Peak Water Demand Study

Probability Estimates for Efficient Fixtures in Single and Multi-family Residential Buildings

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List of Symbols

B	Bathtub
C	Clothes washer
D	Dishwasher
E[x]	Expected value of x
F	Faucet
gpm	Gallon per minute, L^3/T
H	Number of homes
H(n,p)	Dimensionless Hunter's Number
k	Fixture group
K	Number of Identical fixtures
L	Lavatory faucet
M	Number of pulses
n	Number of fixture

N	Number of trace days
p	Probability of fixture use
psi	Pounds per square inch
P_0	Probability of zero busy fixtures
q	Fixture flow rate
$\overline{q}$	Probability-weighted average flow of a busy fixture in the building
$Q_{0.99}$	99^{th} percentile of demand flow, L^3/T
$Q^*_{0.99}$	99^{th} percentile of dimensionless demand flow
S	Shower
t	Average duration of water flow, T
T	Average duration between successive fixture operations, T
T	Toilet
T	Fixture observation window, T
Var[x]	Variance of x
x	Number of busy fixtures
$Z_{0.99}$	99th percentile of the standard normal distribution

Greek

β	Weighting factor
Σ	Summation operation
Π	Product operation
Θ_q	Coefficient of variation of fixture flow in a building

Executive Summary

The probabilistic method for determining peak water supply demand in building plumbing systems has historically been based on the Hunter Fixture Unit method as published in *Methods of Estimating Loads on Plumbing Systems BMS65*. As early as 1974, a US Commission identified the foremost national research need for water supply within buildings as the need for a long-range program to develop an improved computational method for the design and evaluation of water service and distribution systems in buildings. Studies had confirmed that in most building types, the application of Hunter's method results in excessive over-design of the system. The problem of excessiveness was further exacerbated since the Energy Policy Act of 1992 (EPACT) required fixture flow reductions along with other conservation endeavors by the Water Sense program of U.S. Environmental Protection Agency (EPA).

Household plumbing fixture changes need to be identified since the probabilistic method uses the mathematical parameters of a fixture's flow rate, the duration of the flow, and how often the fixture is in use. These fixture parameters have significantly changed since the original Hunter method.

Another problem needing resolve is the absence of congested use of fixtures in residential dwellings. The original Hunter model assumed congested use based on the assumption that there is a queue of people waiting to use each fixture. This assumption is not applicable for single family dwellings.

A probabilistic method using a normal distribution works increasingly well when $np > 5$. This method would necessitate a large number of plumbing fixtures especially if the probability of fixture use is nearing zero. A new computational method is needed where there are a small number of fixtures within a single family dwelling.

To investigate the possibility of an improved computational method that would not result in excessive estimates, a residential end use of water database was created by Aquacraft, Inc. to include the mathematical parameters required for a normal distribution to determine probabilities of fixture use and flow rate patterns. The database included over 1000 single family homes surveyed between the years 2006 and 2011 (some Colorado homes were surveyed in 1996). Common residential indoor fixtures surveyed include toilets, showers, bathtubs, faucets (inclusive of kitchen and lavatory faucets), dishwashers, and clothes washers. The database could also distinguish the efficiency levels (inefficient, efficient, ultra-efficient) for toilets, showers, dishwashers and clothes washers.

Fixture flow rates can be determined by the average flow pulse recorded by the data loggers as the ratio of total volume of water used at the fixture and the total duration of water flowing at the fixture. This expresses a duration-weighted average of all pulses at a given fixture.

Fixture use probabilities can be determined by observing the peak hour of water use. The peak hour of water use is determined from the observed flow pattern as the single hour with the highest average volume of water used in the building. The probability that a fixture is busy is the percentage of time water is flowing from a fixture during a set period of observation. The fixture observation time takes into account the number of days water flow was monitored in each household, and the number of identical fixtures in the building. The underlying assumption is that each identical fixture has an equal probability of use irrespective of the location within a building.

Fixture probabilities were examined by per capita groups, per bedroom groups, and per bathroom groups to determine the correlation with the probability of fixture use. The database revealed a positive correlation between per capita and the probability of fixture use during the fixture peak hour for most of the fixtures. To capture this per capita positive correlation relative to either the number of bedrooms or number of bathrooms, further investigation examined the correlation between per capita and bedroom groups, and per capita and bathroom groups. There was no correlation with either group resulting in abandoning the probability of fixture use per bathroom or per bedroom. Fixture probabilities are most reliable per capita less than or equal to six residents.

Having established fixture p-values and fixture flow rates, a normal distribution developed by Robert Wistort was used to calculate the water supply demand based on a number of different types of fixtures without the use of fixture units. It is a direct analytic method to calculate peak demand. However, this method of distribution was still based on congested use and was not suited for smaller systems such as single family homes where the number of plumbing fixtures is small and the probability of fixture use is slight (when $np<5$).

A modification of the Wistort method using a *zero-truncated binomial distribution* (ZTBD) is proposed for smaller applications. This method is able to extend the Wistort method from large public buildings to small private dwellings. The modification also includes bundling fixtures that operate within a common room such as the bathroom and kitchen. The room group will have a unique p- and q-value. Grouping fixtures in a common room increases the probability of simultaneous multiple fixture use and thus the expected demand flow will increase.

A recommended table of fixture and group specific p-values and q-values are provided as a reference when implementing the modified Wistort method for single and multi-family residential dwellings having efficient plumbing fixtures.

[1] Introduction

1.1 Background and Significance

The peak water demand study originated with IAPMO's Hot Water Task Group when considering the question of pipe sizing during the 2010 Green Plumbing and Mechanical Supplement Code cycle. The problem at hand was the efficient and timely delivery of hot water to the end user. Reducing the hot water pipe size would increase the rate of flow, reduce the volume of water and hence, deliver the hot water to the user more quickly and with less waste. Additionally, reducing the overall pipe size will reduce excess material used in construction that only adds to cost of the project.

The Hot Water Task Group received a draft report on a revision of Hunter's curve (Hunter, BMS65) for efficient plumbing fixtures that resulted in the creation of a new IAPMO Pipe Sizing Task Group to investigate the pipe sizing question from a revisionist point of view. The investigation would begin with the consideration of revising the assumptions and parameters of Hunter's demand estimate model.

A mission statement was developed that charged the IAPMO Pipe Sizing Task Group to investigate if significant water, energy and/or construction cost efficiencies can be achieved by revising the method of estimating the water demand load to be provided for the water distribution system comprising the water service, cold-water distribution and hot water distribution, and to accordingly re-evaluate the minimum required pipe sizes. If so, this Task Group would publish a detailed report of findings that would be used as the basis for code change proposals to the Uniform Plumbing Code that will help to achieve these efficiencies while ensuring continued system efficacy, performance and safety.

In July 2011, IAPMO and the American Society of Plumbing Engineers (ASPE) convened a special task force to revise the methodology for properly sizing plumbing systems in response to the increased use of high-efficiency plumbing fixtures, fixture fittings and appliances and the subsequent decreased demand for water in commercial buildings and residences. In collaboration with IAPMO and ASPE, the Water Quality Research Foundation (WQRF) became a co-sponsor in funding the research project. The Pipe Sizing Task Group was reorganized with three members from ASPE who specialized in statistics and mathematics. An additional member from the University of Cincinnati joined this effort.

1.2 Scope of Work

The Pipe Sizing Task Group readily acknowledged that statistical data was a prerequisite for any investigation toward the development of a probability model to predict peak demands based on the number of plumbing fixtures of different kinds installed in one system. The task group initiated talks with Aquacraft, Inc. to access the largest U.S. database containing residential end uses of water surveys (REUWS), and with sponsorship contracted a specially designed database containing dataset parameters to determine fixture use probabilities. Since the dataset only provided statistics for residential end use, the scope of work was narrowed to single and multi-family residential dwellings.

Since the water supply flow rate to plumbing fixtures is today significantly less than those used in developing the original Hunter's curve, the scope of the work also narrowed to indoor high-efficiency plumbing fixtures with efficiency parameters defined within the database. Residential efficiency fixtures considered in the database are toilets, showers, dishwashers, and clothes washers. Faucets (kitchen and lavatory) were not classified based on efficiency because the intensity and duration of water use at these fixtures depends on the user. Bathtubs were likewise not classified since there are no design benefits to low flow tub spouts (see Figure 2).

1.3 Task Group Objective

The objective of the task group was directed toward developing a statistically based probability model that would predict the peak water demand for single and multi-family dwellings having high efficiency plumbing fixtures. In order to meet this objective, the database was subject to numerous queries to find probabilities of fixture use, peak hours of use, fixture flow rates and efficiency levels, and per capita correlations per number of bedrooms and bathrooms using various sample selections. The task group would also need to develop statistical equations suitable to the datasets provided.

Another objective for the task group is to assess if there would be significant water, energy and/or construction cost efficiencies achieved by implementing the developed method of estimating the peak water demand, and to accordingly re-evaluate the minimum required pipe sizes.

Although this task group has been named Pipe Sizing sub-task group, the effort is narrowed to the development of a statistical model that estimates water supply demand for residential dwellings. Pipe sizes are expected to decrease (comparative to current plumbing codes) where the demand estimates have been reduced (comparative to current codes) especially at the water supply source and inclusive of water meters. However, pressure and velocity limitations also play a significant role in pipe sizing, and fixtures with reduced flow rates are requiring higher operating pressures. This project report will **not** consider the role of system pressure or velocity in pipe design. Its single task is the development of a statistical model to estimate water supply demand for residential dwellings with low-flow fixtures. The only correlation for pipe sizing that can be suggested from this report is between the estimated demand flow rate and the corresponding pipe size limited by velocity requirements in current plumbing codes.

[2] Literature Survey

The post-World War I conservation era spurred by President Warren G. Harding and Secretary of Commerce Herbert Hoover in 1921 resulted in a proliferation of scientific investigations published by the National Bureau of Standards (NBS) for the building and housing industry for the next four decades (Cole 2009). One of the most significant NBS publications for the plumbing industry was a report by Dr. Roy B. Hunter on Methods of Estimating Loads in Plumbing Systems (Hunter 1940). This report explained how the binomial distribution function was applied to different kinds of plumbing fixtures to estimate peak demand for any given plumbing system.

This method of estimating water demand in buildings is still dominate in the United States, and known as Hunter's curve. Dr. Hunter determined the probability that a particular fixture will be operating as p=*t*/T where *t* is the average duration of water flow in seconds and T is the average time between successive operations in seconds. The binomial distribution function derives the probability of simultaneous operation of fixtures out of a total number of fixtures and this analysis was used to determine the maximum number of fixtures that will be operating within the 99th percentile. The probability and flow rate for various types of fixtures were used to determine a weighted average for each fixture type, and this weighted average was expressed in "fixture units", a term coined by Hunter (Cole 2008). In practice, the designer adds all of the fixture units being served by a plumbing system, and the fixture unit total is converted to a flowrate (gpm) by using a graphical curve showing the relation between number of fixture units and flowrate from which the appropriate pipe size can be determined.

The Hunter method is limited by the applicability of the assumption of congested conditions of service which is defined as the maximum practical rate at which fixtures can be used continuously in actual service. In other words, the analysis employed by Hunter is based on the assumption that there is a queue of people waiting to use each fixture. While valid in certain applications where there is a line of people waiting to use the facilities, the assumption is not valid in other applications such as single family homes. This limitation has been a cause of criticism of the pipe sizing method in the plumbing codes when the application of Hunter's method is not valid for the assumption of congested use.

Plumbing research continued at the National Bureau of Standards in the 1950s and 1960s, and in 1962 the United States organized the U.S. National Committee for the International Council for Building Research (USNCCIB) as a counterpart commission to participate in the International Council for Building Research (CIB). CIB was established in 1953 with support of the United Nations to stimulate and facilitate international collaboration and information exchange between government research institutes in building and construction. In 1971 a CIB commission on Water Supply and Drainage for Buildings (CIB W062) was established with a US counterpart commission in 1973. The first position paper published by the US counterpart commission (USNCCIB 1974) identified the foremost national research need for water supply within buildings, namely, the need for a long-range program to develop an improved computational method for the design and evaluation of water service and distribution systems in buildings. Studies had confirmed that in most building types, the application of Hunter's method results in excessive over-design of the system.

From 1971 to the present, CIB W062 has spawned voluminous papers with significant attention toward computational methods to resolve the Hunter method problem. Thomas P. Konen, original member of the U.S. counterpart commission, contributed substantially to the national need for an improved Hunter method. Over a twenty year span, Konen published articles and papers demonstrating a modified Hunter model (Konen and Brady 1974; Konen et al 1976; Konen and Chan 1979; Konen 1980; Konen 1989; Konen 1993; Konen 1995). In 1983, the BOCA Plumbing Code adopted the first major revision of the Hunter method since 1940 based on the proposed modifications of Konen (Galowin 1983). In 1995, the National Standard Plumbing Code adopted further revisions forwarded by Konen based on the performance of low-consumption fixtures (Wagner 1994).

The initial international response to a Hunter re-evaluation came from the United Kingdom, Japan, Brazil, and Sweden (Konen and Gonclaves 1993). The computational approaches to determine design flow rate forwarded by these countries can be categorized as probabilistic or simulation. The probabilistic approach employed a binomial distribution and Poisson distribution based on queuing theory, and the simulation approach employed the Monte Carlo method. Later developments from Brazil employed Fuzzy Logic (Oliveira et al 2009; Oliveira et al 2010) and the Netherlands developed SIMDEUM, a stochastic model based on statistical information on end uses (Pieterse-Quirijns et al 2012).

Outside of the CIB international network, the re-evaluation of Hunter's method has been published in the Journal of American Statistical Association (Connor and Severo 1962), the Journal of American Water Works Association (Chan and Wang 1980), ASHRAE Handbook (ASHRAE 1987), Plumbing Engineer (Konen 1980; Breese 2001), and the Journal of Pipeline Systems Engineering and Practice (Mazumdar et al 2013).

At the 1994 ASPE Convention Robert A. Wistort presented a paper, "A New Look at Determining Water Demand in Buildings: ASPE Direct Analytical Method" (Wistort 1995). In this publication, Wistort describes a procedure for an analytical approach that uses a Normal Distribution to approximate the Binomial Distribution that occurs naturally from the "on or off" nature of plumbing fixtures. This approximation is valid when n, the number of fixtures, is large, and the approximation becomes questionable when n is small.

There are two advantages to Wistort's approach. First, it eliminates the assumption of congested use by giving the designer freedom to choose T, the time between fixture uses, for themselves. Indeed, Hunter's method has built-in values of T that are based on congested use, and they cannot be modified by the designer. Wistort's approach, on the other hand, requires the designer to input T for each fixture type, and, therefore, the designer is free to choose larger values of T representing less frequent use. This is a double edged sword. While the elimination of the assumption of congested use is desirable due its very conservative nature, the designer must be very careful in the selection of T. Wistort's approach is also advantageous in that it eliminates the need for fixture units. While the concept of fixture units was a clever way for Hunter to combine the flow characteristics of different types of fixtures, it has the disadvantage of being a very rough approximation.

[3] Statistical Analysis

3.1 The Database

The database consists of indoor water use measurements collected by Aquacraft Inc., at over 1,000 single-family homes across the United States between 1996 and 2011. The water use data were recorded with a portable data logger connected to the main water supply pipe in each home. The data logger recorded the volume of water flowing through the main pipe every 10 seconds. The recorded flows were analyzed by Aquacraft using their proprietary Trace Wizard software and disaggregated into individual water use events. Each water use was associated with one of the indoor household fixtures or attributed to a leak.

To facilitate queries, the water use data are stored in MS Access format. As shown in Table 1, the database contains two types of information, namely (i) household survey data and (ii) measured flow data. The household survey data reflect characteristics of the home, the residents and the water fixtures; the measure flow data describe the duration, volume, and number of water use events at each fixture group identified in the home. The maximum data-logging period at each home was 14 days. To capture the diurnal variation in indoor residential water use, results for each fixture in each home were summarized on an hourly basis.

Table 1 Survey and measured water use data in the Access database.

Household Survey Data	Measured Flow Data
- Number of residents and age distribution	- Number of times a fixture was used
- Type of fixtures	- Duration of each fixture use event
- Number of each fixture type/group	- Volume of each fixture use event
- Number of renovated or retrofitted fixtures	- Daily observed fixture peak flow
- Number of bedrooms and bathrooms	- Logging dates

The national distribution of 1058 surveyed households is summarized in Table 2. A small percentage (2%) of homes was dropped from the analysis due to vacations or other conditions that gave zero or minimal water use. The remaining 1038 homes had a total of 2,821 occupants who generated nearly 863,000 water use events during 11,385 home-days of monitoring. On a per household basis, this translate to an average of 11 trace days per home, 2.72 residents per home and 831.4 water use events per home. Figure 1 illustrates the geographic location of the 1038 homes that participated in the national water use survey conducted by Aquacraft during the period 1996 to 2011.

Table 2 Homes surveyed.

Location by State		Number of Homes	Number of Occupants	Survey Years
Arizona,	AZ	17	41	2007 - 2009
California,	CA	447	1326	2006 - 2009
Colorado,	CO	206	533	1996, 2007 - 2010
Florida,	FL	32	78	2007 - 2009
Kentucky,	KY	58	128	2007
Nevada,	NV	20	593	2007 - 2009
New Mexico,	NM	237	44	2010 - 2011
Oregon,	OR	24	66	2007 - 2009
Utah,	UT	17	56	2007 - 2009
Responded to survey		1058	2865	-
Invalidated homes		(20)	(44)	-
Analyzed Total		1038	2821	-

Figure 1 Homes from 62 cities in nine States participated in the national water use survey.

https://www.google.com/search?q=map+of+usa&biw=1524&bih=746&source=lnms&tbm=isch&sa=X&sqi=2&ved=0CAYQ_AUoAWoVChMIt7ziv5f3yAIVRCYeCh0t3gv7#tbm=isch&q=map+of+usa+black+and+white&imgrc=3uVkLykiqZs7cM%3A

Six unique fixtures groups were common to most of the 1038 participating homes (see Table 3). Water use falling outside these six categories was lumped into “Other”. For instance, some homes had evaporative coolers, water treatment devices, or unknown fixtures. In addition, leaks were common. Four of the six common fixtures were further classified as ultra-efficient, efficient, or inefficient based on their average volume of water consumed per use or their average fixture flow rate. The criteria for each classification are defined in Figure 2. The database does not differentiate between kitchen faucets and lavatory faucets; therefore, both are included in the fixture group “faucet”. Bathtubs and faucets are not classified based on efficiency because their intensity, duration and volume of water use depend on the user.

Table 3 Six fixture groups were common to most homes.

Fixture Group	Abbreviation	Number of Homes w/ Group	Number of Fixtures in Group	Average Fixtures per Home
Bathtub	B	519	852	1.64
Clothes Washer	C	1002	1002	1.00
Dishwasher	D	722	728	1.01
Faucets	F	1038	4013	3.87
Shower	S	1014	2132	2.10
Toilet	T	1037	2502	2.41

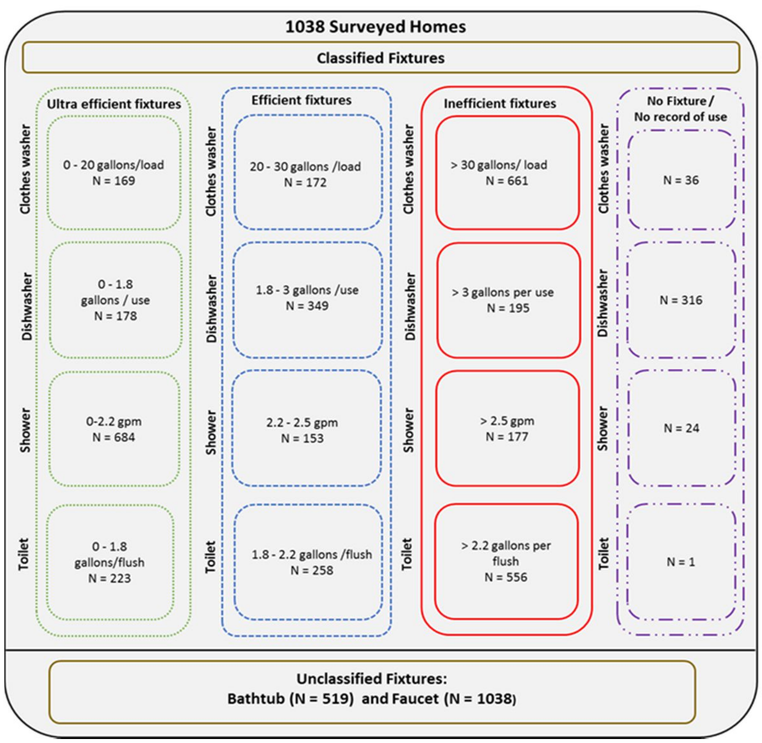

Figure 2 Fixture efficiency criteria and the number of homes within each group.

3.2 Water Use Data

The total volume of water used at each fixture differs due to the fixture function and frequency of use. Table 4 is a breakdown of water use per capita in 1038 households. Toilets had the highest use as gallons per capita daily (GPCD), while dishwashers had the lowest use. The mean daily water use was 60.10 GPCD (72.08 GPCD, including leaks). Nearly 98% of the homes registered some type of leak. Of homes with leaks, 58% had leaks under 10 gallons per day (gpd) while 5% had leaks exceeding 100 gpd. The mean observed leak in a home was 27.2 gpd with a standard deviation of 79.3 gpd. Individual fixture water use as a percentage of the total use is shown in Figure 3. Although leakage accounted for nearly 17% of the volume of daily water use, leaks are not a design factor and, hence, are not considered further in this study.

Table 4 Frequency and volume of water use at 1038 single family homes.

Fixture	Water use events (per capita per day)	Volume (GPCD)
Bathtub	0.08	1.54
Clothes Washer	0.97	14.31
Dishwasher	0.33	0.77
Faucet	22.74	11.87
Shower	0.76	12.59
Toilet	5.80	15.18
Others	9.74	3.84
Leaks	50.11	11.98
Totals (excluding leaks)	40.42	60.10

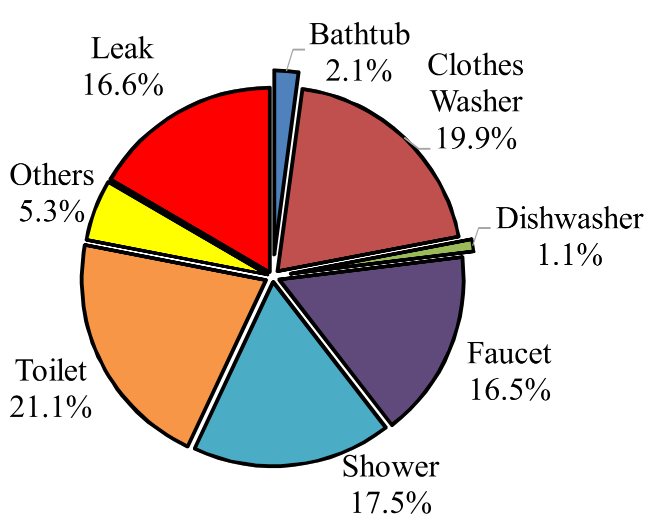

Figure 3 Daily per capita water use at fixtures at 1038 residential households.

