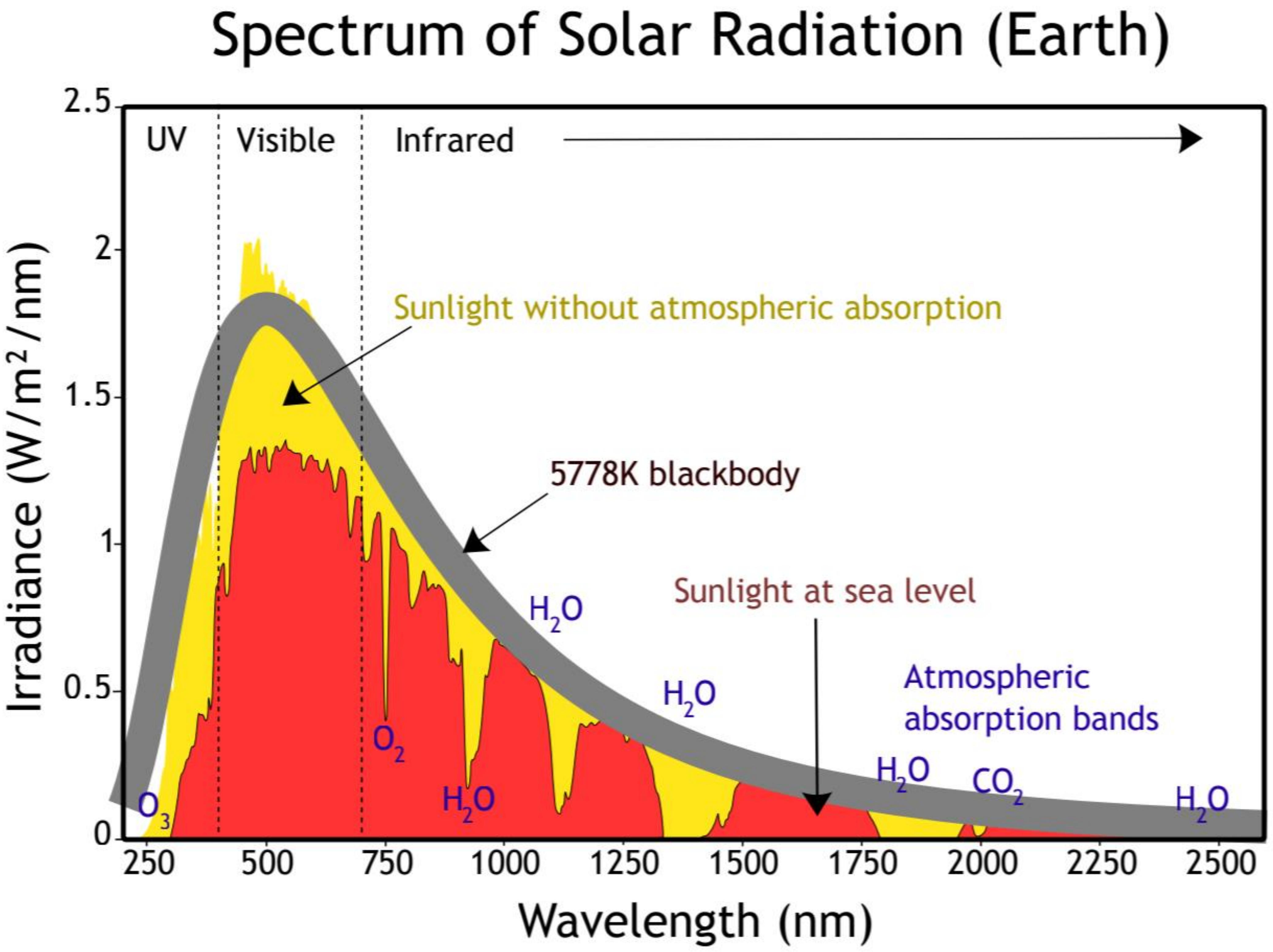
- **Frequencies visible to human eyes are called “visible spectrum”**
- **These frequencies are what we think of as color**



Natural light is a mixture of frequencies

- "White" light is really a mixture of all (visible) frequencies





Additive vs. Subtractive Models of Light

- **We saw the "emission spectrum" for the sun**
  - **How much light is *produced* (by heat, fusion, etc.)**
  - **Useful to compare against other sources of light (e.g. lightbulb)**

Emission Spectrum

Spectrum of various common light sources

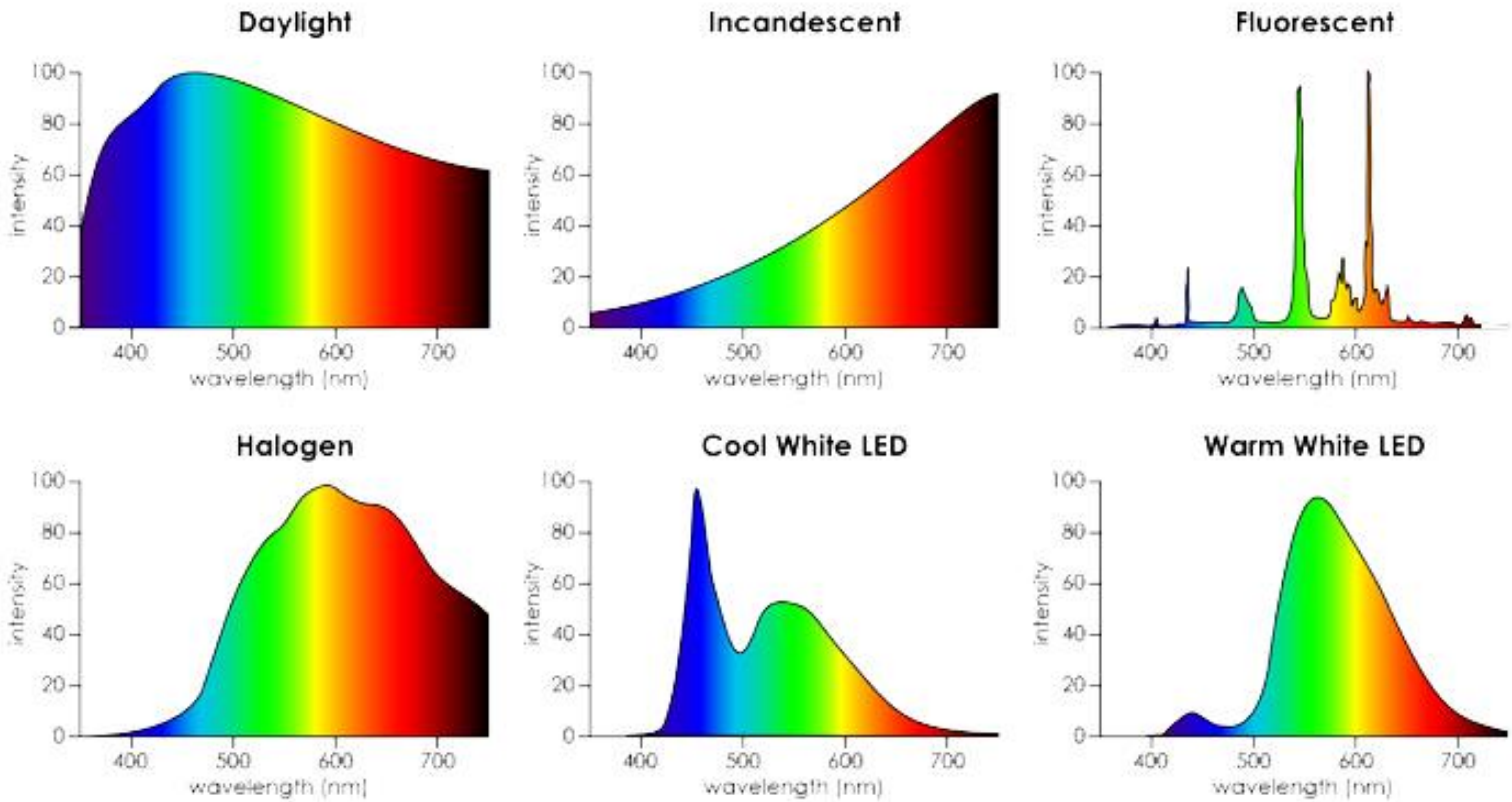


Figure credit:

admesy ADVANCED MEASUREMENT SYSTEMS

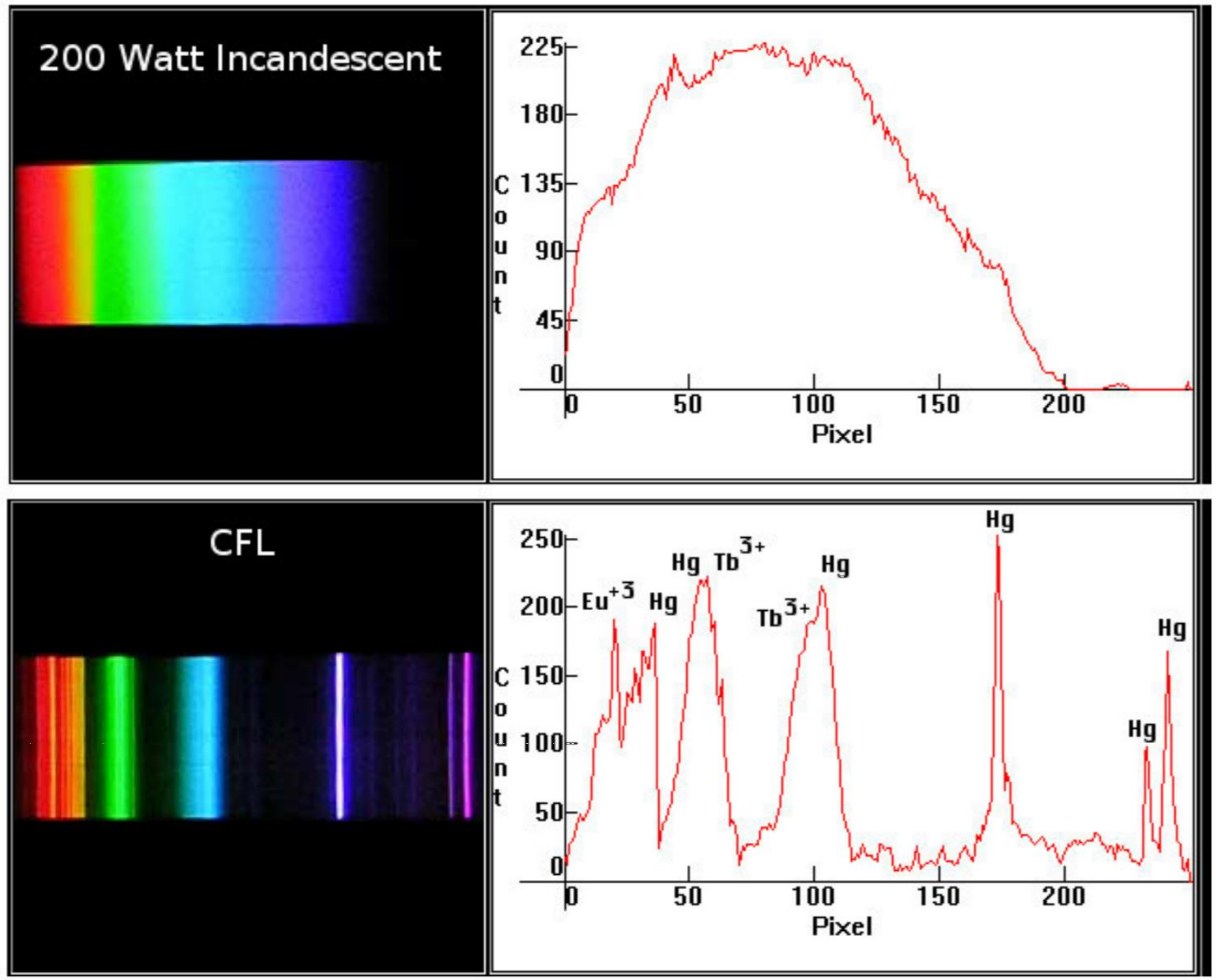
Emission Spectrum – an example

Why so many different kinds of lightbulbs on the market?

"Quality" of light:

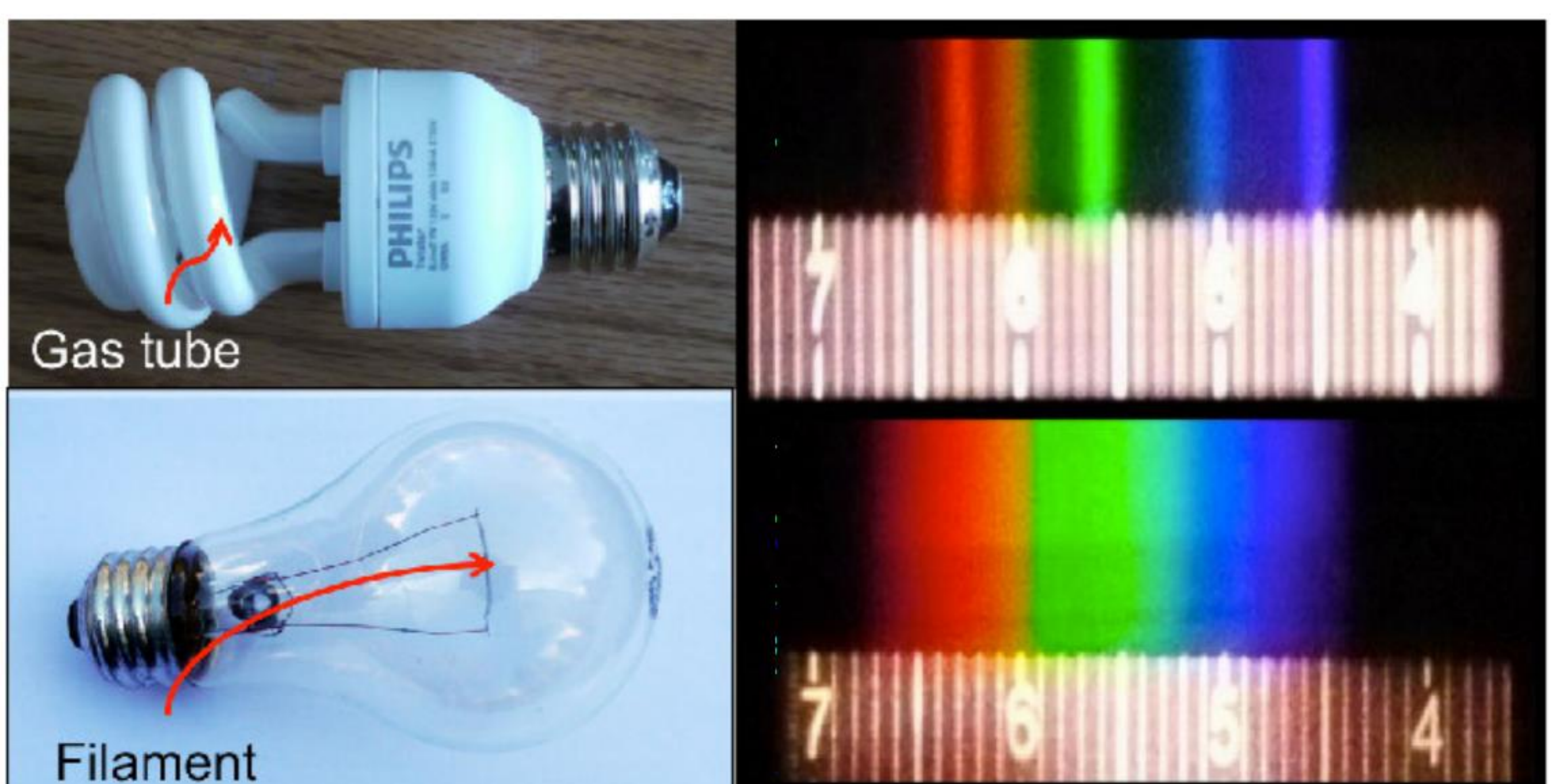
Incandescent:
- +more sun-like
- -power-hungry

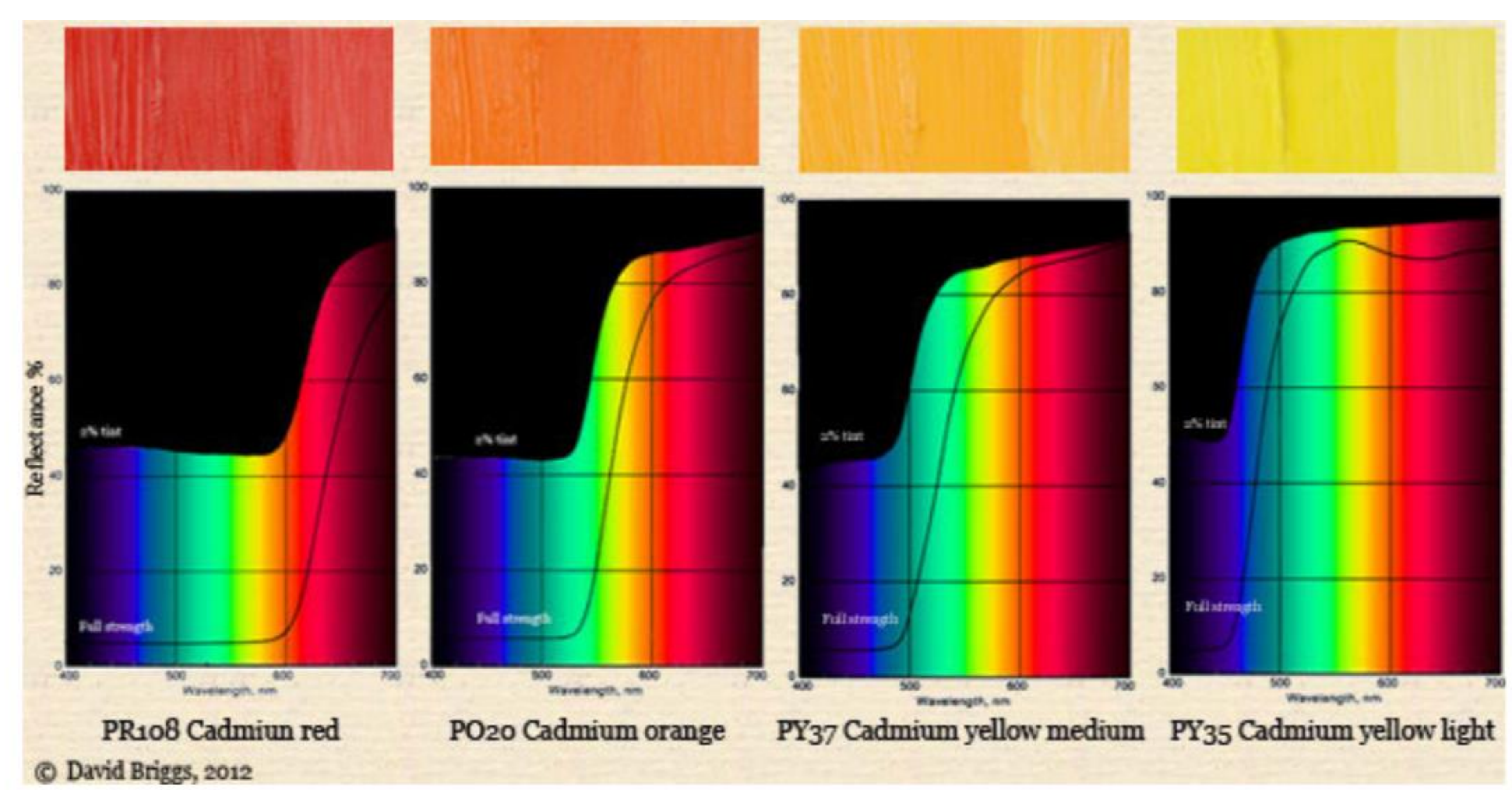
CFL:
- -"choppy" spectrum
- +power efficient



Additive vs. Subtractive Models of Light

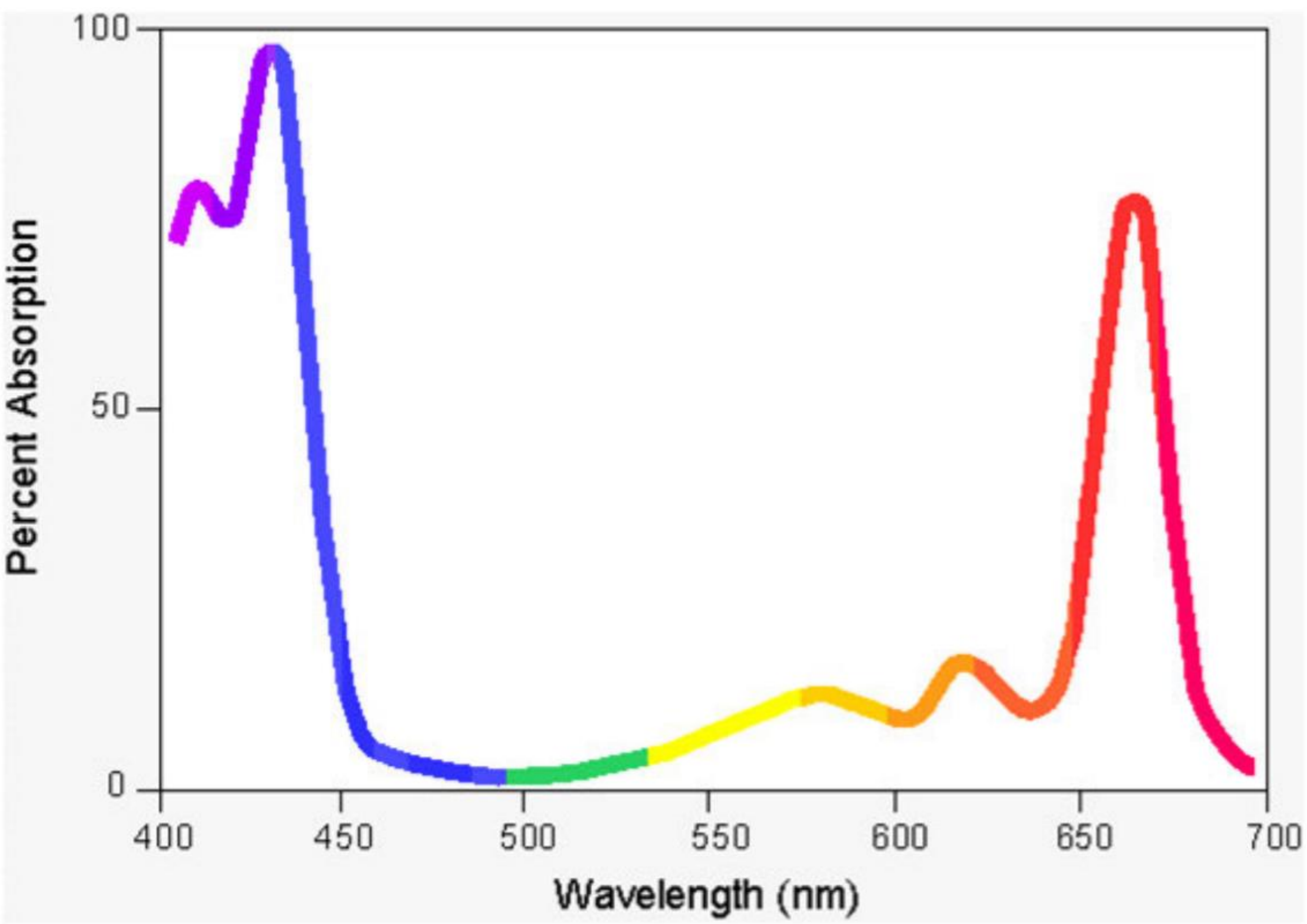
- **We saw the "emission spectrum" for the sun**
  - **How much light is *produced* (by heat, fusion, etc.)**
  - **Useful to compare against other sources of light (e.g. lightbulb)**
- **Another very useful description: "absorption spectrum"**
  - **How much light is *absorbed* (e.g. turned *into* heat)**
  - **Useful to characterize color of paint, ink, etc.**





