Assignment-9: Spectra of Non-Periodic signals

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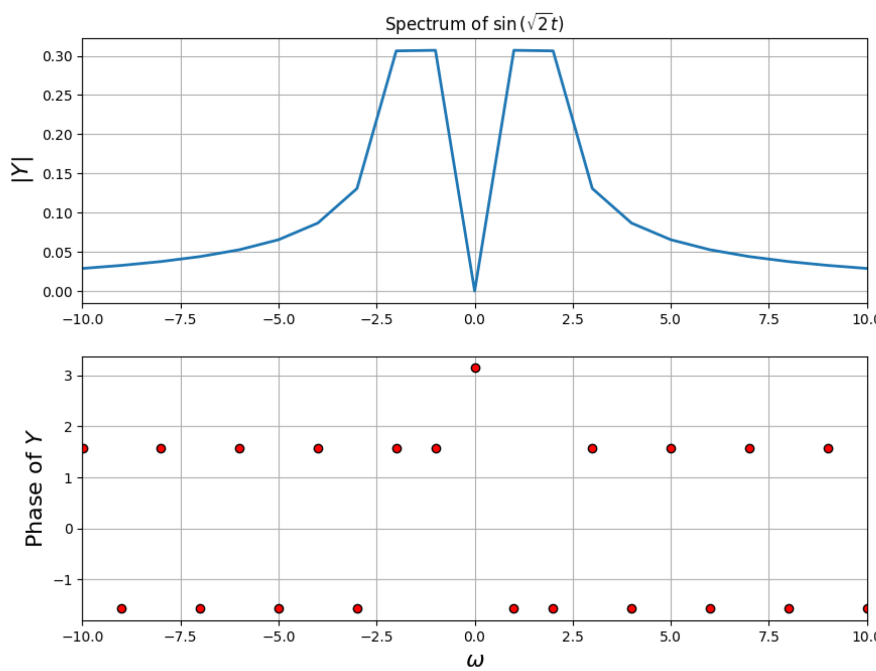
Introduction:

In this assignment, we will proceed further with our discussion on Fourier transform. Here, we will be working on Non-Periodic signals. The concepts we will be going through now are-

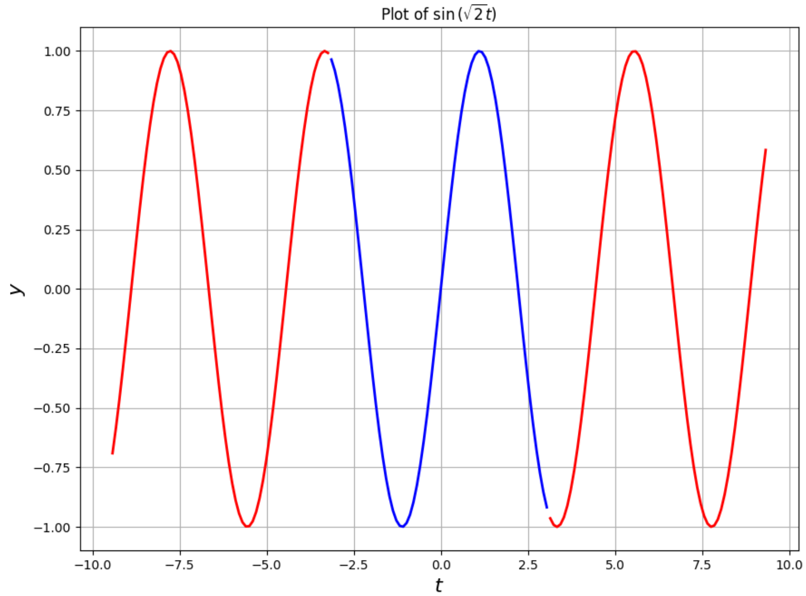
- Plot DFT of Non-Periodic functions
- Improvise it with Hamming Window
- Fit sinusoidal to Windowed signal and extract its frequency and phase
- Analyze the same for noised signal
- Work with Chirped signal and plot its Magnitude and Phase surface plot