3.3 Fixture Flow Rate

Although homes had common fixture groups, the flow rate from each type of fixture in each home differed (See Table A1, Appendix A). The bathtub group had the highest average fixture flow rate (4.47 gpm), while ultra-efficient dishwashers had the smallest average flow rate (0.87 gpm). Within the three fixture groups classified according to volume-based efficiency (see clothes washer, dishwasher, toilet in Figure 2), the fixture average flow rate increased as the fixture efficiency decreased. Similarly, the average volume of water per fixture use increased for all fixture groups as the efficiency of the fixtures decreased (Table A1).

3.4 Probability of Fixture Use

Each fixture group had a different hour of peak use. Shower and toilets had peak use in the morning from 6-8 a.m., clothes washers peaked in late morning, bathtub and faucet use peaked in early evening from 5-6 p.m., while dishwasher use peaked from 8-9 p.m. The maximum hourly probability of a fixture use in an individual home was 0.344 (using Eq. 4.5 in Section 4.2) while the representative maximum hourly probability from grouped homes is 0.089 (using Eq. 4.6 in Section 4.3). When averaged over an entire day, the representative probability of fixture use explained in section 4.3 is less than 1% irrespective of fixture type and fixture efficiency. The clothes washer had the highest probability of fixture use during its peak period followed by the shower, faucet, toilet, bathtub, and dishwasher. Although the hourly probability of fixture use differed by fixture group and fixture efficiency, the efficiency of a fixture does not affect its probability of use. Summary statistics and plots of fixture use probabilities for each group are summarized in Figures A7 through A14 in Appendix A. In what follows, the ultra-efficient and efficient fixture categories are lumped into a single category called “efficient”. The inefficient category is unchanged.

3.4.1 Probability of Fixture Use Per Capita

The relationship between the probabilities of fixture use and the number of residents in homes differed for each fixture (see Figures A7 and A8). There was a similar trend of an increase in probabilities with an increase in the number of residents for homes with 1-5 residents for fixtures like faucets, shower, and toilets. This trend was not observed in homes with 6 or more residents where the small sample size may be a factor in the absence of any detectable relationship in the probability of fixture use and number of residents.

Efficient Fixtures: During the peak hour of each fixture use, the probability of fixture use increased with an increase in the number of residents per household. However, this positive correlation between the number of residents and the hourly probability of fixture use was not evident in the bathtubs and dishwasher.

Inefficient Fixtures: Clothes washer, shower, and toilets had increased probabilities of fixture use as the number of residents in a home increased especially during the peak hour of use. Other inefficient fixtures had no discernable relationship between their hourly probabilities of fixture use and the number of residents in the homes (See Figure A13).

3.4.2 Probability of Fixture Use Per Bedroom

Homes without a bedroom count in their survey response were excluded from this analysis. The sample size of homes with exactly 1 bedroom and greater than 5 bedrooms were less than 2% of the total pool and considered statistically unreliable for this assessment.

Efficient Fixtures: The average number of residents and the fixture count increased as the number of bedrooms in the home increased; however, the hourly probability of fixture use decreased. The correlation between the hourly probabilities of fixture use was present in the toilet, faucet, and clothes washer groups, especially during their peak hours of use.

Inefficient Fixtures: The hourly probability of using toilets and clothes washers was found to be negatively correlated with the number of bedrooms in a home. The use of showers and dishwashers had no correlation with the number of bedrooms.

3.4.3 Probability of Fixture Use Per Bathroom

The sample size for homes with more than 5 bathrooms was less than 2% of the total homes with efficient fixtures, resulting in statistically unreliable results for that group of homes.

Efficient Fixtures: Similar to the bedroom groups, the average number of residents and fixture counts had a positive correlation with the number of bathrooms in a home while the hourly probability of fixture use had negative correlation.

Inefficient Fixtures: Toilets had a negative correlation of probability of fixture use and the number of bathrooms in a home. Other fixtures such as clothes washer, dishwashers, and showers had no correlation with number of bathrooms in a home.

3.4.4 Hour of Peak Water Use

The hour of peak water use at a home can be defined as [i] the hour of maximum water consumption or [ii] the hour with the highest number of water use events. At any given home, these two conditions often occur during the same hour, but their joint occurrence is not assured. The data base was examined to find the hour of peak water use according to both definitions at all 1038 homes. No distinction was made between weekdays and weekends.

The results, depicted in Figure 4, show a similar diurnal distribution of peak water use for both the volume and the number definition. As expected, many homes experienced their peak hour during the morning or early evening. Surprisingly, each hour of the day is represented at least once in the top and bottom plot of Figure 4. This means there is at least one residence in the data base where the hour of peak water use is, for example, consistently at 1 a.m. The 4-hour interval from 6:00 to 10:00 a.m. contains about 50% of the peak hours by volume and 42% of the peak hours by number. The morning rush hour from 7:00 to 8:00 a.m. is the most likely time to experience the hour of peak water use, both by volume and by number of uses.

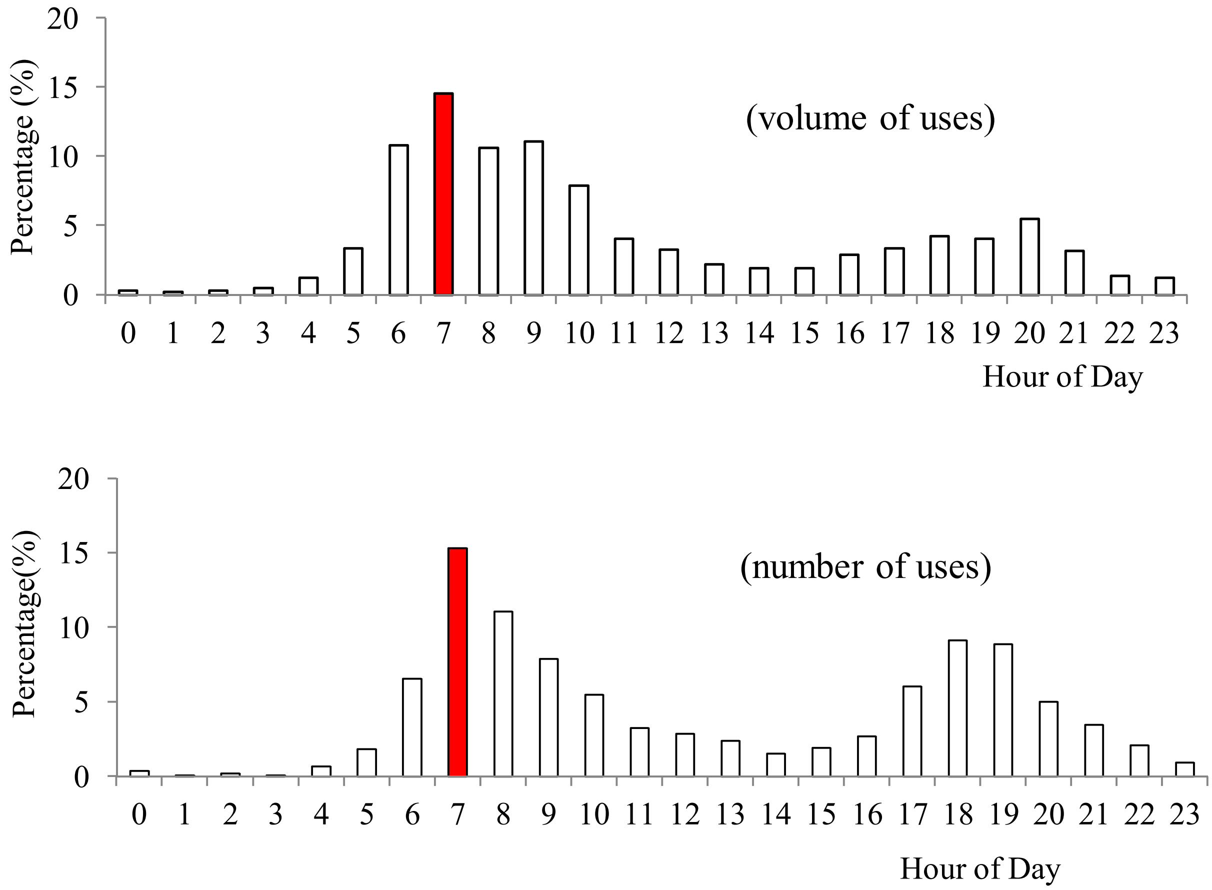

Figure 4 Distribution of observed peak hour in 1038 single family households by volume of water use (top) and number of water use events (bottom).

3.5 Main Assumptions

Following are some key assumptions made in the analysis of the water use data:

1. The flow pattern for a single water use event at any fixture is a rectangular pulse described by two parameters: a constant flow for a fixed duration.
2. Fixtures with identical functions have the same probability of use, irrespective of their location in a building.
3. The kitchen sink and bathroom lavatory are considered to be faucets with the same water use signatures.
4. The plumbing system that exists in each home of the data base is designed properly and operated within typical residential norms.

[4] Method for Estimating

4.1 Estimating Fixture Flow Characteristics

Water use at a fixture generates a flow pulse. To illustrate, Figure 5 shows several hypothetical water use events at a household fixture during a 1-hour window over a 6-day period. The pulse duration is exaggerated. The area of the pulse represents the volume of water used. The pulses on the left side of Figure 5 have unsteady intensities to signify what the data logger may observe. On the right side are the idealized equivalent rectangular pulses derived by setting flow rates at the intensity needed to preserve the volume and the duration of the observed pulse. Two pulses occur on day 2 and again on day 5 during the one-hour monitoring window. Portions of both pulses overlap on day 5, signifying that two identical fixtures operated simultaneously for a brief time in the home. During this period of concurrent water use, the total flow into the home is the sum of the flow rates at the two individual fixtures.

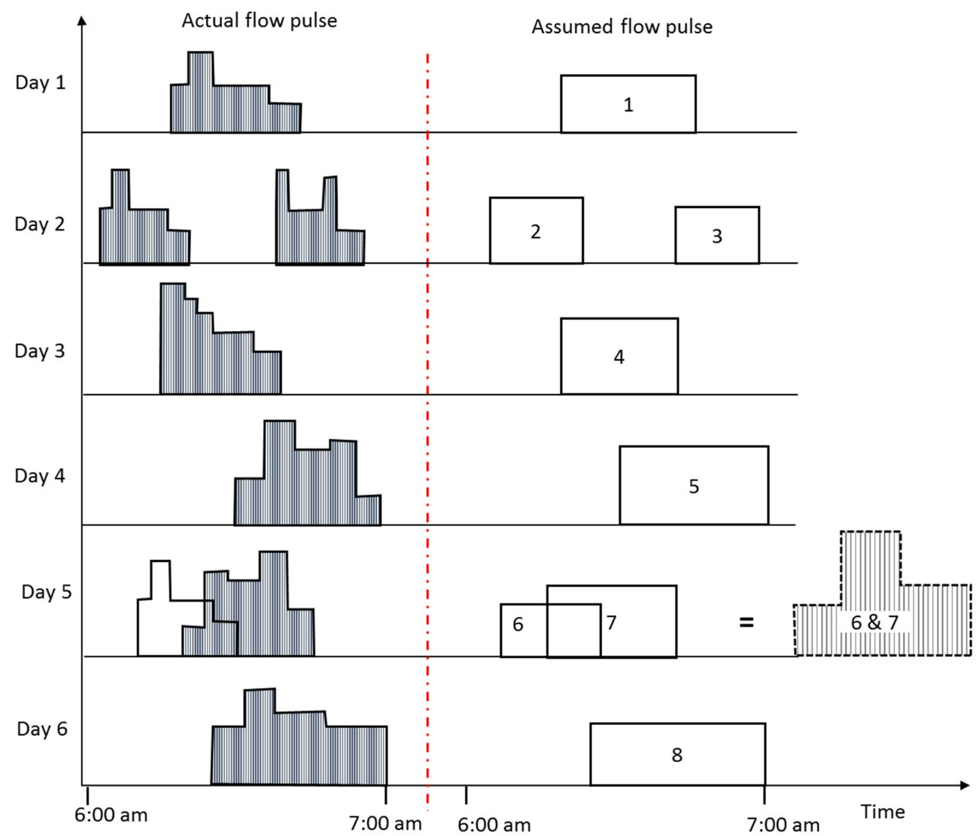

Figure 5 Flow pulses at a fixture during an observation hour for six days. The rectangular pulses have the same duration and volume as their observed shaded companion pulses.

Owing to variations in system pressure, differences in fixture settings, idiosyncrasies in user habits and a host of other factors, the intensity of the rectangular water pulse may vary from use to use at the same fixture. The average rate of flow resulting from water use at a fixture group in a home is computed as the ratio of the total volume used at the fixture to the total duration of time water was flowing at the fixture, or

$$\textit{Average flow rate} = \frac{\textit{Total volume of water used at fixture}}{\textit{Total duration of flowing water at fixture}} \qquad [4.1]$$

This expression is a duration-weighted average of all pulses at a given fixture. To illustrate, Figure 6 depicts graphically how Equation 4.1 obtains the average flow rate for the eight pulses shown in Figure 5. Notice that coincident pulses (6 and 7) do not overlap in the calculation for the average flow rate. An example calculation based on Equation 4.1 for the average flow rate of the eight pulses shown in Figure 6 is presented in Table 5.

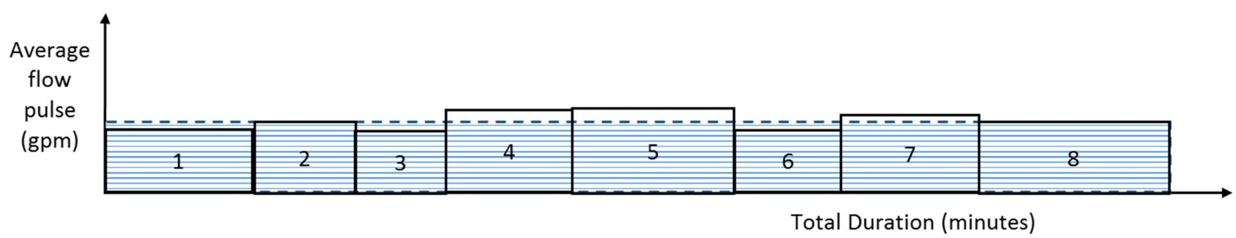

Figure 6 Dashed line is the average flow rate of pulses shown in Figure 5.

Table 5 Example calculation for average flow rate of pulses shown in Figure 6.

Pulse Number	Pulse Intensity (gpm)	Pulse Duration (minutes)	Pulse Volume (gallons)
1	1.81	23.8	43.08
2	2.05	16.1	33.01
3	1.74	13.9	24.19
4	2.25	20.2	45.45
5	2.32	26.1	60.55
6	1.78	17.3	30.79
7	2.12	21.7	46.00
8	2.01	30.2	60.70
Totals:		169.3	343.77
Average Flow Rate at Fixture:		343.77 gallons / 169.3 minutes = **2.03 gpm**	

Other water use characteristics computed for each of the six fixture groups include the average volume per use and average duration per use, given by Eqs [4.2] and [4.3], respectively,

$$Average\ volume\ per\ use = \frac{Total\ volume\ of\ water\ used\ at\ fixture}{Total\ number\ of\ times\ fixture\ was\ used} \qquad [4.2]$$

$$Average\ duration\ per\ use = \frac{Total\ duration\ of\ flowing\ water\ at\ fixture}{Total\ number\ of\ times\ fixture\ was\ used} \qquad [4.3]$$

Note that ratio of the Equation [4.2] to Equation [4.3] also gives the average flow rate at the fixture group as defined in Equation [4.1]. Results of these calculations and other statistical properties of indoor residential water use at the fixture groups are summarized in Appendix A.

Each recorded water use event at a fixture has a corresponding flow duration and flow volume. The relative percentage of fixture use was determined from the total measured data. As shown in Figure 7, water use attributed to leaks accounted for more than half of the 2.5 million water use events and about 70% of time water flowed. However, as summarized in Table 6, the average flow rate of leaks was less than 1% of the demand compared to the other fixtures. Consequently, measured data for leaks and "other" fixture use events were excluded from detailed water use data analysis. Besides, the water supply system for a building is not designed on the basis of anticipated leak events.

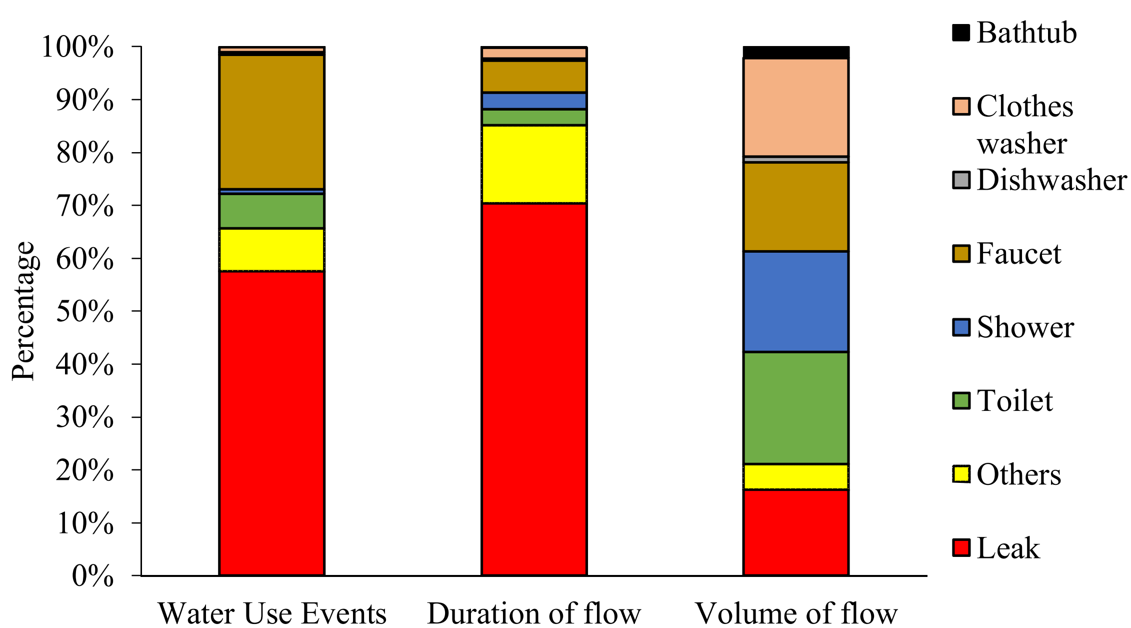

Figure 7 Relative distribution of total measured water use at 1038 households.

(See Table 3 for sample size)

Table 6 Measured and calculated water use for fixtures in 1038 households.

(See Table 3 for sample size)

Data type	Measured at Data Logger (%)			Calculated
Fixture	**Water Use Events**	**Duration of Flow**	**Volume of Flow**	**Average Flow Rate (gpm)**
Bathtub	0.08	0.16	2.15	4.39
Clothes washer	1.03	2.05	18.66	3.06
Dishwasher	0.38	0.35	1.08	1.03
Faucet	25.42	6.08	16.81	0.93
Shower	0.89	3.17	19.01	2.01
Toilet	6.49	3.00	21.22	2.38
Others	8.20	14.83	4.77	0.11
Leak	57.50	70.34	16.31	0.08
Total	100.00	100.00	100.00	

4.2 Estimating Fixture Probabilities (Single Home)

In a 24-hour day, the hourly volume of water consumed in a residence varies. There are hours with a high volume of water consumption when the occupants are waking for a new day or returning from work and school. Similarly, there are hours with a low volume of water consumption when residents are sleeping or away from home. The peak hour of water use is determined from the observed flow pattern as the single hour with the highest average volume of water use in the building (e.g., hour 07 in Figure 8). The possibility that all the fixtures in a building would be busy at the same time is highly unlikely; however, the design building water demand should be representative of water use expected during the peak hour.

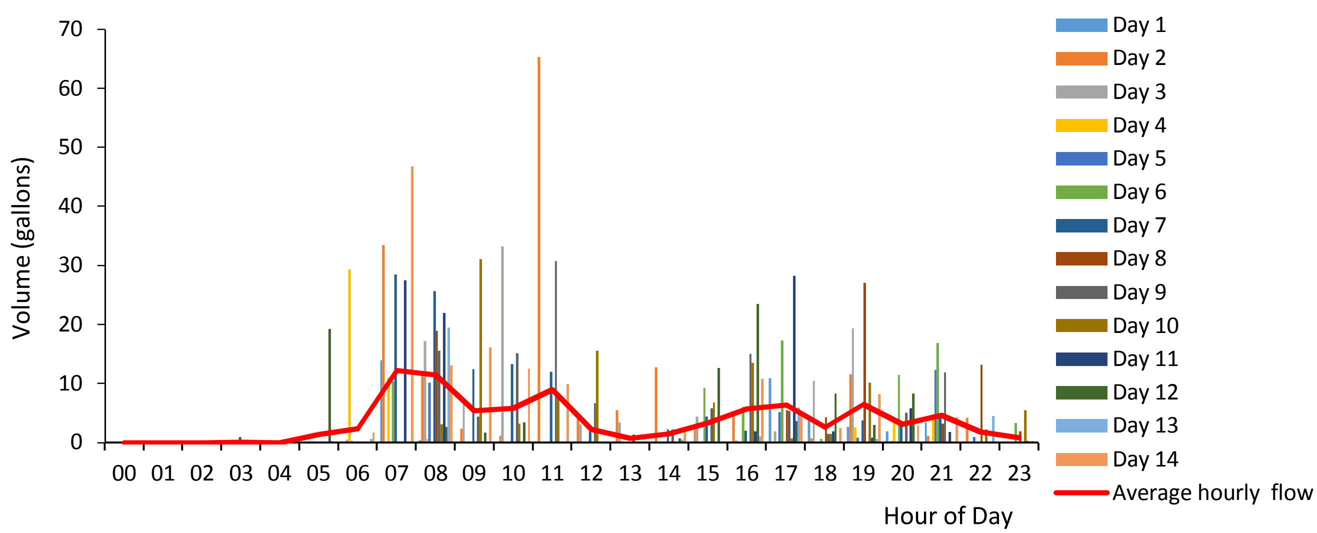

Figure 8 Observed hourly water use in a typical single-family residence over a 2-week period.