Absorption Spectrum

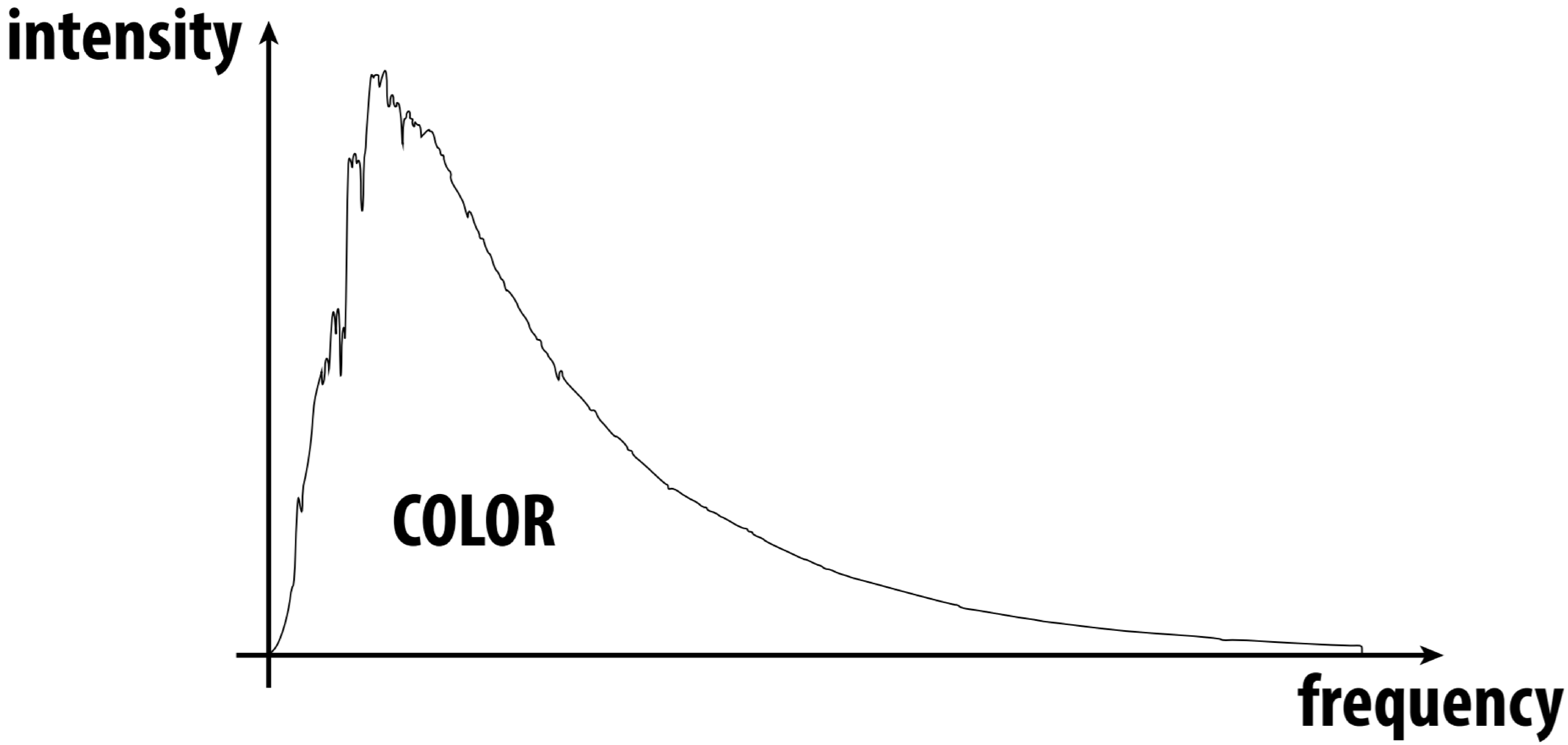
- Emission spectrum is intensity as a function of frequency
- Absorption spectrum is fraction absorbed as function of frequency



Q: What color is an object with this absorption spectrum?

This is the fundamental description of color:

Intensity, emission and absorption as a function of frequency



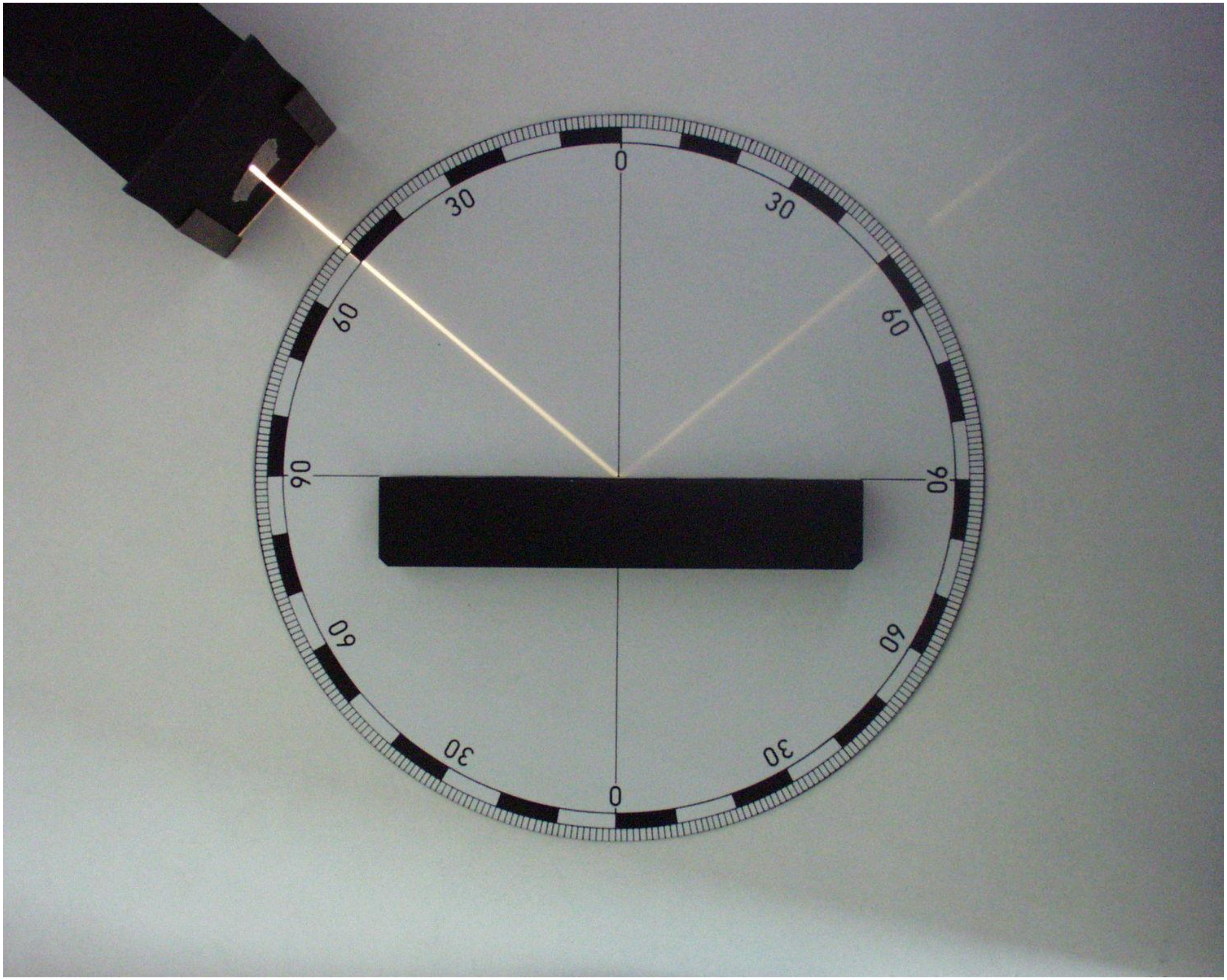
Everything else is just a convenient approximation!

If we think about color this way, then many phenomena we care about become a lot more intuitive

An example: reflection

- **Toy model for what happens when light gets reflected**

Perfect specular reflection



[Zátonyi Sándor]

An example: reflection

- Toy model for what happens when light gets reflected
- Reflection is emission times absorption
  - Light source has emission spectrum f(λ)
  - Surface has reflection spectrum g(λ)
  - Resulting intensity is the product f(λ) * g(λ)

But what about perception?

Q: What color is this dress?

How does electromagnetic radiation (with a given frequency distribution) end up being perceived by a human as a certain color?

The eye

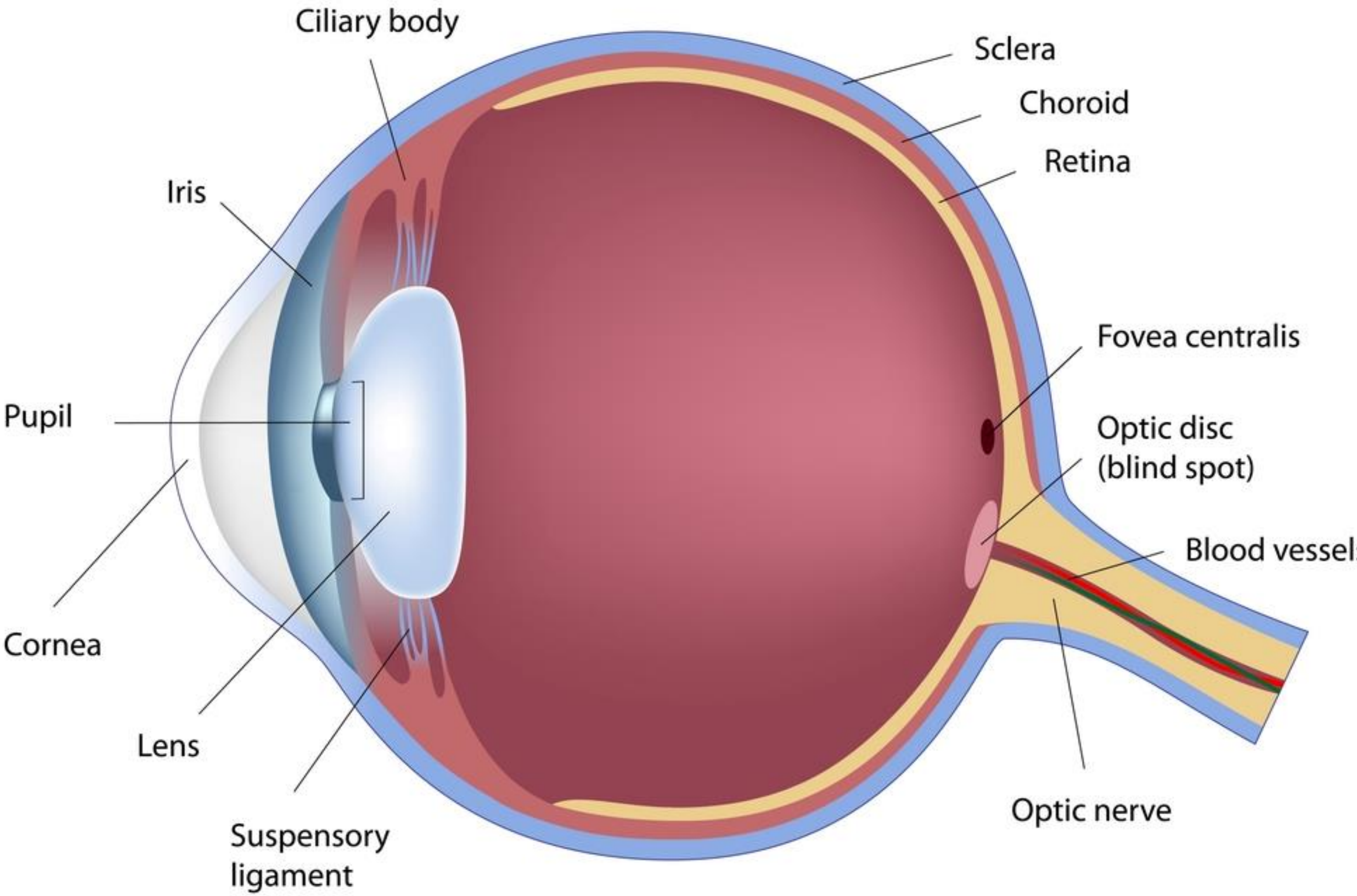


Image credit: Georgia Retina (http://www.garetina.com/about-the-eye)

The eye (optics)

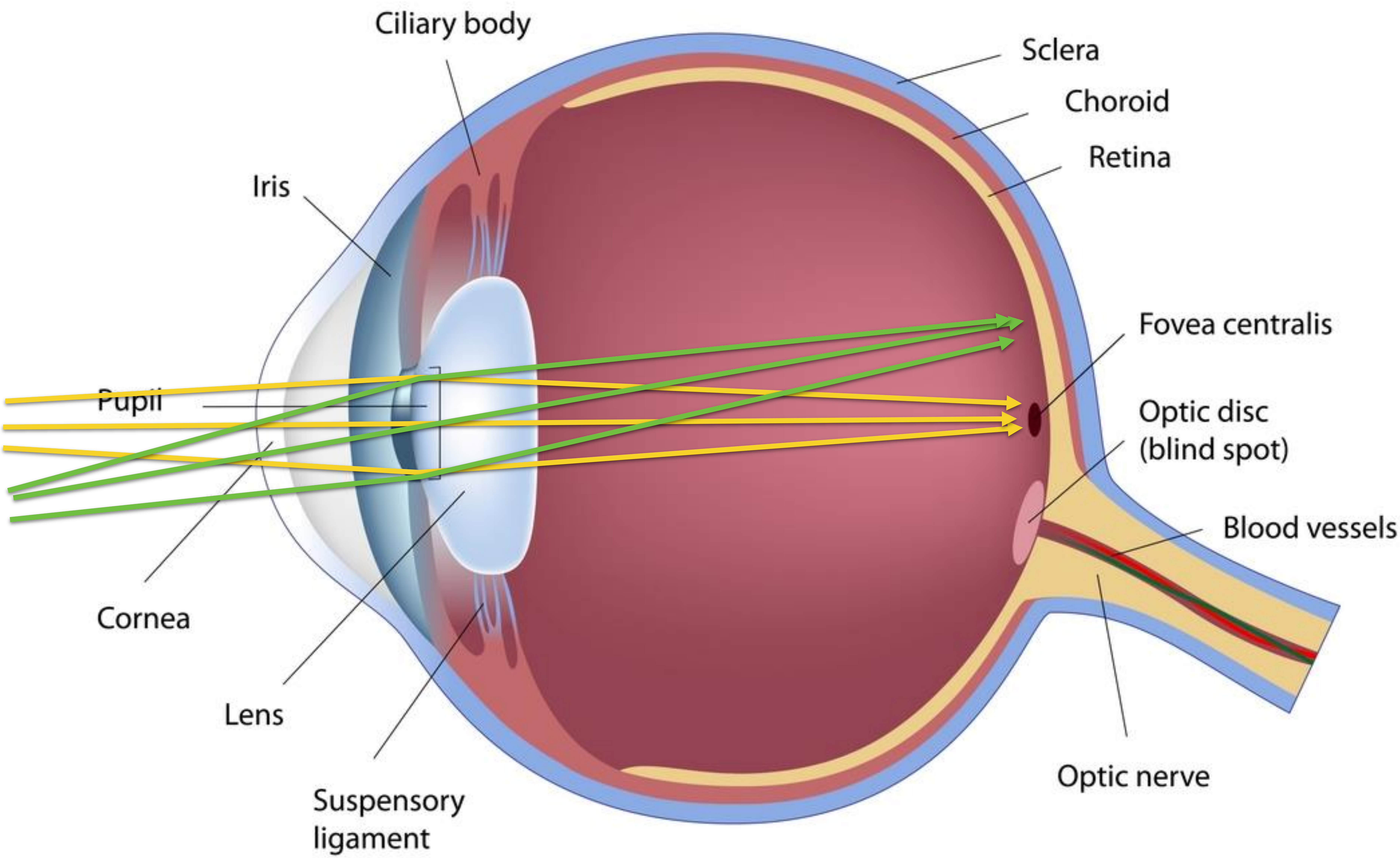


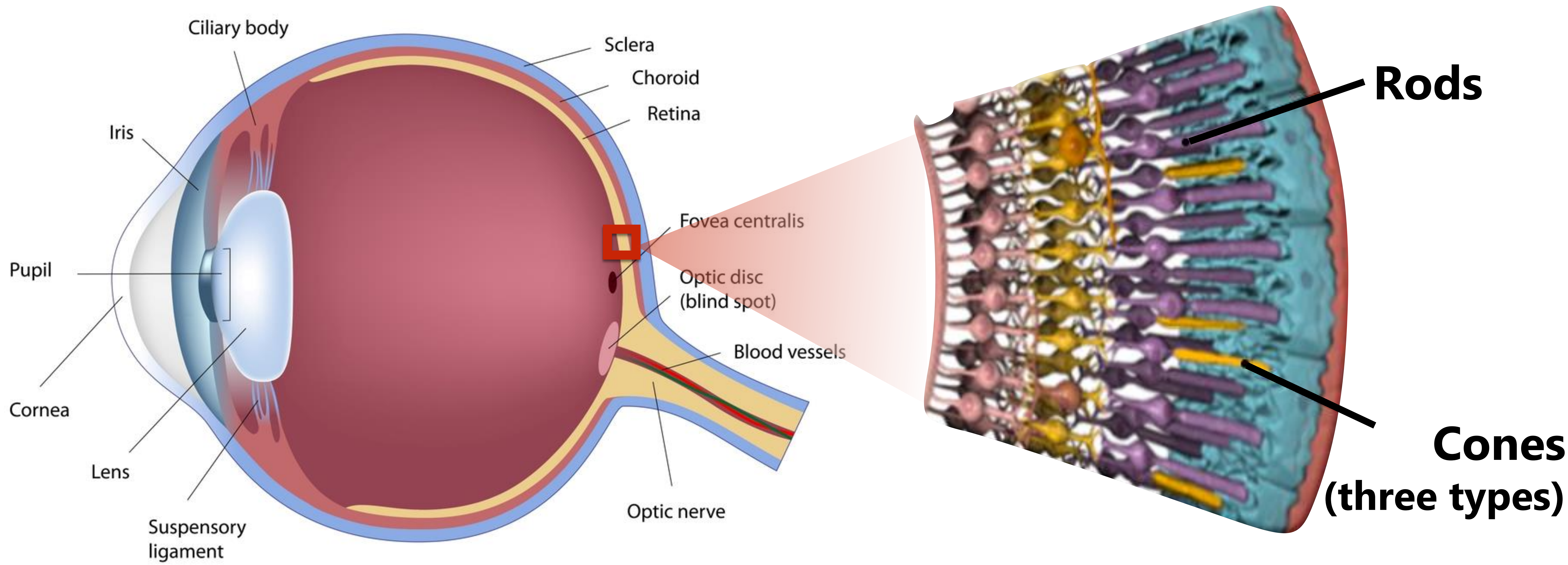
Image credit: Georgia Retina (http://www.garetina.com/about-the-eye)

Photosensor response (eye, camera, ...)

- **Photosensor input: light**
  - Electromagnetic distribution over wavelengths: $\Phi(\lambda)$
- **Photosensor output: a "response"**
  - e.g., a number encoded as an electrical signal
- **Spectral response function:** $f(\lambda)$
  - Sensitivity of sensor to light of a given wavelength
  - Greater $f(\lambda)$ corresponds to a more efficient sensor
  - (when $f(\lambda)$ is large, a small amount of light at wavelength λ will trigger a large sensor response)
- **Total response of photosensor:**

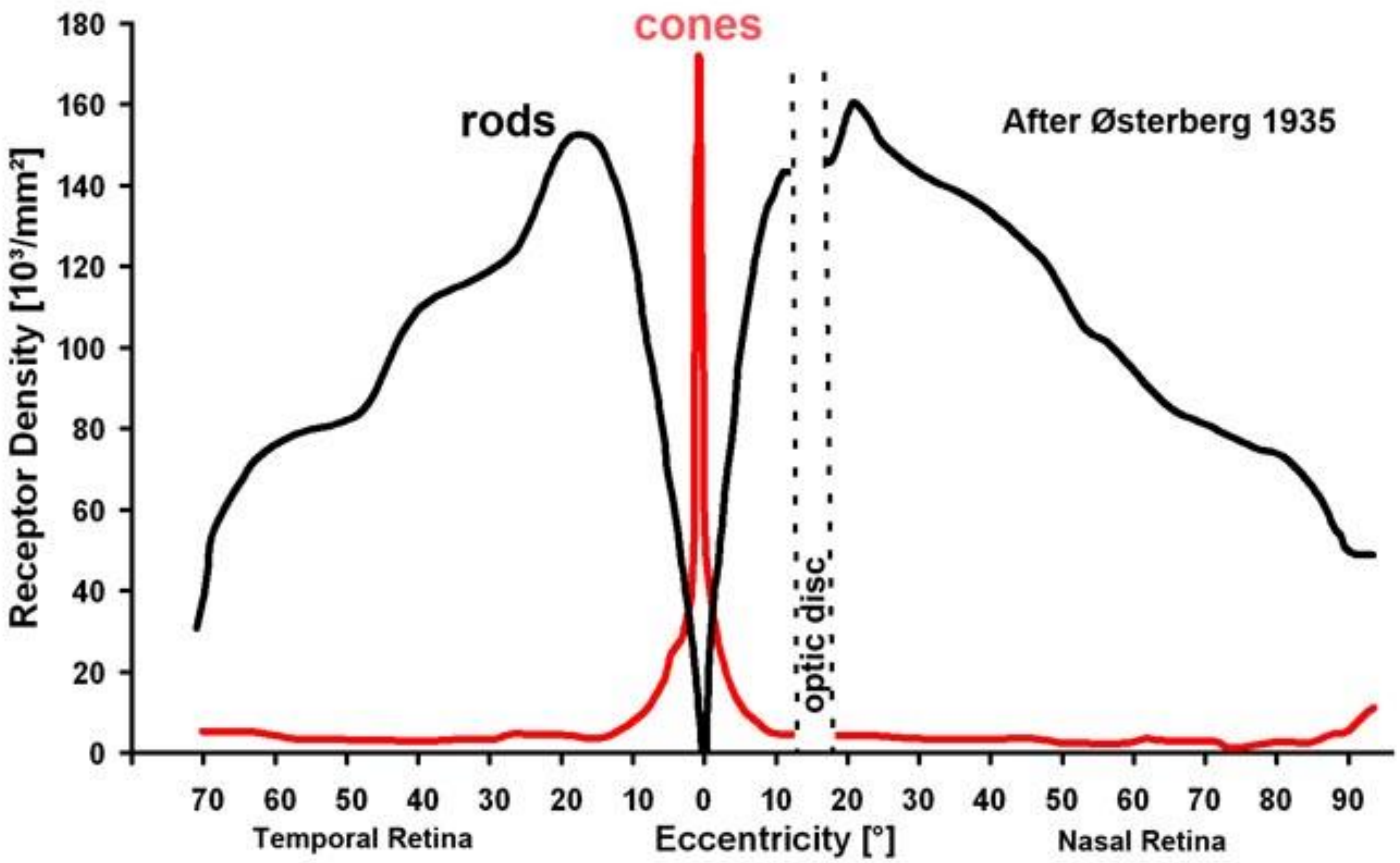
$$R = \int_{\lambda} \Phi(\lambda) f(\lambda) d\lambda$$

