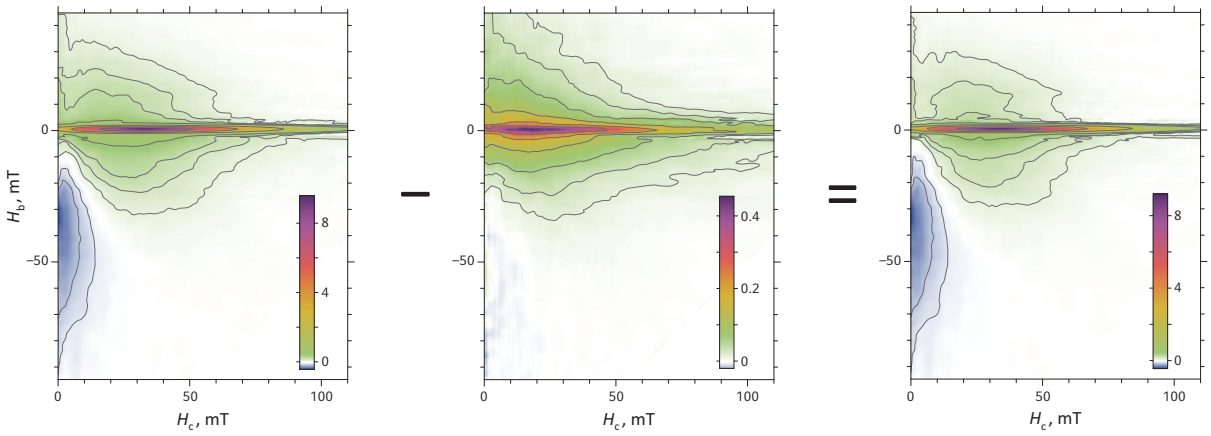
VARIFORC User Manual
Chapter 7:

Combine FORC diagrams



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7.1 LinearCombineFORC highlights

`LinearCombineFORC` is a function of the VARIFORC package that is used for calculating the linear combination of an arbitrary number of corrected FORC measurements (produced by `ImportFORC`) or FORC diagram matrices (produced by `CalculateFORC`). Some application examples are:

- Averaging multiple measurements for signal-to-noise ratio improvements [e.g. *Egli*, 2013],
- Subtraction of a background signal from sample holders or materials in which magnetic particles are dispersed, and
- Calculation of measurement differences, for instance between FORC data obtained from the same sample before and after a chemical treatment used for dissolving a specific magnetic component [e.g. *Ludwig et al.*, 2013].

`LinearCombineFORC` is based on the following steps (Fig. 7.1):

1) Start a system dialog for uploading corrected FORC measurements (i.e. files ending with `CorrectedMeasurements_VARIFORC.frc`, previously produced by `ImportFORC`) or FORC matrices (i.e. files ending with `FORC_VARIFORC.frc`, previously produced by `CalculateFORC` or `IsolateCR`), and assigning to each dataset a linear combination coefficient. The number of files that can be uploaded is unlimited.

2) Check the compatibility of selected data in terms of structure and magnetic units. Corrected measurements must originate from exactly the same FORC protocol (i.e. same field step and measurement range), because interpolation is not used at this stage of FORC processing. FORC matrices must have coincident points (i.e. same grid mesh size and origin) but can extend over different ranges. Linear combinations of FORC data obtained with different measurement protocols can be obtained after calculating individual FORC diagrams over the same grid of points in FORC space with the VARIFORC function `CalculateFORC`. When FORC data with compatible magnetization units (e.g. Am^2/kg and emu/g) are combined, units are converted automatically by the VARIFORC unit management system.

3) Use the linear combination coefficients $c_1, c_2, \ldots, c_n$ entered with step 1 to calculate output data $\mathbf{M}_{out} = c_1\mathbf{M}_1 + c_2\mathbf{M}_2 + \ldots + c_n\mathbf{M}_n$. For example, the difference between two measurements is calculated as $\Delta\mathbf{M} = c_1\mathbf{M}_1 + c_2\mathbf{M}_2$ with $c_1 = +1$ and $c_2 = -1$. In case of measurement averaging, coefficients $c_i = 1/n$ are calculated automatically. Output data are saved with the same format as input files and can be used for further processing with other VARIFORC functions. In case of FORC matrices, the linear combination is calculated for the largest rectangular FORC space that is common to all data.

4) If corrected FORC measurements have been uploaded, linear combinations are calculated automatically for the measurement themselves (i.e. files ending with `_CorrectedMeasu`

rements_VARIFORC.frc), and for the associated measurement differences (i.e. files ending with _CorrectedMeasurementDifferences_VARIFORC.frc). If FORC matrices have been uploaded, linear combinations are calculated automatically for the matrices themselves (i.e. files ending with FORC_VARIFORC.txt), as well as the associated standard errors (i.e. files ending with FORCStandardError_VARIFORC.txt) and coercivity distributions (i.e. files ending with _Backfield_Linear_VARIFORC.txt, _Backfield_Log10_VARIFORC.txt, _Reversal_Linear_VARIFORC.txt, _Reversal_Log10_VARIFORC.frc, _CentralRidge_Linear_VARIFORC.frc, _CentralRidge_Log10_VARIFORC.frc.

Follow the instructions of this manual to enjoy the full functionality of LinearCombineFORC, or refer to the quick VARIFORC guide for using basic options.

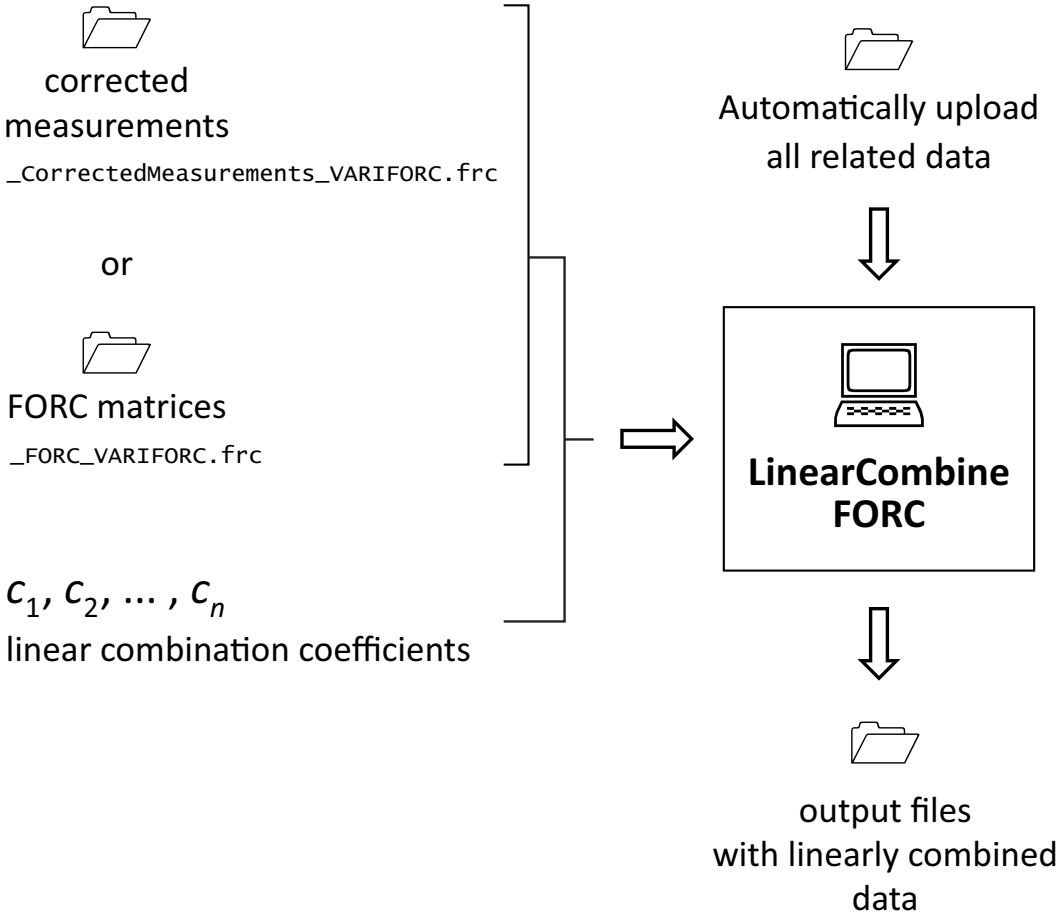


Fig. 7.1: Graphical representation of the LinearCombineFORC workflow.

