Snake Mackerel – An Isogeny Based AKEM

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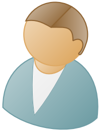
Swiss Isogeny Day 2025

AKEM

Authenticated **K**ey
Encapsulation **M**echanism

Definition

Alice



Bob

Definition

$(\mathsf{sk}_A, \mathsf{pk}_A) \xleftarrow{\$} \mathsf{Gen}$

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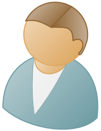
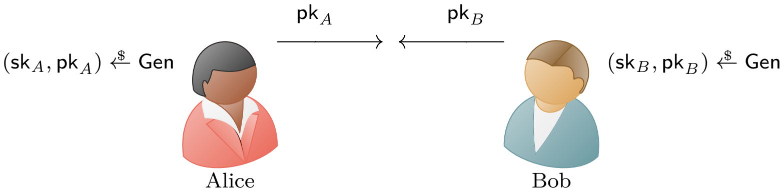


Bob

$(\mathsf{sk}_B, \mathsf{pk}_B) \xleftarrow{\$} \mathsf{Gen}$

Definition



$(\mathsf{sk}_A, \mathsf{pk}_A) \xleftarrow{\$} \mathsf{Gen}$

$\xrightarrow{\mathsf{pk}_A}$

$\xleftarrow{\mathsf{pk}_B}$

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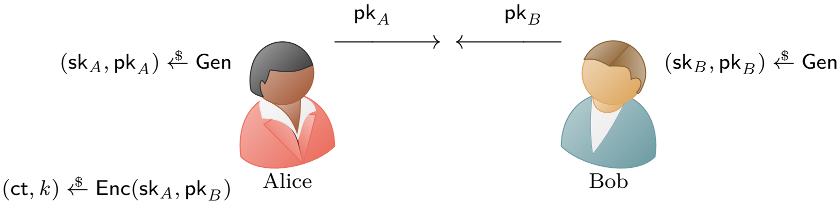

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$(\mathsf{ct}, k) \xleftarrow{\$} \mathsf{Enc}(\mathsf{sk}_A, \mathsf{pk}_B)$

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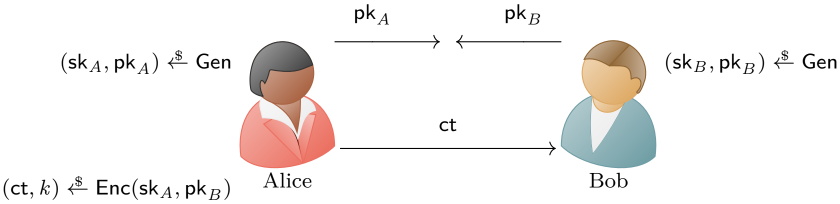

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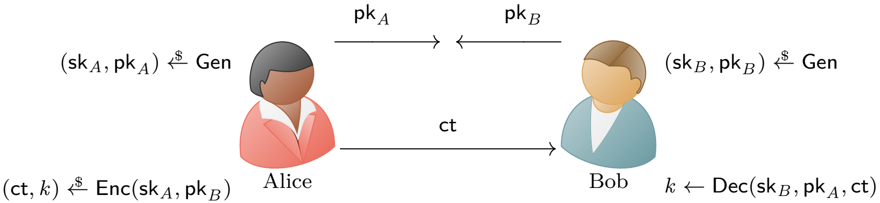

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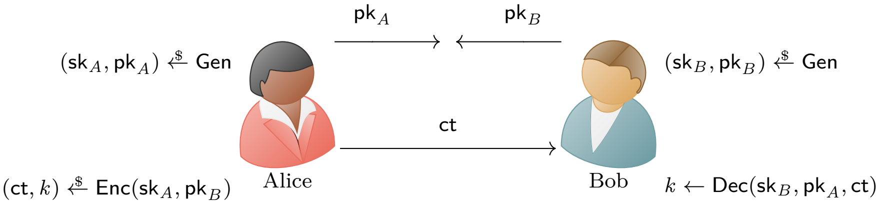

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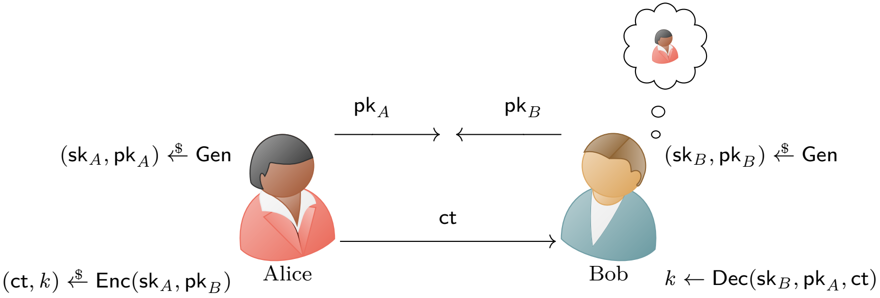

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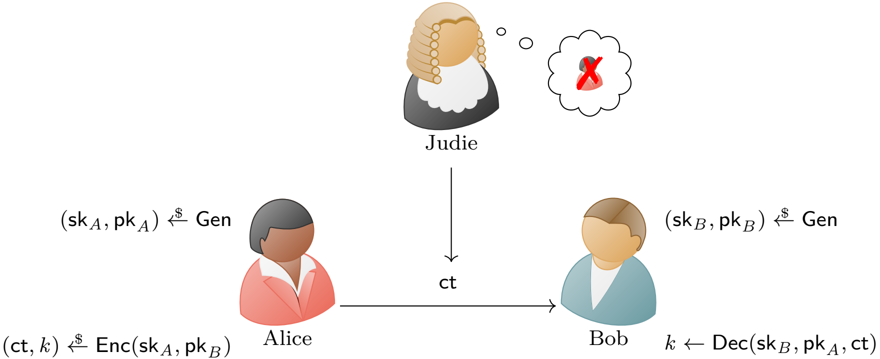

