# EE2703 - Applied Programming Language

## End Semester Exam


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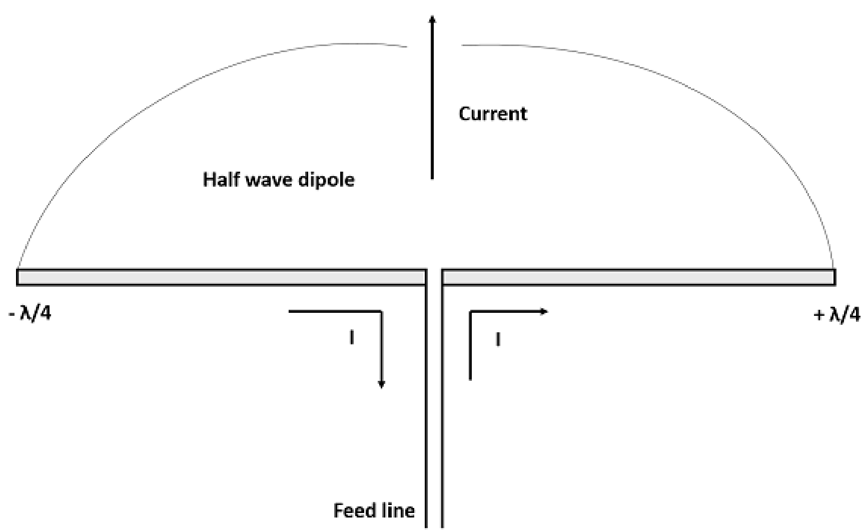
# Contents:

- Current distribution in a dipole antenna
- Finding $P_{ij}$ , $P_B$ (Coefficient matrix of Magnetic vector potential $\overrightarrow{A}_{z,i}$)
- Finding $Q_{ij}$ , $Q_B$ (Coefficients matrix of Magnetic field $\overrightarrow{H}_{\phi}(r, z_i)$)
- Finding Current through Magnetic vector potential method
- Comparison of I vs. z in different methods and variable N

# Dipole Antenna:

It consists of two conductive element i.e., rods or wires and a feeder(connecting the rods). This wire or rod are separated into two sections by an insulator. The radio-frequency (RF) voltage source is applied to the center between the two sections of the dipole antenna. The length of the metal wires is approximately half of the maximum wavelength (i.e.,= $\frac{\lambda}{2}$) of the current passing through it in free space at the frequency of operation.

The current is maximum and voltage is minimum at the center. Conversely, the current is minimum and voltage is maximum at the ends of the dipole antenna. It can either be used as transmitting or receiving antenna. And it is widely used in radio and telecommunications.



# Current through antenna:

From the standard analysis, we assume the current in the antennas to be-

$$I = \begin{cases} Im \times sin(k(l-z)) & 0 \leq z \leq l \\ Im \times sin(k(l+z)) & l \leq z < 0 \end{cases} \tag{1}$$

where,
Im=current injected to the antenna
k=wave number=$\frac{2\pi}{\lambda}$
l=quarter wavelength=$\frac{\lambda}{4}$
z=distance from reference point along the rod

By this way, we get a set of values for the current through the antenna(for N=4), which is-

$$I = \begin{pmatrix} 0.0 \\ 0.3827 \\ 0.7071 \\ 0.9239 \\ 1.0 \\ 0.9239 \\ 0.7071 \\ 0.3827 \\ 0.0 \end{pmatrix}$$

We can clearly see the sinusoidal nature of the current, and we can observe the maximum current in the middle and current minimum current at the ends of the rod.

