Time Value of Money

Time Value of Money

- ***Money has a time value because of the following reasons:***

  (i) Individuals generally prefer current consumption to future consumption.

  (ii) An investor can profitably employ a rupee received today to give him a higher value to be received tomorrow or after a certain period.

  (iii) In an inflationary economy the money received today has more purchasing power than money to be received in future.

Why do Time have value?

- Thus, the fundamental principle behind the concept of time value of money is that a sum of money received today is worth more than if the same is received after some time.
- A corollary to this concept is also the concept that money received in future is less valuable than what it is today.
- For example, if an individual is given an alternative either to receive `10,000 now or after six months; he will prefer `10,000 now. This may be because he may invest this money and earn some interest on it or because he may need money for current consumption or because he may be in a position to purchase more goods with this money than what he is going to get for the same amount after six months.
- Time value of money or time preference for money is one of the central ideas in finance. Individuals as well as business organisations frequently encounter situations involving cash receipts or disbursements over several periods of time.

VALUATION CONCEPTS

- ***The above discussion establishes that there is a preference of money having at present than at a future point of time. This automatically means:***

  (i) that a person will have to pay in future more for a rupee received today; and

  (ii) a person may accept less today for a rupee to be received in future.

- ***The above statements relate to two different concepts.***

List of Interest Formulae

- Single-Payment Compound Amount
- Single-Payment Present Worth Amount
- Equal-Payment Series Compound Amount
- Equal-Payment Series Sinking Fund
- Equal-Payment Series Present Worth Amount
- Equal-Payment Series Capital Recovery Amount
- Uniform Gradient Series Annual Equivalent Amount

Types of Interest

- Simple Interest

  Interest paid (earned) on only the original amount, or principal, borrowed (lent).

- Compound Interest

  Interest paid (earned) on any previous interest earned, as well as on the principal borrowed (lent).

Simple Interest Formula

Formula $SI = P_o(i)(n)$

SI: Simple Interest

P_o: Deposit today (t=0)

i: Interest Rate per Period

n: Number of Time Periods

Simple Interest Example

- Assume that you deposit $1,000 in an account earning 7% simple interest for 2 years. *What is the accumulated interest at the end of the 2nd year?*

- SI = $P_0(i)(n)$ = $1,000(0.07)(2) = $140

Simple Interest (FV)

- What is the Future Value (FV) of the deposit?

$$FV = P_0 + SI = \$1{,}000 + \$140$$

$$= \$1{,}140$$

- *Future Value* is the value at some future time of a present amount of money, or a series of payments, evaluated at a given interest rate.

Compound Value Concept

- *In case of time concept the interest earned on the initial principal becomes a part of the principal at the end of a compounding period, e.g., of `100 is invested at 10% compound interest for two years, the return for the first year will be `10 and for the second year the interest will be received on `110 (i.e., 100 + 10), i.e., `11.*
- *The total amount due at the end of the second year will become `121 (i.e., `100 + 10 + 11). This can be understood better with the following statement.*
- *Statement `1,000 invested at 10% is compounded annually for three years.*

Compounding of Interest over ‘N’ Years

- ***The returns from an investment are generally spread over a number of years. In the early example, the interest has been compounded only for three years. However, if one is required to calculate interest for five-six years, the method given in statement would become tedious.***

Coumpounding

- **Future Worth of a Sum**

  $F = P\,(1 + i)^n$

Where, P → Principal Amount Invested at time 0

F → Future Amount

I → Interest Rate compounded annually

n → Period of Deposit

- **Present Worth of an Amount expected in future**

  $P = F/\,(1 + i)^n$

Future Value Single Deposit (Formula)

$FV_1 = P_0 (1 + i)^1 = \$1{,}000\ (1.07)$
$= \$1{,}070$

Compound Interest

You earned $70 interest on your $1,000 deposit over the first year.

This is the same amount of interest you would earn under simple interest.

Future Value
Single Deposit (Formula)

$$FV_1 = P_0 (1 + i) = \$1{,}000\,(1.07)$$

$$= \$1{,}070$$

$$FV_2 = FV_1 (1 + i)$$

$$= P_0 (1 + i)(1 + i) = \$1{,}000(1.07)(1.07)$$

$$= = P_0 (1 + i)^2 = \$1{,}000(1.07)^2$$

$$= \$1{,}144.90$$

You earned an *EXTRA* *\$4.90* in Year 2 with compound over simple interest.