Question: 1

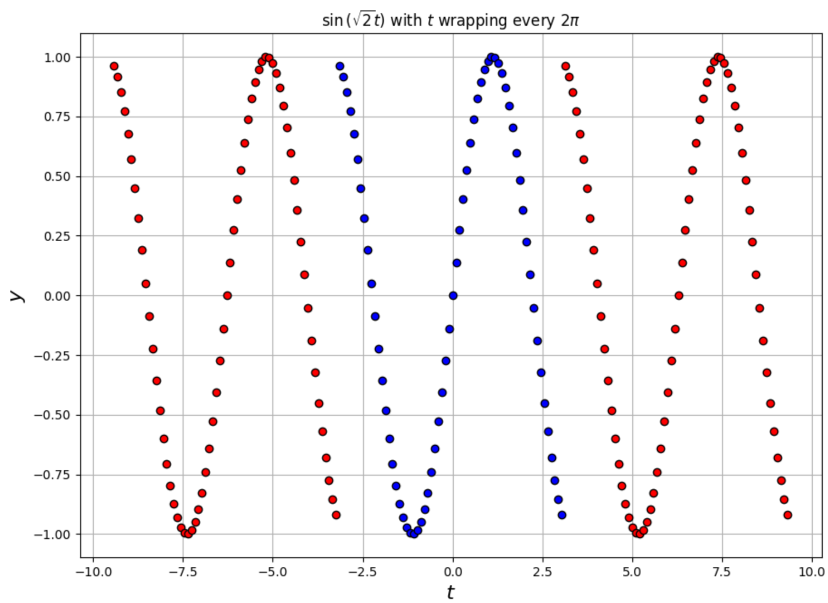
Here is the Spectrum of $\sin(\sqrt{2}t)$ in frequency domain-



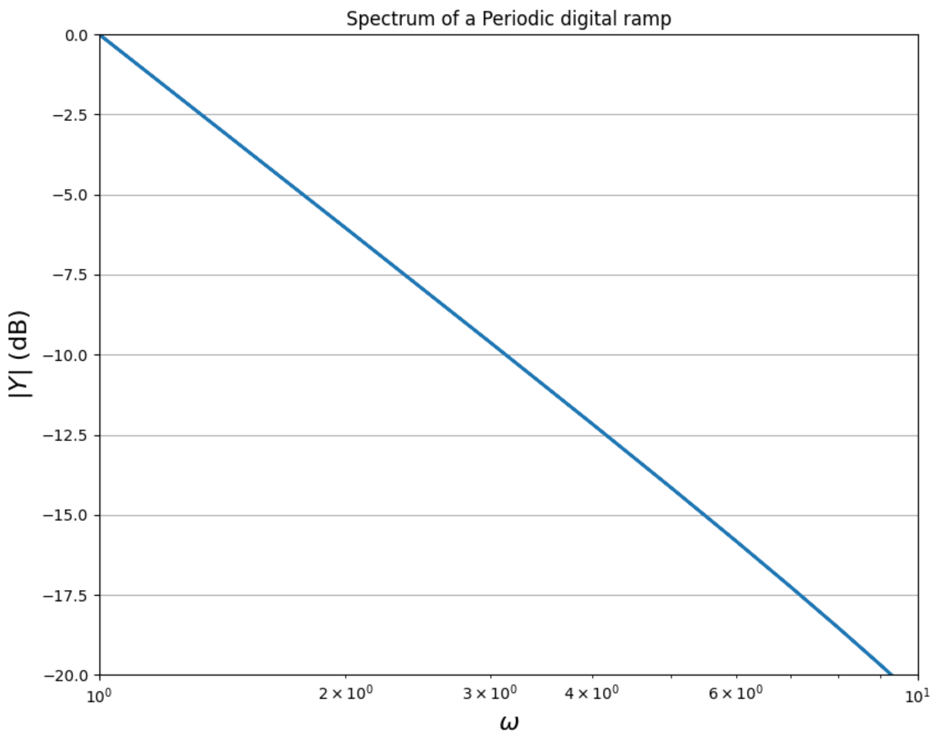
Here is the plot of $\sin(\sqrt{2}t)$ time domain-



As we need the DFT over a finite time interval. We here plot the DFT of the above function in $(-\pi, \pi)$ interval



As we can see in the above plot, the big jumps at nπ will lead to Non-harmonic components in FFT, and cause slow decaying with $\frac{1}{\omega}$. To verify this we will be plotting periodic ramp of it.