***Pseudo code: 1***
$I = Im \times sin(k(l \pm z))$
for $0 \leq z \leq l \rightarrow +$
for $l \leq z < 0 \rightarrow -$

# Ampere's Circuital law:

Around every closed curve, the line integral of the magnetic field B is equal to $\mu_o$ times the net current I threading through the region contained by the curve.

$$\oint H'.dl' = \sum I \tag{2}$$

Applying the same in dipole antenna where $radius = a$, we get-

$$2\pi a H_\phi(z_i) = I_i \tag{3}$$

We convert the same into matrix equation, we get-

$$\begin{pmatrix} H_\phi(z_1) \\ \dots \\ H_\phi(z_{N-1}) \\ H_\phi(z_{N+1}) \\ \dots \\ H_\phi(z_{2N-1}) \end{pmatrix} = \frac{1}{2\pi a} \begin{pmatrix} 1 & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & \dots & 1 \end{pmatrix} \begin{pmatrix} J_1 \\ \dots \\ J_{N-1} \\ J_{N+1} \\ \dots \\ J_{2N-1} \end{pmatrix} = M \times J$$

***Pseudo code: 2***

Create matrix J, order=[$2N - 2 \times 1$], initially define with zeros

Create matrix M = $\frac{1}{2\pi a} \times$ identity matrix of order [2N-2,2N-2]

# Calculating Magnetic vector potential:

Firstly, we will be calculating Magnetic vector potential $\overrightarrow{A}_{z,i}$

$$A_{z,i} = \sum_j I_j \left( \frac{\mu_o}{4\pi} \times \frac{exp(-jkR_{ij})dz'j}{Rij} \right)$$

Writing it in matrix form, we get-

$$A_z, j = \sum jP_{ij}I_j + P_B I_N$$

From the above equation we write the $P_B$ and $P_{ij}$ as,

$$P_B = \frac{\mu_o}{4\pi} \times \frac{exp(-jkR_{iN}) \times dz'_j}{R_{iN}}$$
$$P_{ij} = \frac{\mu_o}{4\pi} \times \frac{exp(-jkR_{ij}) \times dz'_j}{R_{ij}}$$

where,

$$R_{ij} = \sqrt{a^2 + (z_i - z_j)^2}$$

Solving for N=4, we get-

$$R_{ij} = \begin{pmatrix} 0.01 & 0.13 & 0.25 & 0.5 & 0.63 & 0.75 \\ 0.13 & 0.01 & 0.13 & 0.38 & 0.5 & 0.63 \\ 0.25 & 0.13 & 0.01 & 0.25 & 0.38 & 0.5 \\ 0.5 & 0.38 & 0.25 & 0.01 & 0.13 & 0.25 \\ 0.63 & 0.5 & 0.38 & 0.13 & 0.01 & 0.13 \\ 0.75 & 0.63 & 0.5 & 0.25 & 0.13 & 0.01 \end{pmatrix}$$

$$P_B = \begin{pmatrix} 1.27e-08-3.08e-08j \\ 3.53e-08-3.53e-08j \\ 9.20e-08-3.83e-08j \\ 9.20e-08-3.83e-08j \\ 3.53e-08-3.53e-08j \\ 1.27e-08-3.08e-08j \end{pmatrix}$$

$$P_{ij} = \begin{pmatrix} 1.25e{-}06{-}3.93e{-}08j & 9.20e{-}08{-}3.83e{-}08j & 3.53e{-}08{-}3.53e{-}08j & -7.85e{-}12{-}2.50e{-}08j & -7.66e{-}09{-}1.85e{-}08j & -1.18e{-}08{-}1.18e{-}08j \\ 9.20e{-}08{-}3.83e{-}08j & 1.25e{-}06{-}3.93e{-}08j & 9.20e{-}08{-}3.83e{-}08j & 1.27e{-}08{-}3.08e{-}08j & -7.85e{-}12{-}2.50e{-}08j & -7.66e{-}09{-}1.85e{-}08j \\ 3.53e{-}08{-}3.53e{-}08j & 9.20e{-}08{-}3.83e{-}08j & 1.25e{-}06{-}3.93e{-}08j & 3.53e{-}08{-}3.53e{-}08j & 1.27e{-}08{-}3.08e{-}08j & -7.85e{-}12{-}2.50e{-}08j \\ -7.85e{-}12{-}2.50e{-}08j & 1.27e{-}08{-}3.08e{-}08j & 3.53e{-}08{-}3.53e{-}08j & 1.25e{-}06{-}3.93e{-}08j & 9.20e{-}08{-}3.83e{-}08j & 3.53e{-}08{-}3.53e{-}08j \\ -7.66e{-}09{-}1.85e{-}08j & -7.85e{-}12{-}2.50e{-}08j & 1.27e{-}08{-}3.08e{-}08j & 9.20e{-}08{-}3.83e{-}08j & 1.25e{-}06{-}3.93e{-}08j & 9.20e{-}08{-}3.83e{-}08j \\ -1.18e{-}08{-}1.18e{-}08j & -7.66e{-}09{-}1.85e{-}08j & -7.85e{-}12{-}2.50e{-}08j & 3.53e{-}08{-}3.53e{-}08j & 9.20e{-}08{-}3.83e{-}08j & 1.25e{-}06{-}3.93e{-}08j \end{pmatrix}$$