General Future Value Formula

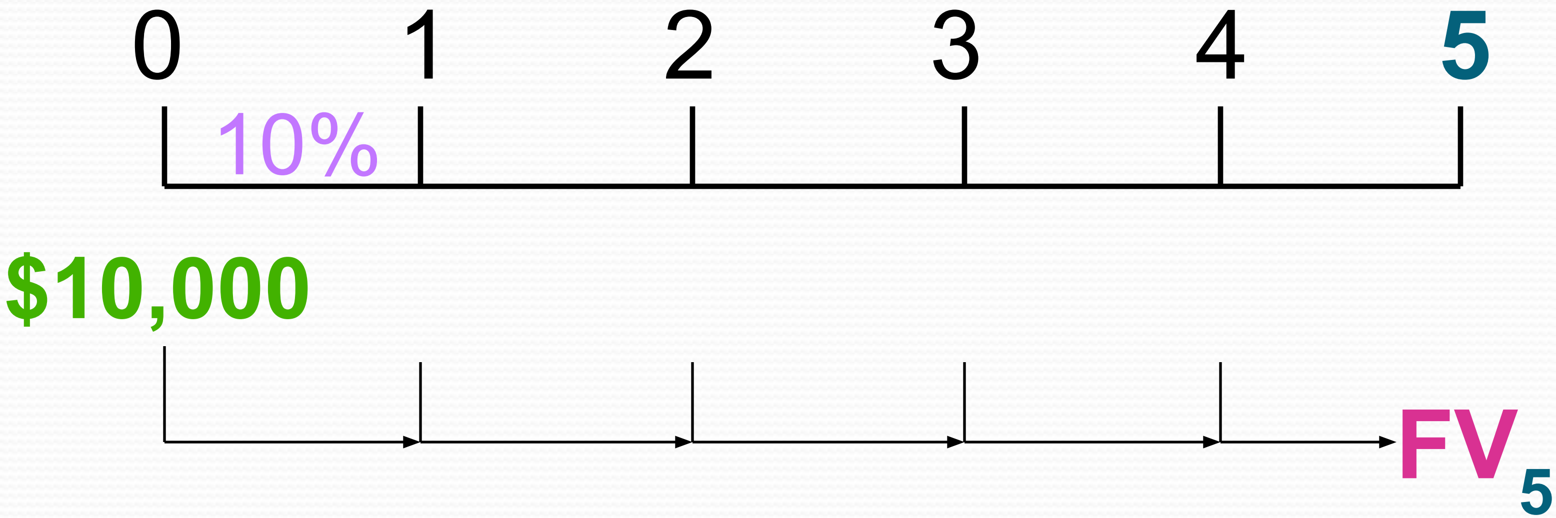
$$FV_1 = P_0(1 + i)^1$$
$$FV_2 = P_0(1 + i)^2$$

General Future Value Formula:

$$FV_n = P_0 (1 + i)^n$$

Problem Example

Julie Miller wants to know how large her deposit of $10,000 today will become at a compound annual interest rate of 10% for 5 years.



Problem Solution

- Calculation based on general formula: $FV_n = P_0 (1 + i)^n$ $FV_5 = \$10,000 (1 + 0.10)^5$ $= \$16,105.10$

Example

- A person deposits a sum of Rs. 20,000/- at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

General Present Value Formula

$$PV_0 = FV_1 / (1 + i)^1$$

$$PV_0 = FV_2 / (1 + i)^2$$

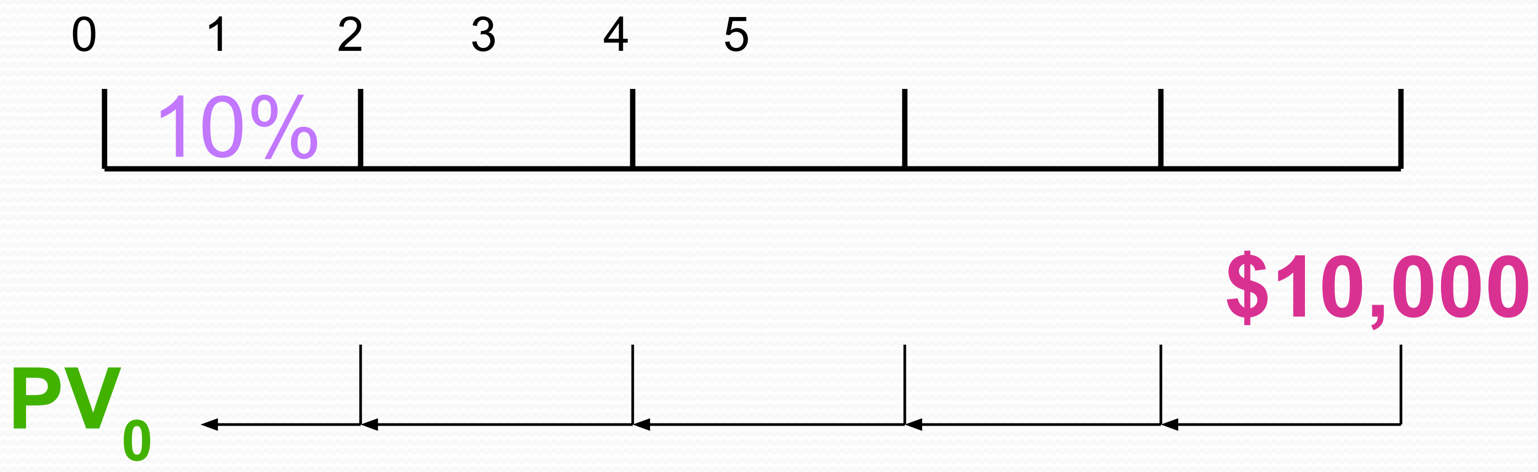
etc.