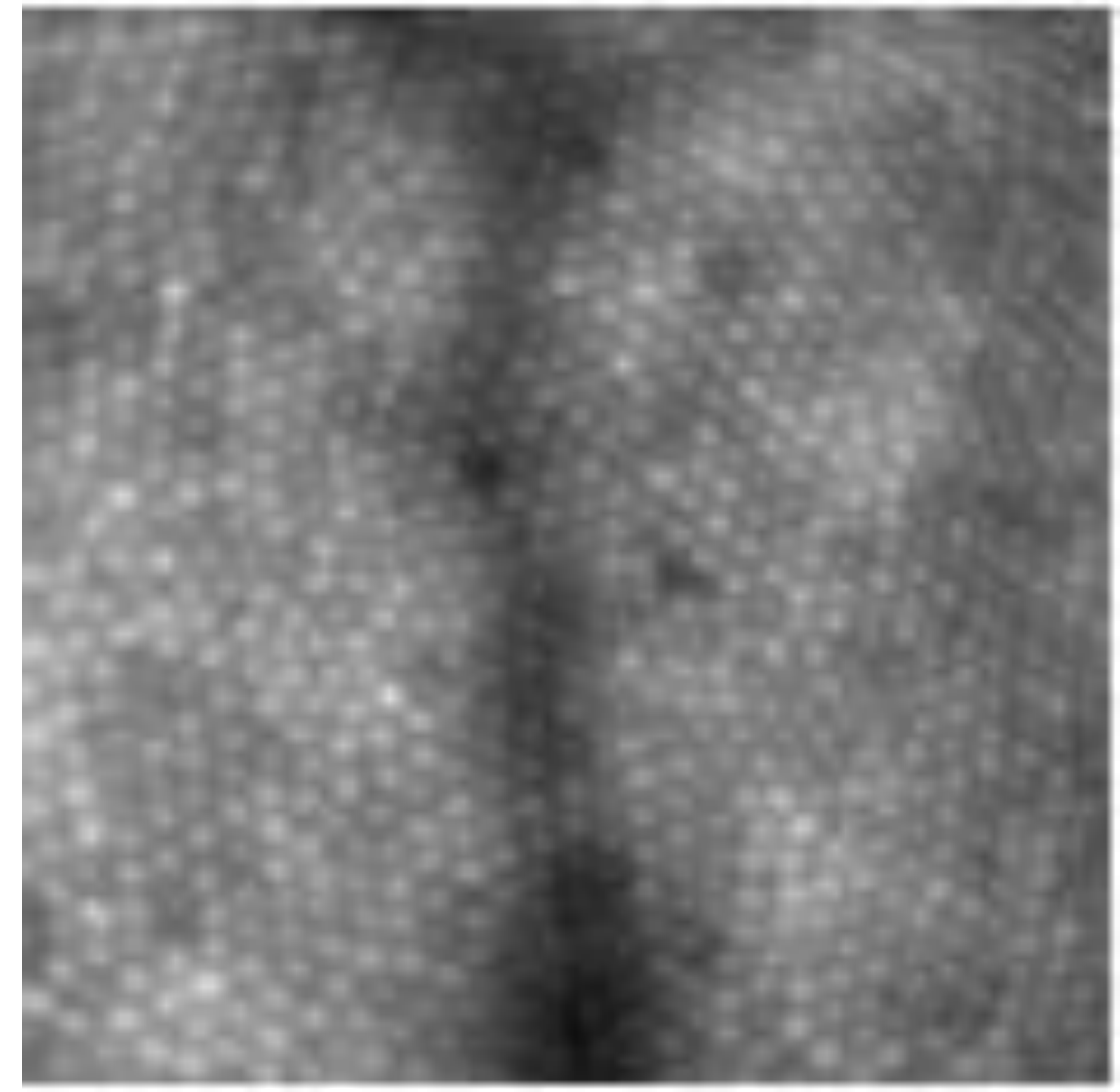
The eye's photoreceptor cells: rods & cones



- Rods are primary photoreceptors (e.g. light-sensitive structures) under dark viewing conditions (scotopic conditions)
  - Approx. 120 million rods in human eye
- Cones are primary receptors under high-light viewing conditions (photopic conditions, e.g., daylight)
  - Approx. 6-7 million cones in the human eye
  - Each of the three types of cone features a different spectral response. This is critical to color vision.

Density of rods and cones in the retina





[Roorda 1999]

- Highest density of cones is in fovea

(best color vision at center of where human is looking)

- Note “blind spot” due to optic disk

Spectral response of cones

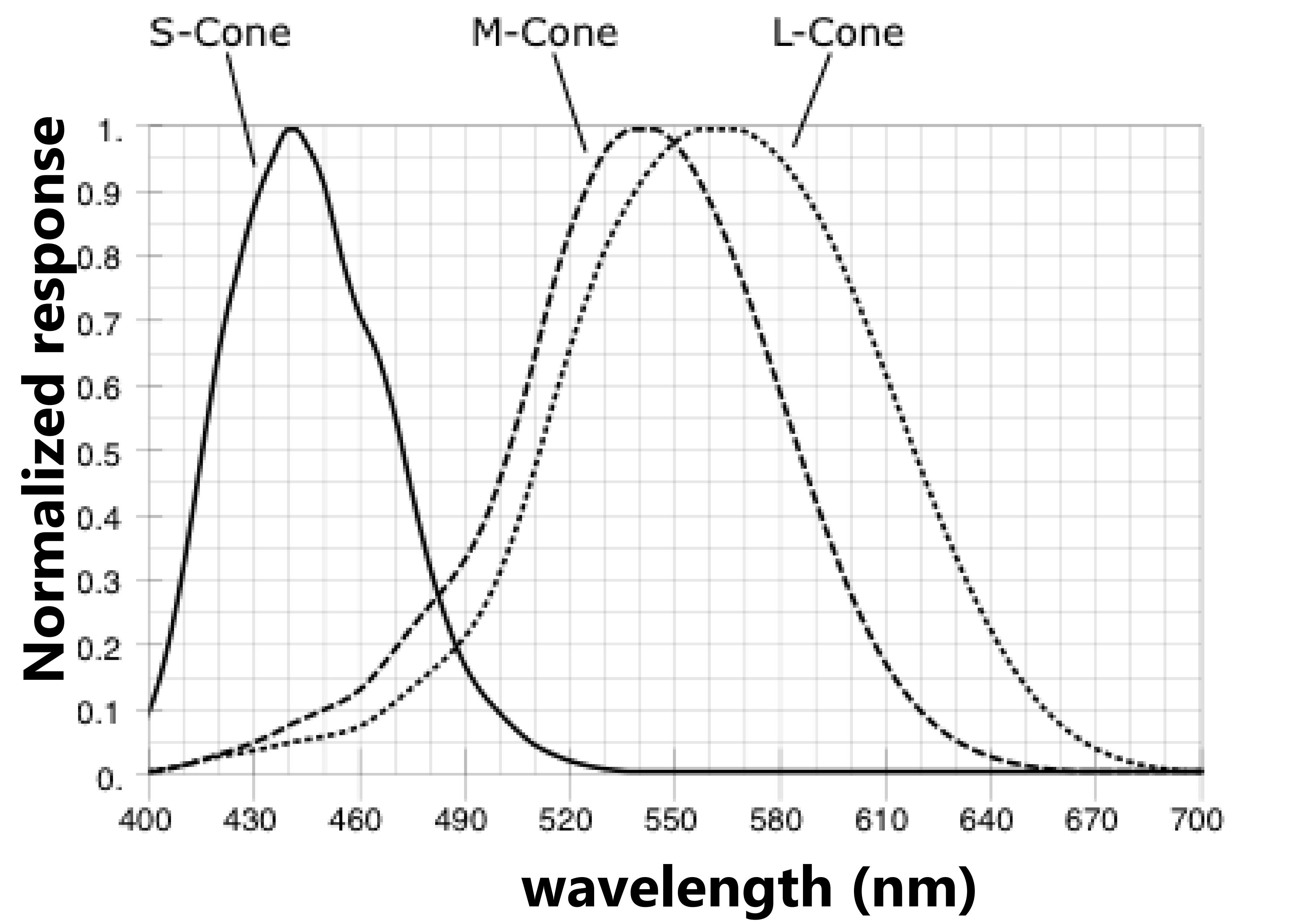
Three types of cones: S, M, and L cones (corresponding to peak response at short, medium, and long wavelengths)

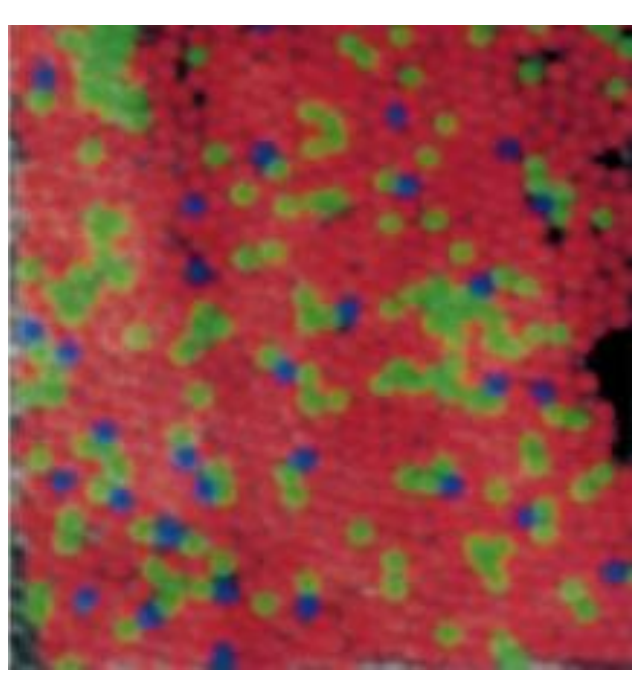
$$S = \int_\lambda \Phi(\lambda) S(\lambda) d\lambda$$

$$M = \int_\lambda \Phi(\lambda) M(\lambda) d\lambda$$

$$L = \int_\lambda \Phi(\lambda) L(\lambda) d\lambda$$

Response functions for S, M, and L cone

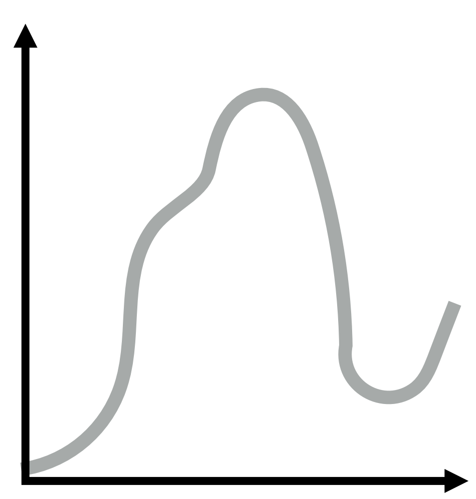




Uneven distribution of cone types in eye
~64% of cones are L cones, ~ 32% M cones

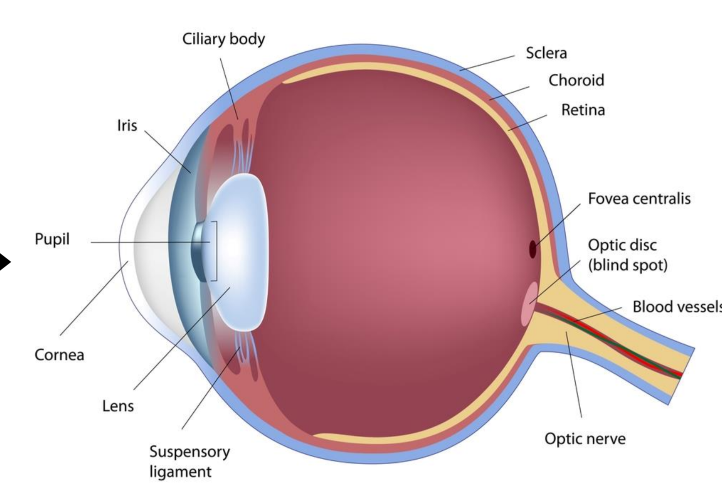
The human visual system

- Human eye does not directly measure the spectrum of incoming light
  - a.k.a. the brain does not receive “a spectrum” from the eye
- The eye measures three response values = (S, M, L) by integrating the incoming spectrum against response functions of S, M, L-cones
- Brain interprets these response functions as colors



Spectrum

$\Phi(\lambda)$



Eye

Focuses light on retina
Cones measure light
(photopic case)

cone responses (S, M, L) carried along optic nerve

Brain

Ok, so color can get pretty complicated!

How do we encode it in a simple(r) way?

Color spaces and color models

- Many ways to specify a color
  - storage
  - convenience

- In general, specify a color from some color space using a color model

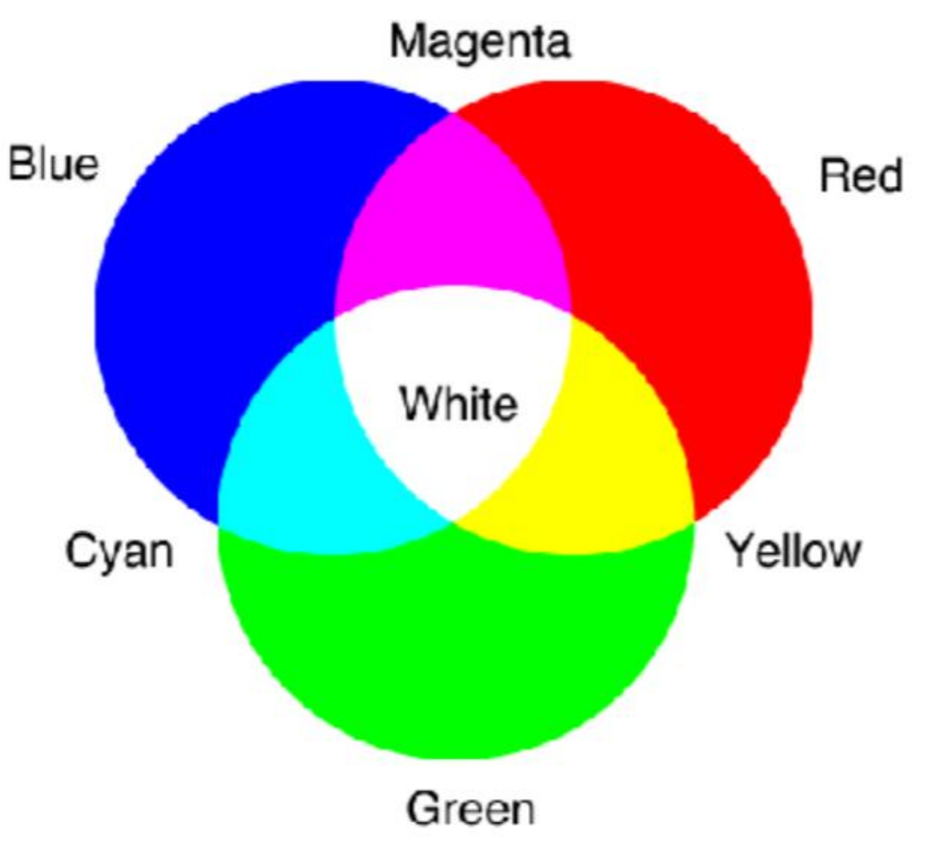
- Color space is like an artists' palette: full range of colors we can choose from

- Color model is the way a particular color in a color space is specified:
  - Artist's palette: "yellow ochre"
  - RBG color model: 204, 119, 34

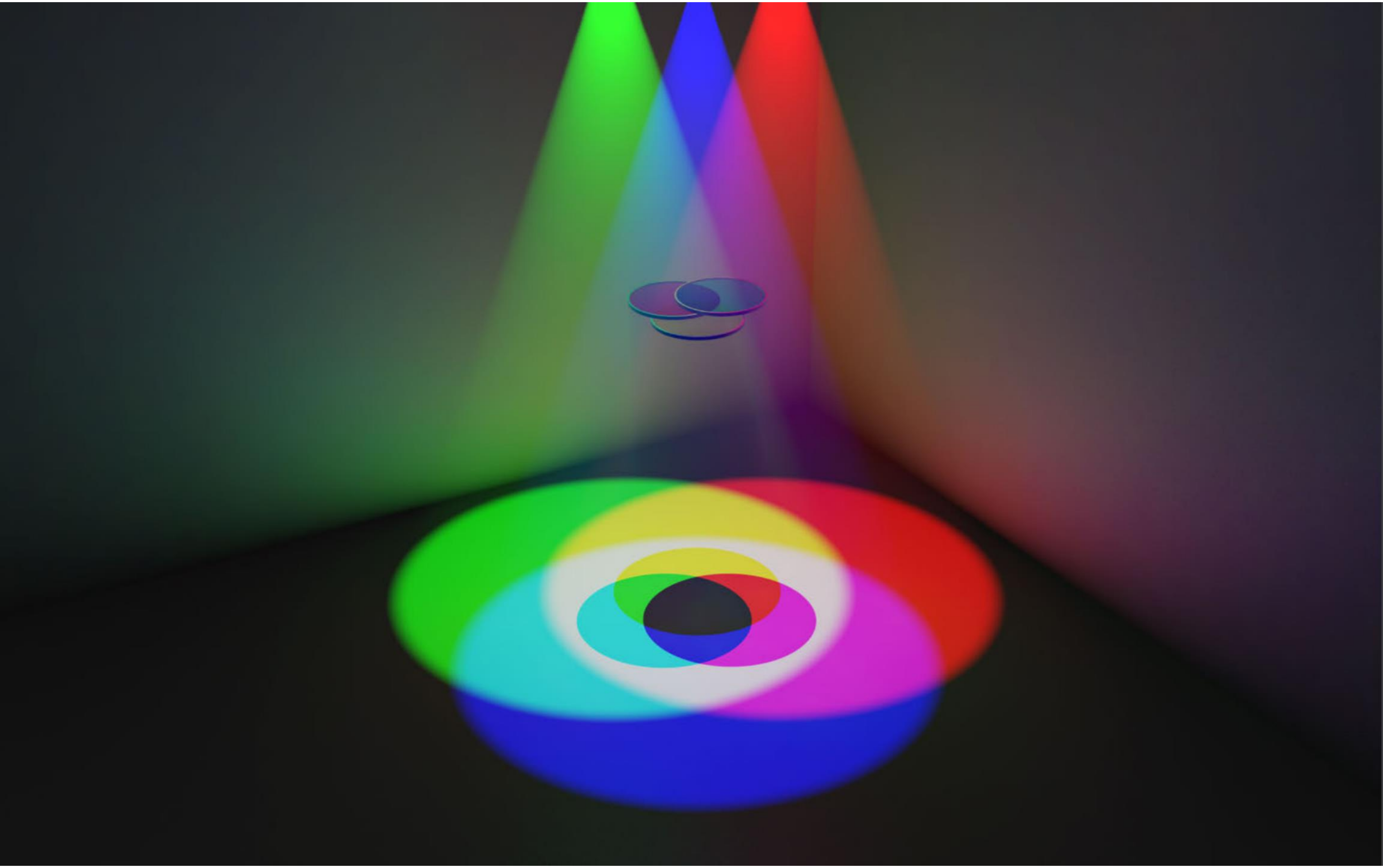
Additive vs Subtractive Color Models

- **Just like we had *emission* and *absorption* spectra, we have *additive* and *subtractive* color models**

- **Additive:**
  - **e.g. used for combining colored lights**
  - **Prototypical example: RGB**

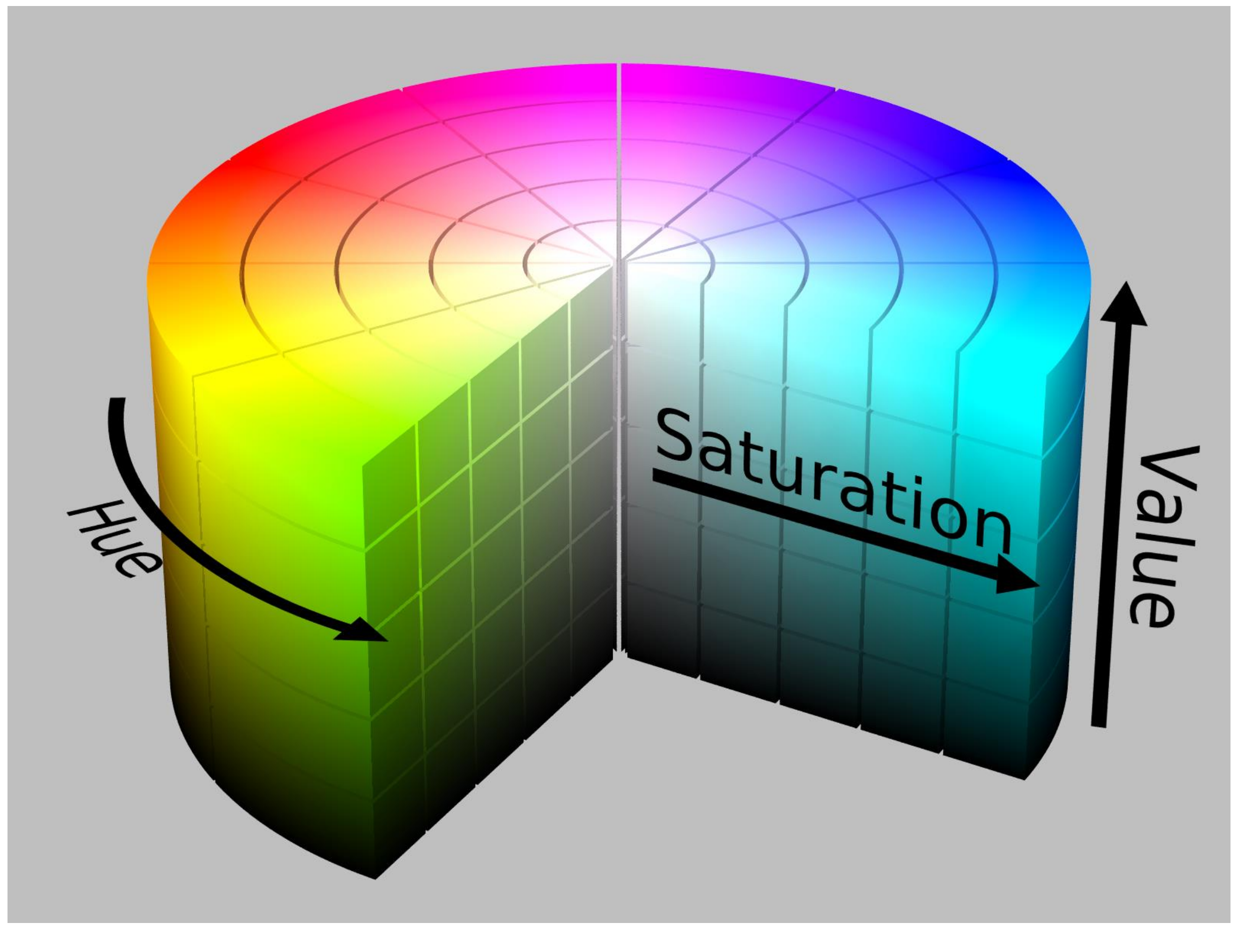


- **Subtractive**
  - **e.g. used for combining paint colors**
  - **Prototypical example: CMYK**



There are many other common color models

- **HSV**
  - **hue, saturation, value**
  - **More intuitive than RGB/CMYK**
  - **Dimensions correspond to natural notions of "characteristics" of color**



- **Y'CbCr**
  - **Y': perceived luminance (intensity)**
  - **Cb: blue-yellow deviation from gray**
  - **Cr: red-cyan deviation from gray**

Practical encoding of Color Values

- How can we encode colors digitally?
- One common encoding: 8bpc hexadecimal values

#ff6600

- Used to specify RGB colors (what does it mean?)