7.2 Using LinearCombineFORC

Like all VARIFORC functions, LinearCombineFORC runs on a Wolfram Mathematica® notebook. A starting copy of this notebook is provided with the VARIFORC installation package under:

```
VARIFORC_Install/Packages/LinearCombineFORC/VARIFORC_LinearCombineFORC.cdf
```

You can copy this file to a different folder for convenience. The Mathematica® notebook stores all processing steps and graphical results for your records. Therefore, you best rename it with reference to the FORC data you want to process.

All LinearCombineFORC notebooks begin with the following command line:

```
Get[FileNameJoin[{$HomeDirectory,"VARIFORC","VARIFORC_LinearCombineFORC_code.txt"}]]
```

for uploading LinearCombineFORC to the computation kernel. This command, as any other command in Mathematica® notebooks, is executed by placing the cursor on it and pressing the keys SHIFT and ENTER at the same time. A copyright message will appear below this line, confirming that LinearCombineFORC has been uploaded successfully.

LinearCombineFORC is then called by the command line

```
VARIFORC`LinearCombineFORC
```

to be executed by pressing the keys SHIFT and ENTER at the same time. At this point, LinearCombineFORC starts a system dialog for:

1) uploading FORC data,
2) uploading linear combination coefficients, and
3) defining output files where results are stored.

The notebook appearance upon performing these steps is shown in Fig. 7.2.

The files required by step 1) should be ready before LinearCombineFORC is started. Unlike other VARIFORC functions, there are no processing options to be uploaded with a parameter file. After terminating the system dialog, LinearCombineFORC proceeds autonomously until the end, without requesting any further action by the user. The processing status is continuously updated by progress messages, e.g.

```
Read E:\VARIFORC\Examples\PelagicCarbonate\Leg138-S0-SF5_FORC_VARIFORC.txt ...
```

warning messages, e.g.

```
WARNING! Different field units encountered.
```

and error messages, e.g.

```
Incompatible magnetization units encountered. Program aborted.
```

❶ Upload source codes

```
In[1]:= Get[FileNameJoin[{$HomeDirectory, "VARIFORC", "VARIFORC_LinearCombineFORC_code.txt"}]]

Function VARIFORC`LinearCombineFORC for importing FORC measurements.
[VARIFORC package v1.0 for Wolfram Mathematica and Mathematica Player Pro.
 © 2014 by Ramon Egli. All rights reserved.]
```

If used for scientific publications and presentations please cite as follows:
Egli, R. (2013). VARIFORC: An optimized protocol for calculating non-regular first-order reversal curve (FORC) diagrams. *Global and Planetary change 110*, 302-320.
http://dx.doi.org/10.1016/j.gloplacha.2013.08.003

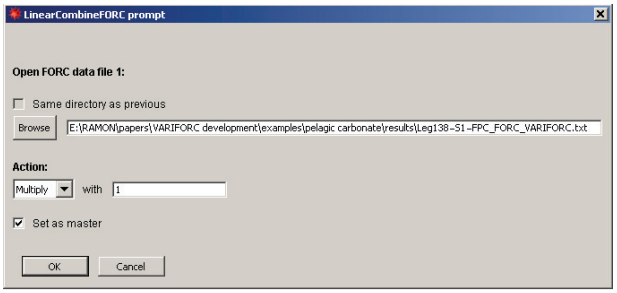
❷ Call CalculateFORC

```
In[2]:= VARIFORC`LinearCombineFORC
```

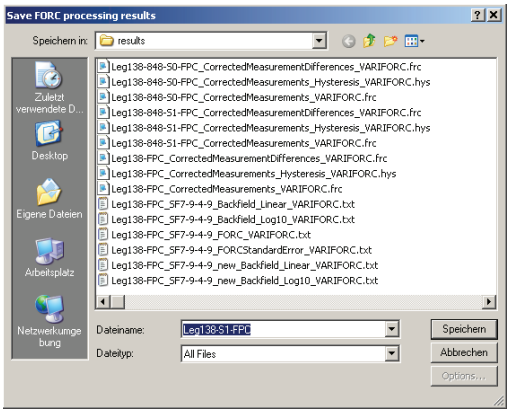
❸ Read auxiliary files

```
Initialization...
```

❹ Upload data files



❺ Define output file(s)



❻ Begin processing

```
master : 1 × E:\VARIFORC\Examples\PelagicCarbonate\Leg138-S0-SF5_FORC_VARIFORC.txt
regular: -1 × E:\VARIFORC\Examples\PelagicCarbonate\Leg138-S1-SF5_FORC_VARIFORC.txt

Read FORC matrices ...
Read E:\VARIFORC\Examples\PelagicCarbonate\Leg138-S0-SF5_FORC_VARIFORC.txt
Read E:\VARIFORC\Examples\PelagicCarbonate\Leg138-S1-SF5_FORC_VARIFORC.txt

Combine FORC matrices ...
Trim E:\VARIFORC\Examples\PelagicCarbonate\Leg138-S0-SF5_FORC_VARIFORC.txt
  to common range of all FORC matrices.

Export results ...
```

Fig. 7.2: Initialization of the LinearCombineFORC notebook.

LinearCombineFORC ends with the following message containing the total computation time:

```
PROGRAM END. Total computation time 1m 5s.
```

You can save the notebook file with all messages shown above for your later records and re-use it.

LinearCombineFORC error messages usually produce a program abort. If one of these messages appears, an error was encountered when reading data files, e.g.:

```
Error encountered in reading FORC matrix file (Grid dimensions). Program aborted.
```

or in combining FORC data, e.g.:

```
Different grid meshes encountered. FORC matrices cannot be combined. Program aborted.
```

Error messages contain a hint about the error source and on how to solve the problem. FORC data reading errors occur only if data files produced by VARIFORC functions have been modified. On the other hand, errors encountered in combining FORC data are produced by incompatibilities related to different measurement protocols, non-coincident FORC matrix meshes, and incompatible units. For example, FORC data expressed in mass-normalized magnetization units (e.g. mAm^2/kg) cannot be combined with other data expressed as magnetic moments (e.g. mAm^2). Such errors are avoided by correctly planning FORC measurements (e.g. using the same FORC protocol if corrected measurements are combined) and FORC processing (e.g. FORC diagram calculations on compatible output grids).

When combining FORC data, take the following general rules into account:

1) FORC measurements should be combined at the earliest possible processing stage, i.e. as drift- and outlier-corrected measurements obtained with the VARIFORC function ImportFORC (file names ending with _CorrectedMeasurements_VARIFORC.frc). For this purpose, measurement should be planned in advance to be obtained with the same FORC measuring protocol. Each set of FORC measurements should be properly normalized with ImportFORC in order to obtain compatible magnetization units (e.g. mass-normalized magnetizations for all datasets). Certain operations are meaningful only with a certain type of units (e.g. magnetic moments vs. mass-normalized magnetizations). See section 7.4 for further details and examples.

2) In some cases, data are better combined as FORC matrices (i.e. FORC_VARIFORC.txt files produced by CalculateFORC). This is the only choice available for combining data obtained with different measurement protocols. It is also the recommended choice when FORC matrices are best obtained with individual processing options (e.g. the combination of measurements with and without a central ridge). See section 7.4 for further details and examples.

3) LinearCombineFORC does not interpolate data, thus avoiding interpolation artifacts. Therefore, field coordinates of FORC measurements or FORC matrix points must be exactly coincident. For this reason, always use “round” numbers when specifying the output grid of FORC diagrams with CalculateFORC, instead of automatic options. For example, an output grid mesh size of 0.5 mT should be used with FORC measurements obtained with field steps of ~0.4-1 mT. On the other hand, the output grid origin can be set automatically

by `CalculateFORC` unless specific requirements need to be met. See section 7.4 for further details and examples.