General Present Value Formula:

$$PV_0 = FV_n / (1 + i)^n$$

Problem Example

Julie Miller wants to know how large of a deposit to make so that the money will grow to $10,000 in 5 years at a discount rate of 10%.



Problem Solution

- Calculation based on general formula: $PV_0 = FV_n / (1 + i)^n$ $PV_0 = \$10{,}000 / (1 + 0.10)^5 = \$6{,}209.21$
-

Example

- A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

Equal-Payment Series Compound Amount

- To find the future worth of n equal payments which are made at the end of every Interest Period till the end of the *n*th Interest period –

Formula: $F = A\dfrac{(1+i)^n - 1}{i} = A(F/A, i, n)$

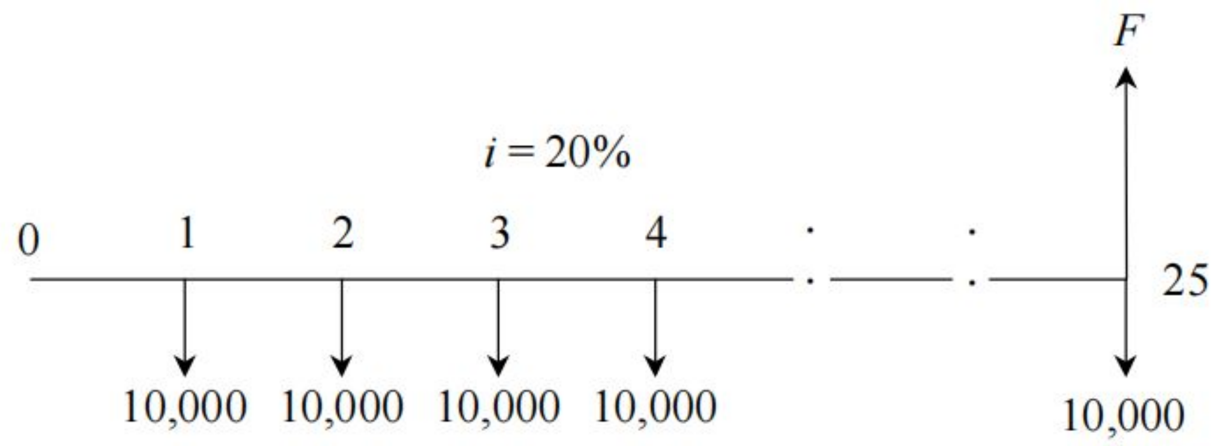
Where, A = Equal amount deposited at the end of each interest period

(F/A, i, n) is called as Equal-Payment Series Compound Amount Factor

Example

A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000/- at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

A = Rs. 10,000
n = 25 years
i = 20%
F = ?



Equal-Payment Series Sinking Fund

- To find the Equivalent amount (A) that should be deposited at the end of every Interest period for n Interest periods to realize a future sum –

