1. Finding the Optimal Hyperplane

Consider the following dataset with two classes:

Point (x_1, x_2)	Class (y)
(2, 3)	+1
(3, 4)	+1
(1, 1)	-1
(2, 1)	-1

(a) Find the equation of the optimal hyperplane assuming a **hard margin** SVM.

(b) Determine the support vectors.

2. Margin Calculation

Given a separating hyperplane equation:

$$3x_1 + 4x_2 - 12 = 0$$

Calculate:

(a) The perpendicular distance from the point $(2, 3)$ to the hyperplane.

(b) The margin if the support vectors are at distances ± 2.5 from the hyperplane.

3. Soft Margin SVM with Slack Variables

For a dataset, an SVM classifier with a soft margin is trained. The loss function includes slack variables ξ_i, and you are given:

- Two misclassified points have slack variables $\xi_1 = 1.5$ and $\xi_2 = 2$.
- The regularization parameter $C = 10$.

Compute the penalty contribution from these misclassified points to the loss function.

4. Kernel Trick: Polynomial and RBF Kernel

Given two points:

$$A(1, 2), \quad B(3, 4)$$

(a) Compute the **polynomial kernel** $K(A, B) = (x^T y + 1)^2$.

(b) Compute the **RBF kernel** with $\gamma = 0.5$ using:

$$K(A, B) = e^{-\gamma ||x-y||^2}$$

5. Lagrange Multipliers and Dual Formulation

Given a simple dataset and the Lagrangian function for SVM, the support vectors have Lagrange multipliers:

$$\alpha_1 = 0.5, \quad \alpha_2 = 0.7, \quad \alpha_3 = 0$$

(a) Find the weight vector w for the classifier if the support vectors are $(1, 2)$ and $(3, 4)$.
(b) Compute the bias term b.

6. Hard Margin SVM Numerical Question

You are given the following linearly separable dataset:

Point (x_1, x_2)	Class (y)
(2, 3)	+1
(3, 3)	+1
(2, 1)	-1
(3, 1)	-1

Tasks:

(a) Find the equation of the optimal separating hyperplane $w_1 x_1 + w_2 x_2 + b = 0$.
(b) Calculate the margin width.
(c) Identify the support vectors.

7. Soft Margin SVM Numerical Question

You are given a dataset that is **not linearly separable** due to some overlap in points:

Point (x_1, x_2)	Class (y)	Slack Variable ξ
(1, 2)	+1	0
(2, 3)	+1	1.2
(3, 3)	+1	0
(1, 1)	-1	0
(2, 1)	-1	1.5

The **regularization parameter** is given as $C = 5$.

Tasks:

(a) Find the total penalty contribution to the loss function from the misclassified points.
(b) Compute the margin width assuming the weight vector $w = (2, 3)$.