Assignment-6: The Laplace Transform

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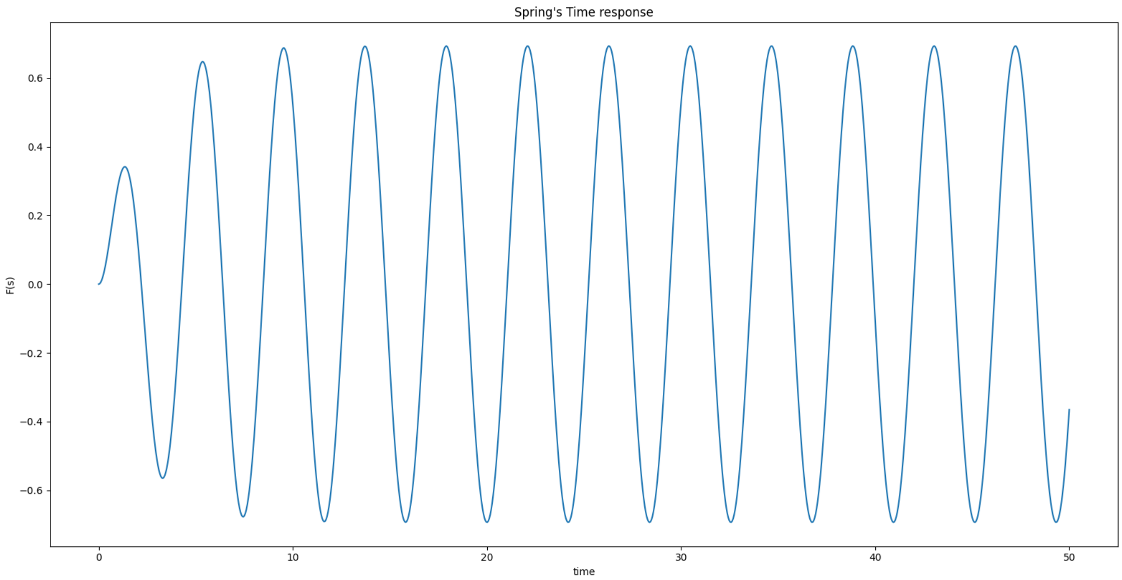
Spring's Time response:

Here we are going to obtain time response of spring system with the given conditions. On solving in the Laplace domain, we get the following equation

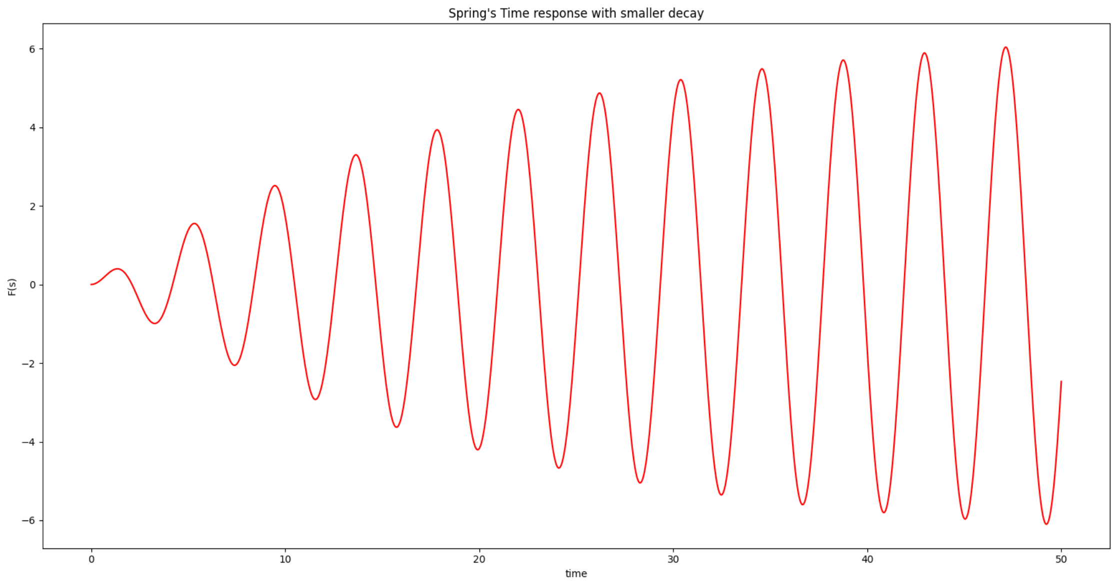
$$F(s) = \frac{s + 0.5}{((s^2 + 0.5)^2 + 2.25)(s^2 + 2.25)} = \frac{s + 0.5}{(s^2 + s + 2.5)(s^2 + 2.25)} \tag{1}$$