Formula: $A = F \dfrac{i}{(1+i)^n - 1} = F(A/F, i, n)$

Where, A = Equal amount deposited at the end of each interest period

(A/F, i, n) is called as Equal-Payment Series Sinking Fund Factor

Example

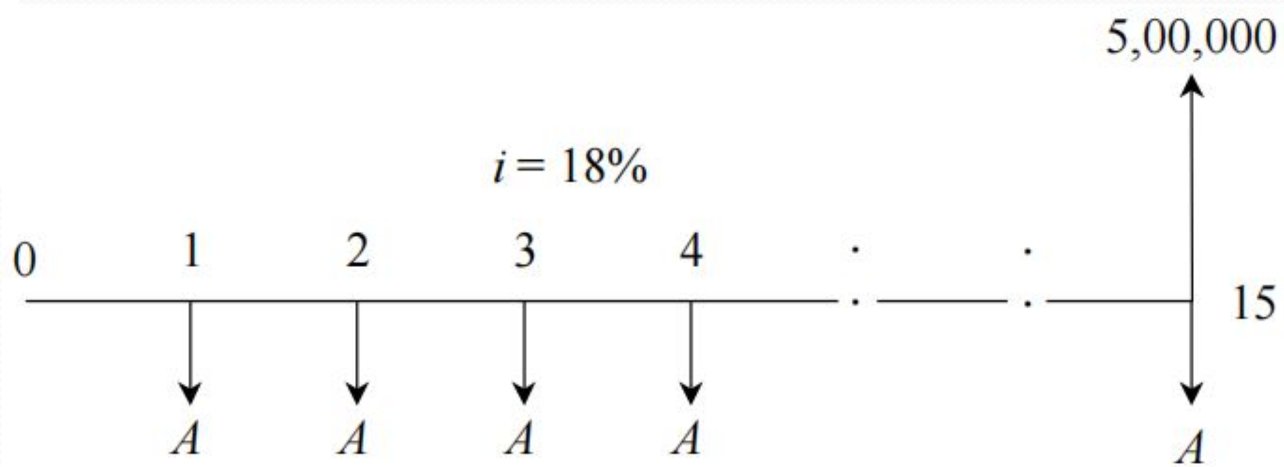
A Company has to replace a present facility after 15 years at an outlay of Rs. 500000/-. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18%, compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

F = Rs. 5,00,000

n = 15 years

i = 18%

A = ?

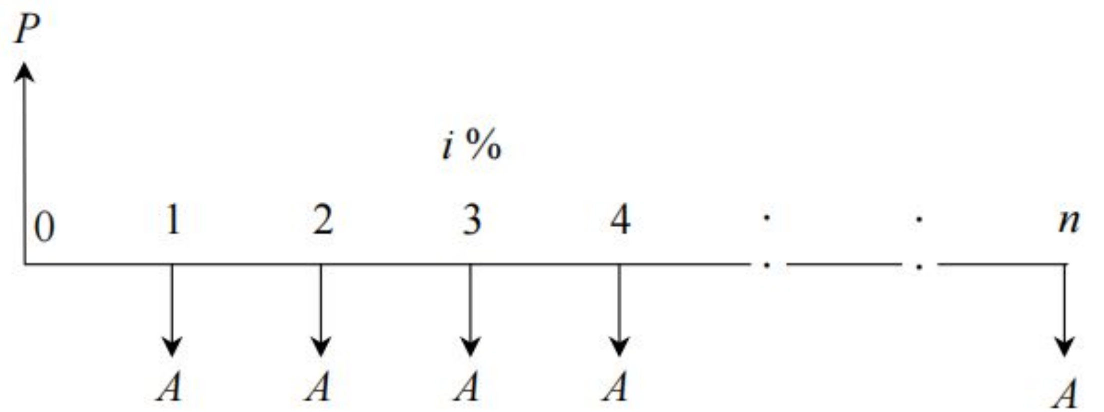


Equal-Payment Series Present Worth Amount

- The objective of this mode of investment is to find the present worth of an equal payment made at the end of every interest period for n interest periods at an interest rate of i compounded at the end of every interest period.
- P = present worth
- A = annual equivalent payment
- i = interest rate
- n = No. of interest periods

$$P = A\,\frac{(1+i)^n - 1}{i(1+i)^n}$$

Cash Flow Daigram

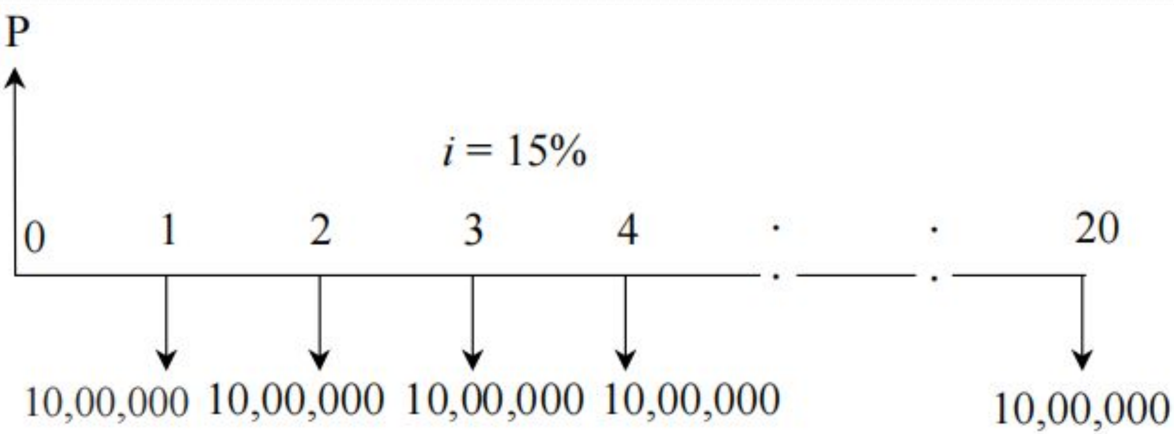


Cash flow diagram of equal-payment series present worth amount.

