Addition:

$\mathbb{F}$ ~ denotes "field"

$\mathbb{R}, \mathbb{C}, \mathbb{Q}$

$\underline{x}, \underline{y} \in \mathbb{F}^n$

$$\underline{x} + \underline{y} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{bmatrix}$$

Linear Combination

$\underline{x}_1, \ldots, \underline{x}_p \in \mathbb{F}^n$

$\alpha_1, \ldots, \alpha_p \in \mathbb{F}$

$$\sum_{i=1}^{p} \alpha_i \underline{x}_i = \alpha_1 \underline{x}_1 + \alpha_2 \underline{x}_2 + \cdots + \alpha_p \underline{x}_p.$$

Binary field $\mathbb{F}_2 = \{0, 1\}$

Addition: XOR

Multiplication: AND

$$\cdots + \alpha_p x_{1,p}$$

$$\cdots + \cdots \alpha_p x_{2,p}$$

$$\cdots \alpha_p x_{n,p}$$

$$\cdots x_{i,j}\,\alpha_j$$

Suppose

$$\underline{x}_i = \begin{bmatrix} x_{1,i} \\ x_{2,i} \\ \vdots \\ x_{n,i} \end{bmatrix}, \quad i = 1, \ldots, p$$

$$\alpha_i \underline{x}_i = \begin{bmatrix} \alpha_i x_{1,i} \\ \alpha_i x_{2,i} \\ \vdots \\ \alpha_i x_{n,i} \end{bmatrix}$$

Li

$$\sum_{i=1}^{p}$$

f

ut of

between

vector.

$\underline{x}_p \Big]_{n \times p}$

$x_{i,j}$

Let $\underline{y} = \sum_{i=1}^{p} \alpha_i \underline{x}_i$

$$\underline{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} \alpha_1 x_{1,1} + \alpha_2 x_{1,2} + \cdots + \alpha_p x_{1,p} \\ \alpha_1 x_{2,1} + \alpha_2 x_{2,2} + \cdots \alpha_p x_{2,p} \\ \vdots \\ \alpha_1 x_{n,1} + \cdots + \alpha_p x_{n,p} \end{bmatrix}$$

$$\therefore y_i = \sum_{j=1}^{p} \alpha_j x_{i,j} = \sum_{j=1}^{p} x_{i,j} \alpha_j$$

Suppos

$\underline{x}_i$

$\alpha_i \underline{x}_i$

We can think of

$\underline{y}$ as the result of

a multiplication between

a matrix & a vector.

$$X = \begin{bmatrix} \underline{x}_1 & \underline{x}_2 & \cdots & \underline{x}_p \end{bmatrix}_{n \times p}$$

$$X_{i,j} = x_{i,j}$$

Let

$$\underline{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$y_i =$

$$\sum_i \alpha_i \underline{x}_i$$

$$\underline{\alpha} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_p \end{bmatrix} \qquad \ldots \qquad \underline{y} = \underline{X} \cdot \underline{\alpha}$$

Observation: A matrix-vector multiplication can be interpreted as a linear combination of a collection of vectors.

The collection of vectors is the columns of the matrix X.

The coefficients of linear combination are the entries of the vector $\underline{\alpha}$.

Ex

$$A = \begin{bmatrix} \underline{a}_1 & \underline{a}_2 & \underline{a}_3 & \underline{a}_4 \end{bmatrix}_{m \times 4}$$

$$\underline{b} = \begin{bmatrix} \vdots \end{bmatrix}$$

$$A\underline{b} = \sum_i \underline{a}_i$$

$\underline{\alpha}$

Observatio

of

between

tor.

$\underline{x}_p \Big]_{n \times p}$

i, j

If $\underline{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

$A\underline{b} = \underline{a}_1 - \underline{a}_4$

Ex

$A = \Big[$

$\underline{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$A\underline{b} = \sum_{i}$