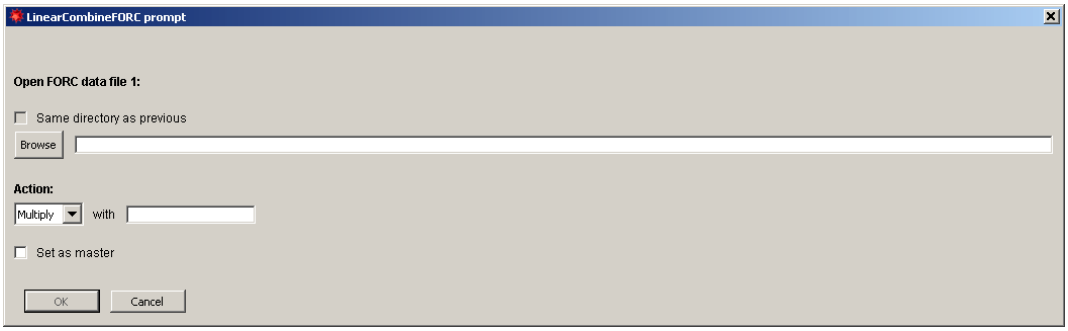
Section 7.4 provides an in-depth discussion of useful linear combination applications.

7.3 Upload of data files and output definition

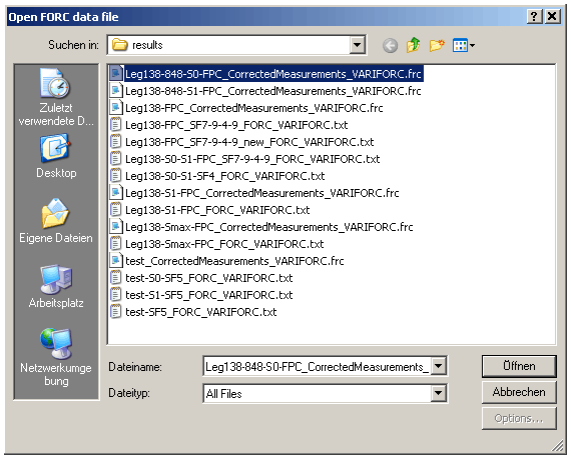
This section describes in detail how data files and corresponding linear combination coefficients can be uploaded with `LinarCombineFORC`, and how an output with linearly combined data is created and exported.

7.3.1 Upload FORC data files

As soon as `LinarCombineFORC` is called with the key combination SHIFT and ENTER, a system dialog is started for uploading FORC data files. The following window prompt will appear



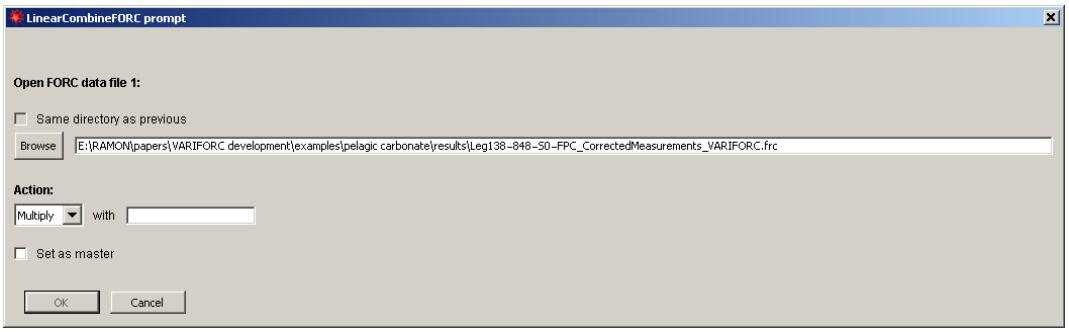
for uploading the first data file. Click on the “Browse” button for file upload and locate the desired data file, e.g.:



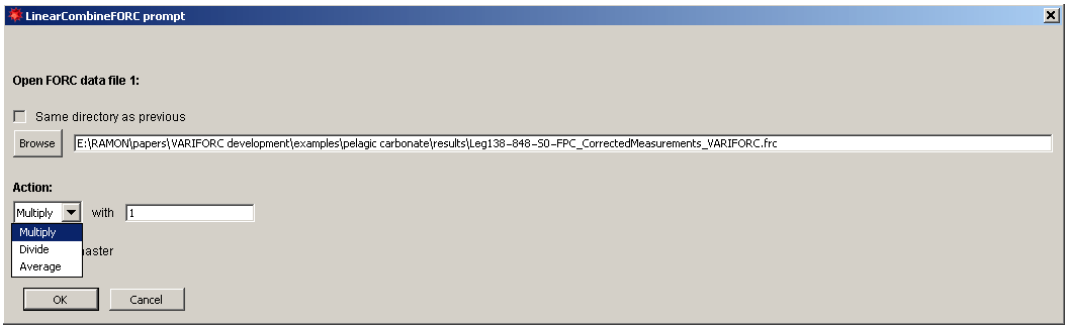
Only two types of files are displayed and selectable: these are: (1) corrected measurement files ending with `_CorrectedMeasurements_VARIFORC.frc` (produced by `ImportFORC`), and (2) FORC matrix files ending with `FORC_VARIFORC.txt` (produced by `CalculateFORC` and other VARIFORC functions such as `IsolateCR`). Related files, such as corrected measurement differences and coercivity distributions, are automatically uploaded and processed together with the chosen ones. These files are always stored *in the same directory* of the chosen file and should never be moved individually to a different place.

- Groups of files produced by VARIFORC functions are characterized by the same name root followed by different endings separated by underscores ("_"). Because such files are needed as a group for further processing, they should always remain in the same directory.
- If `LinearCombineFORC` does not find files associated with the chosen ones, only available data will be processed after giving a warning message. However, further processing of FORC data generated in this manner might be limited and it is always advisable to solve the missing file problem first.

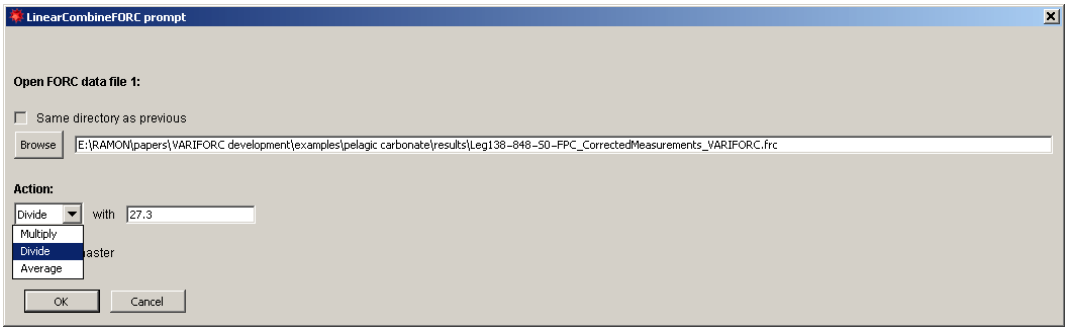
The uploaded file name will appear in the dialog window:



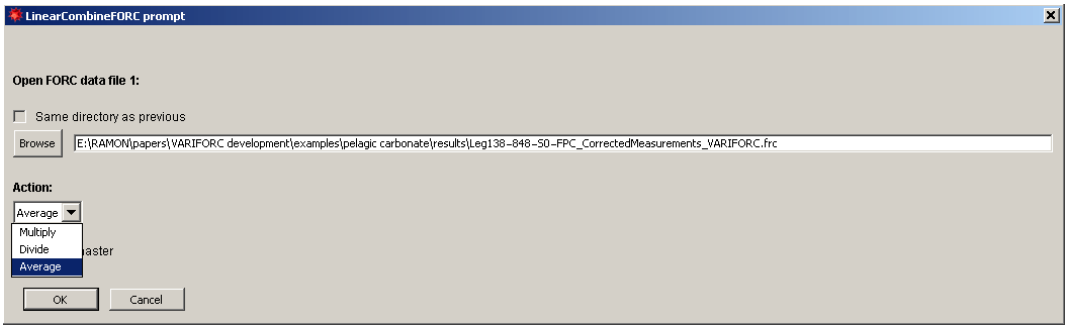
and this choice can be modified by clicking again on the "Browse" button. The starting directory opened with the "Browse" button coincides with the file location of the last `LinearCombineFORC` session, and with a default directory if `LinearCombineFORC` is used for the first time). Once the chosen file is uploaded, you decide what action will be undertaken with the FORC data contained in it, using the options appearing under "**Action:**". The drop-down menu offers three possibilities: (1) multiply with a numerical coefficient that you type in the field to the right, e.g.