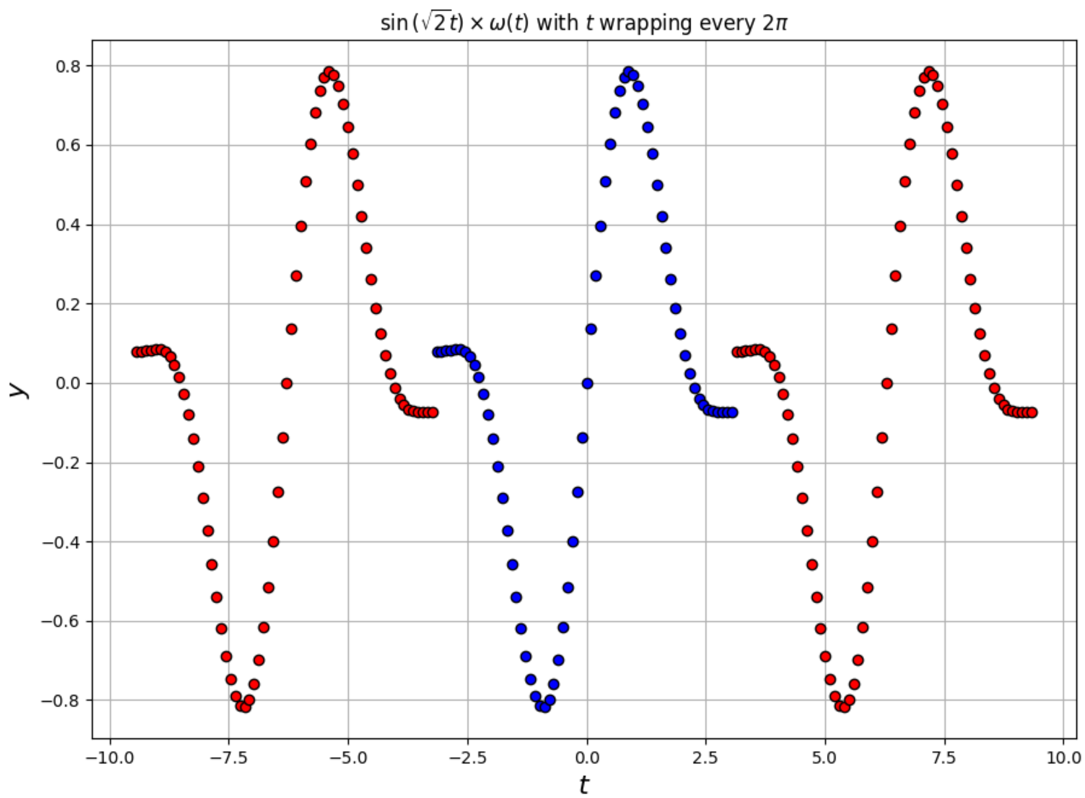
From here we confirm the presence of discontinuities. This is why we need Hamming window.

Windowing:

The hamming window removes the discontinuities by attenuating the high frequency components in DFT of $\sin(\sqrt{2}t)$. The hamming window given is-

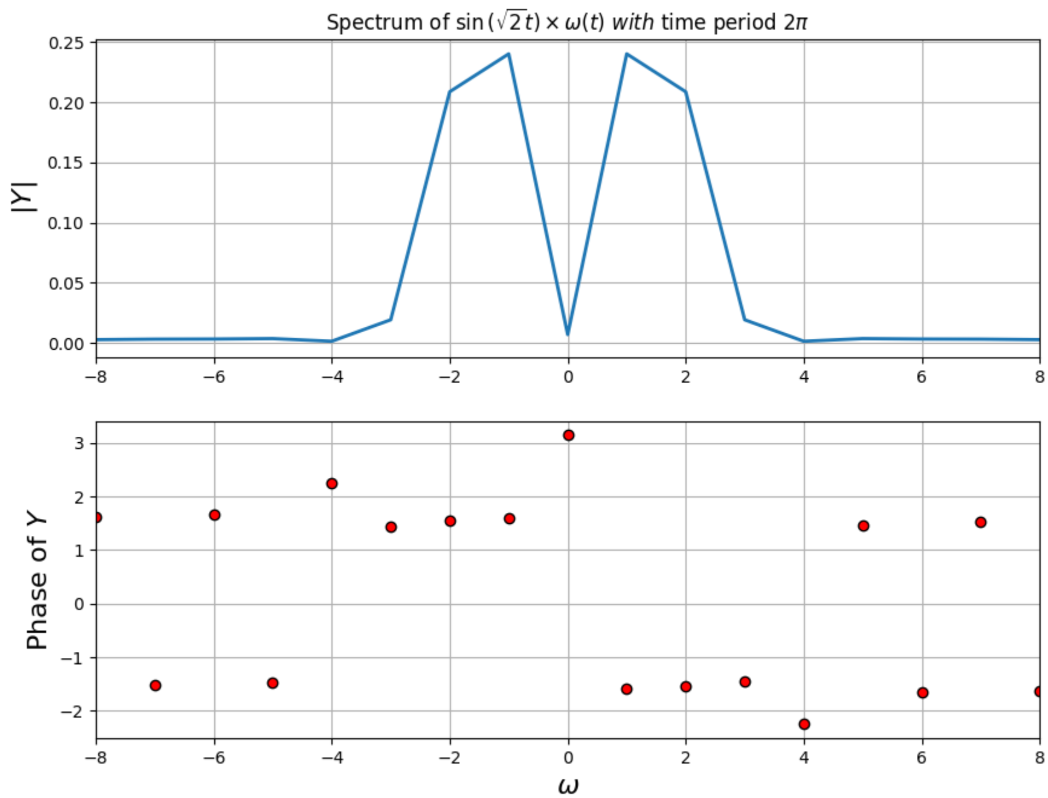
$$\omega[n] = \begin{cases} 0.54 + 0.46 \times cos(\frac{2\pi n}{N-1}) & \text{for } |n| \leq \frac{N-1}{2} \\ 0 & \text{else} \end{cases}$$