Example

A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

A = Rs. 10,00,000
i = 15%
n = 20 years
P = ?



Cash flow diagram of equal-payment series present worth amount.

Solution

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalent amount (A) which is to be recovered at the end of every interest period for n interest periods for a loan (P) which is sanctioned now at an interest rate of i compounded at the end of every interest period

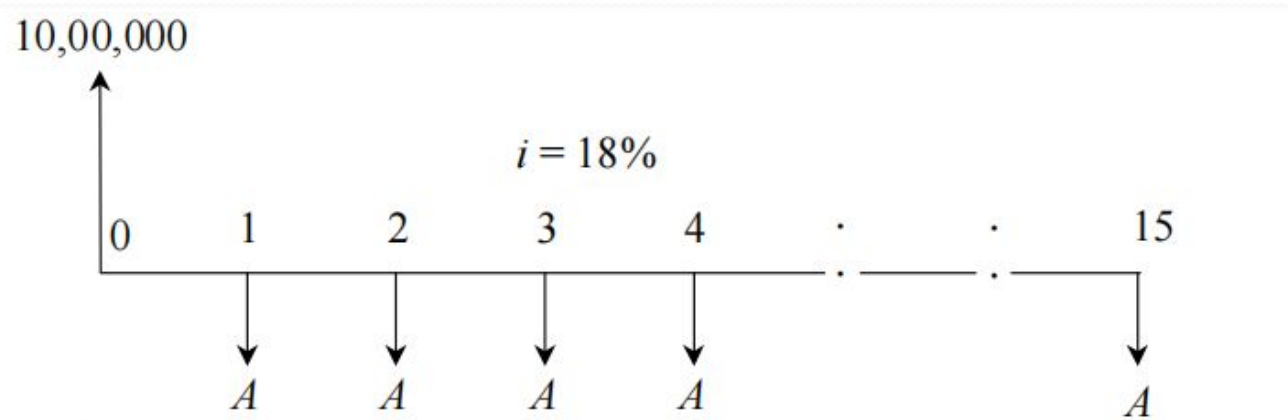
- P = present worth (loan amount)
- A = annual equivalent payment (recovery amount)
- i = interest rate
- n = No. of interest periods

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1}$$

Example-1

- A bank gives a loan to a company to purchase an equipment worth Rs. 1000000 at an interest rate of 18% compunded annually. This amount should be repaid in 15 yearly installments. Find the installment amount that the company has to pay to the bank.

P = Rs. 10,00,000
$i = 18\%$
n = 15 years
$A = ?$



Cash flow diagram of equal-payment series capital recovery amount.

Solution

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1}$$

Uniform Gradient Series Annual Equivalent Amount

- The objective of this mode of investment is to find the annual equivalent amount of a series with an amount A1 at the end of the first year and with an equal increment (G) at the end of each of the following n – 1 years with an interest rate i compounded annually.

Formula: $A = A1 + G\,\frac{(1+i)^n - i.n - 1}{i(1+i)^n - i}$

P = principal amount

n = No. of interest periods

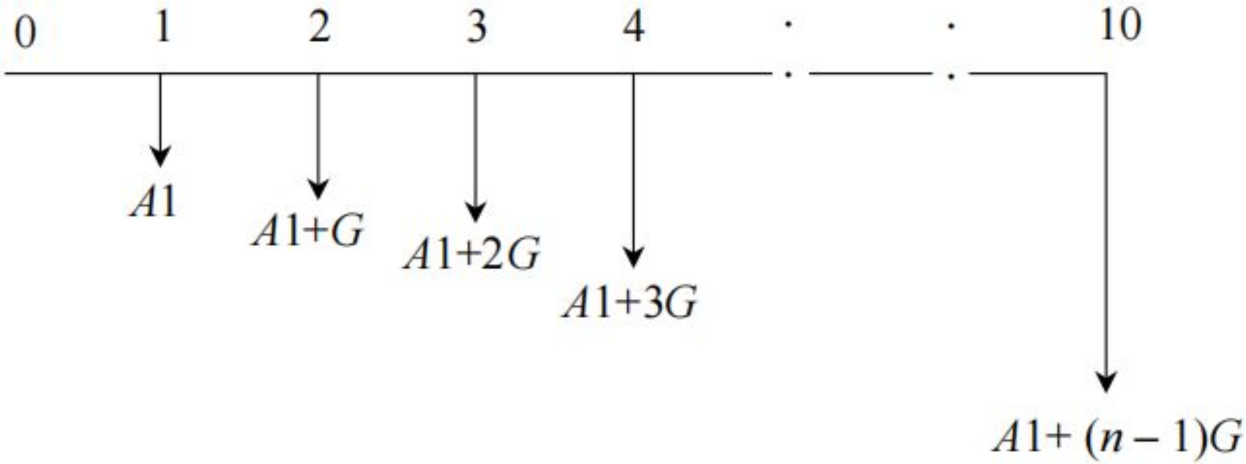
i = interest rate (It may be compounded monthly, quarterly, semiannually or annually)