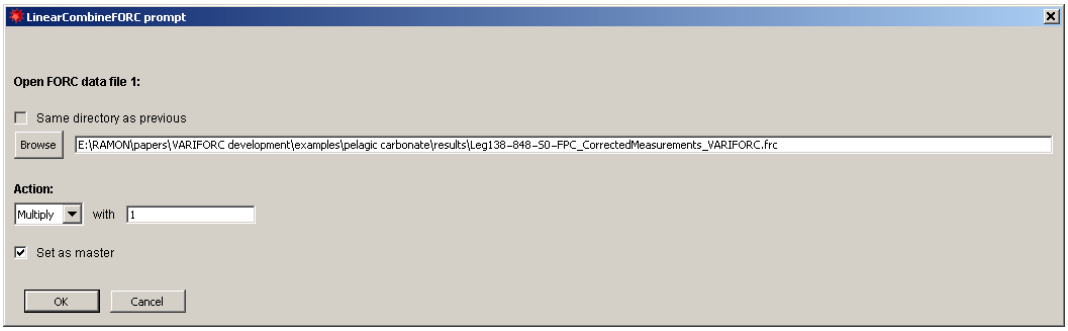
(2) divide by a numerical coefficient that you type in the field to the right, e.g.



and (3) calculate an average with other FORC data, i.e.

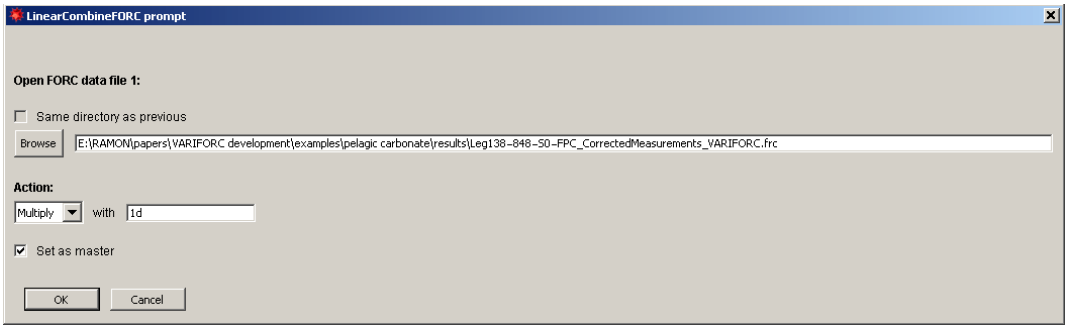


in which case a numerical factor is not asked. Once you have chosen an action and the corresponding numerical factor (if required), the "OK" button on the bottom of the window becomes active and you can click on it to store the selected options and continue with the next file. The last option to be considered before clicking the "OK" button is whether the chosen file should be taken as master or regular file. Click on "Set as master", i.e.

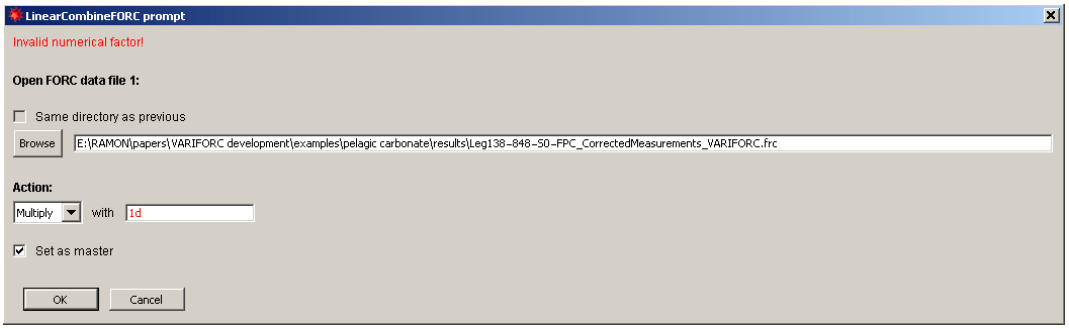


if you want this file to be the master. Differences between master and a regular files are related to how FORC data are combined, and will be explained in detail in section 7.3.3. The choice of a master file is not mandatory, but once you have chosen the master, the choice button will be disabled during further file upload.

A valid numerical factor must be entered with the actions “Multiply” and “Divide”. If you forget to do so, the “OK” button remains disabled. Invalid factors, e.g.

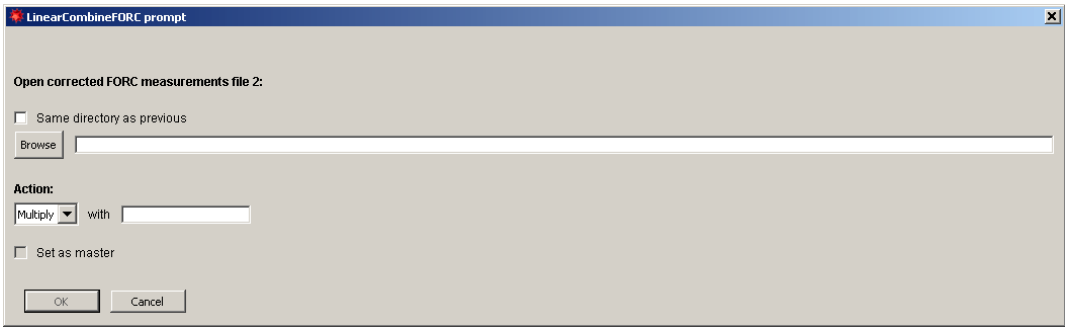


will generate an error message once you click on “OK”:



Valid factors are positive or negative non-zero numbers entered in standard form (e.g. `0.0023`) or with scientific notation (e.g. `2.3e-3`). You can decide to interrupt the input process at any time (for example after realizing that needed files are not available) by clicking on the “Cancel” button.

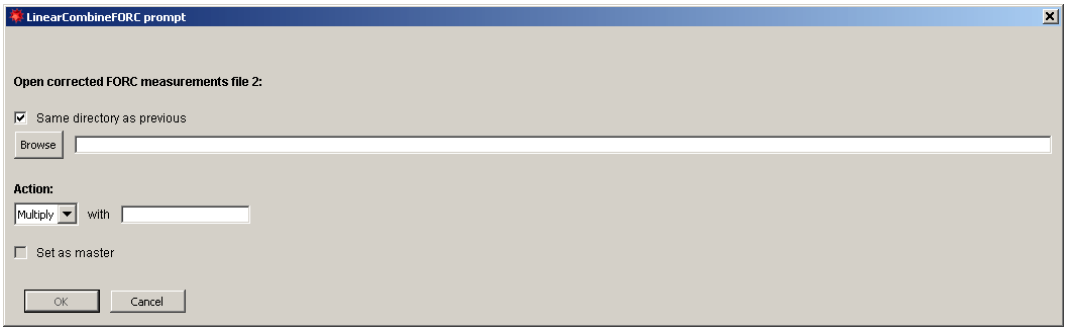
Subsequent FORC data files are entered with the same procedure used for the first one. The window interface remains unchanged, e.g.



but some options limited might be limited by previous choices. For example, only files of the same type as the previously chosen ones can be uploaded, i.e. either corrected measurements (`_CorrectedMeasurements_VARIFORC.frc`) or FORC matrices (`FORC_VARIFORC.txt`). The two data types cannot be mixed. The choice of “**Action:**” options is also limited, depending on previous uploads: if the option used for these uploads was “Multiply” or “Divide”, “Average”

will be excluded and vice-versa. With other words, it is possible to mix “Multiply” and “Divide” options, while “Average” must be applied to all files. Finally, if a master file has already been chosen, the “Set as master” option will be inactive.