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- **Deniability:** Judie cannot be convinced that Alice sent ct

Generic Constructions

- **KEM + (Ring) Signature** (FrodoKEX+ [CHN$^+$24a], Gandalf-AKEM [GJK24])

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Given ID-Scheme ID, a Split Ciphertext KEM KEM requires

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⇒ Reusing the commitment leads to a **more compact scheme** than plain KEM + Signature

⇒ Our **generic** construction SnakeM can be instantiated from **isogenies**

⇒ SnakeM is only **5×** larger than DH-AKEM (64 vs. 296 Bytes) – naive approach 370 Bytes

Snake Mackerel = POKÉ + SQIsignHD

⇢ non-rational

→ secret

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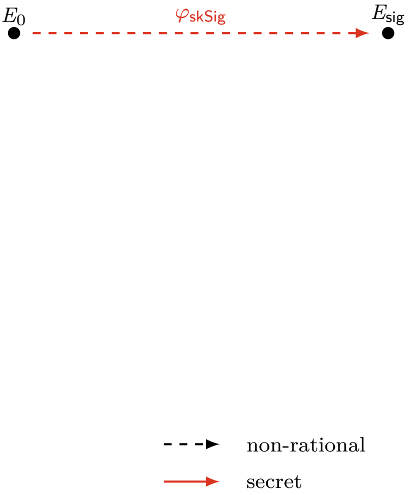

Snake Mackerel = POKÉ + SQIsignHD

$$\begin{tikzcd}
E_0 \arrow[rr, dashed, "\varphi_{\mathsf{skSig}}"] \arrow[dd, dashed, "\varphi_{\mathsf{com}}"] & & E_{\mathsf{sig}} \\
 & & \\
E_{\mathsf{com}} & &
\end{tikzcd}$$

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E_0 \arrow[r, dashed, "\varphi_{\mathsf{skSig}}"] \arrow[d, dashed, "\varphi_{\mathsf{com}}"] & E_{\mathsf{sig}} \arrow[d, "H(E_{\mathsf{com}}, m)"'] \\
E_{\mathsf{com}} & E_{\mathsf{chal}}
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$X_0 \in E_0[D]$

E_0

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$X_1 = [\alpha]\varphi_{\mathsf{skEnc}}(X_0)$ $X_0 \in E_0[D]$

$$\begin{tikzcd}
E_{\mathsf{enc}} & & E_0 \arrow[ll, dashed, "\varphi_{\mathsf{skEnc}}"']
\end{tikzcd}$$

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E_0 \arrow[rr, dashed, "\varphi_{\mathsf{skSig}}"] \arrow[dd, dashed, "\varphi_{\mathsf{com}}"] & & E_{\mathsf{sig}} \arrow[dd] \\
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$$\begin{tikzcd}
X_1 = [\alpha]\varphi_{\mathsf{skEnc}}(X_0) & X_0 \in E_0[D] & & \\
E_{\mathsf{enc}} \arrow[d, "\varphi'"] & E_0 \arrow[l, dashed, "\varphi_{\mathsf{skEnc}}"'] \arrow[d, "\varphi"'] & E_0 \arrow[r, dashed, "\varphi_{\mathsf{skSig}}"] \arrow[d, dashed, "\varphi_{\mathsf{com}}"] & E_{\mathsf{sig}} \arrow[d] \\
E_{\mathsf{ct}} & E_3 & E_{\mathsf{com}} \arrow[r, dashed, "{\sigma = (q, \varphi_{\mathsf{rsp}}(U), \varphi_{\mathsf{rsp}}(V))}"] & E_{\mathsf{chal}} \\
X_3 = [\beta]\varphi'(X_1) & X_2 = [\beta]\varphi(X_0) & &
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E_{\mathsf{ct}} & E_3 \arrow[l, dashed, "\varphi'_{\mathsf{skEnc}}"'] & E_{\mathsf{com}} \arrow[r, dashed, "\sigma = (q, \varphi_{\mathsf{rsp}}(U), \varphi_{\mathsf{rsp}}(V))"] & E_{\mathsf{chal}}
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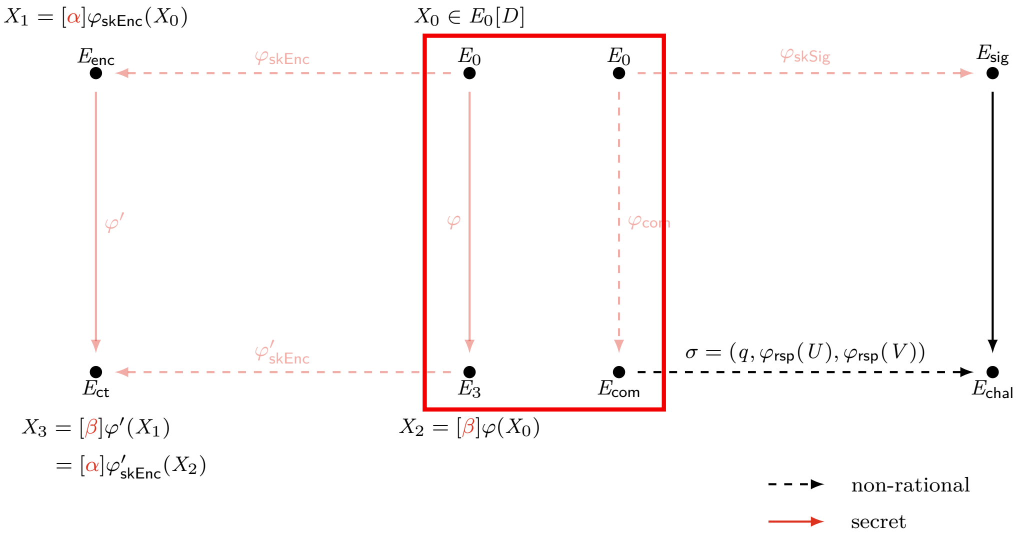