A water fixture has two states. It is either on (running water) or it is off (not running water). A fixture that is "on" is said to be busy; one that is "off" is said to be idle. A good relative frequency estimate of the probability that a fixture is busy is given by the percentage of time that water is flowing from that fixture during a set period of observation. To illustrate the basic idea, consider Figure 9 which shows two water use events at a fixture during an observation window of duration *T*. In this case, the probability *p* of a busy fixture is given by,

$$p = \frac{t_1 + t_2}{T} \qquad [4.4]$$

Since residential water use has strong diurnal variability (Figure 8), each day was divided into 24 one-hour observation windows (hence, *T*=60 minutes). This produced 24 hourly estimates of busy fixture probabilities at each home. To account for homes with two or more identical fixtures and a variable data logging period of up to two weeks, the general expression for estimating the hourly busy fixture probability becomes

$$p = \frac{t_1 + \cdots + t_M}{N \cdot K \cdot T} \qquad [4.5]$$

Here, *M* is the total number of water use pulses attributed to a given fixture group with *K* identical fixtures in a single home monitored for a period of *N* complete days. Equation [4.5] reduces to the simple example of Equation [4.4] if *N*=1, *K*=1 and *M*=2. As expected, the hours with the highest water use volumes also produced the highest busy fixture probabilities.

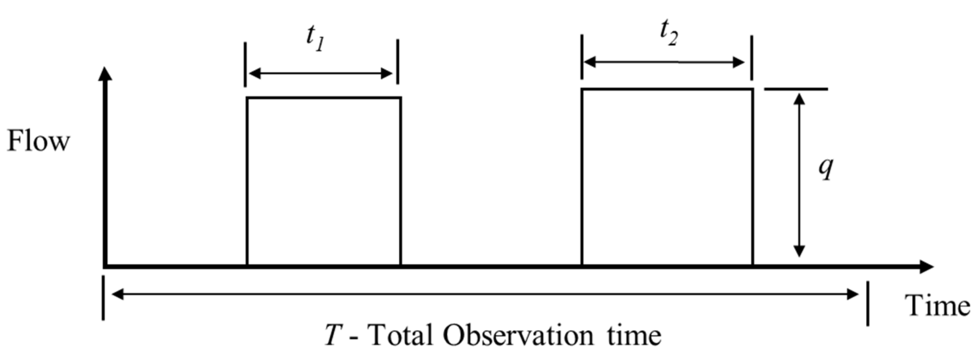

Figure 9 Time intervals needed to estimate the probability of a busy fixture.

4.3 Estimating Fixture Probabilities (Grouped Homes)

The results in Section 4.2 give hourly probabilities of busy fixture use at a *single* home. The water use data base contains hundreds of homes, each with different fixture counts, different monitoring periods ("trace days") and, hence, different hourly p values. How should these estimates of p from single family homes be combined to provide a single representative value?

Homes were first classified into homogeneous categories based on three criteria: number of residents, number of bathrooms and number of bedrooms. To illustrate, suppose there is a group of H homes in a certain category. The corresponding representative hourly value of p from H independent estimates of the probability of fixture use in this category is given by,

$$\bar{p} = \sum_{h=1}^{H} \beta_h p_h \qquad [4.6]$$

where β is a dimensionless weighting factor, $\beta_h = \dfrac{N_h K_h T_h}{\sum_{h=1}^{H} N_h K_h T_h} = \dfrac{N_h K_h}{\sum_{h=1}^{H} N_h K_h}$.

The weighting factor gives greater influence to probability estimates from homes having higher "observation opportunities" (i.e., more fixtures and/or more trace days). The example in Table 7 shows how the weighted average probability of a morning shower (T=60 minutes each day) is found for a group of 5 homes with 3 occupants each. The single home hourly estimates of p computed using Equation [4.5] range from 0.0359 to 0.0892, as shown in Column E. The single representative value is p=0.0526, as the bottom line in Column G. It is the weighted average of the p estimates in Column E.

Table 7 Estimating p for 7-8 a.m. shower use in group of H=5 homes each with 3 occupants.

	A	B	C	D=A*B*T	E = C/D	F= D/Sum(D)	G = E*F
Home ID, h	Shower Fixture Count, K	Monitor Period (days), N	Duration of Busy Fixture (minutes)	Fixture Observation Window (minutes)	Hourly Probability of Fixture Use, p	Weighting Factor, β	Contribution to Weighted Avg Probability Eq [3.6]
1	3	13	84.0	2340	0.0359	0.2727	0.0098
2	2	13	96.5	1560	0.0619	0.1818	0.0112
3	3	14	98.5	2520	0.0391	0.2927	0.0115
4	1	12	64.2	720	0.0892	0.0839	0.0075
5	2	12	108.0	1440	0.0750	0.1678	0.0126
Total	11	64	451.2	8580	--	1.0000	***p* = 0.0526**

4.3.1 Per Capita Groups

The number of residents in a household is one of the characteristics affecting water use in a single-family home. It is hypothesized that the number of users is positively correlated to the frequency of water use in a building and, thus, on the probability of a single fixture in use. The per capita effect on the hourly probability of a busy fixture was calculated for each fixture group and efficiency in homes with 1, 2, 3, 4, 5, 6 and >6 residents.

4.3.2 Per Bedroom Groups

The size of a home is correlated with the number of bedrooms and possibly with the number of occupants. Depending on the style of the house, an increase in the number of bedrooms implies an increase in the number of certain fixtures (e.g., faucet, toilets, etc.). The effect of the number of bedrooms on the hourly probability of fixture use was investigated for homes with 1, 2, 3, 4, 5 and >5 bedrooms.

4.3.3 Per Bathroom Groups

A bathroom group consists of a toilet, a sink, and a bathtub with or without a shower. There is at least one bathroom group in every home. The effect of bathroom groups on the hourly probability of fixture use was investigated for homes with 1, 2, 3, 4, 5 and >5 bathrooms.

4.3.4 Design "p" Value

As shown in Figure 8, water use in a single-family home has a pronounced diurnal pattern. The design "*p*" value is the probability of a busy fixture expected during the peak hour of water use in a building. Depending on the age distribution and occupation of the residents in a home, the hour of peak use can vary from home to home and from fixture to fixture. The peak hours for each home in the database were identified and ranked to determine the most likely peak hour in residential buildings and to focus subsequent water use analysis on the peak hour.

4.4 Estimating Design Flow

4.4.1 *Hunter's Method*

In 1940 Roy Hunter demonstrated how the binomial probability distribution can be used describe the incidence of busy water fixtures in a building. Given a fixture group of n identical fixtures each with probability p of being used, Hunter showed the probability of having exactly x fixtures operating simultaneously out of n total fixtures has a binomial mass function,

$$b(x;n,p)=\binom{n}{x}(p)^{x}(1-p)^{n-x} \qquad x=0,1,\ldots,n \qquad [4.7]$$

Most buildings have a wide assortment of water fixtures. Each fixture group has their own unique values for n and p and, hence, their own distinct version of Equation 4.7. How can the various binomial models be combined to give a single expression for busy fixtures needed in order to estimate the design flow? Hunter recognized that it was not legitimate to simply add the peak flows from each fixture group.

Unfortunately, there is no exact solution to this problem (Butler, 1993). In a clever move, Hunter introduced *fixture units* to effectively collapse the 99th percentile of the binomial mass function to a single curve of peak flow estimate dependent only on the fixture units. The final result, called Hunter's Curve, is the theoretical basis for plumbing codes around the world (Buchberger et al., 2012; IAPMO, 2015).

The most important input for Hunter's method is total fixture units. A chief drawback with fixture units is that they lack an intuitive physical basis. When extending Hunter's Curve to include new fixtures, a big challenge is to find suitable values for the fixture units. Nonetheless, Hunter's fixture unit concept offered an expedient and effective compromise in an era when computations were performed on slides rules and there was an urgent need to develop uniform standards for premise plumbing.

4.4.2 *Wistort's Method*

In 1994 Robert Wistort proposed using the normal approximation for the binomial distribution to estimate peak loads on plumbing system (Wistort, 1994). Similar to Hunter's approach, the number of busy fixtures x is considered to be a random variable with a binomial distribution having a mean $E[x] = np$ and variance $Var[x] = np\ (1-p)$. From the normal approximation, the estimate of the 99th percentile of the flow in the building is

$$Q_{0.99} = \sum_{k=1}^{K} n_k p_k q_k + \left(z_{0.99}\right)\sqrt{\sum_{k=1}^{K} n_k p_k \left(1 - p_k\right) q_k^2} \qquad [4.8]$$

In this expression, n_k is the total number of fixtures in fixture type k, p_k is the probability that a single fixture in fixture type k is operating, q_k is the flow rate at the busy fixture type k and $z_{0.99}$ is the 99th percentile of the standard normal distribution. Besides providing a direct analytic estimate of the design flow, the significant advantage of Wistort's direct method is that it avoids the fixture unit dilemma. In addition, this approach is readily extended to other types of fixtures operating during both congested and non-congested conditions, provided suitable values for p and q are available.

The primary caution with Wistort's method is the behavior at the tails of the probability distribution. The binomial distribution applies to the integers and is bounded ($0 \leq x \leq n$), whereas as the normal distribution applies to the real numbers is unbounded. A poor fit at the tails of the binomial distribution can lead to inaccuracies when estimating the 99th percentile, the nominal standard for design purposes.

In the context of premise plumbing, where p values tend to be small, the normal approximation as defined by the Wistort method in Equation [4.8] will provide accurate results when the condition $np > 5$ is met (Walpole et al., 1998). This corresponds to the region above the blue line in Figure 10. Results from the national database of 1038 single family homes, however, suggest that most households will lie below the blue line in the shaded yellow square defined by the boundaries ($0.01 \leq p \leq 0.10$) and ($1 \leq n \leq 25$).

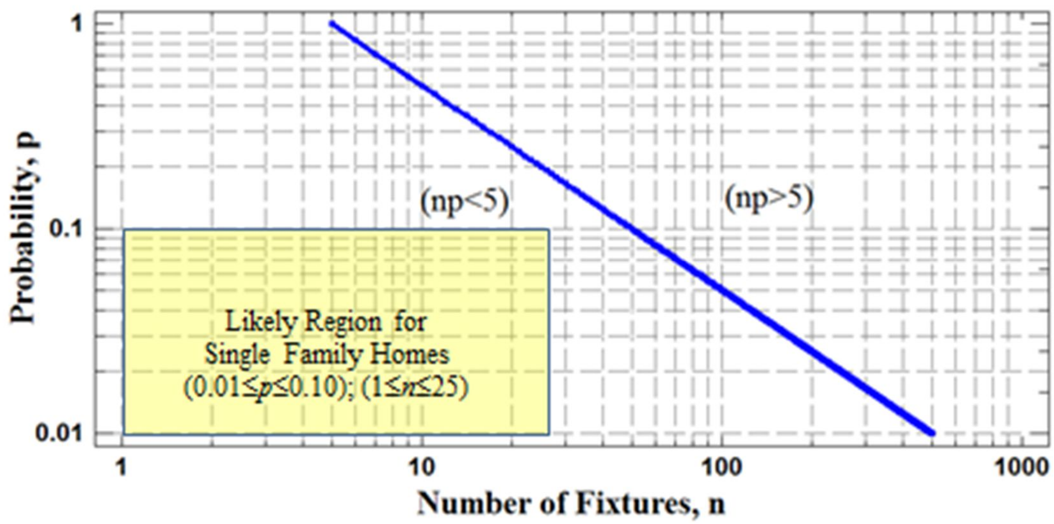

Figure 10 The Wistort method works well in the region above the blue line where $np>5$.

4.4.3 *Modified Wistort Method (Zero-Truncated Binomial Distribution)*

Both Hunter and Wistort focused mainly on water demands in large buildings with high fixture counts. Hunter assumed that during peak periods in public buildings, fixtures would experience congested use (*i.e.*, a queue has formed to use a water fixture). Under these conditions, it was virtually certain that at least one fixture in the building would be drawing water at any instant during the peak period. This convenient premise effectively pulls the distribution of busy fixtures away from the lower binomial boundary of zero. In such cases, the Wistort method for estimating peak flow will work well.

Complications with the binomial model arise in single family homes and other small scale dwellings with few people and few fixtures. In these cases, idle fixtures are the norm even during the period of peak use. The high probability of idle fixtures in the single family home exerts a strong "downward pull" on the mean of the busy fixtures which, in turn, leads to a significant low bias in the estimated peak flow.

To resolve this dilemma, we propose to use a *zero-truncated binomial distribution* (ZTBD) to describe the conditional distribution of busy fixtures in any building including single family homes. As shown in Figure 3.11, the ZTBD (**blue graph**) arises from the parent binomial distribution (**red graph**) by truncating the probability mass at x=0 and rescaling the remaining blue mass spikes to ensure the conditional ZTBD sums to one.

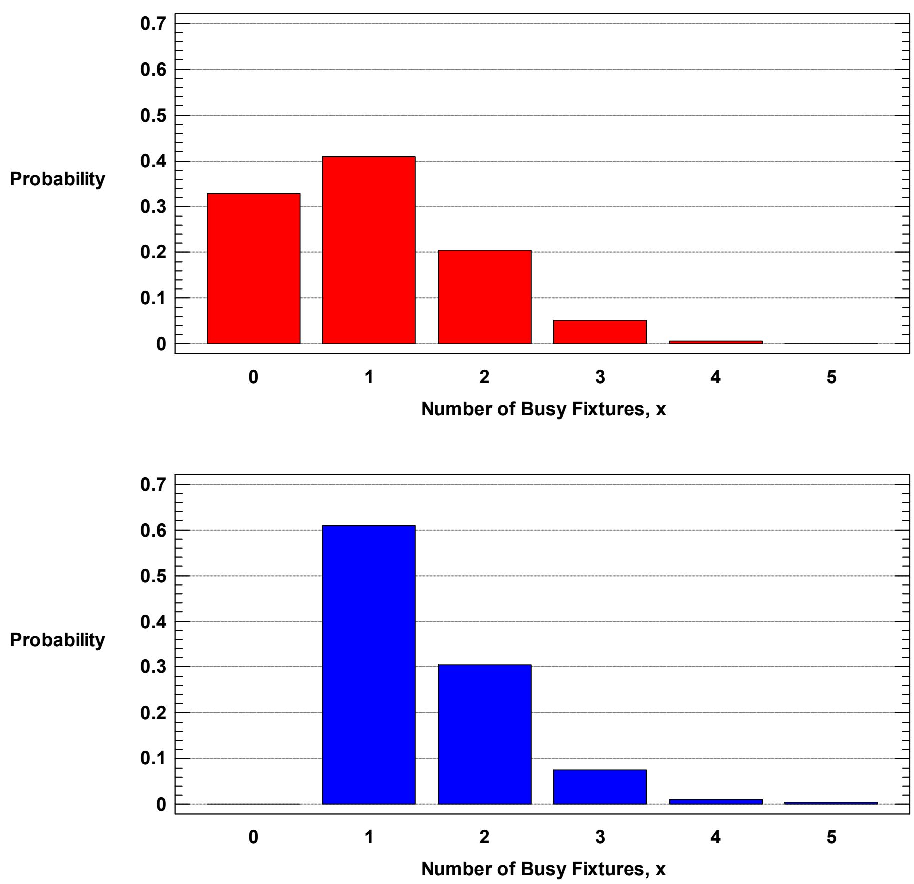

Figure 11 Probability distribution of busy fixtures in a group of n=5 fixtures, each with probability p=0.20 of being in use. Upper (**red**) graph is the standard binomial distribution. Lower (**blue**) graph is the zero-truncated binomial distribution. Since operation of premise plumbing implies running water, only the **blue** graph is appropriate for design purposes.

The theoretical mean and variance of the ZTBD are summarized in Table 8. When $np > 5,\ P_0 \to 0$ and **blue** case 2 reduces to **red** case 1, as expected. In practice, this transition typically requires at least 100 fixtures in the building. The results in Table 8 are for one fixture group; most buildings include a variety of fixture groups. The moments shown in Table 8 can be written to account for any arbitrary combination of fixture groups. Hence, the ZTBD can provide the missing link needed to extend Wistort's method across the full spectrum of application from large public buildings to small private dwellings. For this reason, the ZTBD approach is also referred to as the "Modified Wistort Method".

Table 8 Mean and variance of the number of busy fixtures in a building with *n* identical fixtures, each with probability *p* of being in use.

Case	Probability Model	Mean	Variance	Comment
red 1	binomial distribution	np	$np(1-p)$	Suitable for sizing premise plumbing in large buildings where stagnation during peak periods is unlikely.
blue 2	zero-truncated binomial distribution	$\frac{np}{1-P_0}$	$\frac{np(1-p)}{1-P_0} - P_0\left(\frac{np}{1-P_0}\right)^2$	Suitable for sizing premise plumbing in any building, including single family homes, where stagnation during peak periods is likely.

Note: $P_0 = (1-p)^n$ is probability of stagnation; P_0 is the mass spike above $x = 0$ in **red** graph of Fig 3.11.

Assuming that a normal approximation can be used to describe the upper tail of the ZTBN, the modified Wistort method for multiple fixture groups in a single building is,

$$Q_{0.99} = \frac{1}{1-P_0}\left[\sum_{k=1}^{K} n_k p_k q_k + (z_{0.99})\sqrt{\left[(1-P_0)\sum_{k=1}^{K} n_k p_k (1-p_k) q_k^2\right] - P_0\left(\sum_{k=1}^{K} n_k p_k q_k\right)^2}\,\right] \qquad [4.9]$$

Consider the last fixture on the water supply line in the building. This corresponds to *n*=1 and *k*=1. For this case, Equation [4.9] simplifies to $Q_{0.99} = q$. The design flow is simply the nominal demand of the final fixture. An example is presented in Section 4.5 to demonstrate application of Equation [4.9] to estimate peak water demands a typical single family home.

4.4.4 *Universal Dimensionless Design Equation*

A water fixture cannot discern if it is in a home, a school, a hospital, an office, a restaurant, etc. It is hypothesized, therefore, that a universal design chart for peak water demands will exist if formulated with the proper collection of dimensionless terms. As a starting point, Equation [4.9] can be written in dimensionless form as follows,

$$Q^*_{0.99} = \frac{Q_{0.99}}{\overline{q}} = H(n,p)\left[1+(z_{0.99})\sqrt{\Theta_q^2 - P_0\left(1+\Theta_q^2\right)}\right] \qquad [4.10]$$

where $$\overline{q} = \frac{\sum_{k=1}^{K} n_k p_k q_k}{\sum_{k=1}^{K} n_k p_k} \qquad [4.11]$$

and $$H(n,p) = \frac{\sum_{k=1}^{K} n_k p_k}{1-P_0} \quad with \quad P_0 = \prod_{k=1}^{K}\left(1-p_k\right)^{n_k} \qquad [4.12]$$

and $$\Theta_q = \frac{\sqrt{\sum_{k=1}^{K} n_k p_k \left(1-p_k\right) q_k^2}}{\sum_{k=1}^{K} n_k p_k q_k} \qquad [4.13]$$

In Equation [4.11], $\overline{q}$ is the overall probability weighted mean flow rate at a busy fixture in the building. In Equation [4.12], $H(n,p)$ is the dimensionless "Hunter's Number", representing the expected number of busy fixtures in a building given that at least one fixture is on (*i.e.,* flowing water). The term P_0 represents the probability that all fixtures in the building are off. In Equation [4.13], Θ_q is the coefficient of variation of the flow at all the fixtures in the building, including the zero flow condition. Work is needed to determine whether Equation [4.10] can be formulated as a unique universal dimensionless design expression.

4.5 Example of Estimating Peak Water Demand in a Typical Single Family Home

The proposed modified Wistort method is applied to a typical single family home shown in Figure 12. The home has 11 indoor fixtures. Key parameters (*n*, *p*, *q*) required for estimating the peak water demand are listed in Table 9.

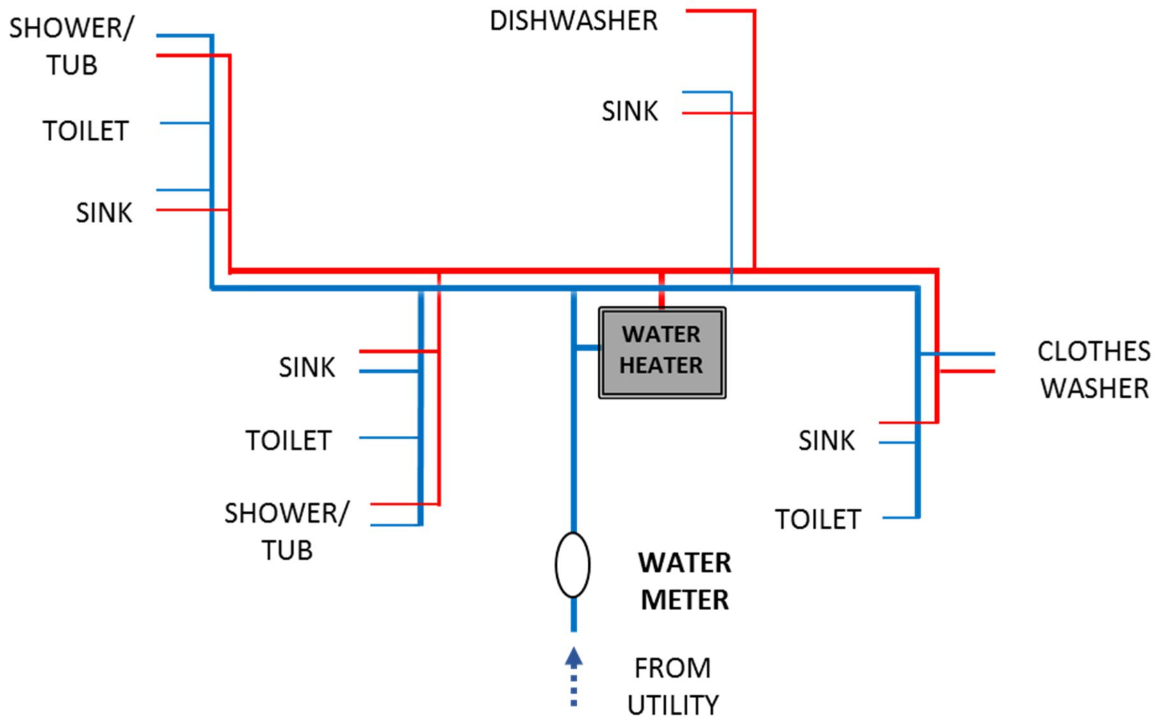

Figure 12 Typical single family home with 2.5 bathrooms.

Table 9 Fixture parameters for the 2.5 bath home shown in Figure 12.

[1] **Fixture Group**	[2] **Fixture Count, *n***	[3] **Probability of Use, *p***	[4] **Design Flow *q* (gpm)**	[5] **Hot Water Fraction**
Shower/Bathtub Combo	2	0.030	4.6	100%
Clothes Washer	1	0.050	4.5	100%
Dishwasher	1	0.005	1.6	100%
Kitchen Faucet	1	0.025	2.2	100%
Lavatory Faucet	3	0.025	1.5	100%
Shower (see Combo)	0	0.025	2.0	100%
Bathtub (see Combo)	0	0.005	7.0	100%
Toilet (1.28 gpf)	3	0.010	4.0	0
TOTAL	11			

The values for the fixture count n are readily obtained from Figure 12. Values for the probability of fixture use p and for the design flow rate q are obtained from the summary Table 15 in Chapter 6. Neither the shower nor the bathtub is listed separately. Instead, both appear jointly as a shower/tub combination. Some of the fixtures are located in a common room where they can be readily accessed by a single user. For instance, the bathroom has four fixtures in close proximity: lavatory faucet, toilet and shower / bathtub combination. Similarly, the kitchen as two fixtures in close proximity: dishwasher and kitchen faucet. Since some or all of these fixtures are often activated in succession by a single user, these fixtures are lumped together and treated as a "Room Group".