Starting from the second upload, the option “Same directory as previous” will become active, e.g.



enabling you to browse for the next file starting from the same directory of the last upload. This option is particularly useful when files are stored in the same directory or in closely related directories.

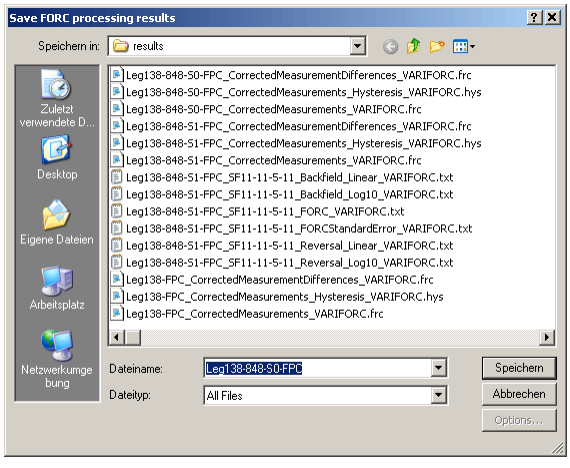
You can upload an unlimited number of FORC data files with the procedure described above. A list of uploaded files and corresponding factors is printed in the Mathematica® notebook and updated after each upload, e.g.

```
master :  1 × E:\VARIFORC\pelagic carbonate\Leg138-S0_CorrectedMeasurements_VARIFORC.frc
regular: -1 × E:\VARIFORC\pelagic carbonate\Leg138-S0_CorrectedMeasurements_VARIFORC.frc
```

In order to stop the file uploading process, click the “Cancel” button when the prompt window appears after choosing the last file.

7.3.2 Choose a root name for the output files

After stopping the upload process, a new window will appear, asking for location a common name root of all output files, e.g.

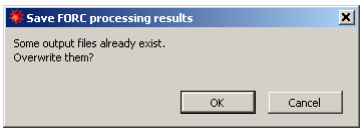


A default location for and root name is proposed on the basis of the master file, or, if a master file has not been chosen, of the first uploaded file. Modify the proposed name according to your requirements, *without adding any VARIFORC extension*, such as `FORC_VARIFORC.txt`. Appropriated VARIFORC extensions will be added automatically by `LinearCombineFORC` according to the type of FORC data being exported.

Uploaded files cannot be overwritten; therefore you have to change the proposed root name or the directory in which results are stored. In case of conflicts between the chosen output file names and uploaded files, a warning message is printed, e.g.

```
Output files root name is in conflict with uploaded FORC matrices.
Choose a root name that does not overwrite uploaded files.
```

and you are asked to enter a different root name. Overwriting of other files that do not serve as data source is possible upon clicking “OK” to the following warning message:



In order to avoid confusions between output files generated by `LinearCombineFORC` and existing FORC data files, it is strongly recommended to include some information about the origin of the new data files in their name. For example, the words “average” and “difference” can be added to recall that these files represent an average or a difference of existing data.

Complete information about data sources is stored in the output files, so that traceability of FORC data is always granted (see section 7.4).

7.3.3 Linear combination of corrected measurement files and the role of master files

Corrected measurement files produced by `ImportFORC` contain a header with essential information about the measurement source, protocol, data structure, units, and normalization, followed by a two-column list of measurements with measurement fields in the first column, and magnetization values in the second column. Each row (H_k, M_k) thus represents a single FORC measurement. Linear combination of corrected measurements taken from different files is possible only for identical measuring protocols, so that the k-th measurement (H_{ik}, M_{ik}) taken from the i-th uploaded file is performed in a field H_{ik} that is the same for all datasets. The values H_{ik} thus represent identical measurement fields and a linear combination of measurements with coefficients $c_1, \ldots, c_n$ is simply given by:

$$M_k^{out} = \sum_{i=1}^{n} c_i M_{ik} \tag{7.1}$$

with the following options for c_i:

(1) c_i coincides with the numerical factor entered together with the corresponding data file if the "Action" option was set to "Multiply" during uploading,

(2) c_i is the reciprocal of the numerical factor entered together with the corresponding data file if the "Action" option was set to "Divide" during uploading,

(3) $c_i = 1/n$ if the "Action" option was set to "Average" during uploading

(see section 7.3.1). In reality, measured fields H_{ik} are not exactly coincident, because of small field control and measurement errors, so that a single set of field values has to be created for the output file. If a master file has been chosen, the set of output fields will coincide with that of the master file, so that output data are generated by

$$(H^{out}, M_k^{out}) = \left(H_{mk}, \sum_{i=1}^{n} c_i M_{ik} \right) \tag{7.2}$$

where m is the index of the master file. This option is recommended in all cases where the main contribution to the output comes from a single data source, which is therefore chosen as master. A typical example is represented by the subtraction of one dataset from another, if measurements to be subtracted involve much weaker magnetizations (e.g. a pelagic carbonate after dissolution single-domain magnetite [*Ludwig et al.*, 2013]). If, on the other hand, all datasets contribute in a similar manner to M_k^{out}, as when averaging identical measurements, a single set of output field is best generated by averaging all measured field values. In this case, averaging will reduce the field scatter produced by field measurement errors. Therefore, a master file *should not be chosen* when combining equivalent FORC data. If a master file has not been chosen, output data are calculated with the following equation:

$$(H^{out}, M_k^{out}) = \left(\frac{1}{n} \sum_{i=1}^{n} H_{ik}, \sum_{i=1}^{n} c_i M_{ik} \right) \tag{7.3}$$

where output field values are obtained from the average of all datasets.

The output file header, which contains supporting information about processing options and summary parameters, is generated with a template taken from the master file, if it has been chosen, and from the first uploaded file otherwise. Processing options are copied as is, while summary parameters, i.e. estimates of the saturation remanence and saturation magnetization, are combined in exactly the same manner as FORC data, if available in all uploaded files. Corresponding standard errors, on the other hand, are calculated according to the equations reported in section 7.3.4.

Measurement protocol, as well as magnetization units, must be compatible in order for uploaded measurement data to be combined correctly. The compatibility of uploaded measurements is checked by LinearCombineFORC before generating output data. Warning messages are generated if uploaded FORC measurements are expressed with different, but compatible field and magnetization units, e.g.

```
WARNING! Different magnetization units encountered. emu/g will be converted to Am2/kg.
```

On the other hand, FORC measurements expressed with incompatible magnetization units (e.g. magnetic moments and mass-normalized magnetizations) cannot be combined, and the following error message is generated:

```
Incompatible magnetization units emu and Am2/kg encountered.
  Data files cannot be combined. Program aborted.
```

Therefore, always ensure that FORC measurements are expressed with compatible magnetization units, i.e. S.I. or c.g.s. units for magnetic moment, and mass- volume- and area-normalized magnetizations, respectively.

Finally, measurement protocol compatibility is verified on the basis of data structure (number of measured curves and number of measurements in each curve), as well as the mean field step sizes reported in file headers. A 1% tolerance is used when comparing mean field step sizes, because of the unavoidable small differences due to finite field control and measurement precisions. This tolerance is sufficient for testing the identity of FORC protocols used with Micromag™ VSM and AGM magnetometers. If a measurement protocol mismatch is detected among uploaded files, the following error message

```
Measurement protocol mismatch. Only identical protocols can be combined. Program aborted.
```

is generated, and further processing is interrupted. In this case, linear combination of corrected measurements is not possible, and FORC matrices should be used instead.

If data structures coincide, but field step differences >1% are detected, the following warning message is produced

```
WARNING! Measurement field steps differ by >1%. Measurement protocols might not coincide.
```

and data processing continues. In this case, however, you should check whether the uploaded files were effectively produced by the same FORC protocol.