F = future amount at the end of year n

A = equal amount deposited at the end of every interest period

G = uniform amount which will be added/subtracted period after period to/ from the amount of deposit A1 at the end of period 1

Cash flow Daigram

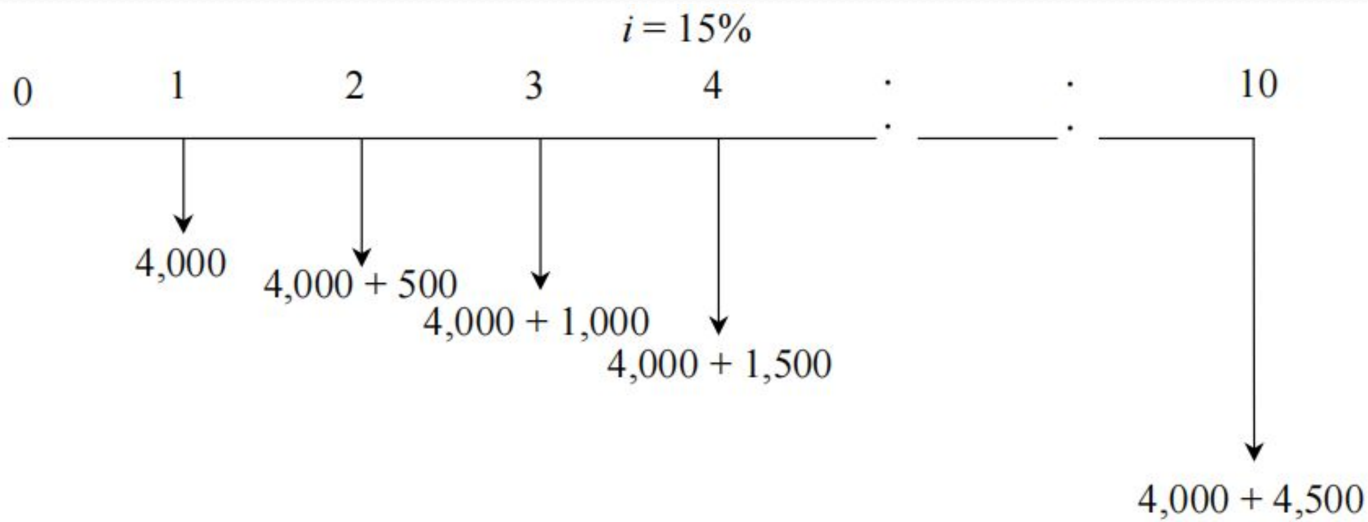


Cash flow diagram of uniform gradient series annual equivalent amount.

Example

A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

$A1$ = Rs. 4,000
G = Rs. 500
i = 15%
n = 10 years
A = ? & F = ?



Solution

- $A = A1 + G\,\frac{(1+i)^n - i.n - 1}{i(1+i)^n - i}$

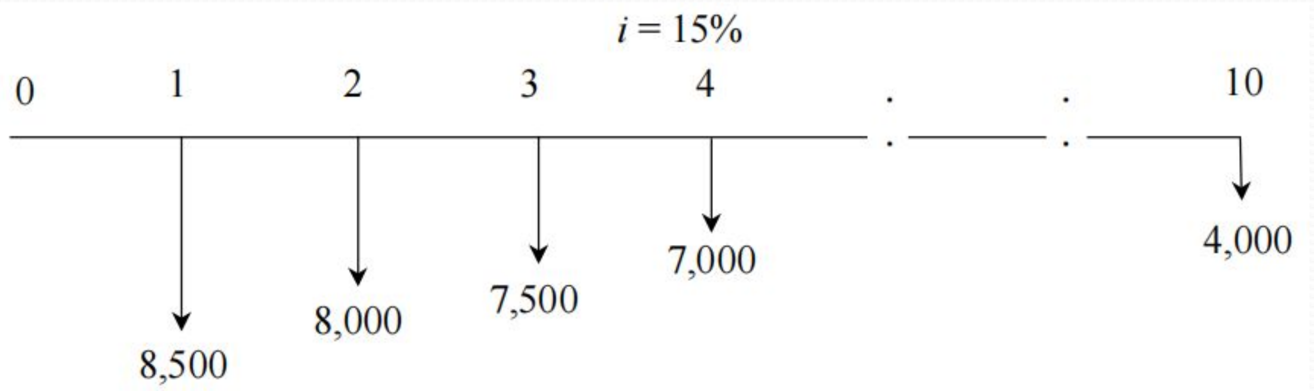
- This is equivalent to paying an equivalent amount of "A" at the end of every year for the next 10 years. The future worth sum of this revised series

$$F = A\,\frac{(1+i)^n - 1}{i}$$

Example-2

A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 8,500 at the end of the first year and thereafter he wishes to deposit the amount with an annual decrease of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