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E_{enc}

φ_{skEnc}

E_0

E_0

φ_{skSig}

E_{sig}

φ'

φ

φ_{com}

$\sigma = (q, \varphi_{\mathsf{rsp}}(U), \varphi_{\mathsf{rsp}}(V))$

$\varphi'_{\mathsf{skEnc}}$

E_{ct}

E_3

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E_{\mathsf{ct}} & E_{\mathsf{com}} \arrow[l, dashed, "\varphi'_{\mathsf{skEnc}}"'] \arrow[r, dashed, "{\sigma = (q, \varphi_{\mathsf{rsp}}(U), \varphi_{\mathsf{rsp}}(V))}"] & E_{\mathsf{chal}}
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$$\begin{aligned} X_3 &= [\beta]\varphi'(X_1) \\ &= [\alpha]\varphi'_{\mathsf{skEnc}}(X_2) \end{aligned}$$

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Compatibility

SQIsignHD and POKÉ use primes $p = c2^a3^b - 1$, but with different **sizes**

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Use **B-SIDH** approach for a more compact scheme with $p \in \mathcal{O}(2^{2\lambda})$

$$(p+1)(p-1) = 2^a ND, \qquad 2^a \in \mathcal{O}(2^{\lambda}), \qquad N = \prod \ell_i \in \mathcal{O}(2^{2\lambda}), \qquad D = q_1 q_2 q_3 \in \mathcal{O}(2^{\lambda})$$

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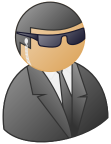
$$p = 2^{133} \cdot 3^6 \cdot 7^2 \cdot 17^4 \cdot 47^2 \cdot 311^2 \cdot 367^2 \cdot 439^2 \cdot 1049^2 \cdot 1373 - 1$$
$$\log p = 247, \qquad \max\{\ell_i\} = 1373, \qquad \max\{\log q_i\} = 39$$

Security

The Best out of Both Worlds?

Confidentiality: Ins-CCA, simplified



Challenger

Adversary

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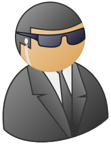

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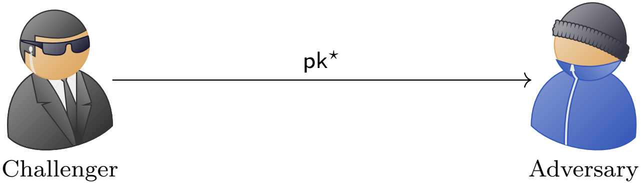

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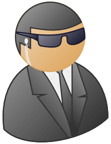

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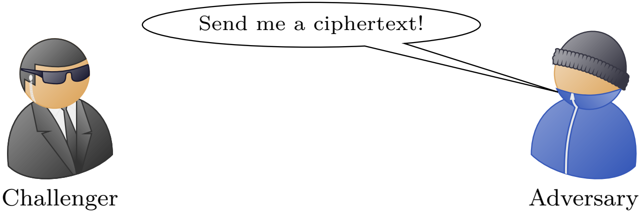

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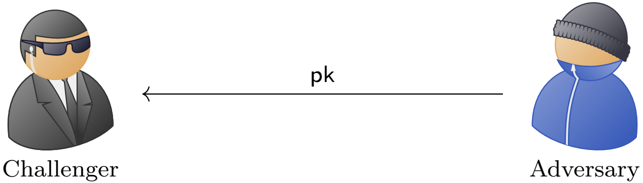

pk

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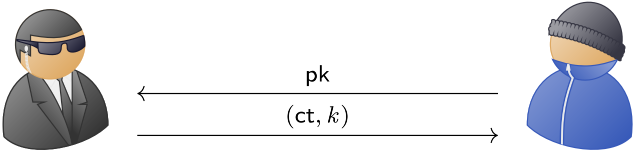

$\xleftarrow{\mathsf{pk}}$

$\xrightarrow{(\mathsf{ct}, k)}$

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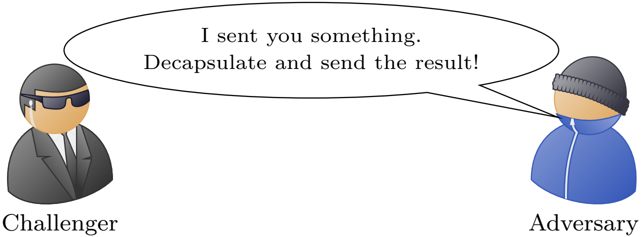

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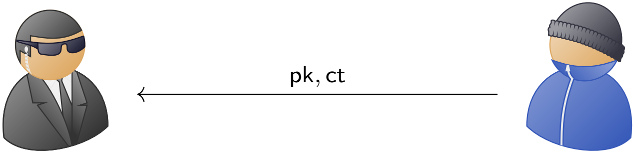

pk, ct

Challenger

Adversary

Confidentiality: Ins-CCA, simplified

$(\mathsf{sk}^\star, \mathsf{pk}^\star) \xleftarrow{\$} \mathsf{Gen}$

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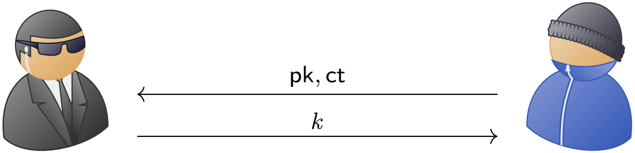

$k \leftarrow \mathsf{Decaps}(\mathsf{pk}, \mathsf{sk}^\star, \mathsf{ct})$

$\xleftarrow{\mathsf{pk}, \mathsf{ct}}$

$\xrightarrow{k}$

Challenger

Adversary

Confidentiality: Ins-CCA, simplified

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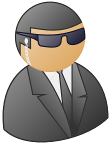
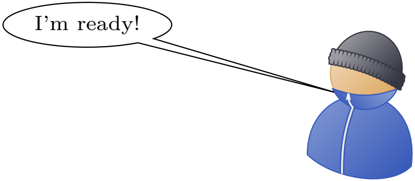