The room group has its own characteristic probability of use p and its own characteristic design flow q which are derived from the basic properties of the member fixtures. The room group is considered to be on if at least one of its member fixtures is busy. For instance, the probability that the kitchen group is on corresponds to the probability that only the dishwasher is running, plus the probability that only the kitchen faucet is running, plus the probability that both the dishwasher and the faucet are running. As shown in Table 10, the probability of having a room group in use is estimated as one minus the probability that all fixtures in the room are off. The individual fixture probabilities of use are taken from Table 9.

Table 10 Determining the probability of use for room groups.

Room Group	Fixtures in Room	Probability of Room Group in Use, p	Design Flow, q
Kitchen Group	[1] Dishwasher [2] Kitchen Faucet	$p = 1 - \left[(1 - p_{[D]})(1 - p_{[K]})\right]$ $p = 1 - \left[(1 - 0.005)(1 - 0.025)\right]$ $p = 1 - 0.970125 = 0.029875 \rightarrow 0.030$	3.8 gpm
Bathroom Group	[1] Lavatory Faucet [2] Toilet [3] Shower/Bath Combo	$p = 1 - \left[(1 - p_{[L]})(1 - p_{[T]})(1 - p_{[C]})\right]$ $p = 1 - \left[(1 - 0.025)(1 - 0.010)(1 - 0.030)\right]$ $p = 1 - 0.9362925 = 0.0637075 \rightarrow 0.065$	7.0 gpm

With only two fixtures in the kitchen group, the design flow for this room is given as the sum of the demands for the dishwasher (1.6 gpm) and the kitchen faucet (2.2 gpm) yielding a total of 3.8 gpm as given in Table 10. The bathroom group has four fixtures: faucet, toilet, shower and bath. The design flow for this room group is obtained as the 99th percentile of the flow distribution that arises from all possible combinations of the busy room fixtures. The 99th percentile is selected to be consistent with the standard set by Hunter in his 1940 analysis of loads on plumbing systems. More work is needed to determine if this threshold is the most appropriate standard for a room group.

If all four fixtures in the bathroom were operated independently, this would lead to 2^4 = 16 different flow possibilities, including the zero demand case. However, the shower and tub are operated as a combination fixture. This implies that at most only one of the two can be running at any instant and, hence, operation of the shower fixture and tub fixture are considered mutually exclusive. Since the bathtub and the shower cannot run simultaneously, this reduces the number flow possibilities from 16 to 12 as listed in Table 11. In Table 11, Case 1 is the zero flow condition. It corresponds to the situation where all the bathroom fixtures are idle. Assuming the fixtures are operated independently, the probability of this case is found as follows,

$$\begin{aligned}P_0 = \Pr[0\ busy\ \ fixtures] &= \Pr[L\ is\ off] \times \Pr[T\ is\ off] \times \Pr[S\,\&\,B\ combo\ is\ off] \\ &= (1-0.025)(1-0.010)(1-0.030) = 0.936293\end{aligned}$$

This result, shown in Column [4] of Table 11 shows that even during the peak period in a single family home, the water fixtures are much more likely to be idle than busy. To illustrate another scenario, consider Case 2 with the lavatory faucet on and all other bathroom fixtures off. This event is given by,

$$\begin{aligned}\Pr[L\ is\ on] &= \Pr[L\ is\ on] \times \Pr[T\ is\ off] \times \Pr[S\,\&\,B\ combo\ is\ off] \\ &= (0.025)(1-0.010)(1-0.030) = 0.024008\end{aligned}$$

Finally, consider Case 4 with the faucet and shower running simultaneously. This event is,

$$\begin{aligned}\Pr[S+L\ are\ on] &= \Pr[S\ is\ on] \times \Pr[L\ is\ on] \times \Pr[T\ is\ off] \times \Pr[B\ is\ off] \\ &= (0.025)(0.025)(1-0.010)(1.000) = 0.000619\end{aligned}$$

Note that if the shower on, the bathtub must be off since use of these two fixtures is mutually exclusive. The flow of 3.5 gpm, given in Column [3], is the sum of the faucet (1.5 gpm) and the shower (2.0 gpm).

Table 11 Estimating design flow for Bathroom Group as 99th percentile of busy-time flow distribution.

[1]	[2]	[3]	[4]	[5]	[6]
Cases	Fixtures	Flow (gpm)	Probability	Conditional (Busy-Time) Probability	Cumulative (Busy-Time) Probability
1	0	0.0	0.936293	--	--
2	L	1.5	0.024008	0.37685	0.37685
3	S	2.0	0.024131	0.37878	0.75563
4	L+S	3.5	0.000619	0.00971	0.76534
5	T	4.0	0.009458	0.14845	0.91379
6	L+T	5.5	0.000243	0.00381	0.91760
7	S+T	6.0	0.000244	0.00383	0.92143
8	B	7.0	0.004826	0.07576	0.99719
9	L+S+T	7.5	0.000006	0.00009	0.99728
10	L+B	8.5	0.000124	0.00194	0.99922
11	T+B	11.0	0.000049	0.00077	0.99999
12	L+T+B	12.5	0.000001	0.00002	1.00000
Sum			1.000000	1.00000	

Note: L = lavatory faucet; S = shower; T = toilet; B = bathtub

All other cases in Table 11 were computed in a manner similar to the examples provided above. As revealed in Column [3], the cases for this bathroom group are presented in ascending order of the flow rate, from a minimum of 0 gpm to a maximum of 12.5 gpm if the lavatory faucet, toilet and bathtub were running simultaneously.

In Column [5] of Table 11, the zero flow condition is removed and the probabilities in Column [4] are rescaled by the busy-time probability, $1-P_0$. This yields conditional probabilities of fixture use given that *at least one* fixture in the bathroom is running. For example, the conditional busy-time probability of use for the bathroom faucet in Case 2 is

$$\Pr\left[L \middle| q>0\right] = \frac{\Pr\left[only\ L\ is\ on\right]}{1-P_0} = \frac{0.024008}{1-0.936293} = 0.37685 \qquad [4.14]$$

The entries in Column [6] of Table 11 are the running sum of the values in Column [5]. Hence, Column [6] is an empirical cumulative probability distribution for the busy-time water demands that may occur in the bathroom group described in Table 9. For code purposes, the 99th percentile of the busy-time distribution is used as the design flow. As highlighted in Table 11, 7.0 gpm is the 99th percentile of the flows expected in this bathroom group.

As summarized in Table 12, the initial fixture count for the single family home depicted in Figure 12 has been reduced from 11 to 6 by bundling into room groups the collective behavior of water fixtures that operate in a common space. The kitchen group includes the dishwasher and kitchen faucet; the bathroom group includes the lavatory faucet, toilet and shower/bathtub combo. This arrangement will now be used in the Wistort and modified Wistort methods to estimate the peak water demand for the single family home using the parameters in Table 12.

Table 12 Fixture parameters used in the Modified Wistort Method to estimate peak water use in a 2.5 bath home (**bold values** appear in Table 13).

[1] Fixture Group	[2] Fixture Count, *n*	[3] Probability of Use, *p*	[4] Design Flow *q* (gpm)
Shower/Bathtub Combo	0	0.030	4.6
Clothes Washer	**1**	**0.050**	**4.5**
Dishwasher	0	0.005	1.6
Kitchen Faucet	0	0.025	2.2
Lavatory Faucet	**1**	**0.025**	**1.5**
Shower (see Combo)	0	0.025	2.0
Bathtub (see Combo)	0	0.005	7.0
Toilet (1.28 gpf)	**1**	**0.010**	**4.0**
Bathroom Group	**2**	**0.065**	**7.0**
Kitchen Group	**1**	**0.030**	**3.8**
TOTAL	6		

The expression to be solved for the 99th percentile of the peak flow at the single family home is,

$$Q_{0.99} = \frac{\sum_{k=1}^{K} n_k p_k q_k}{1-P_0} + (z_{0.99})\sqrt{\frac{\sum_{k=1}^{K} n_k p_k (1-p_k) q_k^2}{(1-P_0)} - P_0 \left(\frac{\sum_{k=1}^{K} n_k p_k q_k}{1-P_0} \right)^2} \qquad [4.15]$$

A three-step process will illustrate the approach. In step one, the idle probability (P_0) is ignored to get the classic binomial estimates of the mean and variance of the total-time demands. The calculations for step one are shown in Table 13. Data in columns [2], [3], [4], and [5] come from Table 12.

Table 13 Step 1 of 3 for modified Wistort estimate of peak water demand

(Not adjusted for busy time).

[1] Group Number (k)	[2] Fixture / Group Type	[3] Count (n)	[4] Probability (p)	[5] Flow (q)	[6] npq Mean (gpm)	[7] $np(1-p)q^2$ Variance $(gpm)^2$	[8] Idle Probability $Po=(1-p)^n$
1	Clothes Washer	1	0.050	4.5	0.225	0.962	0.9500
2	Lavatory Faucet	1	0.025	1.5	0.038	0.055	0.9750
3	Toilet (1.28 gpf)	1	0.010	4.0	0.040	0.158	0.9900
4	Bathroom Group	2	0.065	7.0	0.910	5.956	0.8742
5	Kitchen Group	1	0.030	3.8	0.114	0.420	0.9700
					***(Σ) 1.327**	***(Σ) 7.551**	***(Π) 0.7776**

*Note: **Σ** = summation of five terms in columns [6] and [7] ; **Π** = product of five terms in column [8]

The bottom line in Column [6] of Table 13 is the sum of the mean demand at each fixture,

$$\sum_{k=1}^{K=5} n_k p_k q_k = 1.327 \; gpm$$

The bottom line in Column [7] is the sum of the variance of demand at each fixture,

$$\sum_{k=1}^{K=5} n_k p_k (1-p_k) q_k^2 = 7.551 (gpm)^2$$

These estimates of the mean and the variance of the flow correspond to the results from the classic binomial distribution. The mean demand is low because the probability of zero flow (idle fixtures) is quite high in the single family residence. The bottom line in Column [8] of Table 13 is the product of the idle probabilities at each fixture,

$$\prod_{k=1}^{K=5}(1-p_k)^{n_k} = 0.7776$$

In step two, the estimated mean and variance of the water demands are adjusted to reflect only the busy-time conditions at the household. From Equation [4.15], the busy-time adjustment for the mean demand is

$$E\left[Q|q>0\right]=\frac{\sum_{k=1}^{K}n_k p_k q_k}{1-P_0}=\frac{1.327}{1-0.7776}=5.967\ gpm$$

From Equation [4.15], the busy-time adjustment for the variance of the demand is

$$Var\left[Q|q>0\right]=\frac{\sum_{k=1}^{K}n_k p_k\left(1-p_k\right)q_k^2}{\left(1-P_0\right)}-P_0\left(\frac{\sum_{k=1}^{K}n_k p_k q_k}{1-P_0}\right)^2$$

$$=\frac{7.551}{1-0.7776}-(0.7776)\left(\frac{1.327}{1-0.7776}\right)^2=33.952-27.684=6.268\left(gpm\right)^2$$

In the third and final step, the upper tail of the normal distribution (with $z_{0.99} = 2.33$) is used to provide an estimate of the 99th percentile of the total water demand at the single family home,

$$Q_{0.99}=E[Q]+(2.33)\sqrt{Var[Q]} \qquad [4.16]$$

Results posted in Table 14 show that the design flow estimate from the modified Wistort method is about 50% greater than the original Wistort method and 34% less than the current fixture unit method.

Table 14 Peak flow estimates for 2.5 bath single family home.

Method	Mean Demand $E[Q]$	Standard Deviation $\sqrt{Var[Q]}$	99th Percentile Design Flow
Wistort's Method	1.327 gpm	2.748 gpm	7.73 gpm
Modified Wistort Method	5.967 gpm	2.504 gpm	11.80 gpm
UPC with 24 Fixture Units			18.0 gpm

[5] General Considerations

5.1 Application

The normal distribution of Wistort's method is well suited to predict the probability of busy fixtures in large plumbing systems, especially when the statistic increasingly pulls away from the lower binomial boundary. The statistic only becomes unreliable as it approaches the binomial boundary of zero where $np<5$. The Modified Wistort method discussed in this report shows promise to extend the lower limit to smaller quantities of fixtures. Continued research and verification needs to be performed in order to determine the improved lower limit threshold.

The p-values and q-values (probabilities and flow rates, respectively) in the binomial equation are the two fixture specific variables required to develop an accurate predictor of system flow rates. The probability describes the user's tendency to activate each fixture, and the flow rate is the rate at which each fixture consumes water.

Database queries examining fixture flow rates for fixtures within the efficient boundary markers (Figure 2) suggested that every fixture within an efficient home approximated the industry standard of an efficient fixture. As a result, the efficient fixture flow rates shown in Table 15 are conservative rates showing either the maximum specified fixture performance or a representative value approximating the 95^{th} and 99^{th} percentile of efficient fixture flow rates from the database. This conservatism is also considered a factor of safety in design estimate.

The maximum hourly fixture probability of use per capita was the best representative p-value to use in the Modified Wistort method as shown in Table 15. There is no significant difference in p-values between per capita and bedroom/bathroom groups. The p-values do not weigh heavily in the modified Wistort equation to significantly impact the results as do the q-values.

Having a table of fixture specific p-values and q-values could be used as a handy reference when implementing the modified Wistort method in a simple Excel spreadsheet. The n-value (number of each kind of fixture) would be specific to the particular design of the dwelling. With these three parameters (n, p, q), an estimated supply demand can be calculated for each part of the water distribution system for the whole dwelling and for each supply branch.

As the water distribution system narrows toward a single end use there is no need for a probability model to estimate the demand. The single end use fixture will determine the demand required for its performance. Where there is a high likelihood of two fixtures to be used simultaneously, such as a kitchen sink and dishwasher, the sum of the fixtures flow rate would determine the water supply demand.

5.2 Effect on System Size

Plumbing systems sized by the current Uniform Plumbing Code are commonly known to be oversized. The divergence between code and actual flow rates becomes even more noticeable as the connected fixture units increase. As a result, pipe and equipment sizes within these larger systems are notably oversized. Material and installation costs are greater than necessary. Pumps do not perform at their scheduled duty points. Energy efficiency drops. Meter-based equipment calibrated for higher flow rates suffer from poor accuracy or no readings at all. Water softener resin beds can experience channeling; a scenario where the velocity through the tank is too slow and water channels through the resin resulting in hard water passing through untreated. The solution to each of these issues is an updated plumbing pipe sizing method.

The modified Wistort method has the potential to resolve these discrepancies and increase energy, material, and performance efficiencies. To validate this sizing system, it would be important to obtain data from several new multifamily buildings with efficient fixtures. The data should be strategically collected to confirm the system flow rate predictions at various key locations throughout the system.

Initially the sizing system will be limited to single and multi-family dwellings; however it is anticipated that this method can be expanded to other applications including healthcare, industrial, and commercial applications. Additional data collection would be required to obtain the correct probabilities and flow rates for each building type. There is also the potential for this sizing system to be programmed into BIM design software used in the construction of buildings. The program would recognize the fixtures connected to a given section of pipe and suggest an appropriate pipe size based on the anticipated probable flow rate. The task group is aware of previous attempts to integrate the current code sizing into such software, however results were observed to be incorrect for individual and small quantities of fixtures.

5.3 Sensitivity Analysis

Three key parameters are needed to apply the modified Wistort method as defined in Equation [4.15]: n, p, q. The fixture count, n, is readily determined from the MEP plans in the construction drawings. Current rendering software commonly used in the AEC industry can provide automatic fixture counts.

The probability of fixture use, p, needs to be provided by the design engineer. Some initial guidance is available in Table 12 and Table 15. Although estimates of p values vary by a factor of ten from fixture group to fixture group (0.005 for bathtub to 0.05 for clothes washer) design values of p are likely to be in a fairly narrow range for any give fixture. The nominal flow rate, q, for efficient fixtures often can be obtained from the manufacturer's literature. Additional guidance on fixture flow rates is given in Table 15. In this digital age of big data and information management, it is possible, practical and prudent to establish a national cloud-based archive of p and q values for every fixture in use today.

There is bound to be uncertainty surrounding selection of the appropriate p and q values, particularly in the early phases of adoption. The modified Wistort equation given in Equation [4.15] is much more sensitive to choice of q than to choice of p. To illustrate, Figure 13 shows how the computed value of $Q_{0.99}$ varies with changes in p (blue line) and changes in q (red line). For example, doubling all the fixture flow rates (q) will effectively double the value of $Q_{0.99}$. In comparison, doubling all the fixture probability of use values (p) will produce a 14% increase in the value of $Q_{0.99}$.

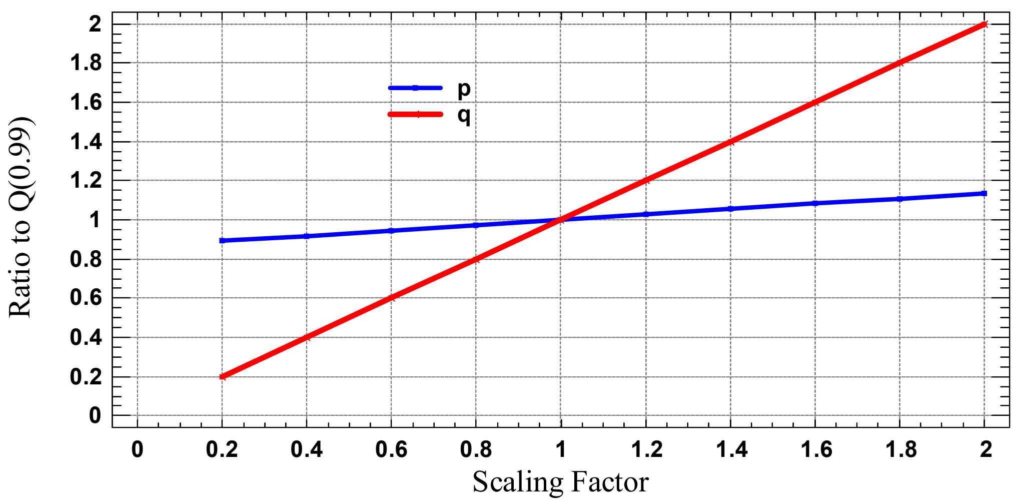

Figure 13 Effect of changing p and q on the value of $Q_{0.99}$ computed from Eq [4.15] for the single family home example presented in Figure 12.

[6] Conclusions and Recommendations

The computational method for estimating water supply demand for single and multi-family dwellings identified in this report as the Modified Wistort Method (MWM) is offered as an improved method to answer the excessive over-design resulting from Hunter's Curve as the current method used in US plumbing codes. This method is effective when the distribution leans toward the lower binomial boundary of zero.

The accuracy of the method is dependent upon the key parameters of fixture use probability and fixture flow rate. As end use of water data improves upon providing the actual characteristics of individual fixture use by the occupants within different types of building as well as displaying the events of simultaneity, so also will the computational model perform with more accurate fixture p-values. The end use of water data used in this report is from the largest available US residential end use of water survey (REUWS) provided by Aquacraft, Inc.

The data requested from Aquacraft's REUWS specified homes having low water consumption fixtures. Although this trimmed a large portion of available data that did not contain efficient fixtures, the sample size was more than adequate to conduct statistical analysis.

The fixture p-values and q-values are sound based upon the data available with the assumptions identified in paragraph 3.5 of this report. This far exceeds the data used in the development of the Hunter method which consisted of a few hotels and government offices.

The modified Wistort method holds promise as an tractable analytical expression to directly estimate the design discharge for a wide spectrum of buildings, ranging from small private dwellings to large public facilities. With proper scaling, the modified Wistort equation can be formulated as a universal dimensionless design expression for determining the peak water demand expected for buildings in the residential, commercial, institutional and other sectors.

A key advantage of the Wistort approach is that it does not rely on mysterious fixture units and it is not calibrated to any particular fixture type. Hence, the dimensionless formulation will remain valid even as water use habits change and fixture types evolve in the future. The modified Wistort method is easily programmed on an electronic spreadsheet, can be offered as a convenient "app" to engineers and inspectors, and is readily incorporated into emerging digital tools (i.e., BIM) of the Architectural-Engineering-Construction industry.

Some specific areas for potential future research include:

- Implement a broad national field program to measure instantaneous peak indoor water demands in the residential sector and at other end users (commercial, institutional, etc).

These data are needed to calibrate and validate the proposed modified Wistort method for estimating peak indoor water demand.

● Analyze data from the national field monitoring program to extend and refine estimates of p and q for a wide range of end uses (residential, commercial, institutional, public, etc).

● Establish a comprehensive "cloud bank" to serve as an on-line digital repository for p and q values for every fixture that is in use today or becomes available in the future.

● Perform detailed in-depth numerical and simulation studies to explore the lower limits of the zero-truncated binomial distribution (used in the modified Wistort method) in order to better understand any limits on its applicability as n gets small ($n \rightarrow 1$) for a wide combination of p and q conditions.

● Investigate the use of other probability distributions (besides the standard normal distribution) as potential candidates to better represent the upper tail of the zero truncated binomial distribution for use in cases where the fixture count n is small.

● Identify an optimal threshold for the design flow to be assigned to a room group (note, the 99^{th} percentile was used in the example presented for the single family home shown in Figure 12). A lower threshold may better accentuate the difference between peak flows expected from hot water demands and from cold water demands.

Expressing the same sentiments as Dr. Hunter, the proof of the adequacy of the proposed method of estimating water supply demand loads expected in residential systems will, in the end, depend on its success in actual trial over a period of years. Yet there is better confidence with better data to recommend the values in Table 15 to be used in the MWM computational model.