We will multiply this hamming window function to DFT of $\sin(\sqrt{2}t)$, we get-



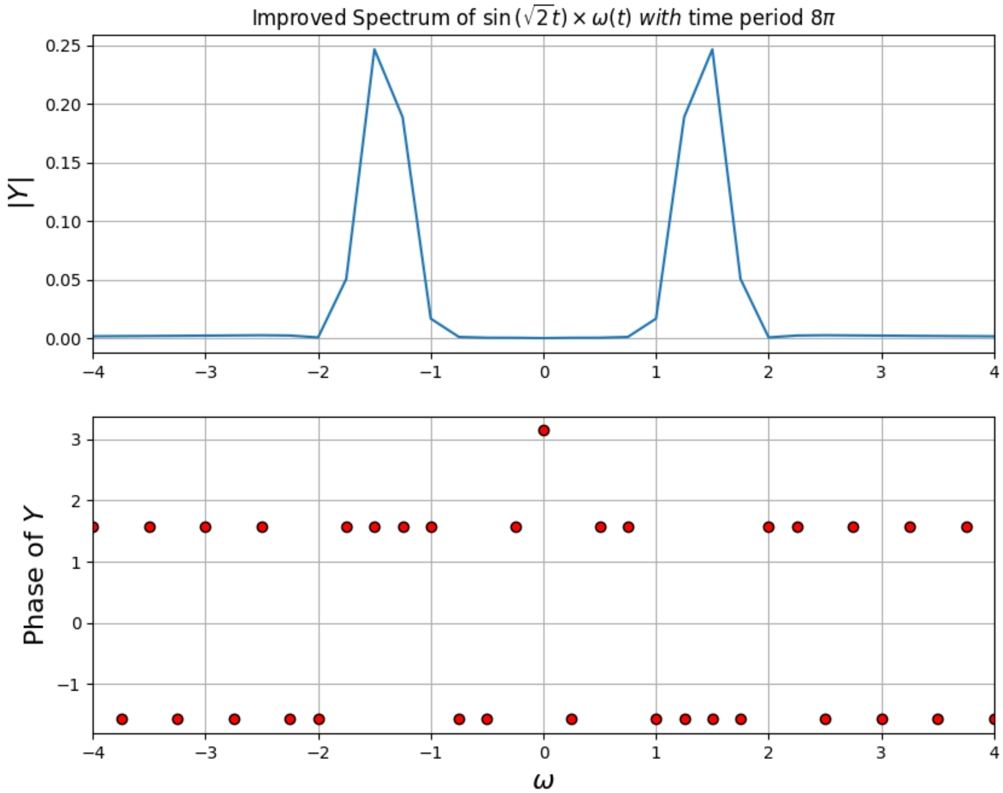
Now, we can see the jump (i.e. discontinuities) is very decreased. So we can proceed to do DFT of $\sin(\sqrt{2}t)$.

This is the spectrum of $\sin(\sqrt{2}t)$ with time period 2π-



Compared to the first plot and the magnitude is greatly improved. But we still have a peak that is two samples wide. But that is because $\sqrt{2}$ lies between 1 and 2, which are the two fourier components available. So, using four times the number of points would give promising results. As at that time $\sqrt{2}$ is not in between 1.25 and 1.5.

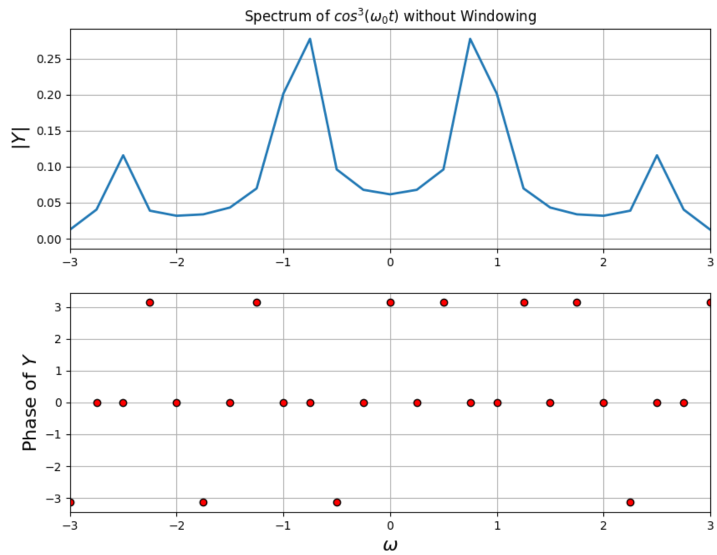
This is the spectrum of $\sin(\sqrt{2}t)$ with time period 8π-



