XNAVMEAS
Range and Range Rate Measurement Function for ODTBX

License

Content

Function name: xnavmeas.m
Calling syntax: [Y,H,R] = xnavmeas(t,X,options)

Description: This function provides range difference and (optionally) range difference rate measurements from input ECI position and velocity vectors to distant, inertially fixed Resident Space Objects (RSO). The range and range rate are based on the LOS vector from the input state to the RSO.

Inputs:

1. Time, *t* (seconds) from epoch (see Options table) 1xN vector
2. Spacecraft ECI position and velocity state *X* (km, km/s, respectively) 6xN matrix, each column *n* is the state vector corresponding to time *t(n).*
3. Options 1x1 structure specifying pulsar and detector characteristics and observation time (see table below) as well as output type for I RSOs.

Outputs:

1. Measurements up to M=2 corresponding to spacecraft range difference and range difference rate (km, km/s respectively) relative to each pulsar specified in the options (1 to I), concatenated into (MI)xN matrix *Y*; each column *n* corresponds to range differences and range difference rates for measurements 1 to I at time *t(n).* Note that range difference rate will only be an output if the useRangeRate option is true.
2. Measurement Jacobian matrices for each pulsar specified in the options concatenated into (MI)x6xN matrix *H*; each page *n* corresponds to (MK)x6 submatrices for each time *t(n).*
3. Measurement Noise Covariance matrices for each pulsar specified in the options concatenated into (MI)xMxN matrix *R*; each page *n* corresponds to (MI)xM submatrices for each time *t(n).*

Table 1 FPS Measurement Options Structure Fields

Name	Description	Units	Valid Values	Default Value
Num_meas	Integer indicating number of desired measurements w.r.t distant RSO	NA	> 1	3
obs_time	Observation (measurement) time (t_{obs}) assumed the same for each measurement	sec	$t_{obs} > 0$	1e5
useRangeRate	Flag to output range rate	NA	True (1) False (0)	True (1)
sigma_r (σ_r)	1xI array of measurement standard deviations corresponding to range measurement noise for each i=1..num_meas RSO	Km	sigma_r ρ 0	Computed from C_1 and obs_time
sigma_rr (σ_{rr})	1xI array of measurement standard deviations corresponding to range rate measurement noise for each i=1..I pulsar	Km/s	sigma_rr ρ 0	Computed from C_1 and obs_time
C_1	1xI array of measurement system attributes	sec^3	> 0	See code
unit_RSO	3xnum_meas array of user-specified ECI unit vectors to RSO. If left empty then default values will be used	NA	Any unit vector	See code

Range and Range Rate Calculation

The range difference represents the time difference between pulses traveling from the distant object (assumed inertially fixed) to the reference point origin and to the spacecraft. For near earth orbits waves travel in nearly parallel paths from resident space objects (RSO) to the spacecraft and reference points because they are so far away. It is assumed here that the reference point will coincide with the ECI origin as shown in Figure 1. As mentioned the RSO are assumed to be stationary w.r.t. ECI for all times. Occultation due to other bodies or spacecraft attitude is not included in this version.