Table 15 Recommended design flow rate and *p* value

Fixture	Flow rate (GPM) Data and Specifications	Flow Rate from Database (GPM) 95th and 99th percentile	Recommended Design Flow Rate (GPM)	Design P value
Shower	2.0@80psi [1]	2.4, 2.5	2.0	0.025
Toilet 1.28 gpf		3.1, 3.7	4.0	0.010
Kitchen faucet	1.8@60psi 2.2 @60psi (temp over-ride)[2]	1.4,1.6	2.2	0.025
Lavatory faucet	1.5 @60psi [1]	1.4, 1.6	1.5	0.025
Laundry faucet (with aerator)	2.0@60psi [3]	1.4, 1.6	2.0	0.025
Hi Flow Tub filler – whirlpool type	20.0 @60psi [4]		20.0	0.005
Tub Filler – standard standalone bathtub		6.8, 8.4	7.0	0.005
Combination Tub/shower			4.6	0.030
Dishwasher	1.3@35psi [5]	1.5, 1.6	1.6	0.005
Clothes Washer – vertical axis – heavy load	3.0-4.0@35psi [5]	4.0, 4.5	4.5	0.050
Clothes Washer – horizontal axis - heavy load	3.0@35psi [5]	4.0, 4.5	4.5	0.050
Bathroom Group – Lavatory, toilet, combination T/S			7.0	0.065
Kitchen Group – Sink , Dishwasher			3.8	0.030

[1] EPA WaterSense Specification

[2] From the 2012 Green Plumbing and Mechanical Supplement

[3] American Standard specification sheet for Product No. 2475.550

[4] American Standard specification sheet for Model R800 Deck mount tub filler

[5] Courtesy of the Association of Home Appliance Manufacturers

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Appendices

Appendix A – Exploratory Data Analysis

The 1038 homes analyzed were built as far back as sometime in the early 1920s. Figure A1 is a distribution of the different decades the surveyed homes were built. About 95% of the homes with no record of the year they were built fall into a group coded as "EPA new homes" that were constructed sometime after the year 2000. Therefore, about 68% of the homes analyzed water data were erected sometime before 1990. The distribution of trace days of the surveyed homes after adjusting for days of minimal water use is as shown in Figure A2 with a an average of 11 trace days per home. In addition, the distribution of number of residents in the surveyed homes is in Figure A3.

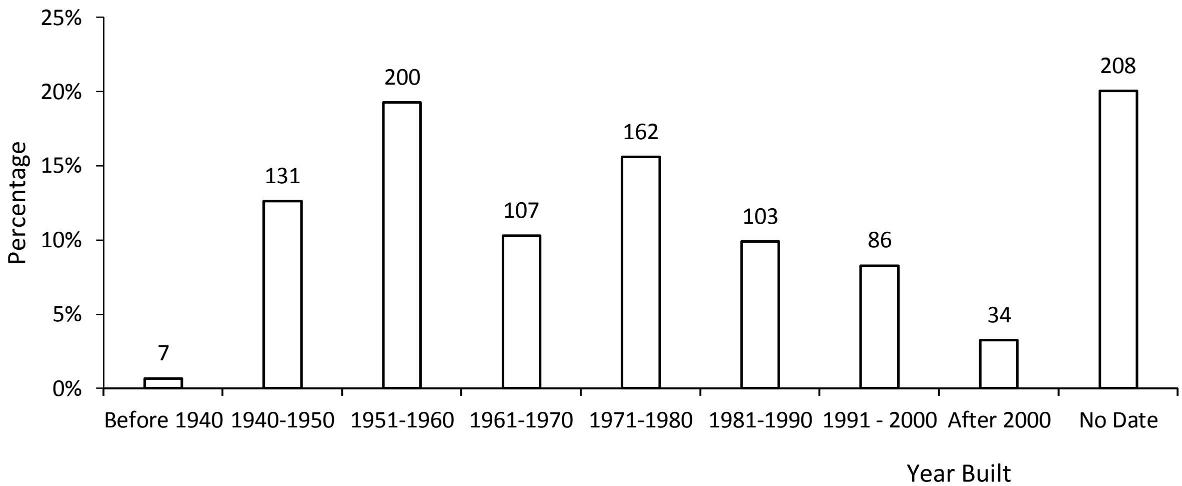


Figure A1: Distribution of surveyed homes with respect to year built

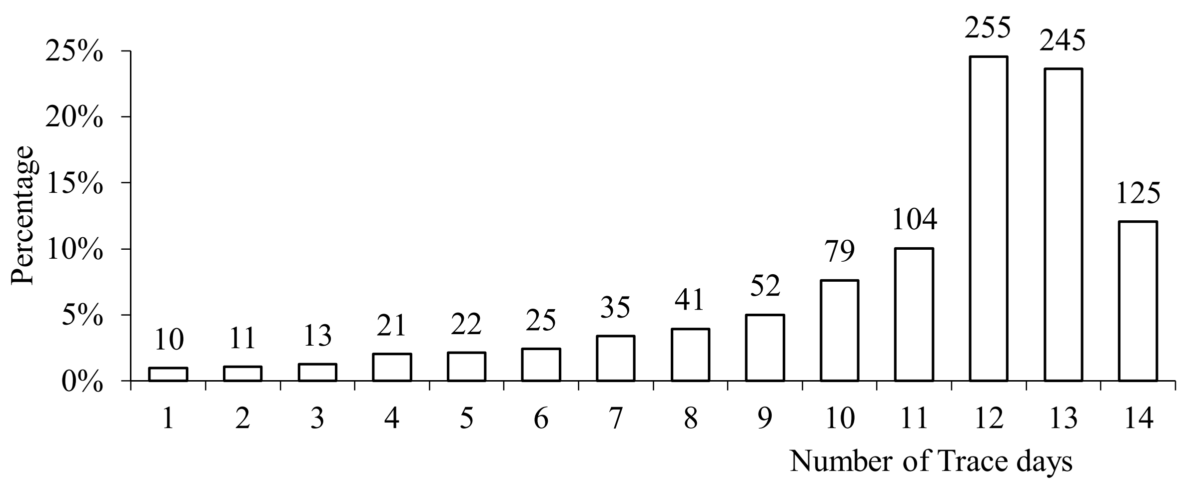


Figure A2: Distribution of valid trace days for 1038 households (Average of 11 days per household)

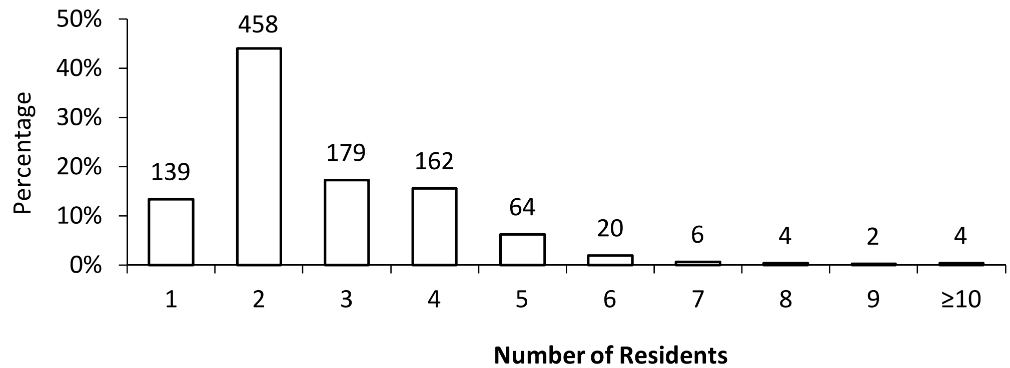


Figure A3: Distribution of occupancy level in 1038 households (Average of 2.72 resident per household)

Box and whisker plots showing some statistical analysis on the distribution of calculated and measured water use details grouped by fixture efficiency. All the fixtures had several outliers for both the measured and calculated water use details especially on the upper end. Other statistical details on flow rate, volume of water used at fixture and duration of fixture use are in Table A1.

Flow rate: Figure A4 shows a wide distribution of flow rates for bathtubs, clothes washer, and toilets. Within each fixture group, the fixture efficiency is inversely proportional to the fixture mean flow rates. Dishwasher and faucets had the lowest flow rate while bathtubs had the highest flow rates.

Volume: In Figure A5, there is a difference between high volume fixtures (bathtubs, clothes washer, and shower) and the low volume fixtures (faucet, dishwasher, and toilets). Similar to the fixture mean flow rate, the fixture efficiency is inversely proportional to the mean volume of water consumed at the fixture.

Duration of use: In Figure A6, the efficiency of the shower was directly proportional to the average duration of shower use (i.e. Homes with inefficient showers had a shorter average bath time than the homes with efficient showers). Only inefficient dishwasher showed a distinct non-overlapping distribution of duration of water use at a fixture compared to other fixtures differentiated by their efficiencies.

Tables A2, A3 and A4 shows average fixture count, average number of residents and the sample size distribution, the different categories of fixture efficiencies in homes grouped by their number of residents, bedrooms and bathrooms respectively. The details in Tables A2, A3 and A4 are the sample distribution for the homes grouped together to determine the hourly probability of fixture use. The hourly probability of fixture use for the different categories of fixture efficiencies in homes grouped by their number of residents, bedrooms, and bathrooms are in Figures A6 – A14. The diurnal pattern of fixture use is especially distinct for the faucets, shower, and Toilets. The statistical details of the 24 hourly probability of fixture use are shown in Table A5.

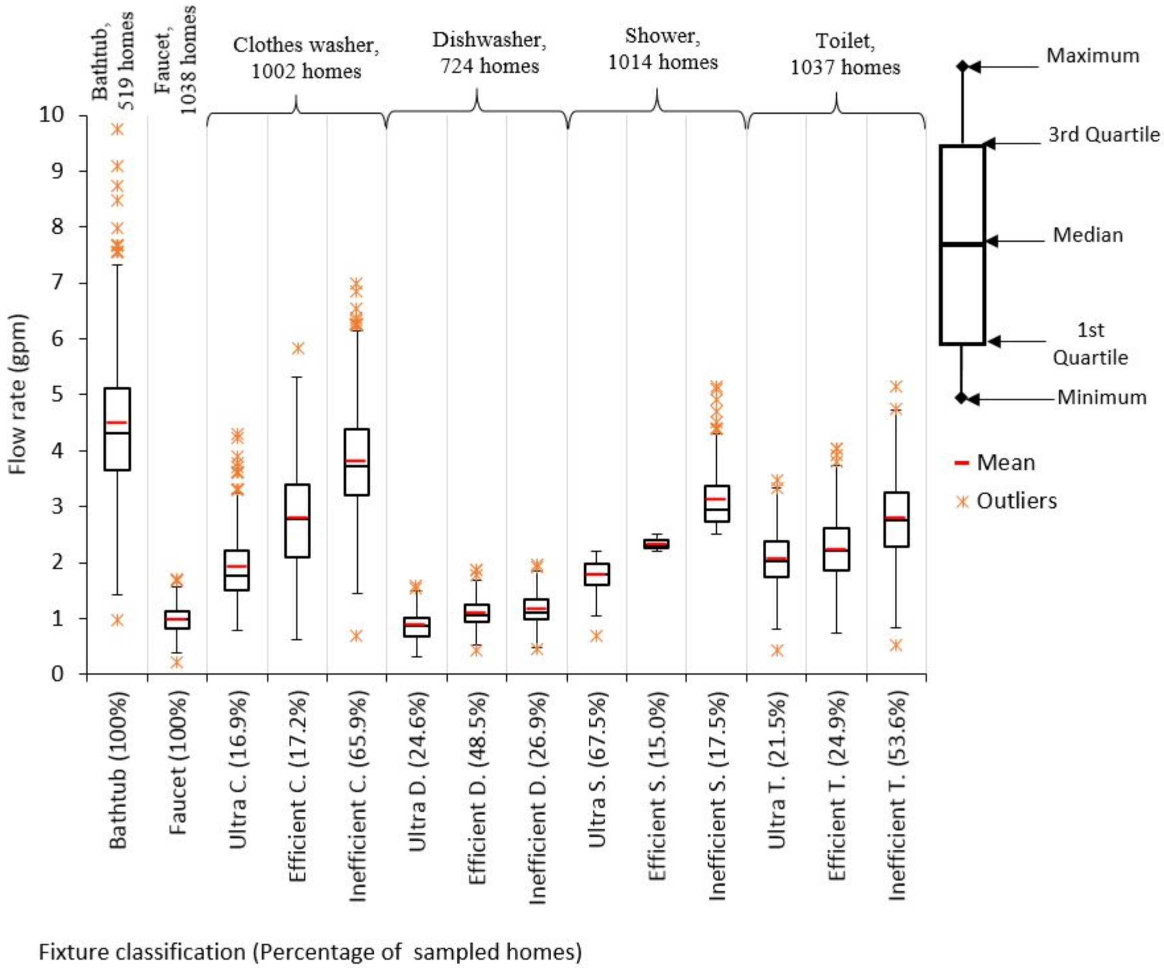


Figure A4: Box and whisker plot of fixture flow rate for each efficiency level

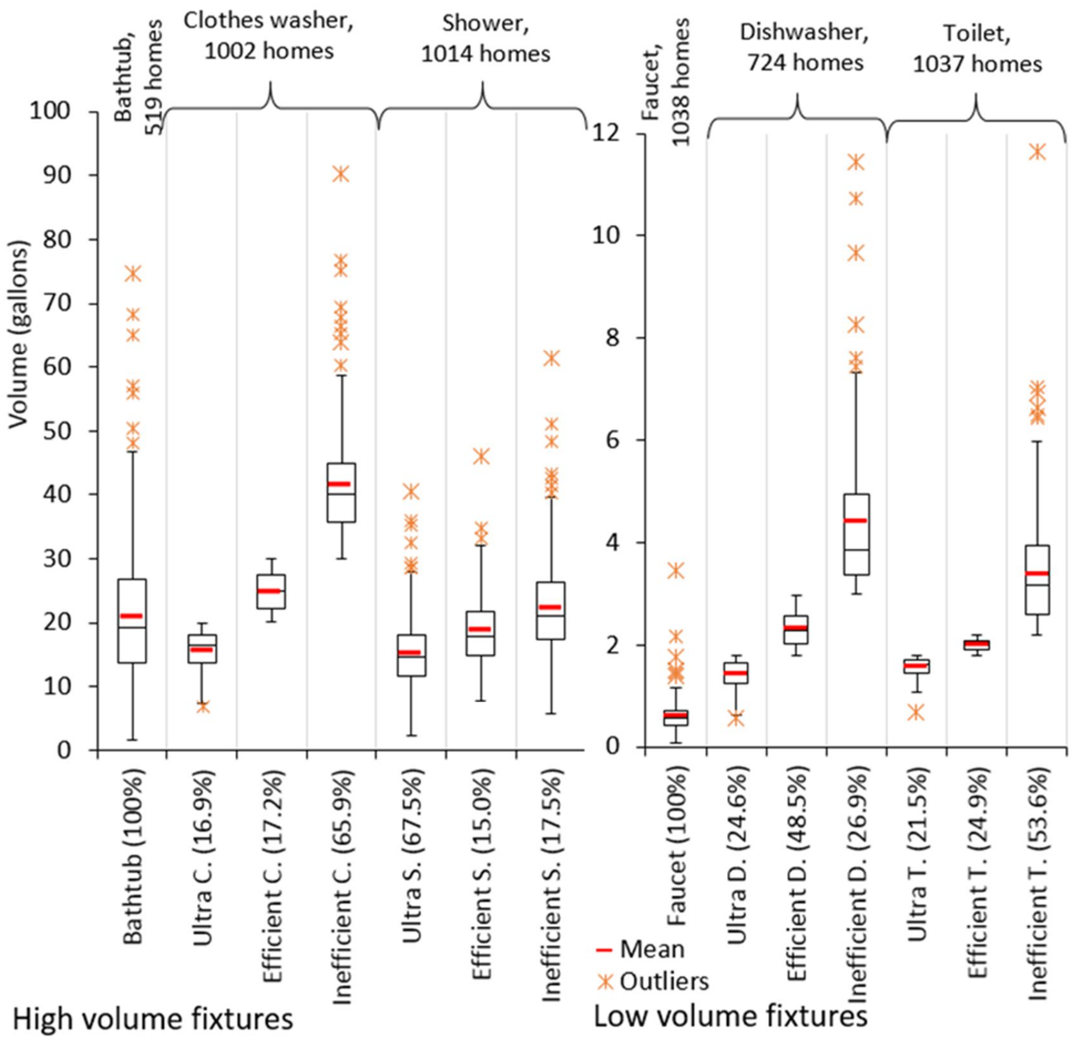


Figure A5: Plot of average daily volume of water use for each fixture efficiency level

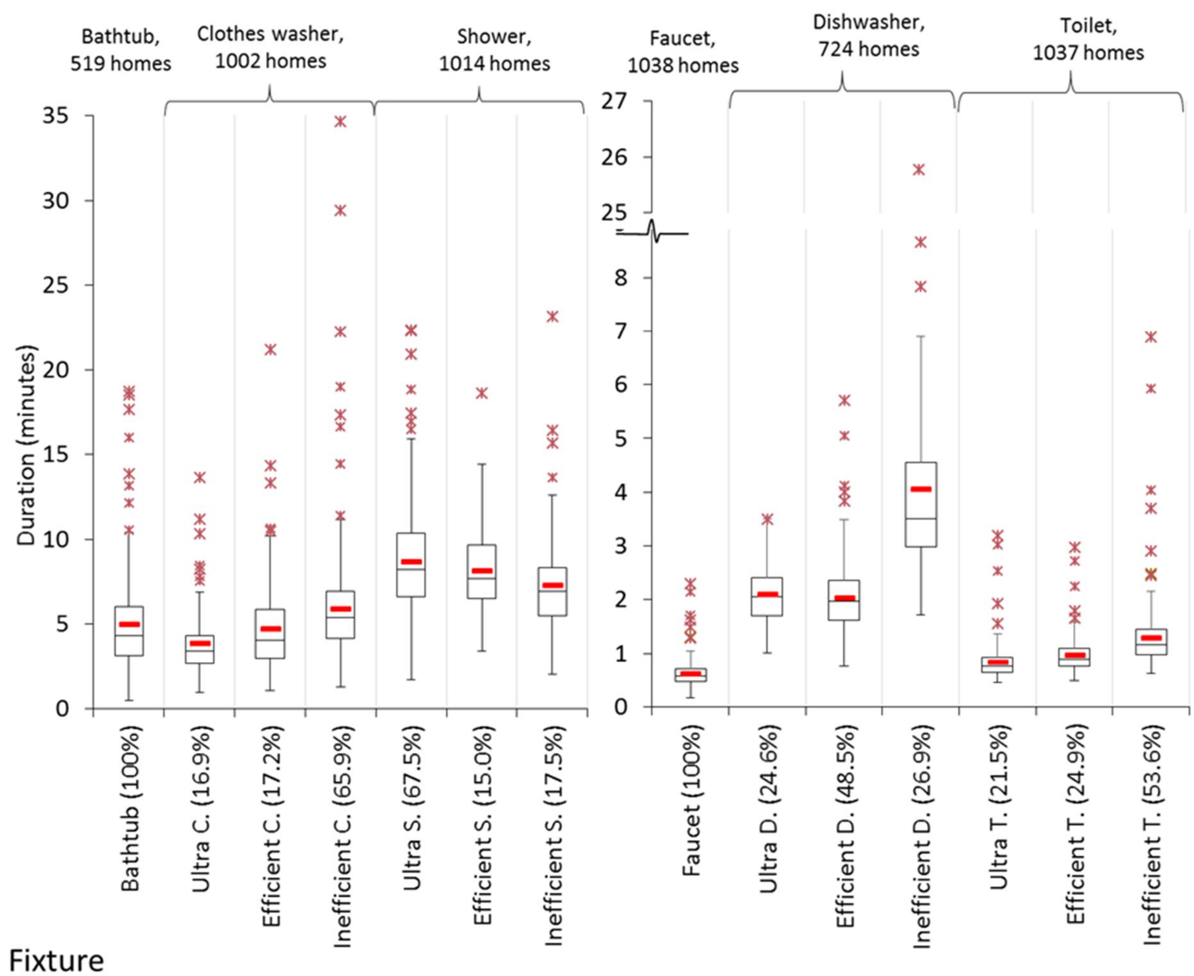


Figure A6: Plot of average duration of fixture use for each efficiency level

Table A1: Statistics on observed duration of flow, volume of flow and flow rates of fixture at various efficiencies

	Duration Per use				Volume Per Use				Flow rate				
Ultra-efficient fixtures													
Fixture	Minimum (minutes)	Maximum (minutes)	Average (minutes)	Standard Deviation (minutes)	Minimum (gallons)	Maximum (gallons)	Average (gallons)	Standard Deviation (gallons)	Minimum (gpm)	Maximum (gpm)	Average (gpm)	Standard Deviation (gpm)	% Homes
Clothes washer	0.94	13.62	3.84	1.85	6.76	19.94	15.82	2.79	0.77	4.30	1.94	0.69	16.9%
Dishwasher	0.75	5.71	1.74	0.53	0.56	1.80	1.43	0.28	0.31	1.58	0.87	0.26	24.6%
Shower	1.69	22.36	8.65	2.92	2.24	40.62	15.18	5.30	0.70	2.20	1.76	0.27	67.5%
Toilet	0.45	3.19	0.84	0.35	0.69	1.80	1.57	0.19	0.43	3.45	2.04	0.53	21.5%
Efficient Fixtures													
Fixture	Minimum (minutes)	Maximum (minutes)	Average (minutes)	Standard Deviation (minutes)	Minimum (gallons)	Maximum (gallons)	Average (gallons)	Standard Deviation (gallons)	Minimum (gpm)	Maximum (gpm)	Average (gpm)	Standard Deviation (gpm)	% Homes
Clothes washer	1.07	21.17	4.70	2.66	20.07	29.92	24.92	3.08	0.63	5.84	2.77	0.88	17.2%
Dishwasher	1.10	5.04	2.20	0.56	1.80	2.98	2.31	0.32	0.41	1.87	1.10	0.24	48.5%
Shower	3.38	18.61	8.11	2.46	7.78	46.10	18.87	5.84	2.20	2.50	2.33	0.09	15.1%
Toilet	0.49	2.98	0.97	0.32	1.80	2.20	1.99	0.11	0.73	4.03	2.23	0.59	24.9%
Inefficient Fixtures													
Fixture	Minimum (minutes)	Maximum (minutes)	Average (minutes)	Standard Deviation (minutes)	Minimum (gallons)	Maximum (gallons)	Average (gallons)	Standard Deviation (gallons)	Minimum (gpm)	Maximum (gpm)	Average (gpm)	Standard Deviation (gpm)	% Homes
Clothes washer	1.26	34.69	5.91	2.98	30.02	90.30	41.57	8.09	0.68	7.00	3.79	0.95	66.0%
Dishwasher	1.72	25.78	4.06	2.22	3.00	11.43	4.40	1.50	0.44	1.95	1.16	0.27	26.9%
Shower	2.03	23.14	7.26	2.64	5.68	61.49	22.56	8.40	2.50	5.17	3.14	0.56	17.5%
Toilet	0.63	6.89	1.29	0.54	2.20	11.65	3.37	1.01	0.52	5.16	2.78	0.75	53.6%
Unclassified Fixtures													
Fixture	Minimum (minutes)	Maximum (minutes)	Average (minutes)	Standard Deviation (minutes)	Minimum (gallons)	Maximum (gallons)	Average (gallons)	Standard Deviation (gallons)	Minimum (gpm)	Maximum (gpm)	Average (gpm)	Standard Deviation (gpm)	% Homes
Bathtub	0.44	18.72	4.99	2.73	1.64	74.60	21.18	10.25	0.96	9.77	4.47	1.22	100.0%
Faucet	0.17	2.29	0.61	0.22	0.07	3.47	0.59	0.26	0.20	1.72	0.97	0.23	100.0%