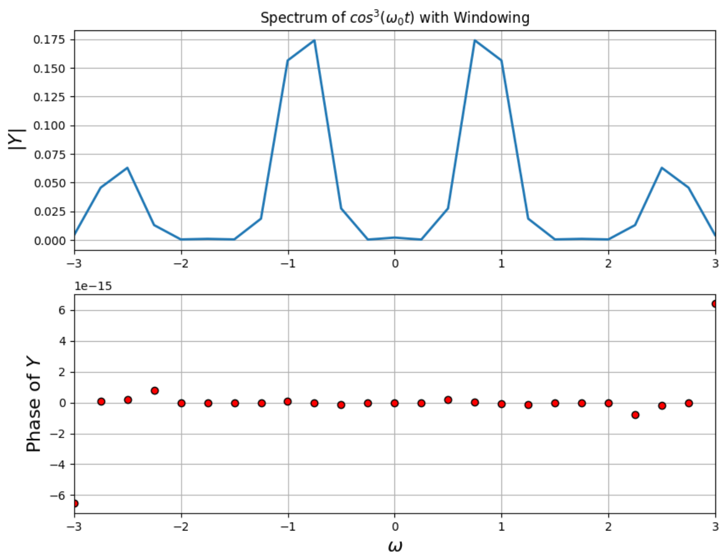
Thus, we get an improved graph which is the result of windowing and choosing right number of points.

Question: 2

The FFT of $cos^3(0.86t)$ without Hamming window



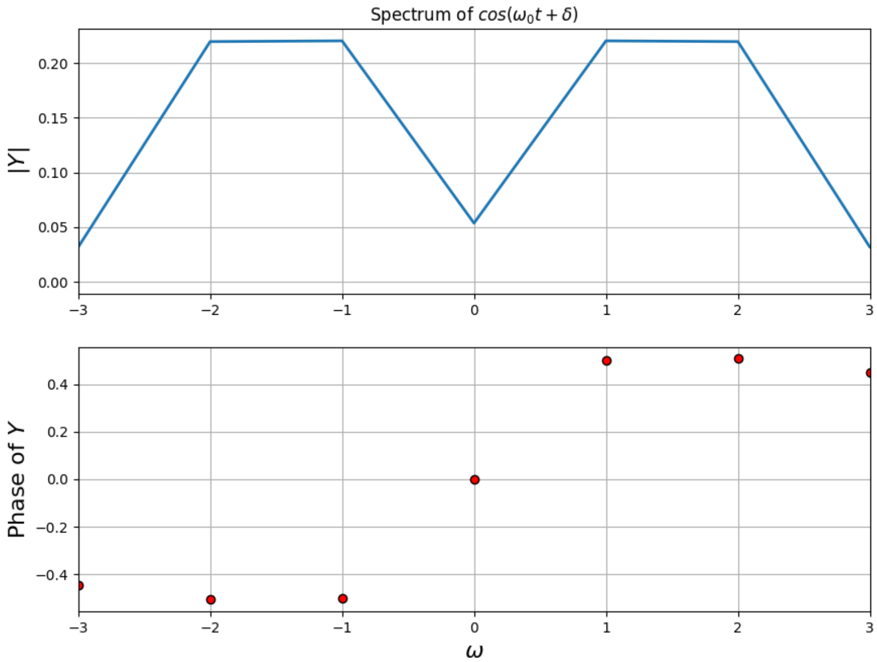
The FFT of $cos^3(0.86t)$ with Hamming window



Here we can clearly notice that after windowing, the higher frequencies are attenuated and hence the peaks are sharper now.

Question: 3

Now from $cos(\omega t + \delta)$ signal, we take 128 samples in time range of $-\pi$ to π , with 0.5< ω_0 < 1.5
Here is the Spectrum of $cos(1.5t + 0.5)$-

