Semester-3RD
(Regular & Back)(Back)
Sub & Code-DMS(MA-2003)
Branch(s)-CSE & IT

AUTUMN END SEMESTER EXAMINATION-2015
DISCRETE MATHEMATICAL STRUCTURES
[MA-2003]

Full Marks:60 **Time:03 Hours**

Answer any six questions including question No.1 which is compulsory.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. Answers all. (2x10)

a. Translate each of the following statements into predicated logical expressions:
(i) At least one of your friends is perfect .
(ii) Every ships have captains .

b. Let $Q(x)$ be the statement $x + 1 > 2x$. If the domain consists of all integers, then find the truth values of
(i) $\exists x\, Q(x)$; and (ii) $\forall x\, Q(x)$.

c. Find the equivalence relation corresponding to the partition set $P = \{$ {a,b}, {c}, {d,e}} of the set $A = \{a, b, c, d, e\}$.

d. Find the least upper bound and greatest lower bound of the POSET $A = \{1,\ 2,\ 3,\ 4,\ 5\}$ with divisibility relation?

e. Find generating functions corresponding to each of the following numeric functions.
(i) $a_n = \frac{1}{n!}; n\geq 1$; and (ii) $a_n = 2^{n+1} - 1; n\geq 0$.

f. Find the number of positive integers not exceeding 100 that are either odd or square of an integer.

g. Let * be the binary operation defined on the set of natural number N as, $\forall\, a, b, \in N;\ a * b = (a, b)$. Is it a monoid? If yes, find its identity element.

h. Define a Ring. Give an example of a finite field.

i. Define Regular graph. If a complete graph has degree 8 for each vertex, then how many edges are there?

j. State the necessary and sufficient conditions for a graph to be an Eulerian graph.

2. (2x4)

a. Prove $n(n + 1)(n + 2)$ is divisible by 3 for $n\in N$ by method of induction.

b. Construct the truth table for $[(p\rightarrow q)\wedge(q\rightarrow r)]\longrightarrow(p\rightarrow r)$. Is it a tautology?

3. (2x4)

a. For the relation $r = \{(a, b)(b, c), (c, d)\}$ on the set $A = \{a, b, c, d\}$; find its corresponding reflexive, symmetric and transitive closures by matrix operation.

b. Does D_{30} under divisibility relation is a complemented lattice? If yes, Draw its Hasse diagram and find complements of each of its elements; where D_{30} is the set of all positive divisors of 30.

4. (2x4)

a. Solve the following recurrence relation by substitution method.

$$a_r - 5a_{r-1} + 6a_{r-2} = r + 1; \text{ for } r \geq 2.$$

b. Solve the following recurrence relation using generating function.

$$a_n - a_{n-1} = n; \quad n > 1; \ a_0 = 1.$$

5. (2x4)

a. Show that $G = \{1, \omega, \omega^2\}$ is a group under the binary operation multiplication. Is it cyclic? If yes, find all of its generators. Here $\omega = \sqrt[3]{1}$.

b. Let $(G, *)$ be an abelian group. Show that for any $a, b \in G$ and $n \in N$

$$(a * b)^n = a^n * b^n.$$

6. (2x4)

a. Write down all the permutations on four symbols 1, 2, 3 and 4. How many of them are even? Which of these permutations are odd?

b. Show that $\left(Z_5; \oplus, \otimes\right)$ is a field where , $\oplus$ and $\otimes$ are addition and multiplication modulo 5 operations on integers respectively.

7. (2x4)

a. State and prove D' Morgan's rules of Boolean algebra.

b. Express the following Boolean function $f(x, y, z)$ in conjunctive and disjunctive normal form.

(x, y, z)	$f(x, y, z)$
$(0, 0, 0)$	1
$(0, 0, 1)$	1
$(0, 1, 0)$	1
$(0, 1, 1)$	0
$(1, 0, 0)$	1
$(1, 0, 1)$	0
$(1, 1, 0)$	0
$(1, 1, 1)$	1

8. (2x4)

a. Are these following graphs Isomorphic? Justify your answer.

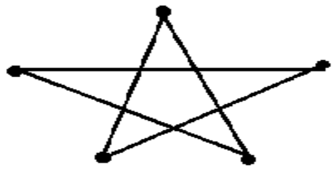


Fig.1

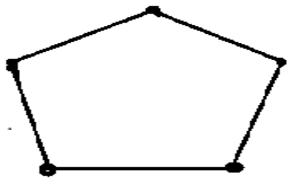
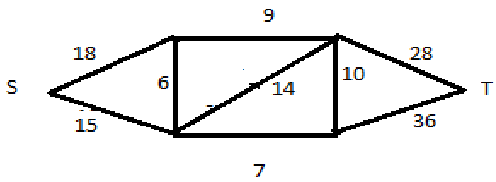


Fig.2

b. Using Dijkstra's algorithm find the shortest path from vertex S to T from the following weighted graph.



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