Spring's Time response with smaller decay:

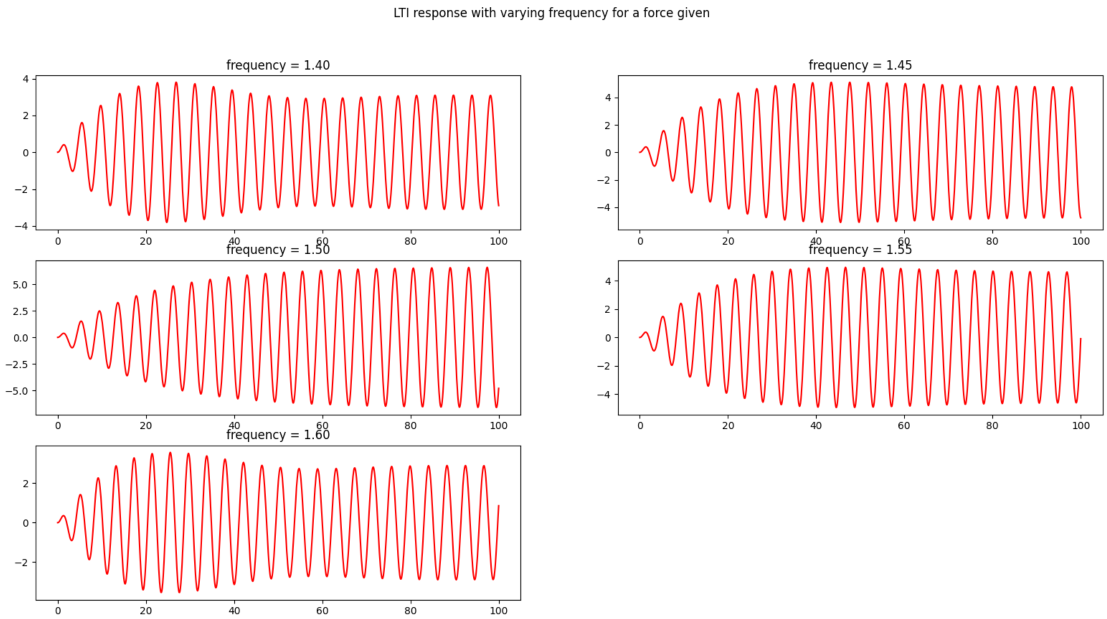
The same problem above is solved here with smaller decay constant, so we get the following transfer equation

$$F(s) = \frac{s + 0.05}{((s^2 + 0.05)^2 + 2.25)(s^2 + 2.25)} = \frac{s + 0.5}{(s^2 + s + 2.2525)(s^2 + 2.25)} \tag{2}$$



LTI response for different frequencies:

From the given input, we find resulting responses for different frequency, thus the plots are-