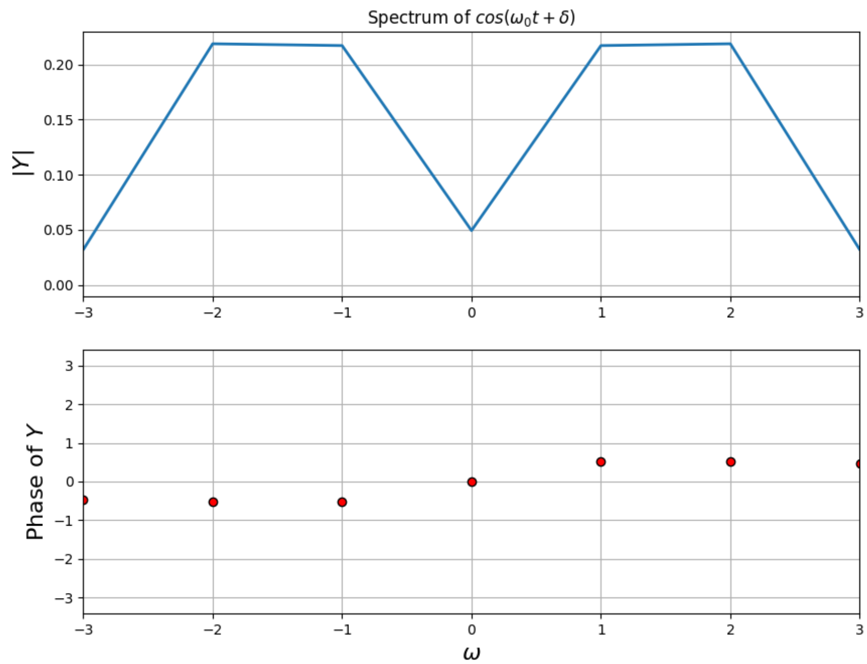


From the signal we find the two peaks at $\pm\ \omega_0$. And estimate ω using weighted average and δ by a window on each half of ω. And we get-

$$\omega_0 = 1.5163$$
$$\delta = 0.5068$$

Question: 4

Now, we repeat the same above process but now we add noise to the signal and windowing it too.
Here is the Spectrum of $cos(1.5t + 0.5)$ with Hamming windowing and added Noise-



Same as the previous time, from the signal we find the two peaks at $\pm\, \omega_0$. And estimate ω using weighted average and δ by a window on each half of ω. This time we get-

$$\omega_0 = 2.0530$$
$$\delta = 0.5038$$

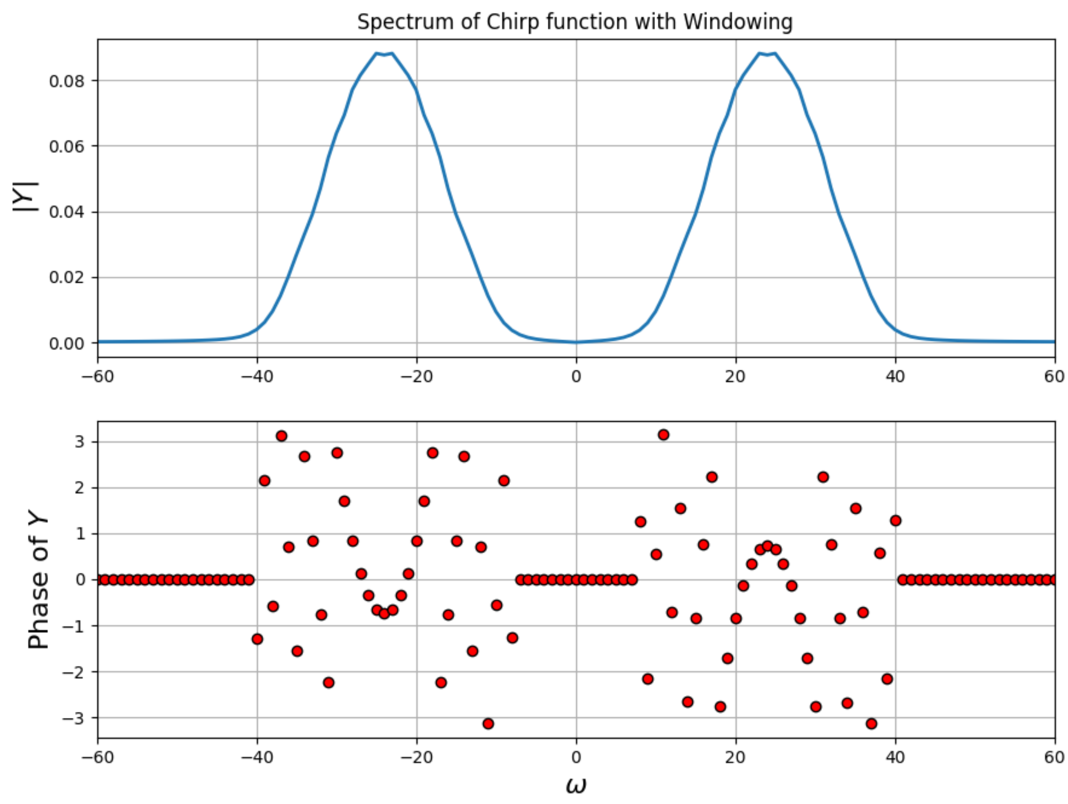
Question: 5

Here we will be plotting DFT of Chirped signal function-

$$f(t) = cos(16t(1.5 + \frac{t}{2\pi}))$$

for t going from π to π in 1024 steps, while frequency continuously changes from 16 to 32 radians per second.