- FORC measurement protocols are checked only on the basis of data structure and field step sizes. This does not ensure that measurement protocols are identical: for example, differences could exist between averaging times, pauses, and actual measurement fields. Therefore, the identity of measurement protocol should be ensured independently of LinearCombineFORC checks, which aim only at ensuring that linear combinations can be calculated without interpolation.

7.3.4. Linear combination of FORC matrices and related coercivity distributions

For each uploaded FORC matrix file with generic name `file_FORC_VARIFORC.txt`, the following files are expected to be found in the same directory:

1) FORC matrix: `file_FORC_VARIFORC.frc`
2) FORC standard error matrix: `file_FORCStandardError_VARIFORC.frc`
3) Backfield coercivity distribution on linear scale: `file_Backfield_Linear_VARIFORC.frc`
4) Backfield coercivity distribution on log10 scale: `file_Backfield_Log10_VARIFORC.frc`
5) Reversal field distribution on linear scale: `file_Reversal_Linear_VARIFORC.frc`
6) Reversal field distribution on log10 scale: `file_Reversal_Log10_VARIFORC.frc`

for the calculation of corresponding output files. All FORC data in these files correspond to FORC coordinates or coercivity fields defined exactly by the FORC matrix grid specified in `file_FORC_VARIFORC.frc`. Therefore, unlike the case of corrected measurements (section 7.3.3), the choice of a master file is not necessary for defining field values, and master FORC matrices serve only to the definition of output file headers. If a master file has not been specified, the first uploaded file is taken for this purpose.

If FORC data associated with the uploaded file names (e.g. coercivity distributions) are missing, `LinearCombineFORC` produces a warning message, e.g.

```
WARNING! At least one linear coercivity distribution file corresponding to selected
  data files is missing. Coercivity distributions will be ignored.
```

and continues processing by ignoring all FORC data of the missing type. If such warning messages are produced, it is advisable to check whether the missing files have been accidentally moved to a different directory.

Before proceeding with data processing, `LinearCombineFORC` checks the compatibility of uploaded FORC data. Two conditions must be fulfilled for compatible data: first, magnetization units should be of the same type (see section 7.3.3), and, second, all FORC matrices and coercivity distributions should be based on perfectly overlapping grids of field values. More precisely, compatible datasets are characterized by a common, non-empty, rectangular FORC region, and data points within this region belong to the same grid (Fig. 7.3). This means that FORC matrix grids must have same mesh size and same origin.

- FORC matrix grids are controlled by the options `INPUT 06` (output mesh size) and `INPUT 07` (output grid origin) of the parameter file read by `CalculateFORC` (see Chapter 4). The FORC space covered by the FORC matrix is specified by the parameters `INPUT 03` and `INPUT 04` of the same parameter file (horizontal and vertical FORC ranges).

- In order to ensure compatible FORC matrices, the `INPUT 06` options `Normal` and `Fine` should be avoided when running `CalculateFORC`, because these options set the grid mesh size to a multiple of the mean field step of measurements, which vary slightly even if same FORC measuring protocols have been chosen. Instead, `INPUT 06` should be specified explicitly by the same numerical value (the mesh size) for all datasets to be combined.
- The origin of the FORC matrix grid is set by `INPUT 07`. The recommended option `Automatic` generates always the same grid origin $(H_{c,0}, H_{b,0}) = (\Delta H, 0)$, where ΔH is the mesh size defined with `INPUT 06`, and can therefore be used for obtaining compatible FORC matrices. If the grid origin is entered explicitly, ensure that the same choice is made for all matrices to be combined.

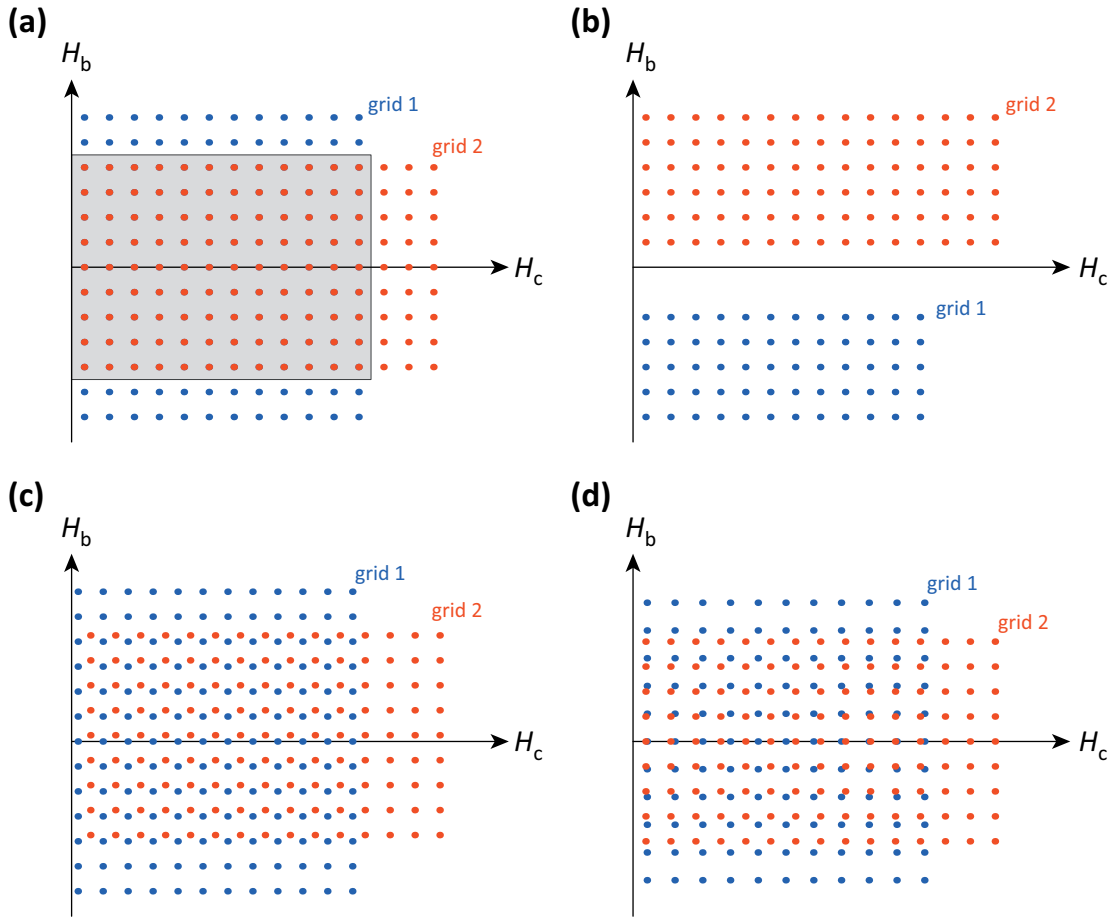


Fig. 7.3: Combination of two FORC matrices with shown grids. **(a)** Compatible grids have coincident points over a non-empty rectangular FORC region (gray rectangle) which represents the output FORC space after linear combination of the two matrices. **(b)** Matrices are incompatible because of non-overlapping FORC spaces. **(c)** Matrices are incompatible matrices because of an offset (mesh size is the same, but grid origins are different). **(d)** Matrices are incompatible because of different mesh sizes.

FORC matrix incompatibilities are signaled by error messages such as

```
Incompatible magnetization units emu and Am2/kg encountered.
  FORC matrices cannot be combined. Program aborted.
```

for unit problems,

```
Different grid meshed encountered. FORC matrices cannot be combined. Program aborted.
```

if mesh sizes do not coincide,

```
Different grid origins encountered. FORC matrices cannot be combined. Program aborted.
```

in case of different grid origins, and

```
A common FORC space does not exist. FORC matrices cannot be combined. Program aborted.
```

for non-overlapping FORC spaces. In these cases, processing is aborted. Similar checks are performed also for the FORC error matrices and the coercivity distributions. These additional checks are in principle not necessary, but are performed in order to exclude incompatibilities due to accidental renaming of unrelated FORC data.