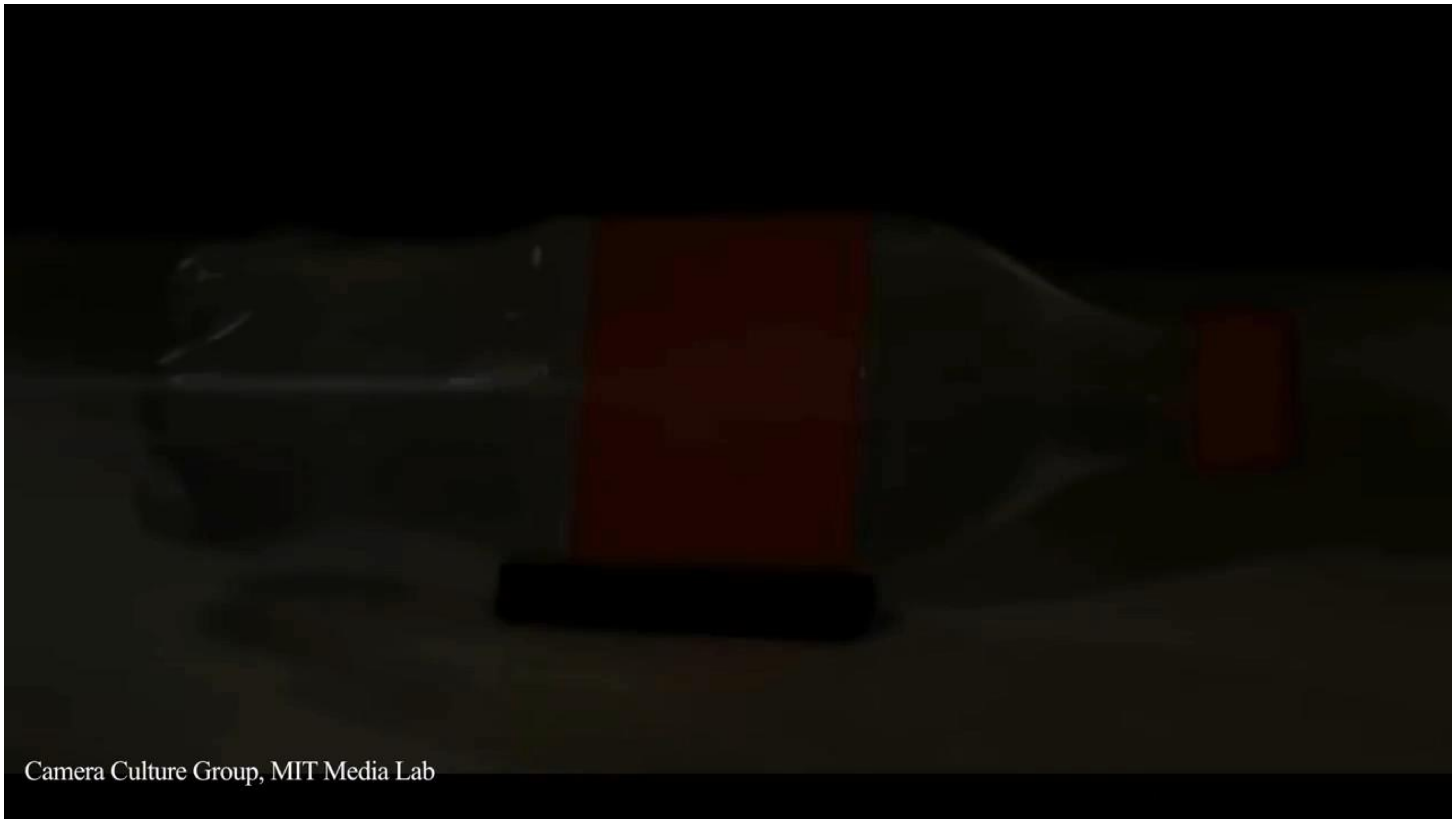
How do we quantify measurements of light/color?

- **Radiometry: system of units and techniques for measuring electromagnetic (EM) radiation**

- **Geometric model of light**
  - **Photons travel in straight lines, represented by rays**

What does light propagation look like?

- **Can't see it with the naked eye!**



- **Can't really see it with cameras either – instead, repeat same experiment many times, take snapshots at slightly different offsets each time.**

How do we quantify measurements of light/color?

- Radiometry: system of units and techniques for measuring electromagnetic (EM) radiation

- Geometric model of light
  - Photons travel in straight lines, represented by rays
  - Good approximation when wavelength << size of objects light interacts with
    - Doesn’t model diffraction, interference, …

- Lots of terminology!
  - We’ll focus mostly on concepts here…

What do we want to measure (and why?)

- Many physical process convert input energy into photons
  - e.g. incandescent lightbulb turns heat into light (blackbody radiation)
  - Nuclear fusion in stars generates photons
- Each photon carries a small amount of energy
- Want some way of recording "amount of energy"
  - Energy of photons hitting an object ~ brightness
  - Film, sensors, sunburn, solar panels, …
  - Some input energy is turned into heat, some into new photons
- Graphics: generally assume "*steady state*" process
  - How long does it take for lighting to reach steady state?

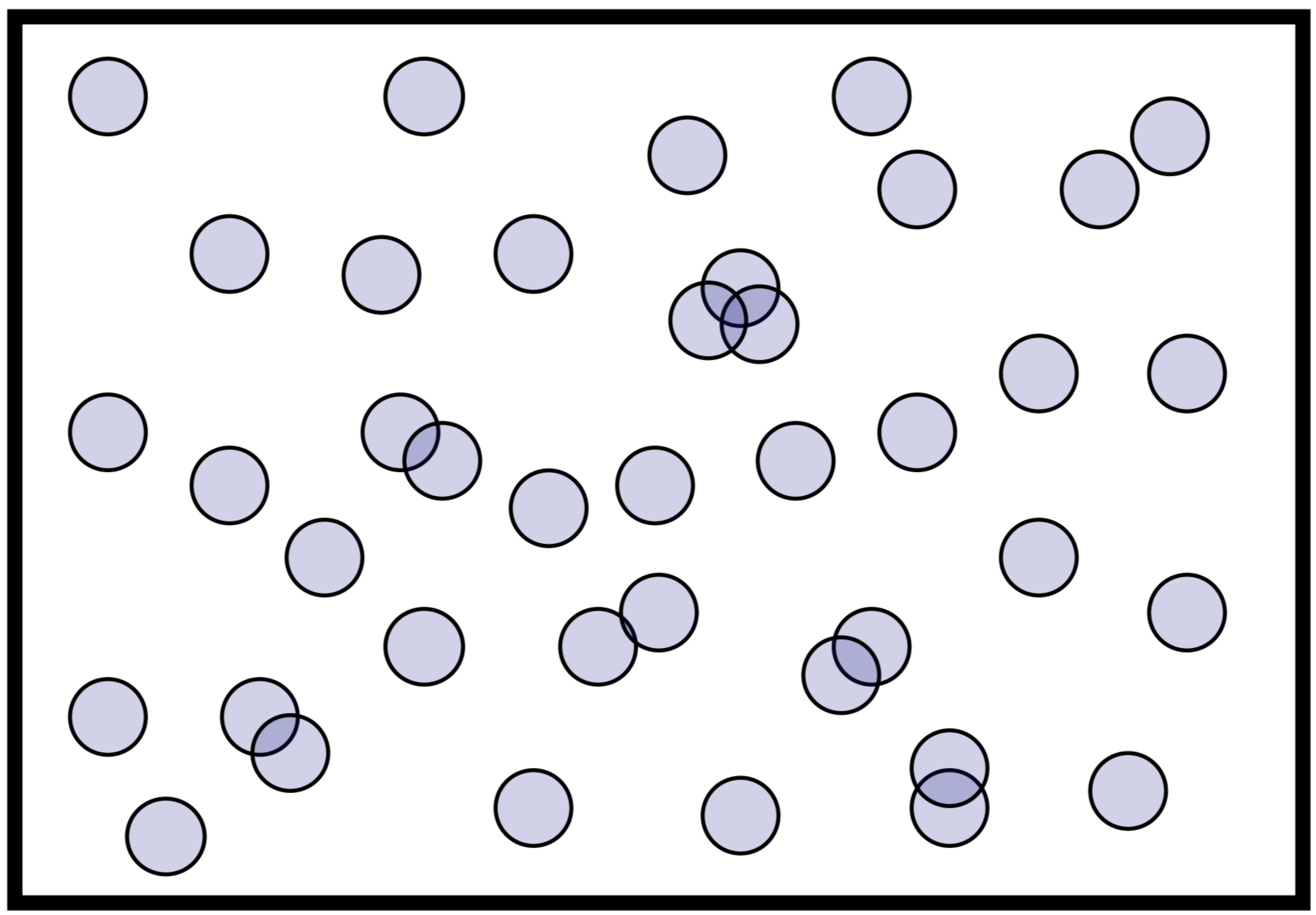
Radiometry

- **Imagine every photon is a little rubber ball hitting the scene:**

- **How can we record this process? What information should we store?**

Radiant energy: total # of hits

- One idea: just store the total number of hits that occur anywhere in the scene, over the complete duration of the scene
- This captures the total energy of all the photons hitting the scene*

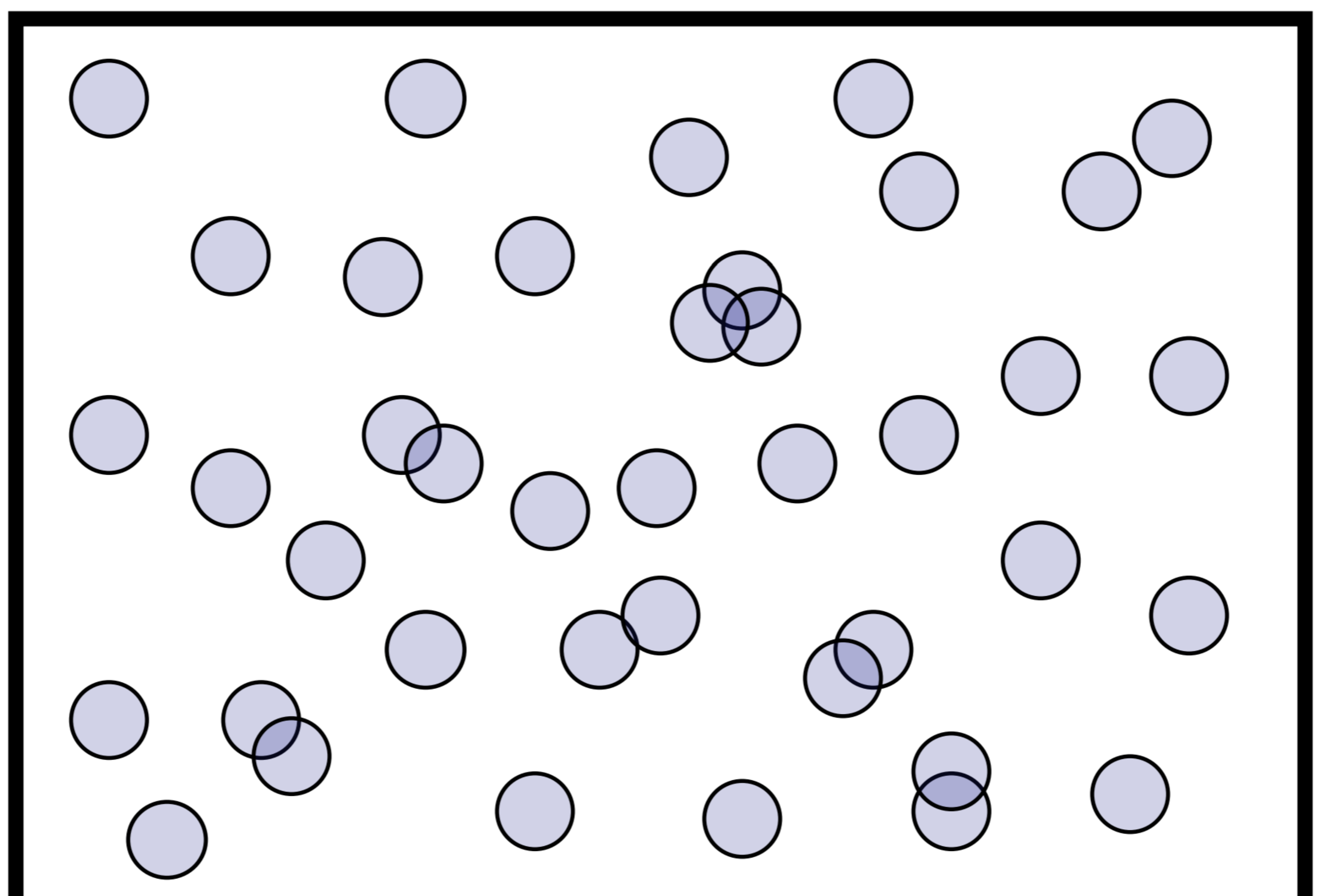


"Radiant energy": 40

*we should really talk about constants/units, but these will not help our conceptual understanding...

Radiant flux: hits per second

- **For illumination phenomena at the level of human perception, usually safe to assume equilibrium is reached immediately**
- **So, rather than record total energy over some arbitrary duration, makes much more sense to record total hits per second**

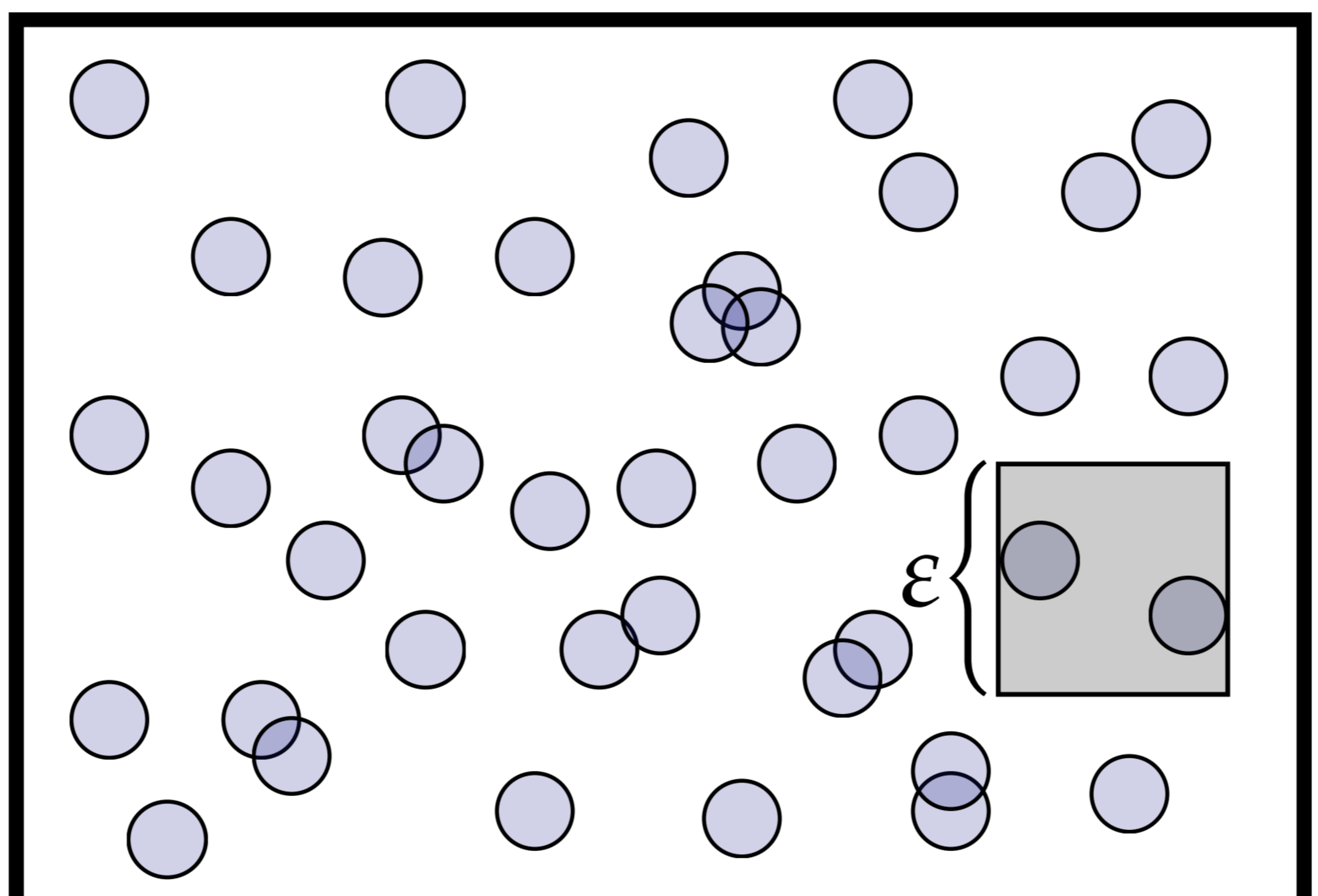


(Takes .05s to display each "hit"!)

Estimate of "radiant flux": 40 hits/2s = 20 hits/s

Irradiance: hits per second per unit area

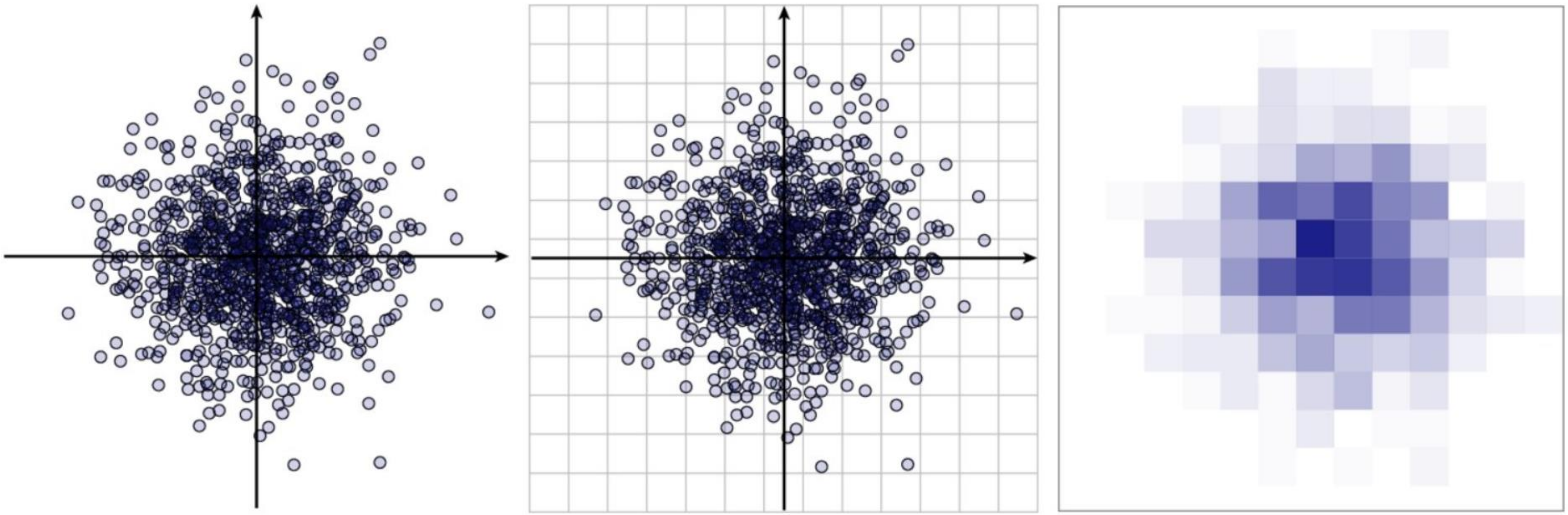
- **To make images, we also need to know where the hits occurred**
- **So, compute hits per second in some "really small" area, divided by area:**



Estimate of "radiant energy density": $2/\varepsilon^2$

Image generation as irradiance estimation

- **From this point of view, our goal in image generation is to estimate the irradiance at each point of an image (or the total radiant flux per pixel):**



Recap…

Radiant Energy (total number of hits)	**Radiant Energy Density** (hits per unit area)
Radiant Flux (total hits per second)	**Radiant Flux Density** **a.k.a. *Irradiance*** (hits per second per unit area)

Measuring illumination: radiant energy

- **How can we be more precise about the amount of energy?**
- **We talked about "number of hits", but do all hits contribute same amount of energy?**
- **Energy carried by a photon:**

Planck's constant

speed of light

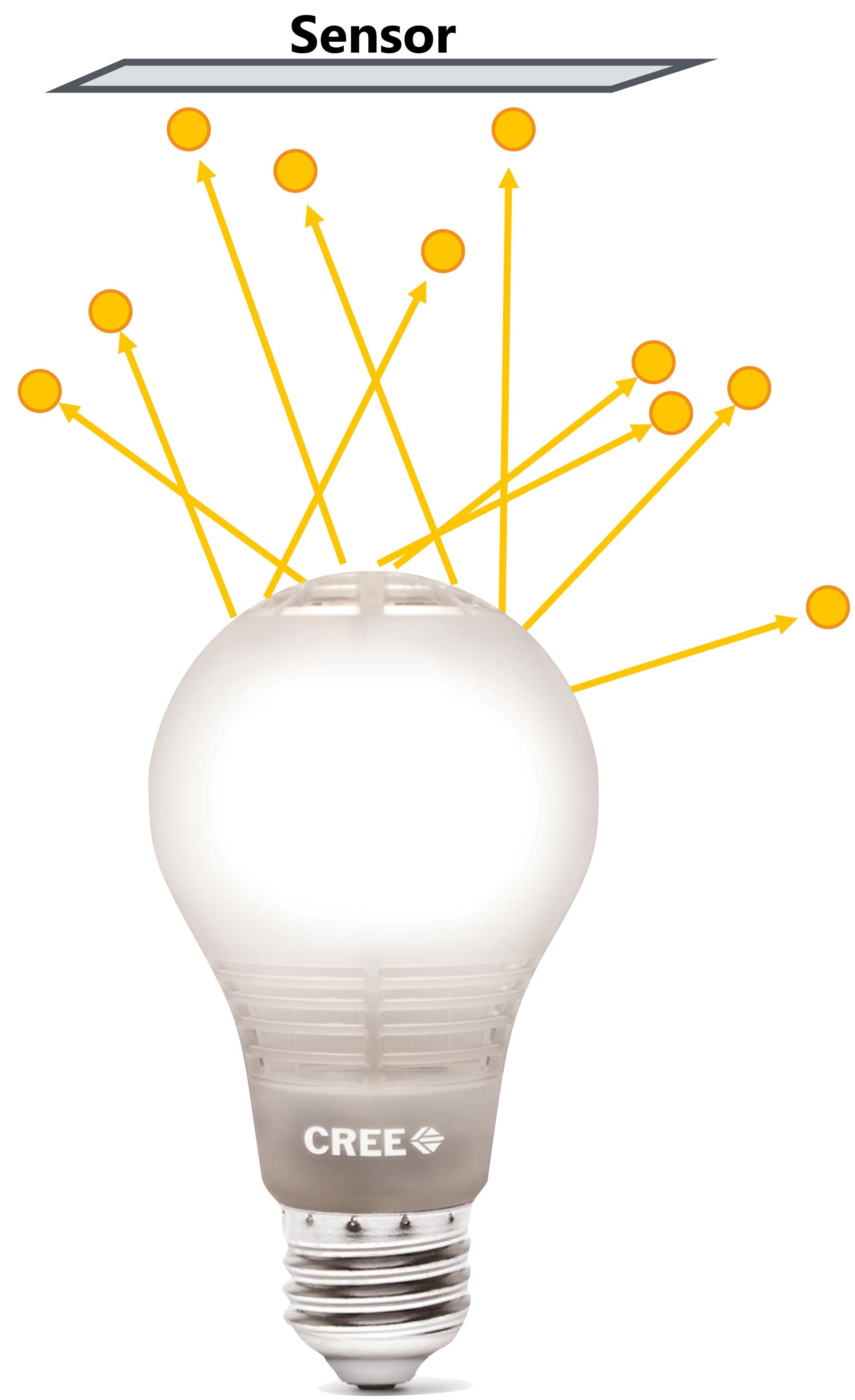
$$Q = \frac{hc}{\lambda}$$

wavelength (color!)