Challenger



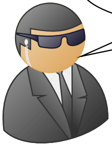
Adversary

Confidentiality: Ins-CCA, simplified

$(\mathsf{sk}^\star, \mathsf{pk}^\star) \xleftarrow{\$} \mathsf{Gen}$

$\mathsf{pk}^\star$

I will send an honest message on your behalf to myself



Challenger

Adversary

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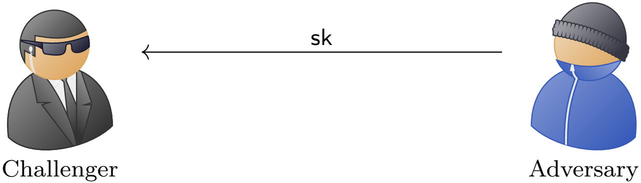


sk

Challenger

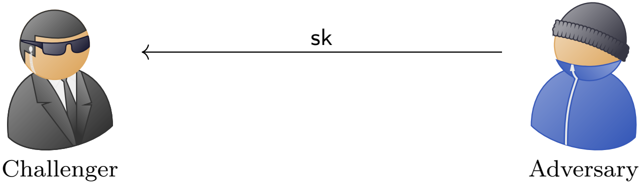
Adversary

Confidentiality: Ins-CCA, simplified

```
$(\mathsf{sk}^\star, \mathsf{pk}^\star) \xleftarrow{\$} \mathsf{Gen}$

$\beta \xleftarrow{\$} \{0, 1\}$
$(\mathsf{ct}, k) \xleftarrow{\$} \mathsf{Encaps}(\mathsf{sk}, \mathsf{pk}^\star)$
**if** $\beta = 1$
    $k \leftarrow \$$
```

$\mathsf{pk}^\star$



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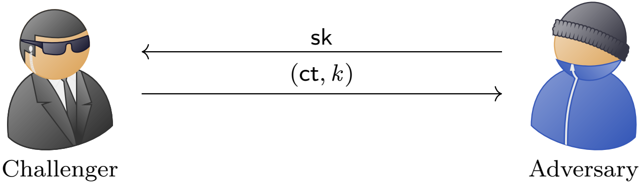