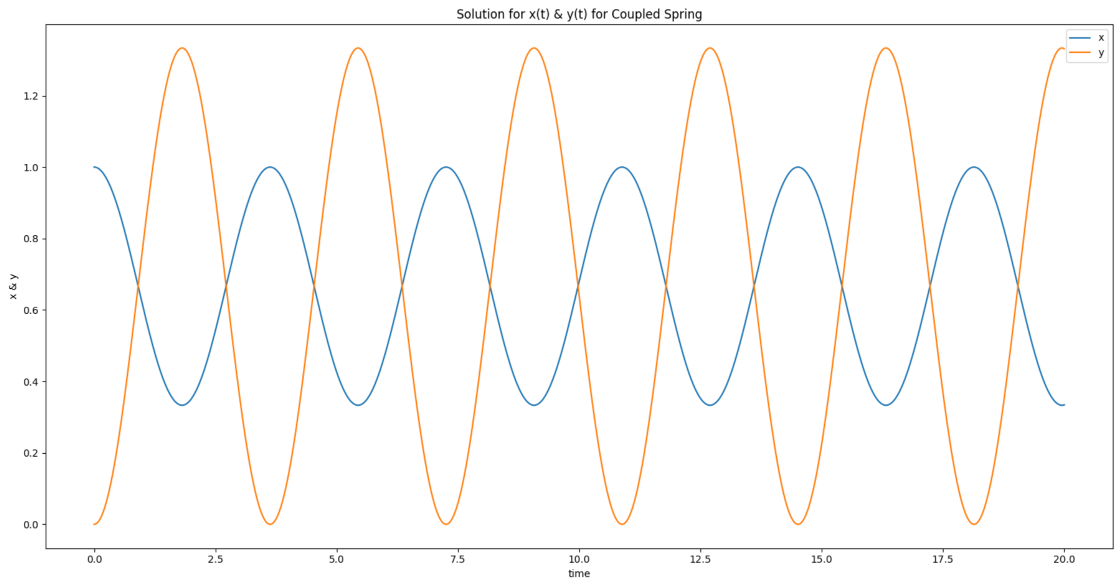


Inference: Here we can clearly notice that, among all the 5 plots, plot with frequency=1.5 (rad/s) is having maximum amplitude.

Time evolution of Coupled Spring problem:

From the given initial conditions of coupled equations, we get the following transfer function for X and Y in Laplace domain.

$$X(s) = \frac{s^2+2}{s^3+3s} \qquad\qquad Y(s) = \frac{2}{s^3+3s} \tag{3}$$



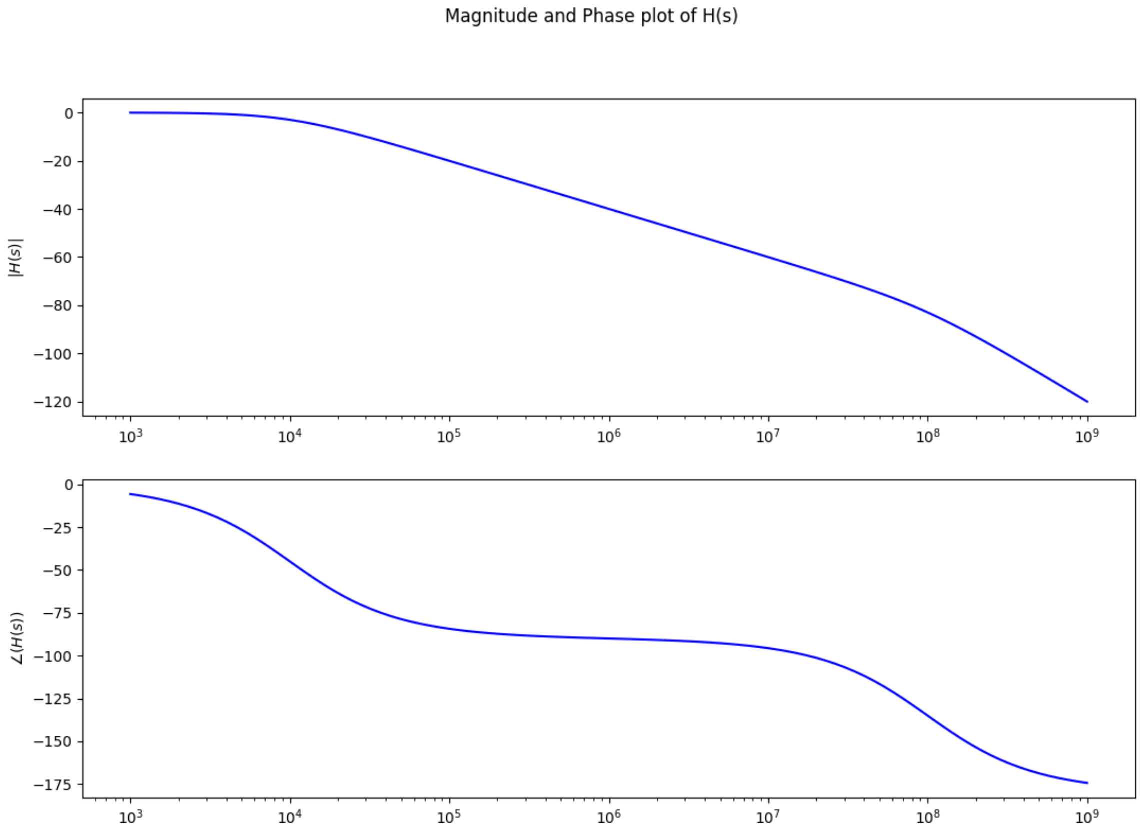
Inference: Here we can notice that plots of Solution of X(t) and Y(t) are sinusoidal with same frequency and phase difference of . And also their amplitudes are 1 and 1.33 respectively.

