Endterm exam

Num. CSE, D-INFK/D-MATH

HS 2015

Prof. R. Hiptmair

Name		Grade
Surname		
Department		
Legi Nr.		
Date	18.12.2015	

1	2	3	4	Total
6	6	12	12	36

- **Keep only writing material and Legi on the table.**
- Keep mobile phones, tablets, smartwatches, etc. **turned off** in your bag.
- Fill this cover sheet first.
- **Turn the cover sheet only when instructed to do so.**
- **Then, put your name and Legi Nr. on each page.**
- **Read the rules carefully.**
- Do not write with red/green colour or with pencil.
- **Make sure to hand-in every sheet.**
- **Duration: 30 min.**
- Additional material: none.

Good luck!

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Rules:

- Motivation for the answers is **not** necessary. Remarks and computations have **no** influence on the total number of points.
- Wrong answers **do not** give negative points.
- Write your solutions in the **predefined boxes**, e.g.

  $\boxed{\text{sample box}}$

- All notes outside the predefined boxes will not be considered.
- Any unclear marking will be considered an error.

1. *Gauss weights* [6 P.]

   Let $\hat{w}_j^n$ and $\hat{c}_j^n$ be the weights and nodes of the Gauss-Legendre quadrature rule on $[0,1]$.

   Express weights w_j^n and nodes c_j^n of a quadrature rule of order $2n$ on a general interval $[a,b]$ in terms of $\hat{w}_j^n$ and $\hat{c}_j^n$:

$$w_j^n = \boxed{(b-a)\hat{w}_j^n}$$

$$c_j^n = \boxed{a + \hat{c}_j^n(b-a)}$$

Scratch space (not evaluated):

2. *Domain of analyticity* [6 P.]

   Determine the domain of analyticity $\mathcal{A}$ of the function

$$f(z) := \coth z = \frac{\cosh z}{\sinh z}.$$

$\mathcal{A} = \boxed{\mathbb{C} \setminus (\pi i \mathbb{Z})}$

(Recall that $\cosh z = (e^z + e^{-z})/2$ and $\sinh z = (e^z - e^{-z})/2$.)

Scratch space (not evaluated):

3. *Dimensions of spaces* [12 P.]

Take $m, k \in \mathbb{N} \setminus \{0\}$ and let $\mathcal{M} = \{a = x_0 < x_1 < \cdots < x_m = b\}$ be a mesh of the interval $[a, b]$. Determine the dimensions of the following real vector spaces in terms of m and k.

$$\dim(\{f \in C^0([a,b]) : f_{|[x_{j-1},x_j]} \in \mathcal{P}_k,\ j = 1,\dots,m\}) = \boxed{mk+1},$$

$$\dim(\{f \in C^k([a,b]) : f_{|[x_{j-1},x_j]} \in \mathcal{P}_k,\ j = 1,\dots,m\}) = \boxed{k+1},$$

$$\dim(\mathcal{P}_k) = \boxed{k+1},$$

$$\dim(\{f \in C^{k-1}([a,b]) : f_{|[x_{j-1},x_j]} \in \mathcal{P}_k,\ j = 1,\dots,m\}) = \boxed{m+k}.$$

Scratch space (not evaluated):

4. *Consistency of an explicit Runge–Kutta method* [12 P.]

The following Butcher scheme describes an explicit Runge-Kutta single step method (RK-SSM):

$$\begin{array}{c|ccc} 0 & 0 & & \\ \alpha & \alpha & 0 & \\ \alpha & 0 & \alpha & 0 \\ \hline 0 & \gamma & \gamma & \gamma \end{array} \tag{1}$$

(a) Write down a correct expression for a single step of size h, in terms of $\mathbf{y}_0$, for the RK-SSM, when applied to the ODE $\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y})$.

$$\mathbf{y}_1 = \boxed{\mathbf{y}_0 + h\gamma\Big(\mathbf{f}(\mathbf{y}_0) + \mathbf{f}\big(\mathbf{y}_0 + h\alpha\mathbf{f}(\mathbf{y}_0)\big) + \mathbf{f}\big(\mathbf{y}_0 + h\alpha\mathbf{f}\left(\mathbf{y}_0 + h\alpha\mathbf{f}(\mathbf{y}_0)\right)\big)\Big)}$$

(b) For what choices of parameters α and γ in $\mathbb{R}$ will the RK-SSM given by (1) be consistent with the autonomous ODE $\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y})$?

$\alpha \in \boxed{\mathbb{R}}$, $\gamma \in \boxed{\left\{\frac{1}{3}\right\}}$.

Scratch space (not evaluated):