$h \approx 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$ (Joules times seconds)

$c \approx 3.00 \times 10^{8}\ \mathrm{m/s}$ (meters per second)

$\lambda \approx 390\text{–}700 \times 10^{-3}\ \mathrm{m}$ (visible)



Measuring illumination: radiant flux (power)

- **Flux: energy per unit time (Watts) received by the sensor (or emitted by the light)**

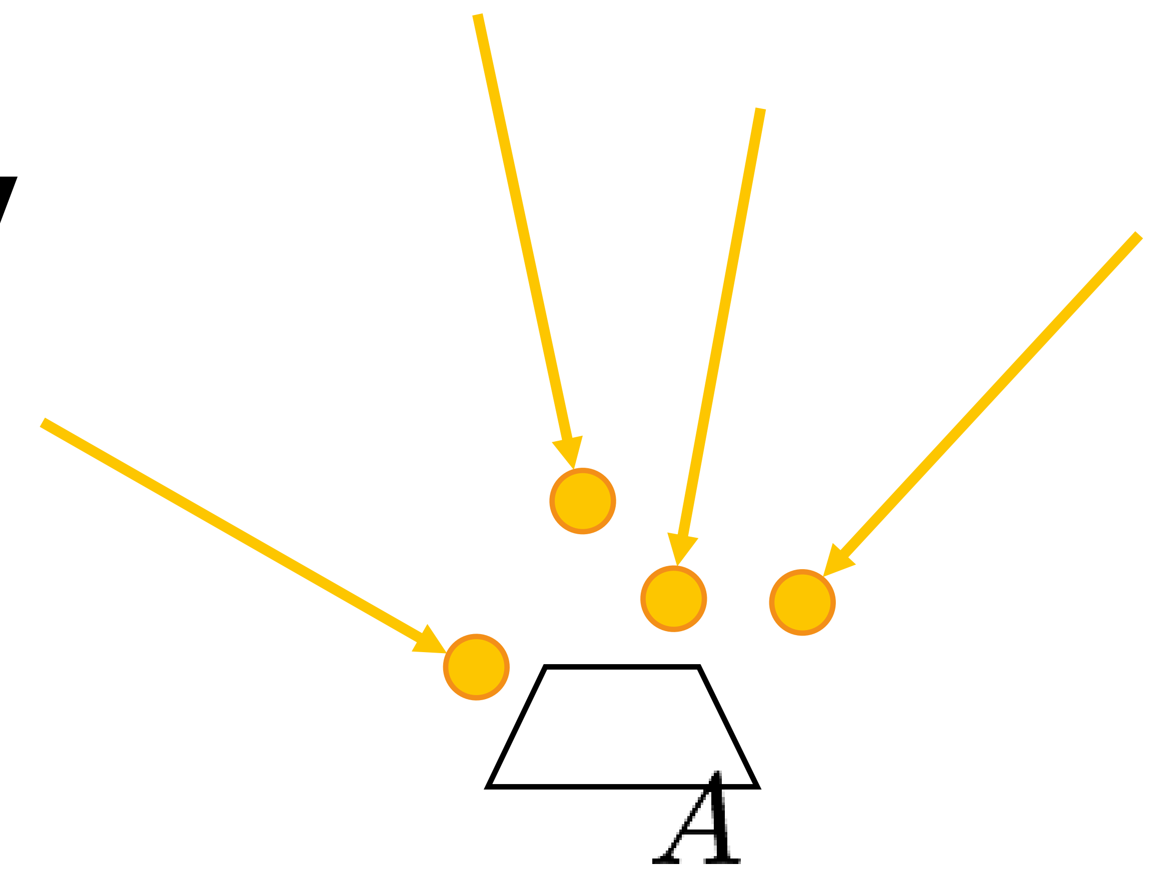
$$\Phi = \lim_{\Delta \to 0} \frac{\Delta Q}{\Delta t} = \frac{\mathrm{d}Q}{\mathrm{d}t} \quad \left[\frac{\mathrm{J}}{\mathrm{s}}\right]$$

- **Can also go the other way: time integral of flux is total radiant energy**

$$Q = \int_{t_0}^{t_1} \Phi(t)\, \mathrm{d}t$$

Measuring illumination: irradiance

- **Radiant flux: time density of energy**
- **Irradiance: area density of flux**



Given a sensor with area *A*, we can consider the average flux over the entire sensor area:

$$\frac{\Phi}{A}$$

Irradiance (E) is given by taking the limit of flux as sensor area becomes tiny:

$$E(\mathrm{p}) = \lim_{\Delta \to 0} \frac{\Delta \Phi(\mathrm{p})}{\Delta A} = \frac{\mathrm{d}\Phi(\mathrm{p})}{\mathrm{d}A} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

What about color?

How might we quantify, say, the amount of green?

Spectral power distribution

- **Describes distribution of energy by wavelength**

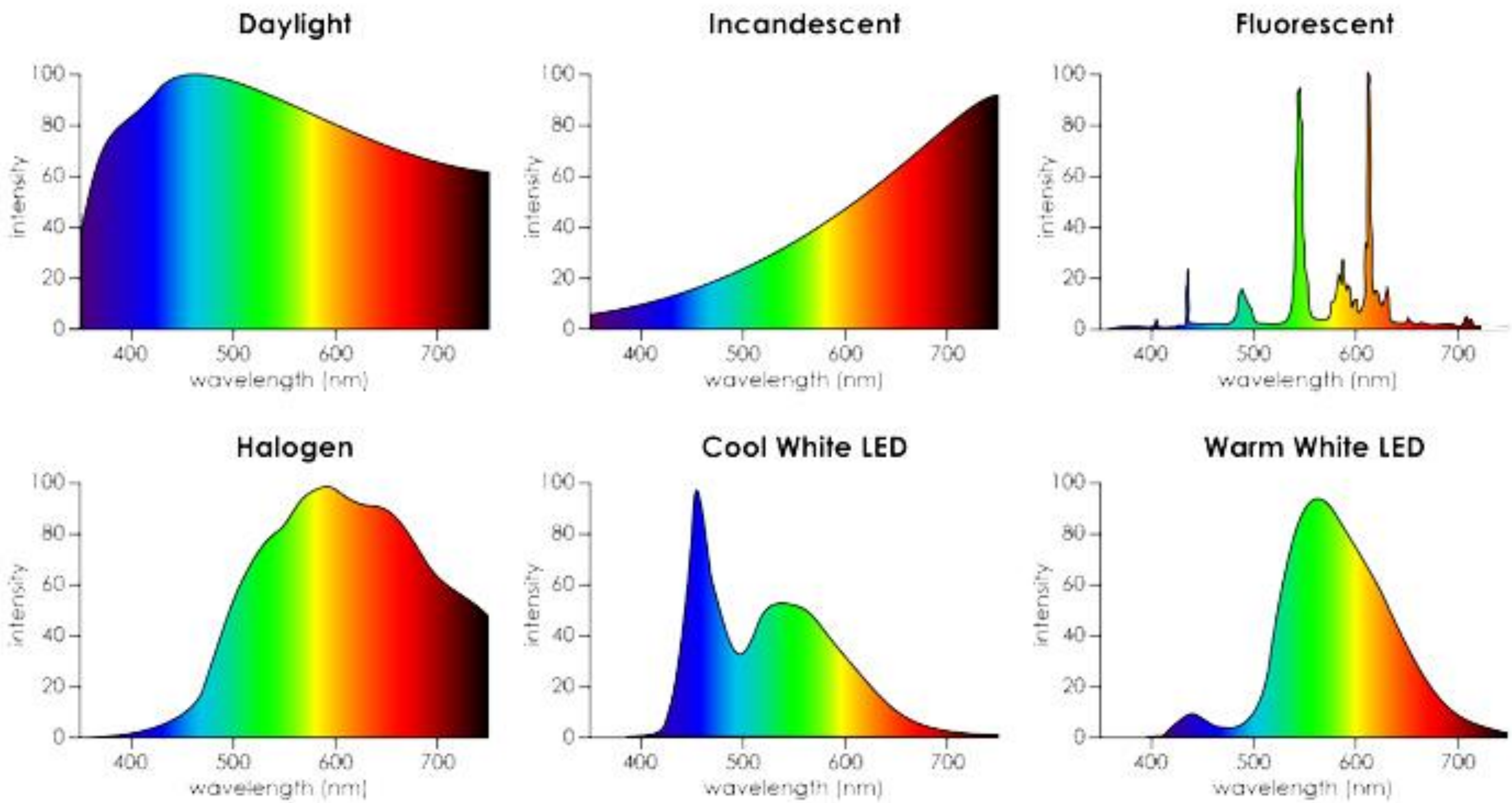


Figure credit:

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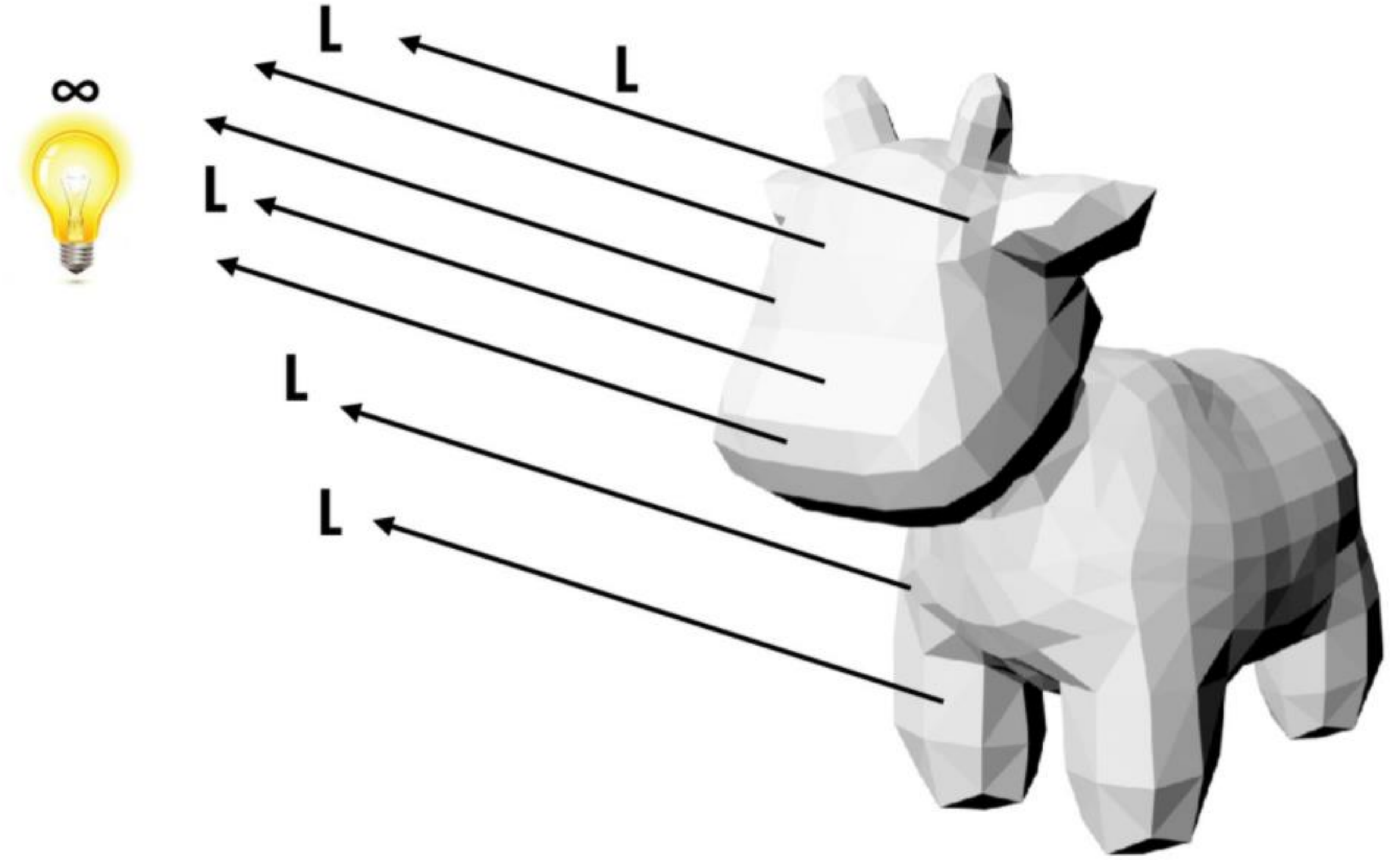
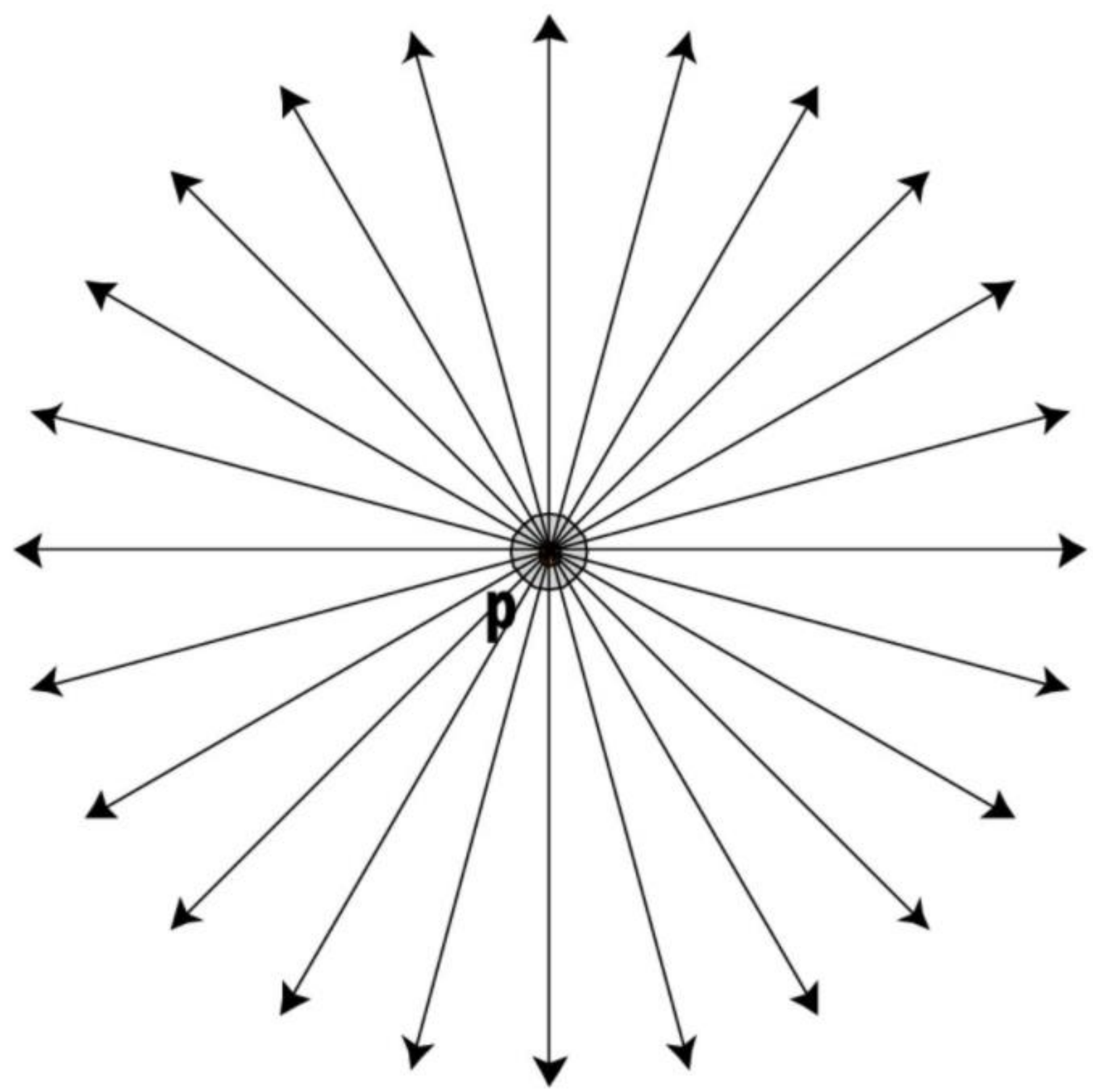
Recall: “directional” lighting

- **Common abstraction: infinitely bright light source “at infinity”**
- **All light directions (L) are therefore identical**

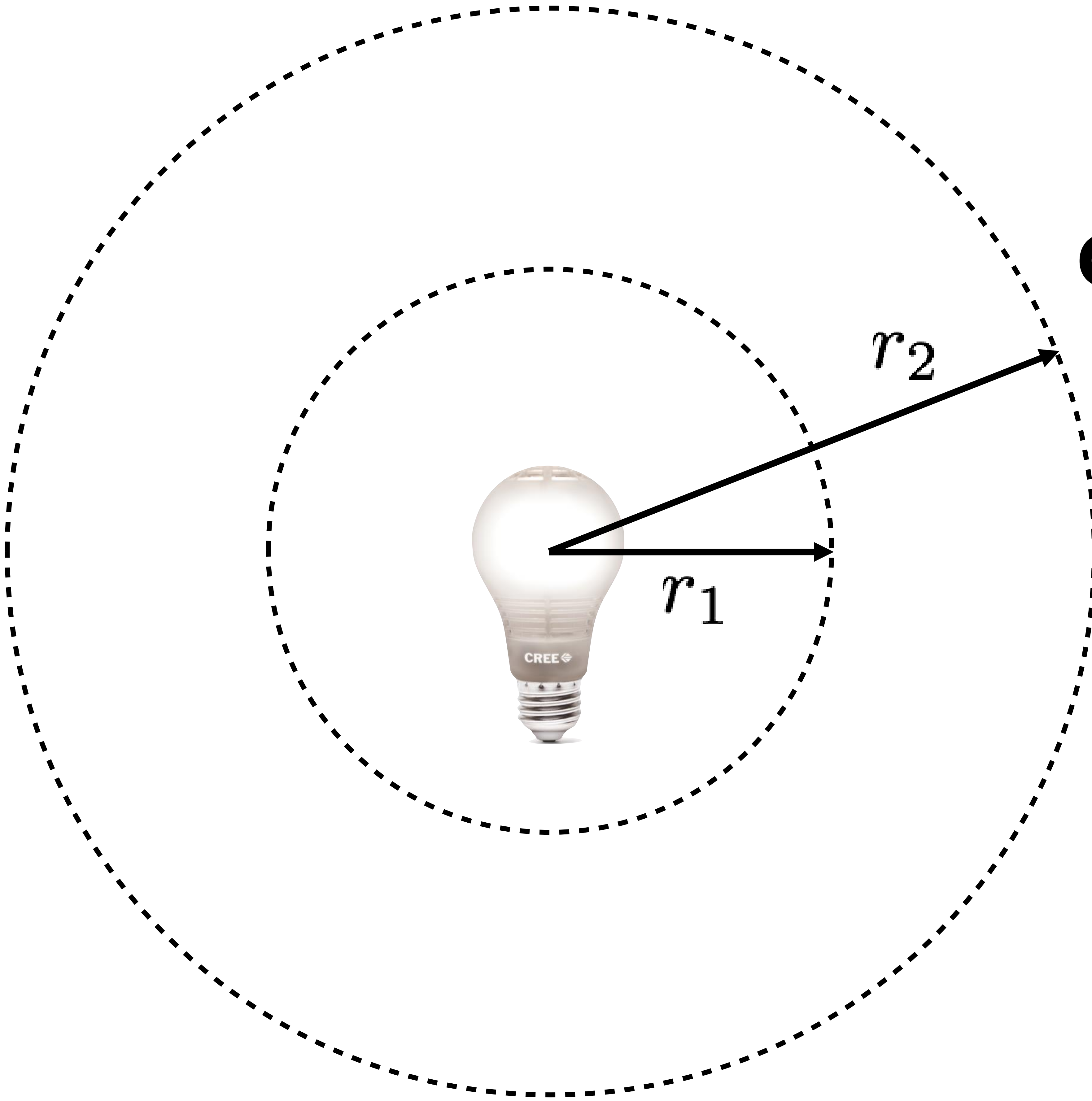


Isotropic point source

- **Slightly more realistic model…**



Irradiance falloff with distance



Assume light is emitting flux Φ in a uniform angular distribution

Q. What is irradiance on surface? Which is brighter?

Compare irradiance at surface of two spheres:

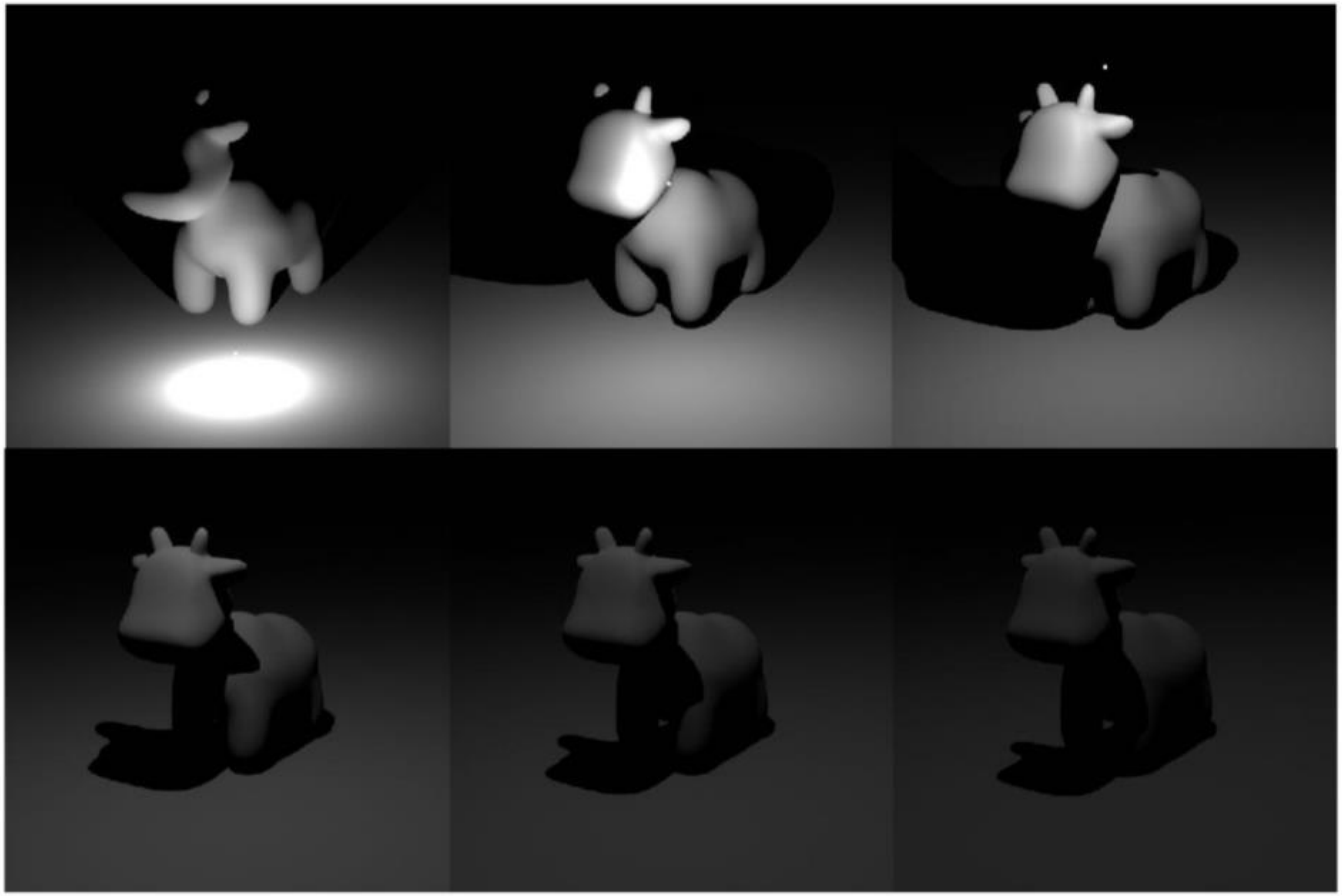
$$E_1 = \frac{\Phi}{4\pi r_1^2} \rightarrow \Phi = 4\pi r_1^2 E_1$$

$$E_2 = \frac{\Phi}{4\pi r_2^2} \rightarrow \Phi = 4\pi r_2^2 E_2$$

$$\frac{E_2}{E_1} = \frac{r_1^2}{r_2^2}$$

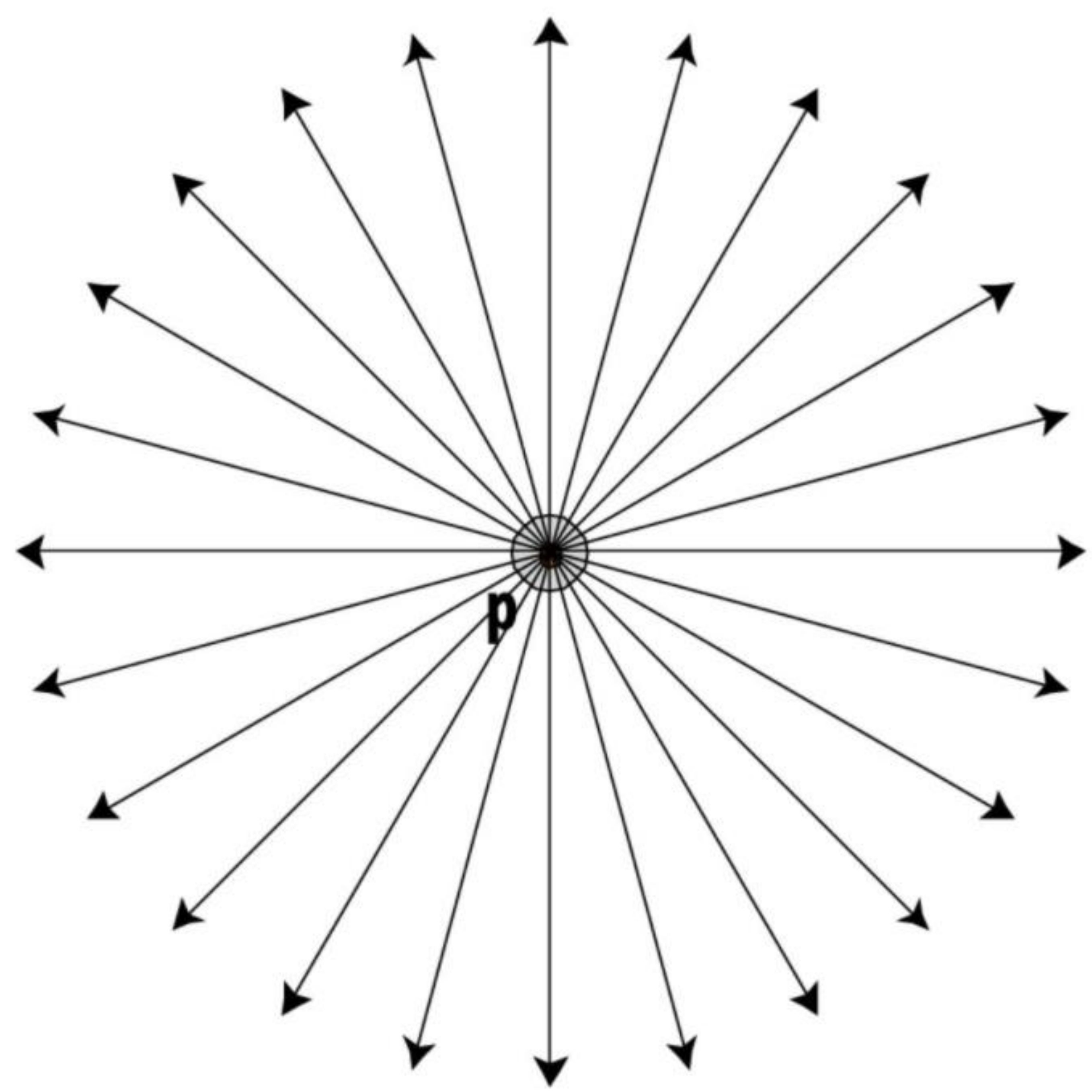
Since same amount of energy is distributed over larger and larger spheres, has to get darker *quadratically* with distance.

Single point light moves in 1m increments



Isotropic point source

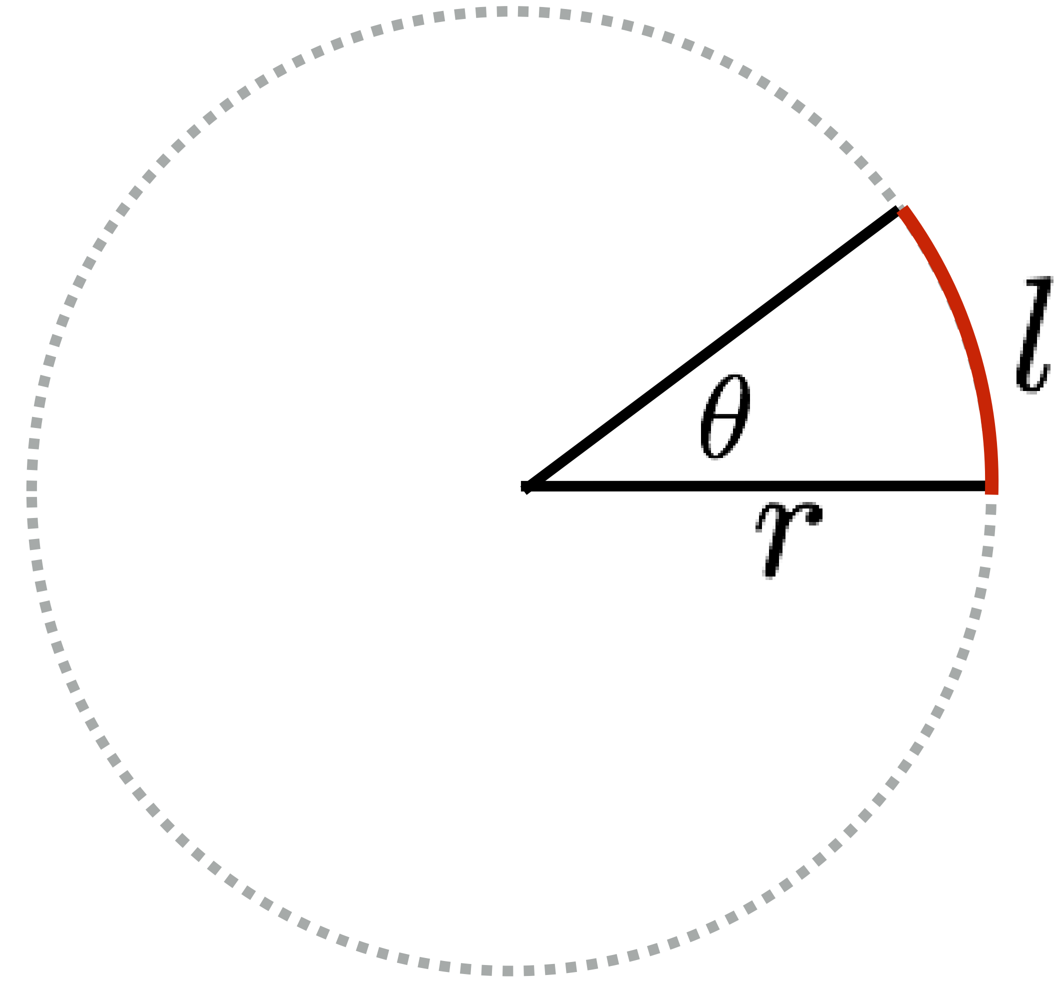
- **Slightly more realistic model…**



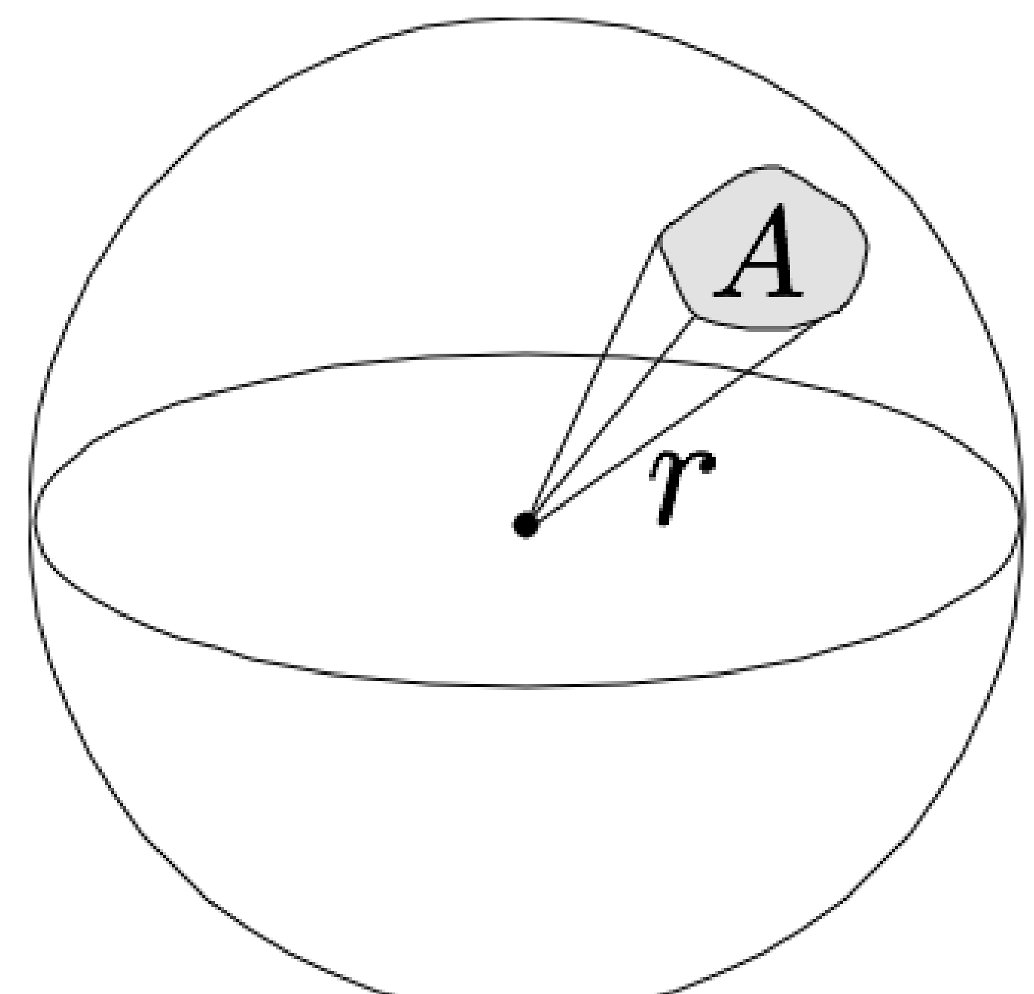
In general, we need to think about directional distribution of irradiance

Angles and solid angles

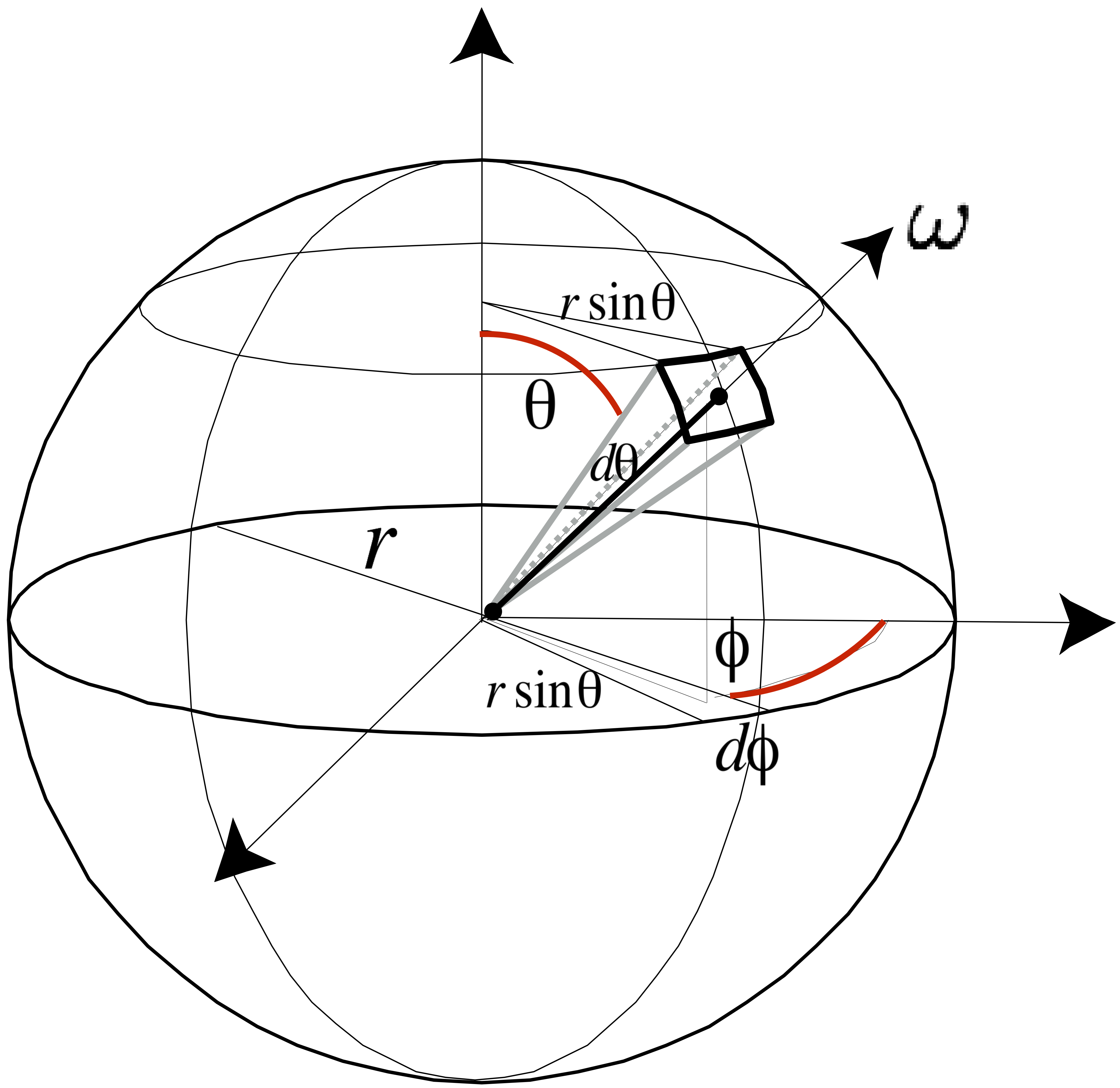
- **Angle: ratio of subtended arc length on circle to radius**
  - $\theta = \frac{l}{r}$
  - **Circle has** 2π **radians**



- **Solid angle: ratio of subtended area on sphere to radius squared**
  - $\Omega = \frac{A}{r^2}$
  - **Sphere has** 4π **steradians**



Differential solid angle



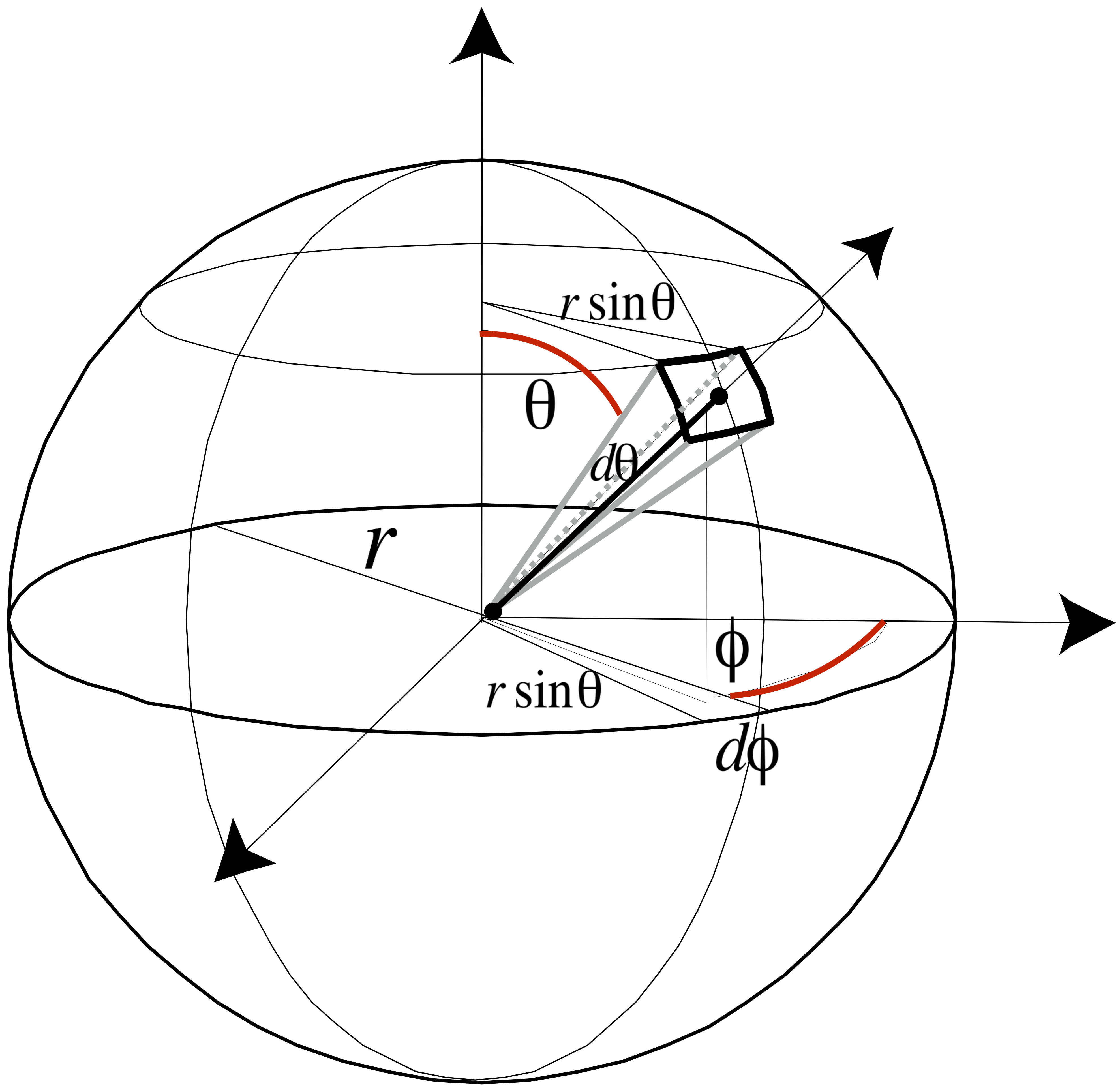
Consider a tiny area swept out by a tiny angle in each direction…

$$\begin{aligned} \mathrm{d}A &= (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi) \\ &= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \end{aligned}$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

Differential solid angle is just that tiny area on the *unit* sphere

Differential solid angle

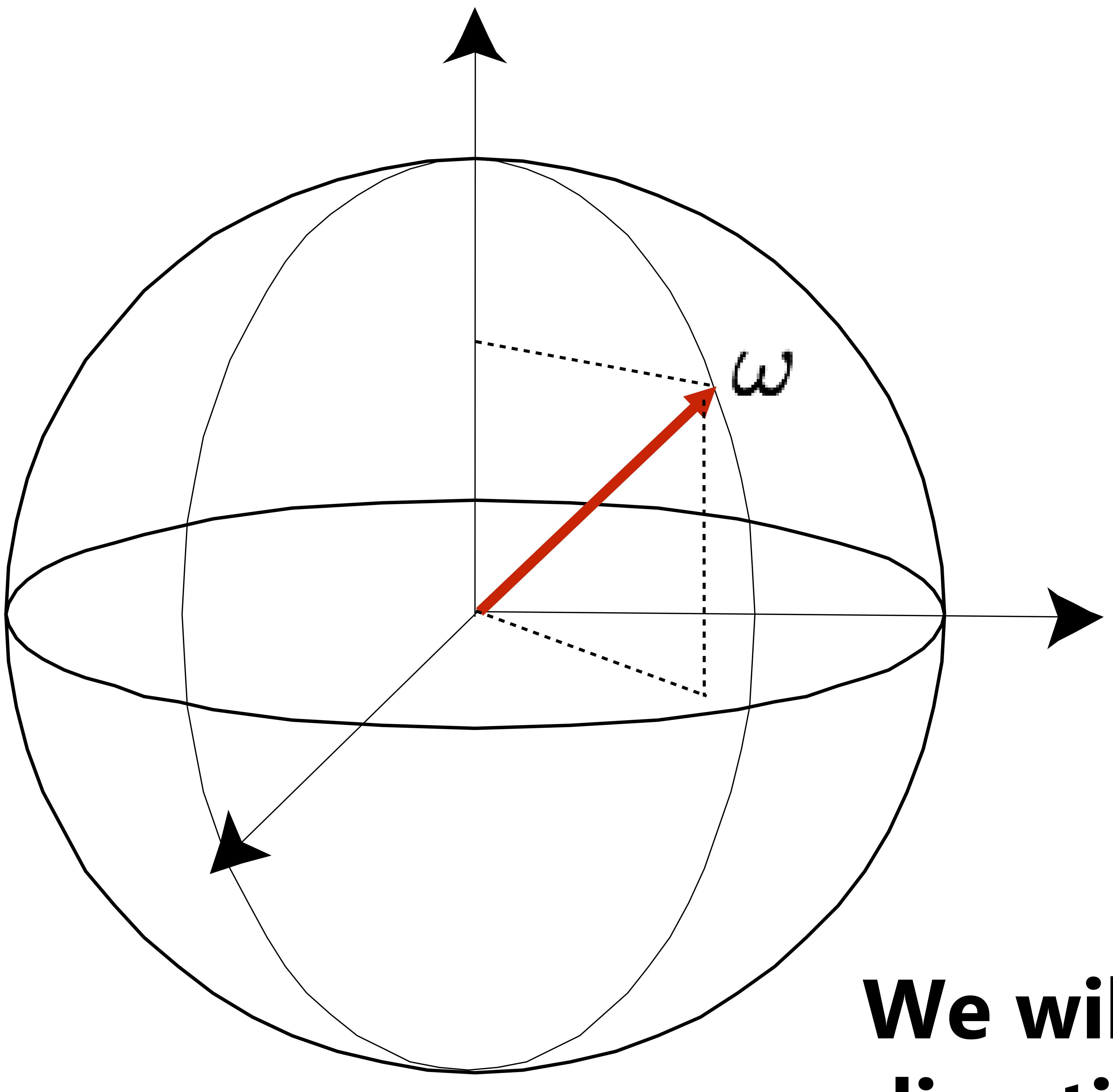


$$\Omega = \int_{S^2} \mathrm{d}\omega$$

$$= \int_0^{2\pi} \int_0^{\pi} \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

$$= 4\pi$$

ω as a direction vector



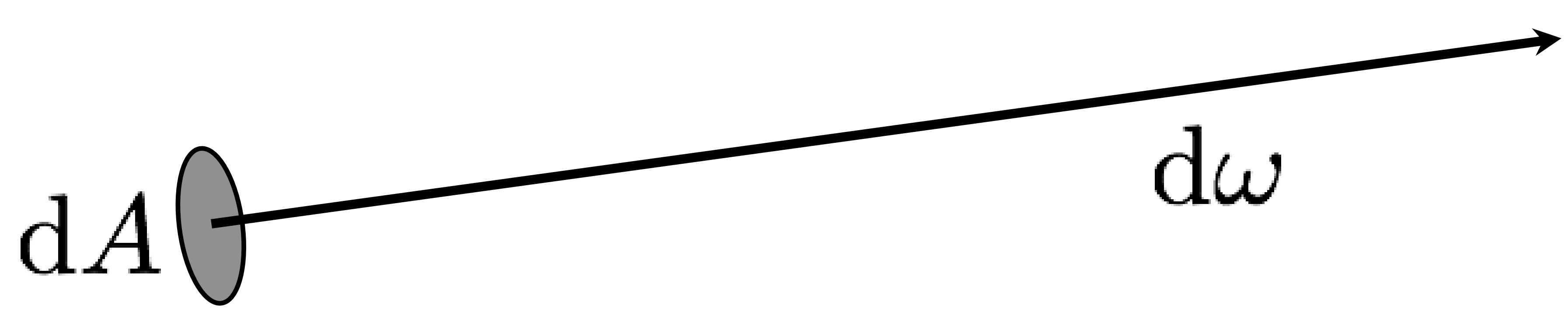
We will use ω to denote a direction vector (unit length)

