Semester-3RD
(Regular & Back)(Back)
Sub & Code-DMS(MA-2003)
Branch(s)-CSE & IT

AUTUMN END SEMESTER EXAMINATION-2017
DISCRETE MATHEMATICAL STRUCTURES
[MA-2003]

Full Marks:60 **Time:03 Hours**

Answer any six questions including question No.1 which is compulsory.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1.		(2×10)
a.	Find the contrapositive of the statement "If it will rain today, we play football" by using propositional logic.	
b.	Using truth table prove that $\sim(p \wedge q) \equiv \sim p \vee \sim q$.	
c.	Determine the truth value of each of the following statements if the domain consists of all integers. (i) $\forall n(n + 1 > n)$ (ii) $\exists n(2n = 3n)$ (iii) $\exists n(n = -n)$ (iv) $\forall n(3n \leq 4n)$	
d.	Find the number of positive integers not exceeding 100 that are neither divisible by 2 nor divisible by 5.	
e.	Find the equivalence relation on the set $A = \{a, b, c, d, e\}$ corresponding to the partition set $P = \{ \{a,b\}, \{c\}, \{d,e\}\}$.	
f.	How many Boolean functions are there of degree 3?	
g.	Find the values of the Boolean function represented by $F(x, y, z) = xy + \bar{z}$.	
h.	Find generating functions corresponding to the numeric function $a_n = (-1)^n; \quad n \geq 0.$	
i.	The set $Z_{20} = \{0, 1, 2, \cdots, 19\}$ under addition and multiplication modulo 20 is a commutative ring. List all zero-divisors in Z_{20}.	
j.	Define complete graph. If a complete graph has degree 8 for each vertex, then how many edges are there?	
2.		(2×4)
a.	Show that the argument form with premises $(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, and $\sim s$ and conclusion $q \rightarrow r$ is valid.	
b.	Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book."	
3.		(2×4)
a.	Use mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for every nonnegative integer n.	

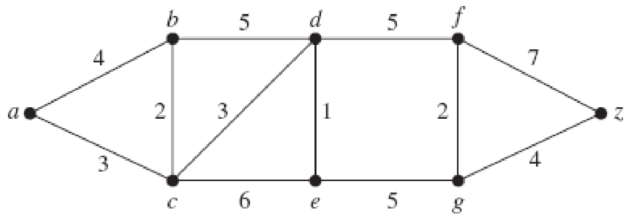
Paper Setter: Dr. Manoranjan Sahoo

Moderator: Dr. T. Panigrahi

b.	Let $A = \{a_1, a_2\, a_3, a_4, a_5\}$ and R be a relation on A given by $R = \{(a_1, a_1), (a_1, a_2), (a_1, a_4), (a_2, a_3), (a_3, a_3), (a_3, a_5), (a_4, a_4), (a_5, a_2)\}$. Find the transitive closure of R using Warshall's algorithm.	
4.	(2×4)	
a.	Let R be a reflexive relation on a set A such that $$(a, b) \in R,\ (a, c) \in R \Longrightarrow (b, c) \in R.$$ Show that R is an equivalence relation.	
b.	Let $S = \{a, b, c\}$.Show that $(P(S), \supseteq)$ is a complemeted lattice? Draw its Hasse diagram and find complements of each of its elements.	
5.	(2×4)	
a.	Find the sum-of-products expansion for the Boolean function $F(x, y, z) = (x + y)\,\bar{z}$.	
b.	The Tower of Hanoi, consists of three pegs mounted on a board together with disks of different sizes. Initially these disks are placed on the first peg in order of size, with the largest on the bottom. The rules of the puzzle allow disks to be moved one at a time from one peg to another as long as a disk is never placed on top of a smaller disk. The goal of the puzzle is to have all the disks on the second peg in order of size, with the largest on the bottom. Let H_n denote the number of moves needed to solve the Tower of Hanoi problem with n disks. Set up a recurrence relation for the sequence $\{H_n\}$ and solve it.	
6.	(2×4)	
a.	What is the solution of the recurrence relation $$a_n = 6a_{n-1} - 9a_{n-2}$$ with initial conditions $a_0 = 1$ and $a_1 = 6$?	
b.	Determine whether the set of positive integers Z^+ with the binary operation * defined by $a * b = gdc(a, b)$ is a semigroup, monoid or nither. If it is monoid specify the identity	
7.	(2×4)	
a.	Let G be the set of all nonzero real numbers and $$a * b = \frac{ab}{2}.$$ Show that $(G, *)$ is an abelean group.	
b.	Let $Z_5 = \{0, 1, 2, 3, 4\}$, $\oplus$ be addition modulo 5 and $\otimes$ be multiplication modulo 5 on Z_5. Show that $(Z_5;\ \oplus,\ \otimes)$ is a field.	
8.	(2×4)	
a.	Define degree of a vertex, bipartite graph, adjacency matrix and Euler path with examples.	

Paper Setter: Dr. Manoranjan Sahoo

Moderator: Dr. T. Panigrahi

b.	Using Dijkstra's algorithm find the shortest path from vertex a to z from the following weighted graph. 	

Paper Setter: Dr. Manoranjan Sahoo

Moderator: Dr. T. Panigrahi