Once the availability and compatibility of FORC data has been checked, linear combinations are calculated for FORC matrices and related coercivity distributions, if available. FORC matrices and coercivity distribution data consist of function values f with standard errors ε, generally representable as $(f_{ik}, \varepsilon_{ik})$ for the k-th element of the i-th dataset. A linear combination of the datasets with coefficients c_i is calculated according to the following equations:

$$M_k^{\text{out}} = \sum_{i=1}^{n} c_i M_{ik}$$
$$(\varepsilon_k^{\text{out}})^2 = \sum_{i=1}^{n} (c_i \varepsilon_{ik})^2 \qquad (7.4)$$

where the output standard error is obtained on the basis of the correct assumption that all datasets are independent from each other.

- FORC matrices containing non-regular contributions (e.g. the central ridge), depend on processing parameters such as the smoothing factors. The linear combination of such matrices require additional considerations. For example, central ridges should be characterized by similar vertical resolutions.
- If possible, averaging of multiple measurements should be performed directly with corrected data produced by `ImportFORC`, instead of FORC matrices, for two reasons: first, FORC matrix calculation is much more time-consuming than dealing with FORC measurements, and, second, proper processing of averaged measurements is simpler because of less measurement noise.

The use of standardized FORC measuring and processing protocols eases the comparison and combination of FORC data. For example, while central ridges with identical vertical profiles are automatically obtained with same measurement and processing protocols, the same result is more difficult to obtain if a difference in measurement resolution must be compensated by appropriately chosen smoothing factors.

7.4 Typical LinearCombineFORC applications

This section discusses some typical applications of `LinearCombineFORC` and gives suggestions about best processing solutions.

7.4.1 Planned averaging

Averaging several FORC measurements is an operation required for improving the quality of measurements obtained from magnetically weak and/or extremely heterogeneous materials. This is because the signal-to-noise ratio of n independent measurements increases by a factor $\sqrt{n}$. For example, a factor 3 improvement is obtained with 9 measurements. The same improvement is *ideally* obtained with a single set of measurements if the averaging time of individual measurements is increased. However, averaging time increases are never as efficient as multiple measurements, because (1) time averaging is affected by correlated noise, and (2) longer measurement times can worsen drift problems to the point that no benefits are obtained. The best weak-sample measuring strategy with Micromag™ VSM or AGM magnetometers consists in repeating the same set of FORC measurements under identical, undisturbed conditions after ~20 min instrument warm-up time.

Multiple measurements offer the possibility of reducing or eliminating some sources of systematic errors. One of them is centering: therefore, it is advisable to re-center the sample before each set of measurements is acquired. In case of geologic materials, small quantities measured with the VSM (few grams) and with the AGM (few milligrams) might not be identical or even representative of the average properties on larger scales. In this case, if enough material is available, multiple samples should be prepared and measured.

As typical example, consider the measurement of weak sediments (e.g. pelagic carbonates). In this case, a certain quantity of sediment material is dried and thoroughly homogenized. The homogenized sediment powder provides the starting material for preparing four ~20 mg specimens, whose masses m_1, m_2, m_3, and m_4 are individually determined. Each sample is measured once with the same FORC protocol, obtaining four identical sets of FORC measurements. Because measurements are performed on different specimens, centering is performed independently for each of them and systematic centering errors are reduced.

The first processing step consists in correcting and normalizing the four measurements with `ImportFORC`. Common output units should be chosen, for instance T for the magnetic field and mAm^2/kg for the magnetization. In this case, specimen masses should be entered in order to convert the measured magnetic moments (e.g. Am^2) into mass magnetizations. Four files containing drift- and outlier-corrected measurements, expressed with appropriated magnetization units, are obtained in this way. There files represent the input of `LinearCombine FORC` for averaging the four FORC measurement sets. This form of measurement averaging can also be performed directly with `ImportFORC` (see chapter 3), provided that all measurements share the same field and magnetization units.

7.4.2 Unplanned averaging

FORC measurements to be averaged might have been obtained from different sources or, for testing reasons, with different measuring protocols. For example, the average magnetic properties of a certain sediment type (e.g. pelagic carbonates) might be based on a collection of FORC measurements that have not been performed with this purpose and are therefore based on different measuring protocols. Certain protocol parameters, such as field step size, averaging time, and pause after reversal, produce intrinsic differences due for instance to timing-sensitive thermal relaxation processes, so that measurements obtained with different protocols might be slightly different. However, such effects can be ignored if averaging is used to analyze FORC features that are representative for a certain group of samples.

FORC measurements obtained with different FORC protocols are typically associated to non-coinciding sets of reversal and measuring field. Direct averaging of such measurements is therefore impossible, unless interpolation is used to redefine measurement points with same reversal and measuring fields. In this case, the best approach consists in processing each set of measurements individually, in order to obtain FORC matrices with coincident grids that can be combined with `LinearCombineFORC`. For this purpose, each set of FORC measurements is processed with `ImportFORC` and `CalculateFORC` in sequence. `CalculateFORC` parameters must be chosen so, that FORC matrices are calculated on perfectly overlapping point grids. Two parameters must be controlled for this purpose: the first is mesh size (`INPUT 06`, see section 4), which should be exactly the same for all datasets. You should therefore check all field step sizes δH of measurements (which are reported by `ImportFORC`), and choose an output mesh size ΔH comprised between 0.5 and 1 times δH. The second parameter is the grid origin (`INPUT 07`, see section 4), which should be either `Automatic` or the same pair of FORC coordinates for all datasets. This ensures that the common FORC space shared by all measurements is occupied by identical grid points.

The combination of FORC measurements obtained with different protocols has certain limits. For example, the maximum attainable resolution is limited by the FORC measurements with largest field steps. Therefore, it makes little sense to combine low- and high-resolution FORC measurements (e.g. $\delta H = 0.5$ mT and $\delta H = 2$ mT) if high-resolution features such as the central ridge are present. Finally, the maximum range of the averaged FORC function is limited by the common FORC space covered by all measurements.

7.4.3 Measurement differences

Differences between FORC measurements performed on the same material before and after a specific treatment (e.g. chemical dissolution of ultrafine magnetite) can be used to infer the intrinsic properties of certain magnetic components [e.g. *Ludwig et al.*, 2013]. Proper normalization is necessary for obtaining meaningful results, because measurements are typically performed on different specimens of the same material before and after treatment.

Normalization is best performed with `ImportFORC`, so that a proper measurement difference is calculated with `LinearCombineFORC` using the coefficients +1 and –1, respectively.

A typical example of this procedure is provided by *Ludwig et al.* [2013] and the corresponding dataset is available for download along with VARIFORC. About 30 g of sediment material taken from a pelagic carbonate core have been dried and thoroughly homogenized by gentle grinding in a mortar. Part of this material has been used to prepare a 27.2 mg sample to be measured with the AGM (Fig. 7.4a). Another portion of the same material has been subjected to a chemical treatment for the selective extraction of single-domain magnetite particles (mostly magnetofossils). Extraction was performed with a citrate-bicarbonate-dithionite (CBD) solution that does not dissolve silicates and the carbonaceous sediment matrix. Therefore, the treated material can be considered identical to the original sediment, except for the absence of single-domain magnetite that was not protected from dissolution because of being included in host minerals. This magnetite is of authigenic origin.

Part of the treated sediment has been used to prepare a 26.4 mg sample for AGM measurements (Fig. 7.4b). Because the relative mass of dissolved particles is negligible, the difference between mass-normalized magnetizations represents the magnetic contribution of the dissolved magnetite (Fig. 7.4c). The two samples, which are magnetically weak, have been measured several times each, in order to increase the signal-to-noise ratio, using the same high-resolution FORC protocol. All FORC measurements obtained from the same sample (e.g. 27.2 mg untreated sediment) have been imported, drift-corrected, normalized, and averaged in a single step with `ImportFORC`. From this point on, there are two possibilities for calculating a FORC diagram corresponding to the difference between measurements of the two samples.

The first possibility, used by *Ludwig et al.* [2013], consists in taking the two corrected measurement files produced by `ImportFORC` and calculate a linear combination with coefficients +1 and –1, respectively, using `LinearCombineFORC`. The output of `LinearCombineFORC`, in form of a corrected measurement file where magnetization values correspond to the difference between mass-normalized magnetizations before and after selective chemical extraction, serve as basis for further processing with other VARIFORC functions, i.e. `CalculateFORC` for obtaining the FORC diagram of extracted particles, and `IsolateCR` for performing a central ridge analysis.

