AUTUMN END SEMESTER EXAMINATION-2019

3rd Semester B.Tech & B.Tech Dual Degree

DISCRETE MATHEMATICAL STRUCTURES
MA 2013

(For 2019 (L.E) & 2018 Admitted Batches)

Time: 3 Hours   Full Marks: 50

Answer any SIX questions.
Question paper consists of four sections-A, B, C, D.
Section A is compulsory.
Attempt minimum one question each from Sections B, C, D.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A

1. (a) Find the inverse and converse of the following statement: [1 × 10]
"Good foods are not cheap"

(b) Let "T" be a tautology and "p" be an arbitrary proposition then find the truth value of $(\sim T) \rightarrow p$.

(c) Determine the truth value of each of the following statements if the domain consists of all integers.
(i) $\forall n(n + 1 > n)$ (ii) $\exists n(2n = 3n)$

(d) Find the equivalence relation corresponding to the partition set $P = \{ \{a,b\}, \{c\}, \{d,e\}\}$ of the set $A = \{a, b, c, d, e\}$.

(e) Find the number of positive integers not exceeding 100 that are divisible by either 4 or 9.

(f) Find generating functions corresponding to the numeric function

$$a_n = n^2; \quad n \geq 1.$$

(g) Write all the permutations defined in the set $A = \{1, 2, 3\}$.

(h) Give example of a zero-divisor in a ring.

(i) Find the inverse of each of the elements of the group $G = \{1, \omega, \omega^2\}$.

(j) Define normal subgroup with an example.

SECTION-B

2. (a) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology. [4]

(b) Show that the argument with the premises $(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, and $\sim s$ and conclusion $q \rightarrow r$ is valid. [4]

3. (a) Use mathematical induction to prove that $n^3 - n$ is divisible by 3 for all positive integer n. [4]

(b) Let $R = \{(a_1, a_1), (a_1, a_2), (a_1, a_4), (a_2, a_3), (a_3, a_3), (a_3, a_5), (a_4, a_4), (a_5, a_2)\}$ be a relation on the set $A = \{a_1, a_2\, a_3, a_4, a_5\}$ Find the transitive closure of R using Warshall's algorithm. [4]

SECTION-C

4. (a) Let R be a reflexive relation on a set A such that [4]

$$(a, b) \in R,\ (a, c) \in R \Rightarrow (b, c) \in R.$$

Show that R is an equivalence relation.

(b) Let $S = \{a, b, c\}$. Show that $[P(S), \subseteq]$ is a complemeted lattice? Draw its Hasse diagram. [4]

5. (a) Find the numeric solution of the recurrence relation $a_n - a_{n-1} = n; \quad n \geq 1$ with $a_0 = 0$ using generating function. [4]

(b) Find the numeric solution of the recurrence relation $a_r - 4a_{r-1} = 8^r$; for $r \geq 1$, with $a_0 = 1$. [4]

6. (a) Prove that the set of positive Integers with divisibility relation is a poset. [4]

(b) Show that $(S_3, \circ)$ is a group, where S_3 is the set of all permutations on a set containing 3 elements, by constructing the composition table. [4]

SECTION-D

7. (a) Let G be a group. Then show that [4]

(i) $ab = ac \Rightarrow b = c$ and

(ii) $(ab)^{-1} = b^{-1}a^{-1}$, $\forall\, a, b, c \in G$.

(b) Let G be the set of all nonzero real numbers and [4]

$$a * b = \frac{ab}{2}.$$

Show that $(G, *)$ is an abelian group.

8. (a) State and prove Lagrange's theorem. [4]

(b) Determine whether the set of positive integers Z^+ with the binary operation * defined by $a * b = gcd\{a, b\}$ is a semigroup or monoid. If it is monoid, specify the identity. [4]

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MAPPING OF QUESTIONS WITH COURSE OUTCOMES AND LEARNING LEVELS

The paper setter /Moderator will provide mapping of Question with Course Outcomes and learning levels in the following format:

Course Name:
Course code:
Examination:

CO1	*Fill in statement*
CO2	*Fill in statement*
CO3	*Fill in statement*
CO4	*Fill in statement*
CO5	*Fill in statement*
CO6	*Fill in statement*

Rows may be added or deleted as necessary

Question number	Course Outcome number	Learning Level (Blooms taxonomy)
Section A		
Q1a		
1b		
1c		
1d		
1e		
1f		
1g		
1h		
1i		
1j		
Section B		
Q2a		
2b		
Q3a		
3b		
Section C		
Q4a		
4b		
Q5a		
5b		
Q6a		
6b		
Section D		
Q7a		
7b		
Q8a		
8b		

Signature of Paper Setter/Moderator