ETH zürich
Departement Informatik

Theoretical Computer Science

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First Midterm Exam

Zurich, November 6th, 2018

Exercise 1

We consider the language

$$L = \{1x \mid x = y1 \text{ for some } y \in \{0,1\}^* \text{ or } x = z00 \text{ for some } z \in \{0,1\}^*\}\,.$$

(a) Construct a nondeterministic finite automaton (in graphical representation) with at most 4 states that accepts L, and describe the idea of your construction informally.

(b) Construct a (deterministic) finite automaton (in graphical representation) with at most 6 states that accepts L.

To this end, you may either apply the power set construction to your NFA from exercise part (a) or you may construct the automaton directly and informally describe the idea of your construction.

5+5 points

Exercise 2

Prove that the following languages are not regular.

(a) $L = \{u\#v \mid u, v \in \{0,1\}^* \text{ and } \mathrm{Number}(v) = 2 \cdot \mathrm{Number}(u)\}$,

(b) $L = \{0^{n^3} \mid n \in \mathbb{N}\}$.

For your proof, you may choose among the following three proof methods, but you are *not* allowed to use the same method for both languages.

(i) Using Lemma 3.12 from the English book (i. e., Lemma 3.3 from the German book, or a direct argumentation about the automaton),

(ii) using the pumping lemma, or

(iii) using the method of Kolmogorov complexity.

Note that, for solutions using the same method for both exercise parts, only exercise part (a) will be graded. **5+5 points**

(please turn the page)

Exercise 3

Let, for all $n \in \mathbb{N} - \{0\}$, the language L_n be defined by

$$L_n = \{x \in \{0,1\}^* \mid |x|_1 \geq n\} .$$

(a) Give a (deterministic) finite automaton (in graphical representation) for L_4 that has at most 5 states. For each state q of your automaton, give the class $\mathrm{Kl}[q]$.

(b) Prove that every (deterministic) finite automaton accepting L_n has at least $n+1$ states.

4+6 points

Exercise 4

Let $n_1, n_2, n_3, \ldots$ be a strictly increasing infinite sequence of positive natural numbers such that, for all $i \in \mathbb{N} - \{0\}$, we have

$$K(n_i) \geq \lceil \log_2 n_i \rceil / 2 .$$

For each $i \in \mathbb{N} - \{0\}$, let q_i denote the largest prime number dividing the number n_i. Prove that the set $Q = \{q_i \mid i \in \mathbb{N} - \{0\}\}$ is infinite. **10 points**