Steady state Transfer function of Two-port network:

Solving the given circuit in Laplace domain, we get the following transfer function-

H(s)= $\dfrac{10^6}{s^2 + 100s + 10^6}$

Here is the Magnitude and Phase response plots of H(s)-

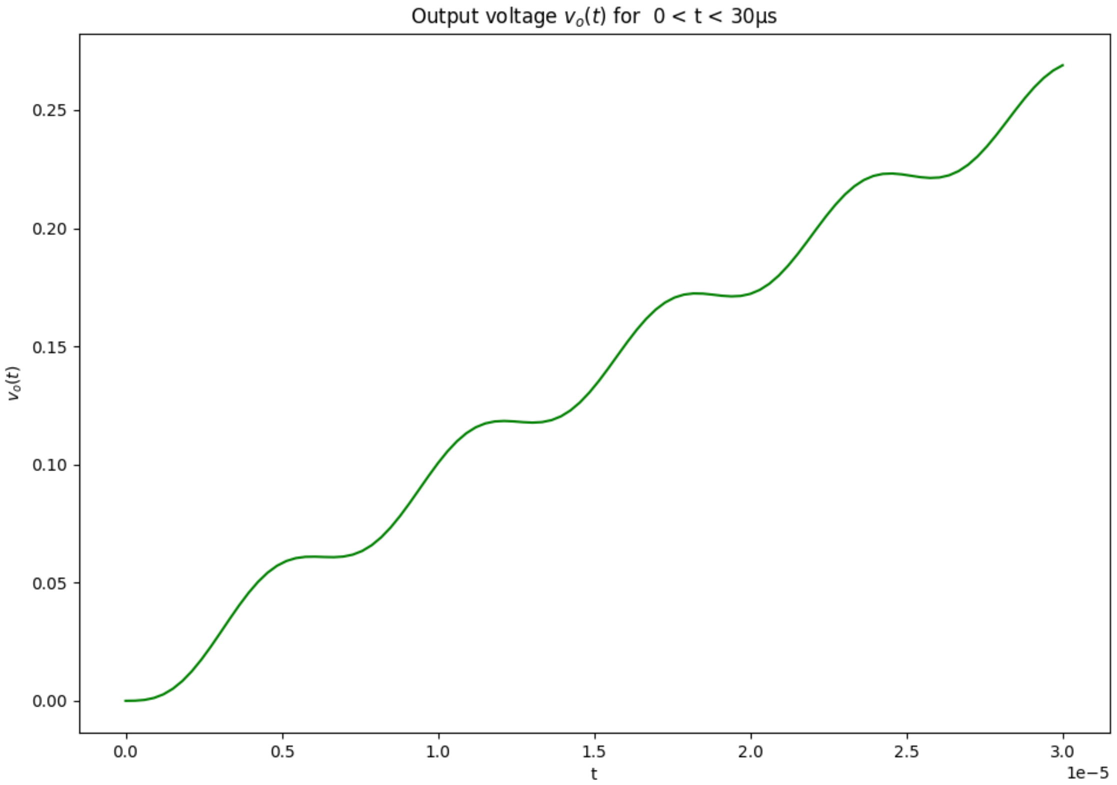


Inference: *signal.bode()* function can be used to plot this.

Two-port network with a Input signal

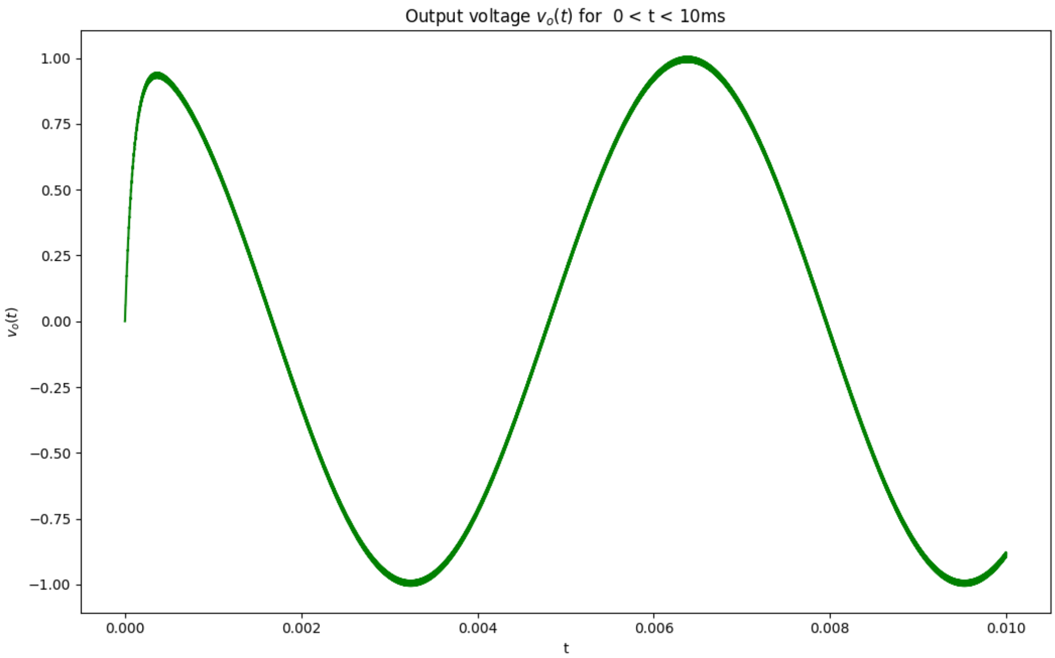
Now, In the same circuit above a input is given to the system,
$v_i(t) = (cos(10^3t) - cos(10^6t)) \times u(t)$

And the output is $\mathrm{v}_o(s) = v_i(s) \times H(s)$
Here we plot $\mathrm{v}_o(t) \quad in \quad 0 < t < 30\mu s$



Inference: *lsim function from scipy.signal* can be used in this plot.

Same as the above question, here that Bode plot is drawn with higher frequency and different range i.e. $0 < t < 10ms$. And also with the given conditions, we get the following plot-



Inference: *lsim function from scipy.signal* can be used in this kind of plots.
From the Bode plot of H(s), we can notice that the system provides unity gain for a low frequency of $10^3 rad/s$.
However, the system dampens a high frequency of $10^6 rad/s, with \quad |H(s)|_{dB} \approx 40$.

And also from the given circuit we find that it is a Low pass filter. Thus, magnitude of oscillations of these frequency is reduced.

Thank you!