$A1 = \text{Rs. } 8{,}500$
$G = -\text{Rs. } 500$
$i = 15\%$
$n = 10 \text{ years}$
$A = ? \ \& \ F = ?$



Solution

- $$A = A1 + G\,\frac{(1+i)^n - i.n - 1}{i(1+i)^n - i}$$

- This is equivalent to paying an equivalent amount of "A" at the end of every year for the next 10 years. The future worth sum of this revised series

$$F = A\,\frac{(1+i)^n - 1}{i}$$

Effective Interest Rate

- Let i be the nominal interest rate compounded annually. But, in practice, the compounding may occur less than a year, i.e. monthly, quarterly, semi-annually, etc. Thus the compounding monthly means that the interest is computed at the end of every month. There are 12 interest periods in a year in the interest is compounded monthly. Under such situation, the formula is –

Formula: Effective Interest Rate (R)= $(1+\frac{i}{c})^{c}-1$

Where, C= The number of interest periods in a year, and

i= The nominal Interest rate.

Example-1

- A person invests a sum of Rs. 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.
- P = Rs. 5,000
- n = 10 years
- i = 12% (Nominal interest rate)
- c=4
- F = ?

Solution

- Effective interest rate, R $= (1 + i/C)^C - 1$
  $= (1 + 12\%/4)^4 - 1$
  $= 12.55\%$, compounded annually.

$$F = P(1 + R)^n = 5{,}000(1 + 0.1255)^{10}$$
$$= \text{Rs. } 16{,}308.91$$

Example:- 2

Find the effective interest rate for 12 months if the nominal rate of interest is 1% per month and compounding occurs monthly. Using this effective rate of interest find conmpound amount at the end of 15 years for an amount of Rs 200000 which is invested now.

Long Questions

1.1 An industrial unit in US has agreed to pay 25,000 $ in royalties at the end of each yaer for next 5 years for the use of a patented product design. If the payments are left in a foreign country, interest on the retained funds will be paid at an annual rate of 15%. What amount will be available in 5 years under these conditions? How large would the uniform annual payments have to be if the patent owners insisted that a minimum of 1,75,000 $ be accumulated in the account by the end of 5 years. [3]

1.2 Returns from an investment will turn down by 150 $ each year for 5 years from a level of 1000 $ at the end of the 1st year. With 7% interest rate, find out annual series amount over the following 6 year period. [2]

1.3 Find the effective interest rate for 12 months if the nominal rate of interest is 1% per month and compounding occurs monthly. Using this effective rate of interest find conmpound amount at the end of 15 years for an amount of Rs 200000 which is invested now. [2]

2.1 If you want to receive ₹ 10 lakhs after 4 years at an interest rate of 8%,how much should you deposit now? If you want to receive double the amount after 4 years, what should be your present investment at the same rate of interest?

3.1 Delisha has to receive Rs.50000 at the end of first year, Rs.70000 at the end of fourth year, Rs.100000 at the end of the seventh year and Rs.80000 at the end of the tenth year from a company from now. How much money Delisha will receive if she decided to get all her money now? The market rate of interest is 10% annual compounding. [3]

Rana deposited money in a savings account in the following pattern. [2]

Year	Amount of deposits ($)
0	4000
1	-----
2	5000
3	6000
4	-----
5	7000
6	8000

Find the maturity amount of his account at the end of the deposit period if money is growing at 8% interest yearly compounding.

3.3 Sanchit deposited $20000 in a bank for 6 years for his sister's study fund. The annual nominal interest rate is 6%. Find the maturity amount of his account when [3]

(i) Compounding is done half yearly.

(ii) Compounding is done quarterly.

3.4 Suppose that you make a series of annual deposits into a bank account that pays 8% interest compounded annually. The initial deposit at the end of the first year is $50000. The deposit amount declines by $1000 in each of the next ten years. How much money would you get immediately after the deposit period? [2]

4.1 Alex deposits a uniform amount of ₹50000 at the end of each year for 20 years. The rate of interest is 7.2% annual compounding. Find the compound amount that Alex will receive at the end of his deposit period. You want the same compound amount at the end of 12 years for your brother's marriage. You will get the same rate as Alex. Decide the annual equivalent amount that you should deposit at the end of every year. [3]

4.2 Robert plans to deposit ₹100000 in the first year in his savings account. He reduces his deposit amount by ₹2000 thereafter for next 11 years. The bank gives 7.5% interest compounded annually. Find the single amount that I should deposit now so that I will get the same future sum as Robert will get at the end of his deposit period, at the same rate of interest. [2]

4.3 You have received the bill amounts of your company as per the following schedule. If these amounts are deposited in your savings account as soon as they are received and grows at the rate of 4% compounded annually, what will be the compounded amount at the end of 7 years in your account? **(3+2)**

End of the Year	Bill amount Received(₹)
1	300000
2	-
3	350000
4	-
5	400000
6	-
7	450000

4.4 Find the maturity amount and compound interest amount you will receive on ₹33280 deposited now for 8 years at a rate of 12.5% compounded annually.

