AUTUMN END SEMESTER EXAMINATION-2018

3rd Semester B.Tech & B.Tech Dual Degree

DISCRETE MATHEMATICAL STRUCTURES

MA 302

(-……..Admitted Batch & Back)

Time: 3 Hours   Full Marks: 60

Answer any SIX questions including question No.1 which is compulsory.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1.			[2 × 10]
	(a)	Find the negation of the statement: $x^2 = 16 \Rightarrow x = 4.$	
	(b)	Using truth table prove that $\sim(p \wedge q) \equiv \sim p \vee \sim q$.	
	(c)	Determine the truth value of each of the following statements if the domain consists of all integers. (i) $\exists n(n = -n)$ (ii) $\forall n(3n \leq 4n)$	
	(d)	Find the equivalence relation on the set $A = \{a, b, c, d, e\}$ corresponding to the partition set $P = \{ \{a,b\}, \{c\}, \{d,e\}\}$.	
	(e)	Find the number of positive integers not exceeding 100 that are neither divisible by 2 nor divisible by 5.	
	(f)	Find generating functions corresponding to the numeric function $a_n = 2^n;\ n \geq 0$.	
	(g)	Find the values of the Boolean function represented by $F(x, y, z) = xy + \bar{z}$.	

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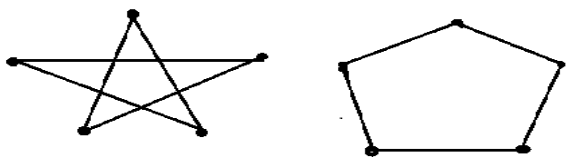
	(h)	Give example of a zero-divisors in a ring.	
	(i)	Define the following terms: (i) Spanning tree and (ii) Complete graph	
	(j)	Find the greatest lower bound and least upper bound of the subset $\{3, 9, 12\}$, in the poset ( ).	
2.			$[2 \times 4]$
	(a)	Show that $\sim(p \lor (\sim p \land q))$ and $\sim p \land \sim q$ are logically equivalent by developing a series of logical equivalences.	
	(b)	Show that the argument form with premises $(p \land t) \to (r \lor s)$, $q \to (u \land t)$, $u \to p$, and $\sim s$ and conclusion $q \to r$ is valid.	
3.			$[2 \times 4]$
	(a)	Use mathematical induction to show that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$.	
	(b)	Let $R = \{(a_1, a_1), (a_1, a_2), (a_1, a_4), (a_2, a_3), (a_3, a_3), (a_3, a_5), (a_4, a_4), (a_5, a_2)\}$ be a relation on the set $A = \{a_1, a_2\, a_3, a_4, a_5\}$ Find the transitive closure of R using Warshall's algorithm.	
4.			$[2 \times 4]$
	(a)	Let *R* be a reflexive relation on a set A such that $(a, b) \in R,\ (a, c) \in R \Longrightarrow (b, c) \in R.$ Show that R is an equivalence relation.	
	(b)	Let $S = \{a, b, c\}$.Show that $(P(S), \subseteq)$ is a complemented lattice? Draw its Hasse diagram and find complements of each of its elements.	
5.		(2×4)	$[2 \times 4]$
	(a)	Find the numeric solution of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$; $n \geq 2$ with $a_0 = 1$ and $a_1 = 6$ using generating function.	

	(b)	Find the numeric solution of the recurrence relation $a_r - 4a_{r-1} = 8^r$; for $r \geq 1$ with $a_0 = 1$.	
6.		(2×4)	$[2 \times 4]$
	(a)	Find the sum-of-products expansion for the Boolean function $F(x, y, z) = (x + y)\,\overline{z}$.	
	(b)	For any Boolean algebra B, prove that $(a + b)(b + c)(c + a) = ab + bc + ca$.	
7.		(2×4)	$[2 \times 4]$
	(a)	Let G be a group and $a, b, c \in G$. Then show that $ab = ac \Rightarrow b = c$, (ii) $(ab)^{-1} = b^{-1}a^{-1}$.	
	(b)	Determine whether the set of positive integers Z^+ with the binary operation $*$ defined by $a * b = gdc(a, b)$ is a semigroup, monoid or nither. If it is monoid specify the identity.	
8.			$[2 \times 4]$
	(a)	Are these following graphs Isomorphic? Justify your answer.  Fig.1 Fig.2	
	(b)	Using Dijkstra's algorithm find the shortest path from vertex a to z from the following weighted graph.	

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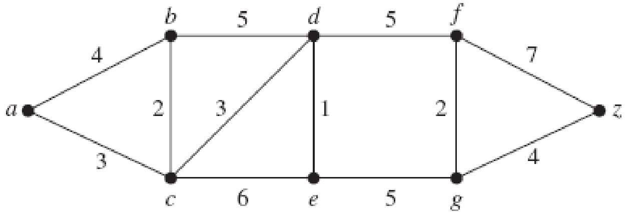

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