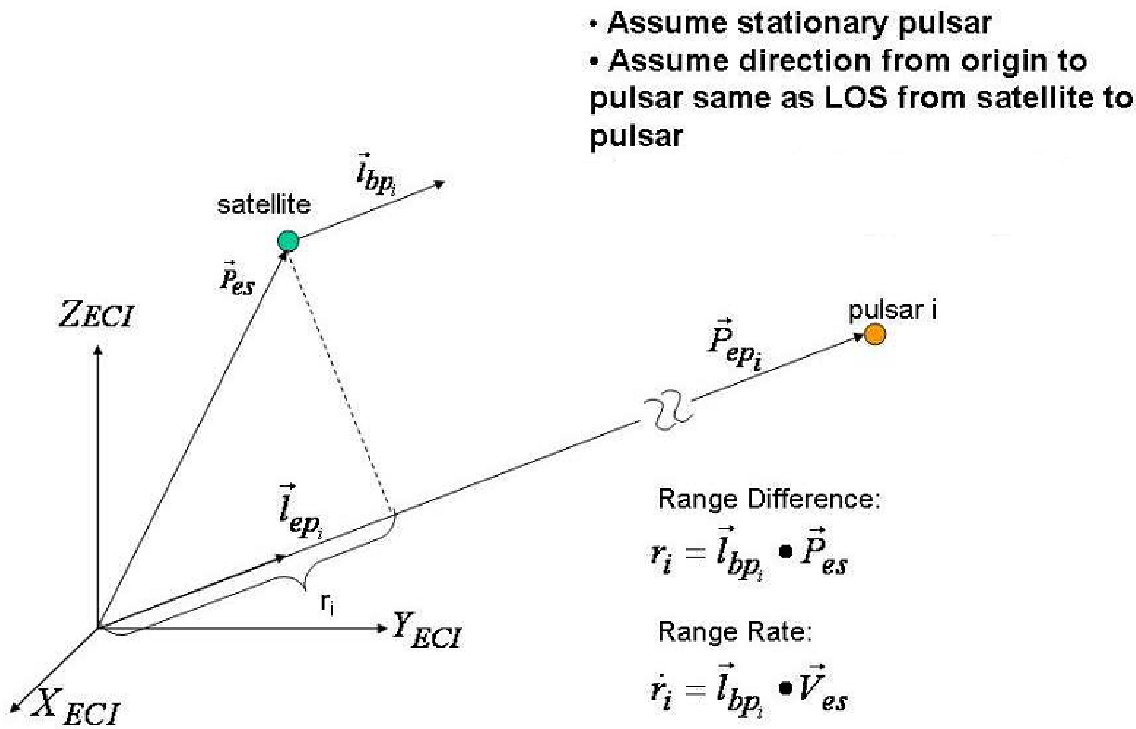

Figure 1. Range and Range Rate Measurement

The range difference is the component of the spacecraft position state in the direction to the RSO:

$$r_i = \vec{l}_{ep_i} \bullet \vec{P}_{es}$$

The range rate is the derivative of the range difference. Given the assumption of stationary objects the range rate is simply the component of the spacecraft velocity in the direction to the RSO:

$$rr_i = \vec{l}_{ep_i} \bullet \vec{V}_{es}$$

The range difference and range difference rate comprise the 2 element vector Y_i for each measurement i at time $t(n)$.

$$Y_i = \begin{bmatrix} r_i \\ rr_i \end{bmatrix}$$

The output matrix Y of size (MI)xN consists of stacked Y_i vectors:

$$Y = \begin{bmatrix} Y_{1_1} \cdots Y_{1_N} \\ \vdots \quad\quad \vdots \\ Y_{i_1} \cdots Y_{i_N} \\ \vdots \quad\quad \vdots \\ Y_{I_1} \ldots Y_{I_N} \end{bmatrix}$$

The measurement Jacobian matrix H_i corresponding to object i is given by the partials of the measurement equations w.r.t the spacecraft state $X(n)$.

$$H_i = \frac{\partial Y_i}{\partial X} = \begin{bmatrix} \vec{l}_{ep_i}{}^T & 0_{1x3} \\ 0_{1x3} & \vec{l}_{ep_i}{}^T \end{bmatrix}, \quad X = \begin{bmatrix} \vec{P}_{es} \\ \vec{V}_{es} \end{bmatrix}$$

The output super matrix H of size (MI)xN consists of stacked Hi matrices:

$$H = \begin{bmatrix} H_{1_1} \cdots H_{1_N} \\ \vdots \quad\quad \vdots \\ H_{i1} \cdots H_{i_N} \\ \vdots \quad\quad \vdots \\ H_{I_1} \cdots H_{I_N} \end{bmatrix}$$

Noise Calculation

The main noise contributor is the pulse time of arrival (TOA) from the RSO. The range noise is simply the TOA multiplied by the speed of light. Measurement noise standard deviations are calculated based on LOS measurement system attributes. The range sigma for measurement i σ_{ri} is calculated using the equations below.

$\sigma_{r\,i} = c\sigma_{TOA\,i}$ where c is the speed of light

$\sigma_{TOA_i} = \dfrac{C_i}{t_{obs}}$ where C_i represents the measurement system attributes

For this preliminary version the range rate noise calculation is approximated to first order by

$$\sigma_{rr_i} = \frac{\sqrt{2}\sigma_{r_i}}{t_{obs}}$$

The measurement noise covariance matrix for measurement i, R_i at time $t(n)$ is given by:

$$R_i = \begin{bmatrix} \sigma_{r\,i}^{\;2} & 0 \\ 0 & \sigma_{rr_i}^2 \end{bmatrix}$$

The format of the output measurement noise covariance matrix for all the specified measurements and times is given by:

$$R = \left[\begin{array}{ccc:ccc:ccc} R_{1_1} & & & R_{1_n} & & & R_{1_N} & & \\ & \ddots & \Theta & & \ddots & \Theta & & \ddots & \Theta \\ & R_{i_1} & & & R_{i_n} & & & R_{i_N} & \\ \Theta & & \ddots & \Theta & & \ddots & \Theta & & \ddots \\ & & R_{I_1} & & & R_{I_n} & & & R_{I_N} \end{array}\right]$$

Each “page” or slice of the composite R matrix corresponds to a particular time t(n).It is composed of individual Ri matrices concatenated along the major diagonal of each page.