AUTUMN END SEMESTER EXAMINATION-2018

3rd Semester B.Tech & B.Tech Dual Degree

DISCRETE MATHEMATICAL STRUCTURES

MA 2003

(-……..Admitted Batch & Back)

Time: 3 Hours Full Marks: 50

Answer any SIX questions including question No.1 which is compulsory.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. [1 × 10]

(a) Find the negation of the statement: $2 + 3 = 5$

(b) Using truth table prove that $\sim(\sim p) \equiv p$.

(c) Determine the truth value of statement $\forall n(3n \leq 4n)$ if the domain consists of all integers.

(d) Find the equivalence relation on the set $A = \{a, b, c\}$ corresponding to the partition set $P = \{\{a, b\}, \{c\}\}$.

(e) Find the number of positive integers not exceeding 100 that are divisible by 2 and divisible by 5.

(f) Find generating functions corresponding to the numeric function $a_n = 2^n$; $n \geq 0$.

(g) Find the values of the Boolean function represented by $F(x, y,) = x + \overline{y}$.

(h) Give example of commutative ring.

(i) What is inverse of 5 in the group of integers under addition?

(j) How many edges are there in a tree with n vertices?

2. [2 × 4]

(a) Show that $\sim(p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent.

(b) Show that the premises

"It is not sunny this afternoon and it is colder than yesterday,"

"We will go swimming only if it is sunny,"

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"If we do not go swimming, then we will take a canoe trip," and
"If we take a canoe trip, then we will be home by sunset"
lead to the conclusion
"We will be home by sunset."

3. $[2 \times 4]$

(a) Use mathematical induction to show that

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

(b) Show that $S = \{1, 2, 3, 4, 5\}$ with divide relation is a poset. Draw its Hasse diagram.

4. $[2 \times 4]$

(a) Find the numeric solution of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$; $n \geq 2$ with $a_0 = 1$ and $a_1 = 6$ using generating function.

(b) Find the numeric solution of the recurrence relation $a_r - 4a_{r-1} = r$; for $r \geq 1$ with $a_0 = 1$.

5. $[2 \times 4]$

(a) State and prove Lagrange's theorem on subgroups.

(b) Let G be a group and $a, b, c \in G$. Then show that

(i) $ab = ac \Rightarrow b = c$, (ii) $(ab)^{-1} = b^{-1}a^{-1}$.

6. $[2 \times 4]$

(a) Give an example on each of the following:
- (i) Non commutative group
- (ii) Normal subgroup
- (iii) Semigroup which is not a group
- (iv) Ring with unit element

(b) What is a zero divisor in a ring? Find all the zero divisors in the ring $R = \{0, 1, 2, 3, 4, 5\}$ with addition and multiplication modulo 6.

7. $[2 \times 4]$

(a) Define the following terms with examples:
- (i) Complete graph
- (ii) Bipartite graph

(b) Are these following graphs Isomorphic? Justify your answer

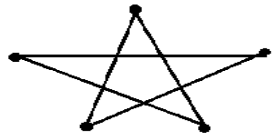


Fig.1

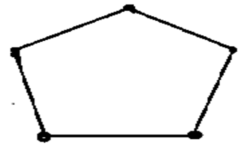
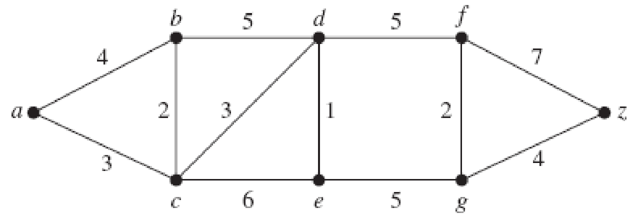


Fig.2

8. $[2 \times 4]$

(a) Find the minimal spanning tree for the following graph



(b) Prove that there exists a unique path joining any two vertices in a tree.

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