Measuring illumination: radiance

- **Radiance is the solid angle density of irradiance**

$$L(\mathrm{p}, \omega) = \lim_{\Delta \to 0} \frac{\Delta E_\omega(\mathrm{p})}{\Delta \omega} = \frac{\mathrm{d}E_\omega(\mathrm{p})}{\mathrm{d}\omega} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{sr}}\right]$$

where E_ω **means that the differential surface area is oriented to face in the direction** ω



In other words, radiance is energy along a ray defined by origin point *p* and direction ω

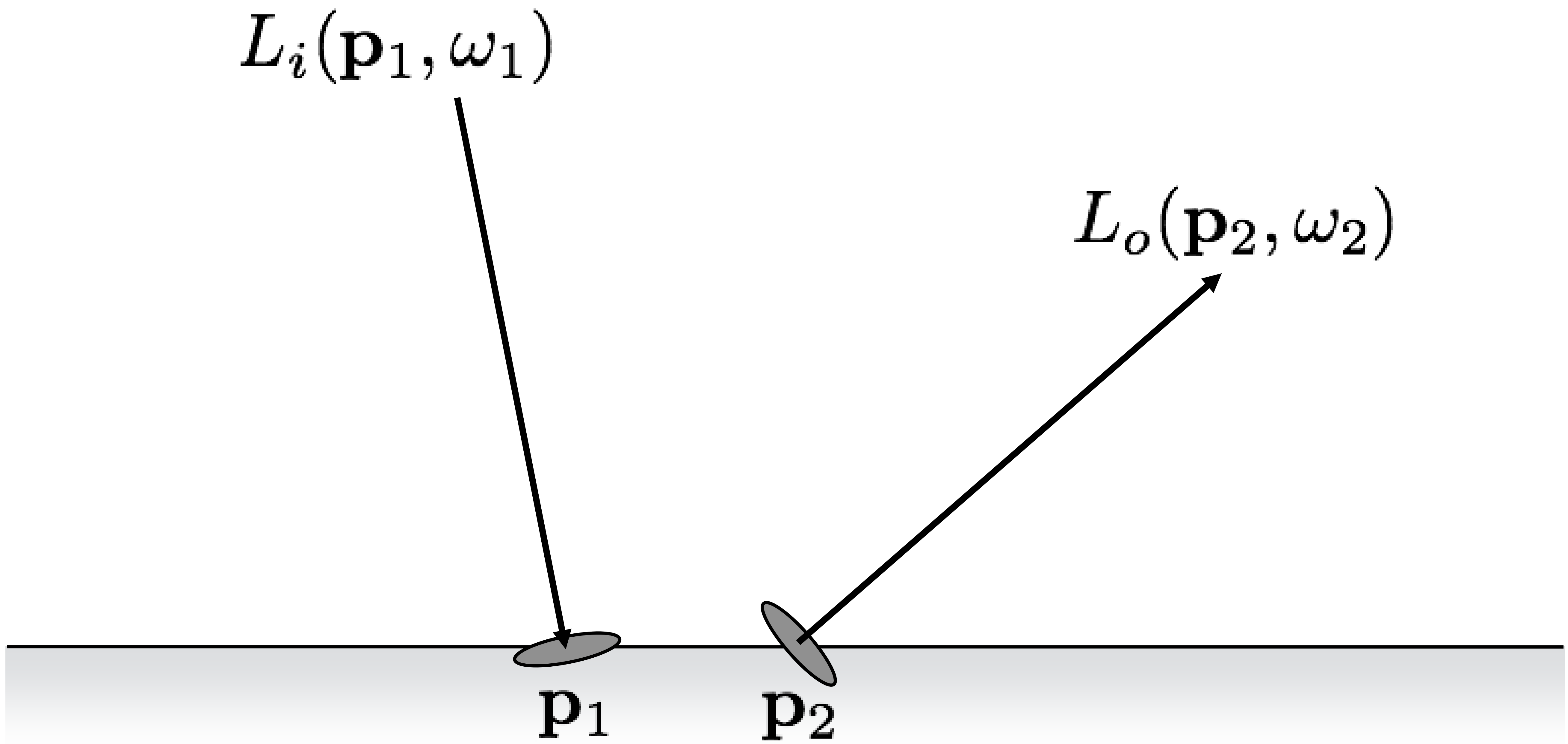
Energy per unit time per unit area per unit solid angle…!

Why do we break energy down to this granularity? (radiance)

Because once we have radiance, we have a complete description of the light in an environment!

Surface radiance

- **Need to distinguish between incident radiance and exitant radiance functions at a point on a surface**



In general: $L_i(\mathbf{p}, \omega) \neq L_o(\mathbf{p}, \omega)$

Properties of radiance

- **Radiance is a fundamental quantity that characterizes the distribution of light in an environment**
  - **Radiance is the quantity associated with a ray**
  - **Rendering is all about computing radiance**

- **Radiance is invariant along a ray in a vacuum**

- **A pinhole camera measures radiance**

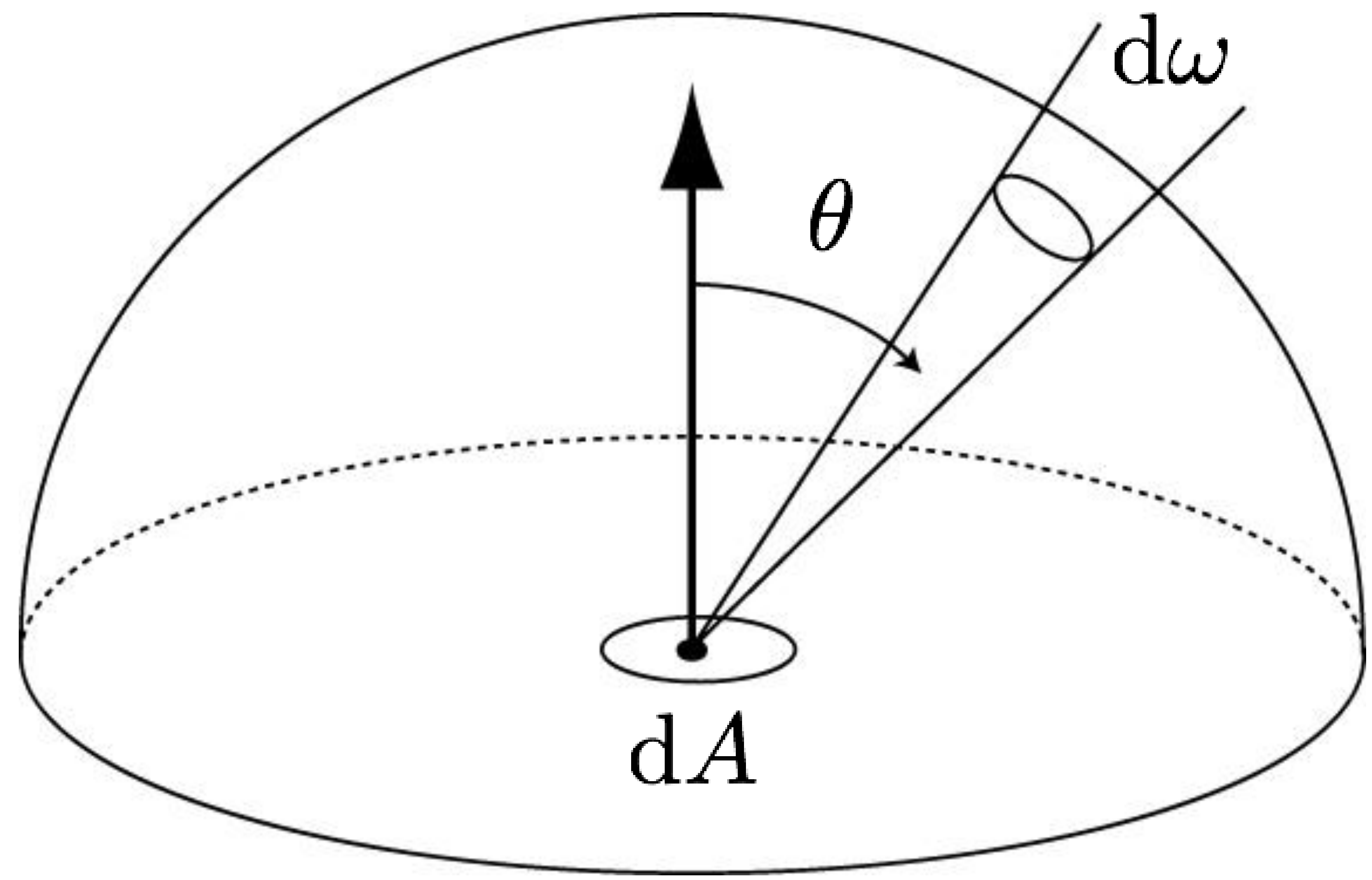
Irradiance from the environment

Computing flux per unit area on surface, due to incoming light from all directions.

$$dE(\mathrm{p}, \omega) = L_i(\mathrm{p}, \omega) \cos\theta \, \mathrm{d}\omega$$

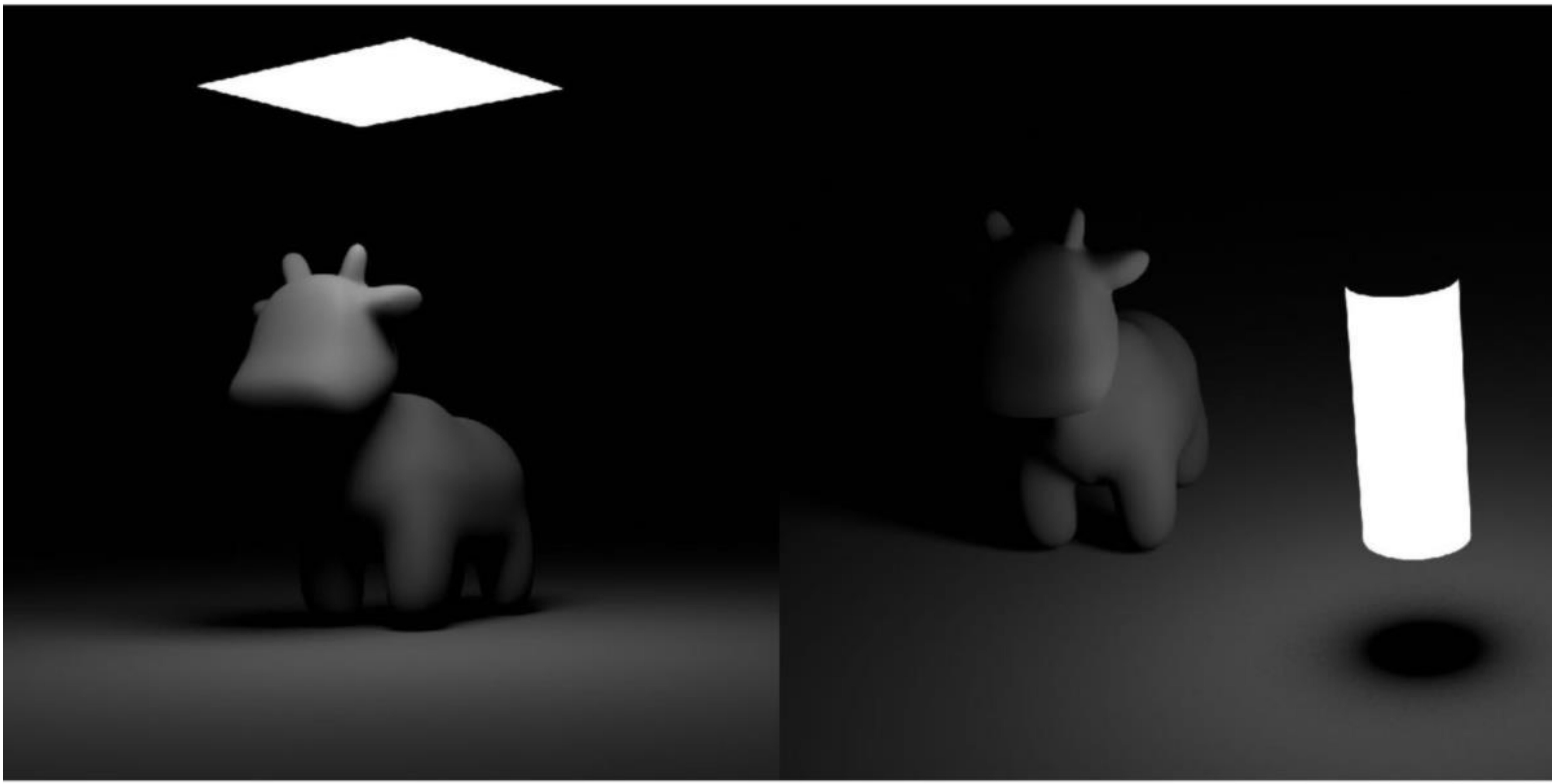
Contribution to irradiance from light arriving from direction ω

$$E(\mathrm{p}, \omega) = \int_{H^2} L_i(\mathrm{p}, \omega) \cos\theta \, \mathrm{d}\omega$$



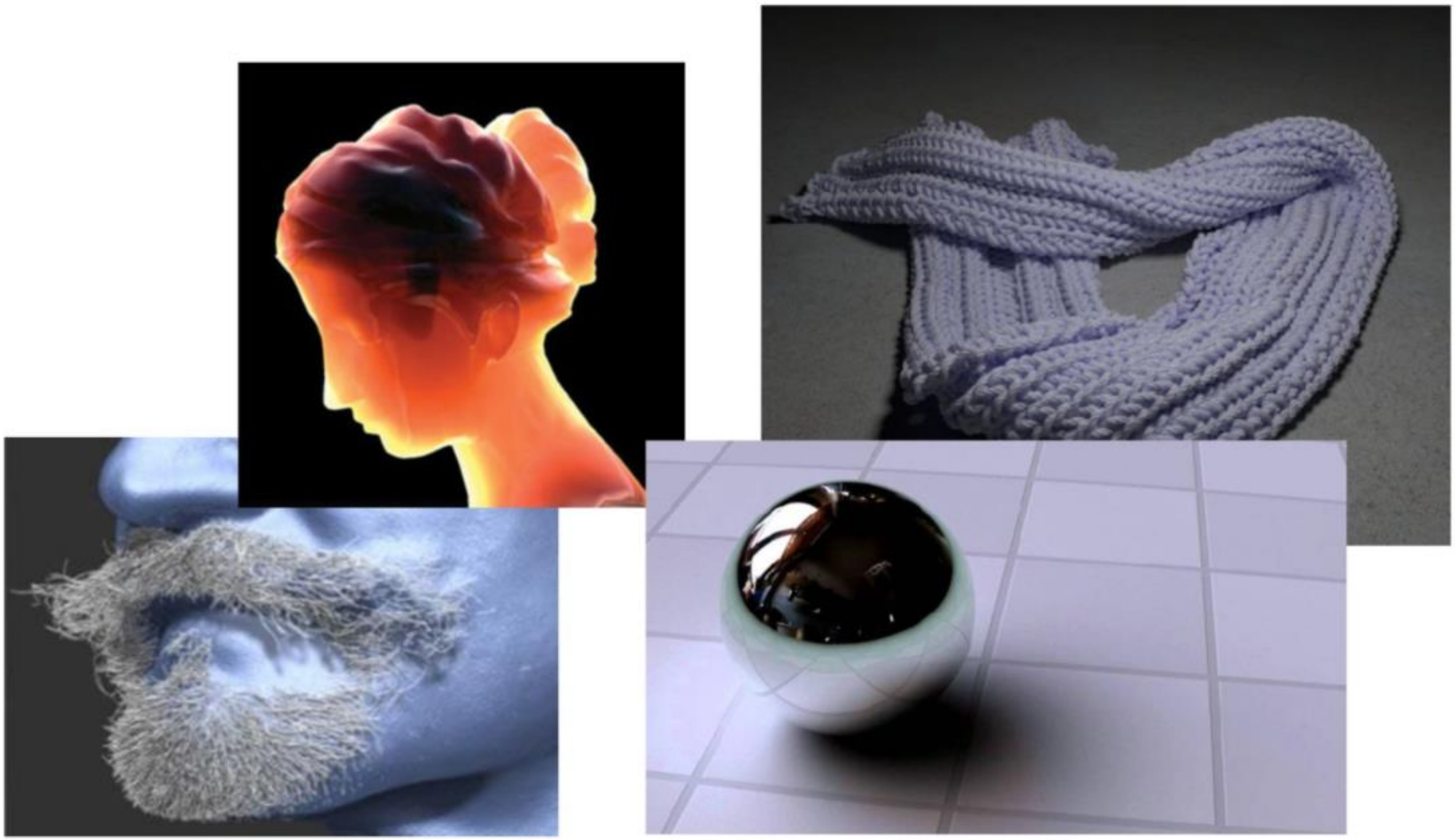
Examples of more sophisticated/realistic light sources

Generally "softer" appearance than point lights:

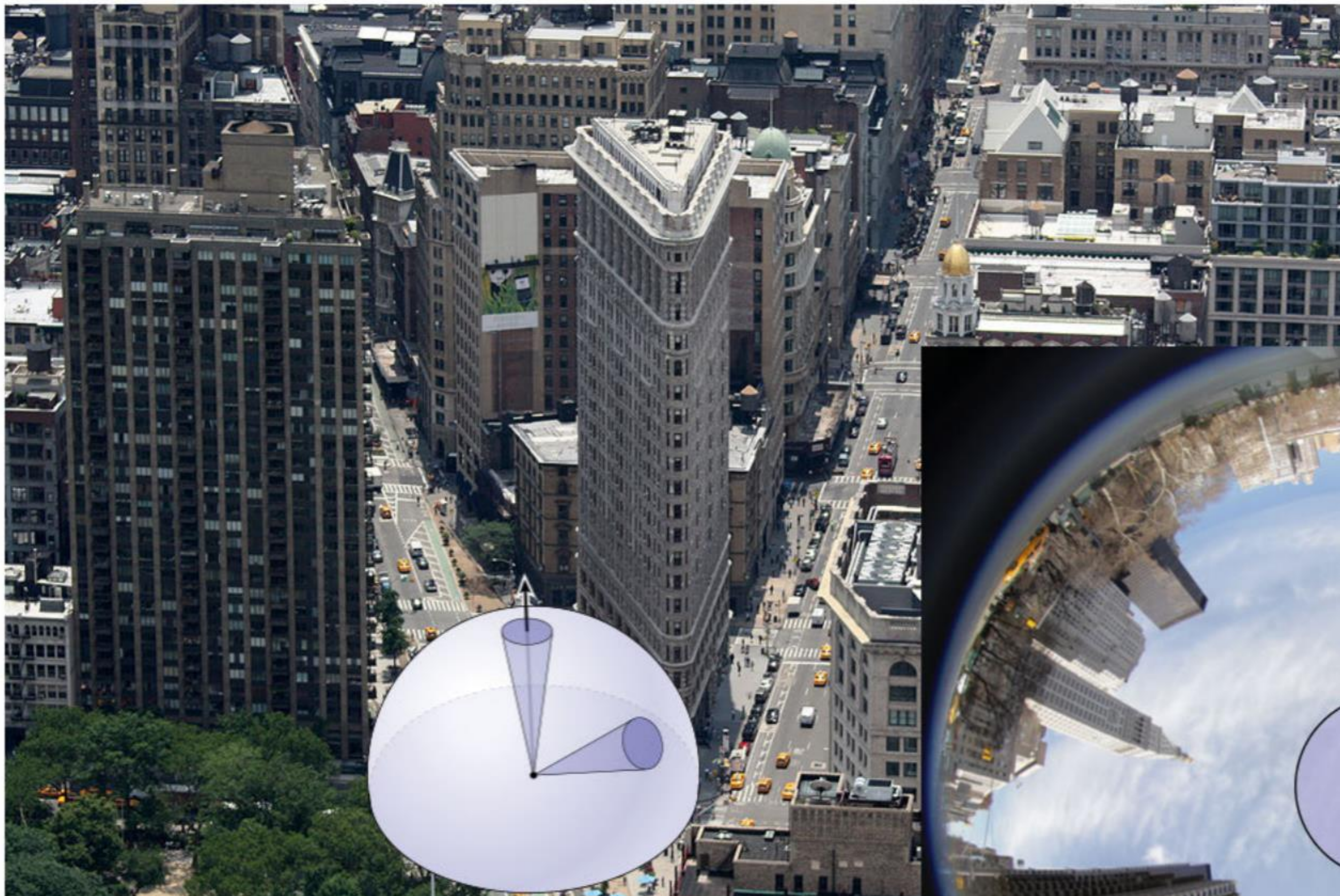


...and better model of real-world lights!

And, towards increasingly realistic renderings



Radiance and irradiance



irradiance

radiance in direction ω

$$E = \int_{\mathrm{H}^2} L(\omega) \cos\theta \, d\omega$$

angle between ω and normal

Incident vs Exitant Radiance

INCIDENT

EXITANT

In both cases: intensity of illumination is highly dependent on *direction* (not just location in space or moment in time).