Here is the Spectrum of windowed Chirped function-

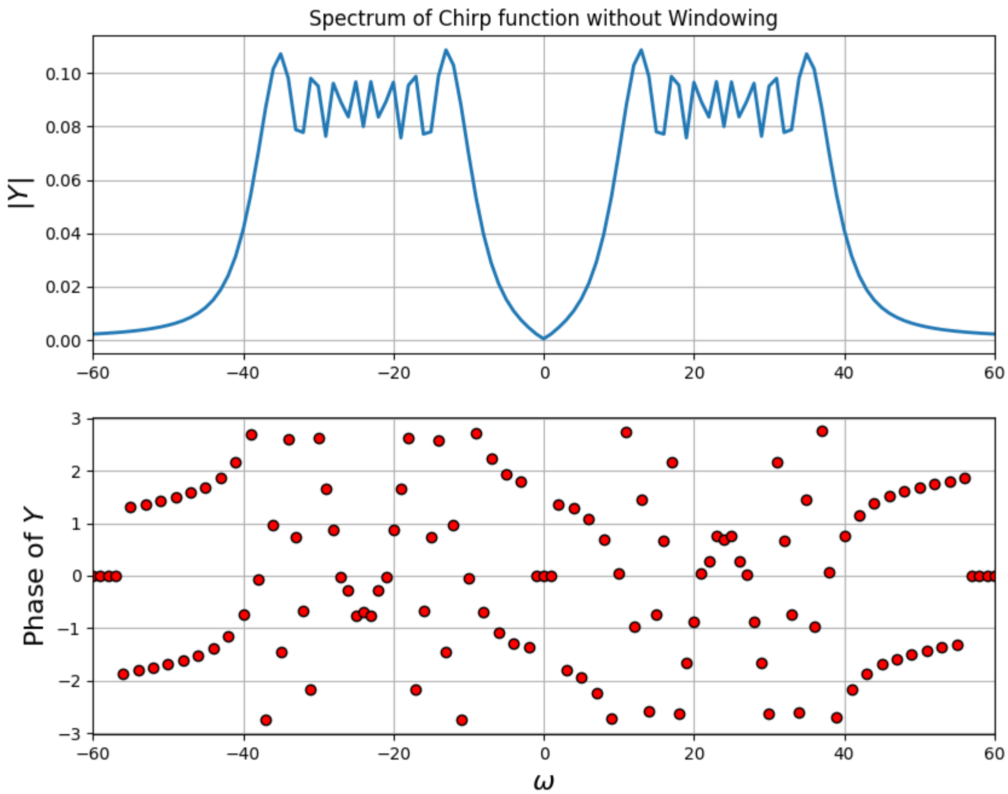


Same as the previous time, we will find ω_0 and δ for this windowed Chirped function and we get-

$$\omega_0 = 24.0084$$

$$\delta = 1.6044$$

Similarly, here is the Spectrum of Non-windowed Chirped function-



Same as the previous time, we will find ω_0 and δ for this Non-windowed Chirped function and we get-

