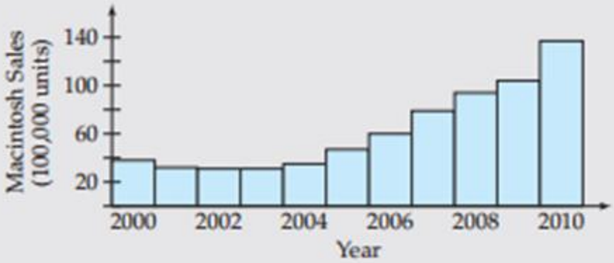
## 1. Penentuan Tingkat Perubahan Sesaat pada Penjualan Tahunan

The following graph shows the annual sales of Apple Macintosh computers between 2000 and 2010 (in hundred thousands of units).




*Source:* Apple

These annual sales are approximated by the function $f(x) = 2x^2 - 9x + 39$, where $x$ stands for *years since 2000*. Find the instantaneous rate of change of this function at $x = 10$ and at $x = 1$ and interpret the results.

**Solution**

The instantaneous rate of change means the *derivative*, so ordinarily we would now take the derivative of the sales function $f(x) = 2x^2 - 9x + 39$. However, this is just what we did in the previous Example, so we will use the result that the derivative is $f'(x) = 4x - 9$. Therefore, we need only evaluate the derivative at the given $x$-values and interpret the results.

$$f'(10) = 4 \cdot 10 - 9 = 31$$ Evaluating $f'(x) = 4x - 9$ at $x = 10$

*Interpretation:* $x = 10$ represents the year 2010 (10 years after 2000). Therefore, in 2010 Macintosh sales were growing at the rate of 31 hundred thousand, or 3.1 million units per year.

$$f'(1) = 4 \cdot 1 - 9 = -5$$ Evaluating $f'(x) = 4x - 9$ at $x = 1$

*Interpretation:* In the year 2001 (corresponding to $x = 1$), Macintosh sales were falling at the rate of 5 hundred thousand, or 500,000 units per year.

## 2. Analisis Marginal

1. Aplikasi Turunan untuk Mendapatkan Biaya Marginal

A company produces a miniature key chain flashlight based on LED (light-emitting diode) technology. The cost function (the total cost of producing $x$ flashlights) is

$$C(x) = 8\sqrt[4]{x^3} + 300 \qquad \text{Cost function}$$

dollars, where $x$ is the number of flashlights produced.

**a.** Find the marginal cost function $MC(x)$.

**b.** Find the marginal cost when 81 flashlights have been produced and interpret your answer.

**Solution**

**a.** The marginal cost function is the derivative of the cost function $C(x) = 8x^{3/4} + 300$, so

$$MC(x) = 6x^{-1/4} = \frac{6}{\sqrt[4]{x}} \qquad \text{Derivative of } C(x)$$

**b.** To find the marginal cost when 81 flashlights have been produced, we evaluate the marginal cost function $MC(x)$ at $x = 81$:

$$MC(81) = \frac{6}{\sqrt[4]{81}} = \frac{6}{3} = 2 \qquad MC(x) = \frac{6}{\sqrt[4]{x}} \text{ evaluated at } x = 81$$

*Interpretation:* When 81 flashlights have been produced, the marginal cost is \$2, meaning that to produce one more flashlight costs about \$2.

*Source:* BereLite

2. Aplikasi Turunan untuk Mencari Rata-rata Biaya Marginal

POD, or *printing on demand*, is a recent development in publishing that makes it feasible to print small quantities of books (even a single copy), thereby eliminating overstock and storage costs. For example, POD-publishing a typical 200-page book would cost \$18 per copy, with fixed costs of \$1500. Therefore, the cost function is

$$C(x) = 18x + 1500 \qquad \text{Total cost of producing } x \text{ books}$$

**a.** Find the average cost function.

**b.** Find the marginal average cost function.

**c.** Find the marginal average cost at $x = 100$ and interpret your answer.

*Source:* e-booktime.com

**Solution**

**a.** The average cost function is

$$AC(x) = \underbrace{\frac{18x + 1500}{x}}_{\substack{\text{Total cost divided} \\ \text{by number of units}}} = \underbrace{18 + \frac{1500}{x}}_{\text{Simplifying}} = \underbrace{18 + 1500x^{-1}}_{\text{In power form}}$$

