ETH zürich
Departement Informatik

Theoretical Computer Science

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Second Midterm Exam

Zurich, December 11th, 2018

Exercise 1

We consider the languages

$$L_\lambda = \{\text{Kod}(M) \mid \lambda \in L(M)\}$$

and

$$L_{\text{all}} = \{\text{Kod}(M) \mid L(M) = \Sigma^*_{\text{bool}}\}\,.$$

(a) Prove that $L_{\text{U}} \leq_{\text{EE}} L_\lambda$.

(b) Prove that $L_{\text{all}} \notin \mathcal{L}_{\text{R}}$.

You may use all results from the lecture and from the exercise sheets. **7+3 points**

Exercise 2

We consider the language

$$L_{\cap\neq\emptyset} = \{\text{Kod}(M_1)\#\text{Kod}(M_2) \mid L(M_1) \cap L(M_2) \neq \emptyset\}\,.$$

(a) Prove that $L_{\cap\neq\emptyset} \in \mathcal{L}_{\text{RE}}$.

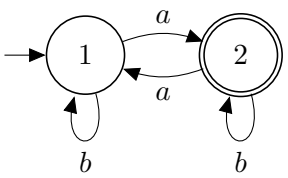
(b) Prove that $L_{\cap\neq\emptyset} \notin \mathcal{L}_{\text{R}}$.

You may use all results from the lecture and from the exercise sheets. **5+5 points**

(please turn the page)

Exercise 3

(a) We consider the finite automaton A as shown below.



Use the dynamic-programming method from the lecture to derive a regular expression α with $L(\alpha) = L(A)$.

(b) Give a regular grammar G satisfying

$$L(G) = \{w \in \{a, b\}^* \mid 2|w|_b \bmod 3 = 2 \text{ or } w \text{ contains the subword } bba\}\,.$$

Give a brief and informal explanation of the idea behind your construction.

5+5 points

Exercise 4

(a) Let $G = (V, E)$ be an undirected graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = \{e_1, e_2, \ldots, e_m\}$.

Give a construction that transforms G into a 3CNF formula Φ that is satisfiable if and only if G contains a vertex cover of size 2.

(b) Let TRIPLE-SAT denote the set of all CNF formulas having *at least* three satisfying assignments. Prove that TRIPLE-SAT is NP-complete.

You may use all results from the lecture and from the exercise sheets.

4+6 points