Table A2: Characteristics of fixtures and homes grouped by the number of residents

Fixtures type & efficiency		No. of residents	1	2	3	4	5	6	>6	Average > 6
Ultra-efficient and efficient fixtures	Clothes washer	Average fixture/home	1	1	1	1	1	1	1	8
		Sample size (341 homes)	8.8%	45.5%	19.1%	17.6%	5.3%	2.6%	1.2%	
		Percentage Ultra Efficient	43.3%	48.4%	55.4%	56.7%	38.9%	33.3%	25.0%	
	Dishwasher	Average fixture/home	1	1.01	1	1	1	1	1.22	9.5
		Sample size (527 homes)	8.5%	44.4%	18.6%	19.0%	5.7%	2.1%	1.7%	
		Percentage Ultra Efficient	28.9%	32.9%	40.8%	35.0%	26.7%	18.2%	33.3%	
	Shower	Average fixture/home	1.82	2.08	2.14	2.30	2.43	2.28	3.38	8.5
		Sample size (837 homes)	11.9%	43.6%	17.0%	16.8%	6.9%	2.2%	1.6%	
		Percentage Ultra Efficient	78.0%	83.0%	84.5%	83.0%	70.7%	77.8%	84.6%	
	Toilet	Average fixture/home	2.02	2.47	2.61	2.66	2.55	2.38	3.38	9.9
		Sample Size (481 homes)	12.9%	43.5%	17.0%	17.3%	6.0%	1.7%	1.7%	
		Percentage Ultra Efficient	51.6%	46.9%	45.1%	39.8%	58.6%	50.0%	25.0%	
Inefficient Fixtures	Fixtures	No. of residents	1	2	3	4	5	6	>6	Average > 6
	Clothes washer	Average fixture/home	1	1	1	1	1	1	1	9.25
		Sample size (661 homes)	15.1%	43.0%	16.8%	15.0%	6.7%	1.7%	1.8%	
	Dishwasher	Average fixture/home	1	1.02	1	1	1	1	1	7.5
		Sample size (195 homes)	12.8%	44.6%	19.5%	12.3%	6.7%	3.1%	1.0%	
	Shower	Average fixture/home	1.58	1.94	2.03	1.81	1.67	3.50	2.33	11
		Sample size (177 homes)	14.7%	46.9%	20.3%	11.9%	3.4%	1.1%	1.7%	
	Toilet	Average fixture/home	1.97	2.23	2.46	2.53	2.80	2.83	3.50	8
		Sample size (556 homes)	13.8%	44.6%	17.4%	14.2%	6.3%	2.2%	1.4%	
Unclassified fixtures	Fixture	No. of residents	1	2	3	4	5	6	>6	Average > 6
	Bathtub	Average fixture/home	1.34	1.57	1.56	1.72	1.95	1.77	2.82	9.5
		Sample size (519 homes)	9.1%	39.9%	17.5%	21.0%	7.9%	2.5%	2.1%	
	Faucet	Average fixture/home	3.37	3.79	4.00	4.10	4.17	4.15	5.06	8.9
		Sample size (1038 homes)	13.4%	44.1%	17.2%	15.6%	6.2%	1.9%	1.5%	

Table A3: Characteristics of fixtures and sampled homes grouped by the number of bedrooms

Fixture type & efficiency		No. of Bedrooms	1	2	3	4	5	>5	No Count	Average > 5
Ultra-efficient and efficient fixtures	Clothes washer	Average Resident/ home	1	2.25	2.58	2.95	3.94	2.83	2.25	6.17
		Average fixture/home	1	1	1	1	1	1	1	
		Sample size (341)	0.3%	11.7%	42.5%	32.6%	10.0%	1.8%	1.2%	
		Percentage Ultra Efficient	100.0%	45.0%	53.8%	53.2%	35.3%	0.0%	25.0%	
	Dishwasher	Average Resident/ home	4	1.98	2.64	2.97	3.76	4	2.76	6.11
		Average fixture/home	1	1	1	1	1	1.33	1	
		Sample size (527)	0.2%	8.2%	40.2%	33.0%	8.0%	1.7%	8.7%	
		Percentage Ultra efficient	0.0%	37.2%	37.7%	33.9%	26.2%	33.3%	19.6%	
	Shower	Average Resident/ home	1.5	2.14	2.51	3	4.05	3.91	2.99	6.09
		Average fixture/home	1	1.57	1.92	2.41	3.08	4.36	2.01	
		Sample size (837)	0.2%	11.6%	41.1%	29.5%	7.2%	1.3%	9.1%	
		Percentage Ultra efficient	100.0%	72.2%	81.4%	84.6%	86.7%	100.0%	78.9%	
	Toilet	Average Resident/ home	1.5	2.12	2.6	2.93	4.3	3.5	2.8	6.125
		Average fixture/home	2	1.73	2.28	2.84	3.47	5.38	2	
		Sample size (481)	0.4%	13.7%	42.0%	31.2%	6.2%	1.7%	4.8%	
		Percentage Ultra efficient	100.0%	47.0%	49.5%	46.0%	30.0%	25.0%	43.5%	
Inefficient Fixtures	Fixture	No. of Bedrooms	1	2	3	4	5	>5	No Count	Average > 5
	Clothes washer	Average Resident/ home	2.67	2.01	2.52	2.94	4.06	4.14	2.86	6
		Average fixture/home	1	1	1	1	1	1	1	
		Sample size (661)	0.5%	12.1%	42.1%	26.6%	5.0%	1.1%	12.7%	
	Dishwasher	Average Resident/ home	1	1.67	2.4	2.81	4.27	0.0	3.16	0
		Average fixture/home	1	1	1.01	1.02	1	0	1	
		Sample size (195)	0.5%	9.2%	46.2%	26.2%	7.7%	0.0%	10.3%	
	Shower	Average Resident/ home	3.0	2.0	2.7	2.8	3.4	2.0	2.0	6
		Average fixture/home	2.0	1.4	1.8	2.2	2.6	4.0	1.7	
		Sample size (177)	1.1%	14.1%	46.9%	23.2%	5.1%	0.6%	9.0%	
	Toilet	Average Resident/ home	3	2.02	2.46	2.96	3.72	3.6	2.78	6
		Average fixture/home	1.5	1.87	2.06	2.61	3.49	3.8	2.47	

	Fixture	No. of Bedrooms	1	2	3	4	5	>5	No Count	Average > 5
		Sample size (556)	0.4%	11.2%	42.1%	25.5%	7.0%	0.9%	12.9%	
Unclassified Fixtures	**Bathtub**	**Avg. Resident/home**	4	2.4	2.68	3.37	4.27	3.75	2.95	6.125
		Avg. fixture/home	1	1.4	1.51	1.79	2.23	2.25	1.32	
		Sample size (519)	0.2%	10.0%	42.8%	29.7%	8.5%	1.5%	7.3%	
	Faucet	**Avg. Resident/home**	2.25	2.05	2.52	2.94	3.97	3.54	2.78	6.08
		Avg. fixture/home	3	3.15	3.55	4.22	5.16	6.15	3.97	
		Sample size (1038)	0.4%	12.3%	42.0%	28.2%	6.6%	1.3%	9.2%	

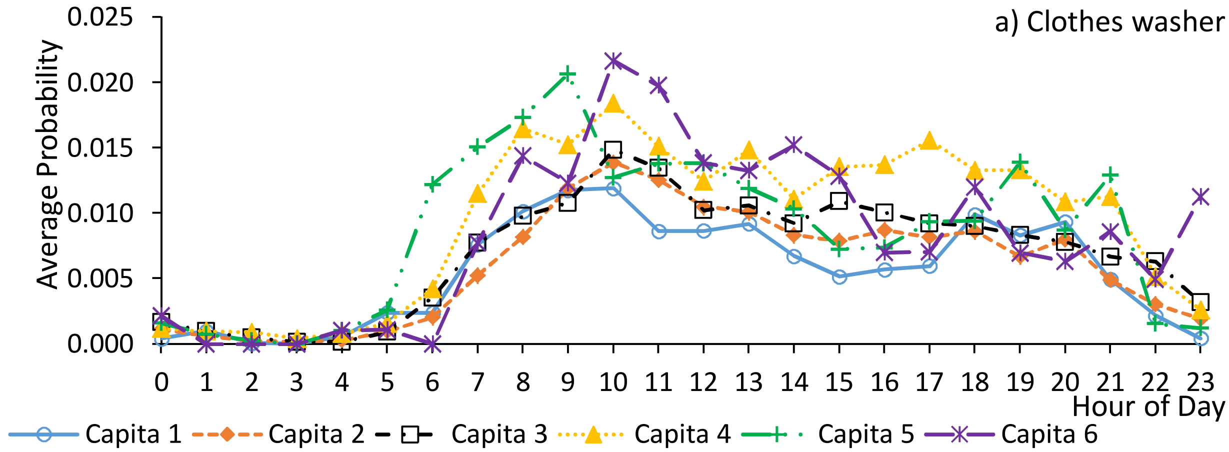
Table A4: Characteristics of fixtures and sampled homes grouped by the number of bathrooms

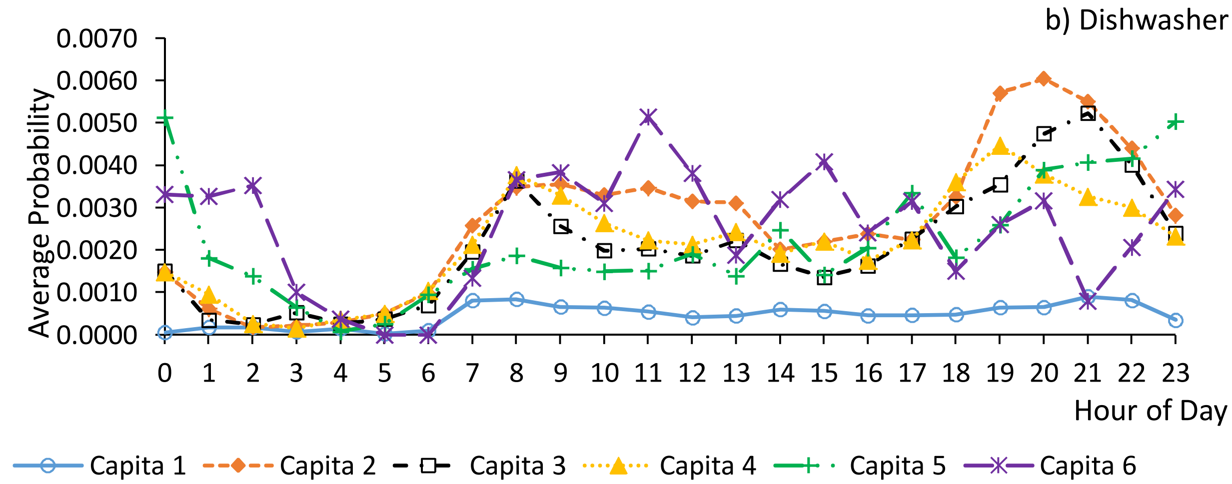
	Fixture type & efficiency	No. of Bathrooms	1	2	3	4	5	>5	Average > 5
Ultra-efficient and efficient fixtures	**Clothes washer**	**Average Resident/ home**	2.64	2.57	3.06	2.76	3	4.5	7
		Average fixture/home	1	1	1	1	1	1	
		Sample size (341)	7.3%	46.6%	35.2%	8.5%	1.2%	1.2%	
		Percentage Ultra Efficient	44.0%	47.2%	55.8%	44.8%	25.0%	50.0%	
	Dishwasher	**Average Resident/ home**	2.06	2.71	2.89	2.92	3.36	4.5	6.7
		Average fixture/home	1	1	1	1	1	1.5	
		Sample size (527)	6.6%	42.9%	38.5%	8.7%	2.1%	1.1%	
		Percentage Ultra Efficient	0.0%	36.7%	31.5%	32.6%	54.5%	16.7%	
	Shower	**Average Resident/ home**	2.69	2.55	2.97	3.02	3.36	4.88	6.5
		Average fixture/home	1.2	1.95	2.37	2.79	3.86	5.125	
		Sample size (837)	9.6%	46.1%	34.8%	6.9%	1.7%	1.0%	
		Percentage Ultra efficient	75.0%	79.8%	77.0%	91.4%	78.6%	100.0%	
	Toilets	**Average Resident/ home**	2.64	2.56	2.88	3.39	2.63	4.14	6.7

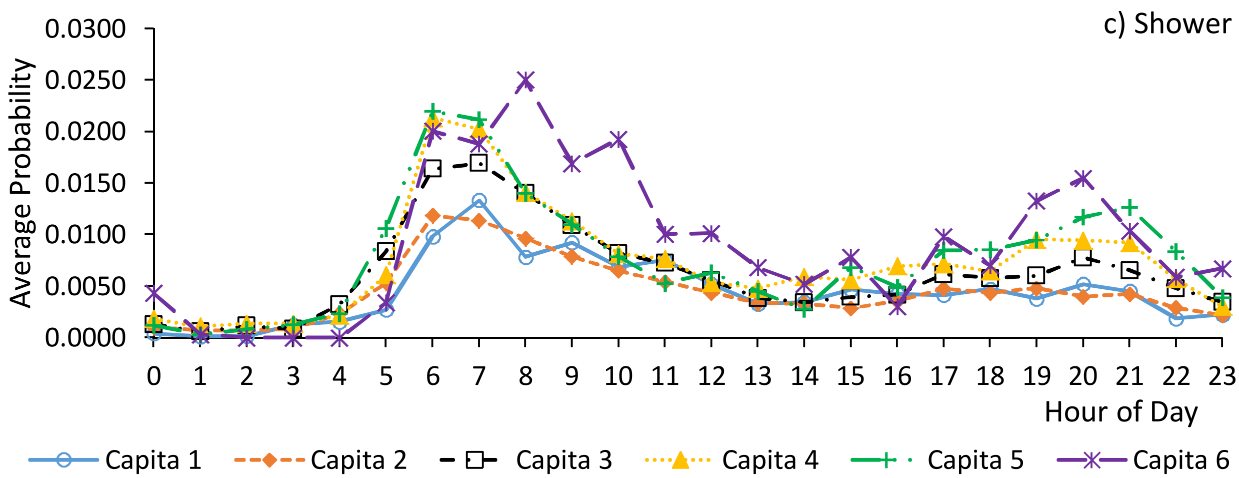
		Average fixture/home	1.04	2.02	2.94	3.82	4.25	6.71	
		Sample size (481)	9.4%	44.1%	37.6%	5.8%	1.7%	1.5%	
		Percentage Ultra efficient	55.6%	49.5%	41.4%	46.4%	50.0%	14.3%	
Inefficient Fixtures	**Fixture**	**No. of Bathrooms**	**1**	**2**	**3**	**4**	**5**	**>5**	**Average > 5**
	Clothes washer	**Average Resident/ home**	2.52	2.5	2.9	3.23	3	4.17	6.17
		Average fixture/home	1	1	1	1	1	1	
		Sample size (661)	12.3%	44.6%	33.9%	6.1%	2.3%	0.9%	
	Dishwasher	**Average Resident/ home**	2.4	2.4	2.9	3.3	2.57	4.5	6.5
		Average fixture/home	1	1	1.01	1.08	1	1	
		Sample size (195)	7.7%	42.6%	38.5%	6.7%	3.6%	1.0%	
	Shower	**Average Resident/ home**	2.35	2.43	2.88	3.18	2	2	6.5
		Average fixture/home	1.1	1.82	2.24	2.18	3	3.5	
		Sample size (177)	17.5%	40.1%	32.2%	6.2%	2.8%	1.1%	
	Toilet	**Average Resident/ home**	2.47	2.46	2.99	2.76	3.27	4.67	6
		Average fixture/home	1.24	2	2.84	3.4	5	5.0	
		Sample size (556)	12.6%	46.4%	30.9%	7.6%	2.0%	0.5%	
Unclassified fixtures	**Fixture**	**No. of Bathrooms**	**1**	**2**	**3**	**4**	**5**	**>5**	**Average > 5**
	Bathtub	**Avg. Resident/home**	3	2.74	3.24	3	3	4.3	6.5
		Avg. fixture/home	1.26	1.46	1.76	1.95	2.5	3.1	
		Sample size (519)	8.3%	43.9%	35.8%	8.1%	1.9%	1.9%	
	Faucet	**Avg. Resident/home**	2.54	2.51	2.93	3.01	3	4.3	6.5
		Avg. fixture/home	2.5	3.31	4.44	5.61	6.53	7.9	
		Sample size (1038)	11.1%	45.3%	34.1%	6.7%	1.8%	1.0%	

Table A5: Statistics on hourly probability of a busy fixture for different fixture efficiency

Group	Capita				Bedroom				Bathroom			
Fixture	Minimum	Maximum	Average	Standard Deviation	Minimum	Maximum	Average	Standard Deviation	Minimum	Maximum	Average	Standard Deviation
Efficient fixtures												
Clothes washer	0.0000	0.0504	0.0084	0.0080	0.0000	0.0465	0.0077	0.0077	0.0000	0.0653	0.0082	0.0092
Dishwasher	0.0000	0.0060	0.0020	0.0015	0.0000	0.0162	0.0019	0.0019	0.0000	0.0065	0.0019	0.0014
Shower	0.0000	0.0250	0.0068	0.0054	0.0000	0.0893	0.0075	0.0105	0.0000	0.0265	0.0059	0.0052
Toilet	0.0010	0.0085	0.0040	0.0017	0.0000	0.0188	0.0092	0.0045	0.0004	0.0121	0.0039	0.0024
Inefficient fixtures												
Clothes washer	0.0000	0.0330	0.0092	0.0072	0.0000	0.0256	0.0089	0.0069	0.0000	0.0257	0.0083	0.0061
Dishwasher	0.0000	0.0169	0.0018	0.0021	0.0000	0.0274	0.0014	0.0025	0.0000	0.0212	0.0010	0.0024
Shower	0.0000	0.0270	0.0055	0.0051	0.0000	0.0183	0.0036	0.0038	0.0000	0.0188	0.0038	0.0040
Toilet	0.0013	0.0120	0.0054	0.0024	0.0010	0.0177	0.0057	0.0029	0.0009	0.0157	0.0053	0.0030
Unclassified fixtures												
Bathtub	0.0000	0.0043	0.0007	0.0007	0.0000	0.0227	0.0009	0.0024	0.0000	0.0030	0.0006	0.0006
Faucet	0.0004	0.0236	0.0070	0.0049	0.0000	0.0183	0.0057	0.0034	0.0002	0.0172	0.0054	0.0037


a) Clothes washer
Average Probability
0.025
0.020
0.015
0.010
0.005
0.000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Hour of Day
Capita 1
Capita 2
Capita 3
Capita 4
Capita 5
Capita 6


b) Dishwasher
Average Probability
0.0070
0.0060
0.0050
0.0040
0.0030
0.0020
0.0010
0.0000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Hour of Day
Capita 1
Capita 2
Capita 3
Capita 4
Capita 5
Capita 6


c) Shower
Average Probability
0.0300
0.0250
0.0200
0.0150
0.0100
0.0050
0.0000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Hour of Day
Capita 1
Capita 2
Capita 3
Capita 4
Capita 5
Capita 6

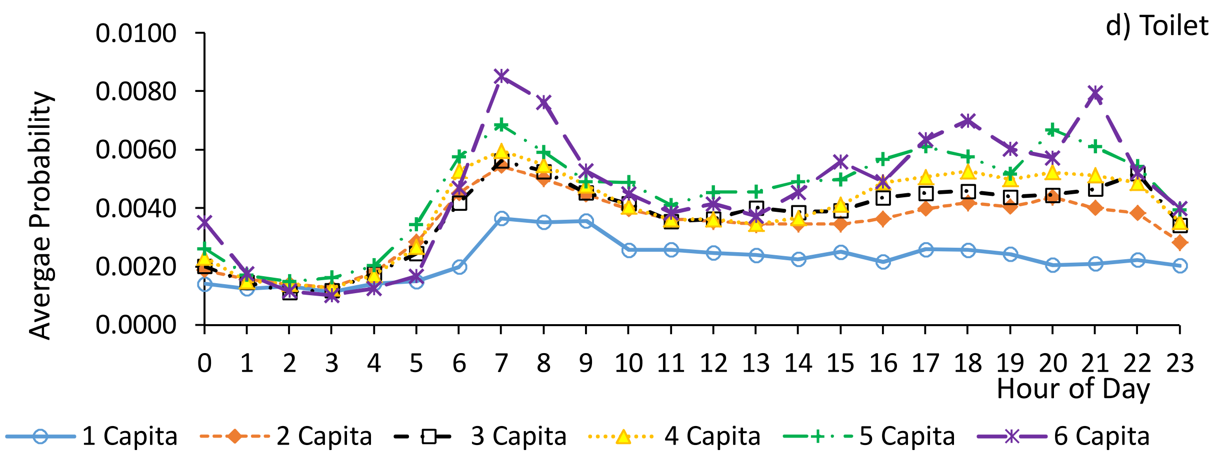
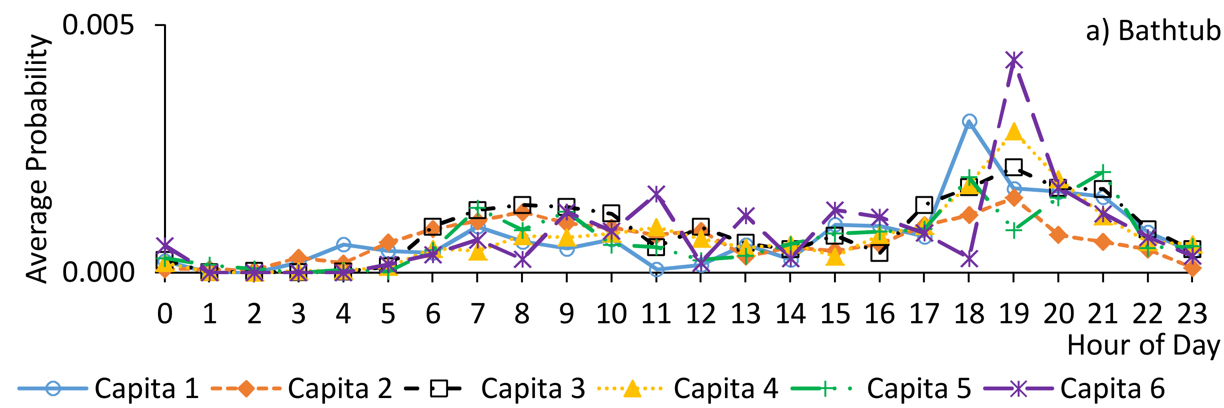


Figure A7: Hourly probability of efficient (ultra-efficient and efficient) fixtures in sampled homes grouped by number of residents for (a) Clothes washer, (b) Dishwasher, (c) Shower, and (d) Toilet