The second possibility for obtaining the same result, consists in calculating the FORC diagrams of the untreated and treated samples first, using `CalculateFORC`. This approach allows one to use different processing options that are optimized to the individual characteristics of the two samples. In the example of Fig. 7.4, larger smoothing factors have been used to process the treated sediment (Fig. 7.4b), because of the much smaller central ridge contribution and lower signal-to-noise ratios. The matrices corresponding to the two FORC diagrams have been finally combined with `LinearCombineFORC`, obtaining the result shown in Fig. 7.4c.

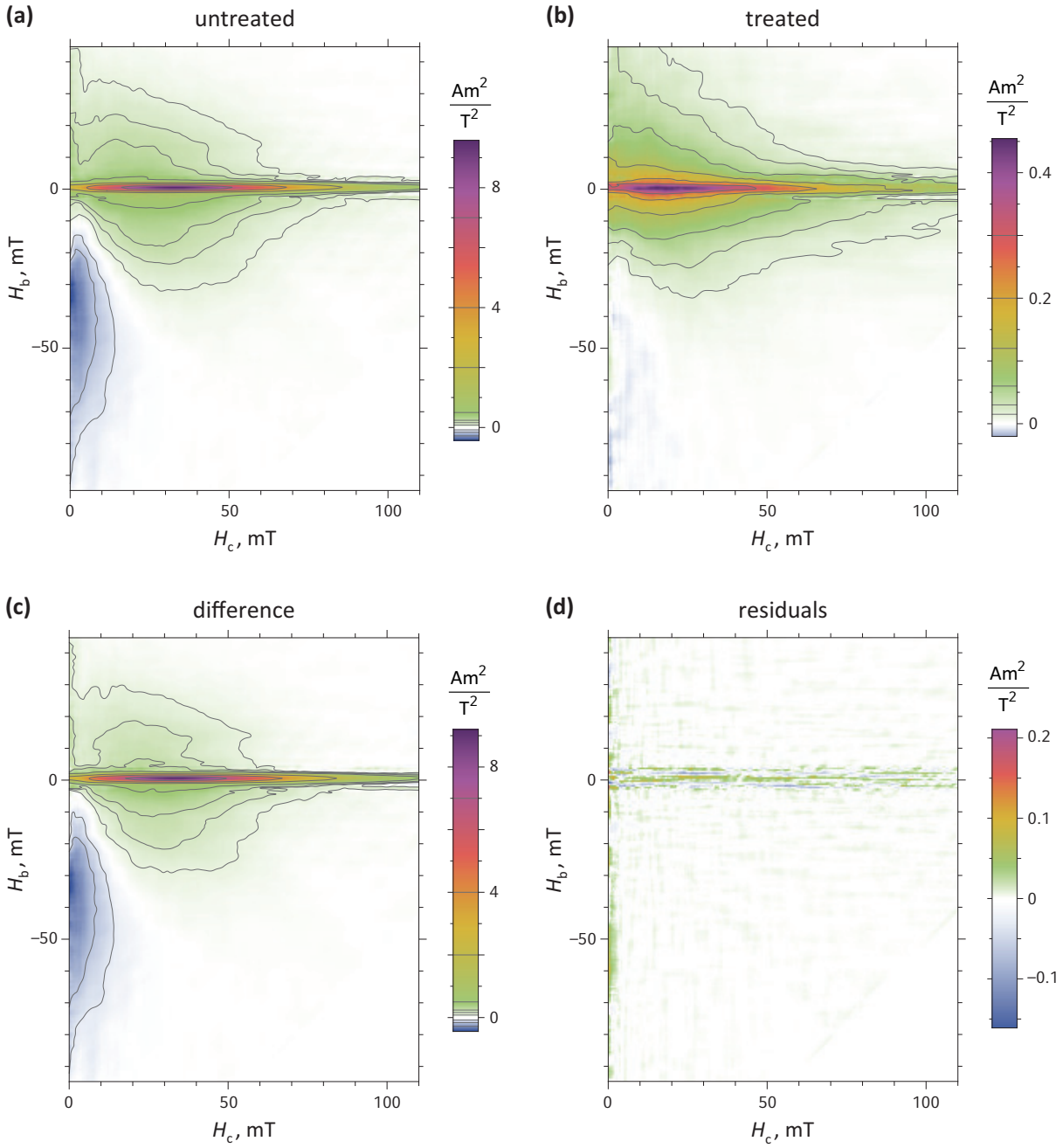


Fig. 7.4: Calculation of FORC measurement differences for a magnetofossil-bearing pelagic carbonate sample (see the downloadable example "pelagic carbonate"). **(a)** FORC diagram resulting from the average of 6 identical measurements of untreated sediment, subsequently processed with a variable smoothing algorithm for simultaneous resolution of the central ridge and remaining low-amplitude FORC features. **(b)** Same as (a) for the same sediment material after selective chemical dissolution of single-domain magnetite. Notice the disappearance of negative FORC amplitudes. **(c)** Difference between the FORC diagrams in (a) and (b), calculated with `LinearCombineFORC`. This is the FORC signature of chemically extracted magnetic particles. **(d)** Difference between the FORC diagram in (c) and a similar FORC diagram obtained directly from measurement differences. Residual FORC amplitudes are due to different processing options used in (a) and (b).

The difference between results obtained with the two abovementioned approaches is shown in Fig. 7.4d in form of residual FORC amplitudes that are at least 2 times smaller than the standard error of the FORC diagrams from which they have been obtained. This means that the two approaches can be considered identical, up to insignificant residual differences arising from the fact that the two FORC diagrams in Fig. 7a,b have not been processed with same parameters.

7.4.4 Background subtraction

Occasionally, it might be desirable to subtract an unwanted "background" contribution arising, from instance, from the sample holder or part of the sample itself. Unwanted contributions of this type should be eliminated at first instance if possible. For example, properly cleaned sample holders might produce a high-field susceptibility contribution due to para- or diamagnetic properties of the holder material, but should not contribute to the FORC diagram. In case background subtraction is unavoidable, it should be performed directly on magnetic moment measurements, *before* normalization by sample mass, volume, or area. Typically, one has a set of FORC measurements for the sample (with sample holder) and for the sample holder alone, respectively. Subtraction of the sample holder contributions is equivalent to the calculation of a measurement difference and can be performed with `LinearCombineFORC` as described in section 7.4.3. The drawback of this approach is that corrected measurements with subtracted background are forcedly expressed in magnetic moment units, which is inconvenient for most applications.

This is one of the rare examples where the unit management system of VARIFORC function is undesirable. For this reason, unit management can be overridden to produce a fictively normalized set of corrected FORC measurements for the sample holder. For example, if the desired output unit is that of a mass-normalized magnetization (e.g. mAm^2/kg), corrected sample holder measurements, originally expressed as magnetic moment, can be transformed to this unit by entering a fictive "sample holder mass" m_{holder} with `INPUT 16` in the parameter file read by `ImportFORC`. The fictive mass is best chosen to be a "round" number of the same order of magnitude of the sample mass m_{sample}, at best a power of 10 (e.g. 1 g or 100 mg). Sample holder measurements, expressed as mass-normalized magnetizations, are subsequently subtracted with `LinearCombineFORC` using the following coefficients: +1 for the sample measurements, and $-m_{holder}/m_{sample}$ for the sample holder.

7.5. Literature

Egli, R. (2013). VARIFORC: an optimized protocol for the calculation of non-regular first-order curves (FORC) diagrams, *Global and Planetary Change*, *210*, 302-320, doi: 10.1016/j.gloplacha.2013.08.003.

Ludwig, P., R. Egli, S. Bishop, V. Chernenko, T. Frederichs, G. Rugel, and S. Merchel (2013). Characterization of primary and secondary magnetite in marine sediment by combining chemical and magnetic unmixing techniques, *Global and Planetary Change*, *110*, 321-339.