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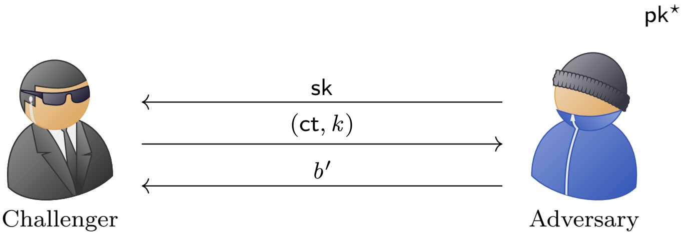
$\textbf{win if } b = b'$
```



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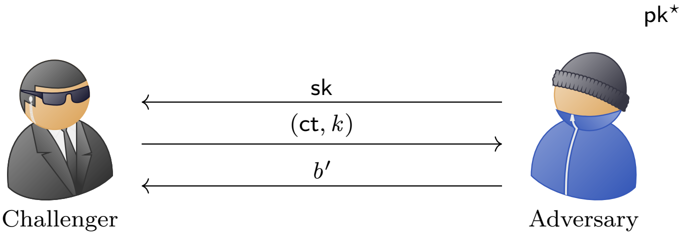

Note

sk is used for the **signature** and should not help to decapsulate the **KEM** part of ct

Confidentiality of SnakeM

Theorem

For any $\mathsf{Ins\text{-}CCA}$ adversary $\mathcal{A}$ against SnakeM, there exist an adversary $\mathcal{B}$ against $\mathsf{OW\text{-}KCA}$ of POKÉ such that

$$\mathsf{Adv}^{\mathsf{Ins\text{-}CCA}}_{\text{SnakeM}}(\mathcal{A}) \leq \mathsf{Adv}^{\mathsf{OW\text{-}KCA}}_{\text{POKÉ}}(\mathcal{B}) + \delta.$$

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$$\mathcal{O}^{\mathsf{kc}}(\mathsf{ct}, k) \to 1 \quad \text{if } \mathsf{ct} \text{ contains key } k$$

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Why $\mathsf{OW\text{-}KCA}$ when POKÉ is $\mathsf{IND\text{-}CCA}$ secure?

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Fujisaki-Okamoto Transform [FO99, HHK17]

$$\mathsf{IND\text{-}CPA} \xRightarrow{\text{T-Transform}} \mathsf{OW\text{-}KCA} \xRightarrow{\text{U-Transform}} \mathsf{IND\text{-}CCA}$$

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- T-Transform makes the encryption randomness **explicit** $\Longrightarrow$ leaks **commitment**

Confidentiality of SnakeM

$$\begin{tikzcd}
E_{\mathsf{enc}} \arrow[d, "\varphi'"] & E_0 \arrow[l, "\varphi_{\mathsf{skEnc}}"'] \arrow[d, "\varphi"'] & E_0 \arrow[r, "\varphi_{\mathsf{skSig}}"] \arrow[d, "\varphi_{\mathsf{com}}"] & E_{\mathsf{sig}} \arrow[d] \\
E_{\mathsf{ct}} & E_3 \arrow[l, "\varphi'_{\mathsf{skEnc}}"'] & E_{\mathsf{com}} \arrow[r, "\sigma = (q, \varphi_{\mathsf{rsp}}(U), \varphi_{\mathsf{rsp}}(V))"] & E_{\mathsf{chal}}
\end{tikzcd}$$

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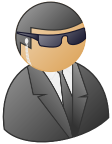
- T-Transform makes the encryption randomness **explicit** $\Longrightarrow$ leaks **commitment**
- We include checks to avoid adaptive attacks like [GPST16, MOXZ24]

Authenticity: Ins-Auth, simplified

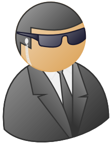


Challenger

Adversary

Authenticity: Ins-Auth, simplified

$(\mathsf{sk}^\star, \mathsf{pk}^\star) \xleftarrow{\$} \mathsf{Gen}$



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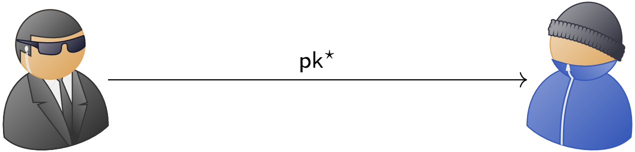

$\mathsf{pk}^\star$

Challenger

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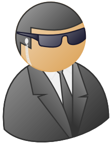

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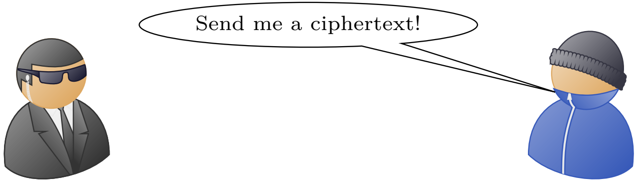

Send me a ciphertext!

Challenger

Adversary

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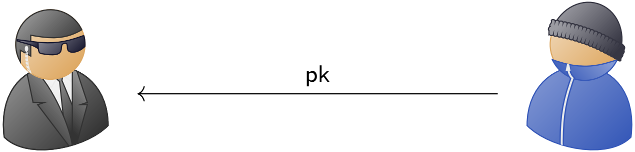
$\mathsf{pk}^\star$



pk

Challenger

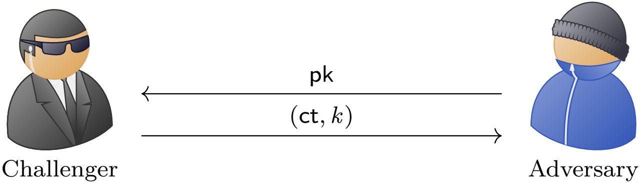
Adversary

Authenticity: Ins-Auth, simplified

$(\mathsf{sk}^\star, \mathsf{pk}^\star) \xleftarrow{\$} \mathsf{Gen}$

$\mathsf{pk}^\star$

$(\mathsf{ct}, k) \xleftarrow{\$} \mathsf{Encaps}(\mathsf{sk}^\star, \mathsf{pk})$



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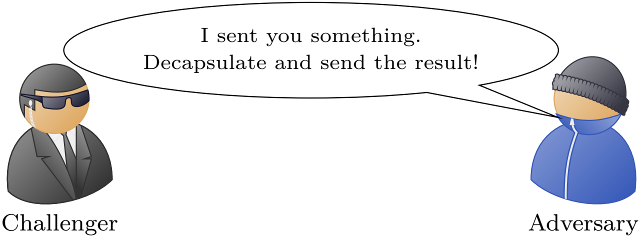

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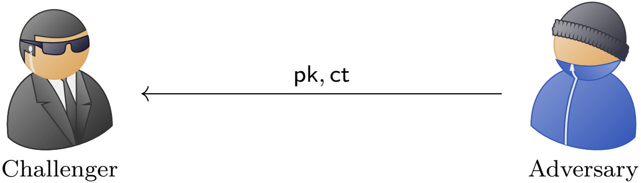

pk, ct

Challenger

Adversary

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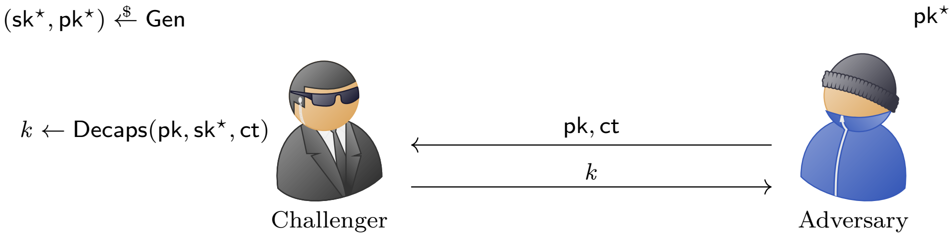

$(\mathsf{sk}^\star, \mathsf{pk}^\star) \xleftarrow{\$} \mathsf{Gen}$

$\mathsf{pk}^\star$

$k \leftarrow \mathsf{Decaps}(\mathsf{pk}, \mathsf{sk}^\star, \mathsf{ct})$

pk, ct

k

Challenger

Adversary

Authenticity: Ins-Auth, simplified

$(\mathsf{sk}^\star, \mathsf{pk}^\star) \xleftarrow{\$} \mathsf{Gen}$

$\mathsf{pk}^\star$

Authenticity: Ins-Auth, simplified

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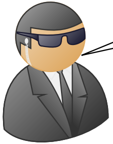
Try to send a fresh ciphertext on my behalf!



Challenger

$\mathsf{pk}^\star$

Adversary

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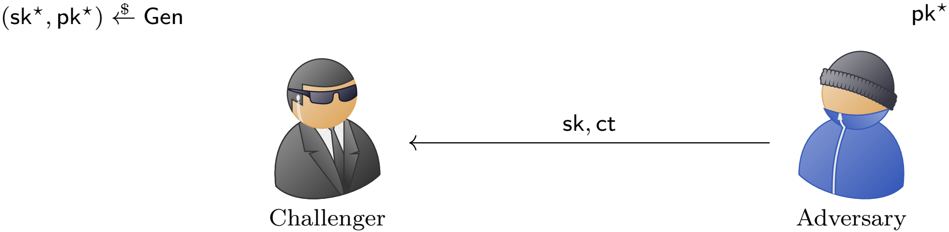

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$\mathsf{pk}^\star$

sk, ct

Challenger

Adversary

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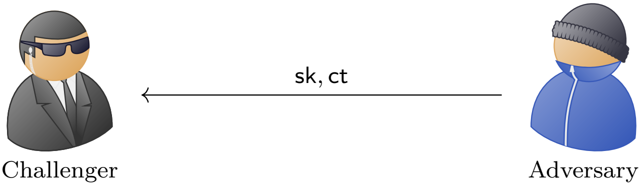
```
$(\mathsf{sk}^\star, \mathsf{pk}^\star) \xleftarrow{\$} \mathsf{Gen}$
if $\mathsf{ct}$ not fresh:
    abort
$k \xleftarrow{\$} \mathsf{Decaps}(\mathsf{pk}^\star, \mathsf{sk}, \mathsf{ct})$
win if $k \neq \bot$
```

$\mathsf{pk}^\star$



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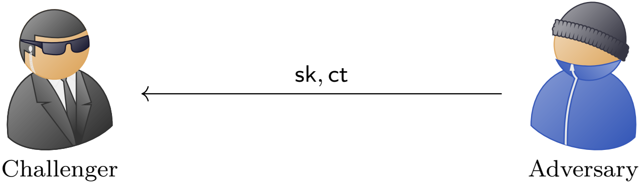

Note

An honest Decaps checks the **signature** against $\mathsf{pk}^\star$ and returns $\bot$ if the signature is invalid

Non-Malleability: Return of the Lollipop

Observation

For Ins-Auth the signature needs to be **non-malleable**

$$\mathsf{ct} = (\mathsf{ct}_{\mathsf{KEM}}, \sigma) \qquad \Longrightarrow \qquad \mathsf{ct}' = (\mathsf{ct}_{\mathsf{KEM}}, \sigma')$$

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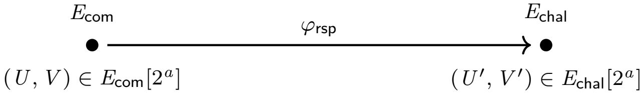
In SQIsignHD, the signature is **interpolation data** $\sigma = (q, U', V')$


E_{com}
φ_{rsp}
E_{chal}
$(U, V) \in E_{\mathsf{com}}[2^a]$
$(U', V') \in E_{\mathsf{chal}}[2^a]$

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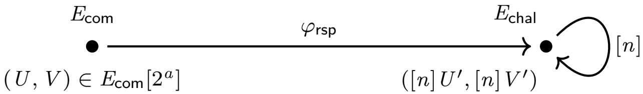

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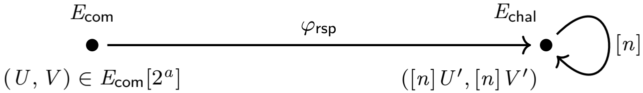

- If $n^2 q \leq 2^a$, then $\sigma' = (\mathbf{n^2} q, [\mathbf{n}]U', [\mathbf{n}]V')$ is a valid signature too