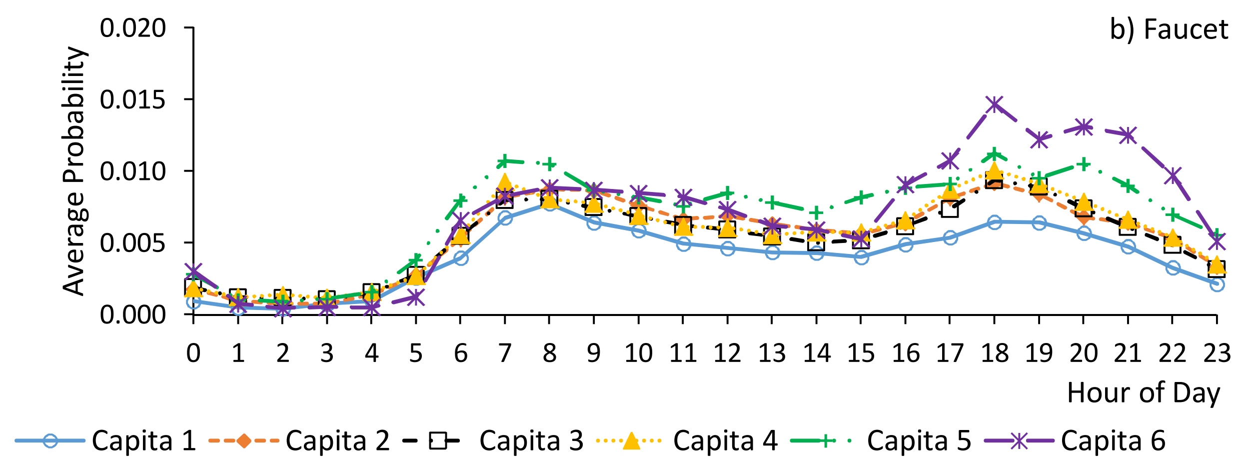
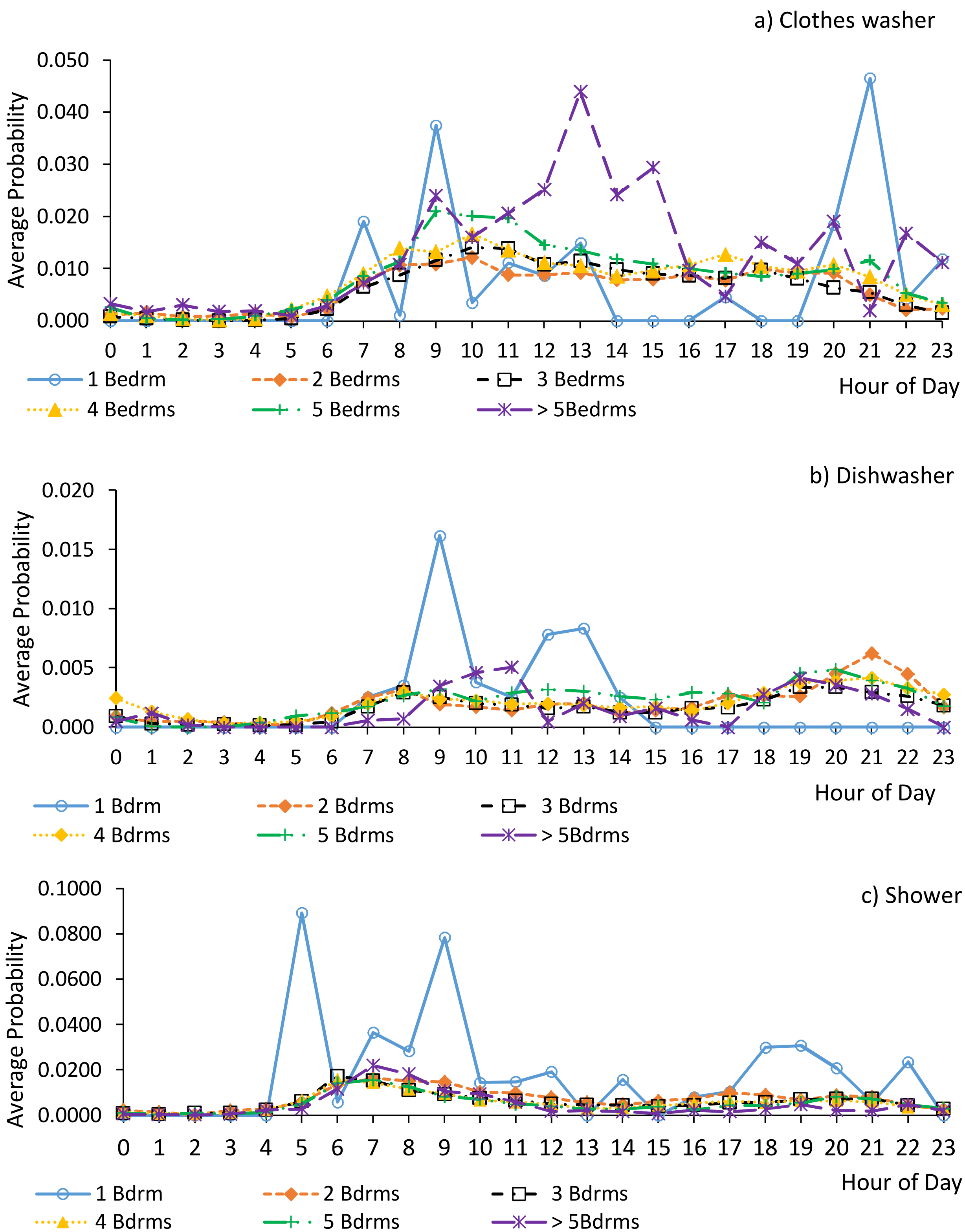


Figure A8: Hourly probability of unclassified fixtures in sampled homes grouped by number of residents for (a) Bathtub, and (b) Faucet


a) Clothes washer
Average Probability
0.050
0.040
0.030
0.020
0.010
0.000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Hour of Day
1 Bedrm
2 Bedrms
3 Bedrms
4 Bedrms
5 Bedrms
> 5Bedrms
b) Dishwasher
Average Probability
0.020
0.015
0.010
0.005
0.000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Hour of Day
1 Bdrm
2 Bdrms
3 Bdrms
4 Bdrms
5 Bdrms
> 5Bdrms
c) Shower
Average Probability
0.1000
0.0800
0.0600
0.0400
0.0200
0.0000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Hour of Day
1 Bdrm
2 Bdrms
3 Bdrms
4 Bdrms
5 Bdrms
> 5Bdrms

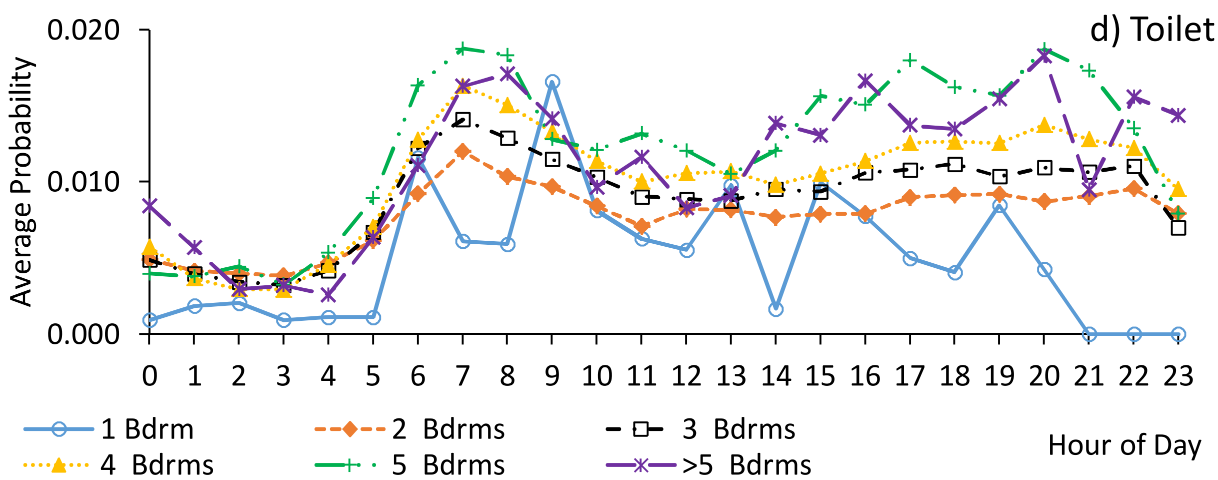


Figure A9: Hourly probability of efficient (ultra-efficient and efficient) fixtures in sampled homes grouped by number of bedrooms for (a) Clothes washer, (b) Dishwasher, (c) Shower, and (d) Toilet

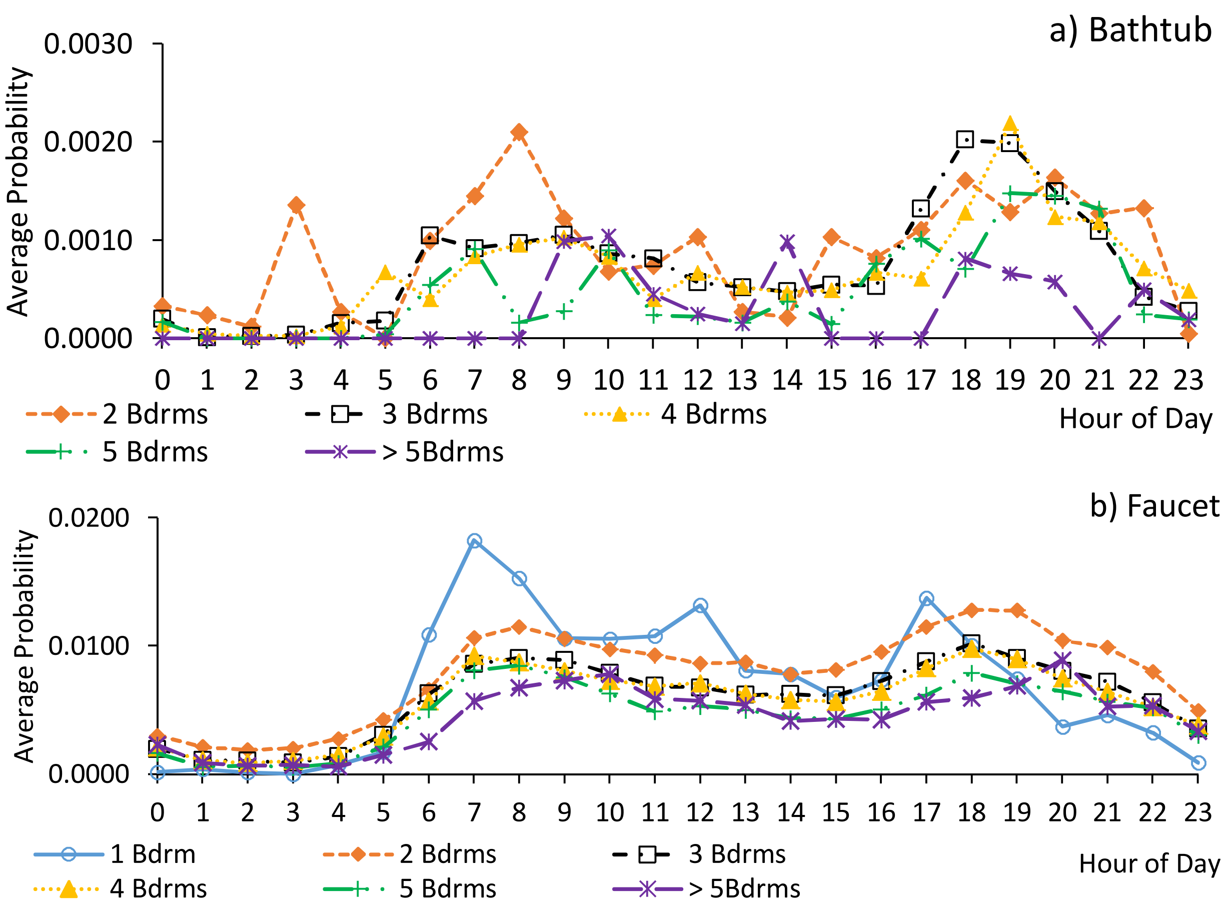
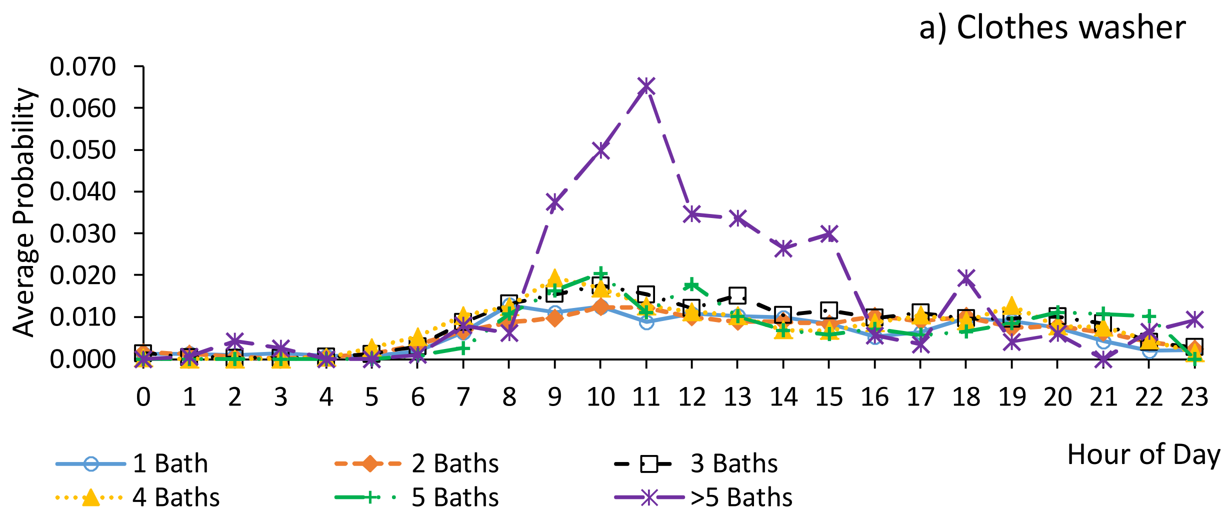
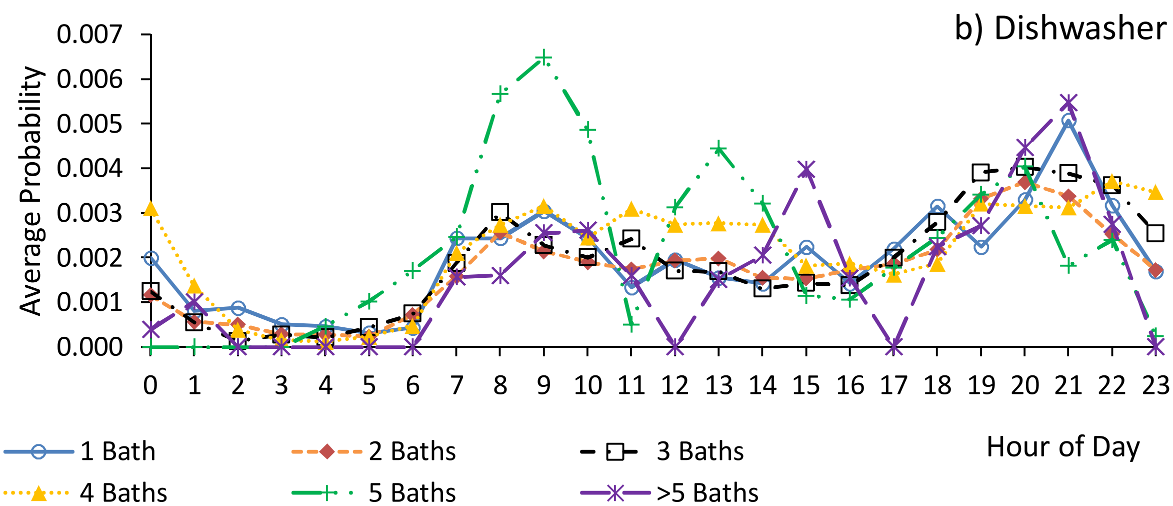
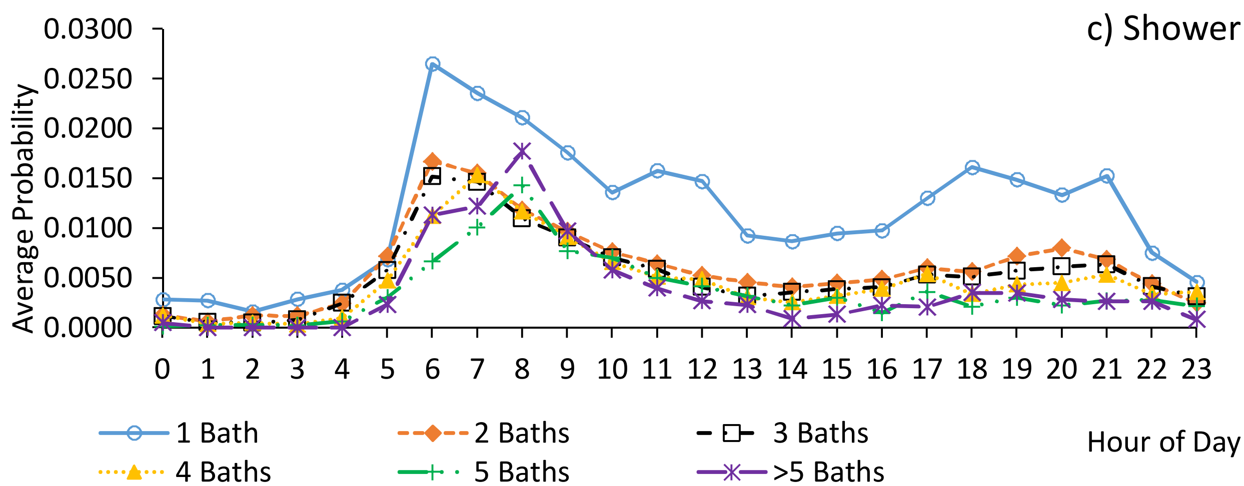


Figure A10: Hourly probability of unclassified fixtures in sampled homes grouped by number of bedrooms for (a) Bathtub, and (b) Faucet


a) Clothes washer
Average Probability
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Hour of Day
1 Bath
2 Baths
3 Baths
4 Baths
5 Baths
>5 Baths


b) Dishwasher
Average Probability
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Hour of Day
1 Bath
2 Baths
3 Baths
4 Baths
5 Baths
>5 Baths


c) Shower
Average Probability
0.0300
0.0250
0.0200
0.0150
0.0100
0.0050
0.0000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Hour of Day
1 Bath
2 Baths
3 Baths
4 Baths
5 Baths
>5 Baths

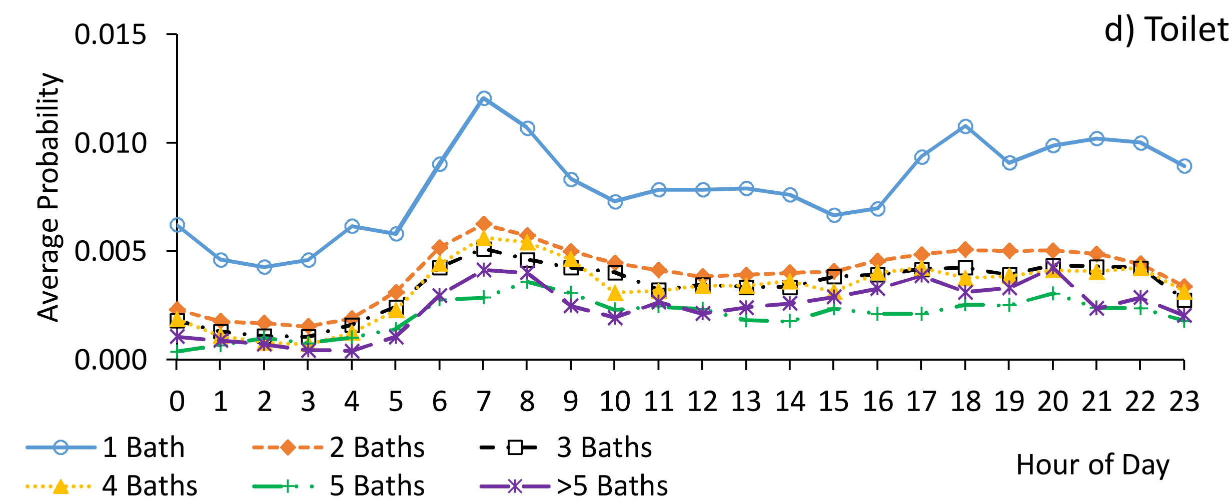


Figure A11: Hourly probability of efficient (ultra-efficient and efficient) fixtures in sampled homes grouped by number of bathrooms for (a) Clothes washer, (b) Dishwasher, (c) Shower, and (d) Toilet