The rendering equation

- **Core functionality of photorealistic renderer is to estimate radiance at a given point, in a given direction**
- **Summed up by the rendering equation (Jim Kajiya)**

angle between incoming direction and normal (Lambert's Law)

outgoing/observed radiance

emitted radiance (e.g., light source)

$$L_o(\mathrm{p}, \omega_o) = L_e(\mathrm{p}, \omega_o) + \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_o)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \, \mathrm{d}\omega_i$$

point of interest

incoming radiance

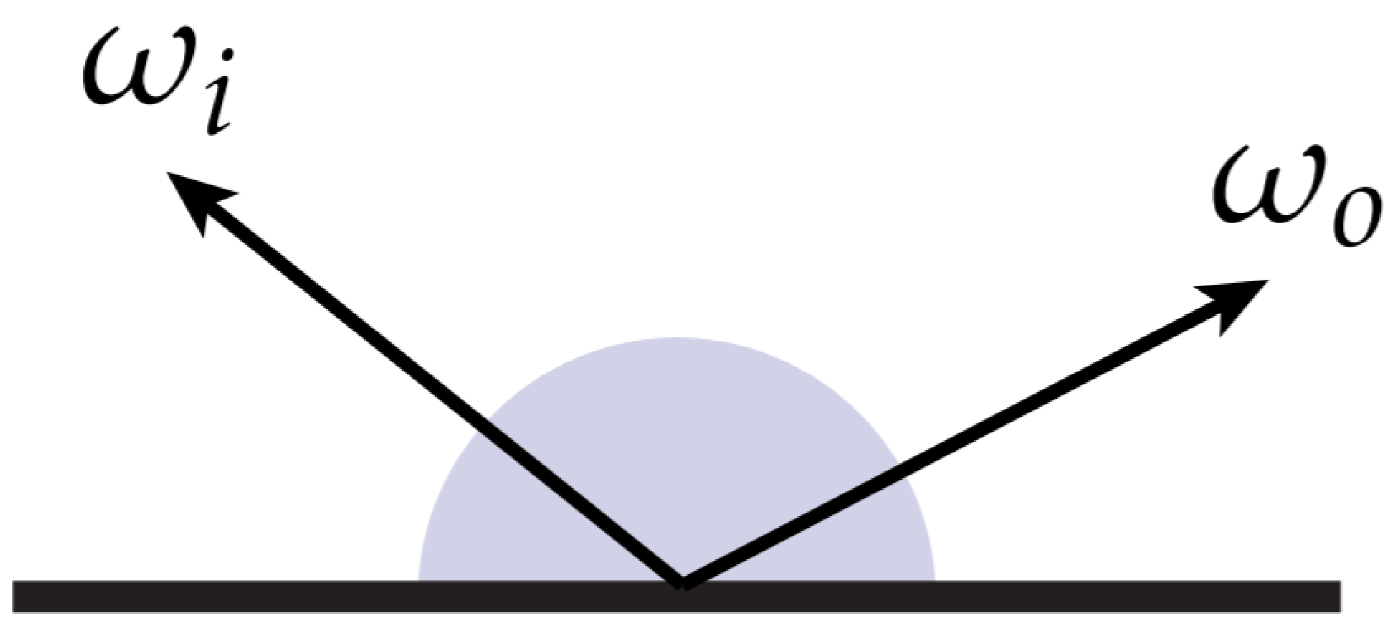
direction of interest

scattering function

all directions in hemisphere

Key challenge: to evaluate incoming radiance, we have to compute yet another integral. I.e., rendering equation is recursive.

Scattering function



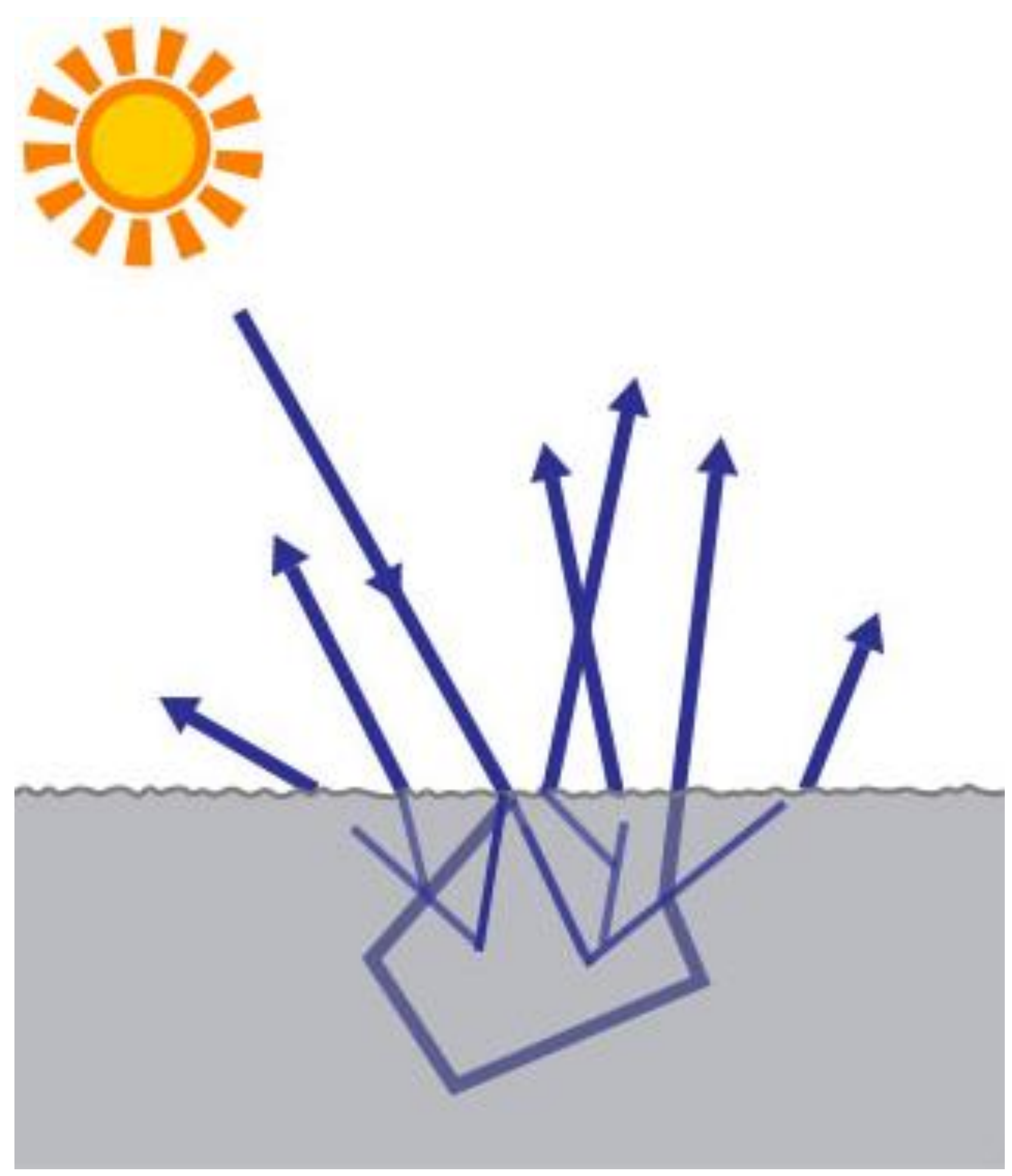
How does *reflection* of light affect the outgoing radiance?

$$L_o(\mathbf{p},\omega_o) = \int_{\mathcal{H}^2} f_r(\mathbf{p},\omega_i,\omega_o) L_i(\mathbf{p},\omega_i) \cos\theta \, \mathrm{d}\,\omega_i$$

Reflection function

- **Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident (same) side without change in frequency**

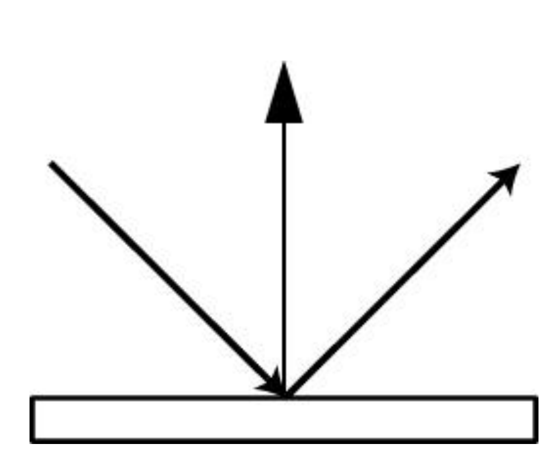
- **Choice of reflection function determines surface appearance**



Some basic reflection functions

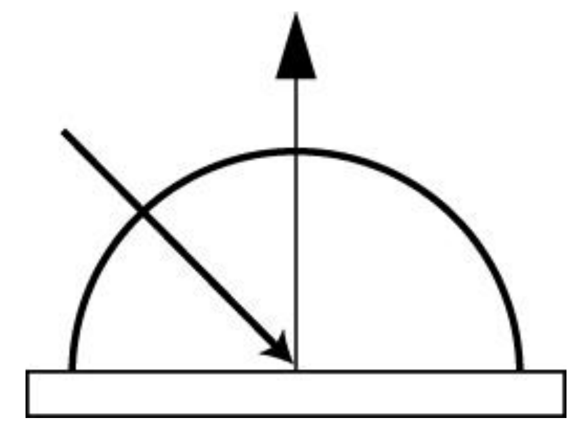
- **Ideal specular**

  Perfect mirror



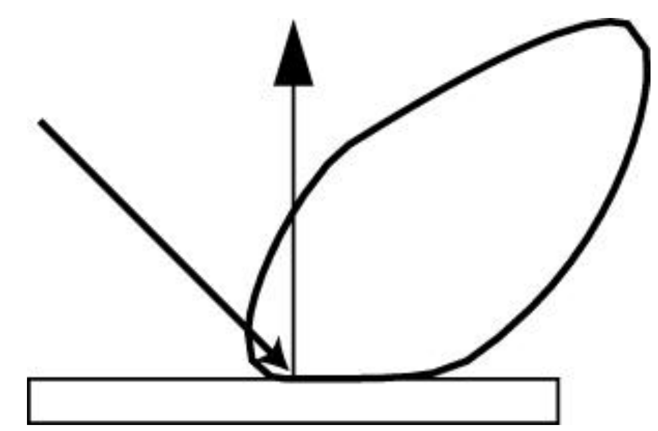
- **Ideal diffuse**

  Uniform reflection in all directions



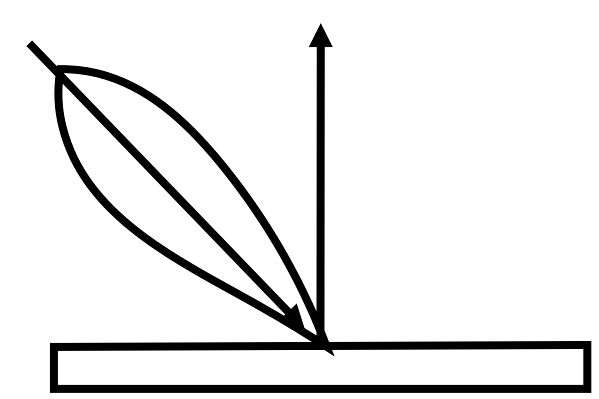
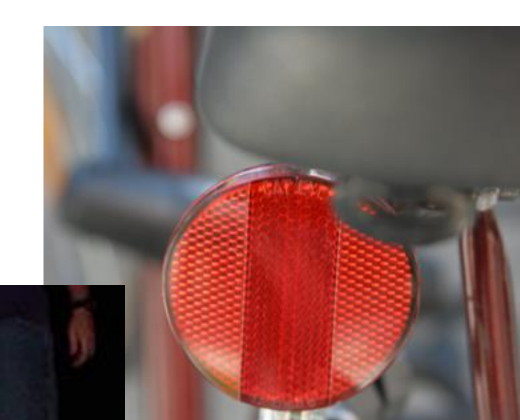

- **Glossy specular**

  Majority of light distributed in reflection direction



- **Retro-reflective**

  Reflects light back toward source

Diagrams illustrate how incoming light energy from given direction is reflected in various directions.

Materials: diffuse

Materials: plastic

Materials: red semi-gloss paint

Materials: Ford mystic lacquer paint

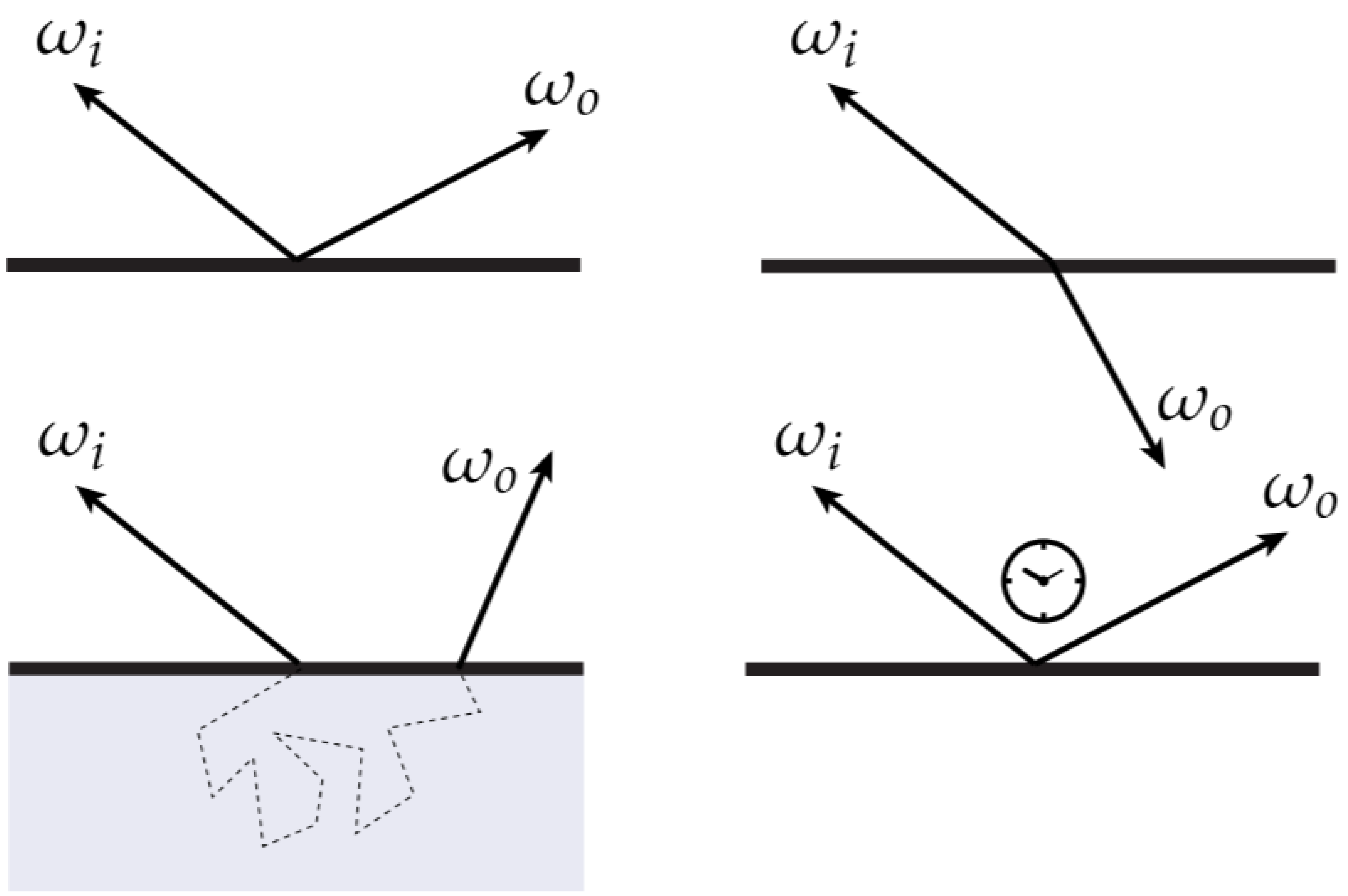
Materials: mirror

Materials: gold

Materials

Models of scattering

- How can we model "scattering" of light?
- Many different things could happen to a photon:
  - bounces off the surface
  - transmitted through surface
  - bounces around inside surface
  - absorbed and re-emitted
  - ...



- What goes in must come out!

Scattering off a surface: the BRDF

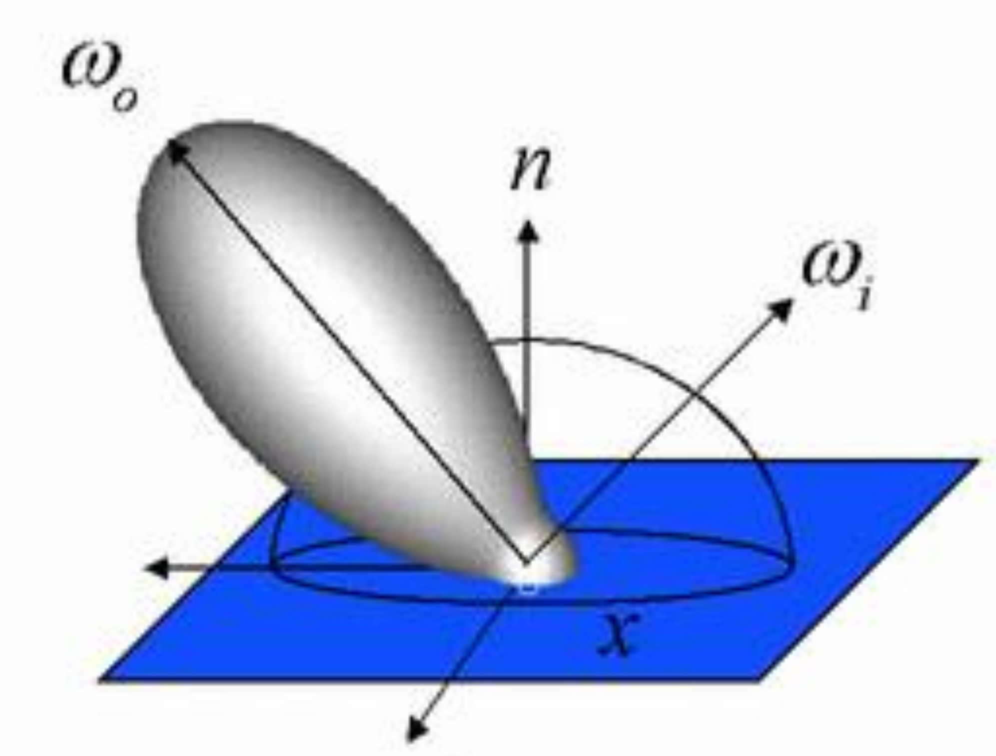
- **"Bidirectional reflectance distribution function"**
- **Encodes behavior of light that "bounces off" surface**
- **Given incoming direction ω_i, how much light gets scattered in outgoing direction ω_o?**
- **Describe as a distribution: $f_r(\omega_i \rightarrow \omega_o)$**

$$f_r(\omega_i \rightarrow \omega_o) \geq 0$$

why *less* than or equal?

$$\int_{\mathcal{H}^2} f_r(\omega_i \rightarrow \omega_o)\ \cos\theta\, d\omega_i \leq 1$$

where did the rest of the energy go?!



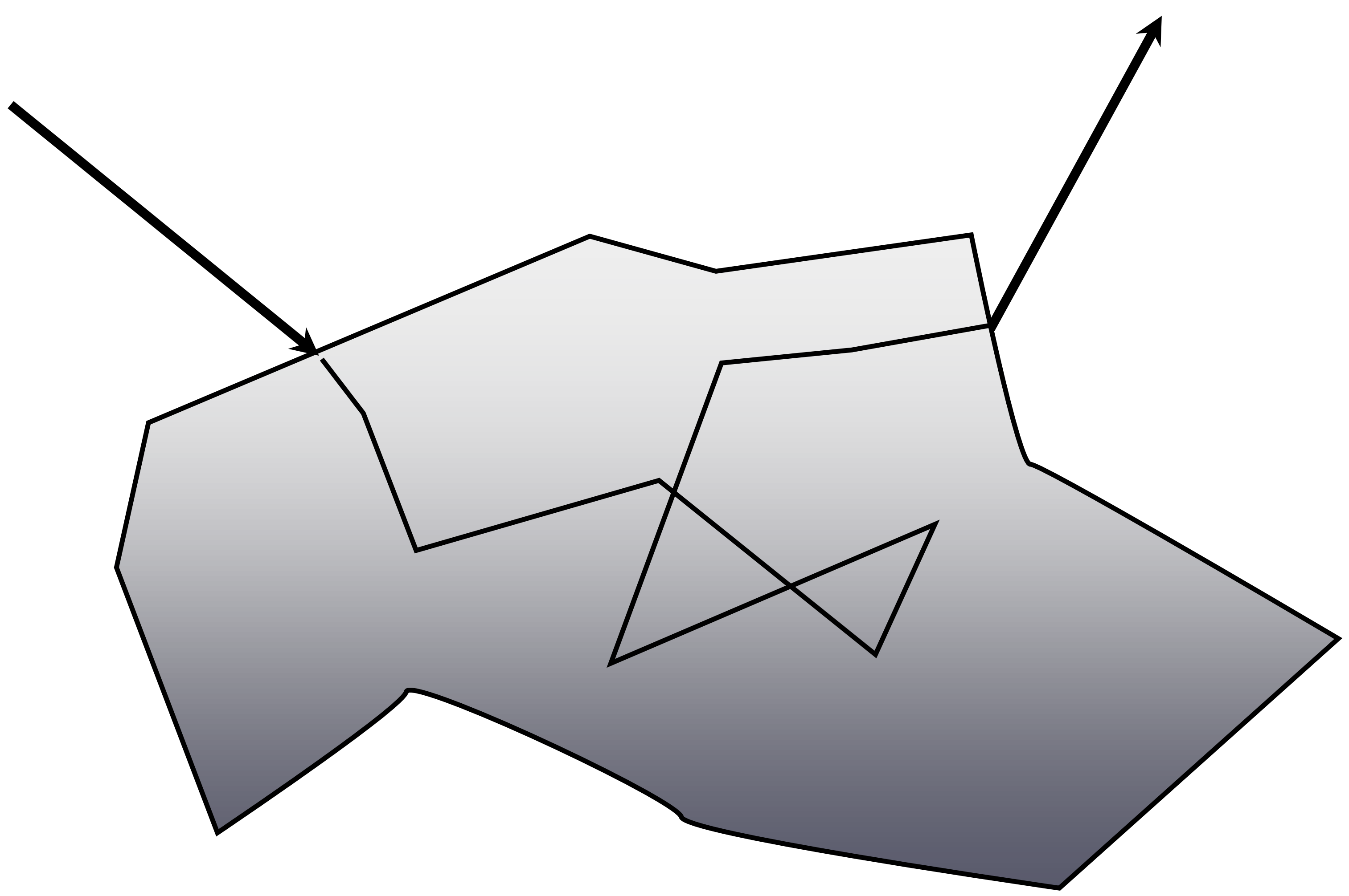
$$f_r(\omega_i \rightarrow \omega_o) = f_r(\omega_o \rightarrow \omega_i)$$

"Helmholtz reciprocity"

Q: Why should Helmholtz reciprocity hold? Think about little mirrors…

Reflection & Subsurface scattering

- **Visual characteristics of many surfaces caused by light entering at different points than it exits**
  - **Violates a fundamental assumption of the BRDF**



[Jensen et al 2001]

[Donner et al 2008]

BRDF

BSSRDF