**b.** The *marginal* average cost is the derivative of average cost. We could use the Quotient Rule on the first expression above, but it is easier to use the Power Rule on the last expression:

$$MAC(x) = \frac{d}{dx}(18 + 1500x^{-1}) = -1500x^{-2} = -\frac{1500}{x^2}$$

**c.** Evaluating at $x = 100$:

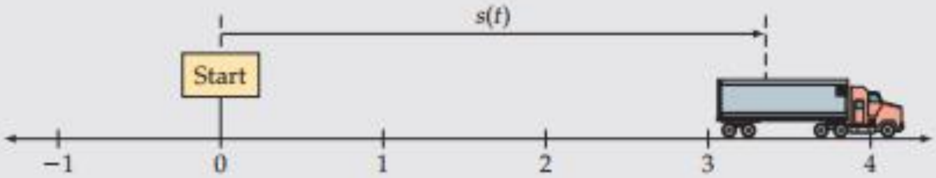
$$MAC(100) = -\frac{1500}{100^2} = -\frac{1500}{10{,}000} = -0.15 \qquad -\frac{1500}{x^2} \text{ at } x = 100$$

*Interpretation:* When 100 books have been produced, the average cost per book is decreasing (because of the negative sign) by about *15 cents per additional book produced*. This reflects the fact that while *total* costs rise when you produce more, the *average cost per unit* decreases, because of the economies of mass production.

## 3. Kecepatan dan Percepatan

A delivery truck is driving along a straight road, and after $t$ hours its distance (in miles) east of its starting point is

$$s(t) = 24t^2 - 4t^3 \quad \text{for } 0 \le t \le 6$$




**a.** Find the velocity of the truck after 2 hours.

**b.** Find the velocity of the truck after 5 hours.

**c.** Find the acceleration of the truck after 1 hour.

**Solution**

**a.** To find velocity, we differentiate distance:

$v(t) = 48t - 12t^2$ — Differentiating $s(t) = 24t^2 - 4t^3$

$v(2) = 48 \cdot 2 - 12 \cdot (2)^2$ — Evaluating $v(t)$ at $t = 2$

$= 96 - 48 = 48$ miles per hour — Velocity after 2 hours

**b.** At $t = 5$ hours:

$v(5) = 48 \cdot 5 - 12 \cdot (5)^2$ — Evaluating $v(t)$ at $t = 5$

$= 240 - 300 = -60$ miles per hour — Velocity after 5 hours

What does the negative sign mean? Since distances are measured *eastward* (according to the original problem), the "positive" direction is east, so a negative velocity means a *westward* velocity. Therefore, at time $t = 5$ the truck is driving *westward at 60 miles per hour* (that is, back toward its starting point).

**c.** The acceleration is

$a(t) = 48 - 24t$ — Differentiating $v(t) = 48t - 12t^2$

$a(1) = 48 - 24 = 24$ — Acceleration after 1 hour

Therefore, after 1 hour the acceleration of the truck is 24 mi/hr$^2$ (it is "speeding up").

## 4. Memprediksi Pertumbuhan Populasi

United Nations demographers predict that $t$ years from the year 2010 the population of the world will be:

$$P(t) = 6900 + 198\,t^{2/3}$$

million people. Find $P'(8)$ and $P''(8)$ and interpret these numbers.

**Solution** The derivative is

$$P'(t) = 132t^{-1/3} \qquad \text{Derivative of } P(t)$$

so

$$P'(8) = 132(8)^{-1/3} = 132 \cdot \frac{1}{2} = 66 \qquad \text{Evaluating at } t = 8$$

*Interpretation:* In 2018 ($t = 8$ years from 2010) the world population will be growing at the rate of 66 million people per year.

The second derivative is

$$P''(t) = -44t^{-4/3} \qquad \text{Derivative of } P'(t) = 132t^{-1/3}$$

$$P''(16) = -44(8)^{-4/3} \qquad \text{Evaluating at } t = 8$$

$$= -44 \cdot \frac{1}{16} = -\frac{44}{16} = -\frac{11}{4} = -2.75$$

The fact that the first derivative is positive and the second derivative (the rate of change of the derivative) is negative means that the growth is continuing but more slowly.

*Interpretation:* After 8 years, the growth rate is *decreasing* by about 2.75 million people per year each year. In other words, in the following year the population will continue to grow, but at the slower rate of (rounding 2.75 to 3) about $66 - 3 = 63$ million people per year.

*Source:* U.N. Department of Economic and Social Affairs