5.1 A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs 30,000 at the end of the first year and thereafter he wishes to deposit Rs 30,000 at the end of the first year and there after he wishes to deposit the same amount with an annual decrease of Rs 2000 for the next 9 years with an interest rate of 18%. Find the total amount at the end of 10^{th} year of the above series.

5.2 A person invests a sum of Rs 50,000 in a bank at a nominal interest rate of 15% for 20 years. The compounding is quarterly. Find the maturity amount of the deposit after 20 years.

5.3 Assume you save 4000 dollars per year and deposit it at the end of the year in an imaginary saving account (or some other investment) that gives you 6% interest rate (per year compounded annually), for 20 years. How much money will you have at the end of the 20th year?

6.1 Suppose that someone wants to be able to withdraw Rs.30000 at the end of 4 years and withdraw Rs.40000 at the end of 8 years, leaving a total sum of zero in their bank account after the last withdrawal. If they earn 3% on their balances, how much must they deposit today to satisfy their withdrawal needs?

6.2 A person plans to deposit ₹100000 in the first year in his savings account. He reduces his deposit amount by ₹2000 thereafter for next 12 years. The bank gives 7.5% interest compounded annually. Find the maturity value of the deposit.

6.3 If Rs.50000 is deposited in a savings account at present for 15 years and the account draws interest at 8% per year compounded annually, find the value of the account at the end of 15 years. If the same amount is to be generated as annual deposits, how much has to be that yearly deposit?

Solution:-

1.1 F = A(F/A, 15,5) = 1,68,558 $ A= F(A/F, 15,5) = 25,956 $

1.2 A = $ 1000 - $ 150 (A/G, 7, 6) = $654.67

1.3 R= 12.68% F= P (F/P, 12.68, 15) = Rs 11, 98,760.90

2.1 F = 10,00,00, i = 8%, n = 4 P = 10,00,000[1 / (1+0.08)4] = 7,35,029.852

If received amount will be doubled , then P = 20,00,000[1 / (1+0.08)4] = 14,70,058

3.1 P=50000/ (1.1)1 +70,000/(1.1)4 +1,00,000/ (1.1)7 +80,000/(1.1)10 = 175424.76

3.2 F= 4000(1.08)6 +5000(1.08)4 +6000(1.08)3 +7000((1.08)1 +8000= 36268.21

3.3 R=(1+ 0.06/2)2 -1=0.0609 F= 20000(1.0609)6 = 28515.22

R=(1+ 0.06/4)4 -1= 0.0613636 F= 20000(1.0613636)6 = 28590.064

3.4 A=50,000-1000 (A/G, 8%. 11) A= 45760.497 F= 45760.497 (F/A, 8%, 11) F= 761705.78

4.1 A=50000 and i= 7.2% F= A(F/A,i,n) for 20 years = ₹2095099.55

A= F(A/F,i,n) for 12 years = 2095099.55(0.00606878) = ₹12,714.6982

4.2 P= [100000+2000(A/G, 7.5%,12)] x [P/A, 7.5%, 12) = ₹8,45,626.325

4.3 F= 3lakh(1.04)7 + 350000(1.04)5 + 400000(1.04)3 + 450000 = ₹ 1720553.64

4.4 F= 33280(1.125)8 = ₹ 85389.31 Compound interest = ₹52109.31

Solution:-

5.1 By using Gradient formula, A= Rs 23,609.14 (A1= 30,000, G=2000, i= 0.18,n=10 years)

F= A(1+ i)n -1 /n = Rs 555314.57

5.2 R= (1 + i/c)c -1 = 16.98% F= P(1+R)N = Rs 11, 51, 336.691

5.3 F=20,000, c=2, n=5 R=(1+i/c)c -1 = 10.25% P=F/ (1+i)n = 12,278$

6.1 P1= F/ (1+i) ^n= 26654.82 P2= F/ (1+i) ^n= 31578.116 P= P1+P2= 58232.9565

6.2 Gradient series A= 90699.47 F= 1671280.24

6.3 F=50000 (1.08) = 158608.46 A= 158608.46{0.08/ (1.08) ^15-1} = 5847.32