AUTUMN END SEMESTER EXAMINATION-2018

......3rd...... Semester B.Tech & B.Tech Dual Degree

DISCRETE MATHEMATICS

MA-2003

Regular-2017 Admitted Batch)

Time: 3 Hours | Full Marks: 50

Answer any SIX questions including question No.1 which is compulsory.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. [1x10]

(a) Find the contrapositive of the statement "The home team wins whenever it is raining."

(b) Find the truth value of $\exists x\, P(x)$ if $P(x)$: $x > 3$, where the domain consists of all real numbers.

(c) Write the generating function for the sequence $\{a_k\}$ with $a_k = k + 1$.

(d) Let $G = (Z, +)$, $H = (3Z, +)$. Find all the left coset of H in G.

(e) Write all the even permutations on the set $A = \{1, 2, 3\}$.

(f) Determine whether the degree sequence 3, 3, 3, 3, 2 is graphic or not?

(g) Define Integral Domain and give an example.

(h) Let $G = (Z, +)$ and a mapping $\emptyset: G \rightarrow G$ be defined by $\emptyset(x) = x + 1$, $x \in G$. Examine if $\emptyset$ is a homomorphism or not?

(i) Define Bipartite graph with an example.

(j) Find the Minimum Spanning Tree of the following graph given below.

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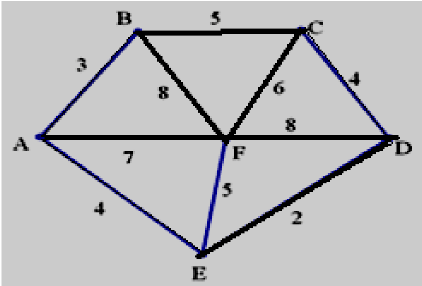


2. (a) Show that $((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$ is a tautology using the truth table. [4+4]

(b) Use Rules of inference to show that the hypotheses "Randy works hard", "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" leads to the conclusion "Randy will not get the job."

3. (a) Use mathematical induction to show that for all non-negative integers n; [4+4]

$$1 + 2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 1.$$

(b) Let $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $\left(a_0 = 2, a_1 = 5\right)$. Use strong induction to prove that $a_n = 2^n + 3^n$.

4. (a) Find the solution of the Recurrence relation [4+4]

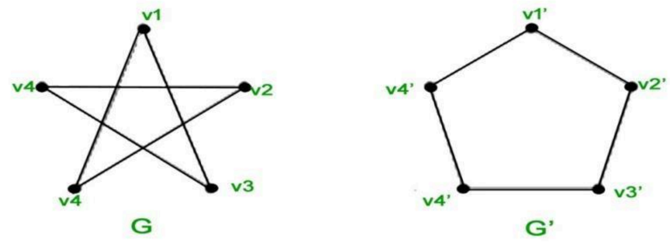
$$a_n = 4a_{n-1} - 3a_{n-2} + 2^n + 3.$$

(b) Use generating function to solve the following recurrence relation $a_k = 3a_{k-1} + 2, a_0 = 1$.

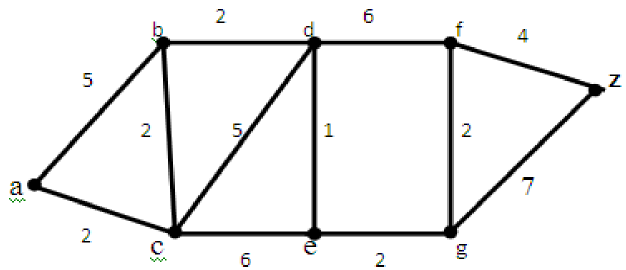
5. (a) State and prove Lagrange's theorem. [4+4]

(b) Show that if $a^2 = e$ for all a in a group $G = (A, *)$, then G is commutative.

6. (a) Verify whether that the graphs G and G' are isomorphic or not? [4+4]



(b) Find the Shortest path from vertex a to vertex z in the given graph using Dijkstra's Algorithm.



7. (a) Find the transitive Closure of the relation R on the set $A = \{a, b, c, d, e\}$, where [4+4]

$$R = \{(a, c), (b, d), (c, a), (d, b), (e, d)\}.$$

(b) Draw the Hasse diagram of the poset $(\{3, 5, 9, 15, 24, 45\}, |)$. Find the maximal, minimal, greatest and least element of the given poset.

8. (a) Find the CNF and DNF of the following Boolean function $f(x, y, z) = (x + y)(\overline{x} + z)(y + \overline{z})$. [4+4]

(b) Let $(A, +, \times)$ be a Ring such that $a \times a = a$ for all a in A. Show that

(i) $a + a = 0$ for all a, where 0 is the additive identity.

(ii) The binary operation $\times$ is commutative.

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