***Pseudo code: 3***

for $0 \leq i, j \leq 2N$

Use this, $R_{ij} = \sqrt{a^2 + (z_i - z_j)^2}$

$P_{ij} = \frac{\mu_o}{4\pi} \times \frac{exp(-jkR_{ij}) \times dz'_j}{R_{ij}}$

find matrix $R_{ij}, order = [2N-2, 2N-2], \implies$ ignore all sides and middle cross rows and columns

find matrix $R_{iN}, order = [2N-2, 1], \implies$ ignore first, middle and last elements

find matrix $P_{ij}, order = [2N-2, 2N-2], \implies$ ignore all sides and middle cross rows and columns

find matrix $P_B, order = [2N-2, 1], \implies$ ignore first, middle and last elements

# Calculating Magnetic field vector:

Secondly lets calculate Magnetic field vector $\overrightarrow{H}_\phi(r, z_i)$)-

$$H_\phi(r, z) = -\frac{1}{\mu}\frac{\partial A_z}{\partial r}$$

Now, let's expand this and convert it into matrix form and solve further-

$$H_\phi(r, Z_i) = \sum P_{ij} \times \frac{a}{\mu 0}(\frac{jk}{R_{ij}} + \frac{1}{R_{ij}^2})I_j + P_B\frac{a}{\mu 0}(\frac{jk}{R_{iN}} + \frac{1}{R_{iN}^2})I_m$$

$$H_\phi(r, z_i) = \sum Q_{ij}I_j + Q_{Bi}I_m$$

From the above equation, we can write $Q_B$ and $Q_{ij}$-

$$Q_{ij} = P_{ij} \times \frac{a}{\mu_o}(\frac{jk}{R_{ij}} + \frac{1}{R_{ij}^2})$$

$$Q_B = P_B \times \frac{a}{\mu_o}(\frac{jk}{R_{iN}} + \frac{1}{R_{iN}^2})$$

Solving for N=4, we get-

$$Q_B = \begin{pmatrix} 0.0028 - 0.0009j \\ 0.008 - 0.001j \\ 0.0542 - 0.001j \\ 0.0542 - 0.001j \\ 0.008 - 0.001j \\ 0.0028 - 0.0009j \end{pmatrix}$$

$$Q_{ij} = \begin{pmatrix} 9.9521e{+}01{-}0.001j & 5.4208e{-}02{-}0.001j & 8.0201e{-}03{-}0.001j & 1.2493e{-}03{-}0.0008j & 5.8288e{-}04{-}0.0007j & 2.2597e{-}04{-}0.0006j \\ 5.4208e{-}02{-}0.001j & 9.9521e{+}01{-}0.001j & 5.4208e{-}02{-}0.001j & 2.7723e{-}03{-}0.0009j & 1.2493e{-}03{-}0.0008j & 5.8288e{-}04{-}0.0007j \\ 8.0201e{-}03{-}0.001j & 5.4208e{-}02{-}0.001j & 9.9521e{+}01{-}0.001j & 8.0201e{-}03{-}0.001j & 2.7723e{-}03{-}0.0009j & 1.2493e{-}03{-}0.0008j \\ 1.2493e{-}03{-}0.0008j & 2.7723e{-}03{-}0.0009j & 8.0201e{-}03{-}0.001j & 9.9521e{+}01{-}0.001j & 5.4208e{-}02{-}0.001j & 8.0201e{-}03{-}0.001j \\ 5.8288e{-}04{-}0.0007j & 1.2493e{-}03{-}0.0008j & 2.7723e{-}03{-}0.0009j & 5.4208e{-}02{-}0.001j & 9.9521e{+}01{-}0.001j & 5.4208e{-}02{-}0.001j \\ 2.2597e{-}04{-}0.0006j & 5.8288e{-}04{-}0.0007j & 1.2493e{-}03{-}0.0008j & 8.0201e{-}03{-}0.001j & 5.4208e{-}02{-}0.001j & 9.9521e{+}01{-}0.001j \end{pmatrix}$$