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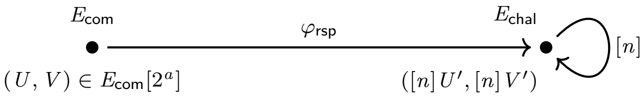

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$$\begin{tikzcd}
E_{\mathsf{com}} \arrow[rr, "\varphi_{\mathsf{rsp}}"] & & E_{\mathsf{chal}} \arrow[loop right, "{[n]}"] \\
(U, V) \in E_{\mathsf{com}}[2^a] & & ([n]\,U', [n]\,V')
\end{tikzcd}$$

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⇒ Non-Malleable version of SQIsignHD?

Non-Malleable SQIsignHD

It would be desirable to **check cyclicity** of HD-represented isogenies

Non-Malleable SQIsignHD

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Bad News

So far, a generic cyclicity check seems out of reach

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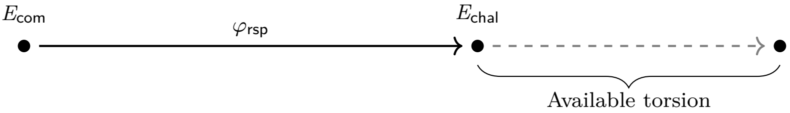
Idea:

- During signing: require **minimum length** $q \geq 2^a / \log p$



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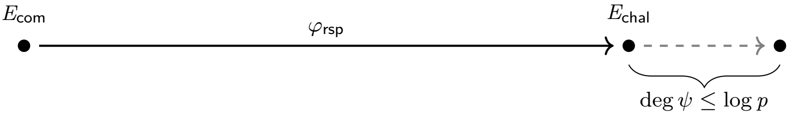


Non-Malleable SQIsignHD

It would be desirable to **check cyclicity** of HD-represented isogenies

Bad News

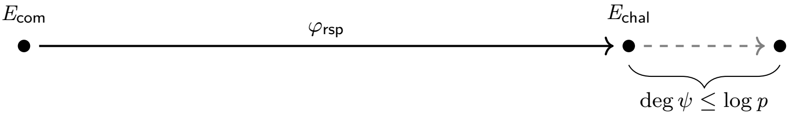
So far, a generic cyclicity check seems out of reach

Good News

We don't need a generic cyclicity check!

Idea:

- During signing: require **minimum length** $q \geq 2^a / \log p$
- During verification: evaluate φ_{rsp} on $E_{\mathsf{com}}[n]$ for all prime $n \leq \sqrt{\log p}$ and see if it vanishes



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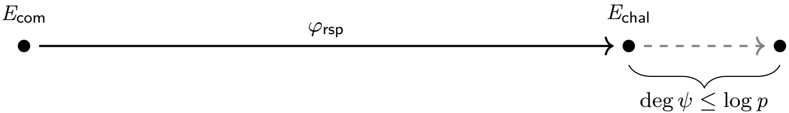

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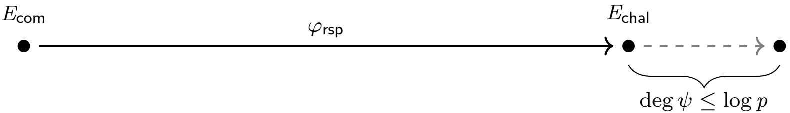

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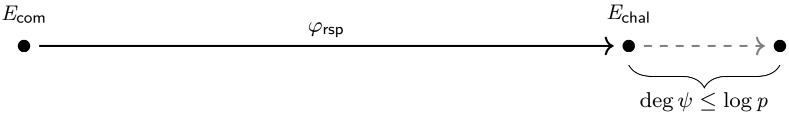

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Non-Malleability of SQIsignHD

For any NM adversary $\mathcal{A}$ against a *slight modification* of SQIsignHD, there exist adversaries $\mathcal{B}$ against OneEnd and $\mathcal{C}$ against Cyclic RUGDIO indistinguishability (CR-IND) such that

$$\mathsf{Adv}^{\mathsf{NM}}(\mathcal{A}) \leq \mathsf{Adv}^{\mathsf{OneEnd}}(\mathcal{B}) + q_{\mathsf{Trans}} \cdot \mathsf{Adv}^{\mathsf{CR\text{-}IND}}(\mathcal{C}).$$

Authenticity of SnakeM

Theorem

For any $\mathsf{Ins\text{-}Aut}$ adversary $\mathcal{A}$ against SnakeM, there exist an adversary $\mathcal{B}$ against $\mathsf{SS\text{-}Enc}$ and an adversary $\mathcal{C}$ against $\mathsf{NM\text{-}Enc}$ such that

$$\mathsf{Adv}^{\mathsf{Ins\text{-}Aut}}_{\mathrm{SnakeM}}(\mathcal{A}) \leq +\mathsf{Adv}^{\mathsf{SS\text{-}Enc}}_{\mathrm{POKÉ},\mathrm{SQIsignHD}}(\mathcal{B}) + \mathsf{Adv}^{\mathsf{NM\text{-}Enc}}_{\mathrm{POKÉ},\mathrm{SQIsignHD}}(\mathcal{C}) + \delta.$$

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- NM: Given $\mathsf{pk}_{\mathsf{ID}}$ and transcripts $\mathcal{T} = \{(\mathsf{com}_i, \mathsf{chal}_i, \mathsf{rsp}_i)\}$, compute $(\mathsf{com}', \mathsf{chal}', \mathsf{rsp}') \notin \mathcal{T}$

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$$\begin{tikzcd}
E_0 & E_{\mathsf{sig}} \arrow[d, "\varphi_{\mathsf{chal}}"] \\
E_{\mathsf{com}} \arrow[r, "\sigma = (q, \varphi_{\mathsf{rsp}}(U), \varphi_{\mathsf{rsp}}(V))"] & E_{\mathsf{chal}}
\end{tikzcd}$$

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(P_3, Q_3) \in E_{\mathsf{ct}} & (P_2, Q_2) \in E_{\mathsf{com}} \arrow[rr, "\sigma = (q, \varphi_{\mathsf{rsp}}(U), \varphi_{\mathsf{rsp}}(V))"] & & E_{\mathsf{chal}}
\end{tikzcd}$$

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$$\begin{tikzcd}
(P_2, Q_2) \in E_{\mathsf{enc}} \arrow[d, dashed, "\varphi'_{\mathsf{com}}"] \arrow[dr, phantom, "\text{Adversary can decrypt :(}"] & (P_0, Q_0) \in E_0 \arrow[l, dashed, "\varphi_{\mathsf{skEnc}}"'] \arrow[d, dashed, "\varphi_{\mathsf{com}}"] & E_{\mathsf{sig}} \arrow[d, "\varphi_{\mathsf{chal}}"] \\
(P_3, Q_3) \in E_{\mathsf{ct}} & (P_2, Q_2) \in E_{\mathsf{com}} \arrow[l, dashed, "\varphi'_{\mathsf{skEnc}}"'] \arrow[r, "\sigma = (q, \varphi_{\mathsf{rsp}}(U), \varphi_{\mathsf{rsp}}(V))"] & E_{\mathsf{chal}}
\end{tikzcd}$$

Compactness – Is It Worth It?

Scheme (variant)	Confidentiality	Authenticity	Deniability	PQ	Size (in bytes)	
					ct	pk
Group-based						
DH-AKEM [ABH$^+$21]	**Ins-CCA**	**Out-Aut**	**DR***	✗	32	32
Zheng [Zhe97, BSZ02]	**Ins-CCA**	**Ins-Aut**	**HR***	✗	64	64
Lattice-based						
EtStH-AKEM (BAT + Antrag) [AJKL23]	**Ins-CCA**	**Out-Aut**	—	✓	1 119	1 417
NIKE-AKEM (Swoosh) [AJKL23]	**Ins-CCA**	**Out-Aut**	**DR***	✓	$>$ 221 184	$>$ 221 184
EaNtH-AKEM (BAT + Swoosh)	**Ins-CCA**	**Out-Aut**	**DR***	✓	473	$>$ 221 705
FrodoKEX+ [CHN$^+$24b]	**IND-1BatchCCA**	**UNF-1KCA**	**DR**	✓	72	21 300
Den. AKEM (BAT + Gandalf) [GJK24]	**Ins-CCA**	**Out-Aut**	**HR & DR**	✓	1 749	1 417
Isogeny-based						
EtStH-AKEM (POKÉ + SQIsignHD) [AJKL23]	**Ins-CCA**	**Out-Aut**	—	✓	493	432
NIKE-AKEM (CSIDH) [AJKL23]	**Ins-CCA**	**Out-Aut**	**DR***	✓	256†	256†
EaNtH-AKEM (POKÉ + CSIDH)	**Ins-CCA**	**Out-Aut**	**DR***	✓	384	624
Den. AKEM (POKÉ + Erebor) [GJK24]	**Ins-CCA**	**Out-Aut**	**HR & DR**	✓	740	432
SnakeM	**Ins-CCA**	**Ins-Aut**	**HR**	✓	296	368

Open Questions

Cryptanalysis

- $\mathsf{OW\text{-}KCA}$ of POKÉ + Countermeasures
- Additional Enc oracle in SS and NM

Other Constructions

- Though there are already some ideas...

Better Security Proof

- Reduce $\mathsf{NM\text{-}Enc}$ and $\mathsf{SS\text{-}Enc}$ to (more) standard assumptions
- Maybe in an Algebraic Isogeny Model

Questions?

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✉ research@meers.org

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SnakeM in Detail