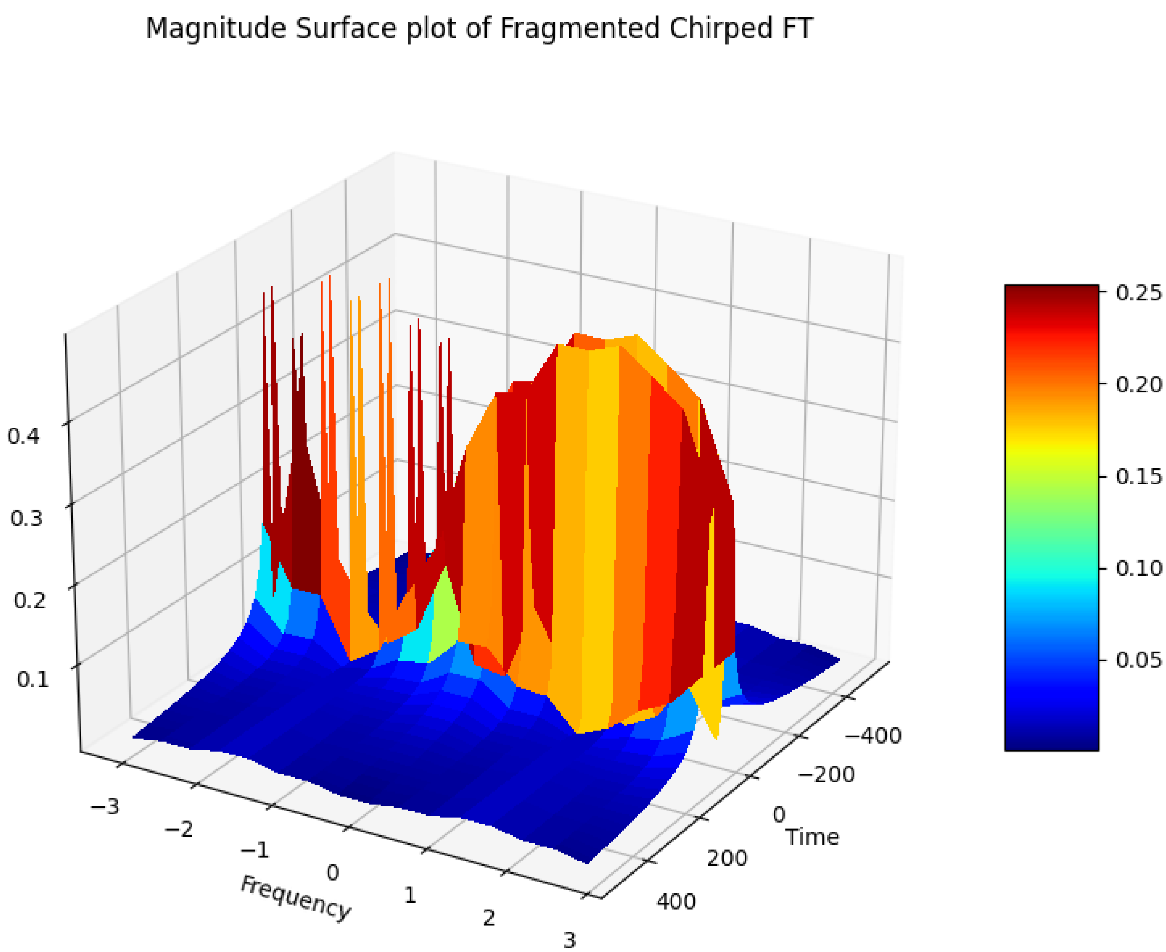
$$\omega_0 = 24.5193$$

$$\delta = 1.3646$$

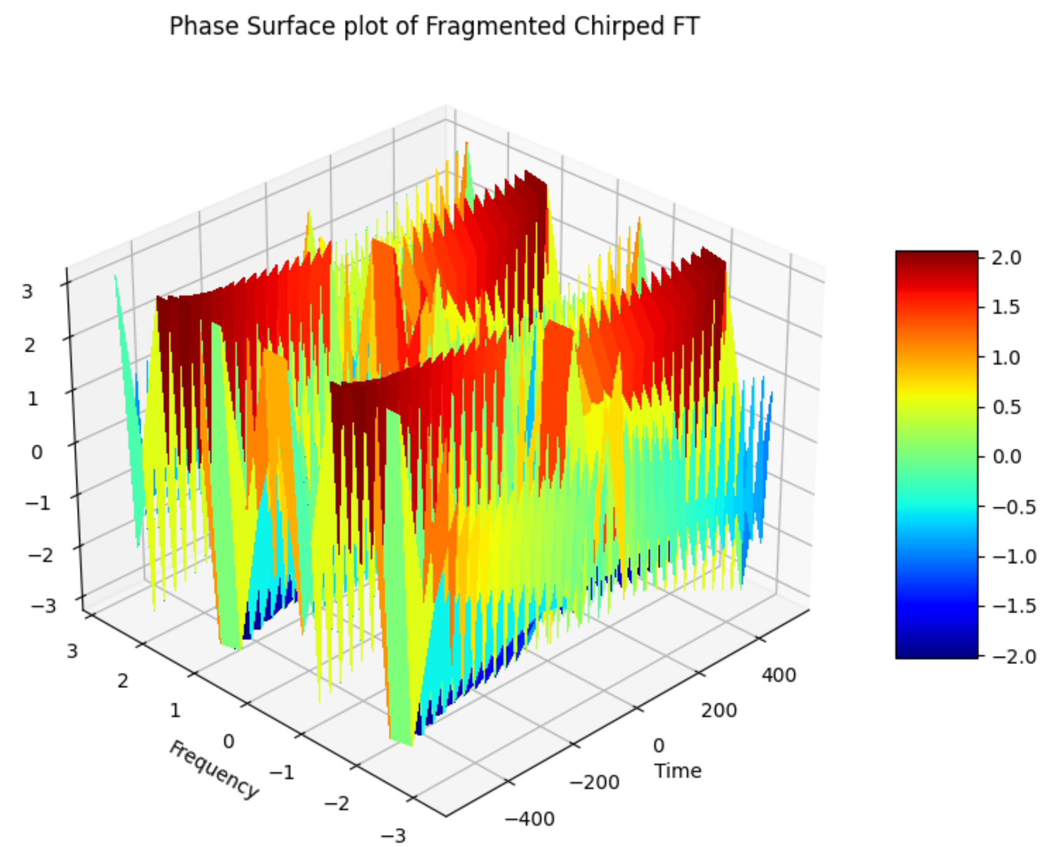
Question: 6

From the same chirped signal, now we will break the 1024 vector into pieces that are 64 samples wide. And then extract the DFT of each and store as a column in a 2D array. Then plot the array as a surface plot to show the variation of Frequency vs. Time.

Here is the Magnitude surface plot of Fragmented Chirped siganl-



Here is the Phase surface plot of Fragmented Chirped siganl-



Conclusion:

We can plot and analysis precise DFT of Non-periodic signal in Python. Thus, we learnt a lot of interesting things in this assignment, and proving once again that Python is compatible with various signal processing problems.

Thank you!