***Pseudo code: 4***

for $0 \le i, j \le 2N$

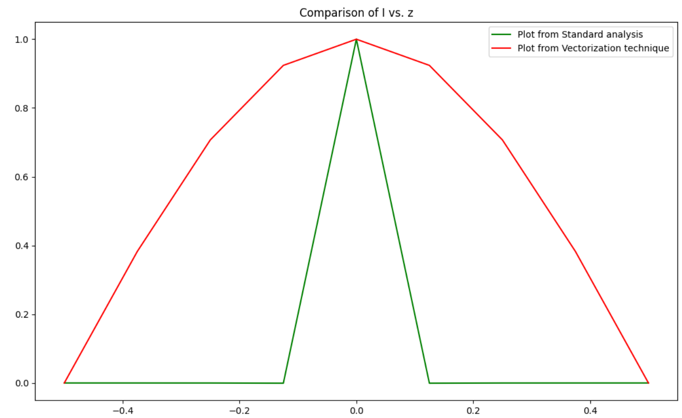
From this common formula, $Q_{ij} = P_{ij} \times \frac{a}{\mu_o}(\frac{jk}{R_{ij}} + \frac{1}{R_{ij}^2})$

find matrix $Q_{ij}, order = [2N-2, 2N-2]$, $\implies$ ignore all sides and middle cross rows and columns

find matrix $Q_B, order = [2N-2, 1]$, $\implies$ ignore first, middle and last elements

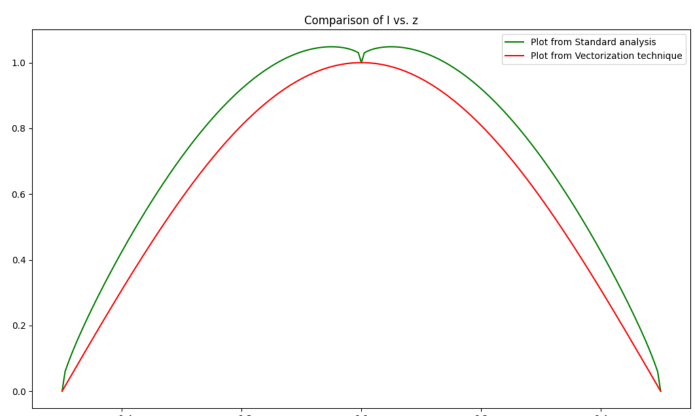
# Comparison of current plots:

For N=4,




Here we can see a large deviation from the actual current distribution, it is because number of divisions along the length which we take is very small.

For N=100,




Here relatively we can see very less deviation from the actual graph. And also we can see a abnormal dip in the middle of the rod or wire. It is caused because the vectorisation method which we did is just a good approximation from the boundary conditions we had, but not a accurate one.

***Pseudo code: 5***

From $(M - Q) \times J = Q_B \times I_M$ find J
Incorporate boundary conditions (add first, middle, last elements) to J and call it $J_final$
Plot (J_final vs. z) and (I vs. z) for N=4 and 100

# Conclusion:

We analysed Current distribution in a dipole antenna through two methods. And we even saw the comparison of plots between them too. On the way, we did dealt with lot of interesting matrices creation. Thus, Current in half-wave dipole antenna varies sinusoidally along the length of the rod covering half wavelength.

**Thank you!**