```
$\underline{\mathsf{SnakeM.Gen}}$
$(\mathsf{sk}_{\mathsf{KEM}}, \mathsf{pk}_{\mathsf{KEM}}) \xleftarrow{\$} \mathsf{KEM.Gen}$
$(\mathsf{sk}_{\mathsf{ID}}, \mathsf{pk}_{\mathsf{ID}}) \xleftarrow{\$} \mathsf{ID.Gen}$
$\mathsf{s} \xleftarrow{\$} \{0,1\}^{\eta}$
$\mathsf{sk} \leftarrow (\mathsf{sk}_{\mathsf{KEM}}, \mathsf{sk}_{\mathsf{ID}}, \mathsf{s})$
$\mathsf{pk} \leftarrow (\mathsf{pk}_{\mathsf{KEM}}, \mathsf{pk}_{\mathsf{ID}})$
$\textbf{return } (\mathsf{sk}, \mathsf{pk})$

$\underline{\mathsf{SnakeM.Encaps}(\mathsf{sk}_{\mathsf{SND}}, \mathsf{pk}_{\mathsf{RCV}})}$
$\textbf{parse } \mathsf{sk}_{\mathsf{SND}} = (\cdot, \mathsf{sk}_{\mathsf{ID}}, \cdot)$
$\textbf{parse } \mathsf{pk}_{\mathsf{RCV}} = (\mathsf{pk}_{\mathsf{KEM}}, \cdot)$
$\mathsf{pk}_{\mathsf{ID}} \leftarrow \mathsf{derive}(\mathsf{sk}_{\mathsf{ID}})$
$\mathsf{pk}_{\mathsf{SND}} \leftarrow \mathsf{derive}(\mathsf{sk}_{\mathsf{SND}})$
$(\mathsf{com}, R) \xleftarrow{\$} \mathsf{ID.Com}$    \\ $\mathsf{com} = \mathsf{ct}_0$
$(\mathsf{ct}_1, K) \xleftarrow{\$} \mathsf{KEM.Encaps}_1(\mathsf{pk}_{\mathsf{KEM}}, R)$
$(\mathsf{chl}, \mathsf{pad}) \leftarrow \mathsf{G}(\mathsf{pk}_{\mathsf{ID}}, \mathsf{com}, \mathsf{pk}_{\mathsf{RCV}}, \mathsf{ct}_1, K)$
$\mathsf{rsp} \xleftarrow{\$} \mathsf{ID.Rsp}(\mathsf{sk}_{\mathsf{ID}}, \mathsf{com}, \mathsf{chl}, R)$
$\mathsf{ct}_{\mathsf{rsp}} \leftarrow \mathsf{rsp} \oplus \mathsf{pad}$
$\mathsf{ct} \leftarrow (\mathsf{com}, \mathsf{ct}_1, \mathsf{ct}_{\mathsf{rsp}})$
$\mathsf{k} \leftarrow \mathsf{H}(K, \mathsf{com}, \mathsf{ct}_1, \mathsf{rsp}, \mathsf{pk}_{\mathsf{SND}}, \mathsf{pk}_{\mathsf{RCV}})$
$\textbf{return } (\mathsf{ct}, \mathsf{k})$