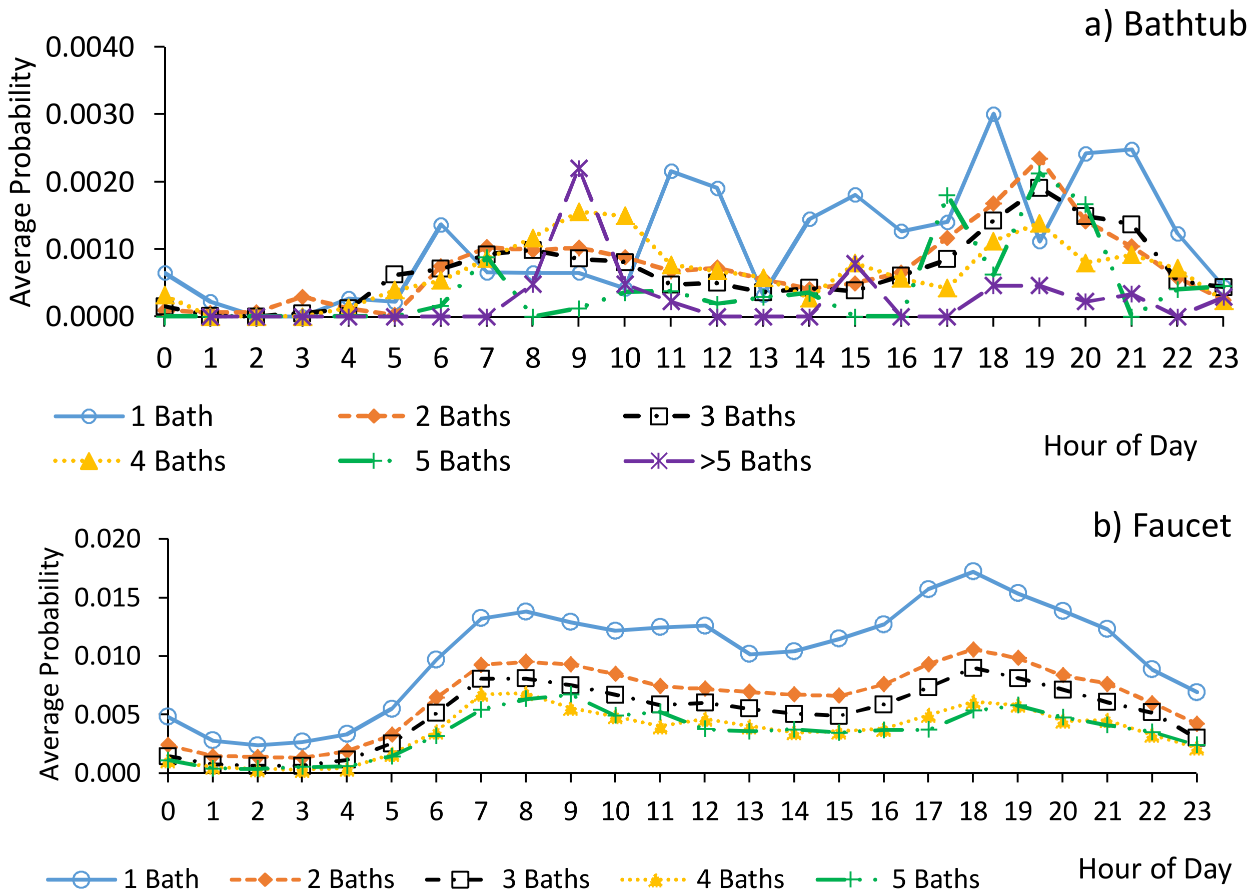
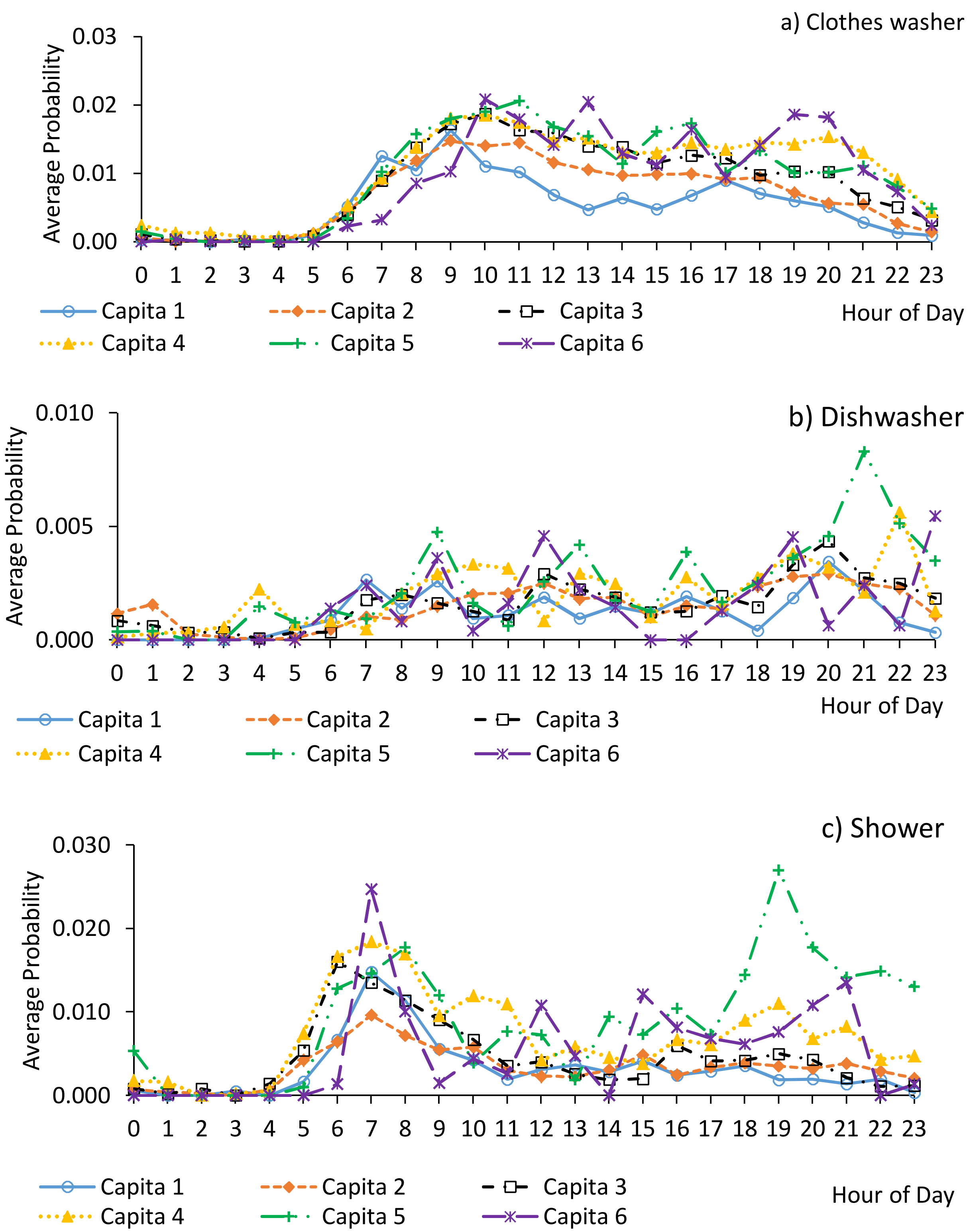


Figure A12: Hourly probability of unclassified fixtures in sampled homes grouped by number of bathrooms for (a) Bathtub, and (b) Faucet



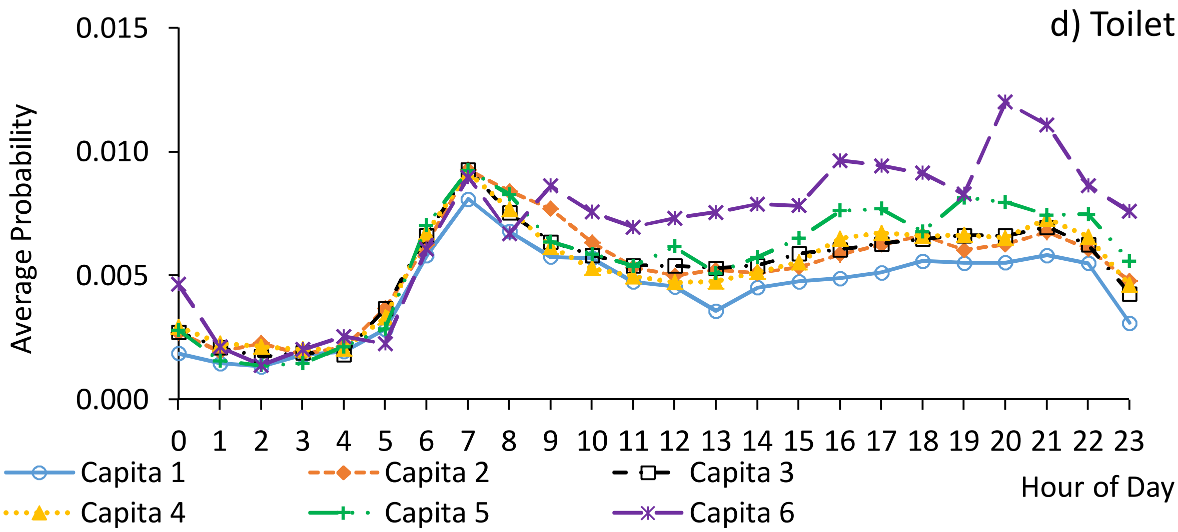
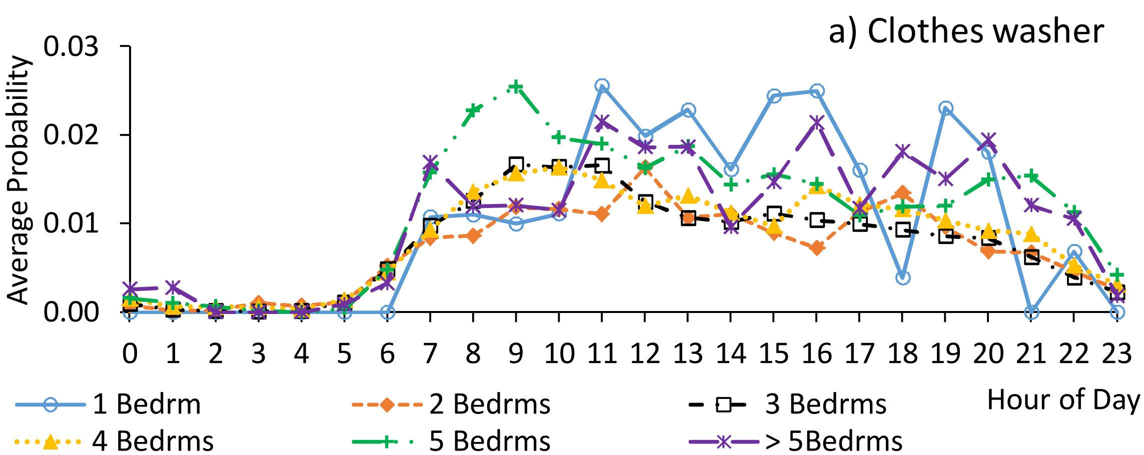
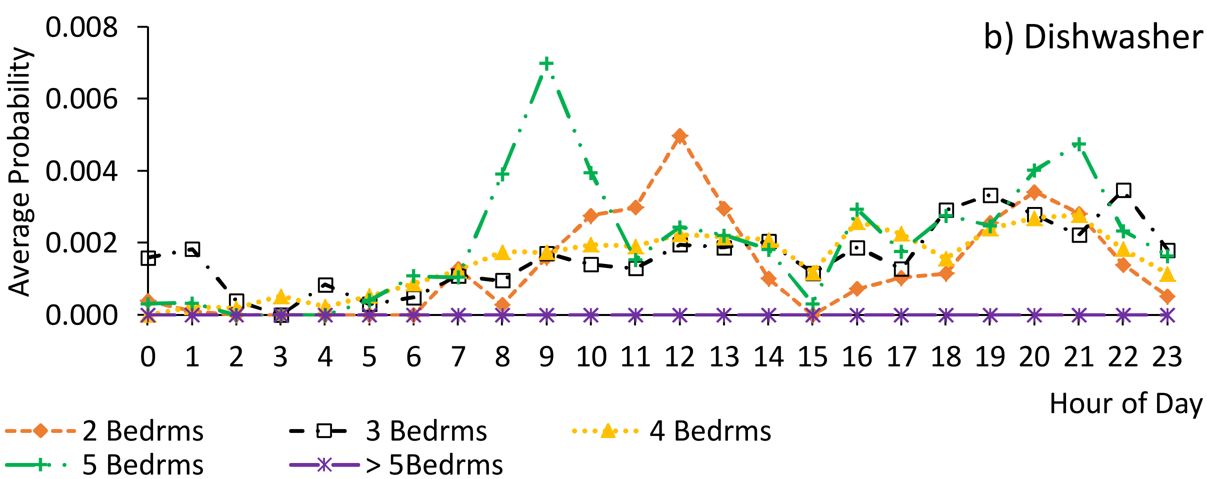


Figure A13: Hourly probability of inefficient fixtures in sampled homes grouped by number of residents for (a) Clothes washer, (b) Dishwasher, (c) Shower, and (d) Toilet





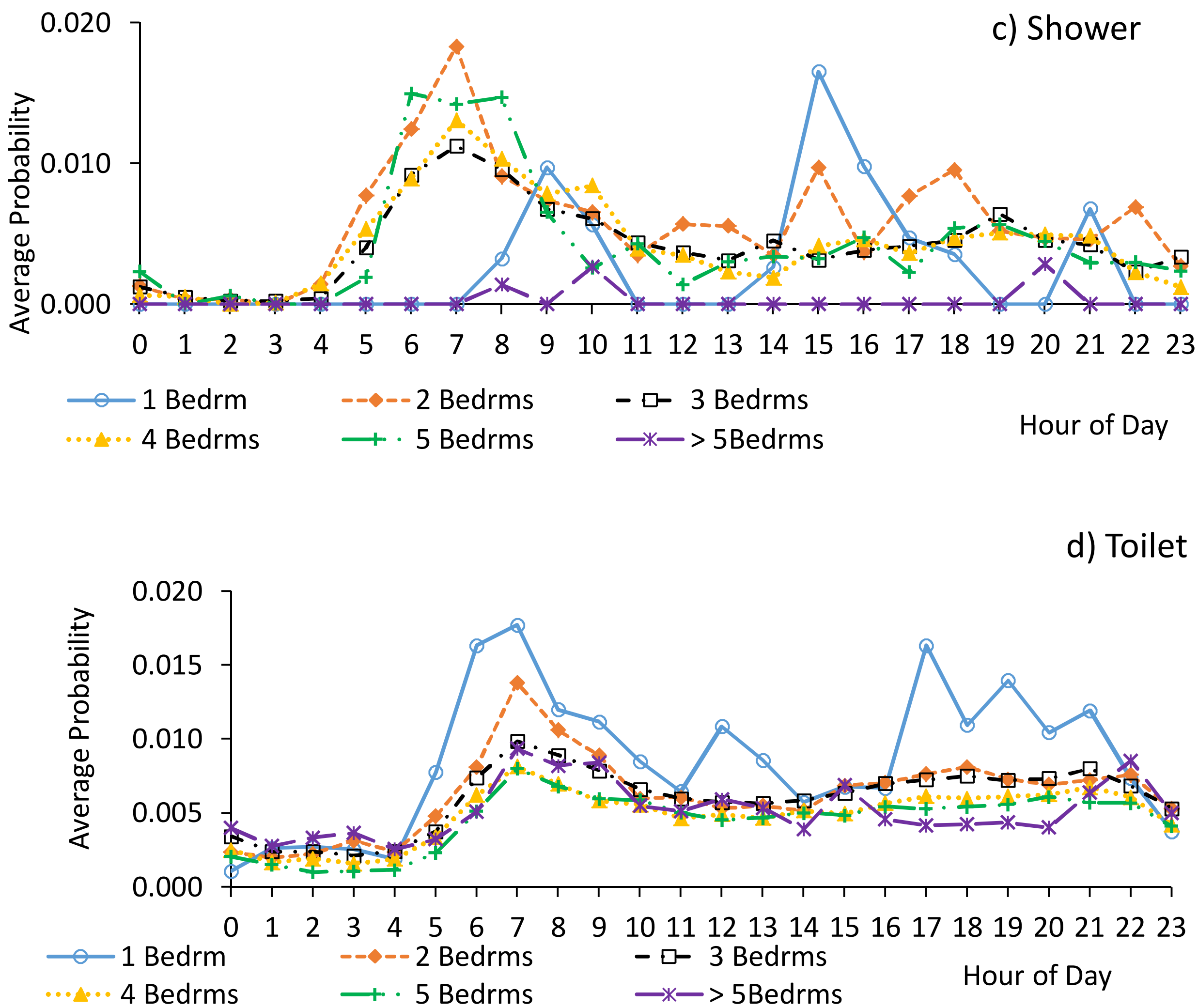
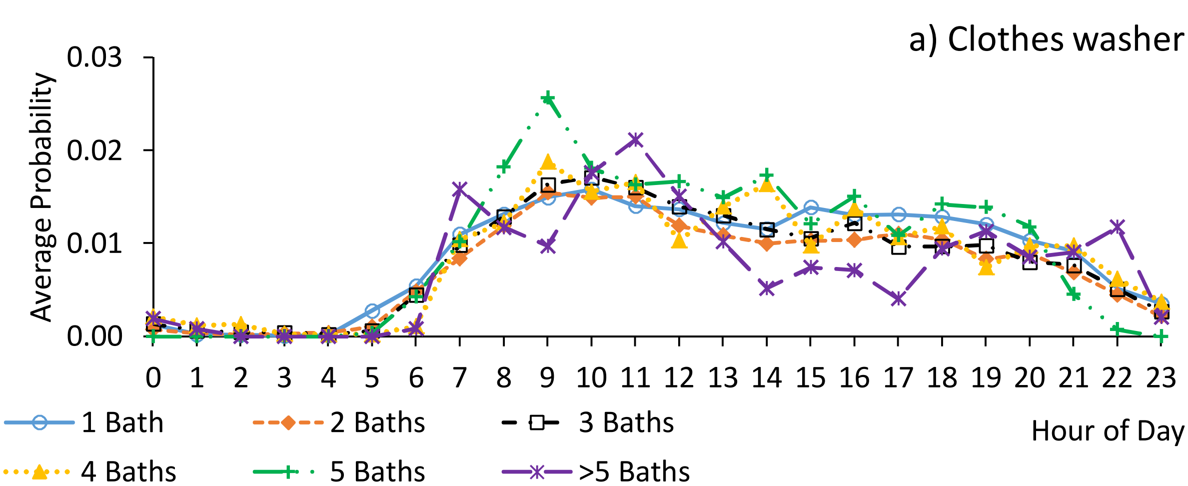


Figure A14: Hourly probability of inefficient fixtures in sampled homes grouped by number of bedrooms for (a) Clothes washer, (b) Dishwasher, (c) Shower, and (d) Toilet



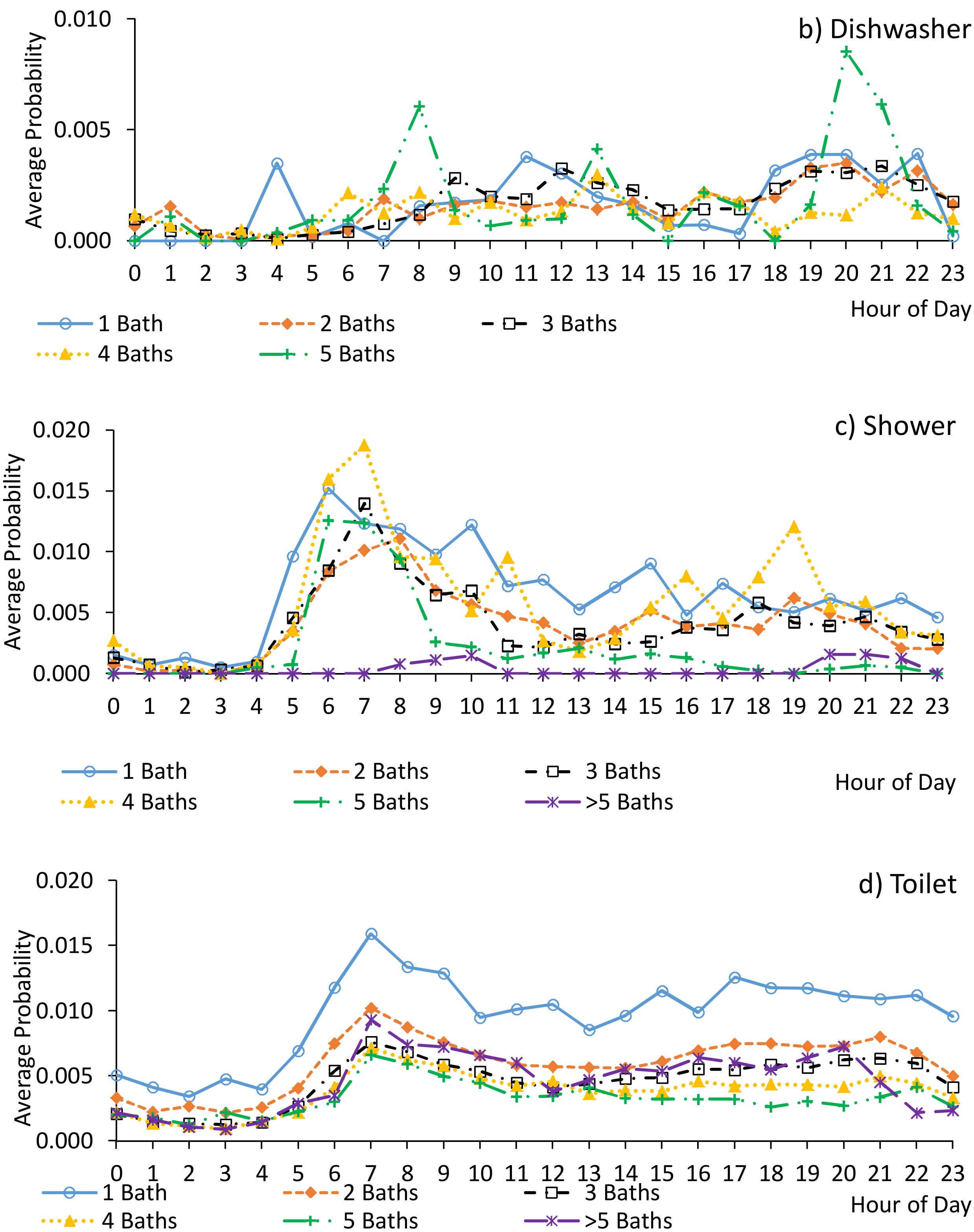


Figure A15: Hourly probability of inefficient fixtures in sampled homes grouped by number of bathrooms for (a) Clothes washer, (b) Dishwasher, (c) Shower, and (d) Toilet

Appendix B – Members of the Task Group

Steven Buchberger, Ph.D., is Professor and Head of the Department of Civil Engineering, Architectural Engineering and Construction Management at the University of Cincinnati. He is a registered professional engineer in Colorado. His teaching and research focus on reliability-based design in water resources and urban hydrology. Professor Buchberger has advised 60 graduate students at UC. He served as Associate Editor of *ASCE Journal of Water Resources Planning and Management* for ten years and was the Chief Editor of two special issues on Water Distribution Systems Analysis.

Toritseju Omaghomi is a PhD student in Environmental Engineering at the University of Cincinnati. Her Masters in Environmental Engineering was an analysis of methods for estimating water demand in buildings and a Bachelor's degree in Agricultural Engineering focused on soil and water conservation. Her research interest includes water conservation, modelling and integrated engineering of water resource at a regional and household scale. Her current research involves sustainability concerns associated water conservation measures implemented in buildings and the transition to a sustainable design method for water supply in buildings.

Timothy Wolfe is a professional engineer with 12 years of experience in building design consulting. He currently works for TRC Worldwide Engineering – MEP, LLC. as a senior mechanical engineer responsible for managing the plumbing department. Tim has been a member of ASPE since 2011 and is actively involved with the Central Indiana Chapter including serving on the board as the Vice President, Legislation officer from 2013-2015.

Jason Hewitt is a professional engineer with 9 years' experience designing high rise building along the west coast. He currently works for CB Engineers as the Seattle office manager. He is a founding member of the Seattle ASPE Chapter and currently on the board as VP of Legislative.

Daniel Cole is the Technical Services Manager at International Association of Plumbing and Mechanical Officials (IAPMO). He is a licensed Journeyman Plumber in the State of Illinois and member of the American Society of Plumbing Engineers (ASPE). He has published several articles on Hunter's curve and the fixture unit methodology. His research focus is on the plumbing investigations performed at the National Bureau of Standards (now NIST) with a particular interest in the work of Roy B. Hunter. He also has a focus on promoting water efficiency standards for the built environment by acting as liaison to IAPMO's Green Plumbing and Mechanical Code Supplement (GPMCS) Technical Committee and the Water Efficiency and Sanitation Standard (WEStand) Technical Committee.