$\underline{\mathsf{SnakeM.Decaps}(\mathsf{pk}_{\mathsf{SND}}, \mathsf{sk}_{\mathsf{RCV}}, \mathsf{ct})}$
$\textbf{parse } \mathsf{pk}_{\mathsf{SND}} = (\cdot, \mathsf{pk}_{\mathsf{ID}})$
$\textbf{parse } \mathsf{sk}_{\mathsf{RCV}} = (\mathsf{sk}_{\mathsf{KEM}}, \cdot, \mathsf{s})$
$\textbf{parse } \mathsf{ct} = (\mathsf{com}, \mathsf{ct}_1, \mathsf{ct}_{\mathsf{rsp}})$
$\mathsf{pk}_{\mathsf{RCV}} \leftarrow \mathsf{derive}(\mathsf{sk}_{\mathsf{RCV}})$
$K \leftarrow \mathsf{KEM.Decaps}(\mathsf{sk}_{\mathsf{KEM}}, \mathsf{com}, \mathsf{ct}_1)$
$\textbf{if } K = \bot$    \\ Decaps may fail
    $K \leftarrow \mathsf{s}$
$(\mathsf{chl}, \mathsf{pad}) \leftarrow \mathsf{G}(\mathsf{pk}_{\mathsf{ID}}, \mathsf{com}, \mathsf{pk}_{\mathsf{RCV}}, \mathsf{ct}_1, K)$
$\mathsf{rsp} \leftarrow \mathsf{ct}_{\mathsf{rsp}} \oplus \mathsf{pad}$
$\textbf{if } \mathsf{ID.Ver}(\mathsf{pk}_{\mathsf{ID}}, \mathsf{com}, \mathsf{chl}, \mathsf{rsp}) = 1 :$
    $\mathsf{k} \leftarrow \mathsf{H}(K, \mathsf{com}, \mathsf{ct}_1, \mathsf{rsp}, \mathsf{pk}_{\mathsf{SND}}, \mathsf{pk}_{\mathsf{RCV}})$
    $\textbf{return } \mathsf{k}$
$\textbf{return } \bot$
```